Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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Exercise Sheet #9

Advanced Algorithms (WS 2022/23)

Exercise 1 – Rank structure

Suppose we want to build the rank structure for a bit string of length $n = 256$.

a) Let A be the array that stores the cumulative rank of chunks and let B be the array that stores the cumulative rank of subchunks. Compute the lengths of A and B; show your calculations. **3 Points**

b) Explicitly write the first 10 rows of the lookup table for the bit strings of length $\frac{1}{2} \log n$. One row may be build up like this:

$$[\texttt{bitstring}][\texttt{rank}(0)][\texttt{rank}(1)][\texttt{rank}(2)]\ldots$$

You can write the query answers $\texttt{rank}(j)$ in decimal notation. **3 Points**

Exercise 2 – Operations of rank structure

Give pseudocode for the following methods in a rank structure:

- $\texttt{predecessor}(i)$, which returns the index of the predecessor of the element indexed by i;
- $\texttt{successor}(i)$, which returns the index of the successor of the element indexed by i.

3 Points

Exercise 3 – The child operation on succinct binary trees

Consider the succinct representation of binary trees from the lecture. Let i be the index (position of its representative 1 in the bit string) of a vertex v that has two children. Prove that the indices of the children of v are then given by $2\texttt{rank}(i)$ and $2\texttt{rank}(i) + 1$. **5 Points**

Exercise 4 – Operations on LOUDS

Consider the LOUDS representation of a tree from the lecture. Let `i` be the index of a vertex v (that is, the position of its "1" in the LOUDS bit string). Give pseudocode for the following methods, which should run in constant time:

- `outDegree(i)`, which returns the outdegree of v;
- `childNum(i, j)`, which returns the index of the j-th child of v if it exists and otherwise -1;
- `isRoot(i)`, which returns whether vertex v is the (real) root; and
- `isLeaf(i)`, which returns whether vertex v is a leaf.

6 Points

Please hand in your solutions on Wuecampus until the beginning of the next lecture, that is 14:15 on Wednesday, January 11.