Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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Johannes Zink
Oksana Firman

Exercise Sheet #8

Advanced Algorithms (WS 2022/23)

Exercise 1 – Ski rental

Consider the following randomized strategy for the ski rental problem. Instead of a coin, we throw a fair 3-sided die. If it shows 1 we buy skis on the $M/3$-th day, if it shows 2 we buy skis on the $2M/3$-th day, otherwise we buy skis on the M-th day.

a) Show that this strategy is in expectation not better than the 2-competitive deterministic ones. **4 Points**

Now we change our strategy IV from the lecture and buy skis on the $M/2$-th day only if we throw HEAD twice in a row and otherwise we buy skis on the M-th day.

b) What is the expected competitive ratio of this strategy? **3 Points**

Exercise 2 – Online bin packing

In the offline bin packing problem, you are given a set of items each with a size in $[0, 1]$. The goal is to pack these items into bins of size 1, such that no bin is overpacked and the total number of used bins is minimized. In the online version, the items arrive in a stream-like fashion. Each item has to be packed immediately and later repacking is not allowed. We consider the *FirstFit* strategy. Here, the item is packed in the first bin where it still fits. If it fits in no bin, a new bin is opened.

a) Prove that FirstFit is 2-competitive (up to some additive constant). **4 Points**

b) Show with an example that it can not be better than 3/2-competitive. **2 Points**

Exercise 3 – Online tree coloring game

In the game of online tree coloring, a *forest warden* and a *painter* play against each other. The game starts with an empty graph G_0. In turn i, the forest warden adds a vertex v_i to G_{i-1} and any edges from v_i to vertices in G_{i-1}. The resulting graph G_i has to be a forest, that is, a set of trees. The painter then has to assign a color to v_i such that G_i is properly colored, that is, no two adjacent vertices have the same color.

a) Show that there is a strategy with which the forest warden can enforce an arbitrarily large number of colors. **5 Points**

b) What is the best the forest warden can hope for, if the degree of vertex v_i in G_i may be at most k? Explain your answer. **2 Points**