

Exercise Sheet #5

Advanced Algorithms (WS 2022/23)

Exercise 1 – Pathwidth

We consider the cactus graph G given in Figure 1.

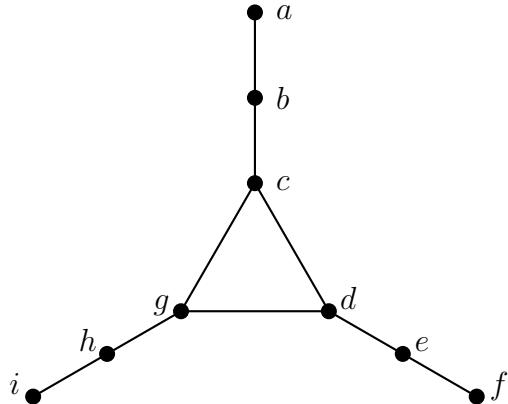


FIGURE 1: Graph G for Exercise 1.

- a) Give a path decomposition of G with the minimum width. What is the pathwidth of G ? **2 Points**
- b) Show that your solution of (a) is correct, i.e., that there is no a path decomposition with a smaller width. **3 Points**

Exercise 2 – Computing a maximum-weight independent set

We consider the graph C with weighted vertices given in Figure 2. The weights of the vertices are specified in the figure.

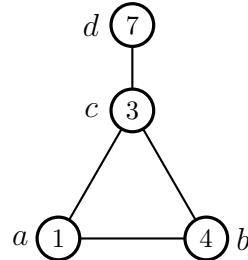


FIGURE 2: Graph C for Exercise 2.

- a) Give a *nice* path decomposition of C with minimum width. What is the pathwidth of C ? **3 Points**
- b) Employ the algorithm from the lecture to find the weight of a maximum-weight independent set of C using the nice path decomposition from (a). Describe the intermediate steps of the computation. In particular, add all entries $D[\cdot, \cdot]$ of the dynamic program. **5 Points**

Exercise 3 – MAXCUT in graphs of bounded pathwidth

In the fourth lecture, we studied approximation algorithms for the MAXCUT problem.

Show that the unweighted version of this problem (where all edge weights are 1) is FPT with respect to the pathwidth. You may assume that you are given a nice path decomposition of width k . **7 Points**