Lehrstuhl für Informatik I
– Algorithmen und Komplexität –
Universität Würzburg

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# Exercise Sheet #3

# Advanced Algorithms (WS 2022/23)

### Exercise 1 – Lucky vertex order for GREEDY VERTEX COLORING

Show that there exists an ordering $v_1, \dots, v_n$ of the vertices of G such that the algorithm GREEDYVERTEXCOLORING from the lecture computes an optimal solution. (Finding such an order remains of course hard.) **5 Points**

### Exercise 2 – Min Vertex Coloring in planar graphs

We consider the subproblem of Min Vertex Coloring where the input is restricted to planar graphs. By the famous four color theorem every planar graph can be colored with just four colors. Eventhough the proof involvs a case distinction with 1476 cases (or 633 in an improved version), the proof is constructive and gives a polynomial-time algorithm that colors every planar graph with four colors. However, the decision problem of whether a planar graph is 3-colorable is NP-complete.

Using the 4-color algorithm as a black box, describe an approximation algorithm for planar graphs with additive approximation guarantee 1 that also computes a coloring. **4 Points**

### Exercise 3 – Nearest Addition Algorithm for Metric TSP

Prove that the NEARESTADDITIONALGORITHM from the lecture is a 2-approximation algorithm for Metric TSP.

*Hint:* Take a look at Prim's algorithm. **5 Points**

**Exercise 4 – Non-approximability of Multiprocessor Scheduling**

Show that there can be no approximation algorithm $\mathcal{A}$ for MUTLIPROCESSORSCHEDULING, with $|\text{ALG}(I) - \text{OPT}(I)| \leq k$ for a constant $k$ (unless $P = NP$).

*Hint 1:* How could such an algorithm, if it existed, be used to solve this NP-hard problem?
*Hint 2:* Scale the problem instance. **6 Points**

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Please hand in your solutions on Wuecampus until the beginning of the next lecture, that is 14:15 on Wednesday, November 9.