

## Homework Assignment #10

### Approximation Algorithms (Winter Semester 2022/23)

#### Exercise 1

Show that assuming  $P \neq NP$ , there exists no approximation algorithm for MINIMUMDEGREESPANNINGTREE with ratio  $\alpha < 3/2$ .

[3 points]

#### Exercise 2

Let  $G = (V, E)$  be a graph with an (unknown) Hamiltonian path and let  $n = |V|$ . Give a polynomial-time algorithm that finds a simple path of length  $\Omega\left(\frac{\log n}{\log \log n}\right)$ .

[5 points]

#### Exercise 3

In the lecture, we learned about a local search algorithm that finds a spanning tree with a provable upper bound on the maximum degree. To prove the efficiency of the algorithm, we used the potential function

$$\Phi(T) = \sum_{v \in V(G)} 3^{\deg_T(v)},$$

where  $T$  is the current spanning tree.

Prove that the potential function decreases with every edge flip by at least factor  $\frac{2}{27n^3}$ , i.e., that  $\Phi(T') \leq (1 - \frac{2}{27n^3})\Phi(T)$ , where  $T'$  is the tree after the edge flip. [5 points]

#### Exercise 4

Two students, Peter and Susi, study the approximation algorithm for MINIMUMDEGREESPANNING-TREE from the lecture. This algorithm finds a spanning tree with maximum degree at most  $2 \cdot \text{OPT} + \ell$ , where  $\ell := \lceil \log_2 n \rceil$ . Peter and Susi want to improve the quality of this result and argue as follows:

*Peter:* „If we want to find better results, then we have to choose a smaller  $\ell$ ! By this formula, this guarantees a smaller maximum degree of the spanning tree!“

*Susi:* „But shouldn't we choose a larger  $\ell$ ? That allows more flips, and shouldn't that give us a better result?“

a) Settle the dispute between Peter and Susi. Generalize the result from the lecture by allowing values  $\ell := \lceil \log_b n \rceil$  for arbitrary  $b > 1$ . For  $b = 2$ , you should obtain the result from the lecture as a special case. [4 points]

*Peter:* „Then we agree! But what does that mean for the running time?“

b) How does the choice of  $\ell$  (or  $b$ ) affect the running time of the algorithm? [3 points]