Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

Würzburg, December 2, 2022

Prof. Dr. Alexander Wolff
M.Sc. Marie Diana Sieper

Homework Assignment #6
Approximation Algorithms (Winter Semester 2022/23)

Exercise 1 – METRIC-k-CLUSTER

Let $G = (V, E)$ be a complete graph with edge weights $c\colon E \to \mathbb{Q}_{\geq 0}$ that satisfy the triangle inequality. Let k be a positive integer. We want to find a partition of V into k sets of vertices $V_1, \dots, V_k$, called *clusters*, such that the weight of the most expensive intra-cluster edge is minimized. In other words, we have to minimize

$$\max_{1 \leq i \leq k,\ u,v \in V_i} c(u, v).$$

a) Devise a factor-2 approximation algorithm for this problem. **[7 points]**

b) Show that under the assumption P $\neq$ NP, there exists no factor-$(2 - \varepsilon)$ approximation algorithm for this problem, where $\varepsilon > 0$.

Suggestion: Use the hardness of the graph coloring problem: Given a graph $G = (V, E)$ and a parameter k, can the vertices in V colored with at most k colors such that no two adjacent vertices share a color? **[6 points]**

Exercise 2 – Greedy for METRIC-k-CENTER

Consider the following greedy algorithm for METRIC-k-CENTER.

Algorithm 1: Greedy-Metric-k-Center($G = (V, E; c), k$)

```
Pick an arbitrary vertex v ∈ V
S ← {v}
while |S| < k do
    u ← Vertex with c(u, S) = max_{v∈V} c(v, S)
    S ← S ∪ {u}
```

Show that this algorithm is a factor-2 approximation for METRIC-k-CENTER. **[7 points]**

Please hand in your solutions via WueCampus until 12:00 on Friday, December 16.