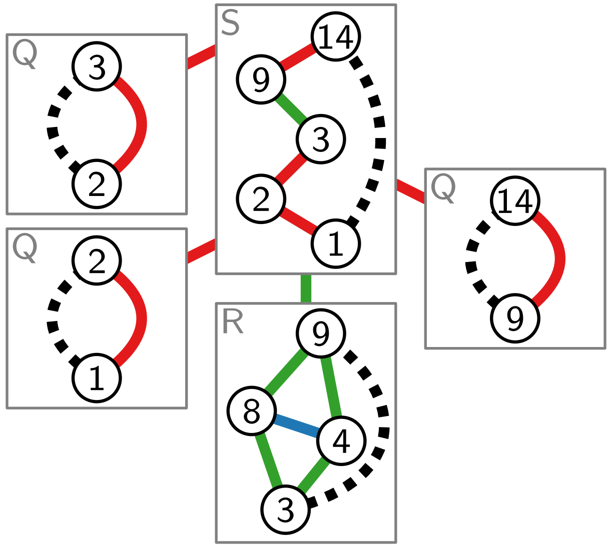
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

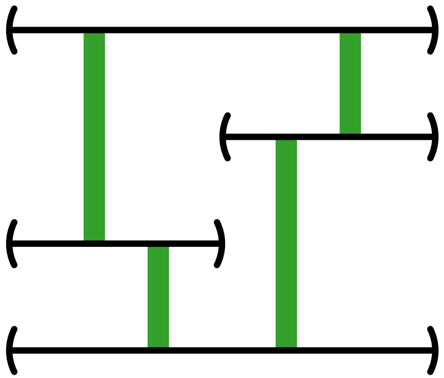
Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension



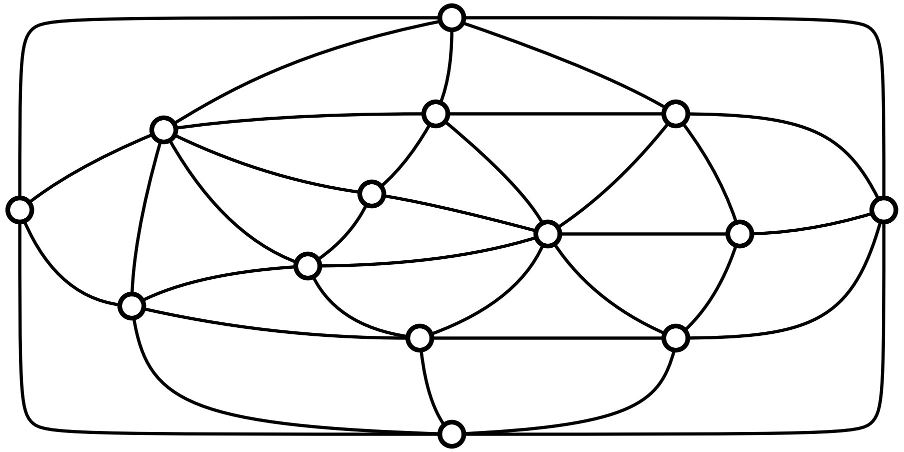
Part I:
Problem Definition

Alexander Wolff



Partial Representation Extension Problem

Let $G = (V, E)$ be a graph.



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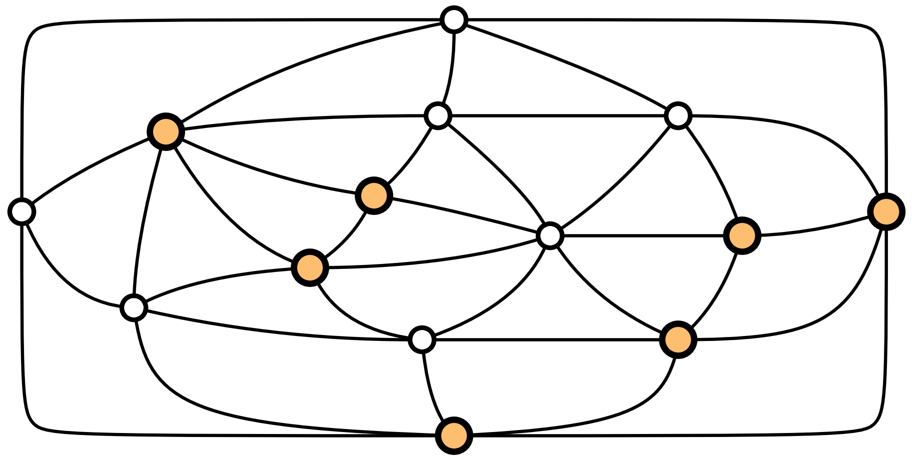
Let $G = (V, E)$ be a graph.

Let $V' \subseteq V$



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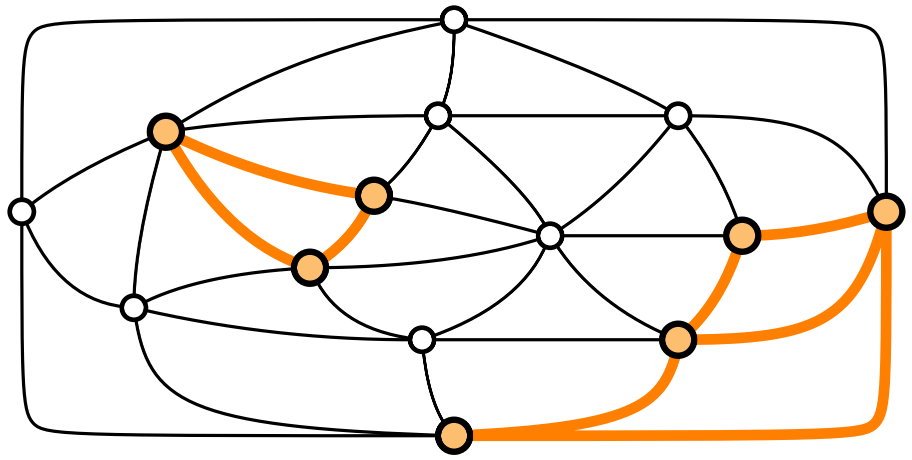
Let $V' \subseteq V$ and $H = G[V']$



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Let $G = (V, E)$ be a graph.

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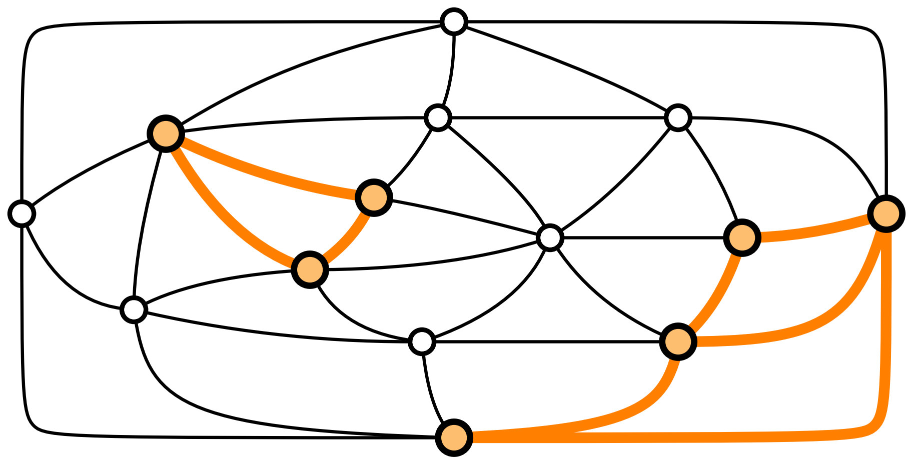
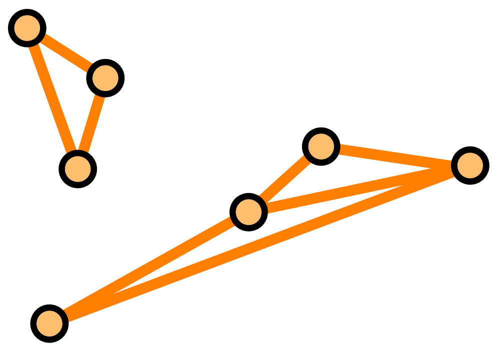
Let Γ_H be a representation of H.





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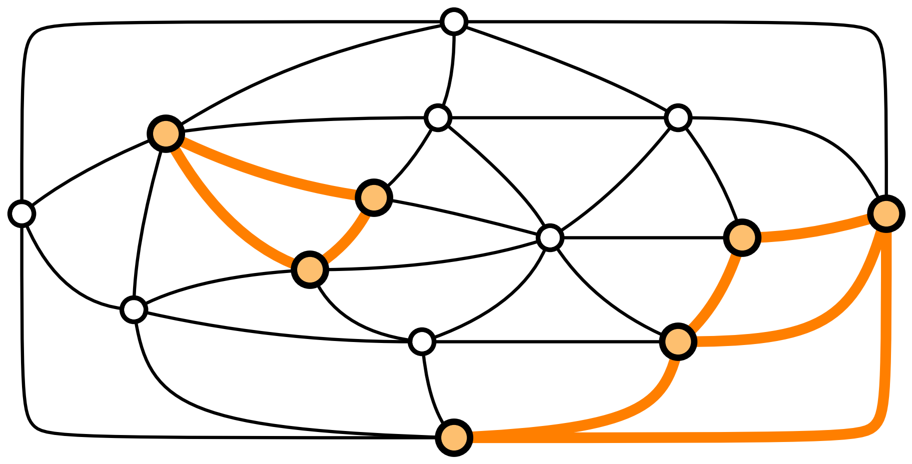

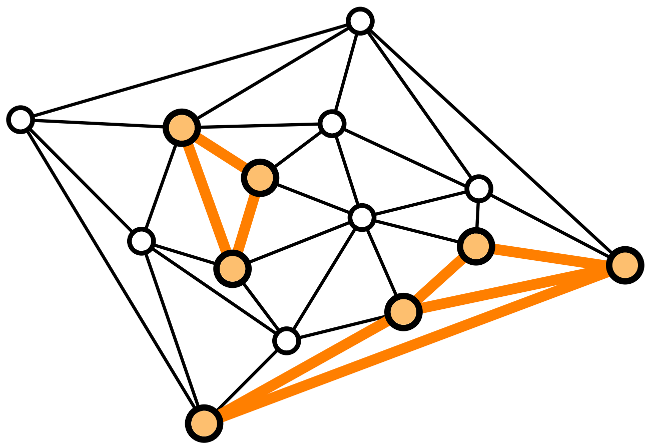

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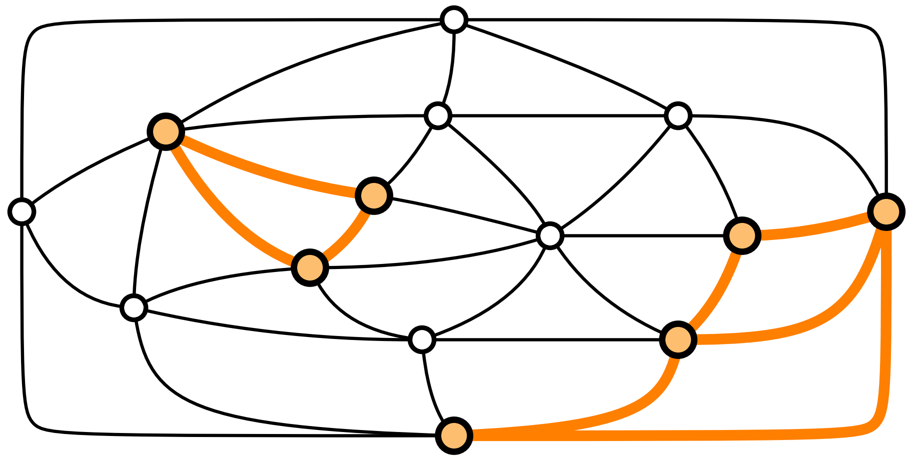

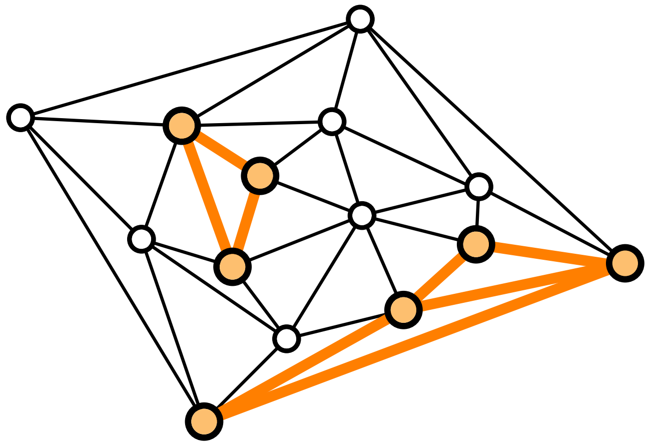

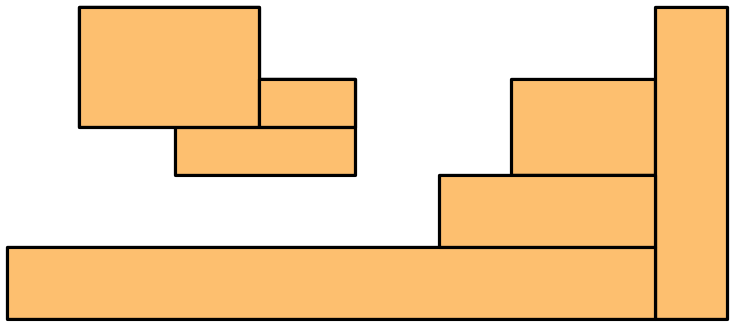

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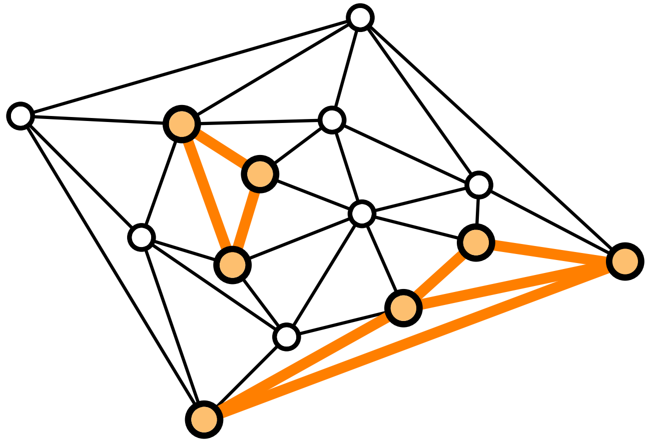

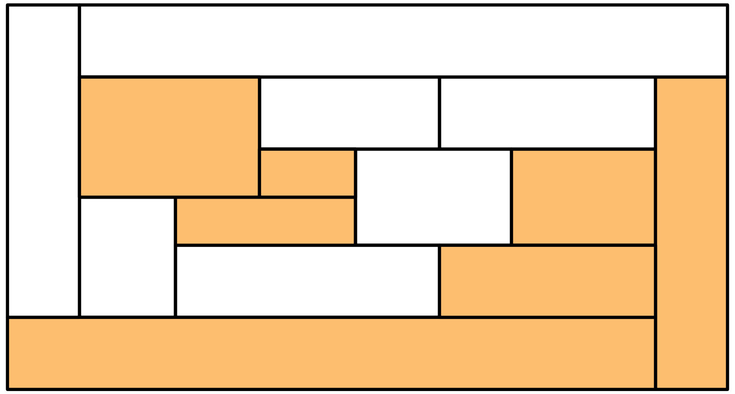

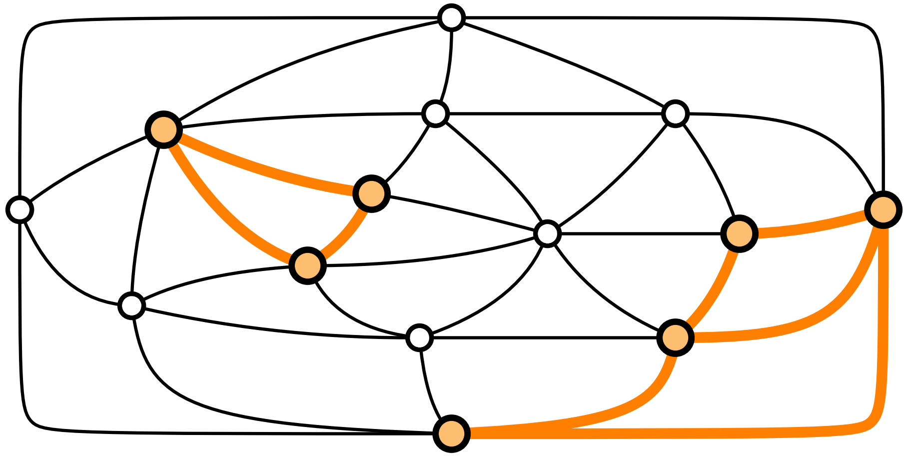

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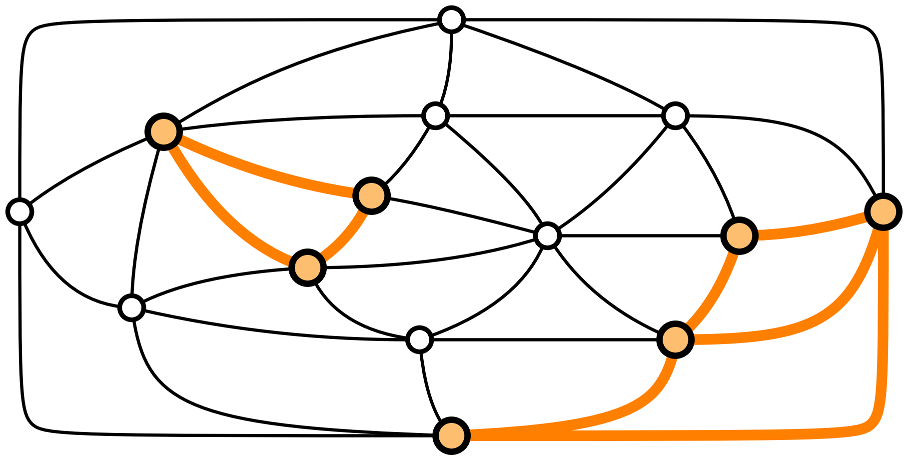

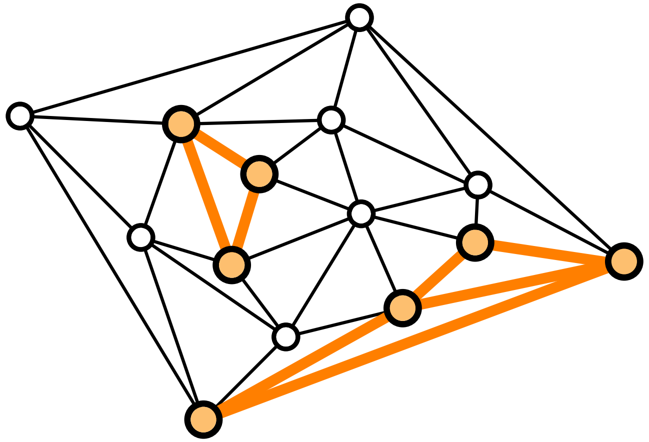

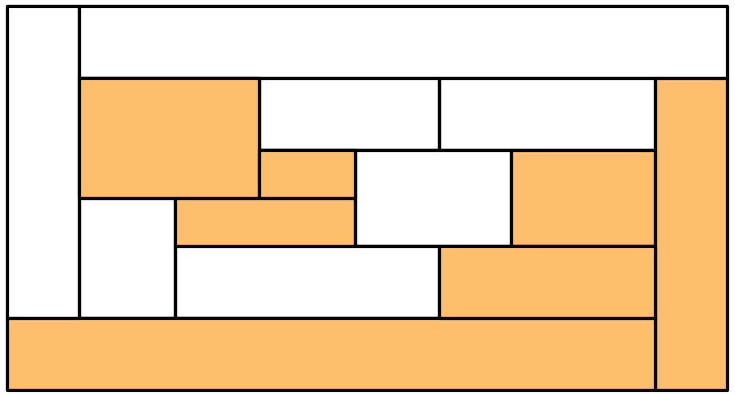

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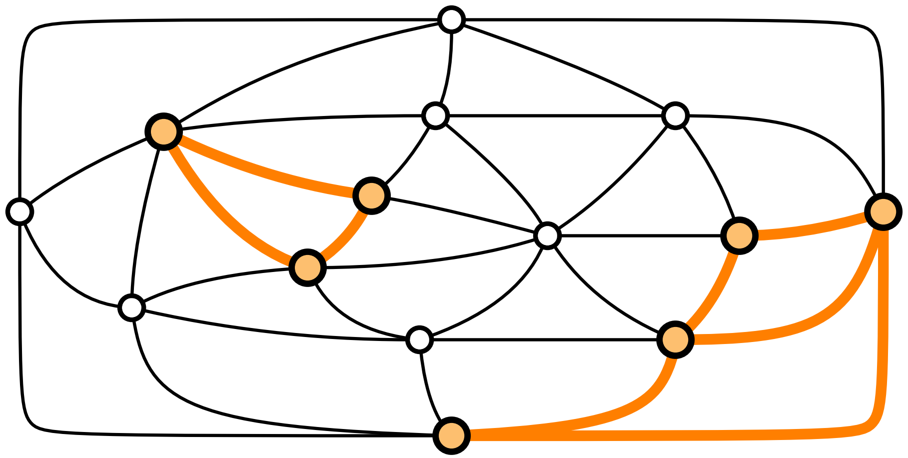
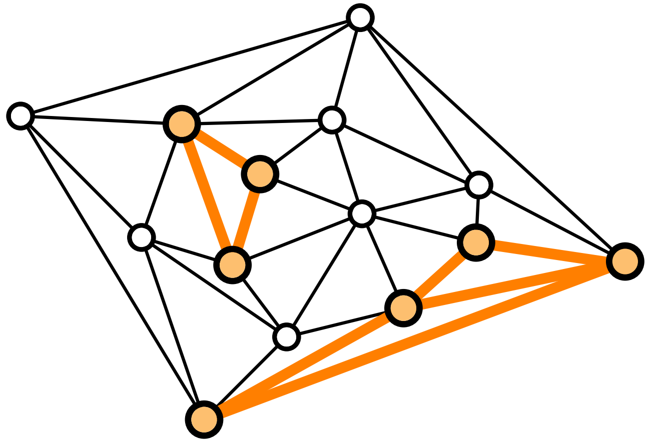
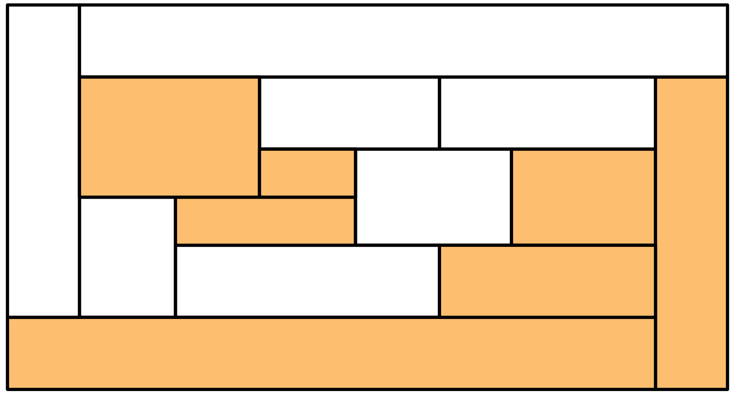
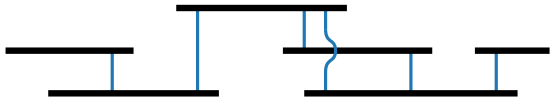






Polytime for:

- (unit) interval graphs



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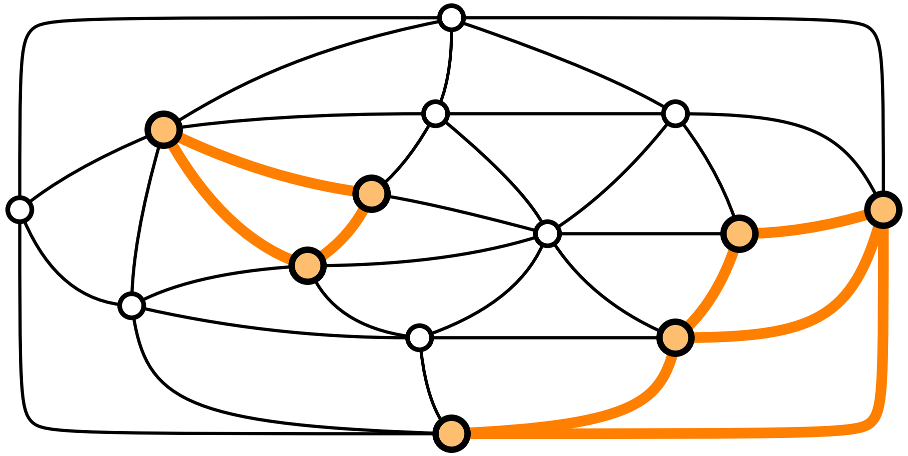

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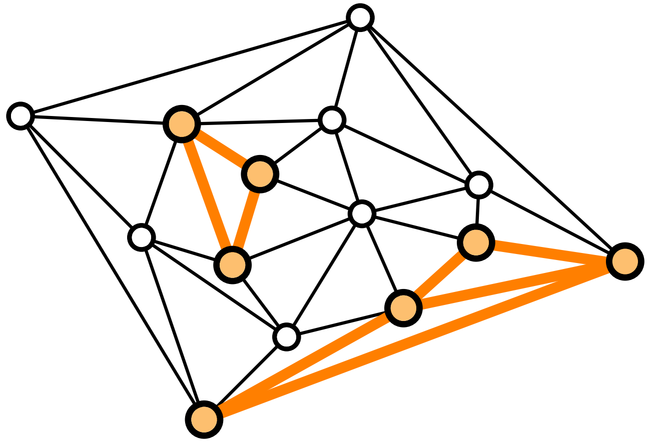

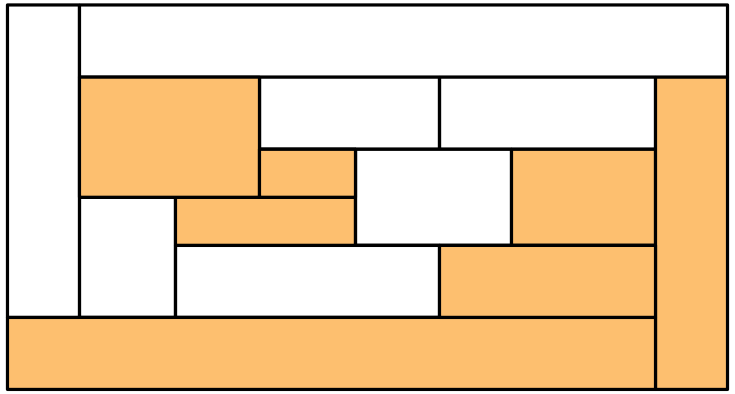

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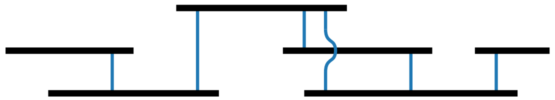

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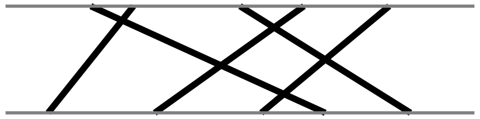

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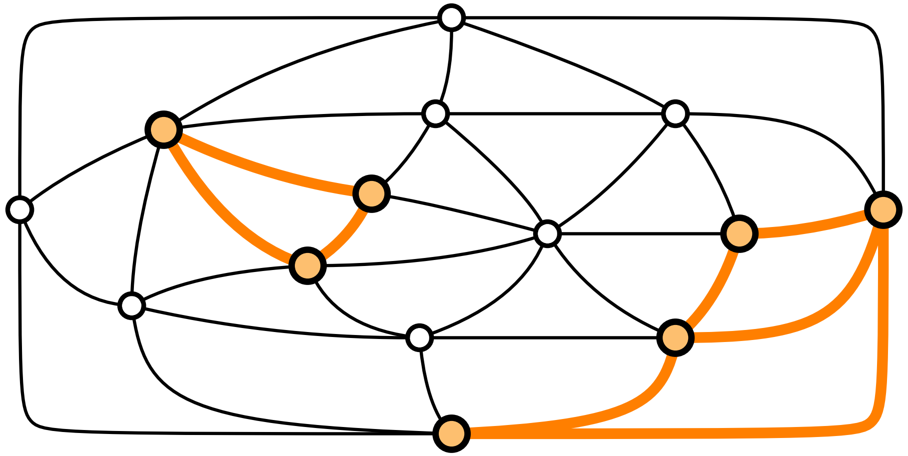

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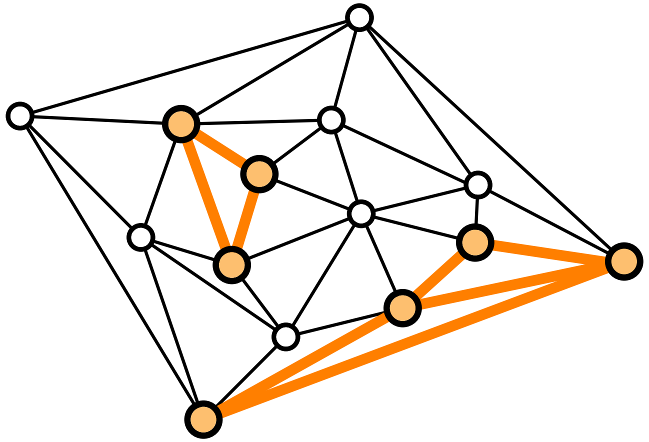

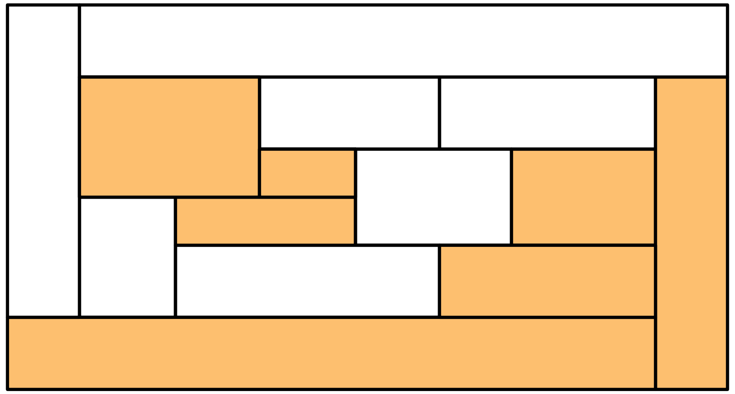

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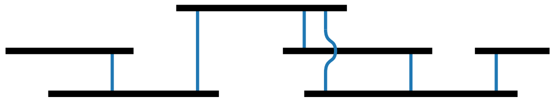

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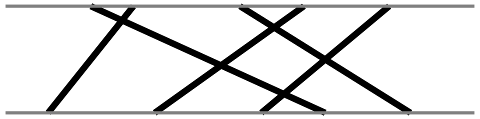
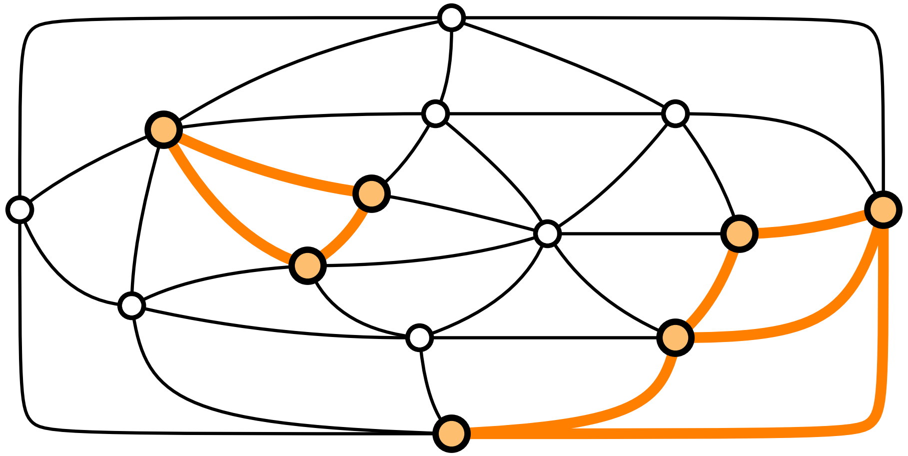


- circle graphs

Partial Representation Extension Problem



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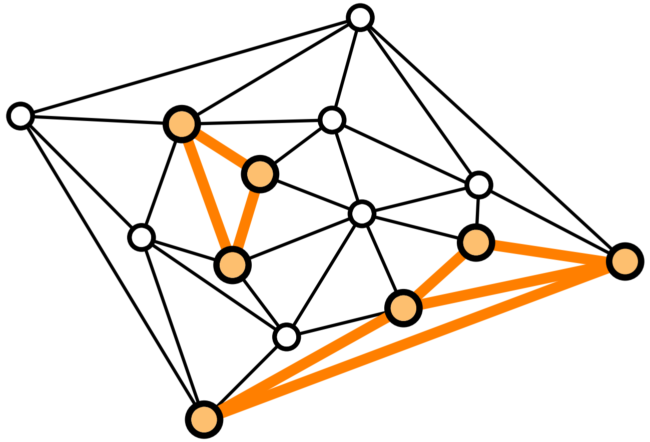

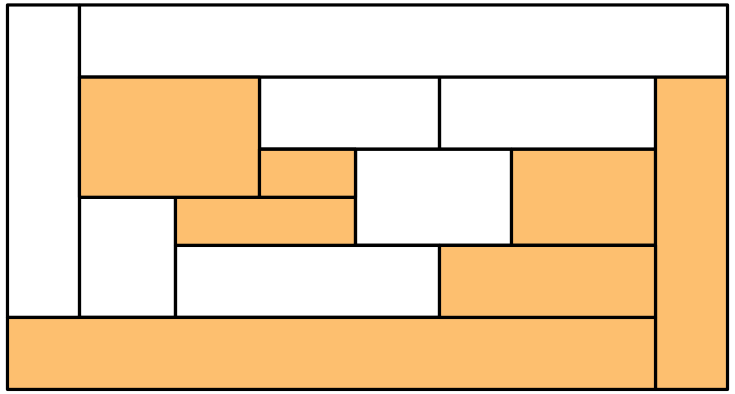

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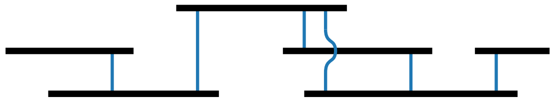

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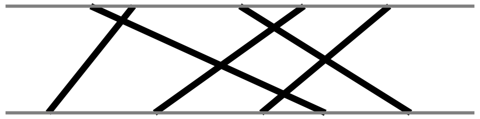

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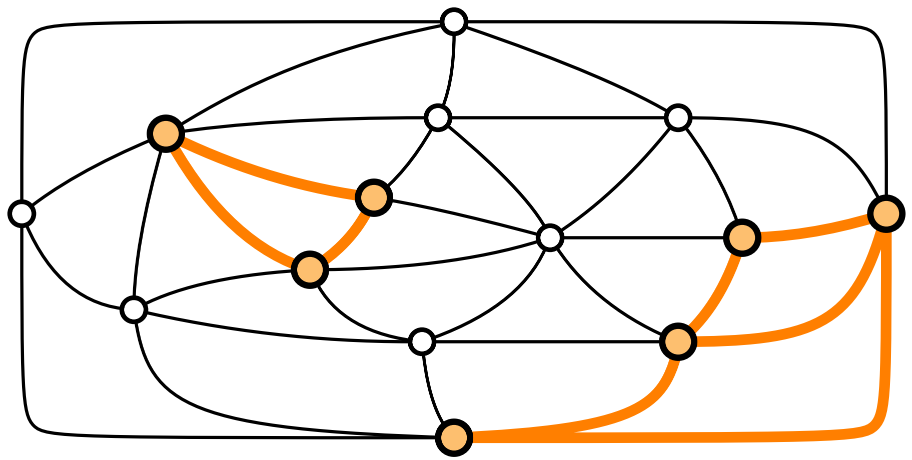

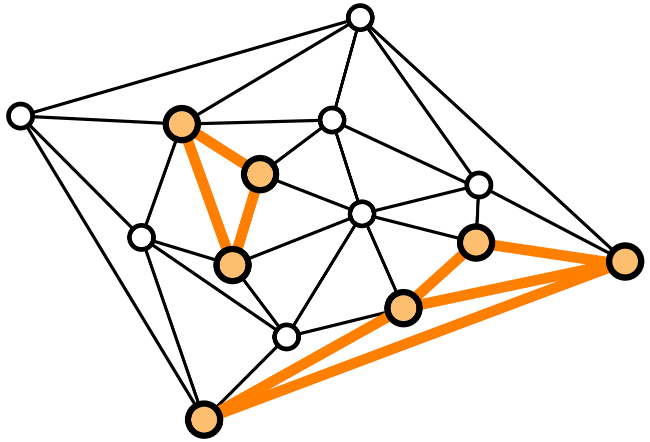

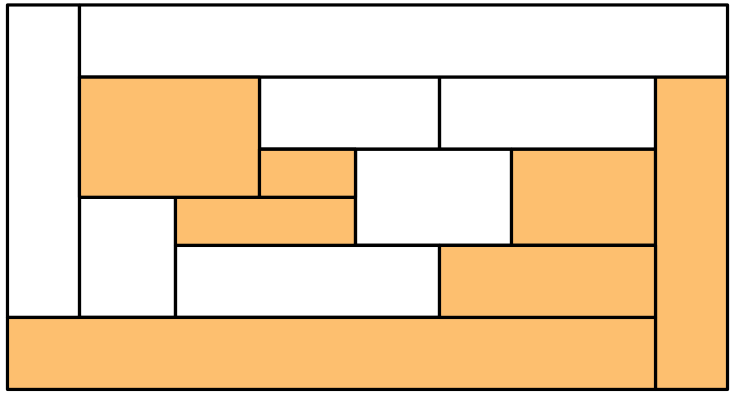

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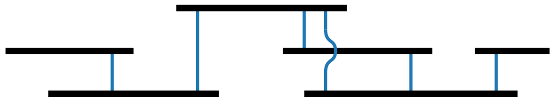

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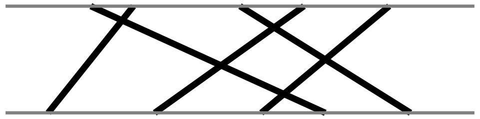

- circle graphs

NP-hard for:

- planar straight-line drawings

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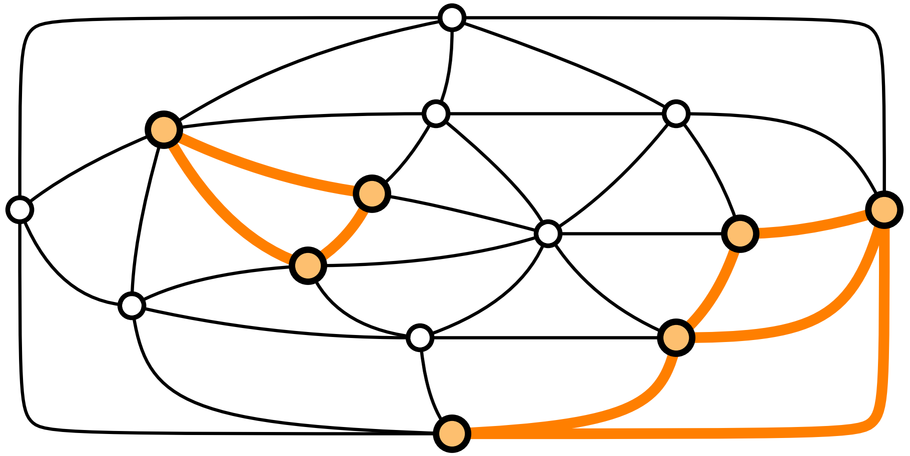

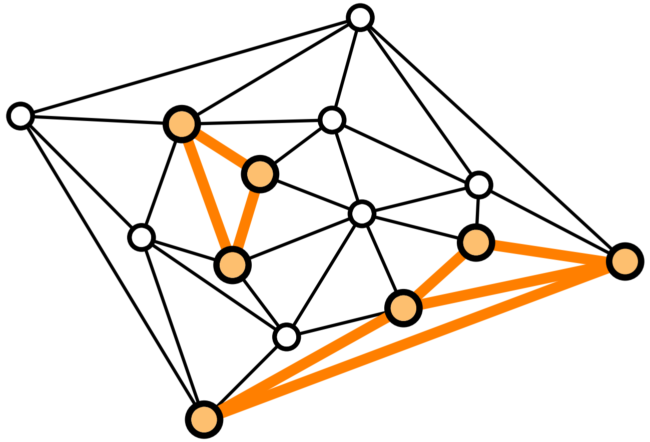

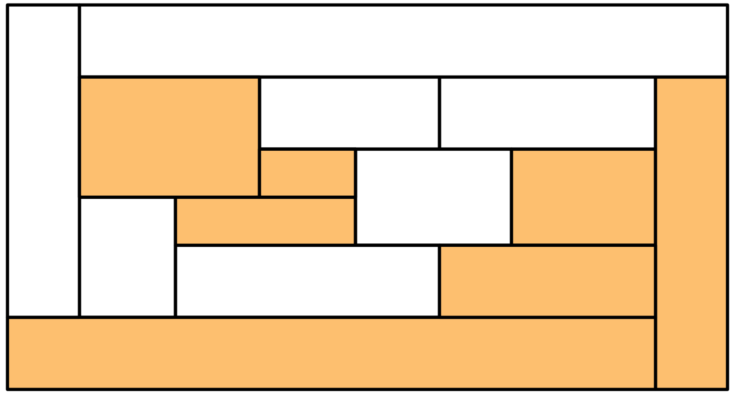

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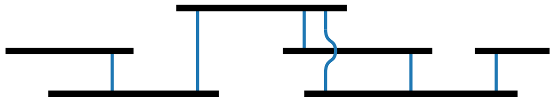

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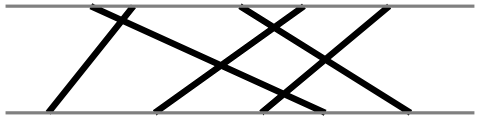

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NP-hard for:

- planar straight-line drawings
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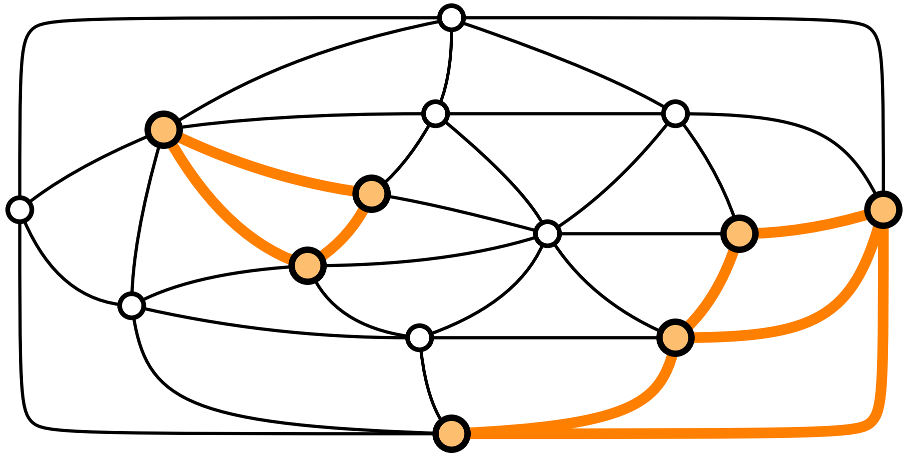

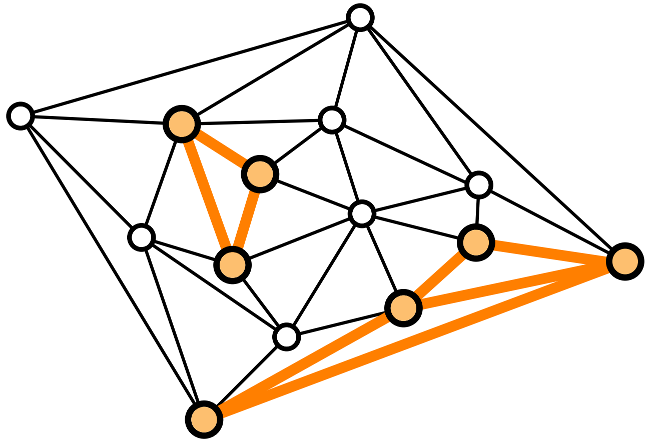

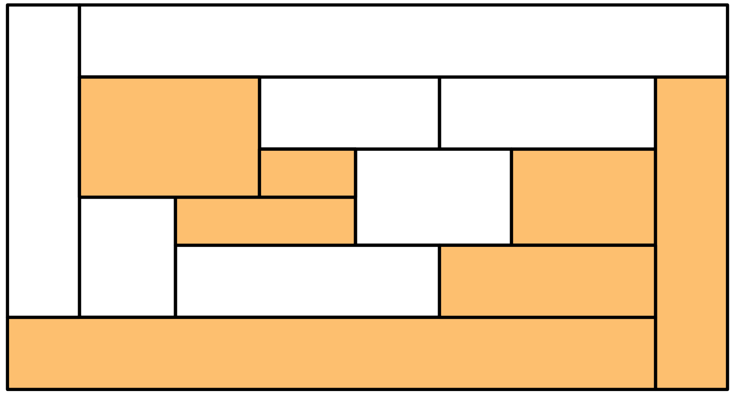

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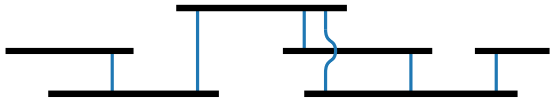

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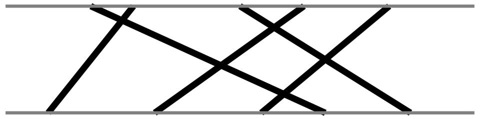

- circle graphs

NP-hard for:

- planar straight-line drawings
- contacts of
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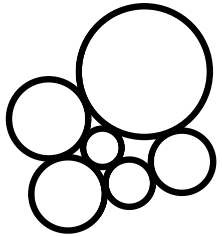

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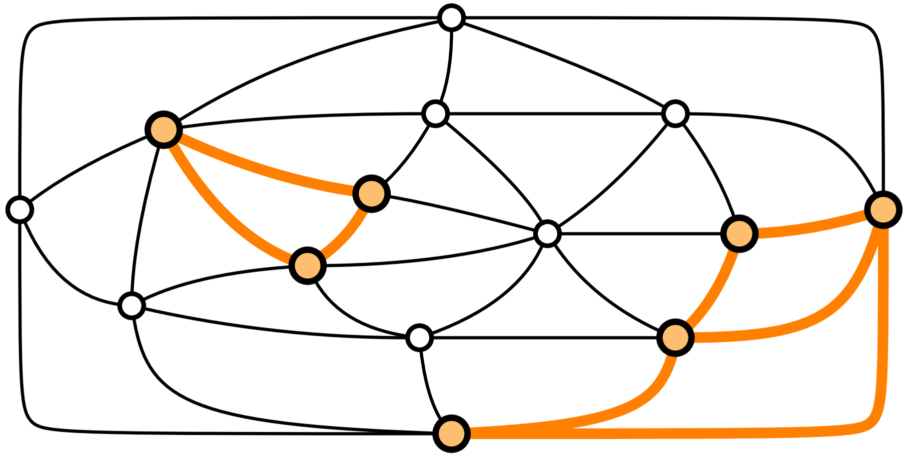

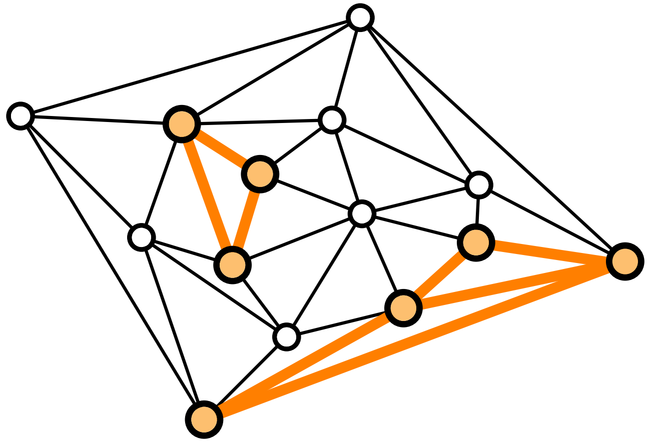

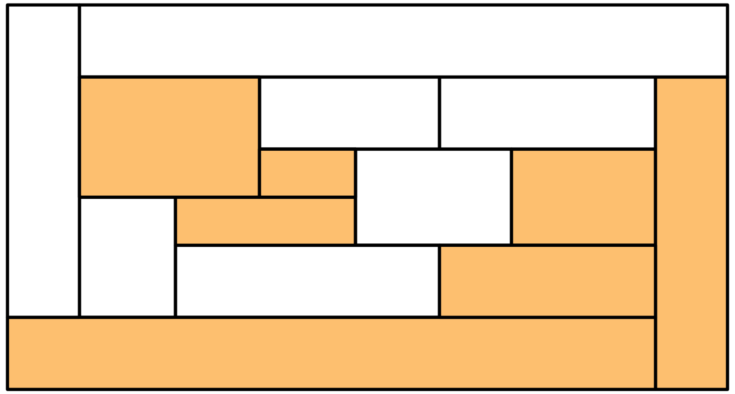

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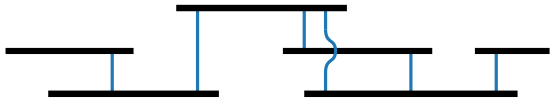

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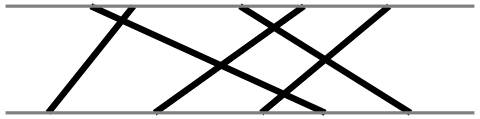

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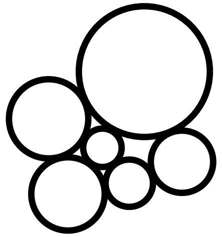

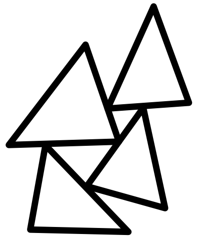

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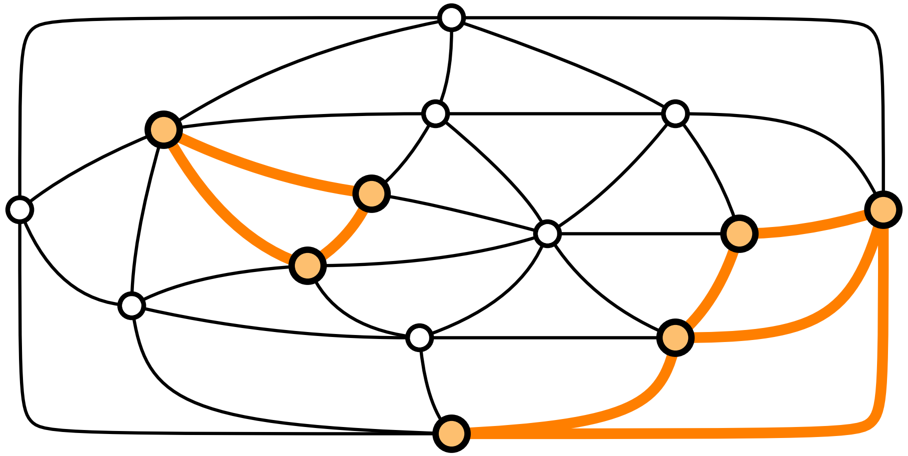

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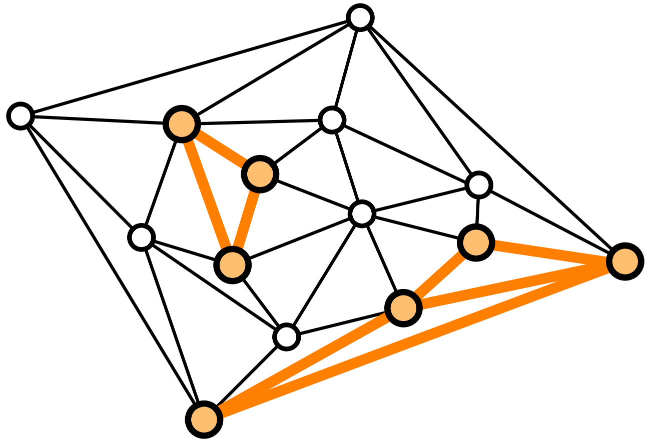

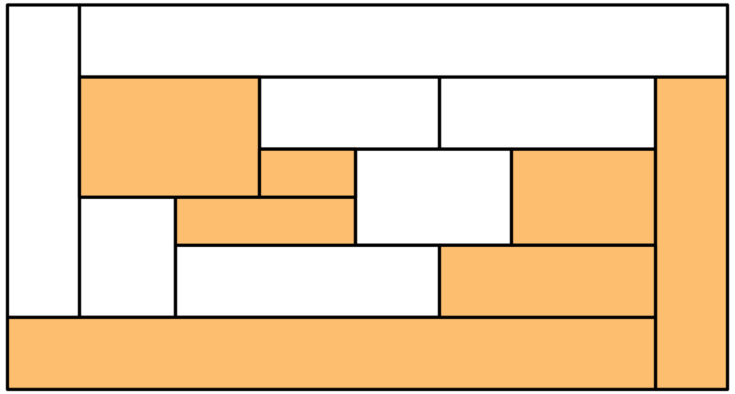

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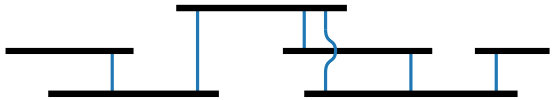
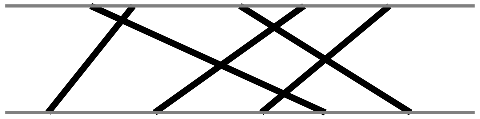


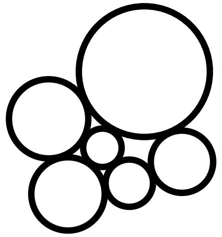
- permutation graphs

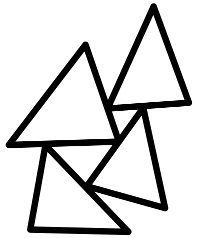


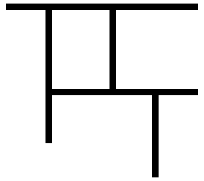
- circle graphs

NP-hard for:

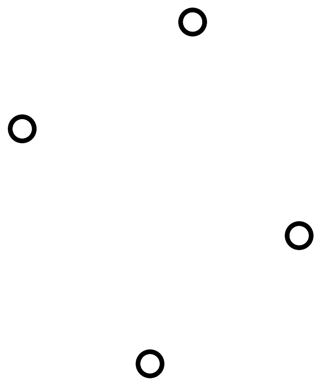
- planar straight-line drawings
- contacts of
  - disks
  - triangles
  - orthogonal segments







Bar Visibility Representation



Bar Visibility Representation

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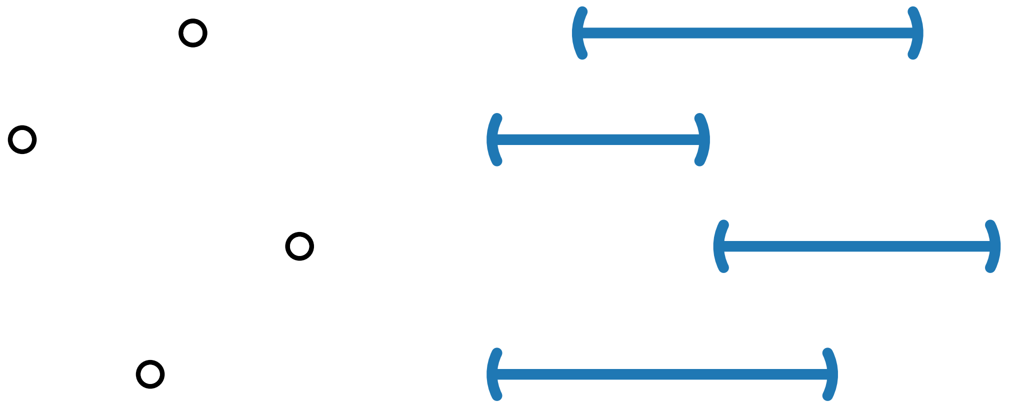
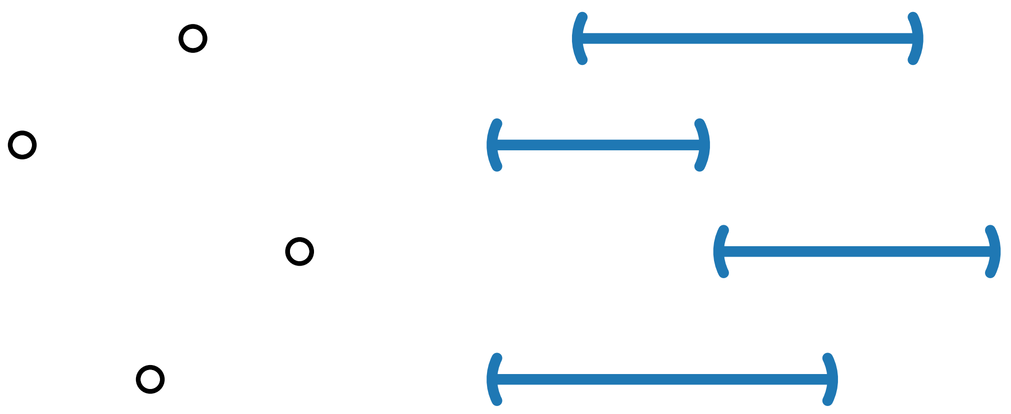


Bar Visibility Representation

- Vertices correspond to horizontal open line segments called **bars**.
- **Edges** correspond to unobstructed vertical lines of sight.



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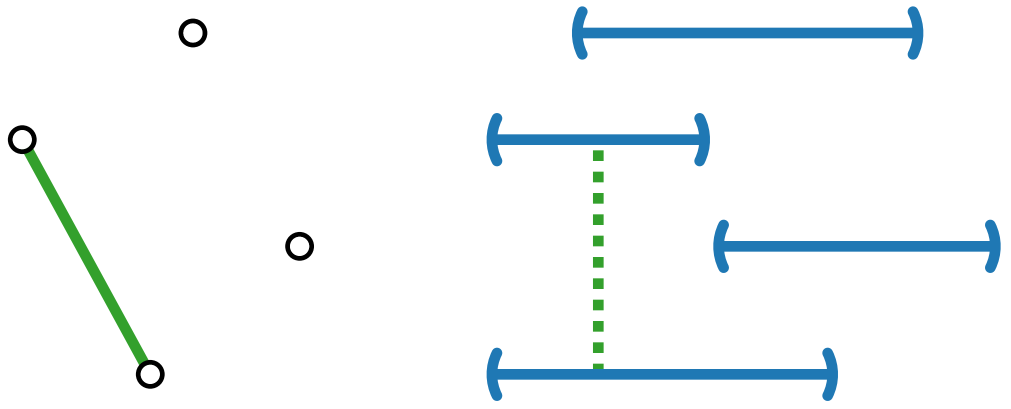

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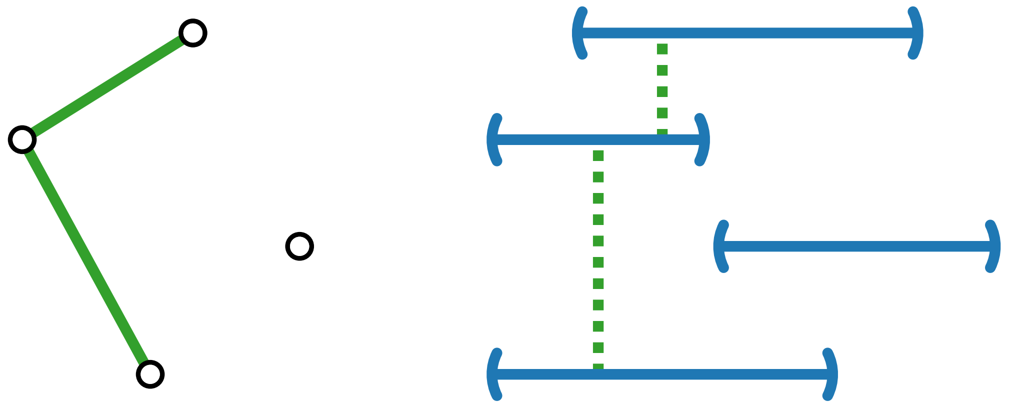

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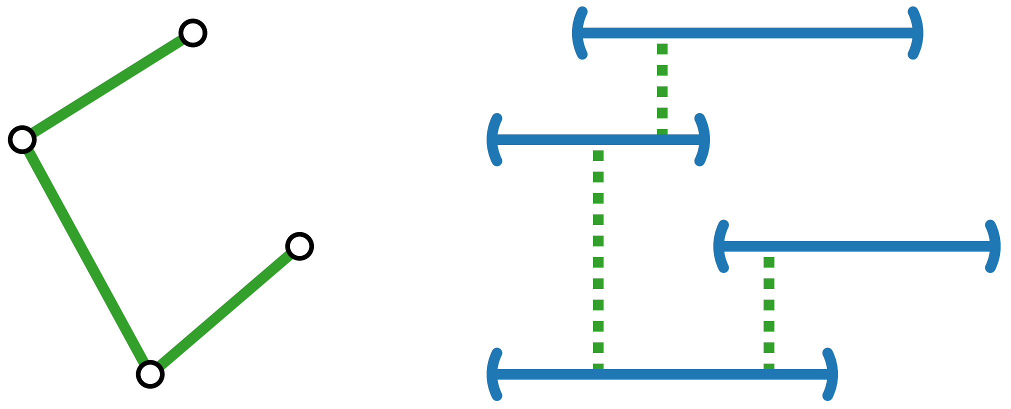

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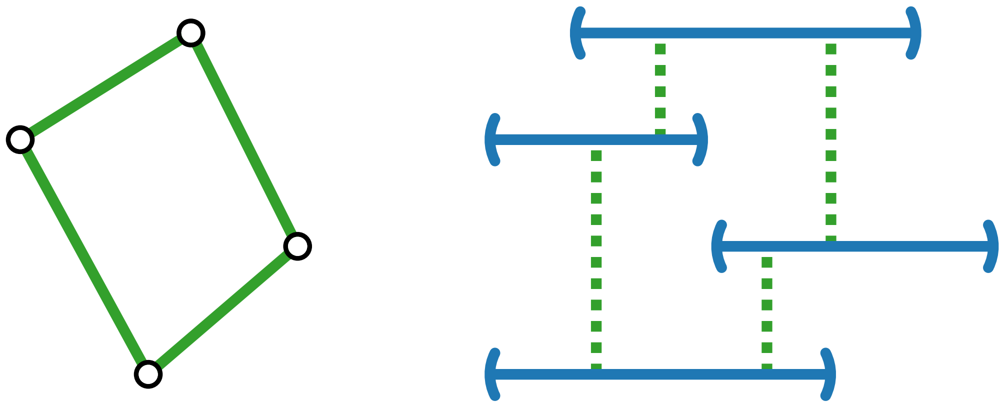

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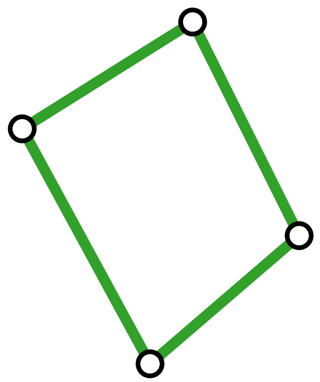

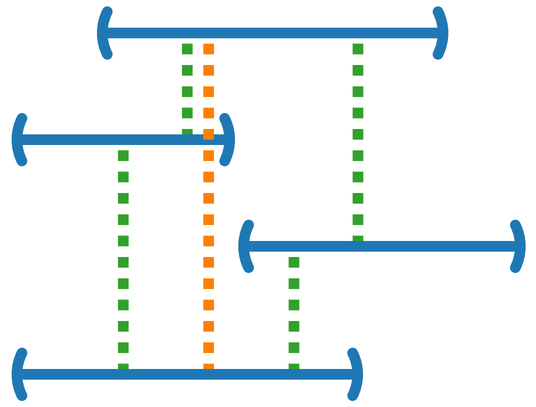

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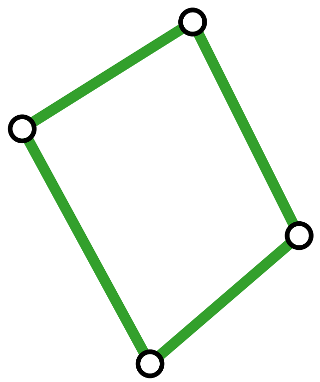

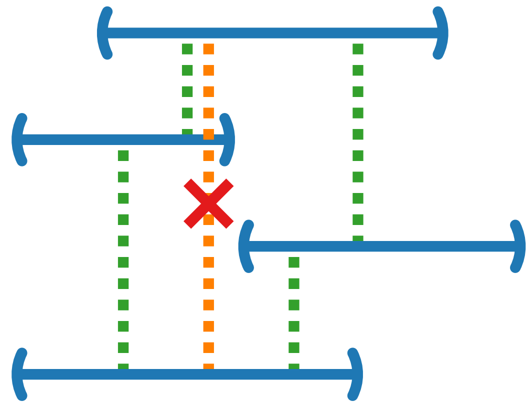

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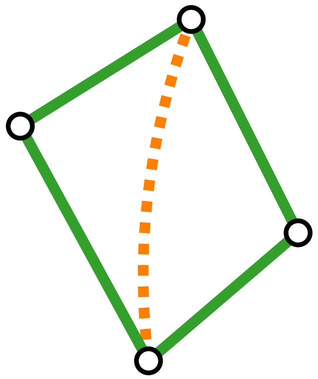

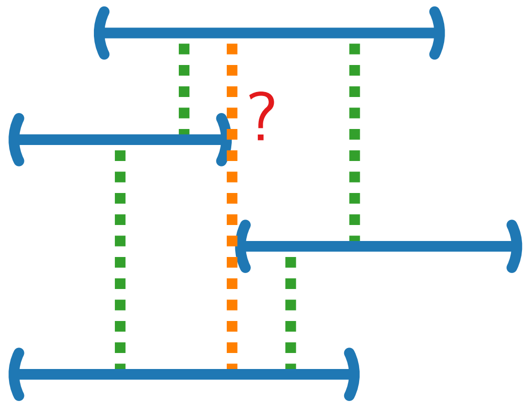

Bar Visibility Representation

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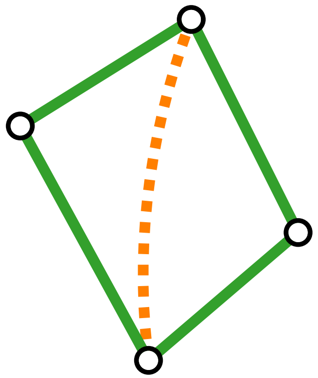

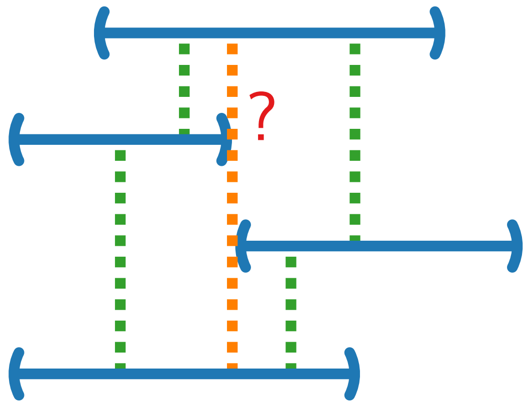

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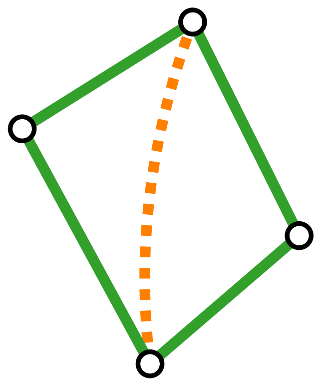

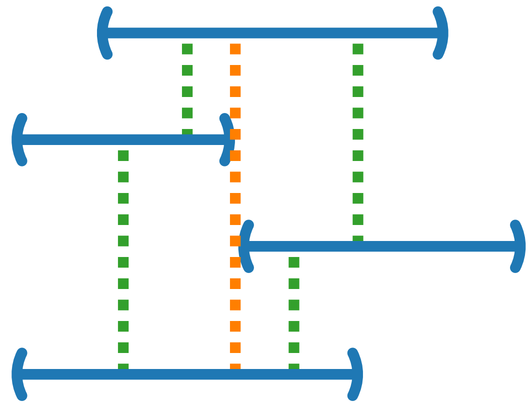

Models.

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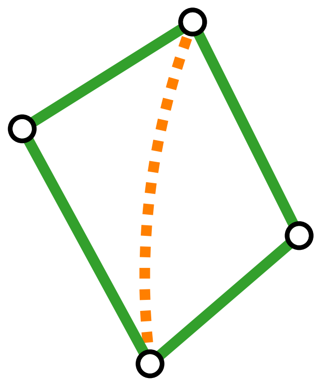

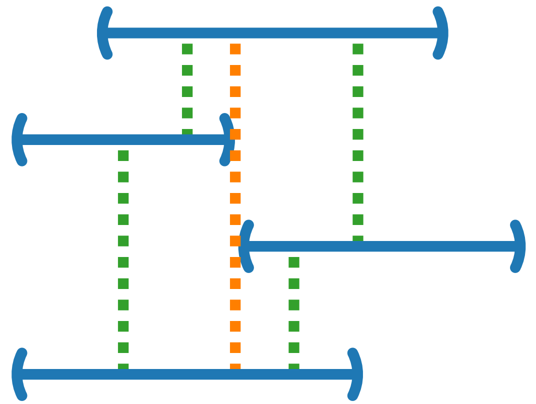

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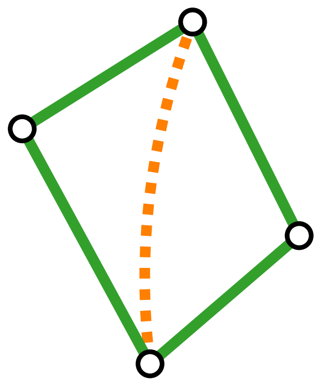

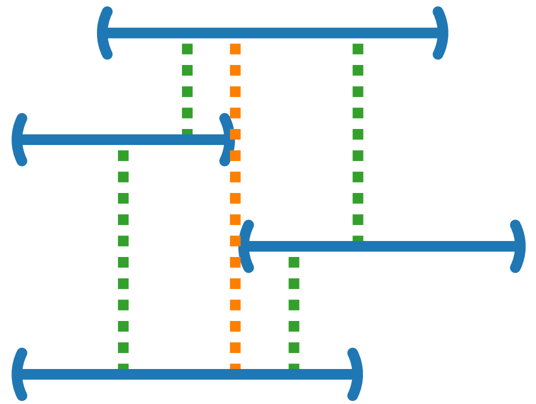

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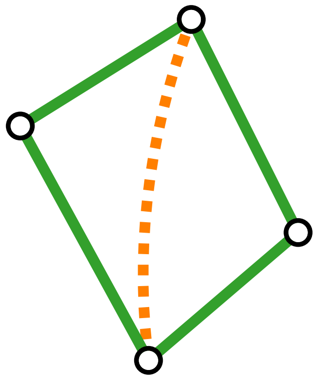

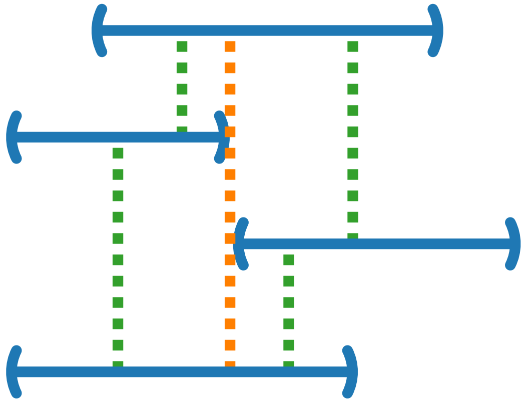

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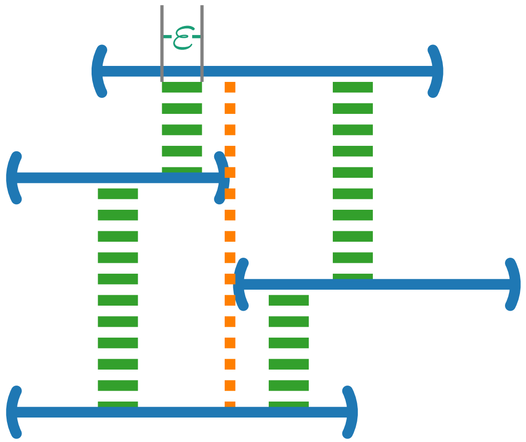

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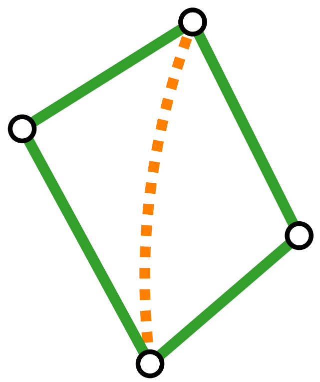

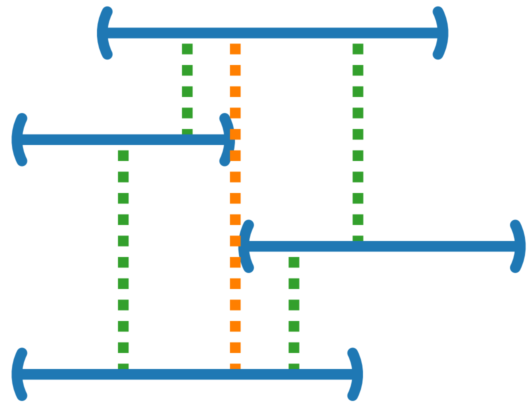

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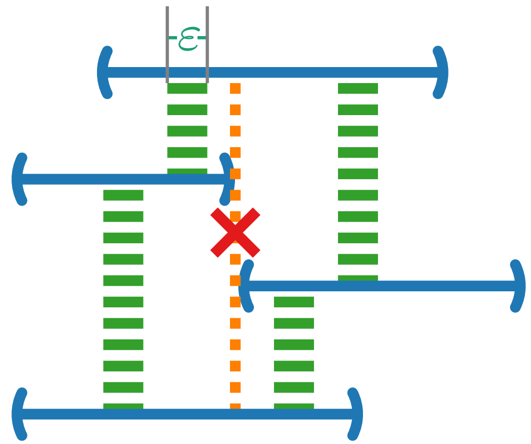

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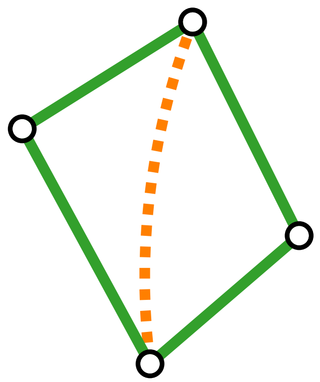

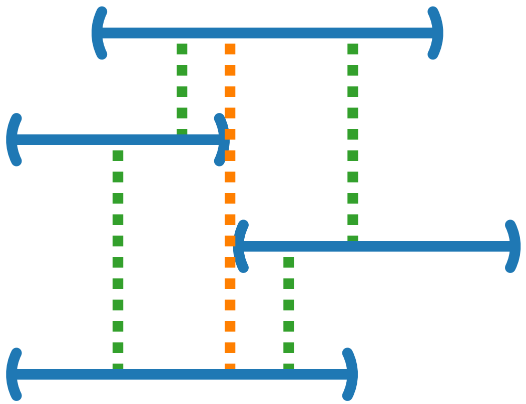

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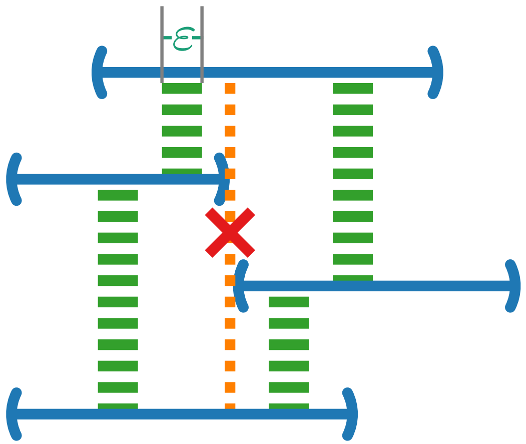

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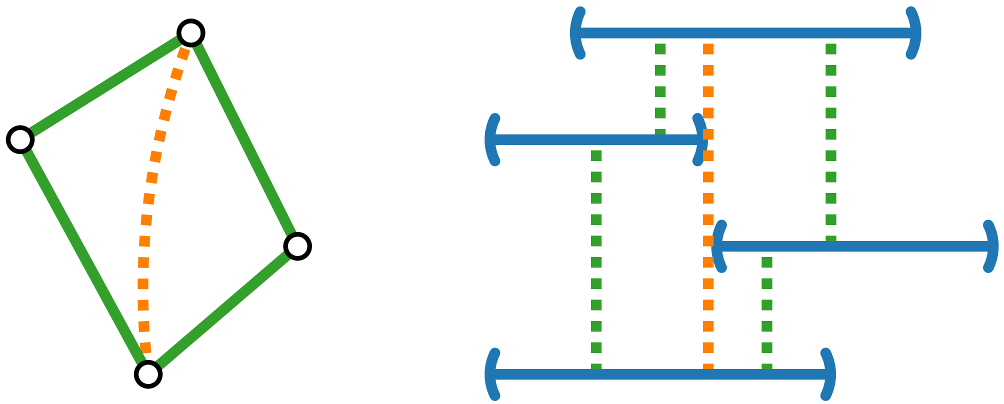

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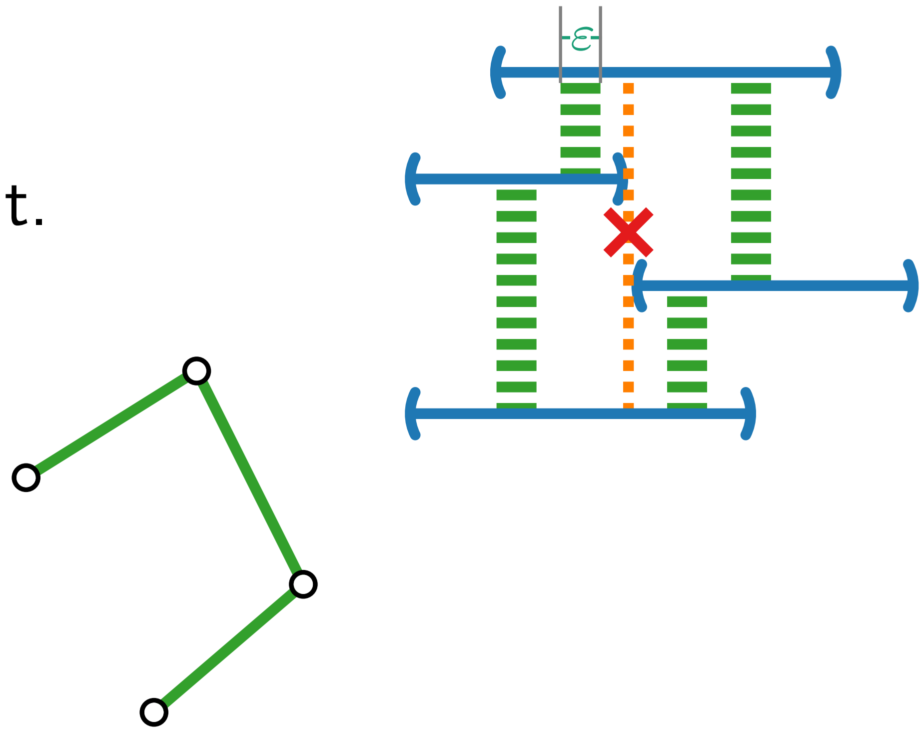
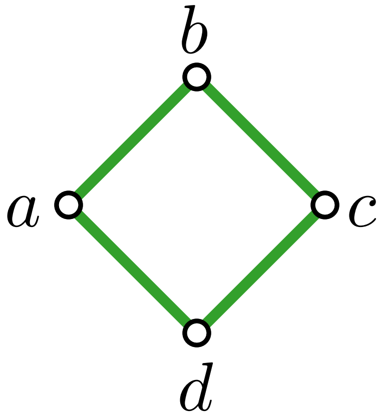
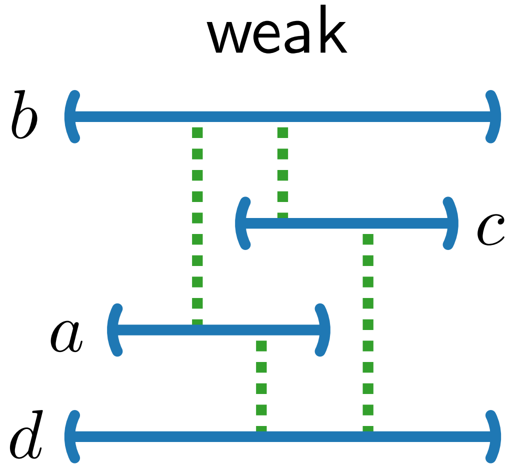


Problems





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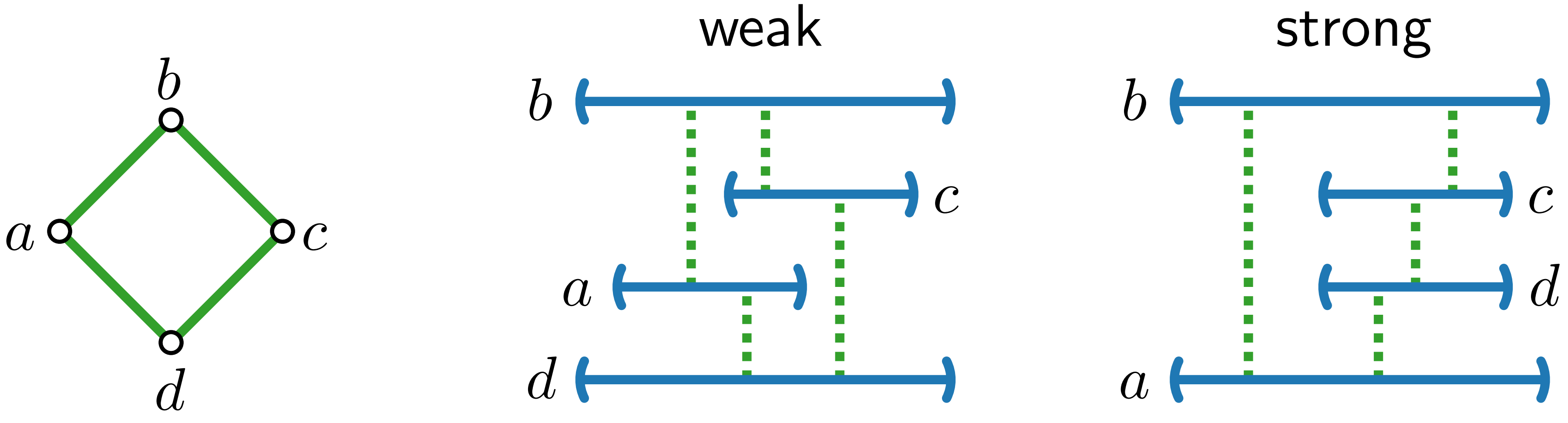

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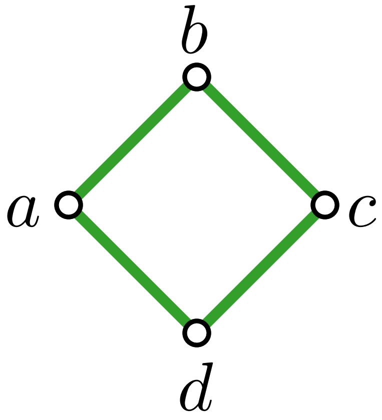

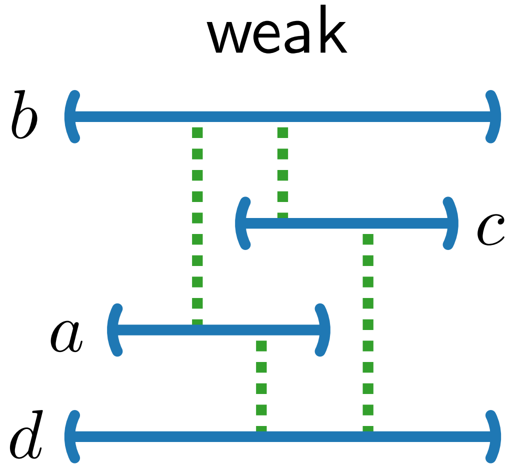

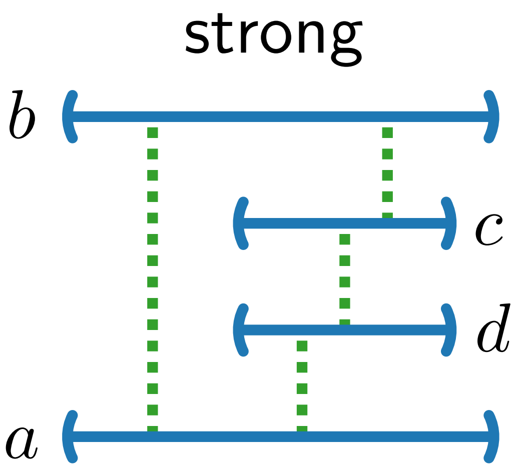

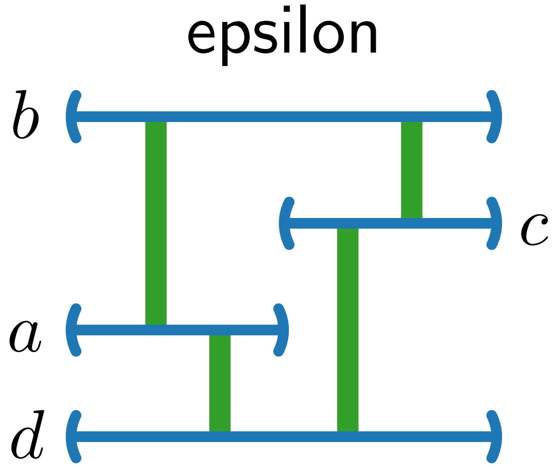

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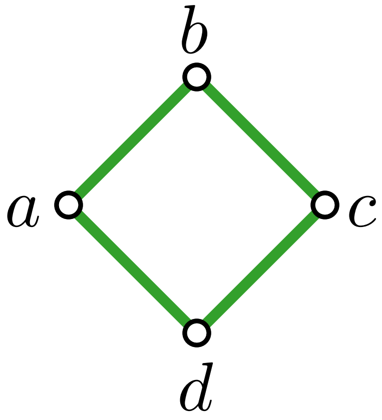

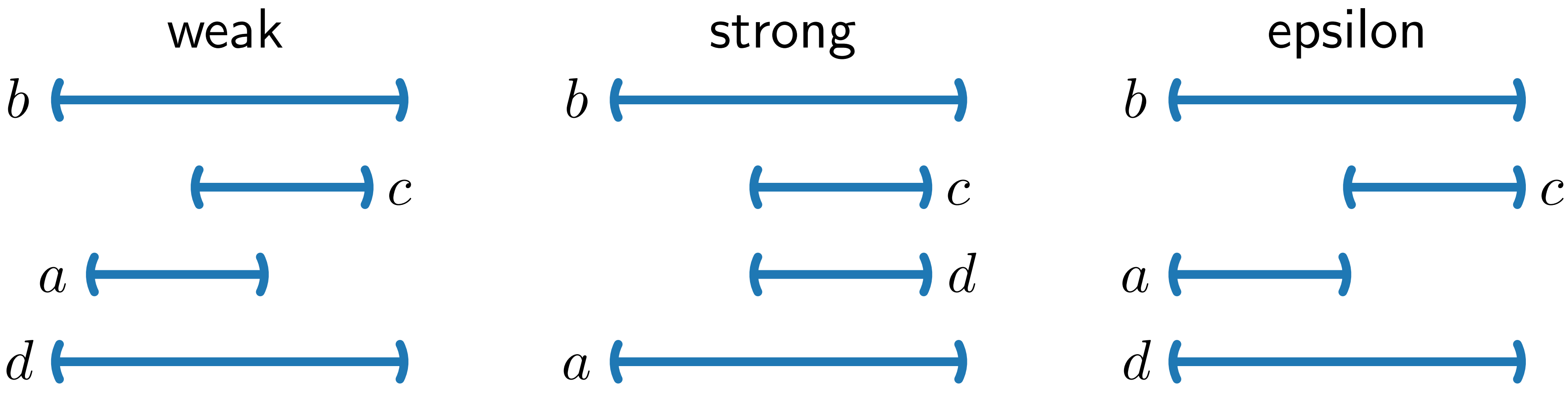

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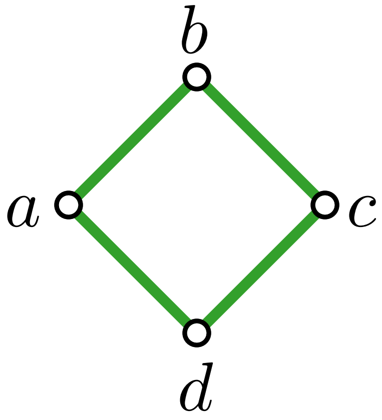

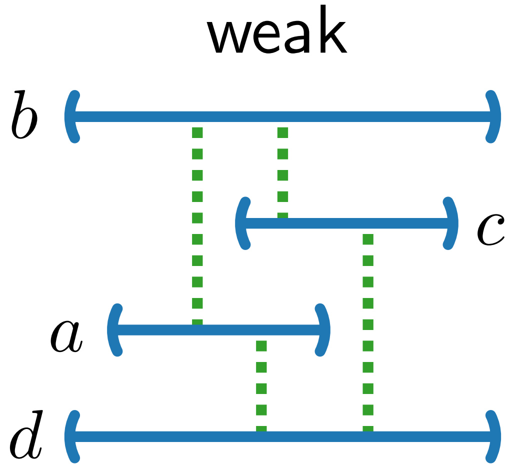

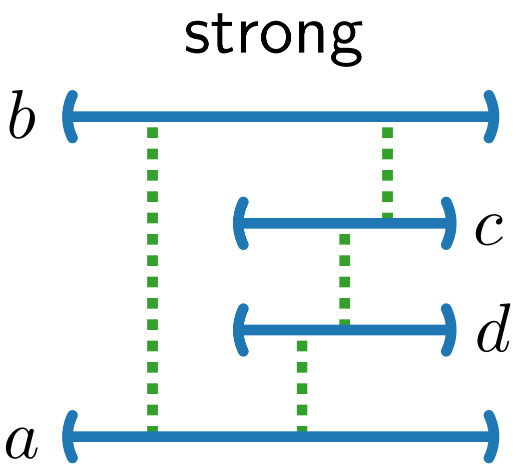

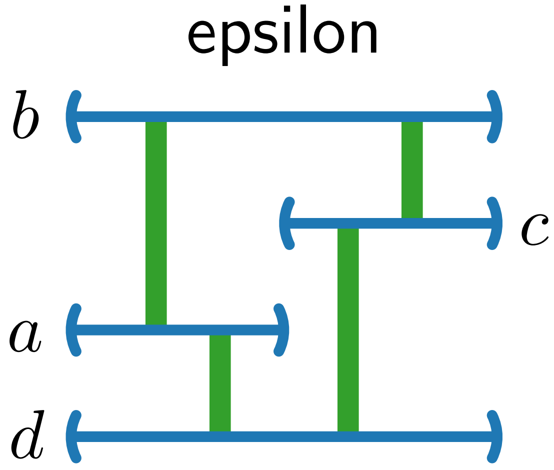

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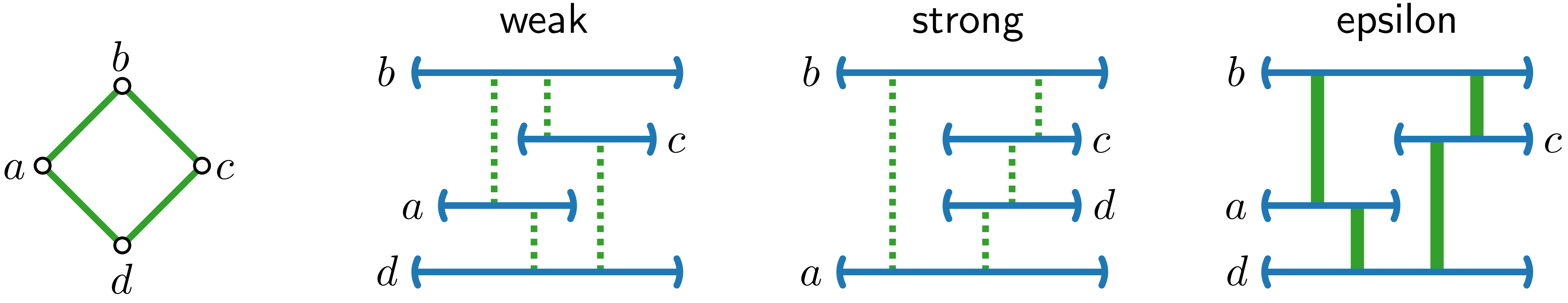
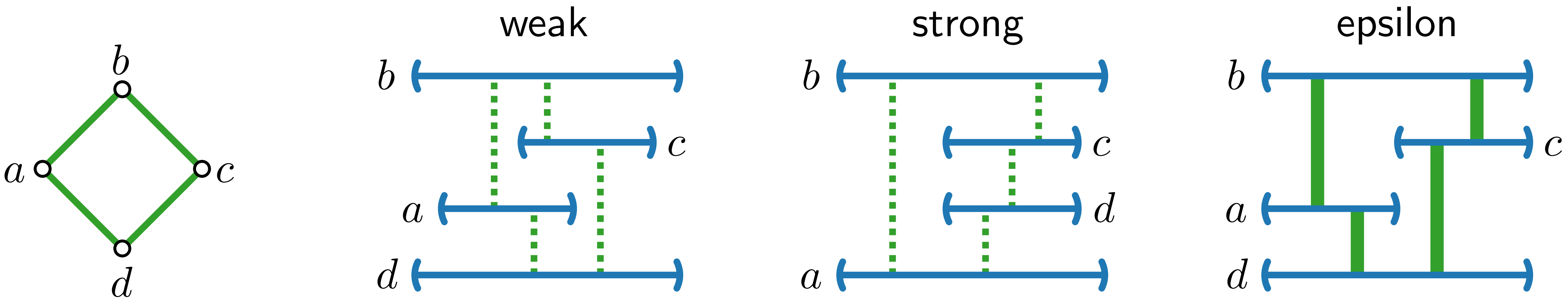


Recognition Problem.
Given a graph G, **decide** whether there exists a weak/strong/ε bar visibility representation ψ of G.

Problems



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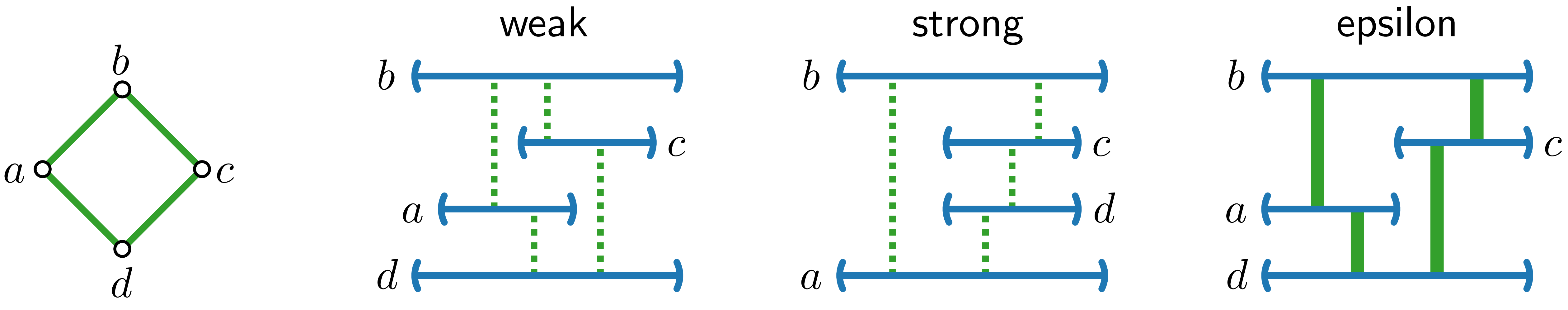

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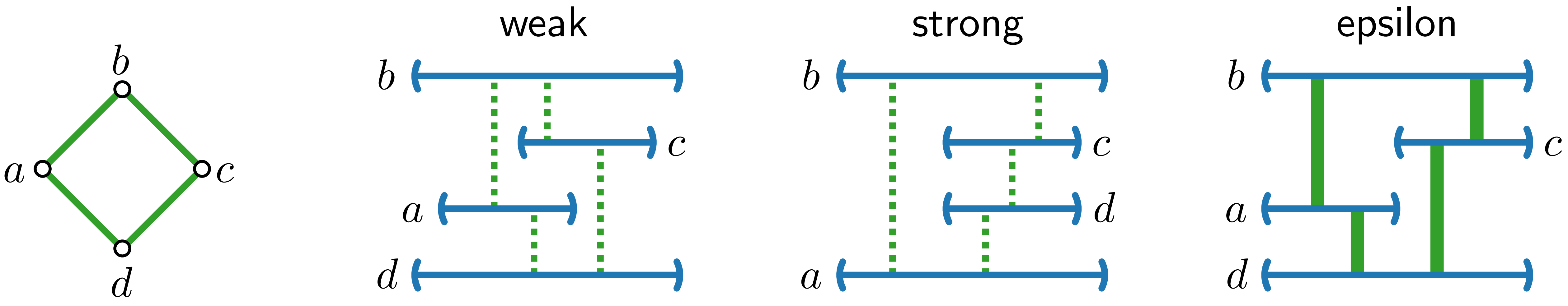
Partial Representation Extension Problem.
Given a graph G and a **set of bars** ψ' **of** $V' \subset V(G)$, **decide** whether there exists a weak/strong/ε bar visibility representation ψ of G **where** $\psi|_{V'} = \psi'$ (and **construct** ψ if a representation exists).

Background



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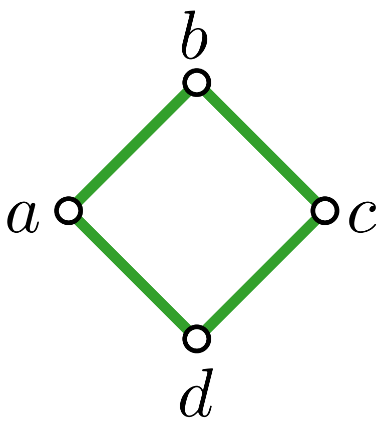

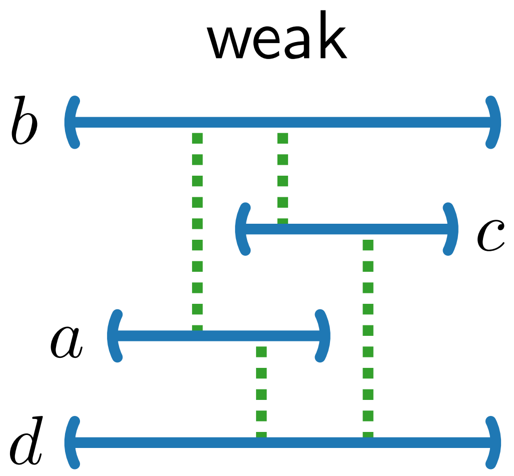

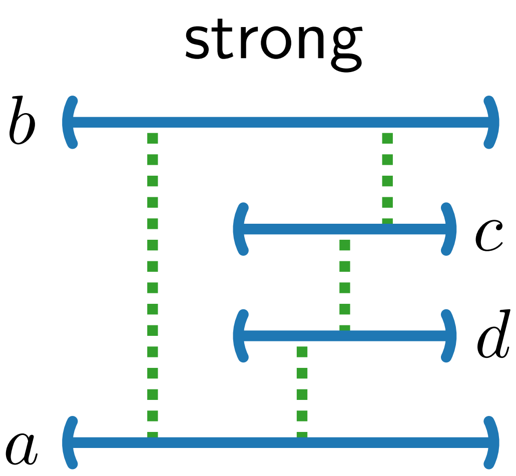

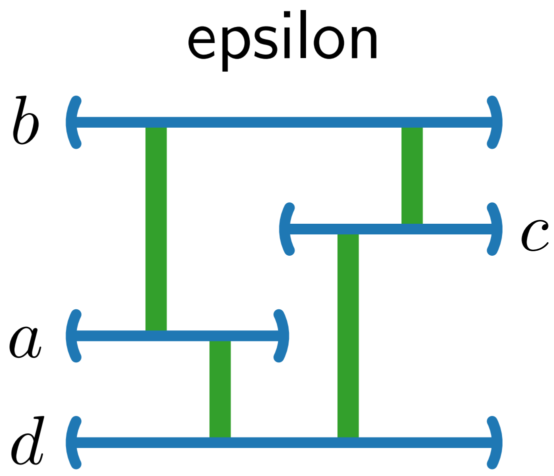

Weak Bar Visibility.

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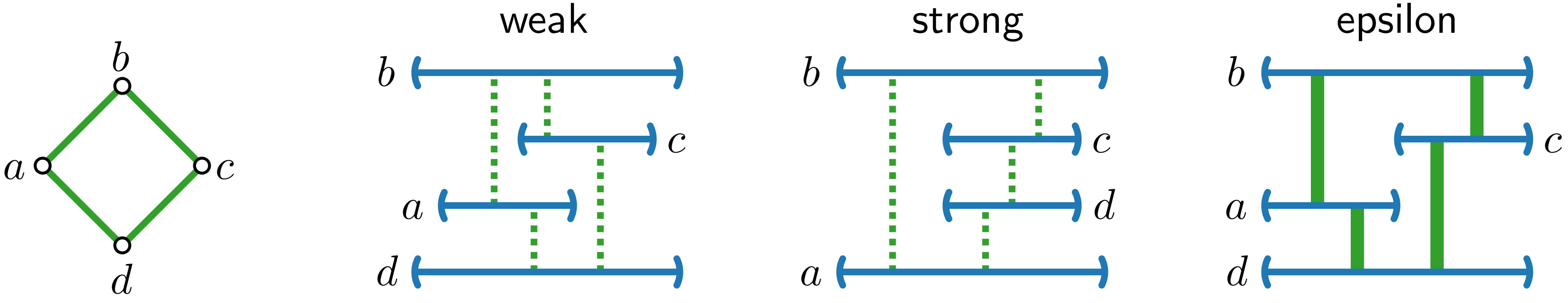

Weak Bar Visibility.

- All planar graphs. [Tamassia & Tollis '86; Wismath '85]

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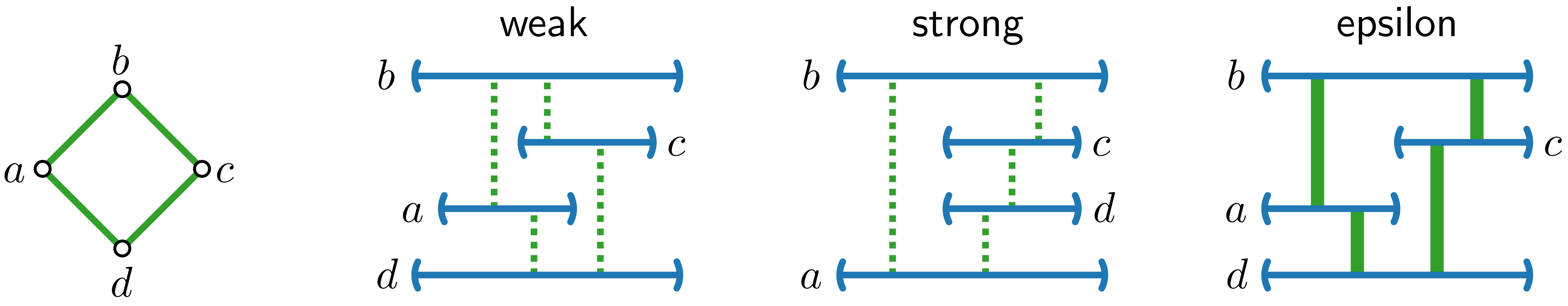

Weak Bar Visibility.

- All planar graphs. [Tamassia & Tollis ’86; Wismath ’85]
- Linear time recognition and construction [T&T ’86]

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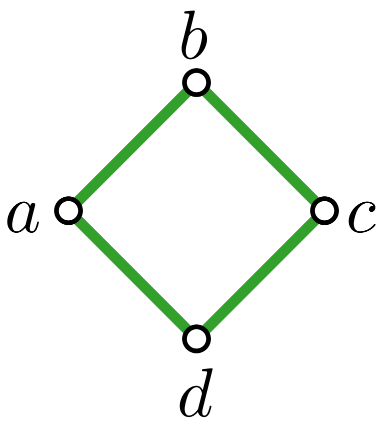

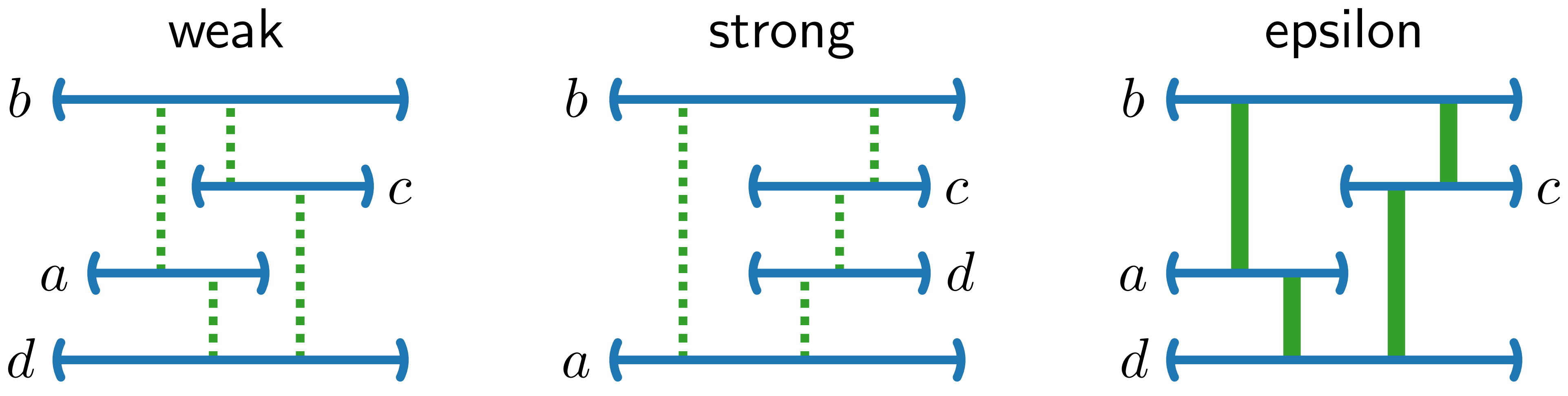

Weak Bar Visibility.

- All planar graphs. [Tamassia & Tollis '86; Wismath '85]
- Linear time recognition and construction [T&T '86]
- Representation Extension is NP-complete [Chaplick et al. '14]

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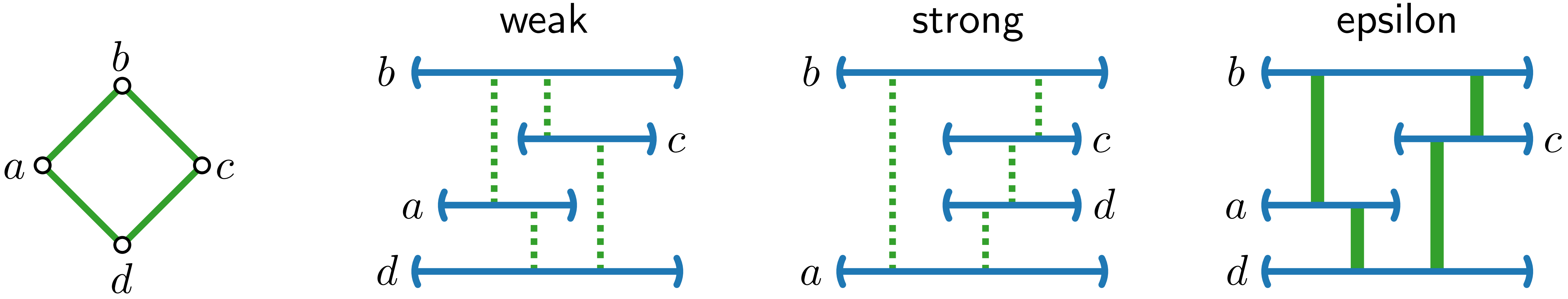

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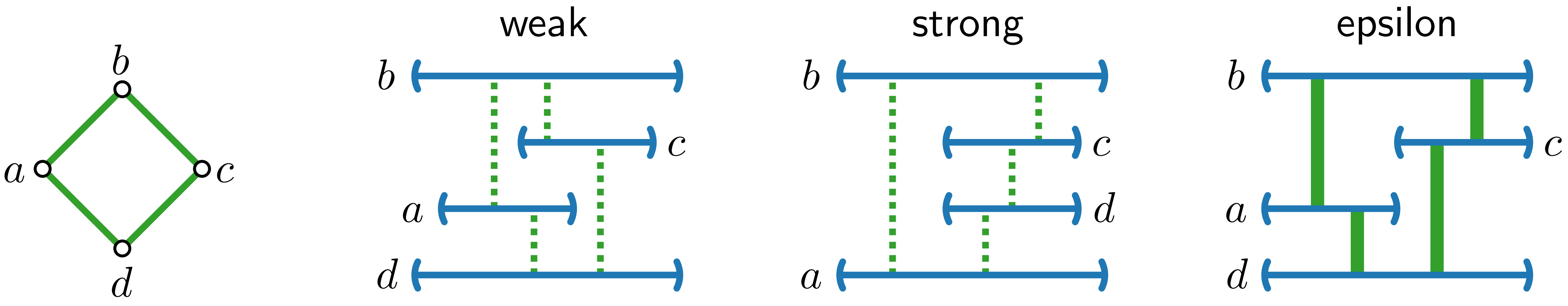

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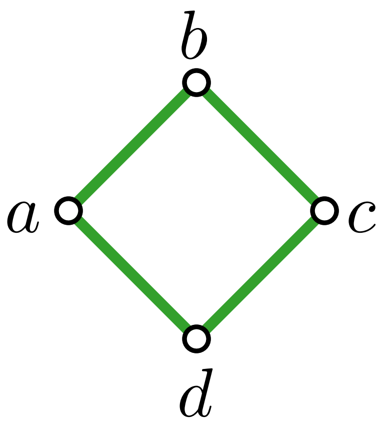
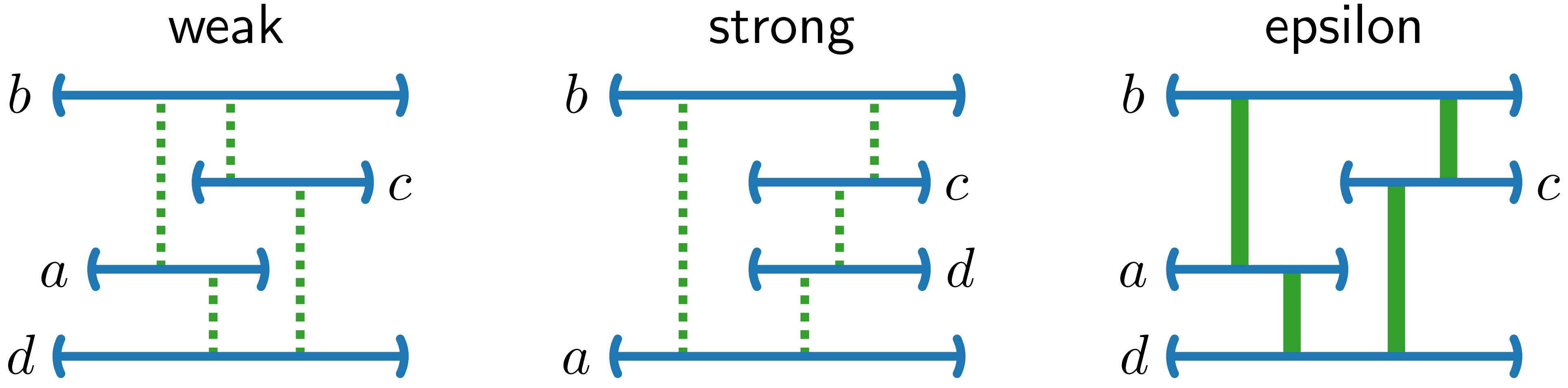
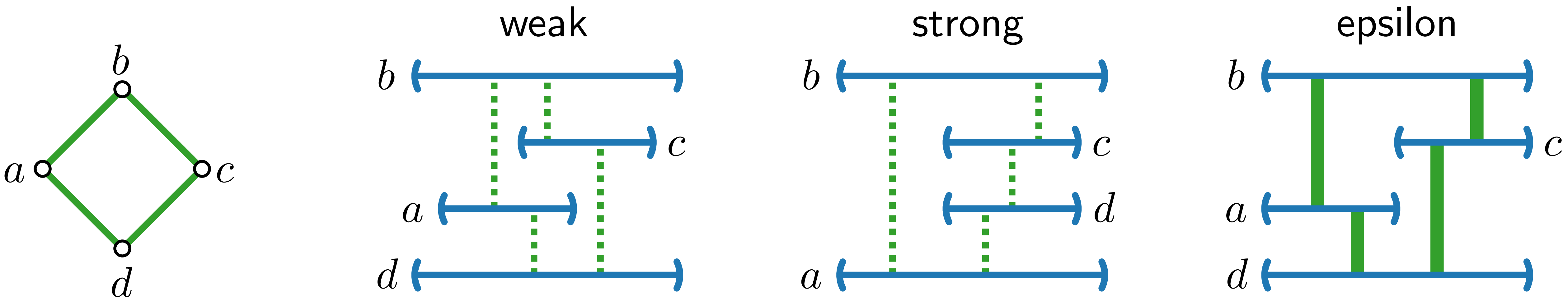




ε-Bar Visibility.

Background



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- Planar graphs that can be embedded with all **cut vertices** on the outerface. [T&T '86, Wismath '85]

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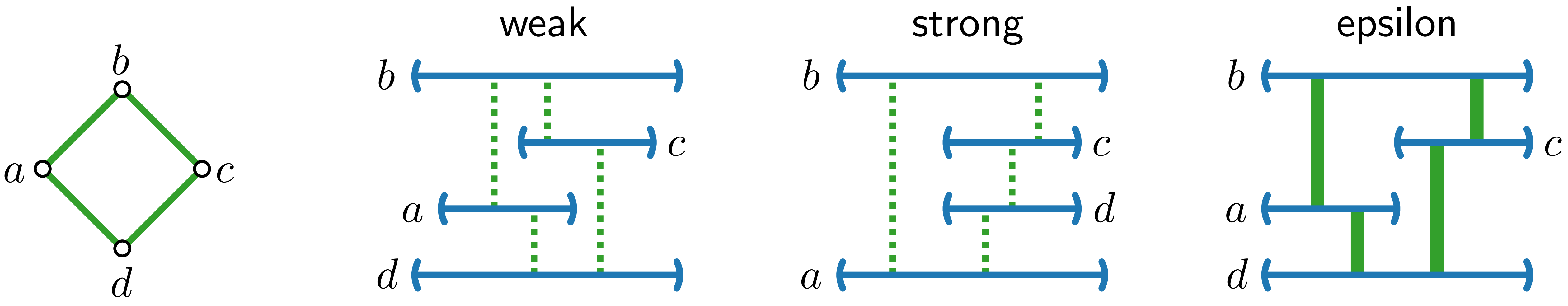

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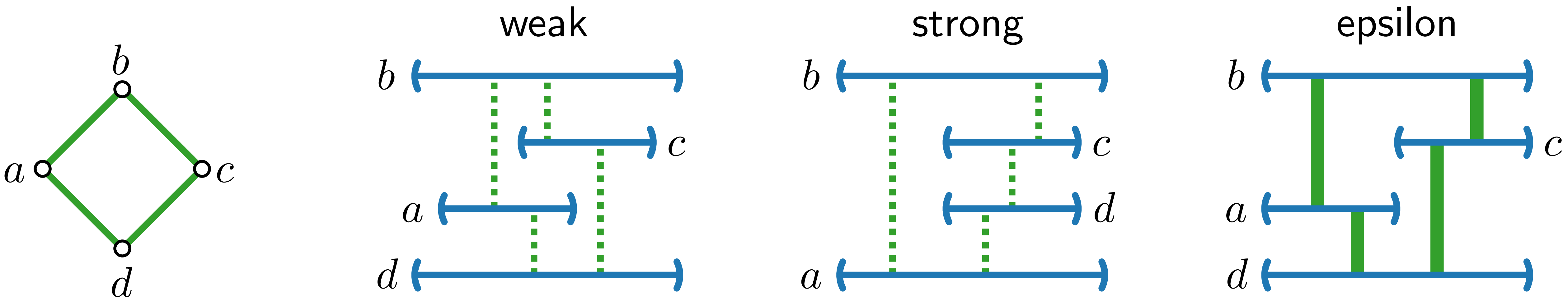

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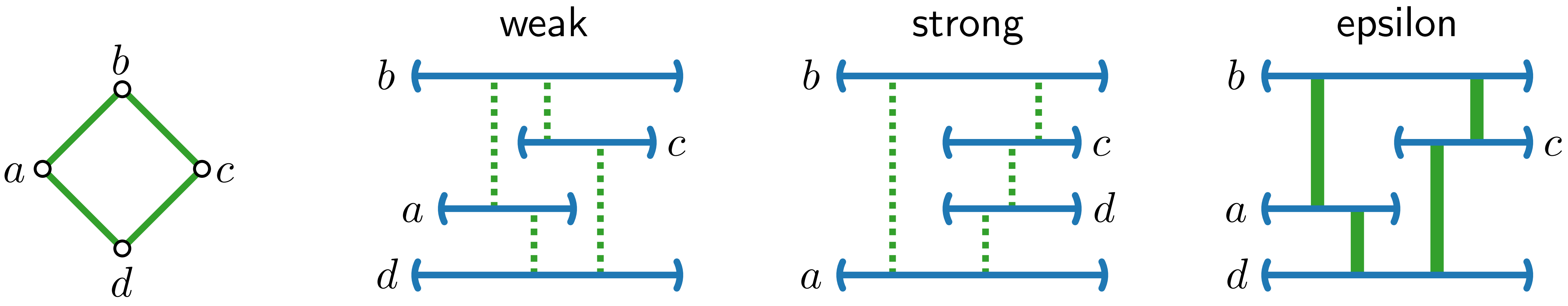

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Julius-Maximilians-
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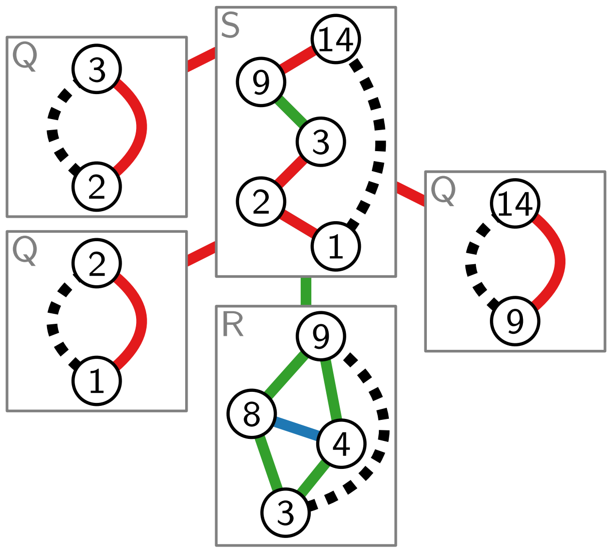
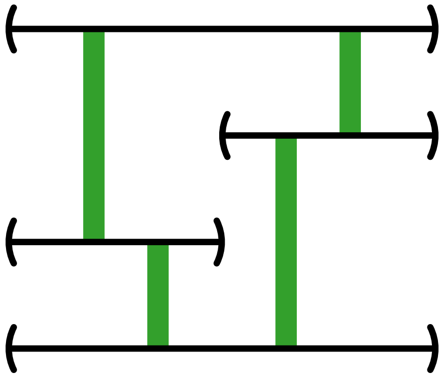
WÜRZBURG

Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension



Part II:
Recognition & Construction



Alexander Wolff

ε-bar Visibility and st-Graphs

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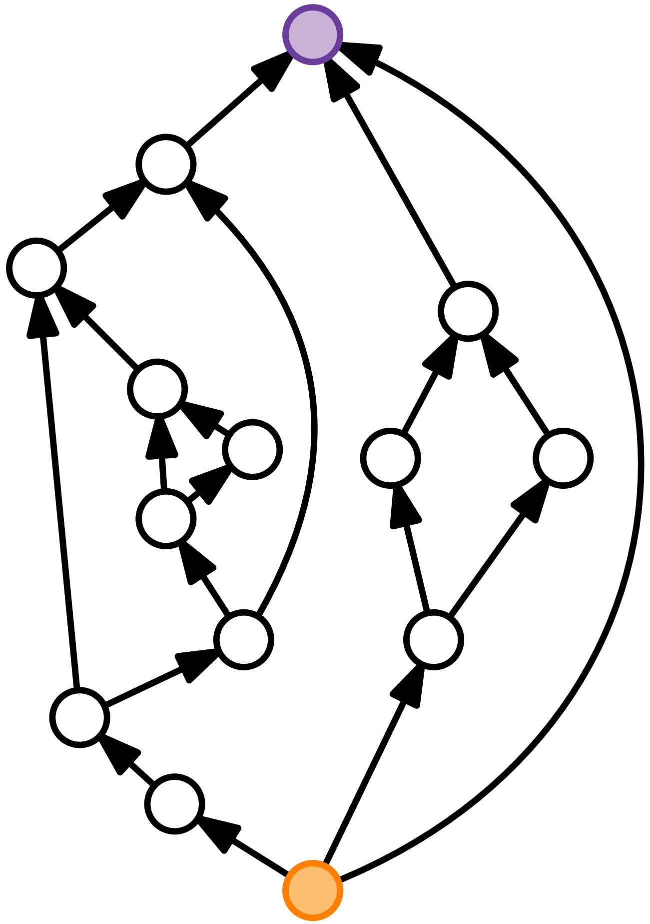

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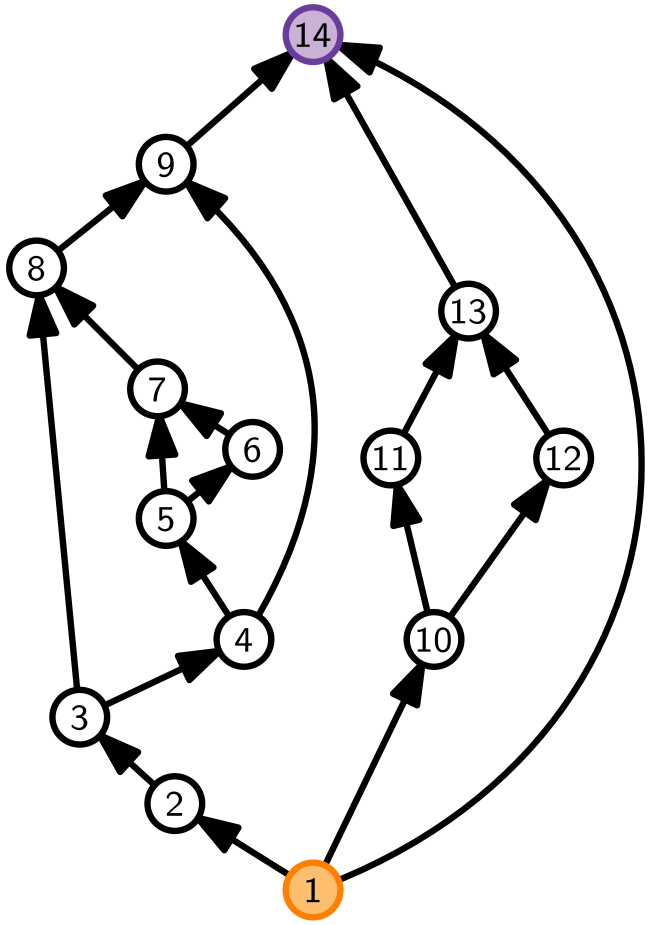

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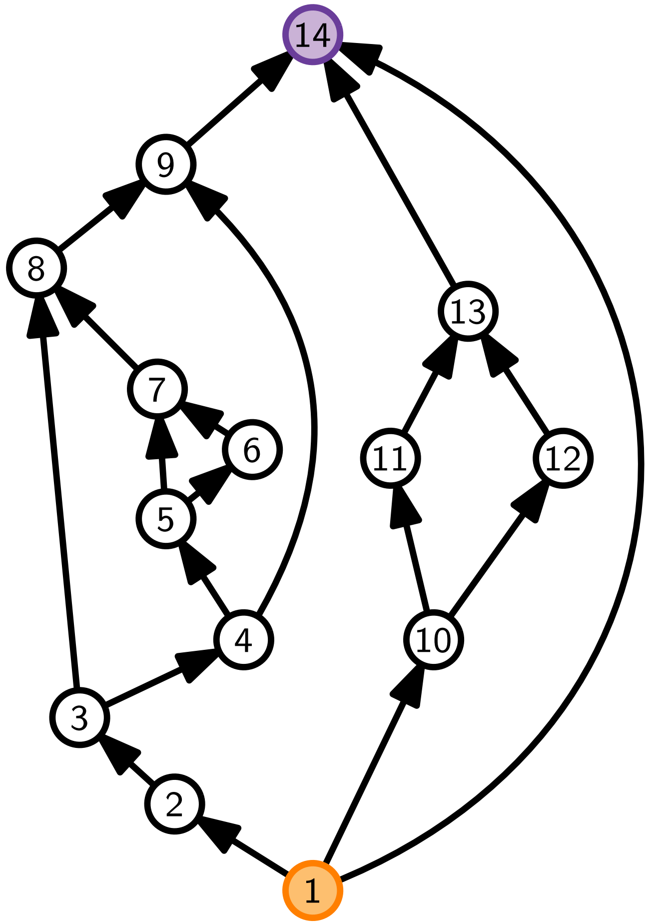

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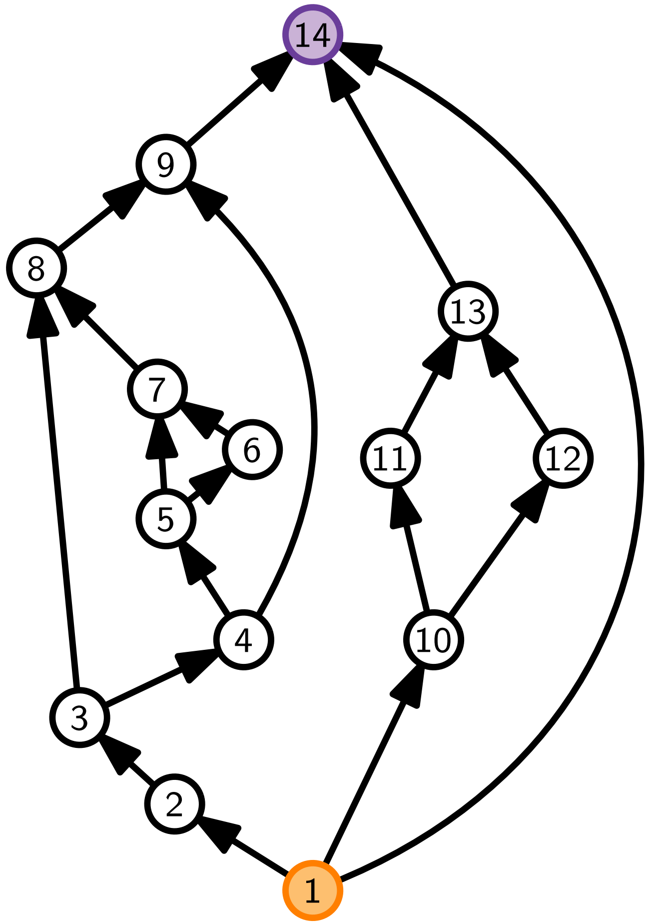

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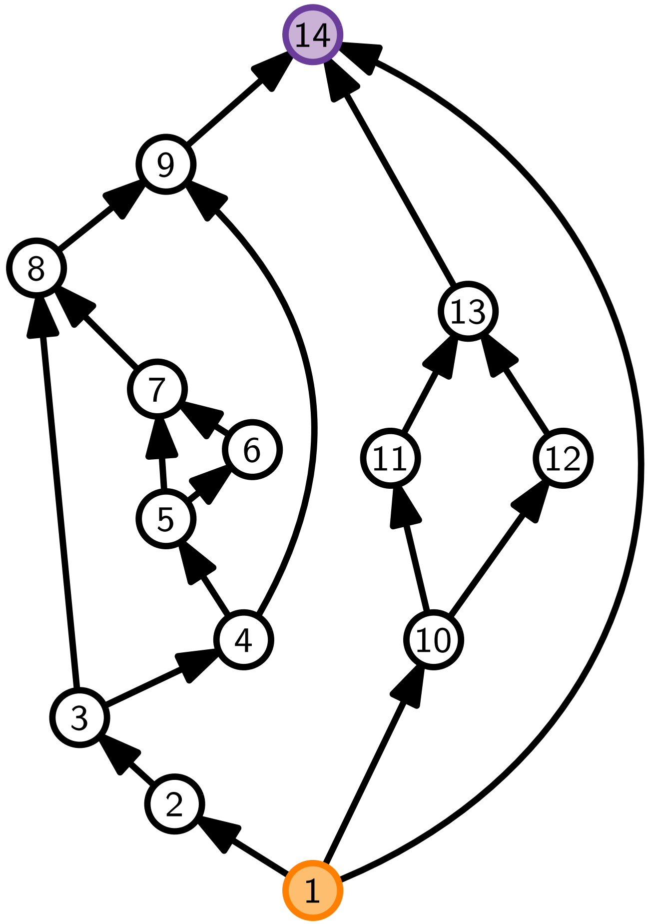

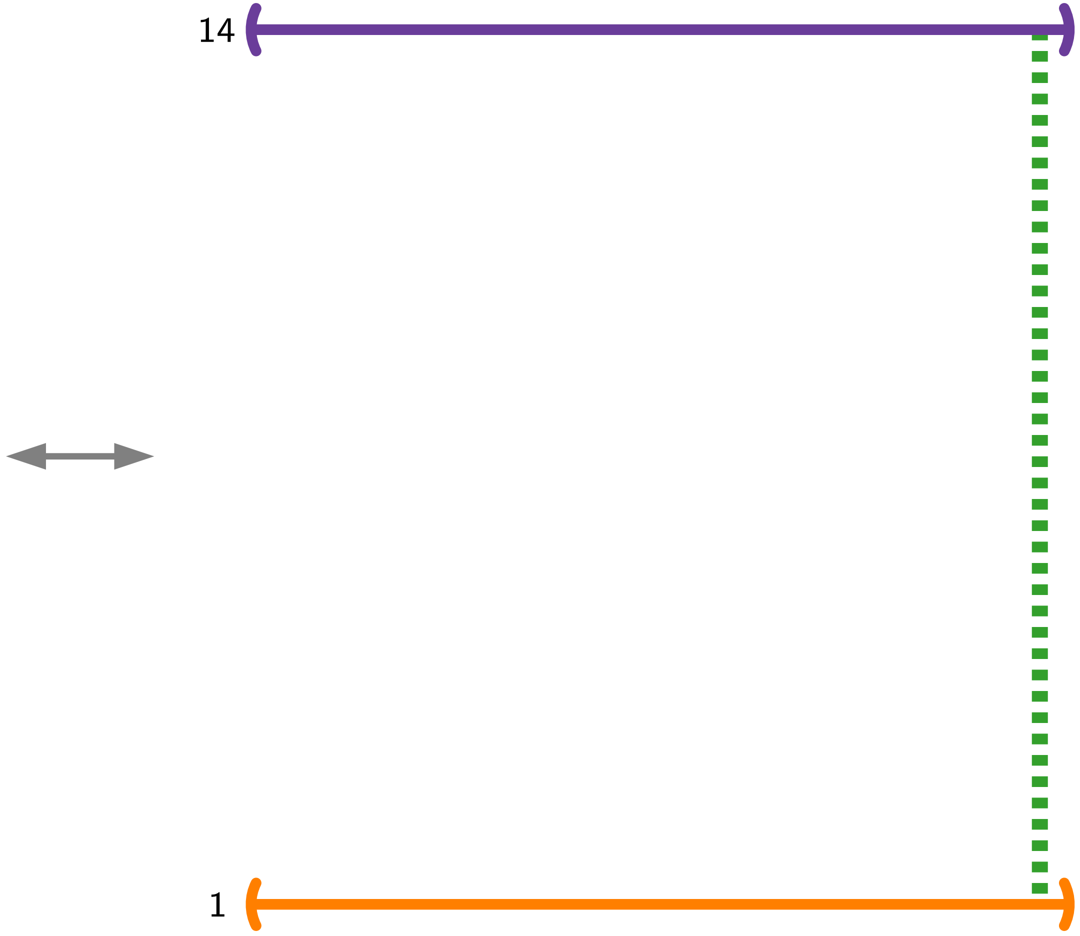

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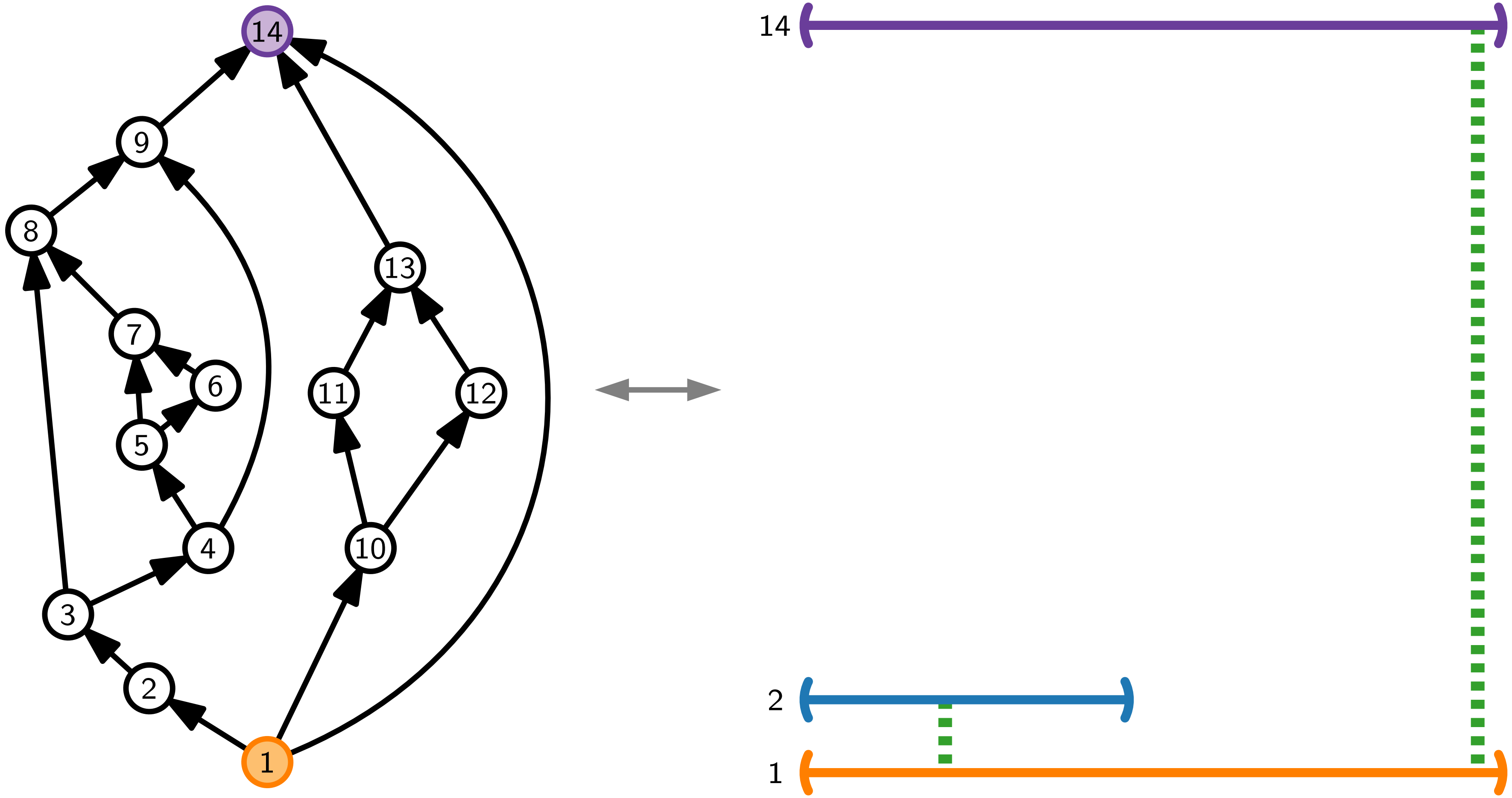

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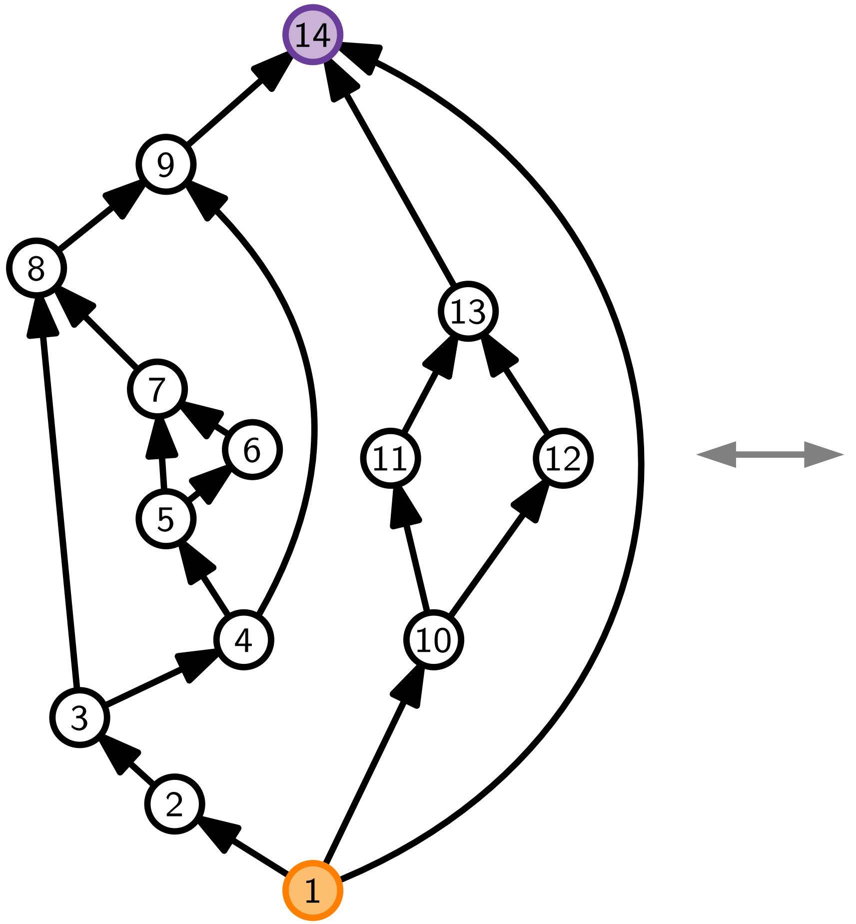

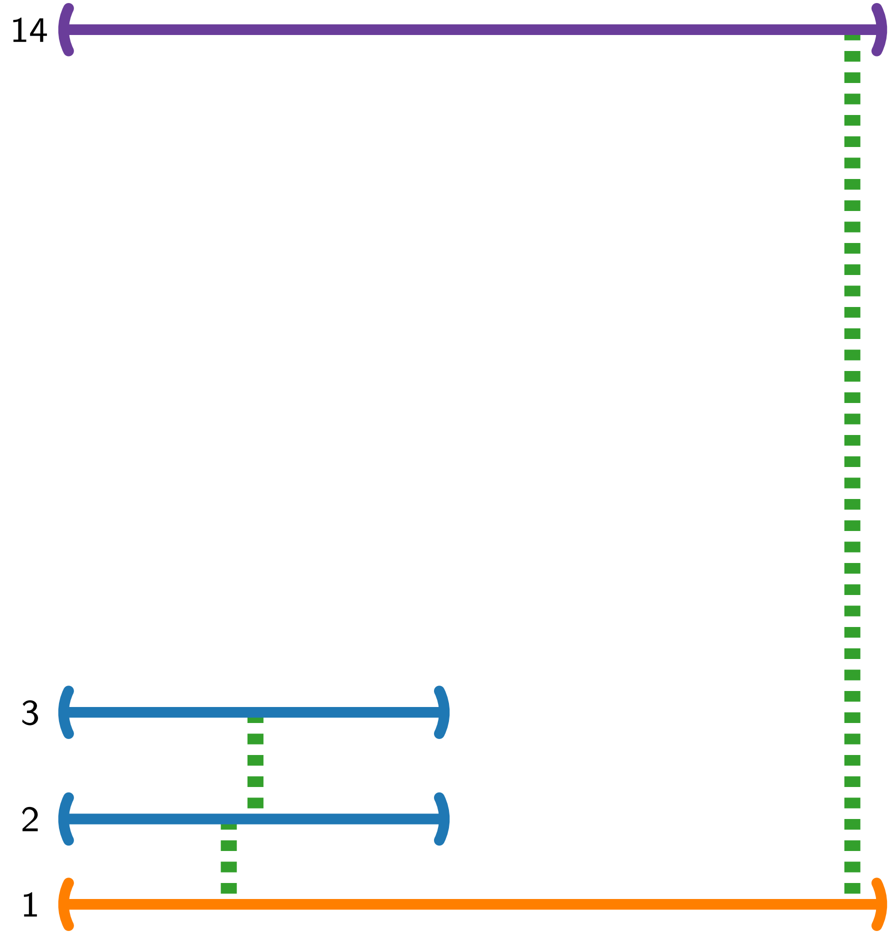

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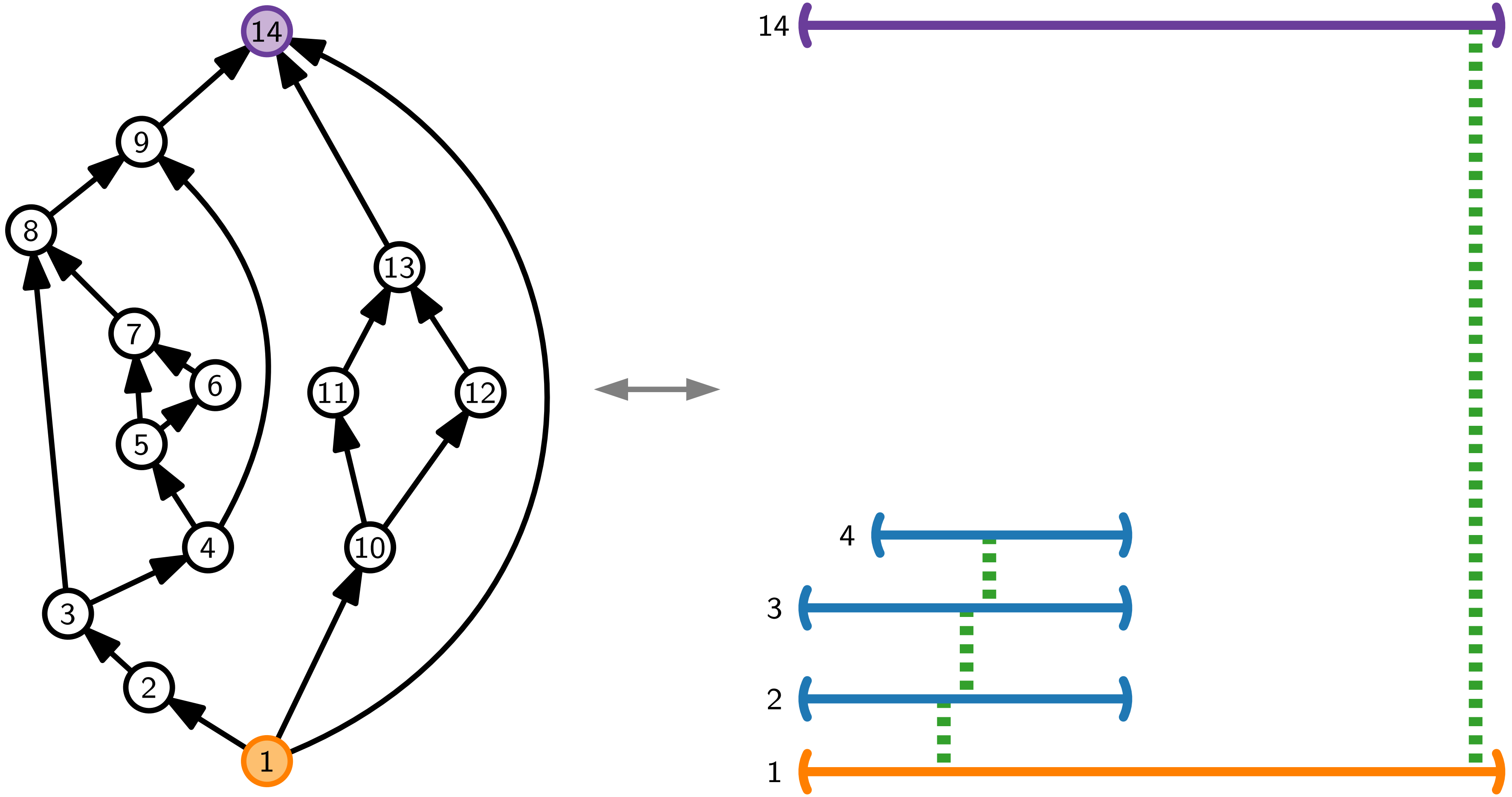

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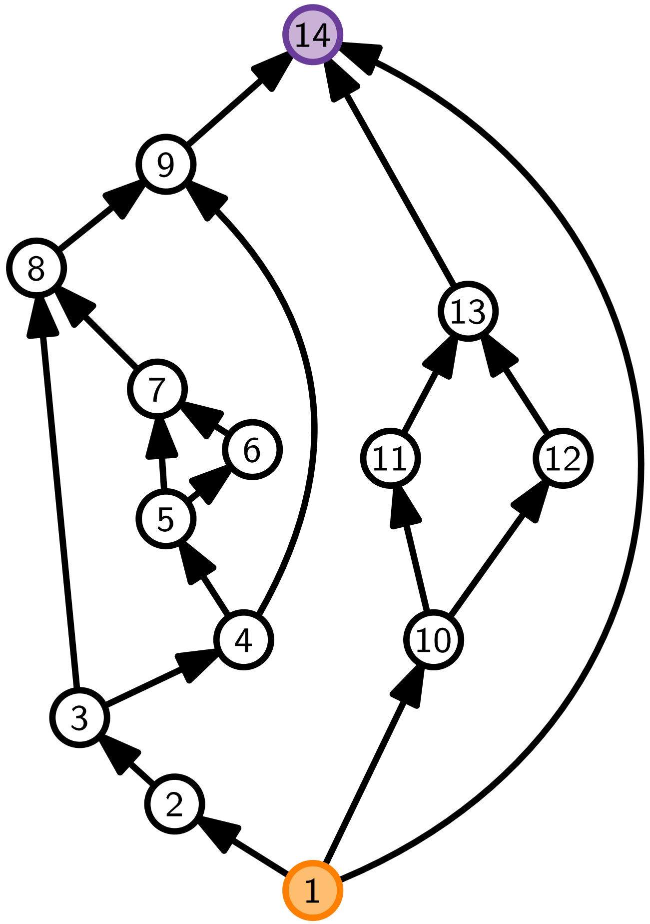

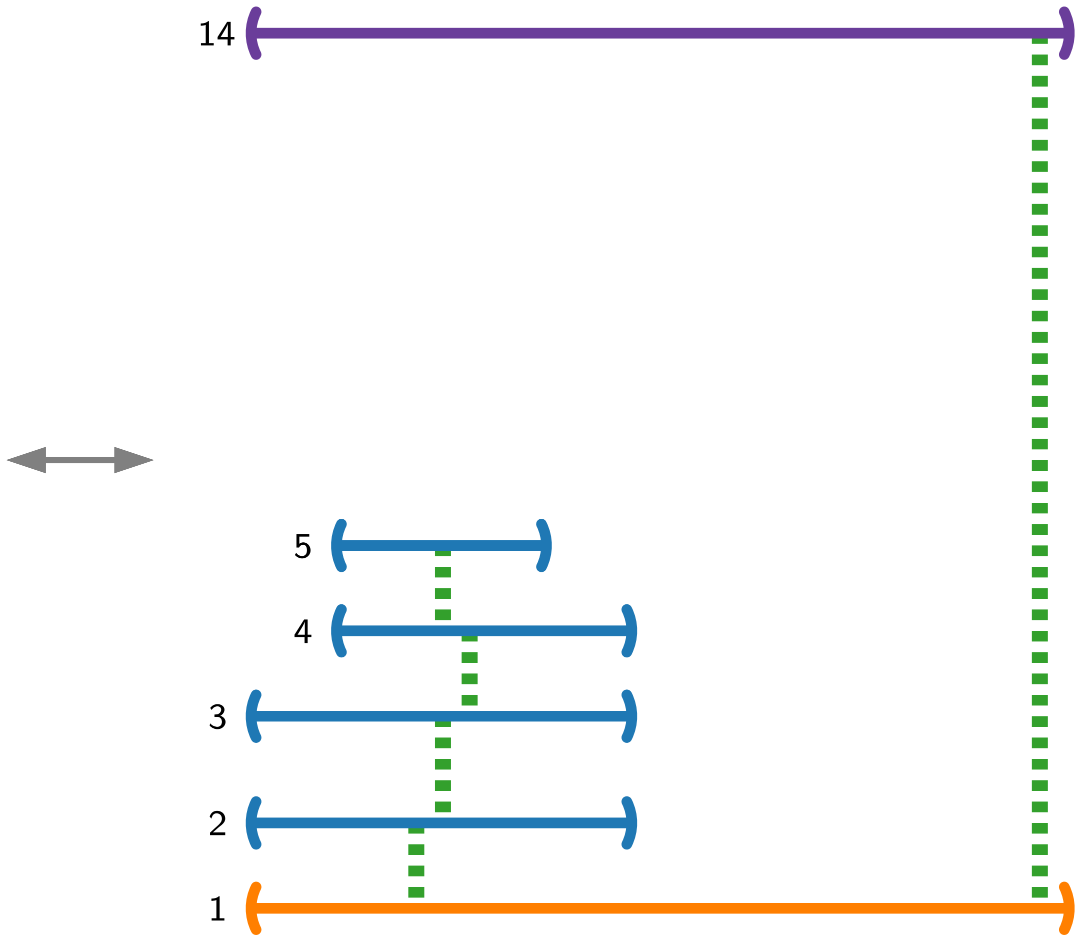

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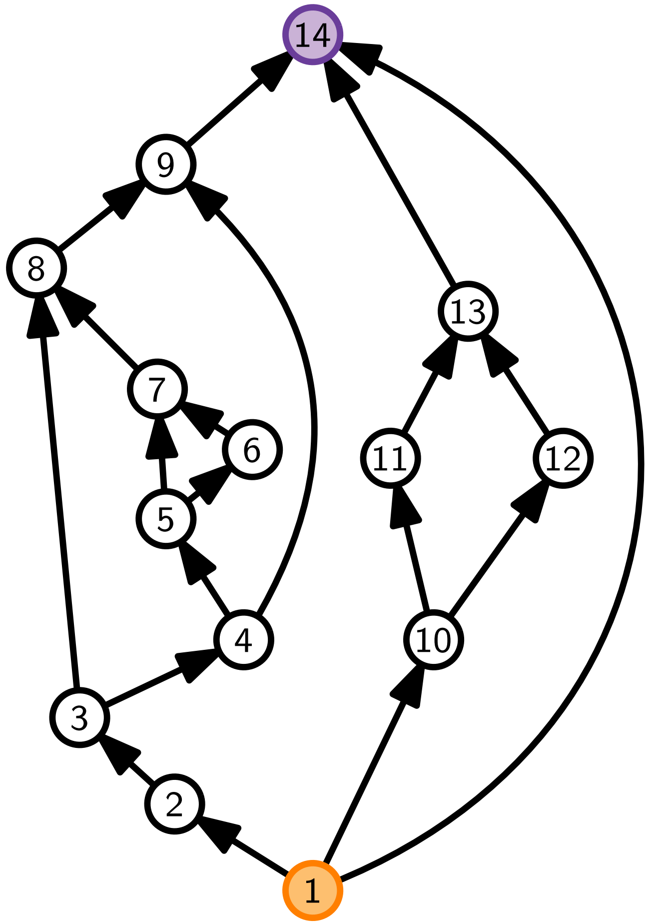

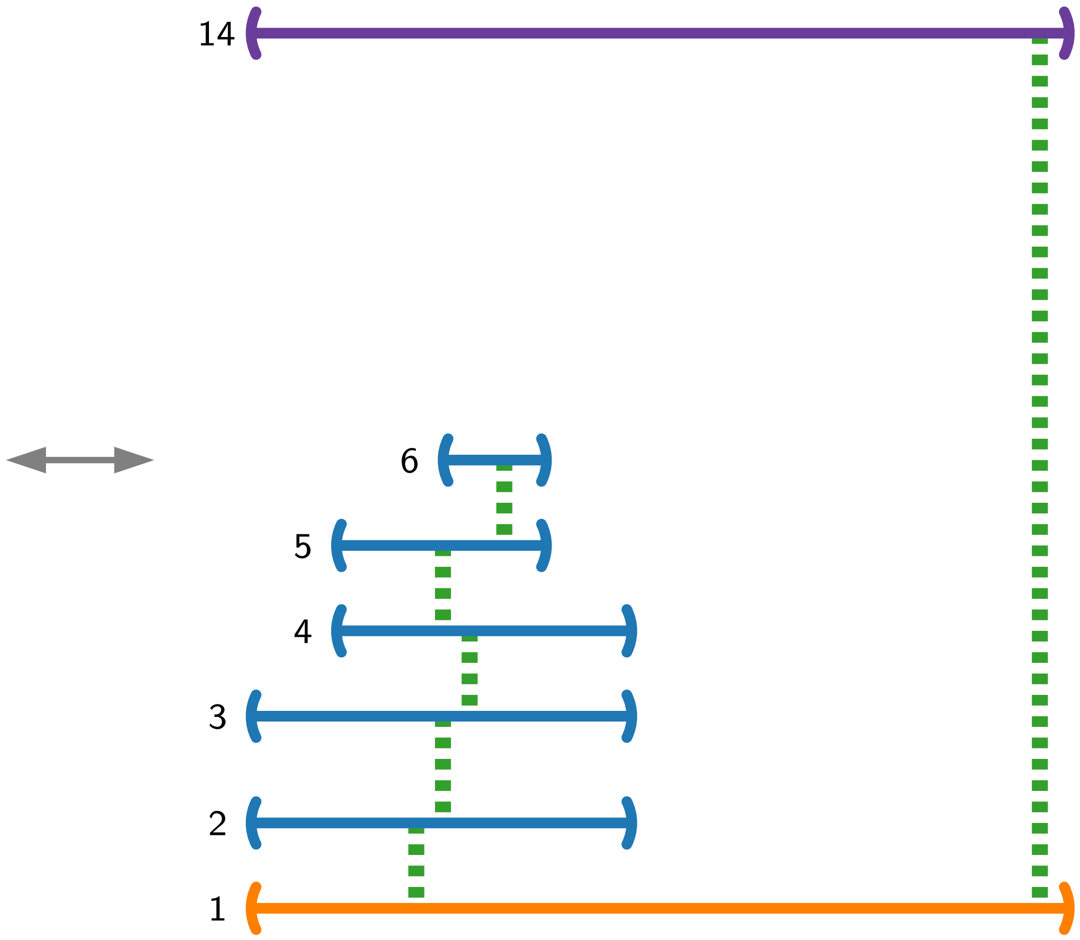

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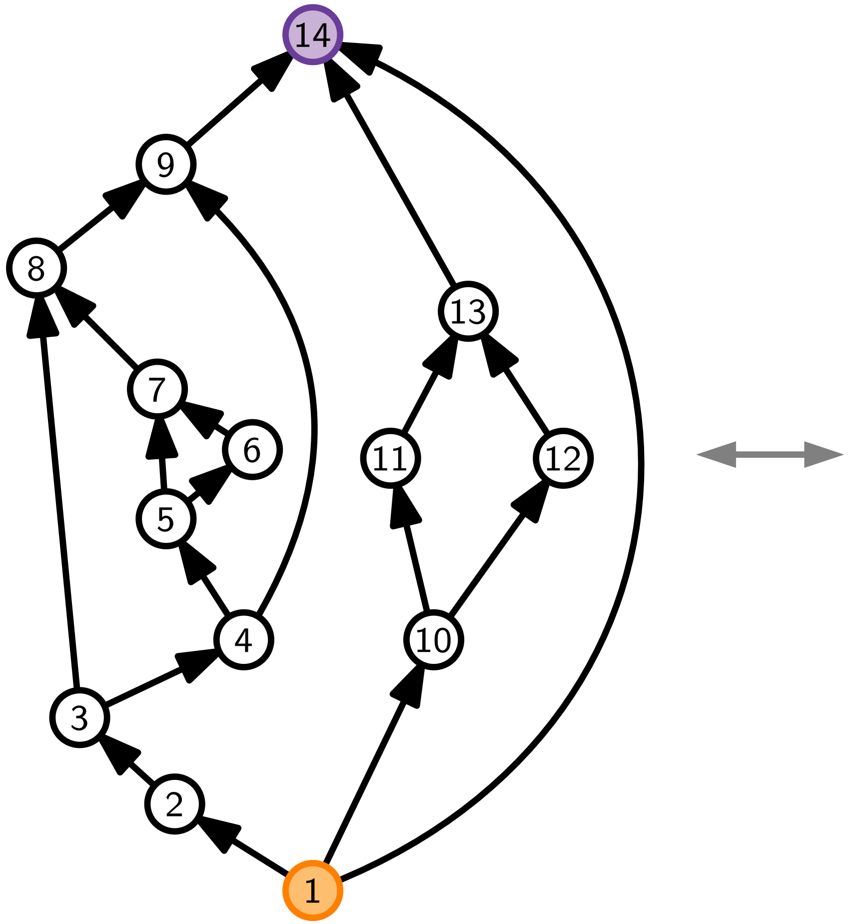

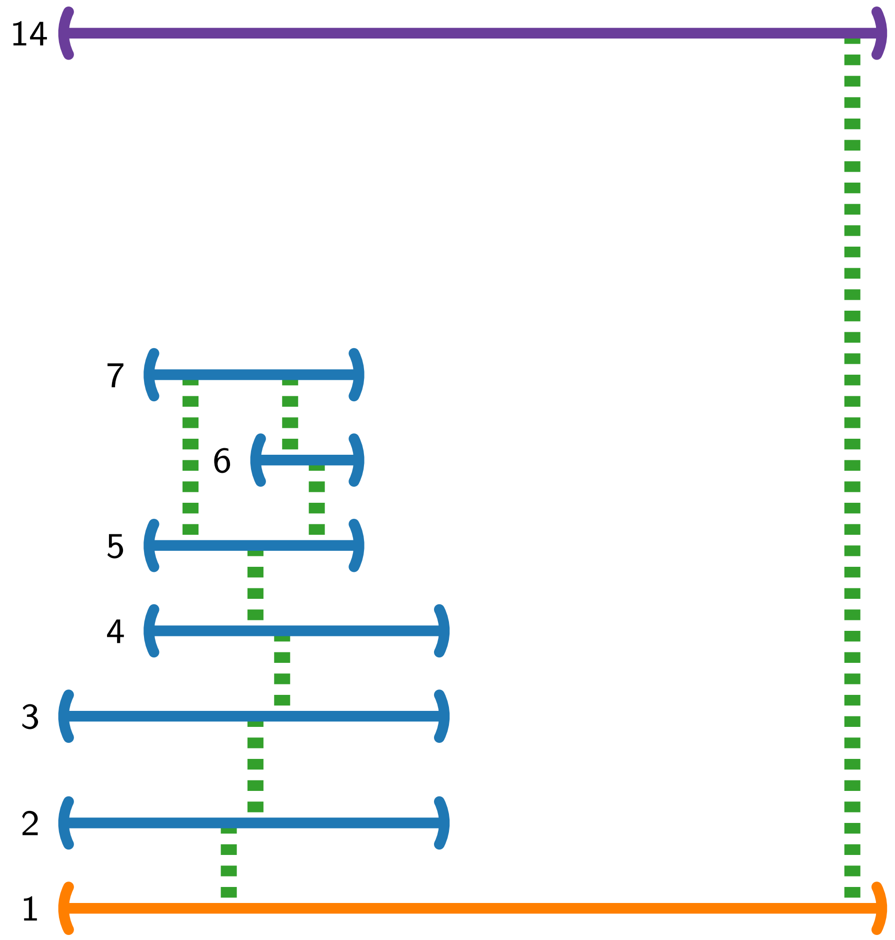

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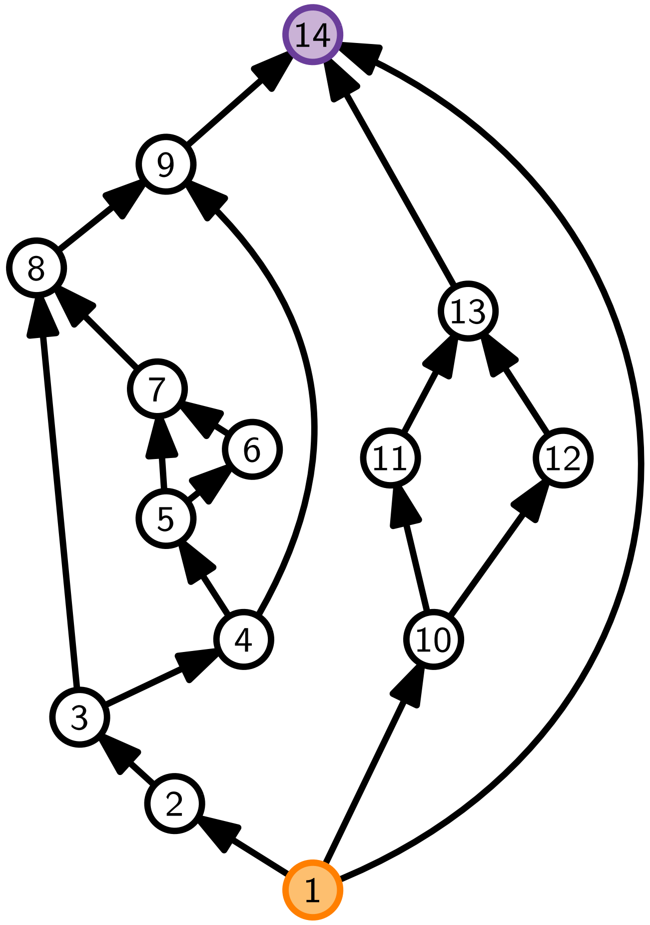

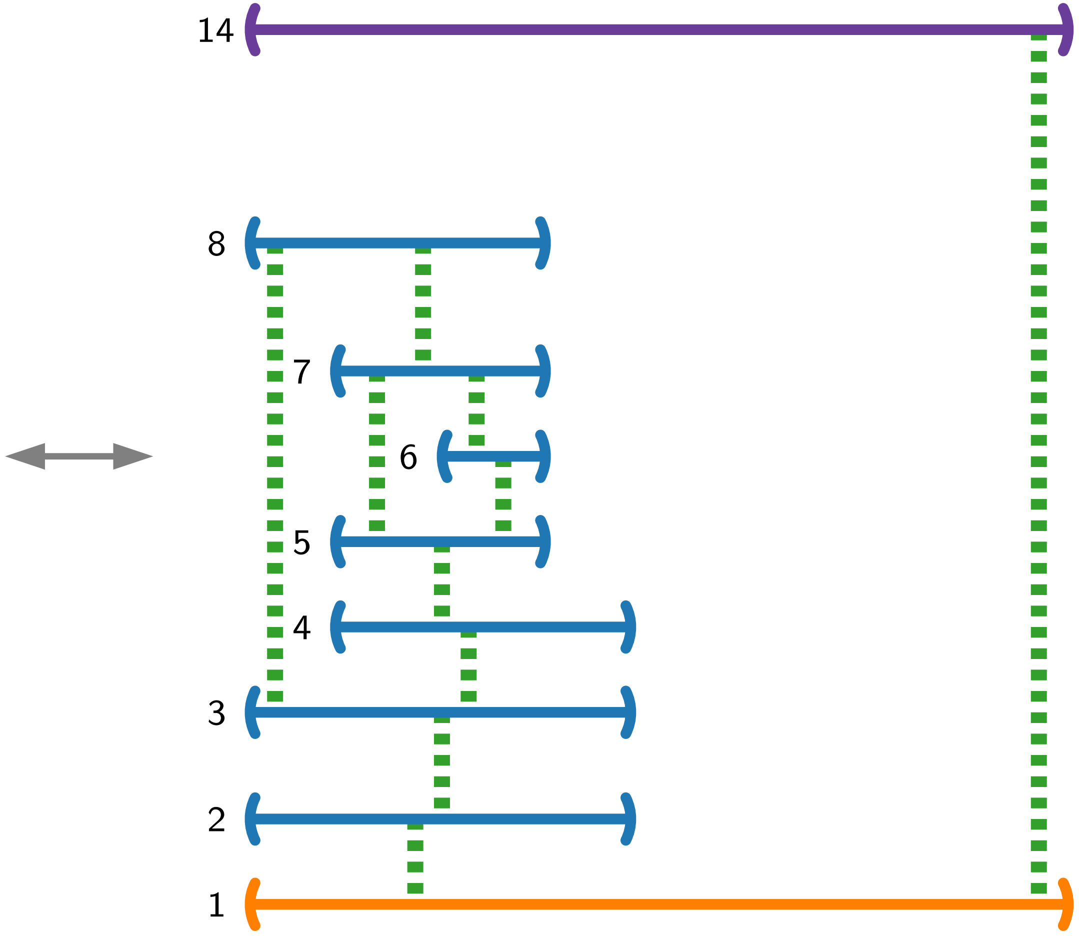

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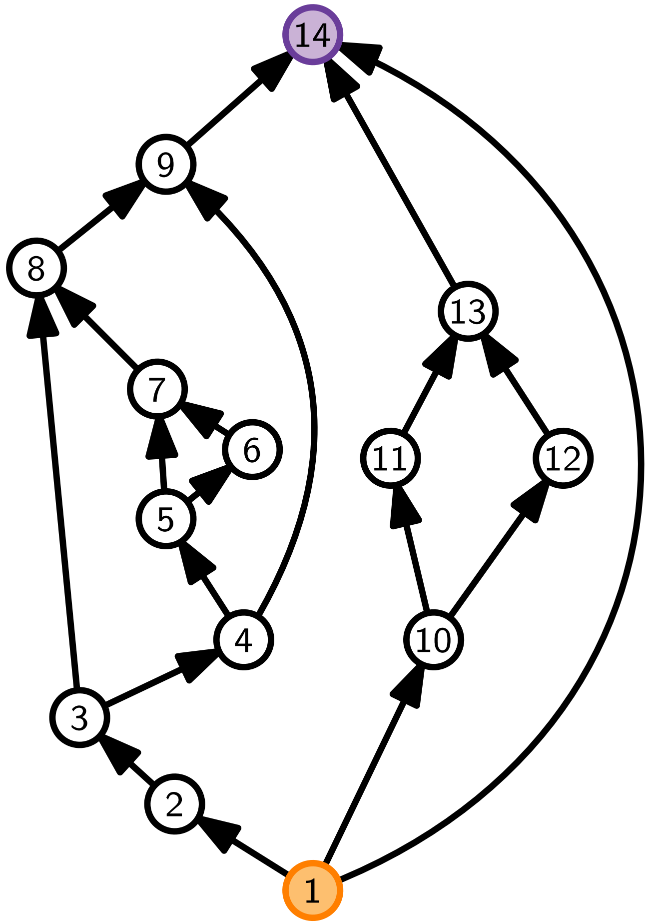

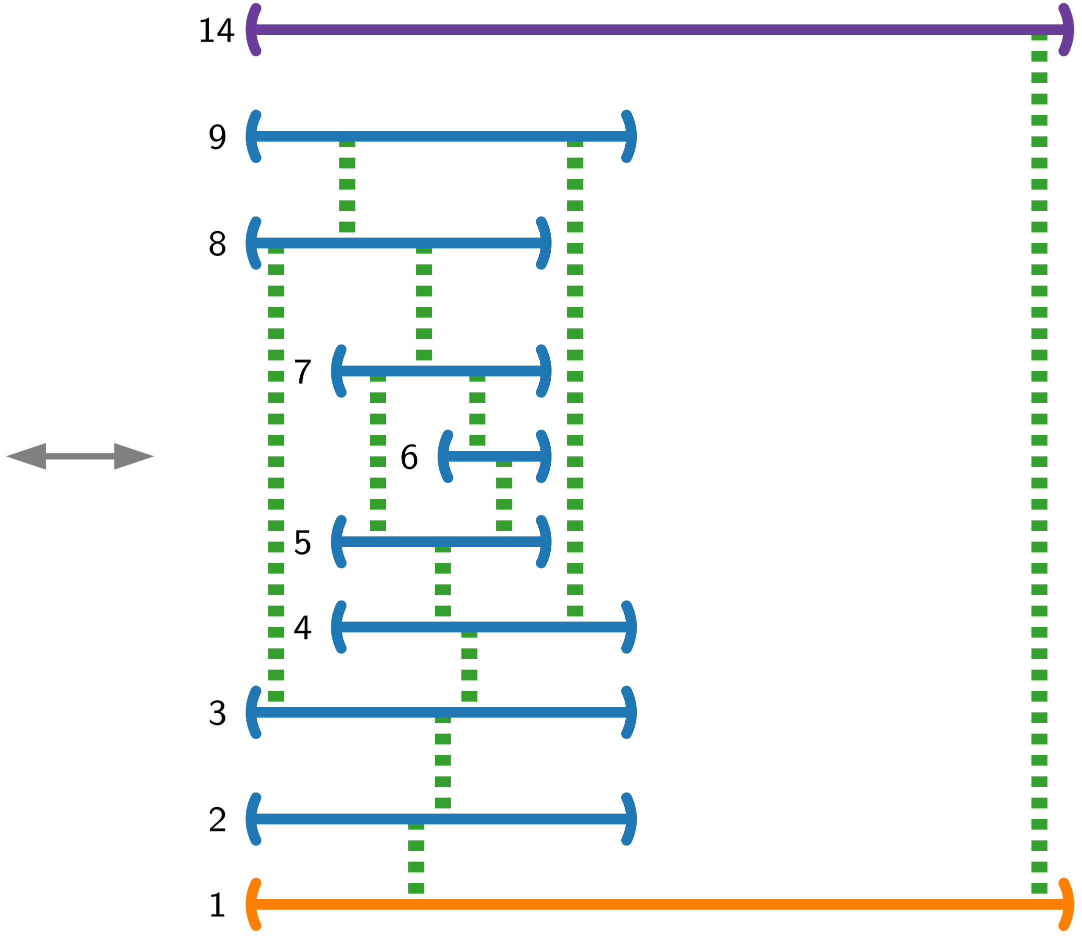

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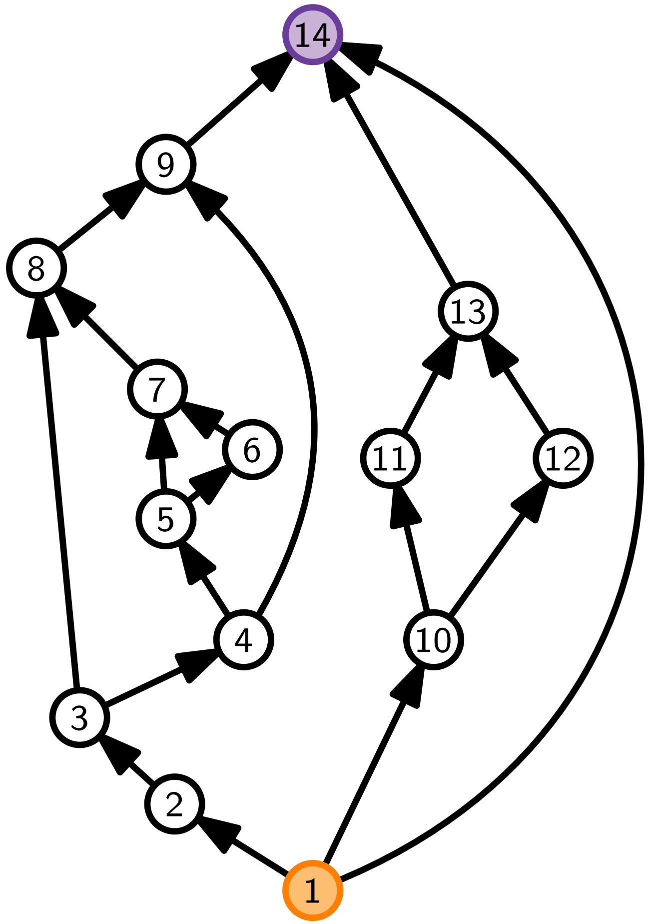

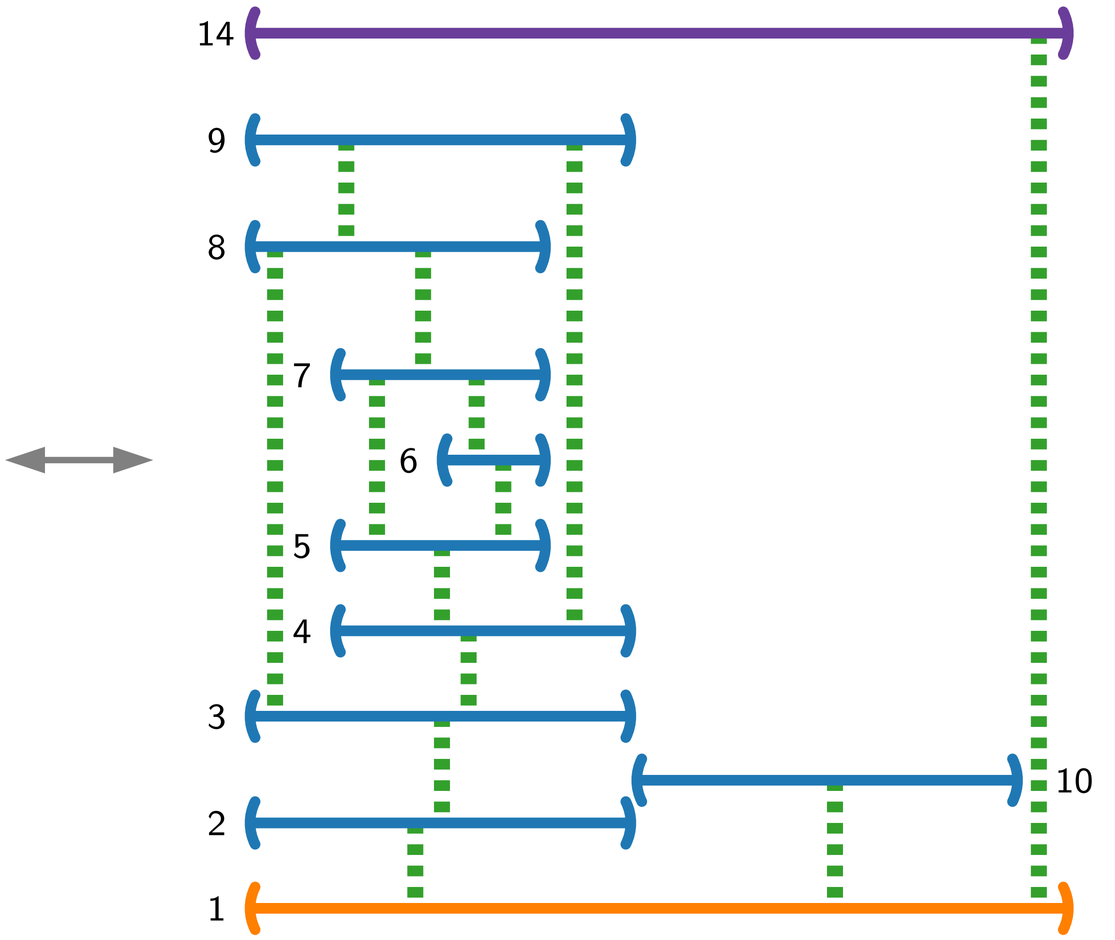

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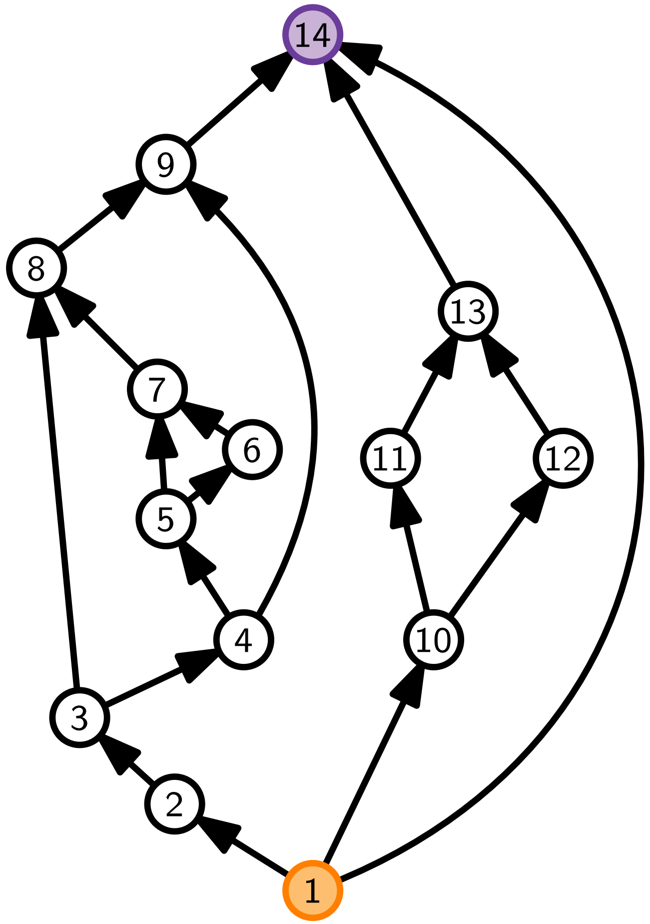

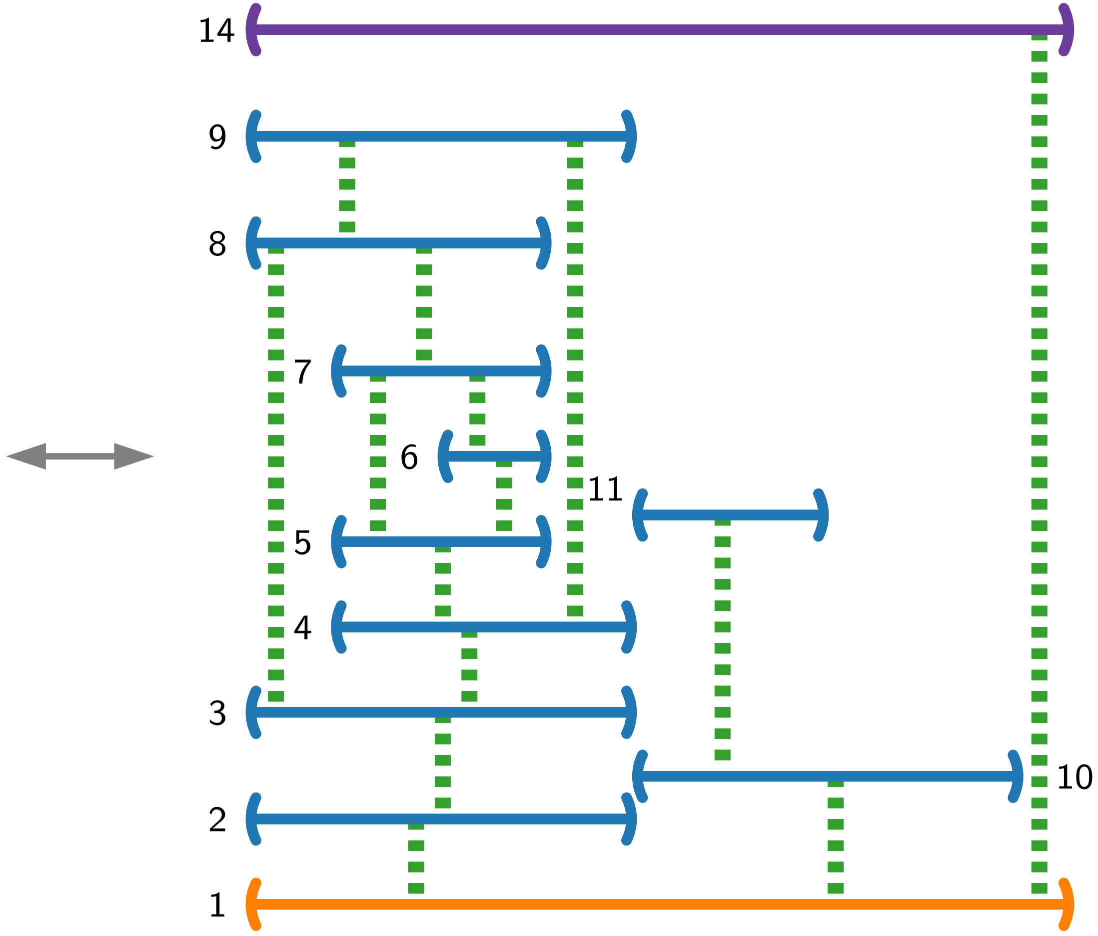

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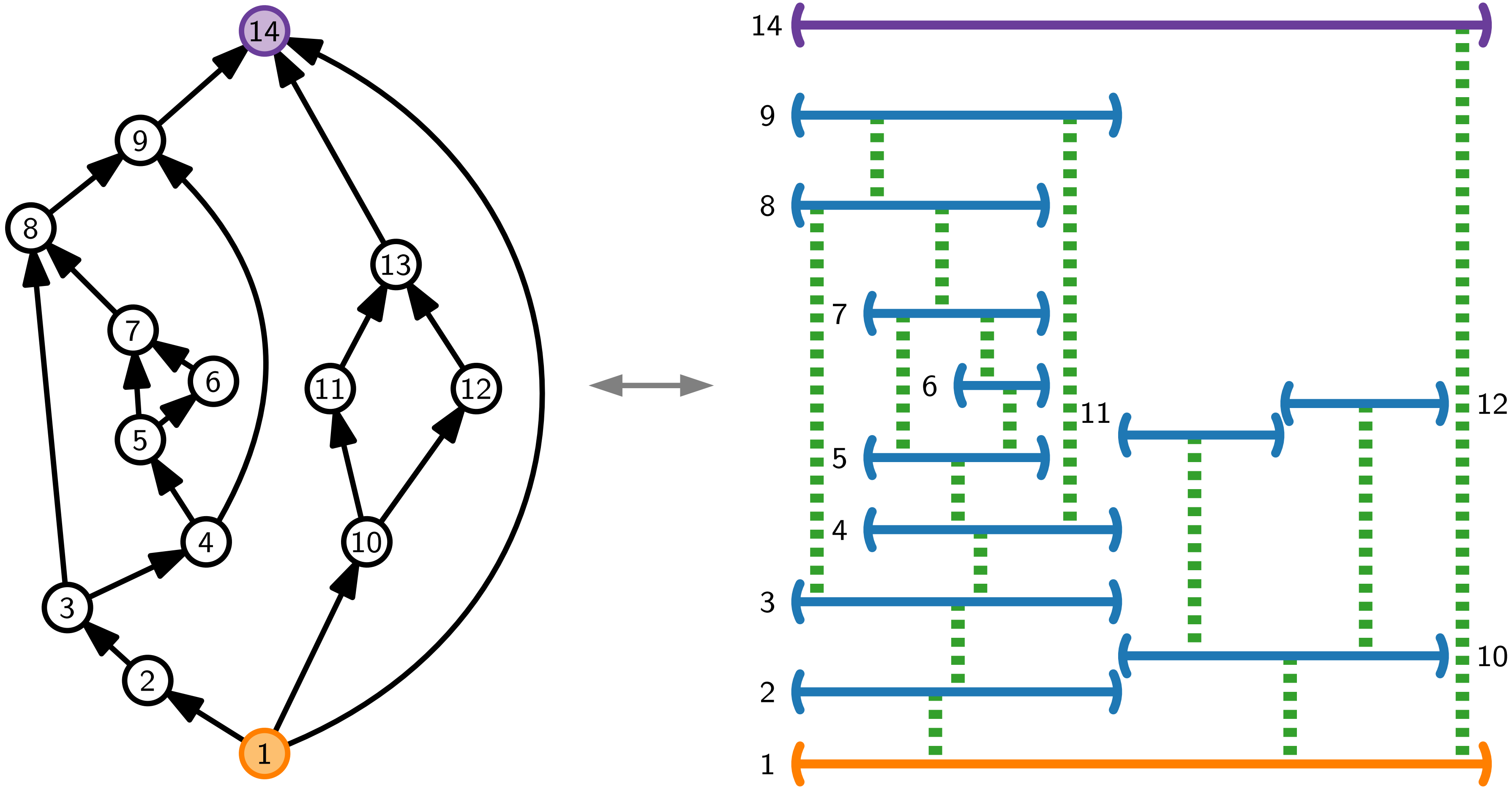

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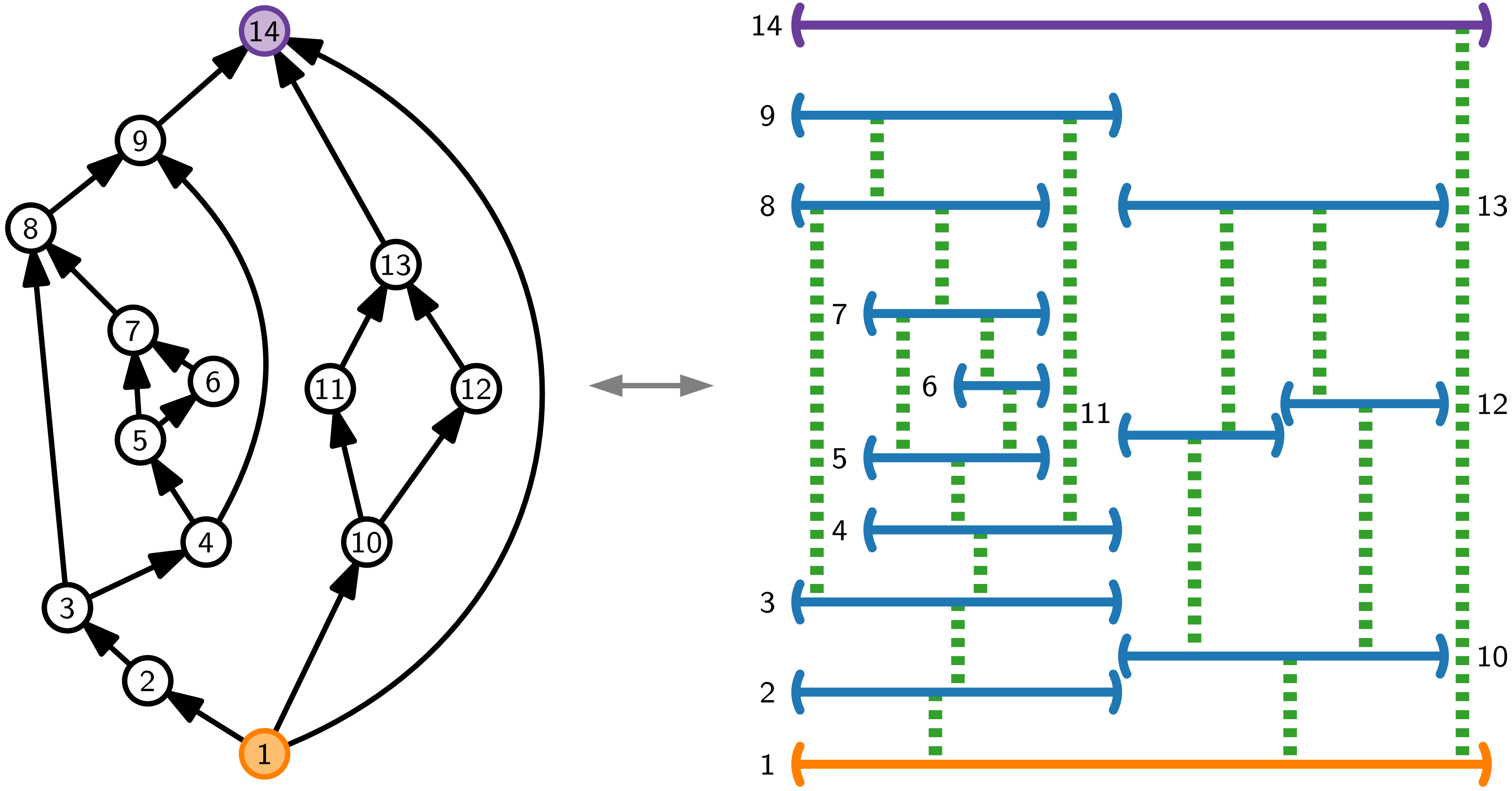

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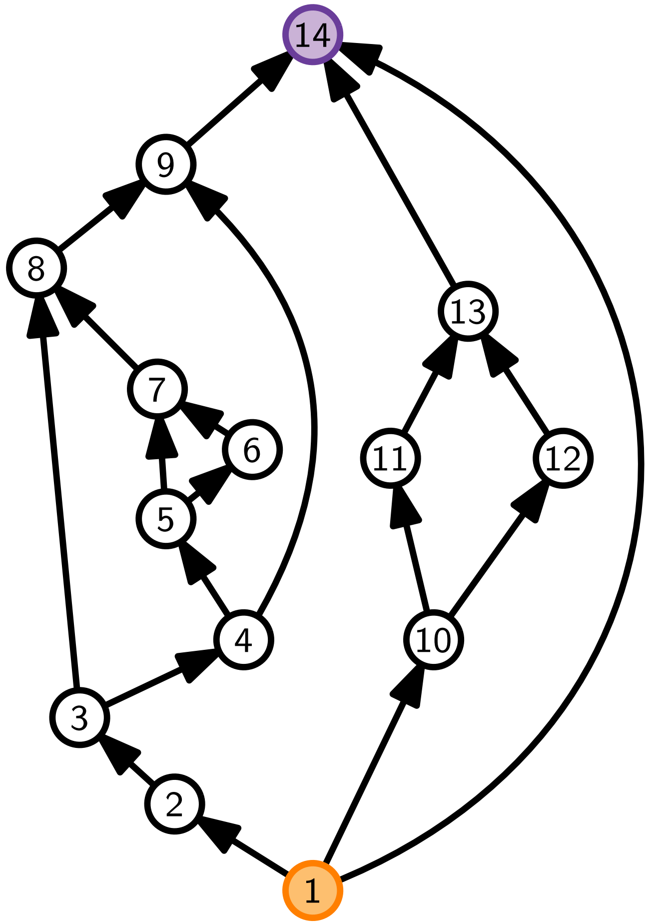

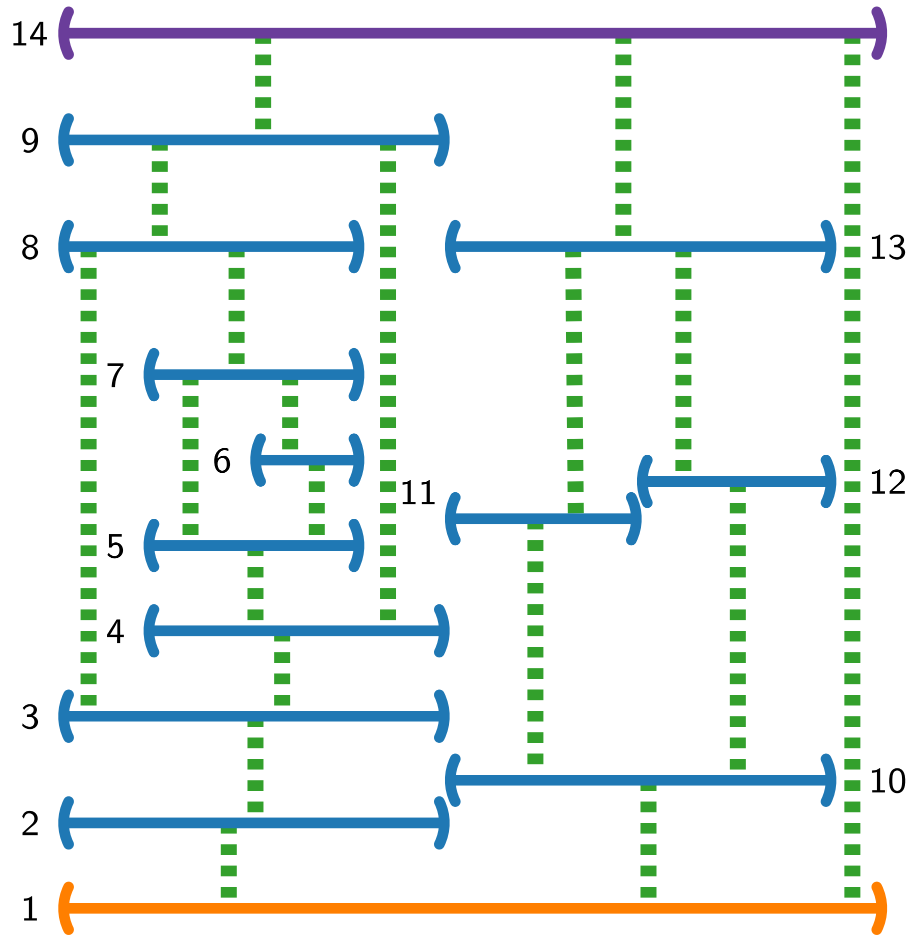

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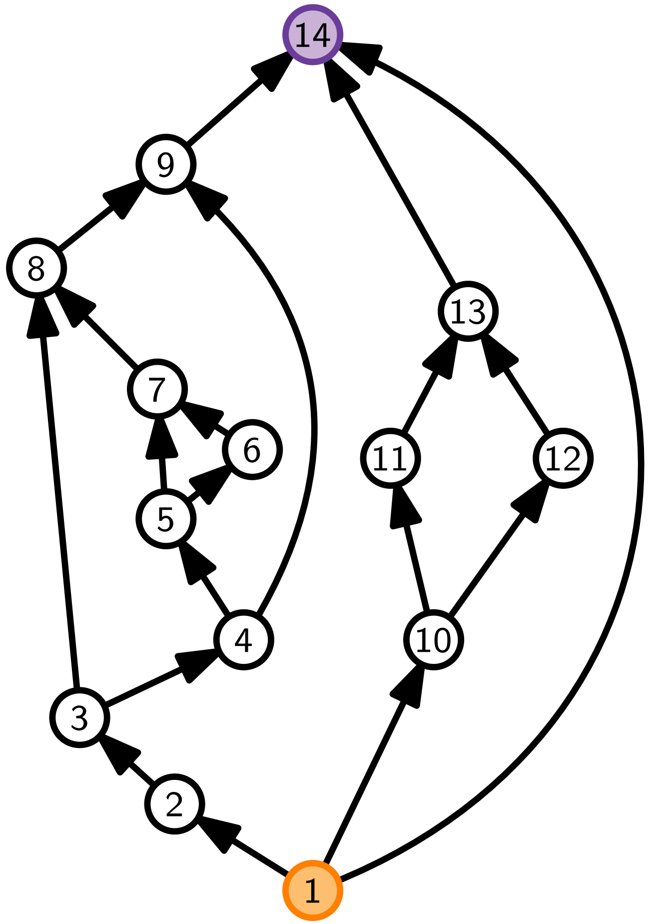
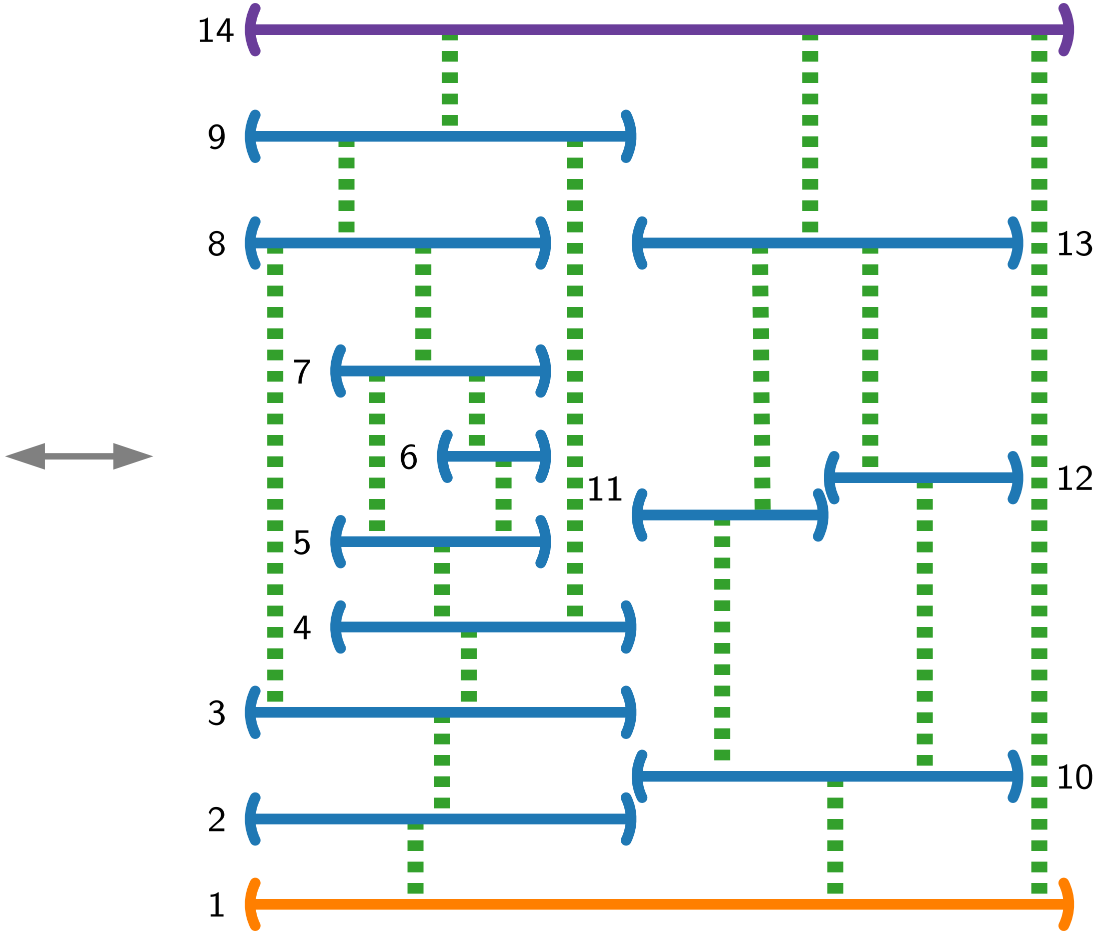
Testing whether an acyclic planar **di**graph has a weak bar visibility representation is NP-complete.





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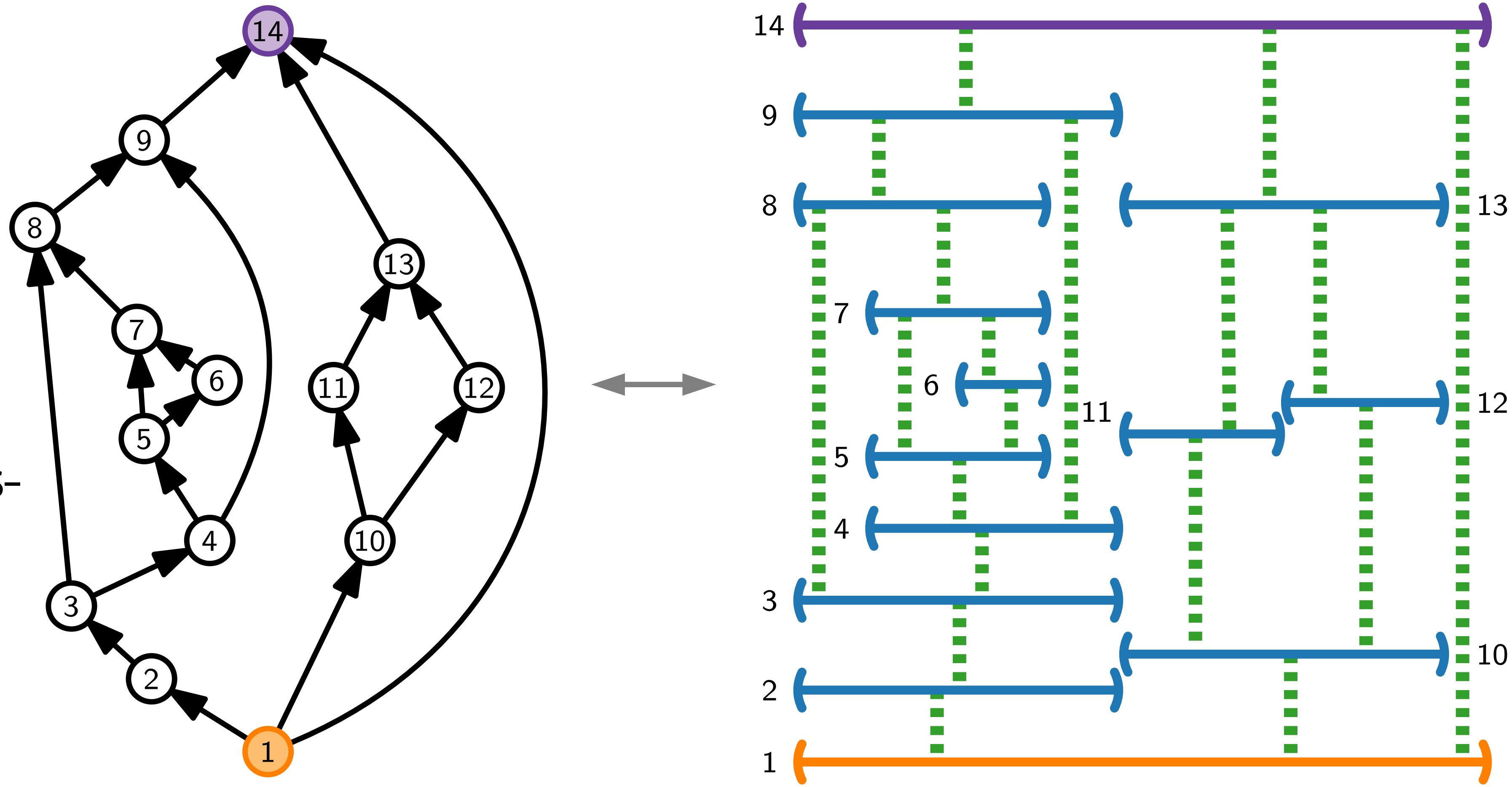
- This is upward planarity testing! [Garg & Tamassia '01]



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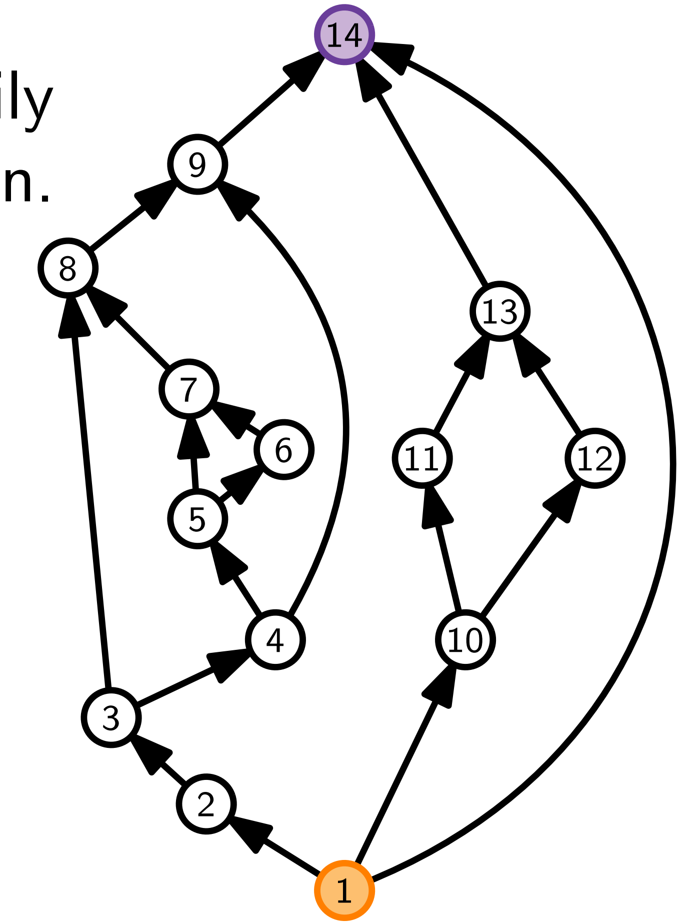

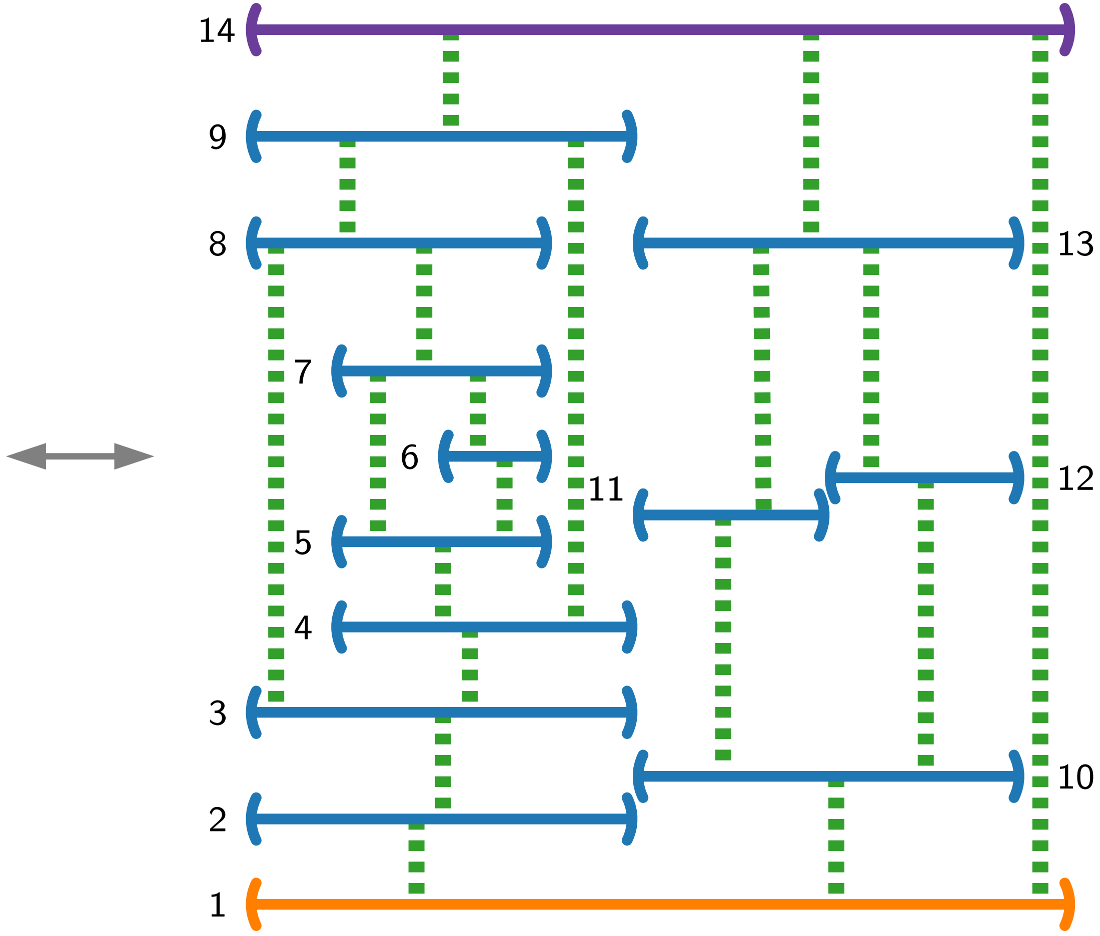

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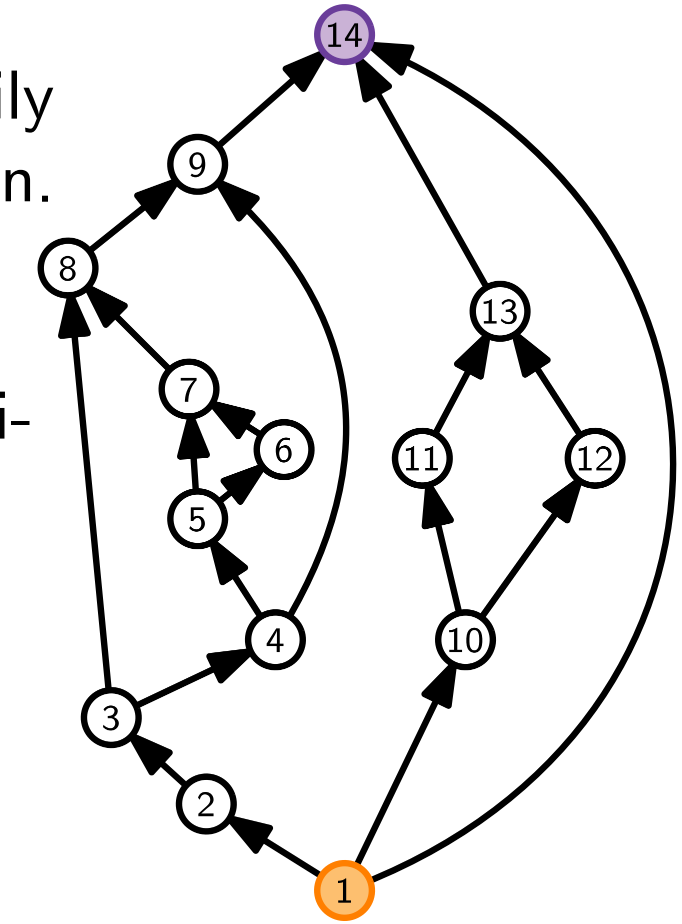

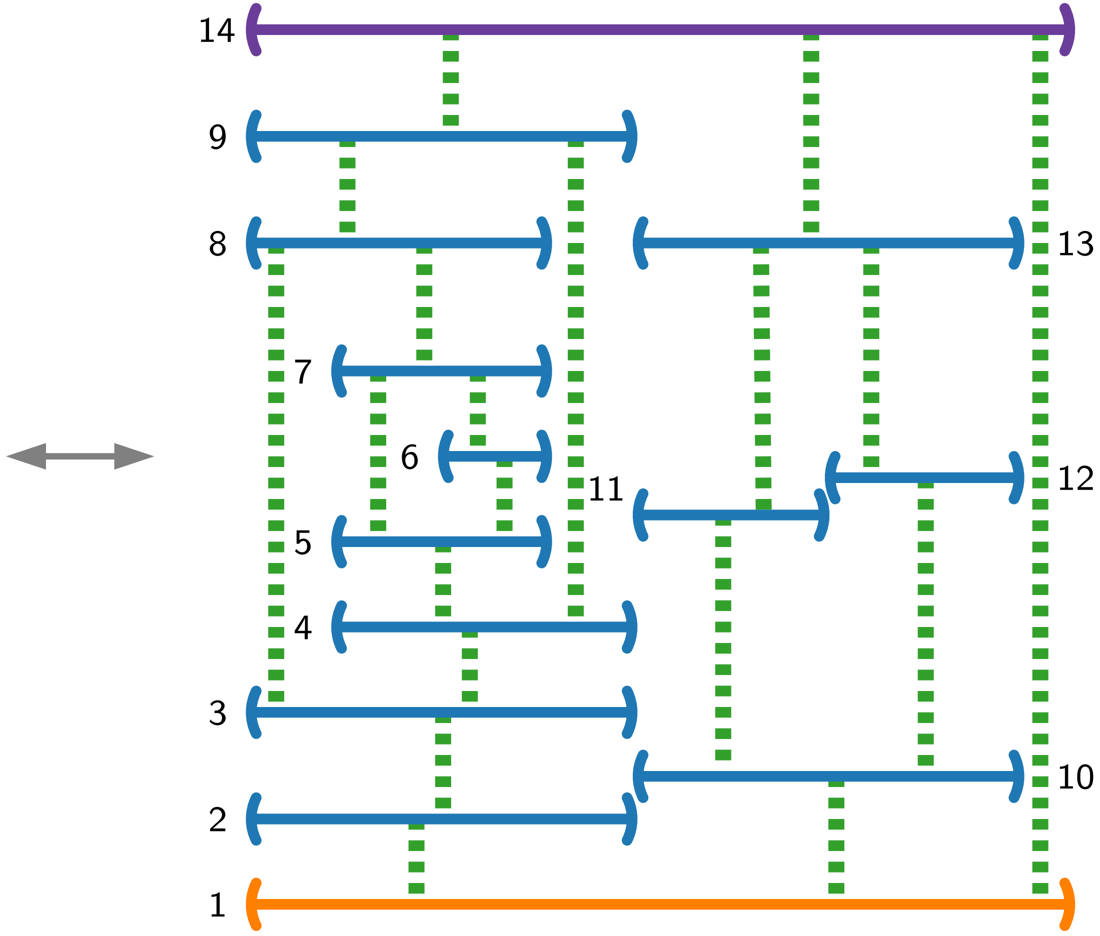

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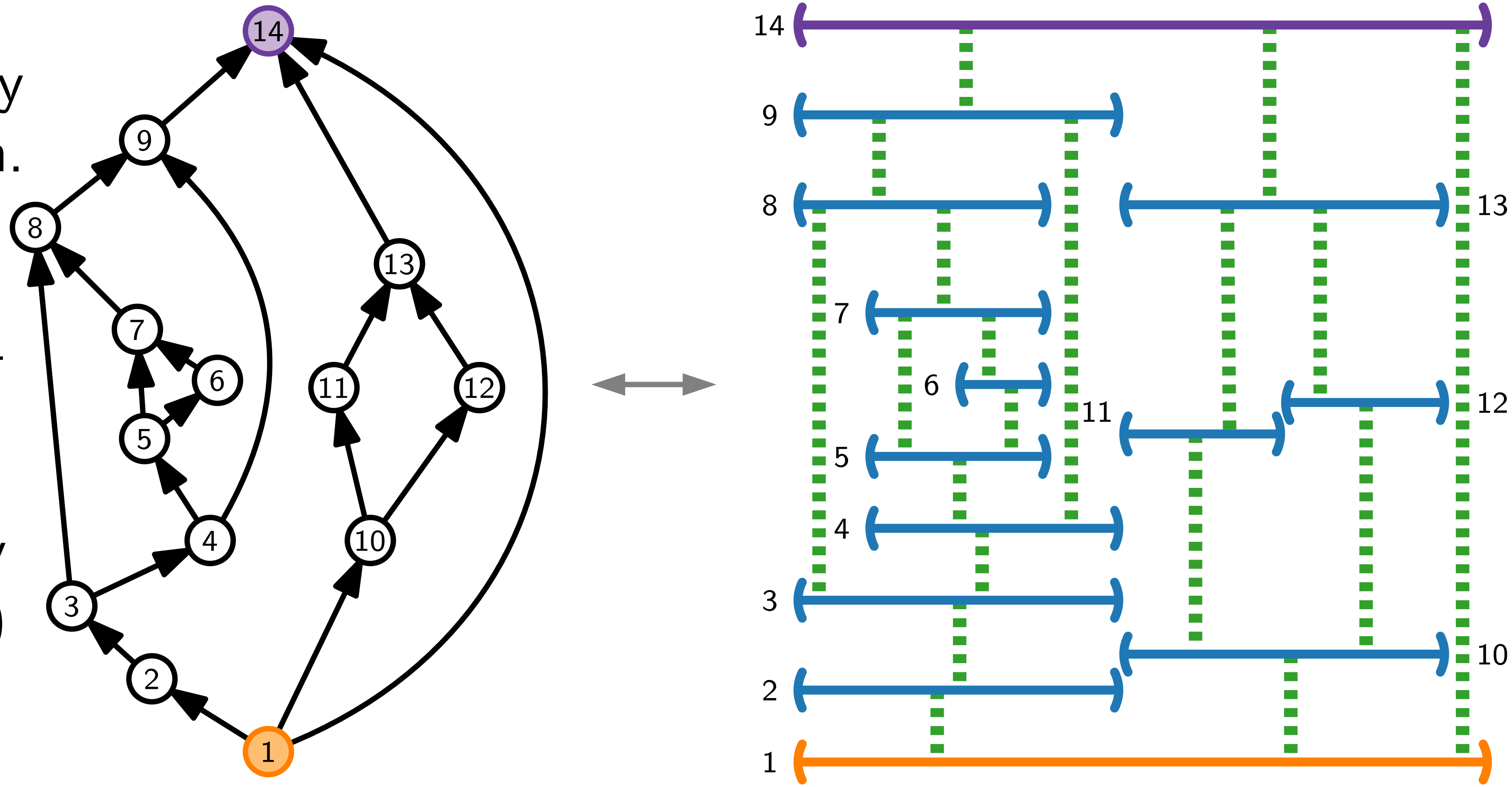
- In a **rectangular** bar visibility representation $\psi(s)$ and $\psi(t)$ span an enclosing rectangle.



Results and Outline

Theorem 1. [Chaplick et al. '18]
Rectangular ε-Bar Visibility Representation Extension can be solved in $\mathcal{O}(n \log^2 n)$ time for st-graphs.

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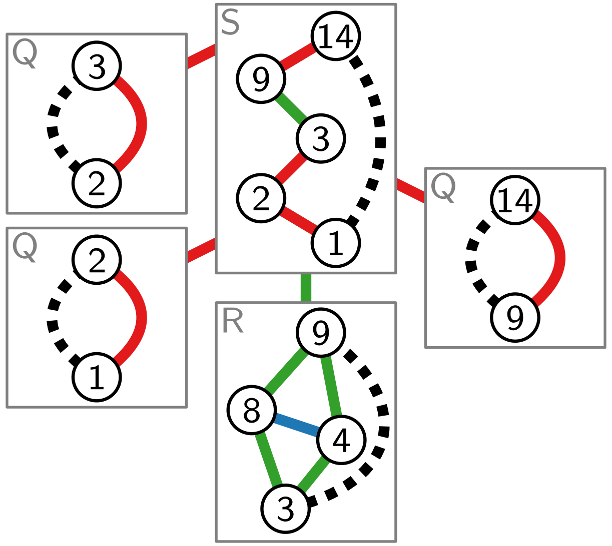
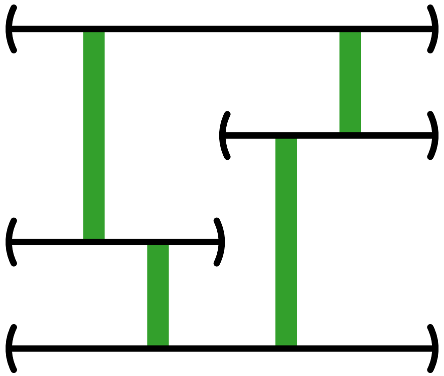
Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension



Part III:
SPQR-Trees



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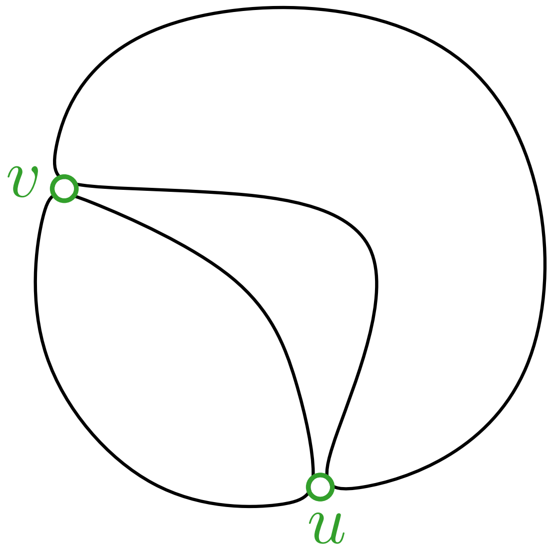

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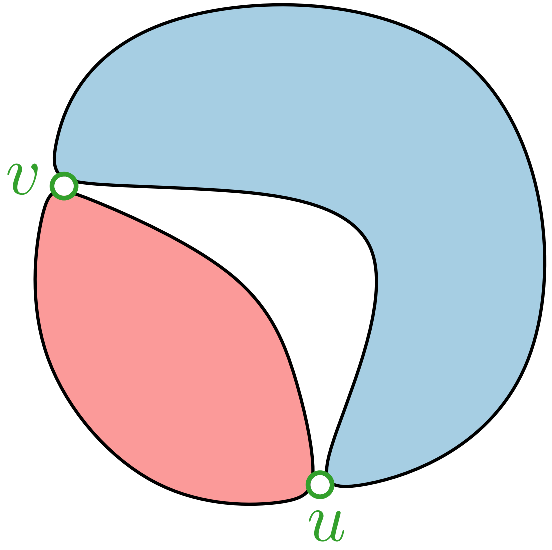

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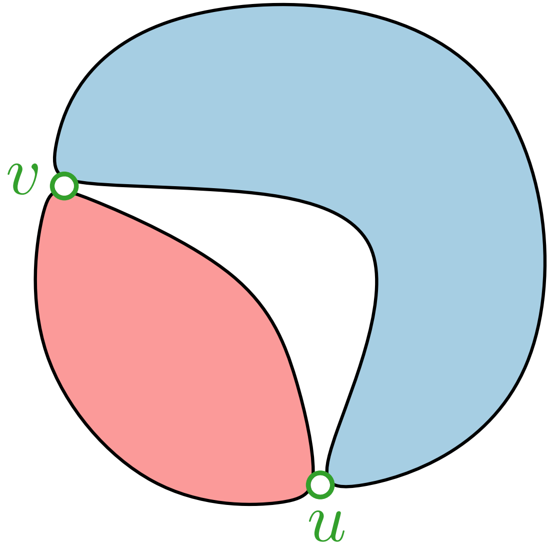

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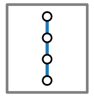

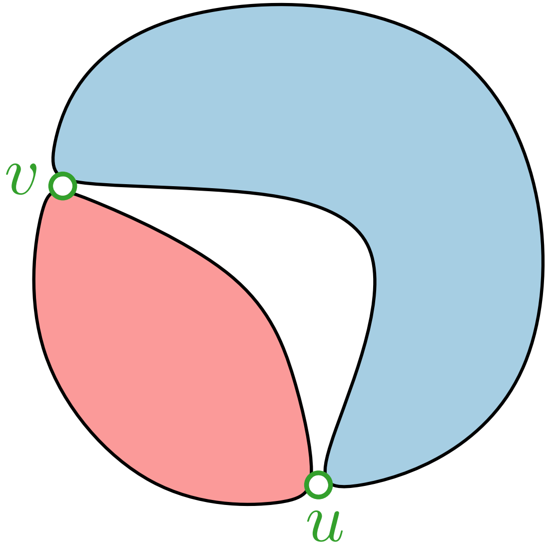

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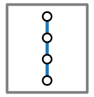
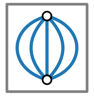
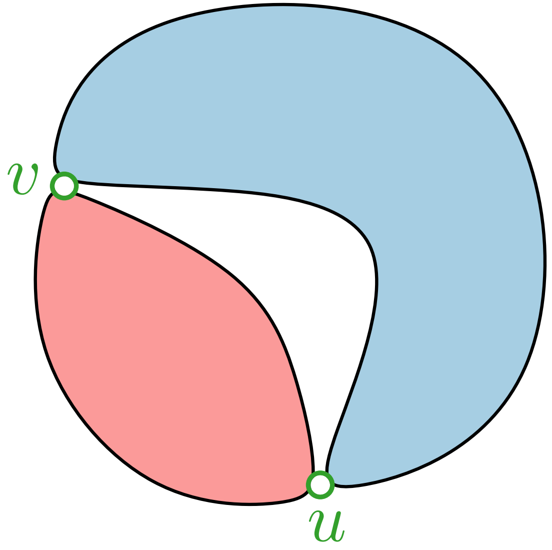
  - **S**-nodes represent a series composition 
  - **P**-nodes represent a parallel composition 



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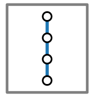
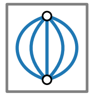
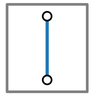
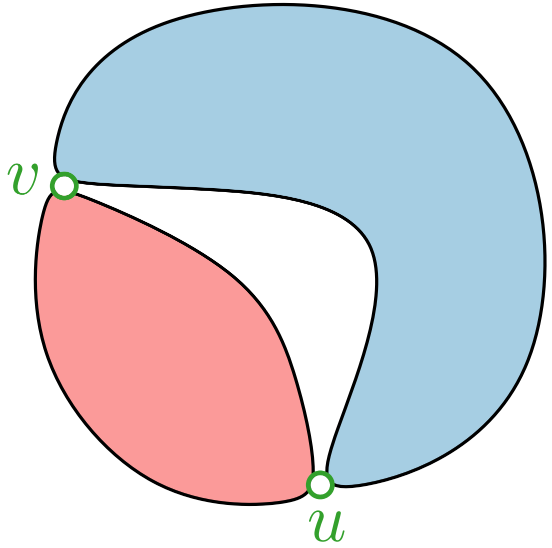
  - **S**-nodes represent a series composition 
  - **P**-nodes represent a parallel composition 
  - **Q**-nodes represent a single edge 



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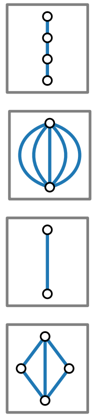

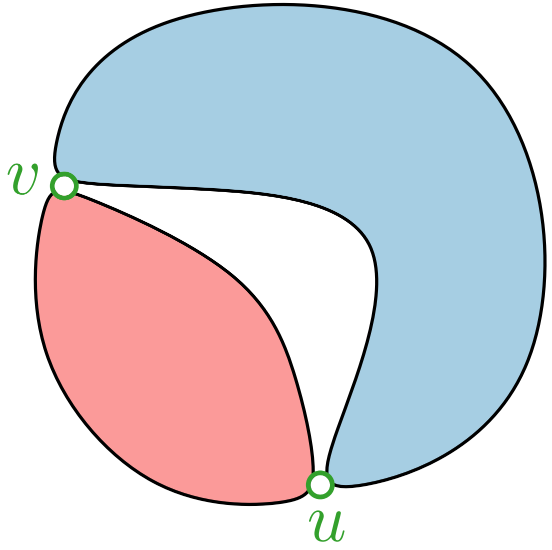

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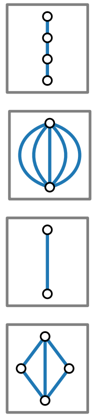

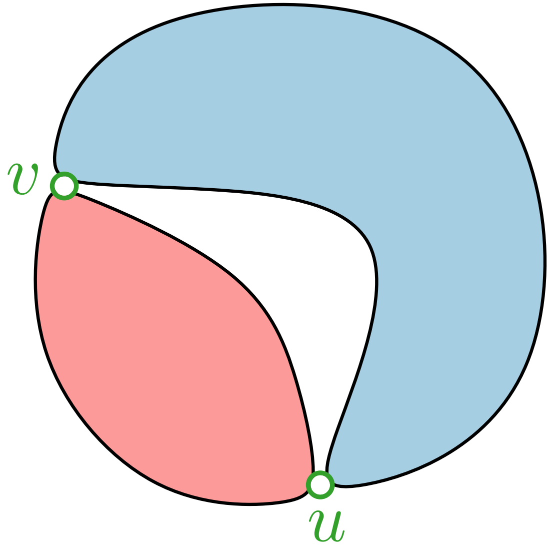

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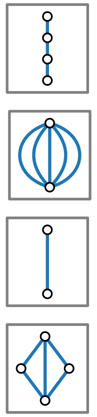

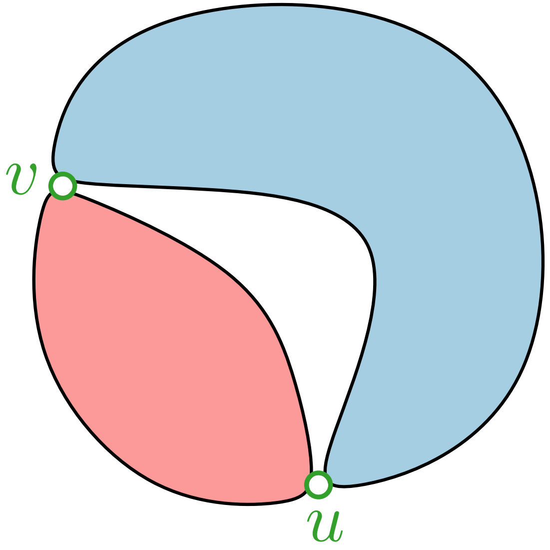

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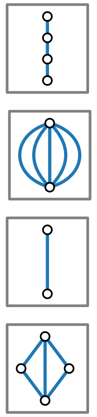

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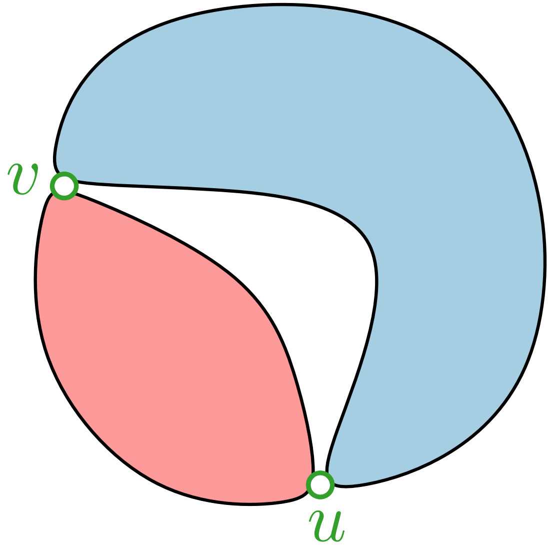
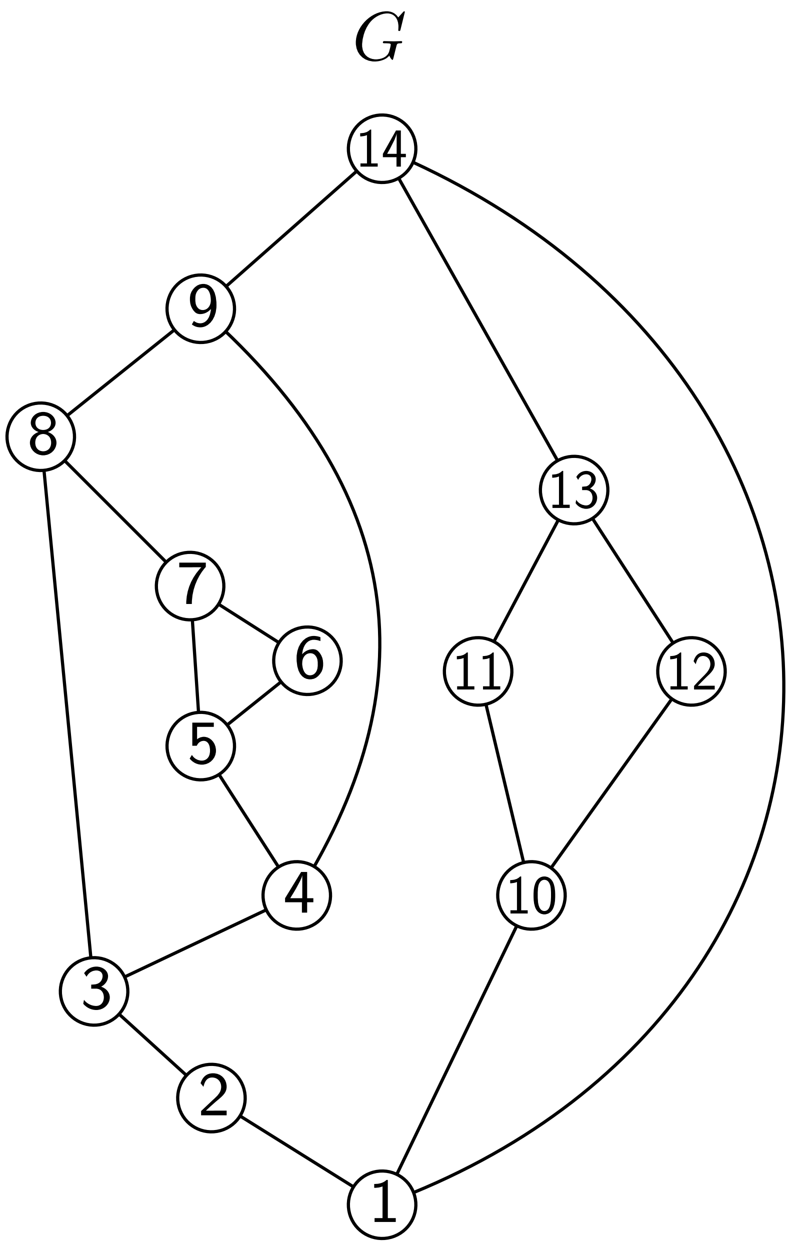
- T can be computed in $\mathcal{O}(n)$ time. [Gutwenger, Mutzel '01]



SPQR-Tree Example



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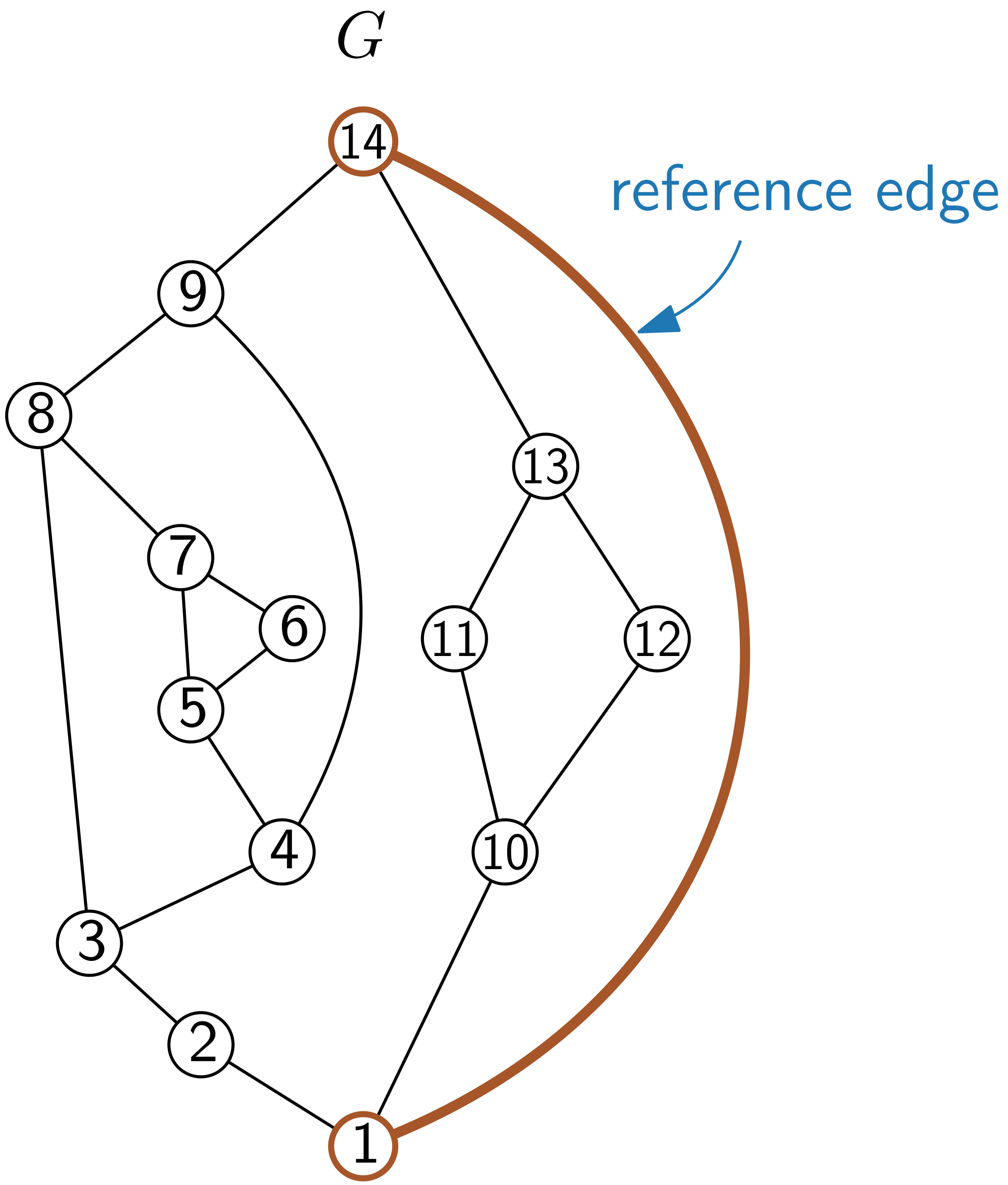

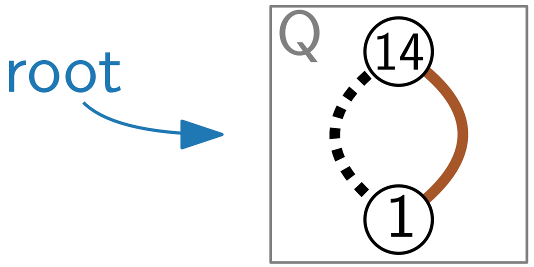

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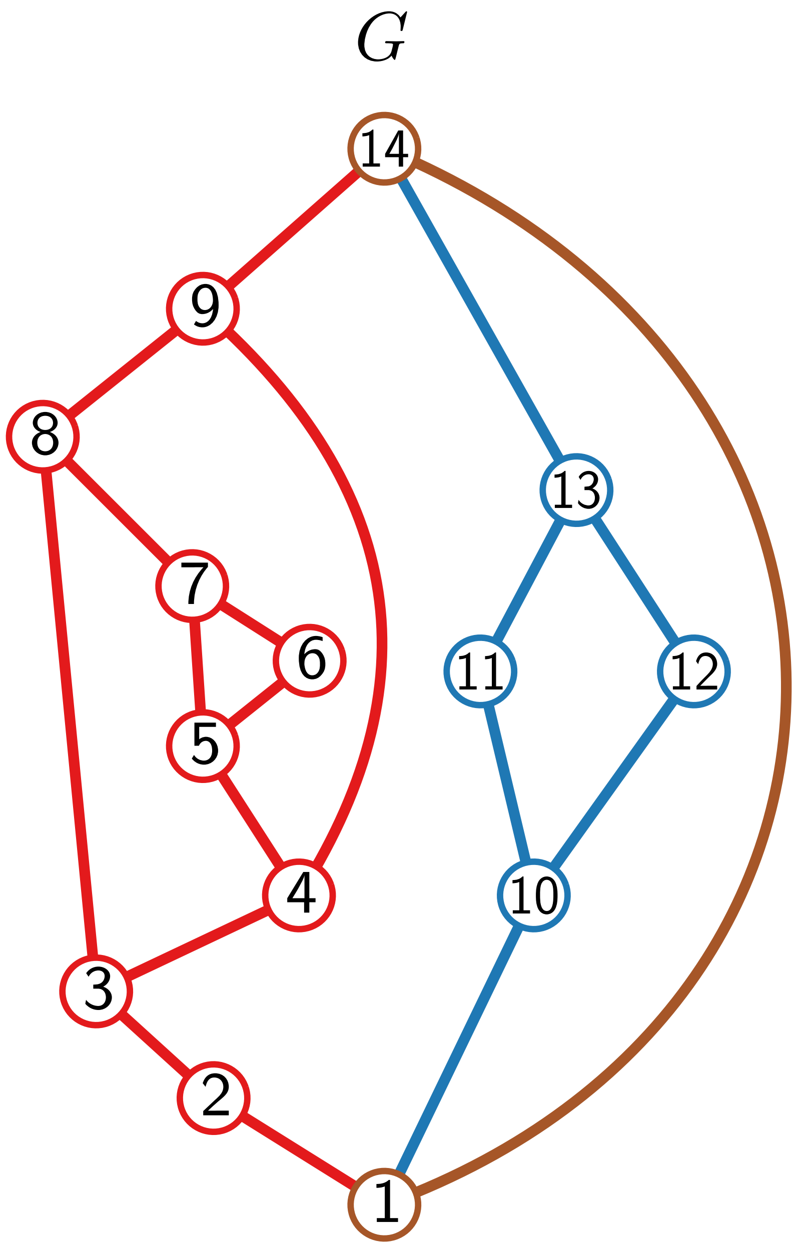

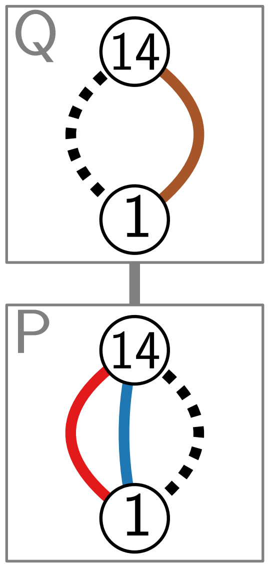

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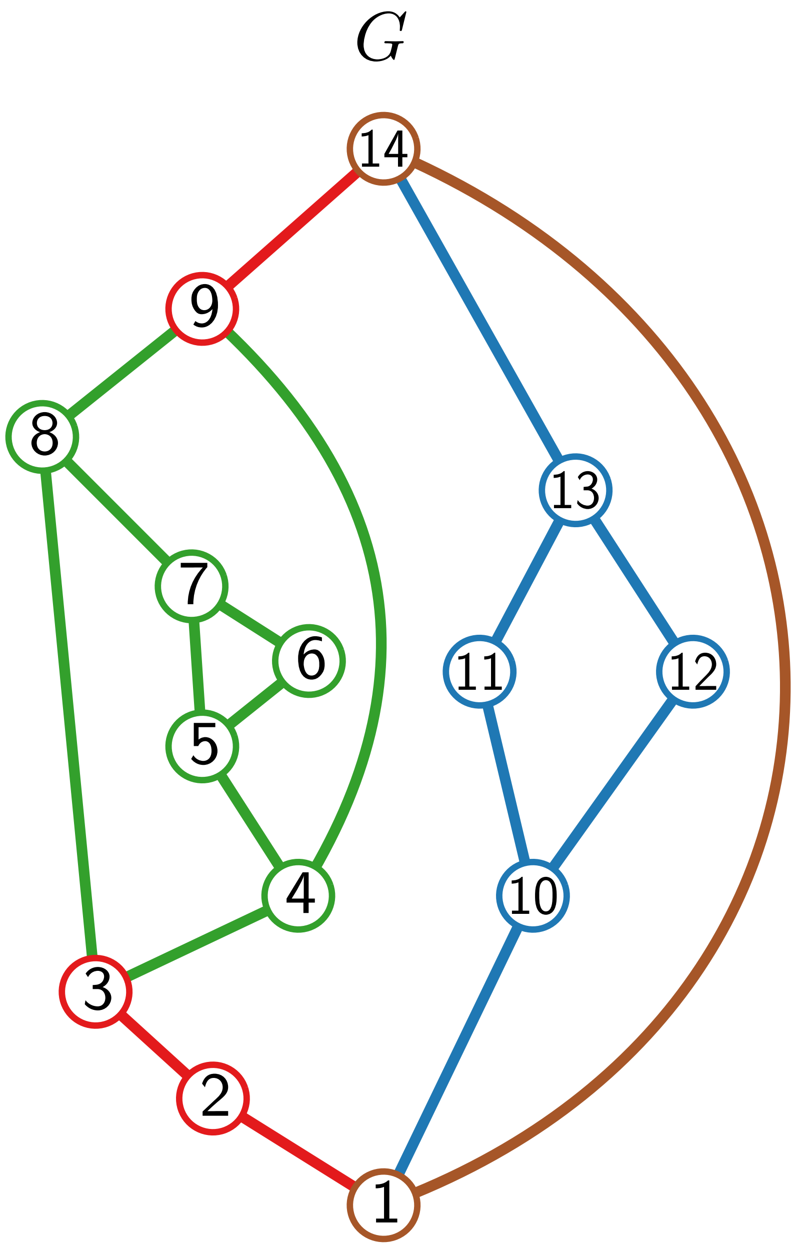

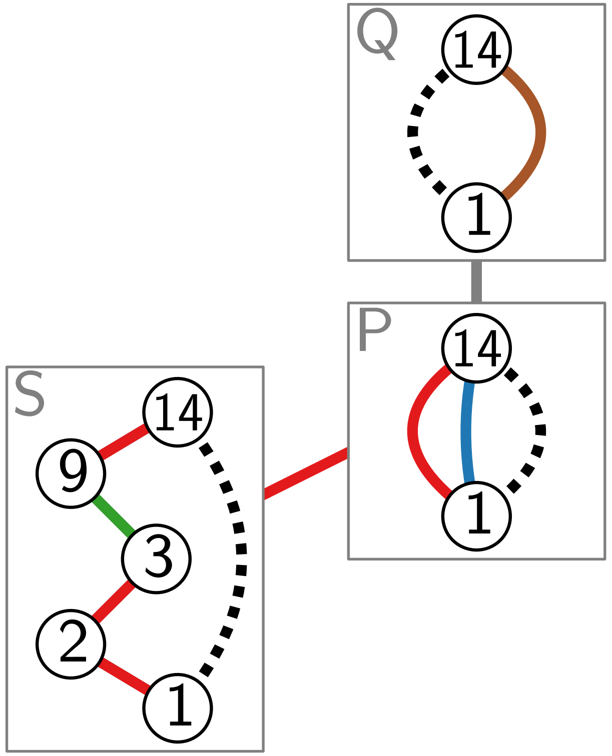

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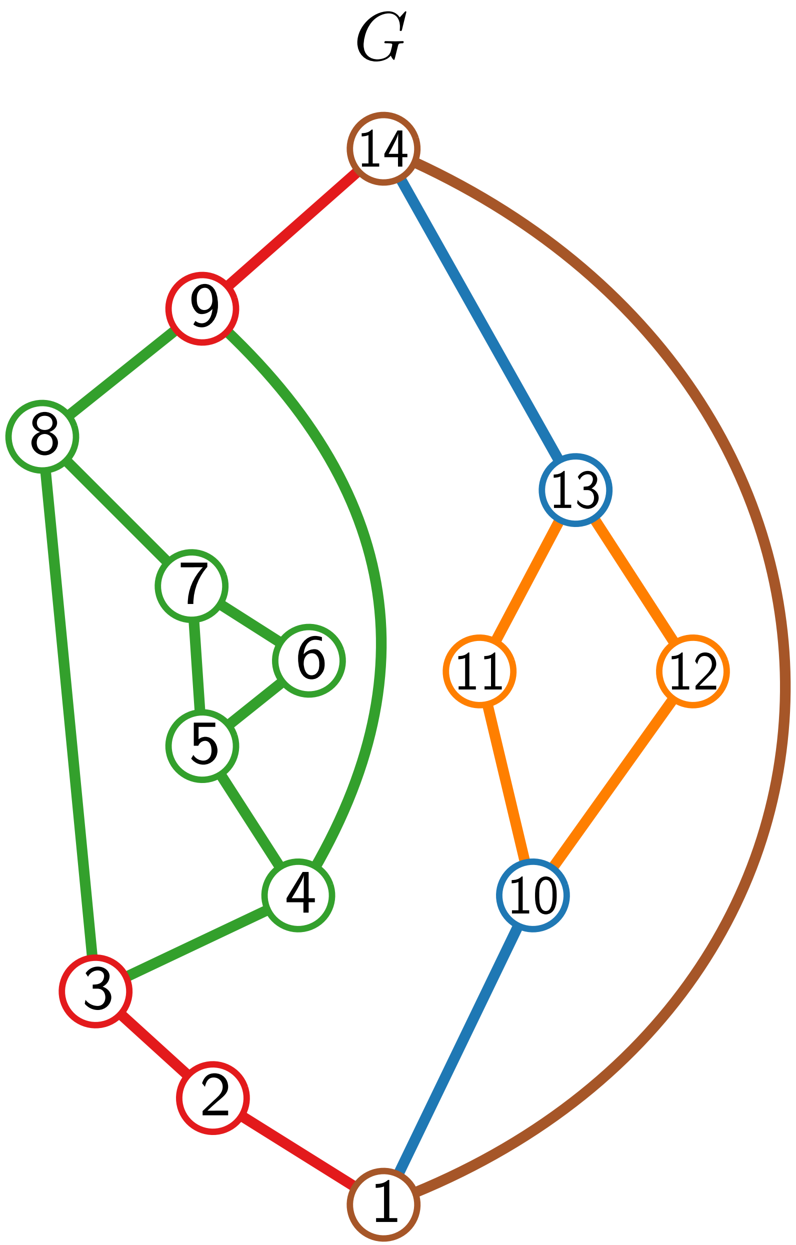

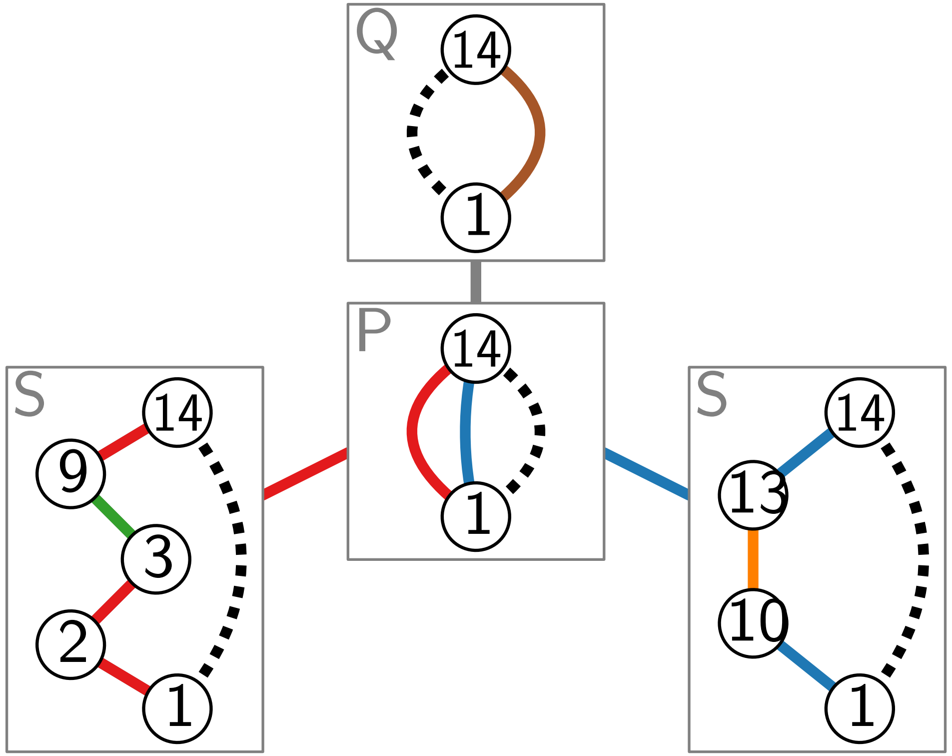

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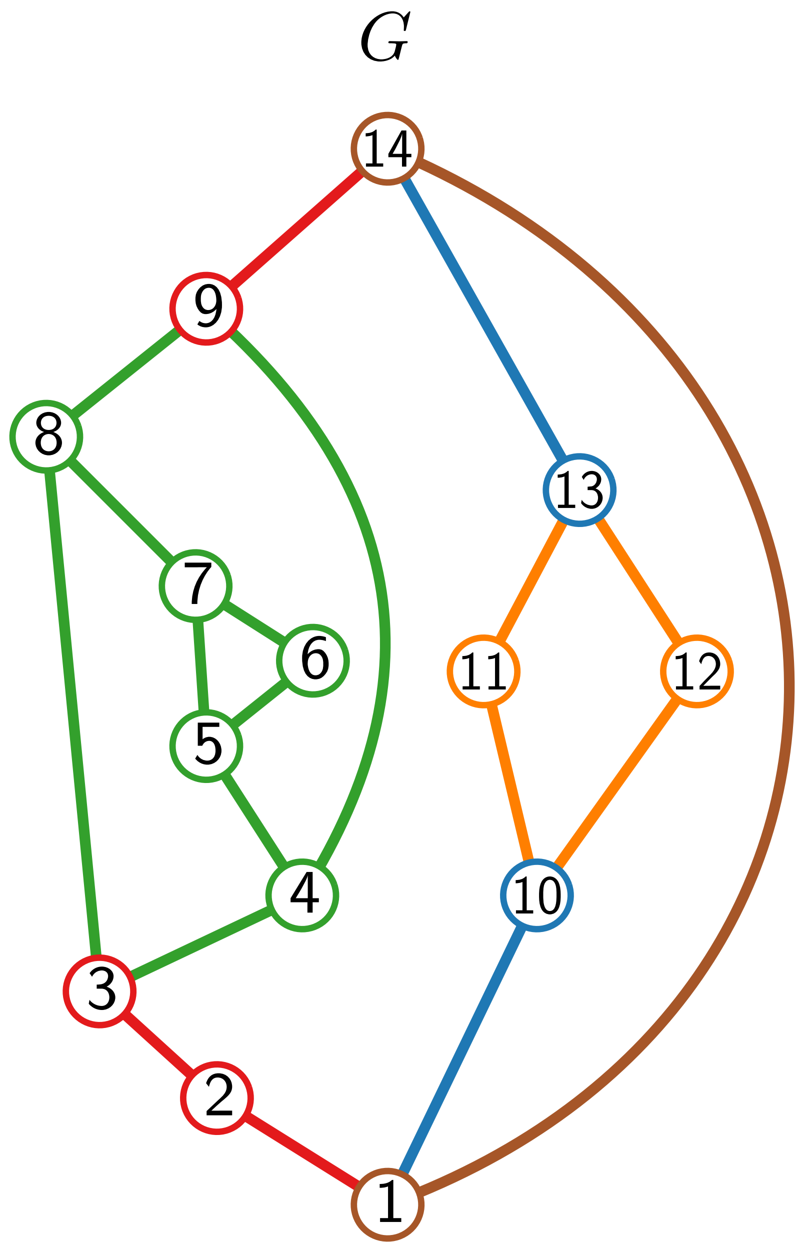

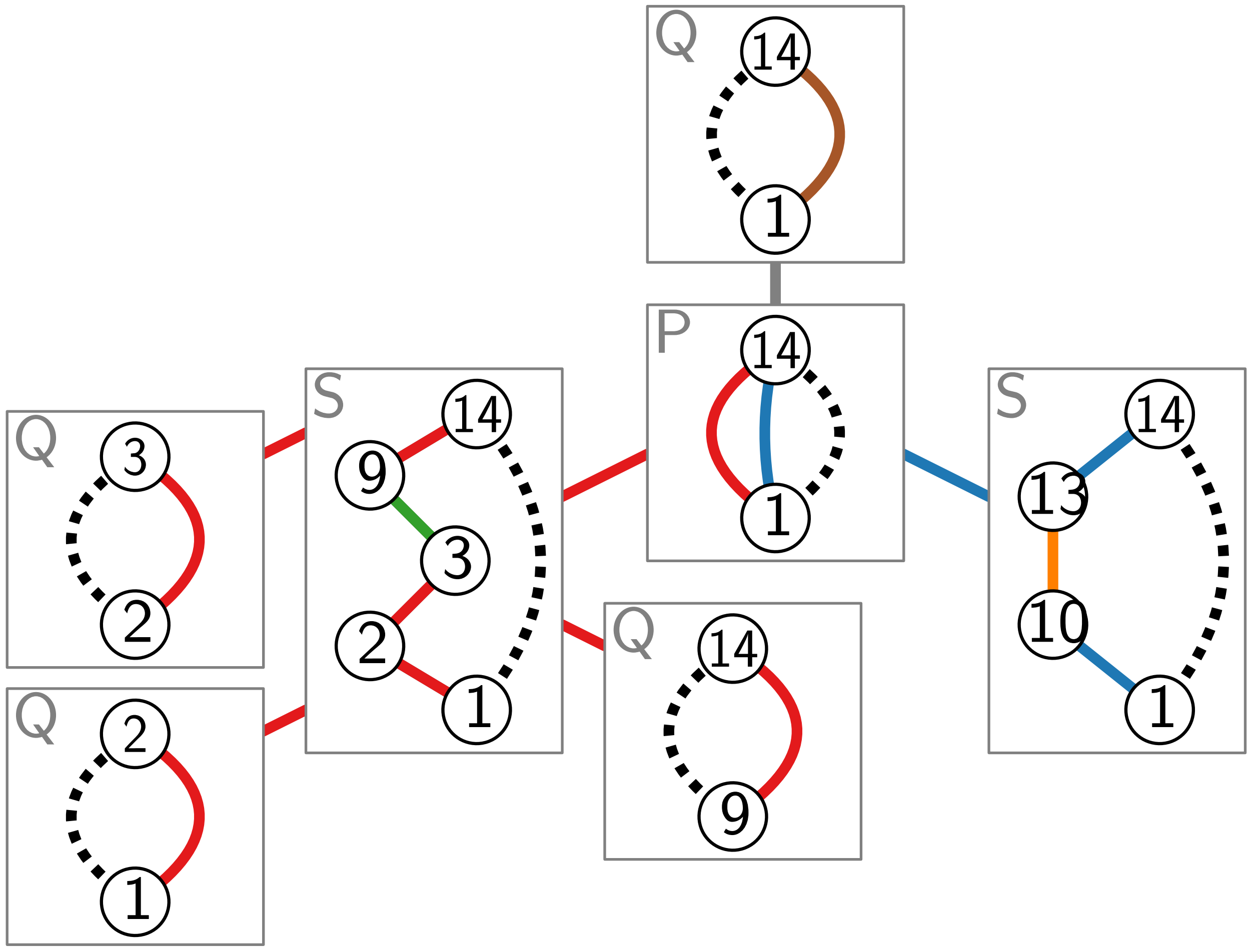

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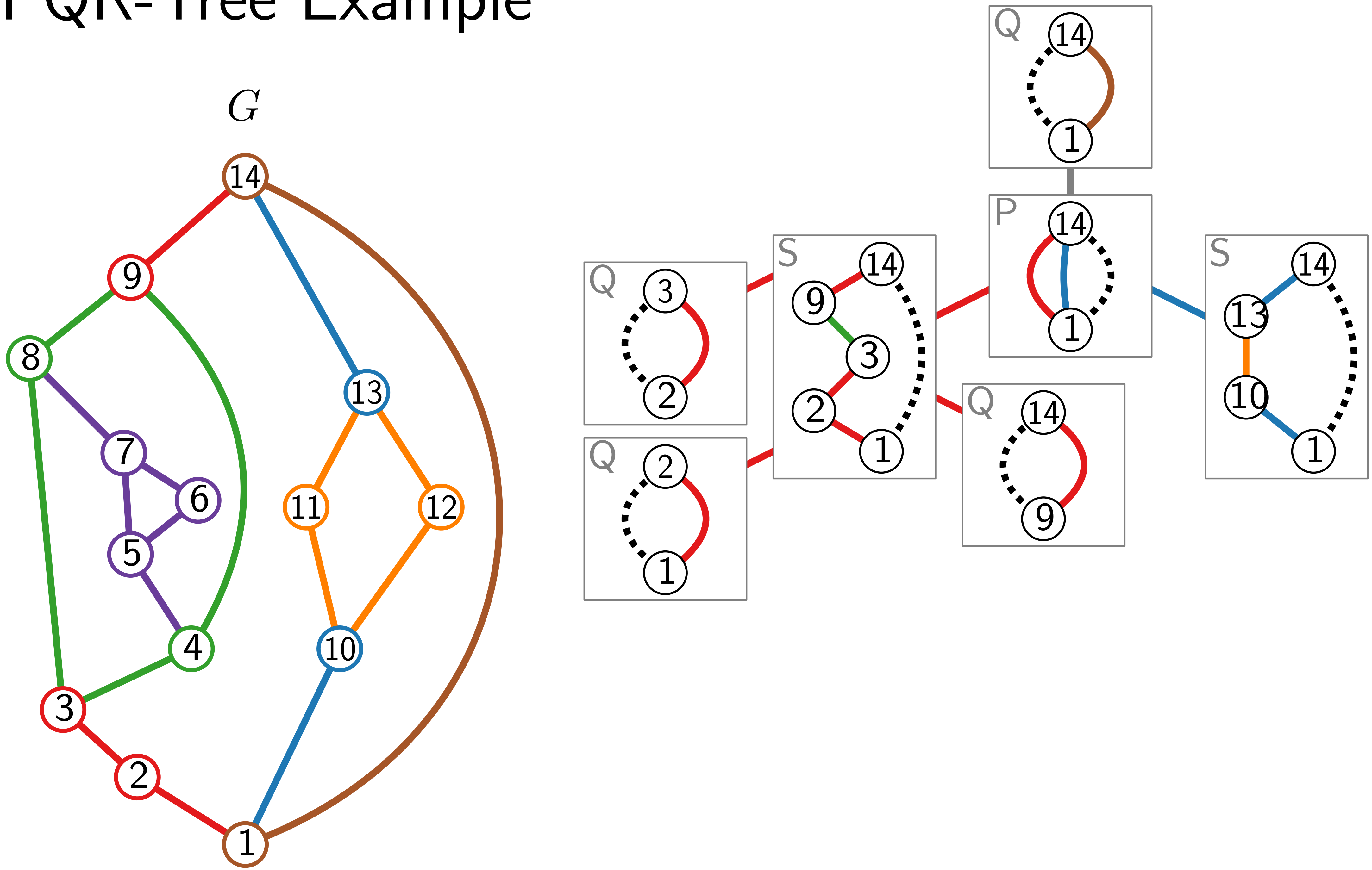

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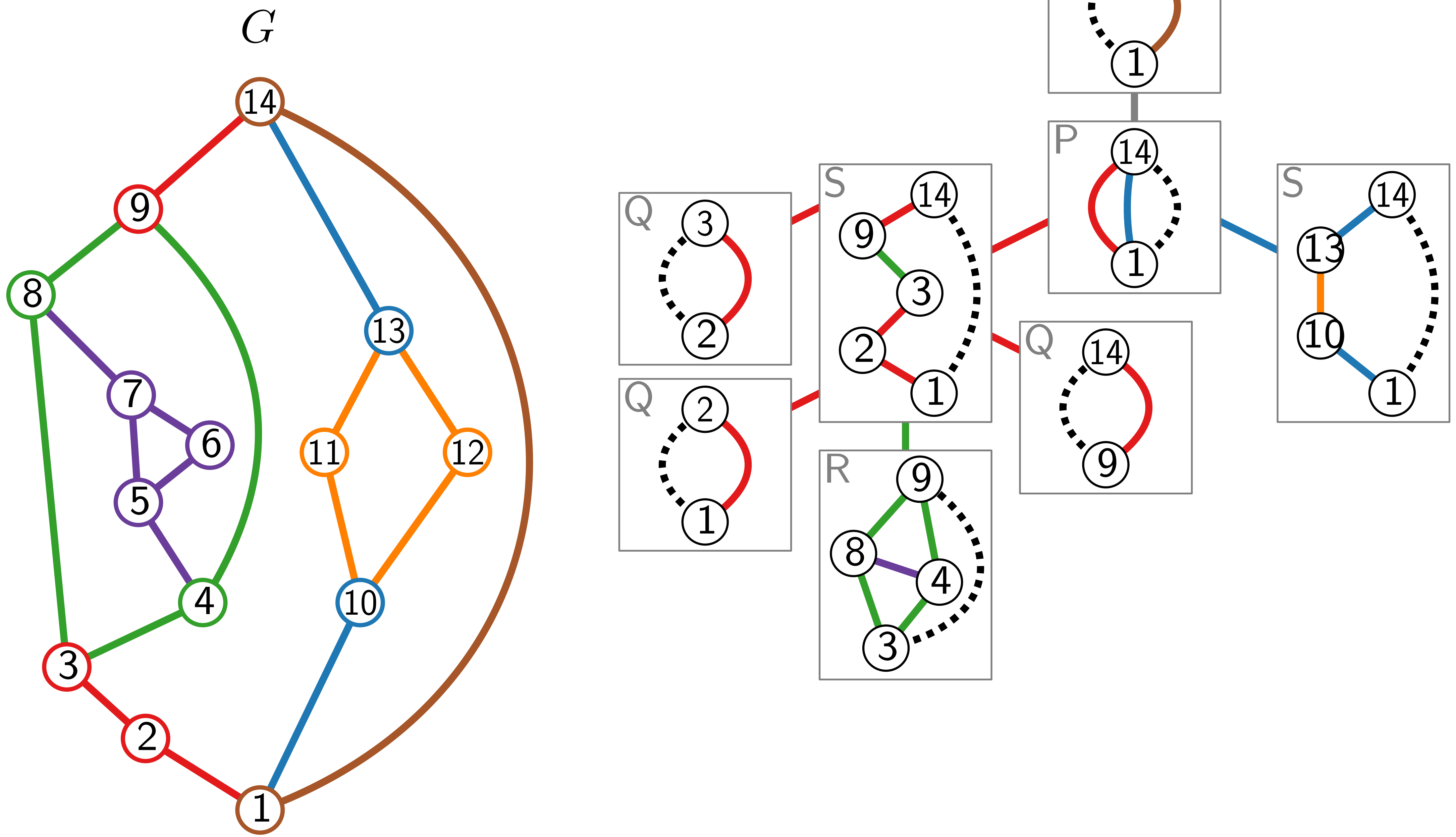

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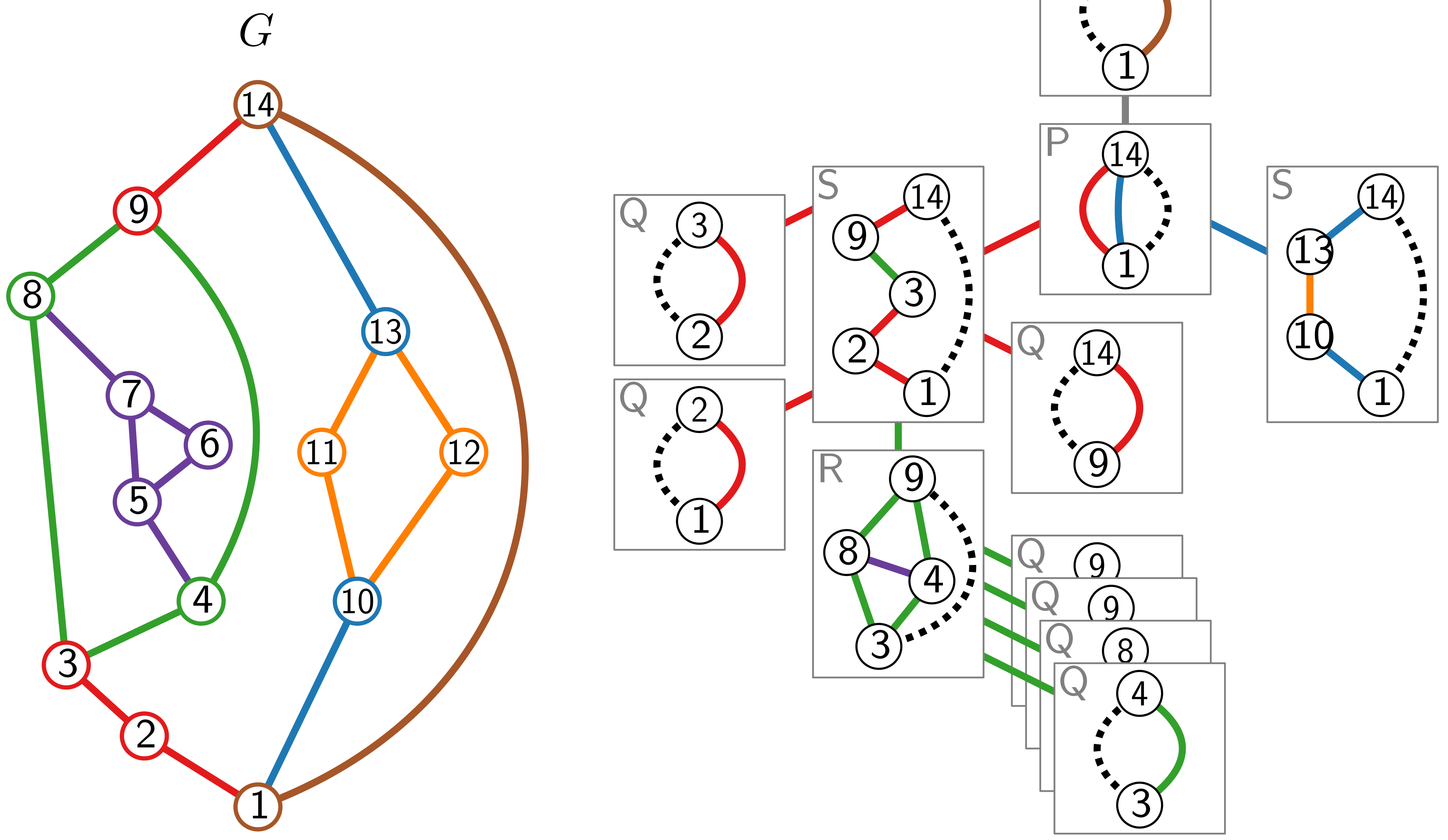

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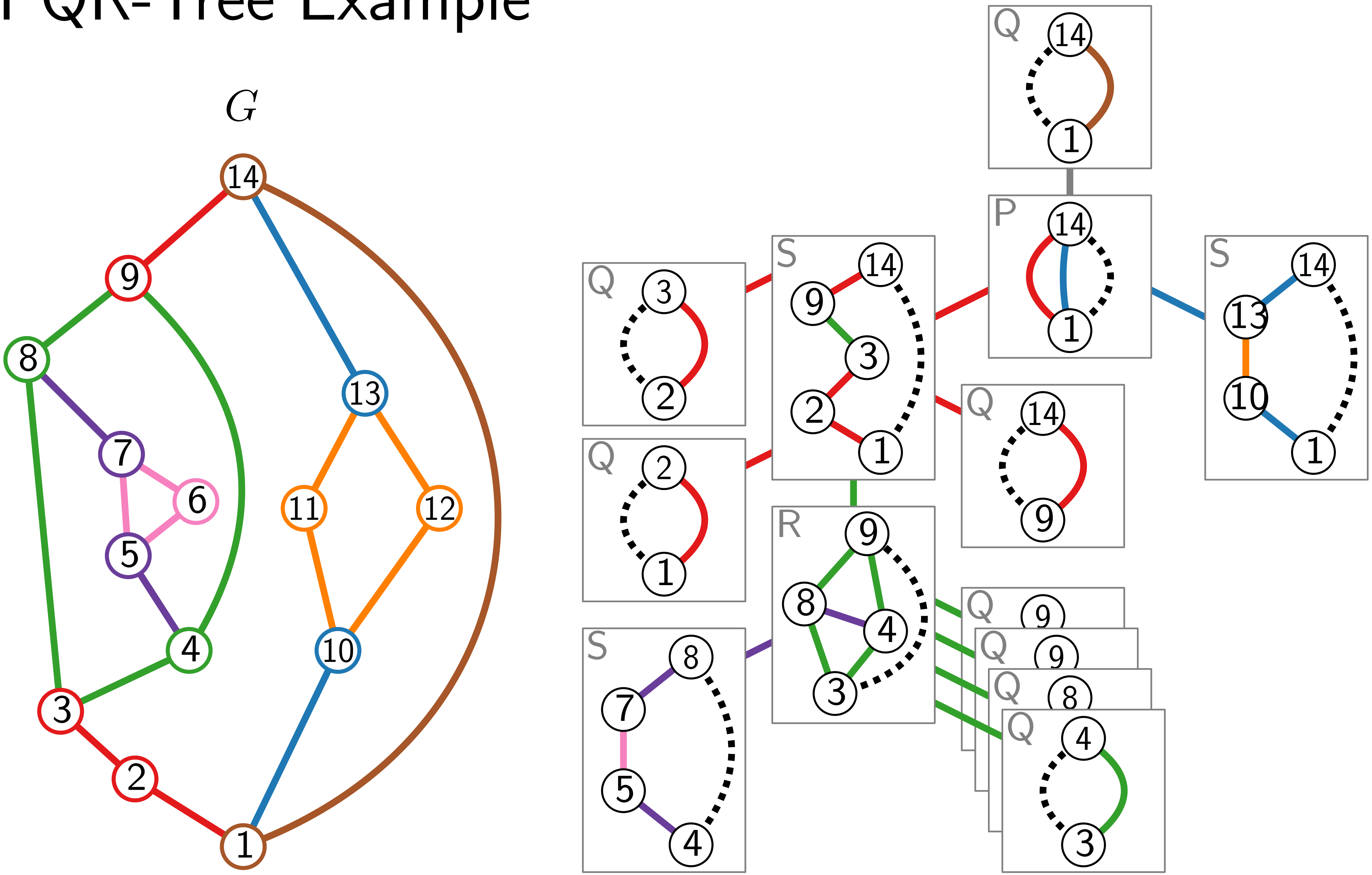

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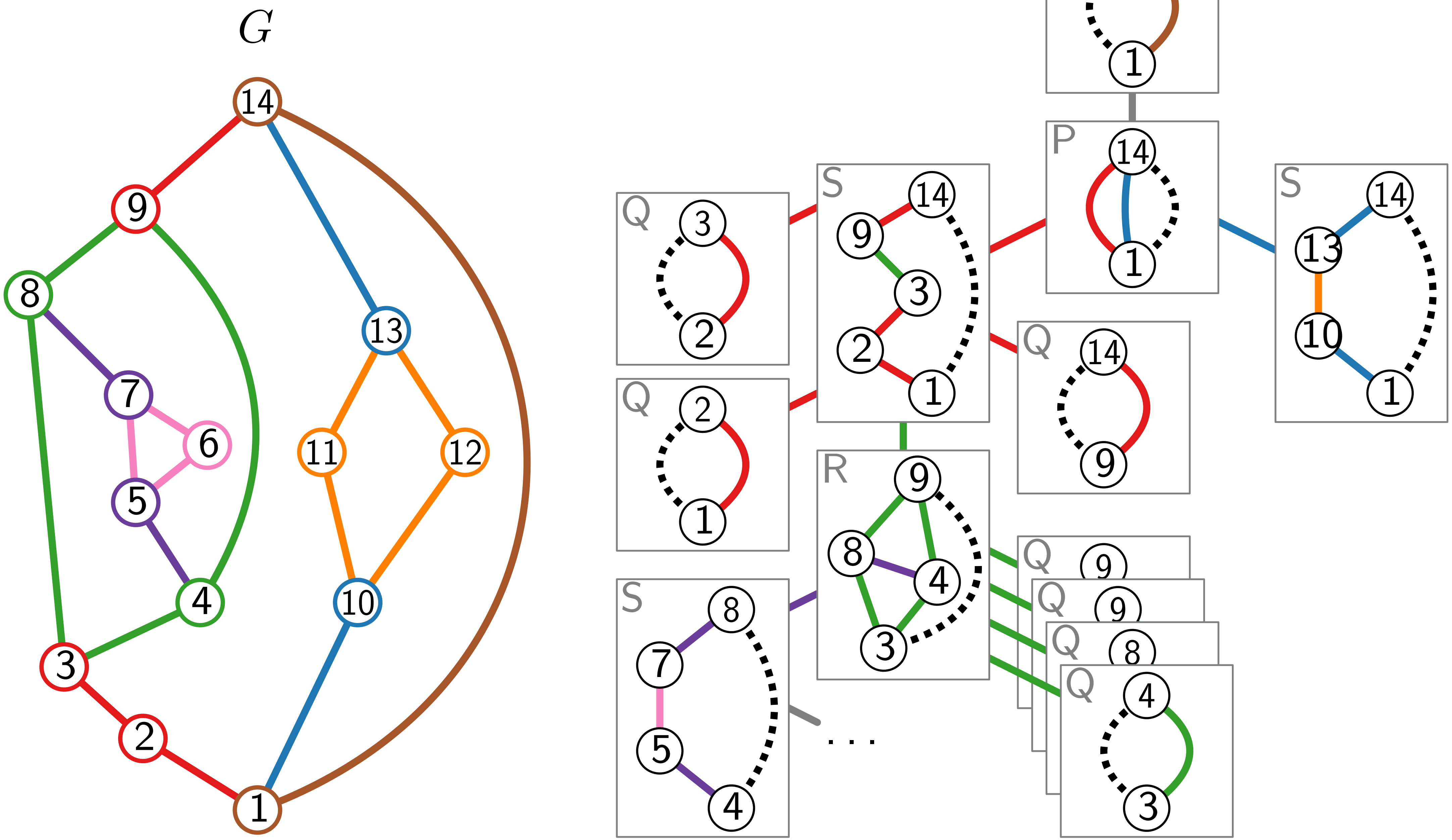

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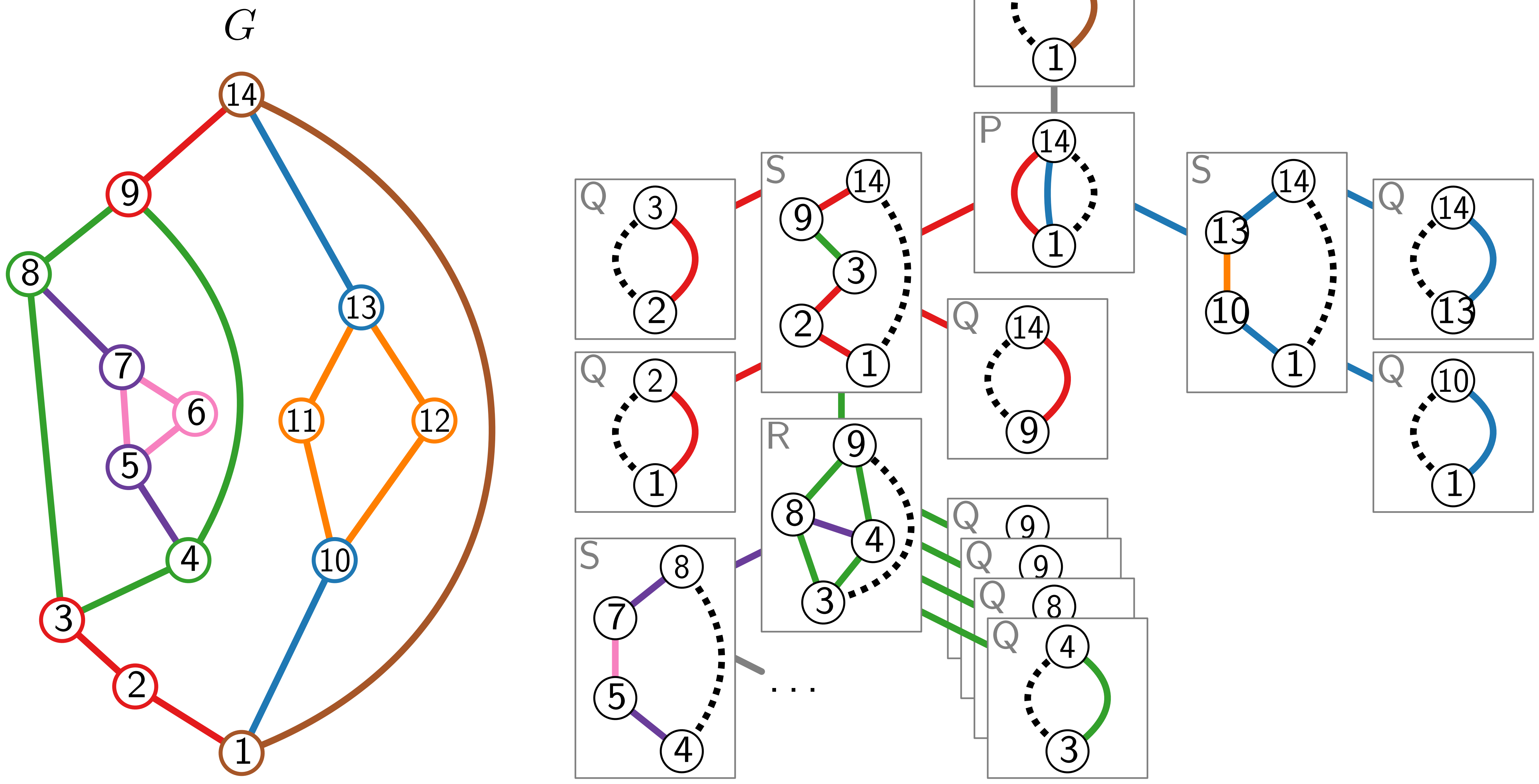

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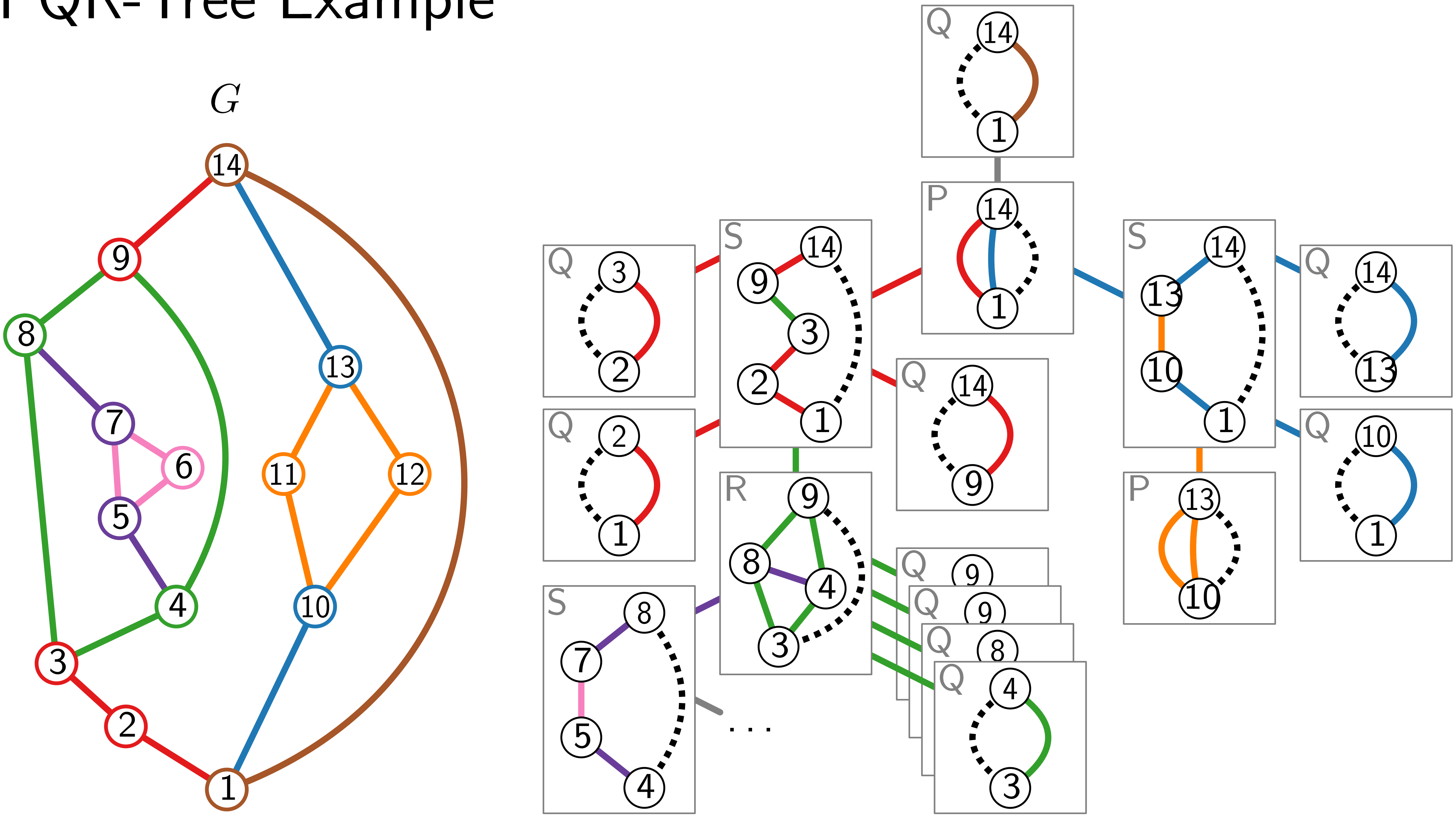

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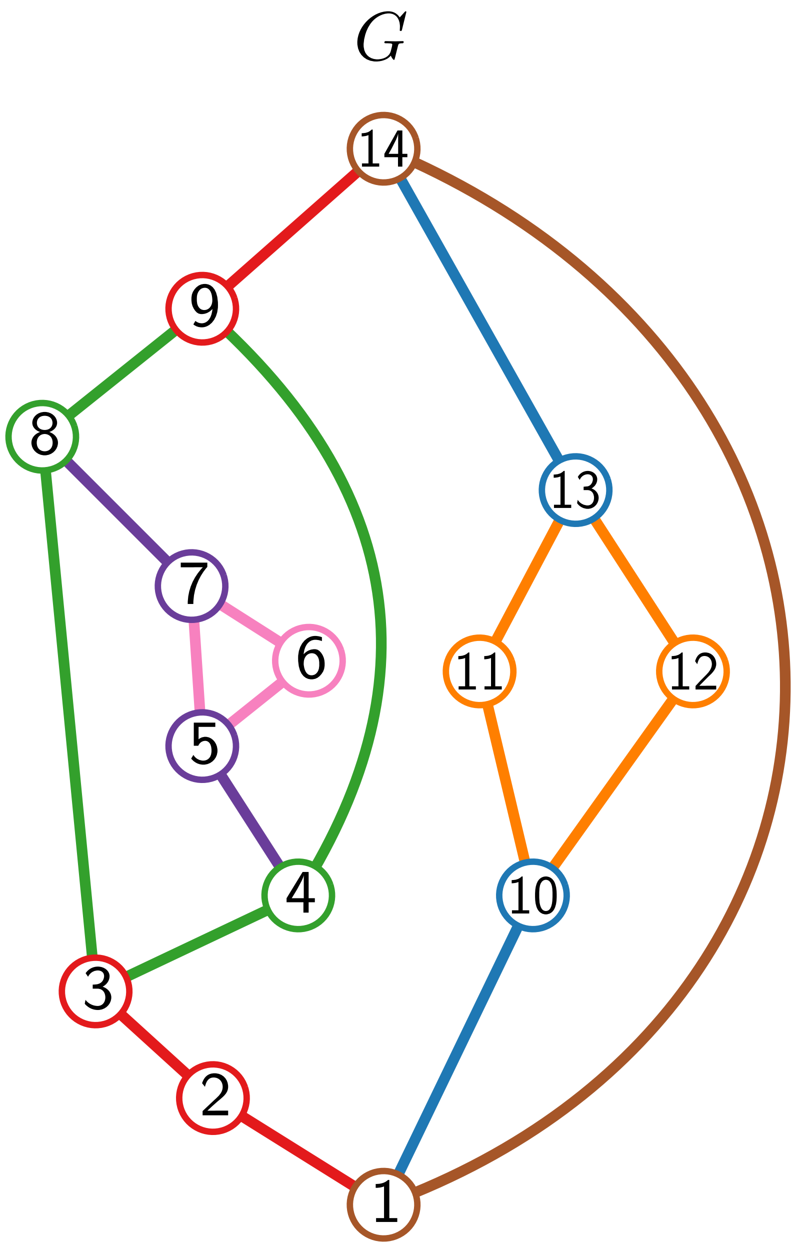

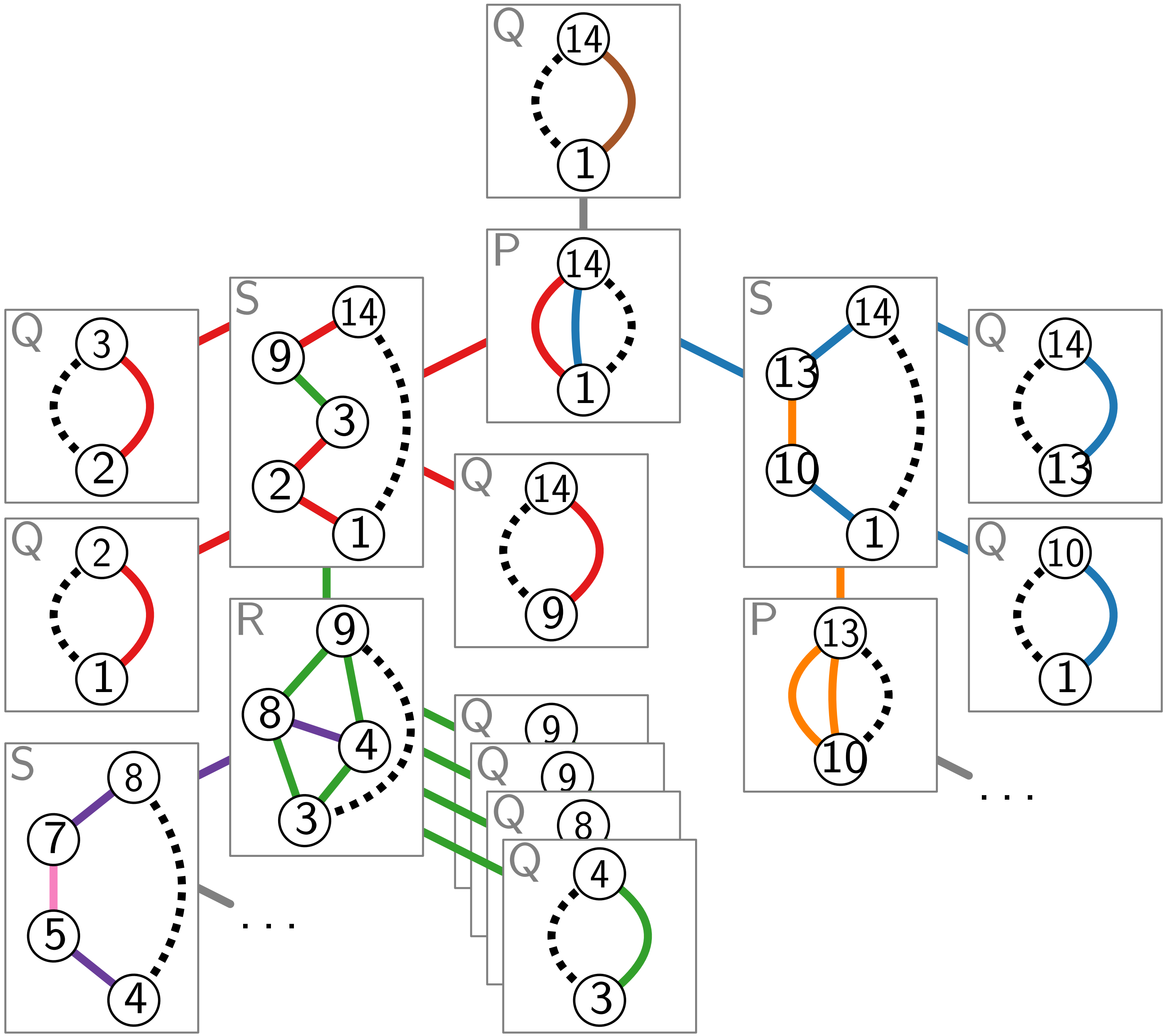

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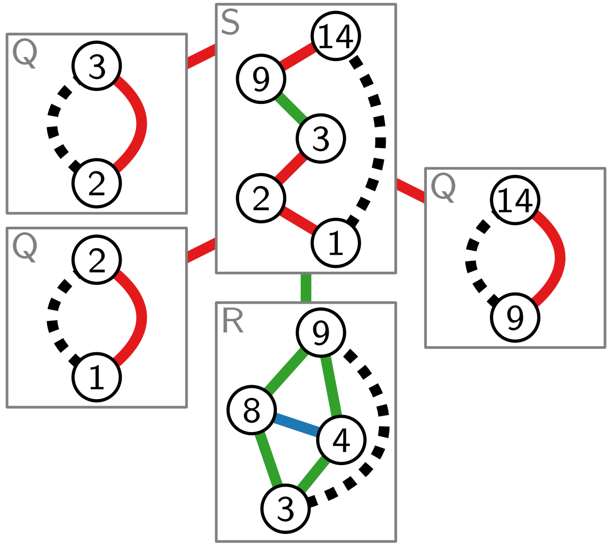
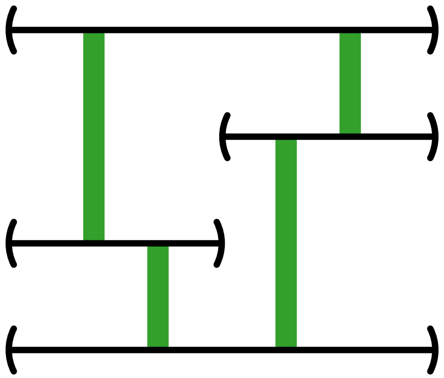
Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension



Part IV:
Rectangular
Representation Extension



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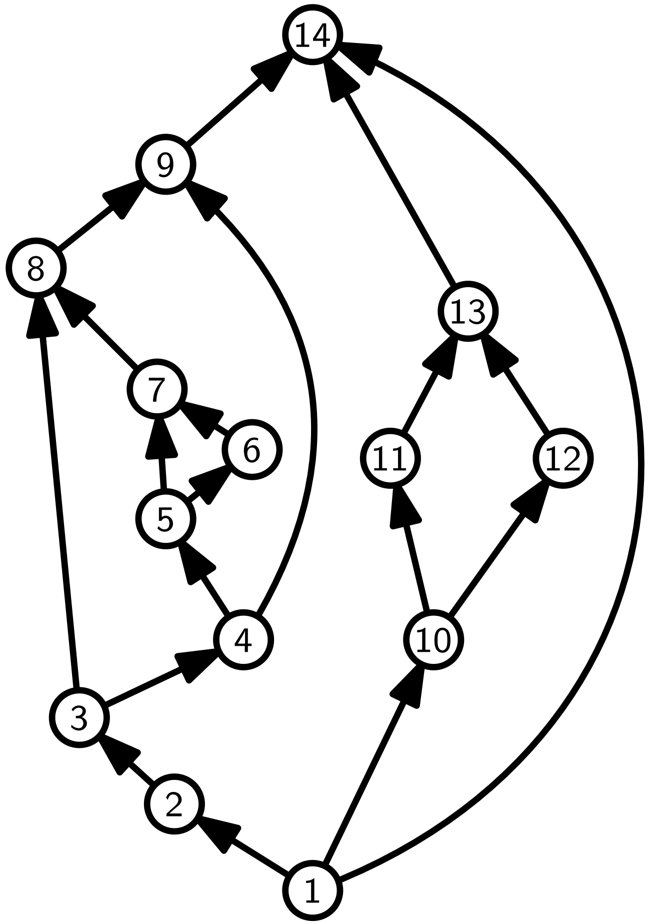
Representation Extension for st-Graphs

Theorem 1'.
Rectangular ε-Bar Visibility Representation Extension can be solved in $\mathcal{O}(n^2)$ time for st-graphs.



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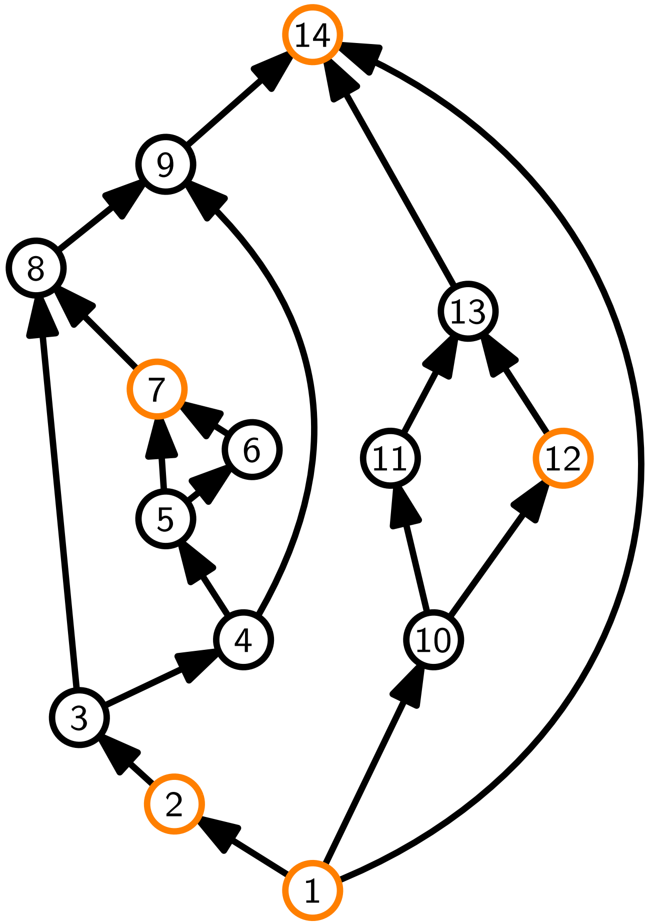

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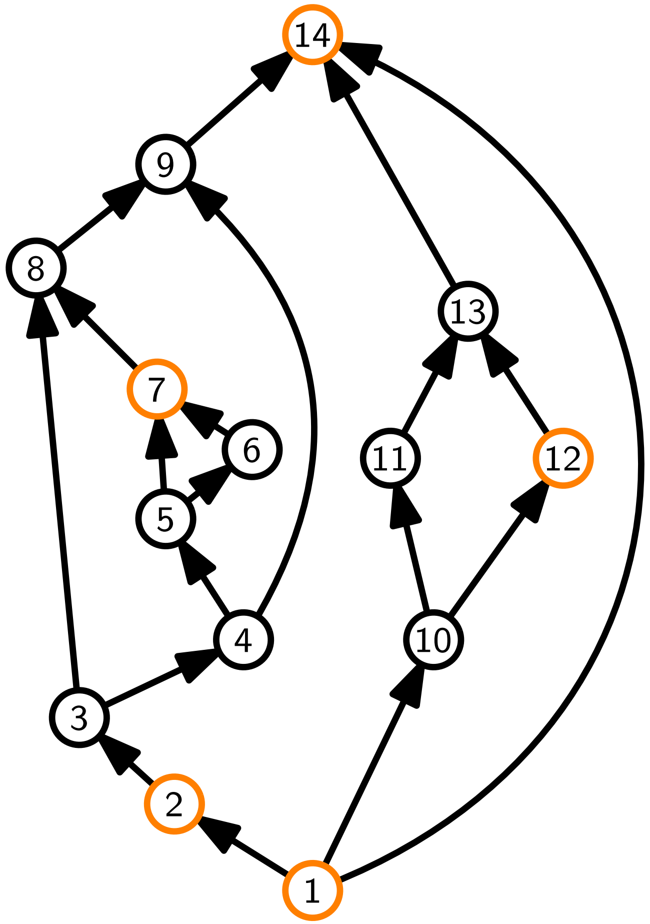

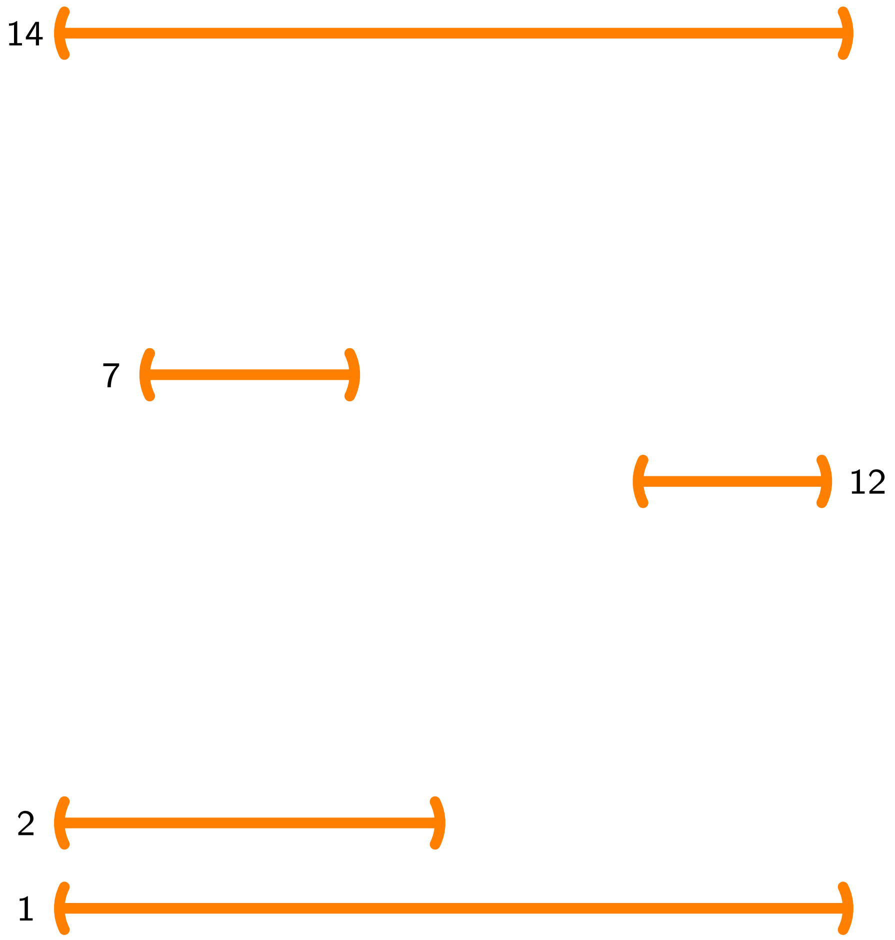

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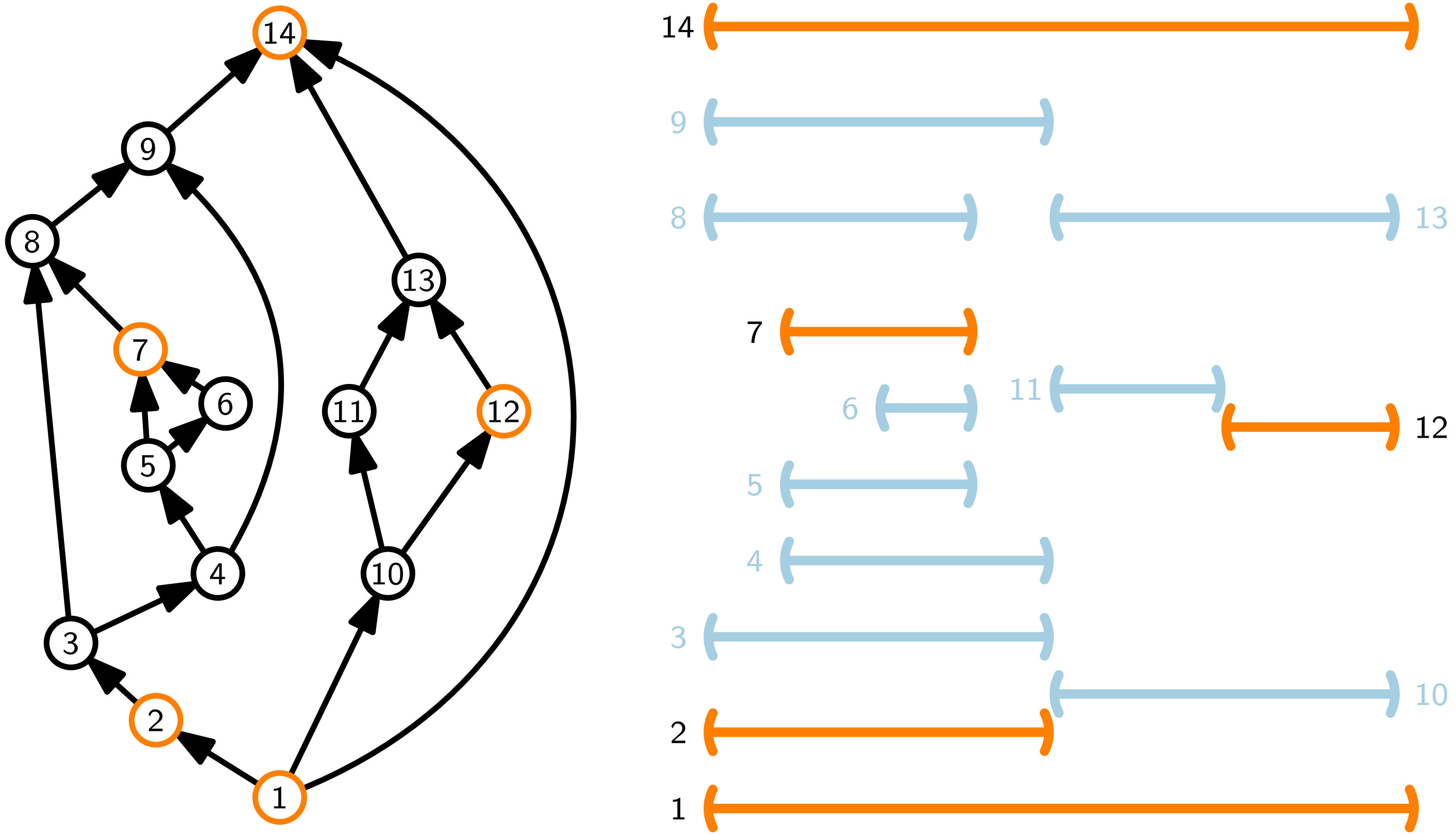

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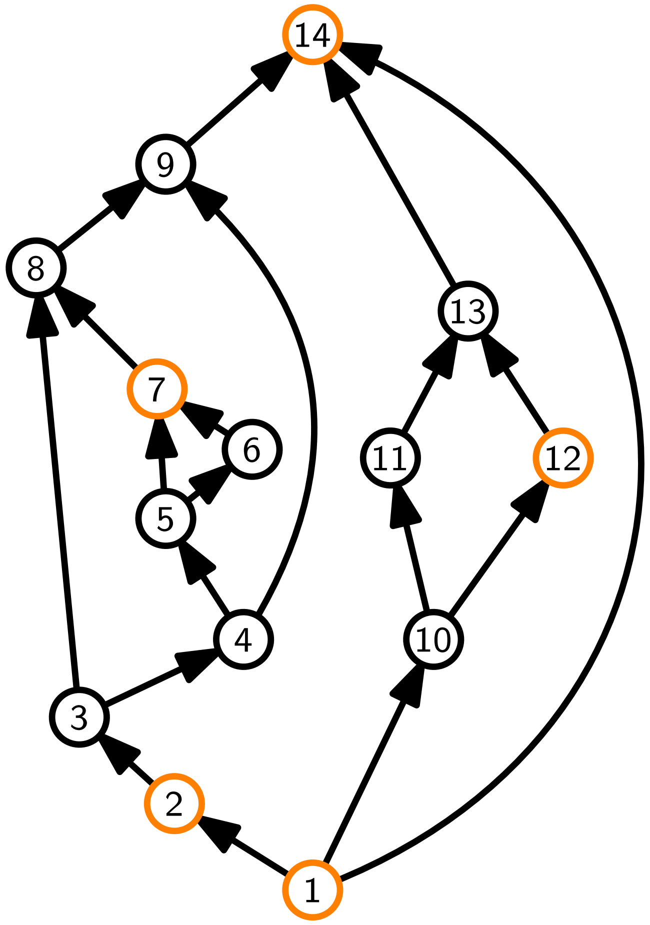

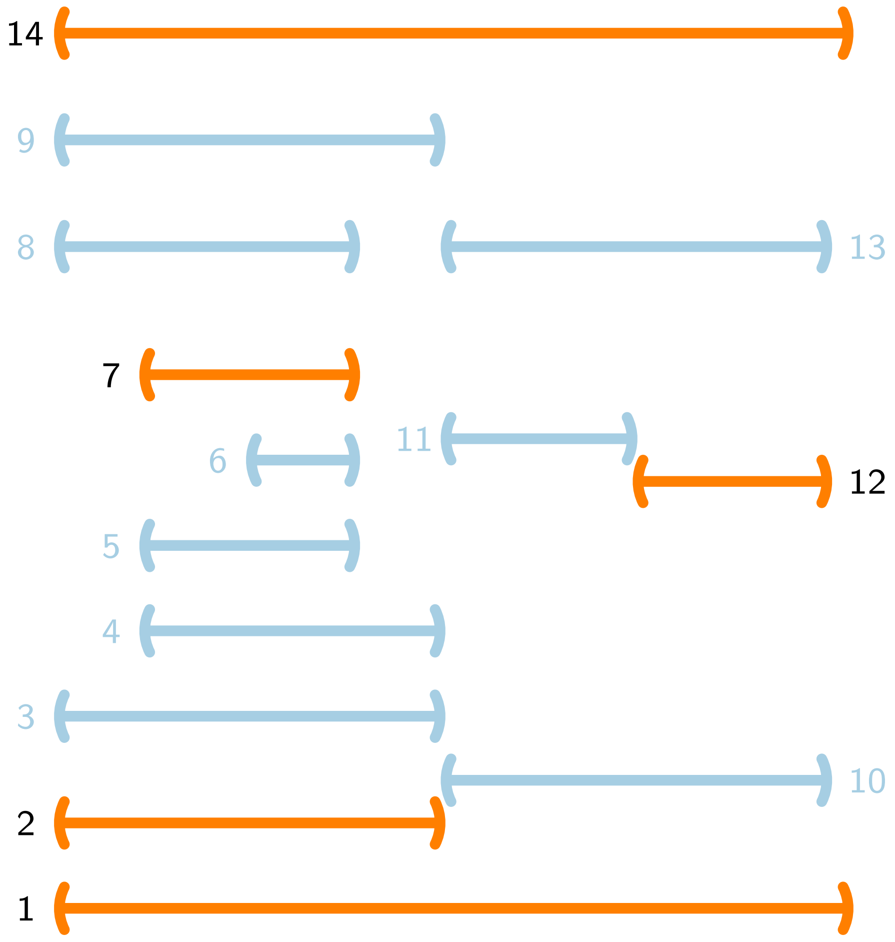

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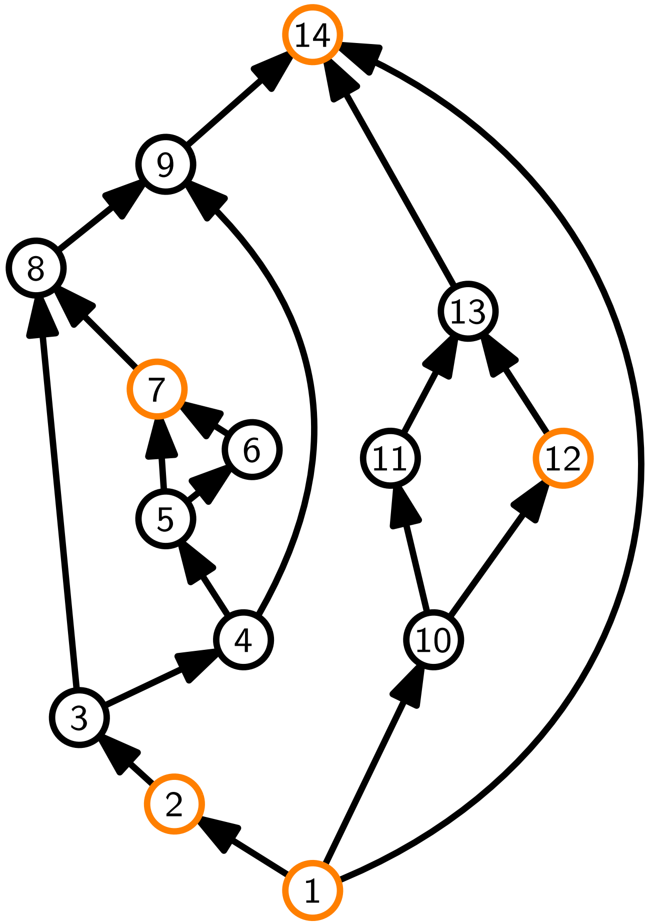

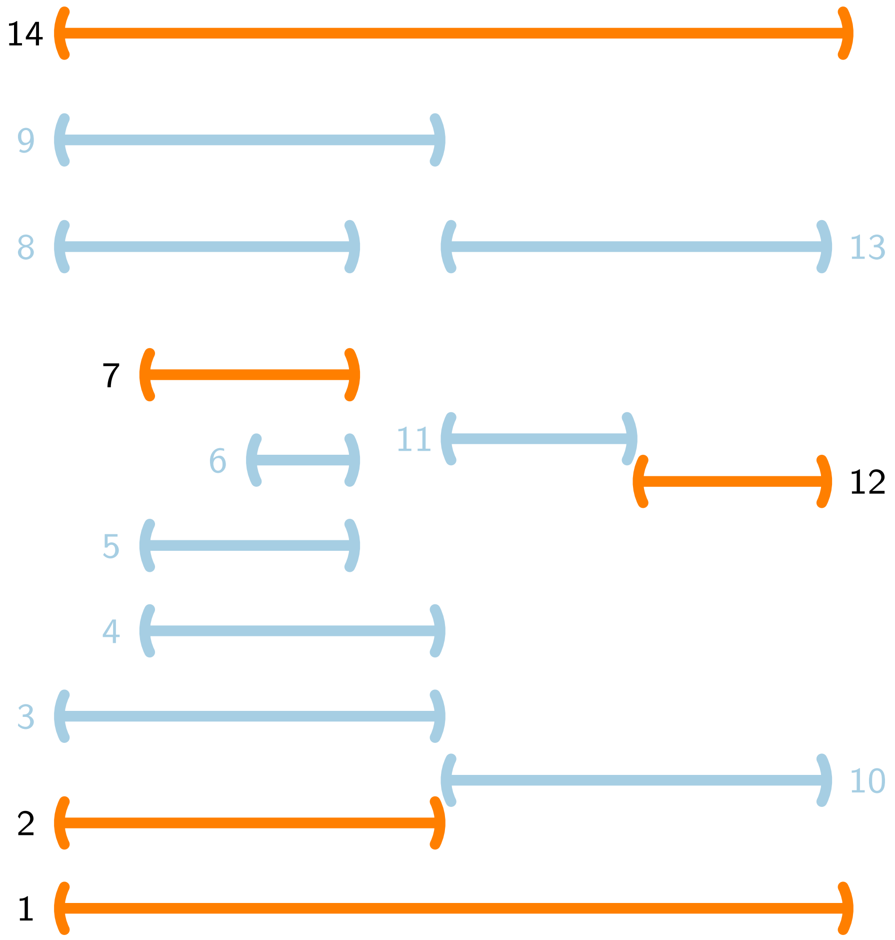

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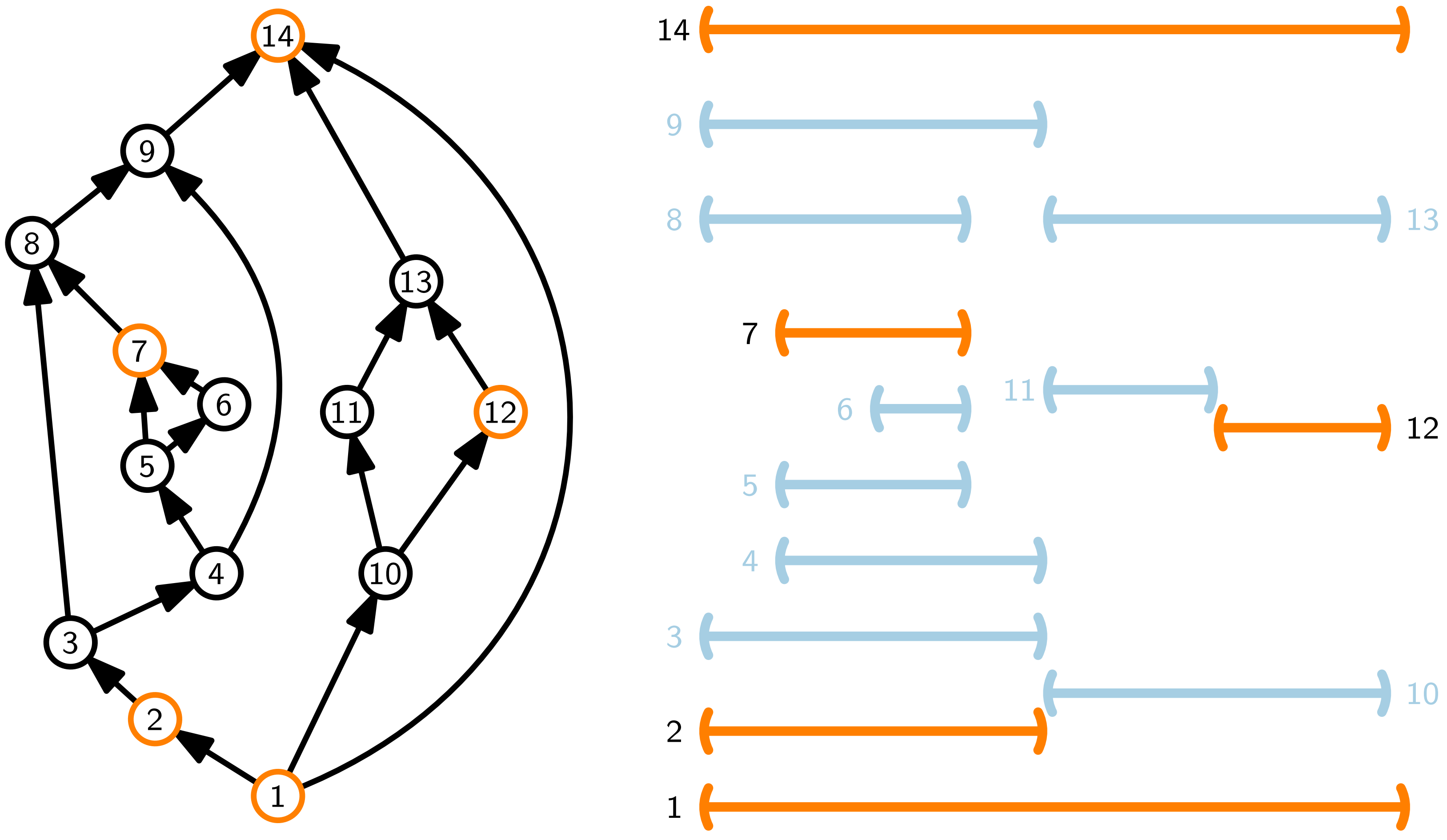

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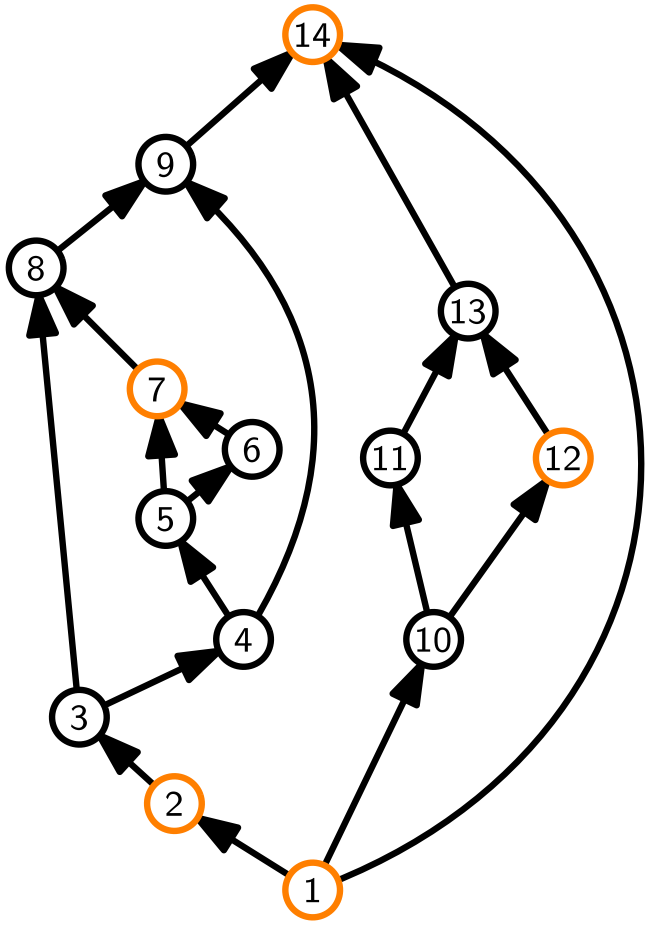

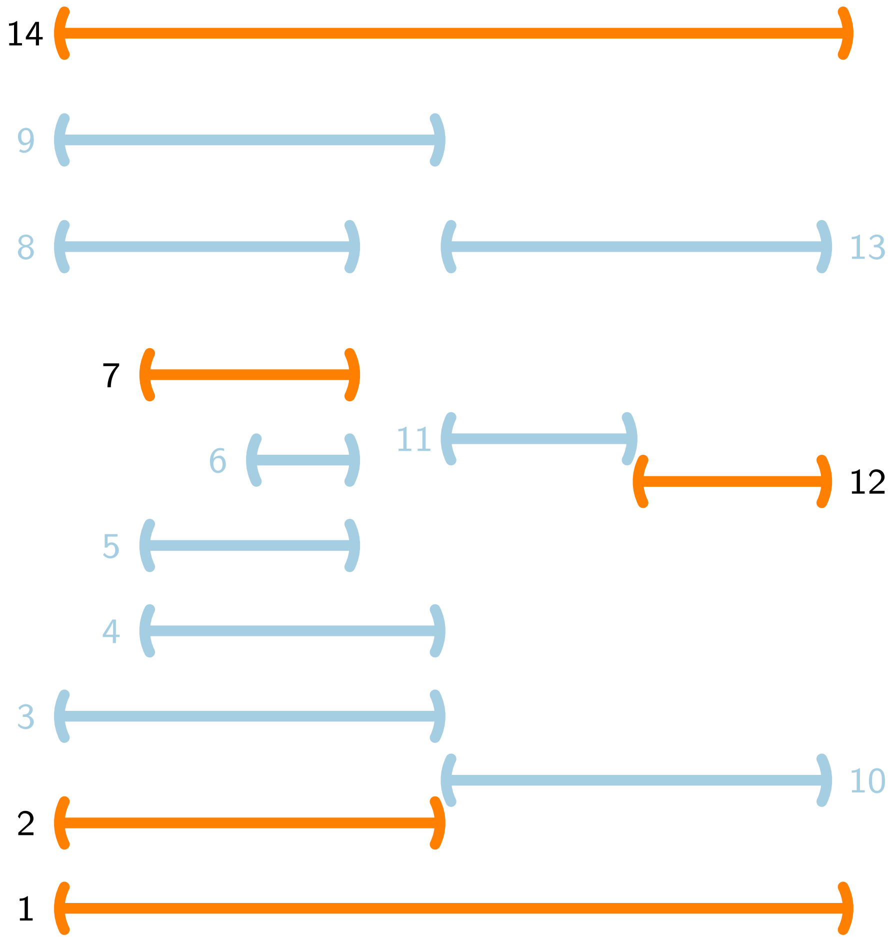

- Simplify with assumption on y-coordinates
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- Solve problems for **S**-, **P**-, and **R**-nodes
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Proof idea. The relative positions of **adjacent** bars must match the order given by y.
So, we can adjust the y-coordinates of any solution to be as in y by sweeping from bottom to top.

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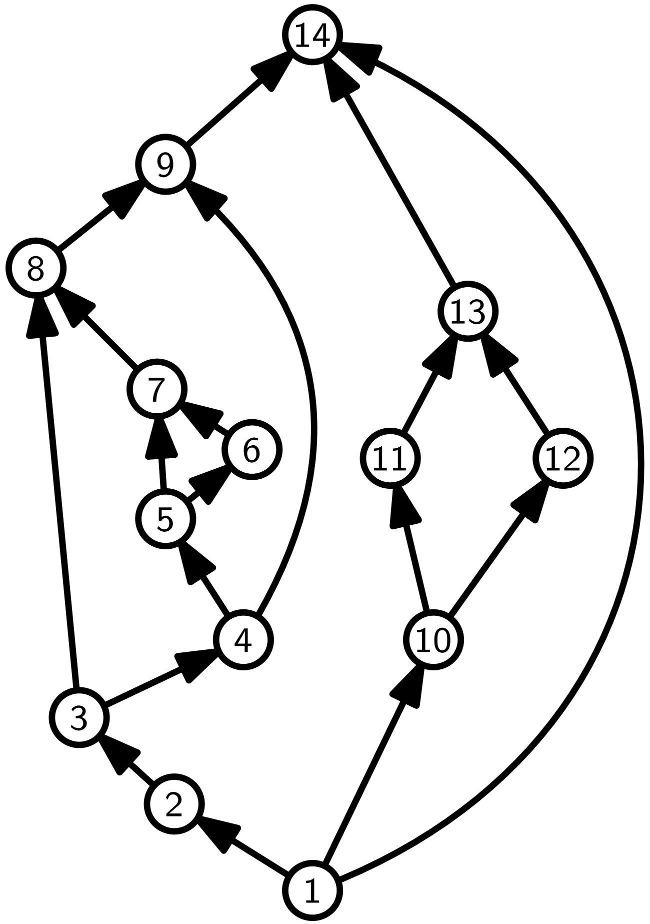
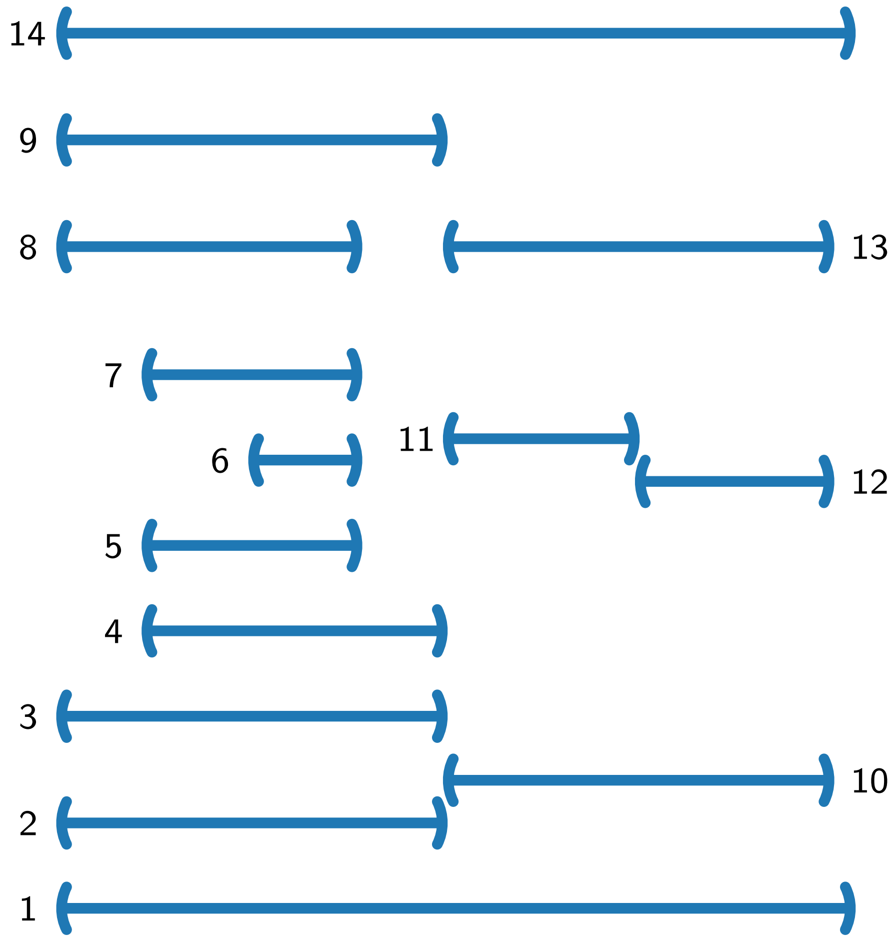
G has a representation extending ψ'
where the y-coordinates of the bars are as in y.

We can now assume that all y-coordinates are given!

Proof idea. The relative positions of **adjacent** bars must match the order given by y.
So, we can adjust the y-coordinates of any solution to be as in y by sweeping from bottom to top.

But why do SPQR-Trees help?





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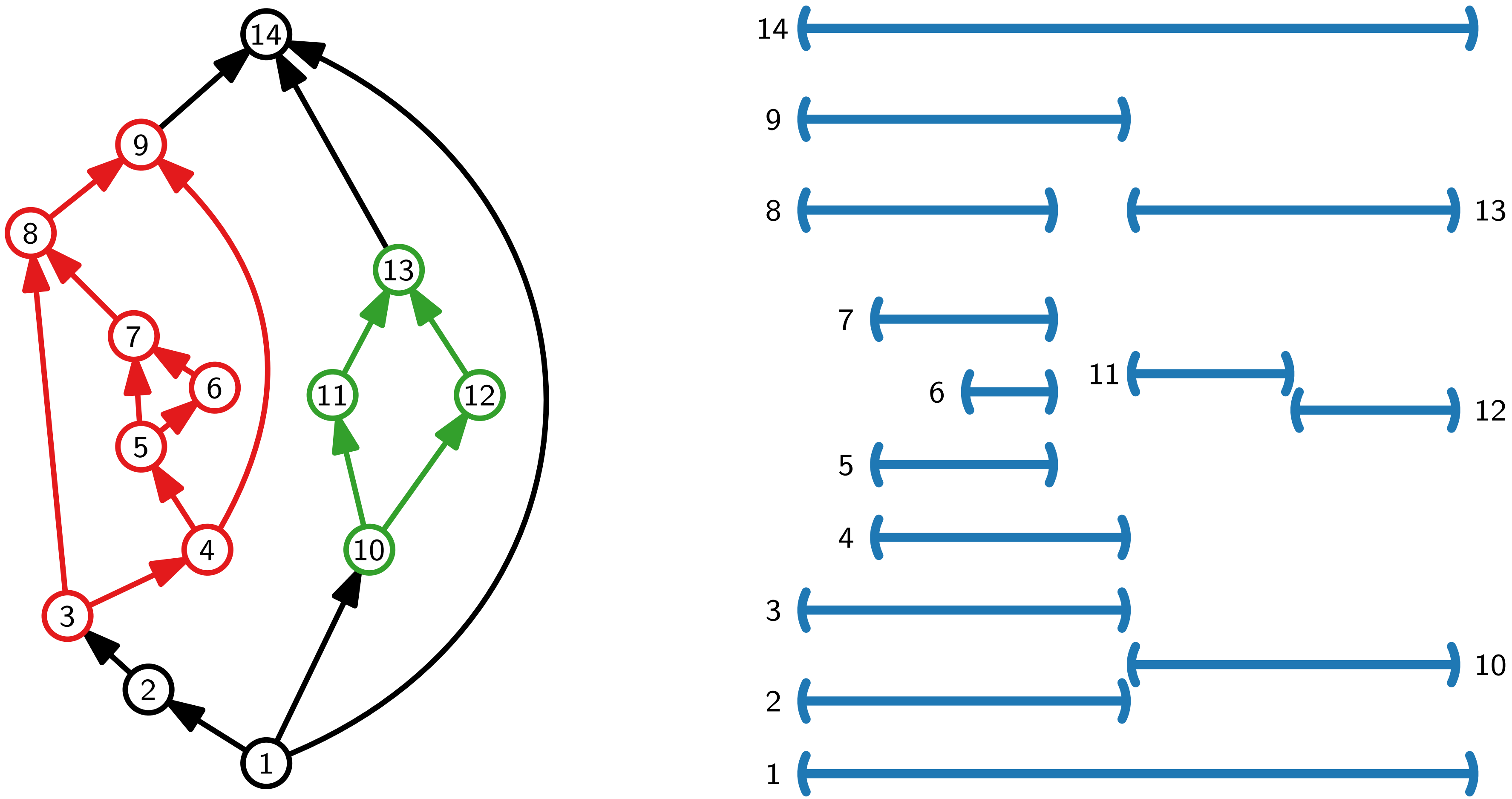

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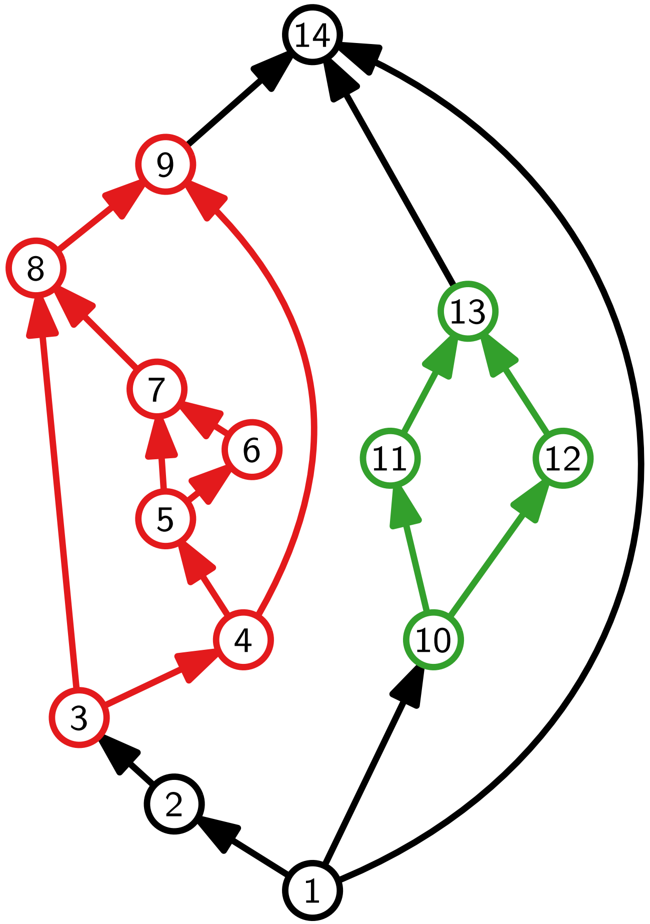

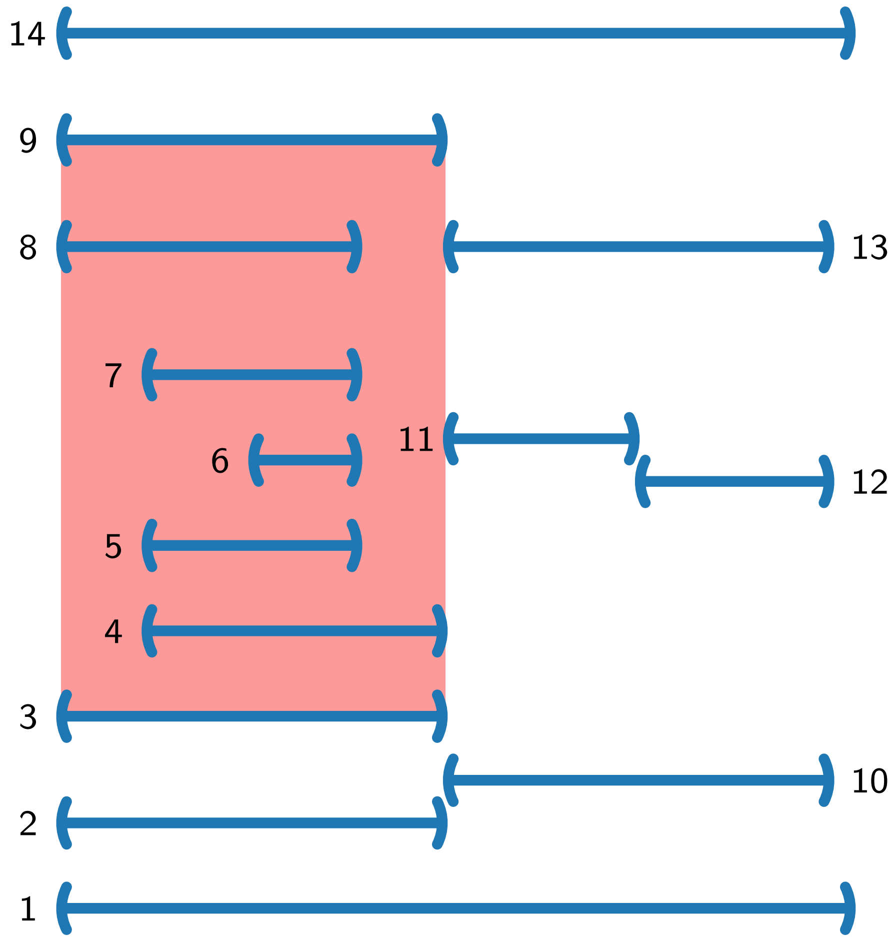

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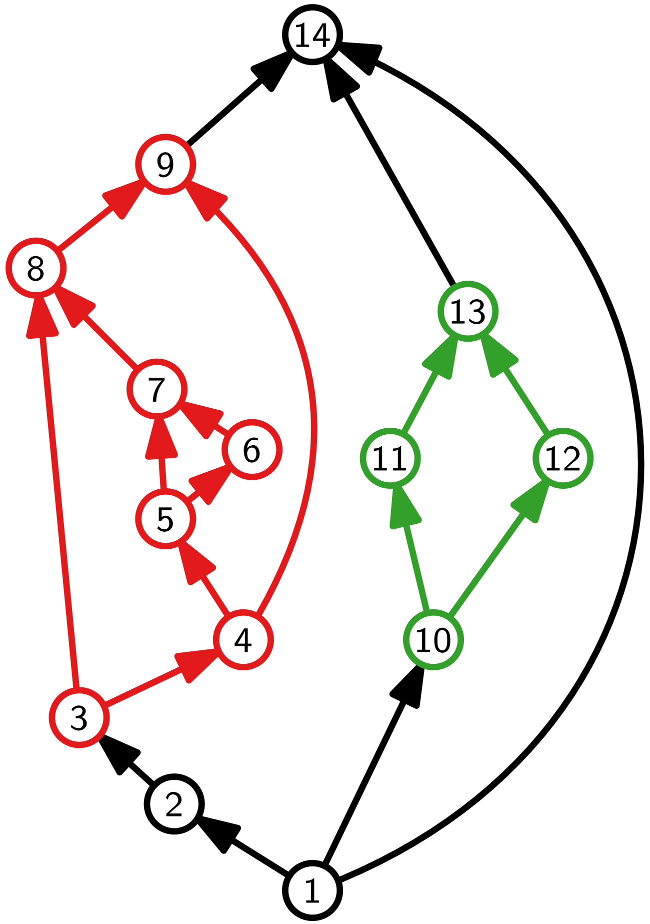
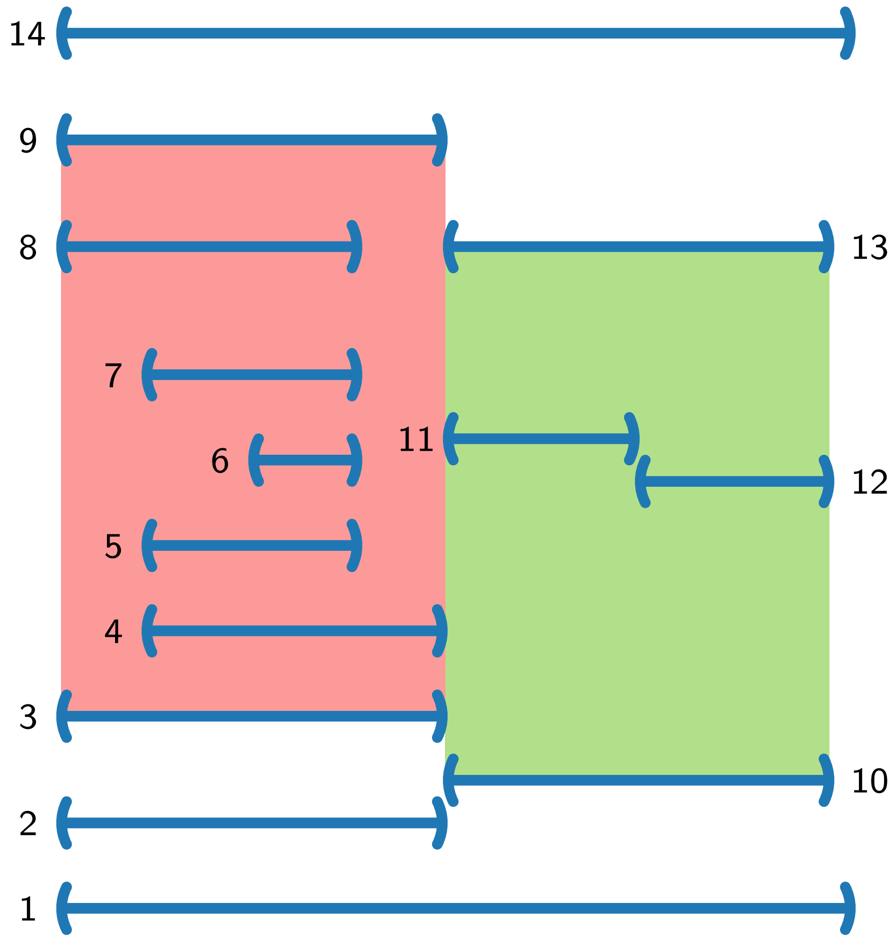
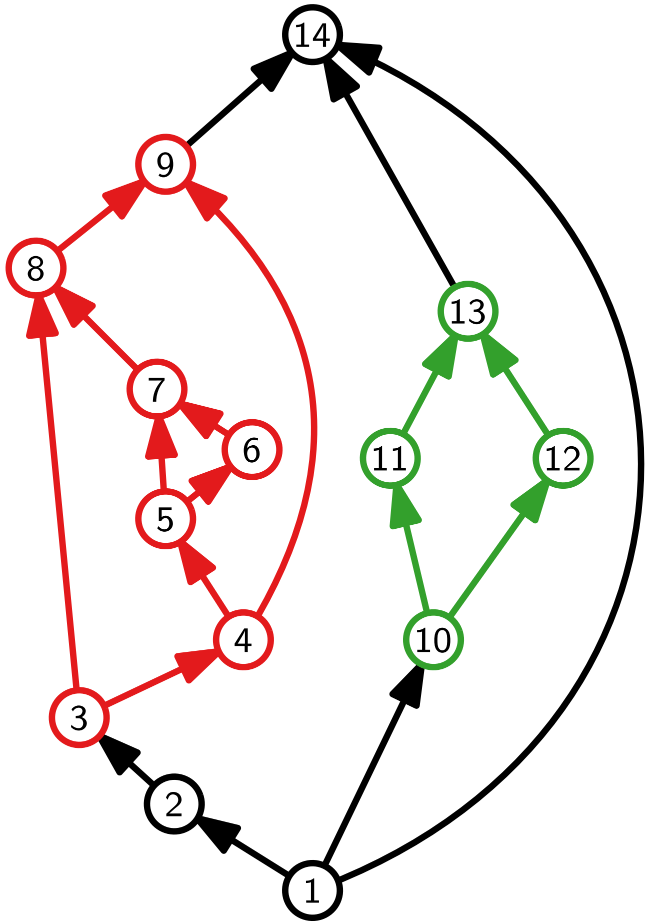
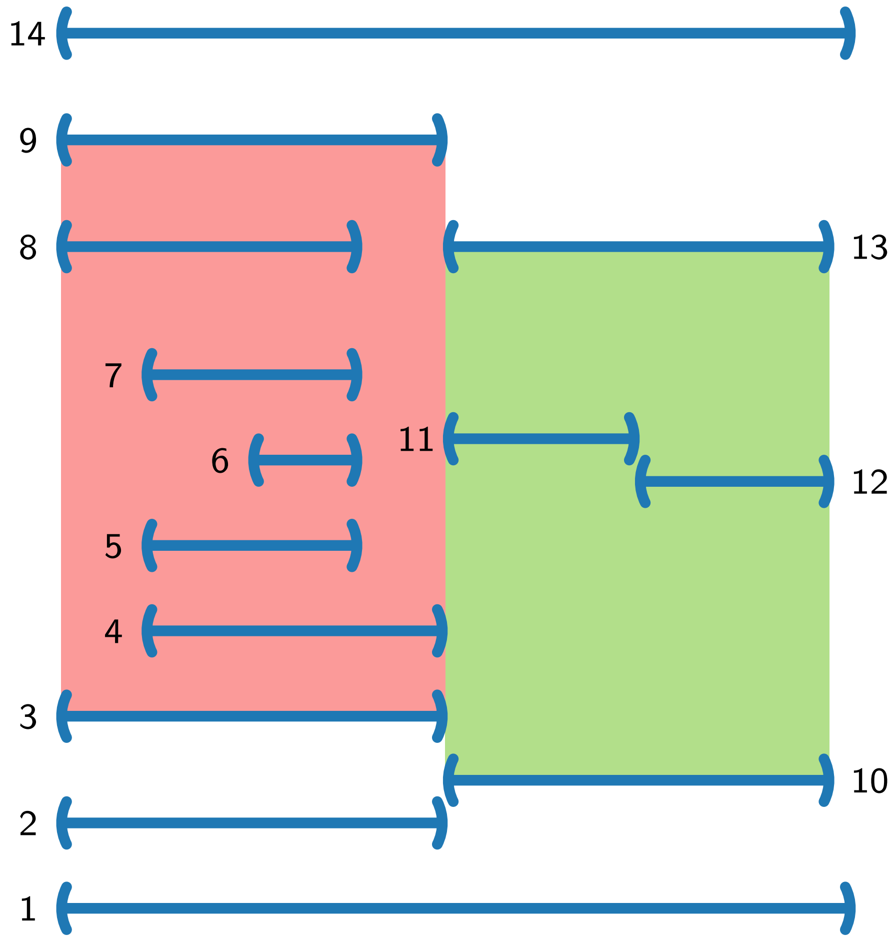




But why do SPQR-Trees help?

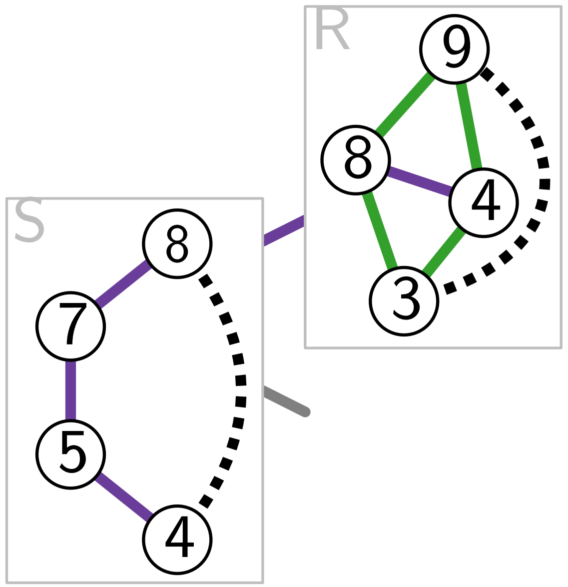
Lemma 2.
The SPQR-tree of an st-graph G induces a recursive **tiling** of any ε-bar visibility representation of G.



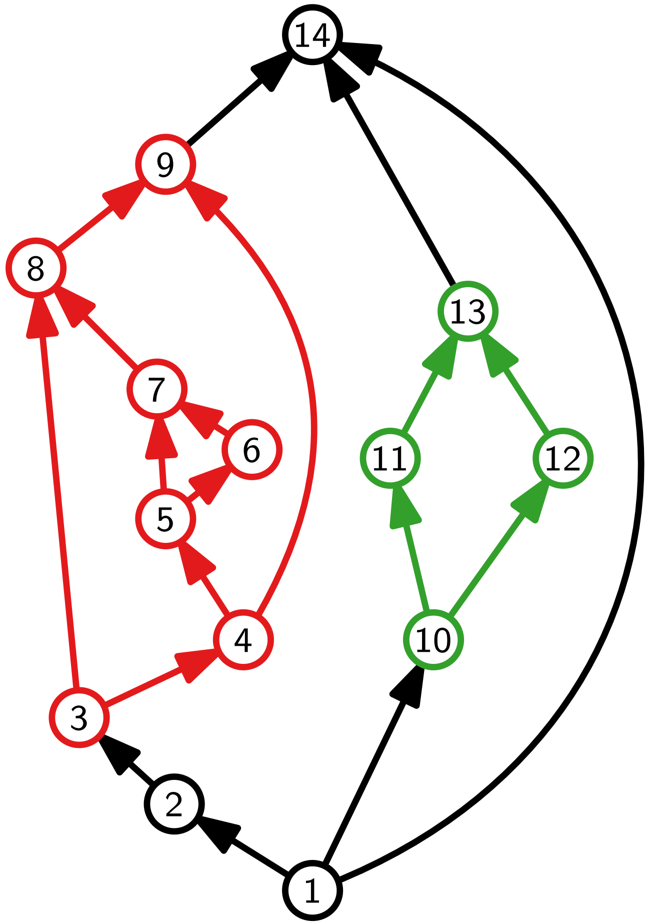


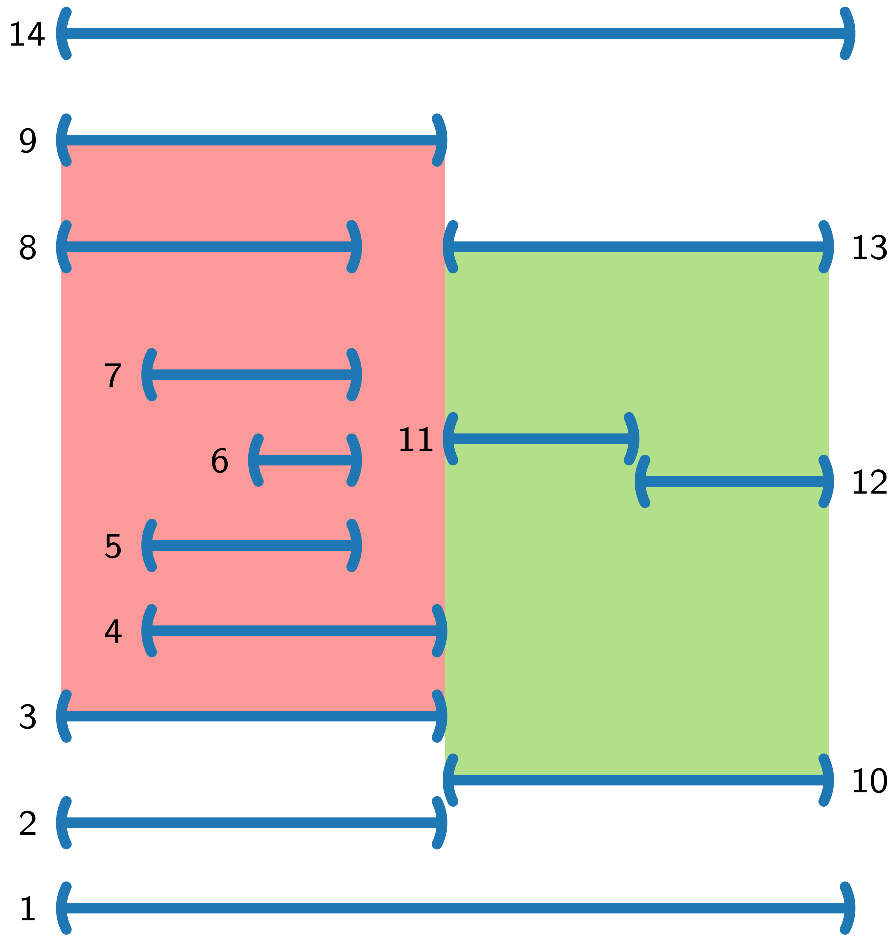
But why do SPQR-Trees help?

Lemma 2.
The SPQR-tree of an st-graph G induces a recursive **tiling** of any ε-bar visibility representation of G.



Solve tiles bottom-up





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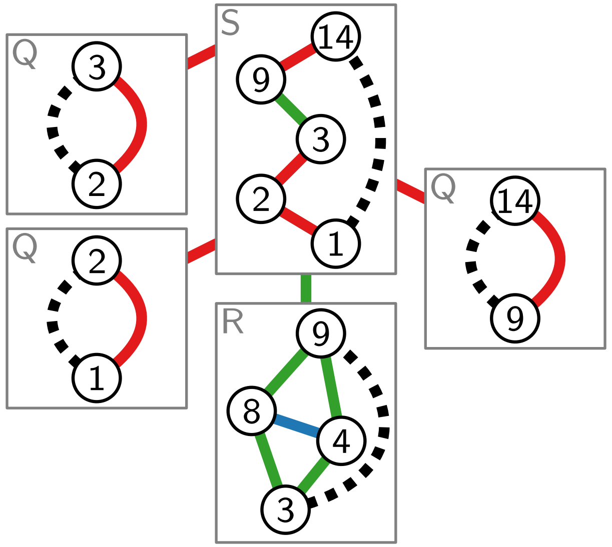
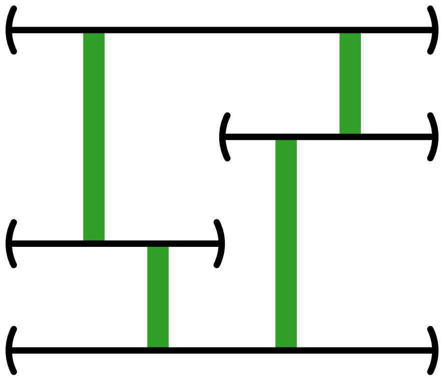
Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension



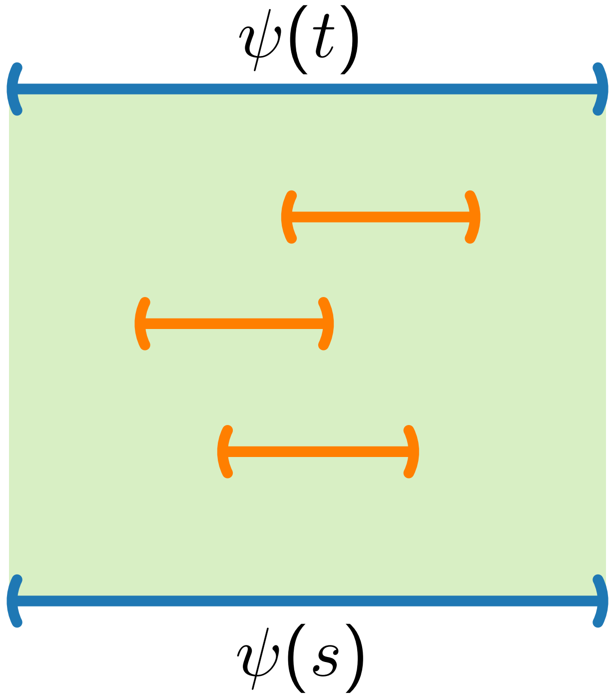
Part V:
Dynamic Program

Alexander Wolff



Tiles

Convention. **Orange** bars are from the partial representation



Tiles

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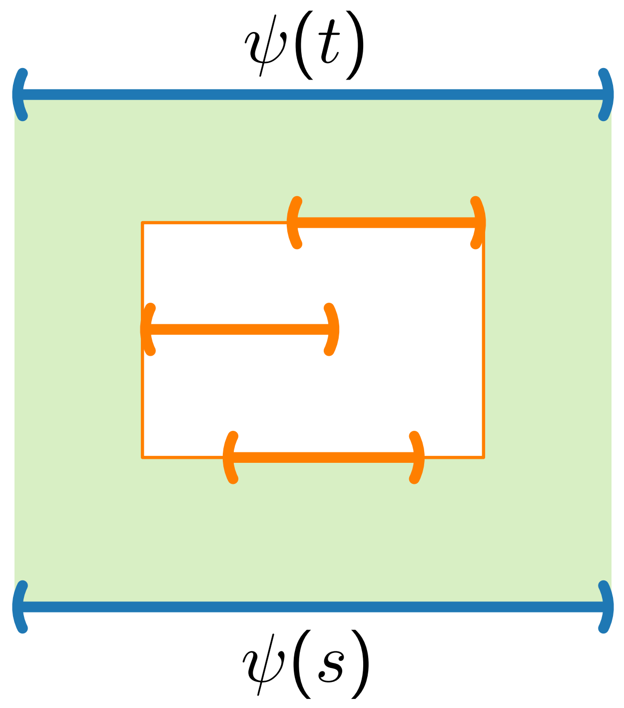

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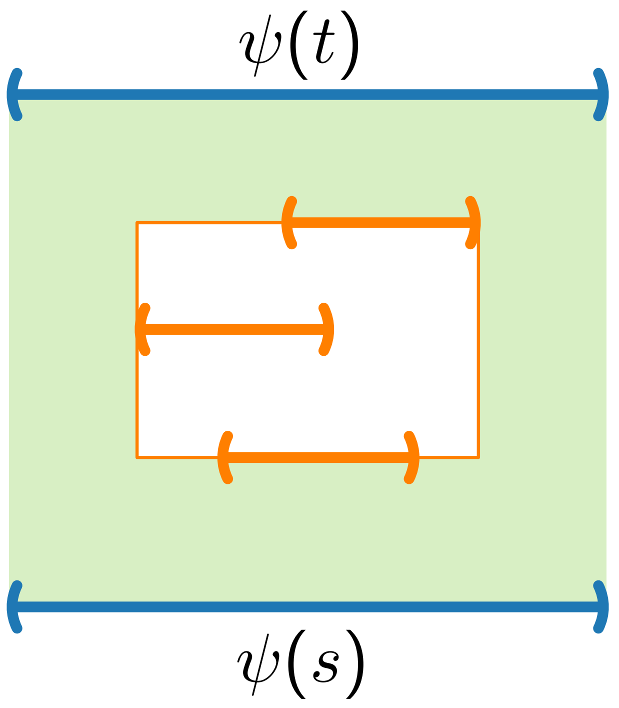
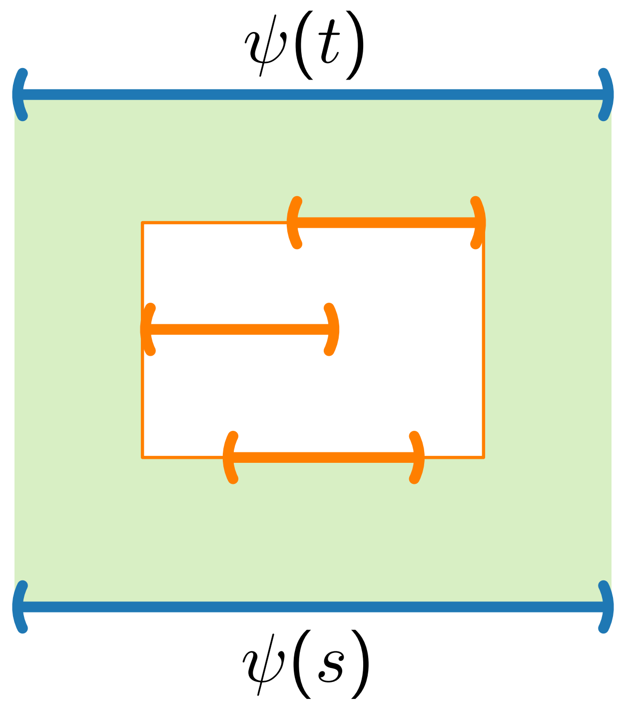


Observation.
The bounding box (tile) of any solution ψ **contains** the bounding box of the partial representation.

Tiles

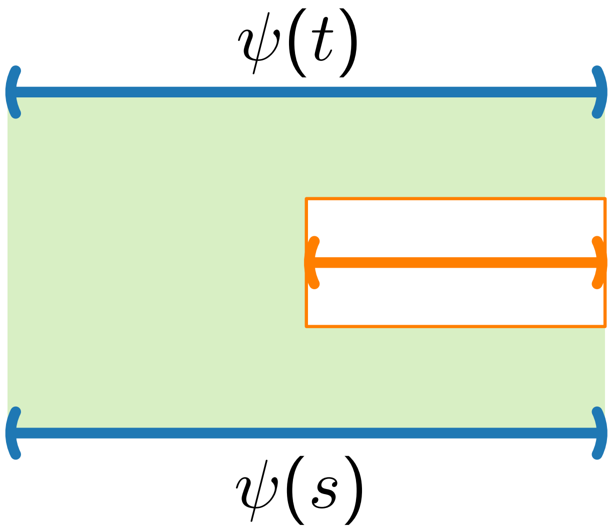
Convention. **Orange** bars are from the partial representation



Observation.
The bounding box (tile) of any solution ψ **contains** the bounding box of the partial representation.

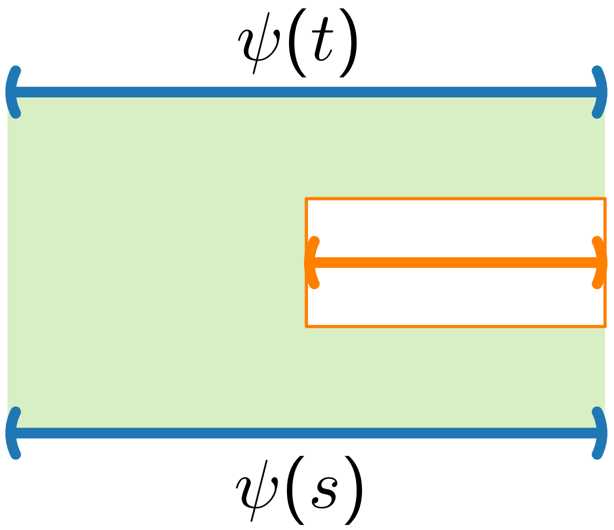
How many different **types** of tiles are there?

Types of Tiles



- Right **F**ixed – due to the orange bar
- Left **L**oose – due to the orange bar

Types of Tiles



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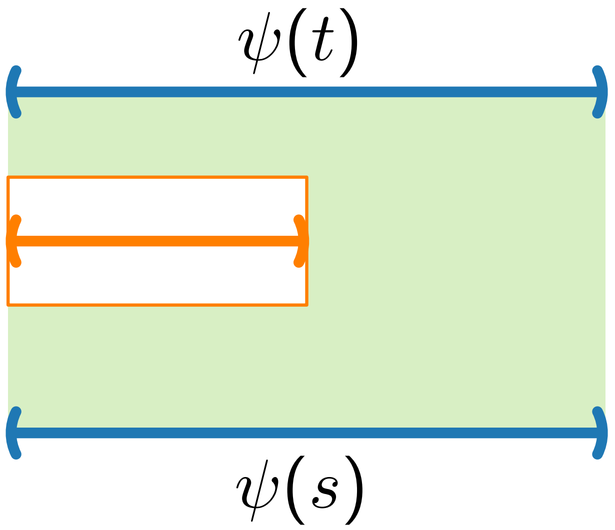

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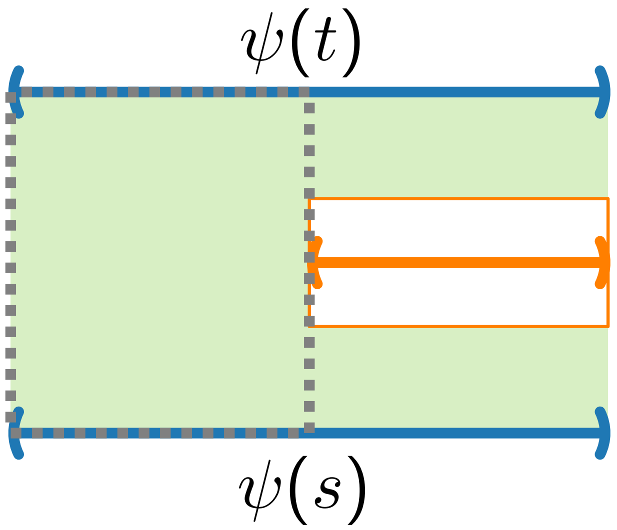

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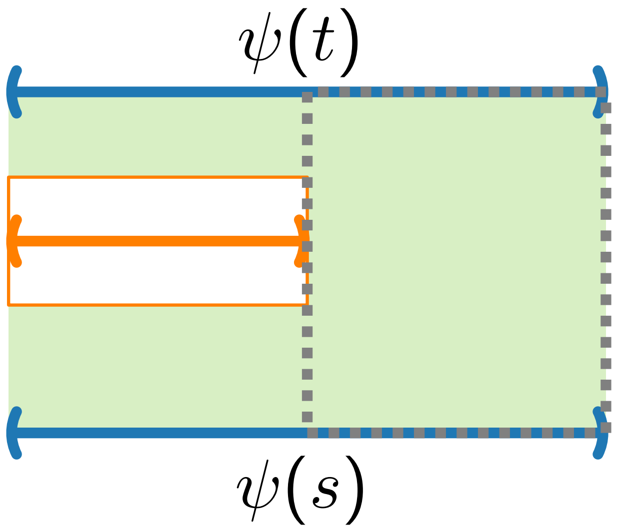

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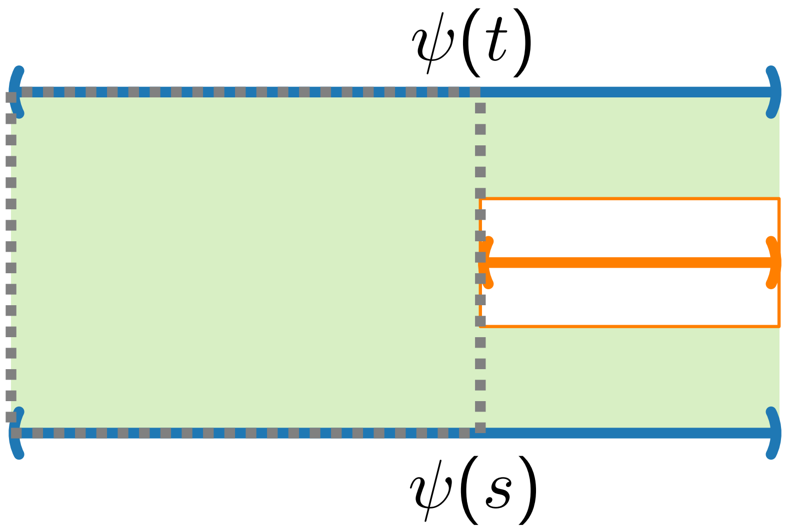

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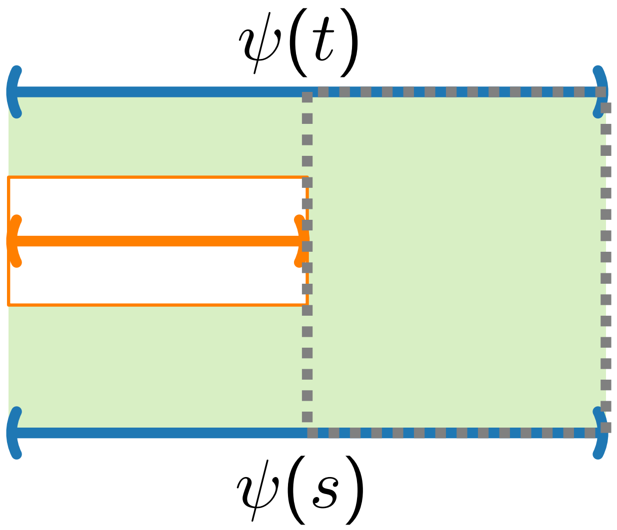

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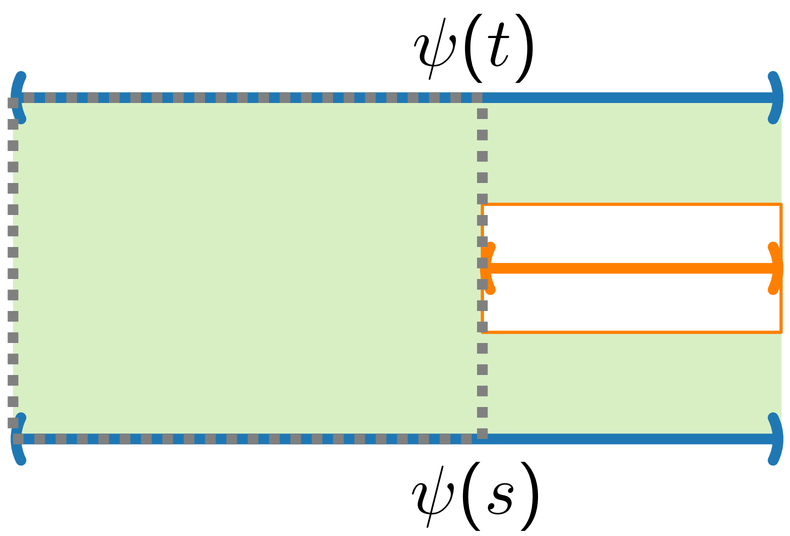

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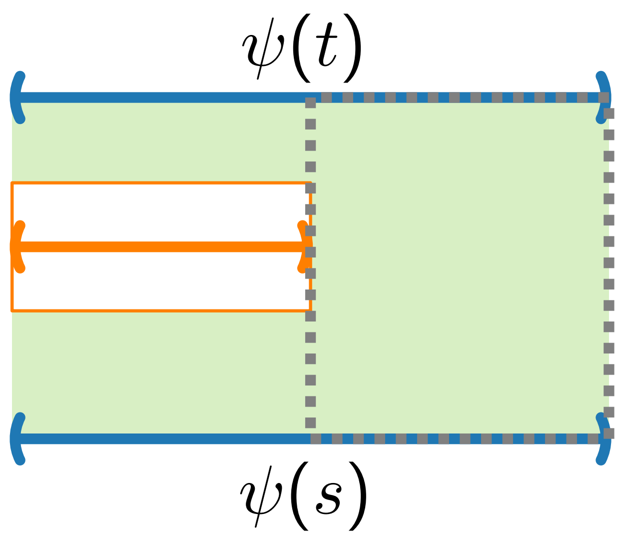

- Left **F**ixed – due to the orange bar
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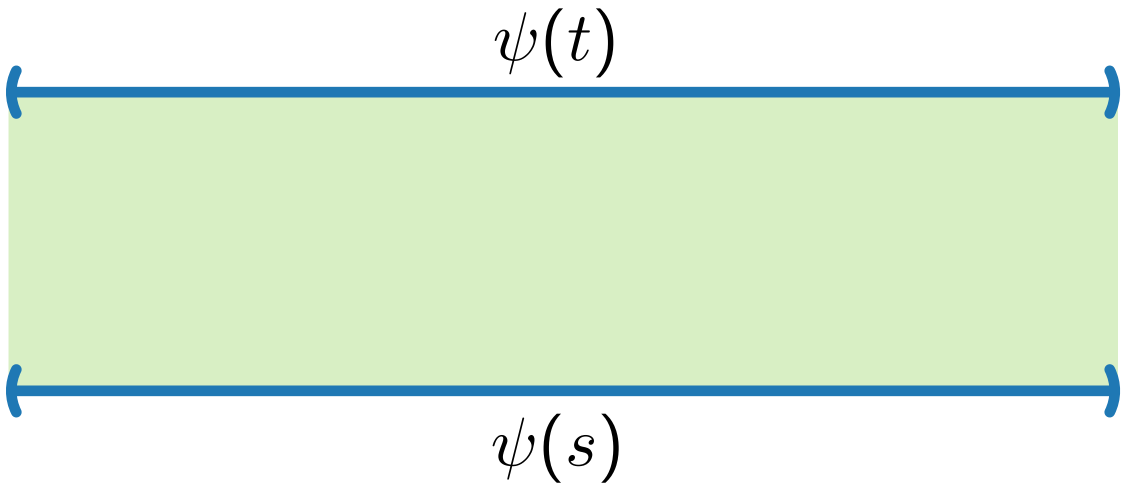
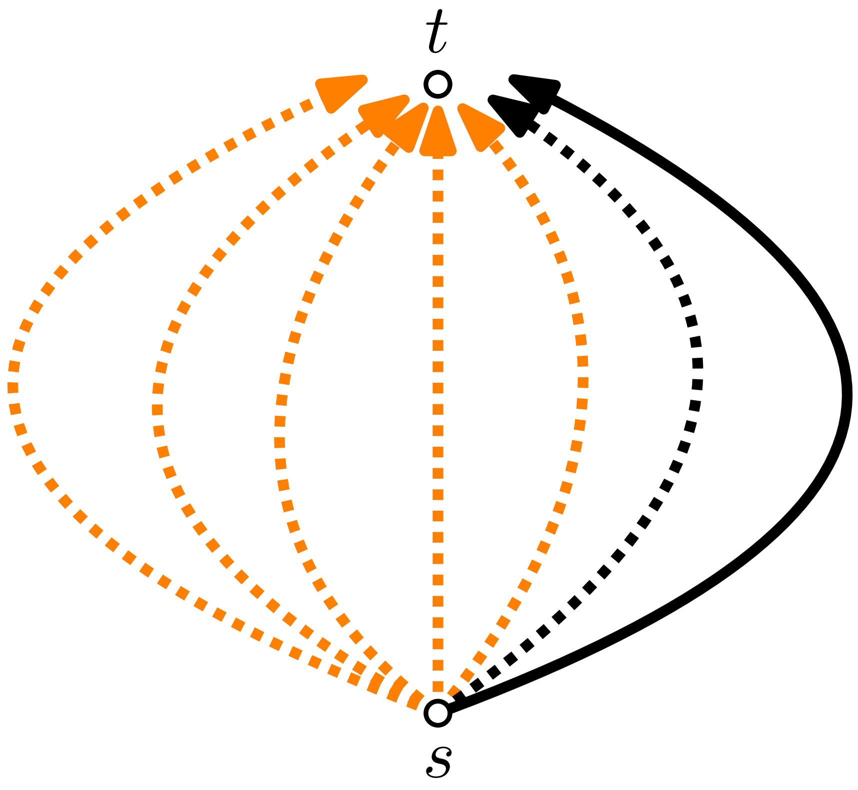
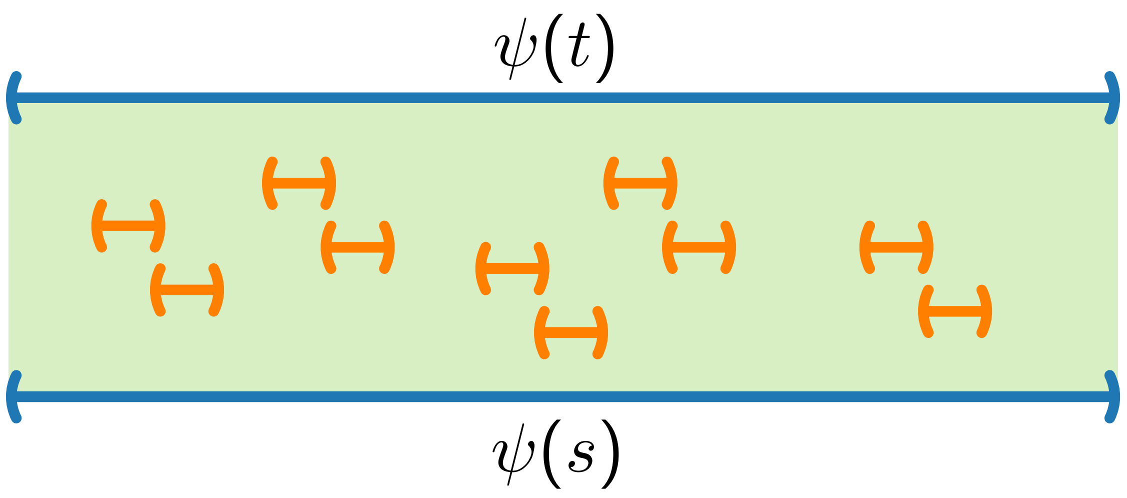
Four different types: **FF, FL, LF, LL**

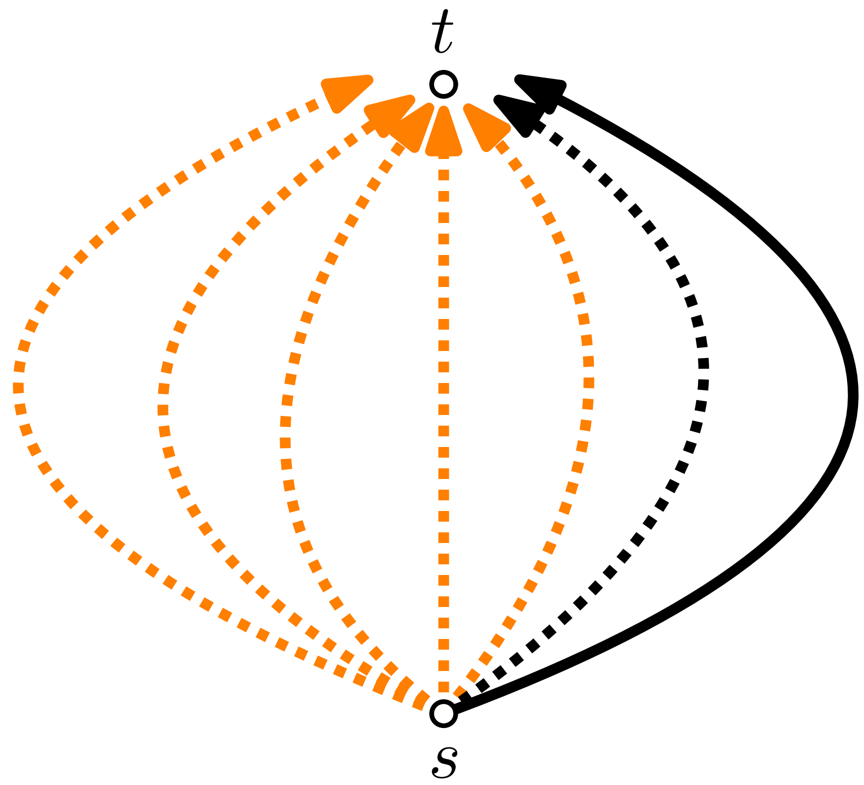
P-Nodes





P-Nodes





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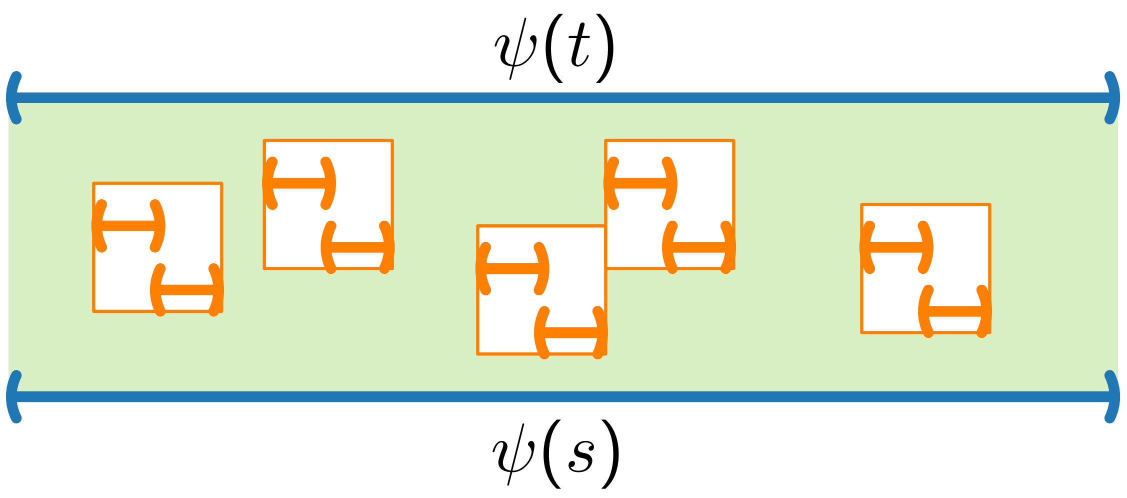

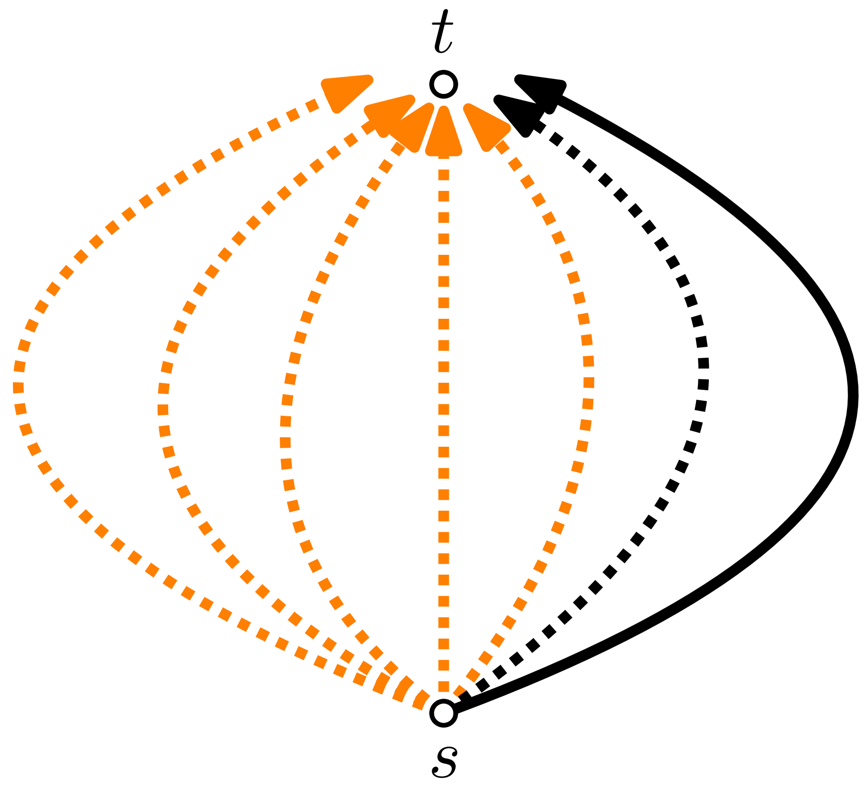

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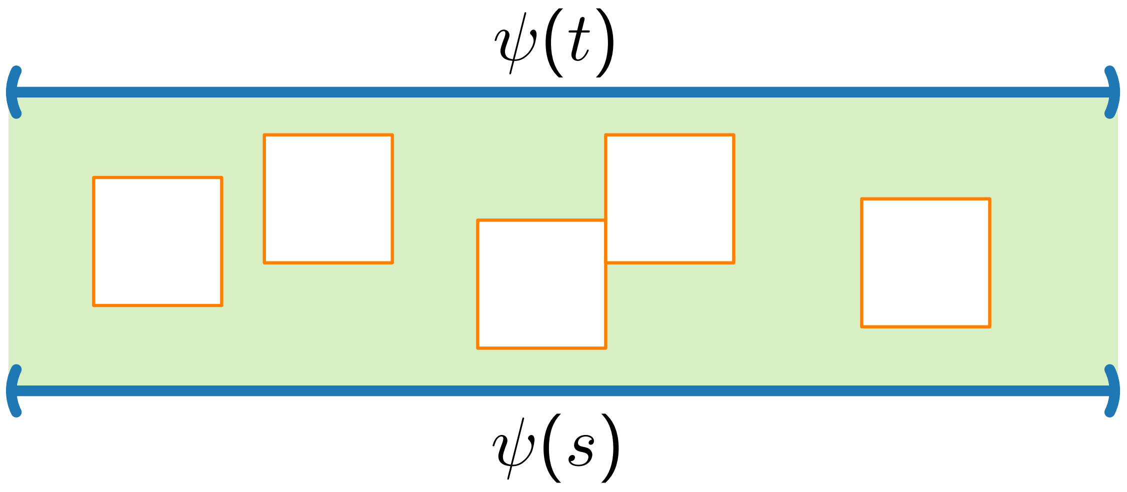

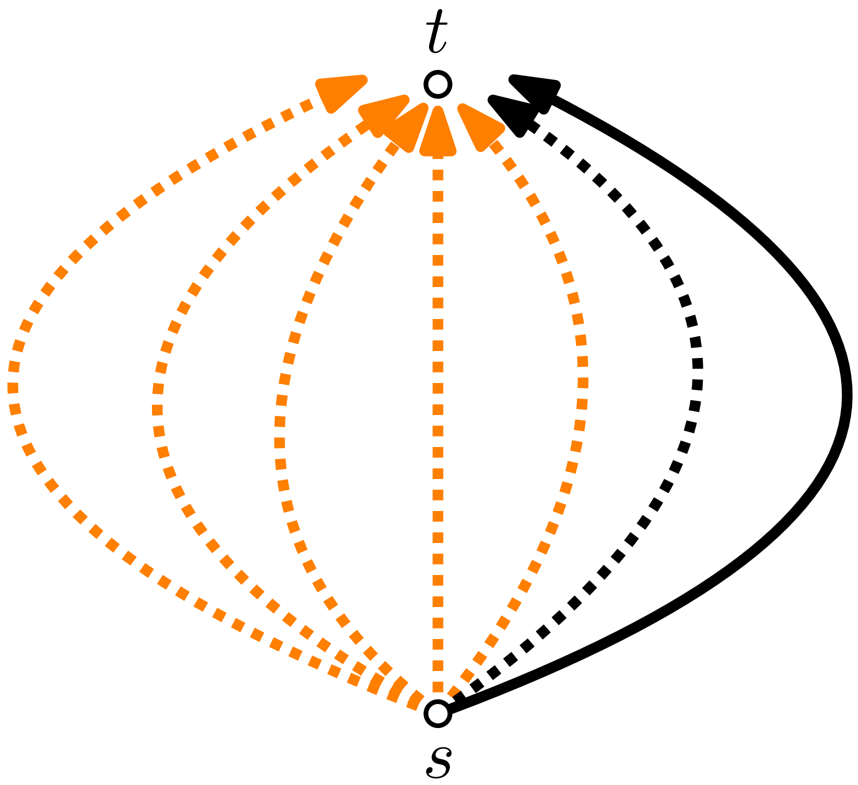

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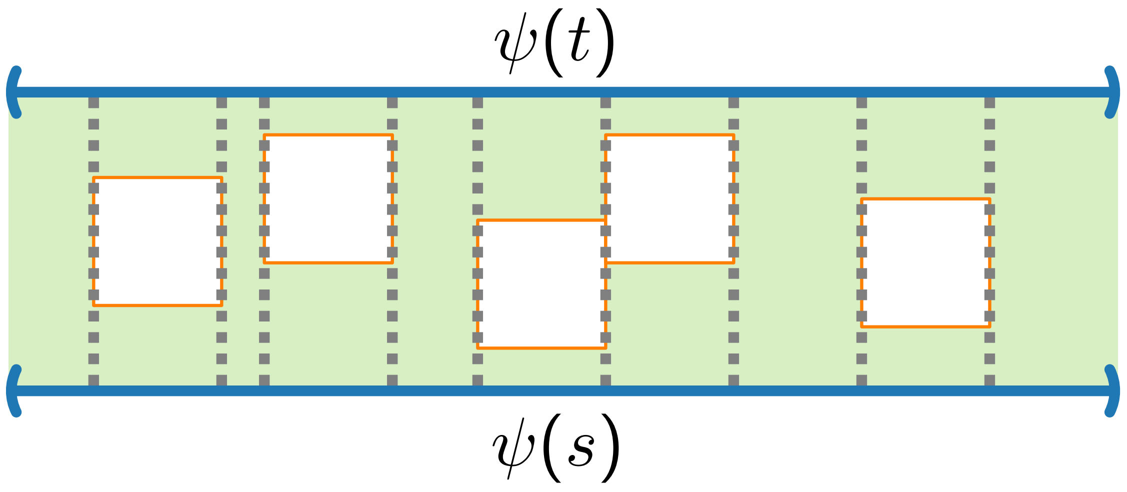

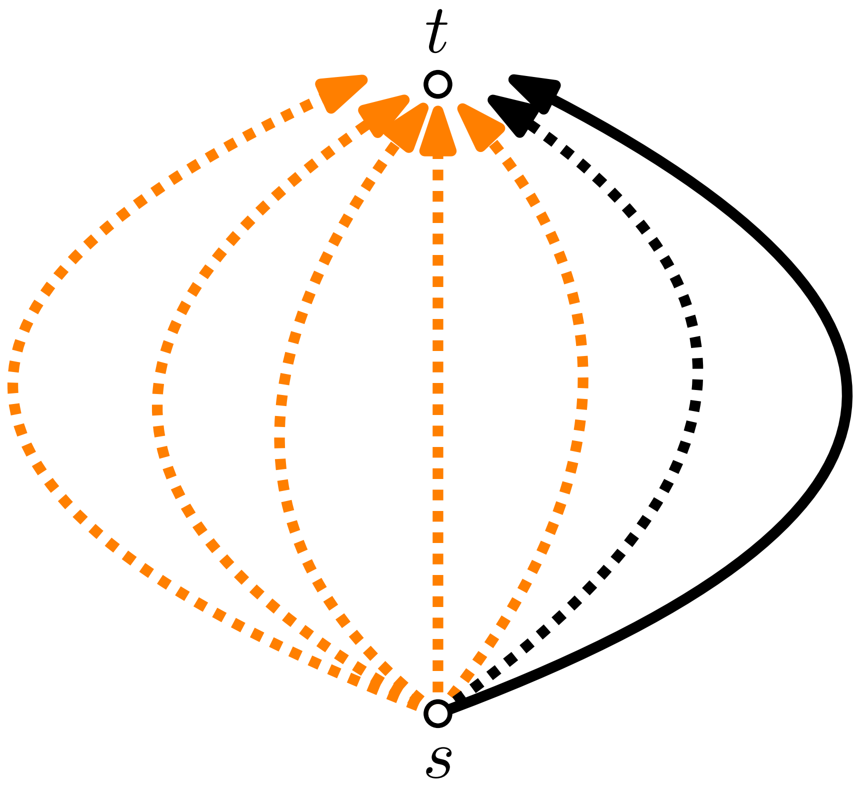

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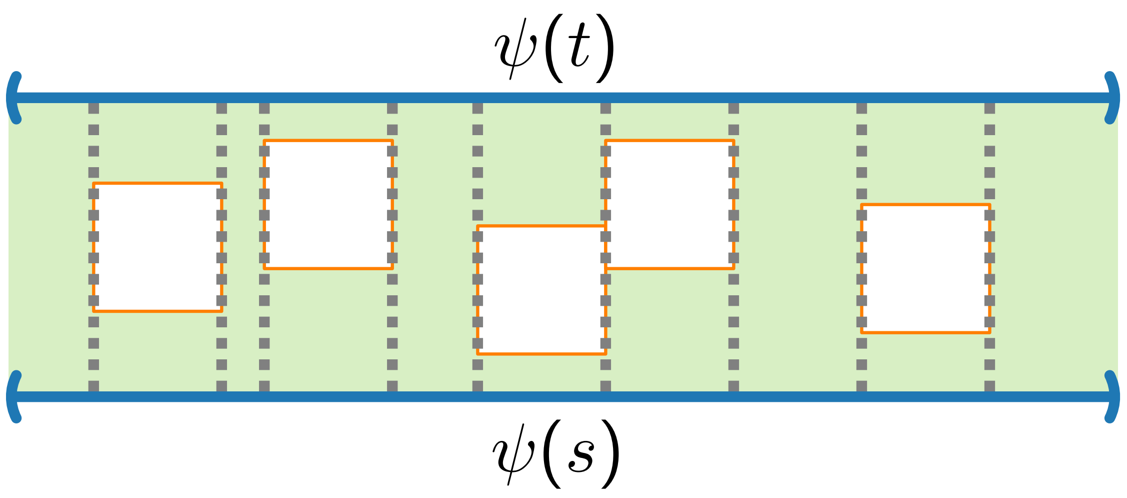
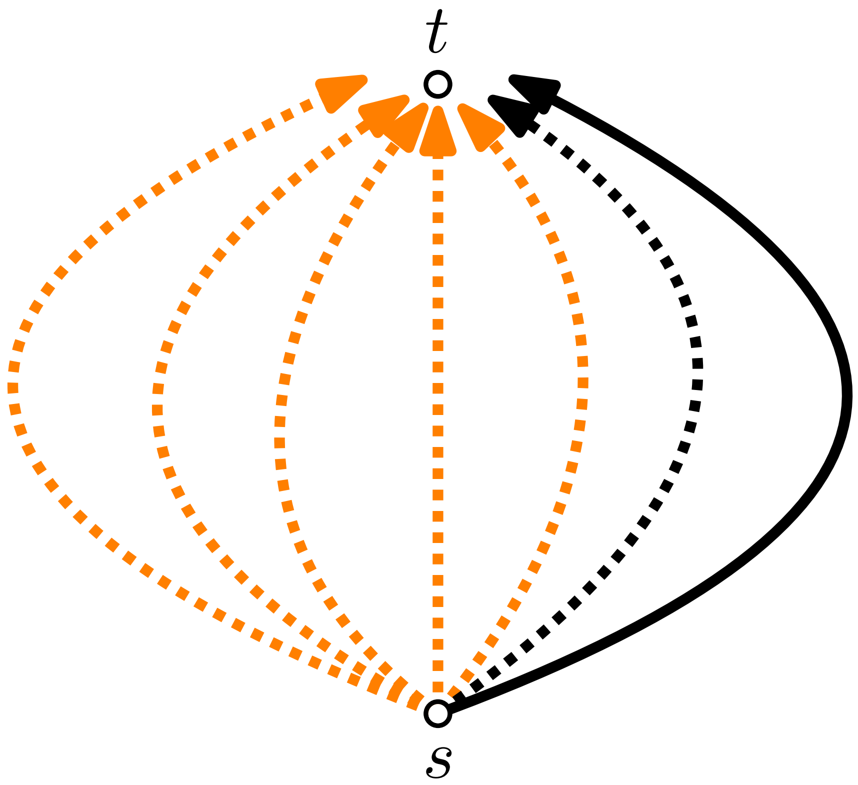
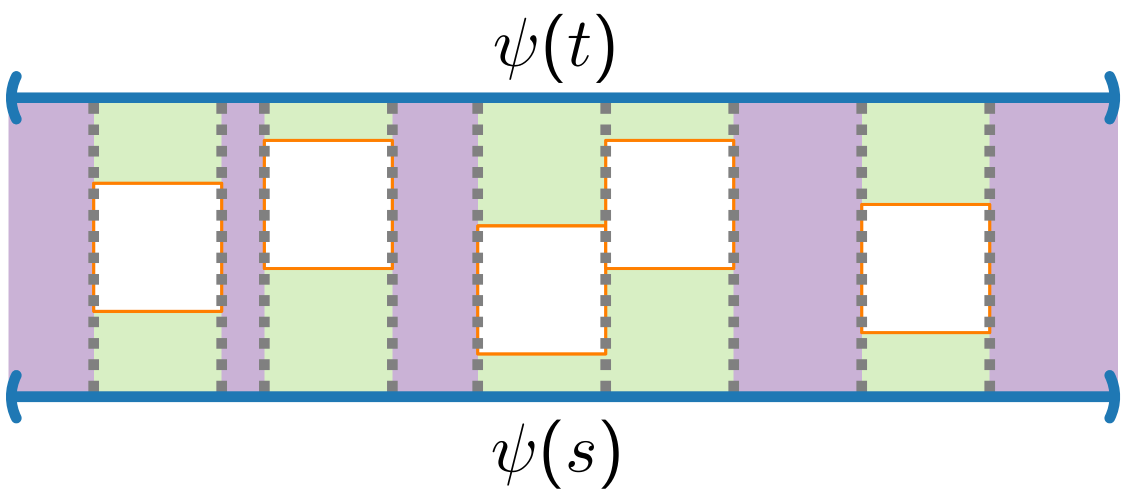
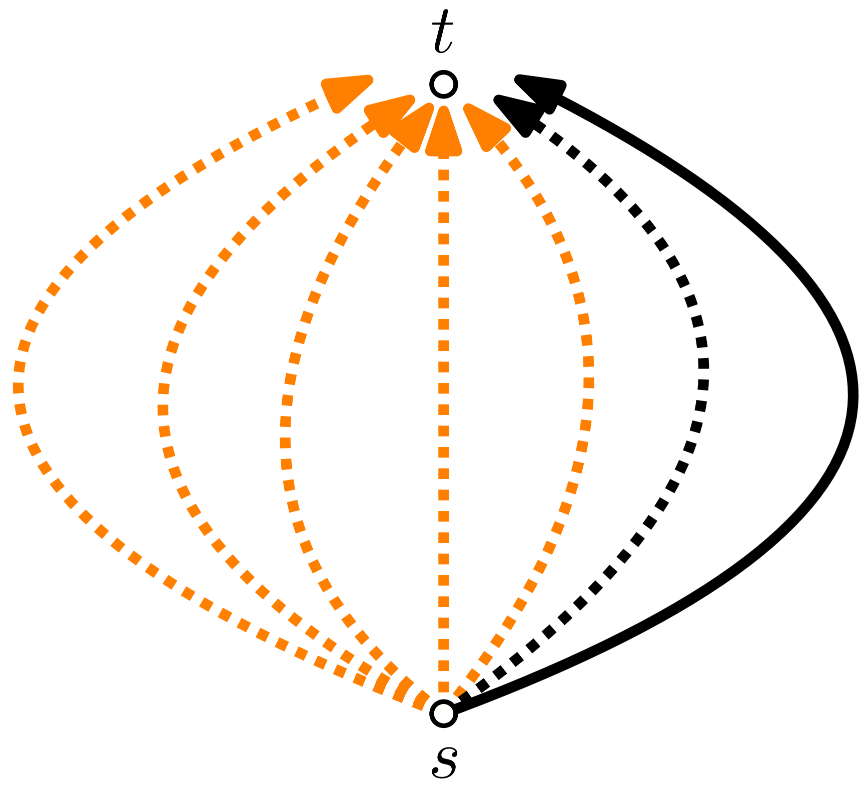




- Children of **P**-node with prescribed bars occur in given left-to-right order

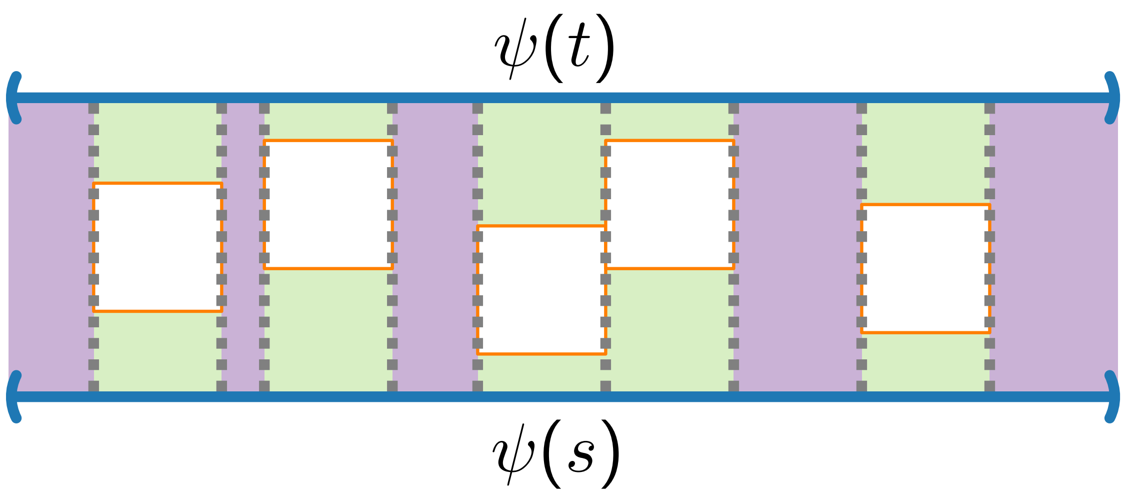
P-Nodes

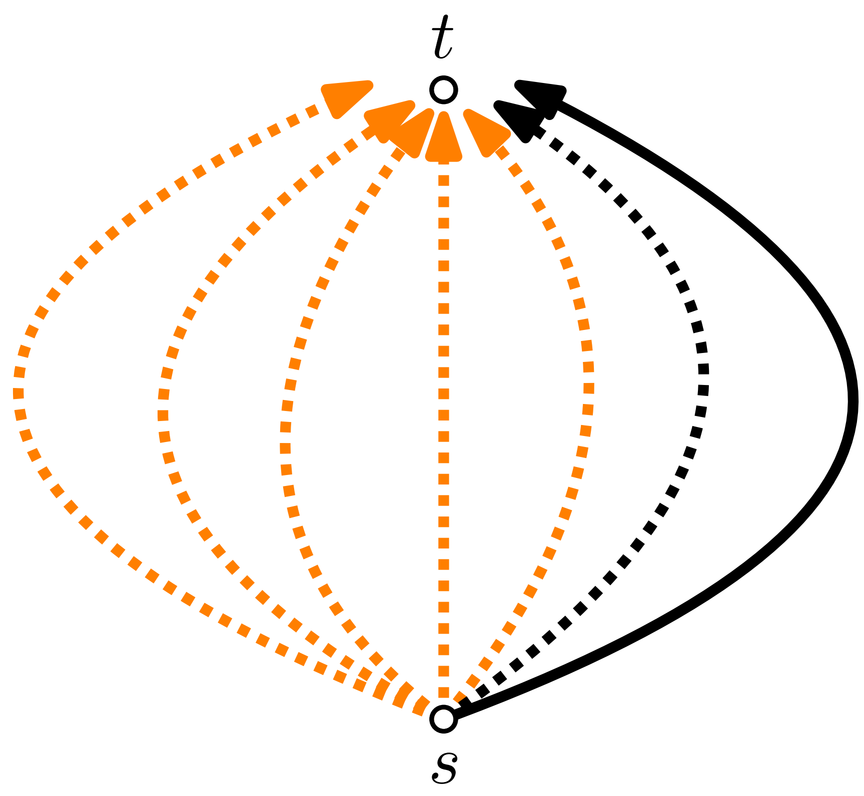




- Children of **P**-node with prescribed bars occur in given left-to-right order
- But there might be some gaps. . .

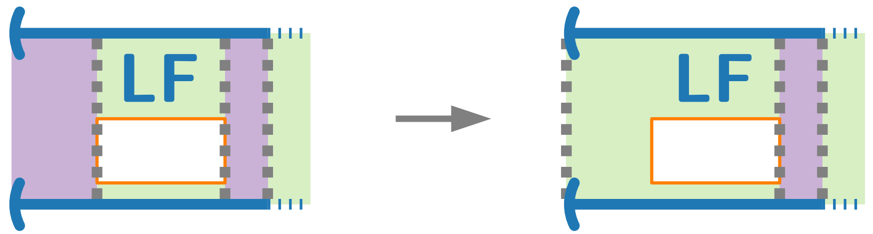
P-Nodes



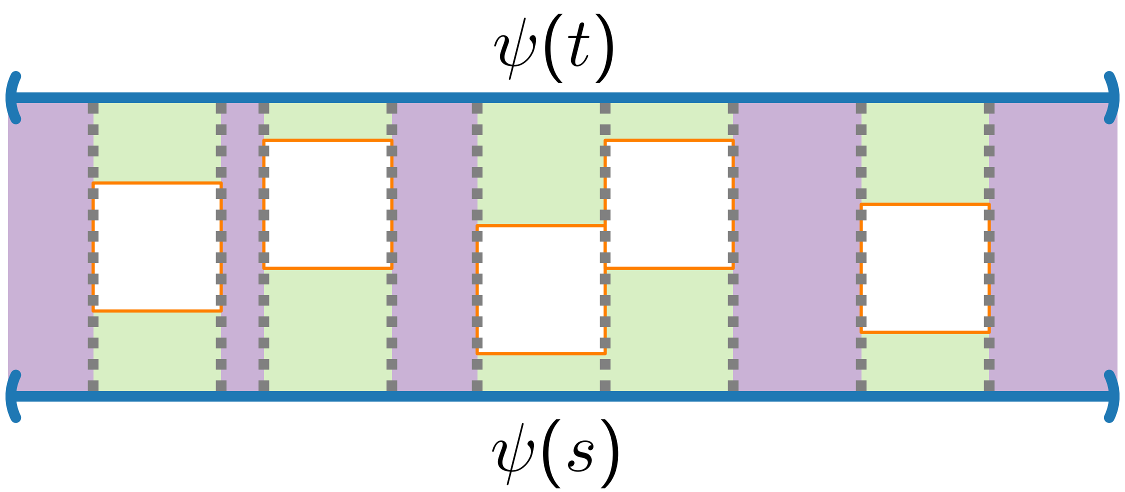


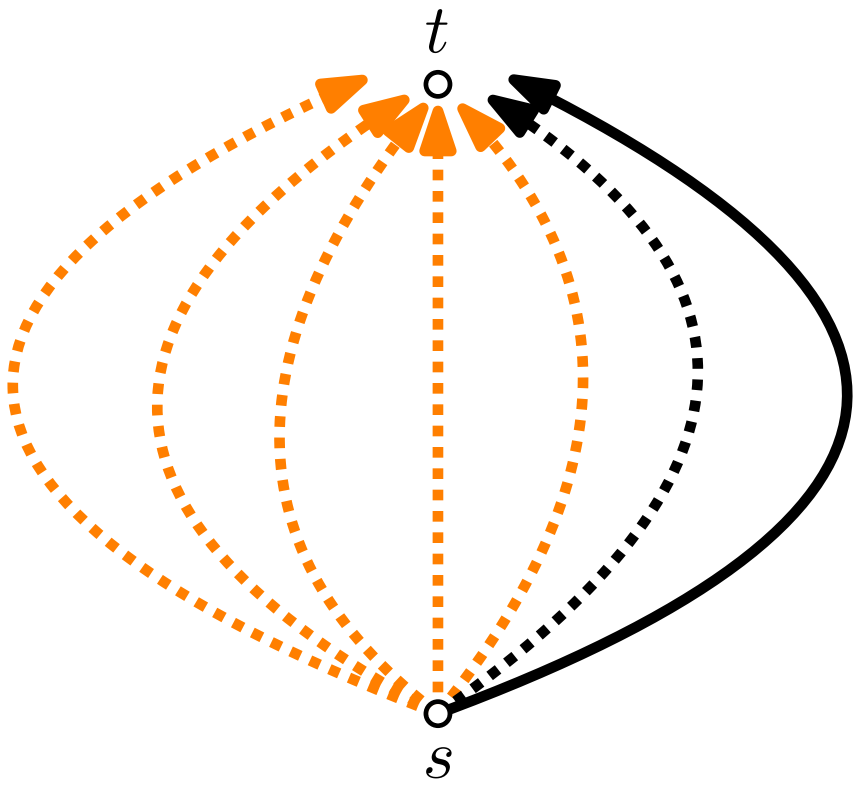
- Children of **P**-node with prescribed bars occur in given left-to-right order
- But there might be some gaps. . .

Idea.
Greedily *fill* the gaps by preferring to "stretch" the children with prescribed bars.



P-Nodes





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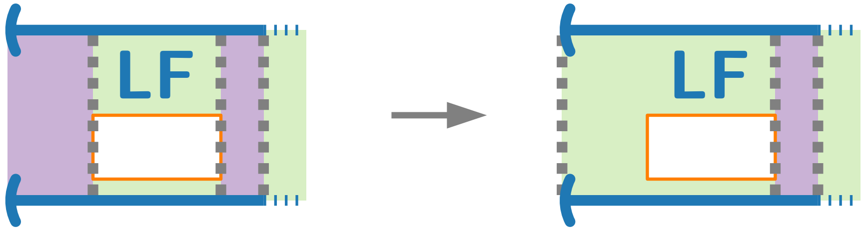
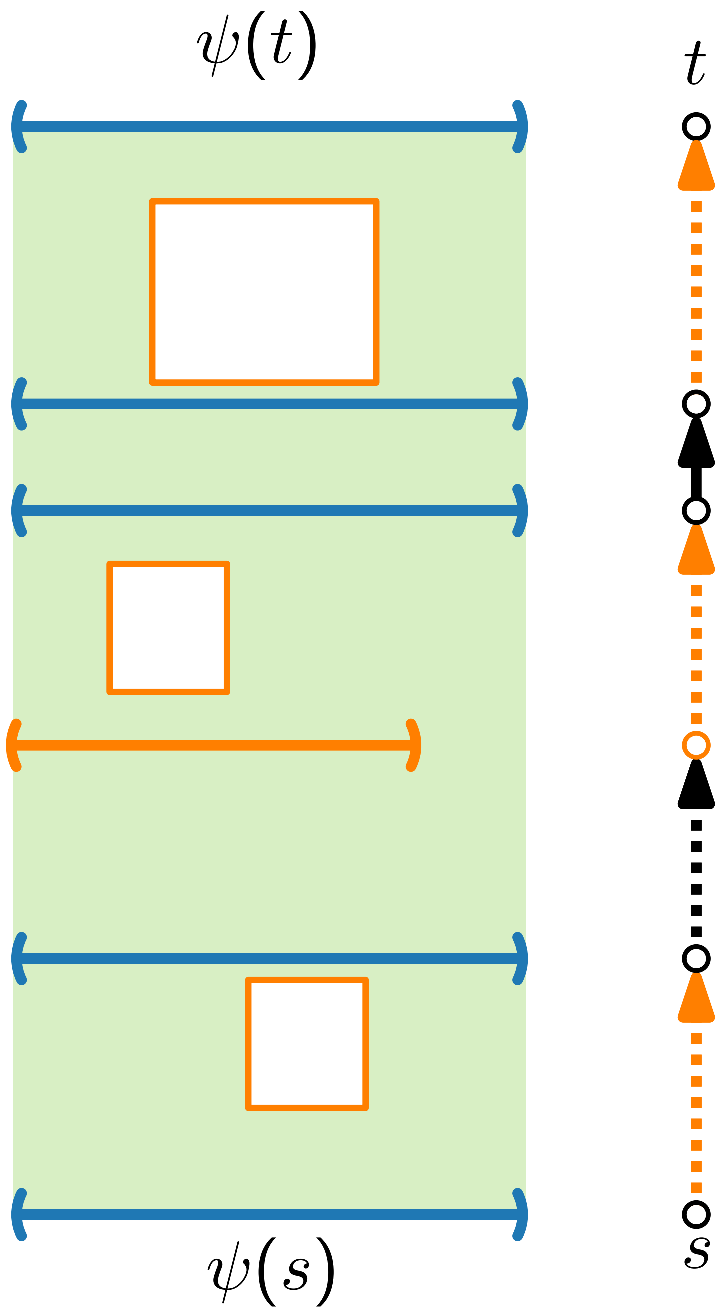
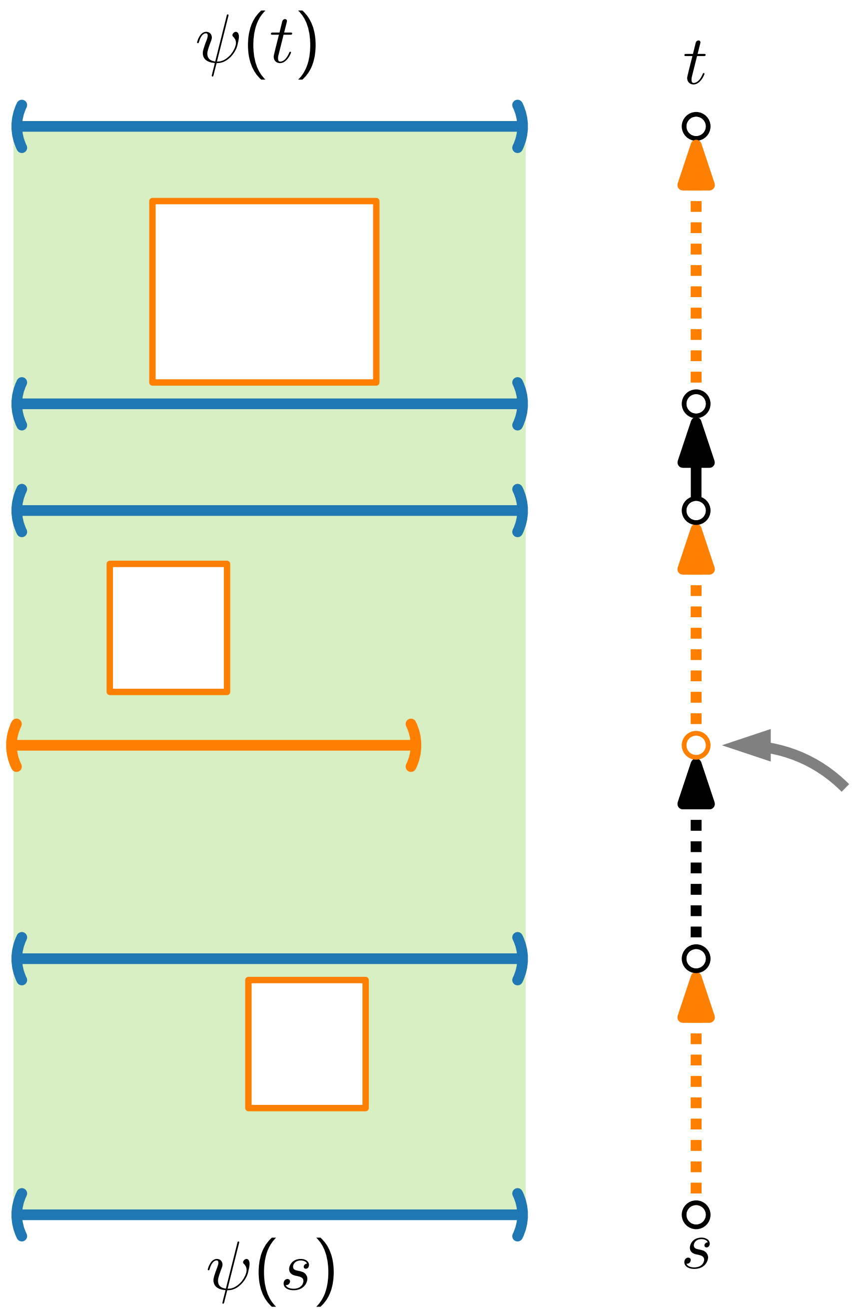


Outcome.
After processing, we must know the valid types for the corresponding subgraphs.

S-Nodes

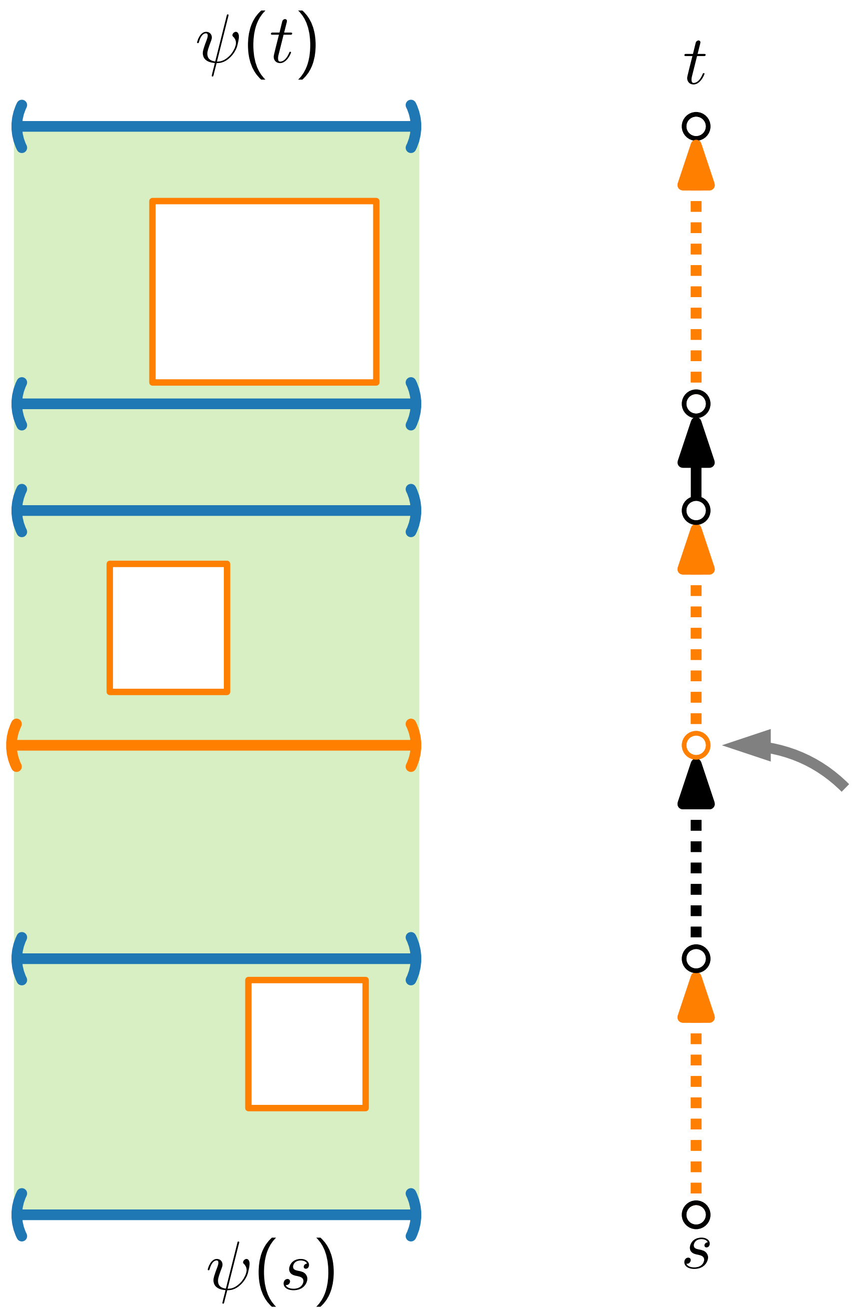


S-Nodes



This fixed vertex means we can only make a Fixed-Fixed representation!

S-Nodes



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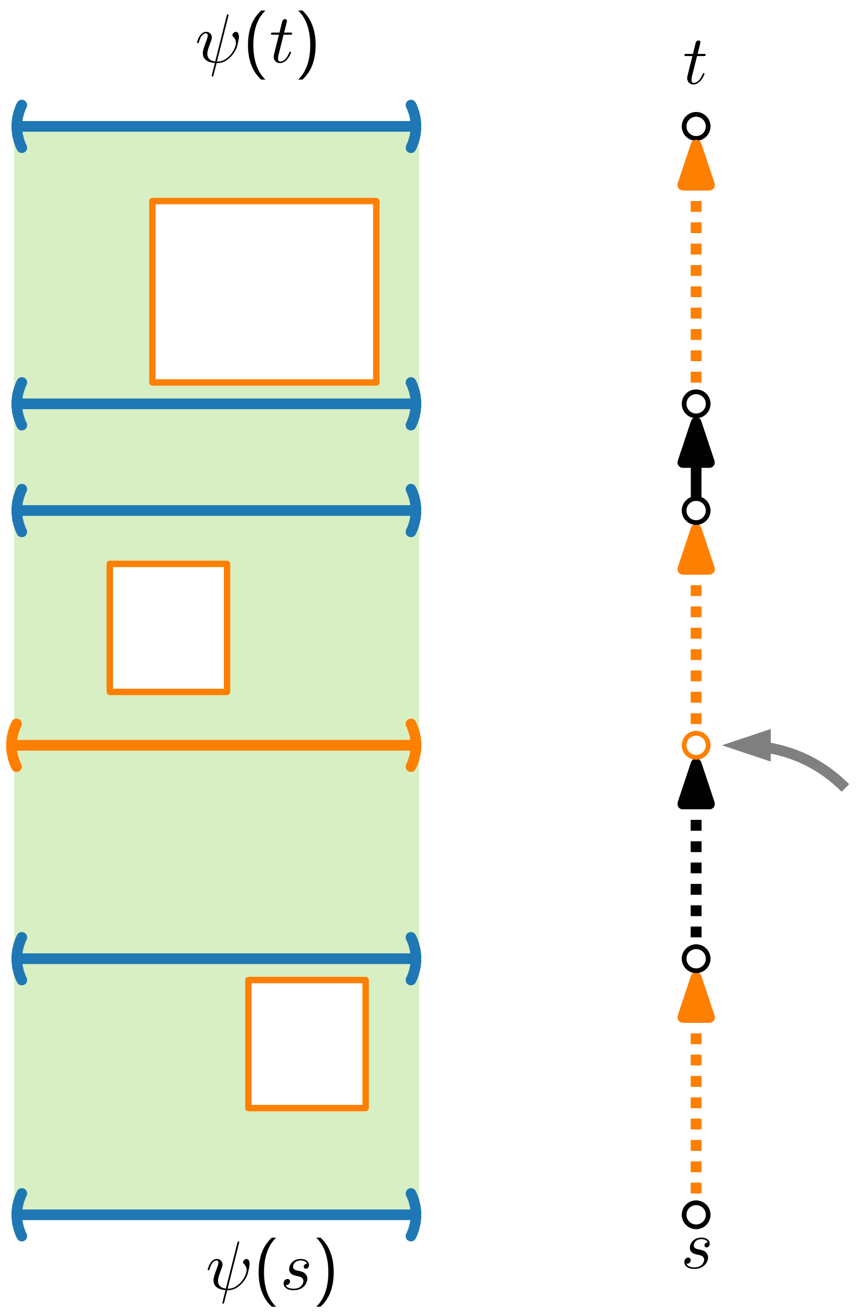

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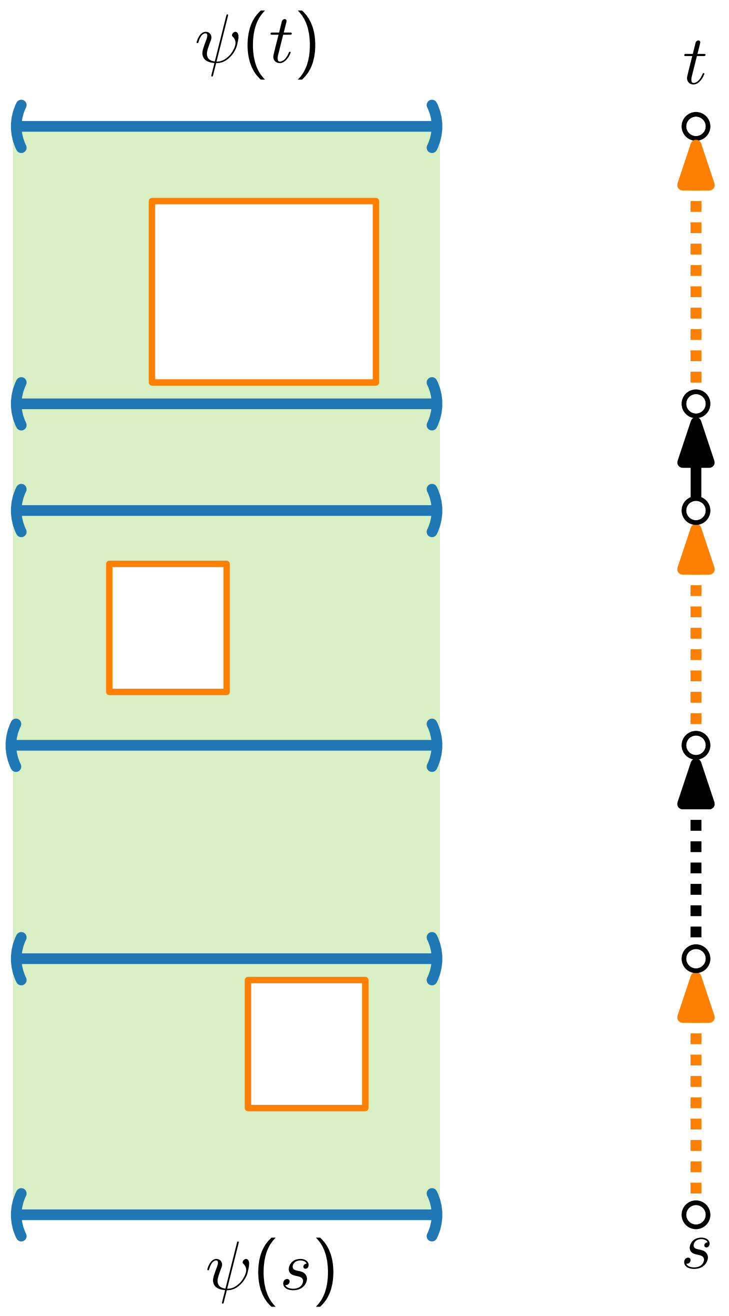
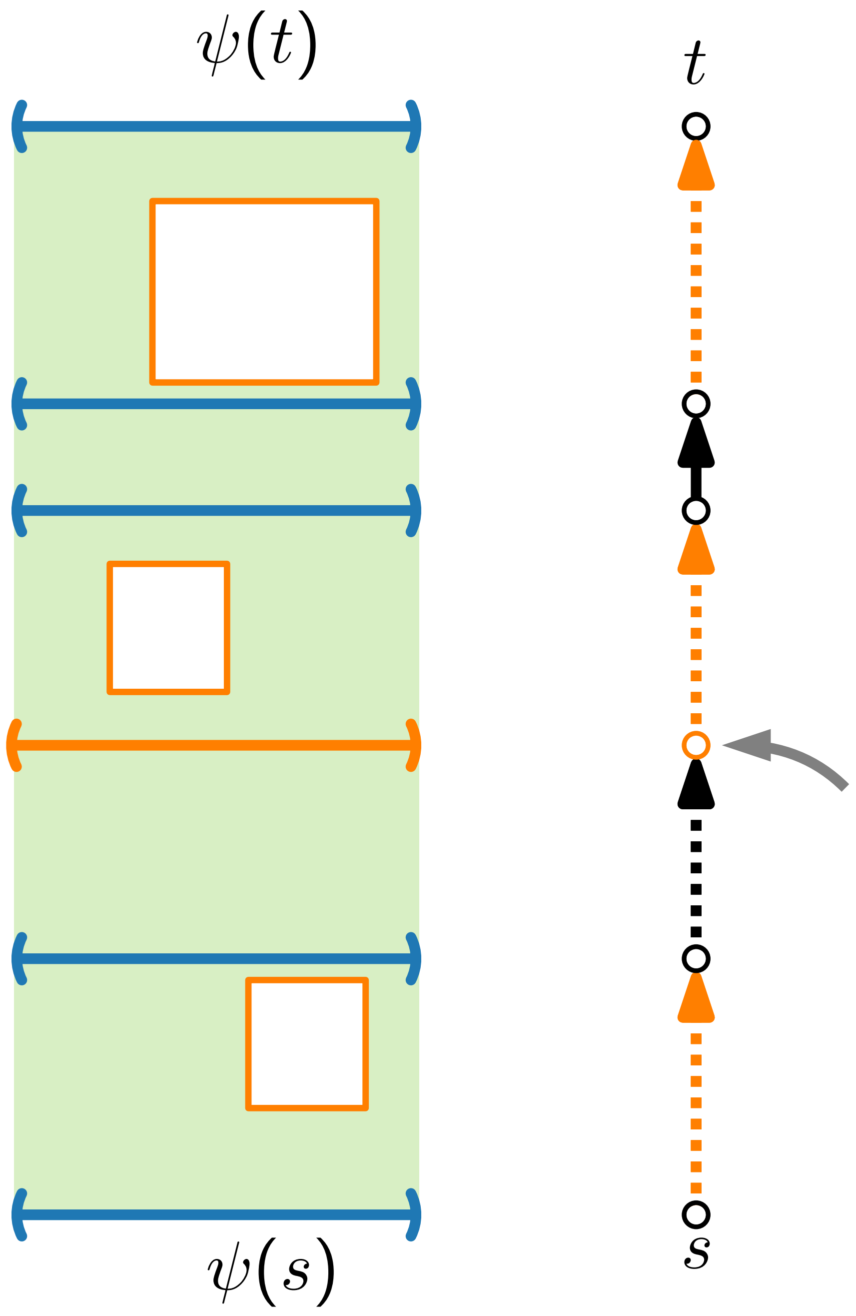
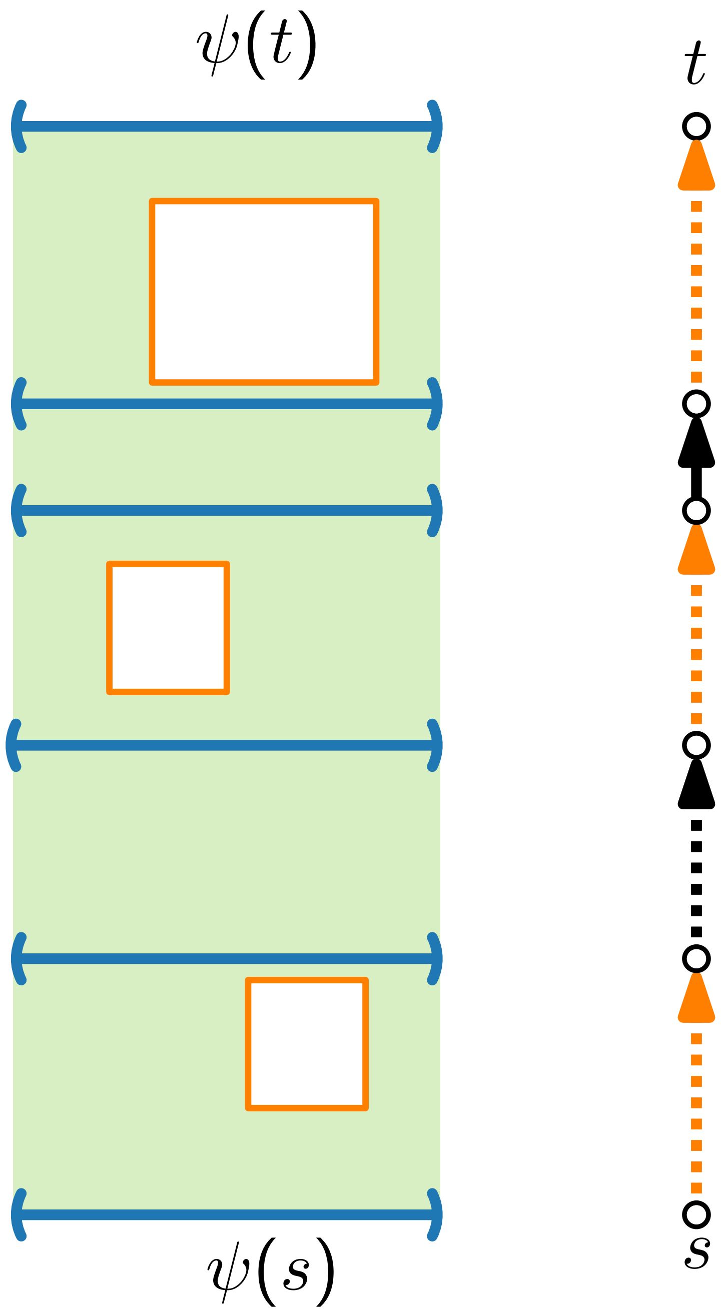


S-Nodes

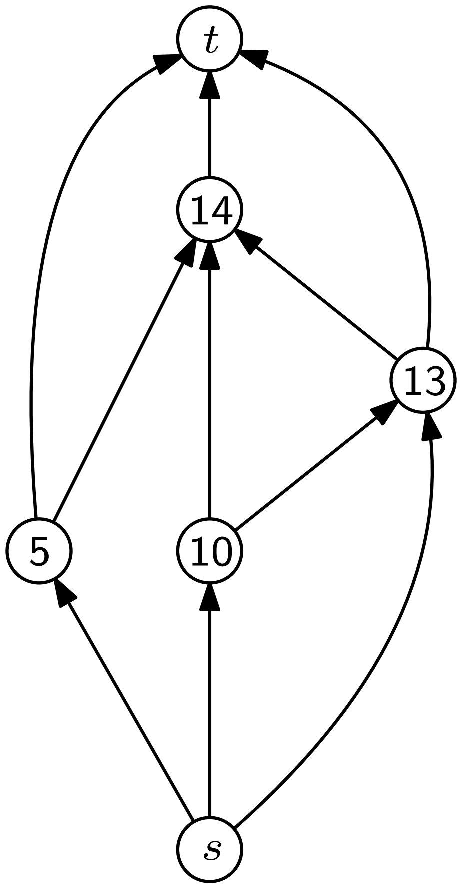


Here we have a chance to make all (**LL, FL, LF, FF**) types.

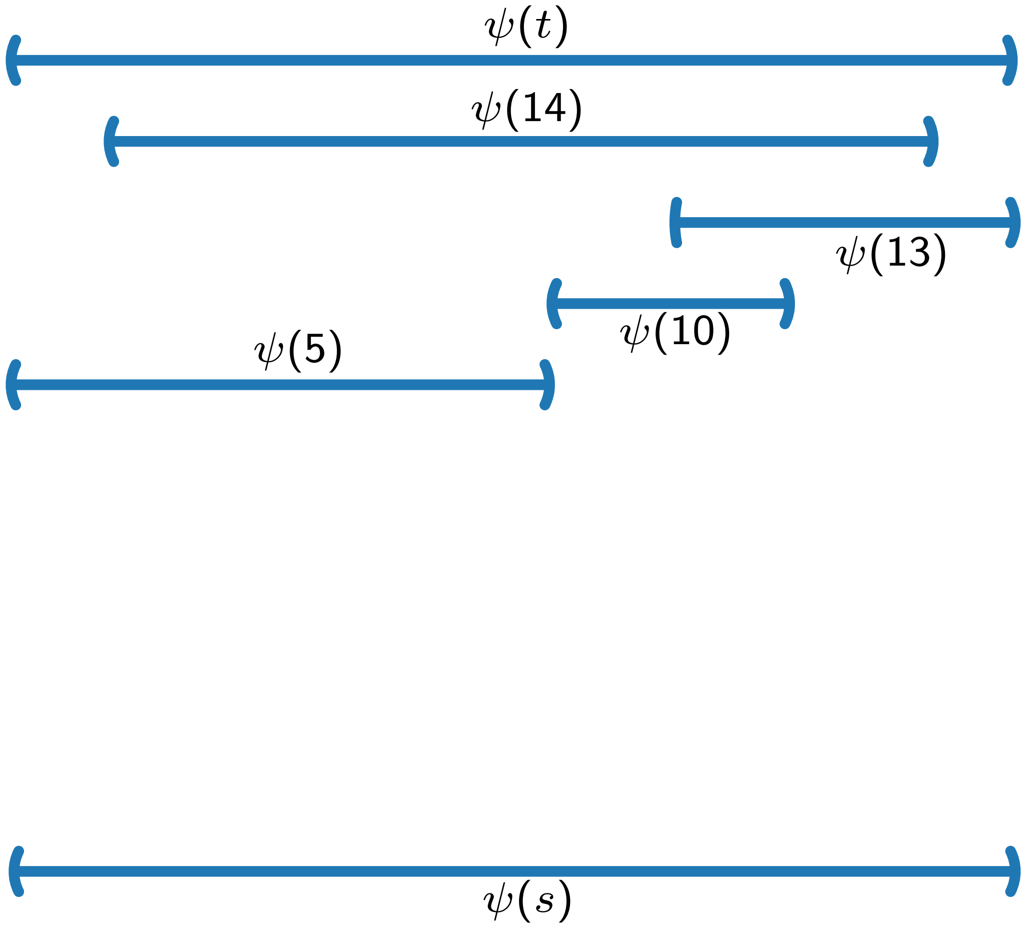
This fixed vertex means we can only make a Fixed-Fixed representation!

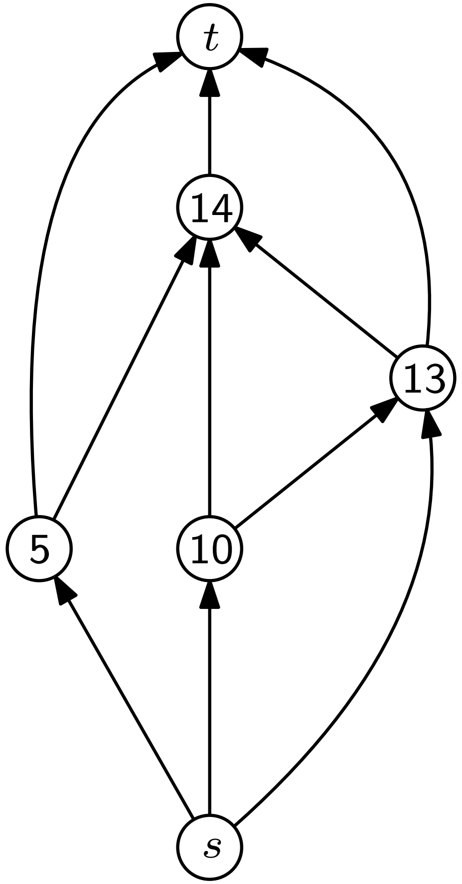


R-Nodes

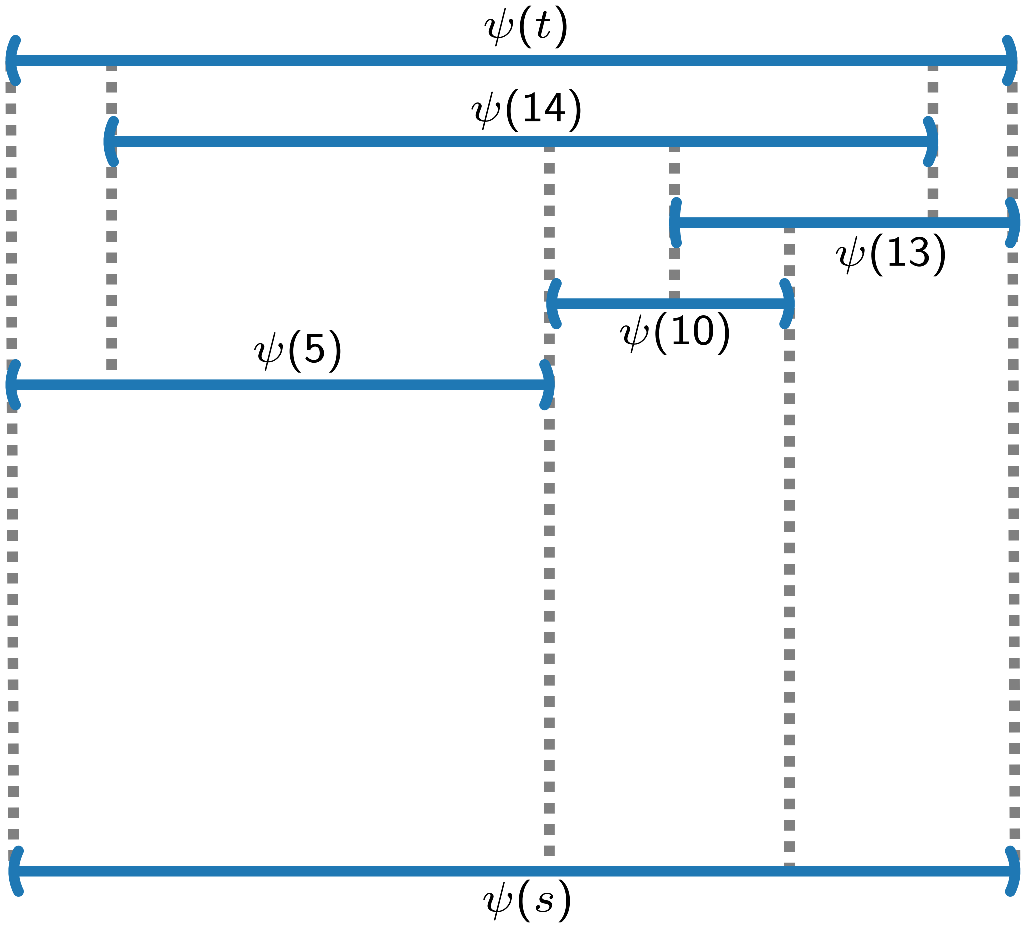


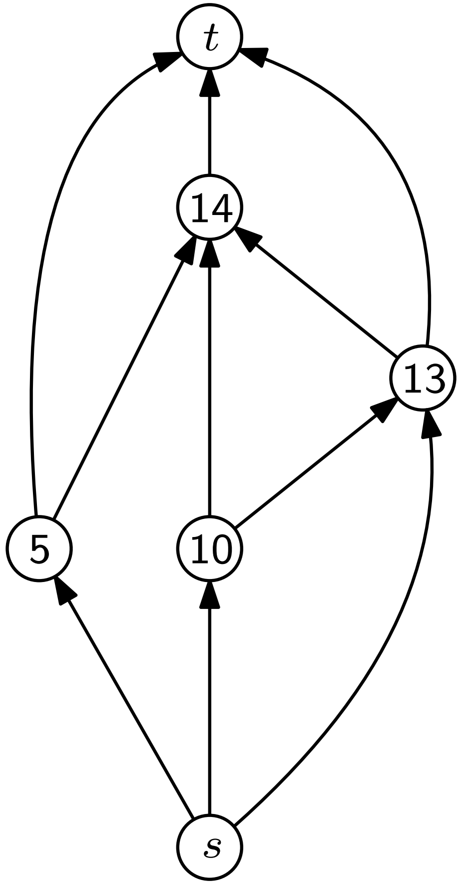
R-Nodes





R-Nodes





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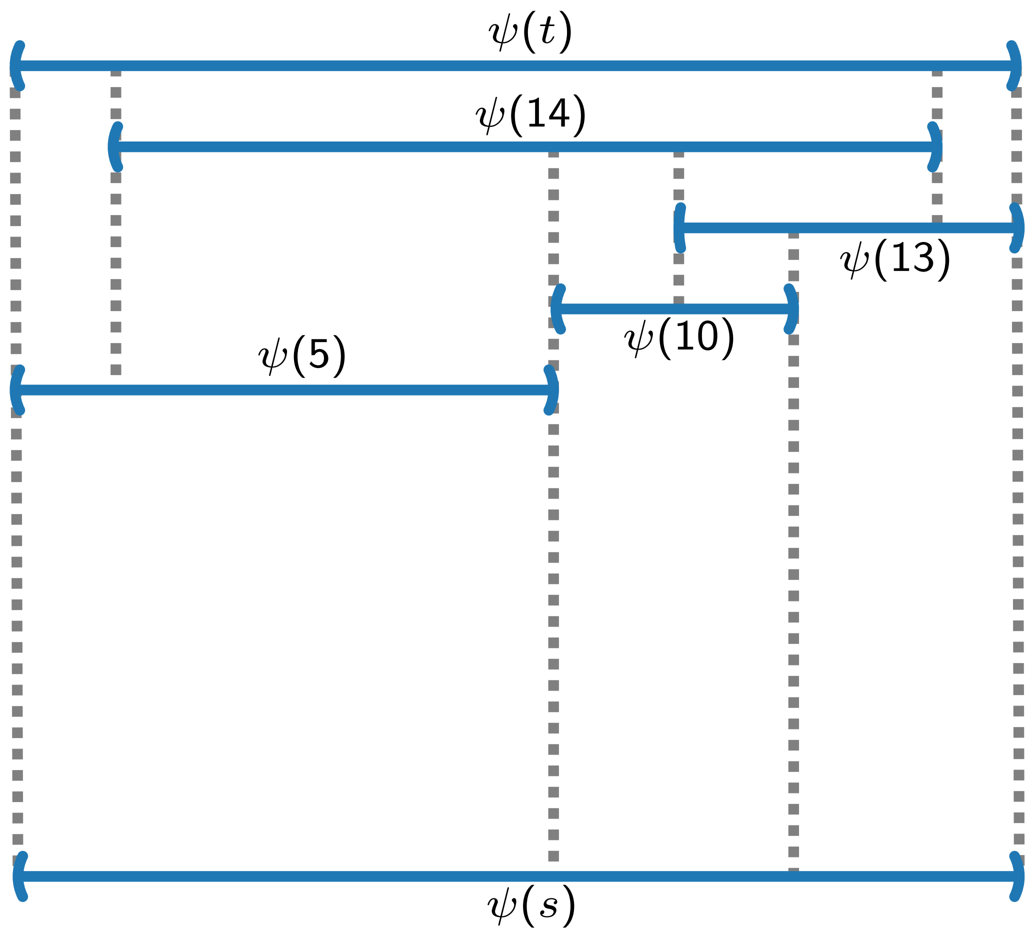
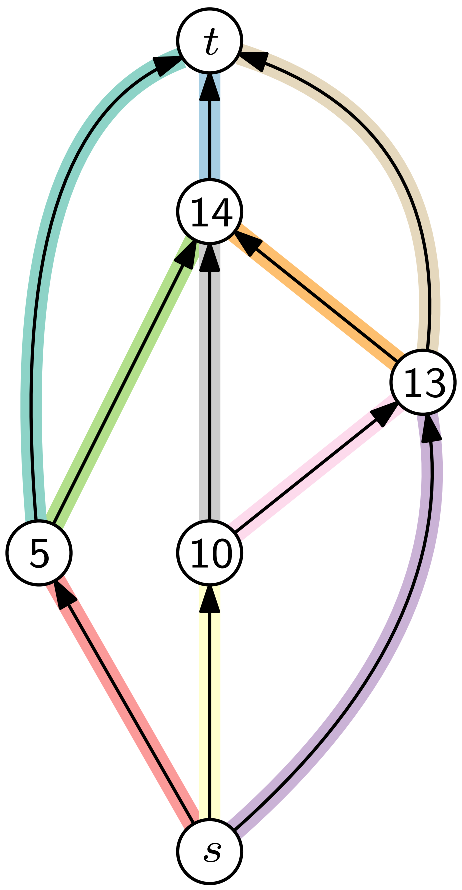
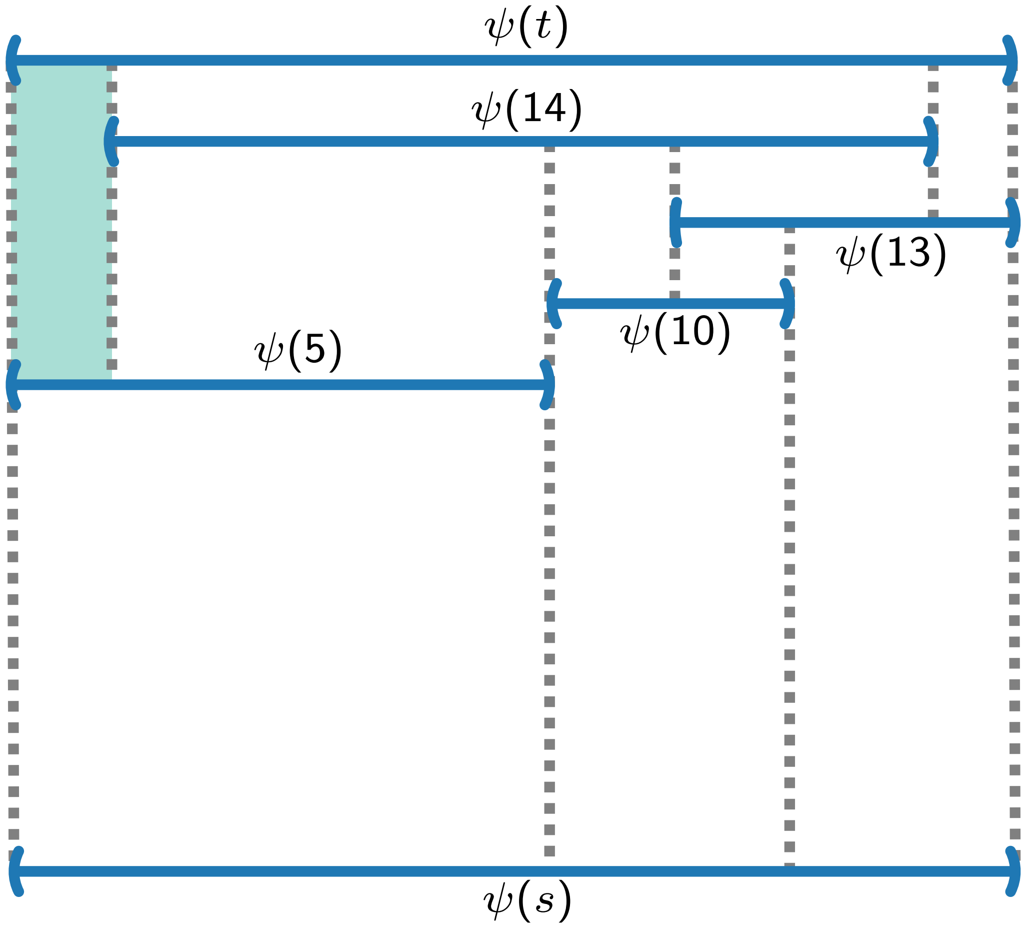
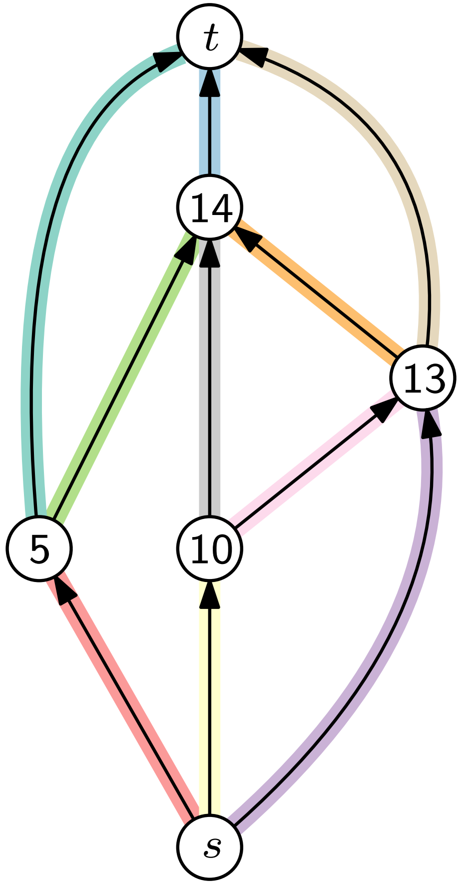




R-Nodes





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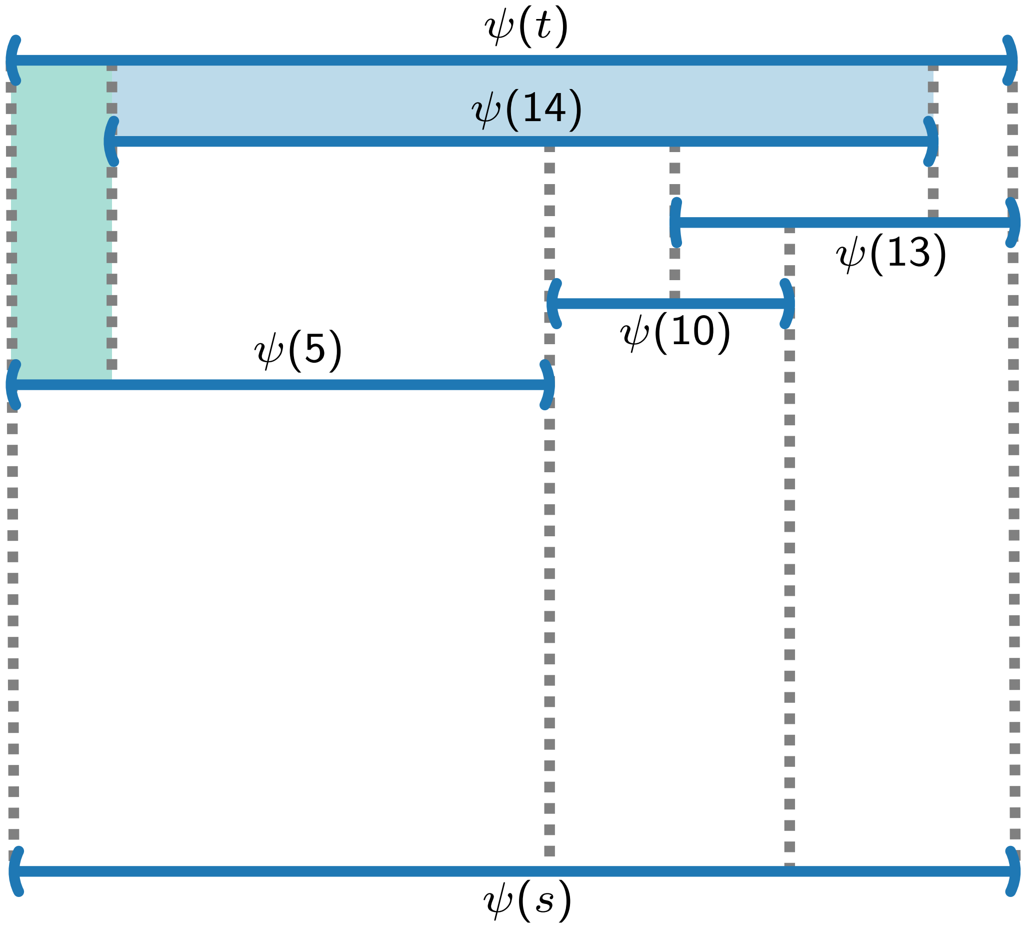
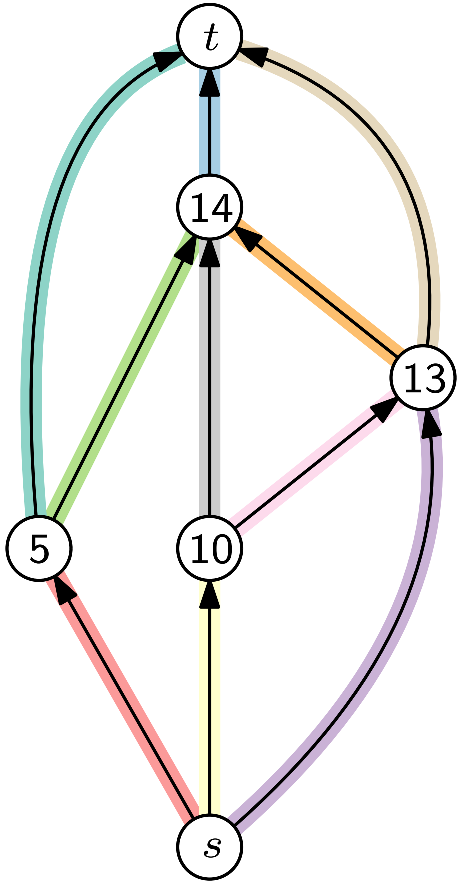
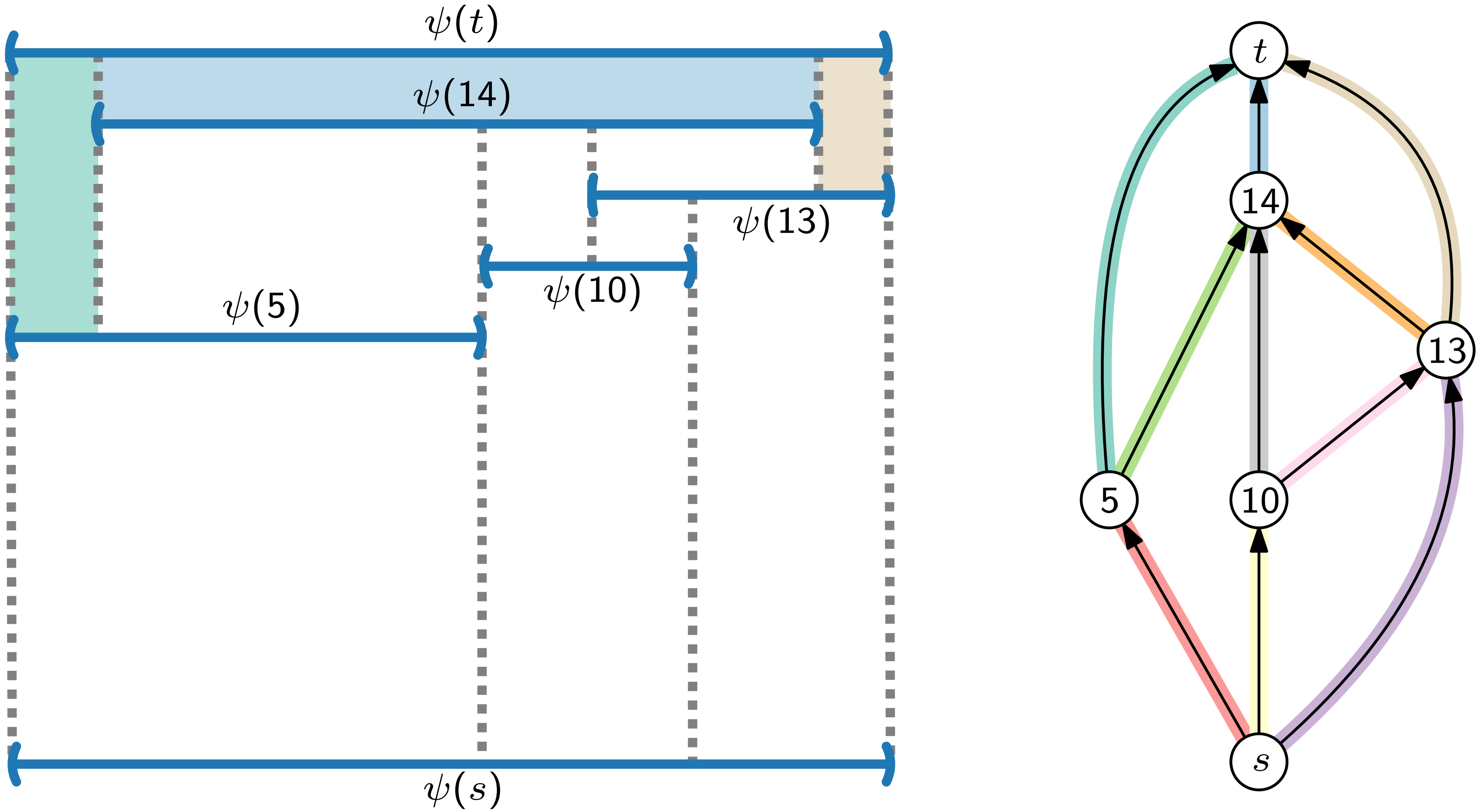
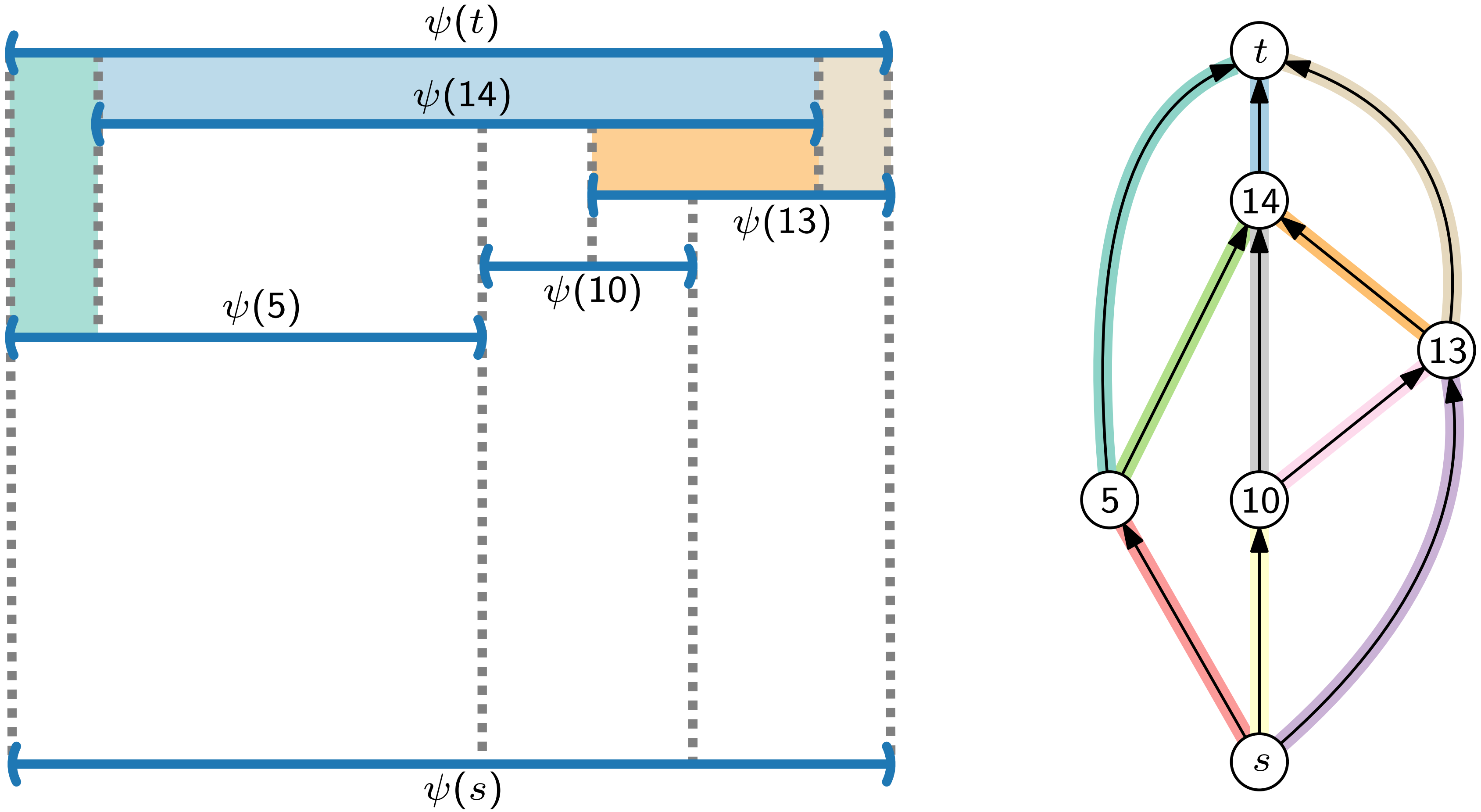




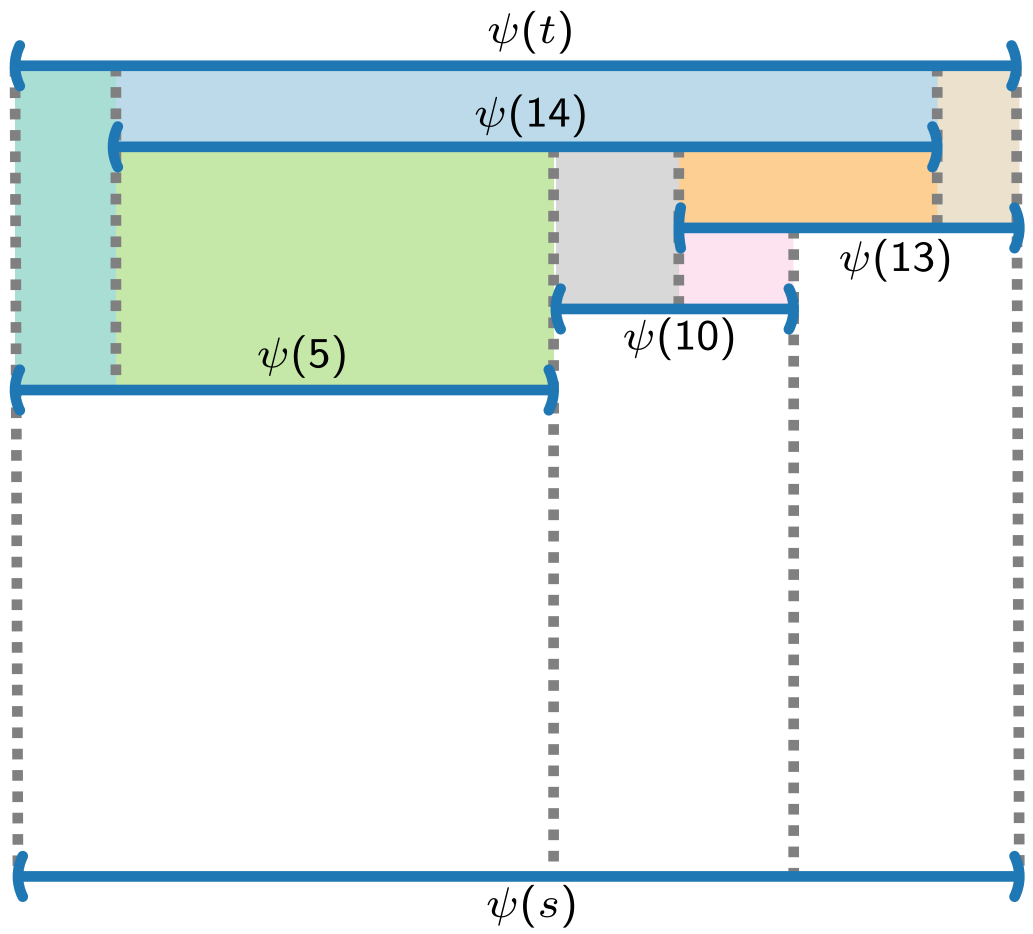
R-Nodes

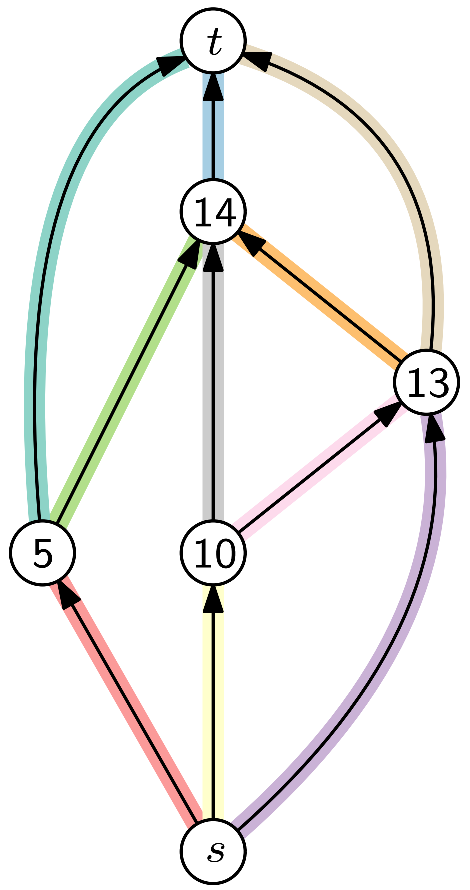


R-Nodes

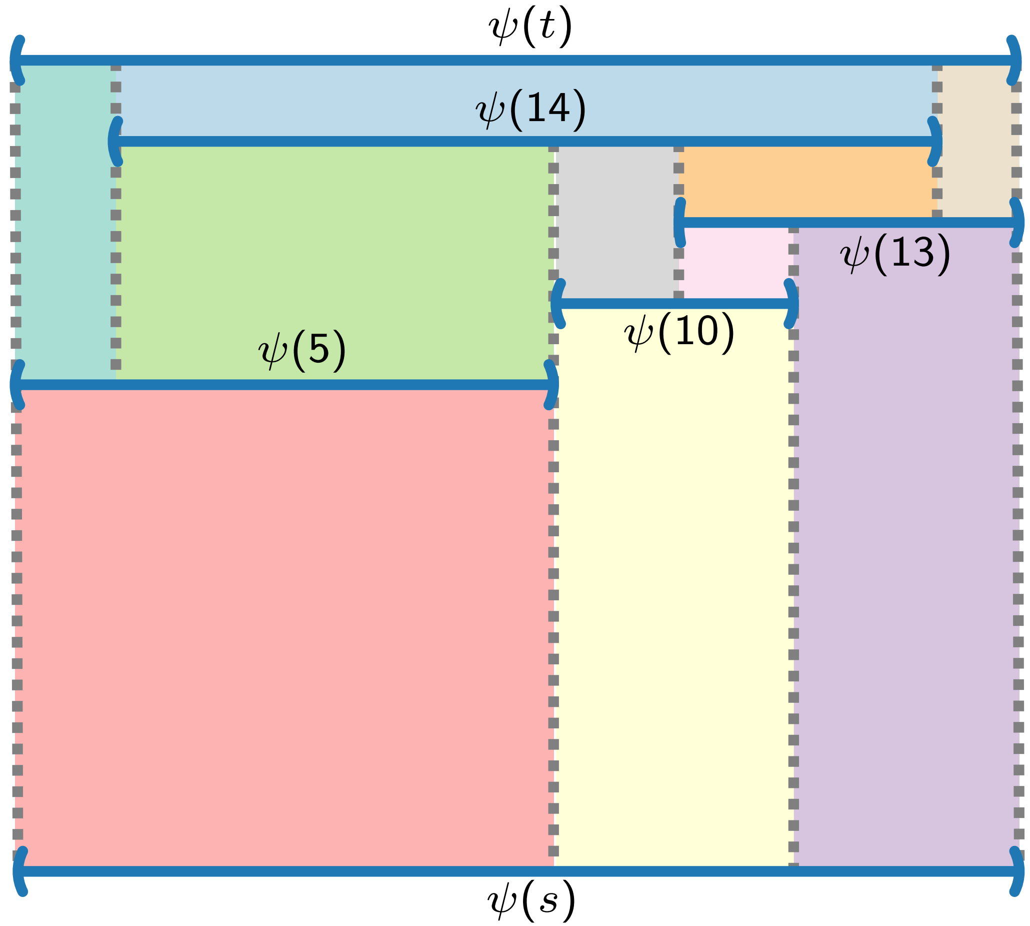


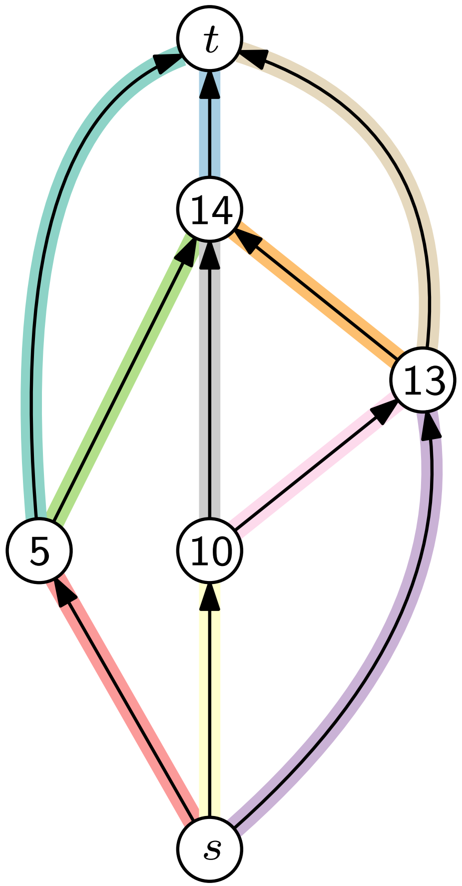
R-Nodes



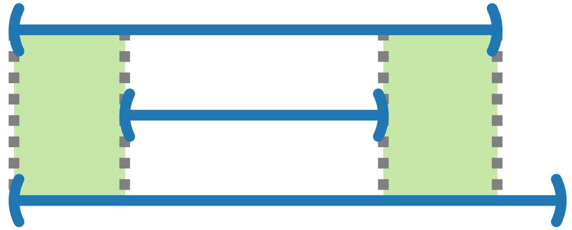


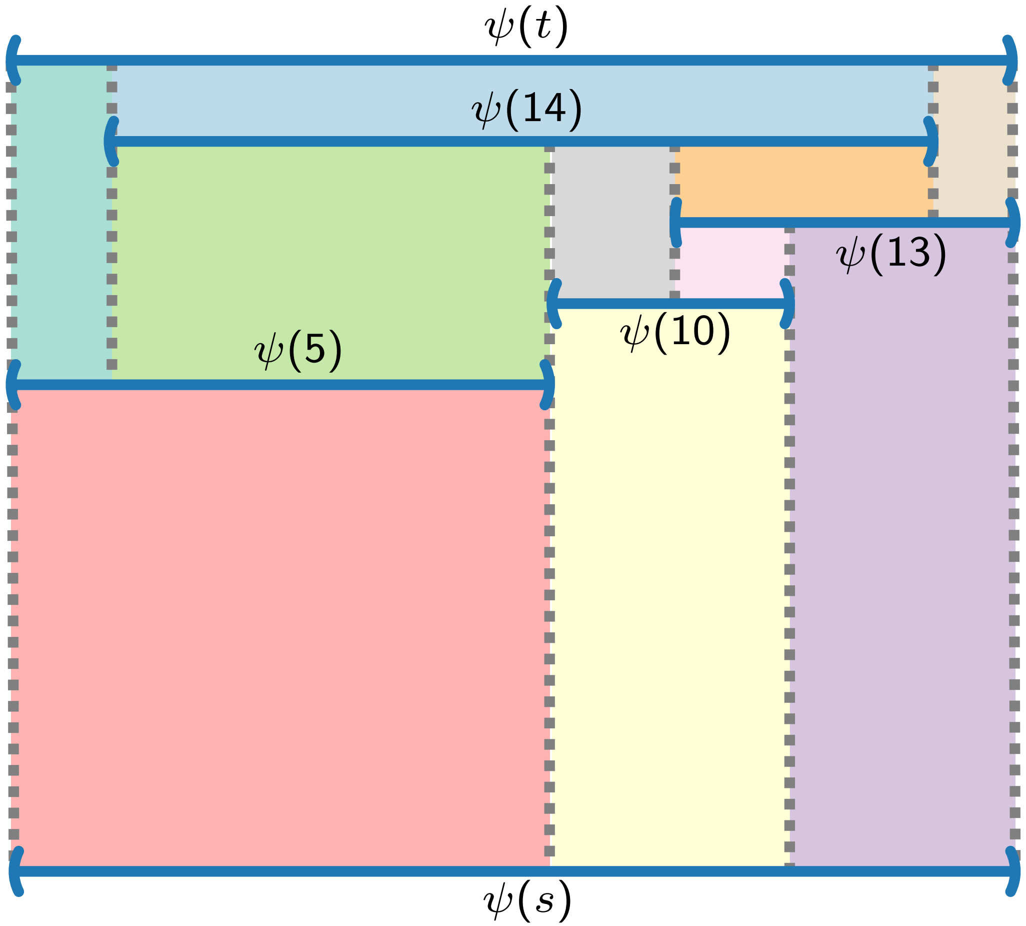
R-Nodes

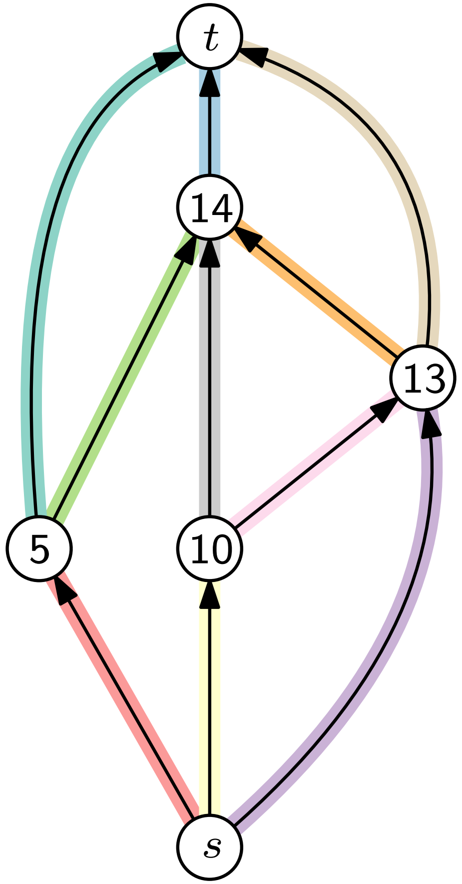




R-Nodes







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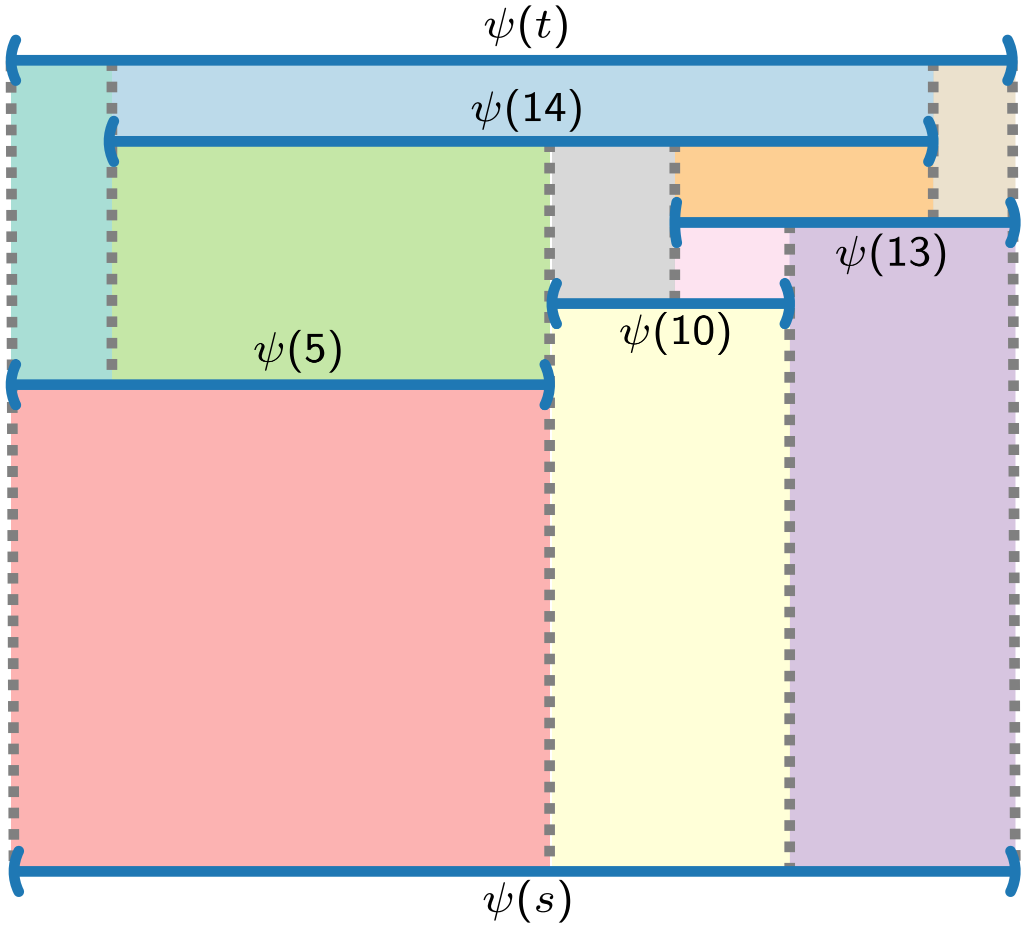

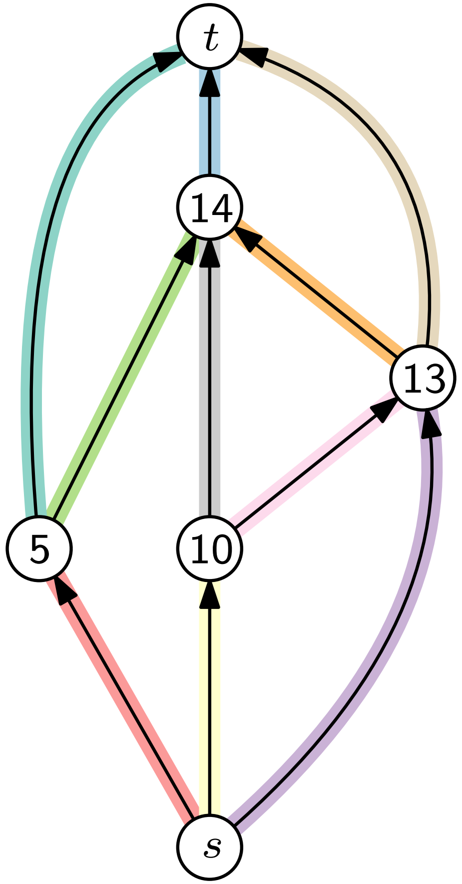

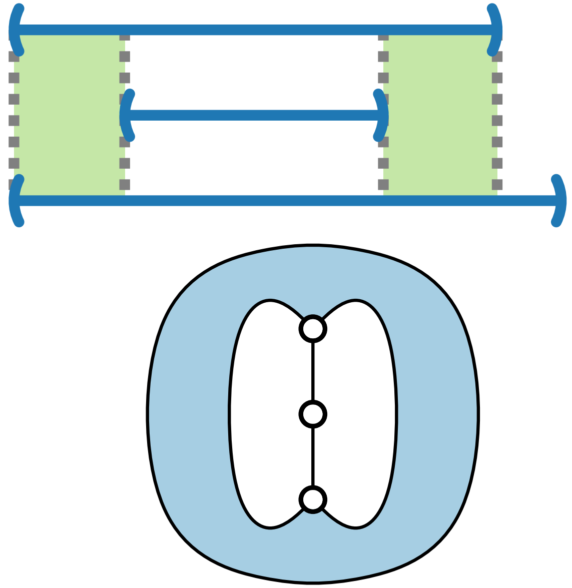

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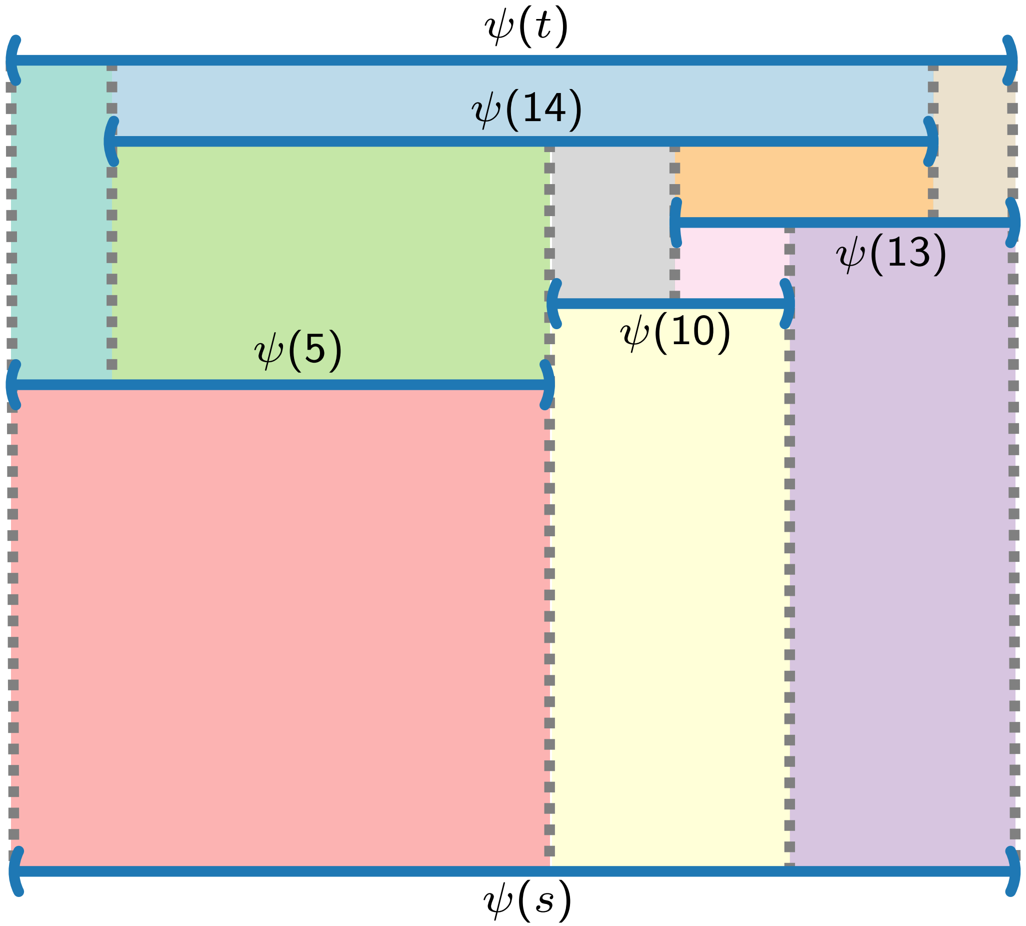
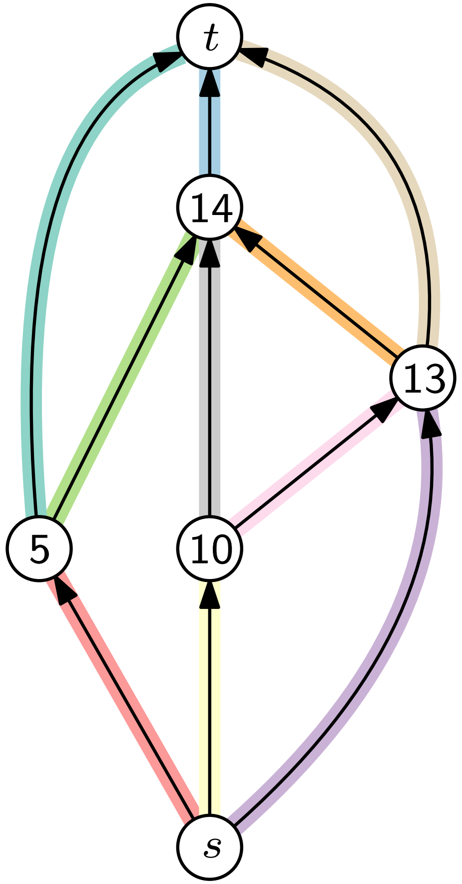
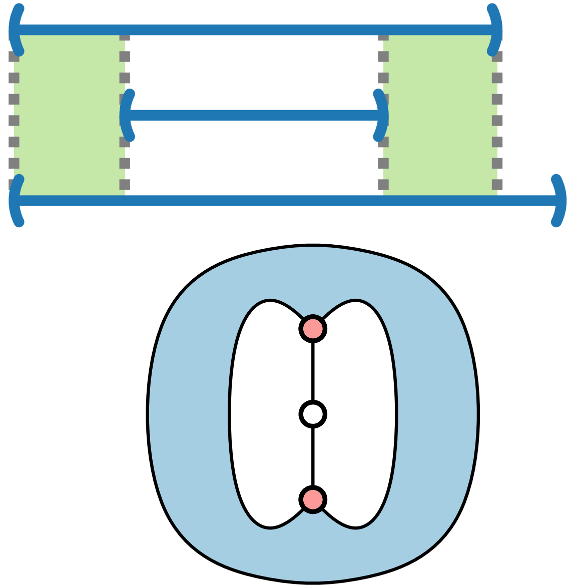
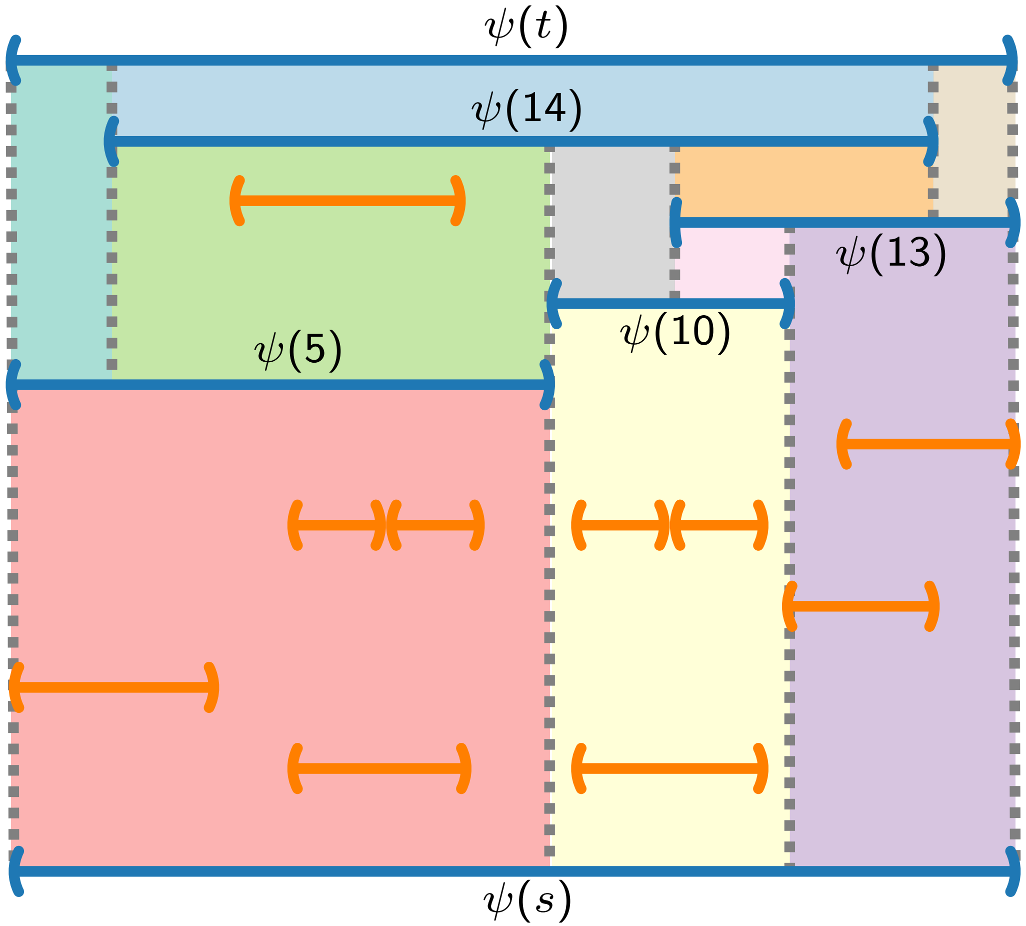


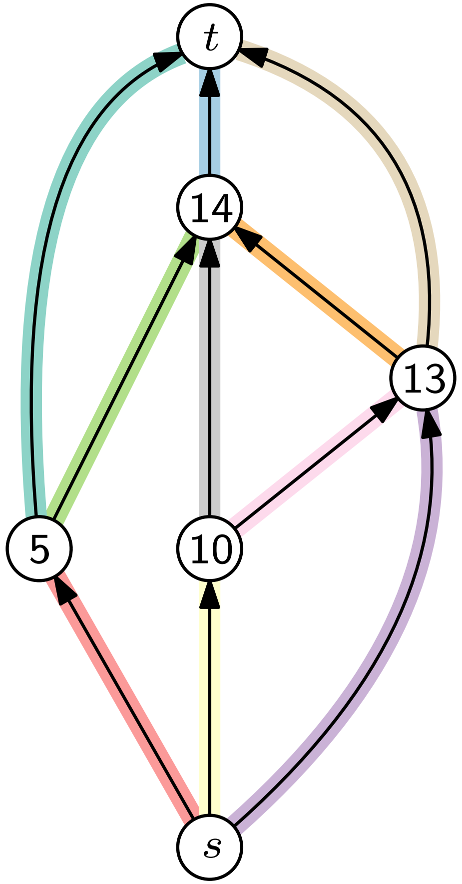


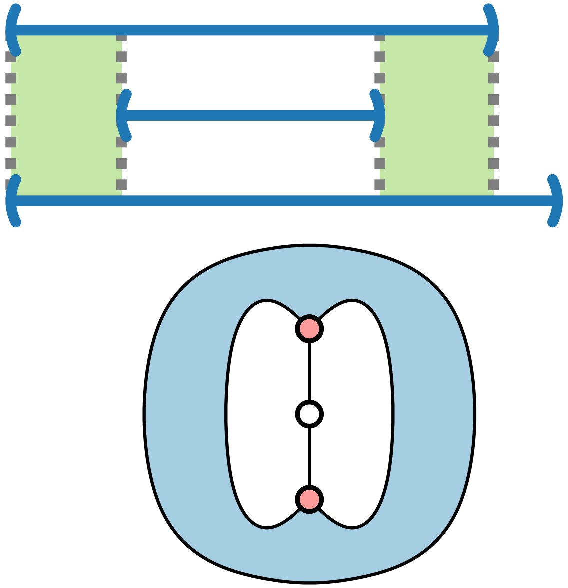


separation pair!

R-Nodes



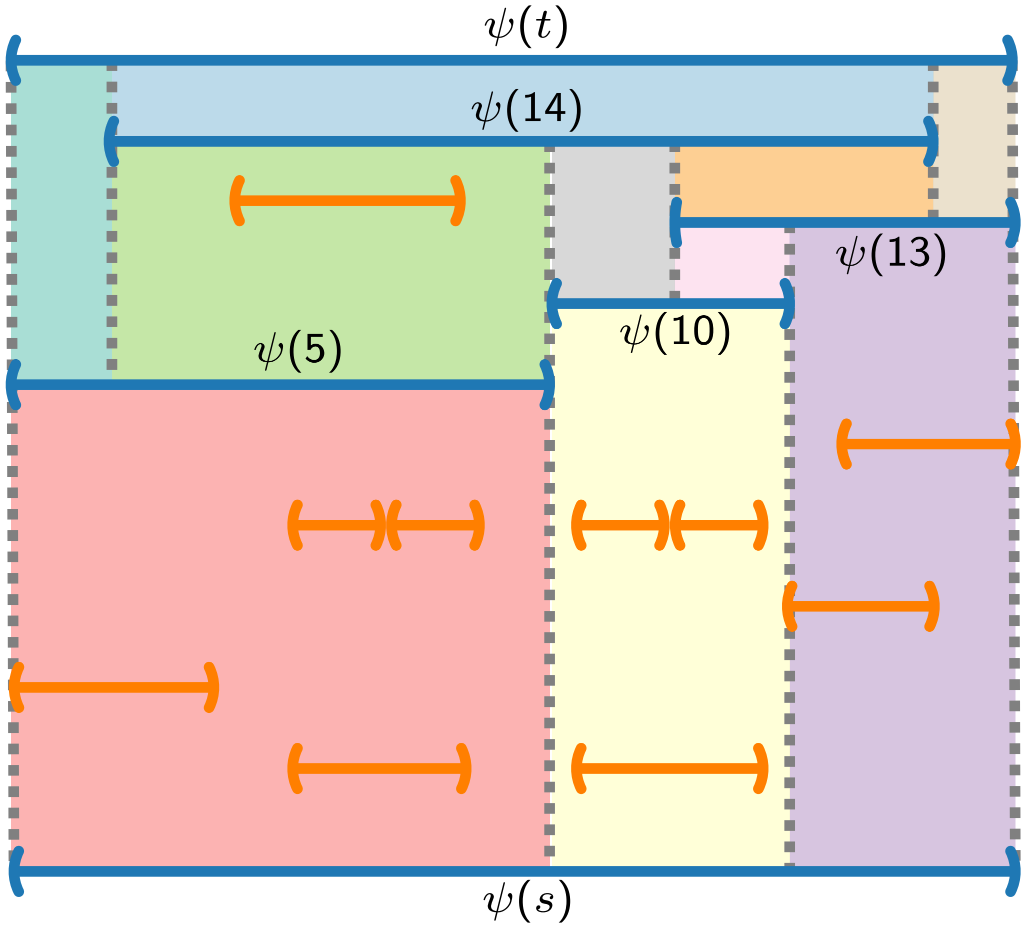


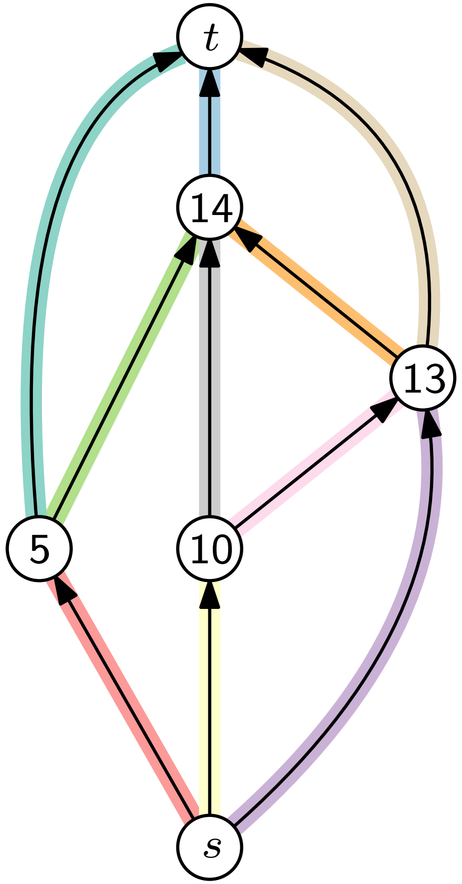


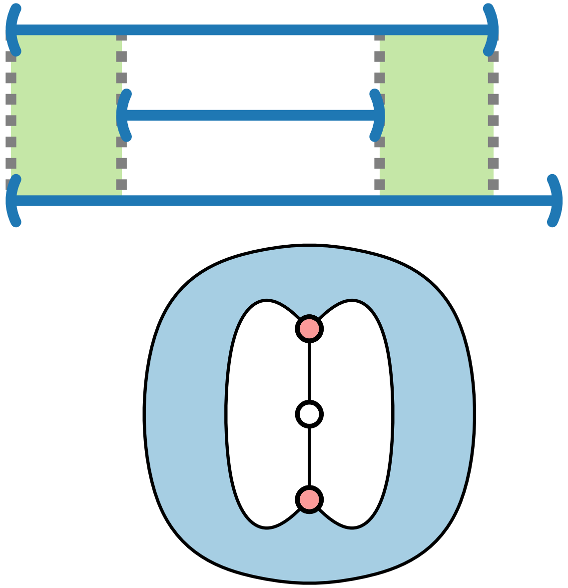
separation pair!

R-Nodes

- for each child (edge) e:







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- for each child (edge) e:
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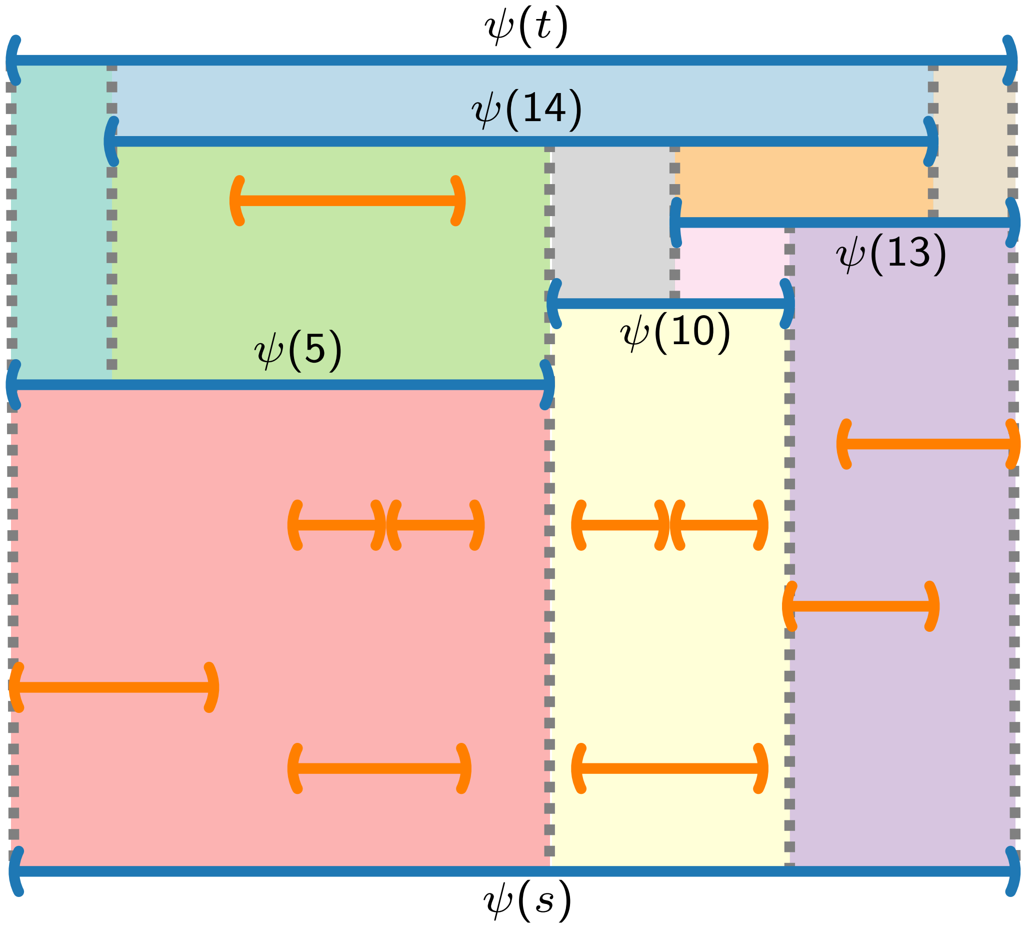

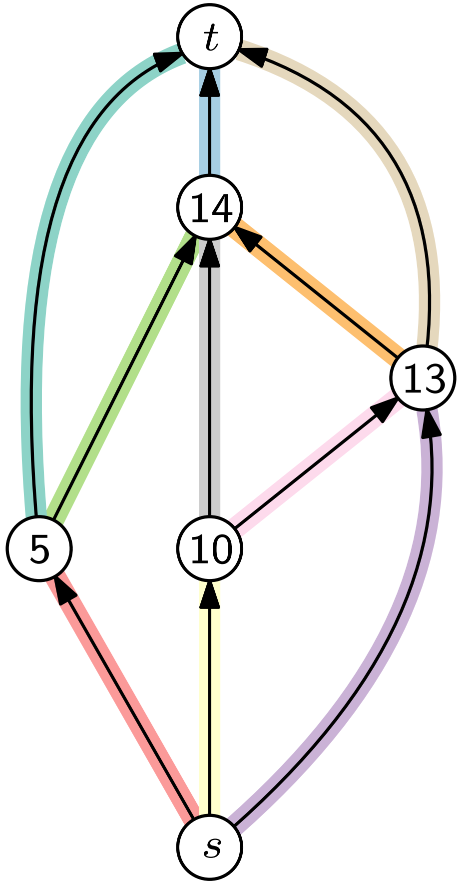

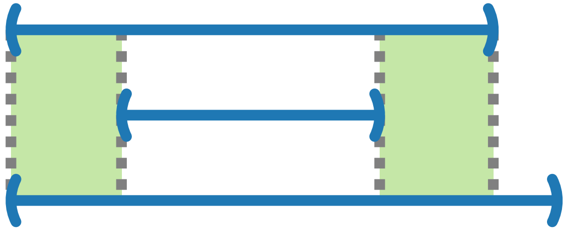

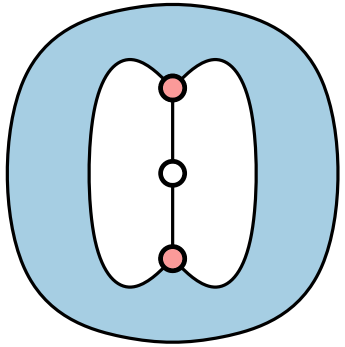

separation pair!

R-Nodes with 2-SAT Formulation

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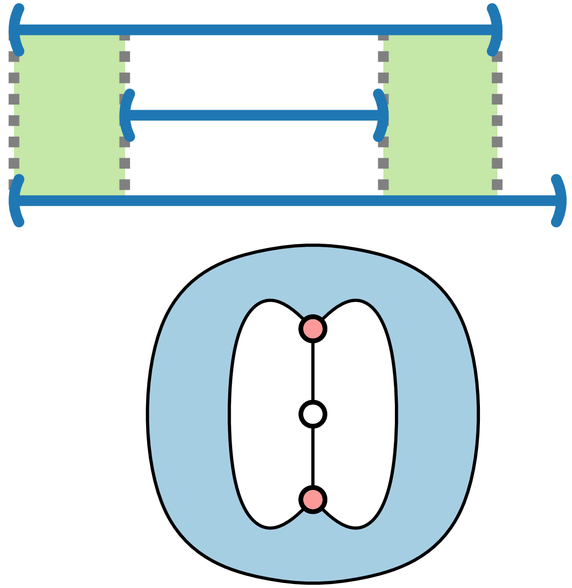

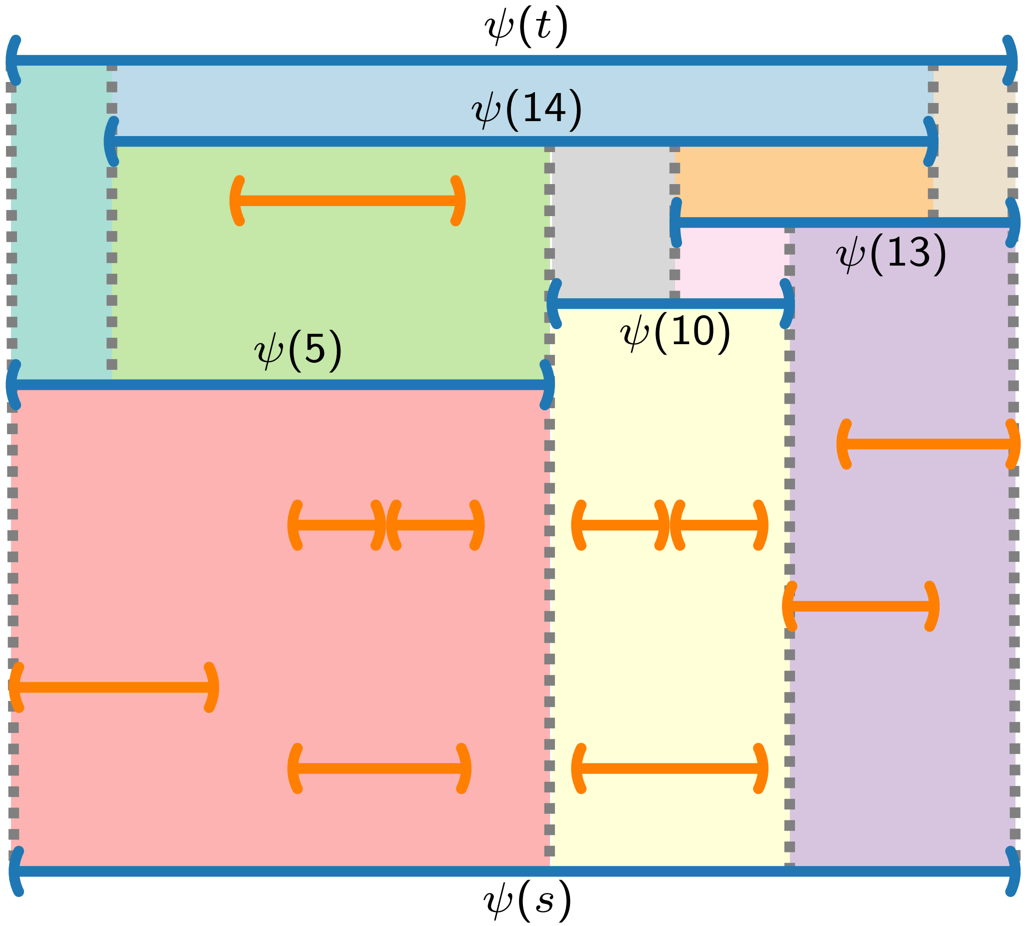

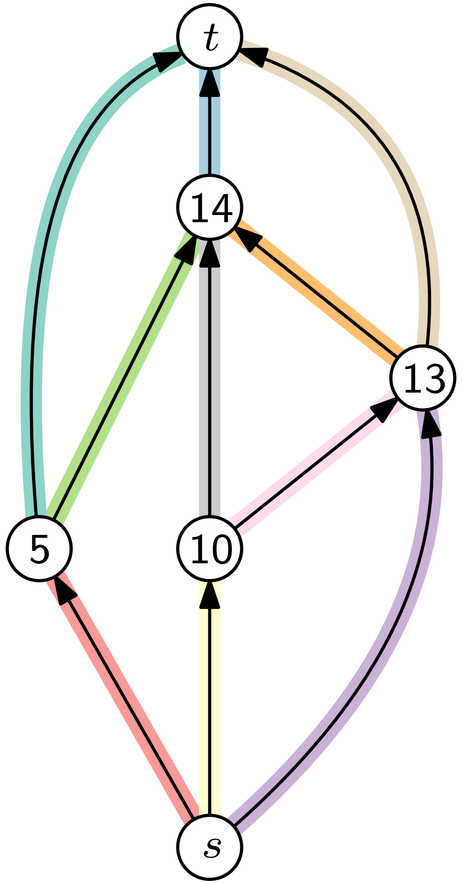

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R-Nodes with 2-SAT Formulation

- for each child (edge) e:
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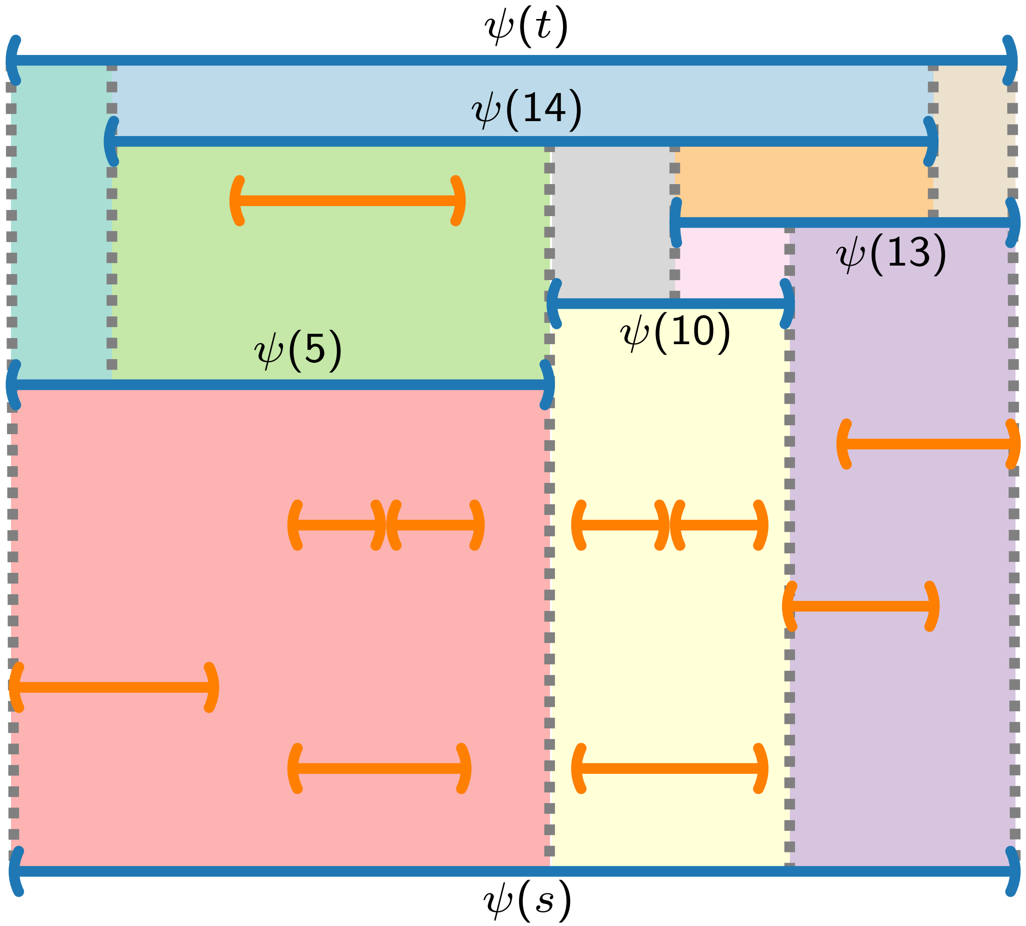

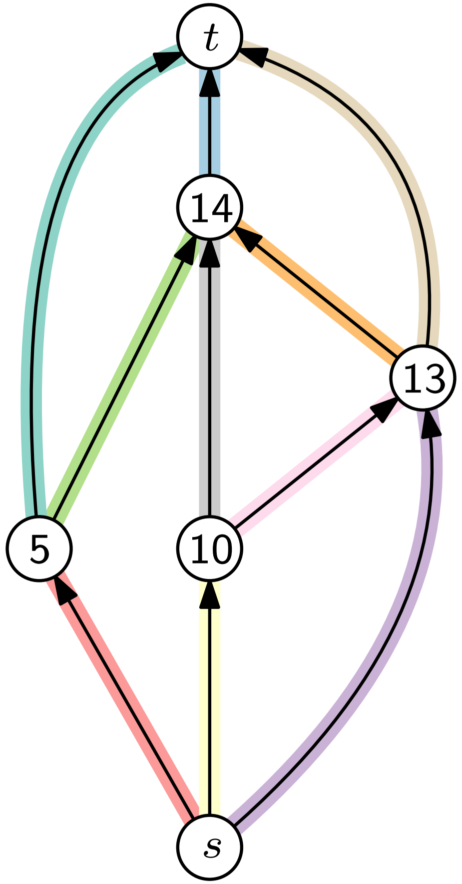

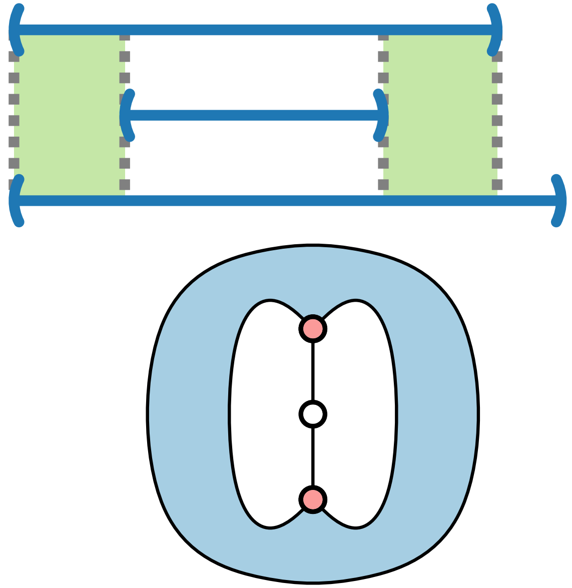

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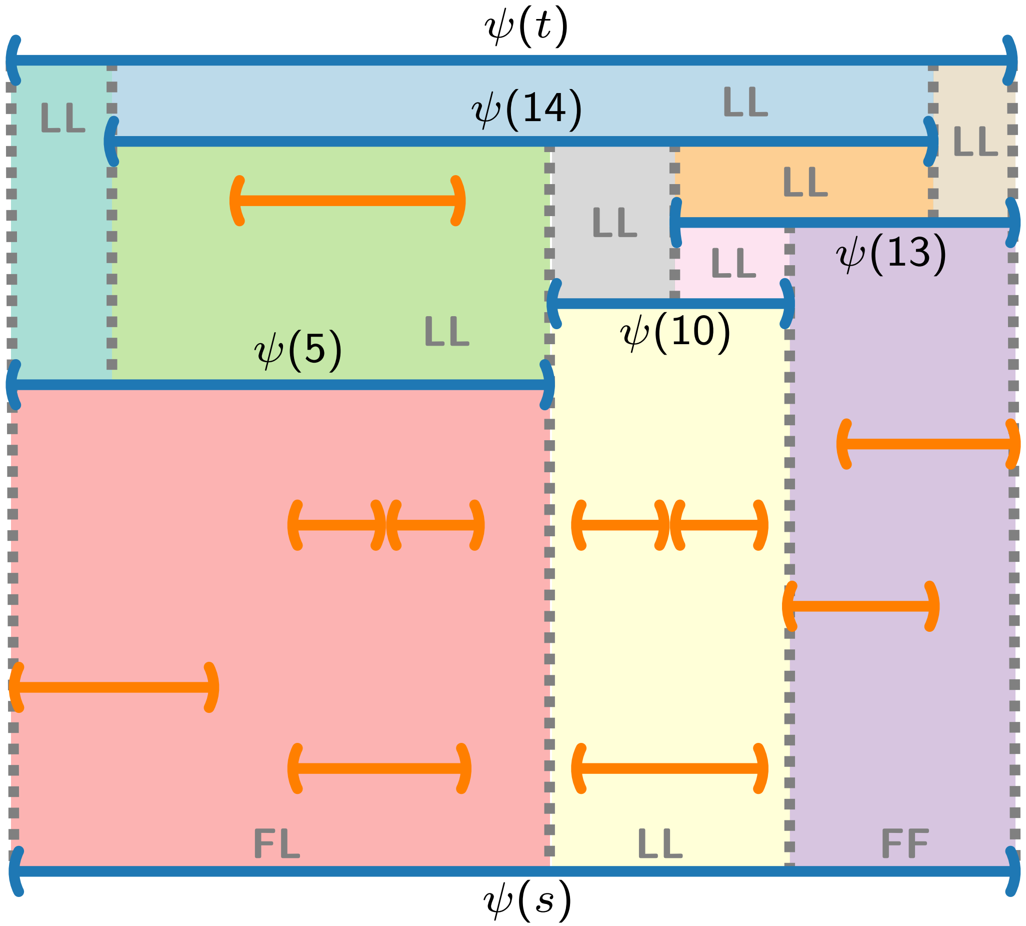

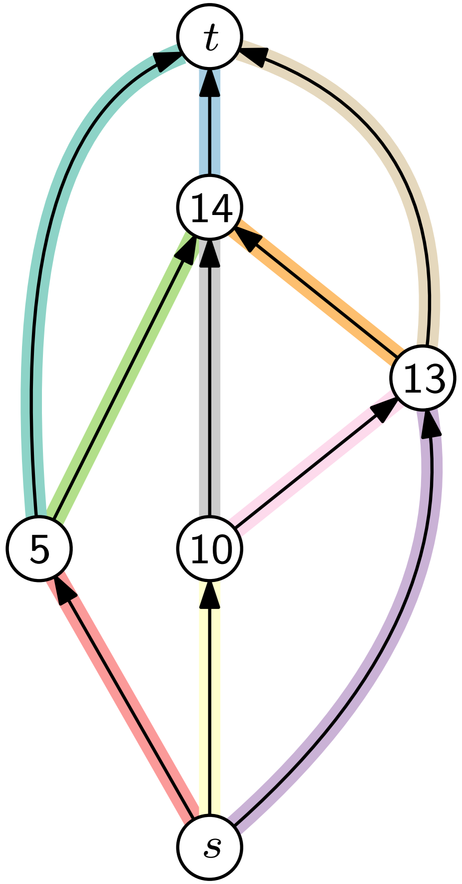

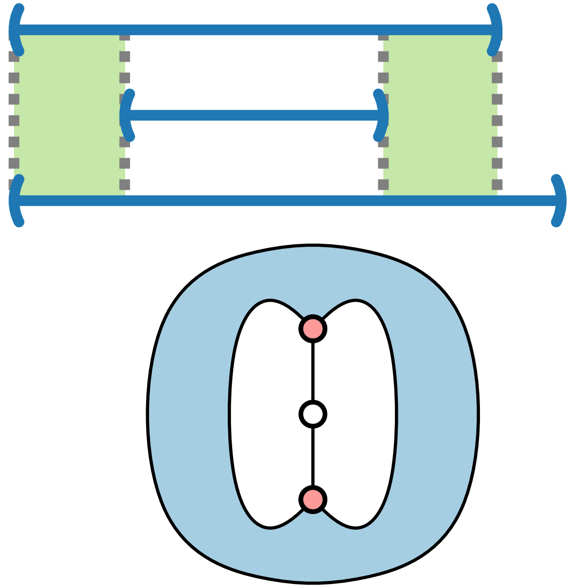

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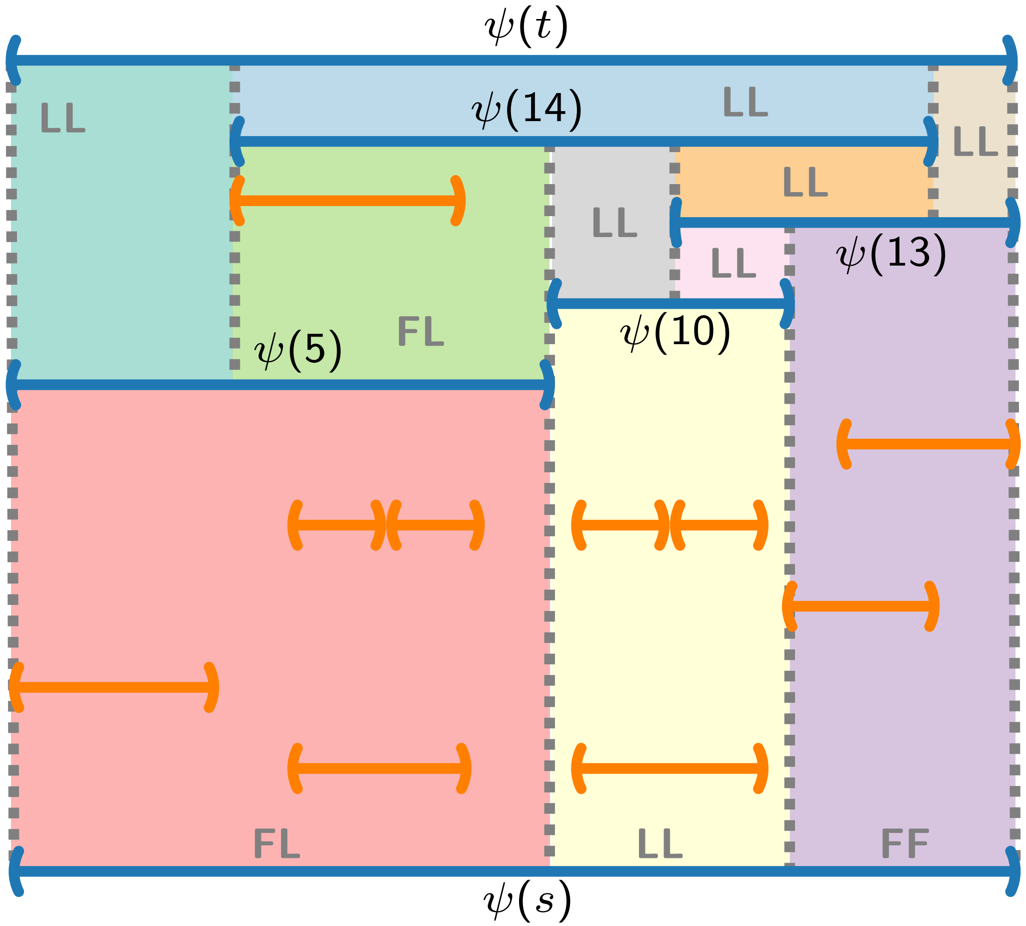

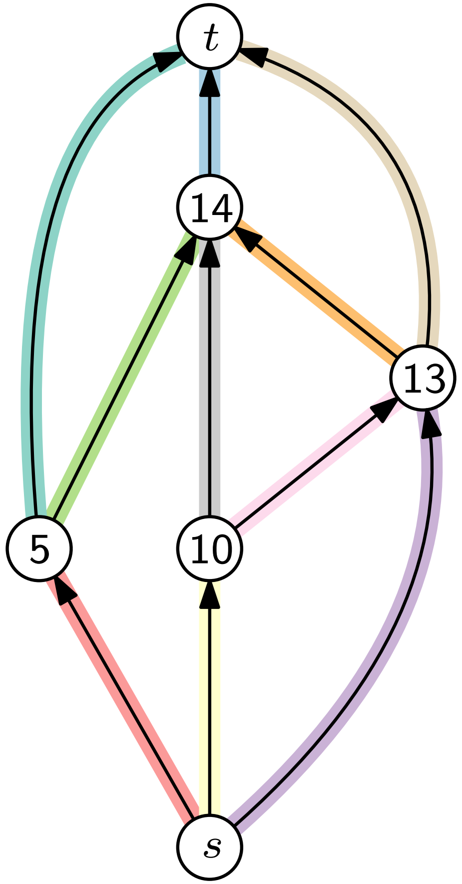

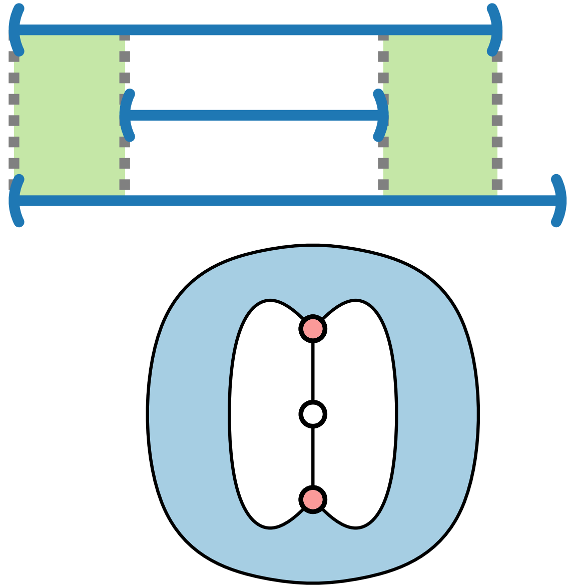

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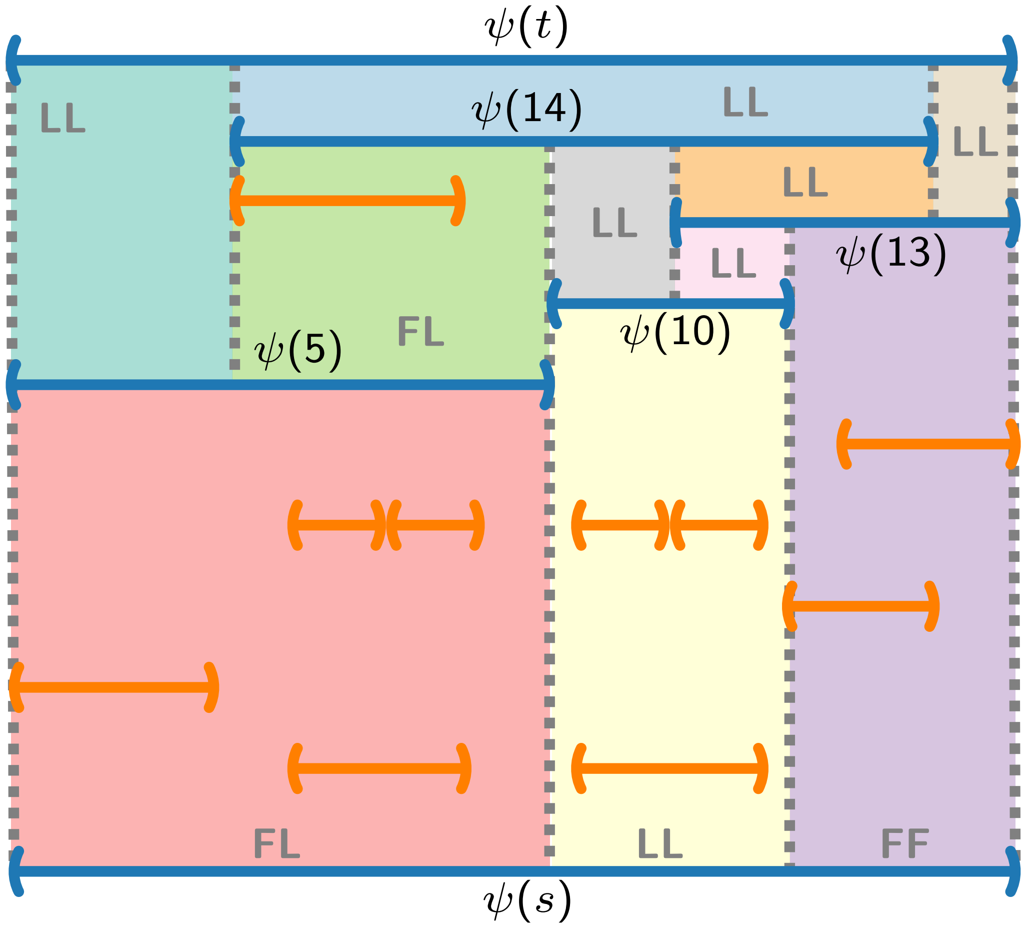

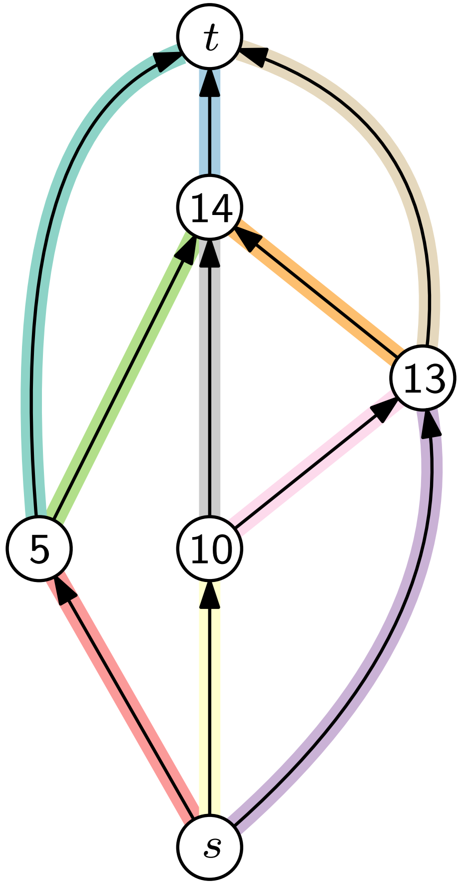

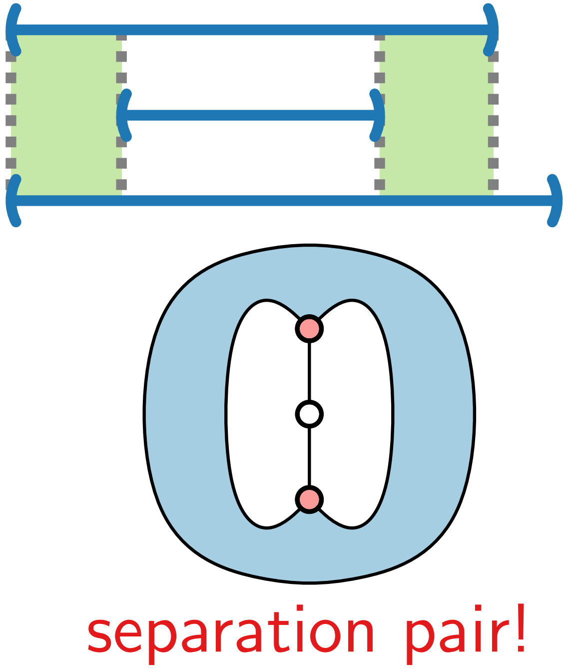

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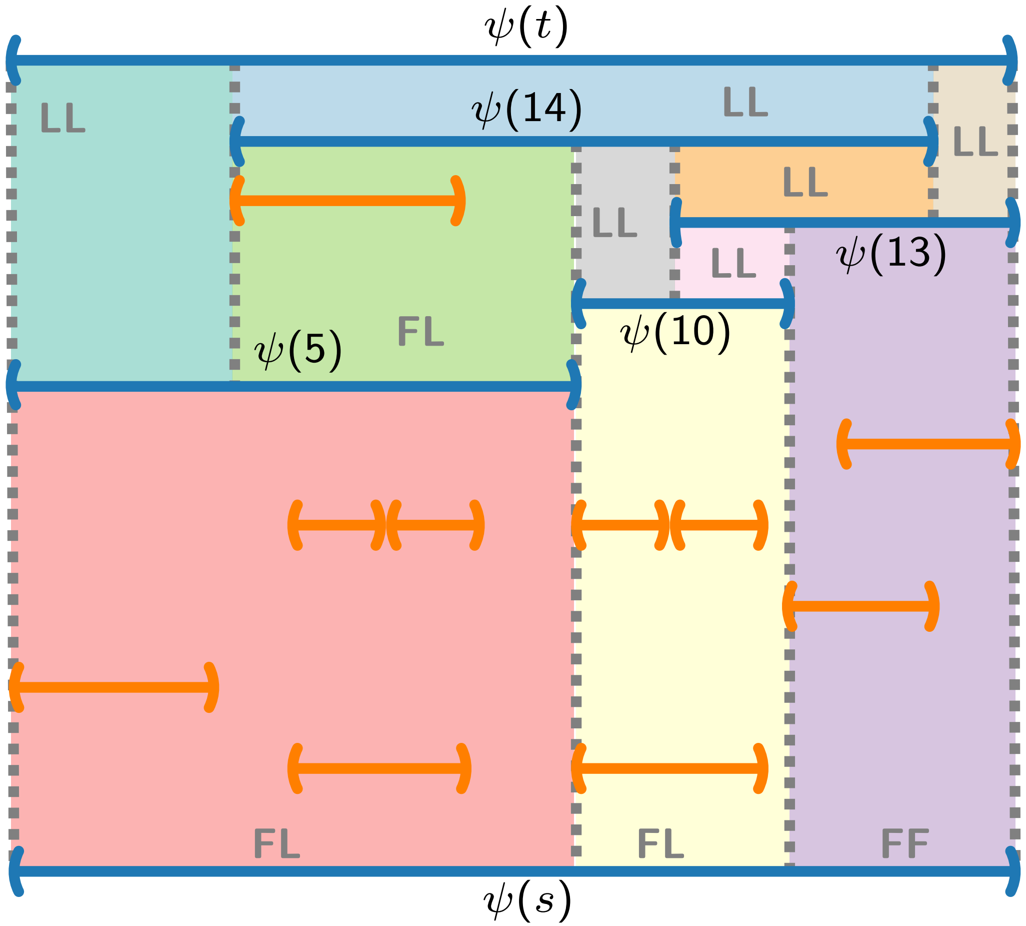

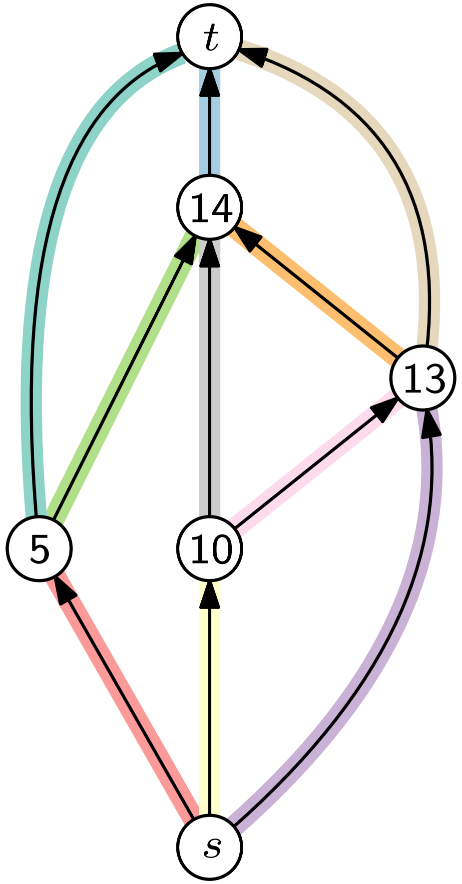

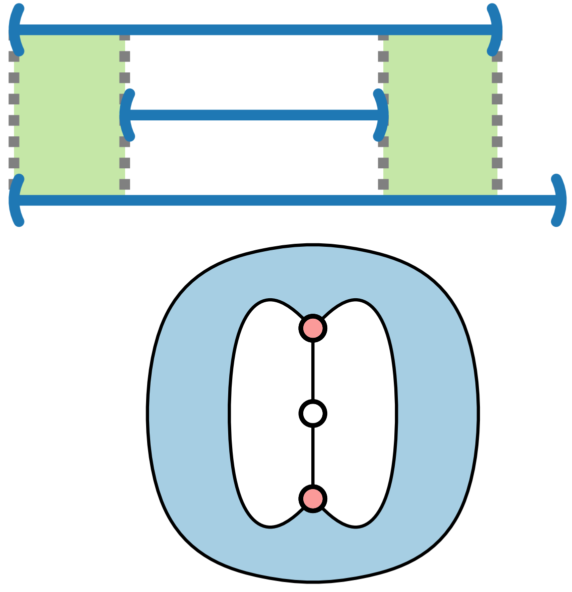

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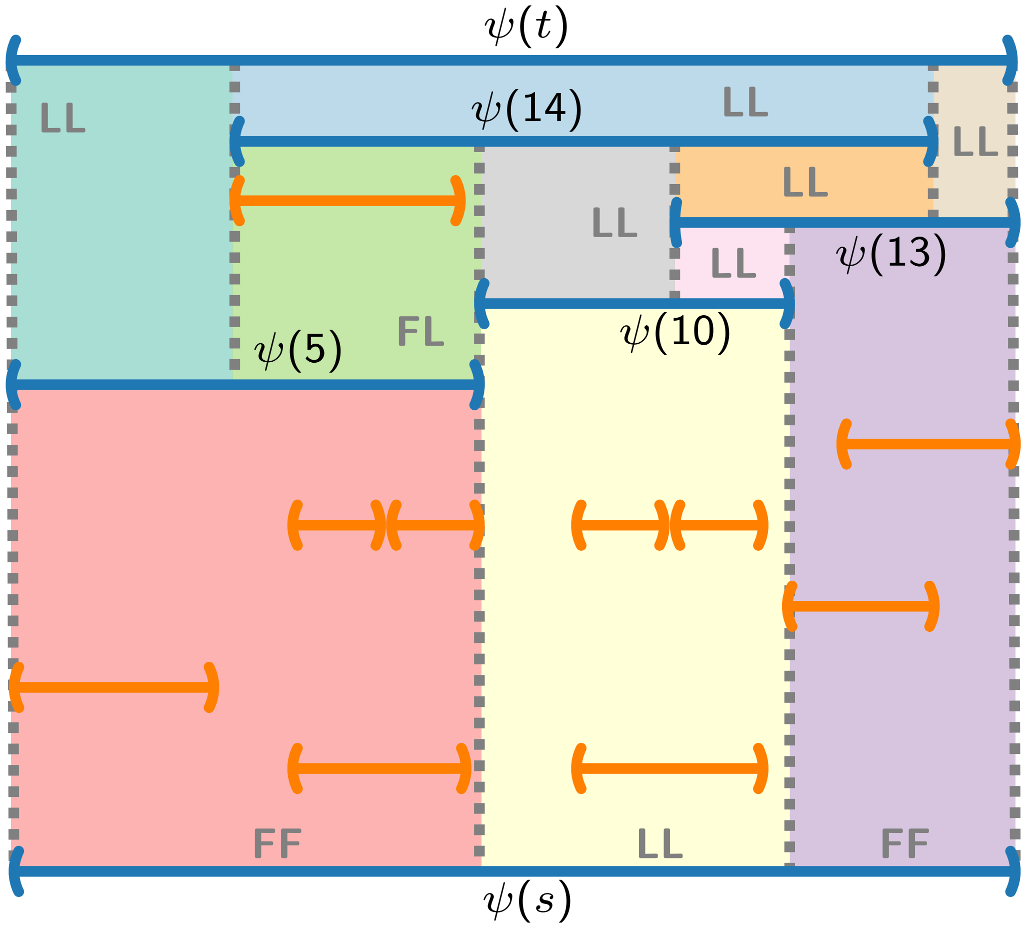

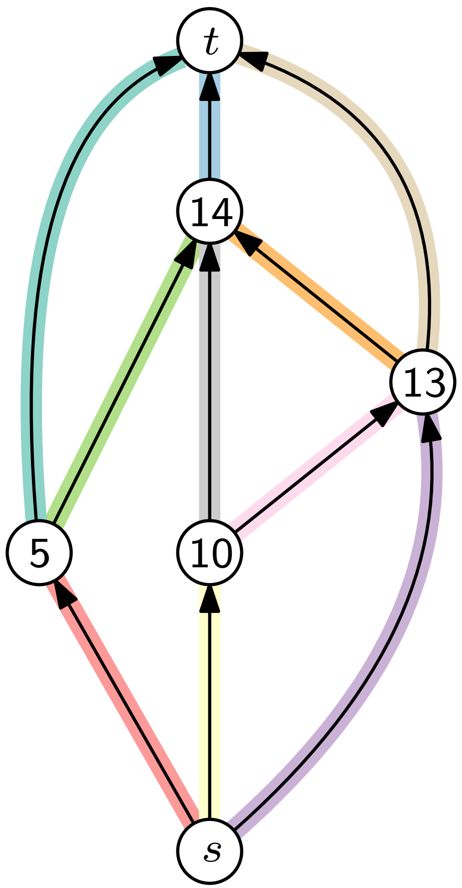

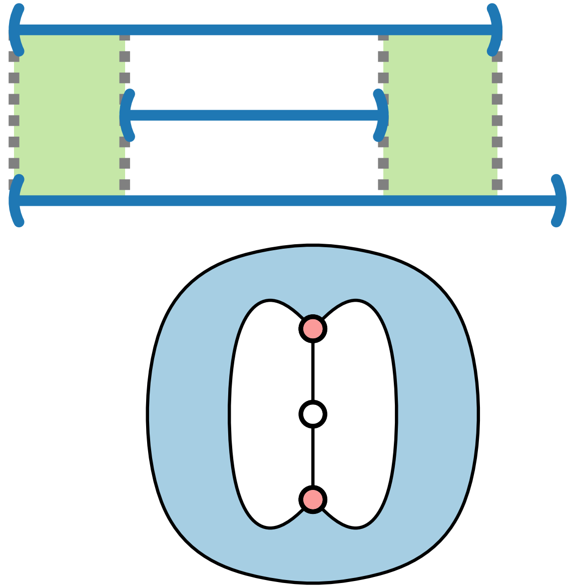

separation pair!

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- for each child (edge) e:
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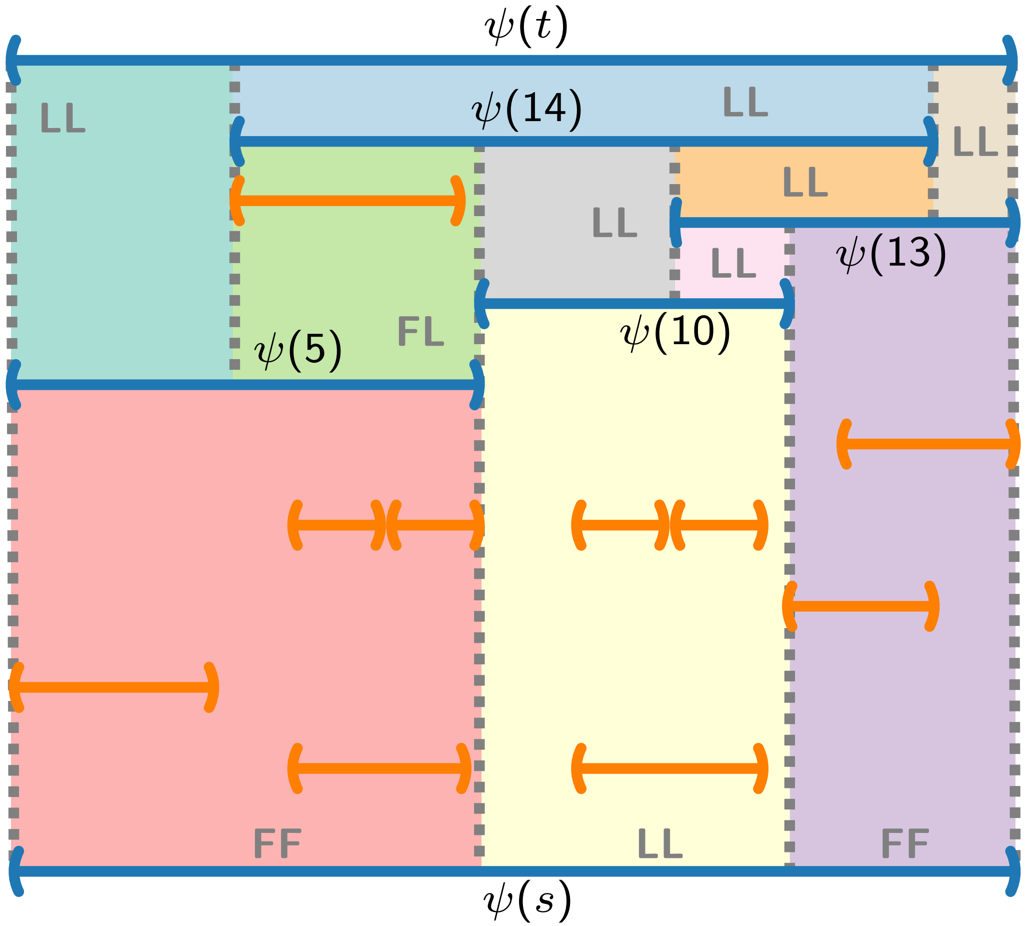
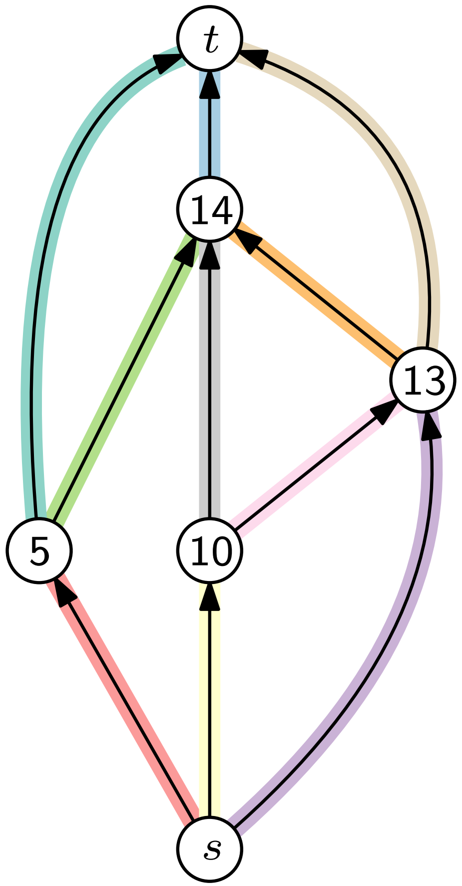
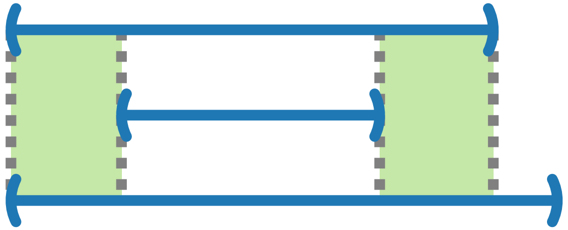
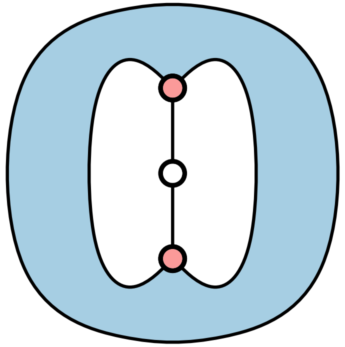
  - 2 variables l_e, r_e encoding fixed/loose type of its tile
  - consistency clauses – $O(n^2)$ many,









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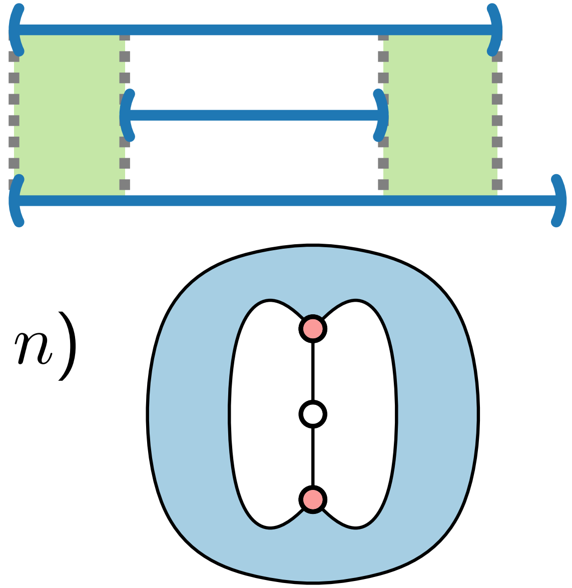
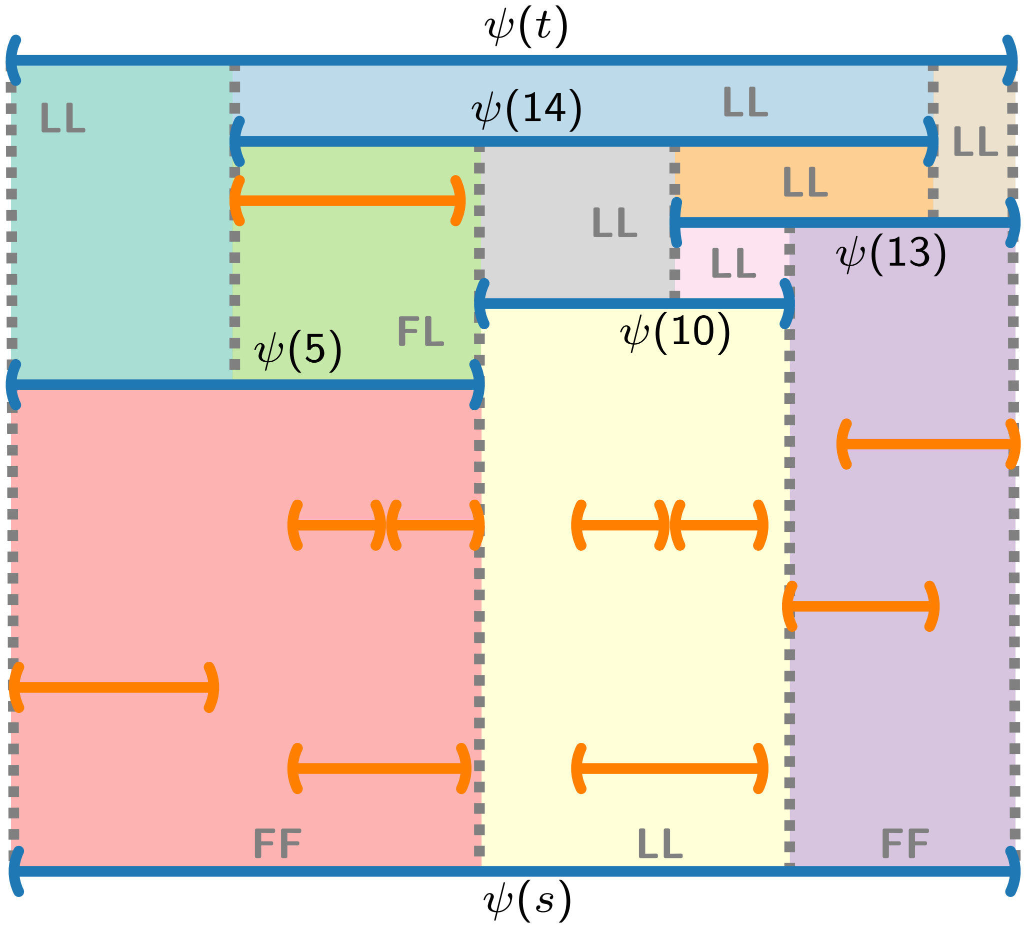
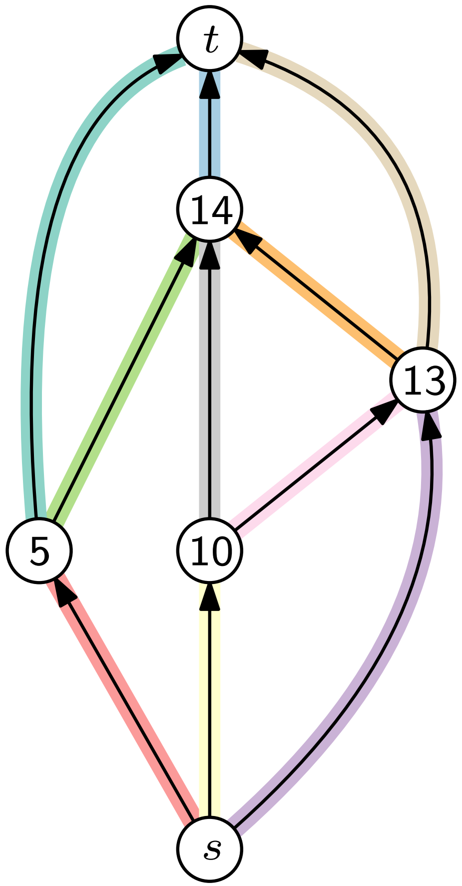
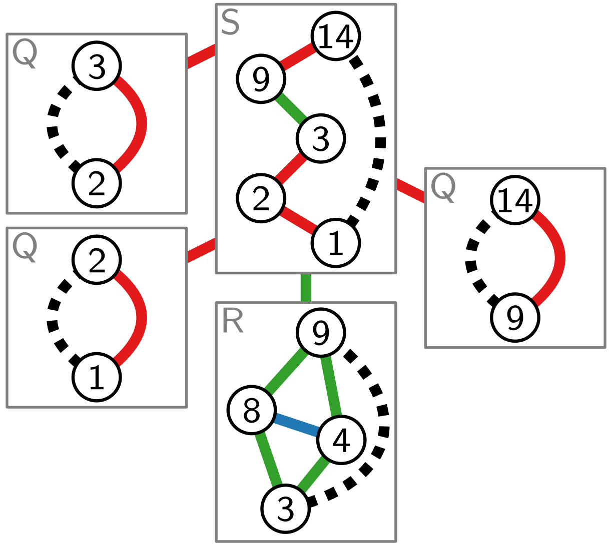




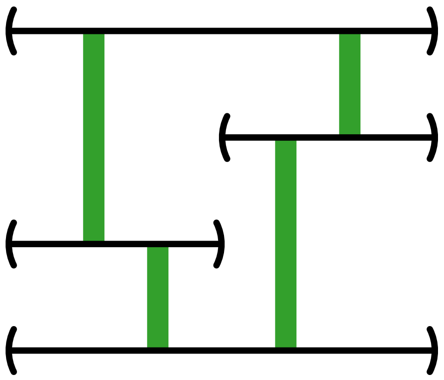


Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension



Part VI:
NP-Hardness of the General Case



Alexander Wolff

NP-Hardness of RepExt in the General Case

Theorem 2.
ε-Bar Visibility Representation Extension is NP-complete.

- Reduction from Planar Monotone 3-SAT

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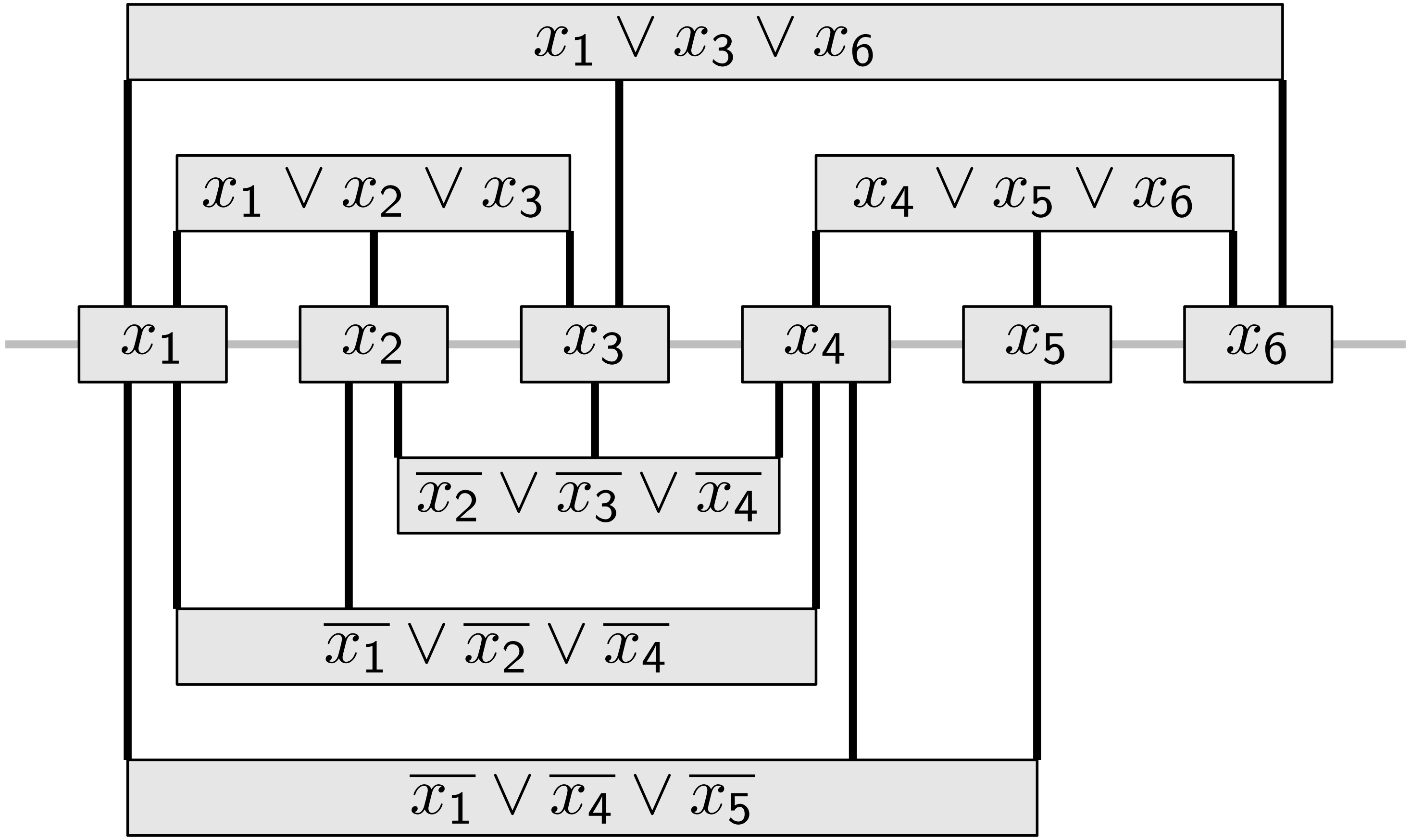

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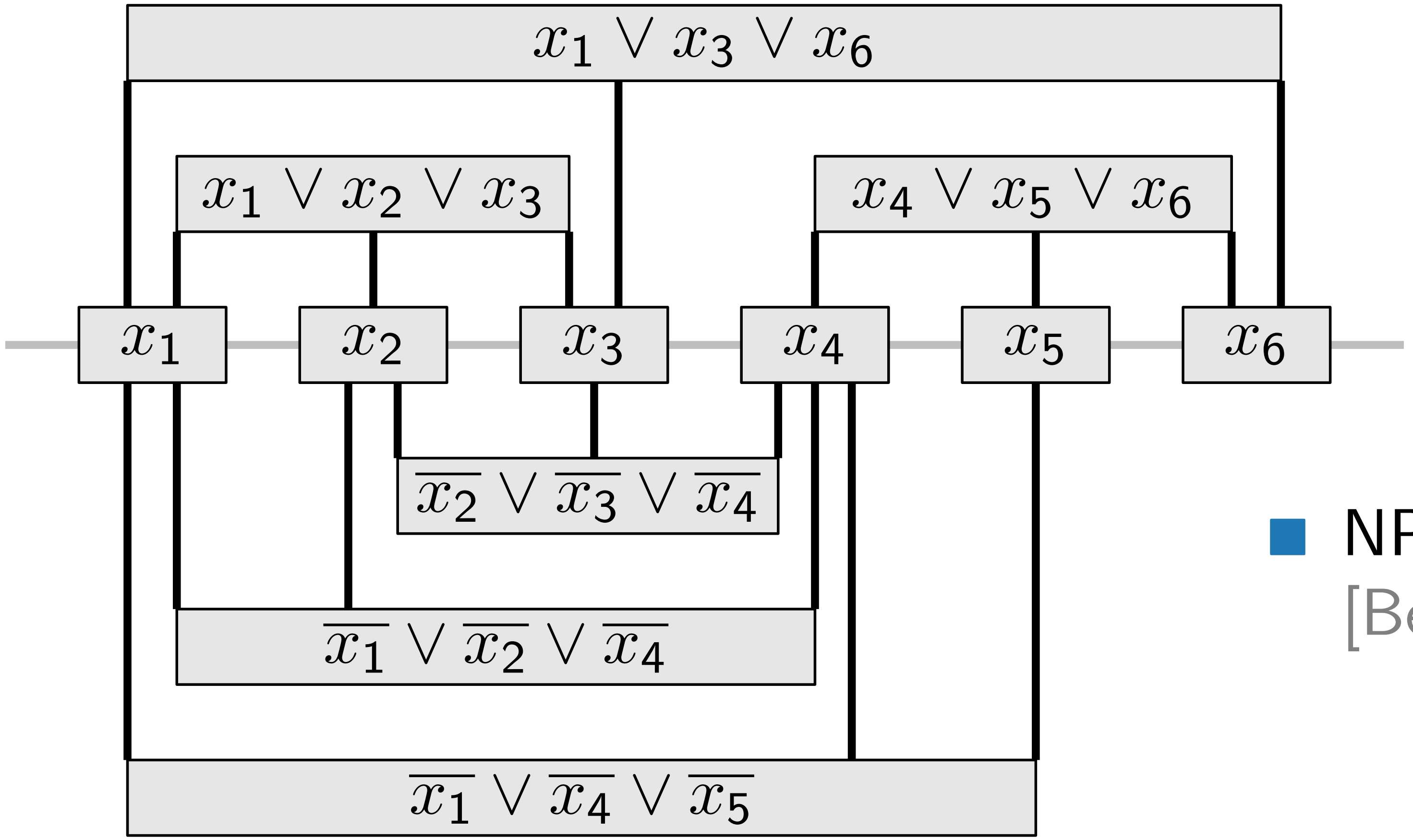

- NP-complete
  [Berg & Khosravi '10]

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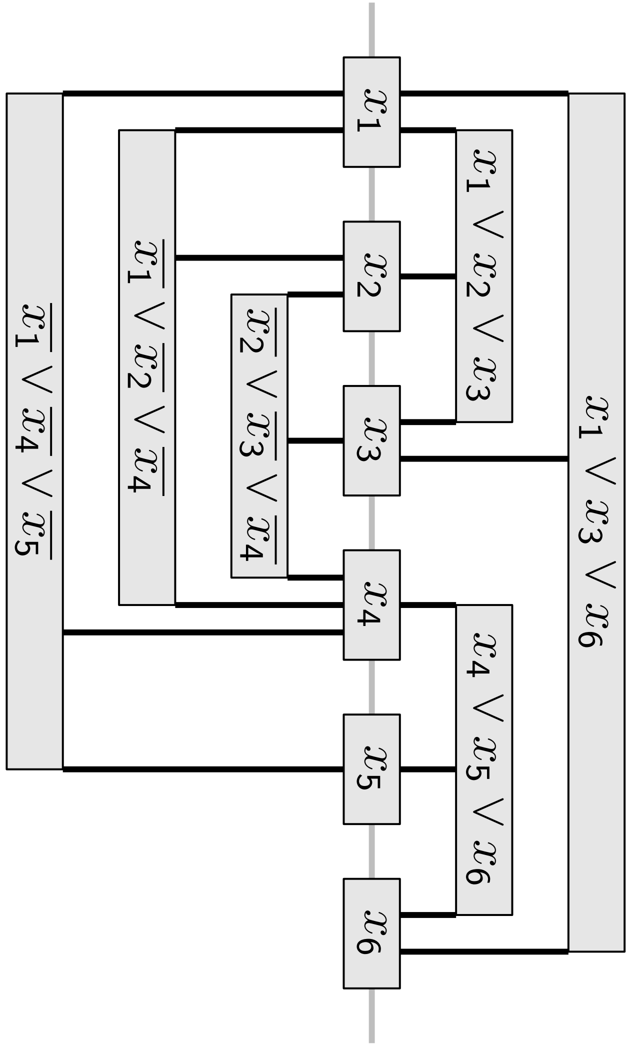

- NP-complete
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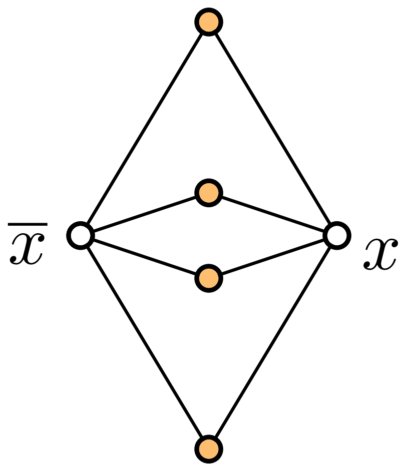
Variable Gadget



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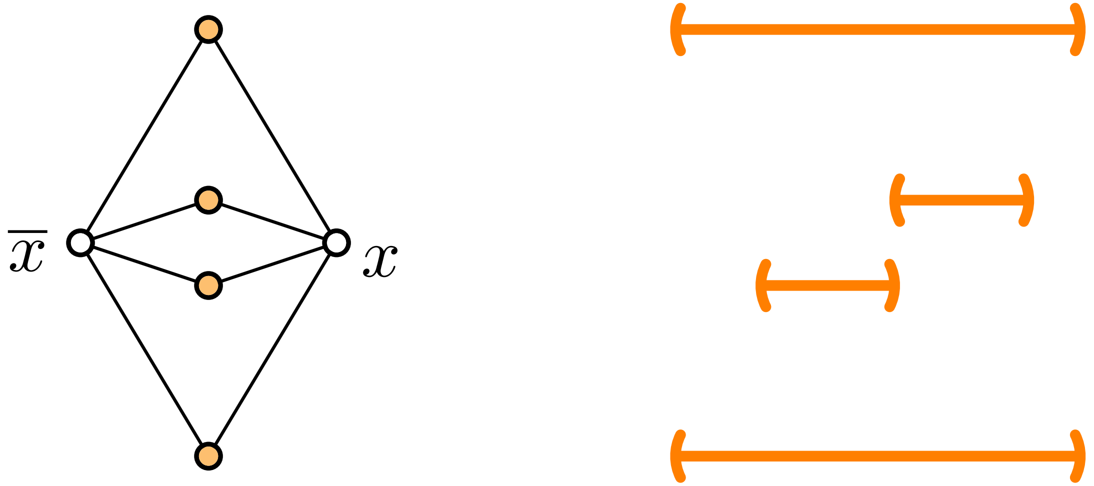

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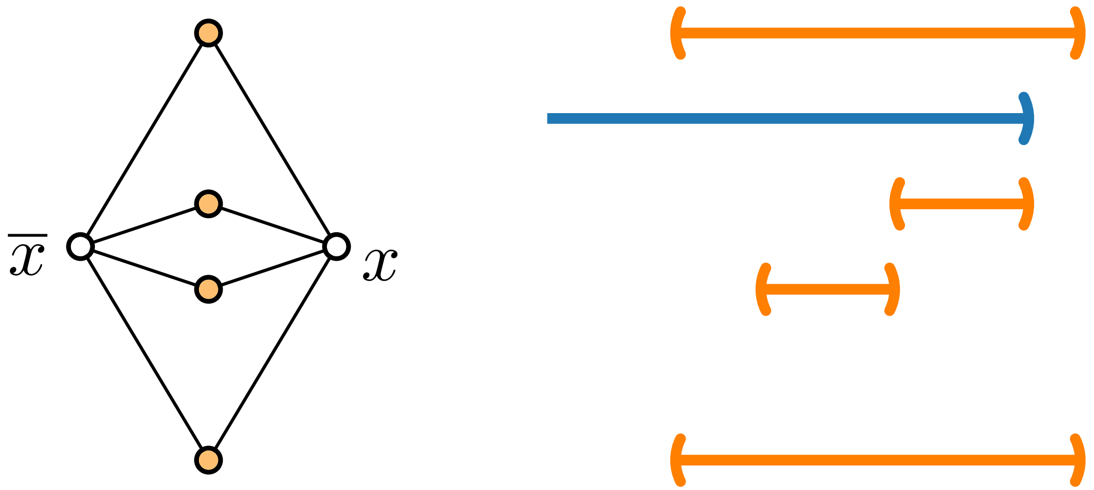

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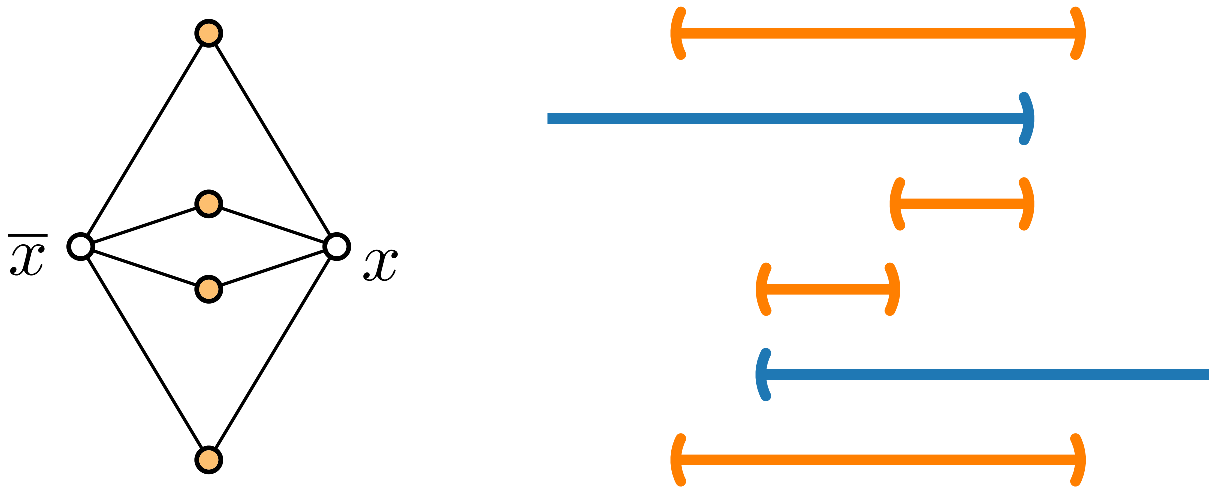

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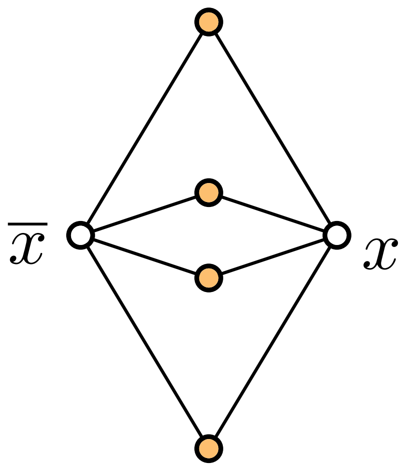

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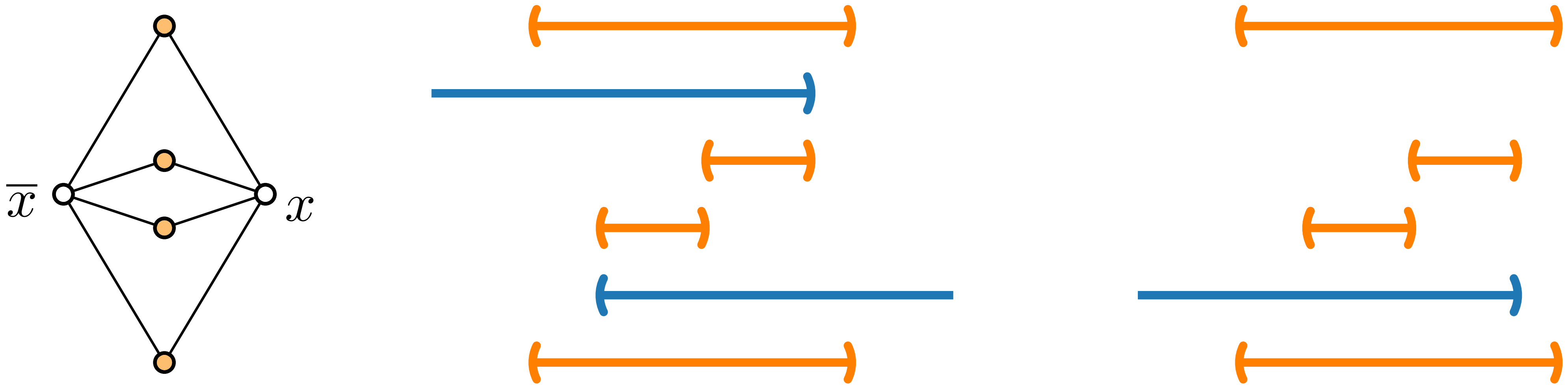

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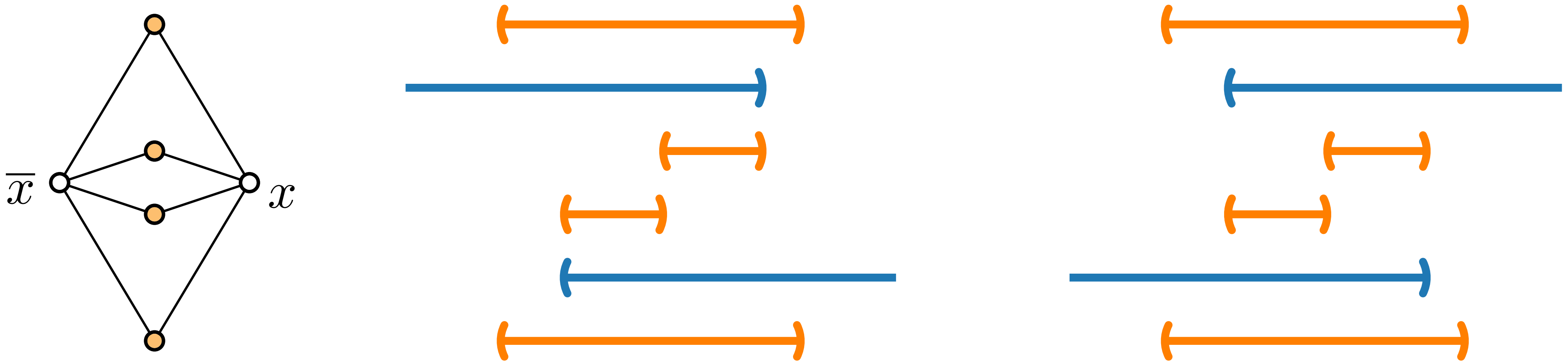

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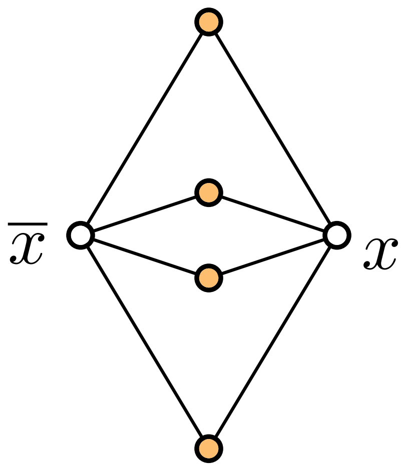

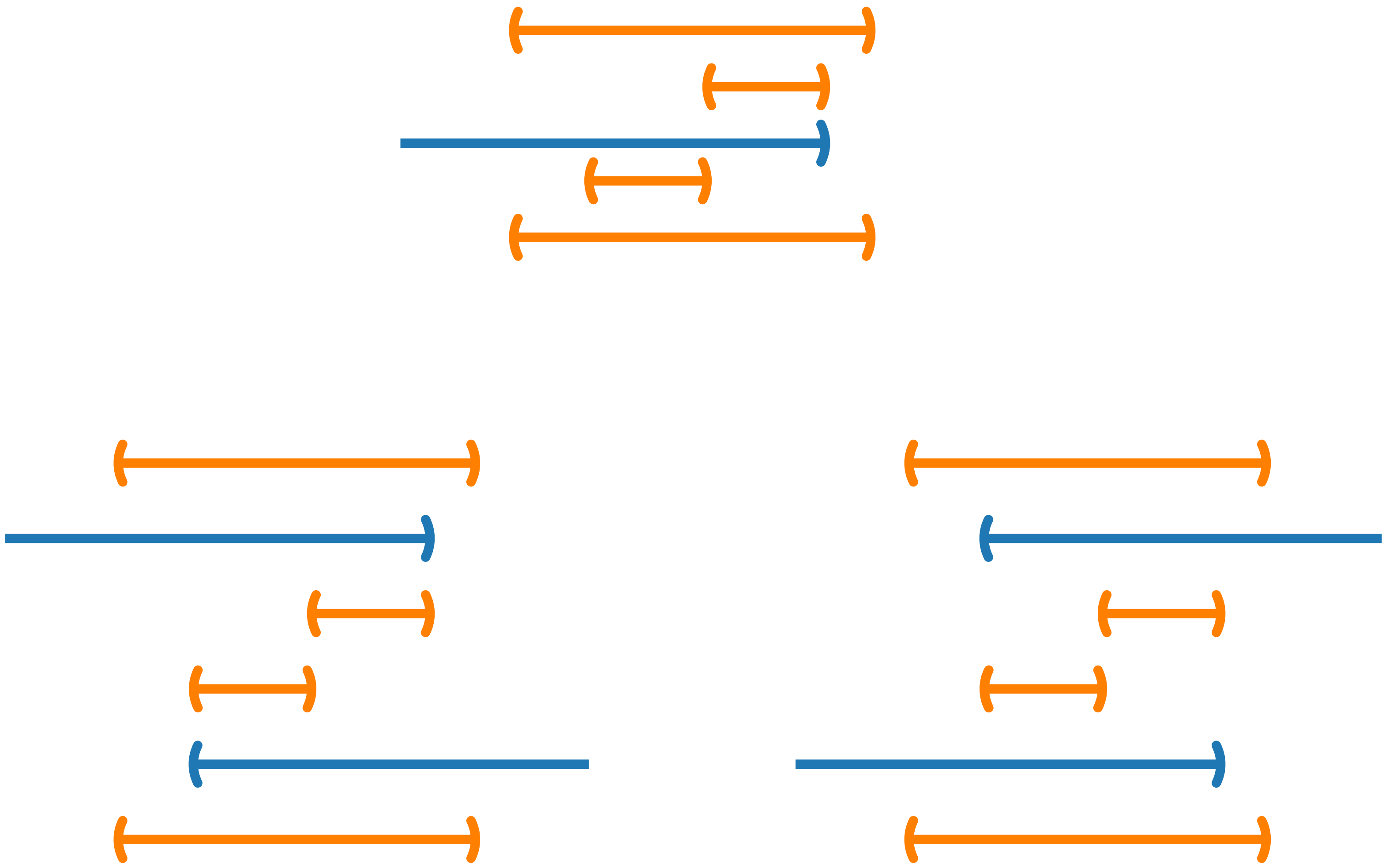

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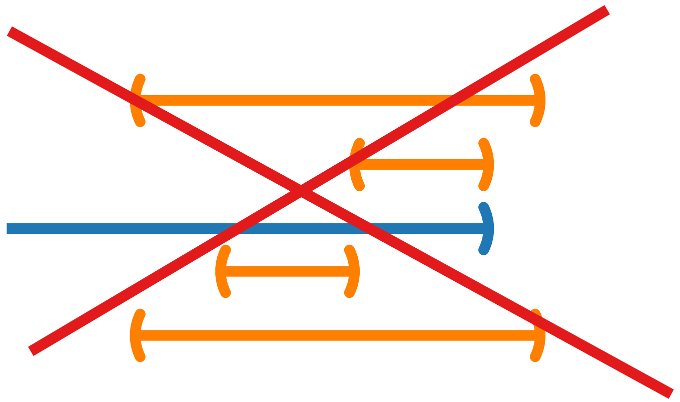
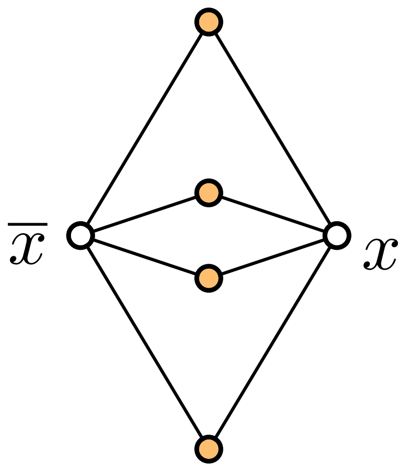
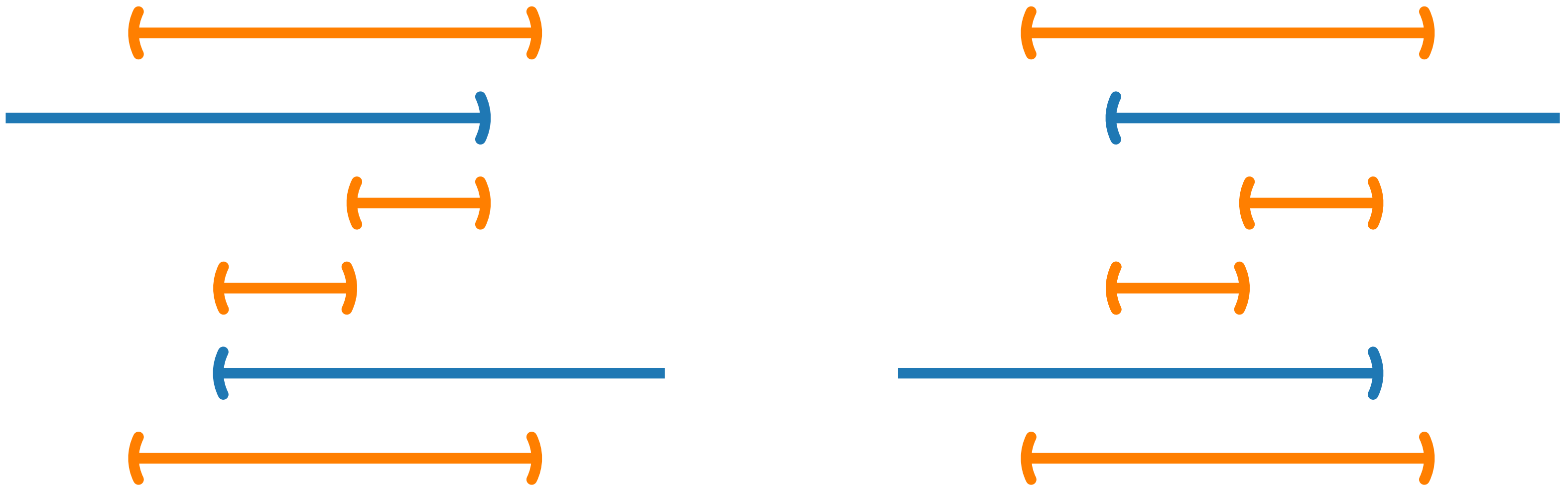
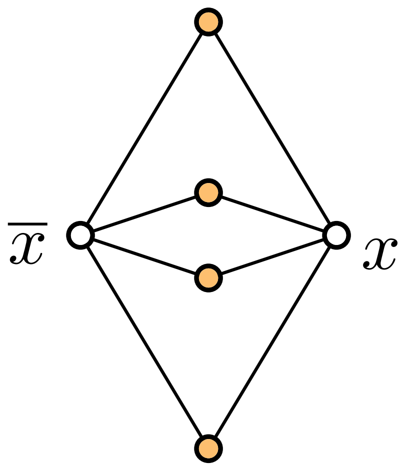
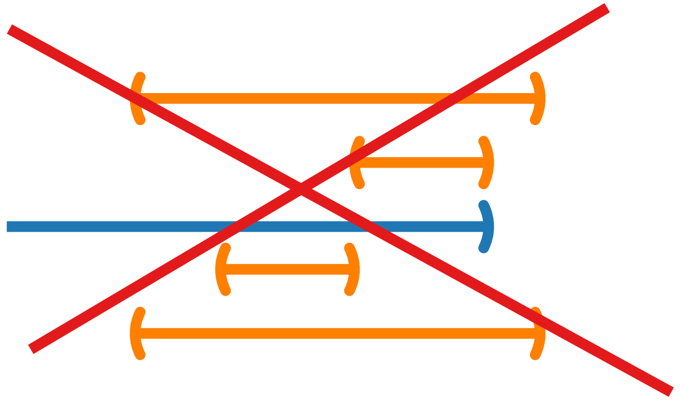


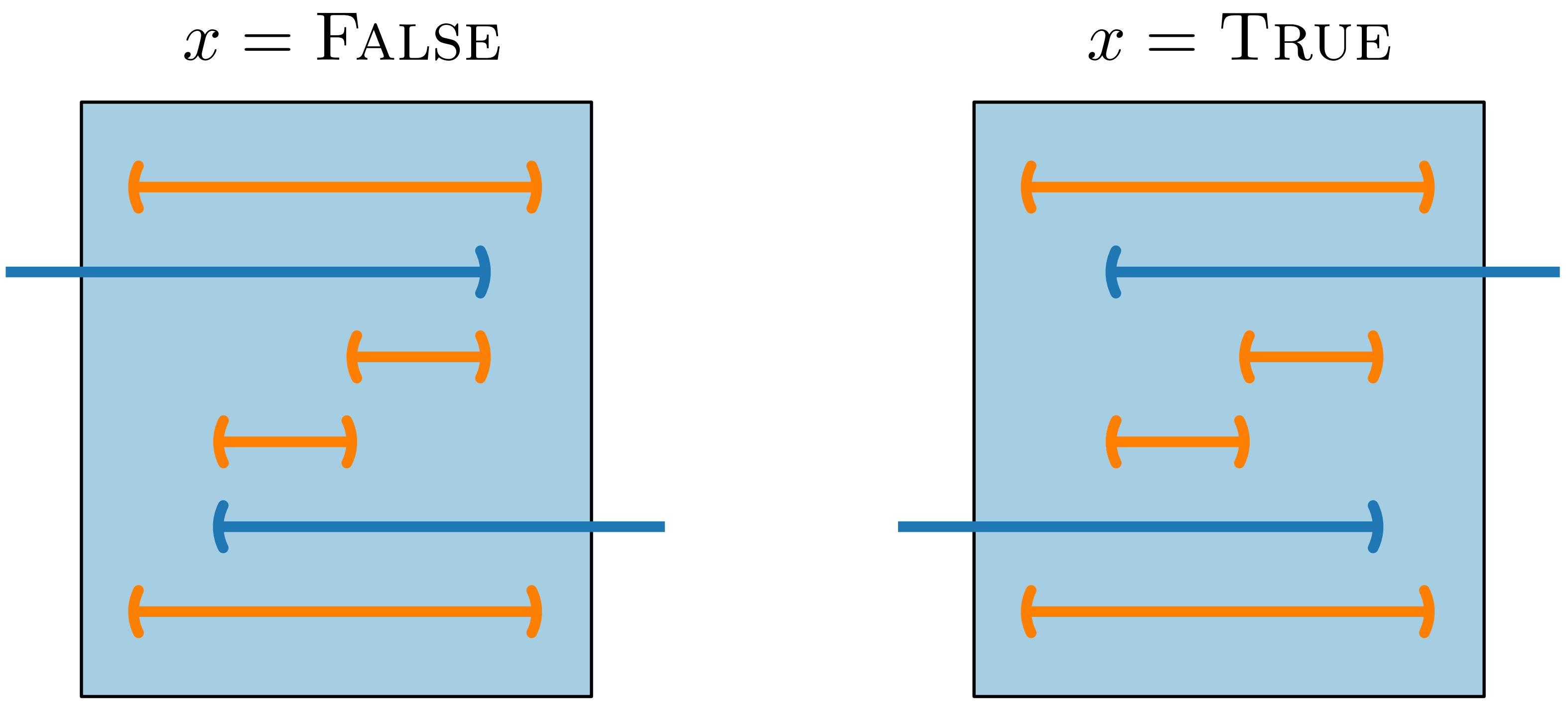




Variable Gadget







Clause Gadget

$x \vee y \vee z$

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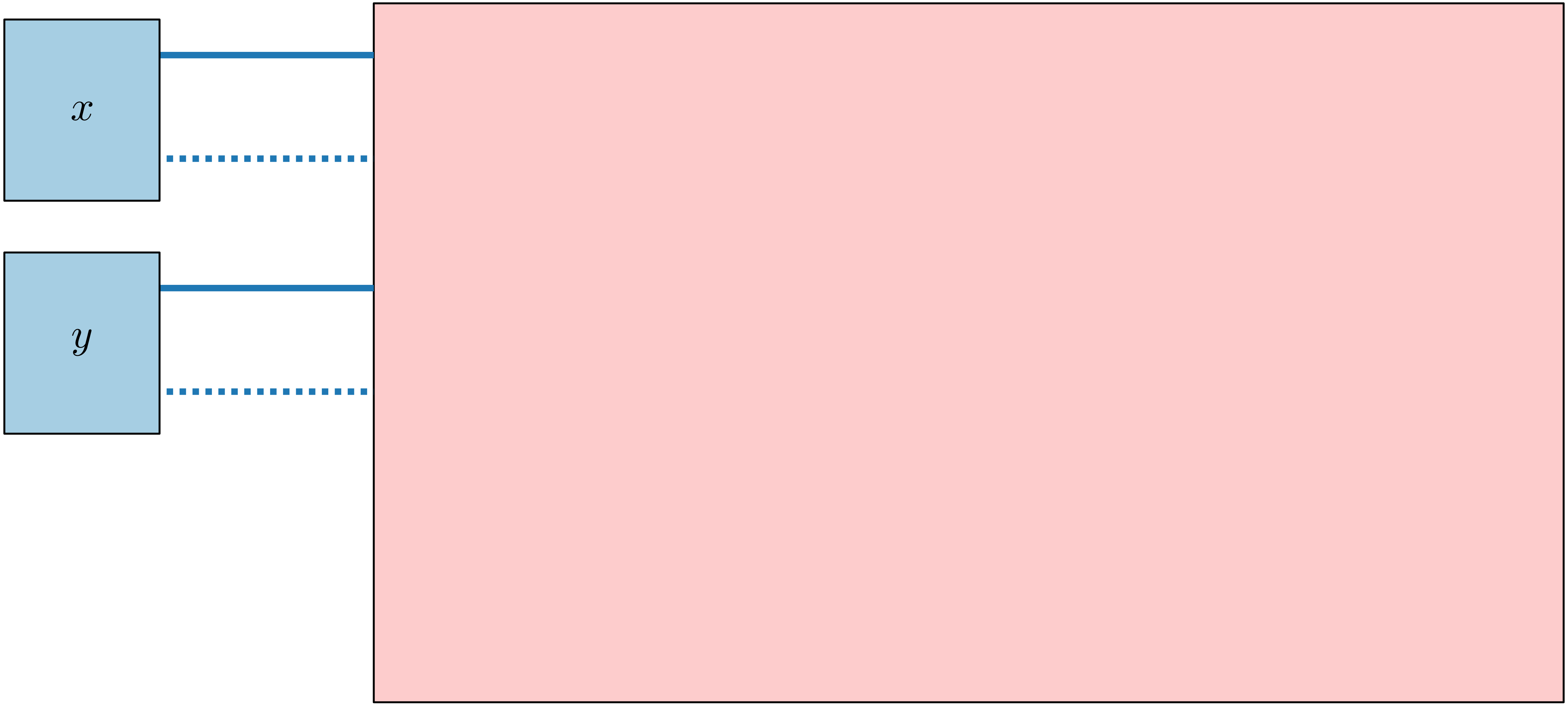

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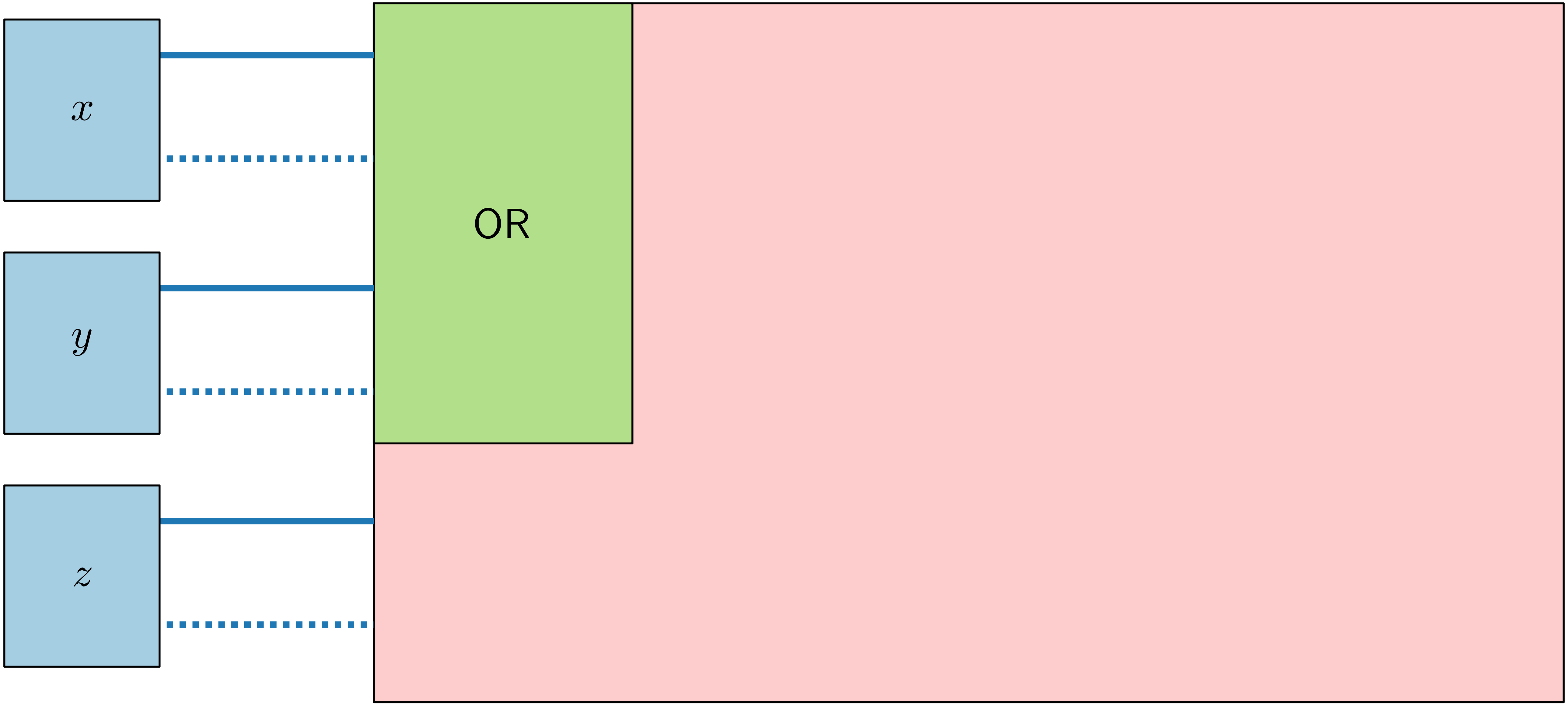

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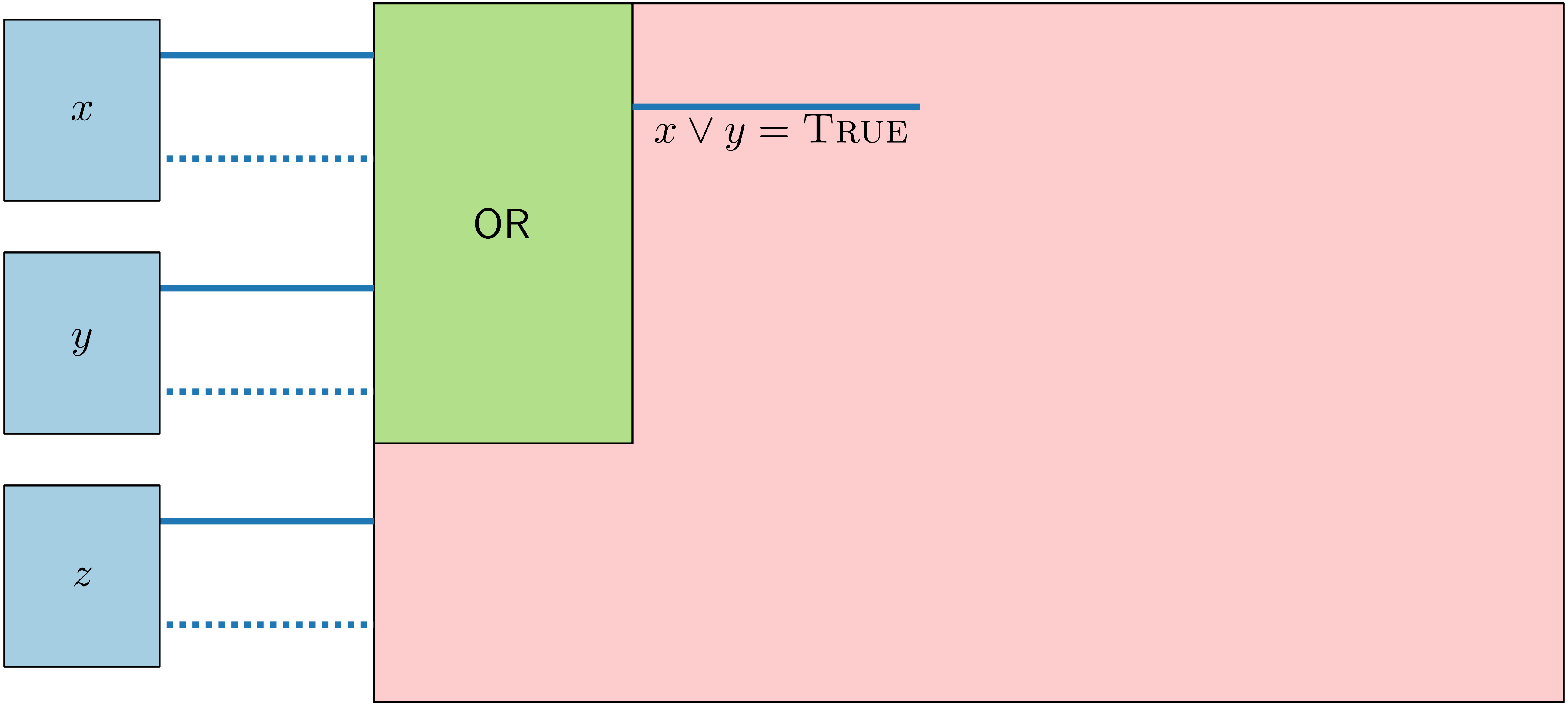

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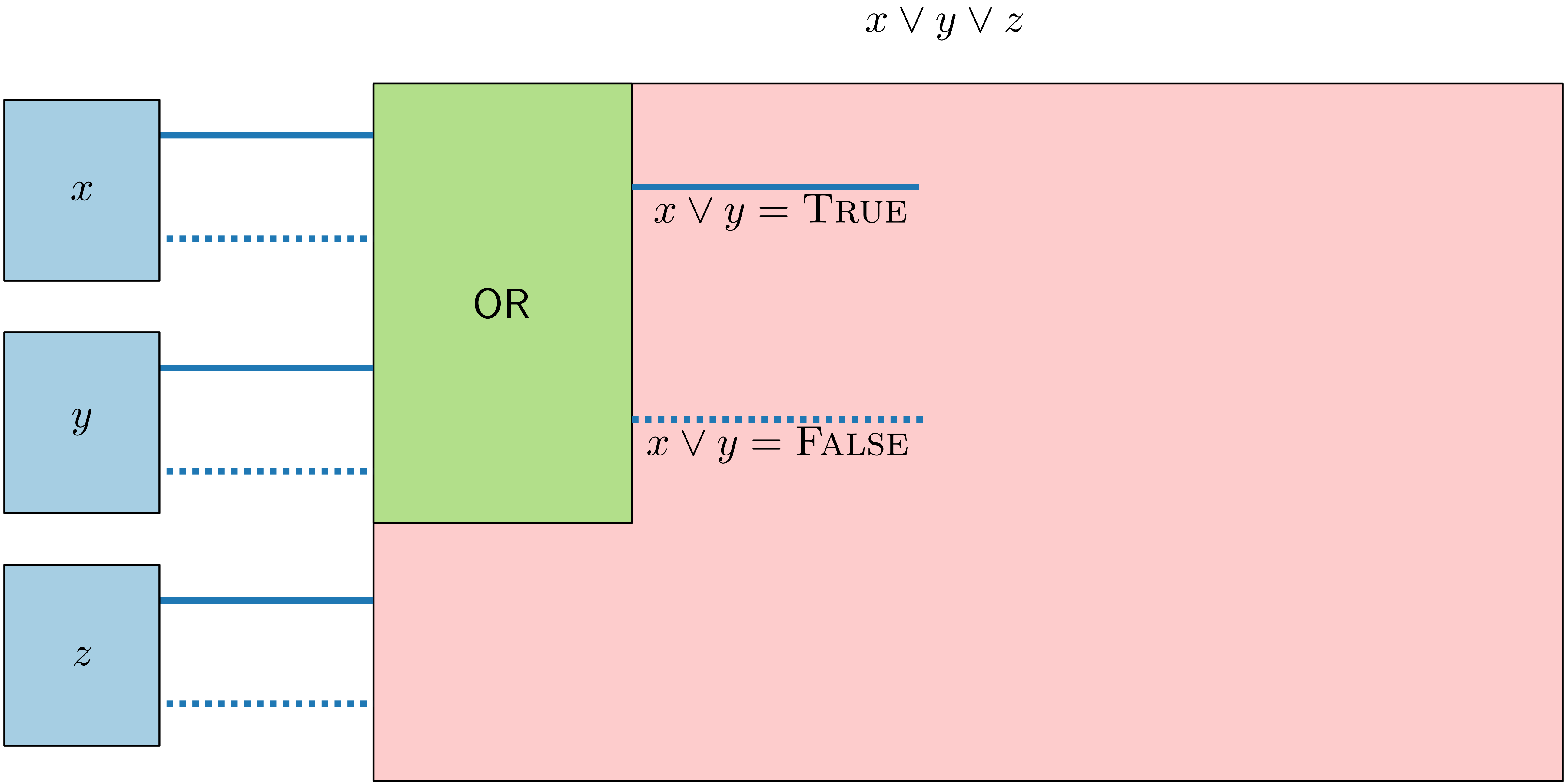

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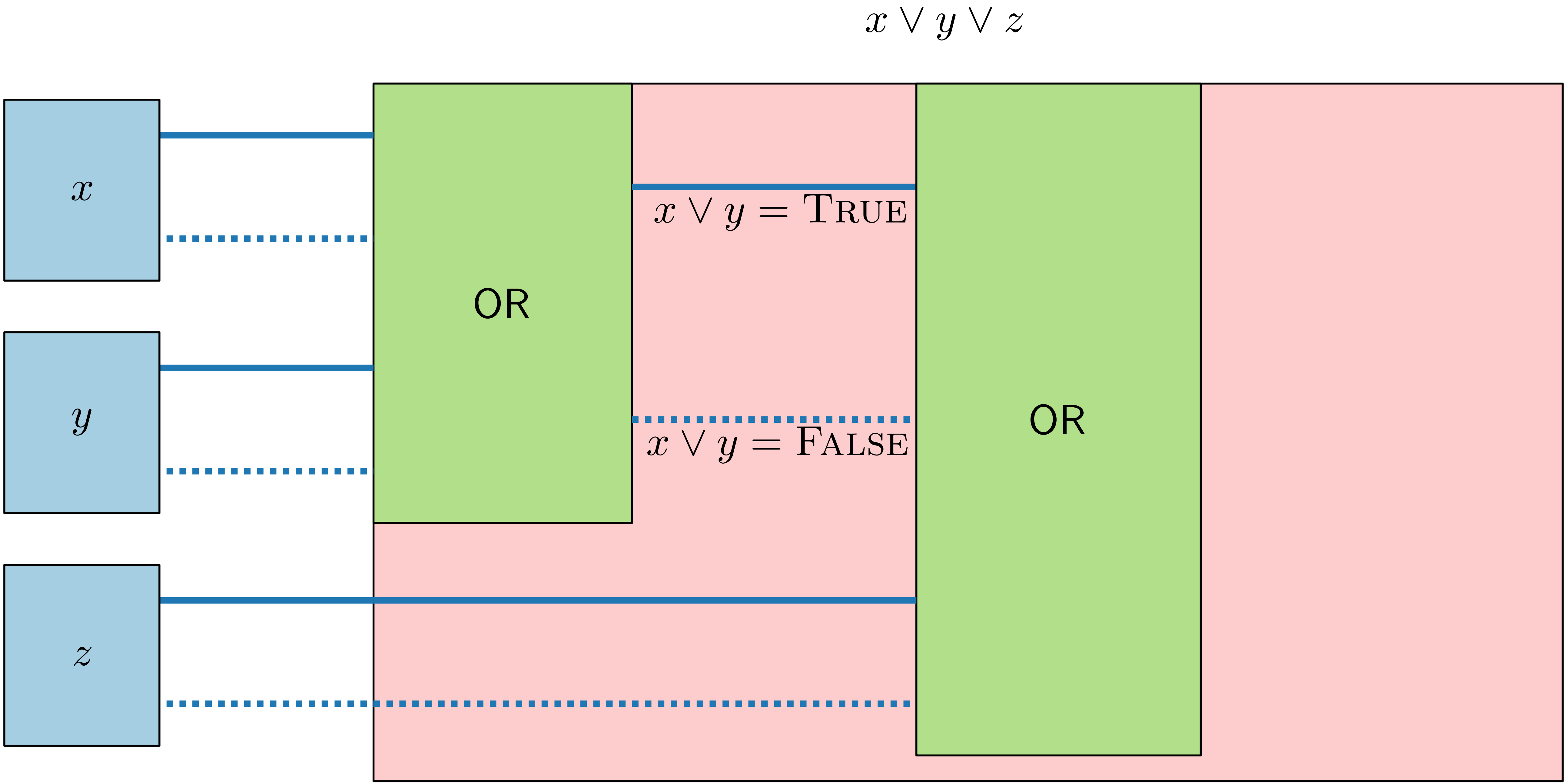

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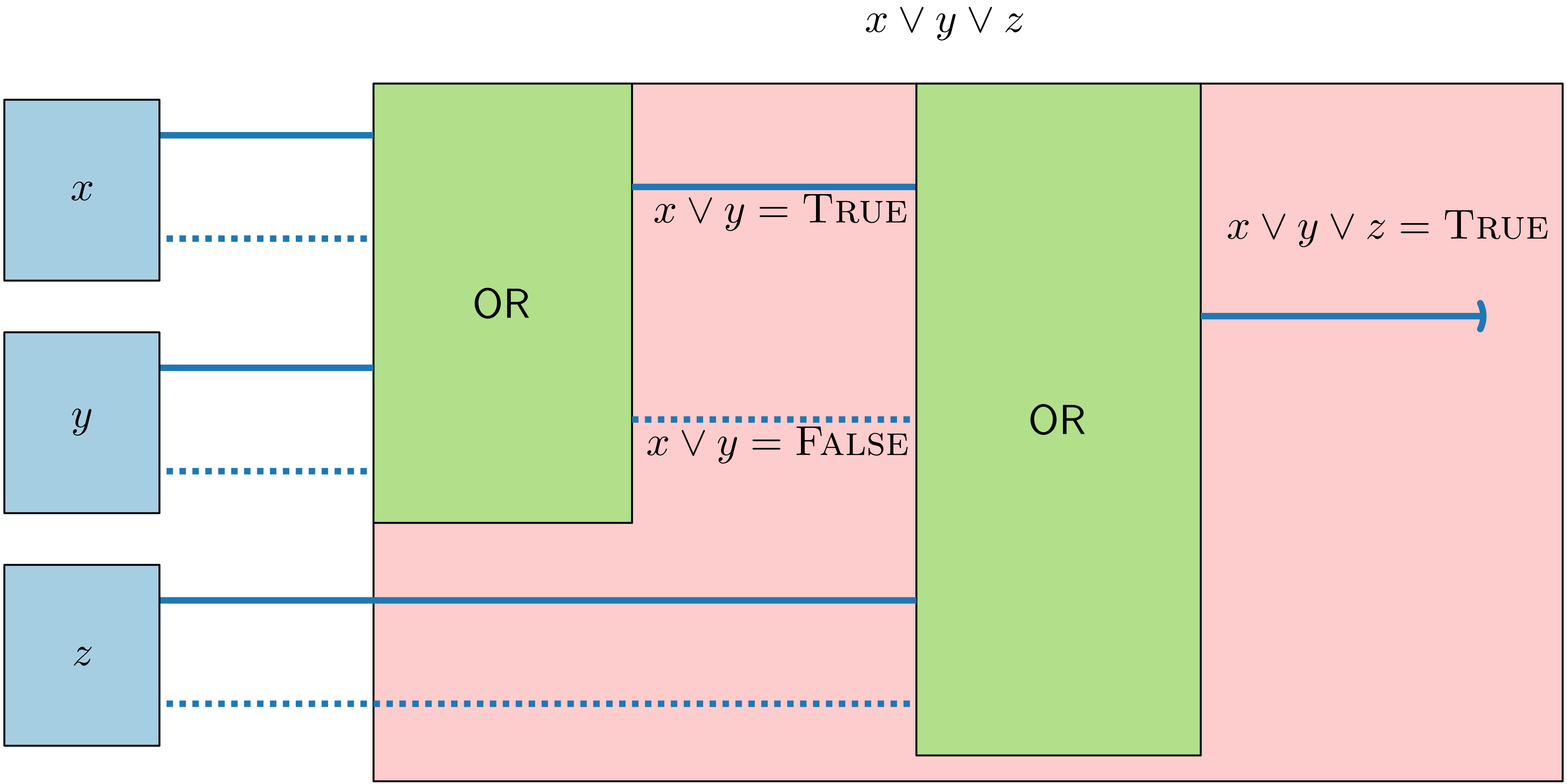

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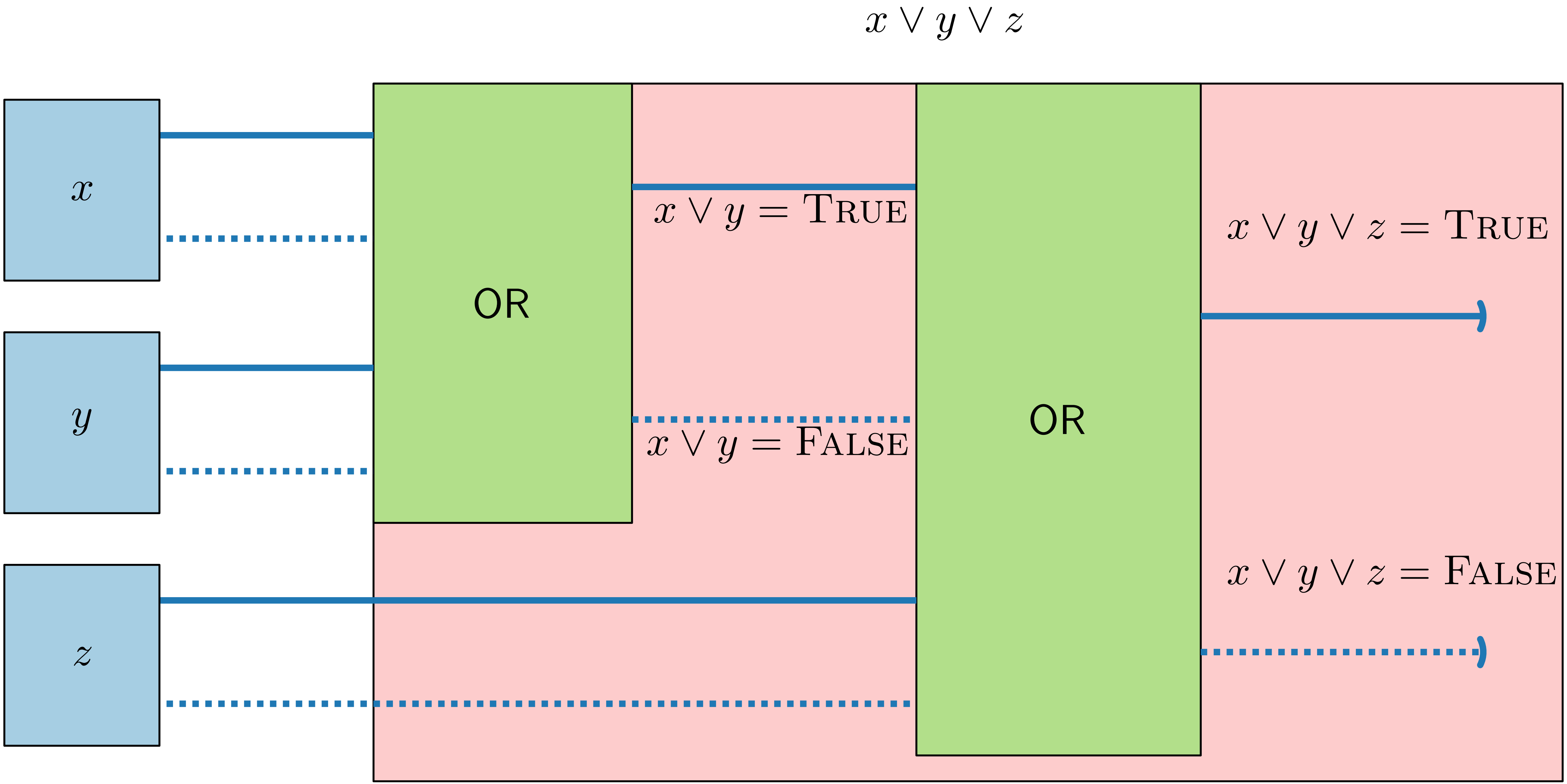

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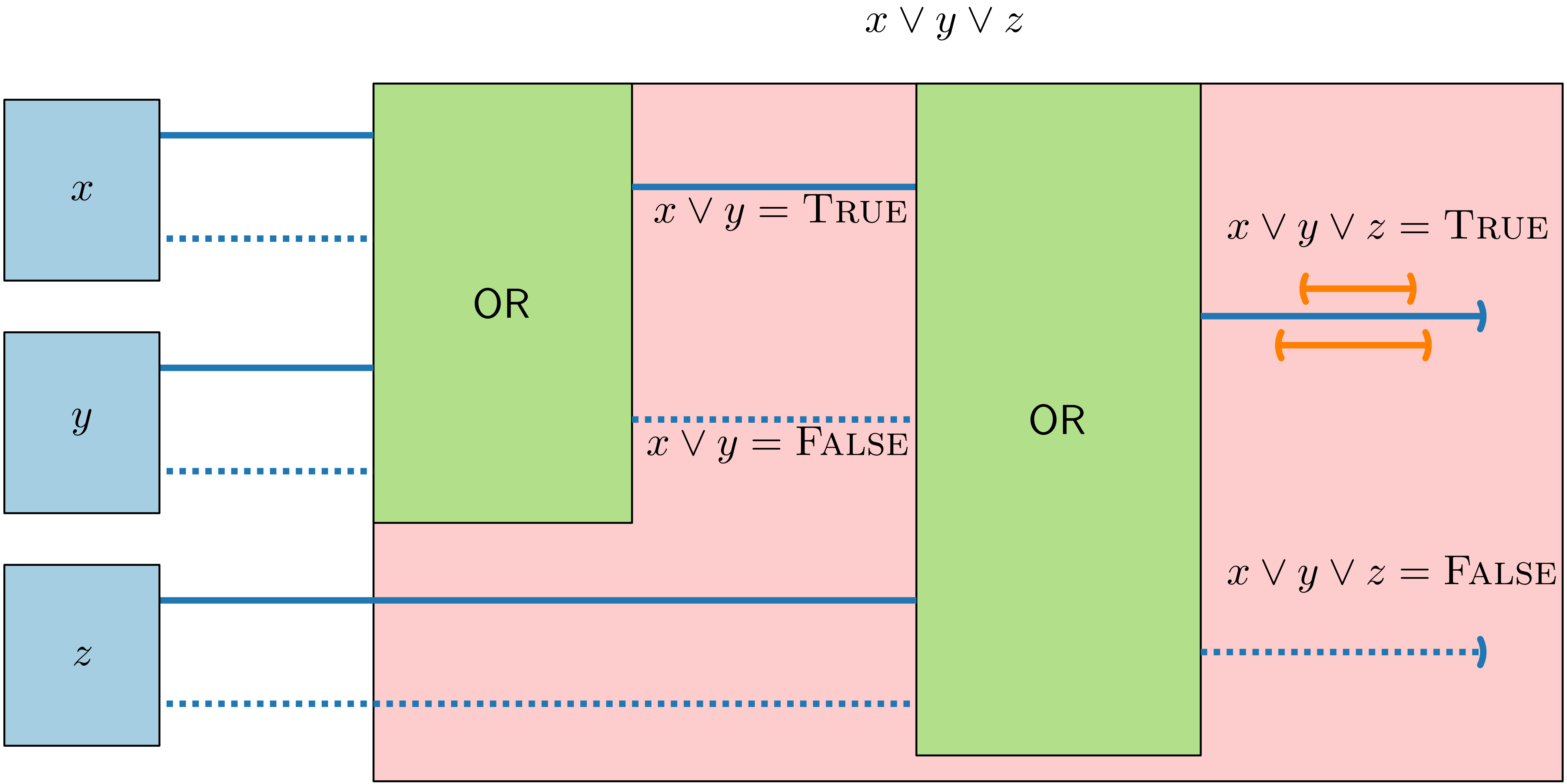

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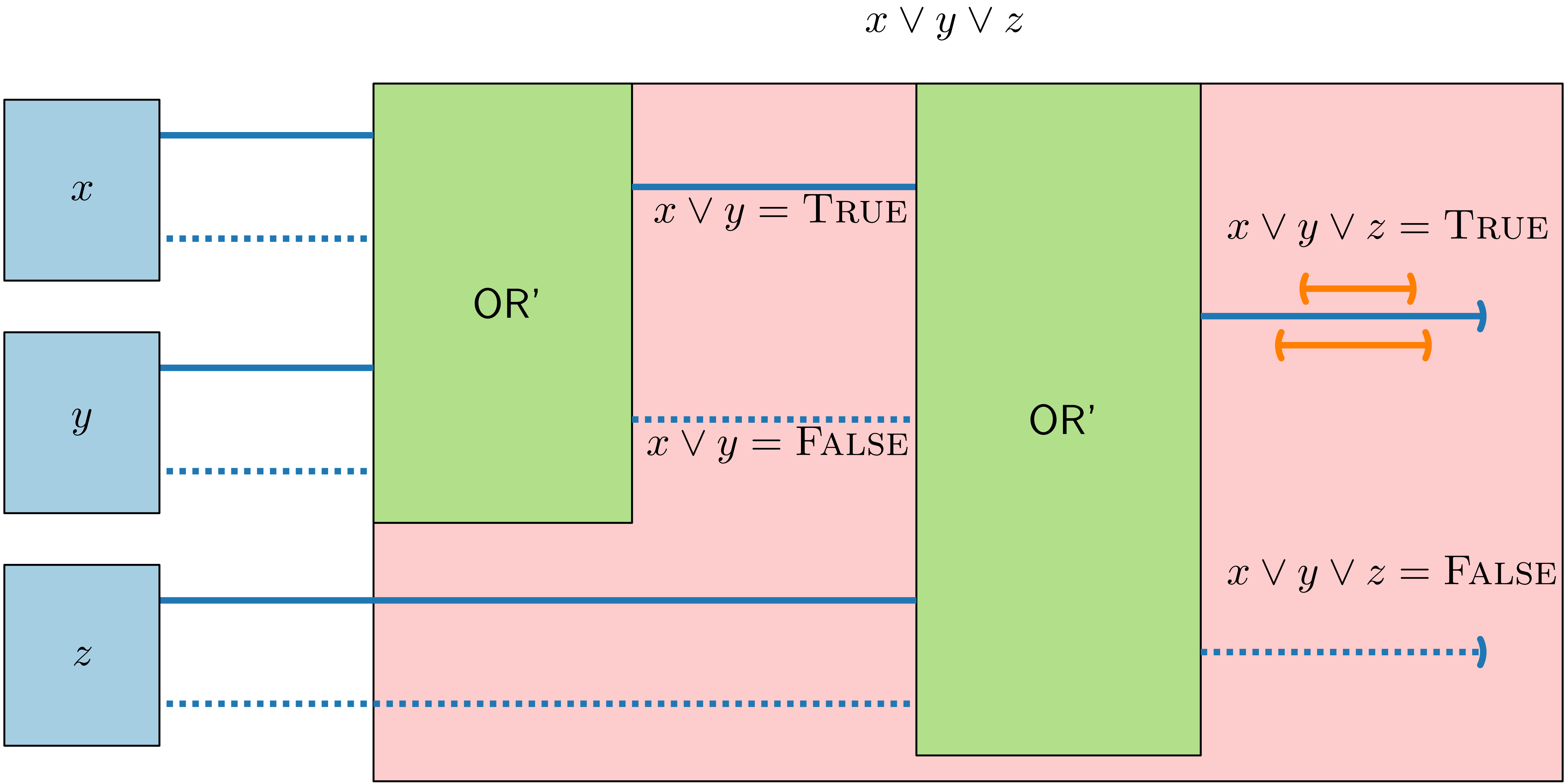

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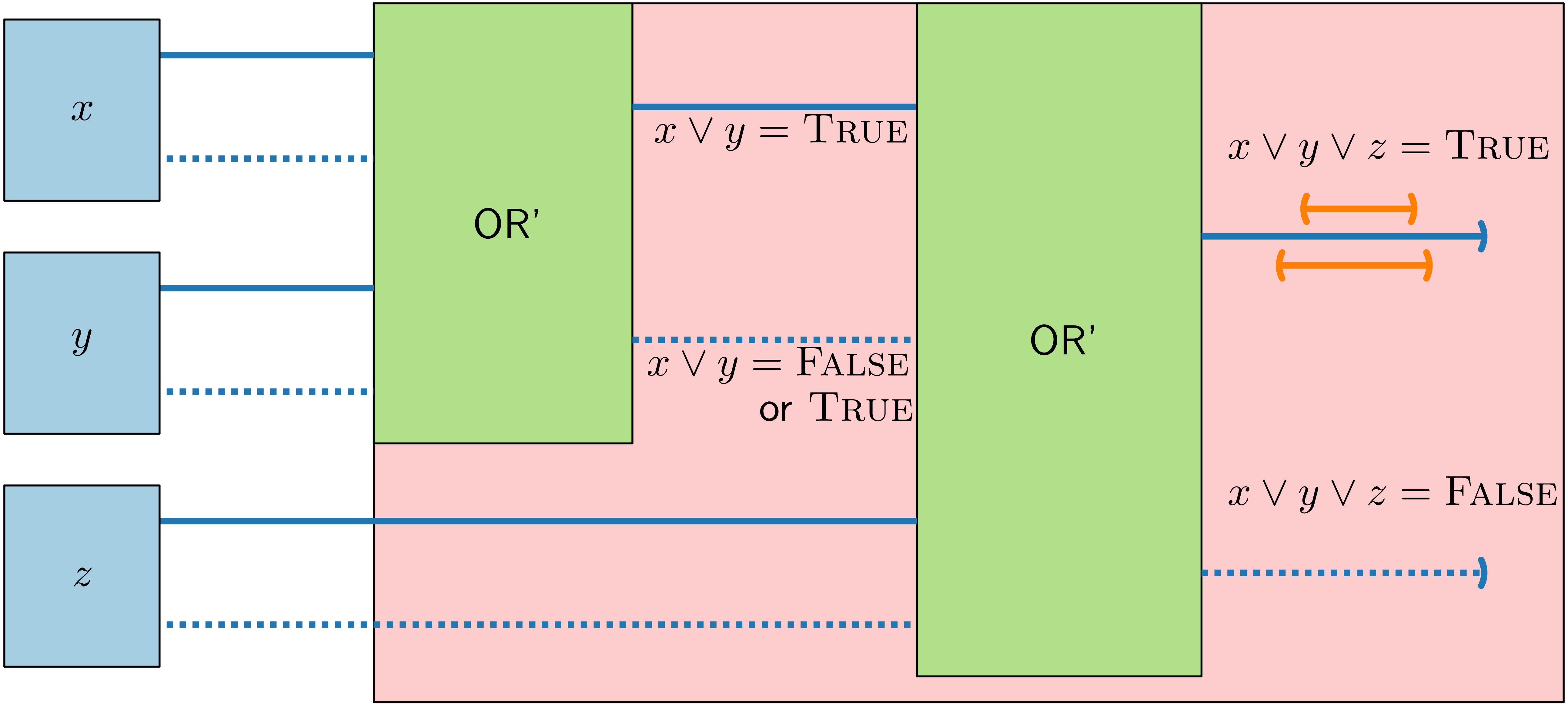

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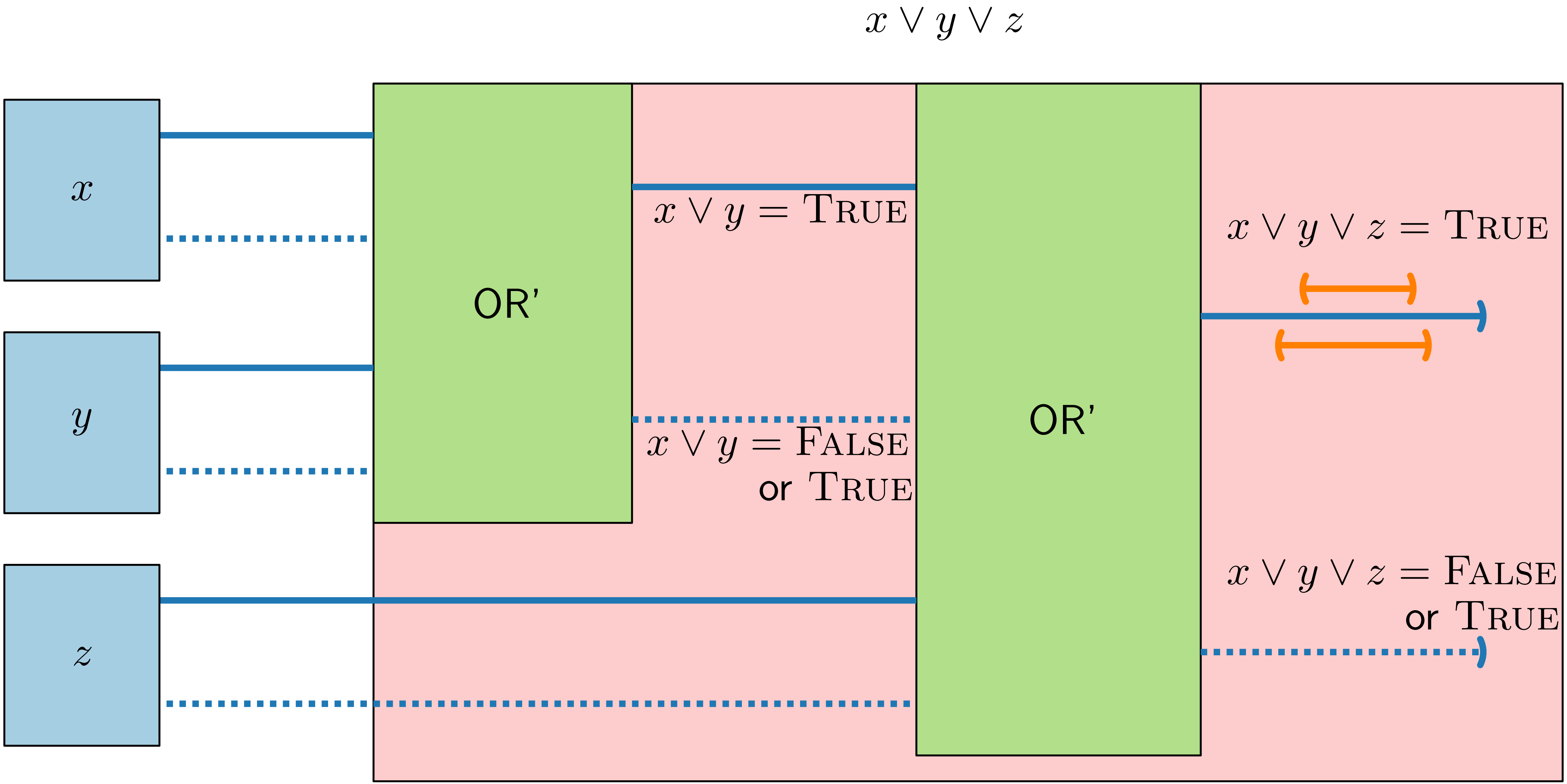

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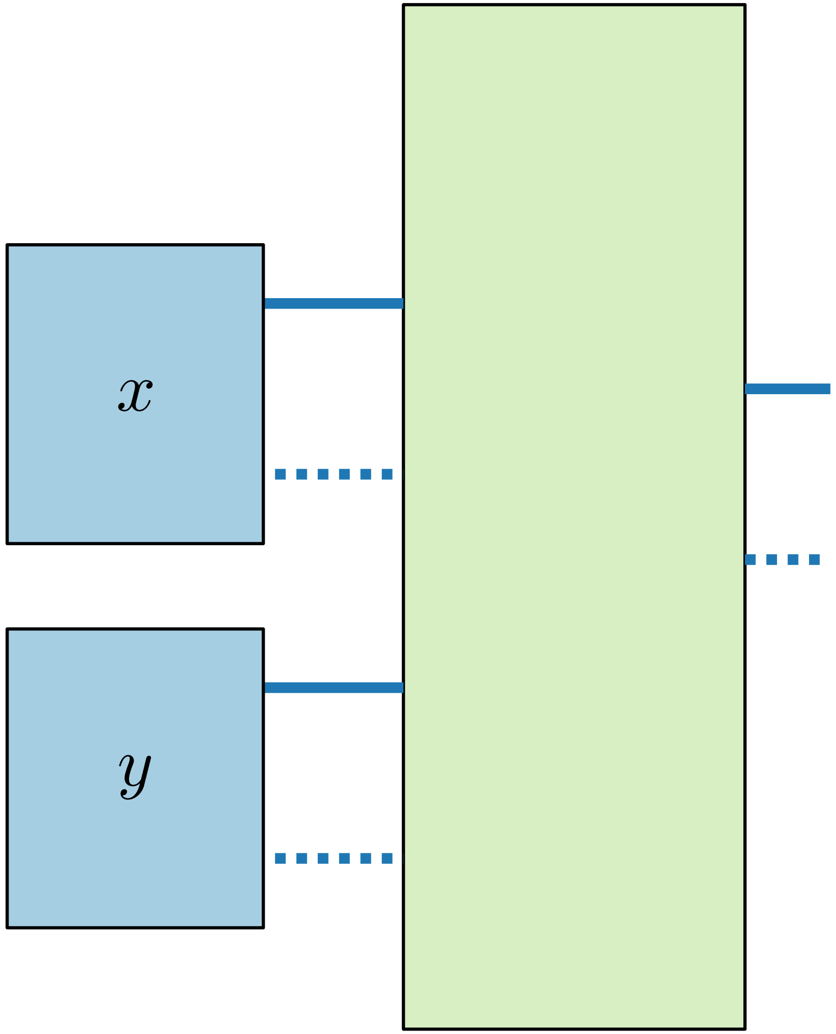

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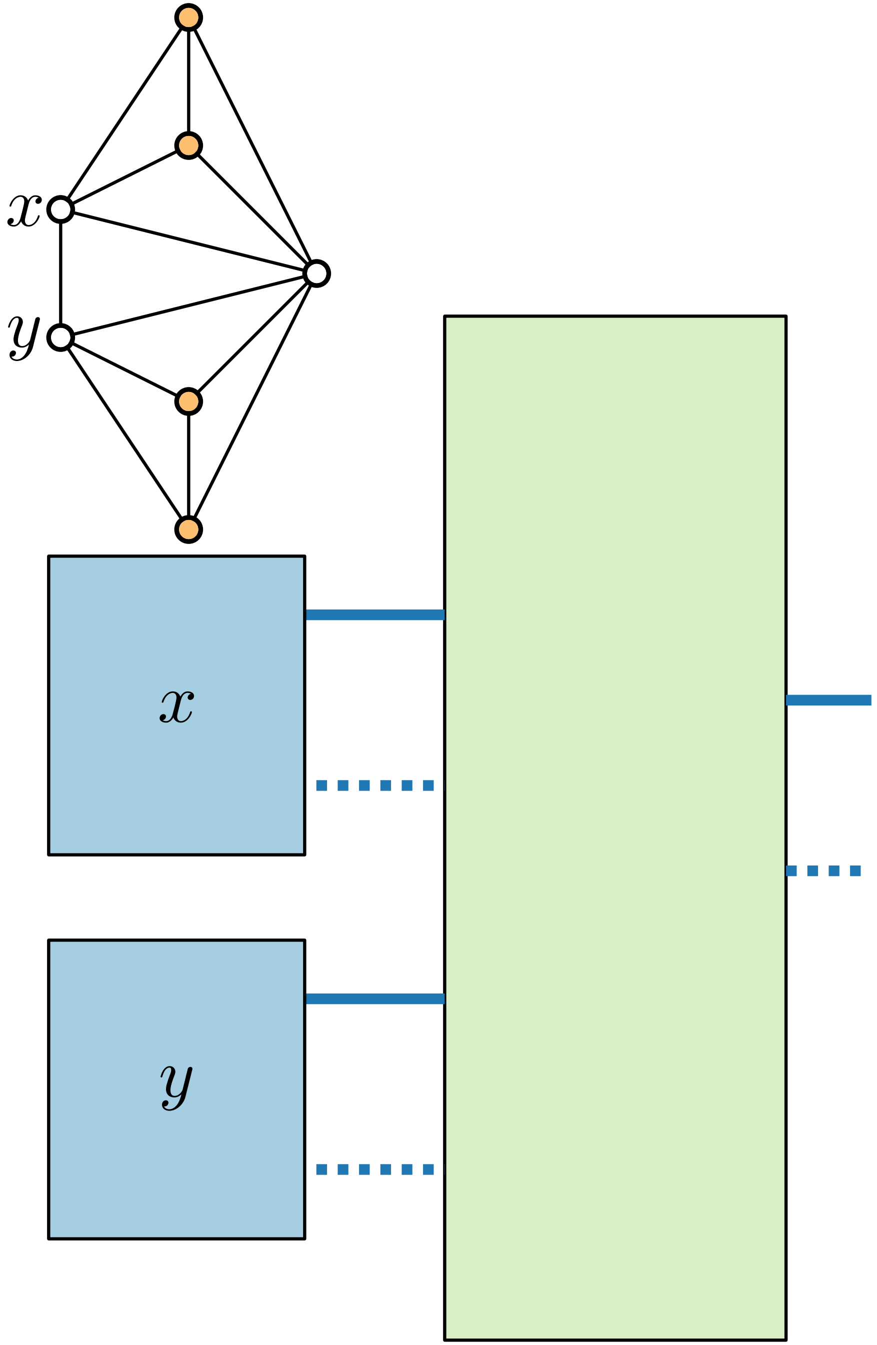

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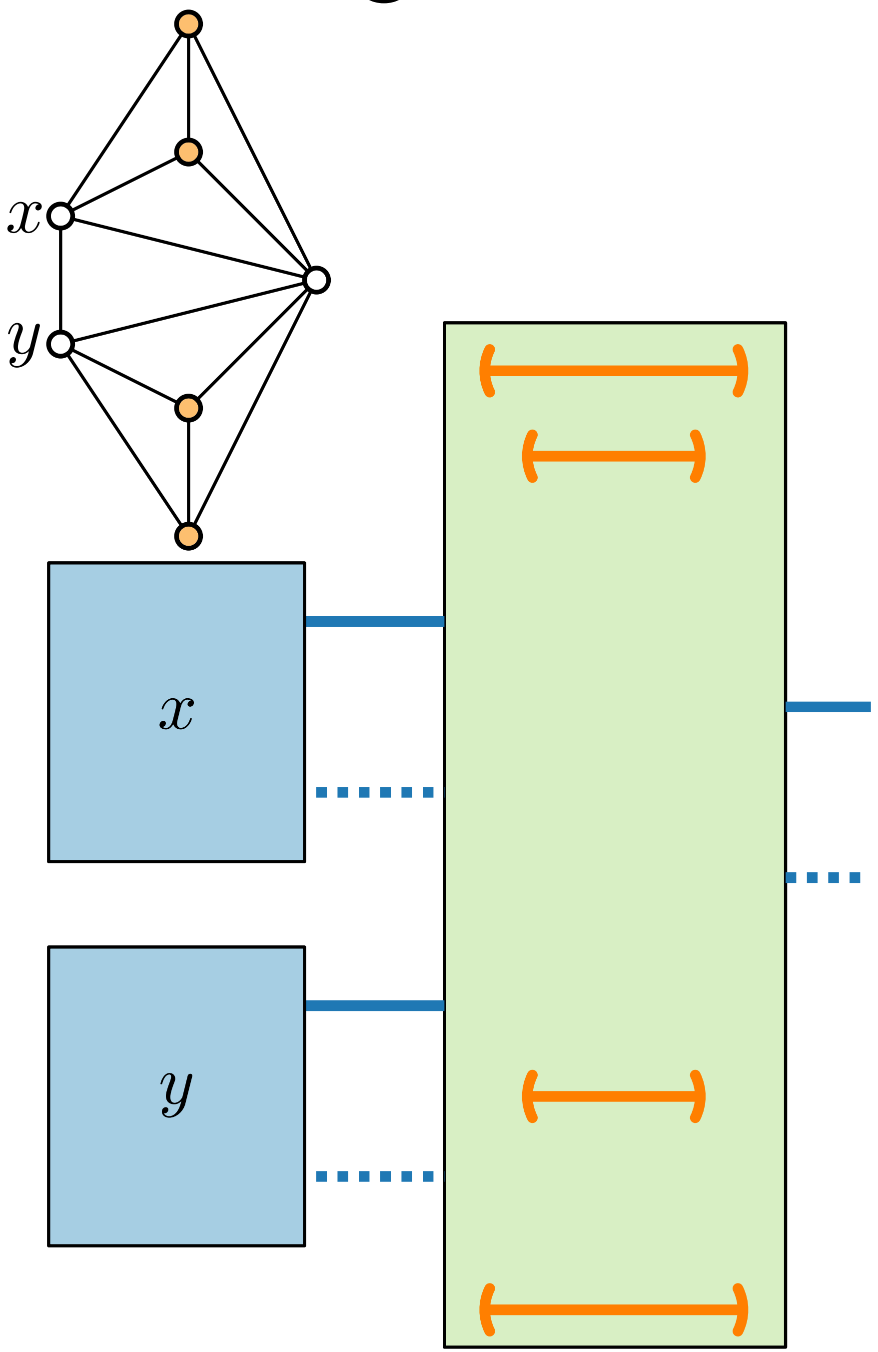

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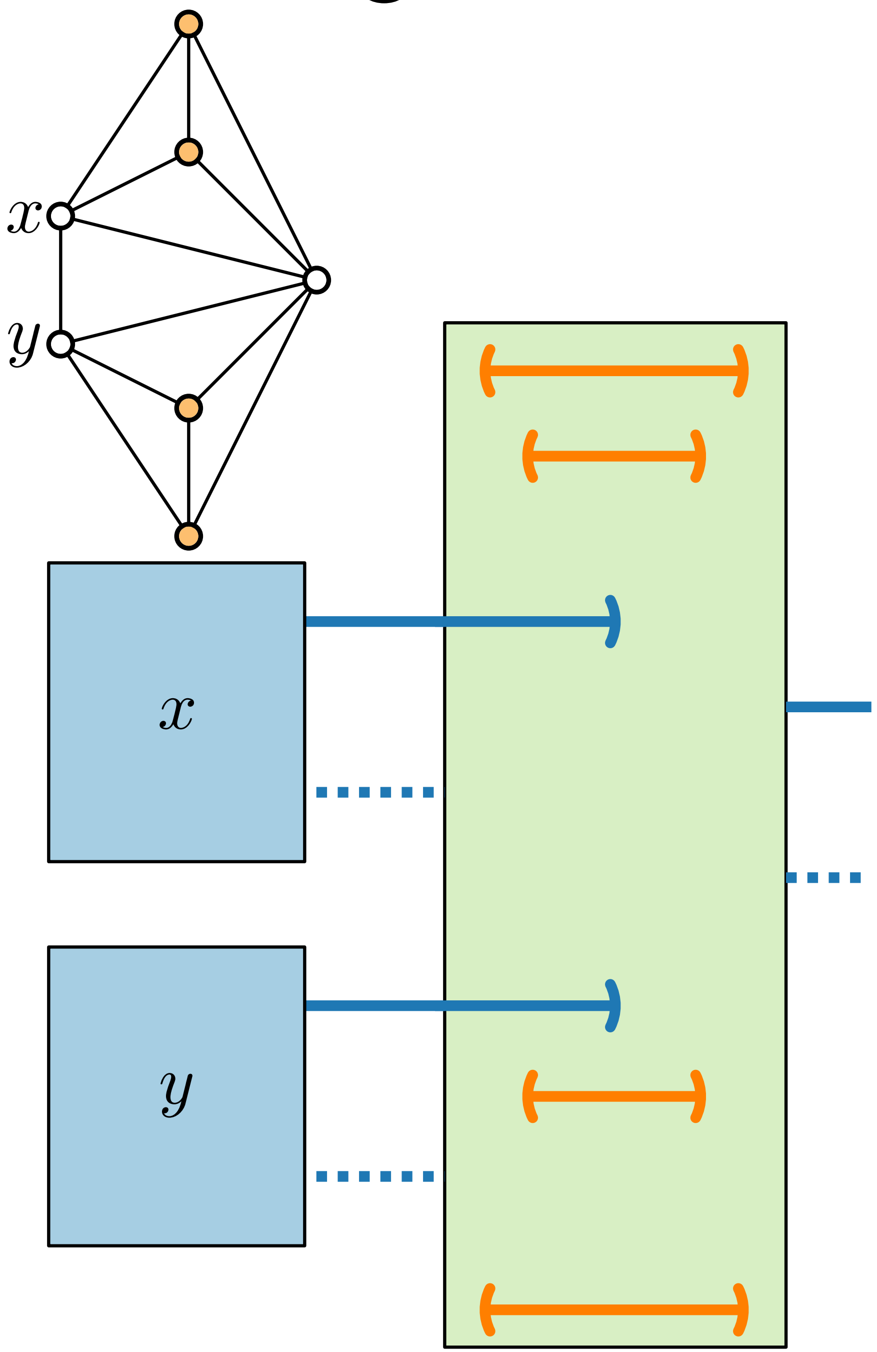

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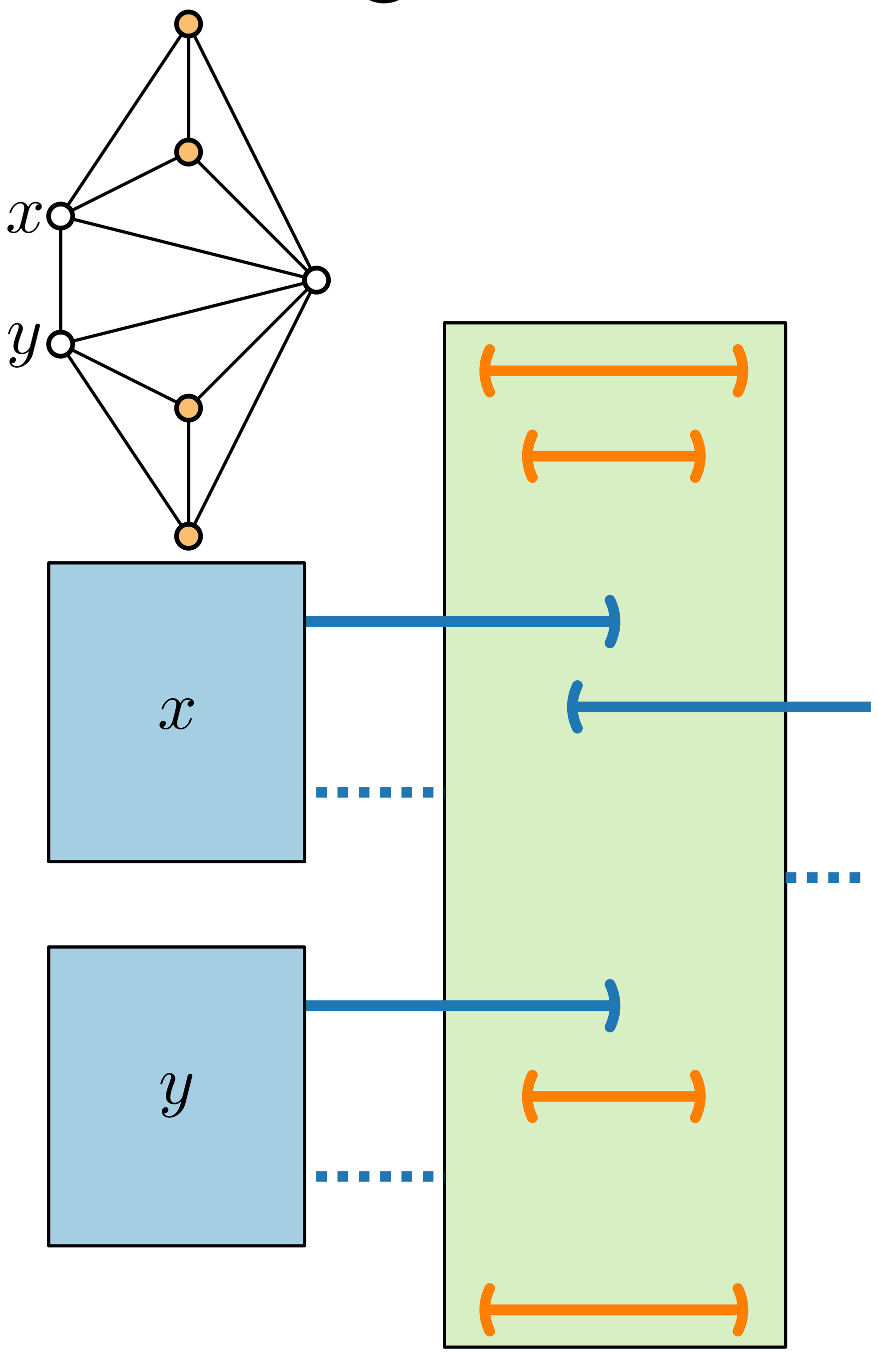

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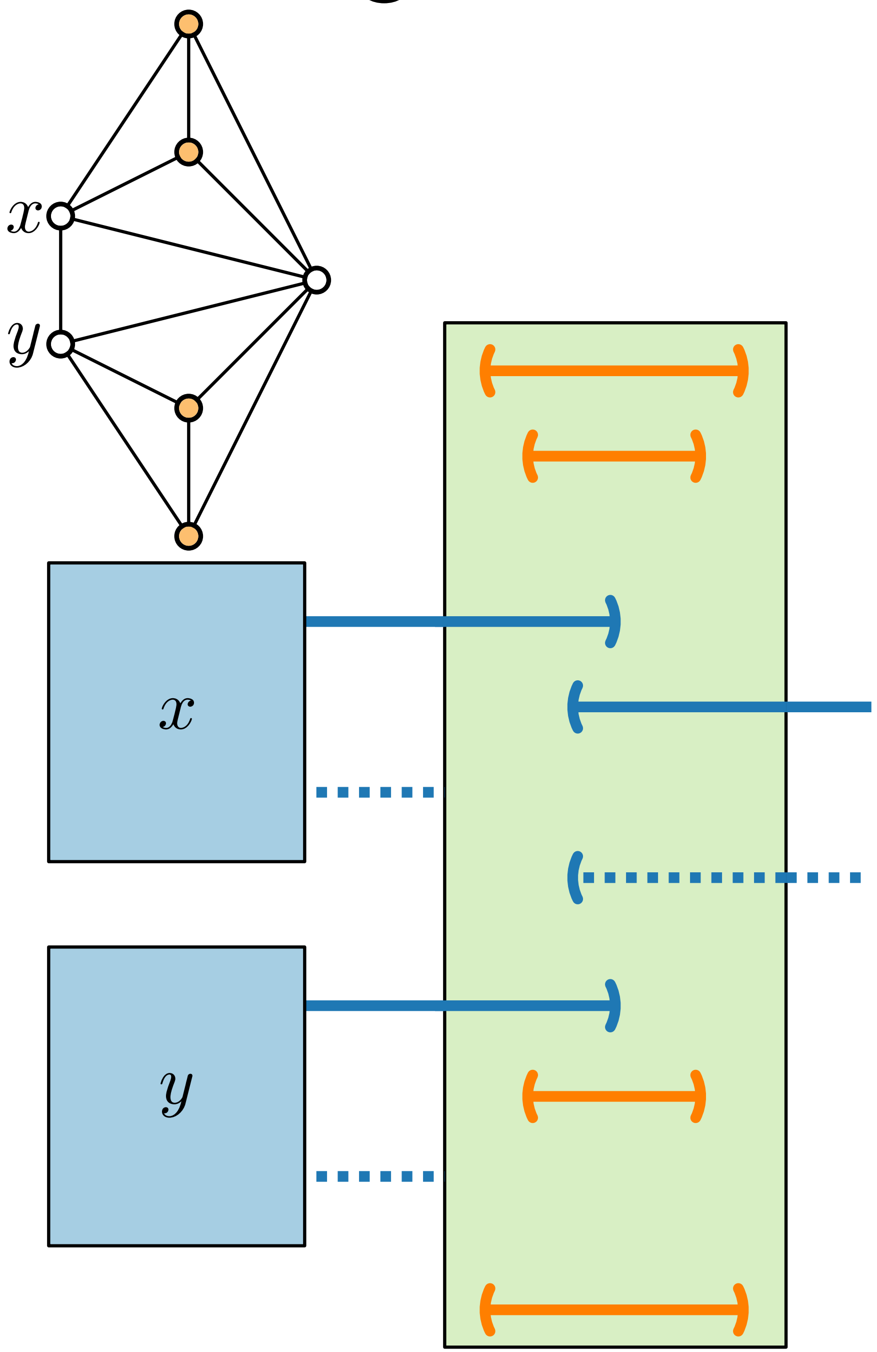

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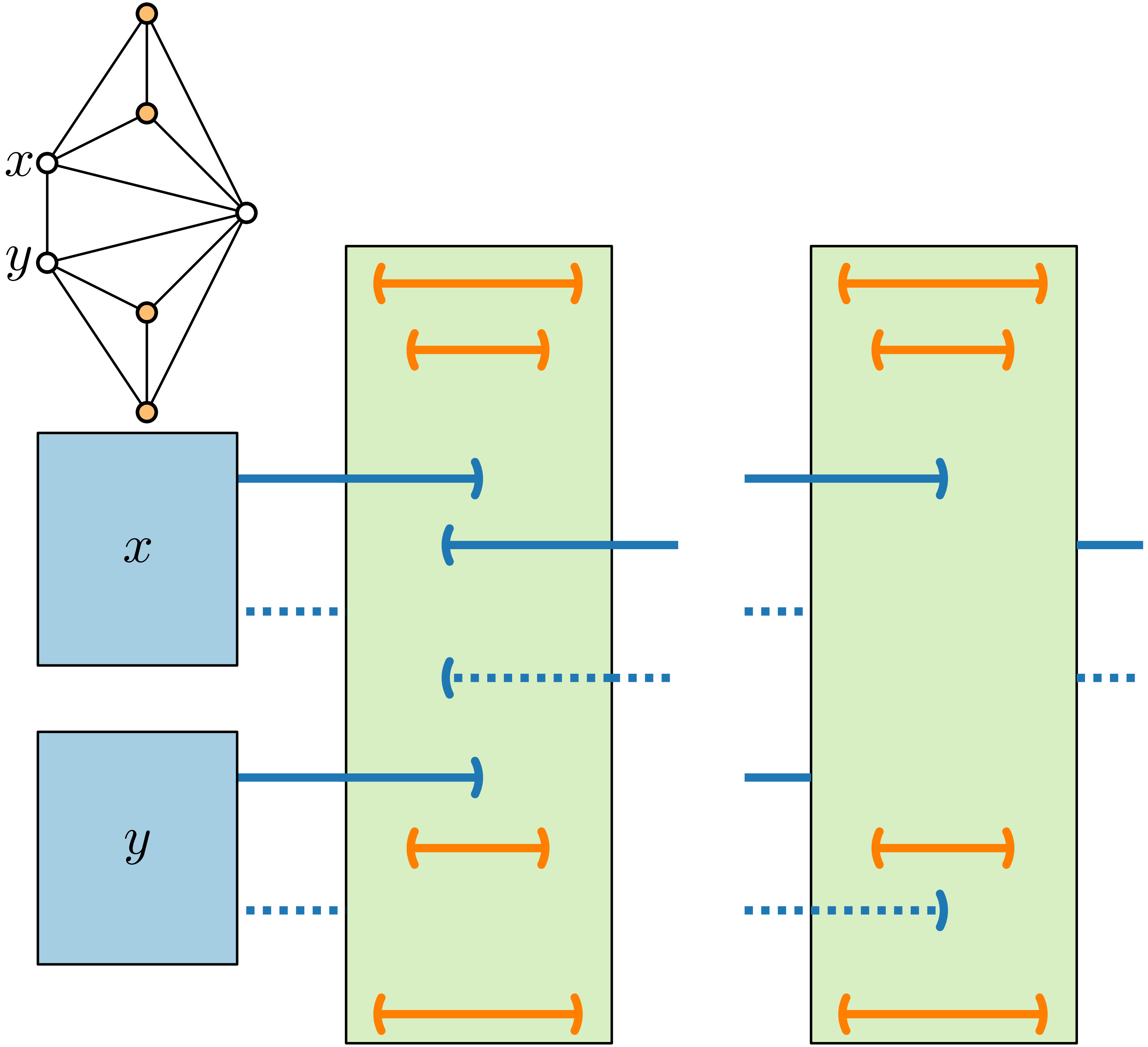

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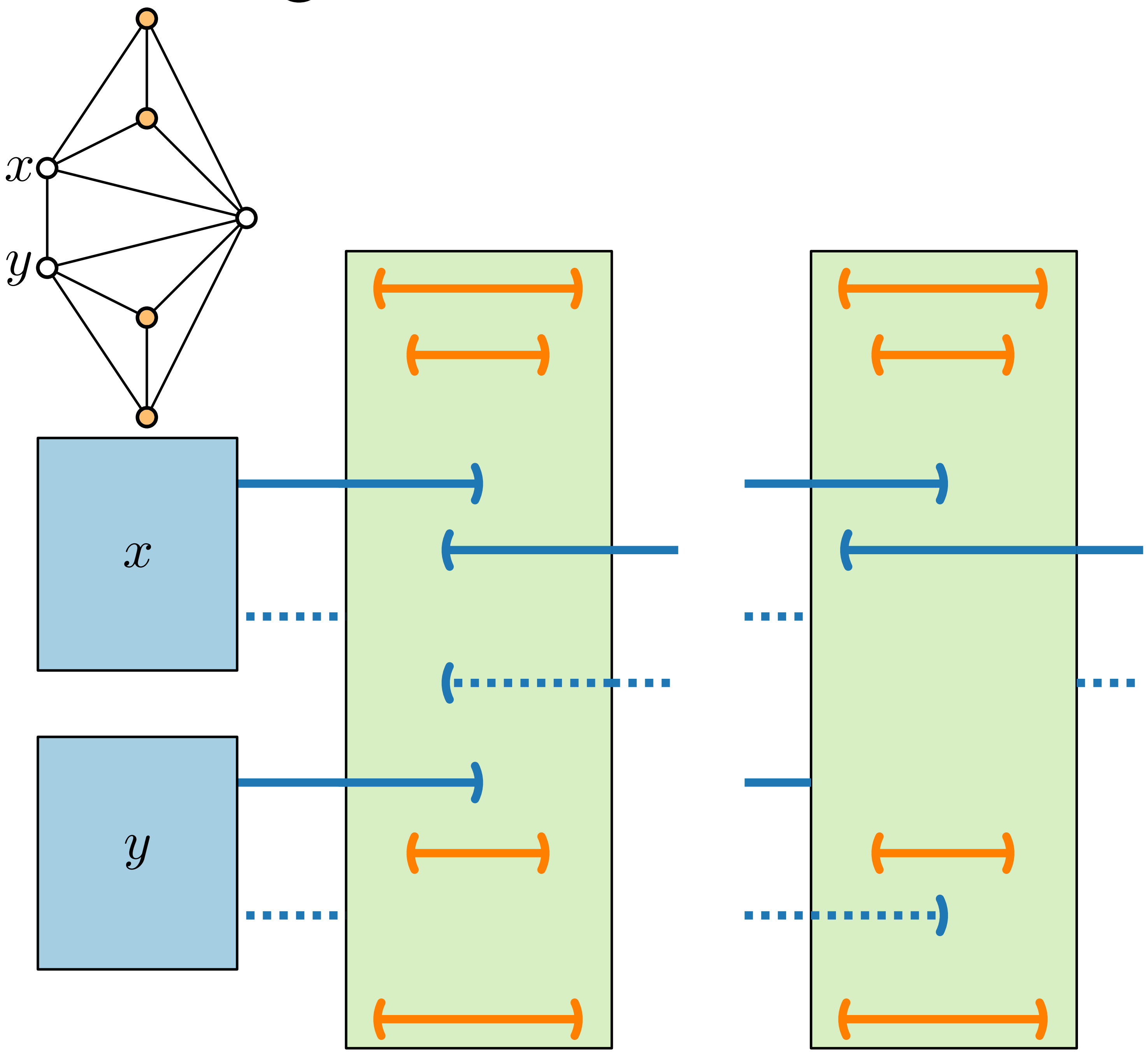

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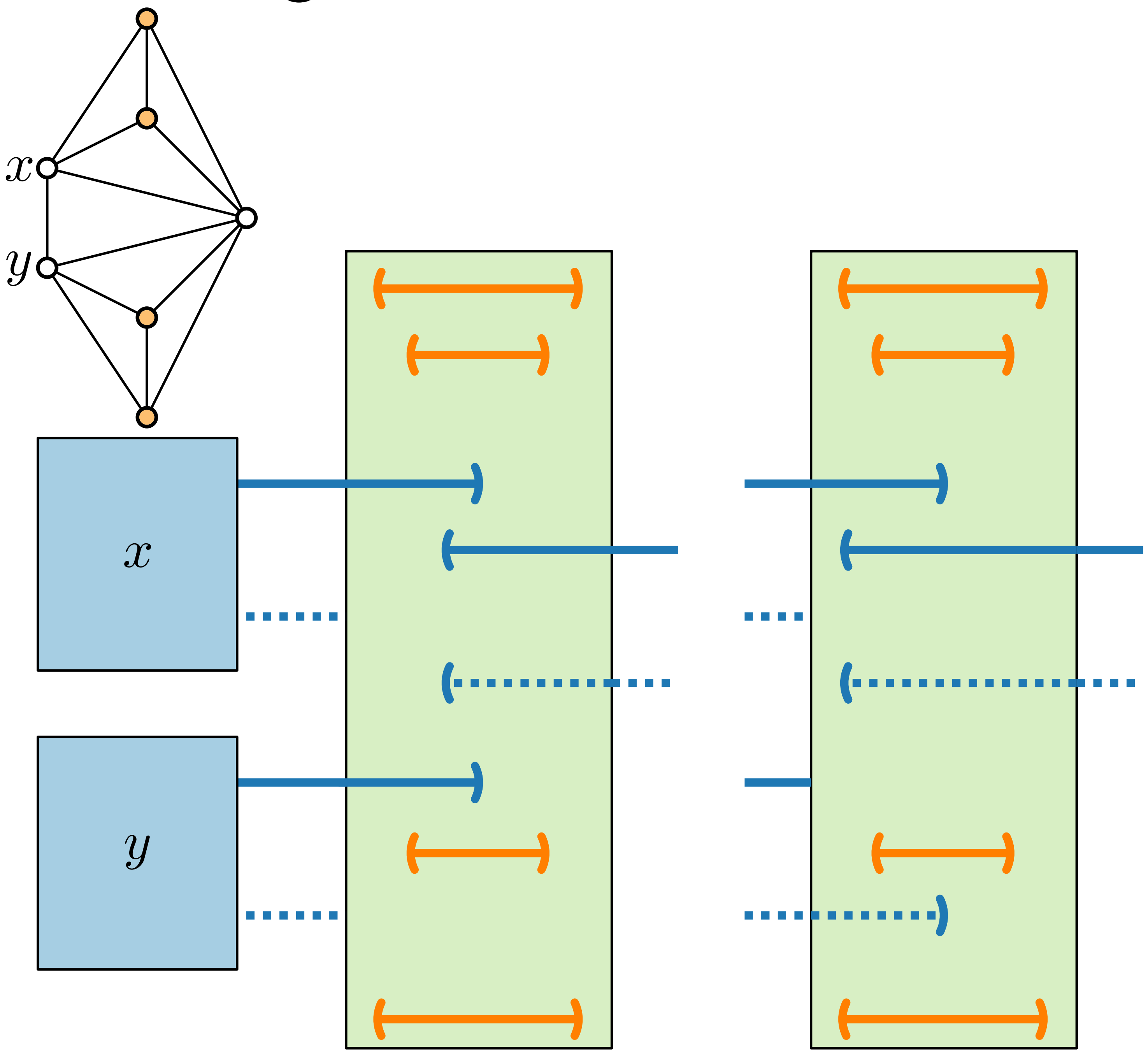

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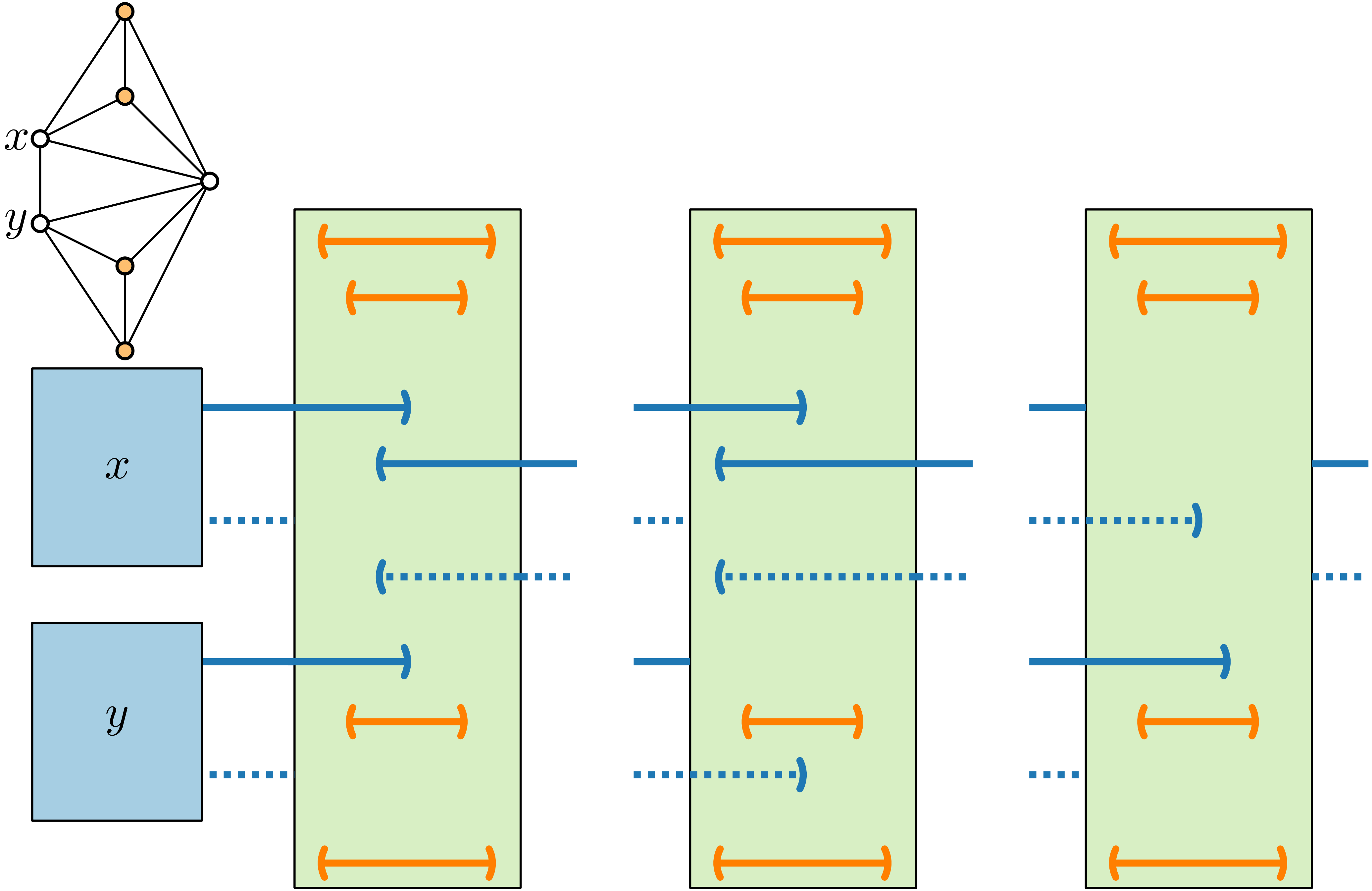

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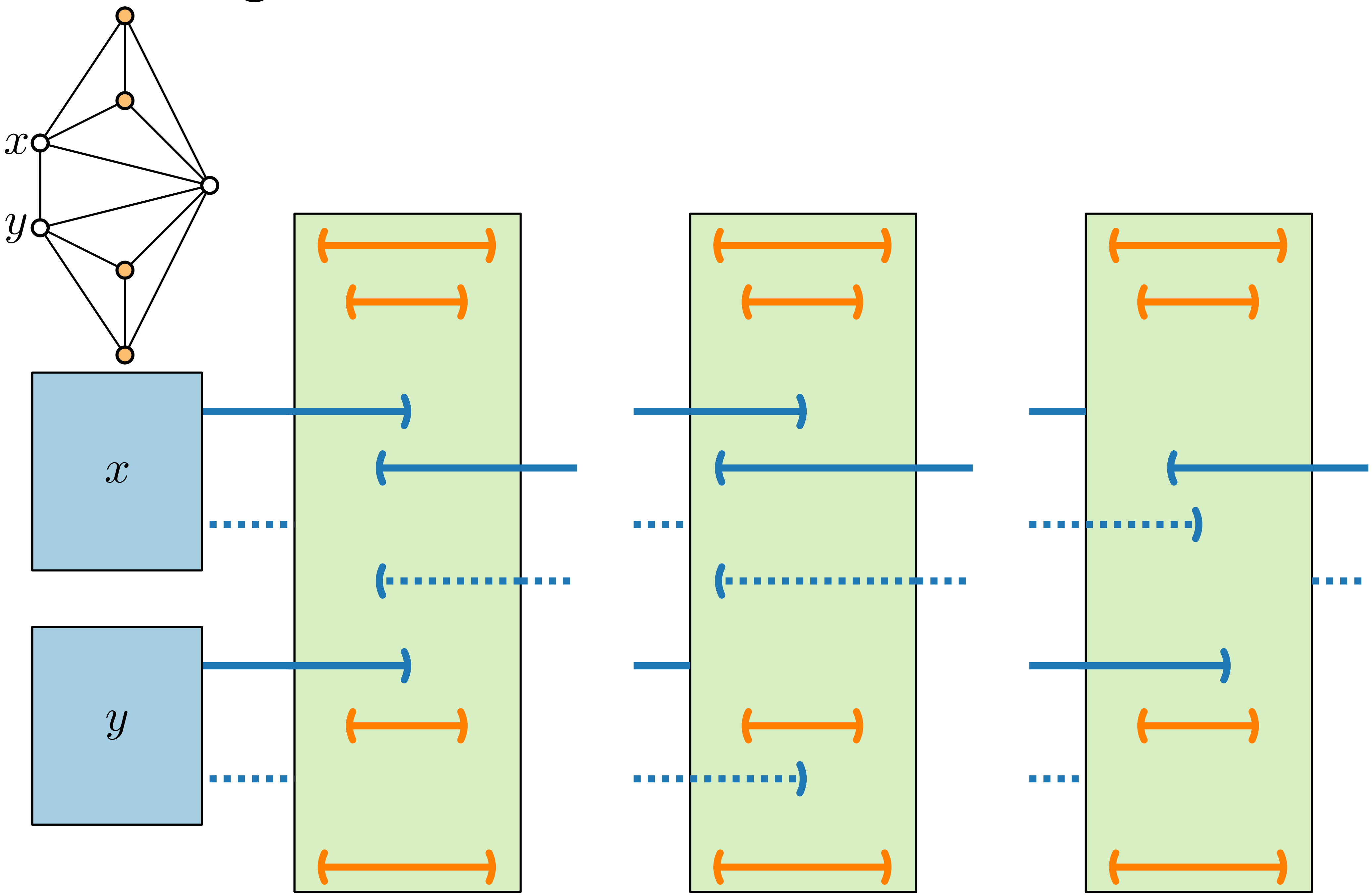

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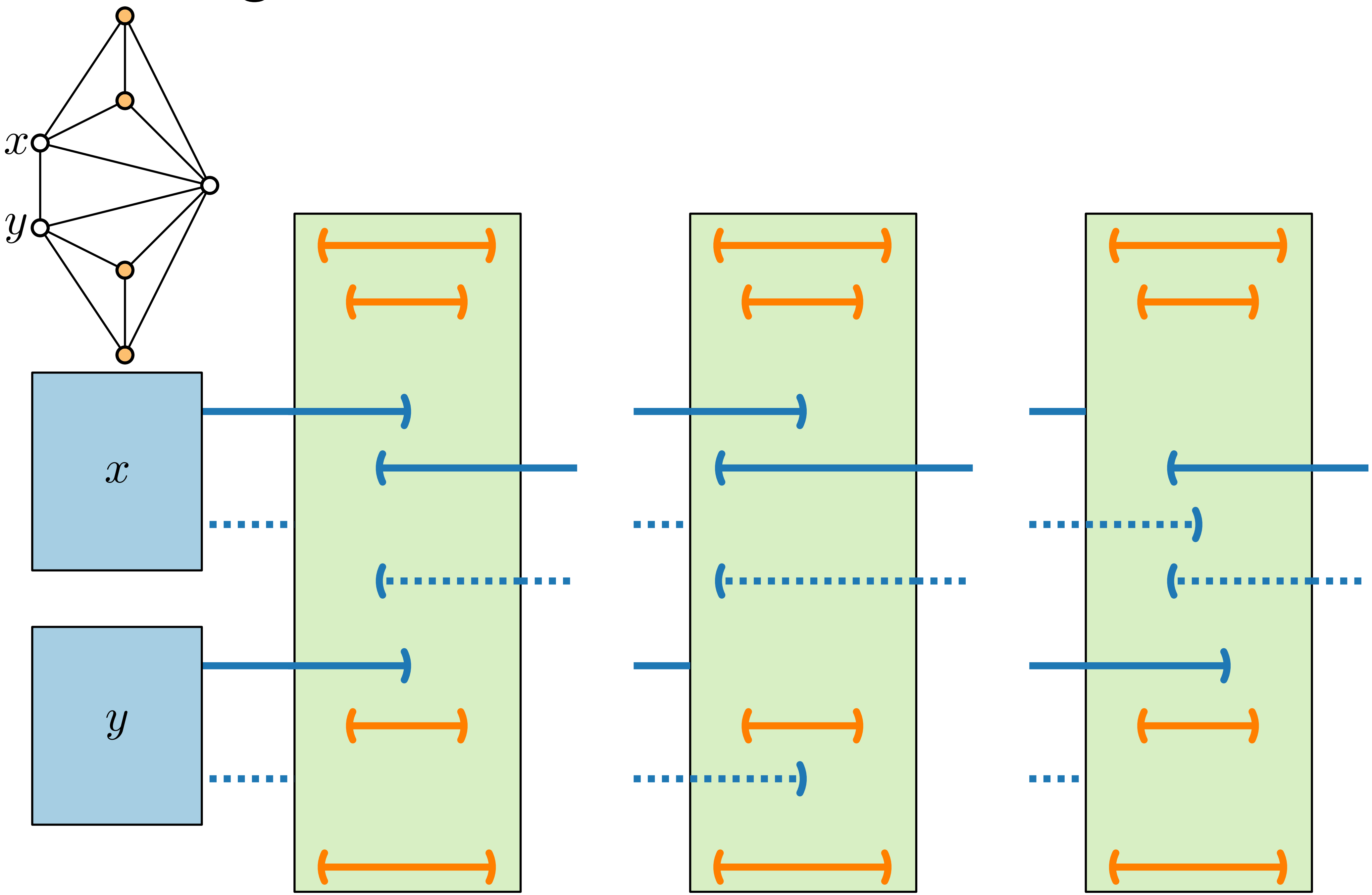

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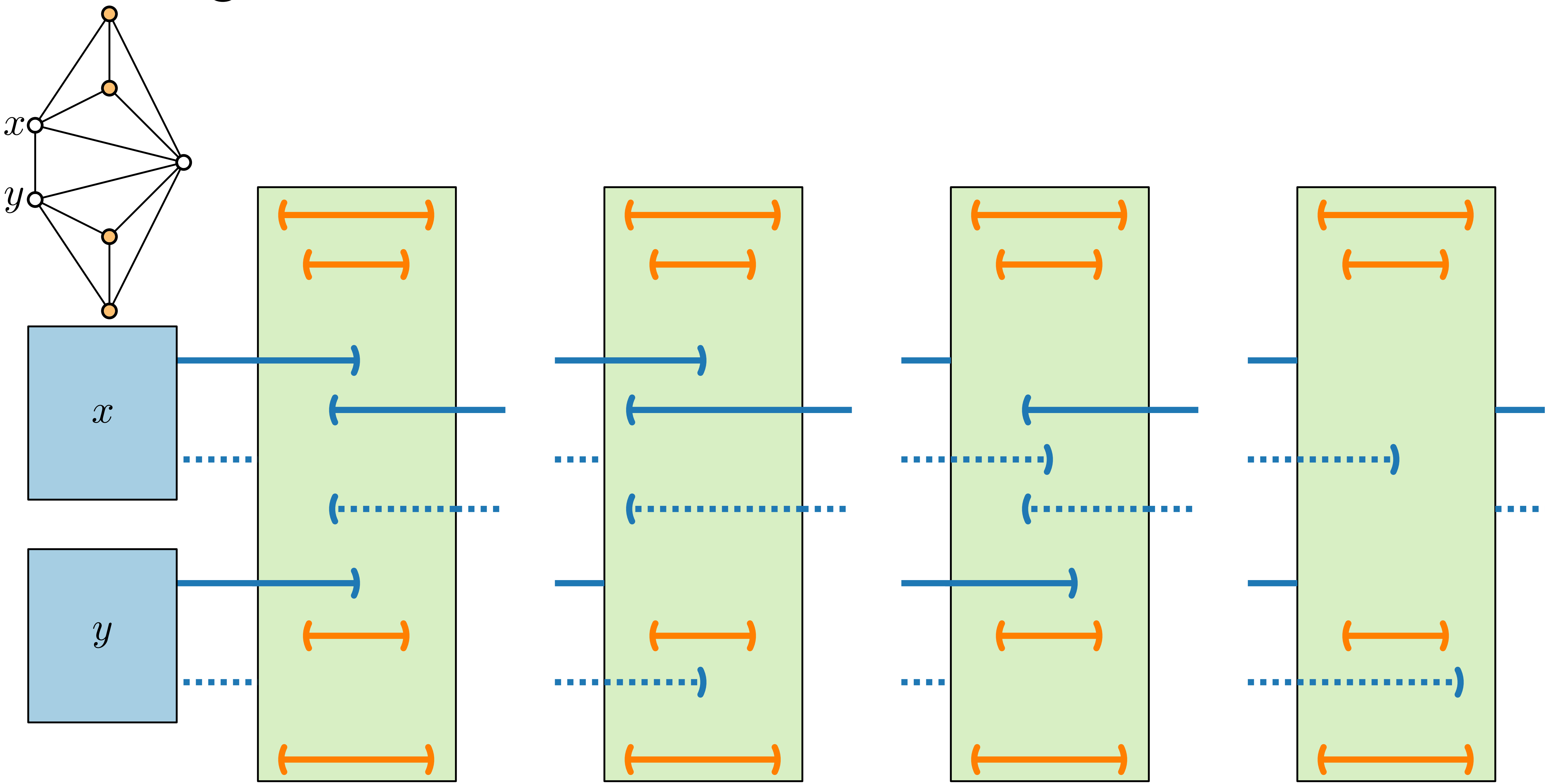

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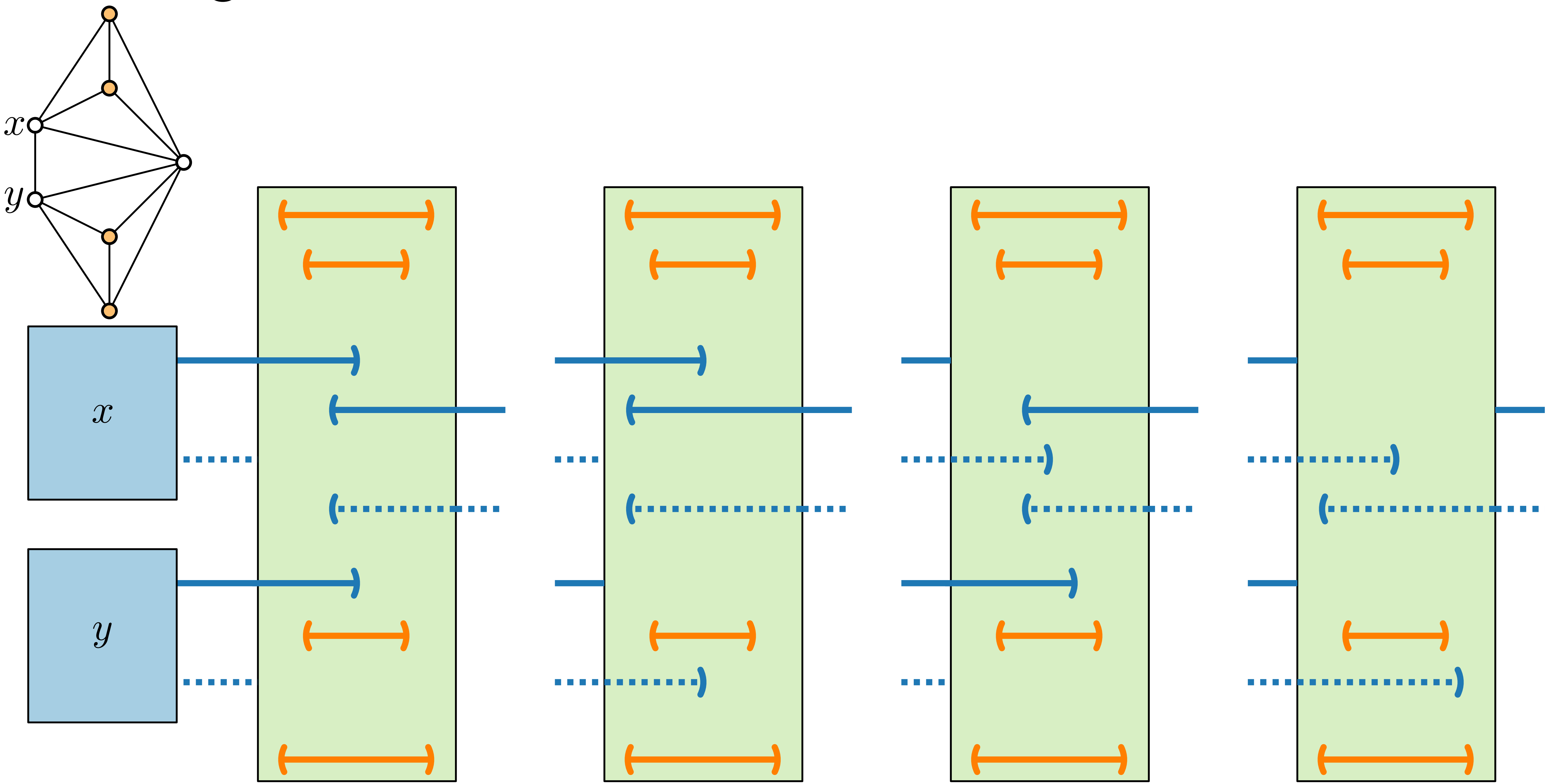

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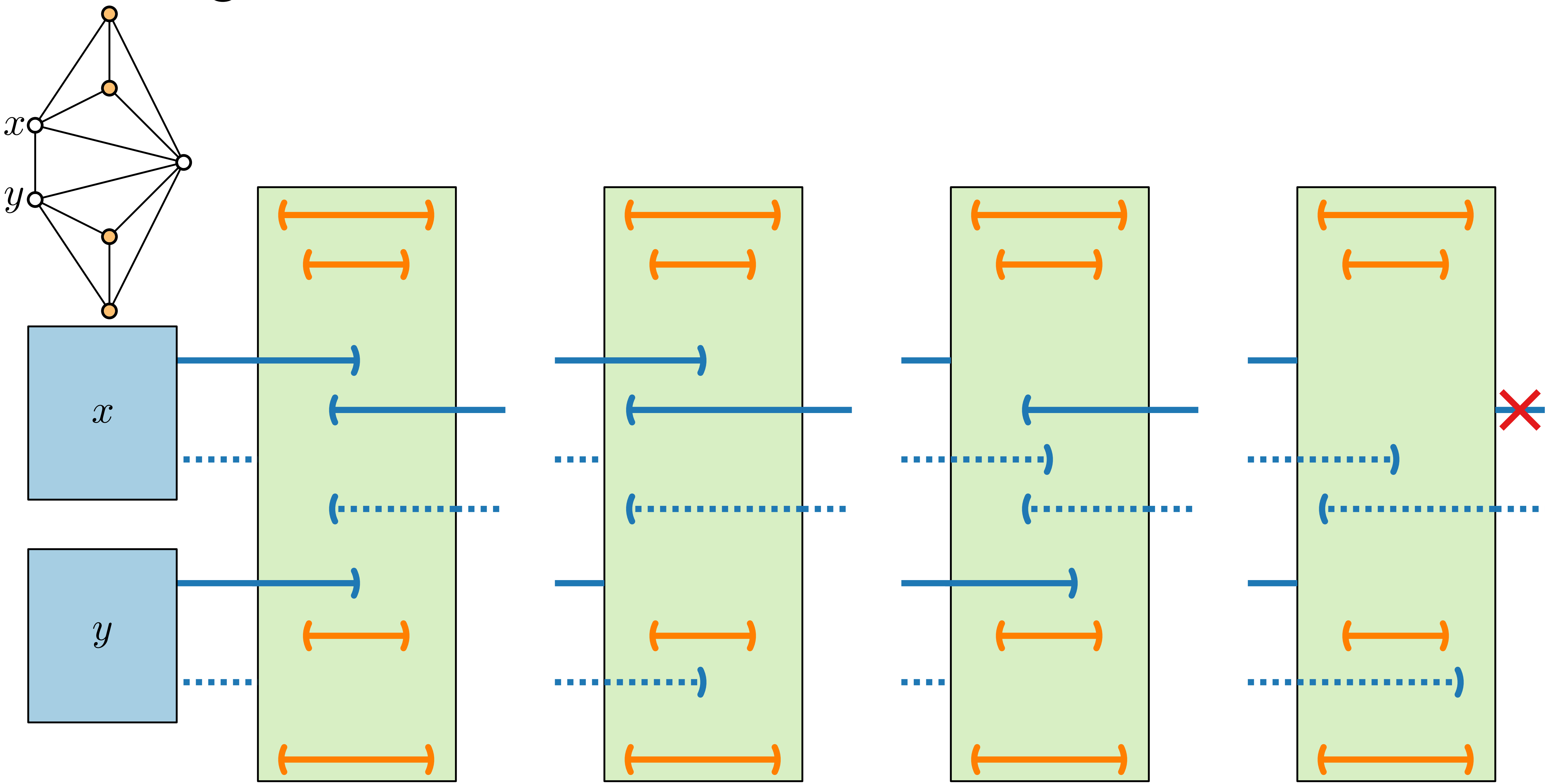

Discussion

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- Can **Strong** Bar Visibility Recognition / Representation Extension can be solved in polynomial time for st-graphs?

Literature

Main source:

- [Chaplick, Guśpiel, Gutowski, Krawczyk, Liotta '18]
  The Partial Visibility Representation Extension Problem

Referenced papers:

- [Gutwenger, Mutzel '01] A Linear Time Implementation of SPQR-Trees
- [Wismath '85] Characterizing bar line-of-sight graphs
- [Tamassia, Tollis '86] Algorithms for visibility representations of planar graphs
- [Andreae '92] Some results on visibility graphs
- [Chaplick, Dorbec, Kratchovíl, Montassier, Stacho '14]
  Contact representations of planar graphs: Extending a partial representation is hard