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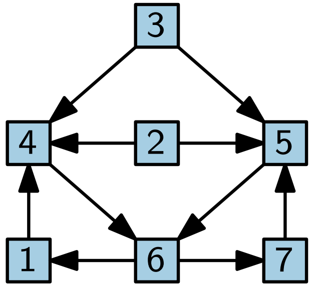
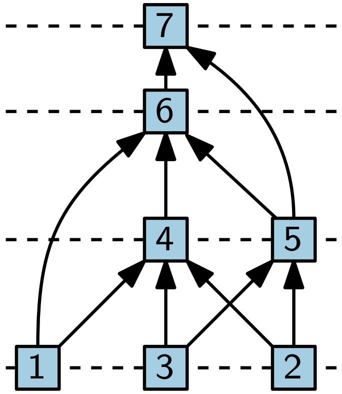
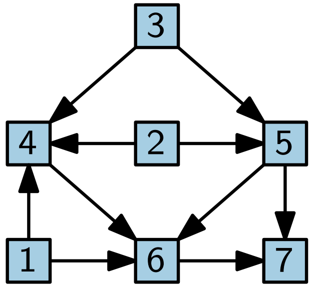
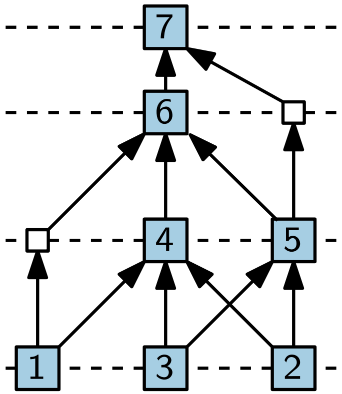


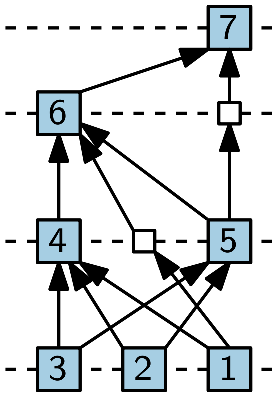


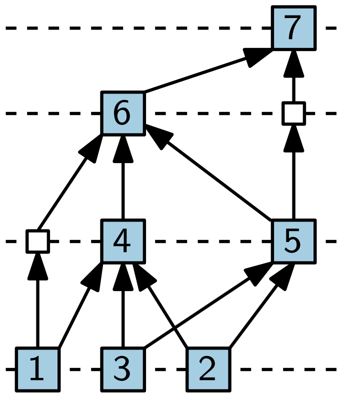
Visualization of Graphs





Lecture 8:
Hierarchical Layouts:
Sugiyama Framework

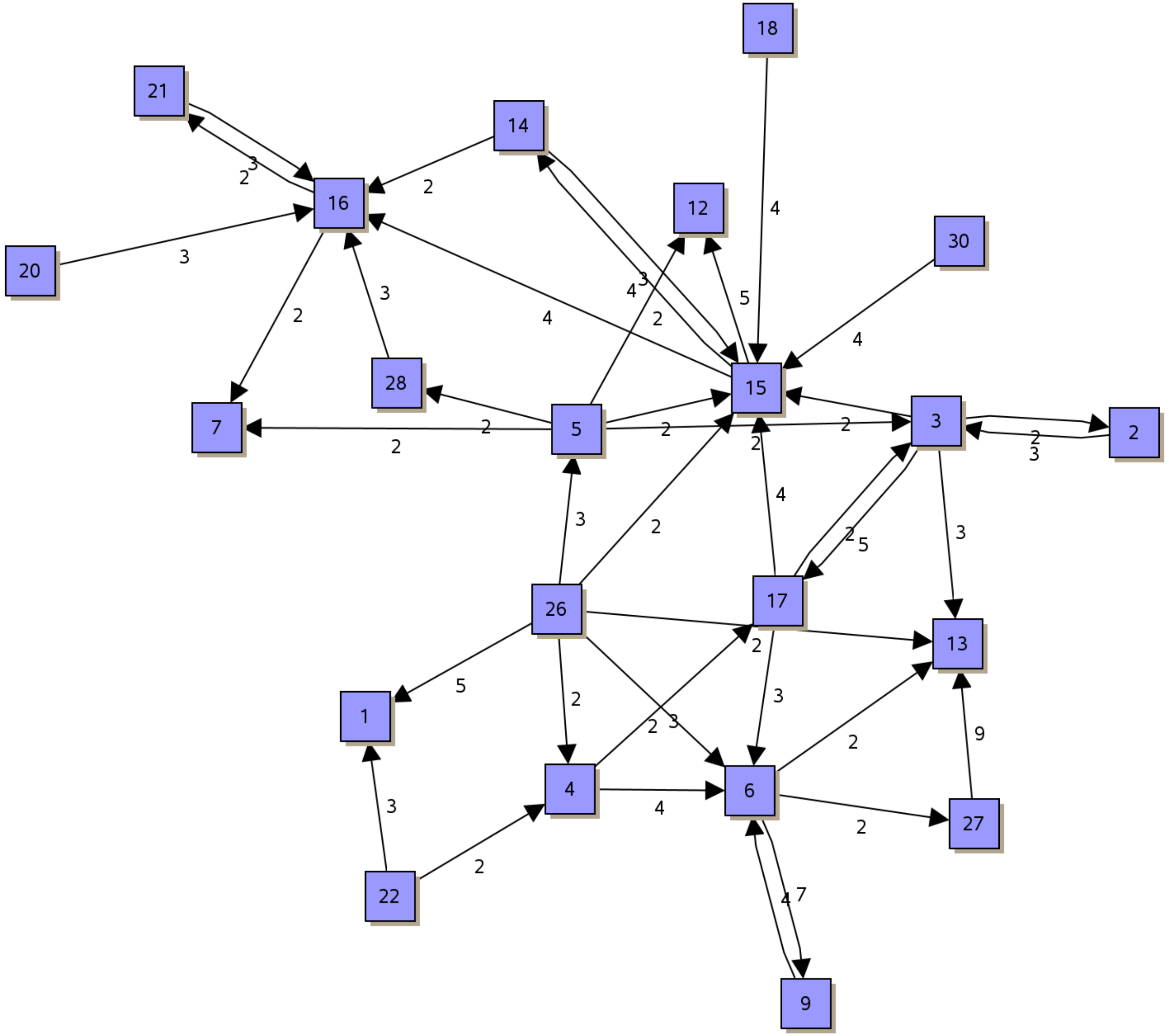




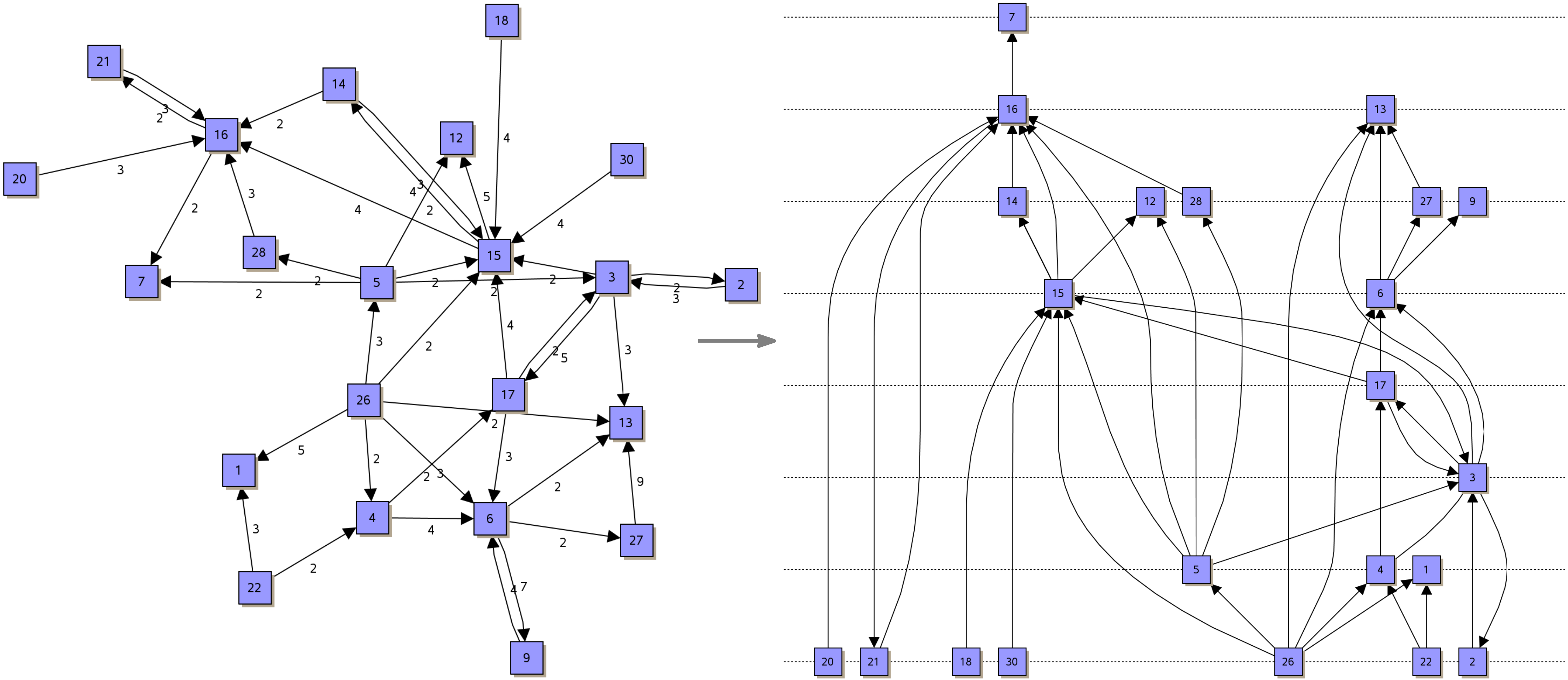
Part I:
The Framework

Tim Hegemann

Hierarchical Drawings – Motivation



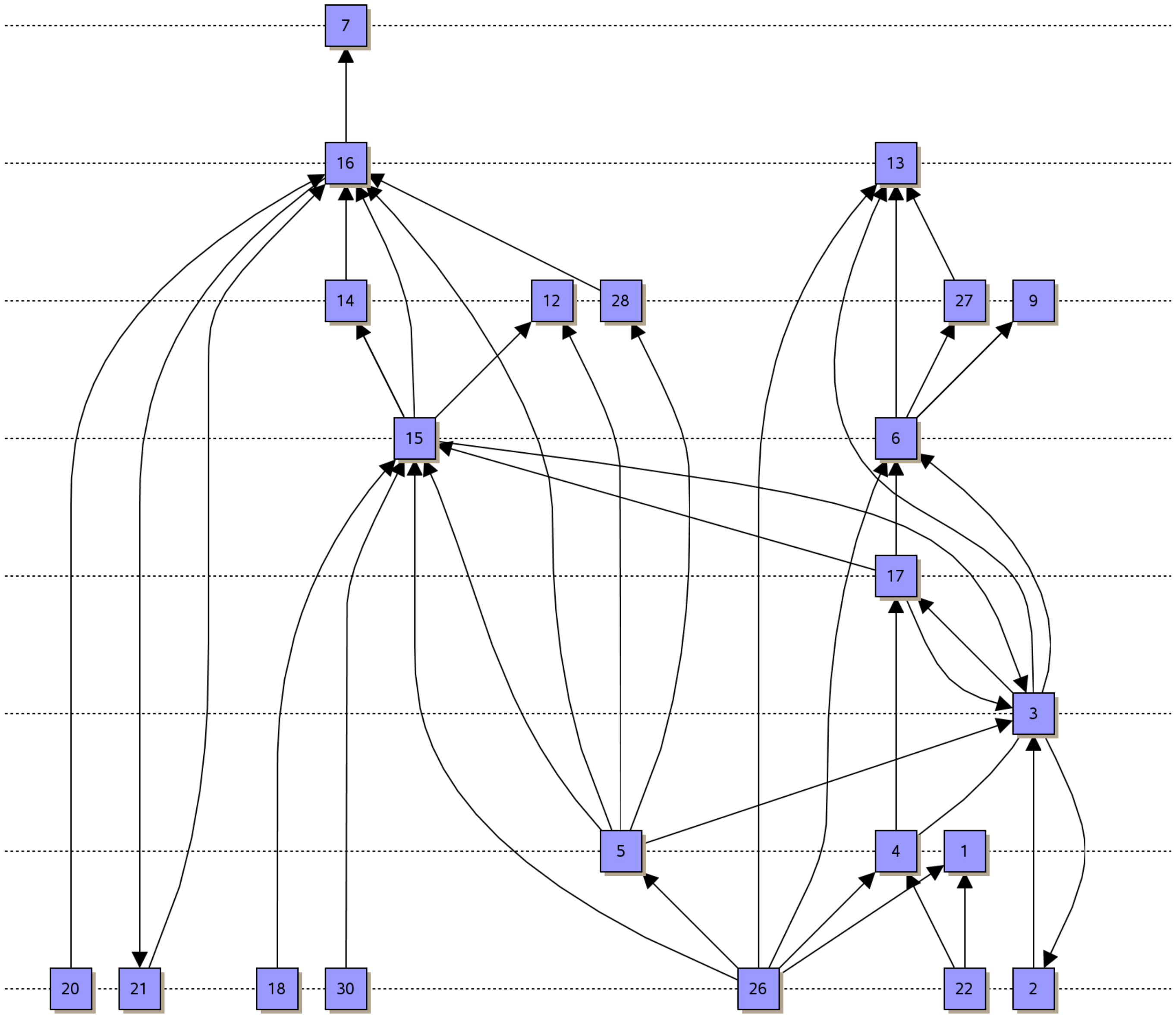
Hierarchical Drawings – Motivation



Hierarchical Drawing

Problem Statement.

- Input: digraph $G = (V, E)$
- Output: drawing of G that “closely” reproduces the hierarchical properties of G



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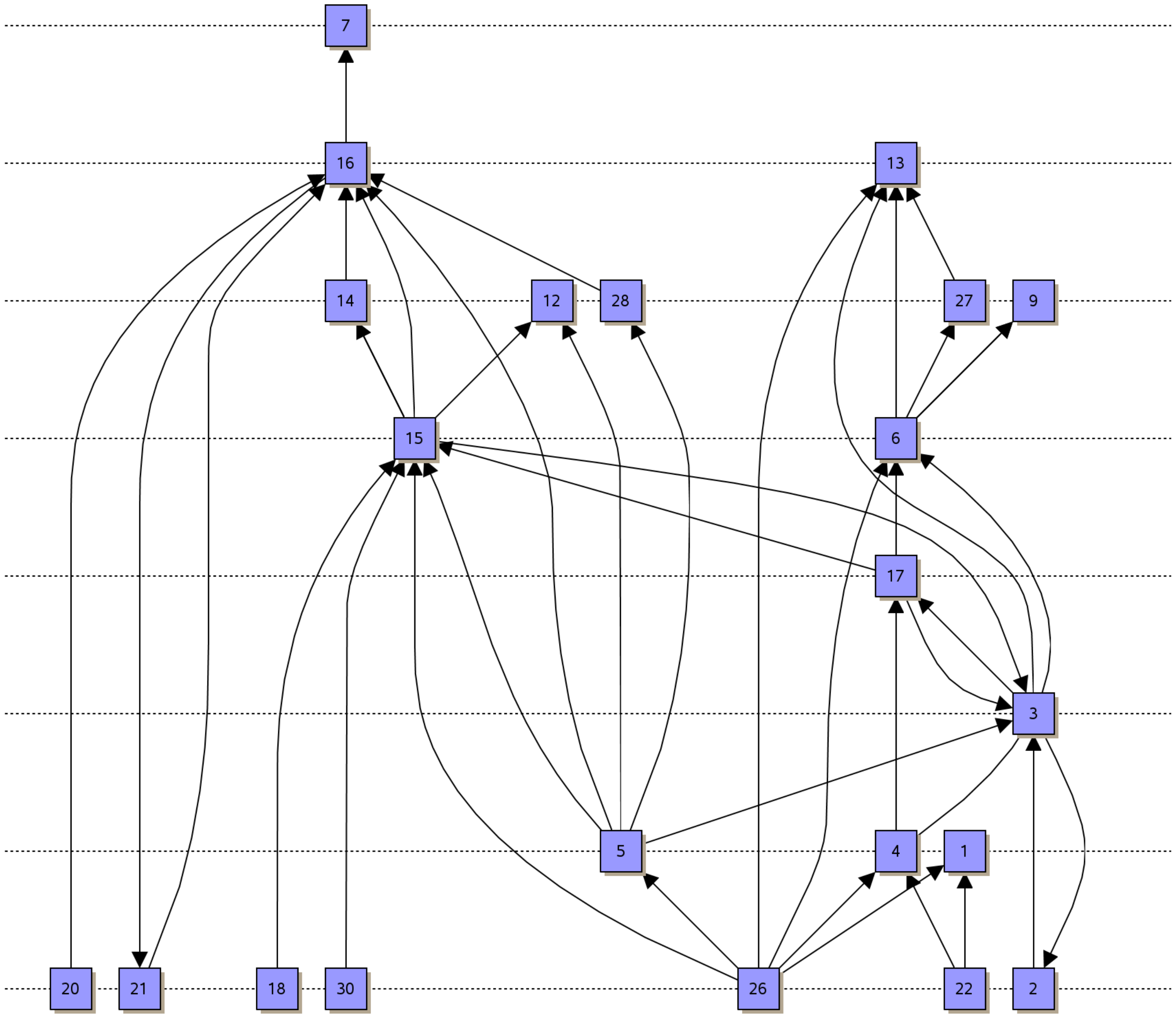
Desirable Properties.



Hierarchical Drawing

Problem Statement.

- Input: digraph $G = (V, E)$
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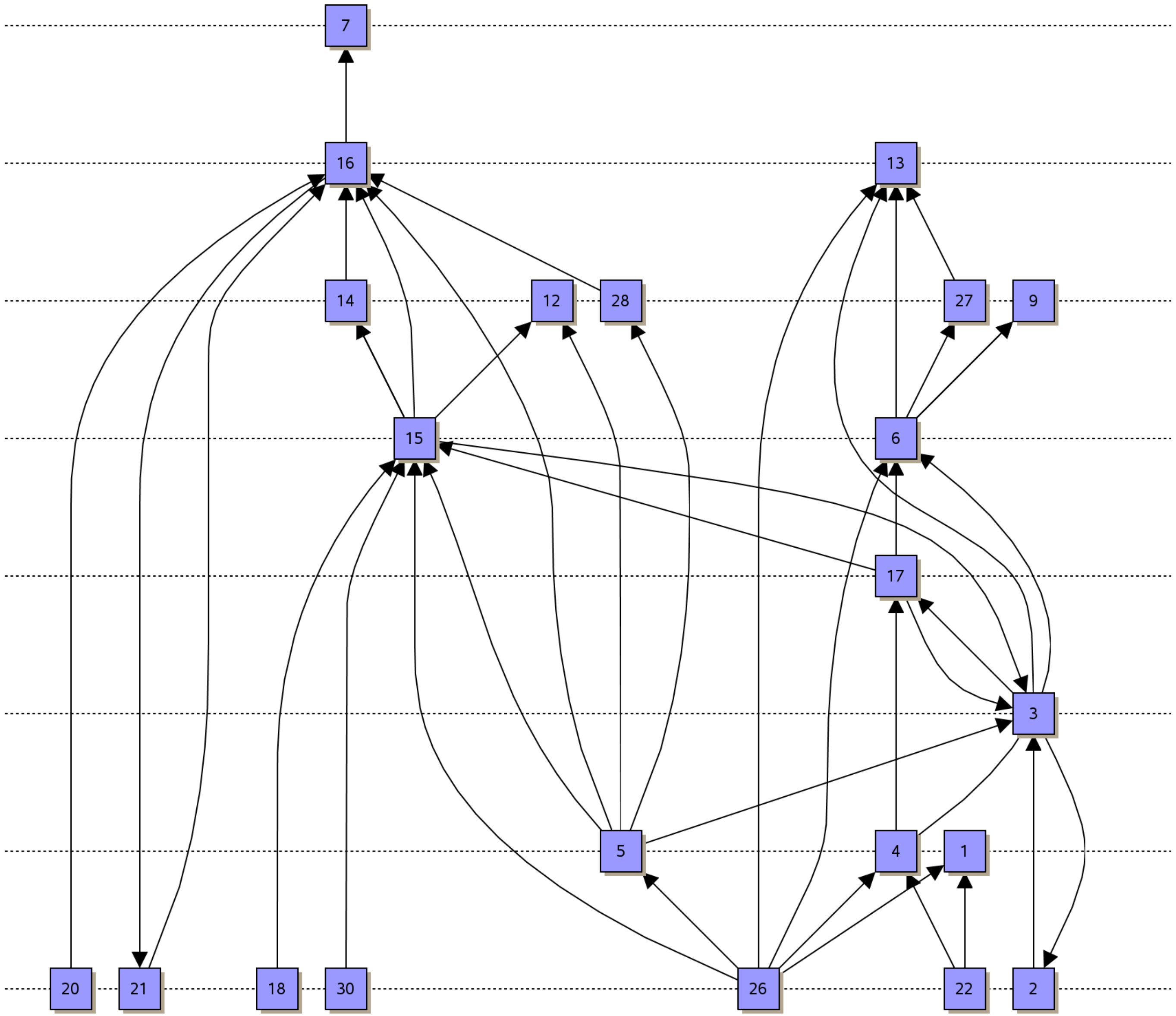
Desirable Properties.

- vertices occur on (few) horizontal lines



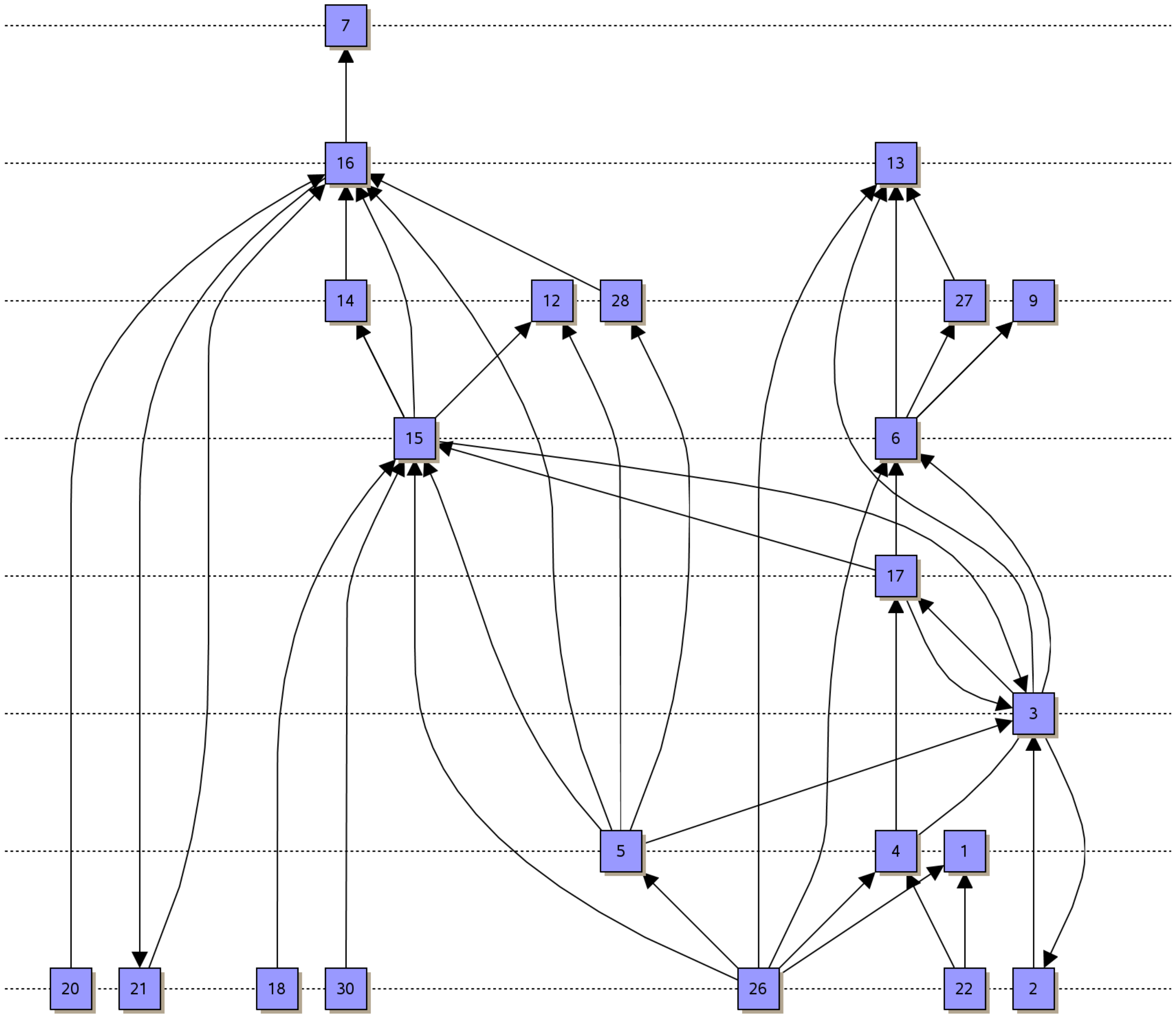
Hierarchical Drawing

Problem Statement.

- Input: digraph $G = (V, E)$
- Output: drawing of G that "closely" reproduces the hierarchical properties of G

Desirable Properties.

- vertices occur on (few) horizontal lines
- edges directed upwards



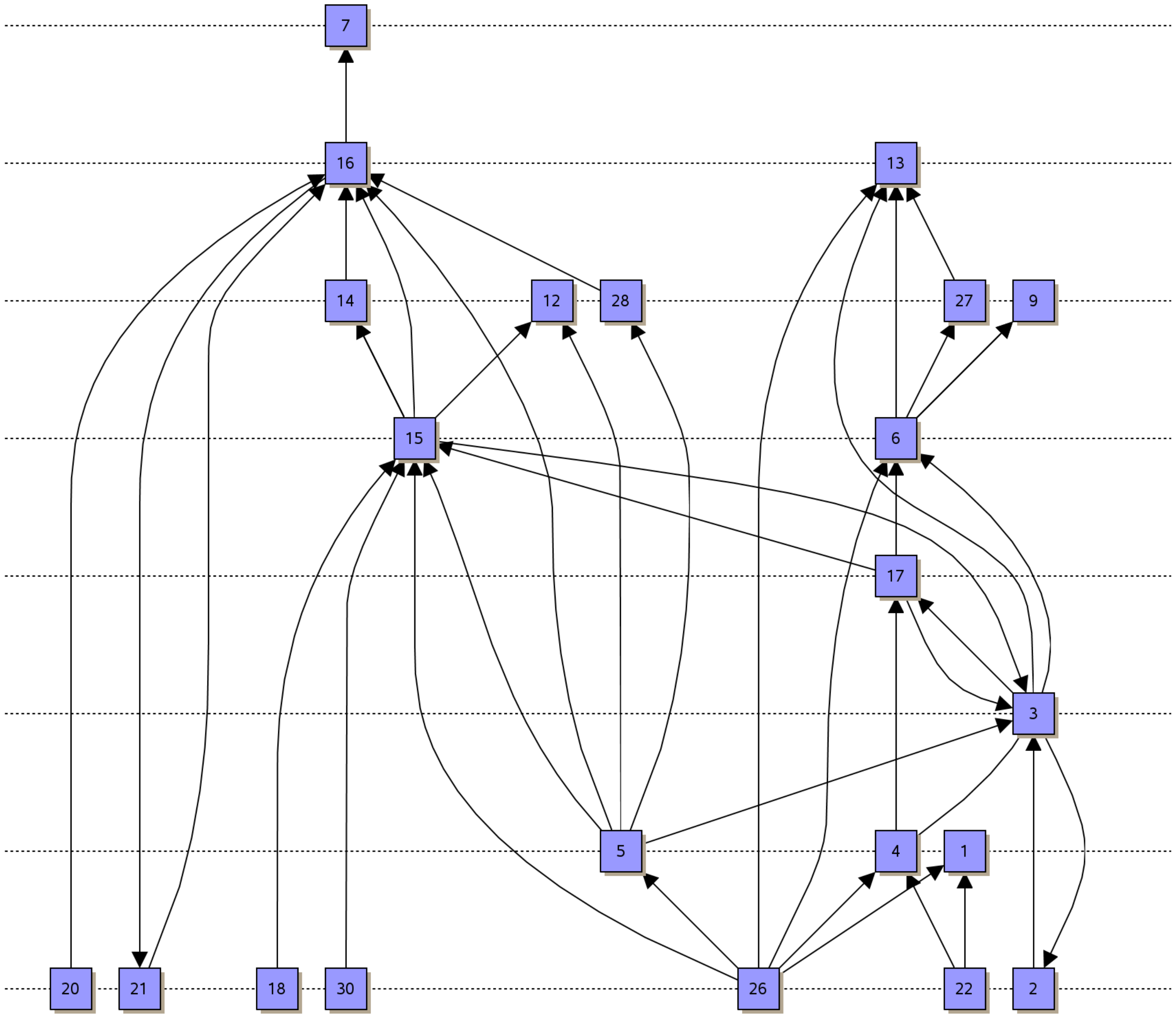
Hierarchical Drawing

Problem Statement.

- Input: digraph $G = (V, E)$
- Output: drawing of G that "closely" reproduces the hierarchical properties of G

Desirable Properties.

- vertices occur on (few) horizontal lines
- edges directed upwards
- edge crossings minimized



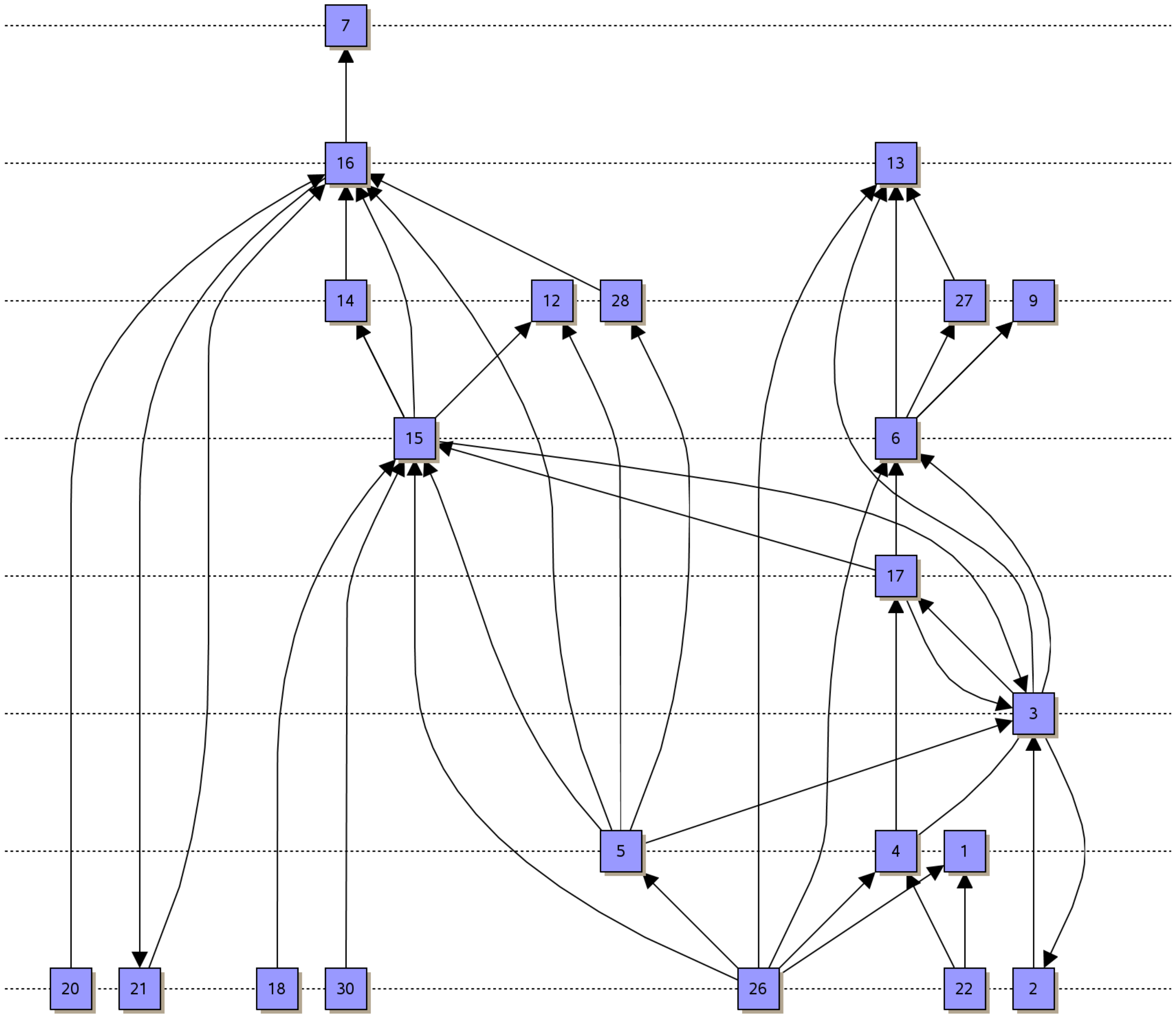
Hierarchical Drawing

Problem Statement.

- Input: digraph $G = (V, E)$
- Output: drawing of G that "closely" reproduces the hierarchical properties of G

Desirable Properties.

- vertices occur on (few) horizontal lines
- edges directed upwards
- edge crossings minimized
- edges as short as possible



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Desirable Properties.

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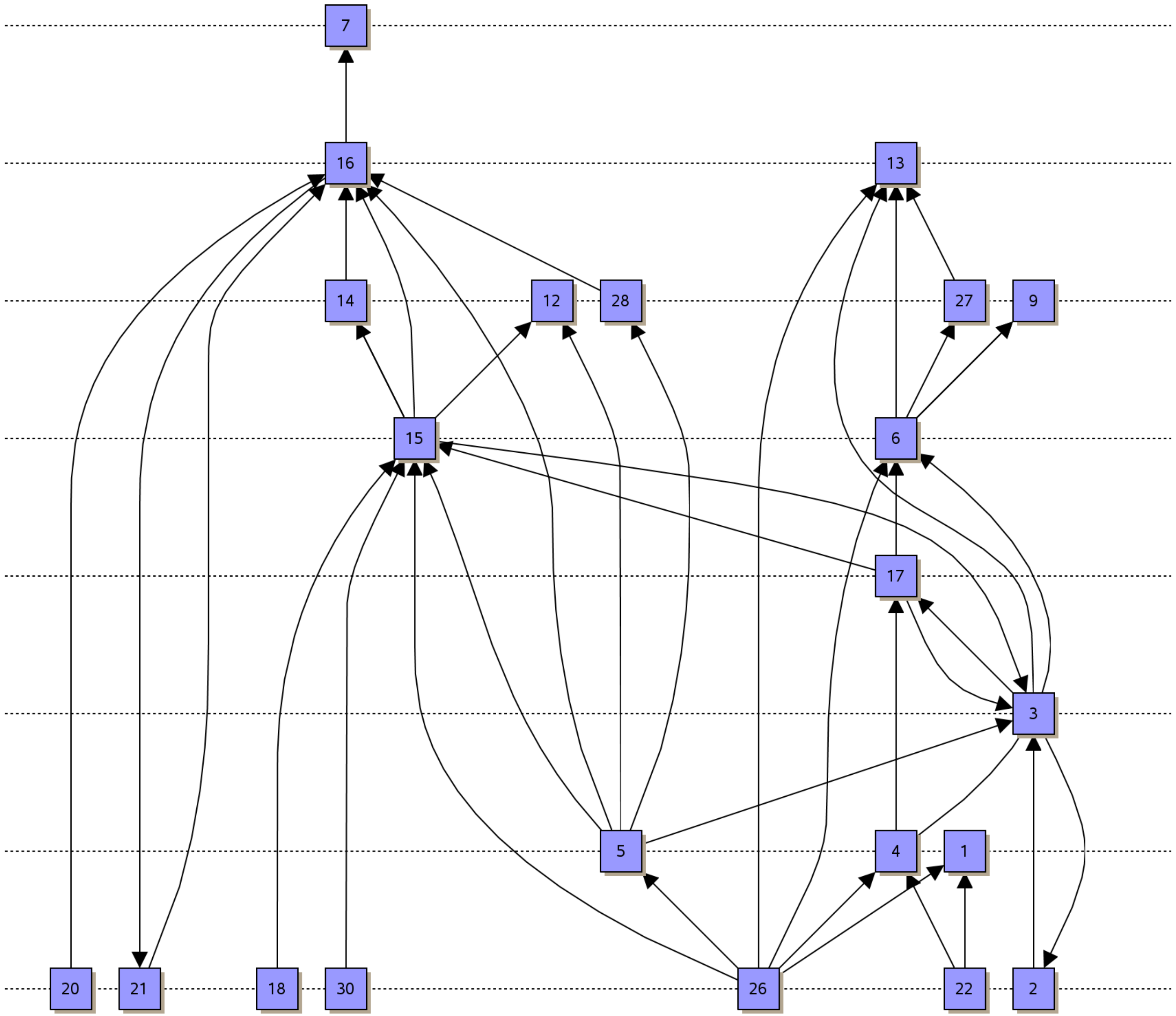
- vertices evenly spaced



Hierarchical Drawing

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Desirable Properties.

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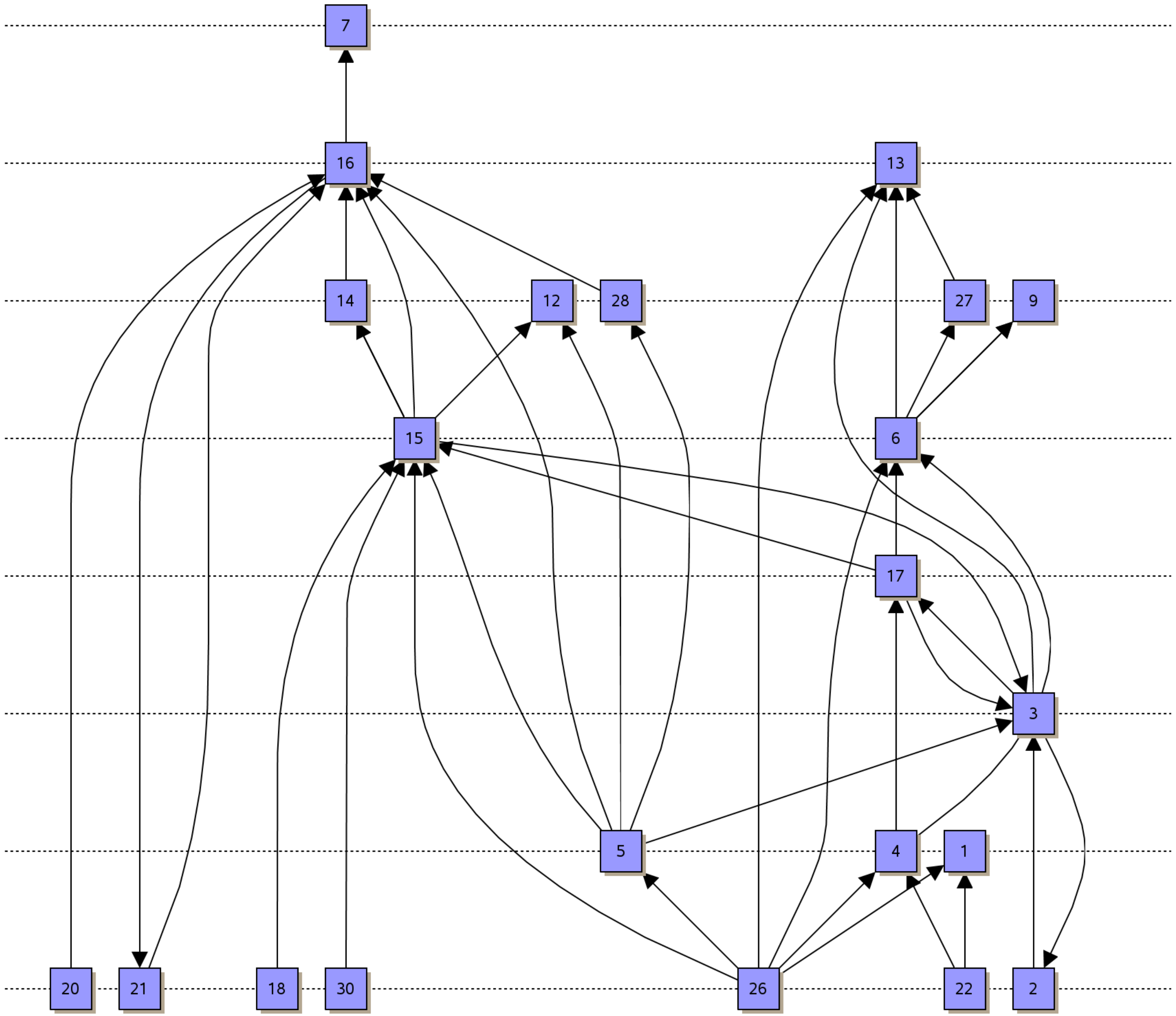
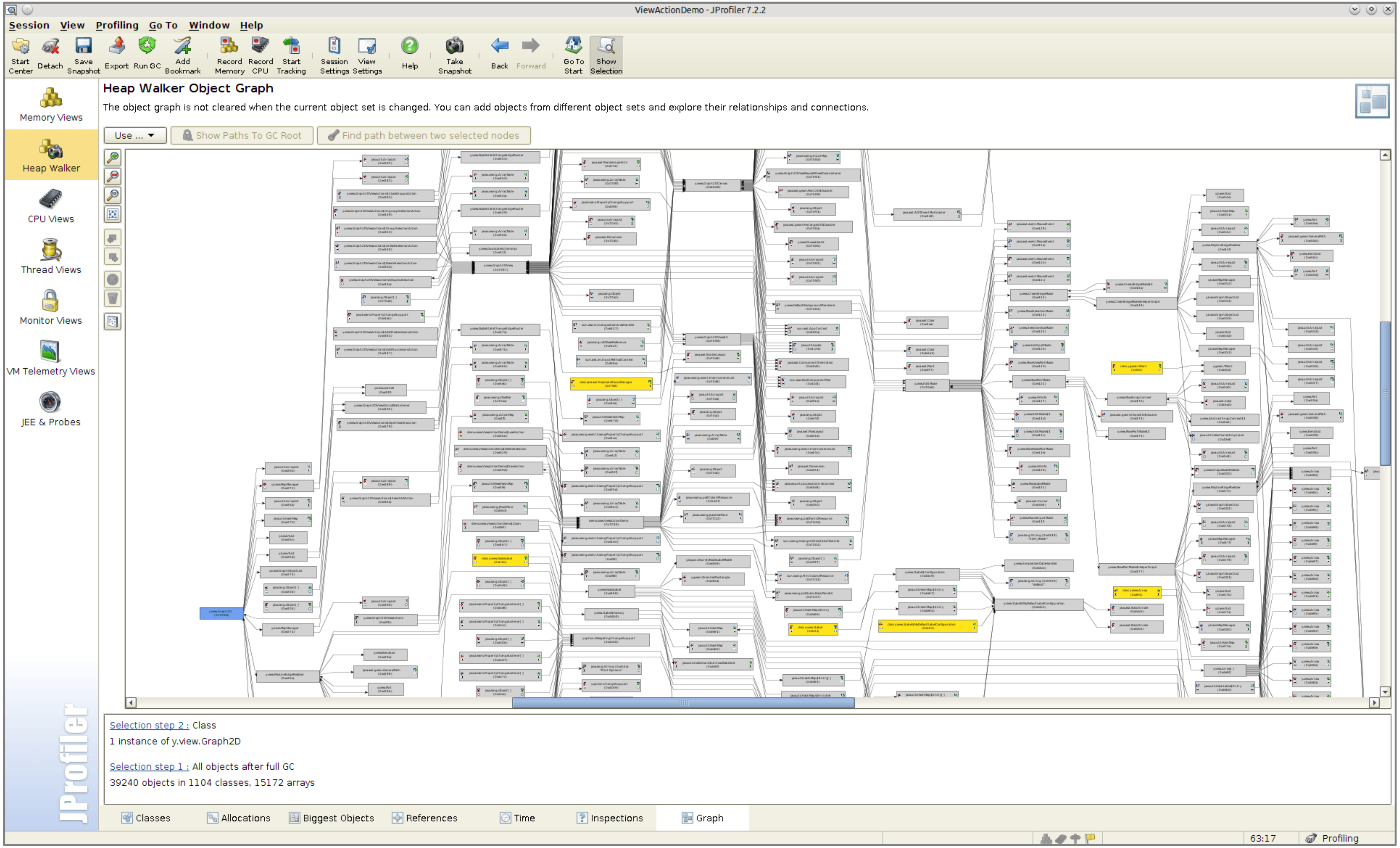
- edges directed upwards
- edge crossings minimized
- edges as short as possible
- vertices evenly spaced

Criteria can be contradictory!



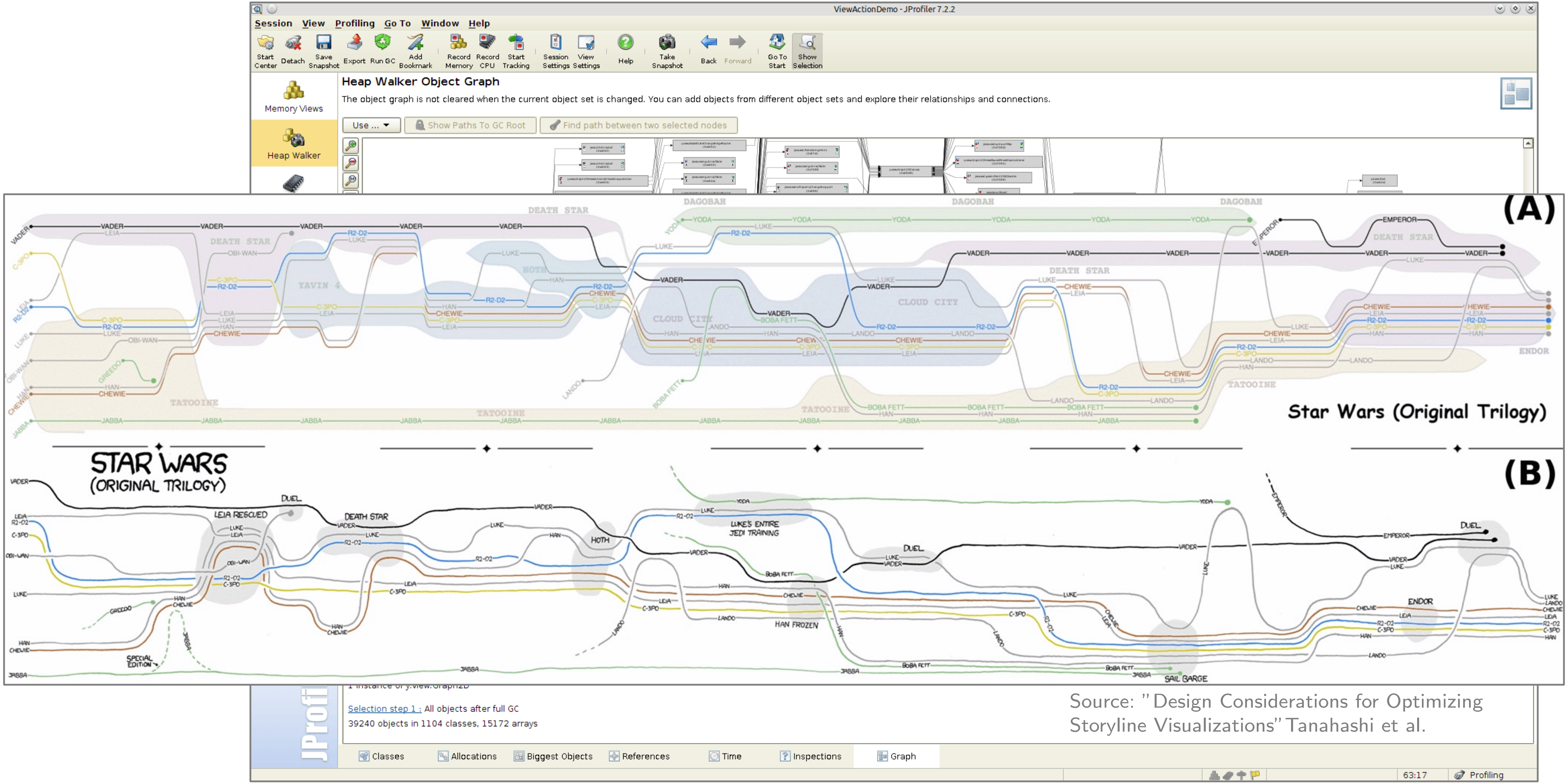
Hierarchical Drawing – Applications

yEd Gallery: Java profiler JProfiler using yFiles



Hierarchical Drawing – Applications

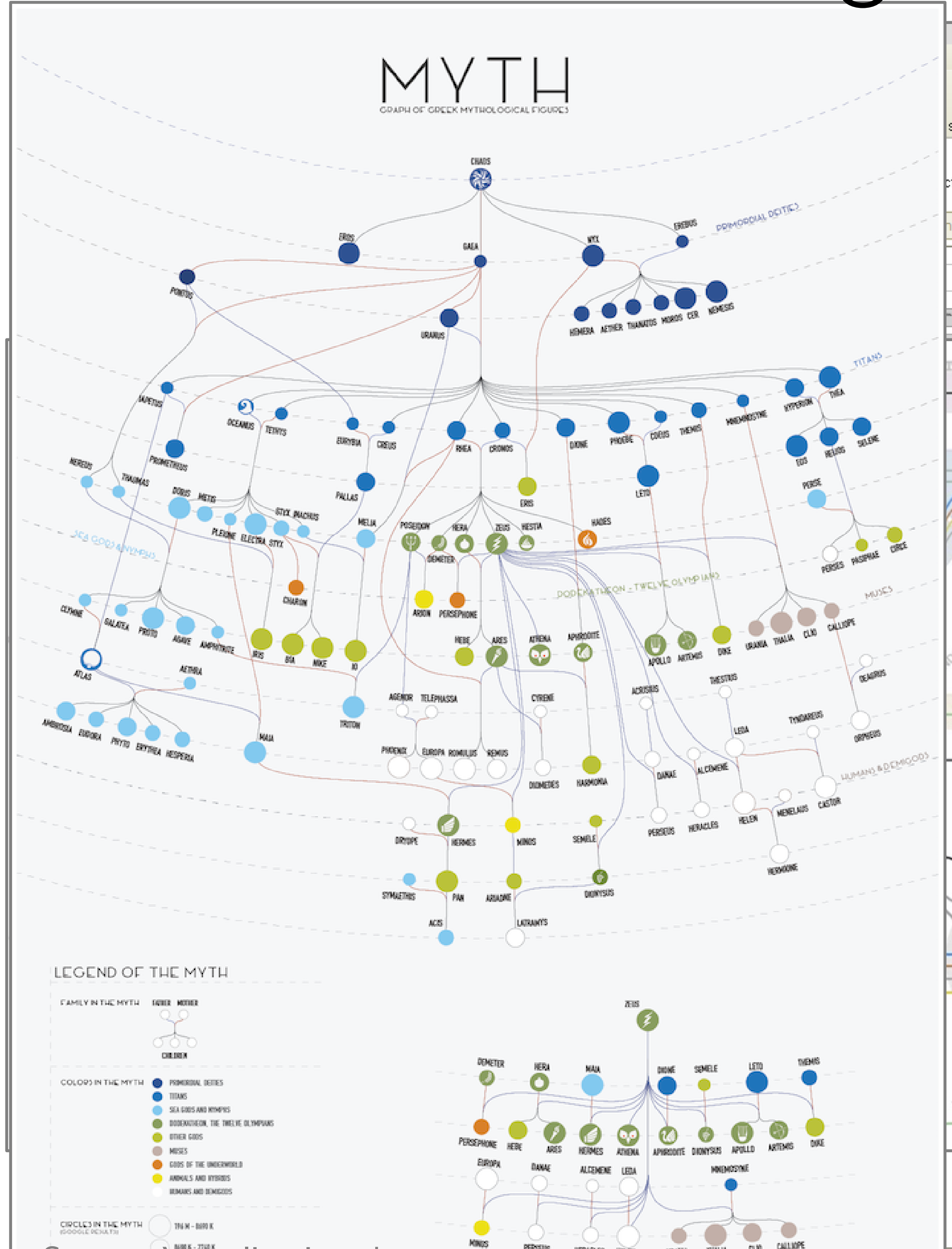
yEd Gallery: Java profiler JProfiler using yFiles



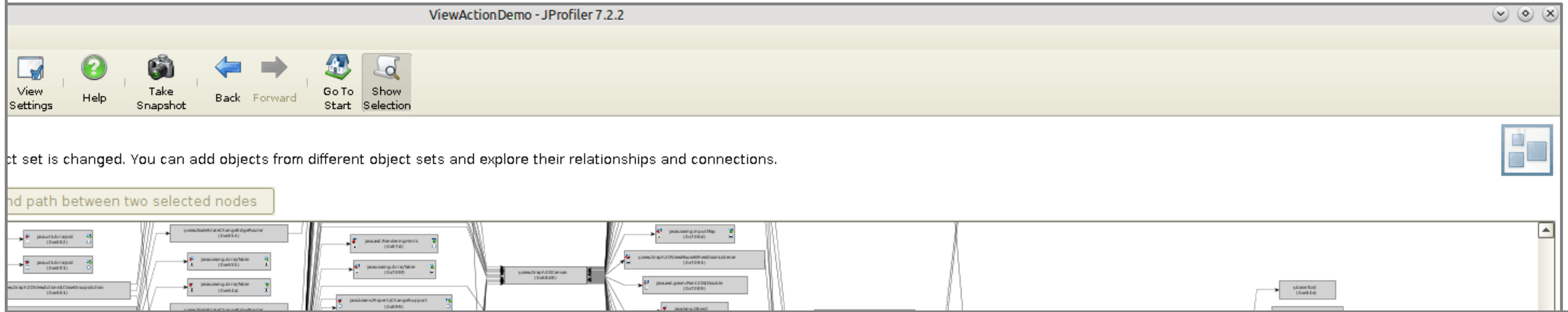
Source: "Design Considerations for Optimizing Storyline Visualizations" Tanahashi et al.

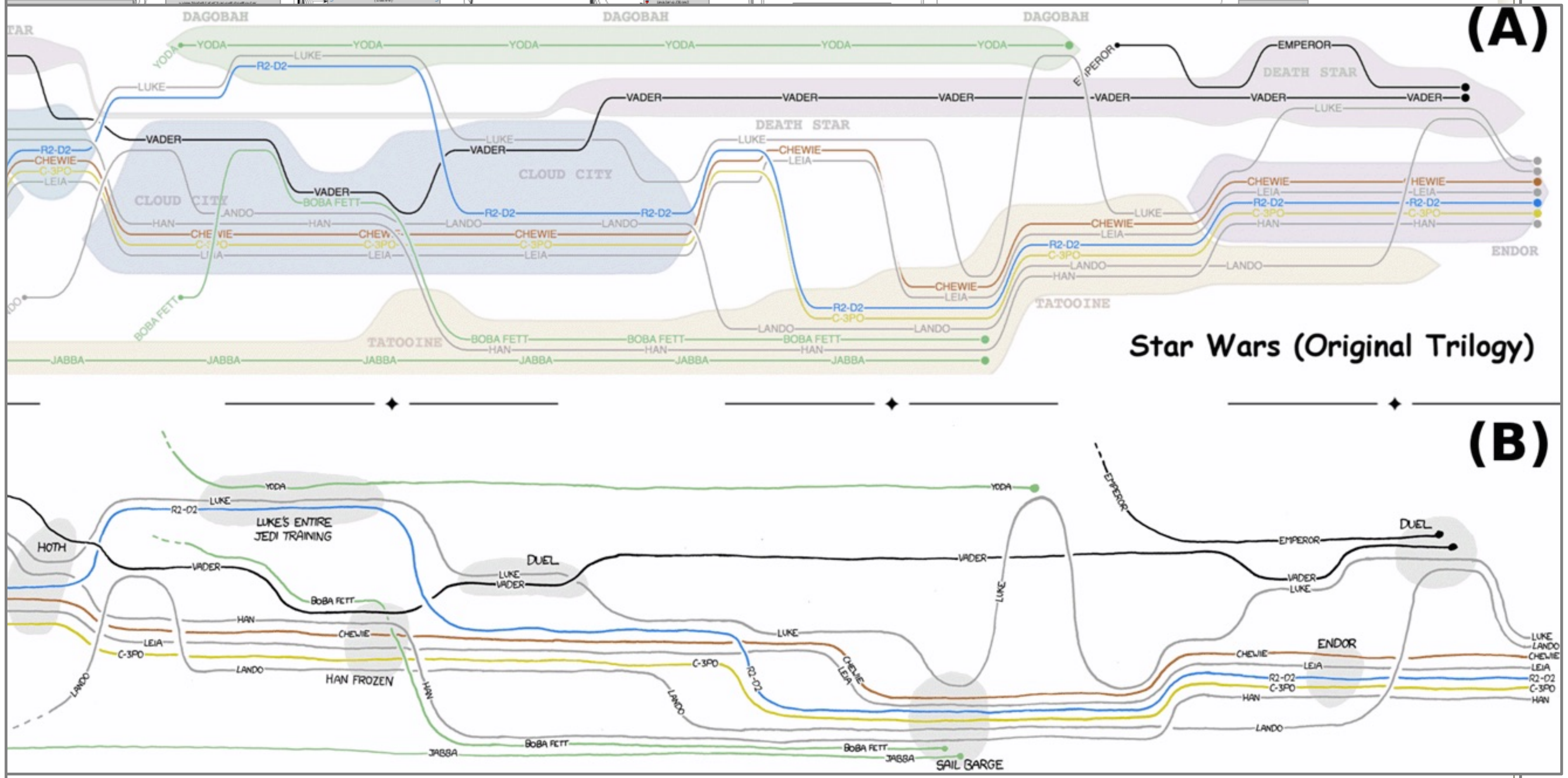
Hierarchical Drawing – Applications

yEd Gallery: Java profiler JProfiler using yFiles



Source: Visualization that won the Graph Drawing Contest 2016. Klawitter & Mchedlidze





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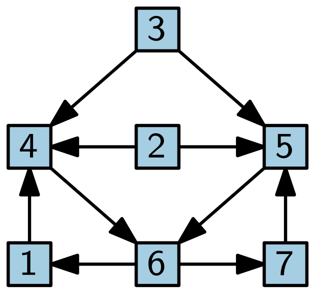
Classical Approach – Sugiyama Framework

[Sugiyama, Tagawa, Toda '81]

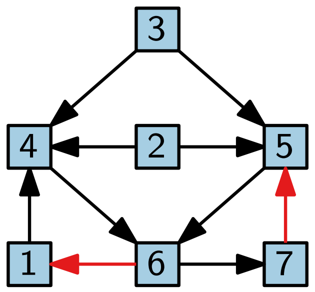
Input



Classical Approach – Sugiyama Framework

[Sugiyama, Tagawa, Toda '81]

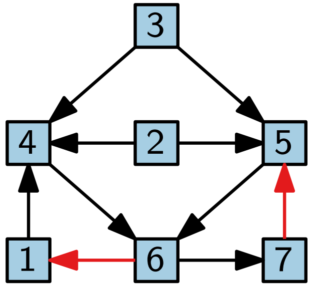
Input

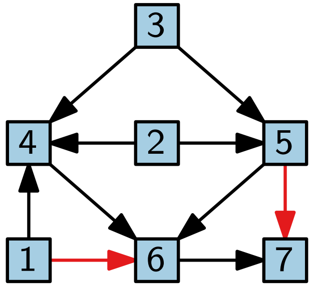


Classical Approach – Sugiyama Framework

[Sugiyama, Tagawa, Toda '81]

Input → Cycle Breaking

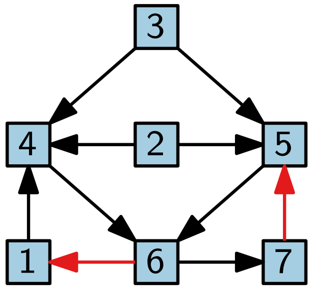


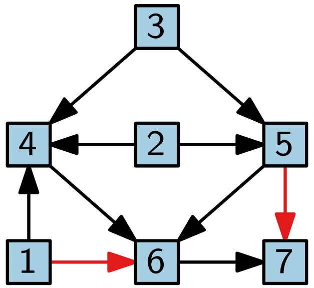


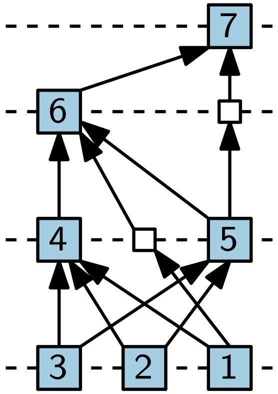
Classical Approach – Sugiyama Framework

[Sugiyama, Tagawa, Toda '81]

Input → Cycle Breaking → Leveling

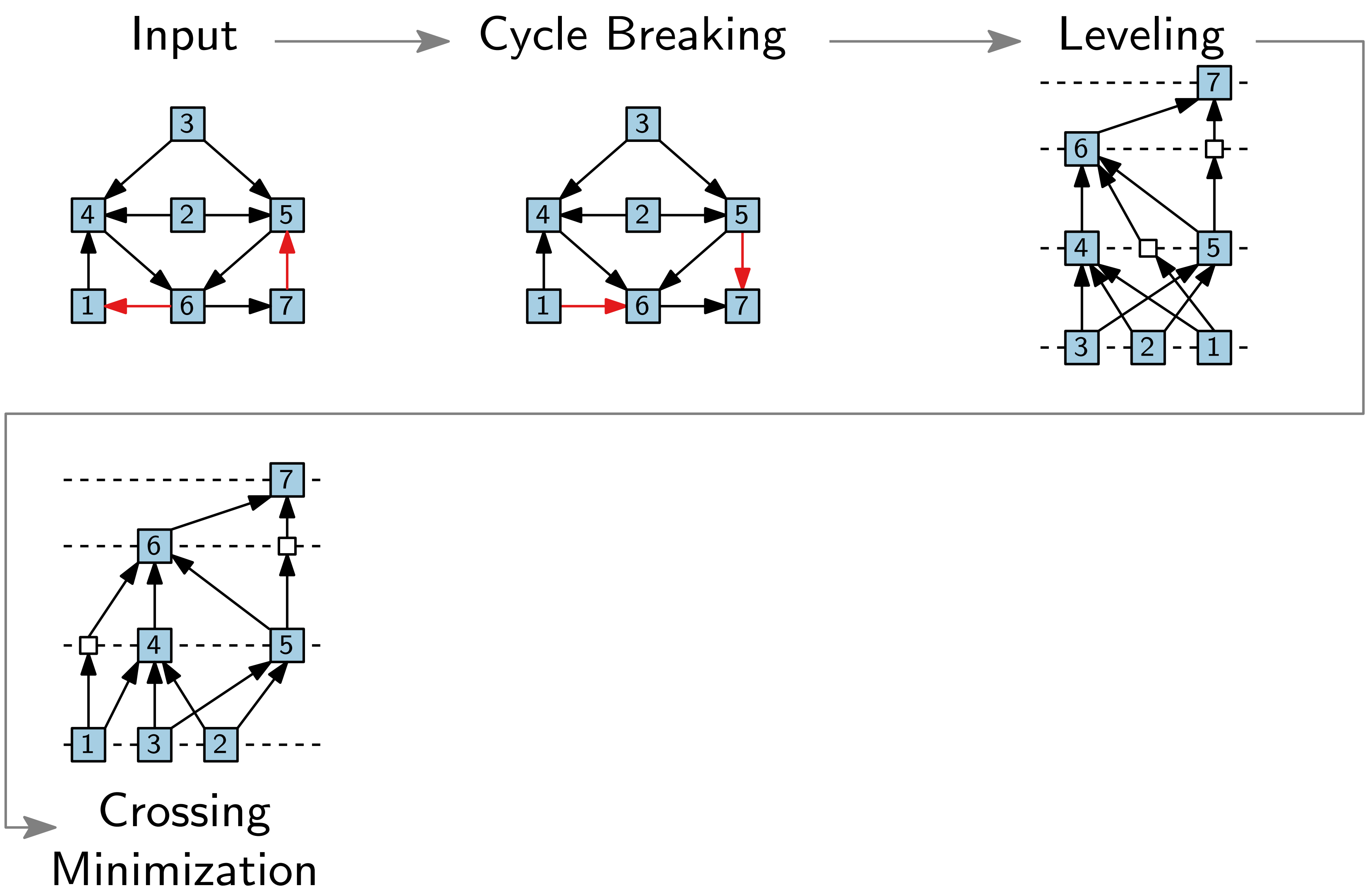






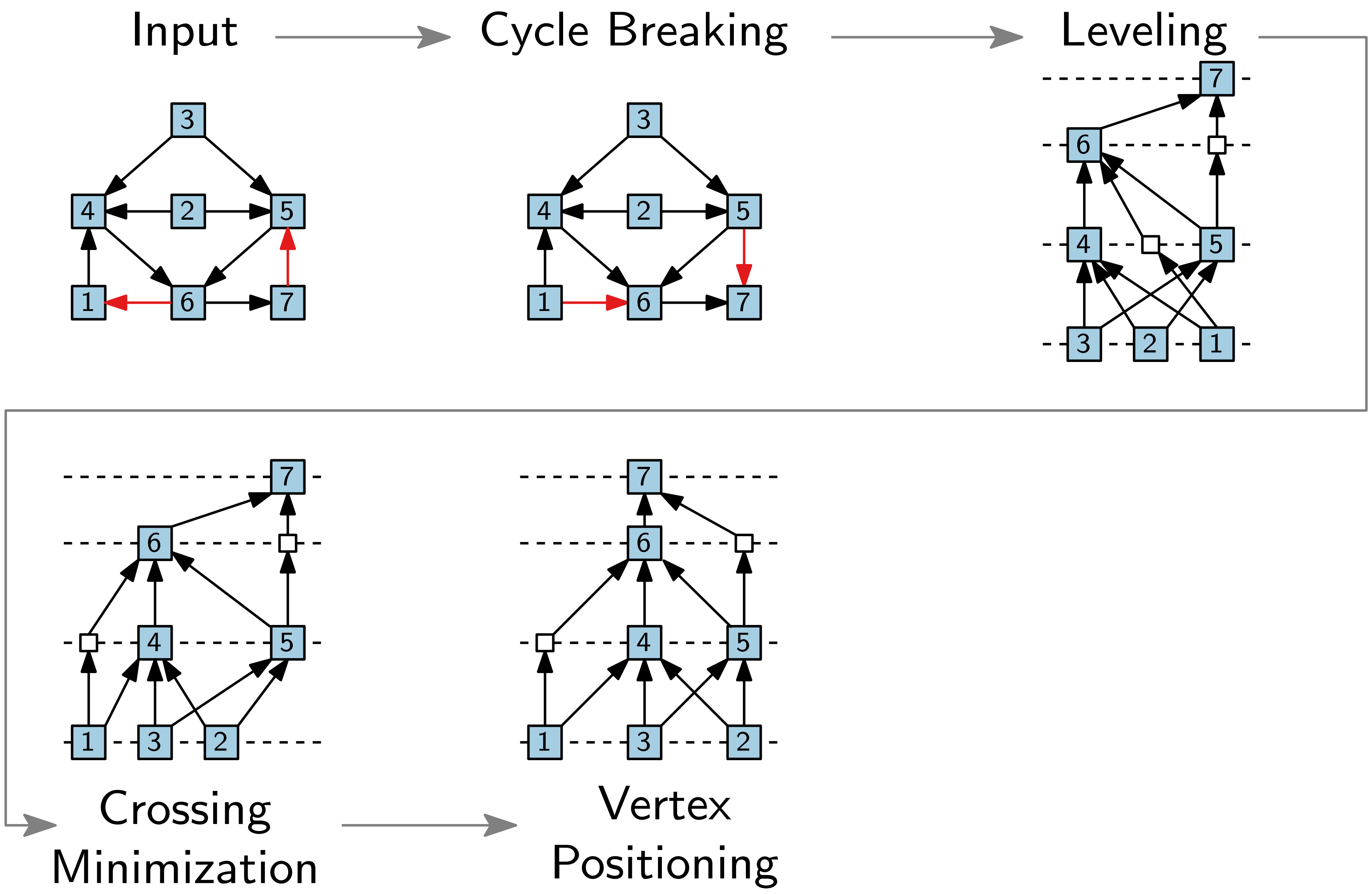
Classical Approach – Sugiyama Framework

[Sugiyama, Tagawa, Toda '81]



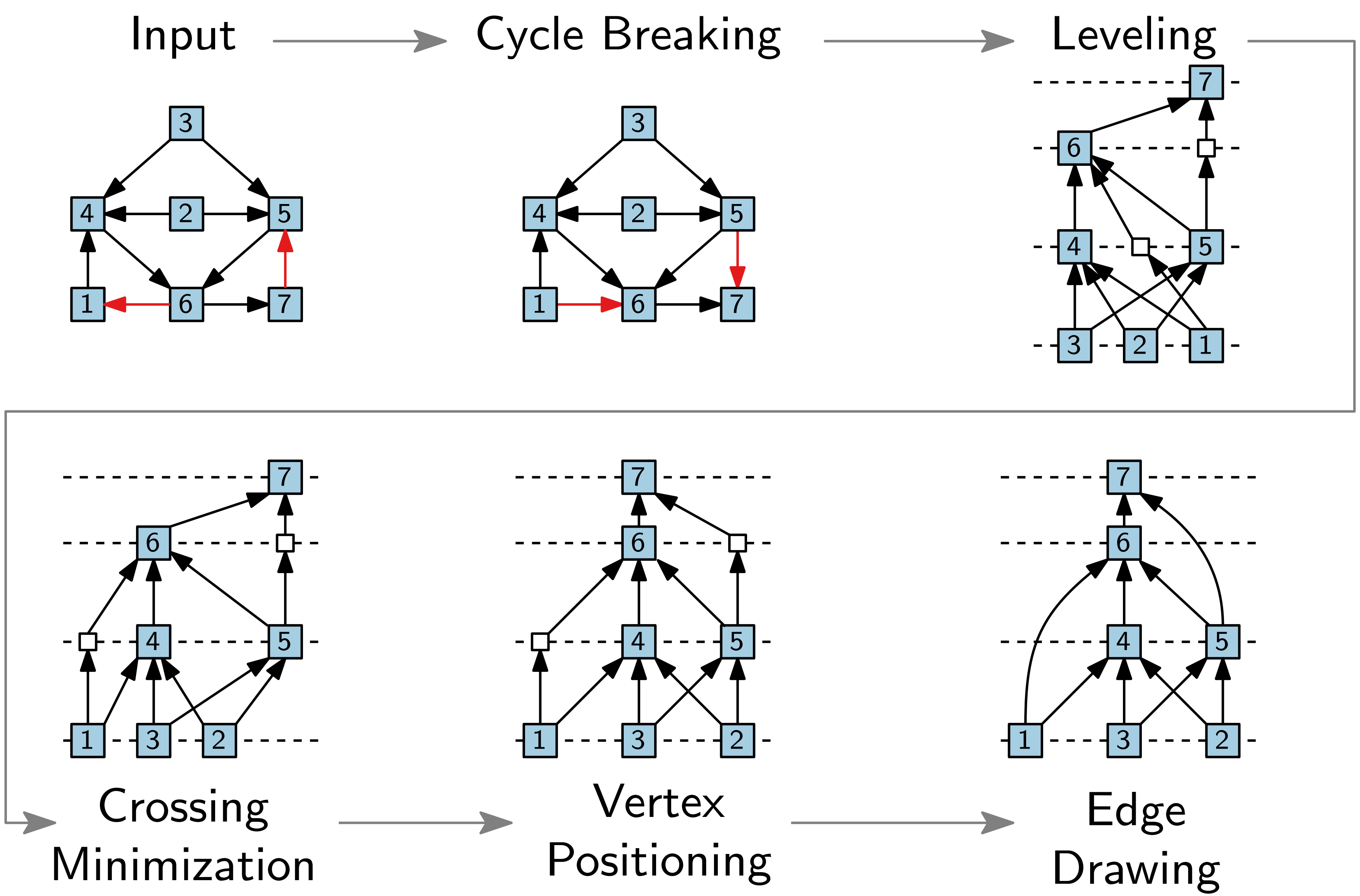
Classical Approach – Sugiyama Framework

[Sugiyama, Tagawa, Toda ’81]



Classical Approach – Sugiyama Framework

[Sugiyama, Tagawa, Toda ’81]



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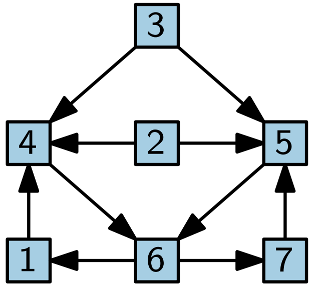
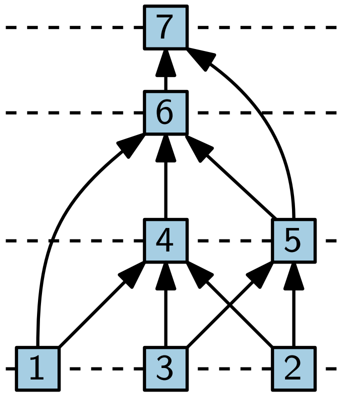
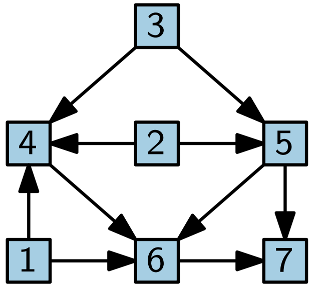
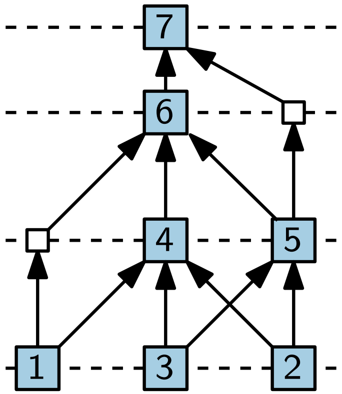


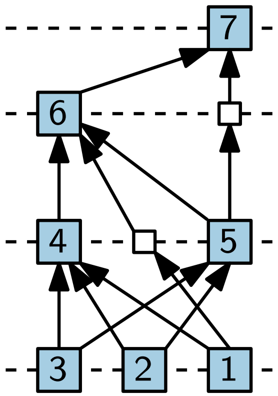


Visualization of Graphs

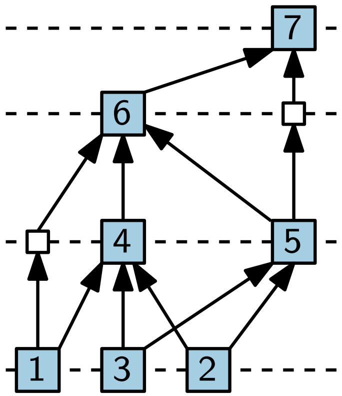


Lecture 7:
Hierarchical Layouts:
Sugiyama Framework



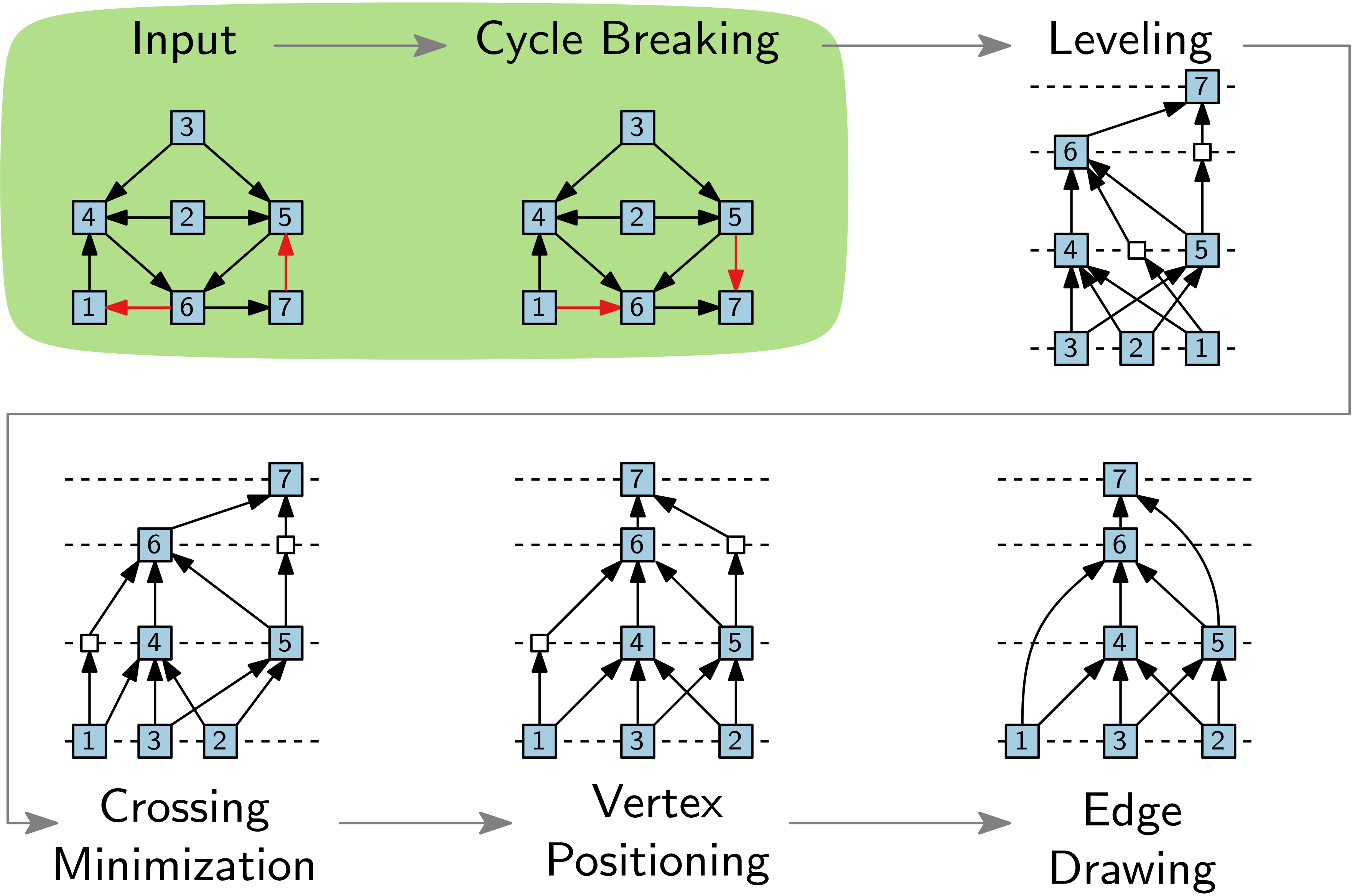


Part II:
Cycle Breaking

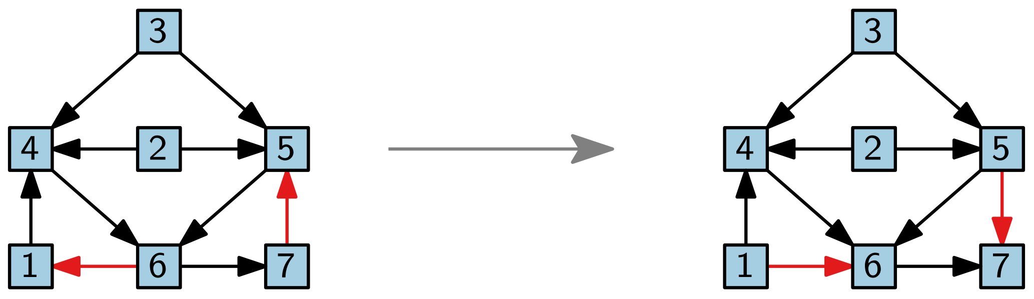


Jonathan Klawitter

Step 1: Cycle breaking

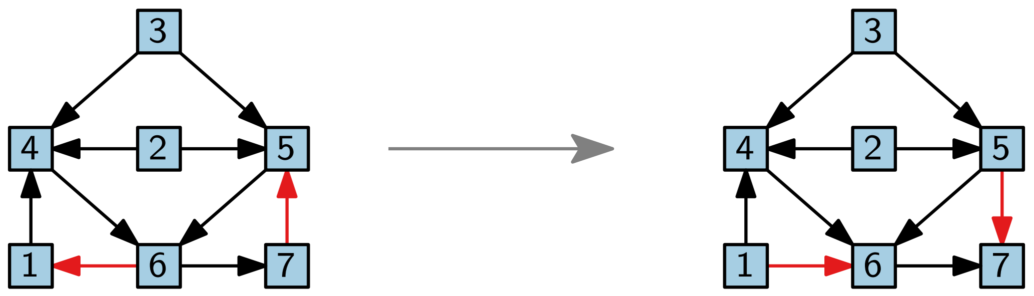


Step 1: Cycle breaking



Approach.

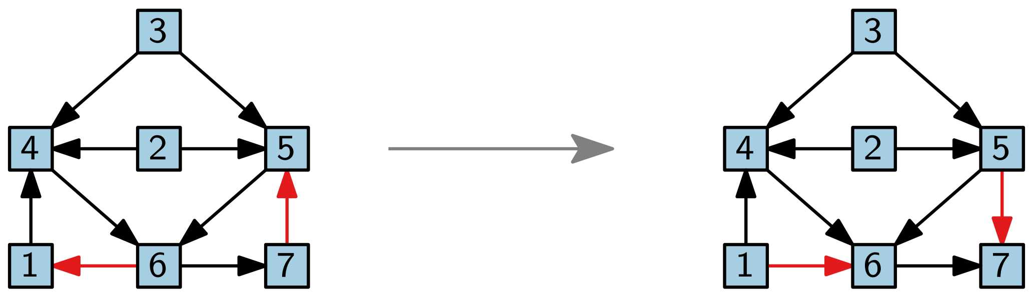
Step 1: Cycle breaking



Approach.

- Find minimum set $E^{\star}$ of edges which are not upwards.

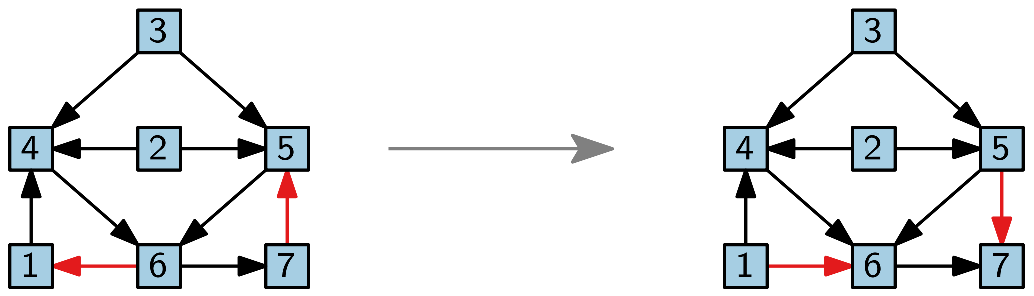
Step 1: Cycle breaking



Approach.

- Find minimum set $E^\star$ of edges which are not upwards.
- Remove $E^\star$ and insert reversed edges.

Step 1: Cycle breaking



Approach.

- Find minimum set $E^\star$ of edges which are not upwards.
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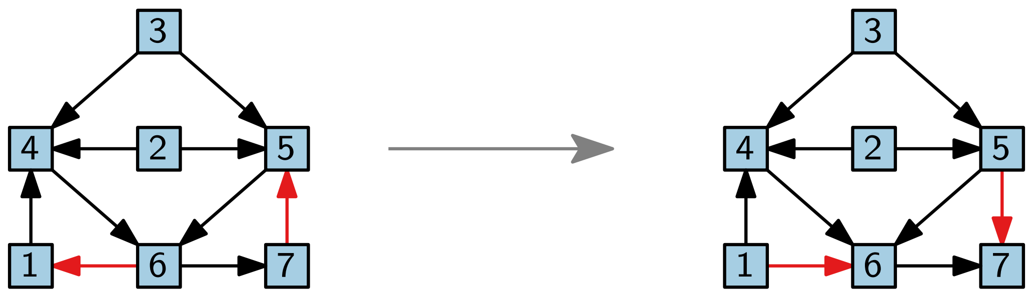
Problem MINIMUM FEEDBACK ARC SET **(FAS).**

Step 1: Cycle breaking



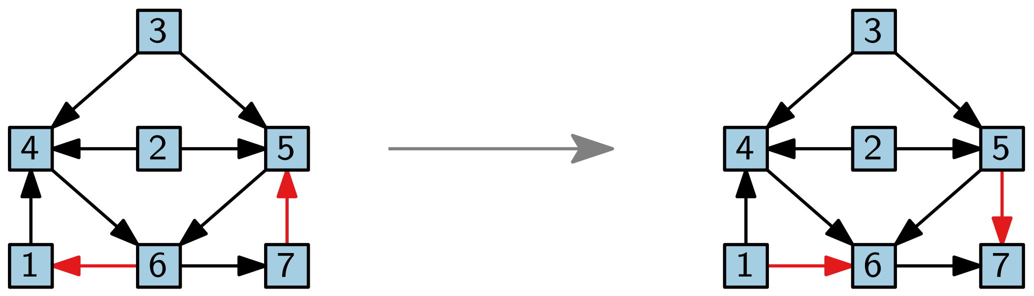
Approach.

- Find minimum set $E^\star$ of edges which are not upwards.
- Remove $E^\star$ and insert reversed edges.

Problem Minimum Feedback Arc Set **(FAS).**

- Input: directed graph $G = (V, E)$
- Output:

Step 1: Cycle breaking



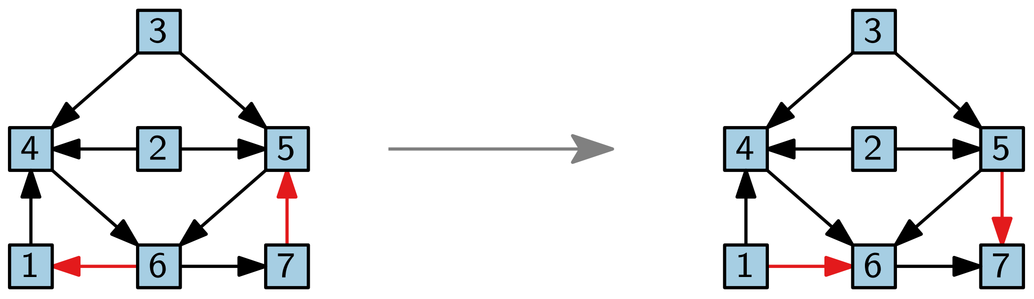
Approach.

- Find minimum set $E^\star$ of edges which are not upwards.
- Remove $E^\star$ and insert reversed edges.

Problem Minimum Feedback Arc Set (FAS).

- Input: directed graph $G = (V, E)$
- Output: min. set $E^\star \subseteq E$, so that $G - E^\star$ acyclic

Step 1: Cycle breaking



Approach.

- Find minimum set $E^\star$ of edges which are not upwards.
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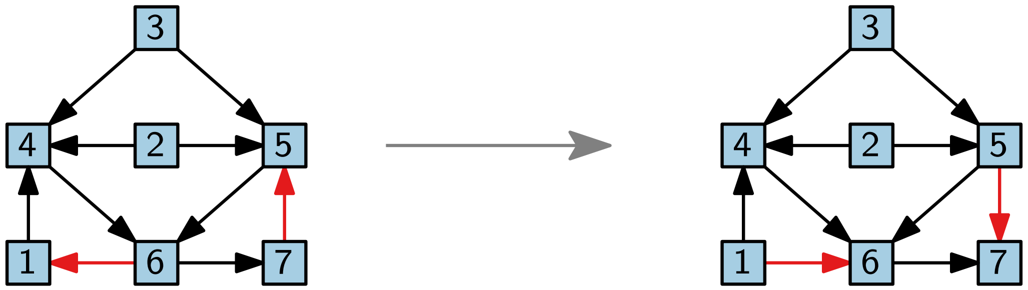
Problem Minimum Feedback ~~Arc~~ Set (F~~A~~S).

- Input: directed graph $G = (V, E)$
- Output: min. set $E^\star \subseteq E$, so that $\cancel{G - E^\star}$ acyclic
  $G - E^\star + E^\star_r$

Step 1: Cycle breaking



Approach.

- Find minimum set $E^\star$ of edges which are not upwards.
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Problem Minimum Feedback ~~Arc~~ Set **(F~~A~~S).**

- Input: directed graph $G = (V, E)$
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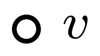
  $G - E^\star + E^\star_r$

. . . NP-hard ☹

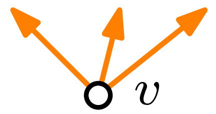
Heuristic 1

[Berger, Shor '90]



Heuristic 1

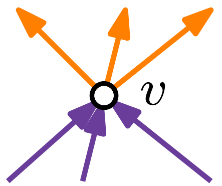
[Berger, Shor '90]



$$N^{\rightarrow}(v) \quad := \quad \{(v,u) | (v,u) \in E\}$$

Heuristic 1

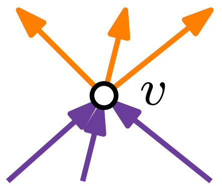
[Berger, Shor '90]



$$
\begin{aligned}
N^{\rightarrow}(v) &:= \{(v,u) | (v,u) \in E\} \\
N^{\leftarrow}(v) &:= \{(u,v) | (u,v) \in E\}
\end{aligned}
$$

Heuristic 1

[Berger, Shor '90]

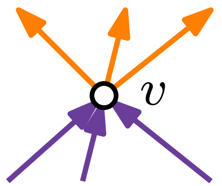


$$
\begin{aligned}
N^{\rightarrow}(v) &:= \{(v,u) | (v,u) \in E\} \\
N^{\leftarrow}(v) &:= \{(u,v) | (u,v) \in E\} \\
N(v) &:= N^{\rightarrow}(v) \cup N^{\leftarrow}(v)
\end{aligned}
$$

Heuristic 1

[Berger, Shor '90]

GreedyMakeAcyclic(Digraph $G = (V, E)$)



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\end{aligned}
$$

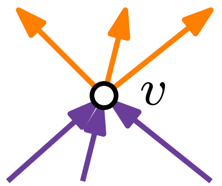
Heuristic 1

[Berger, Shor '90]

GreedyMakeAcyclic(Digraph $G = (V, E)$)

$E' \leftarrow \emptyset$

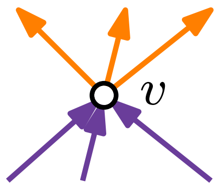
return (V, E')



$$N^{\rightarrow}(v) := \{(v,u) | (v,u) \in E\}$$
$$N^{\leftarrow}(v) := \{(u,v) | (u,v) \in E\}$$
$$N(v) := N^{\rightarrow}(v) \cup N^{\leftarrow}(v)$$

Heuristic 1

[Berger, Shor '90]



GreedyMakeAcyclic(Digraph $G = (V, E)$)

$E' \leftarrow \emptyset$

foreach $v \in V$ **do**

return (V, E')

$$N^{\rightarrow}(v) := \{(v,u) | (v,u) \in E\}$$

$$N^{\leftarrow}(v) := \{(u,v) | (u,v) \in E\}$$

$$N(v) := N^{\rightarrow}(v) \cup N^{\leftarrow}(v)$$

Heuristic 1

[Berger, Shor '90]

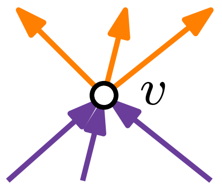
GreedyMakeAcyclic(Digraph $G = (V, E)$)

$E' \leftarrow \emptyset$

foreach $v \in V$ **do**

  if $|N^{\rightarrow}(v)| \geq |N^{\leftarrow}(v)|$ **then**

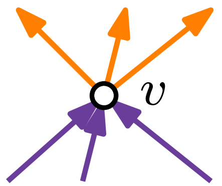
return (V, E')



$$N^{\rightarrow}(v) := \{(v,u) | (v,u) \in E\}$$
$$N^{\leftarrow}(v) := \{(u,v) | (u,v) \in E\}$$
$$N(v) := N^{\rightarrow}(v) \cup N^{\leftarrow}(v)$$

Heuristic 1

[Berger, Shor '90]



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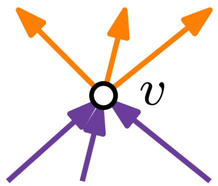

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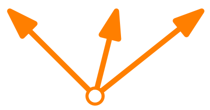

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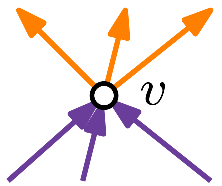

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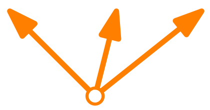

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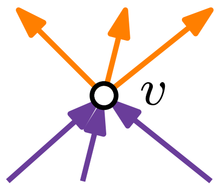

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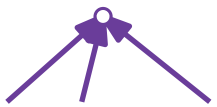

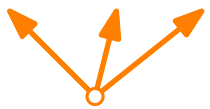

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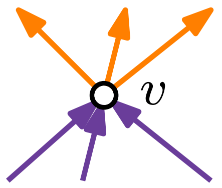

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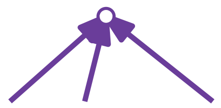

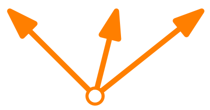

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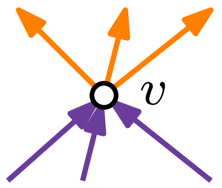

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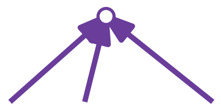

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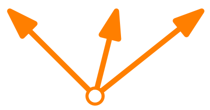

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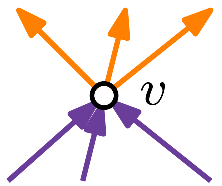

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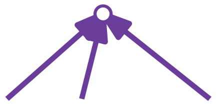

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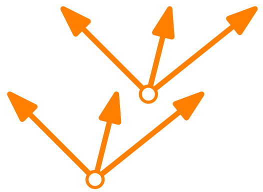

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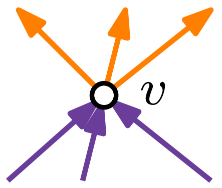

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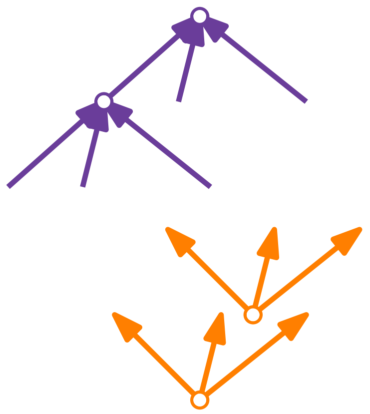

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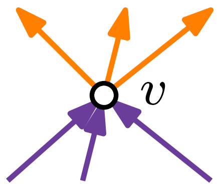

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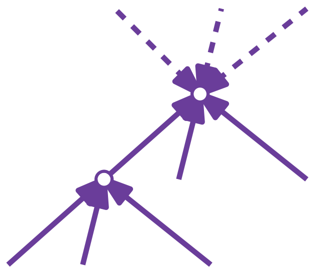

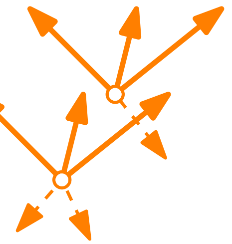

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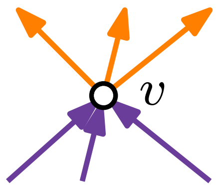

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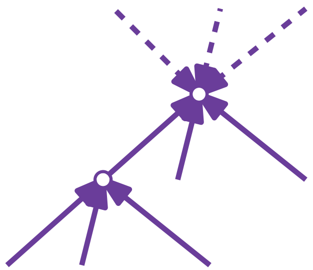

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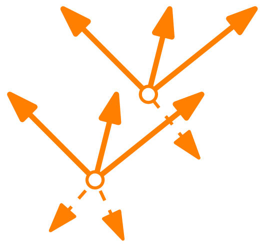

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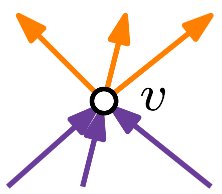

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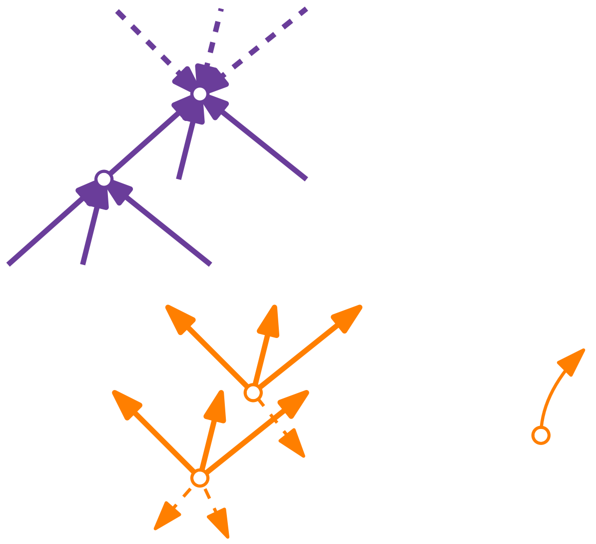

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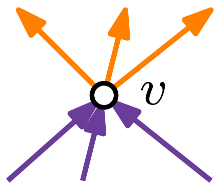

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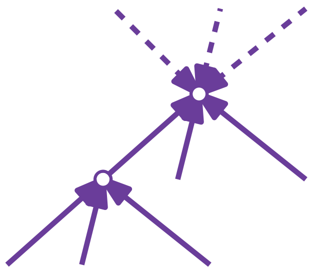

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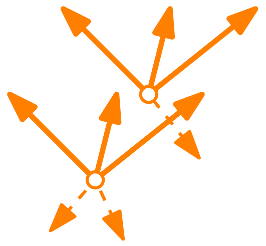

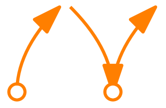

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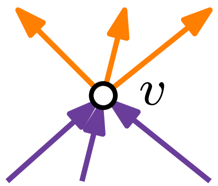

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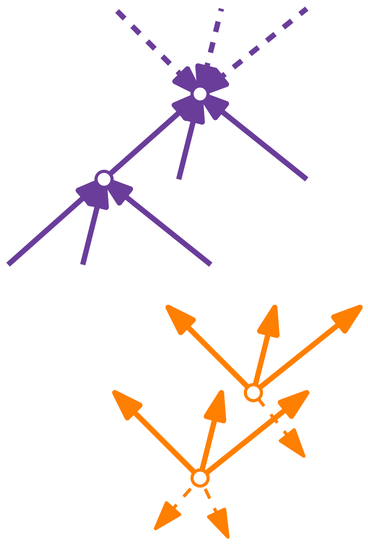

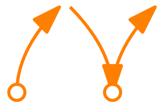

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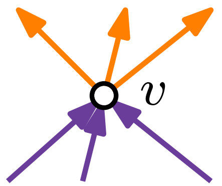

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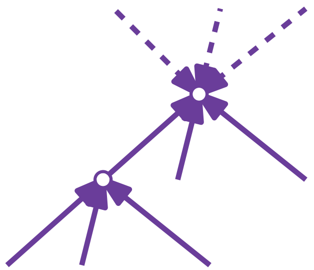

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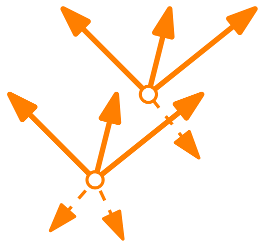

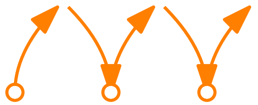

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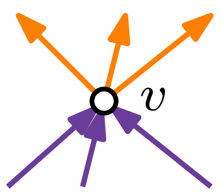

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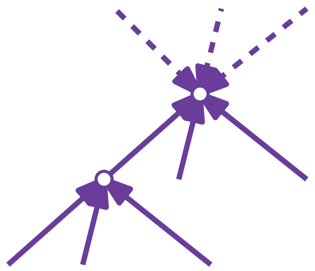

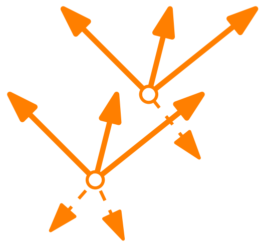

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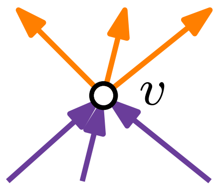

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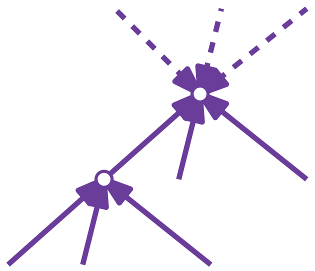

- $G' = (V, E')$ is a DAG
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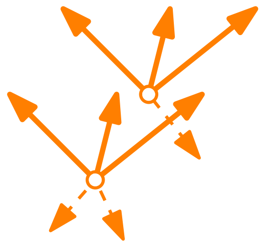
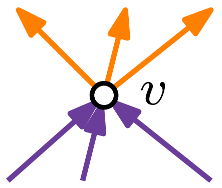


Heuristic 1

[Berger, Shor '90]



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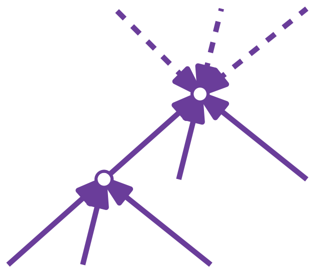

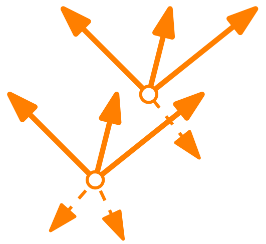

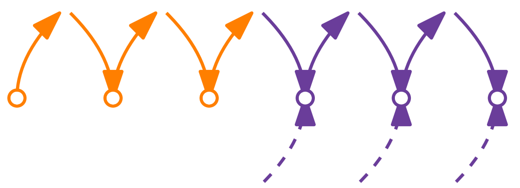

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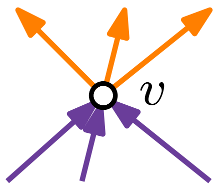

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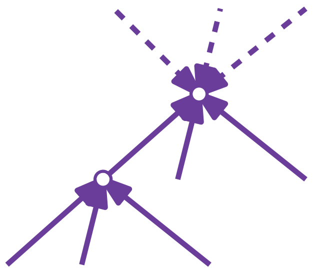

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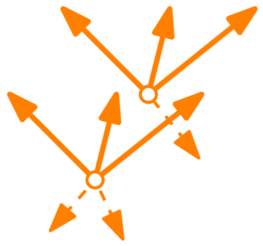

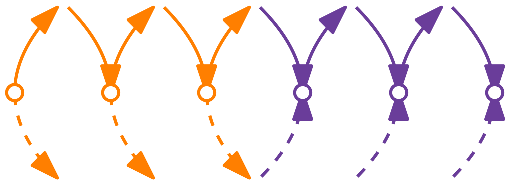

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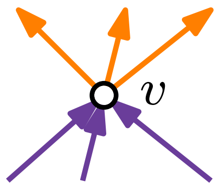

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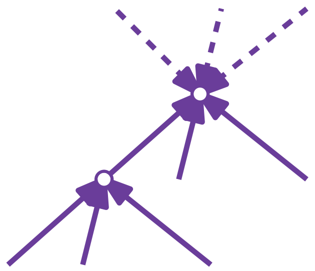
- Time:



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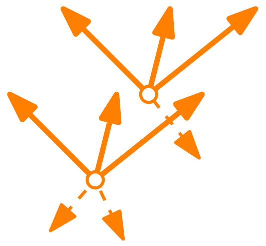

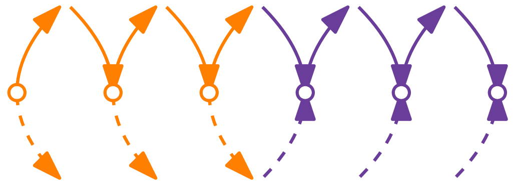

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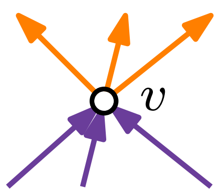

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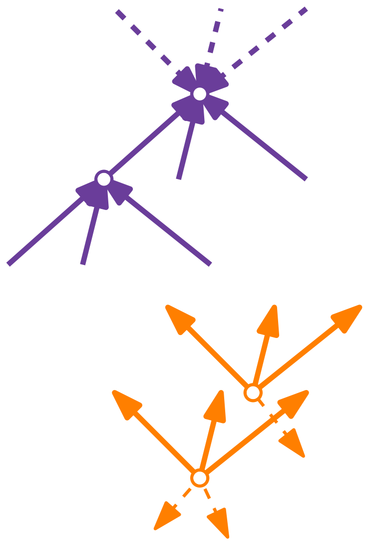
- Time: $\mathcal{O}(n+m)$



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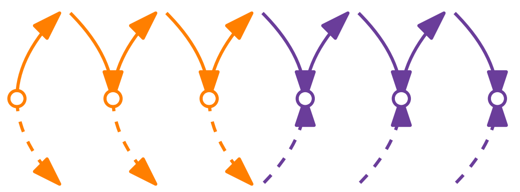

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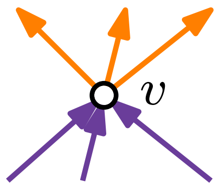

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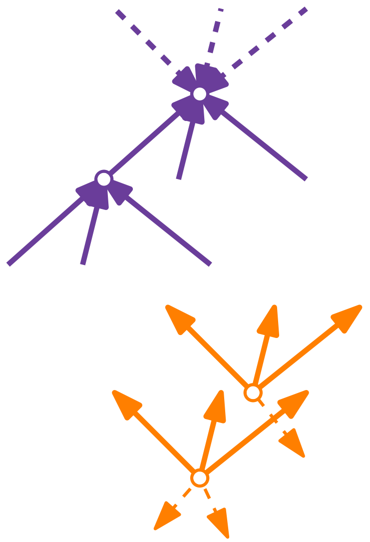
- Time: $\mathcal{O}(n+m)$
- Quality guarantee: $|E'| \geq$



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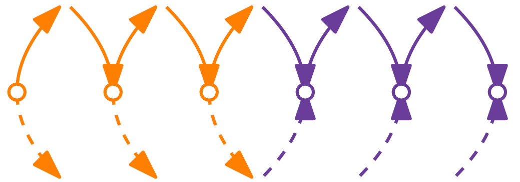

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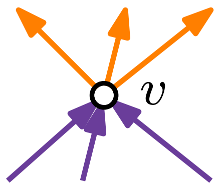

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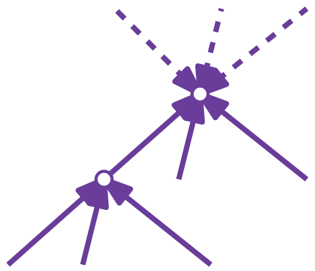

- Time: $\mathcal{O}(n+m)$
- Quality guarantee: $|E'| \geq |E|/2$
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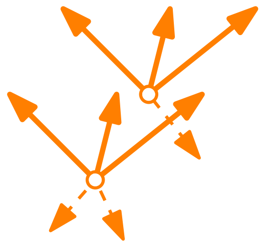

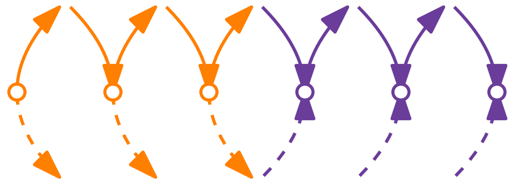

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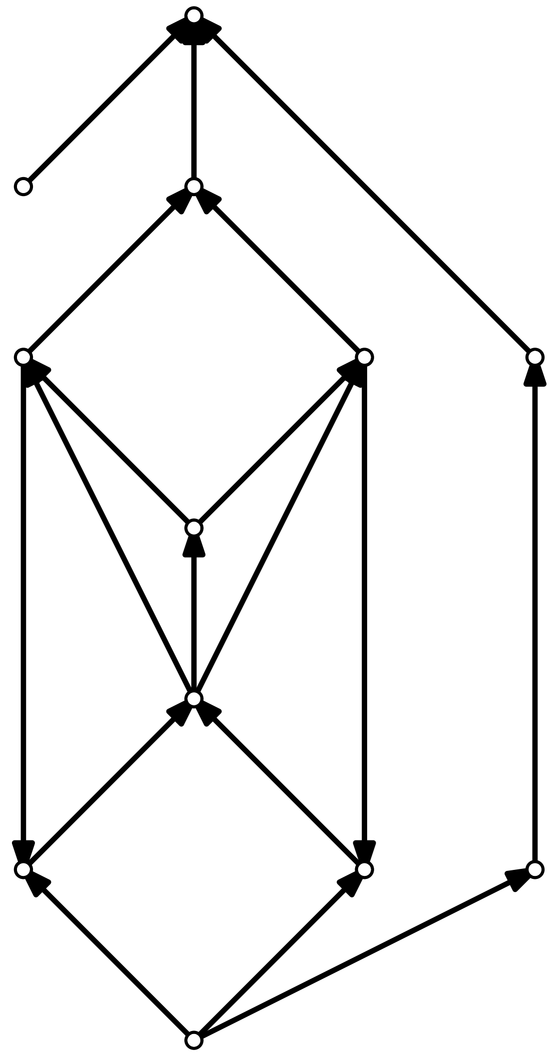
[Eades, Lin, Smyth '93]



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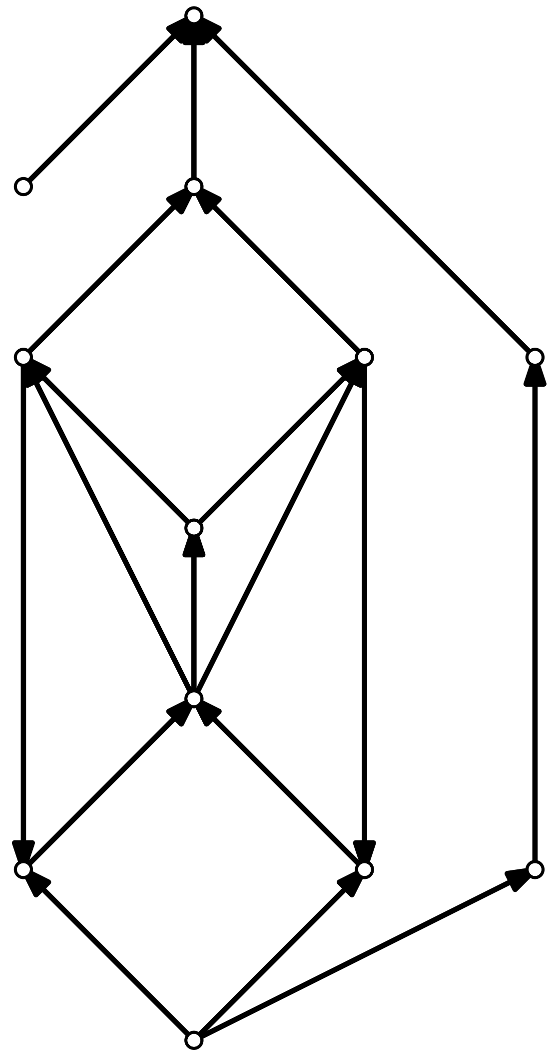

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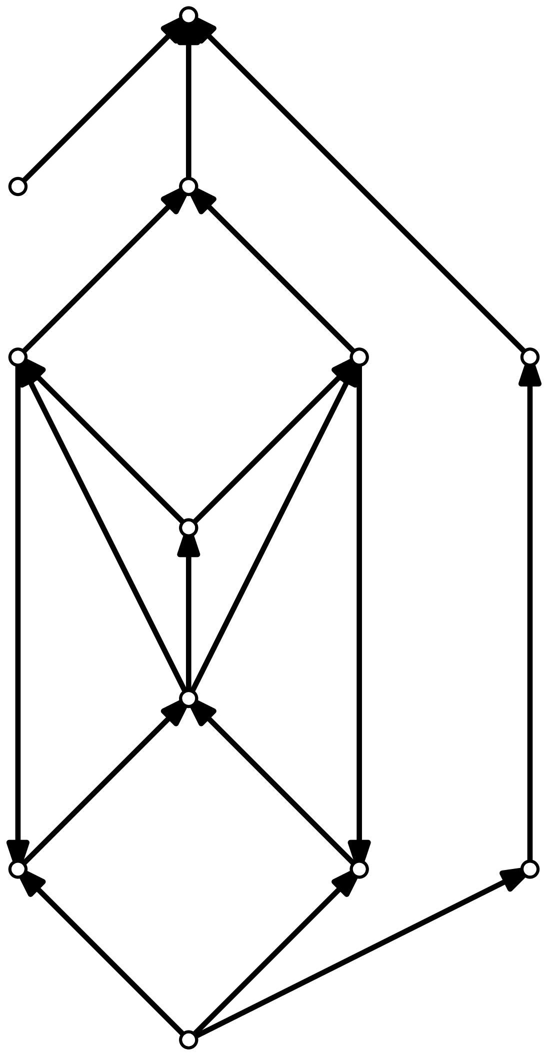

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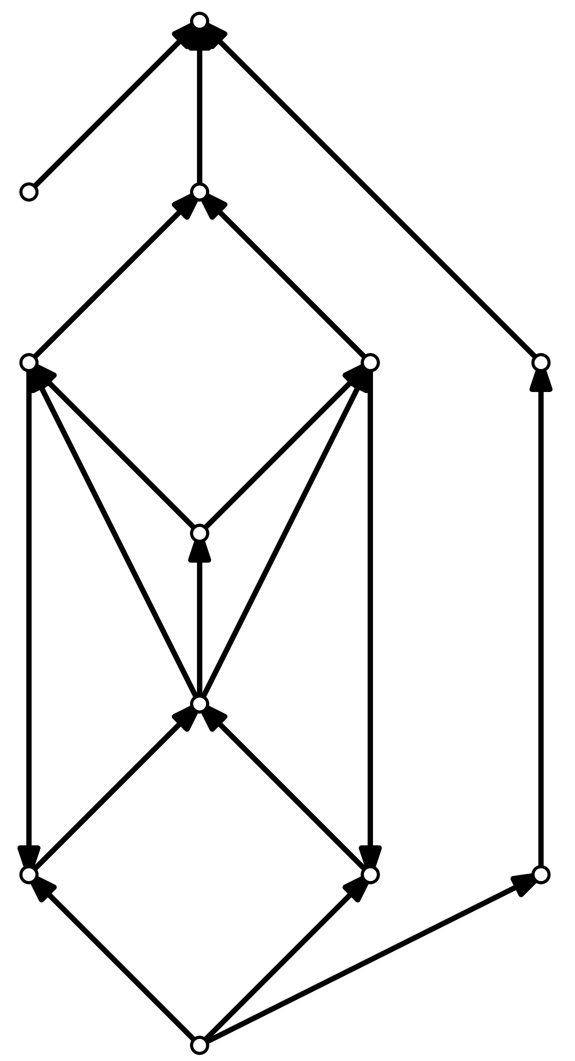

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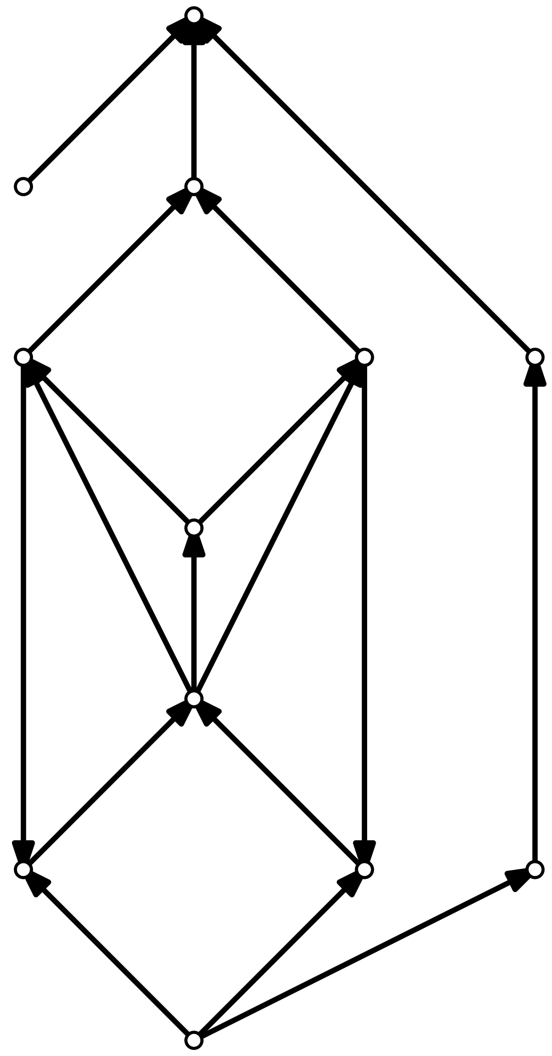

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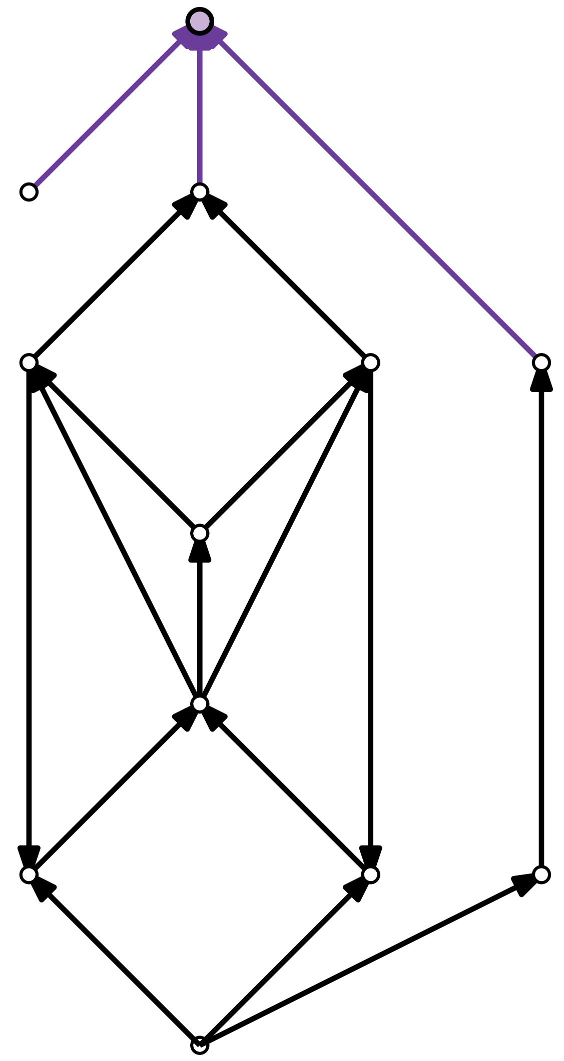

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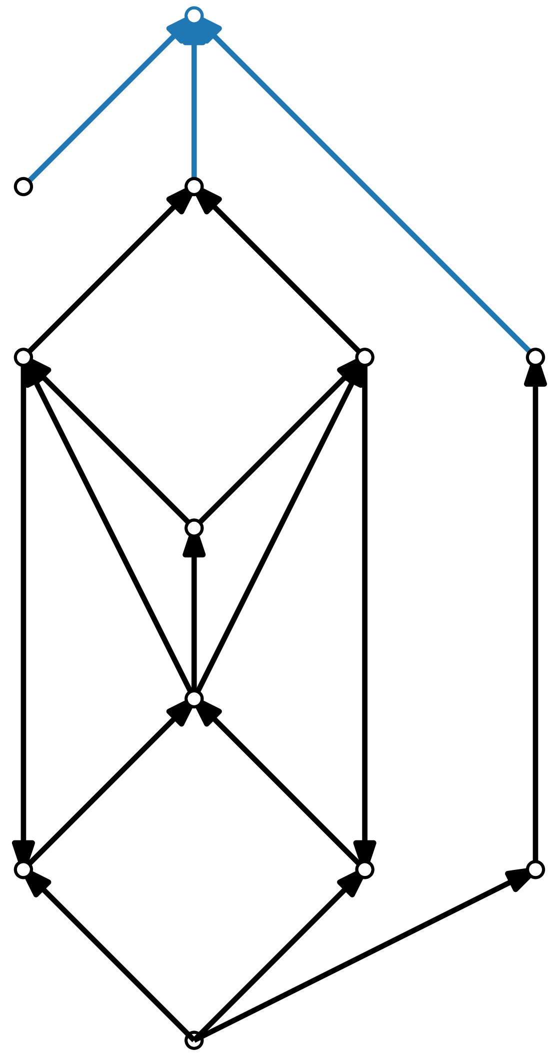

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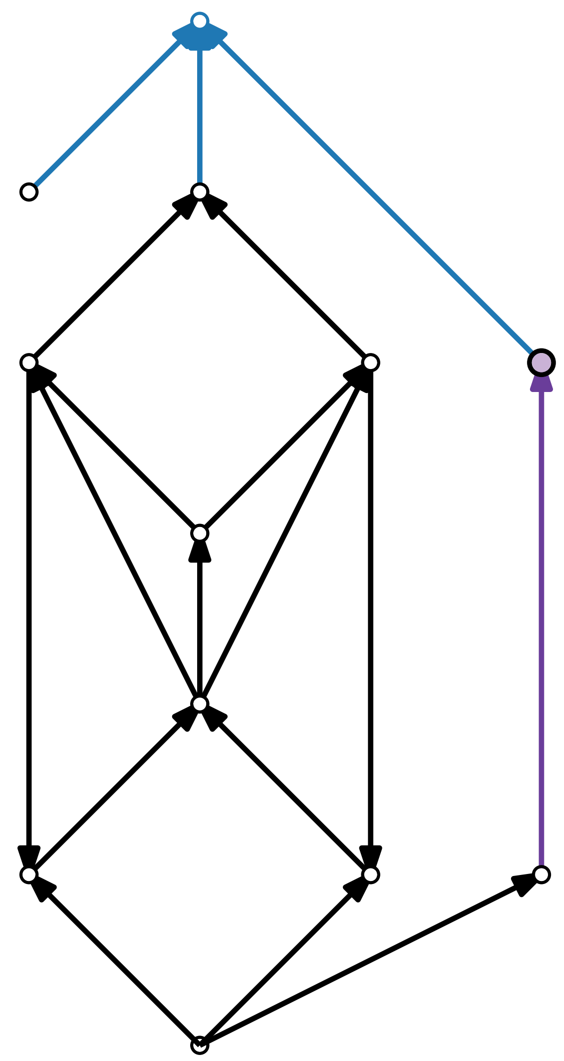

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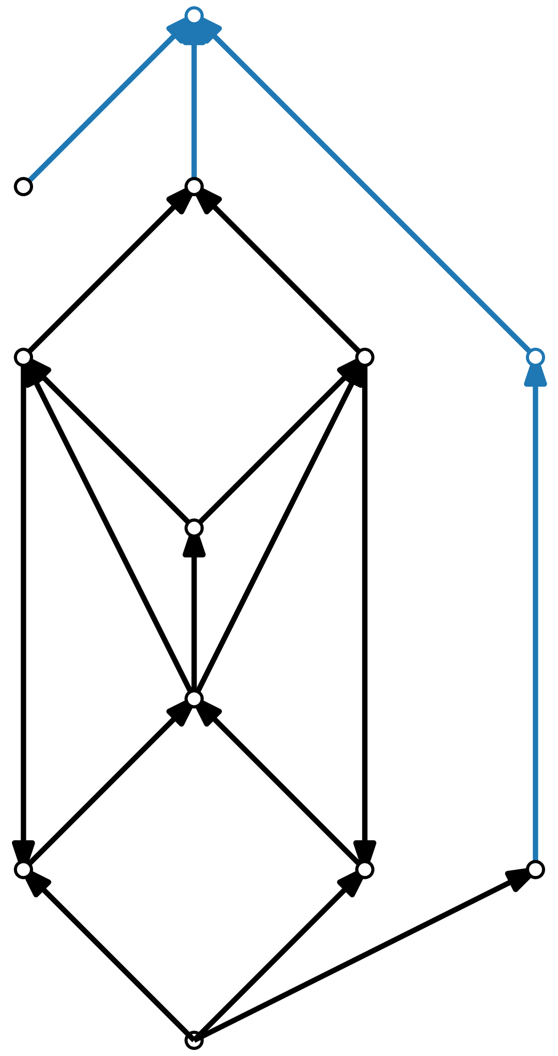

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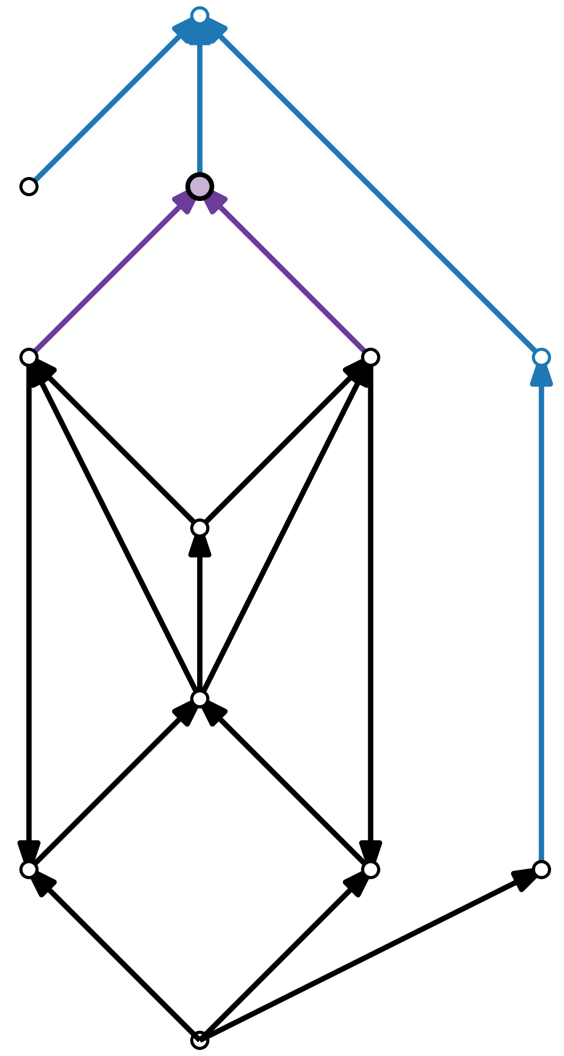

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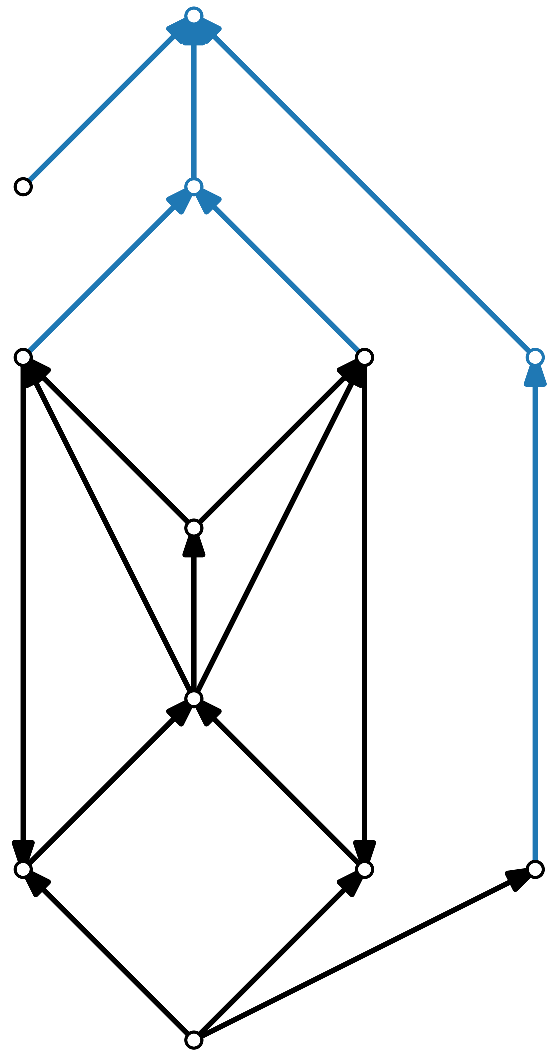

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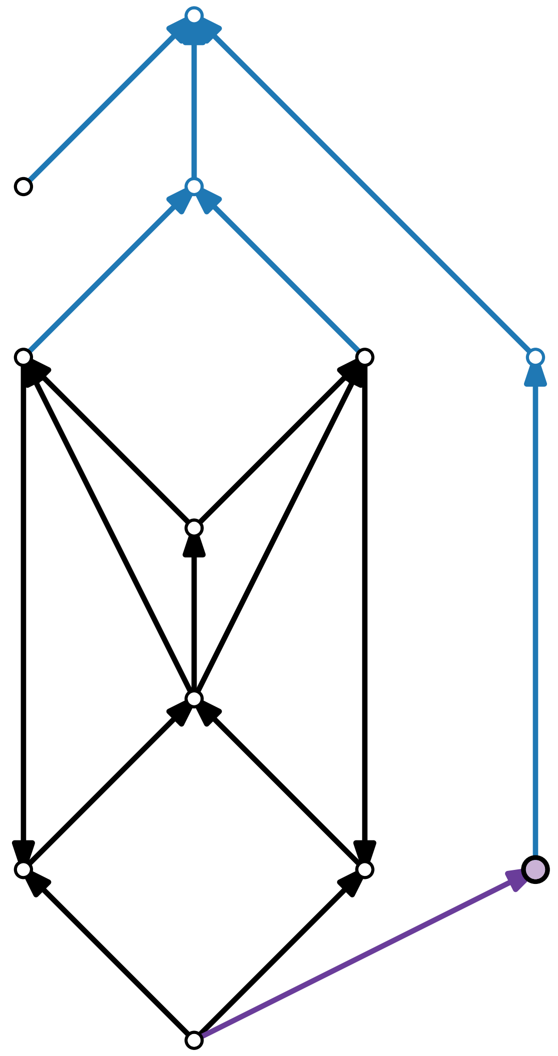

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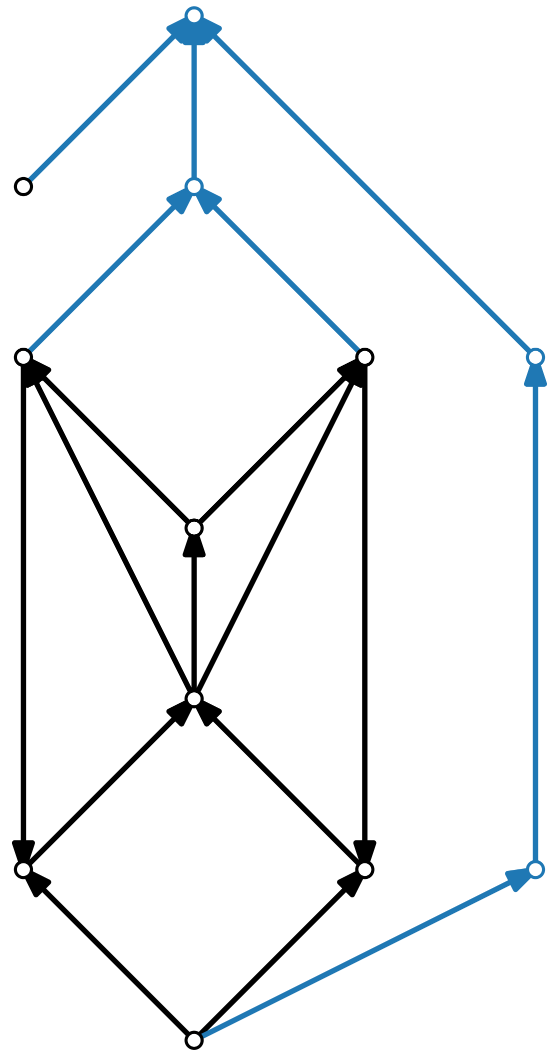

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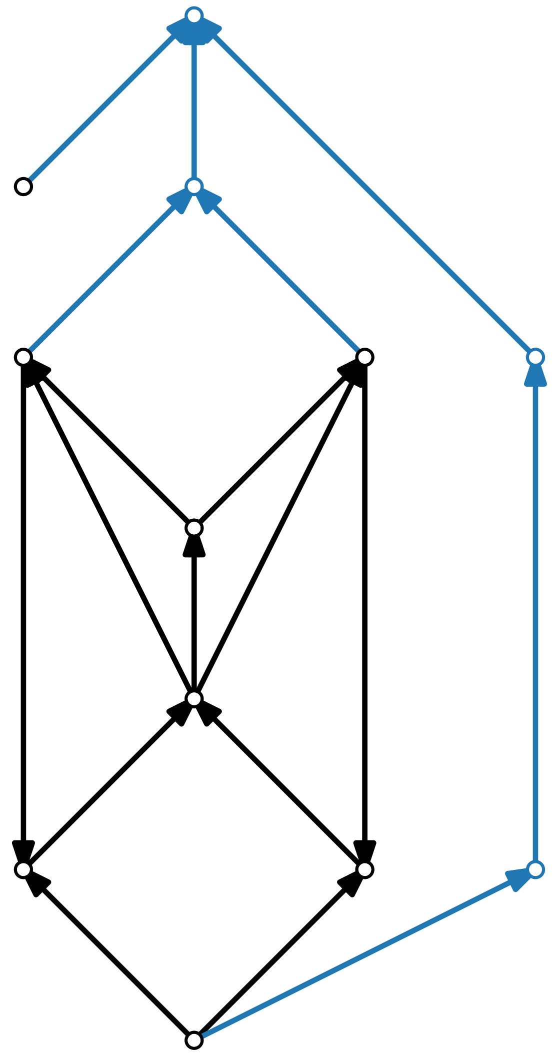

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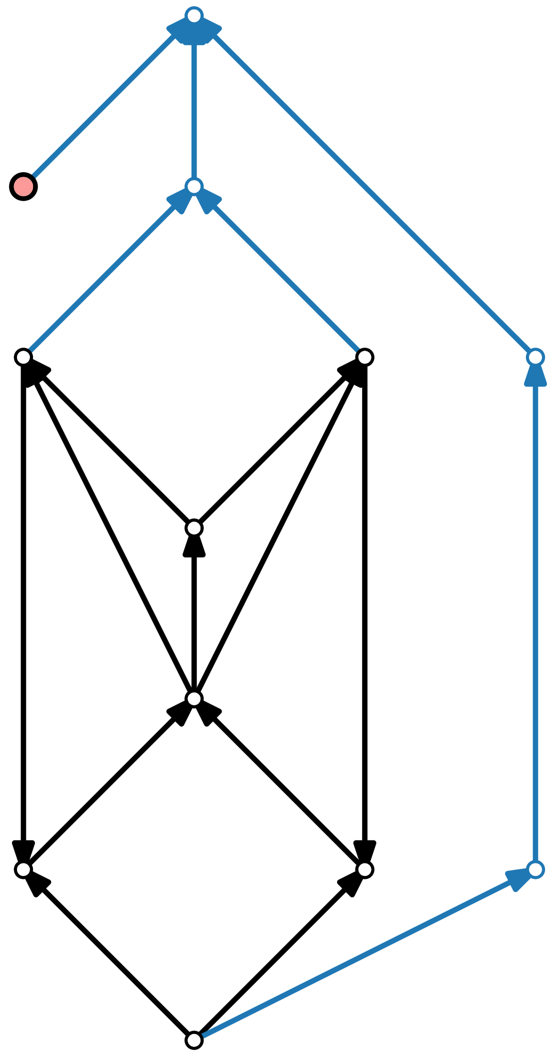

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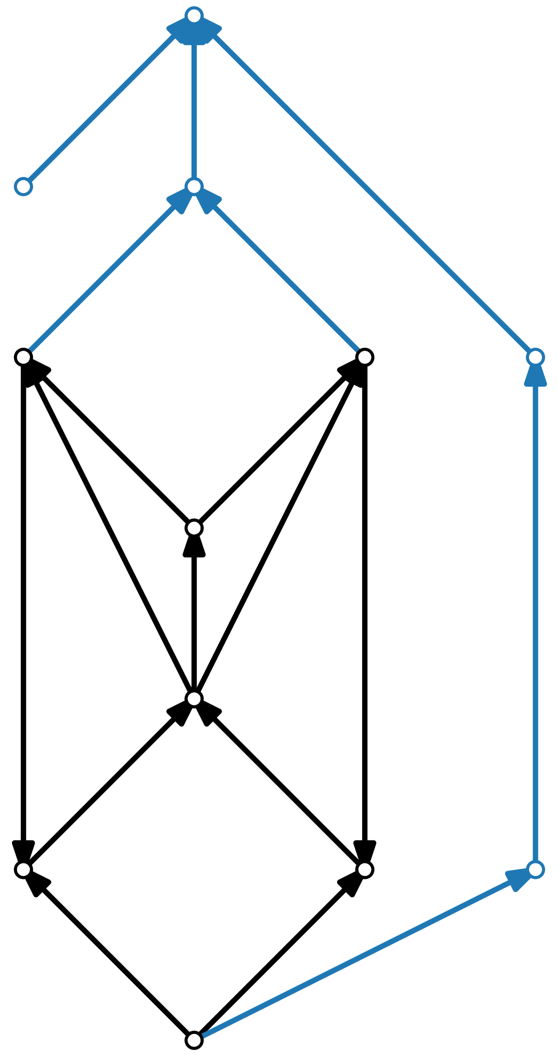

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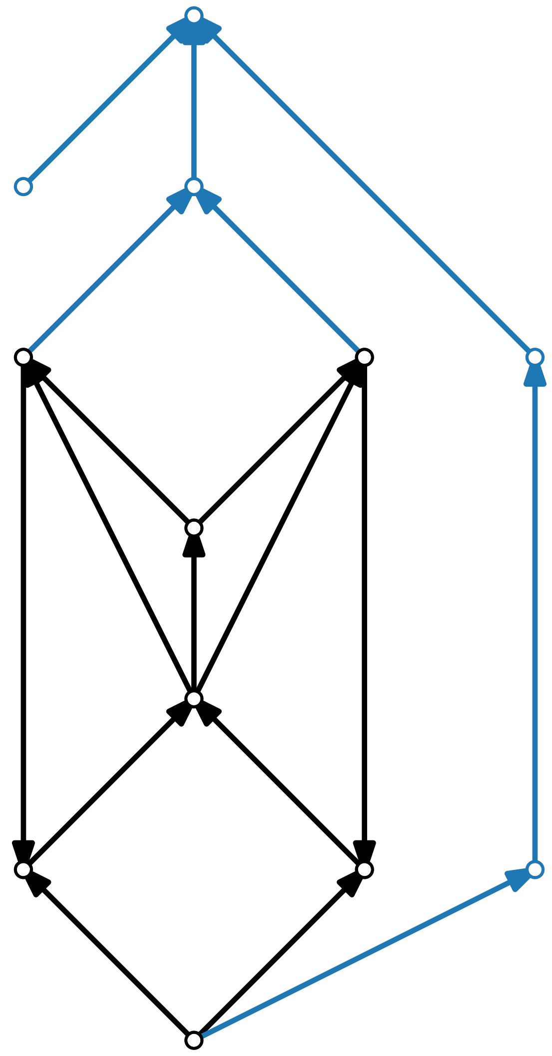

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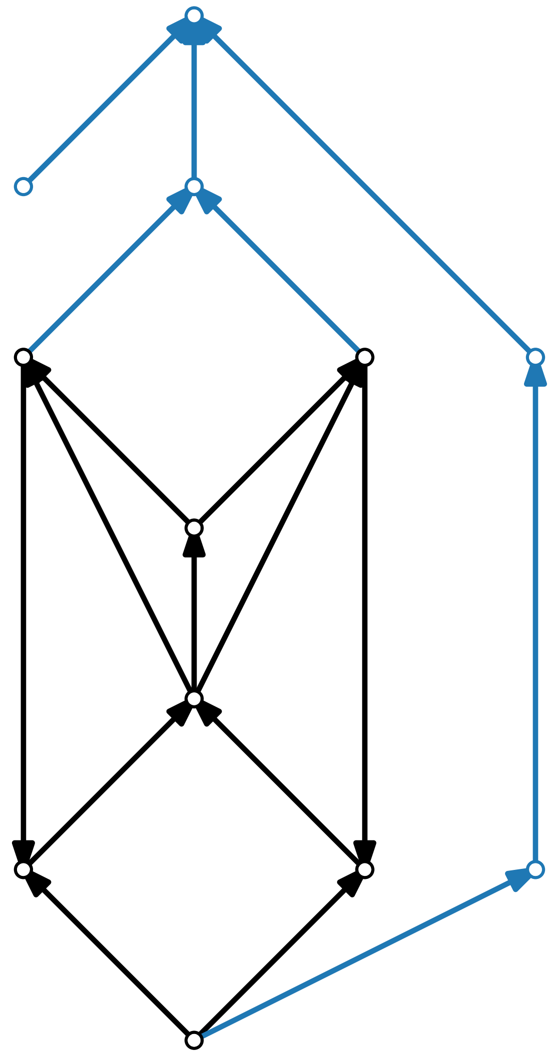

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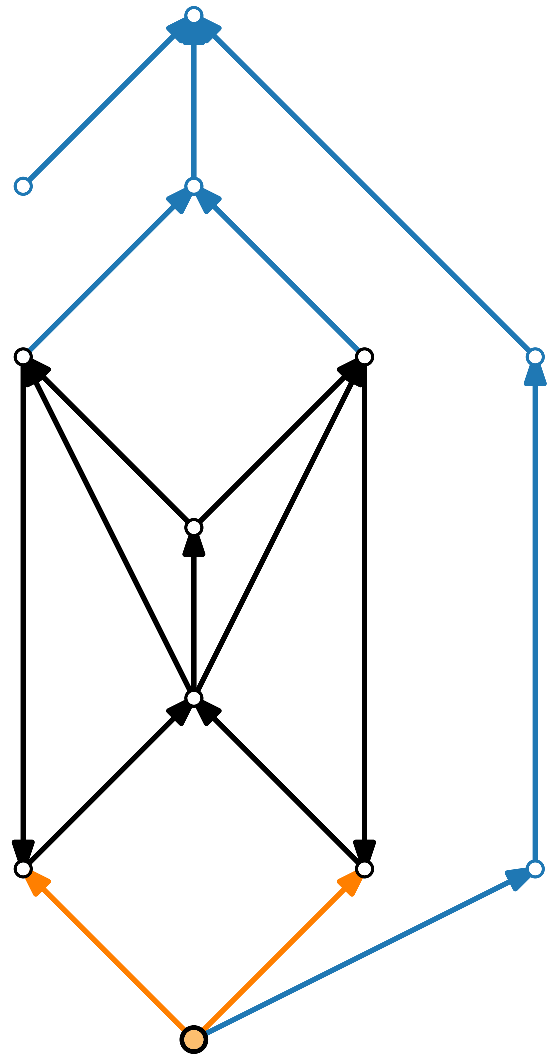

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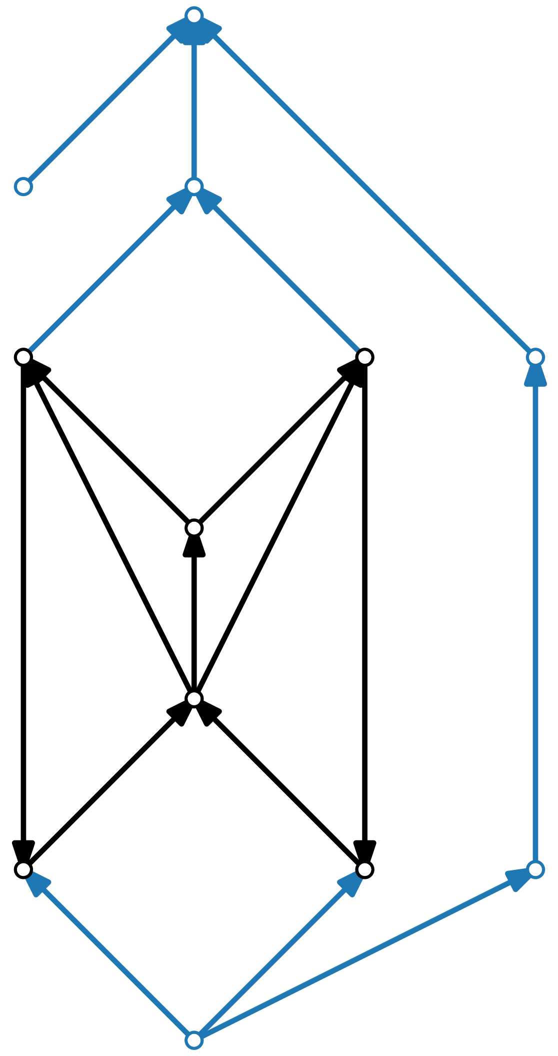

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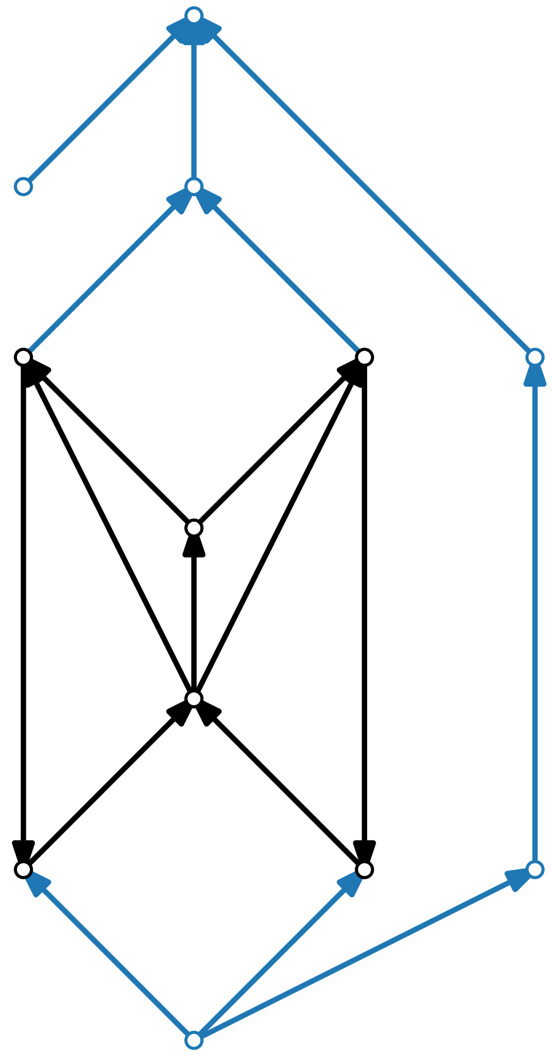

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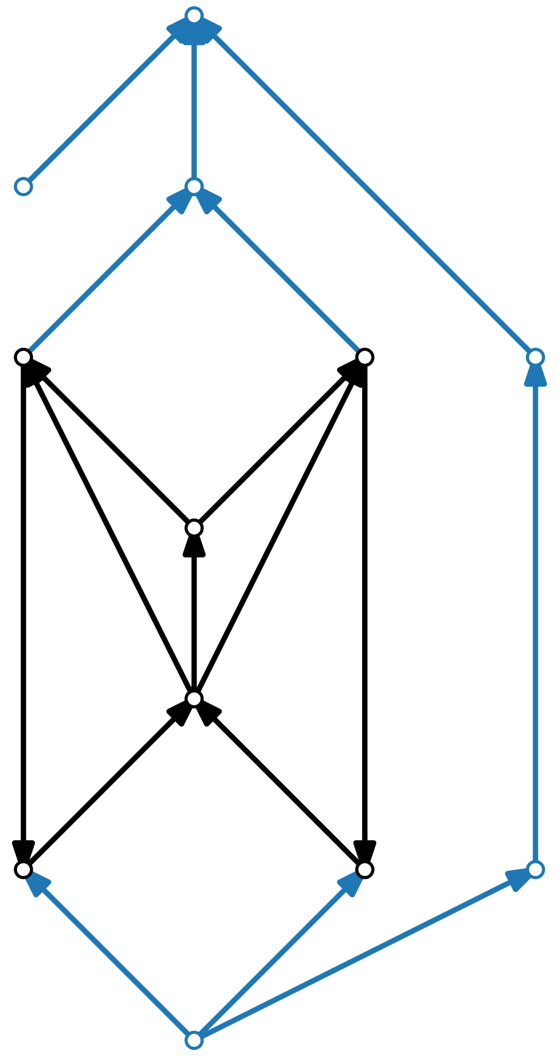

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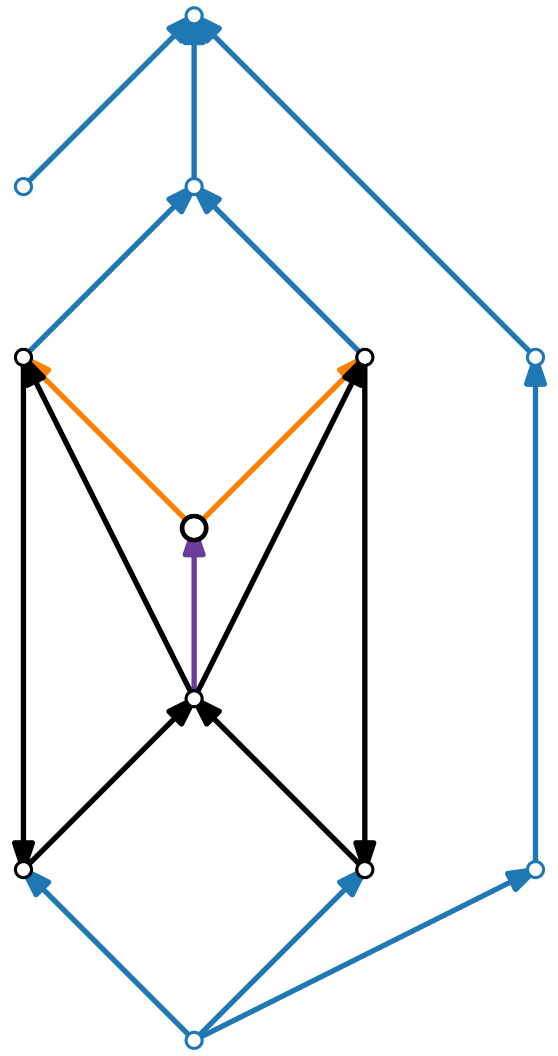

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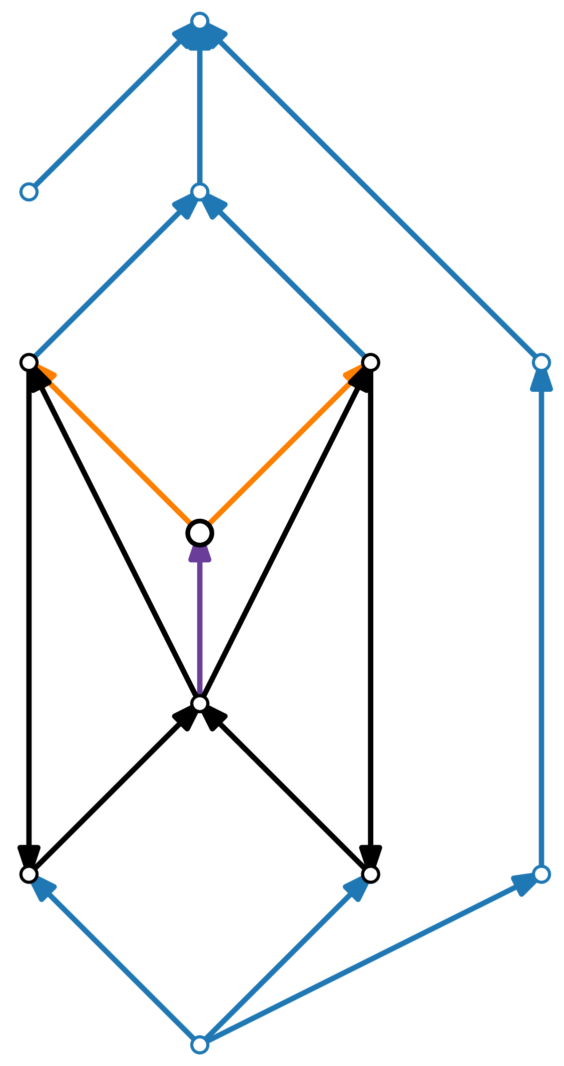

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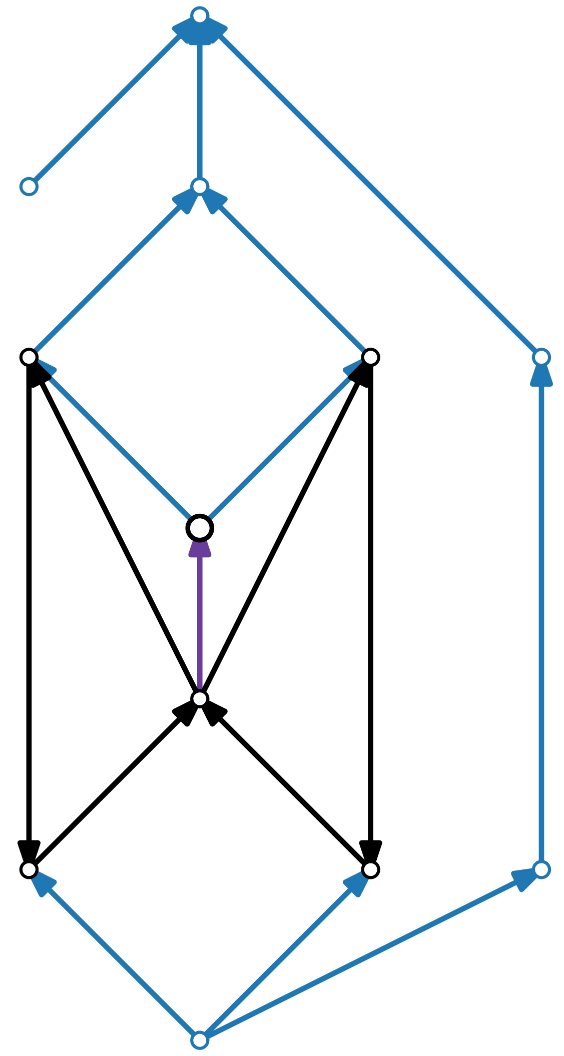

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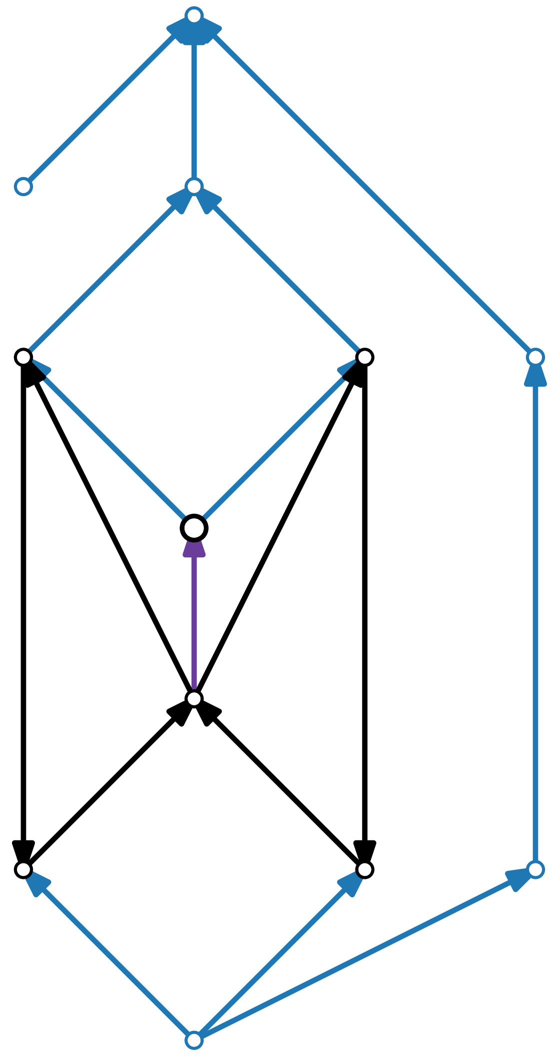

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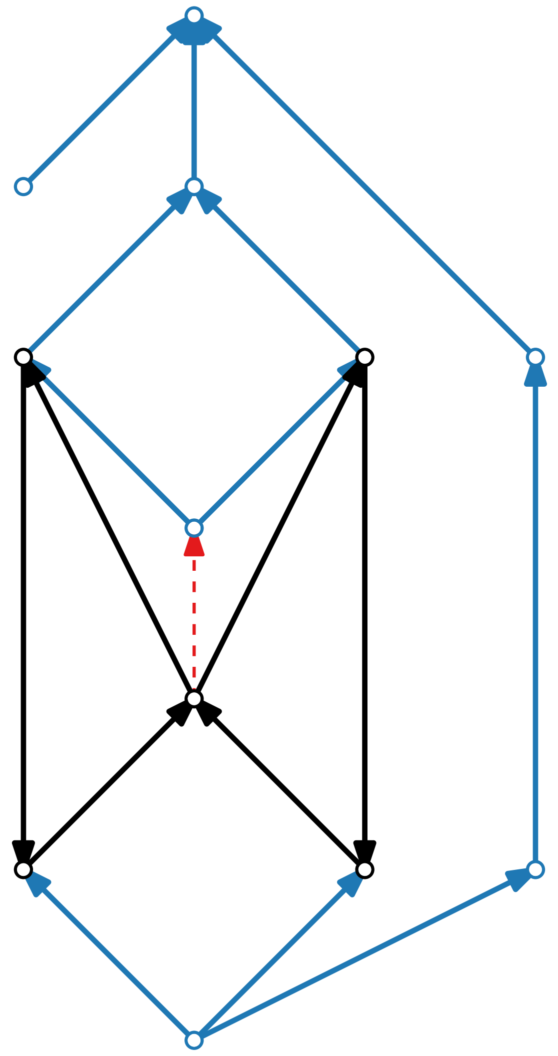

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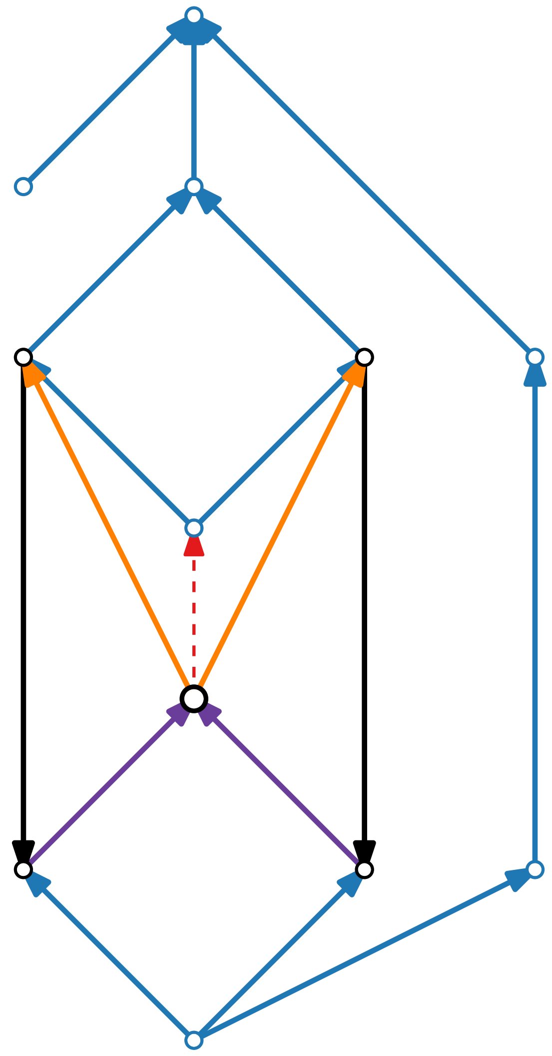

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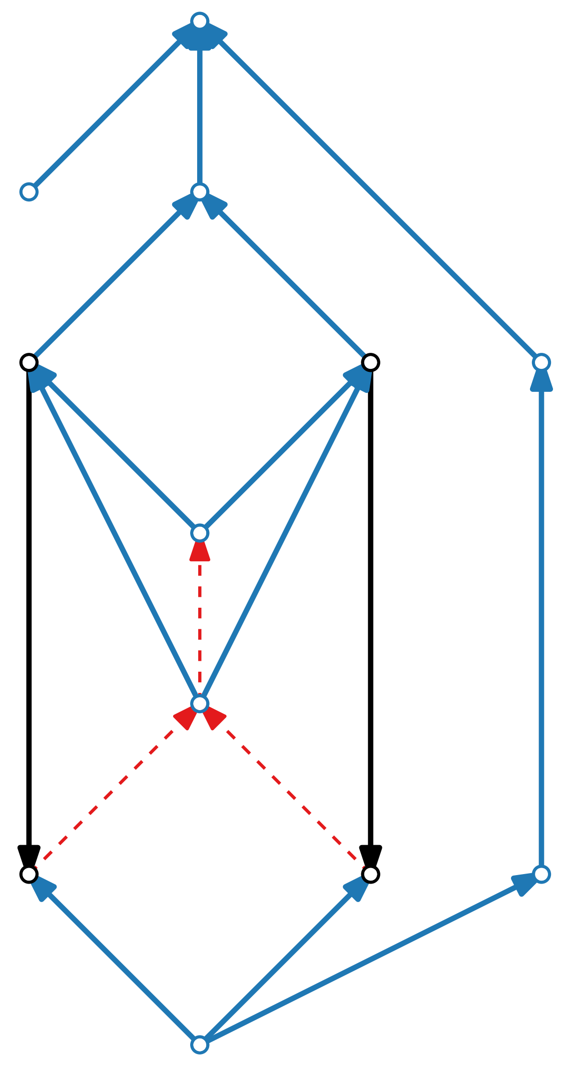

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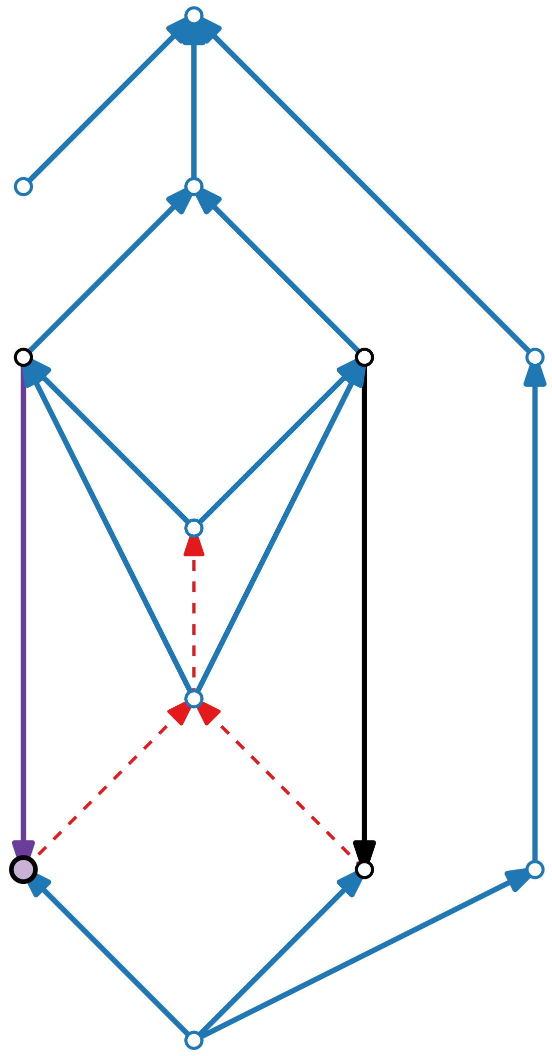

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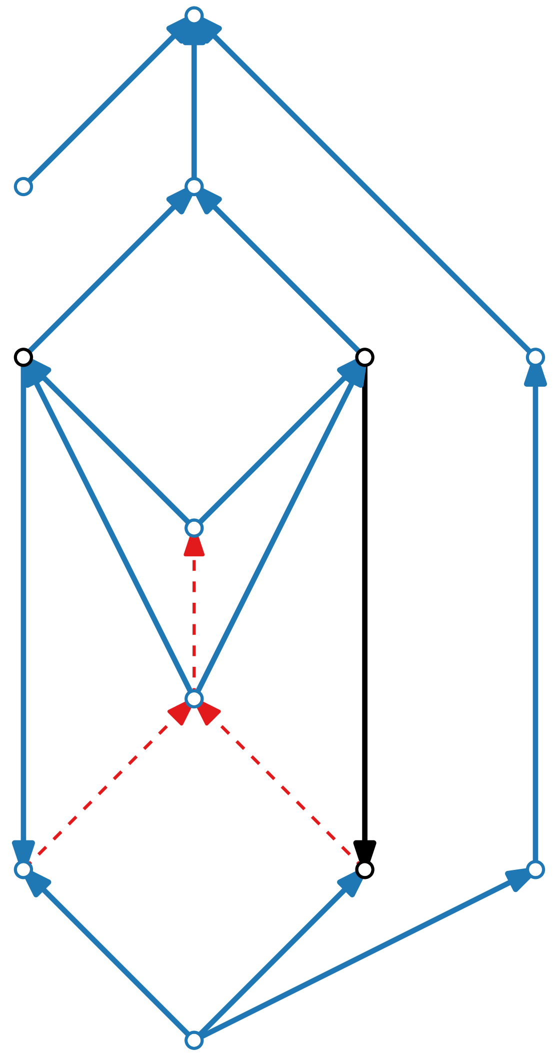

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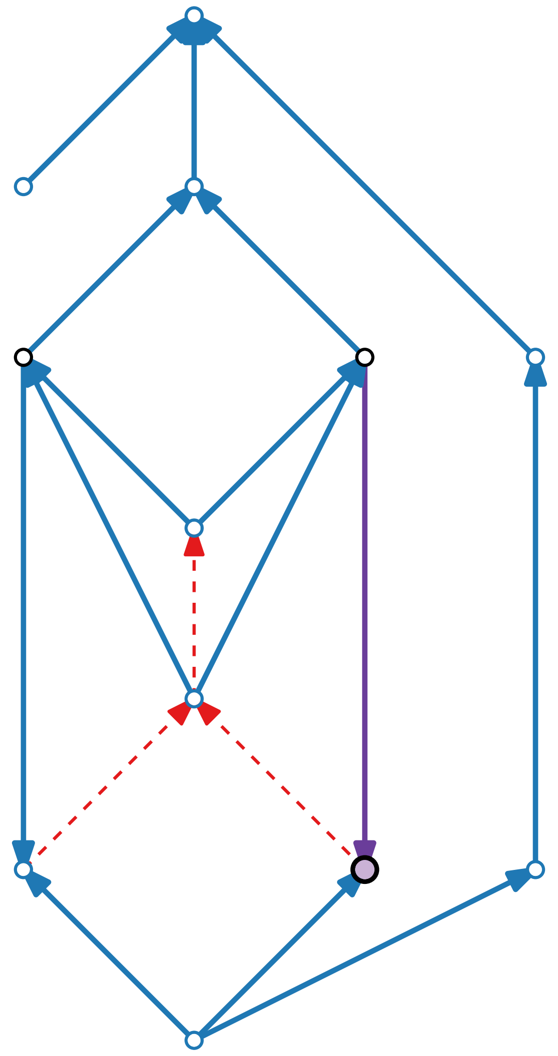

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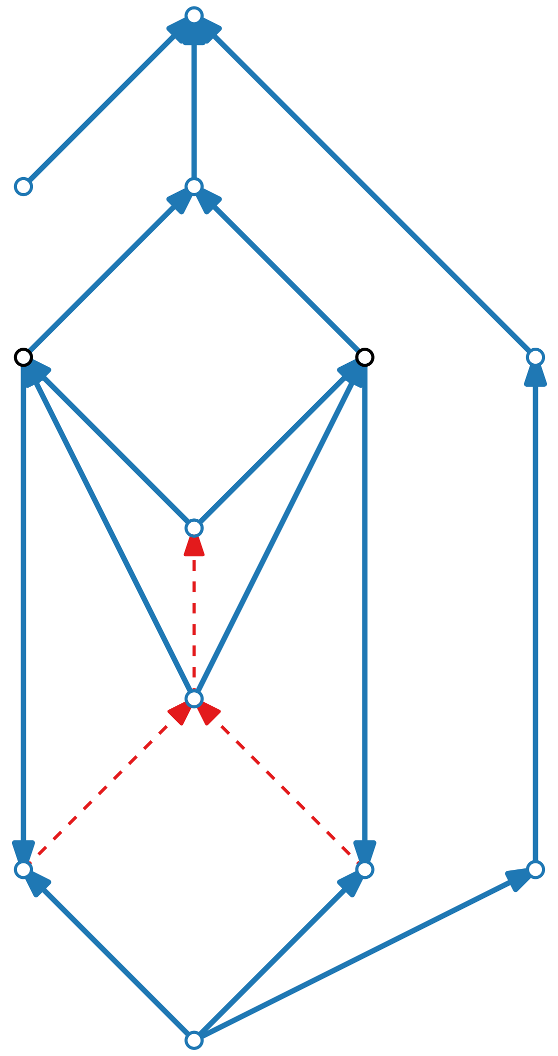

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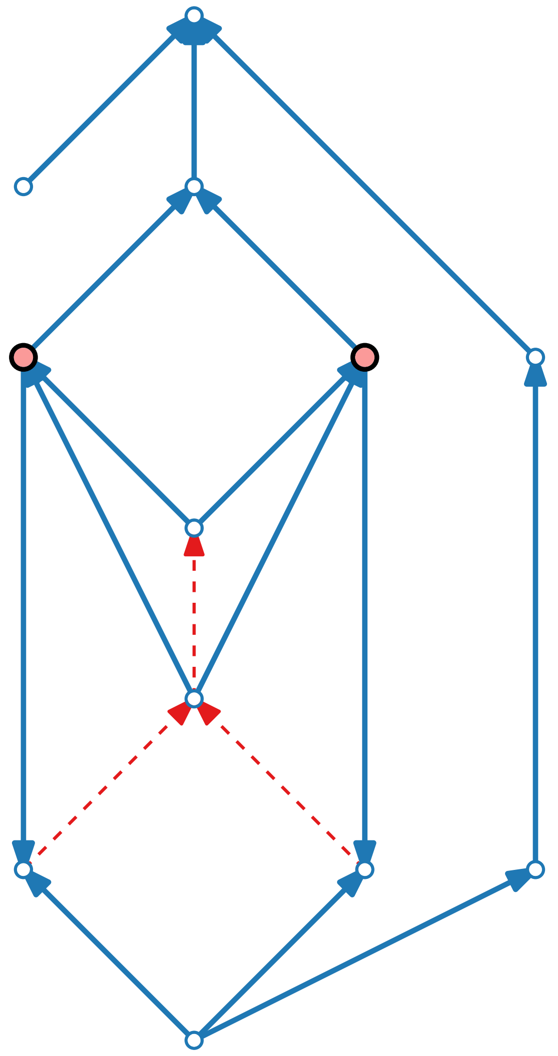

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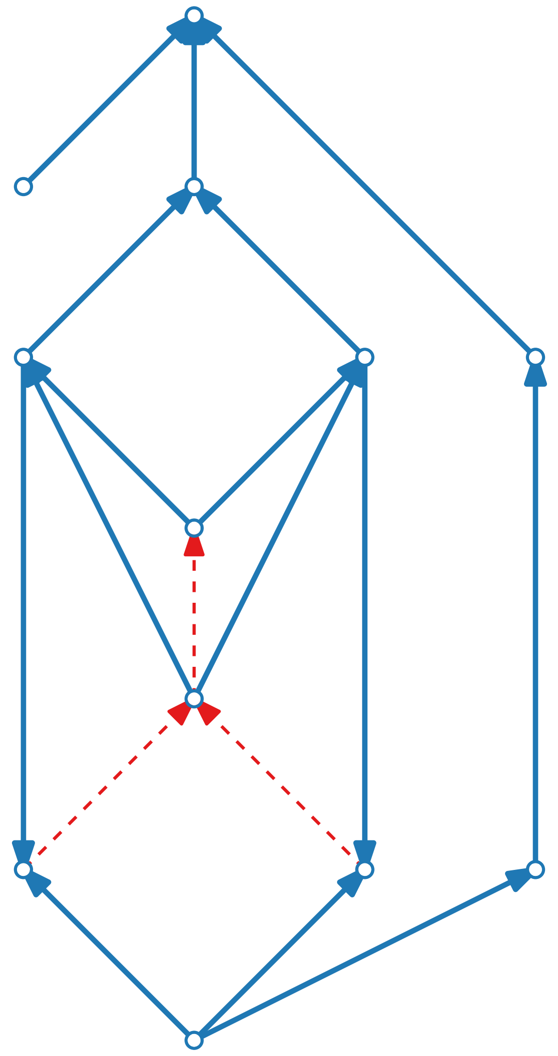

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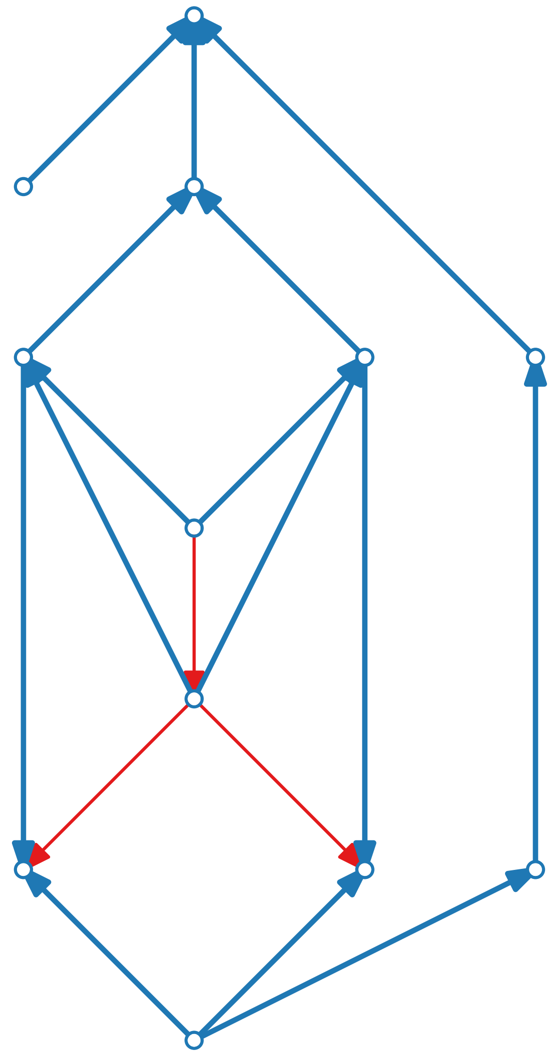

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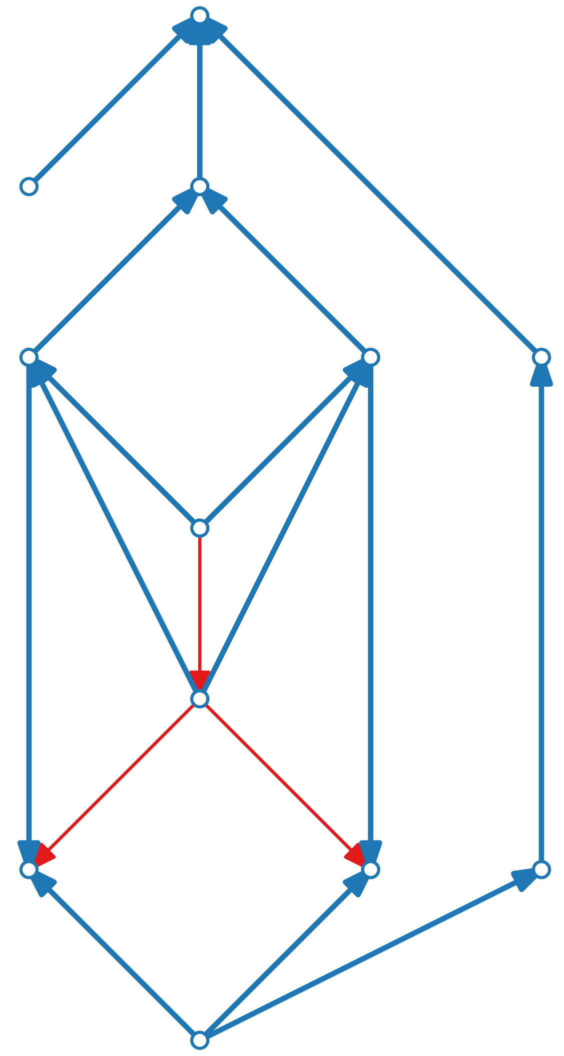
- Time: $\mathcal{O}(n+m)$



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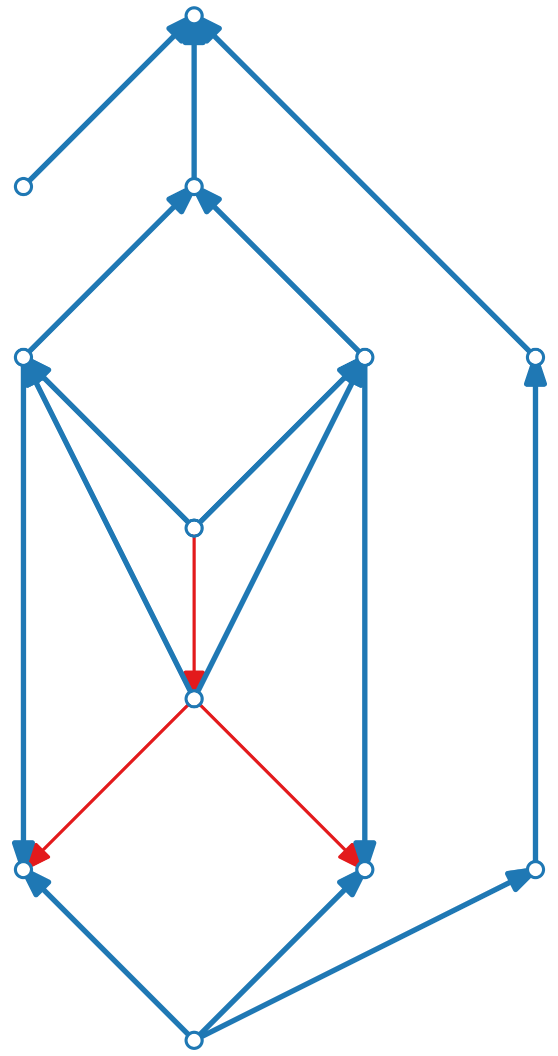
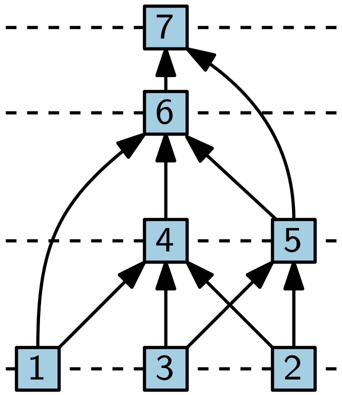
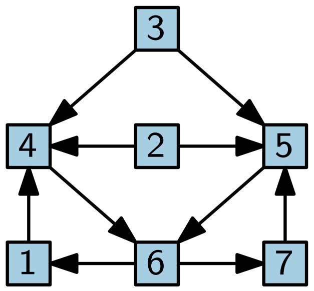
- Time: $\mathcal{O}(n+m)$
- Quality guarantee: $|E'| \geq |E|/2 + |V|/6$

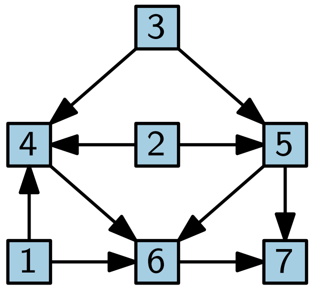


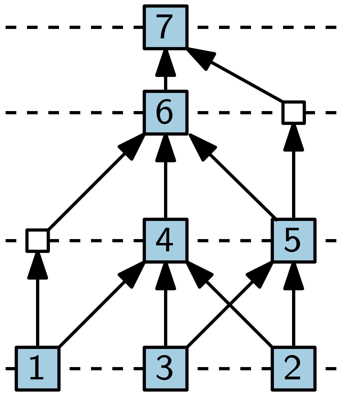


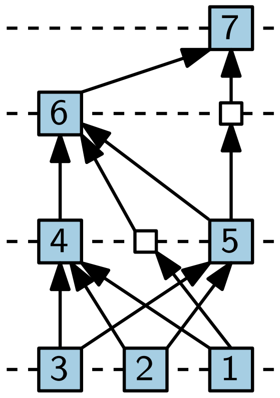


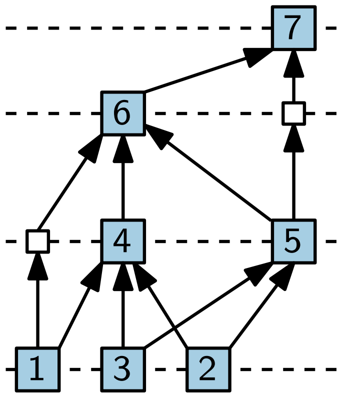
Visualization of Graphs

Lecture 7:
Hierarchical Layouts:
Sugiyama Framework





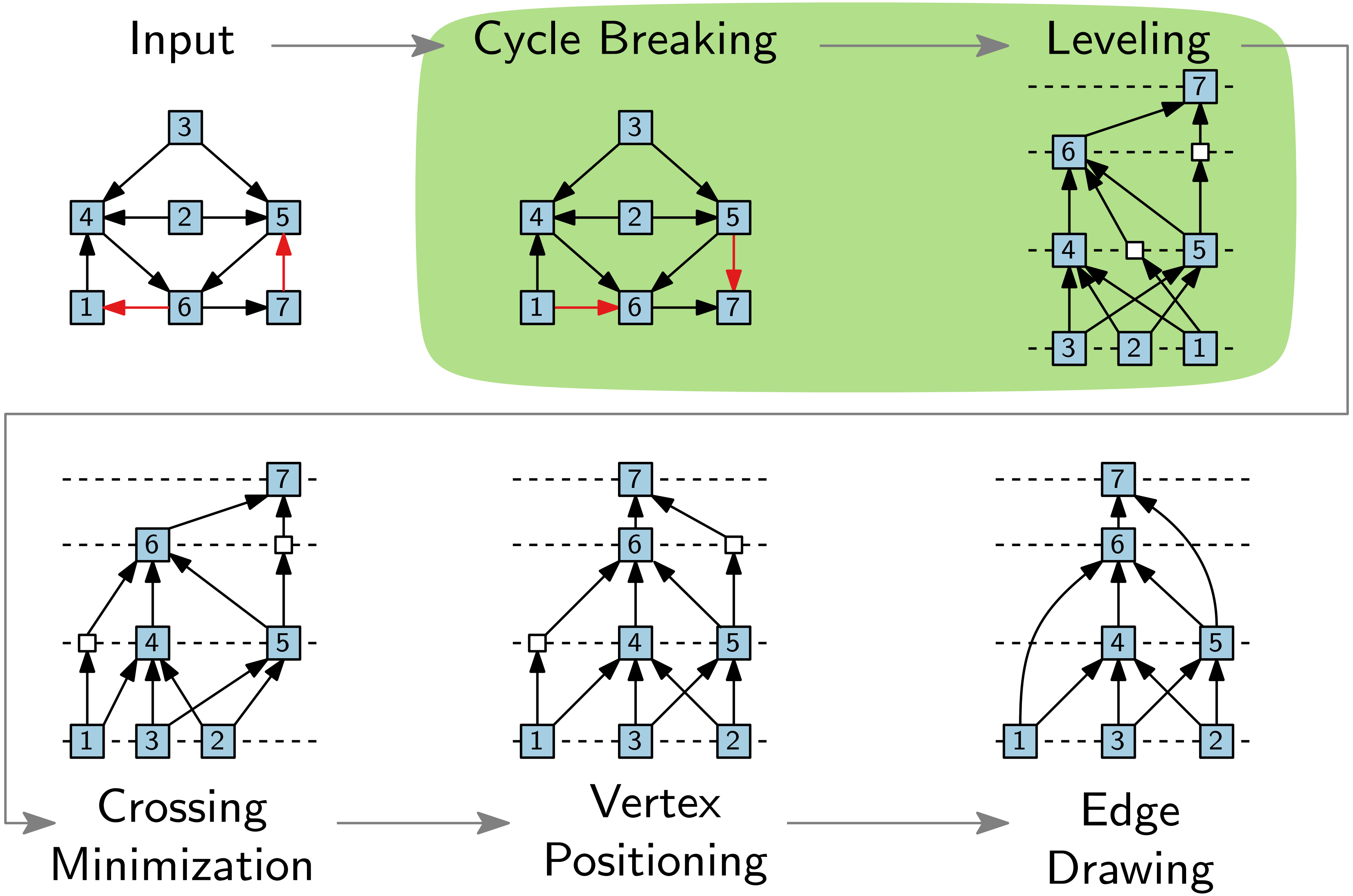




Part III:
Leveling

Jonathan Klawitter

Step 2: Leveling



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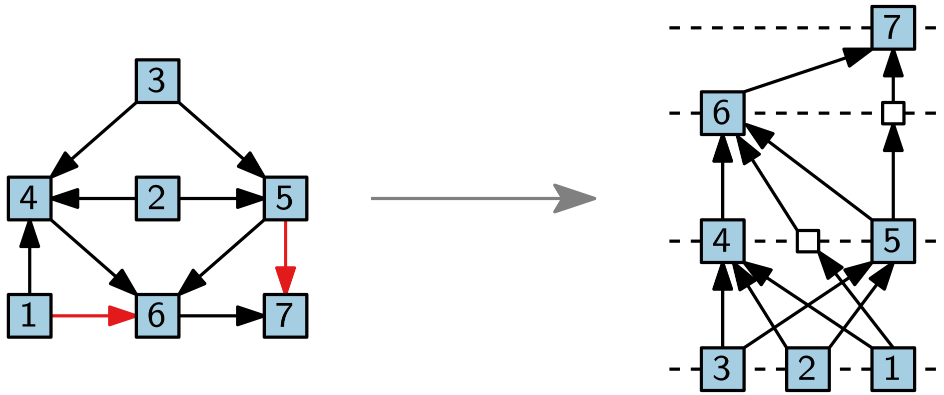
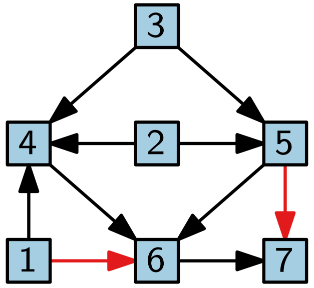
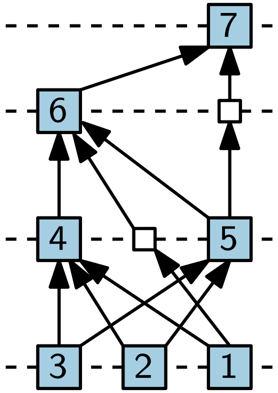


Problem.

Step 2: Leveling

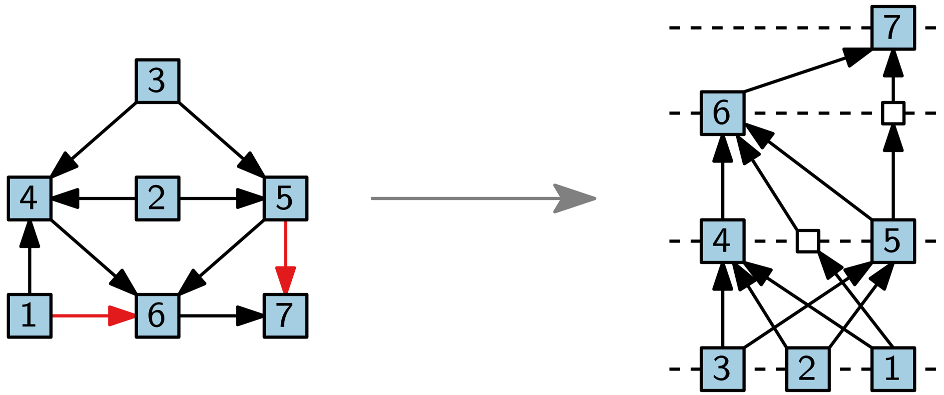




Problem.

- Input: acyclic digraph $G = (V, E)$

Step 2: Leveling



Problem.

- Input: acyclic digraph $G = (V, E)$
- Output: Mapping $y\colon V \to \{1, \ldots n\}$,
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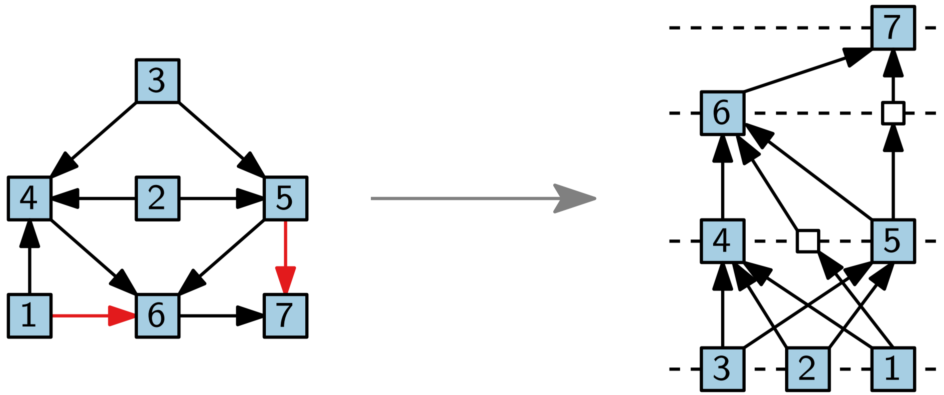

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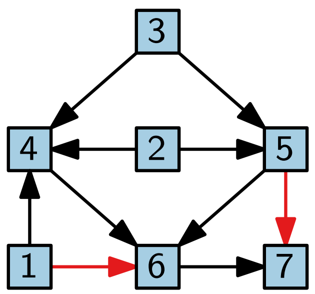

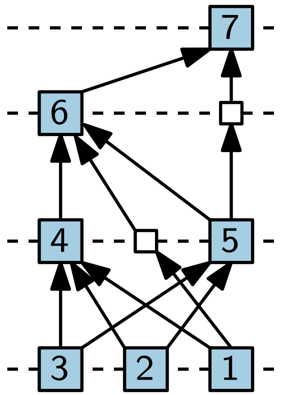

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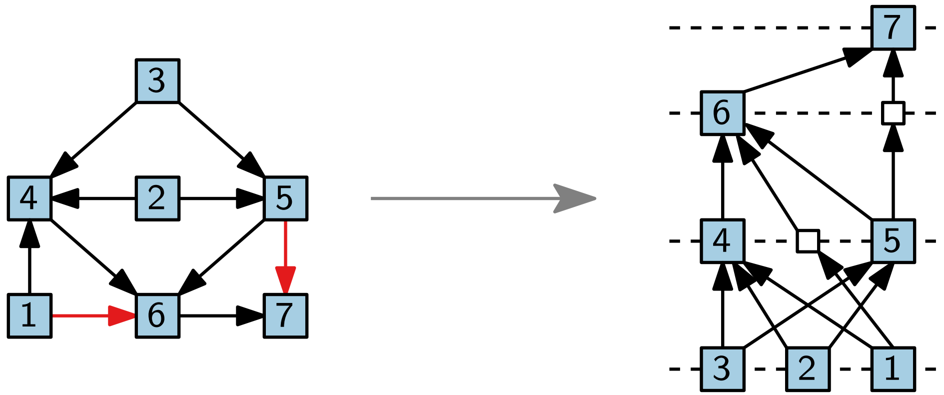

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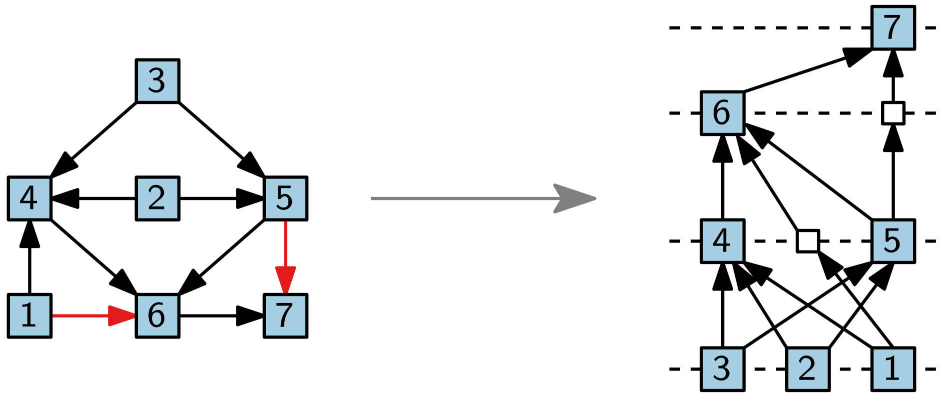

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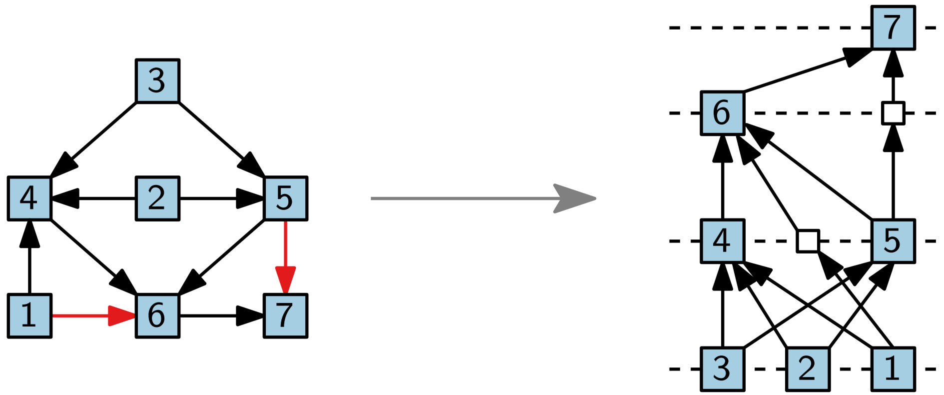
Step 2: Leveling



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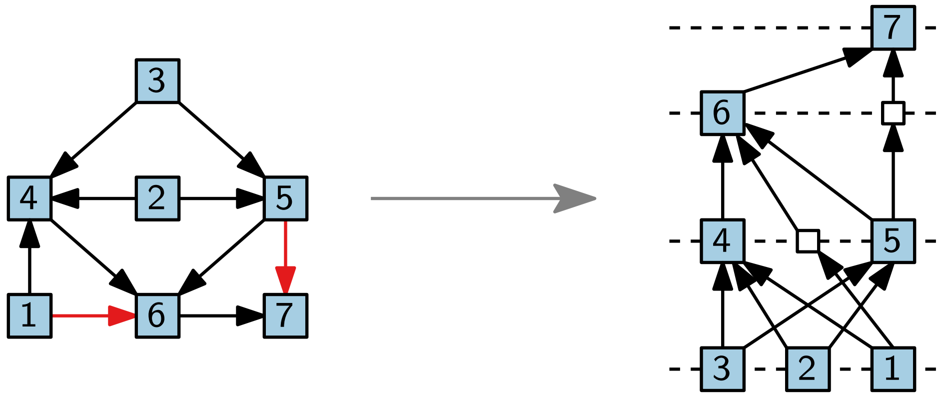
- number of layers, i.e. $|y(V)|$
- length of the longest edge, i.e. $\max_{uv \in E} y(v) - y(u)$

Step 2: Leveling



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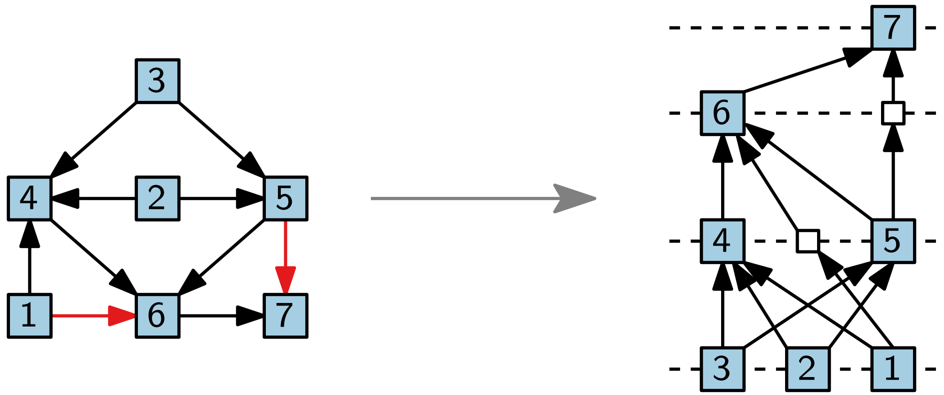
- number of layers, i.e. $|y(V)|$
- length of the longest edge, i.e. $\max_{uv \in E} y(v) - y(u)$
- width,

Step 2: Leveling



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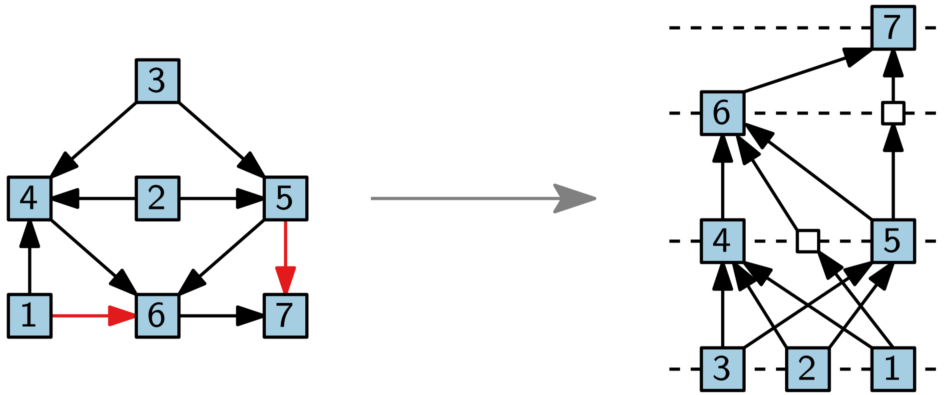

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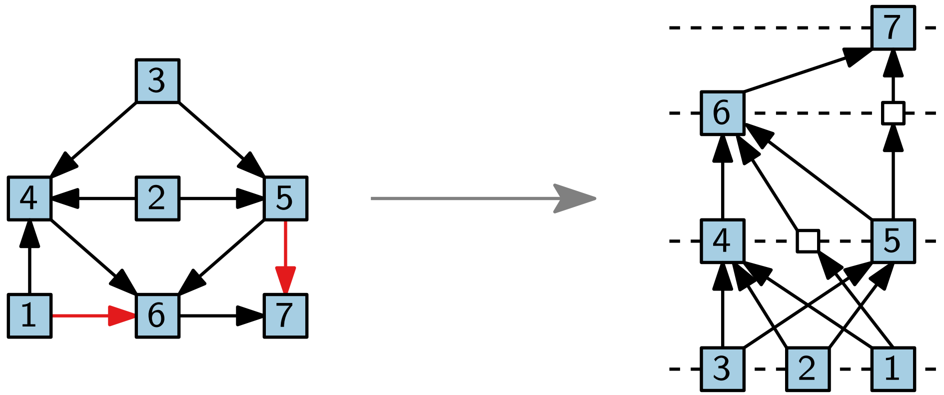

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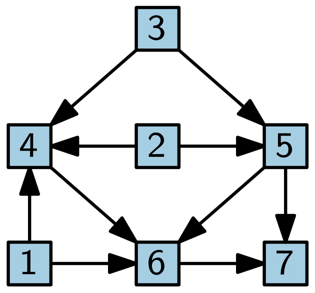
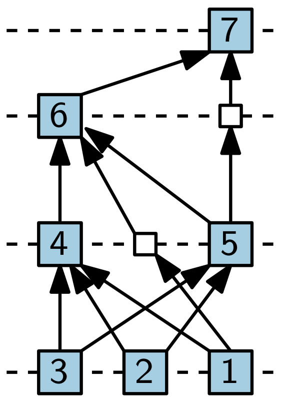
- total edge length, i.e. number of dummy vertices

Min Number of Layers

Algorithm.





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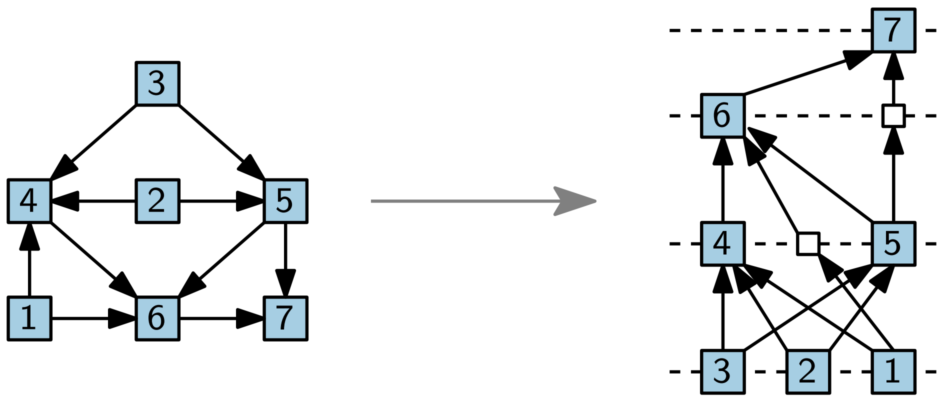
  set $y(q) := 1$



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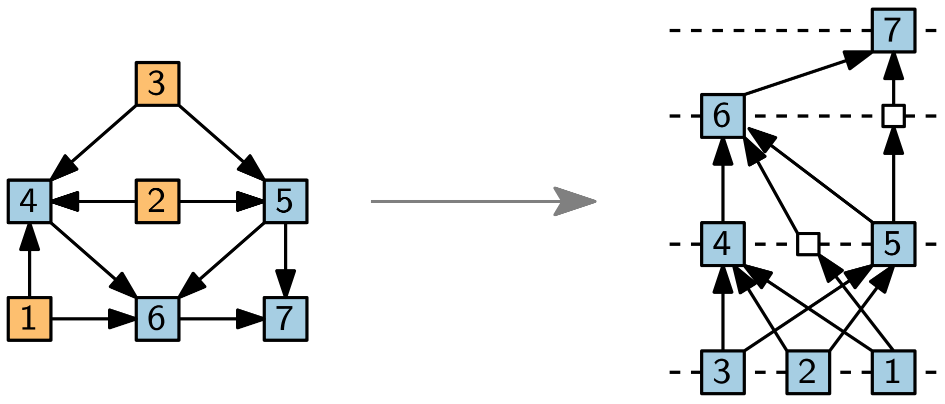

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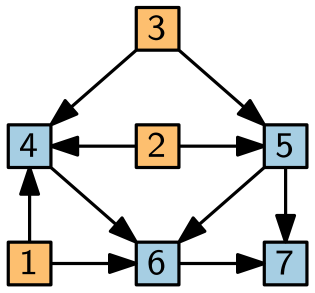

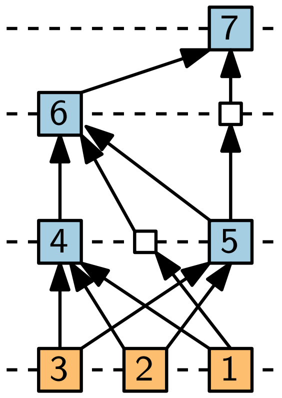

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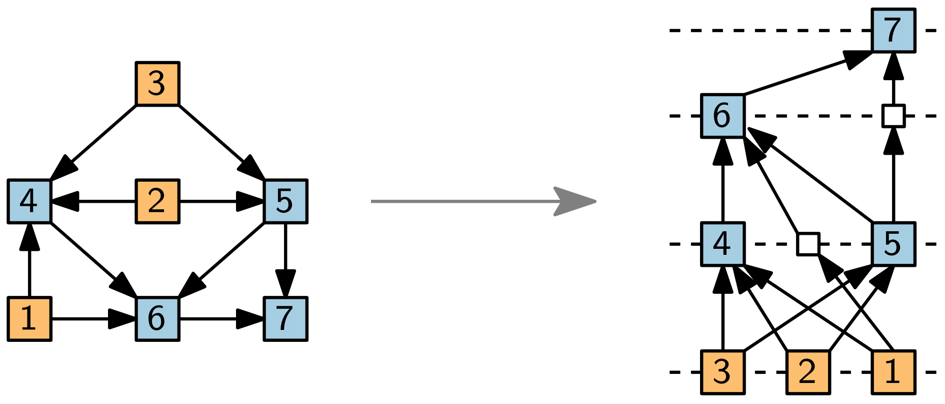

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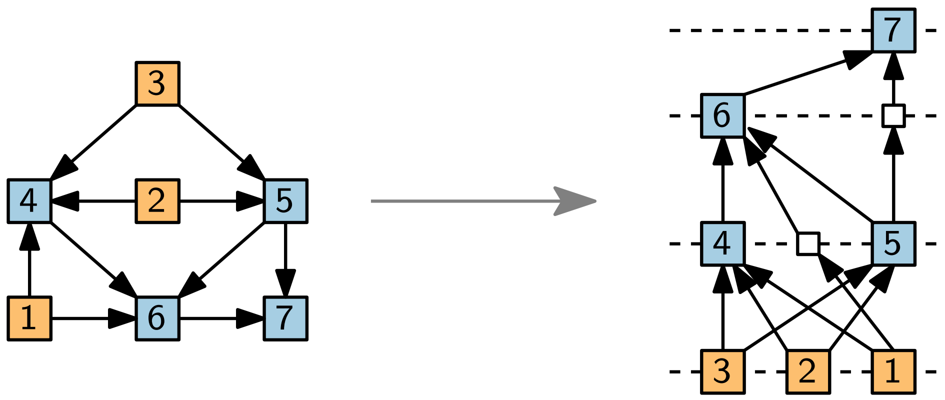

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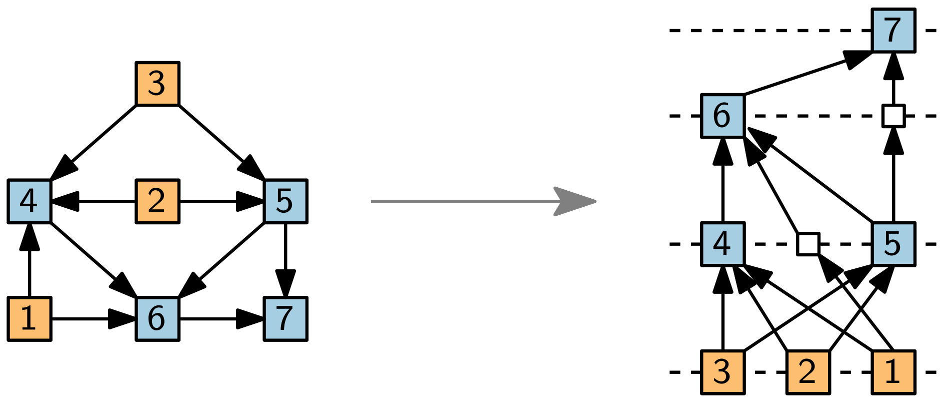

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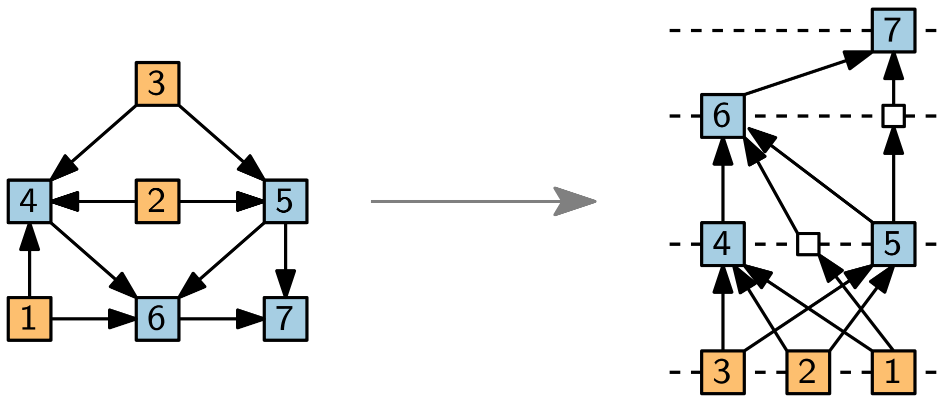

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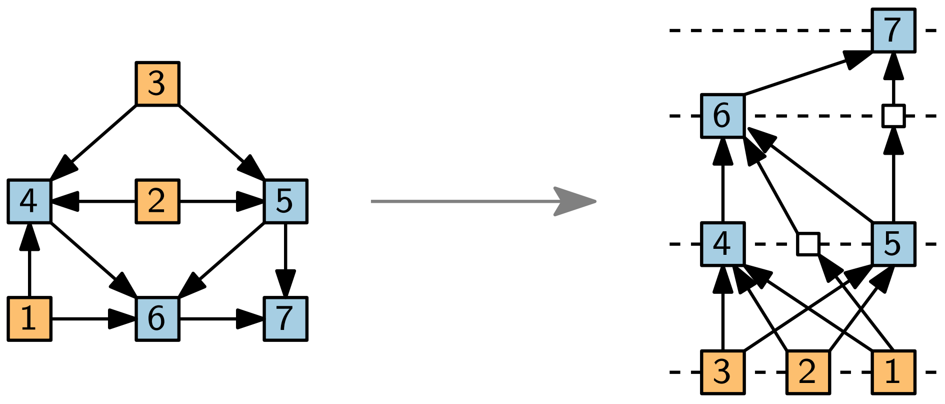

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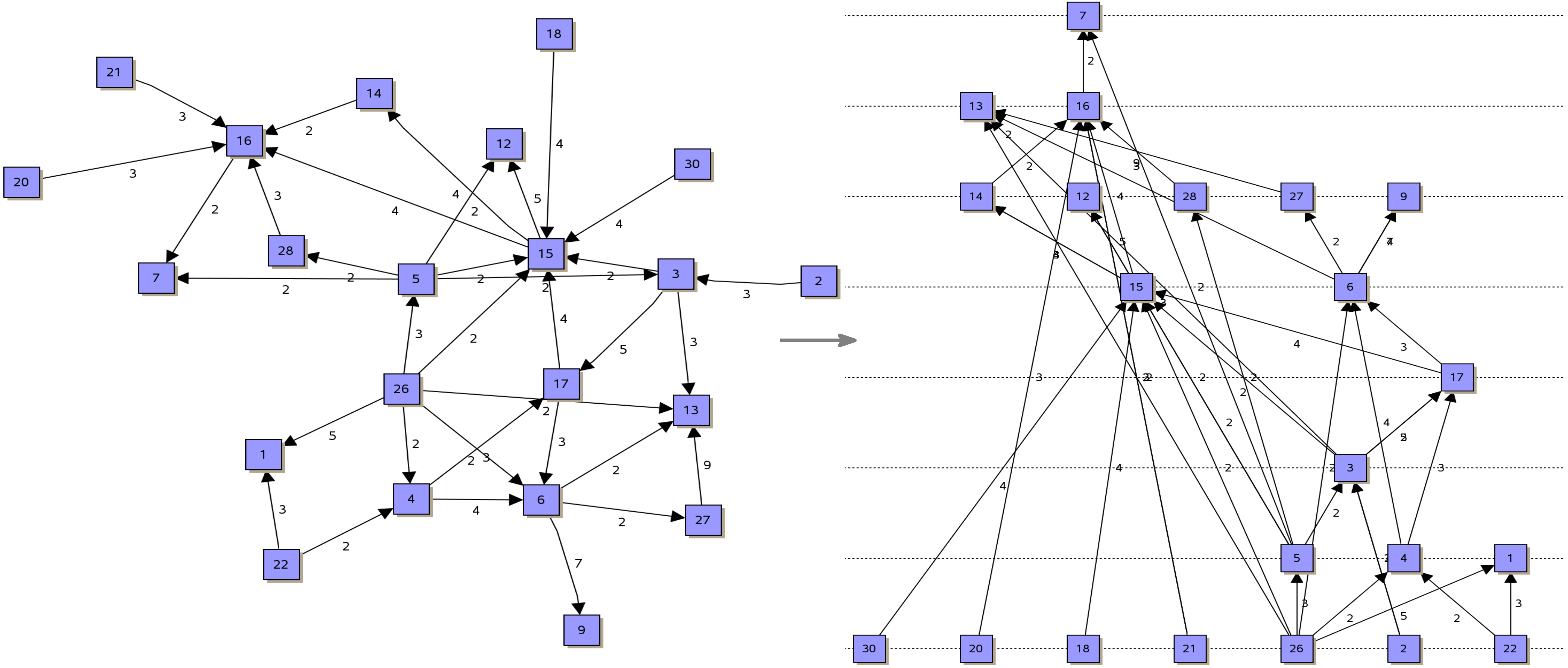
  ...which is optimal!

- Can be implemented in linear time with recursive algorithm.

Example


21
14
18
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12
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7
5
3
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26
2
22

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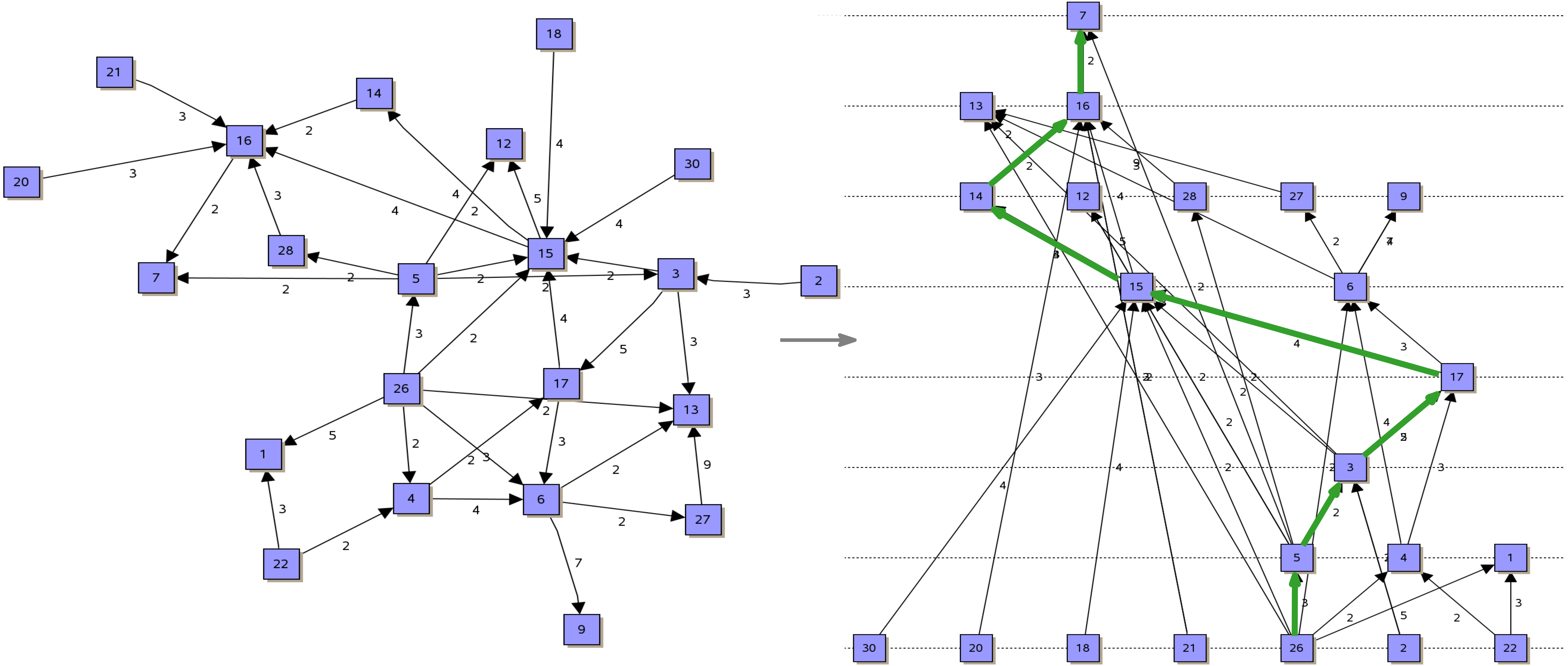

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One can show that:

- Constraint-matrix is **totally unimodular**

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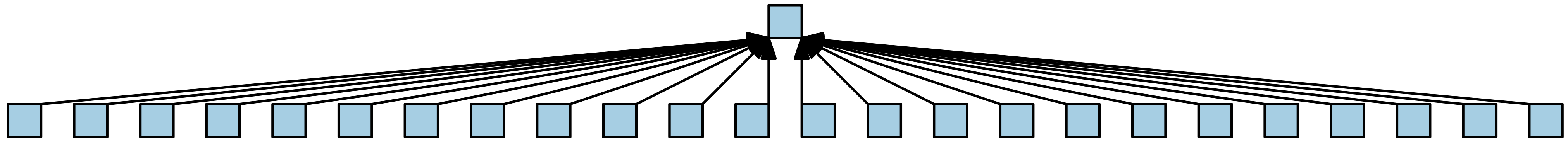
- The total edge length can be minimized in polynomial time

Width



Drawings can be very wide.

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- as long as there are free machines and available jobs, take the first available job and assign it to a free machine

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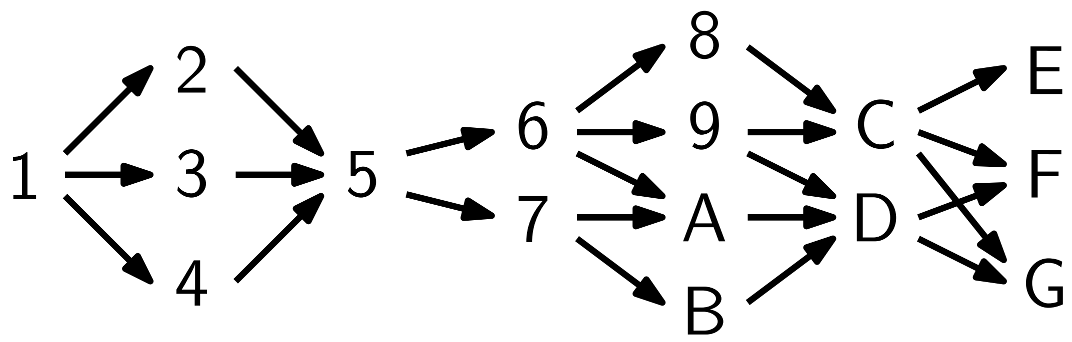
Input: Precedence graph (divided into layers of arbitrary width)



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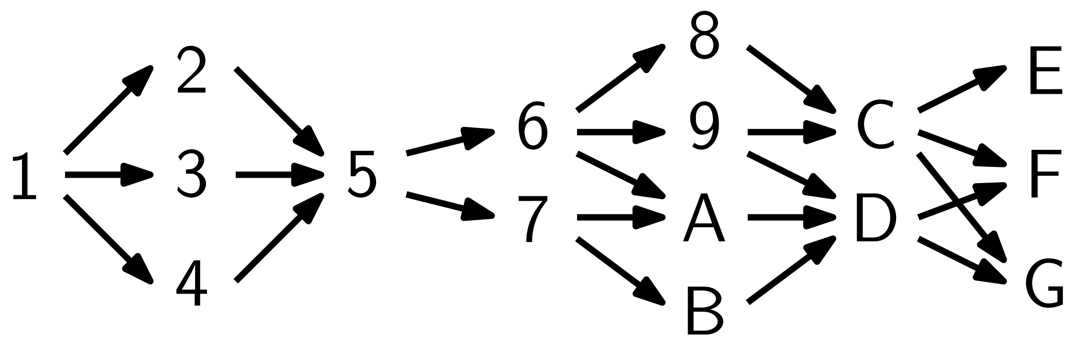

Number of Machines is $W = 2$.

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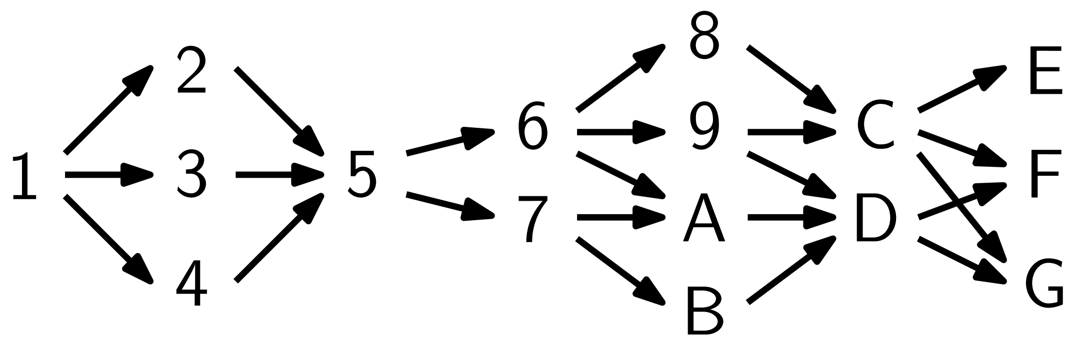

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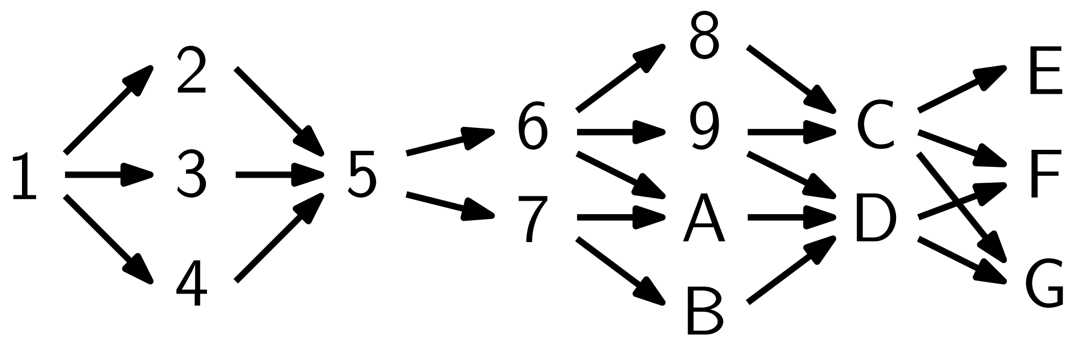

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Output: Schedule

M_1										
M_2										
t	1	2	3	4	5	6	7	8	9	10

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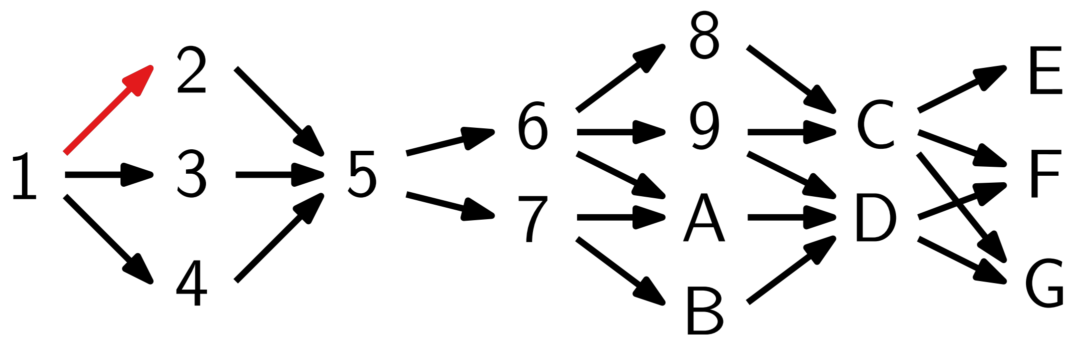

Number of Machines is $W = 2$.

Output: Schedule

M_1	1									
M_2	–									
t	1	2	3	4	5	6	7	8	9	10

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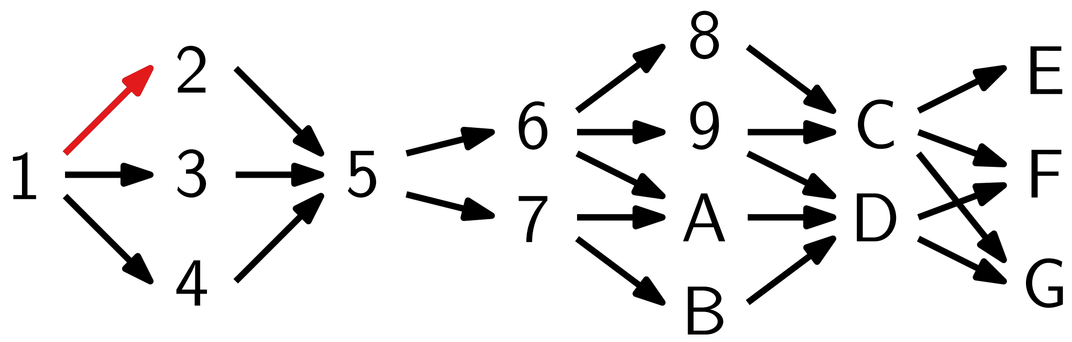

Number of Machines is $W = 2$.

Output: Schedule

M_1	1	2								
M_2	–	3								
t	1	2	3	4	5	6	7	8	9	10

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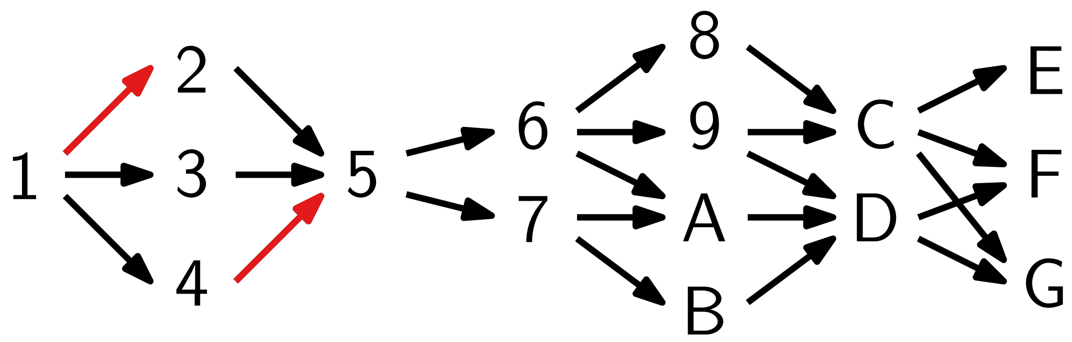

Number of Machines is $W = 2$.

Output: Schedule

M_1	1	2	4							
M_2	–	3	–							
t	1	2	3	4	5	6	7	8	9	10

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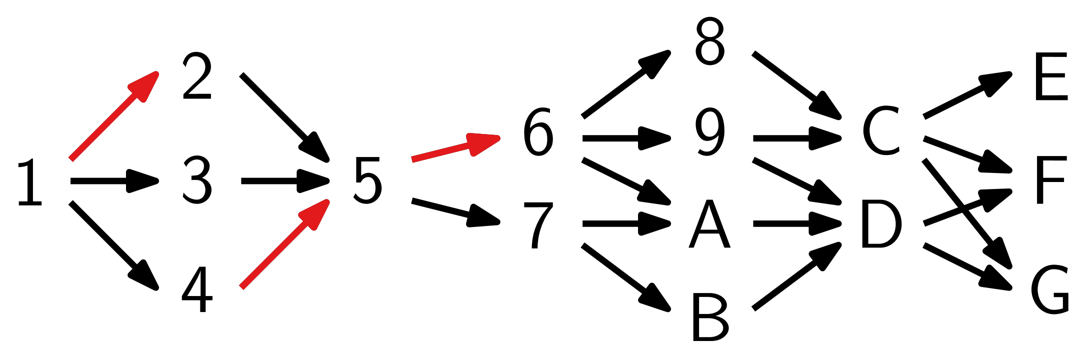

Number of Machines is $W = 2$.

Output: Schedule

M_1	1	2	4	5						
M_2	–	3	–	–						
t	1	2	3	4	5	6	7	8	9	10

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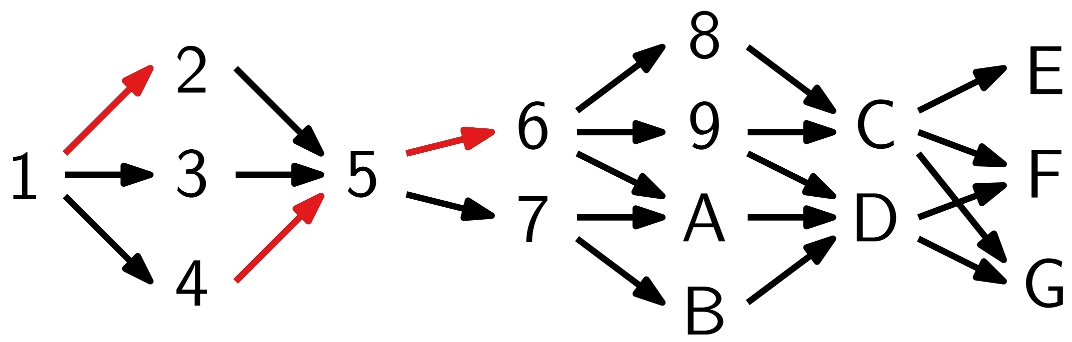

Number of Machines is $W = 2$.

Output: Schedule

M_1	1	2	4	5	6					
M_2	–	3	–	–	7					
t	1	2	3	4	5	6	7	8	9	10

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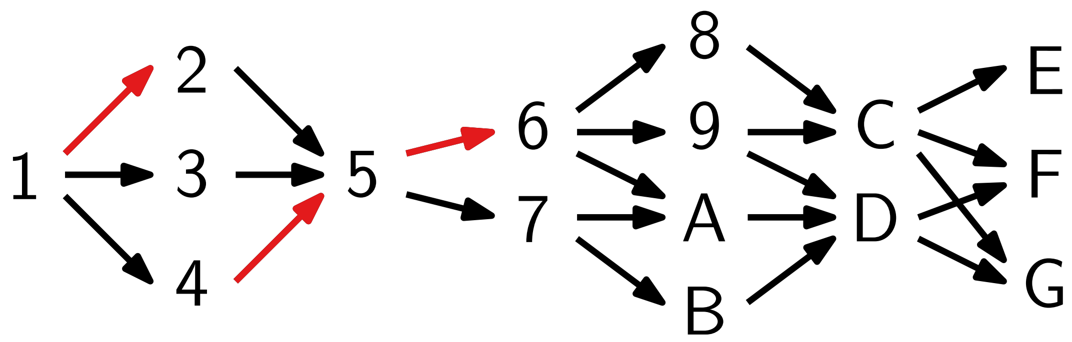

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M_1	1	2	4	5	6	8				
M_2	–	3	–	–	7	9				
t	1	2	3	4	5	6	7	8	9	10

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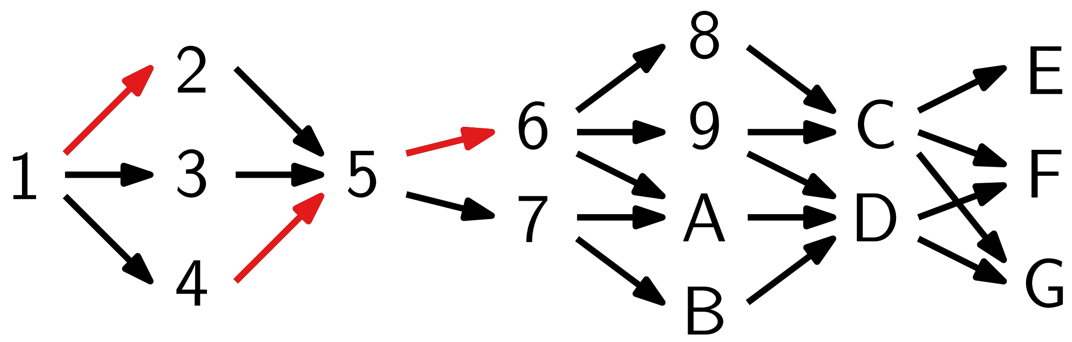

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Output: Schedule

M_1	1	2	4	5	6	8	A			
M_2	–	3	–	–	7	9	B			
t	1	2	3	4	5	6	7	8	9	10

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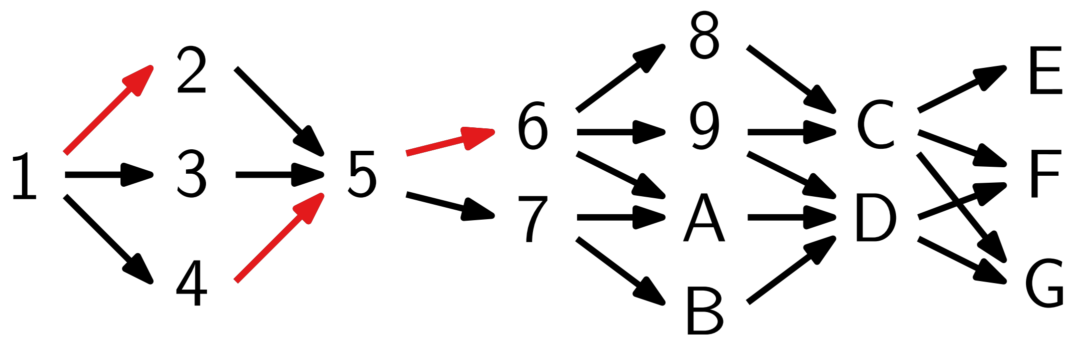

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Output: Schedule

M_1	1	2	4	5	6	8	A	C		
M_2	–	3	–	–	7	9	B	D		
t	1	2	3	4	5	6	7	8	9	10

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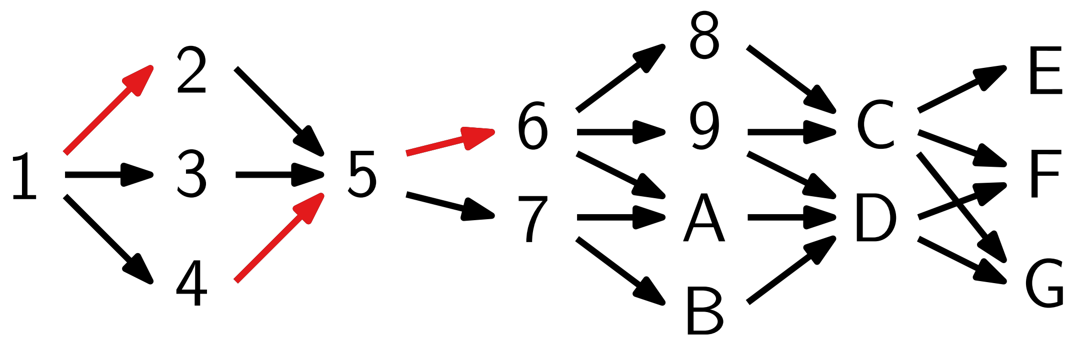

Number of Machines is $W = 2$.

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M_1	1	2	4	5	6	8	A	C	E	
M_2	–	3	–	–	7	9	B	D	F	
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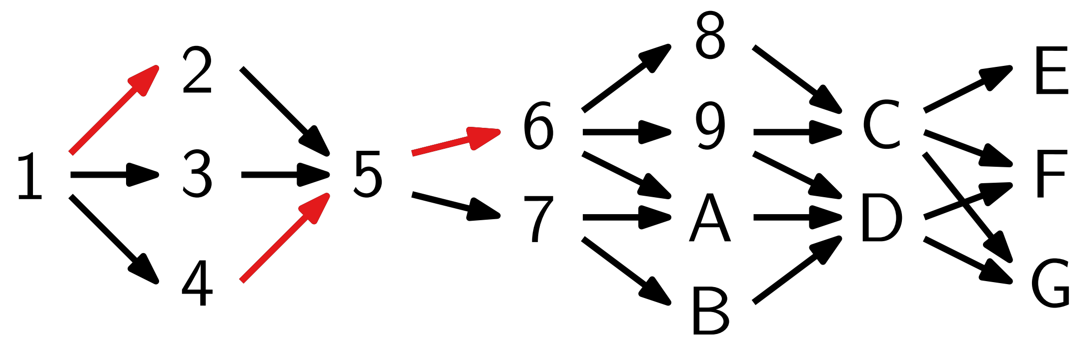

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M_1	1	2	4	5	6	8	A	C	E	G
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t	1	2	3	4	5	6	7	8	9	10

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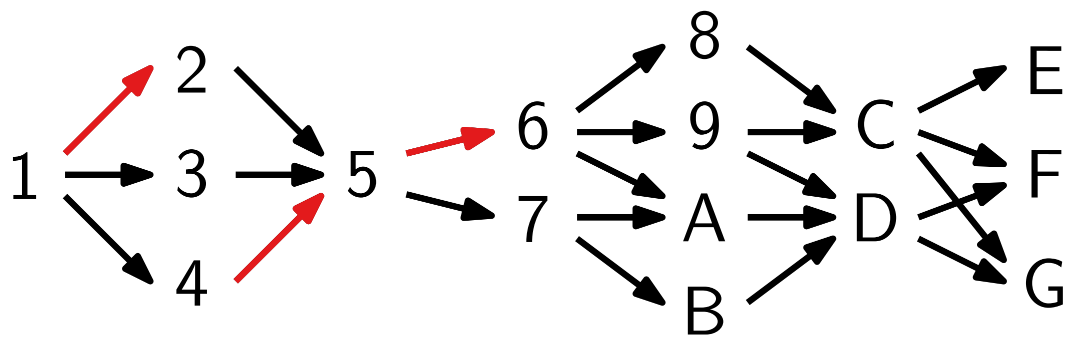

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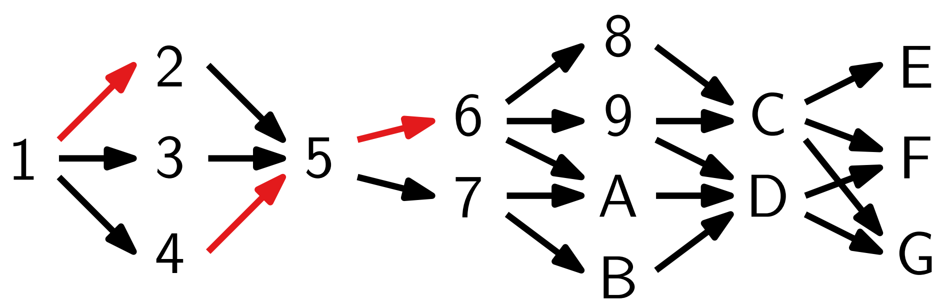
Question: Good approximation factor?

Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



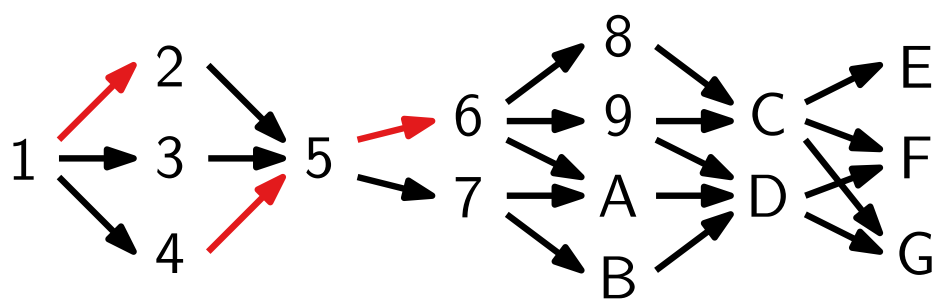
Schedule

M_1	1	2	4	5	6	8	A	C	E	G
M_2	–	3	–	–	7	9	B	D	F	–
t	1	2	3	4	5	6	7	8	9	10

„The art of the lower bound“

Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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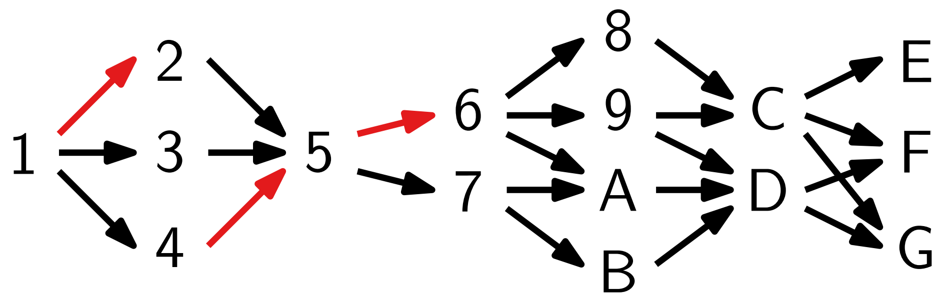
„The art of the lower bound“

OPT $\geq$

Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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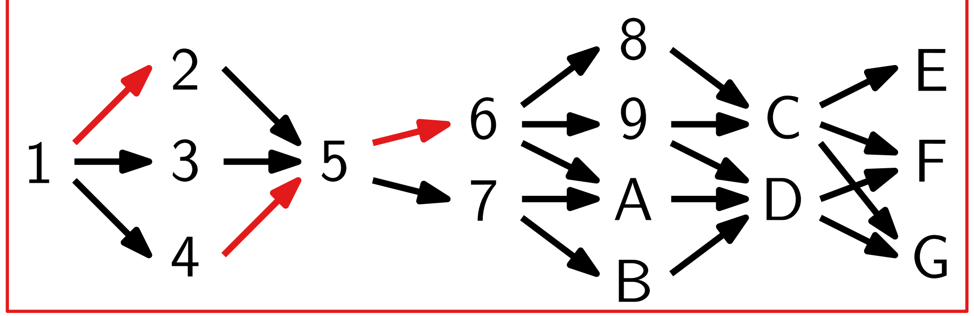
„The art of the lower bound"

OPT $\geq \lceil n/2 \rceil$

Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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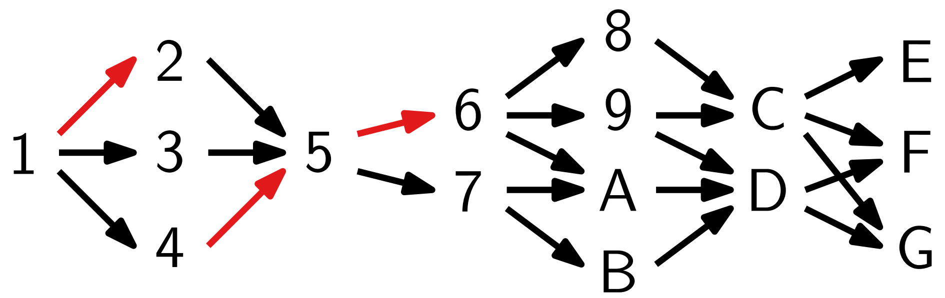
„The art of the lower bound“

OPT $\geq \lceil n/2 \rceil$ and OPT $\geq$

Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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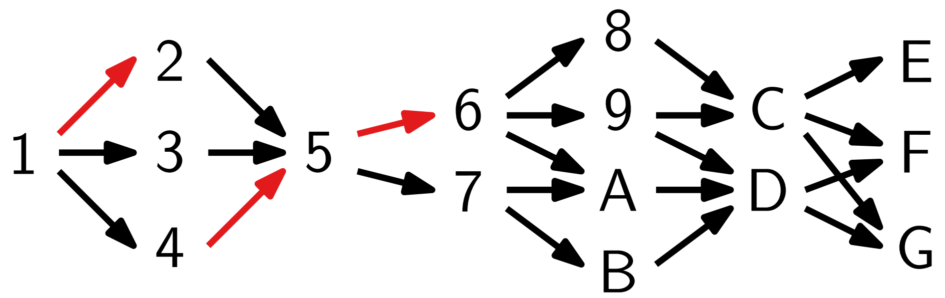
„The art of the lower bound“

OPT $\geq \lceil n/2 \rceil$ and OPT $\geq \ell :=$ Number of layers of $G_<$

Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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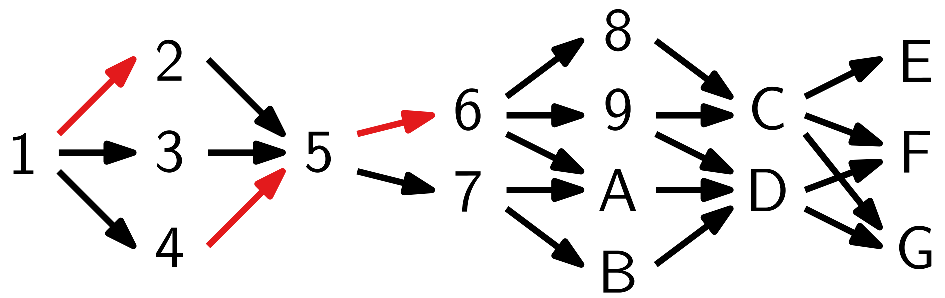
„The art of the lower bound"

OPT $\geq \lceil n/2 \rceil$ and OPT $\geq \ell :=$ Number of layers of $G_<$

Goal: measure the quality of our algorithm using the lower bounds

Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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„The art of the lower bound"

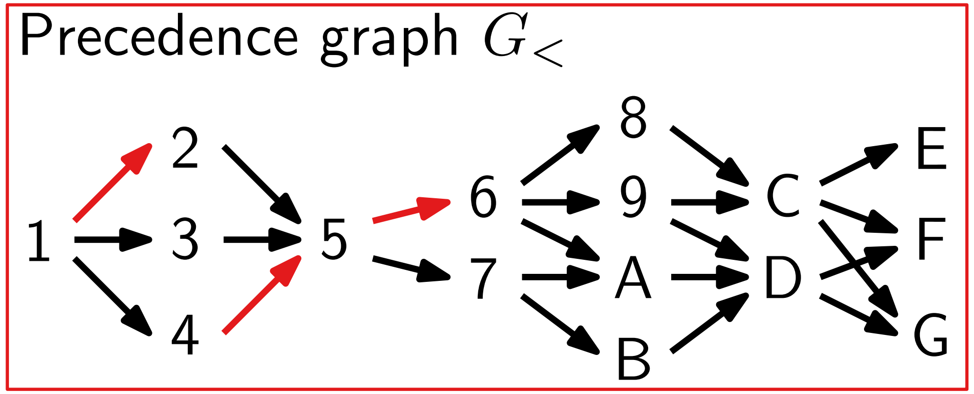
OPT $\geq \lceil n/2 \rceil$ and OPT $\geq \ell :=$ Number of layers of $G_<$

Goal: measure the quality of our algorithm using the lower bounds

Bound. ALG $\leq$

Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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Approximating PCMPS - Analysis for $W = 2$

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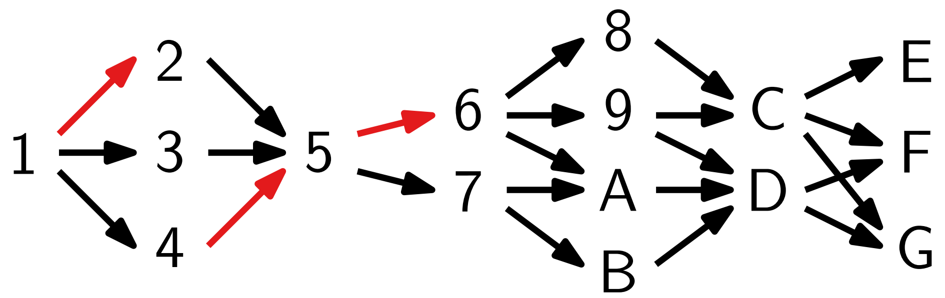

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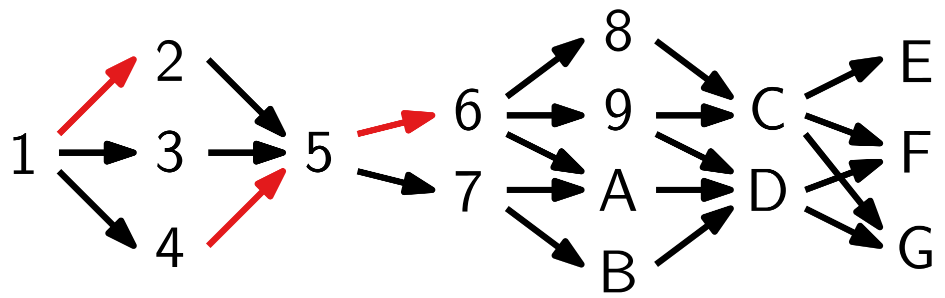
Goal: measure the quality of our algorithm using the lower bounds

Bound. ALG $\leq$

insertion of pauses (–) in the schedule
(except the last) maps to layers of $G_<$

Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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„The art of the lower bound"

OPT $\geq \lceil n/2 \rceil$ and OPT $\geq \ell :=$ Number of layers of $G_<$

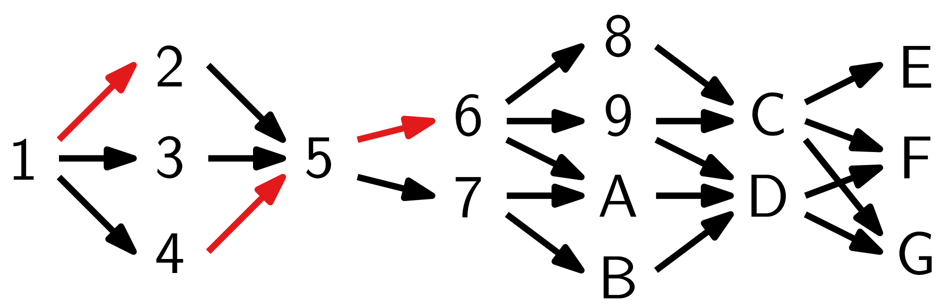
Goal: measure the quality of our algorithm using the lower bounds

Bound. ALG $\leq \left\lceil \frac{n+\ell}{2} \right\rceil$

insertion of pauses (–) in the schedule
(except the last) maps to layers of $G_<$

Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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„The art of the lower bound“

OPT $\geq \lceil n/2 \rceil$ and OPT $\geq \ell :=$ Number of layers of $G_<$

Goal: measure the quality of our algorithm using the lower bounds

Bound. ALG $\leq \left\lceil \frac{n+\ell}{2} \right\rceil \approx$

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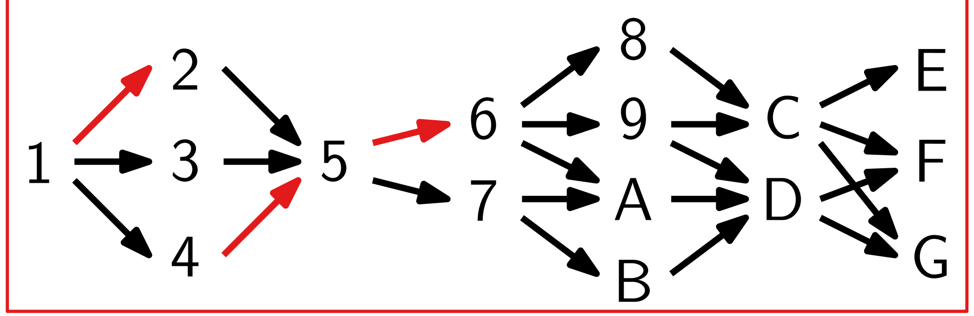
Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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„The art of the lower bound"

OPT $\geq \lceil n/2 \rceil$ and OPT $\geq \ell :=$ Number of layers of $G_<$

Goal: measure the quality of our algorithm using the lower bounds

Bound. ALG $\leq \left\lceil \frac{n+\ell}{2} \right\rceil \approx \lceil n/2 \rceil + \ell/2$

insertion of pauses (–) in the schedule
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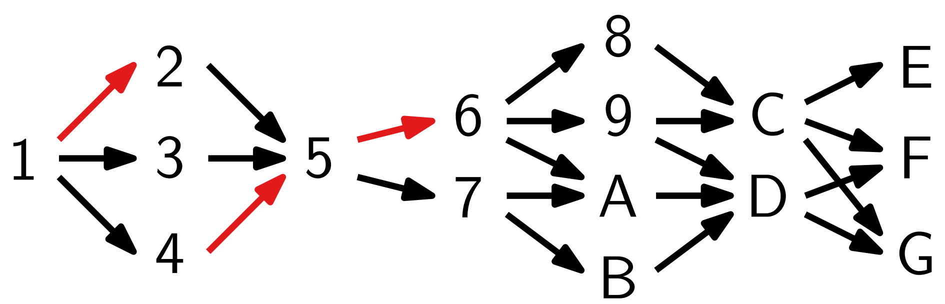
Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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Goal: measure the quality of our algorithm using the lower bounds

Bound. ALG $\leq \left\lceil \frac{n+\ell}{2} \right\rceil \approx \lceil n/2 \rceil + \ell/2 \leq$

insertion of pauses (–) in the schedule
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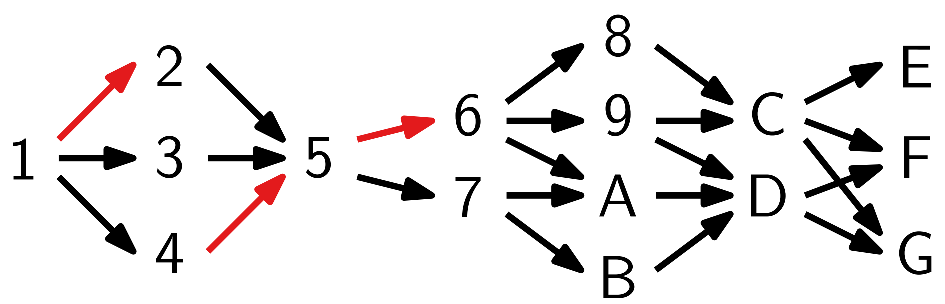
Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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„The art of the lower bound“

OPT $\geq \lceil n/2 \rceil$ and OPT $\geq \ell :=$ Number of layers of $G_<$

Goal: measure the quality of our algorithm using the lower bounds

Bound. ALG $\leq \left\lceil \frac{n+\ell}{2} \right\rceil \approx \lceil n/2 \rceil + \ell/2 \leq 3/2 \cdot$ OPT

insertion of pauses (–) in the schedule
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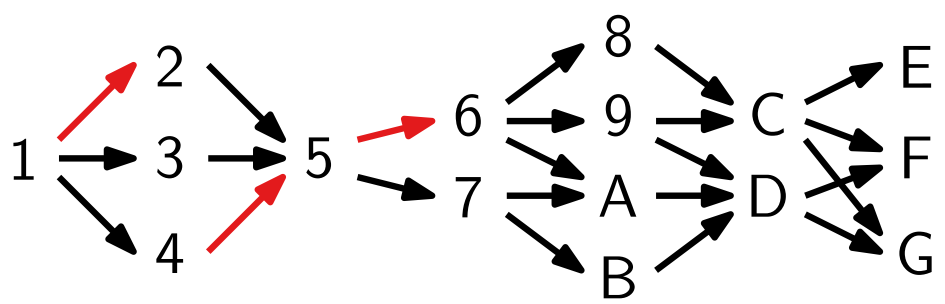
Approximating PCMPS - Analysis for $W = 2$

Precedence graph $G_<$



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„The art of the lower bound“

OPT $\geq \lceil n/2 \rceil$ and OPT $\geq \ell :=$ Number of layers of $G_<$

Goal: measure the quality of our algorithm using the lower bounds

$\leq (2 - 1/W) \cdot$ OPT in general case

Bound. ALG $\leq \left\lceil \frac{n+\ell}{2} \right\rceil \approx \lceil n/2 \rceil + \ell/2 \leq 3/2 \cdot$ OPT

insertion of pauses (–) in the schedule
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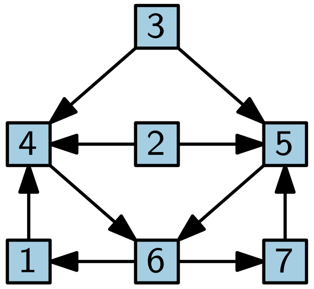
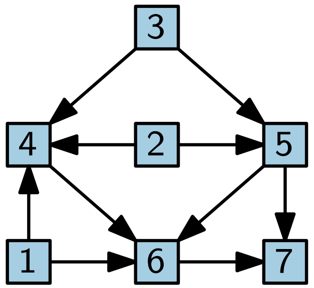
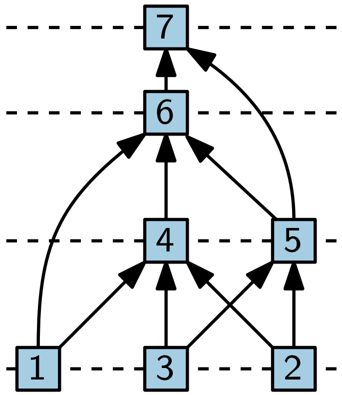
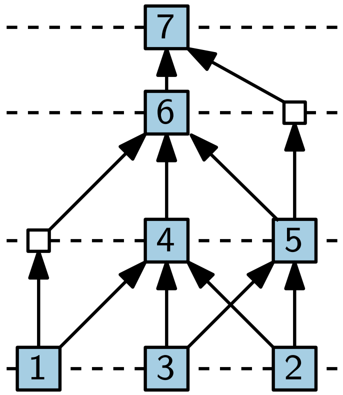




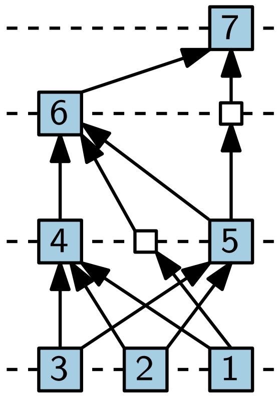
Visualization of Graphs

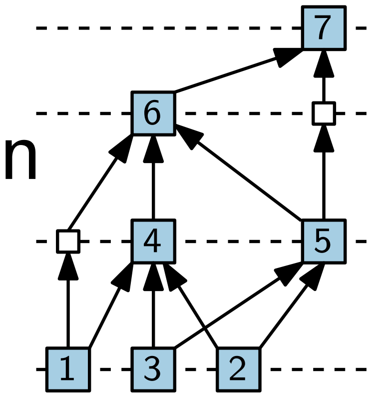


Lecture 7:
Hierarchical Layouts:
Sugiyama Framework



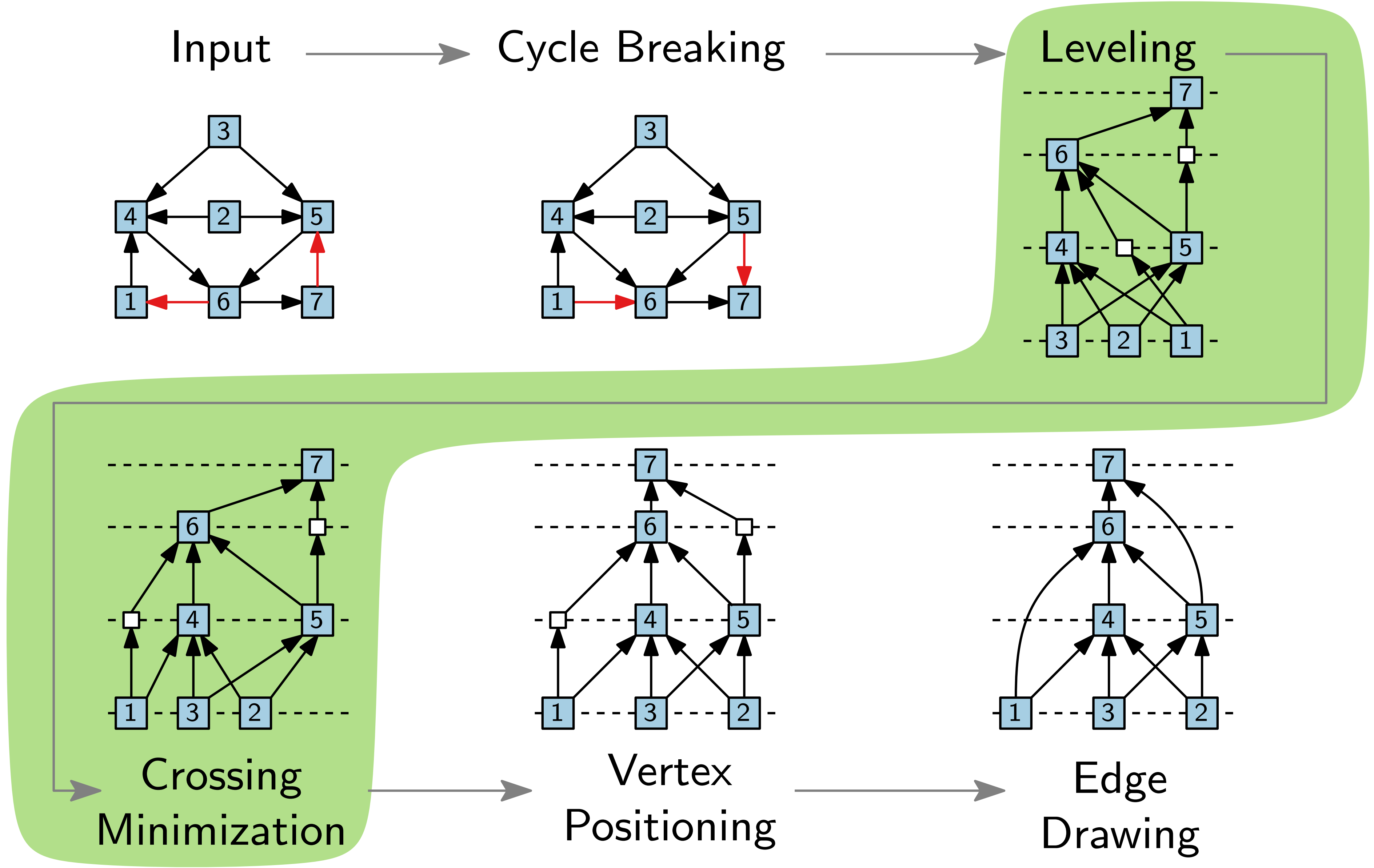
Part IV:
Crossing Minimisation



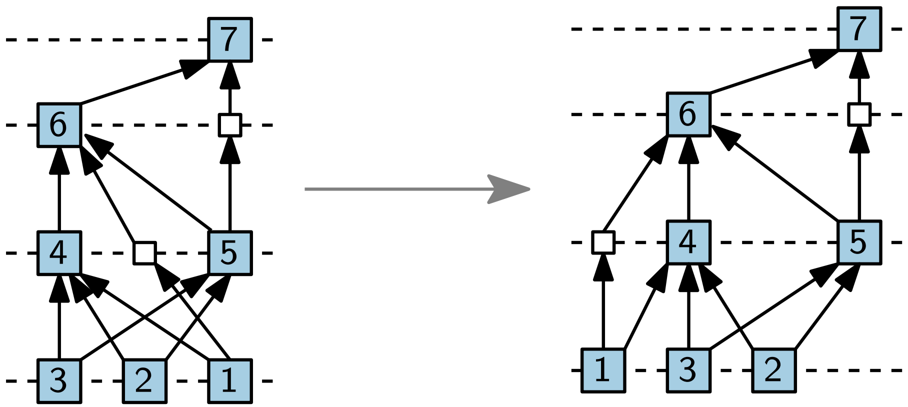


Jonathan Klawitter

Step 3: Crossing Minimization

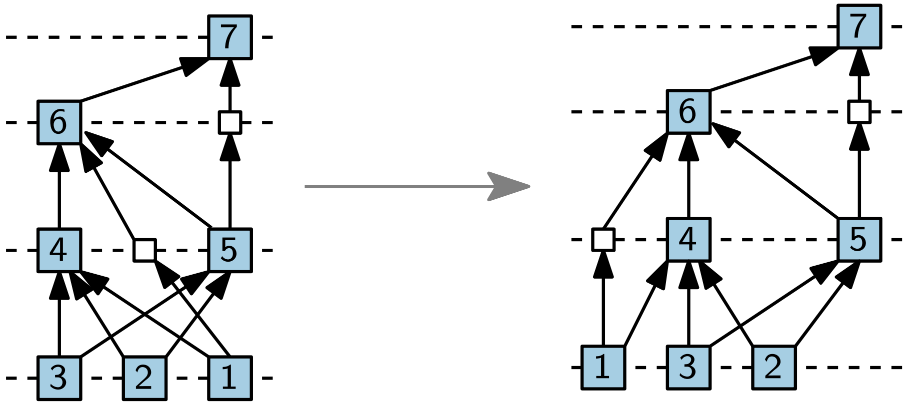


Step 3: Crossing Minimization



Problem.

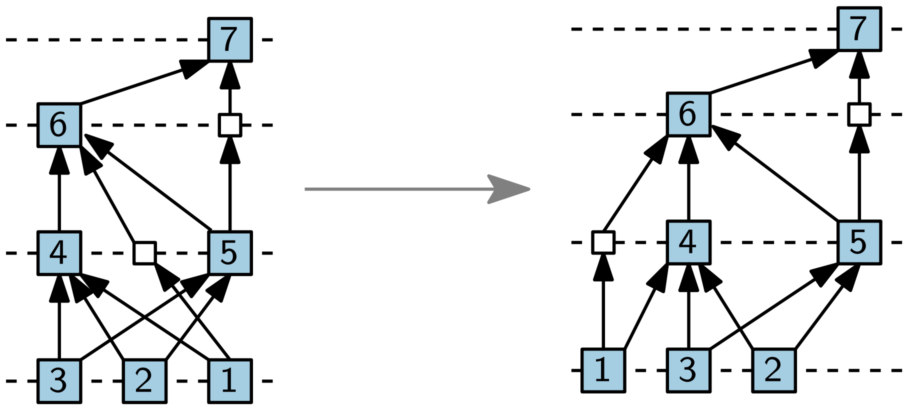
Step 3: Crossing Minimization



Problem.

- Input: Graph G, layering $y\colon V \to \{1, \ldots, n\}$

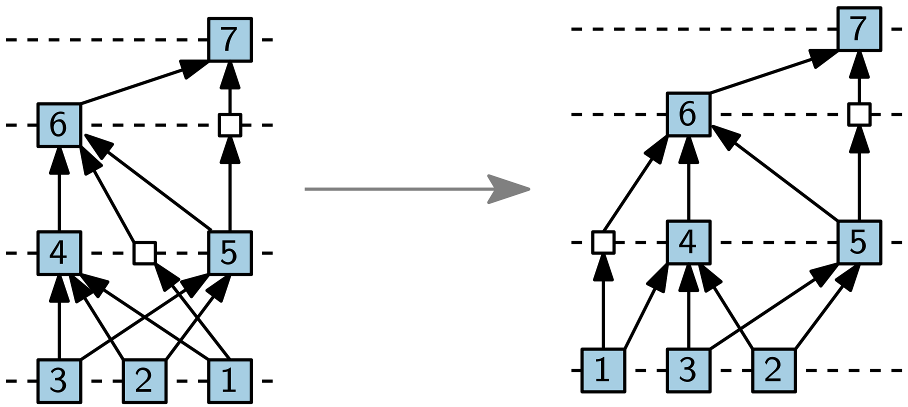
Step 3: Crossing Minimization



Problem.

- Input: Graph G, layering $y \colon V \to \{1, \ldots, n\}$
- Output: (Re-)ordering of vertices in each layer so that the number of crossings in minimized.

Step 3: Crossing Minimization



Problem.

- Input: Graph G, layering $y \colon V \to \{1, \ldots, n\}$
- Output: (Re-)ordering of vertices in each layer so that the number of crossings in minimized.

- NP-hard, even for 2 layers [Garey & Johnson '83]

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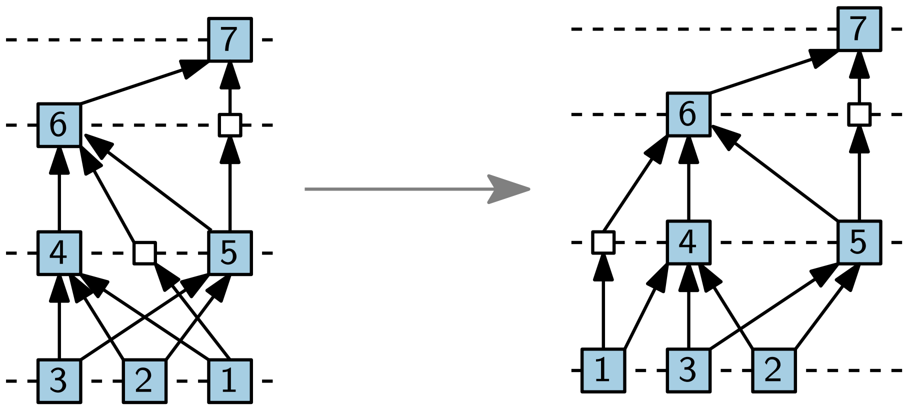

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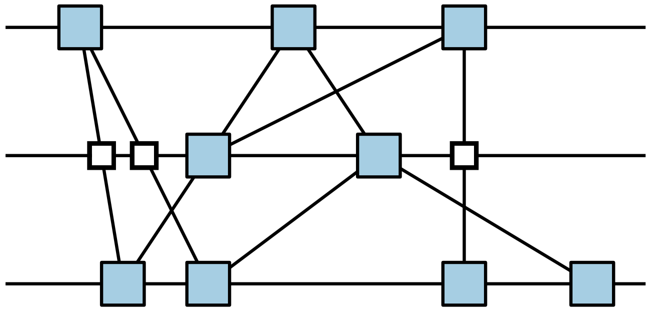
- Output: (Re-)ordering of vertices in each layer so that the number of crossings in minimized.

- NP-hard, even for 2 layers [Garey & Johnson '83]
- hardly any approaches optimize over multiple layers :(

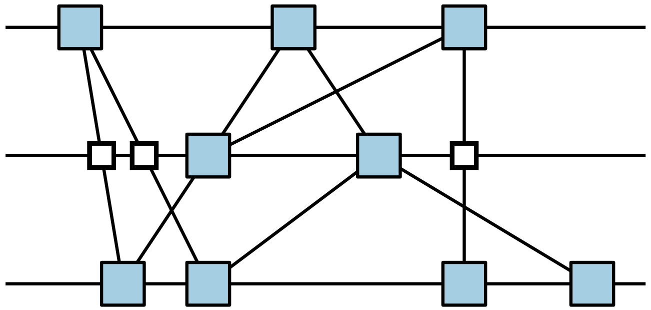
Iterative Crossing Reduction – Idea



Iterative Crossing Reduction – Idea

Observation.

The number of crossings only depends on permutations of adjacent layers.



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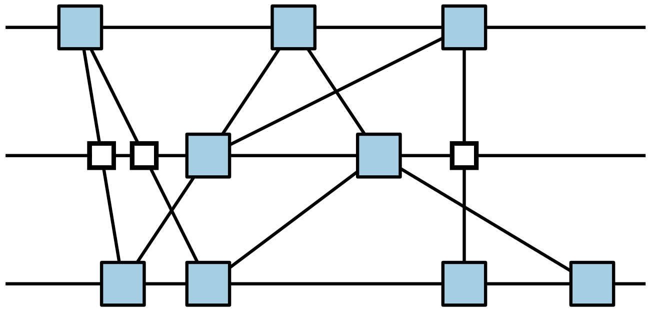

- Add dummy-vertices for edges connecting "far" layers.

Iterative Crossing Reduction – Idea

Observation.
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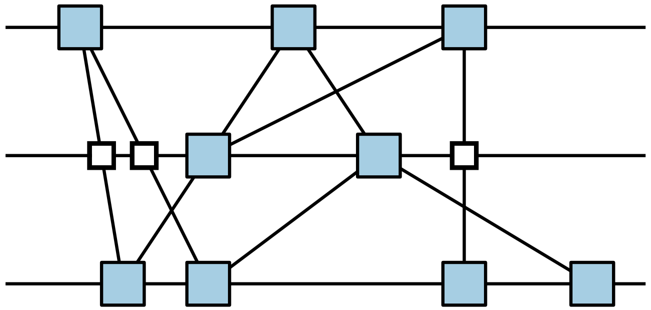

- Add dummy-vertices for edges connecting “far” layers.
- Consider adjacent layers $(L_1, L_2), (L_2, L_3), \ldots$ bottom-to-top.

Iterative Crossing Reduction – Idea

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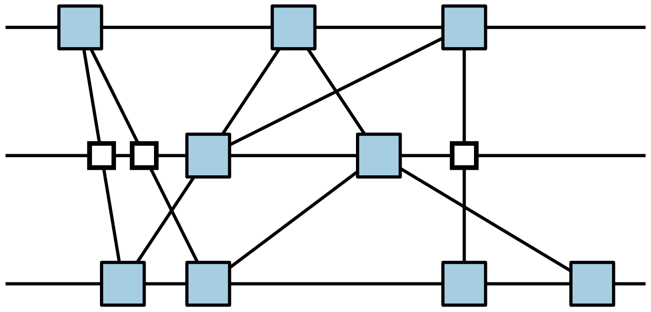

- Add dummy-vertices for edges connecting “far” layers.
- Consider adjacent layers $(L_1, L_2), (L_2, L_3), \ldots$ bottom-to-top.
- Minimize crossings by permuting L_{i+1} while keeping L_i fixed.

Iterative Crossing Reduction – Algorithm

(1) choose a random permutation of L_1

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(2) iteratively consider adjacent layers L_i and L_{i+1}

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(3) minimize crossings by permuting L_{i+1} and keeping L_i fixed

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(4) repeat steps (2)–(3) in the reverse order (starting from L_h)

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(5) repeat steps (2)–(4) until no further improvement is achieved

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Iterative Crossing Reduction – Algorithm

one-sided crossing minimization

(1) choose a random permutation of L_1

(2) iteratively consider adjacent layers L_i and L_{i+1}

(3) minimize crossings by permuting L_{i+1} and keeping L_i fixed

(4) repeat steps (2)–(3) in the reverse order (starting from L_h)

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One-Sided Crossing Minimization

Problem.

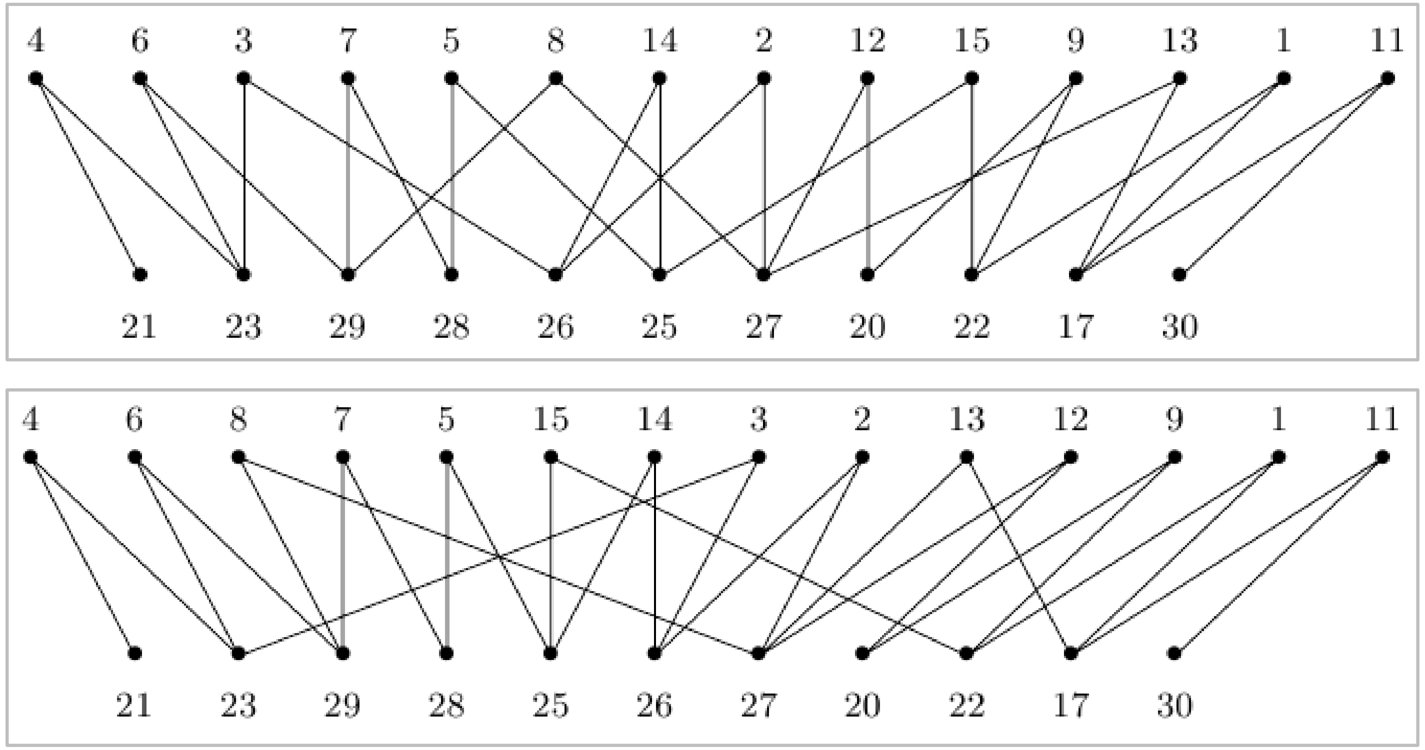

Abb. aus [Kaufmann und Wagner: Drawing Graphs]
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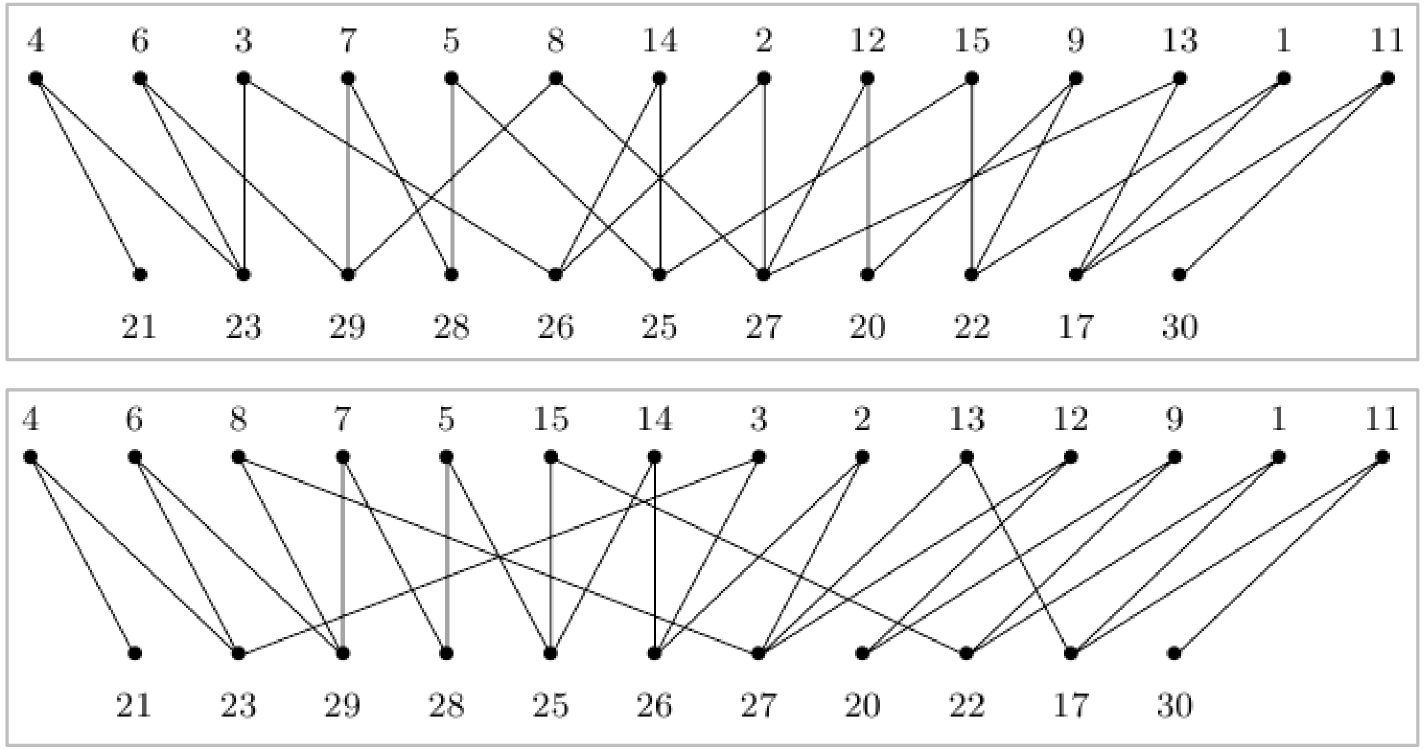
One-Sided Crossing Minimization

Problem.

- Input: bipartite graph $G = (L_1 \cup L_2, E)$, permutation π_1 on L_1



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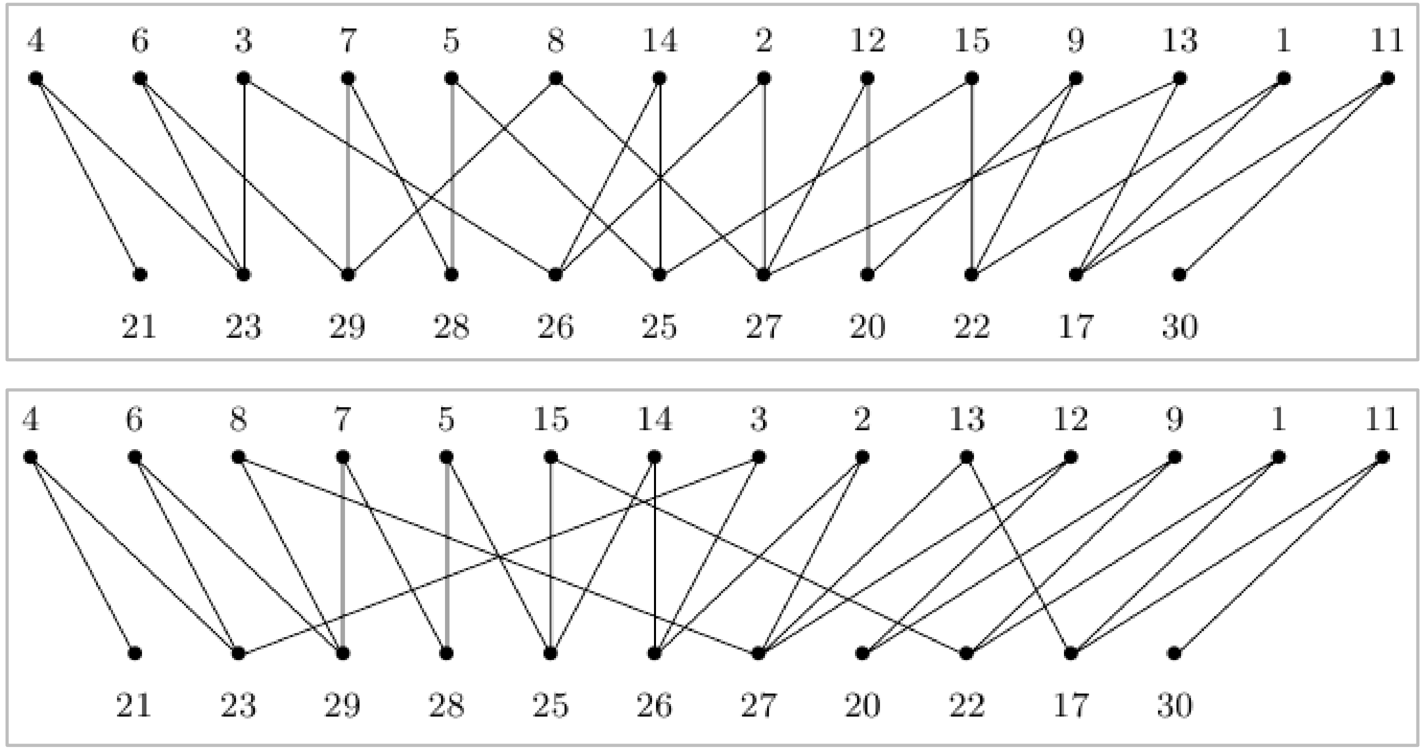
(c) Springer-Verlag

One-Sided Crossing Minimization

Problem.

- Input: bipartite graph $G = (L_1 \cup L_2, E)$, permutation π_1 on L_1
- Output: permutation π_2 of L_2 minimizing the number of edge crossings.



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- Input: bipartite graph $G = (L_1 \cup L_2, E)$, permutation π_1 on L_1
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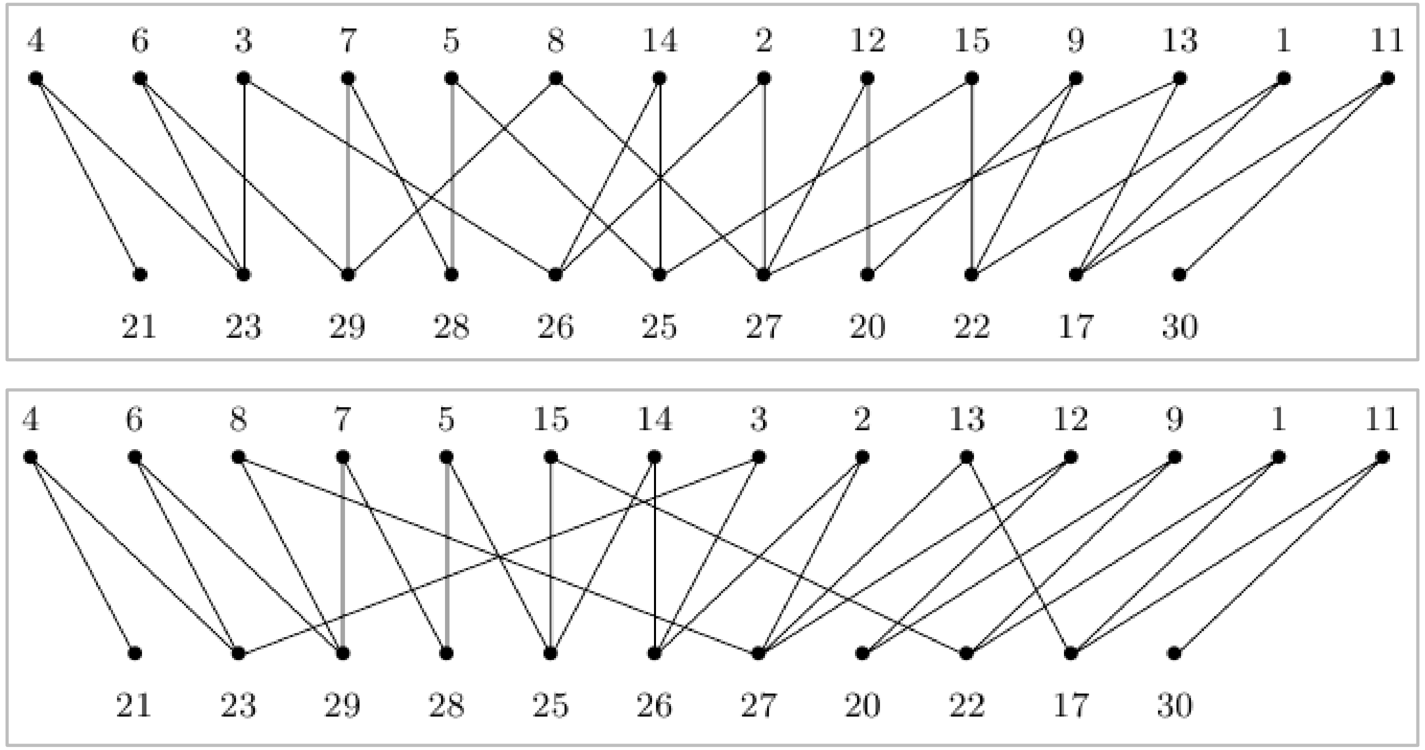
One-sided crossing minimization is NP-hard.
[Eades & Whitesides '94]



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Algorithms.

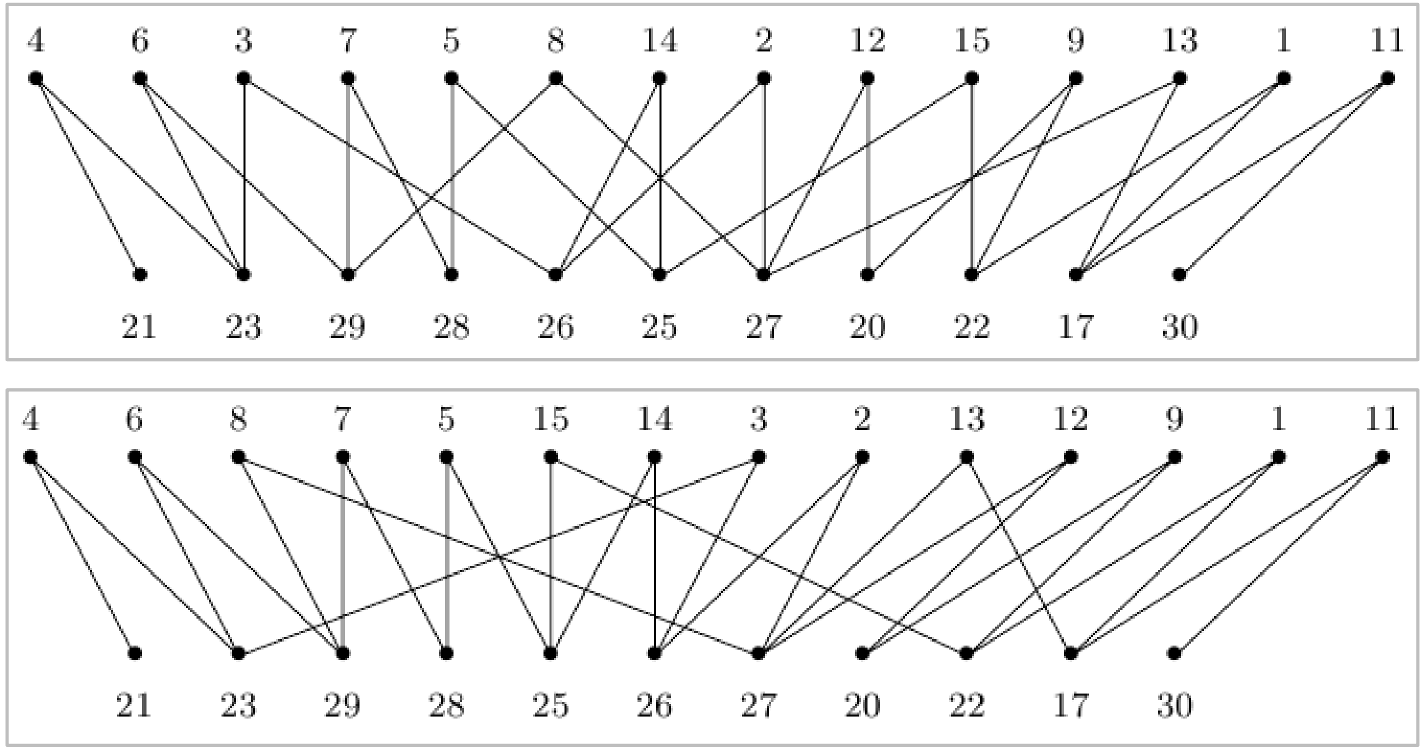

Abb. aus [Kaufmann und Wagner: Drawing Graphs]
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One-Sided Crossing Minimization

Problem.

- Input: bipartite graph $G = (L_1 \cup L_2, E)$, permutation π_1 on L_1
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One-sided crossing minimization is NP-hard.
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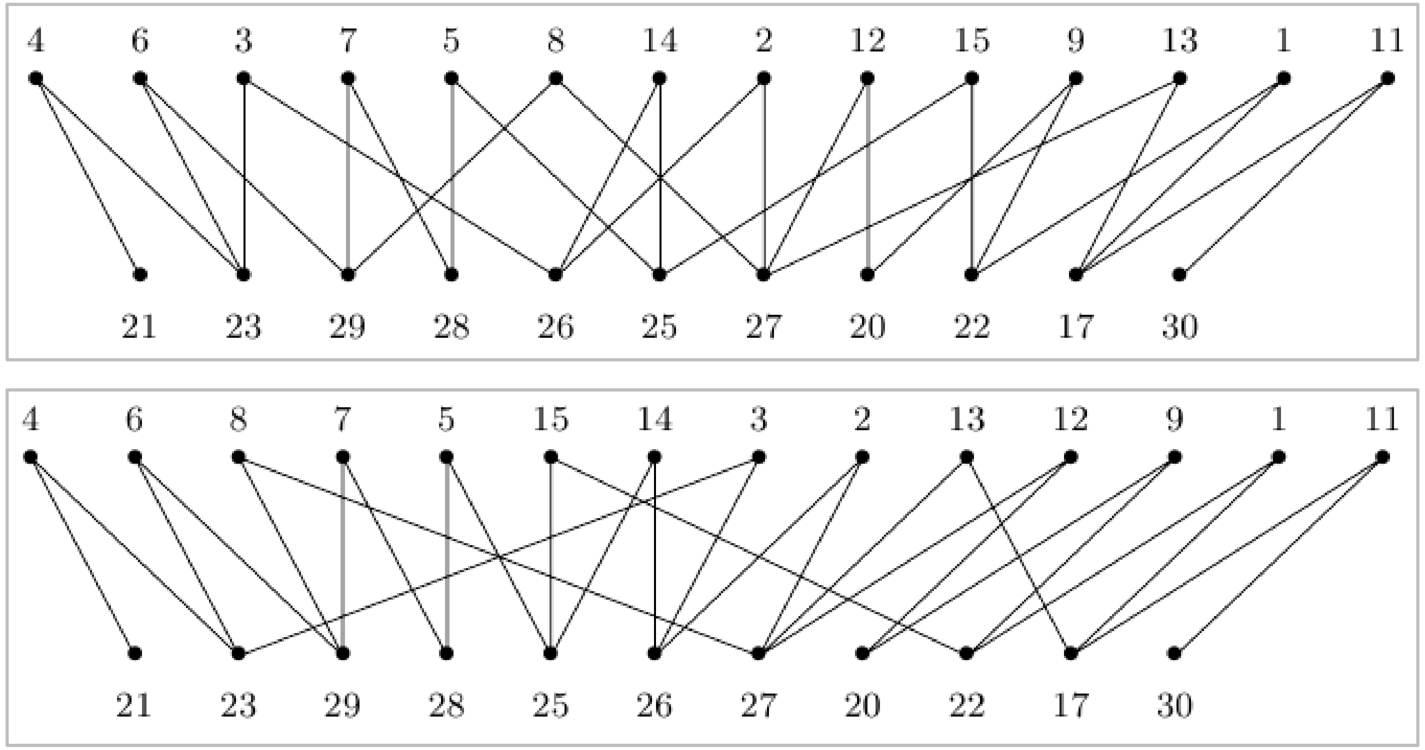

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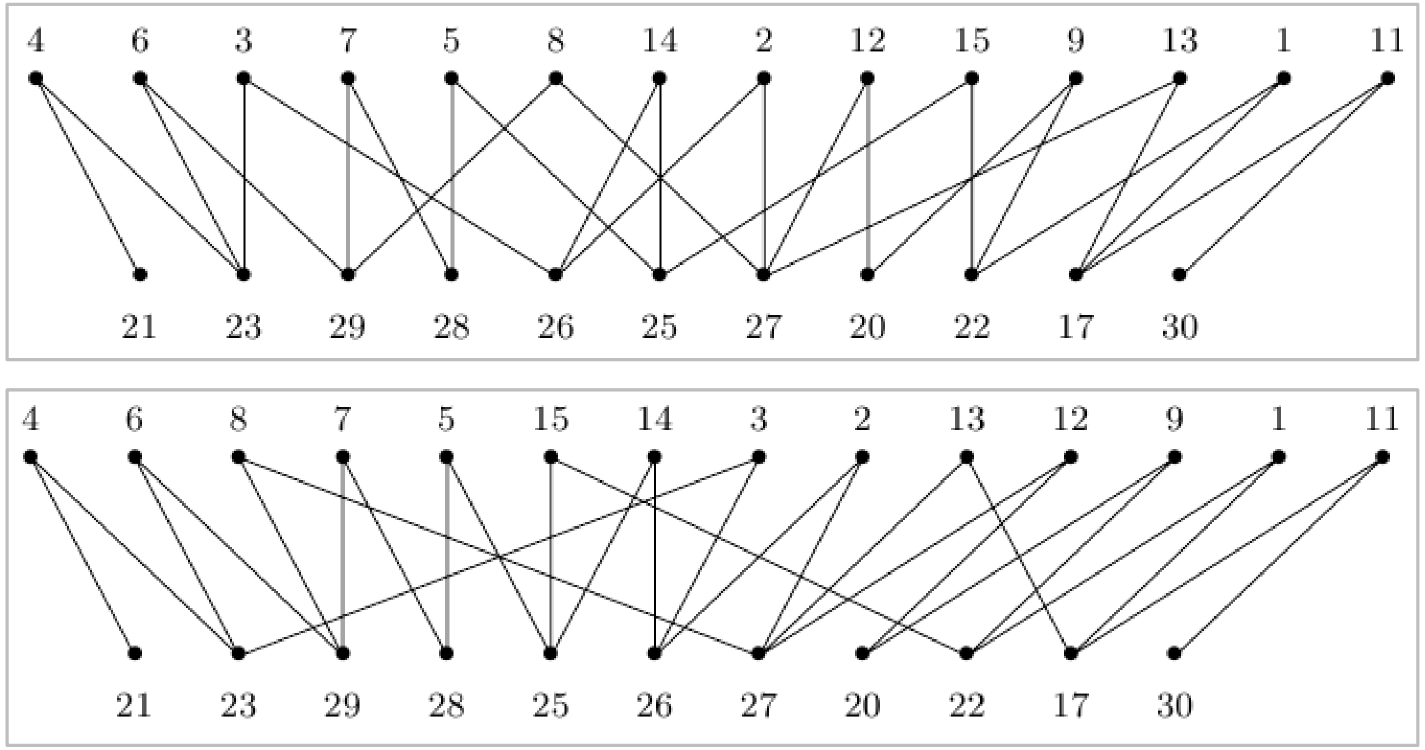

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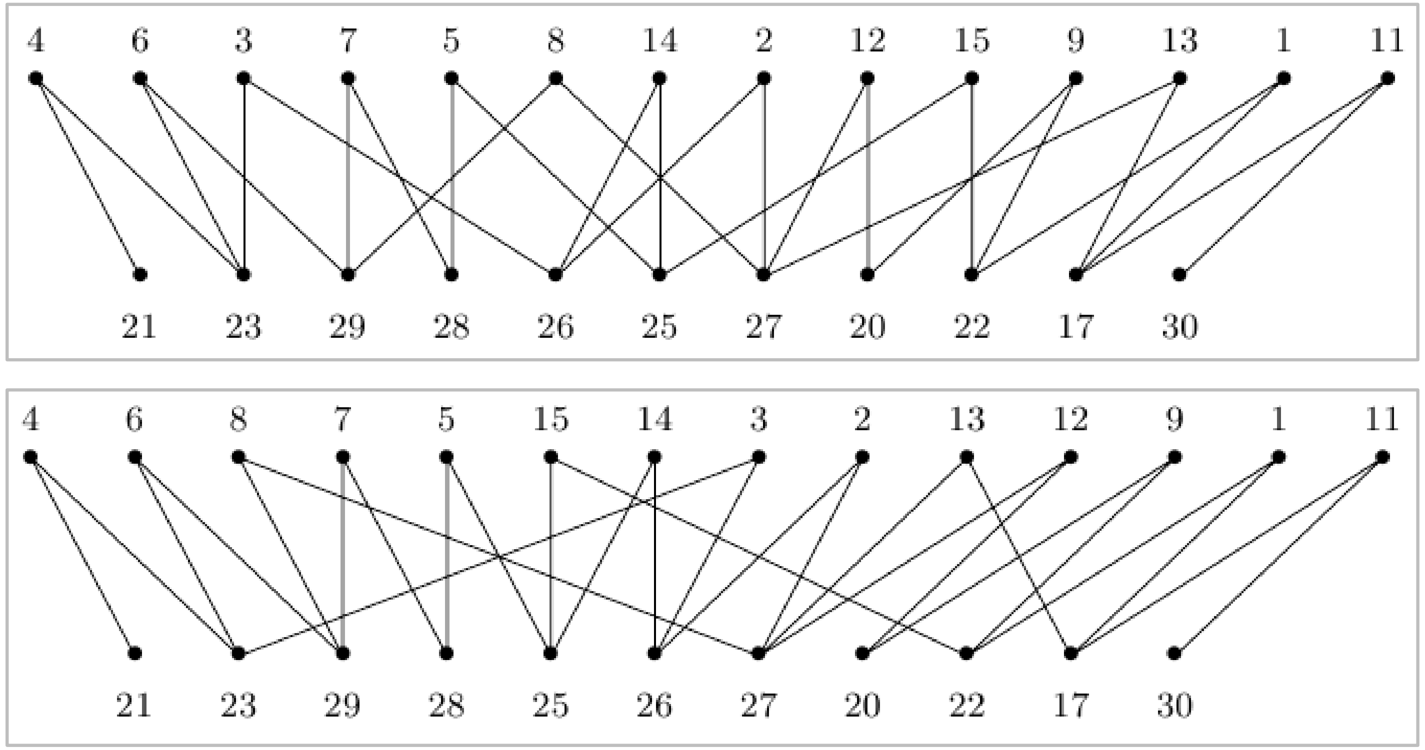

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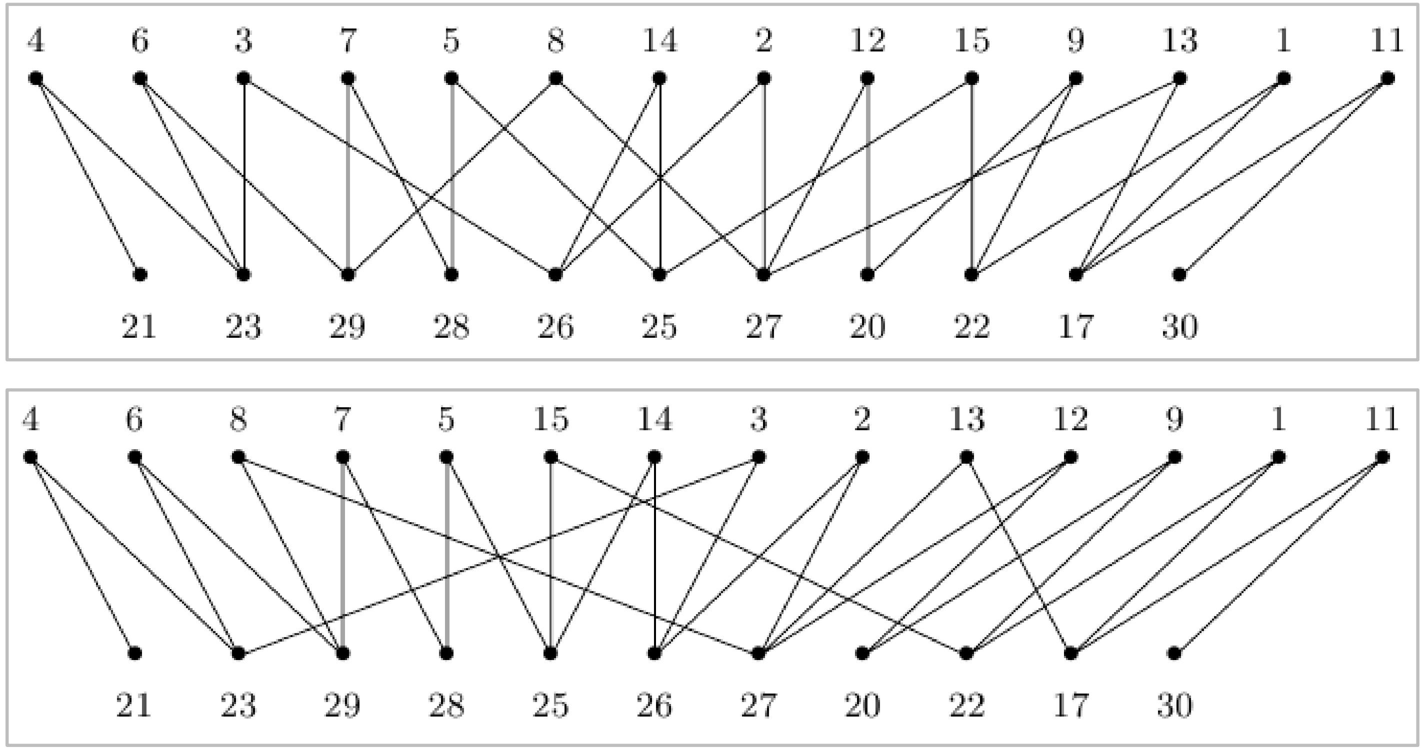

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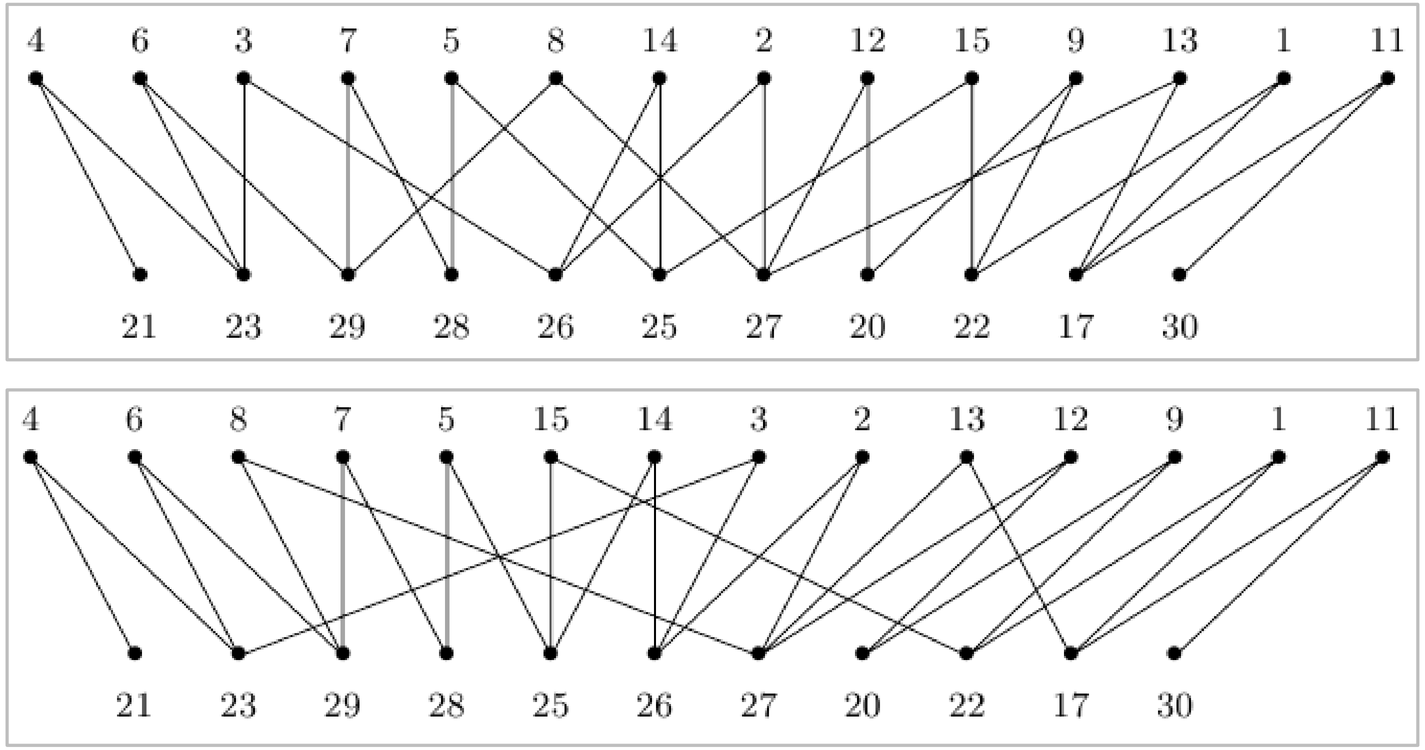

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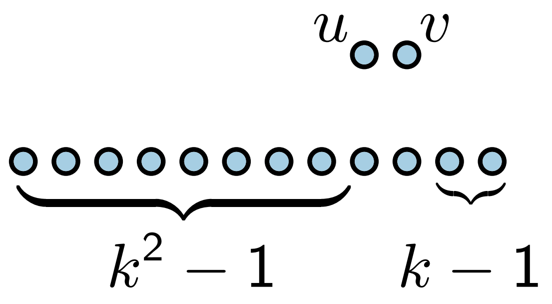

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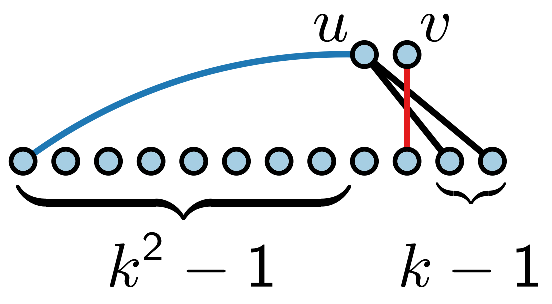

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[Eades & Wormald '94]

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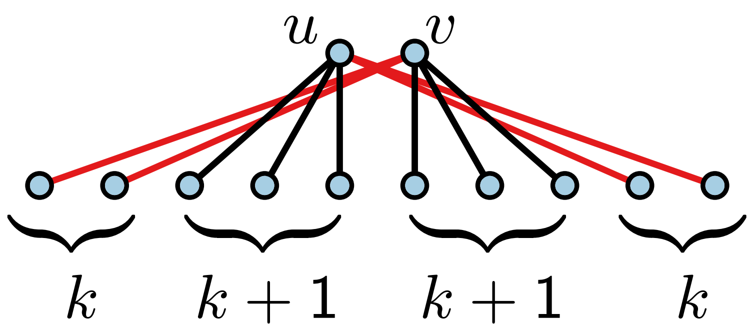

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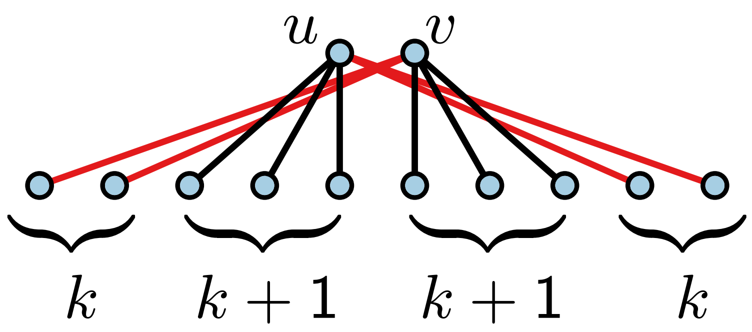

$2k(k+1) + k^2$ vs. $(k+1)^2$

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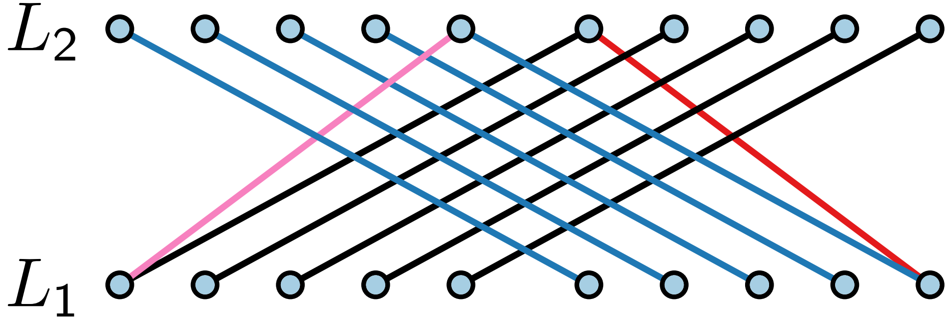

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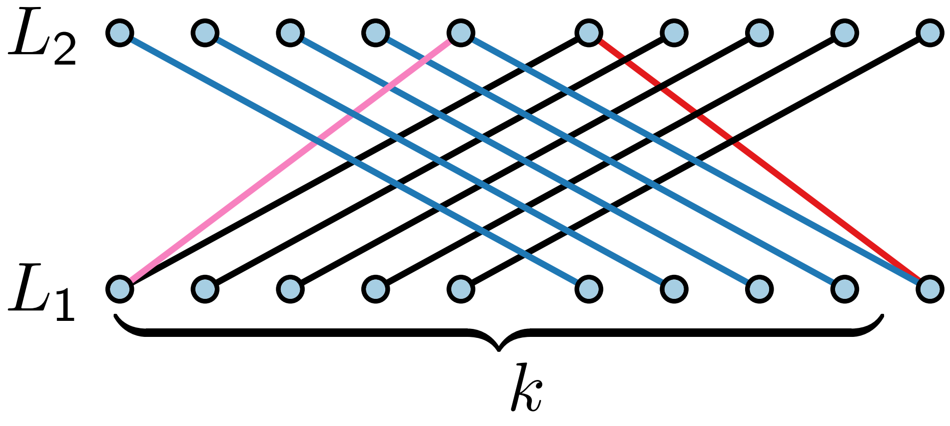

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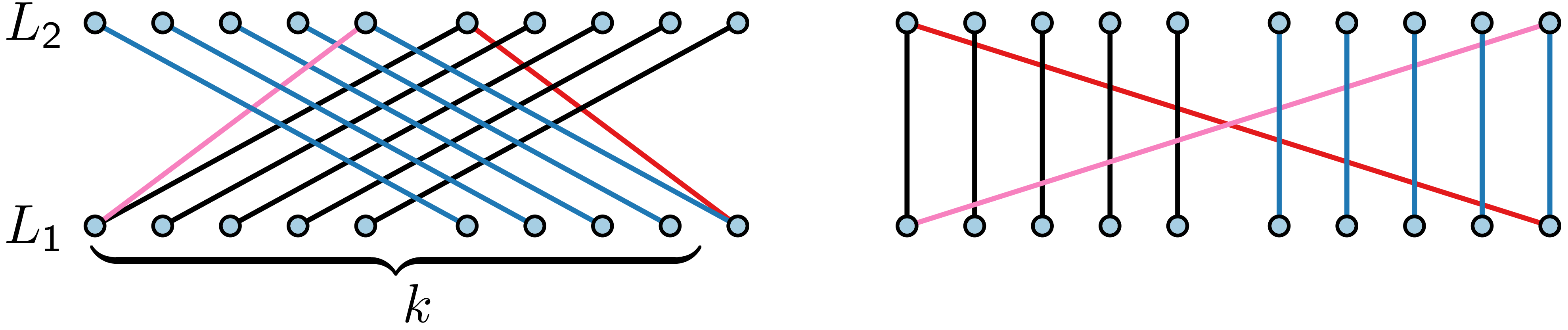

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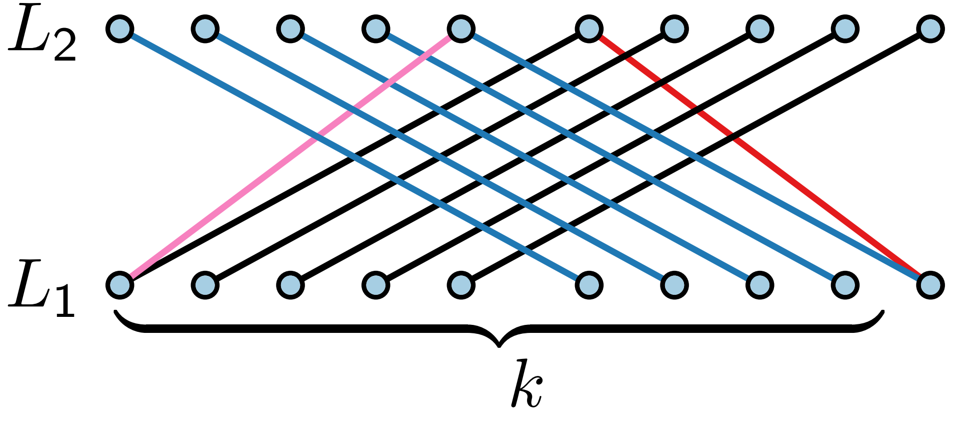
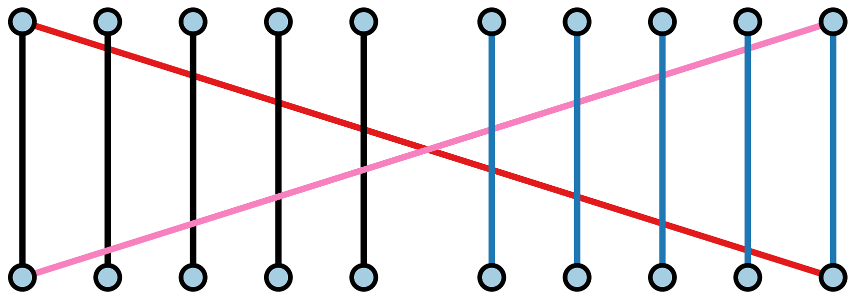




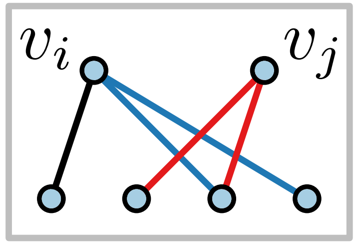
$\approx k^2/4$

$\approx 2k$

Integer Linear Program

[Jünger & Mutzel, '97]

- Constant $c_{ij} :=$ # crossings between edges incident to v_i or v_j when $\pi_2(v_i) < \pi_2(v_j)$



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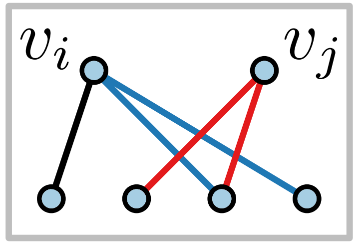

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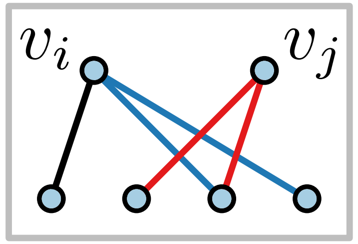

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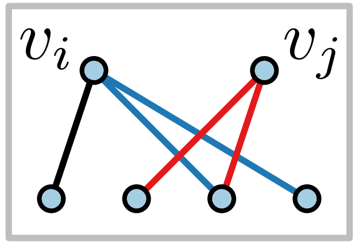

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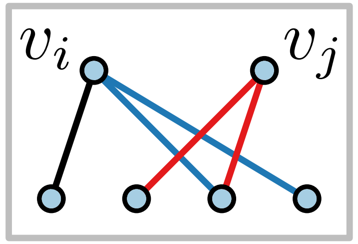

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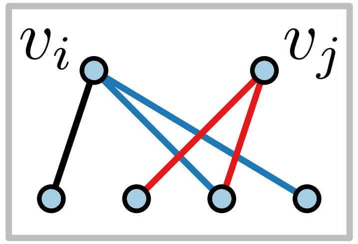

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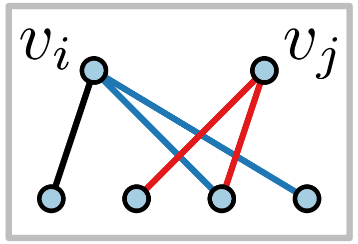

$$x_{ij} = \begin{cases} 1 & \text{when } \pi_2(v_i) < \pi_2(v_j) \\ 0 & \text{otherwise} \end{cases}$$

- The number of crossings of a permutations π_2

$$\text{cross}(\pi_2) = \sum_{i=1}^{n_2-1} \sum_{j=i+1}^{n_2} (c_{ij} - c_{ji}) x_{ij} + \sum_{i=1}^{n_2-1} \sum_{j=i+1}^{n_2} c_{ji}$$

Integer Linear Program

[Jünger & Mutzel, '97]

- Constant $c_{ij} := \#$ crossings between edges incident to v_i or v_j when $\pi_2(v_i) < \pi_2(v_j)$

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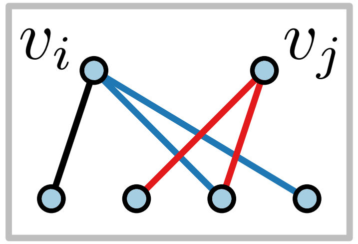

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Integer Linear Program

- Minimize the number of crossings:

$$\text{minimize} \sum_{i=1}^{n_2-1} \sum_{j=i+1}^{n_2} (c_{ij} - c_{ji}) x_{ij}$$

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$$0 \leq x_{ij} + x_{jk} - x_{ik} \leq 1 \qquad \text{for } 1 \leq i < j < k \leq n_2$$

Integer Linear Program

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- Transitivity constraints:

$$0 \leq x_{ij} + x_{jk} - x_{ik} \leq 1 \qquad \text{for } 1 \leq i < j < k \leq n_2$$

i.e., if $x_{ij} = 1$ and $x_{jk} = 1$, then $x_{ik} = 1$

Integer Linear Program

- Minimize the number of crossings:

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i.e., if $x_{ij} = \overset{1}{\underset{0}{}}$ and $x_{jk} = \overset{1}{\underset{0}{}}$, then $x_{ik} = \overset{1}{\underset{0}{}}$

Integer Linear Program

- Minimize the number of crossings:

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i.e., if $x_{ij} = \substack{1\\0}$ and $x_{jk} = \substack{1\\0}$, then $x_{ik} = \substack{1\\0}$

Properties.

Integer Linear Program

- Minimize the number of crossings:

$$\text{minimize} \sum_{i=1}^{n_2-1} \sum_{j=i+1}^{n_2} (c_{ij} - c_{ji}) x_{ij}$$

- Transitivity constraints:

$$0 \leq x_{ij} + x_{jk} - x_{ik} \leq 1 \qquad \text{for } 1 \leq i < j < k \leq n_2$$

i.e., if $x_{ij} = \genfrac{}{}{0pt}{}{1}{0}$ and $x_{jk} = \genfrac{}{}{0pt}{}{1}{0}$, then $x_{ik} = \genfrac{}{}{0pt}{}{1}{0}$

Properties.

- Branch-and-cut technique for DAGs of limited size

Integer Linear Program

- Minimize the number of crossings:

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$$0 \leq x_{ij} + x_{jk} - x_{ik} \leq 1 \qquad \text{for } 1 \leq i < j < k \leq n_2$$

i.e., if $x_{ij} = \begin{matrix}1\\0\end{matrix}$ and $x_{jk} = \begin{matrix}1\\0\end{matrix}$, then $x_{ik} = \begin{matrix}1\\0\end{matrix}$

Properties.

- Branch-and-cut technique for DAGs of limited size
- Useful for graphs of small to medium size

Integer Linear Program

- Minimize the number of crossings:

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Properties.

- Branch-and-cut technique for DAGs of limited size
- Useful for graphs of small to medium size
- Finds optimal solution

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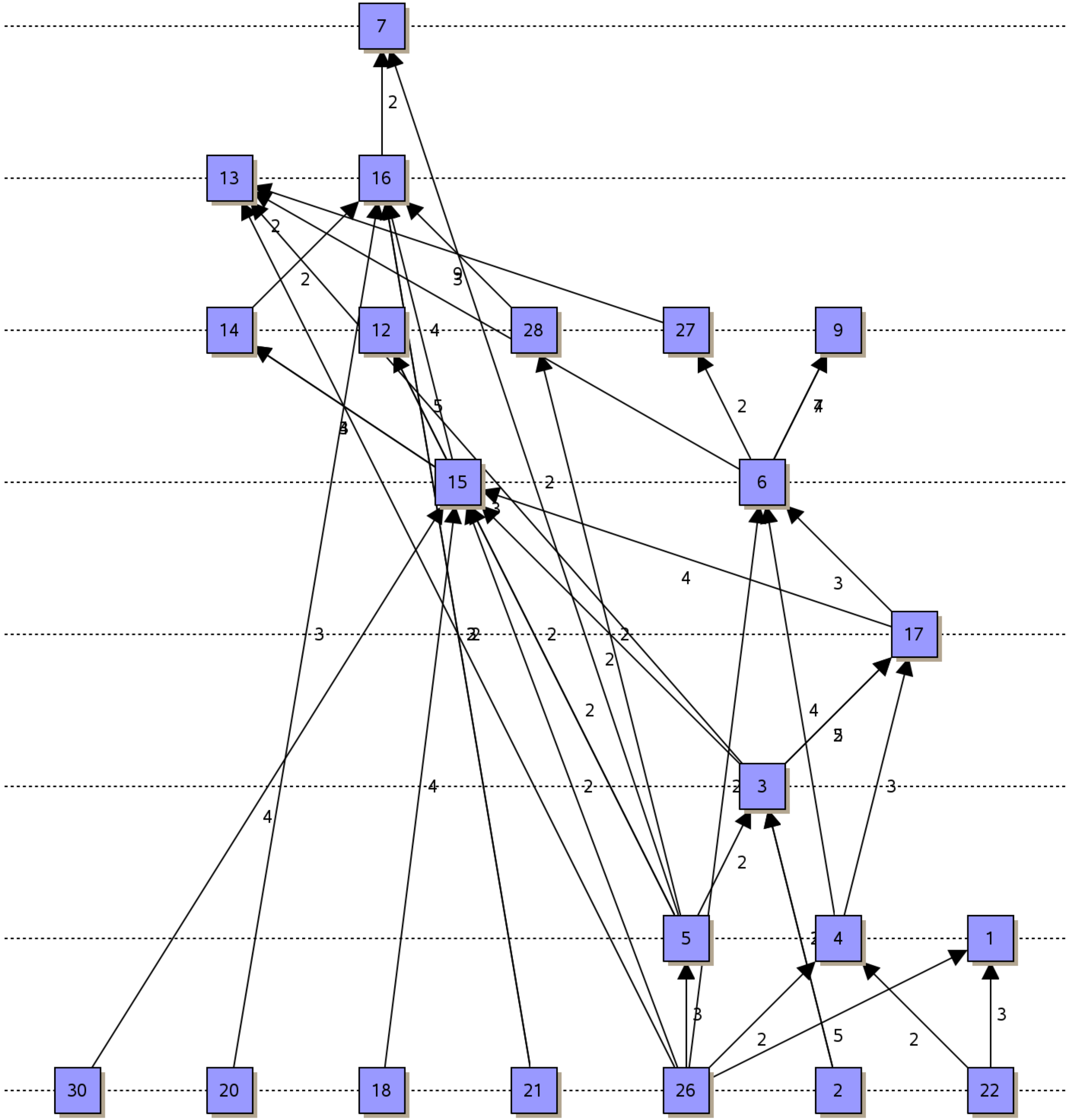
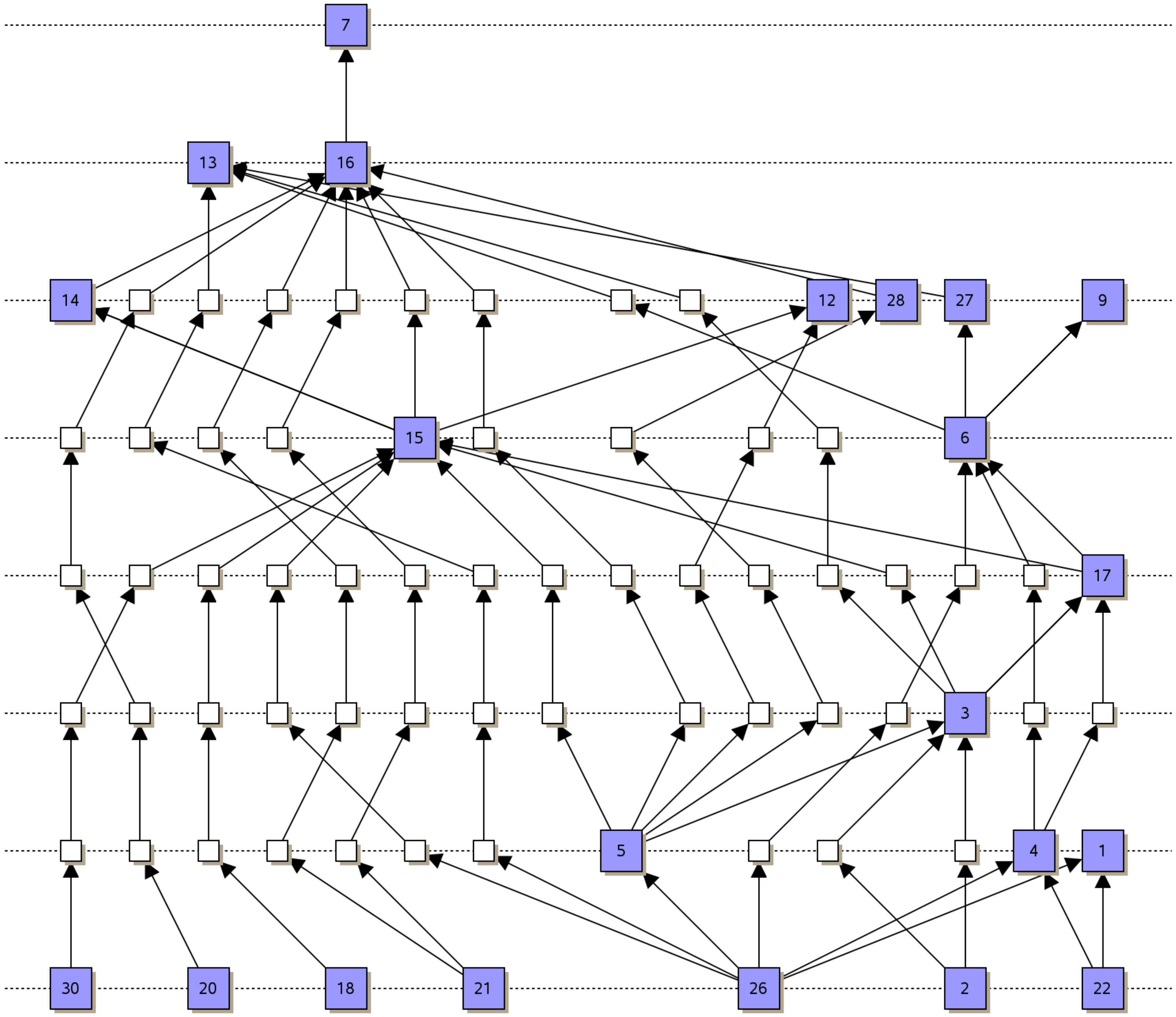
Properties.

- Branch-and-cut technique for DAGs of limited size
- Useful for graphs of small to medium size
- Finds optimal solution
- Solution in polynomial time is not guaranteed

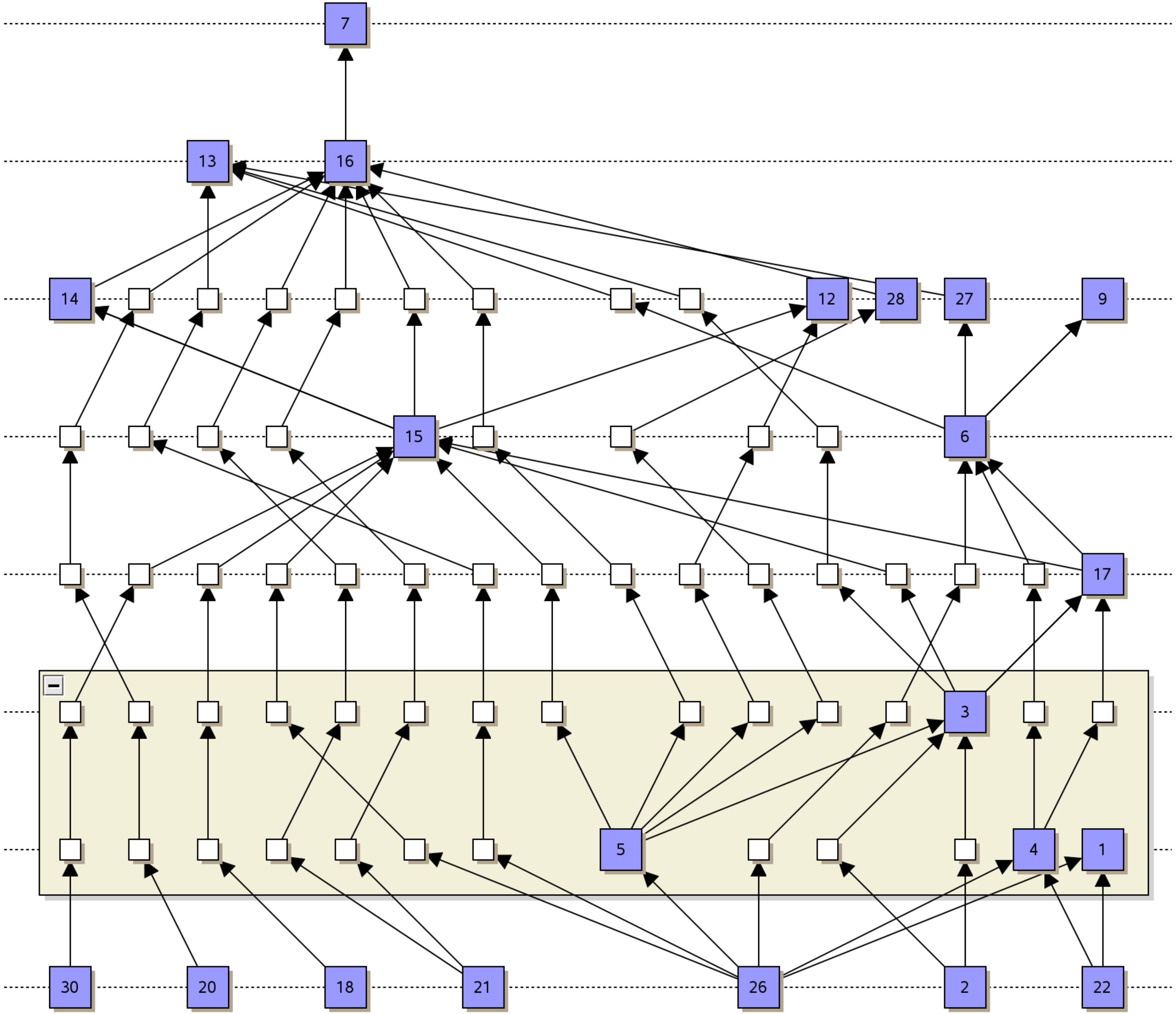
Iterations on Example



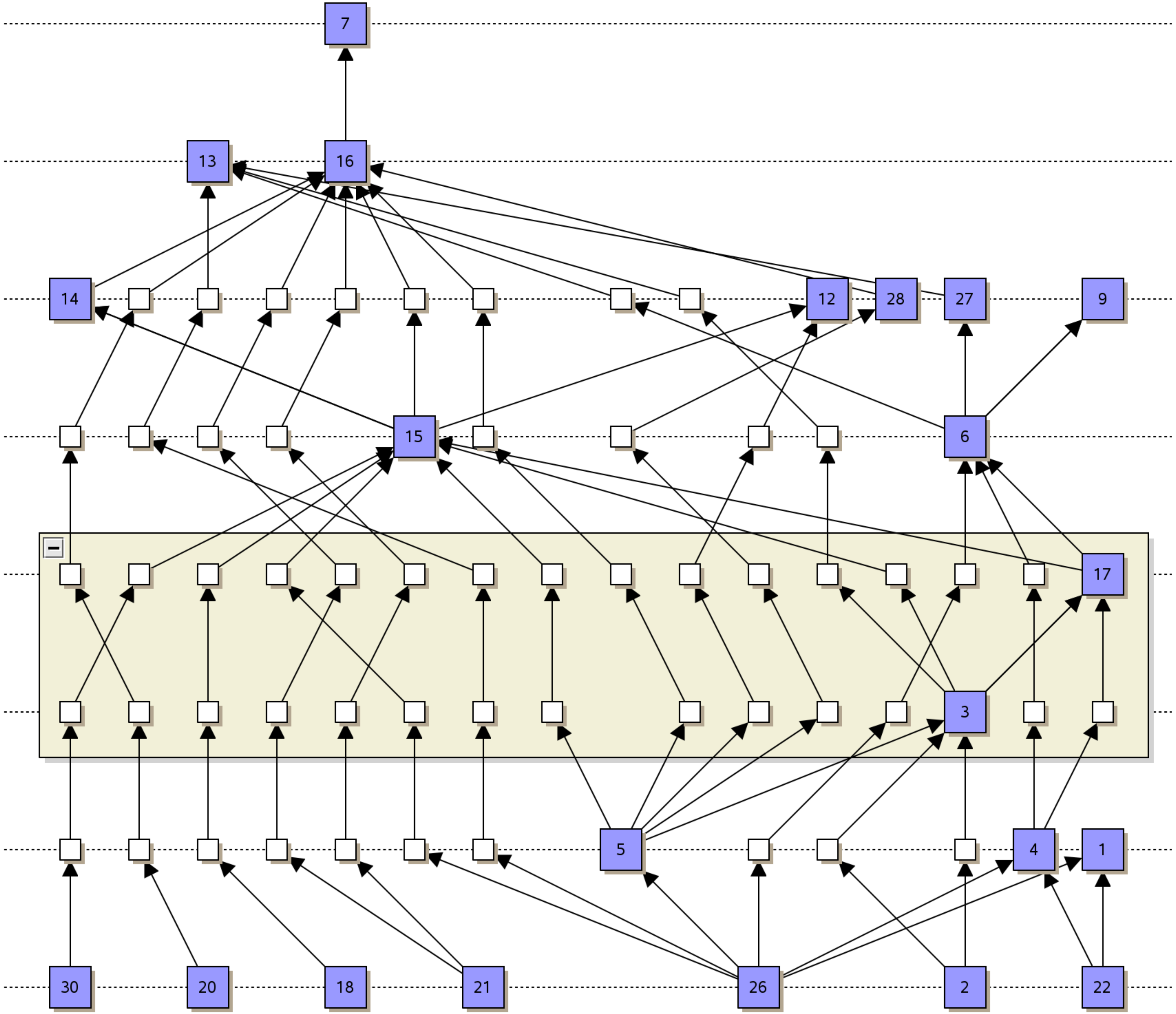
Iterations on Example



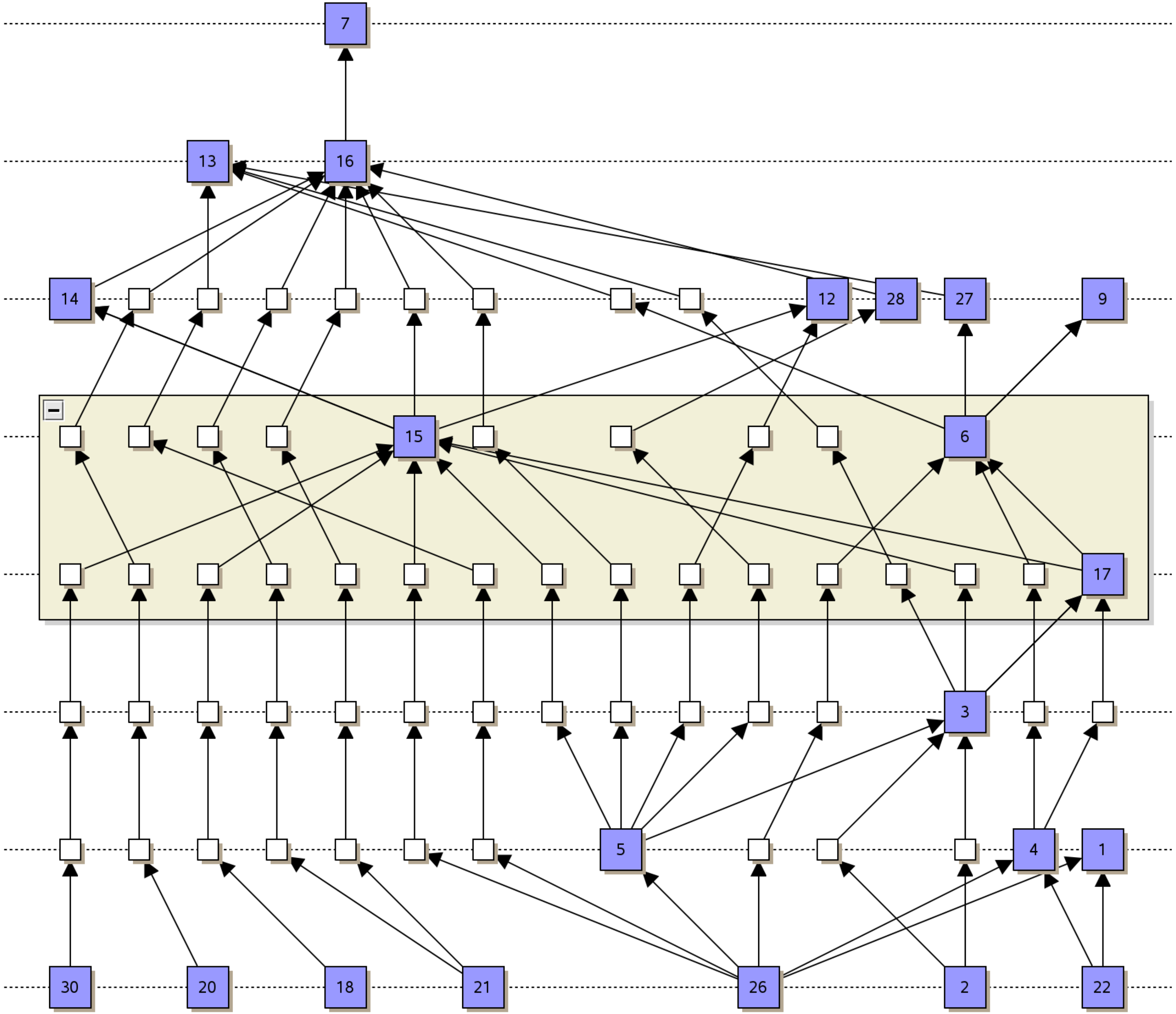
Iterations on Example



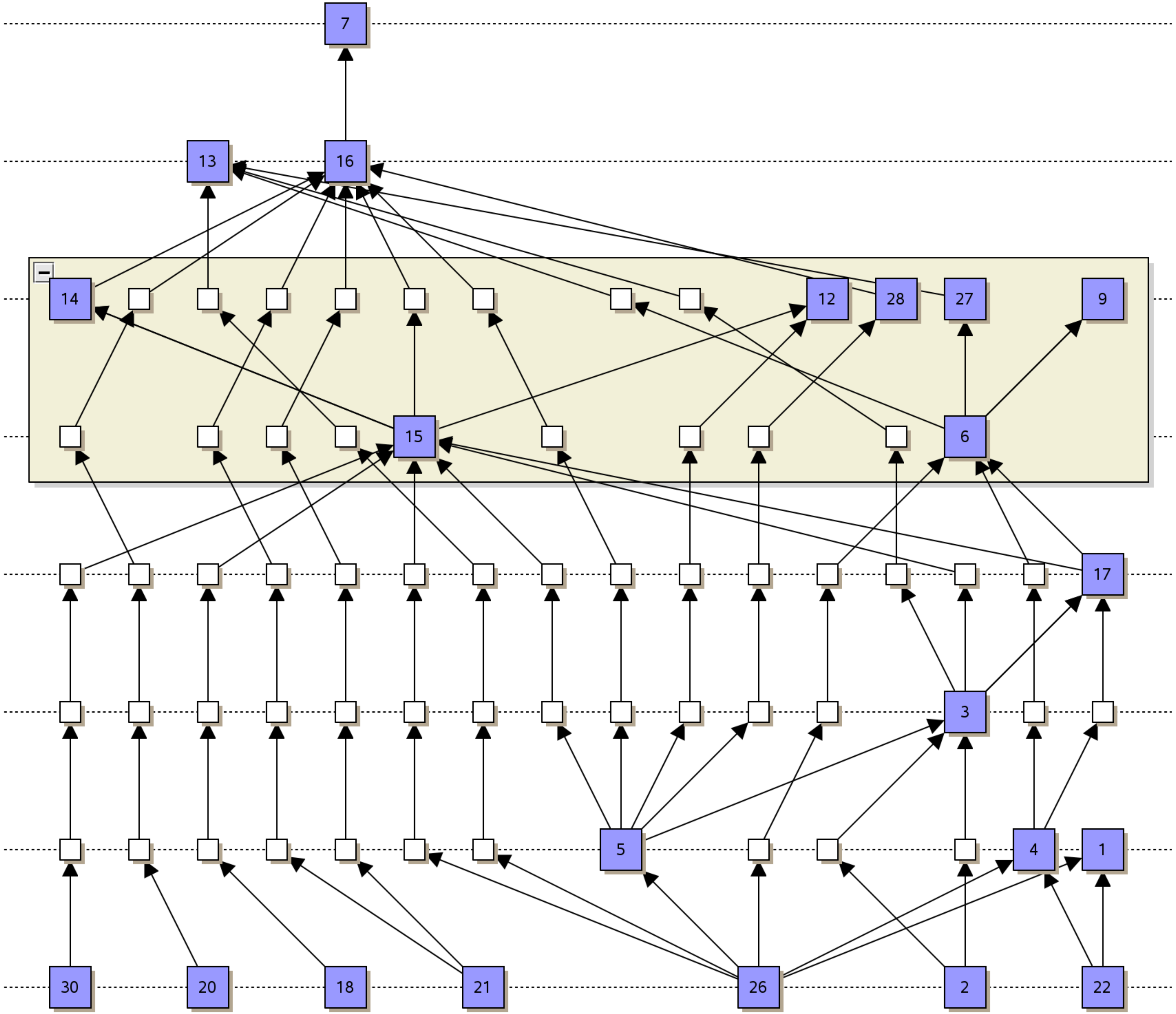
Iterations on Example



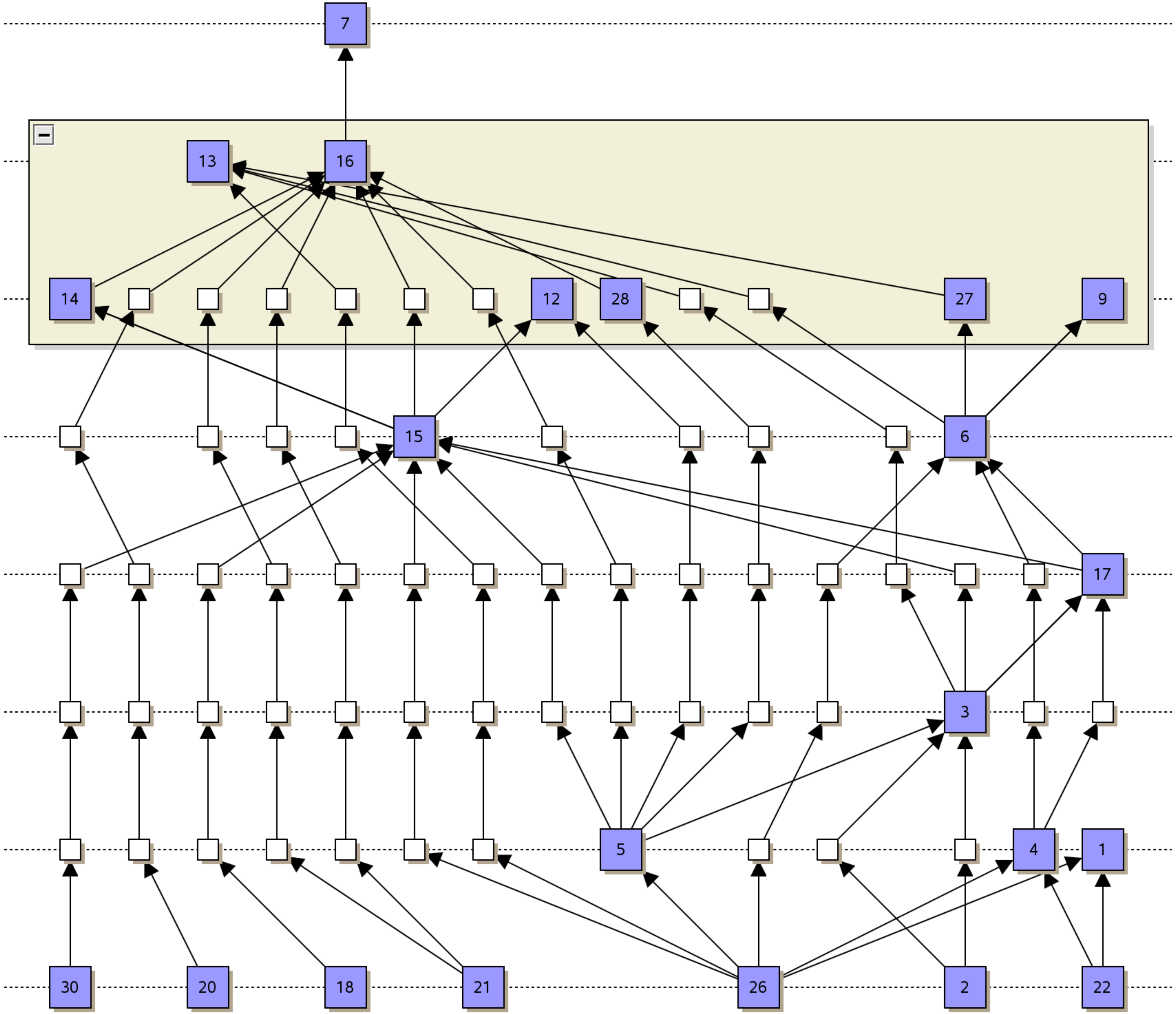
Iterations on Example



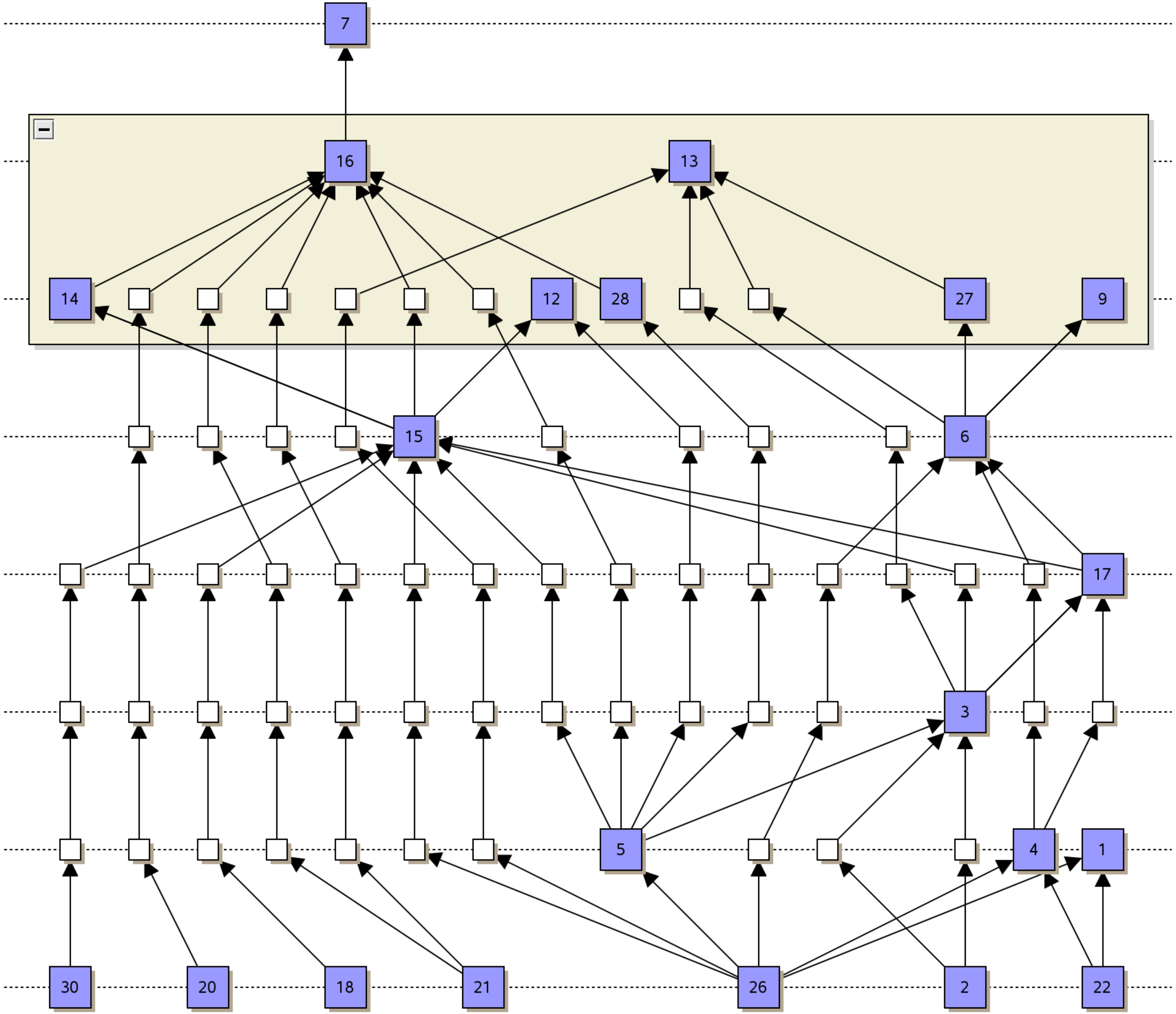
Iterations on Example



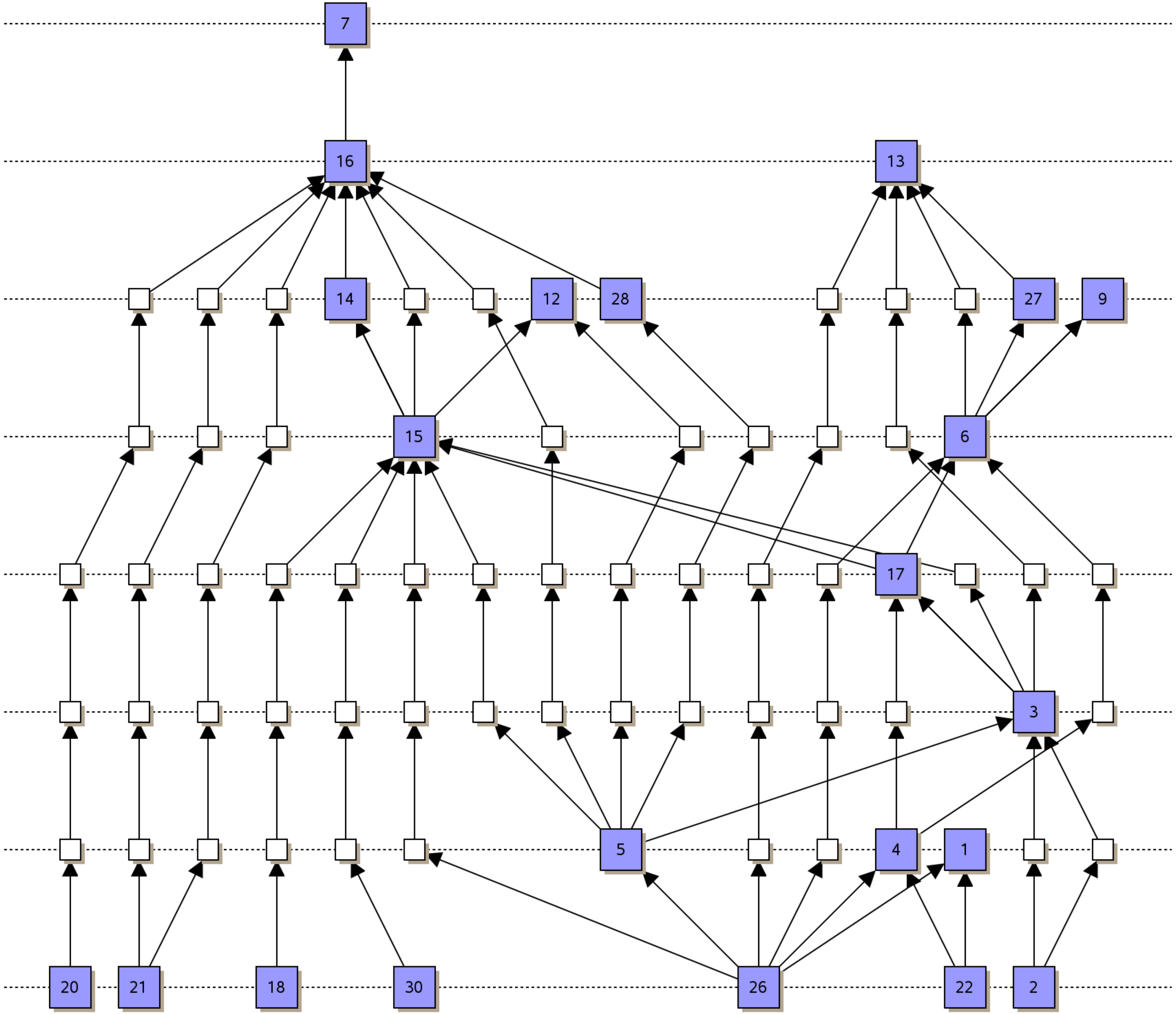
Iterations on Example



Iterations on Example



Iterations on Example



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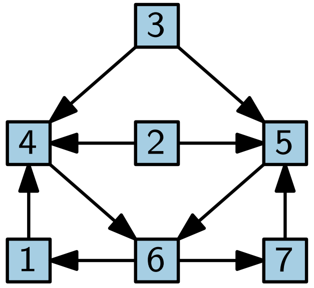
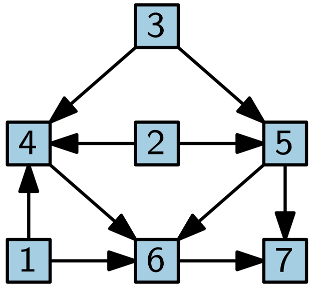
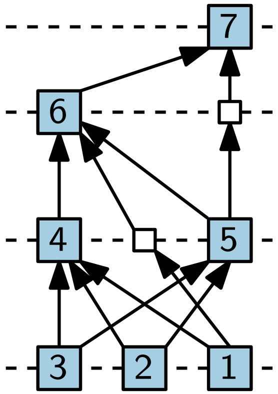




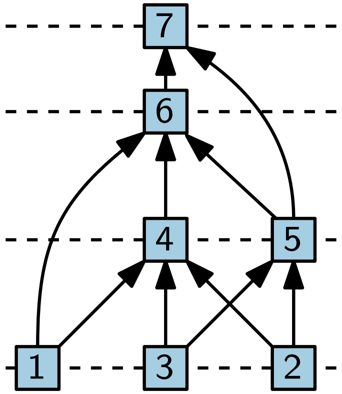


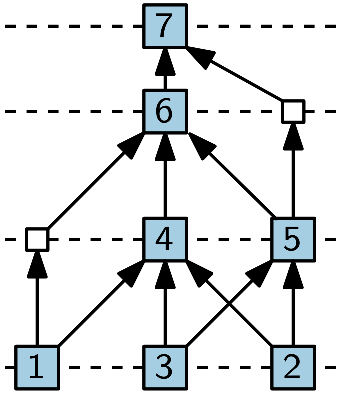
Visualization of Graphs

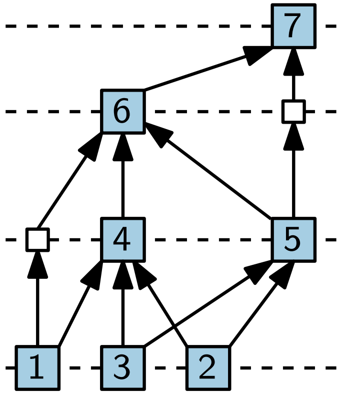
Lecture 7:
Hierarchical Layouts:
Sugiyama Framework

Part V:
Vertex Positioning & Drawing Edges

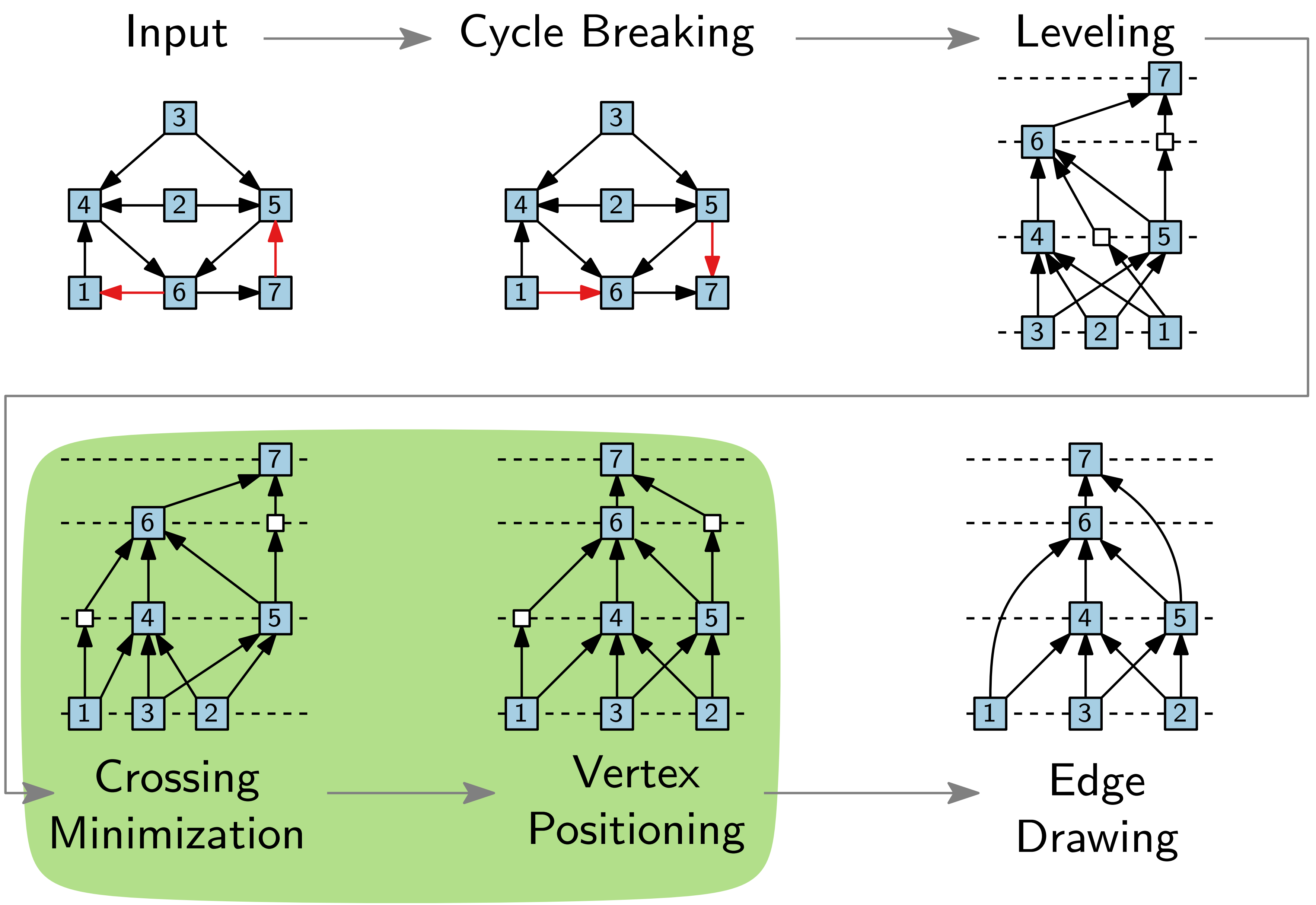
Jonathan Klawitter



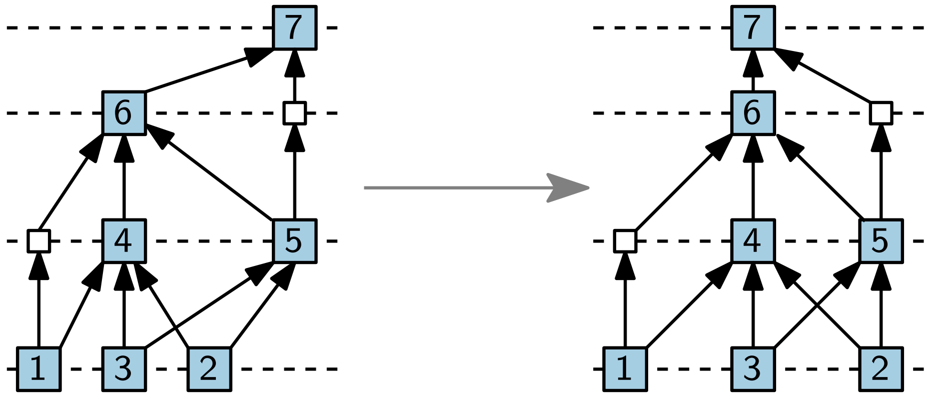




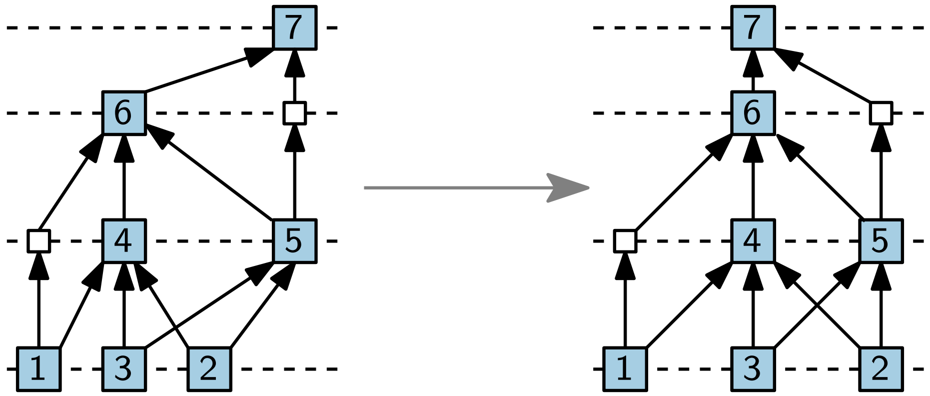
Step 4: Vertex Positioning



Step 4: Vertex Positioning



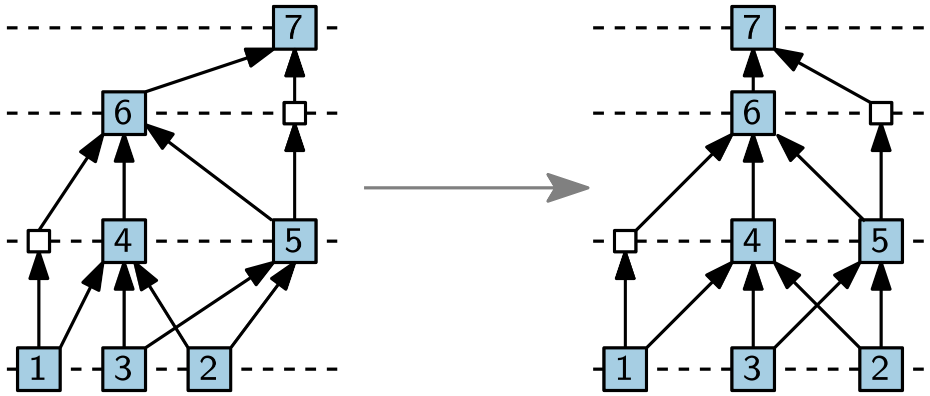
Step 4: Vertex Positioning



Goal.

Paths should be close to straight, vertices evenly spaced

Step 4: Vertex Positioning

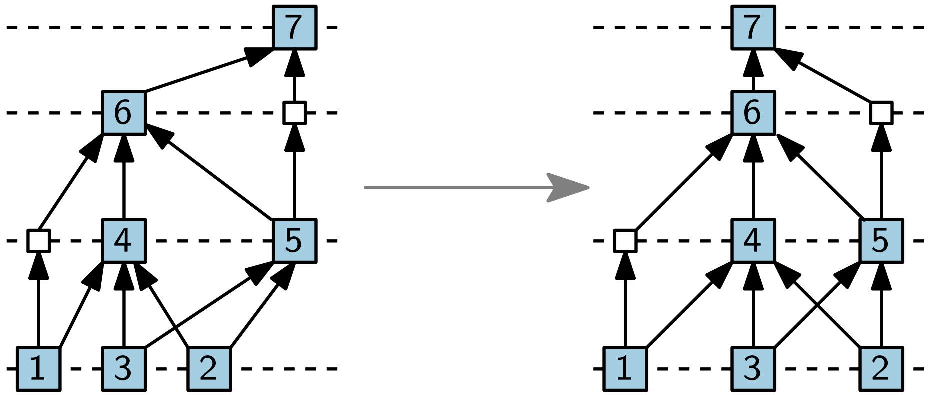


Goal.

Paths should be close to straight, vertices evenly spaced

- **Exact:** Quadratic Program (QP)

Step 4: Vertex Positioning



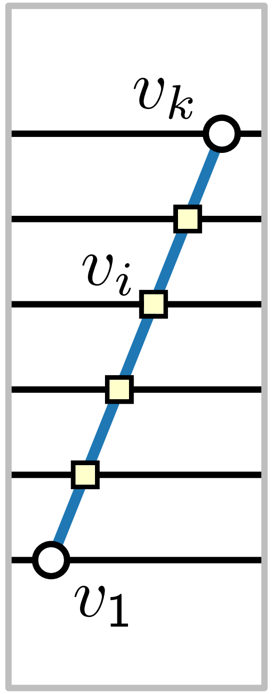
Goal.

Paths should be close to straight, vertices evenly spaced

- **Exact:** Quadratic Program (QP)
- **Heuristic:** Iterative approach

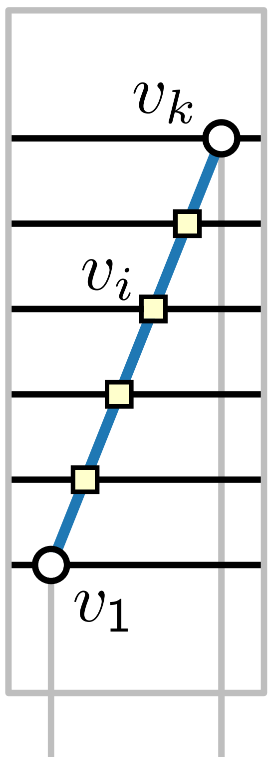
Quadratic Program

- Consider the path $p_e = (v_1, \ldots, v_k)$ of an edge $e = v_1 v_k$ with dummy vertices: $v_2, \ldots, v_{k-1}$



Quadratic Program

- Consider the path $p_e = (v_1, \ldots, v_k)$ of an edge $e = v_1 v_k$ with dummy vertices: $v_2, \ldots, v_{k-1}$
- x-coordinate of v_i according to the line $\overline{v_1 v_k}$ (with equal spacing):



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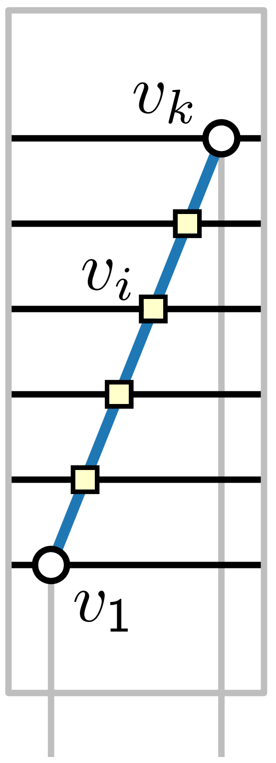
$$\overline{x(v_i)} =$$



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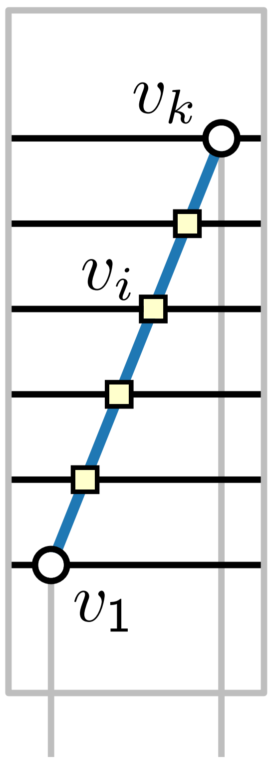
$$\overline{x(v_i)} = x(v_1) +$$



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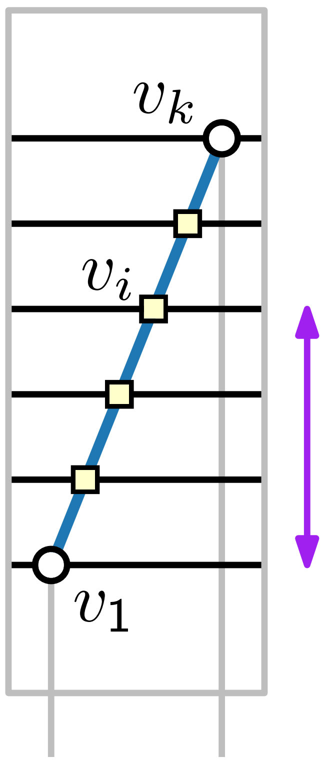
$$\overline{x(v_i)} = x(v_1) + \frac{i-1}{}$$



Quadratic Program

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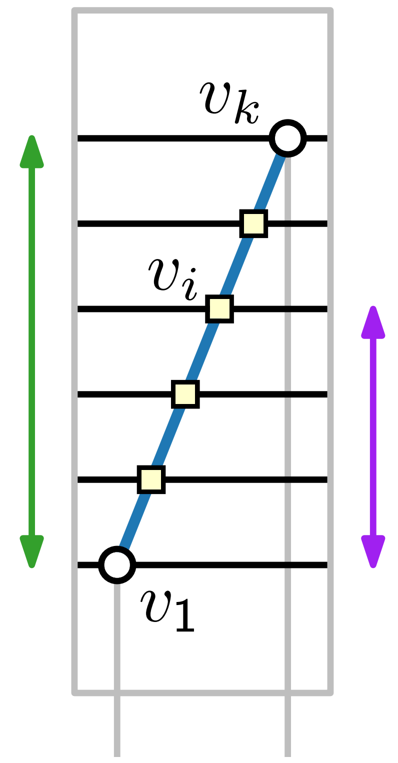

Quadratic Program

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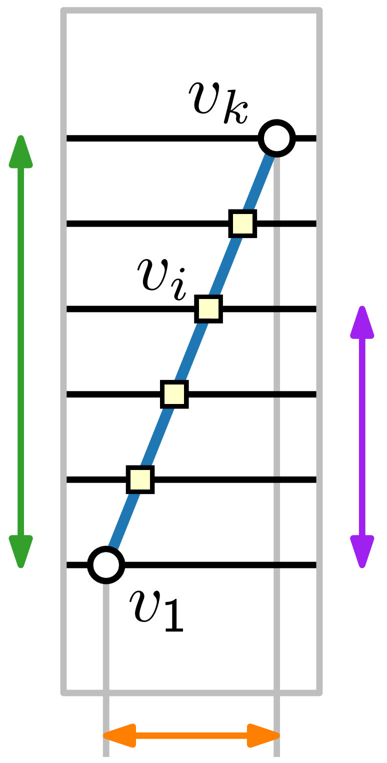

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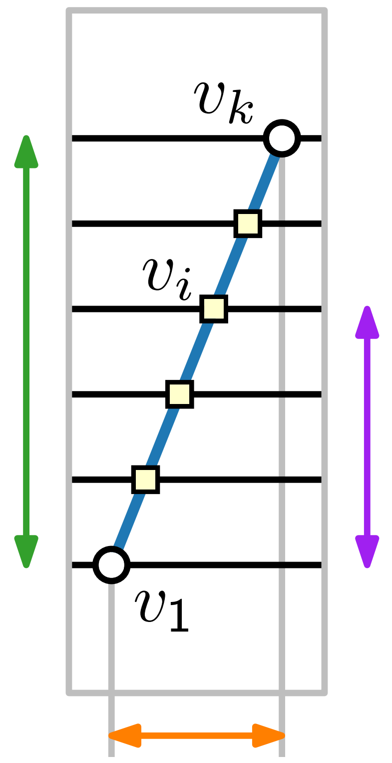
$$\overline{x(v_i)} = x(v_1) + \frac{i-1}{k-1}\left(x(v_k) - x(v_1)\right)$$

- Define the deviation from the line



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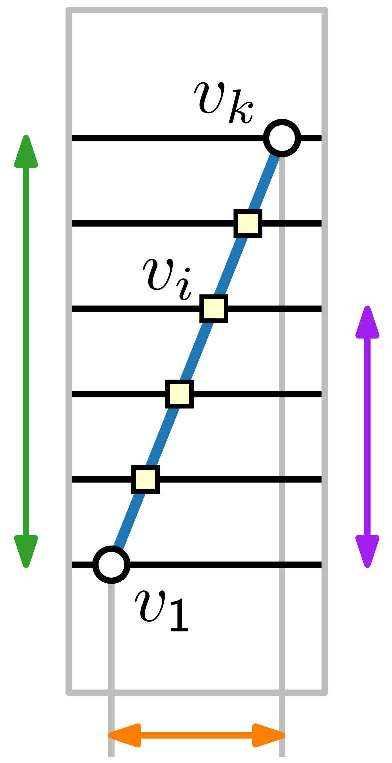
- Define the deviation from the line

$$\mathrm{dev}(p_e) :=$$



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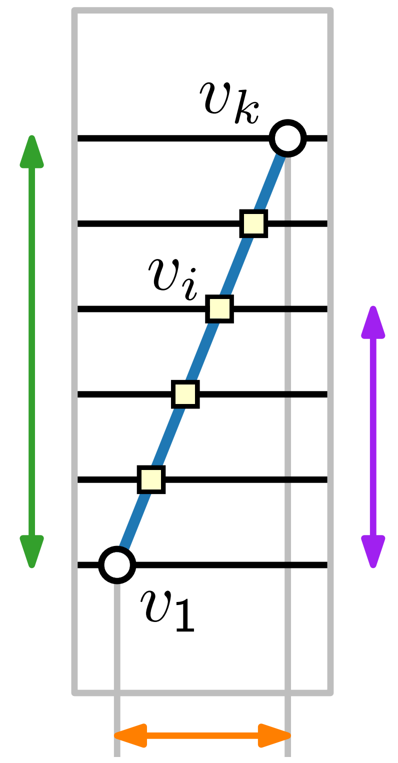
$$\mathrm{dev}(p_e) := \sum_{i=2}^{k-1}$$



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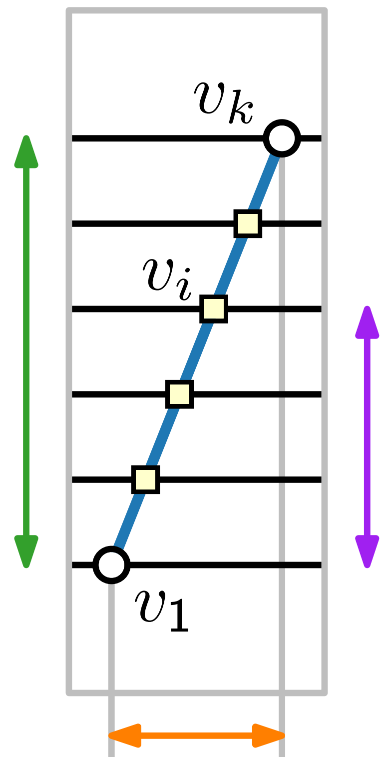
$$\mathrm{dev}(p_e) := \sum_{i=2}^{k-1} \left(x(v_i) - \overline{x(v_i)}\right)$$



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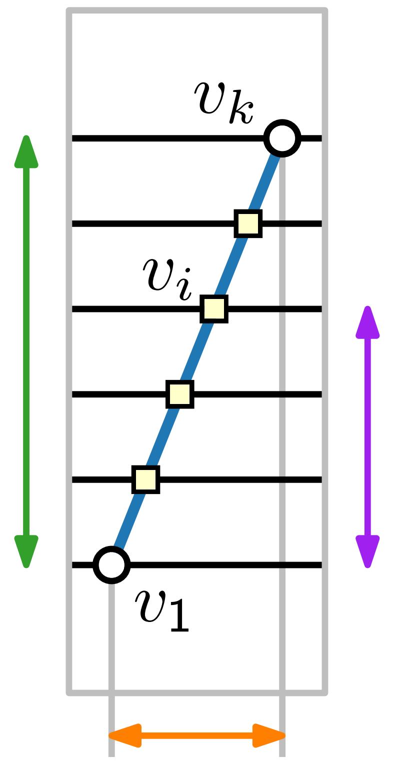
- Define the deviation from the line

$$\mathrm{dev}(p_e) := \sum_{i=2}^{k-1} \left(x(v_i) - \overline{x(v_i)}\right)^2$$



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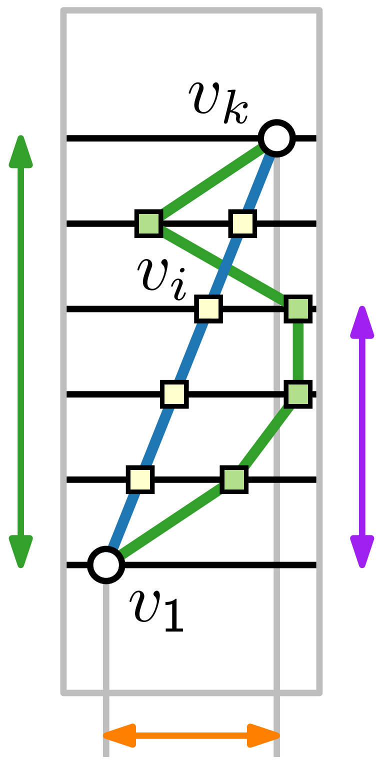

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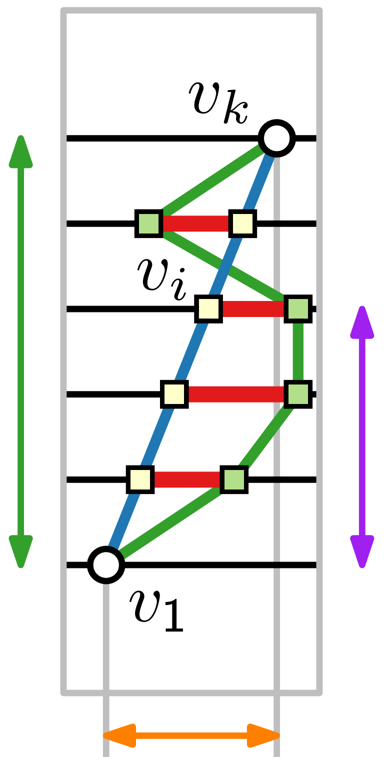

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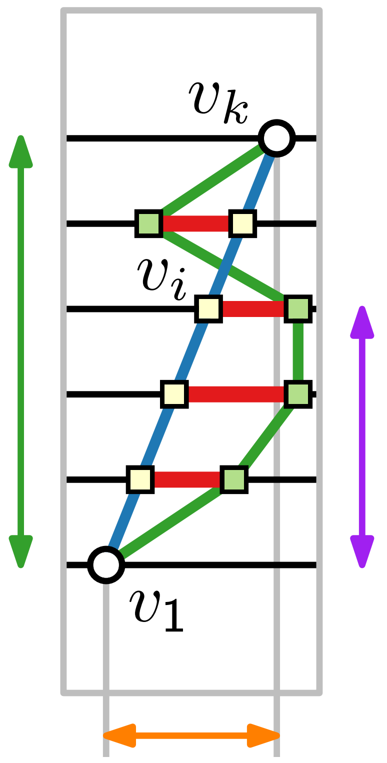
- Objective function:



Quadratic Program

- Consider the path $p_e = (v_1, \dots, v_k)$ of an edge $e = v_1 v_k$ with dummy vertices: $v_2, \dots, v_{k-1}$
- x-coordinate of v_i according to the line $\overline{v_1 v_k}$ (with equal spacing):

$$\overline{x(v_i)} = x(v_1) + \frac{i-1}{k-1}\left(x(v_k) - x(v_1)\right)$$

- Define the deviation from the line

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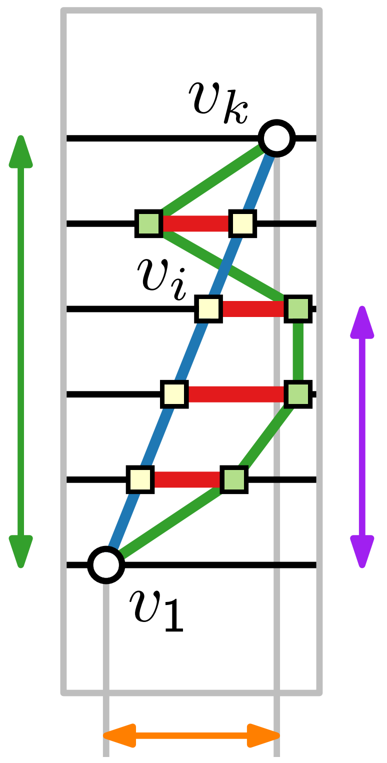
- Objective function: $\min \sum_{e \in E} \mathrm{dev}(p_e)$



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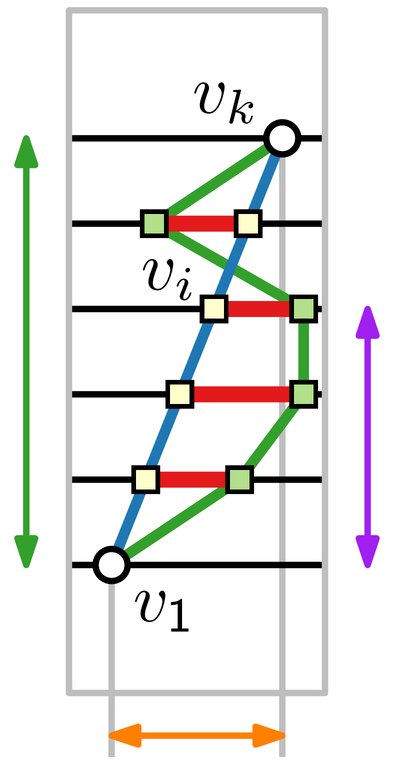
- Constraints for all vertices v, w in the same layer with w right of v:



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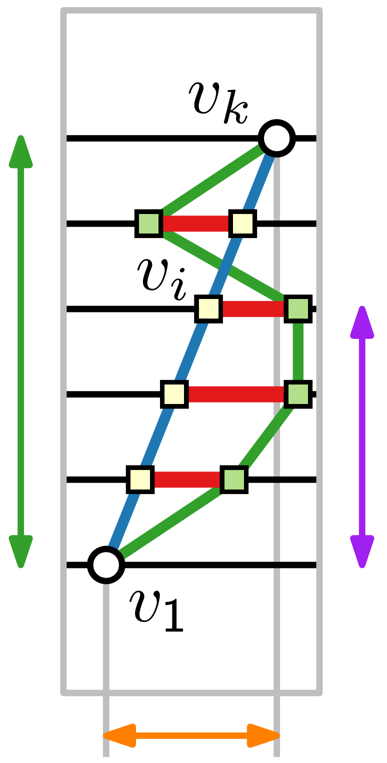

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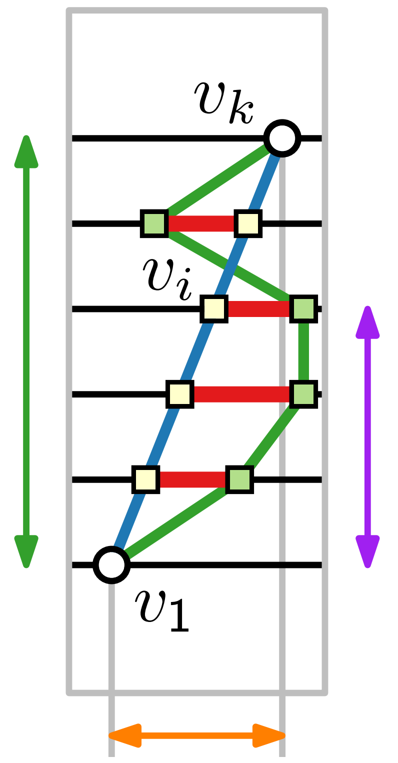

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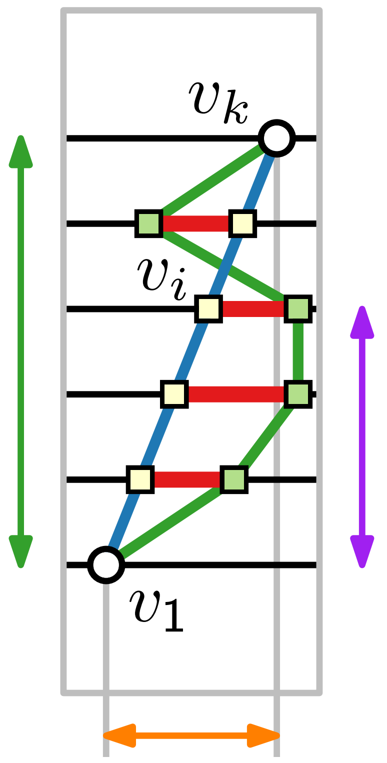
min. horizontal distance



- QP is time-expensive

Quadratic Program

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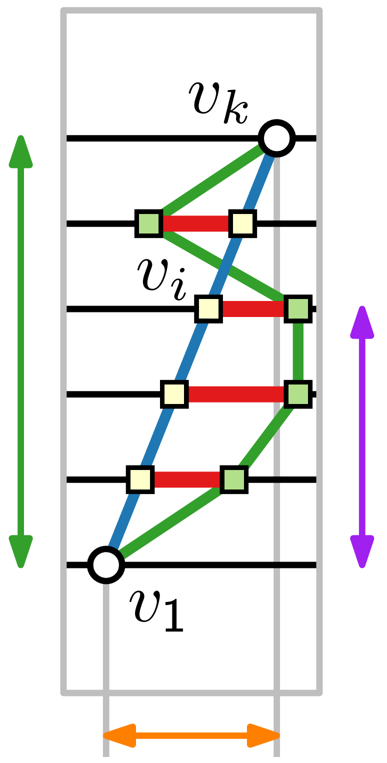

- QP is time-expensive
- width can be exponential

Iterative Heuristic

- Compute an initial layout

Iterative Heuristic

- Compute an initial layout
- Apply the following steps as long as improvements can be made:

Iterative Heuristic

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  1. Vertex positioning,

Iterative Heuristic

- Compute an initial layout
- Apply the following steps as long as improvements can be made:

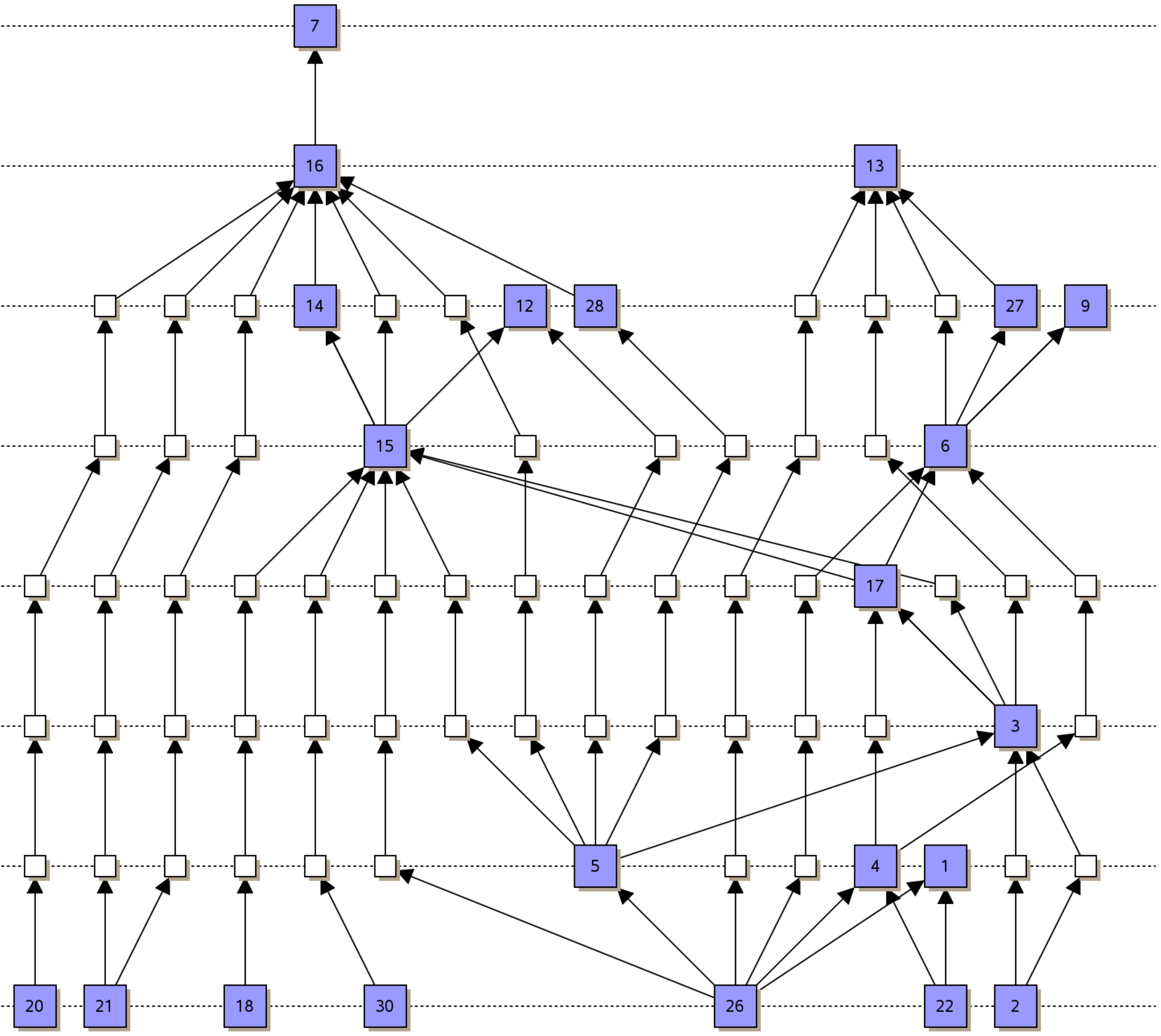
  1. Vertex positioning,
  2. edge straightening,

Iterative Heuristic

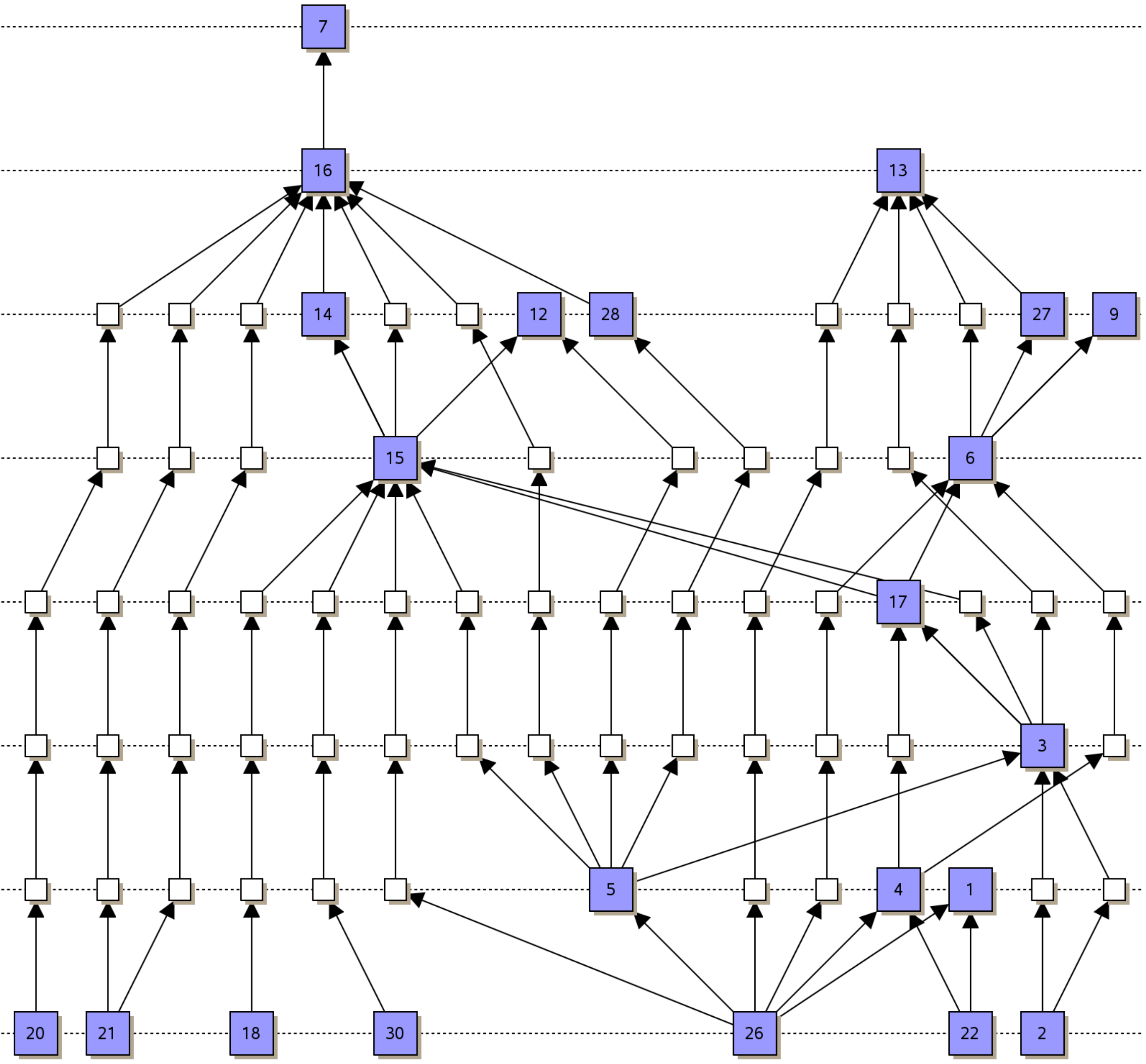
- Compute an initial layout
- Apply the following steps as long as improvements can be made:

   1. Vertex positioning,
   2. edge straightening,
   3. Compactifying the layout width.

Example



Example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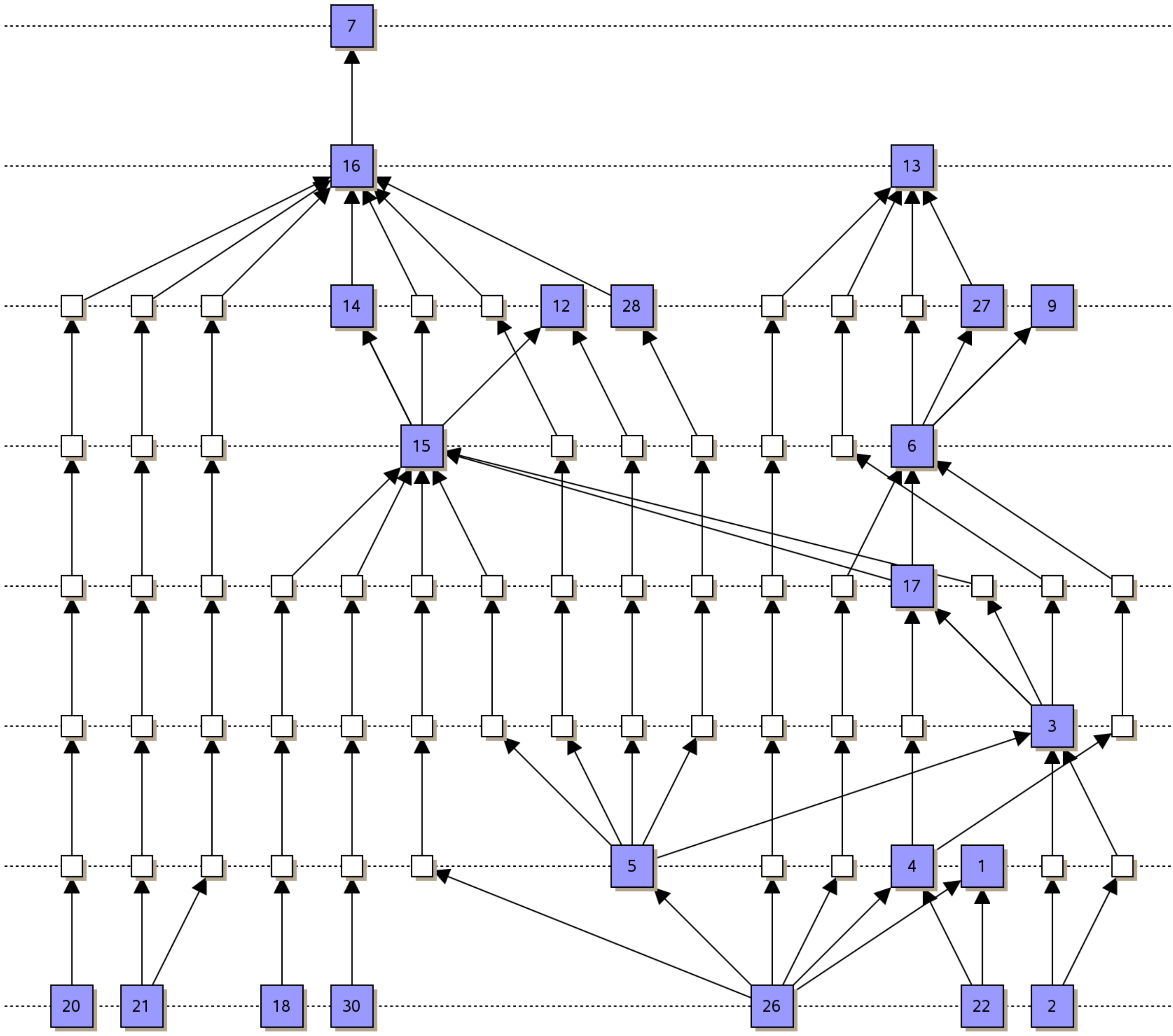
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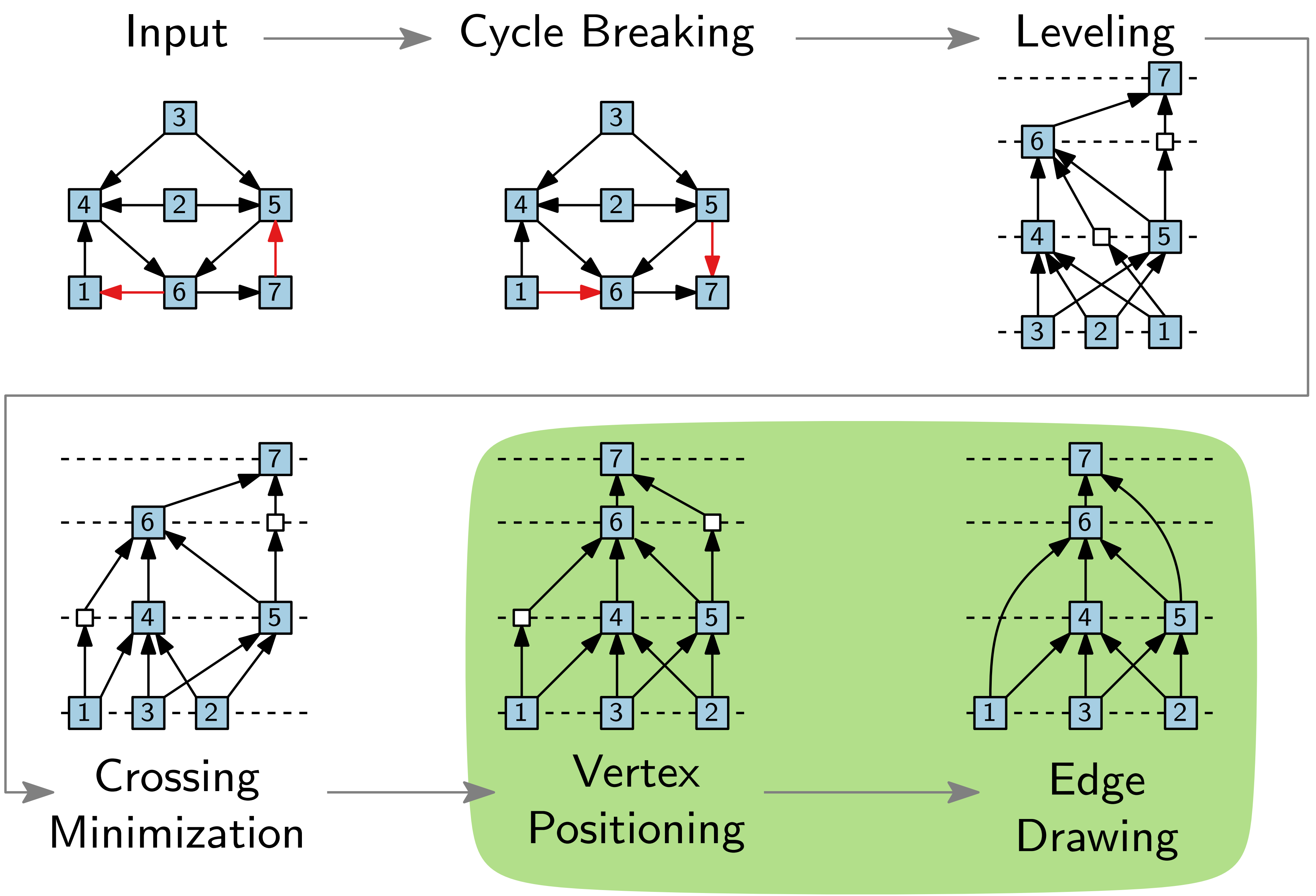
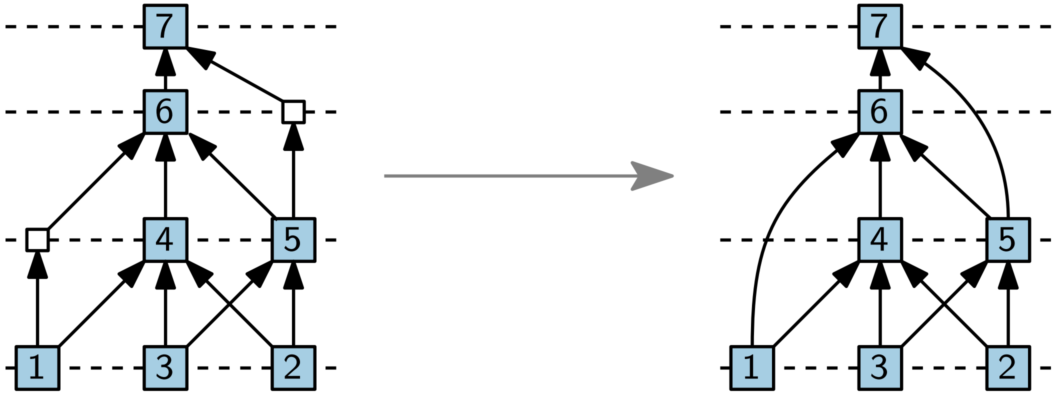
Step 5: Drawing Edges



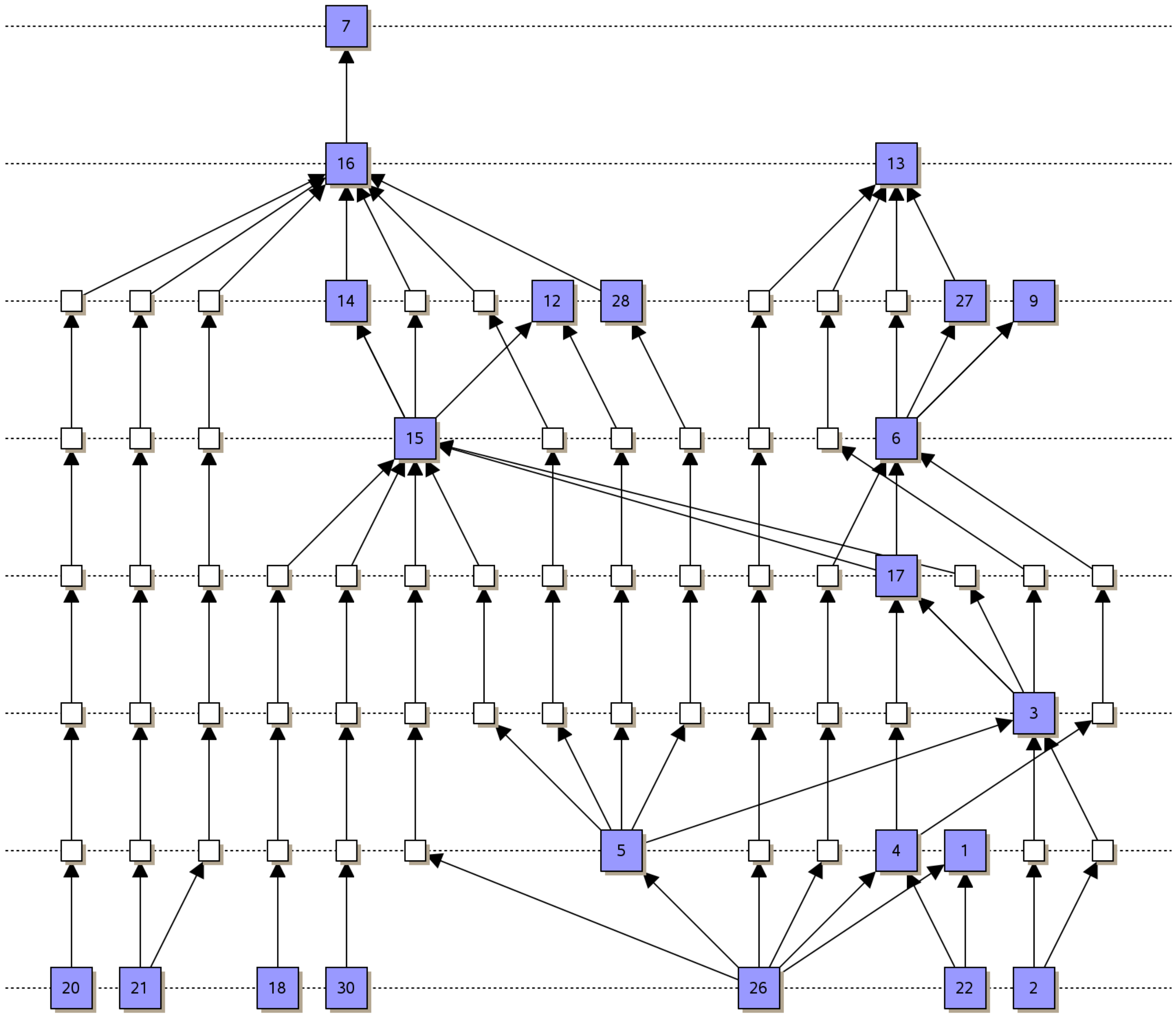
Step 5: Drawing Edges



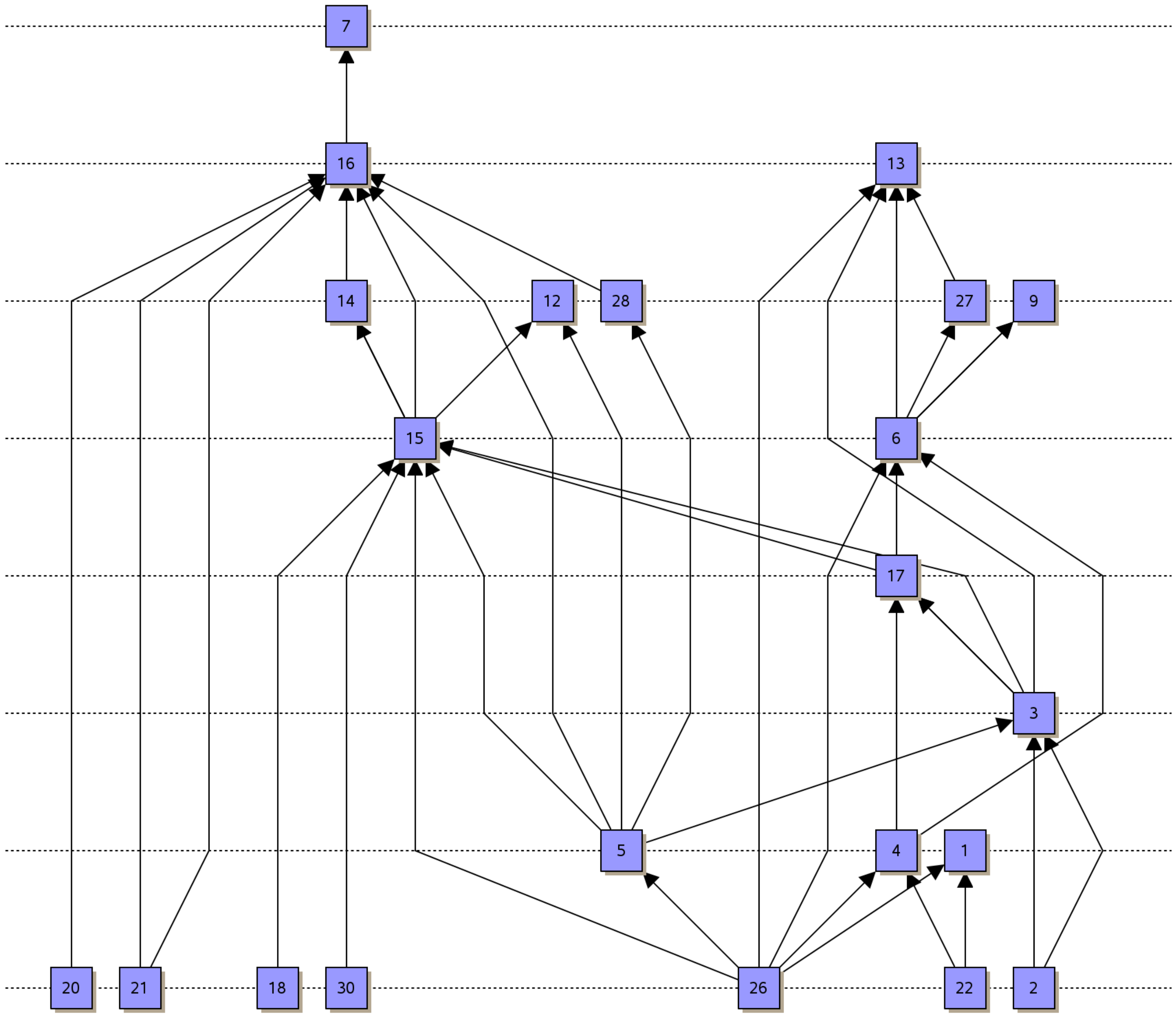
Possibility.

Substitute polylines by Bézier curves

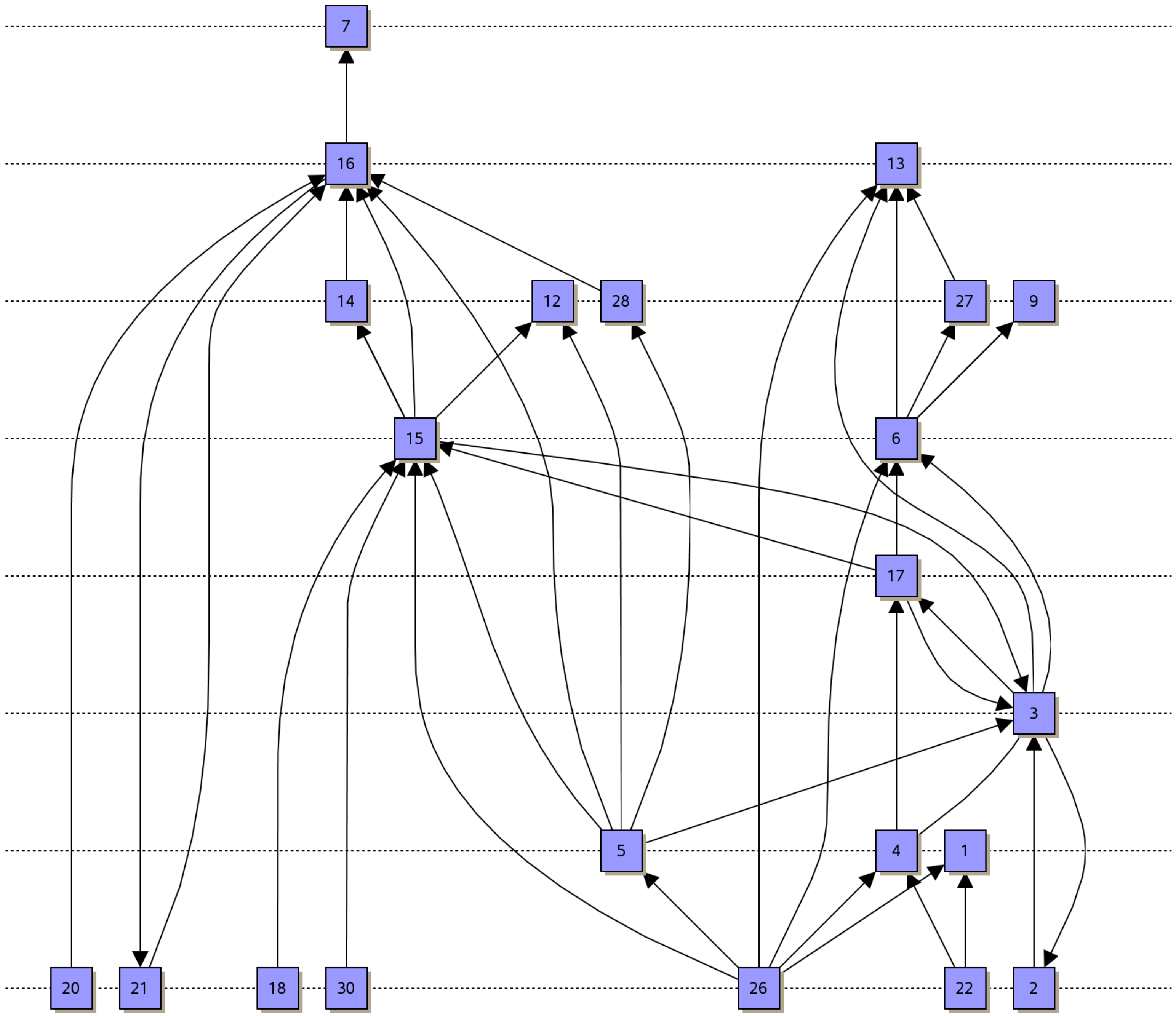
Example



Example

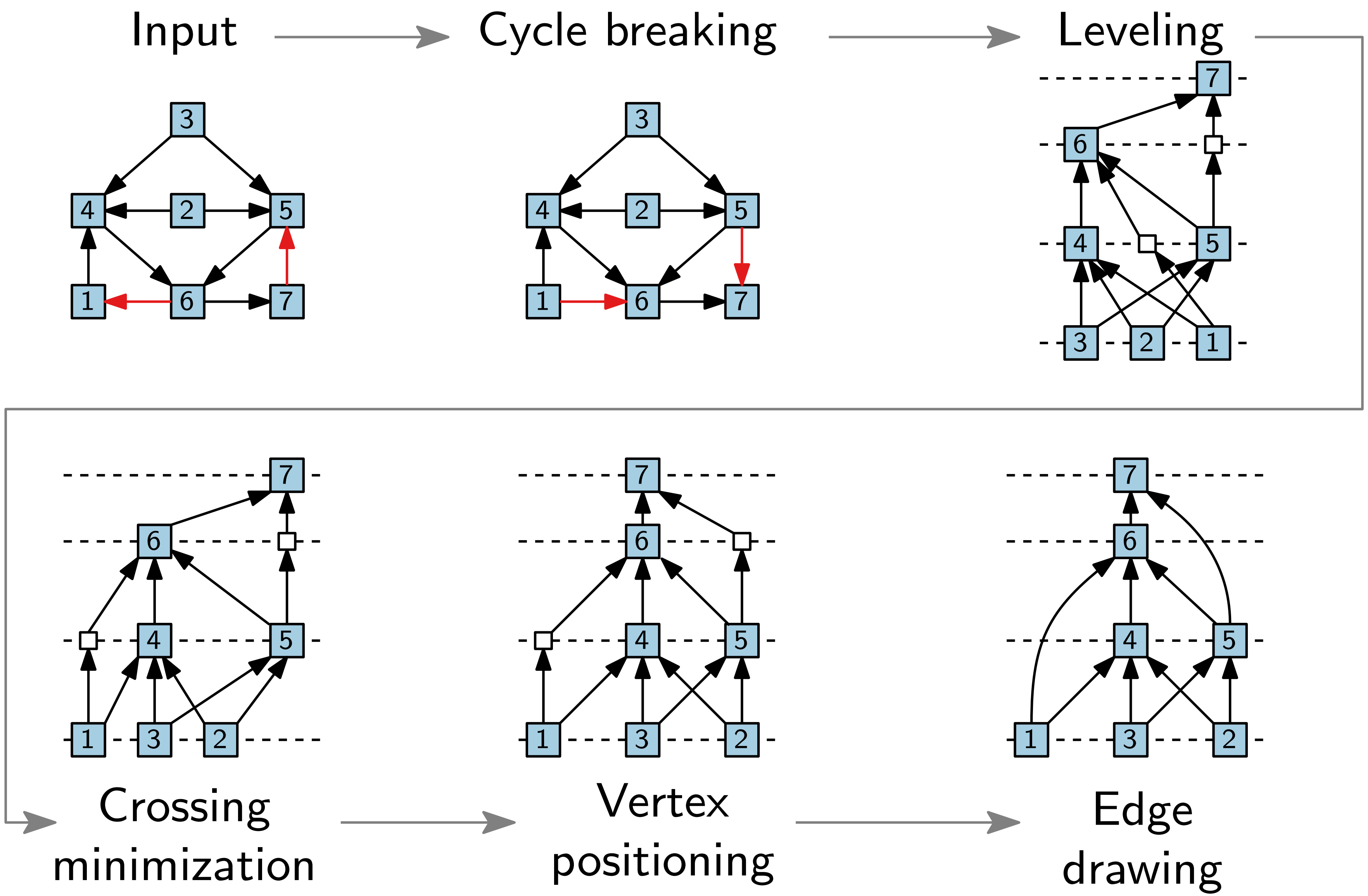


Example



Classical Approach – Sugiyama Framework

[Sugiyama, Tagawa, Toda ’81]



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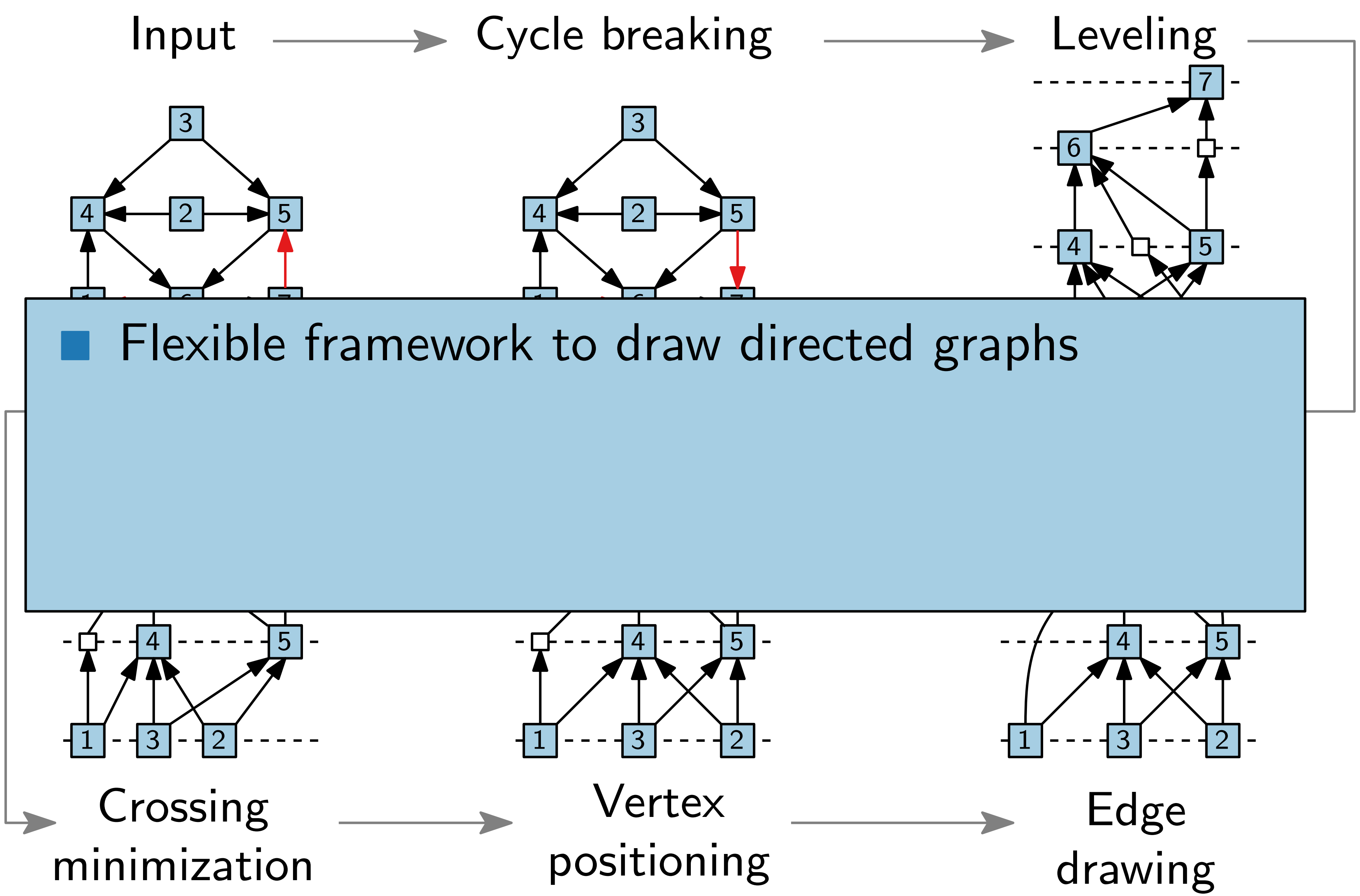

- Flexible framework to draw directed graphs

Classical Approach – Sugiyama Framework

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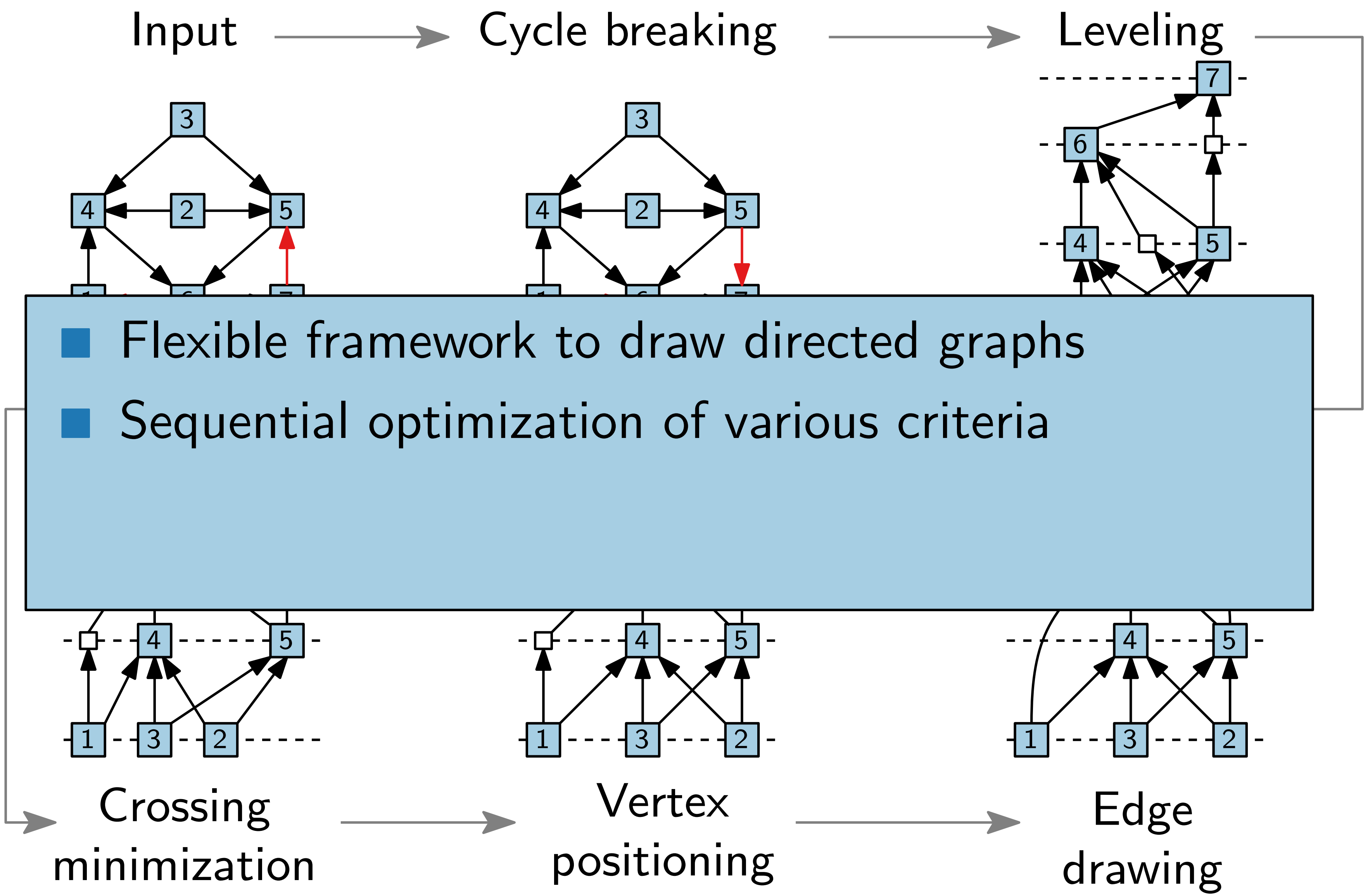

- Flexible framework to draw directed graphs
- Sequential optimization of various criteria

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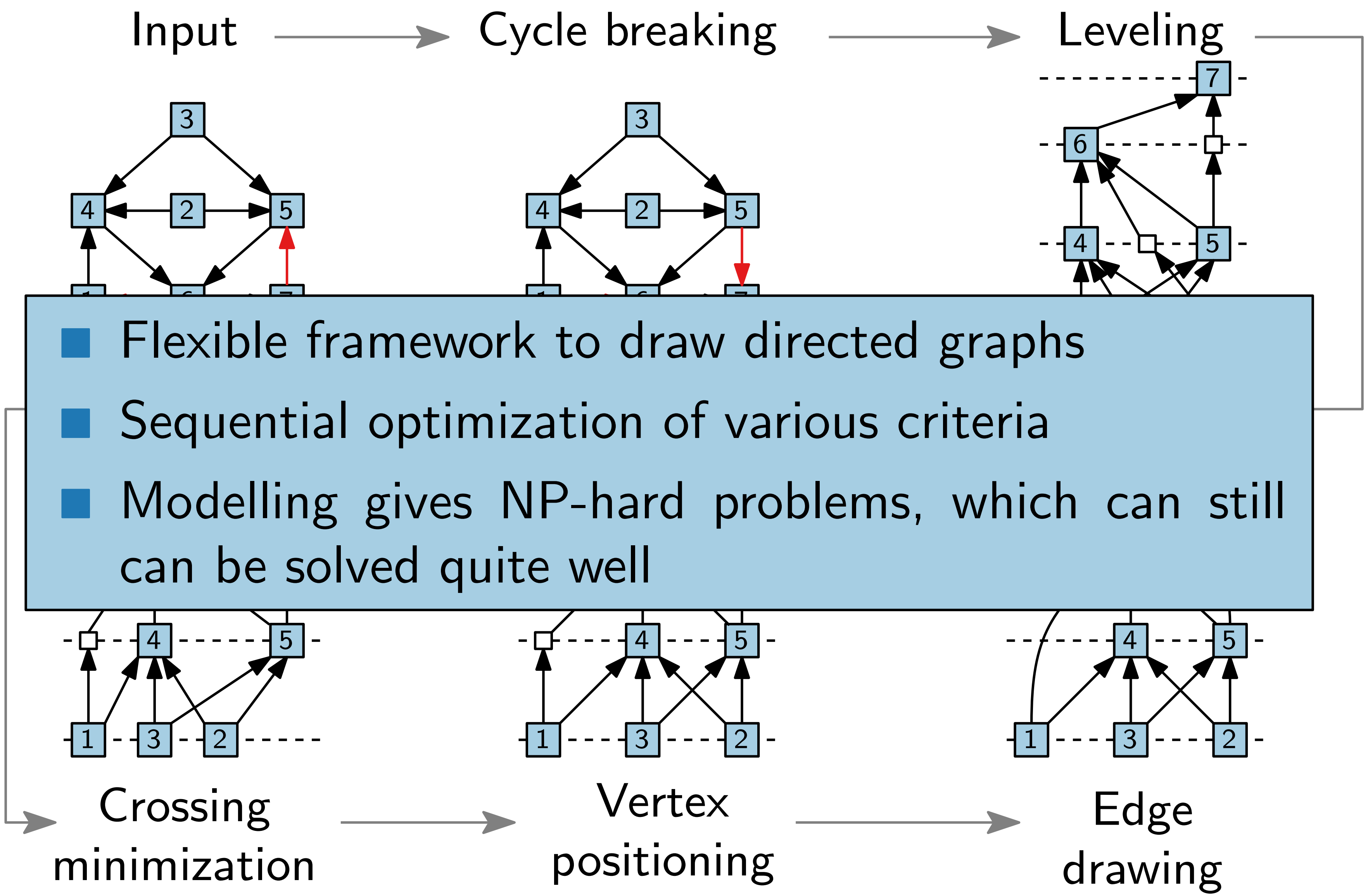

- Flexible framework to draw directed graphs
- Sequential optimization of various criteria
- Modelling gives NP-hard problems, which can still can be solved quite well

Literature

Detailed explanations of steps and proofs in

- [GD Ch. 11] and [DG Ch. 5]

based on

- [Sugiyama, Tagawa, Toda '81] Methods for visual understanding of hierarchical system structures

and refined with results from

- [Berger, Shor '90] Approximation alogorithms for the maximum acyclic subgraph problem
- [Eades, Lin, Smith '93] A fast and effective heuristic for the feedback arc set problem
- [Garey, Johnson '83] Crossing number is NP-complete
- [Eades, Whiteside '94] Drawing graphs in two layers
- [Eades, Wormland '94] Edge crossings in drawings of bipartite graphs
- [Jünger, Mutzel '97] 2-Layer Straightline Crossing Minimization: Performance of Exact and Heuristic Algorithms