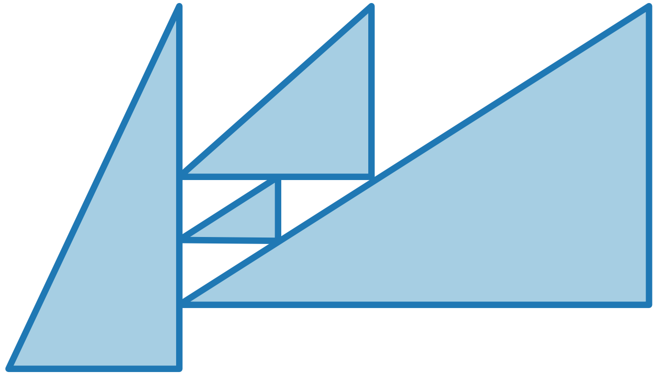
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

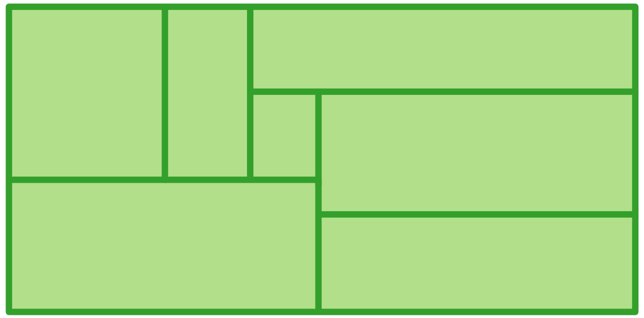
Visualization of Graphs

Lecture 7:
Contact Representations of Planar Graphs:
Triangle Contacts and Rectangular Duals



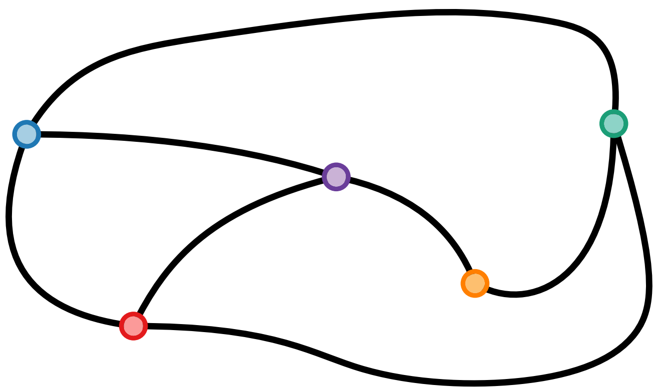
Part I:
Geometric Representations

Alexander Wolff



Intersection Representation of Graphs

In an **intersection representation** of a graph,
– each vertex is represented by a set



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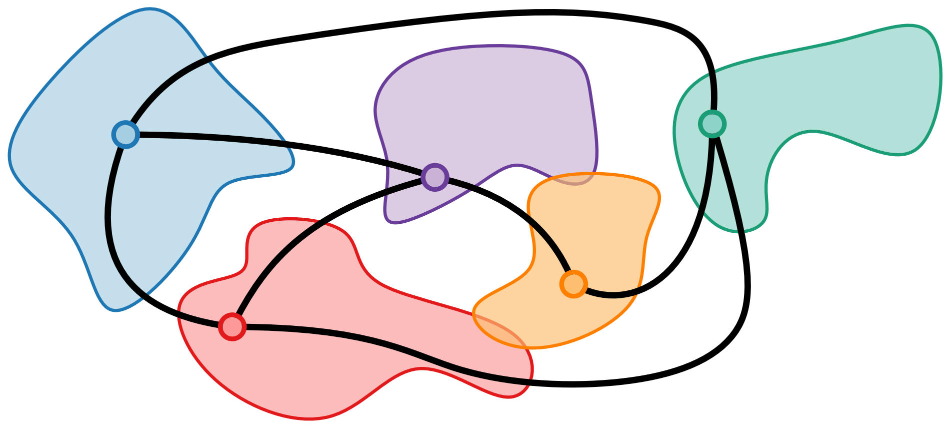

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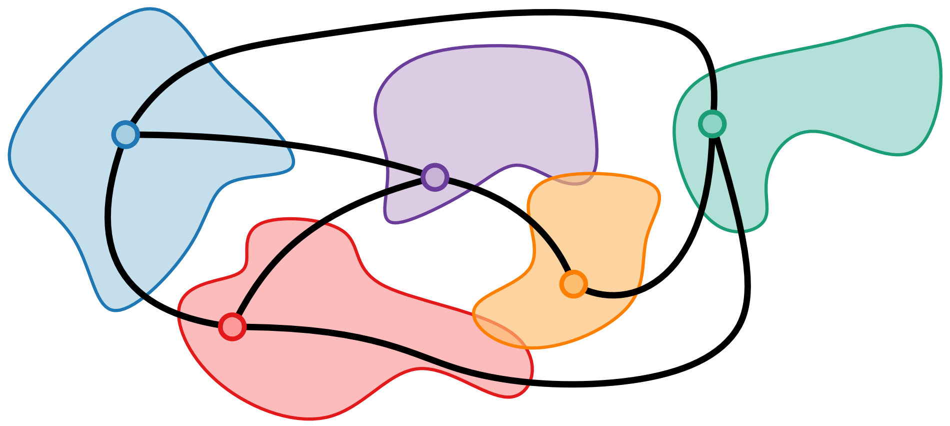

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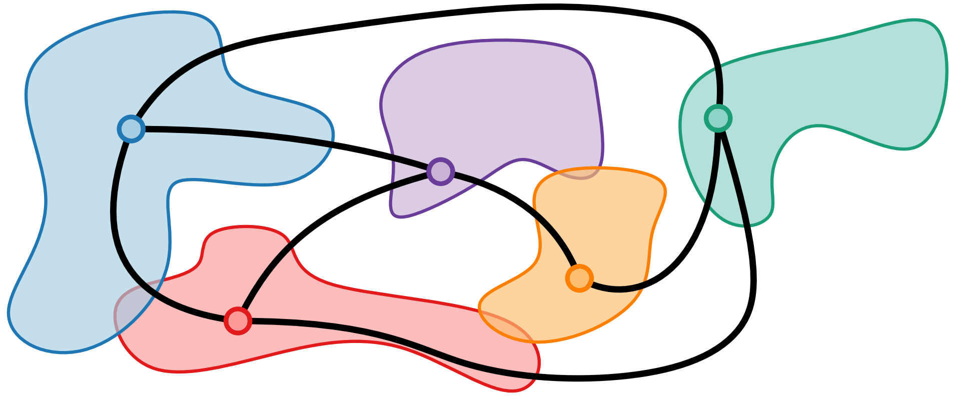

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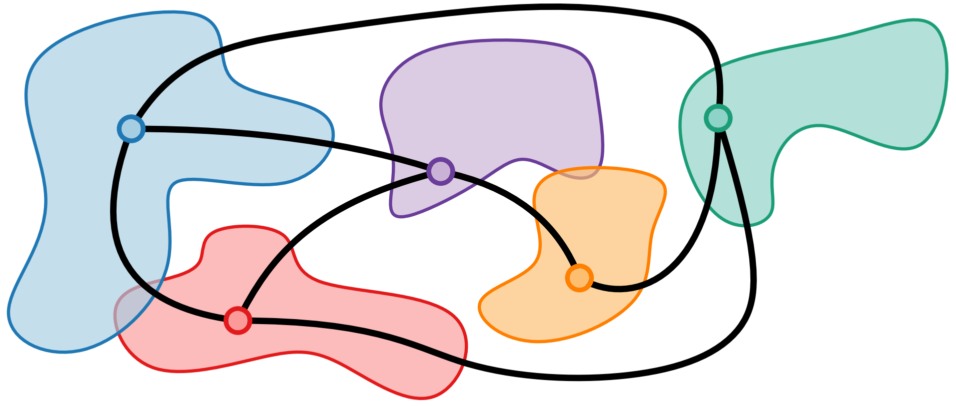

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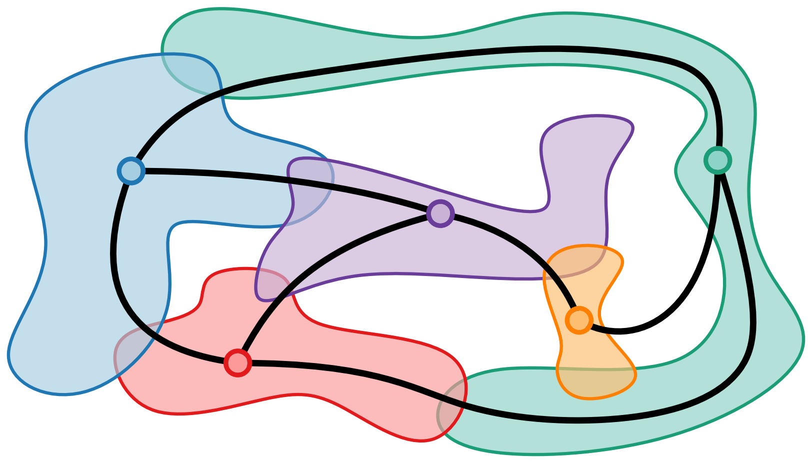

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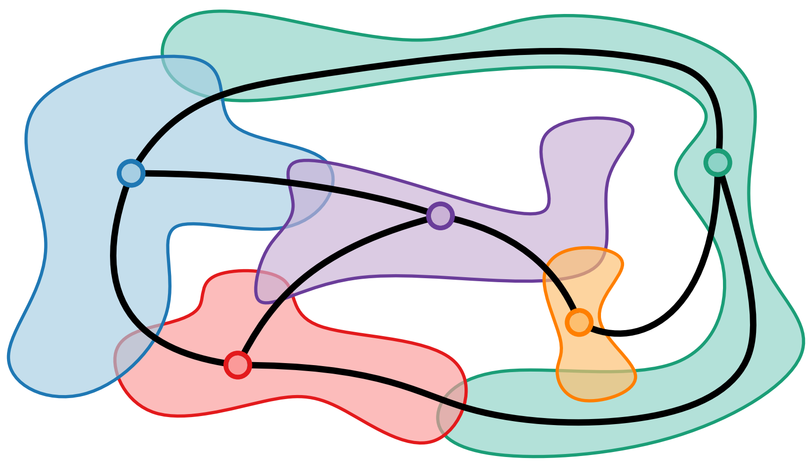
For a collection $\mathcal{S}$ of sets,
the **intersection graph** $G(\mathcal{S})$ of $\mathcal{S}$
has vertex set $\mathcal{S}$ and edge set
$\{\{S, S'\} \colon S, S' \in \mathcal{S}, S \neq S',$ and $S \cap S' \neq \emptyset\}$.



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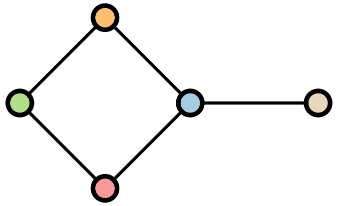

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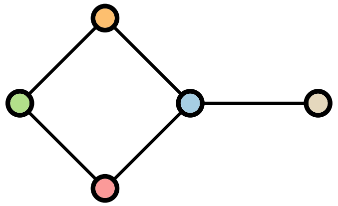

Represent each vertex v by a geometric object $S(v)$

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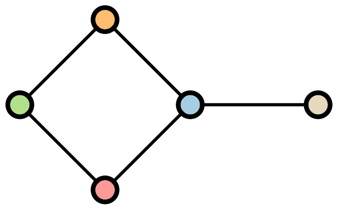

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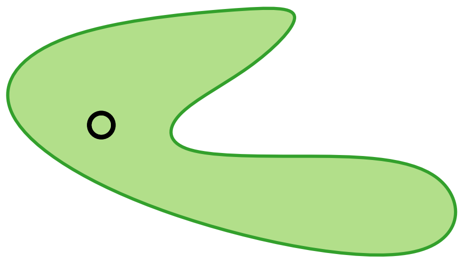

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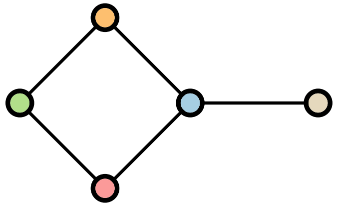

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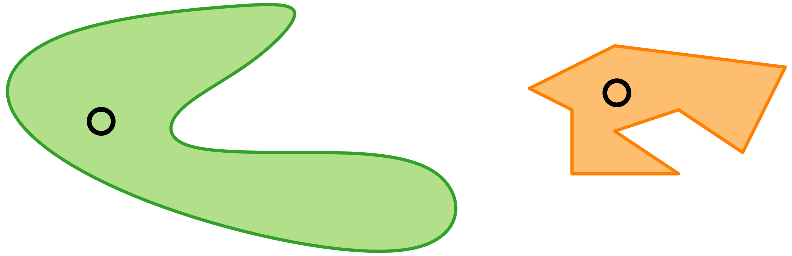

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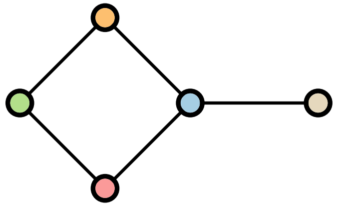

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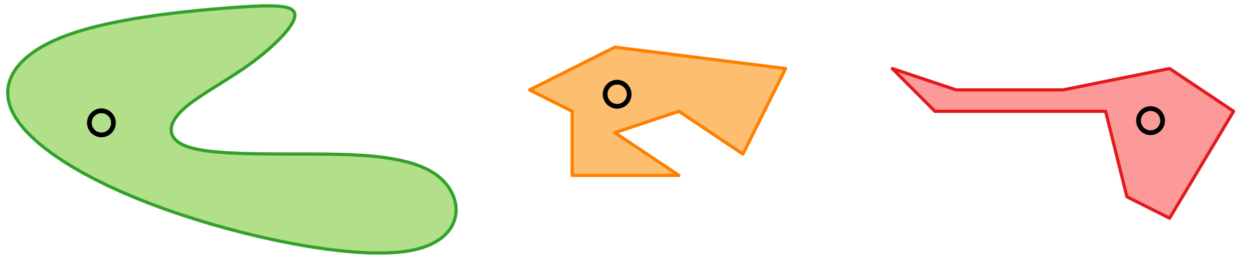

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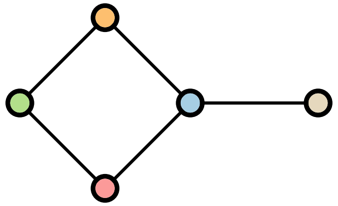

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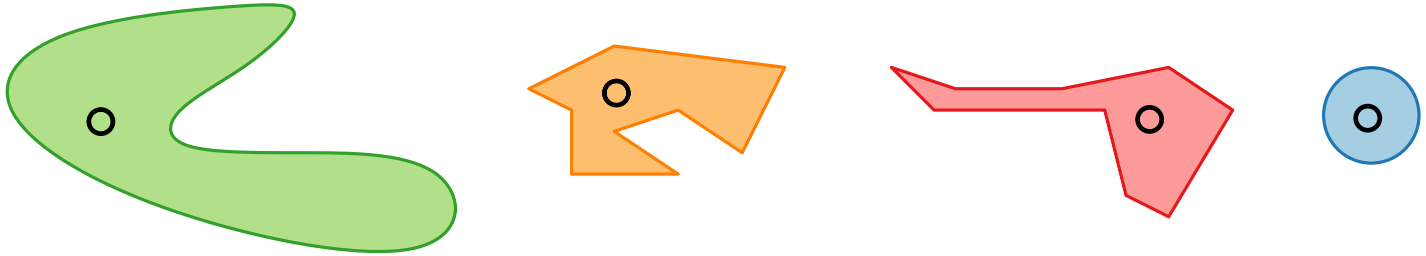

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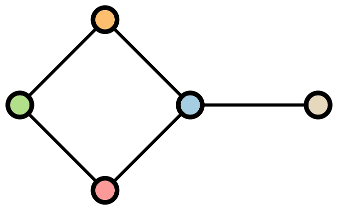

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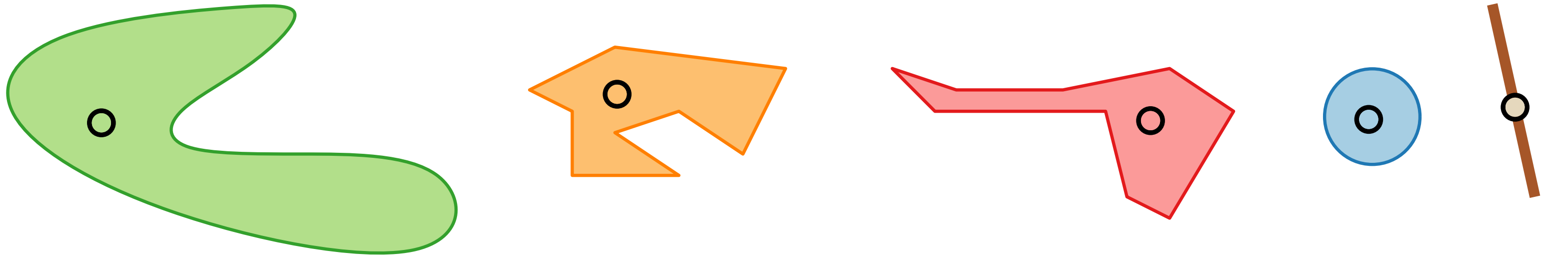

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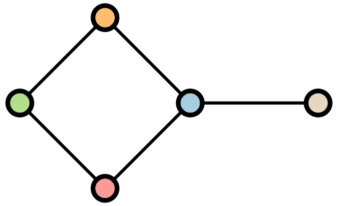

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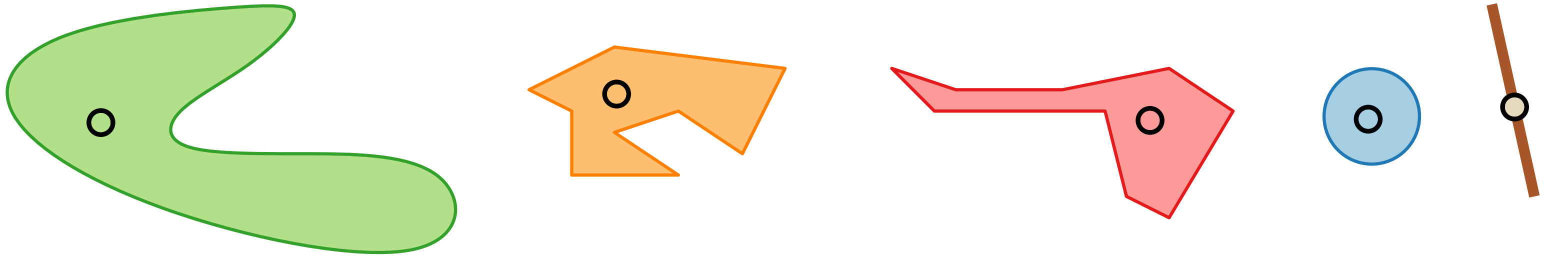

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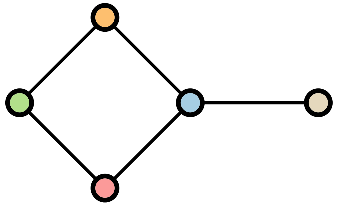

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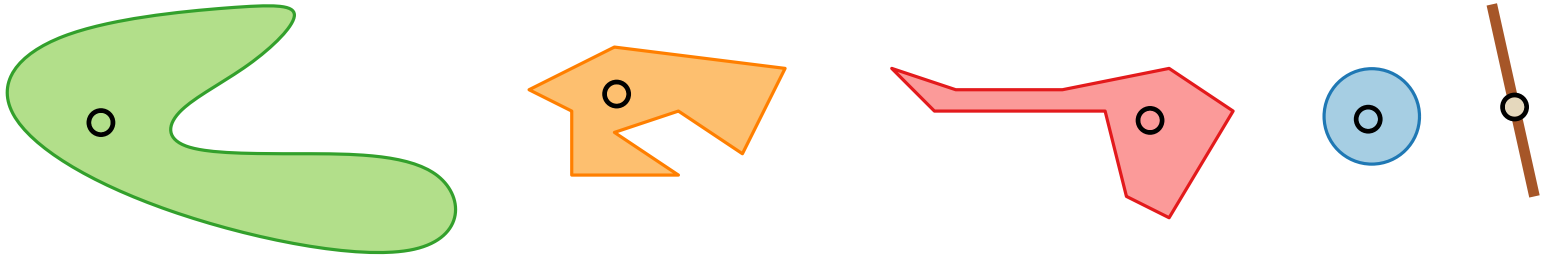

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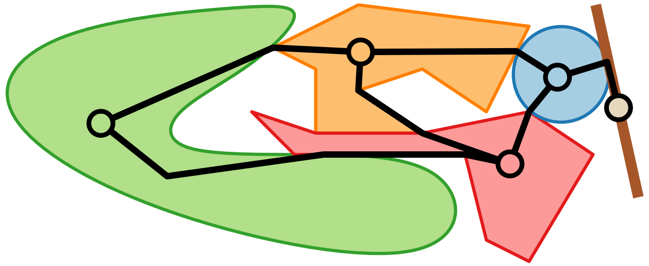

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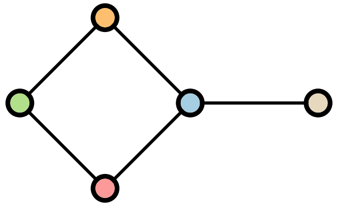

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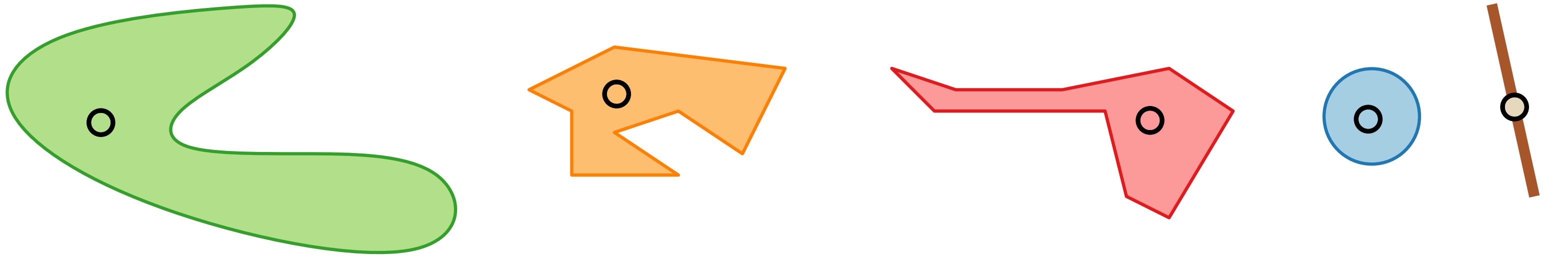

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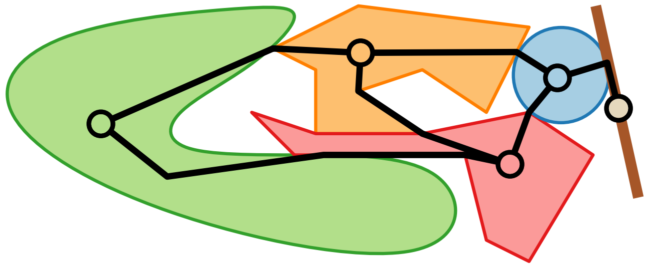

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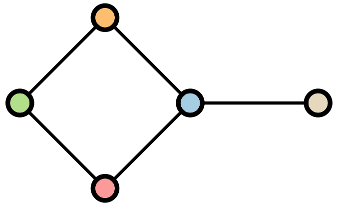

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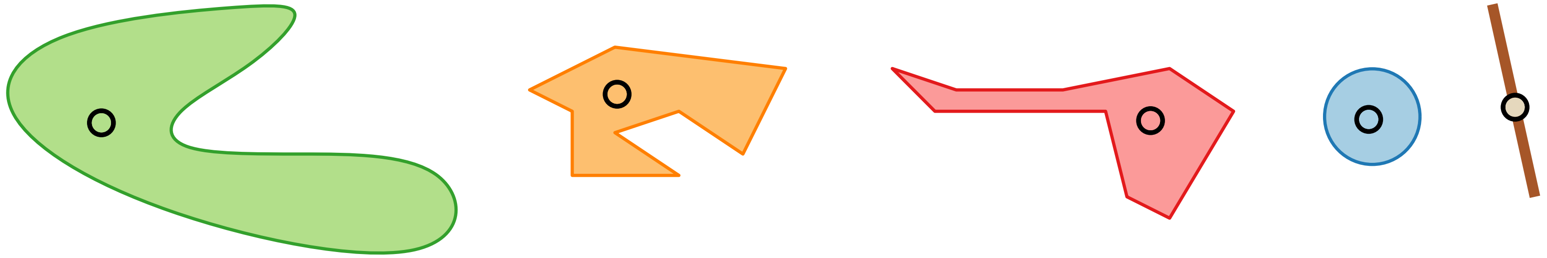

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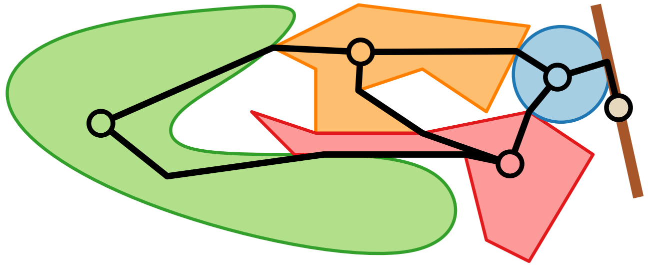

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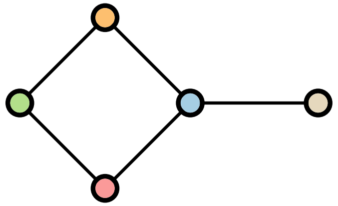
Let G be a graph.  Let $\mathcal{S}$ be a family of geometric objects (e.g., disks).

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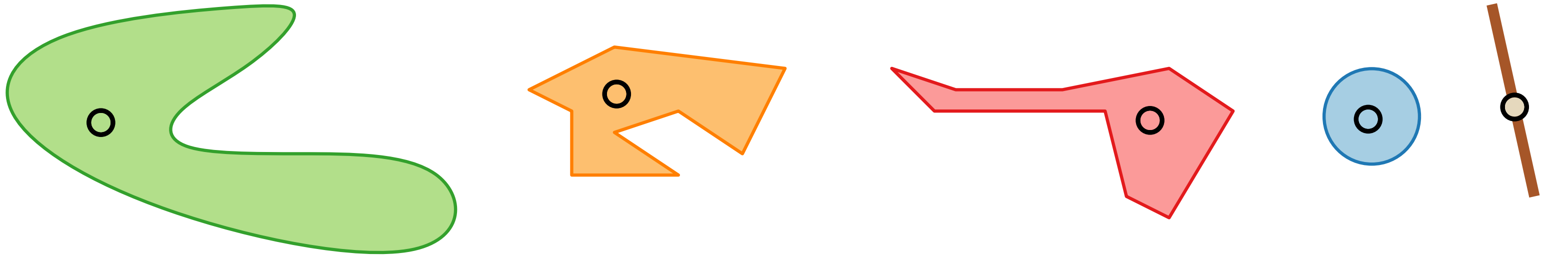

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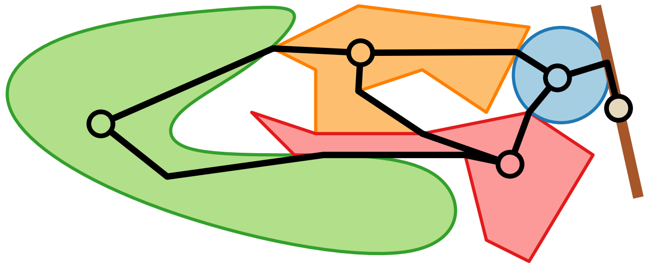

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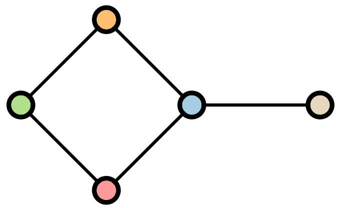

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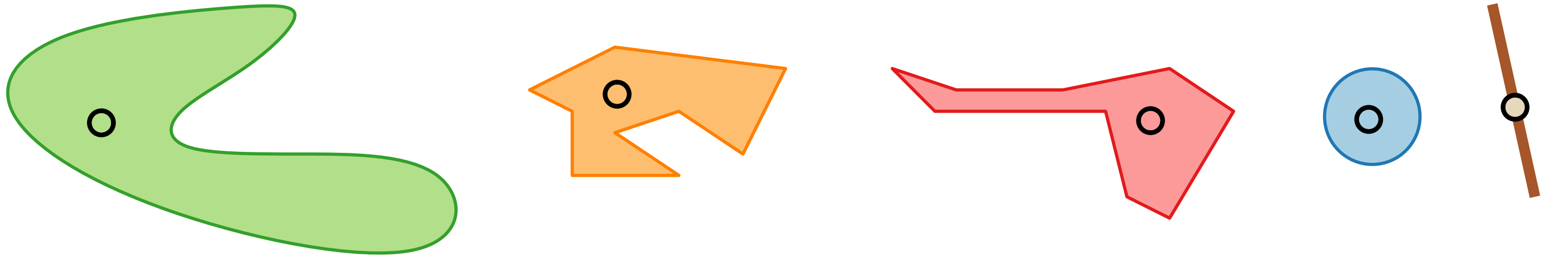

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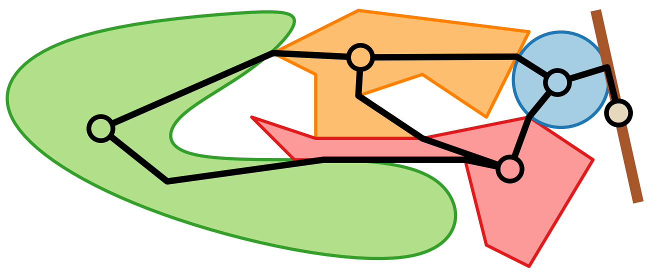

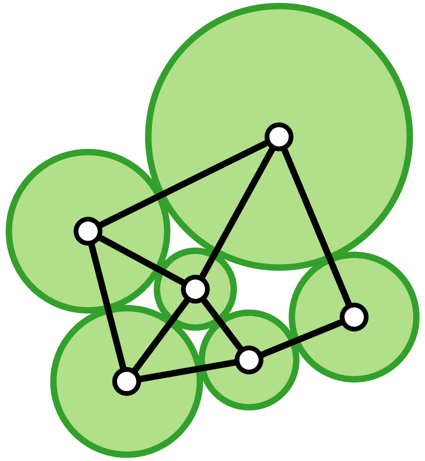

disks

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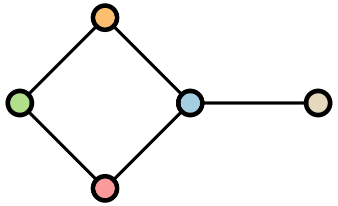
Let G be a graph.  Let $\mathcal{S}$ be a family of geometric objects (e.g., disks).

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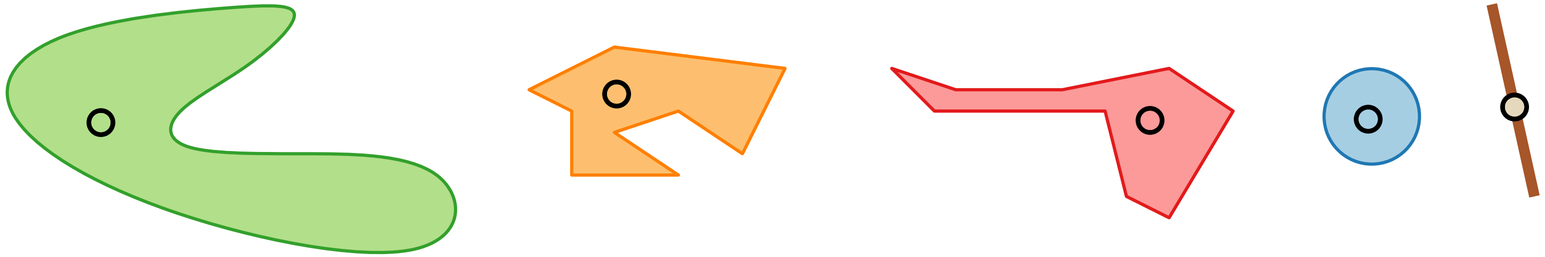

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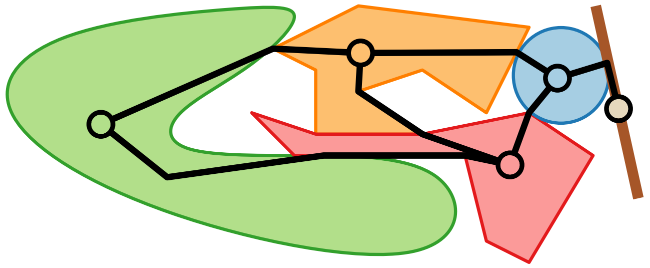

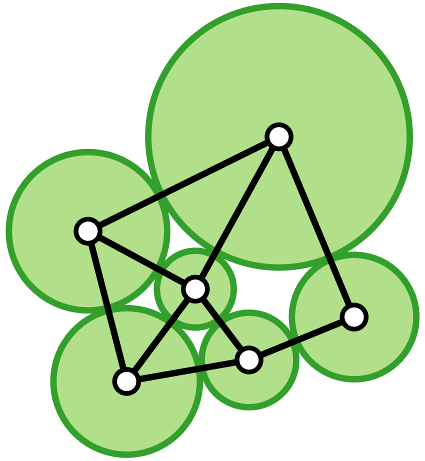

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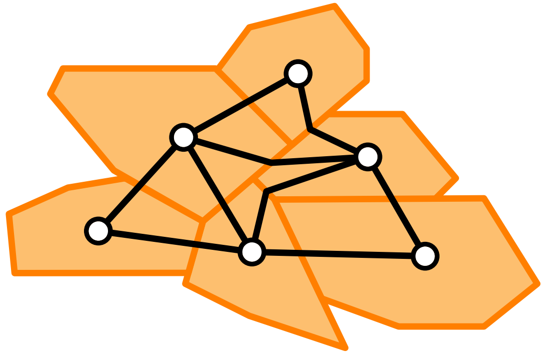

polygons

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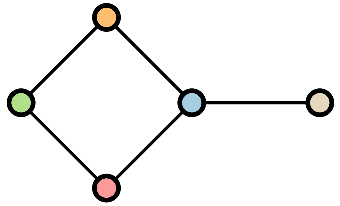

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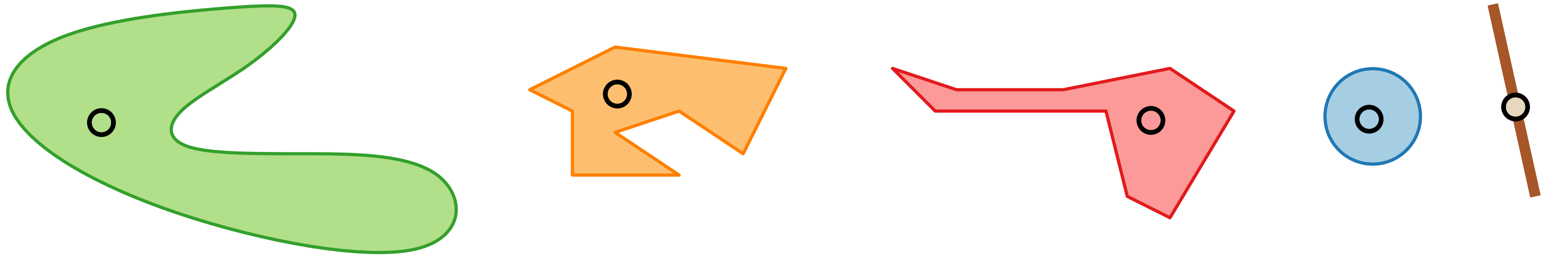
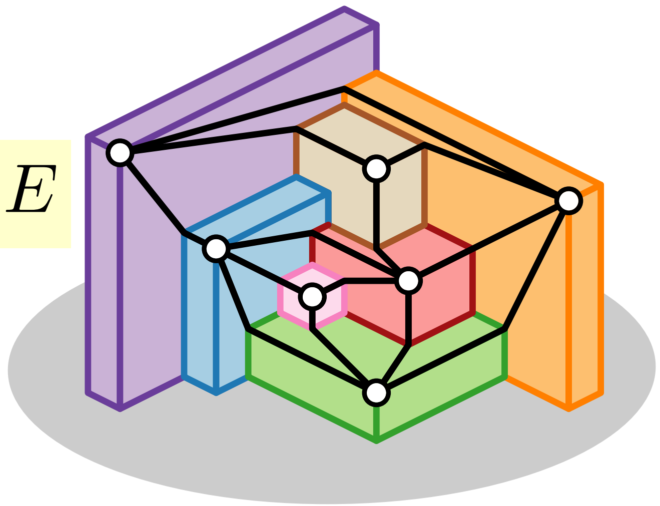


rectangular cuboids



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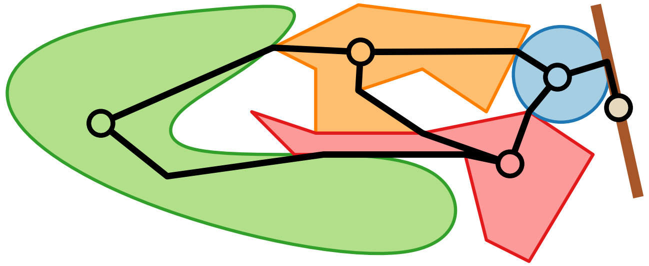

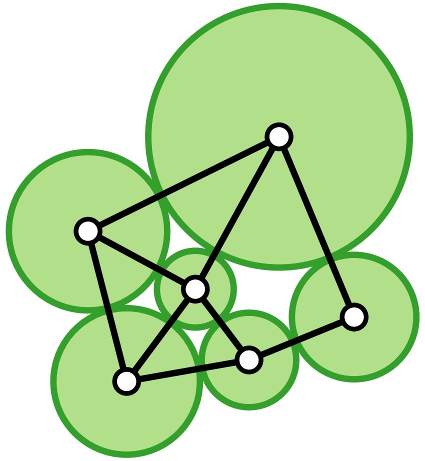

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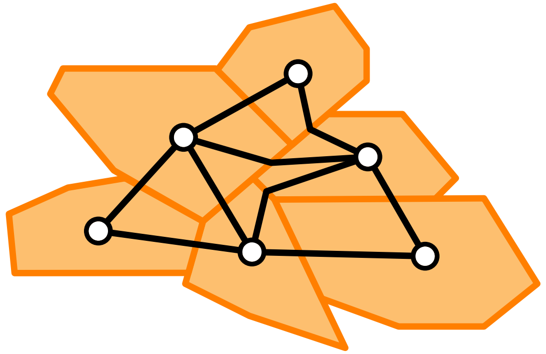

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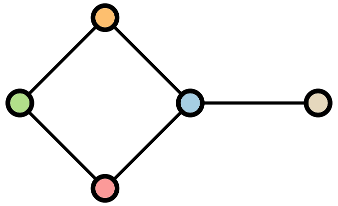

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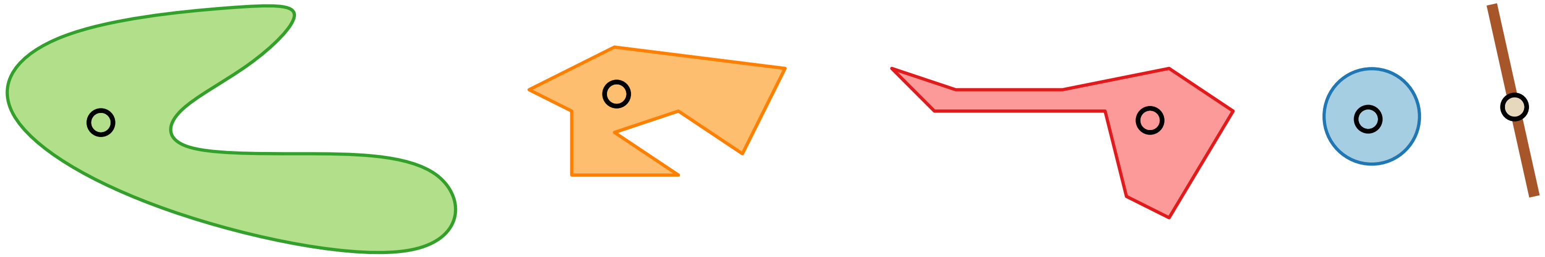
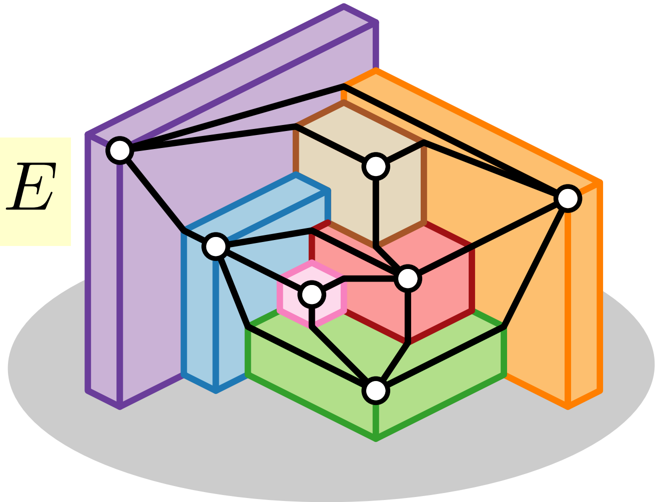


rectangular cuboids



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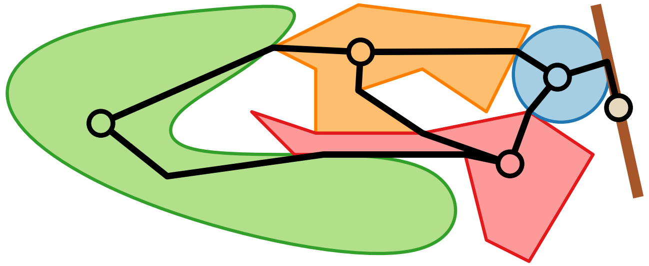
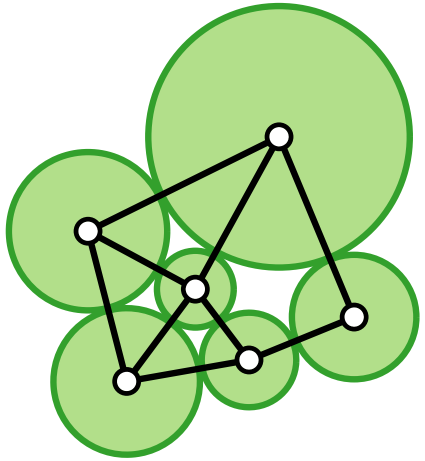


G is planar



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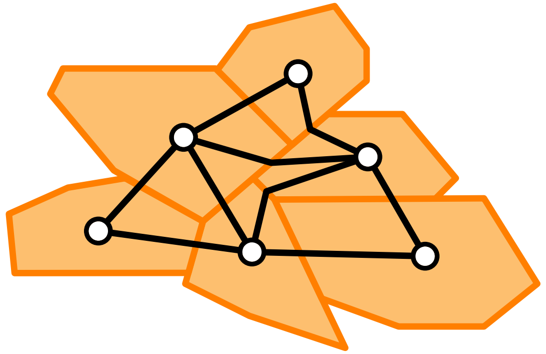

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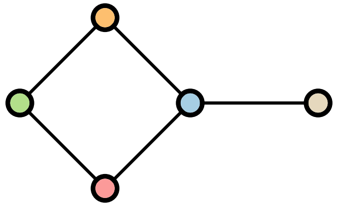

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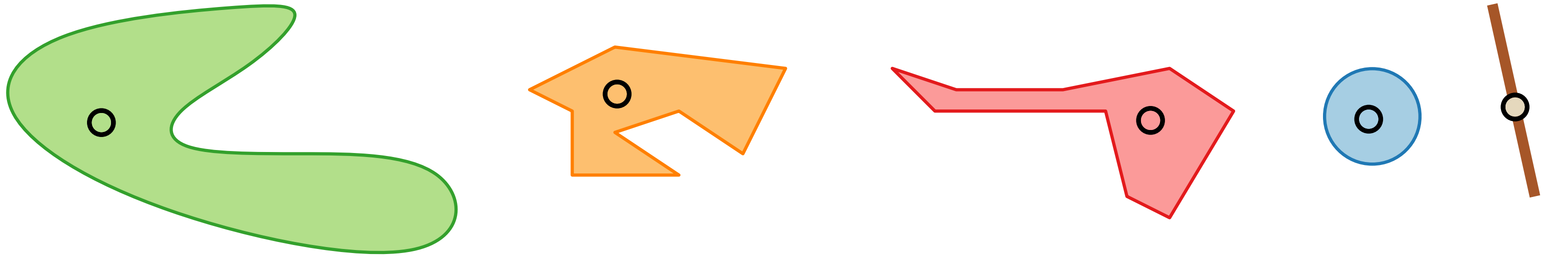
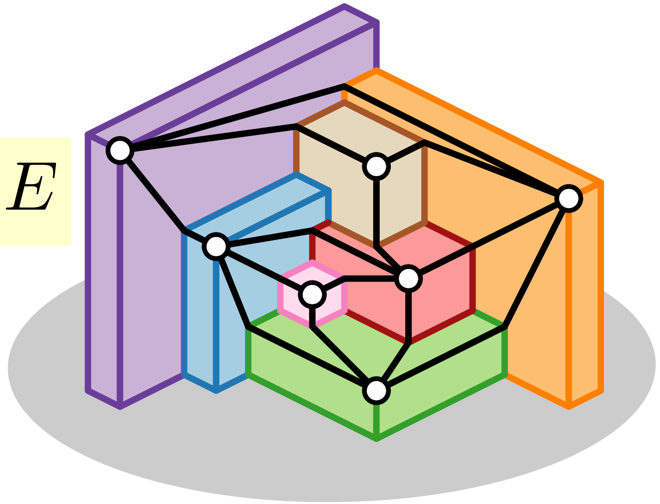


rectangular cuboids



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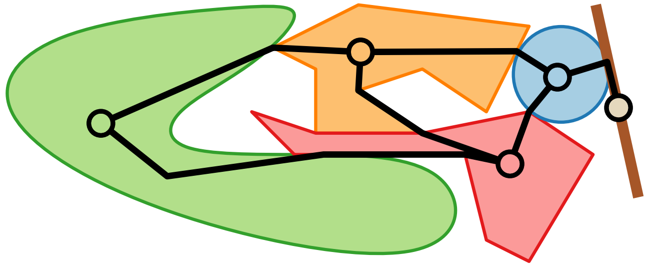

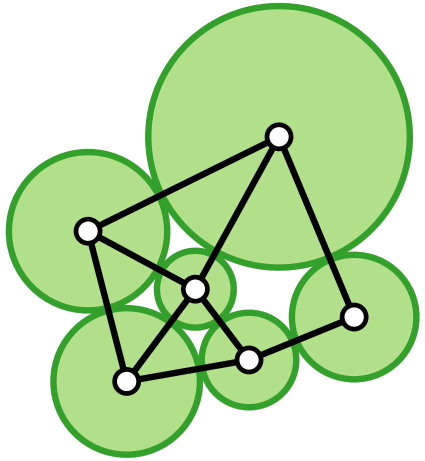

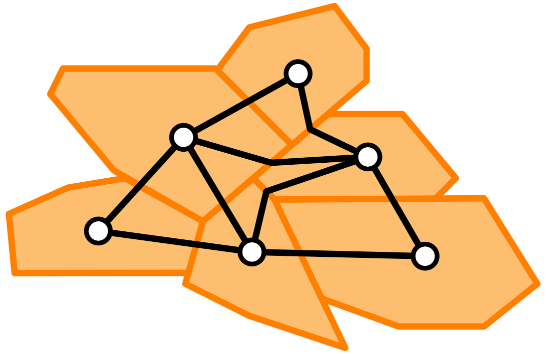

G is planar → disks [Koebe 1936]

polygons

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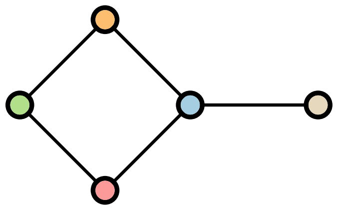
Let G be a graph.  Let $\mathcal{S}$ be a family of geometric objects (e.g., disks).

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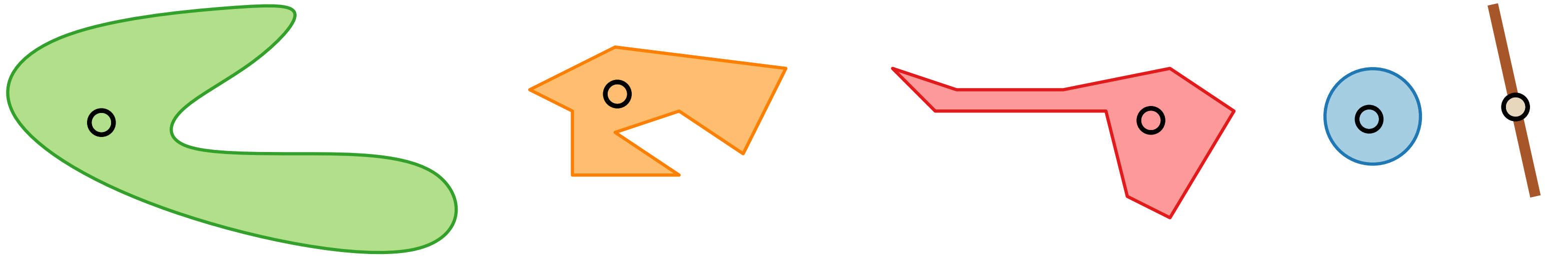
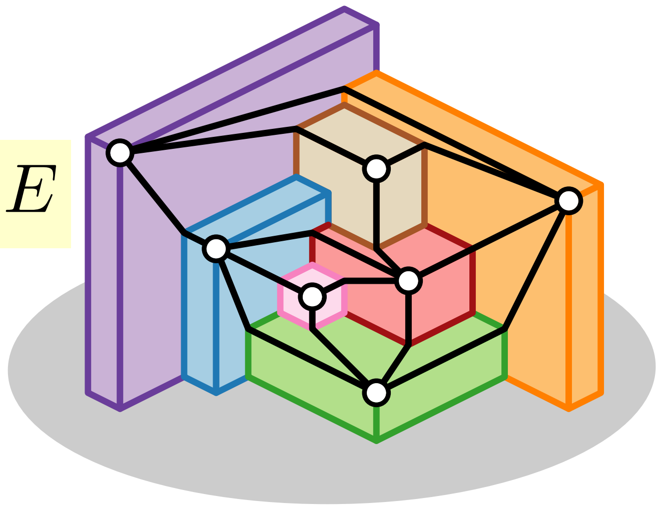
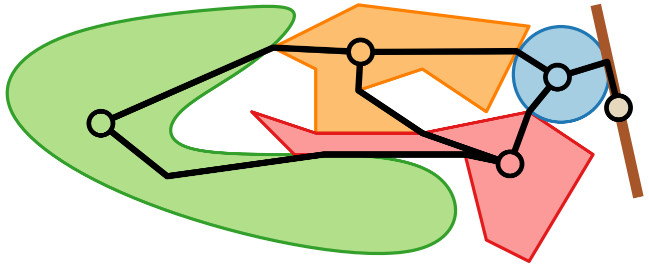
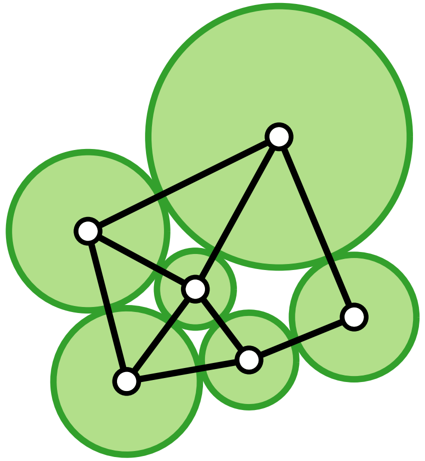


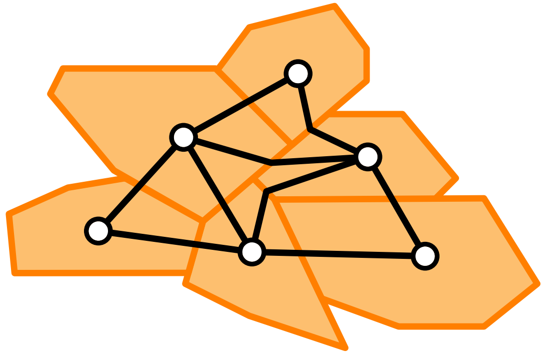
rectangular cuboids



In an **$\mathcal{S}$-contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$





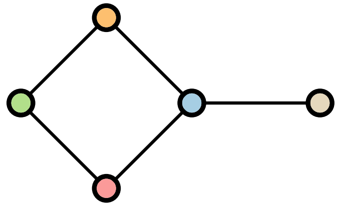


G is planar → disks → polygons

[Koebe 1936]

Contact Representation of Graphs

Let G be a graph.



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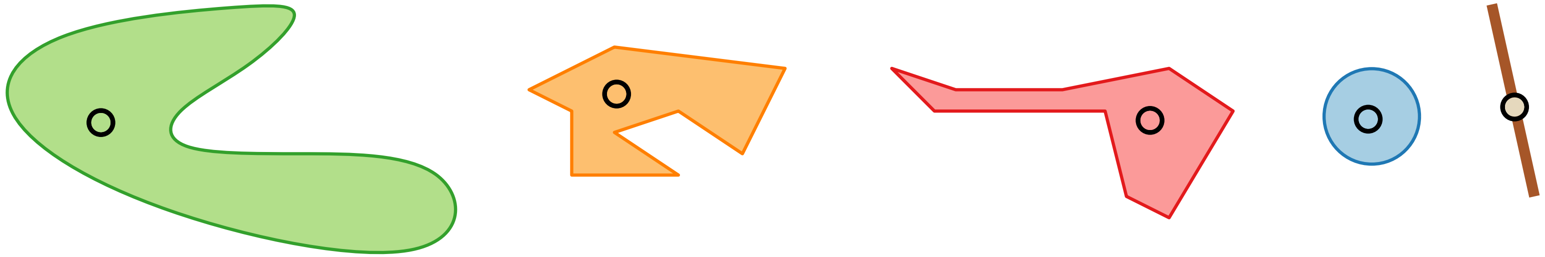

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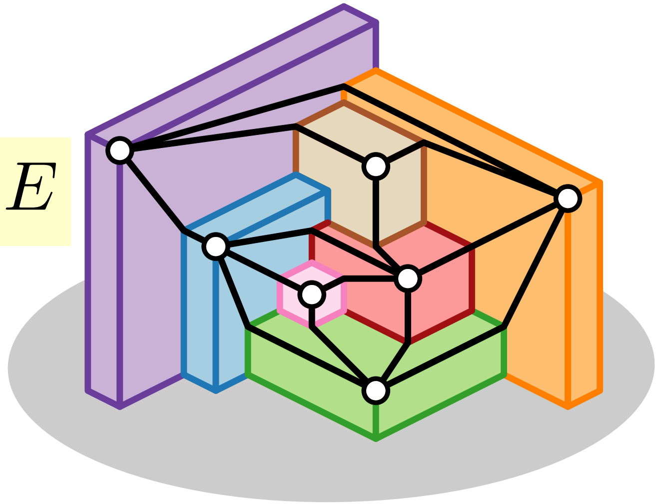

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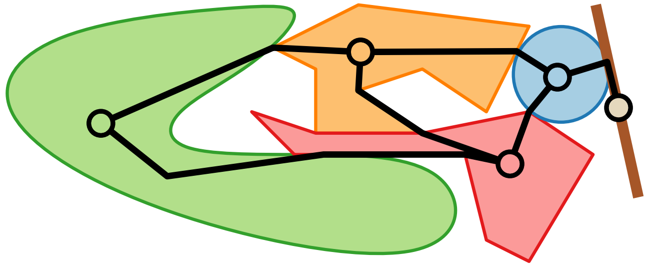

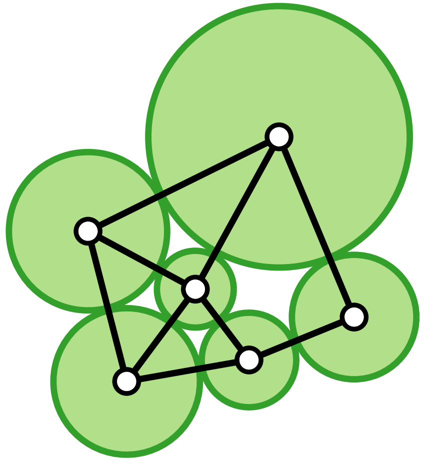

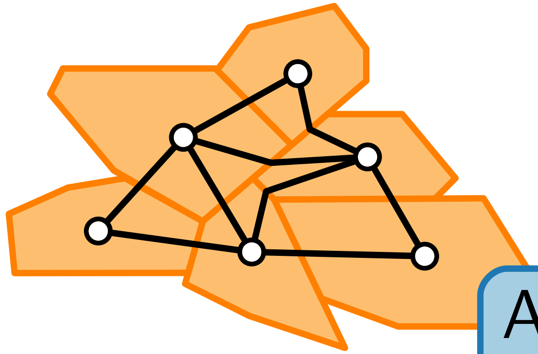

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A contact representation is an intersection representation with interior-disjoint sets.

Contact Representation of Planar Graphs

Is the intersection graph of a contact representation
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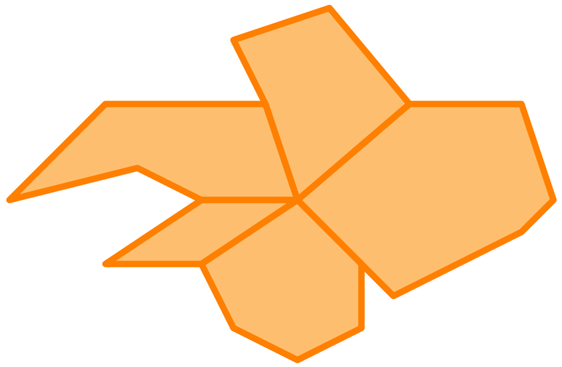

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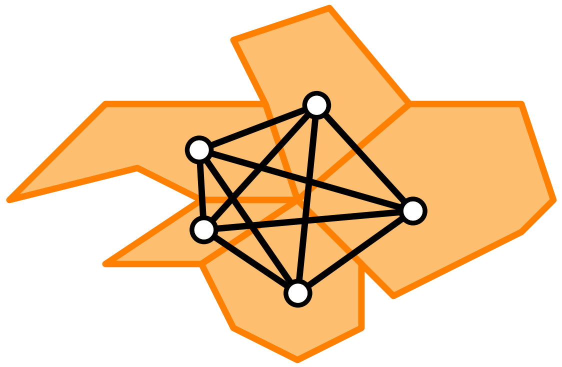

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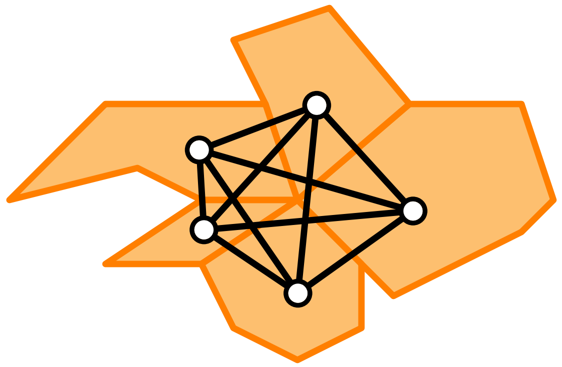

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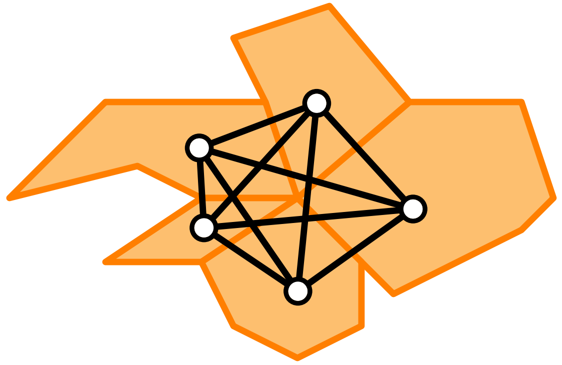

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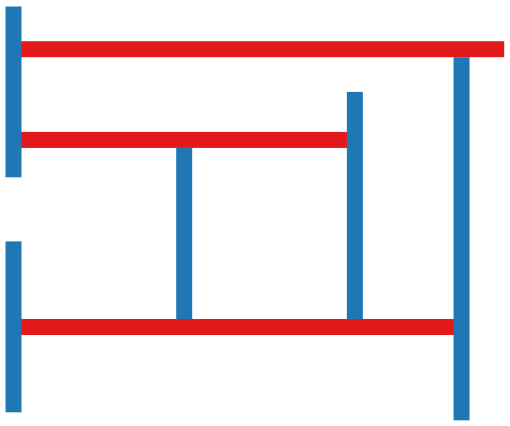

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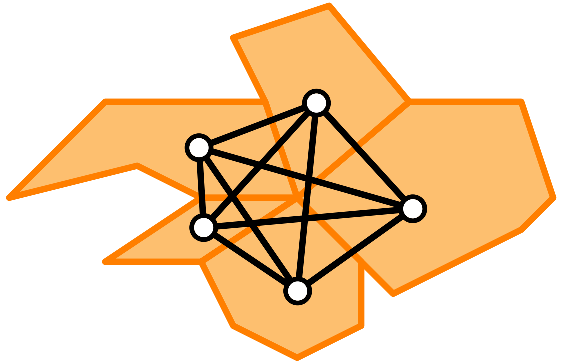

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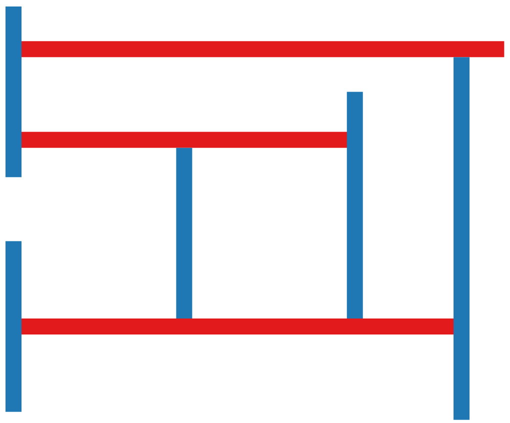

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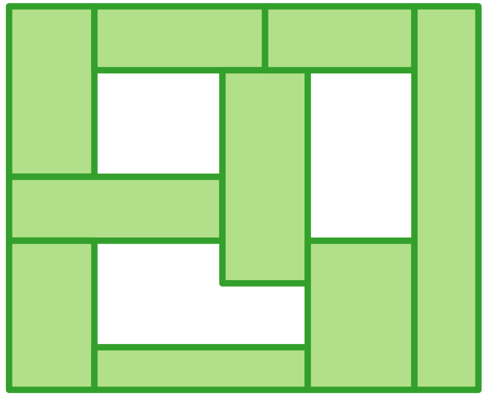

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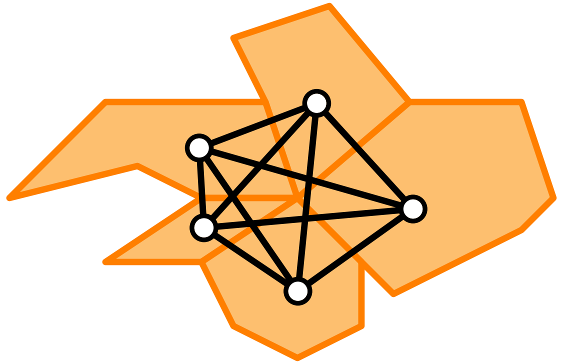

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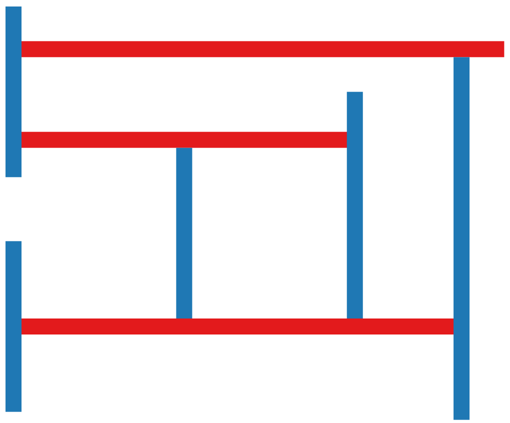

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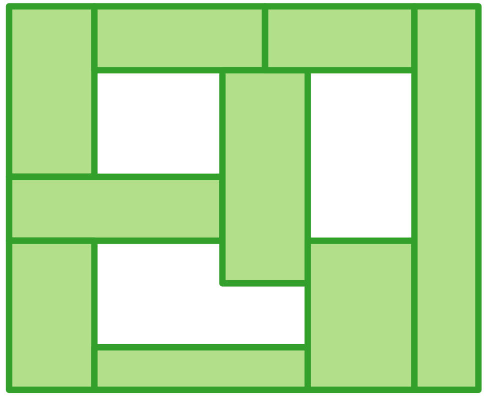

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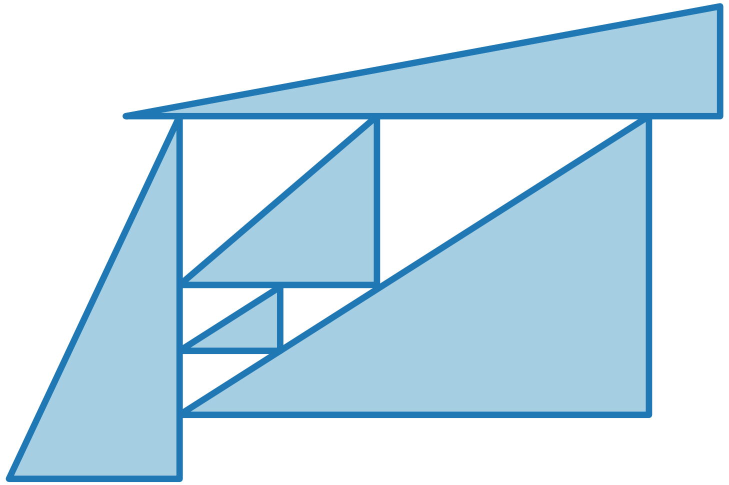

planar triangulations

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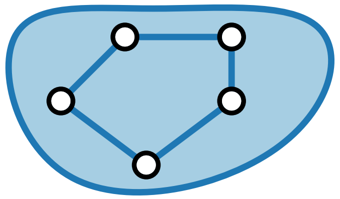

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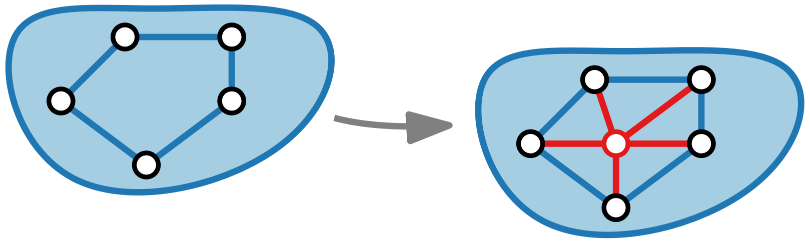

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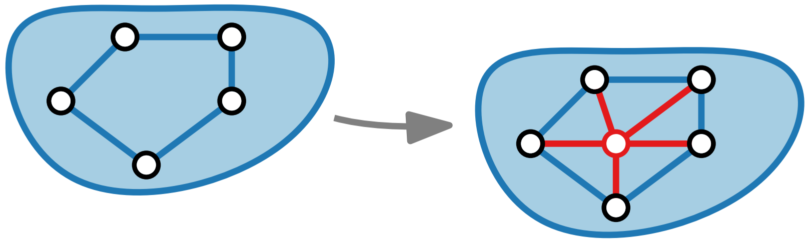

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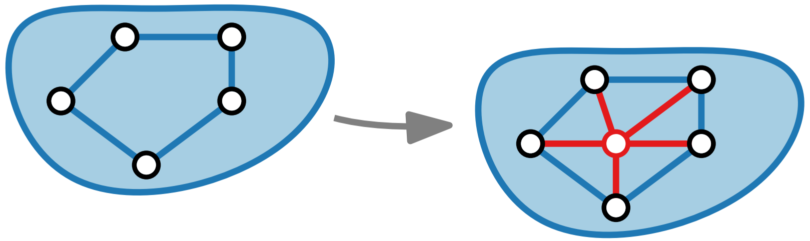

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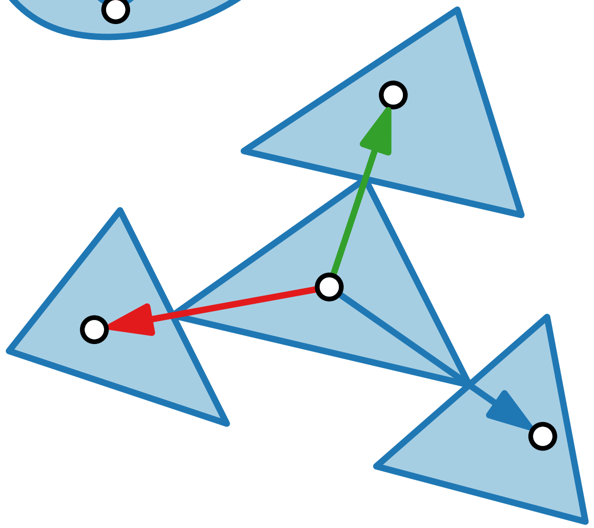

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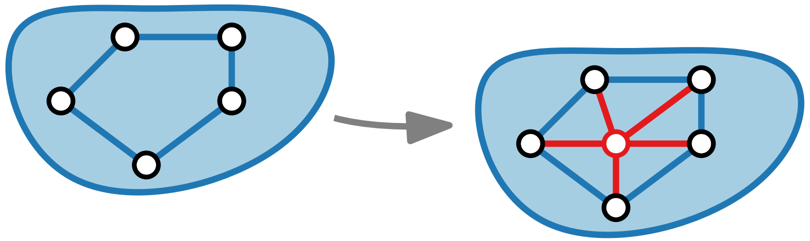

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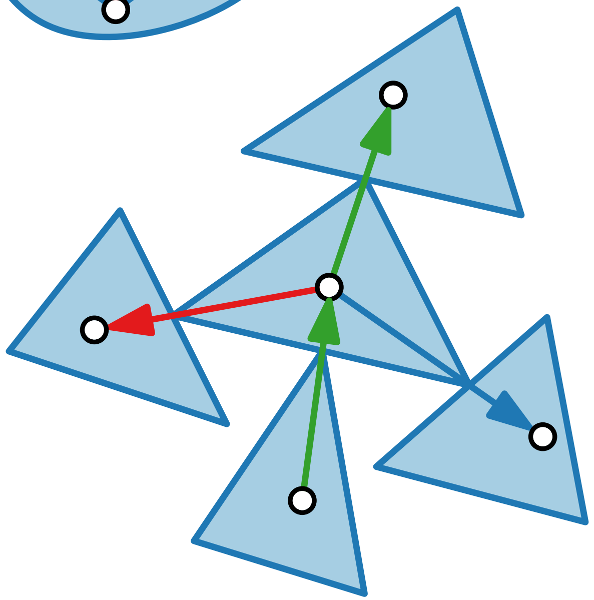

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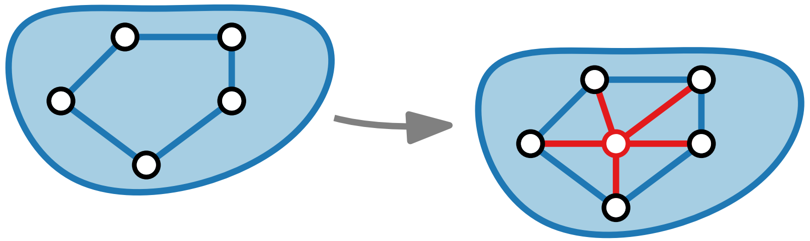

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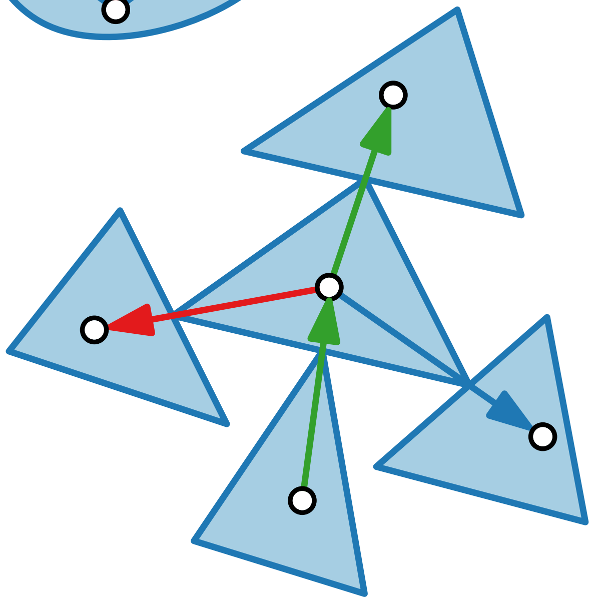

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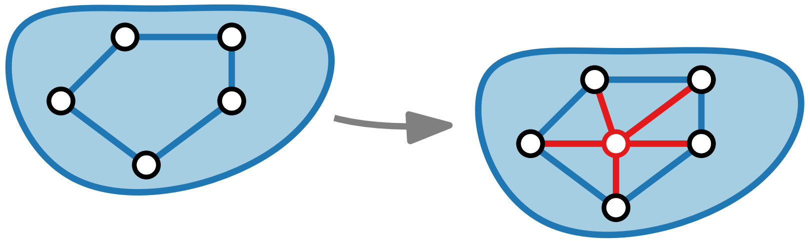
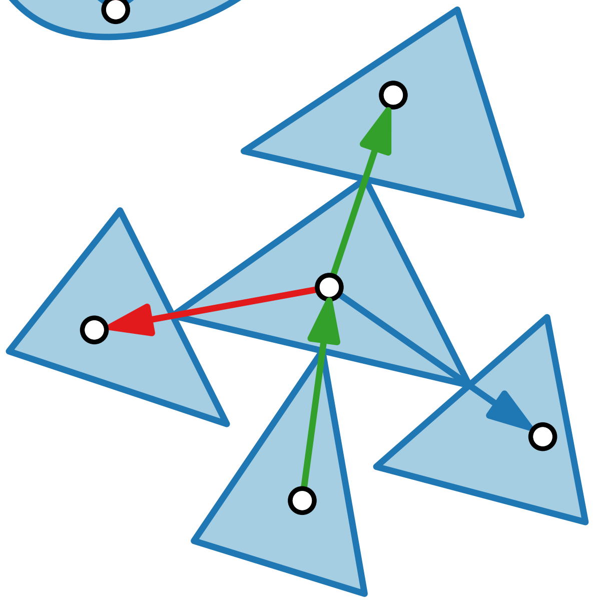
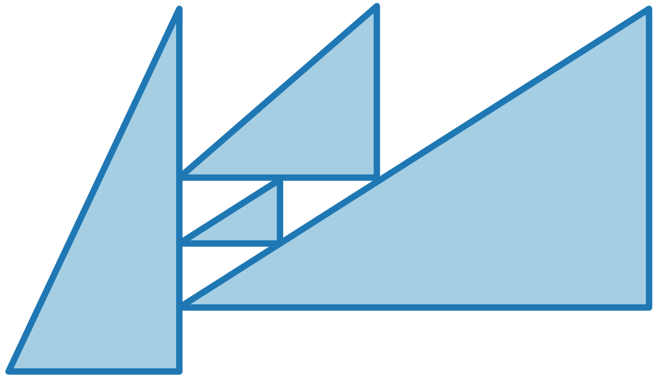


- Describe contact representation combinatorically.
  - Which objects touch each other in which way?
- Compute combinatorical description.
- Show that combinatorical description can be used to construct drawing.



This Lecture

Representation with right-triangles and corner contact:



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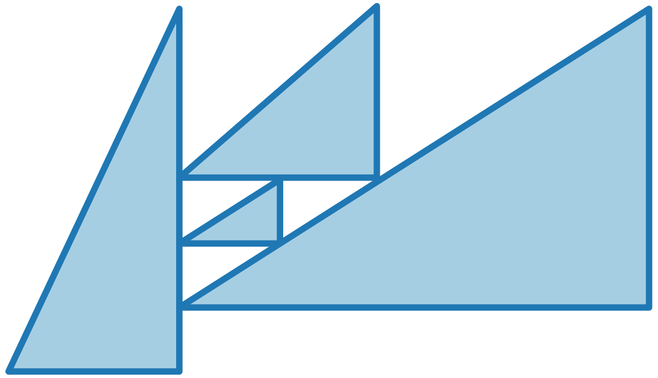

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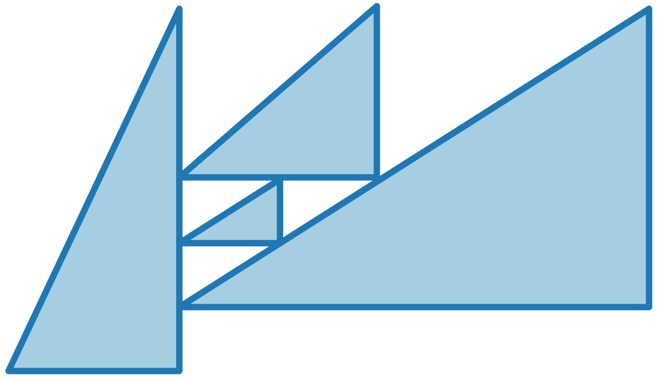

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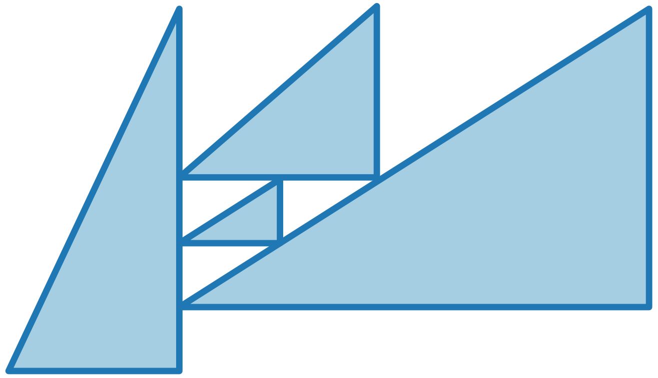

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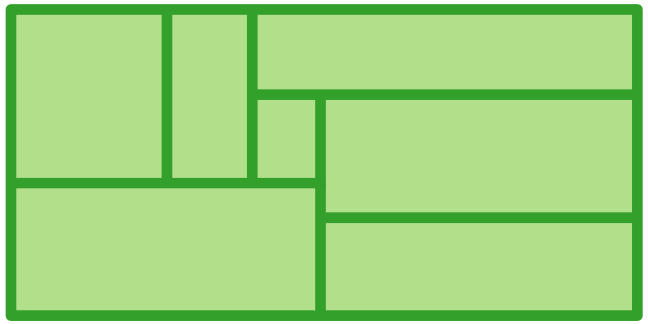

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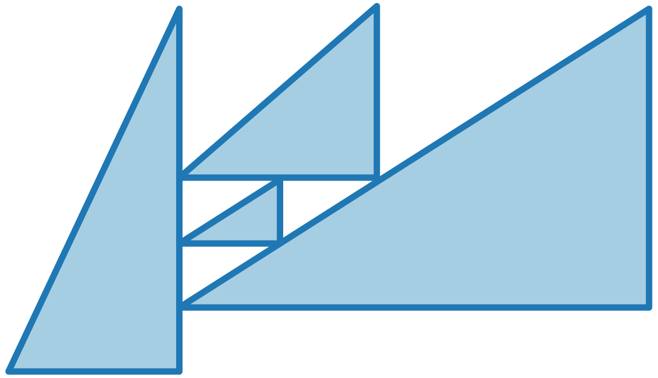

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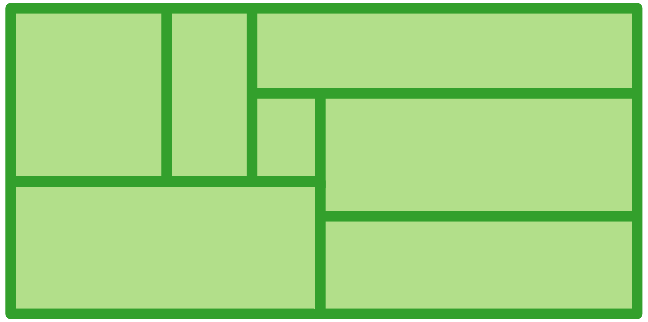

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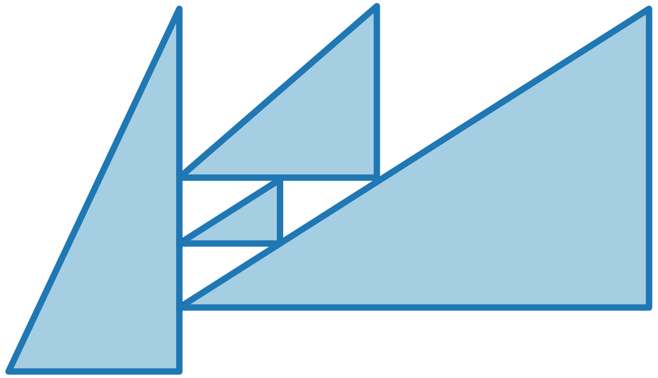

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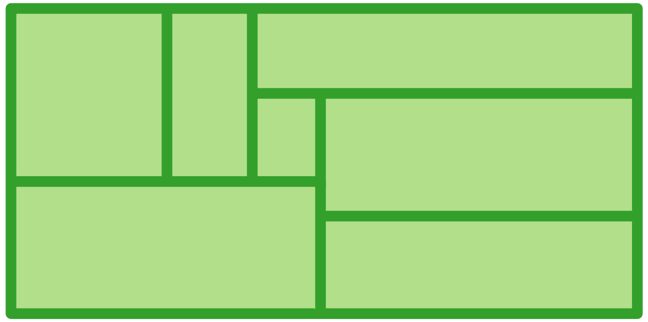
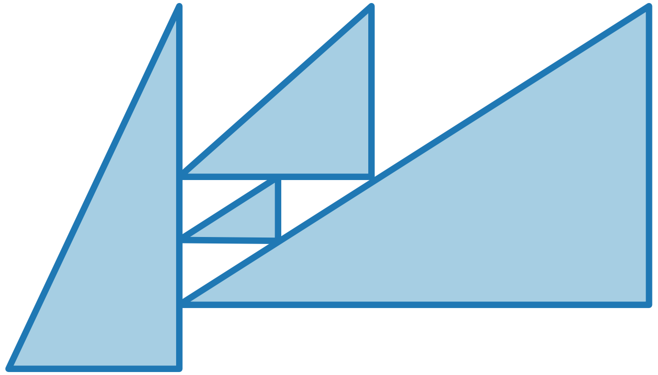


Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

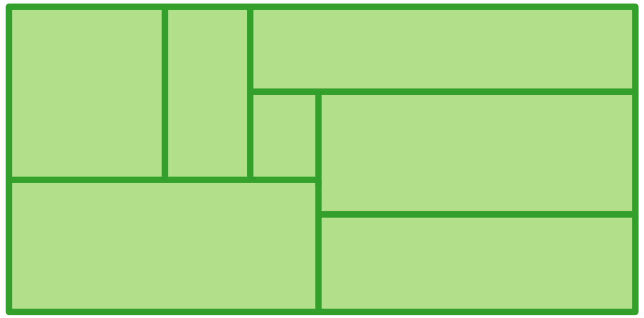
Visualization of Graphs

Lecture 7:
Contact Representations of Planar Graphs:
Triangle Contacts and Rectangular Duals



Part II:
Triangle Contact Representations

Alexander Wolff



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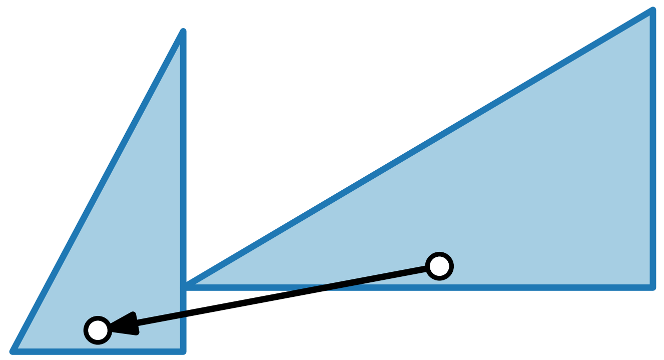

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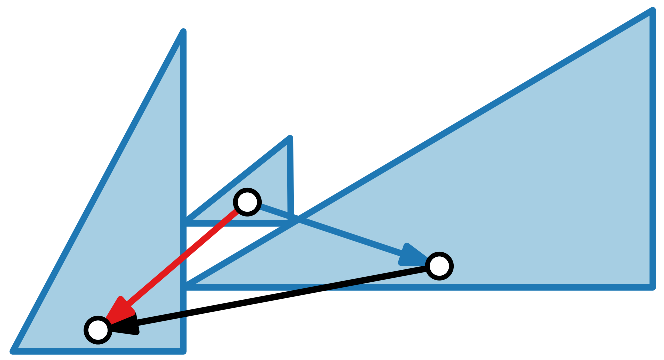

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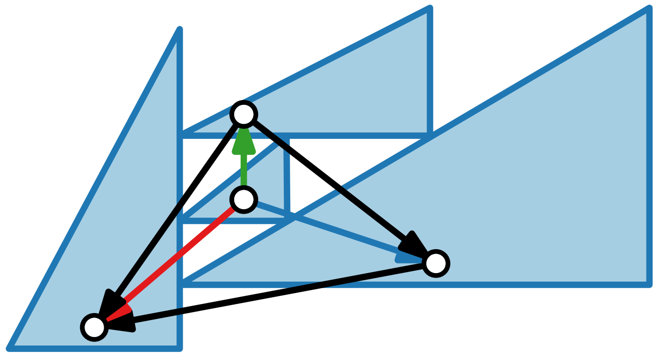

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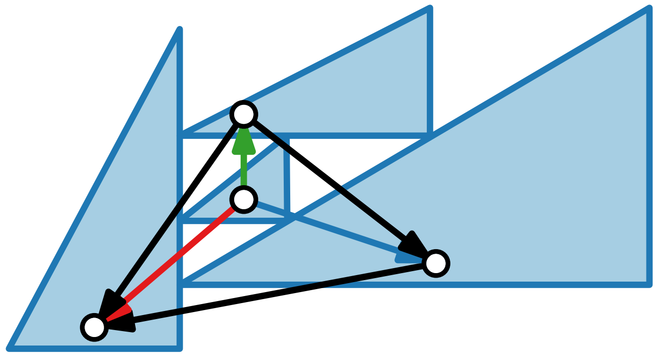

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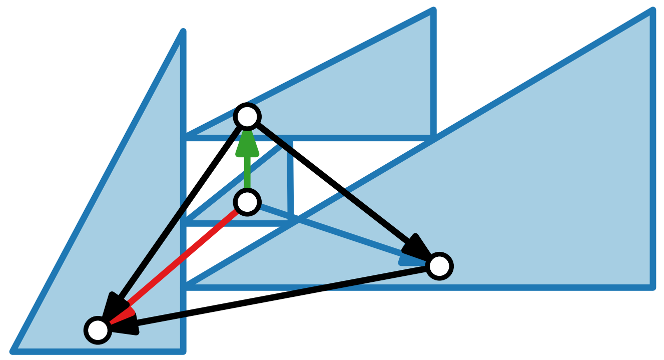

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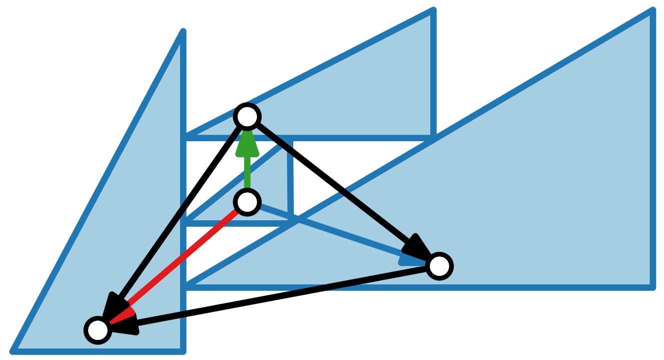

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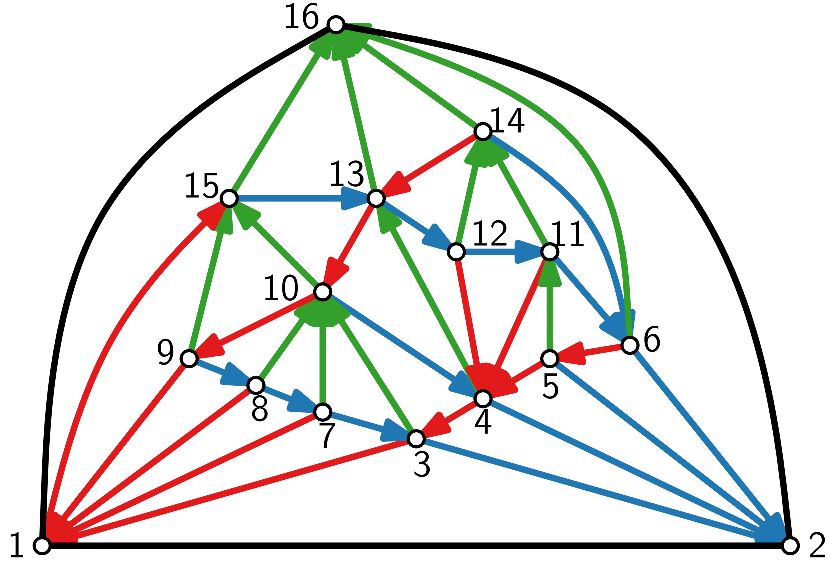
- Outgoing edges in Schnyder forest indicate corner contacts.

Triangle Contact Representation Example



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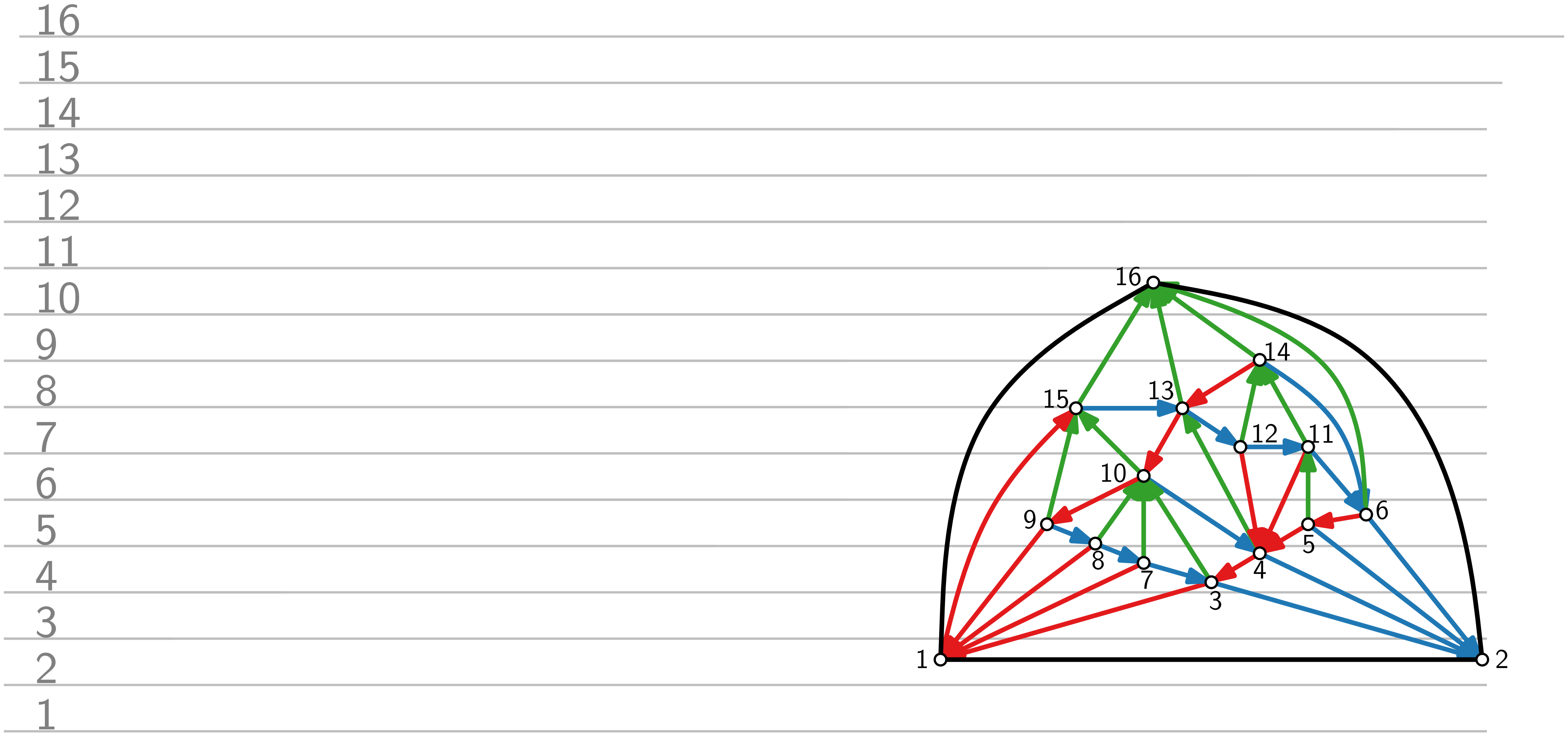

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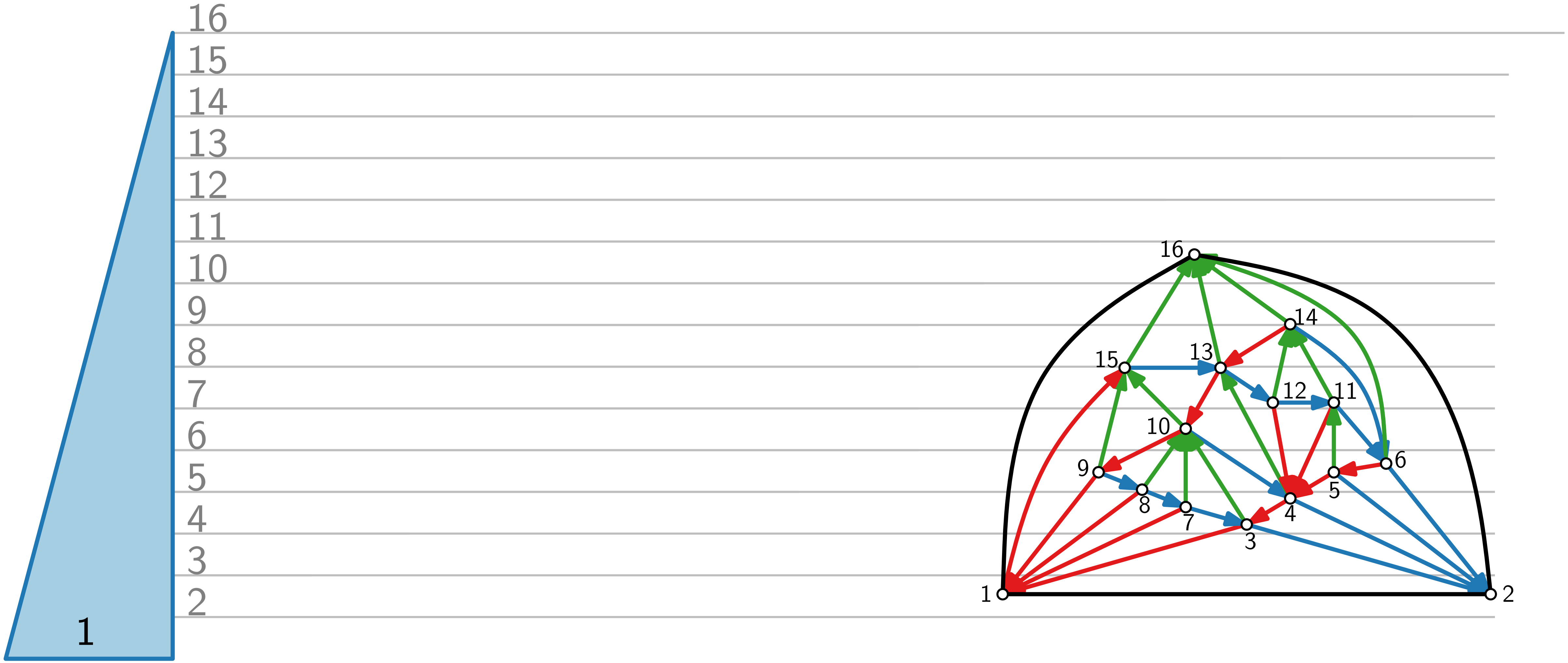

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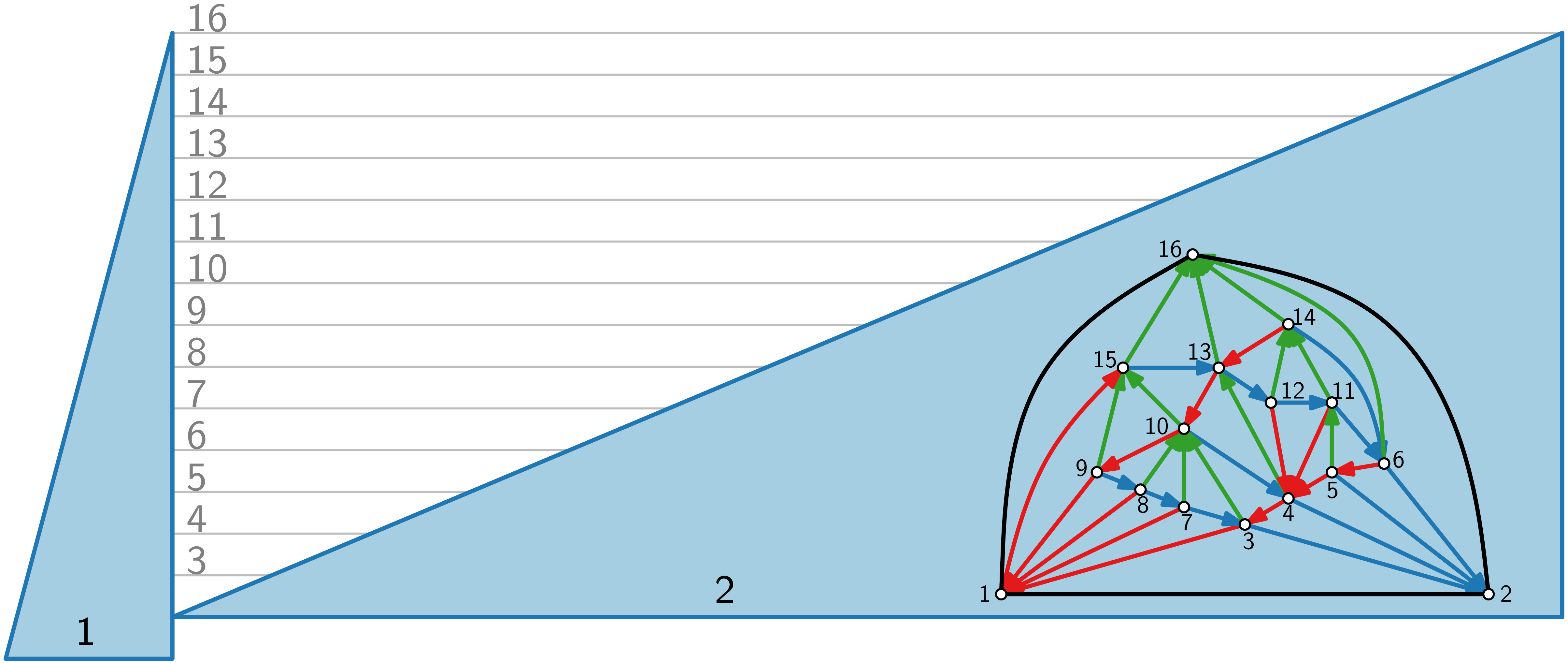

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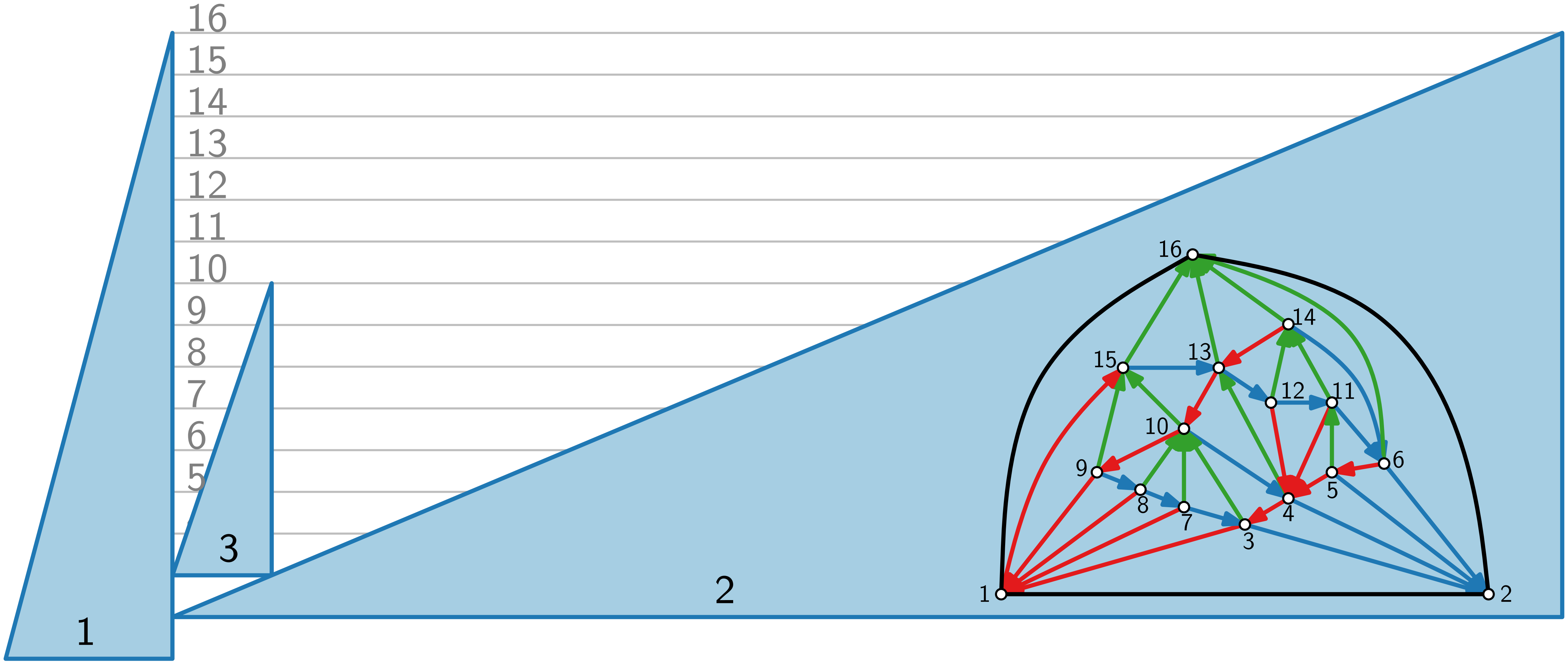

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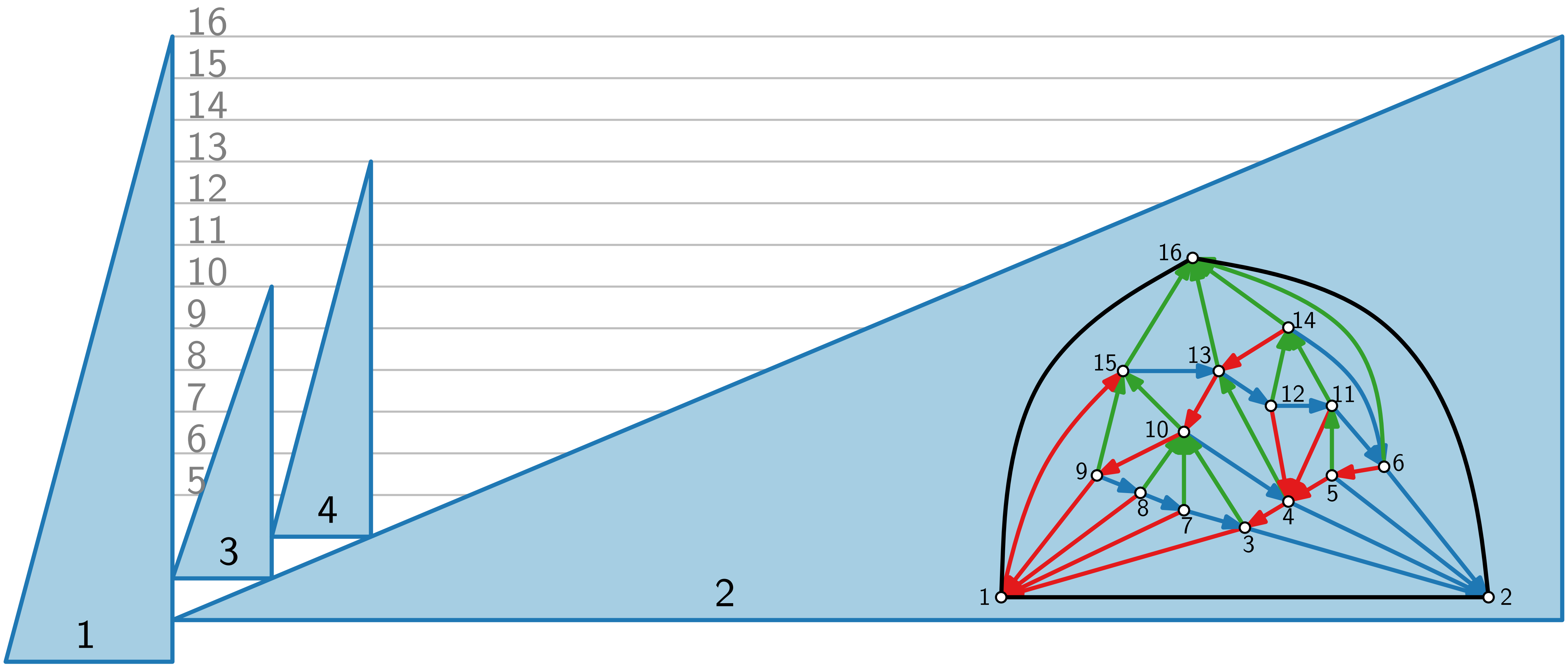

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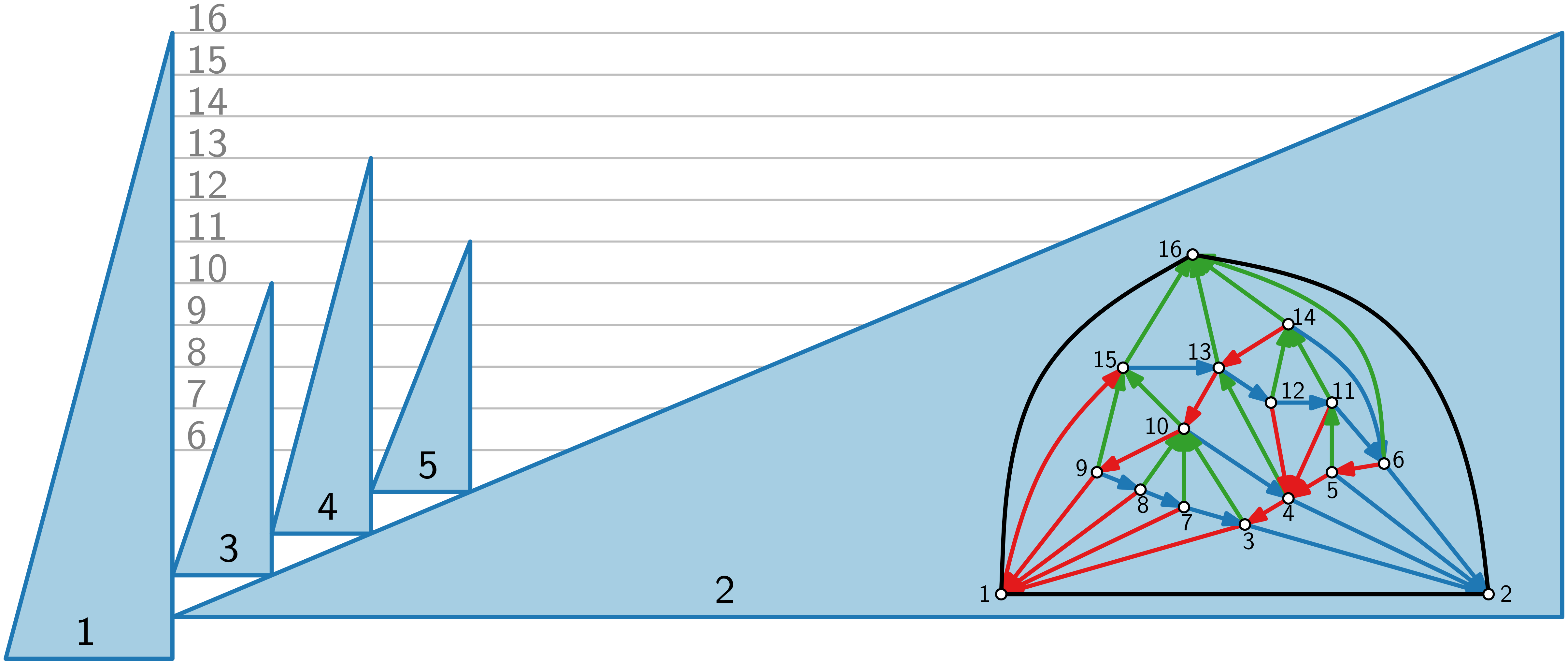
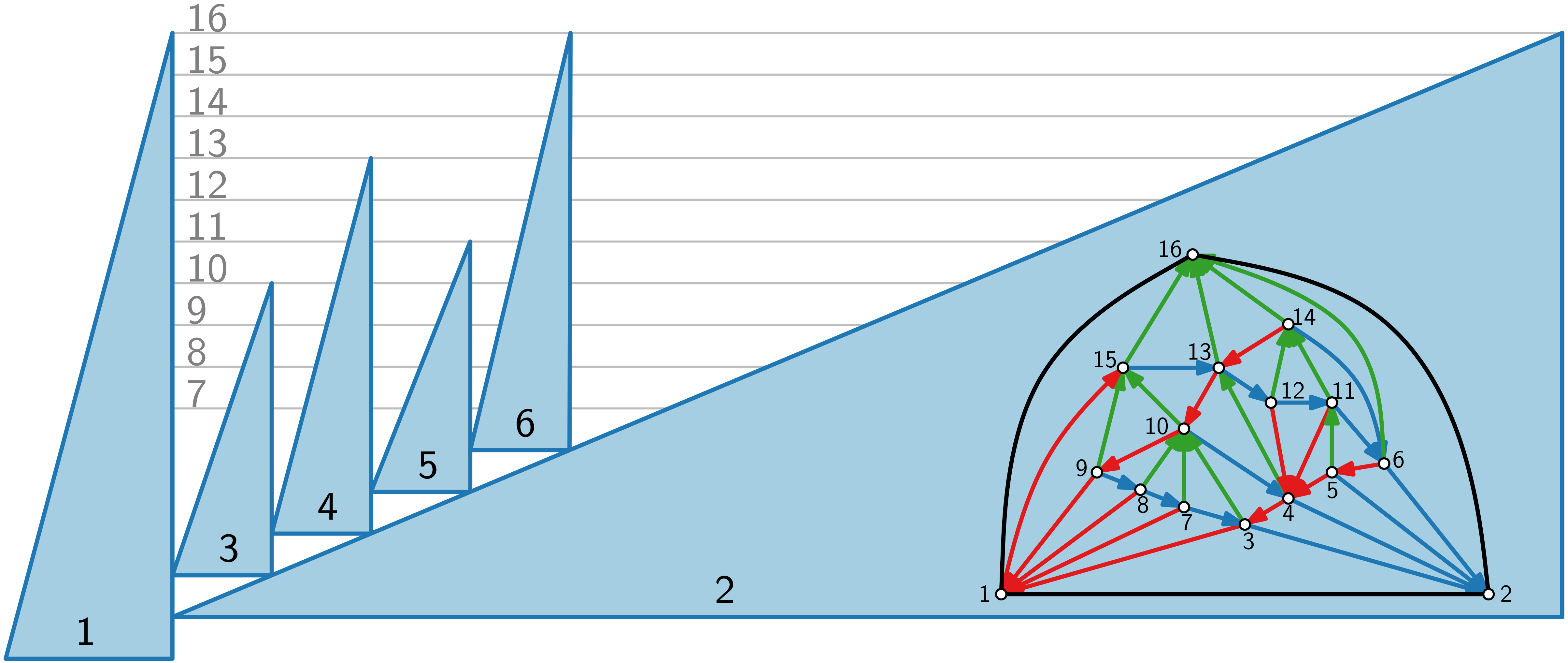


Triangle Contact Representation Example



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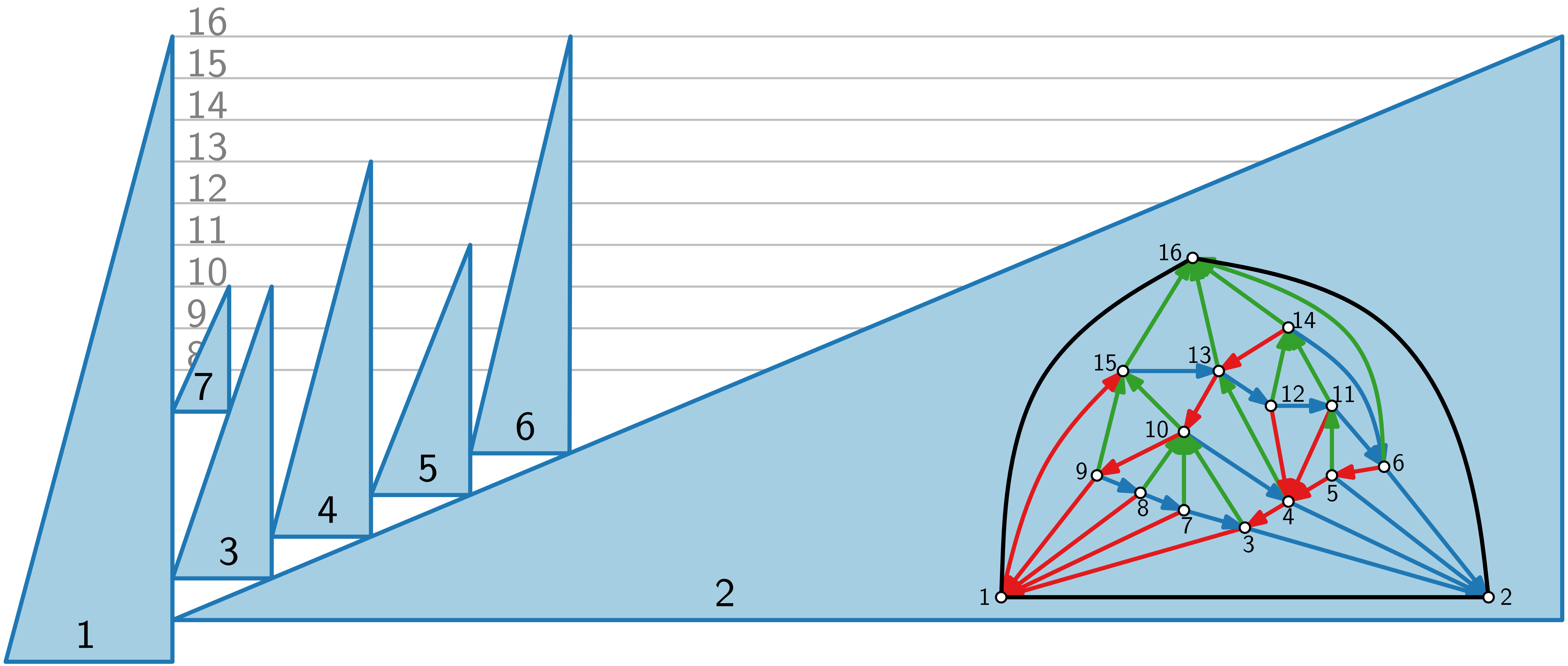

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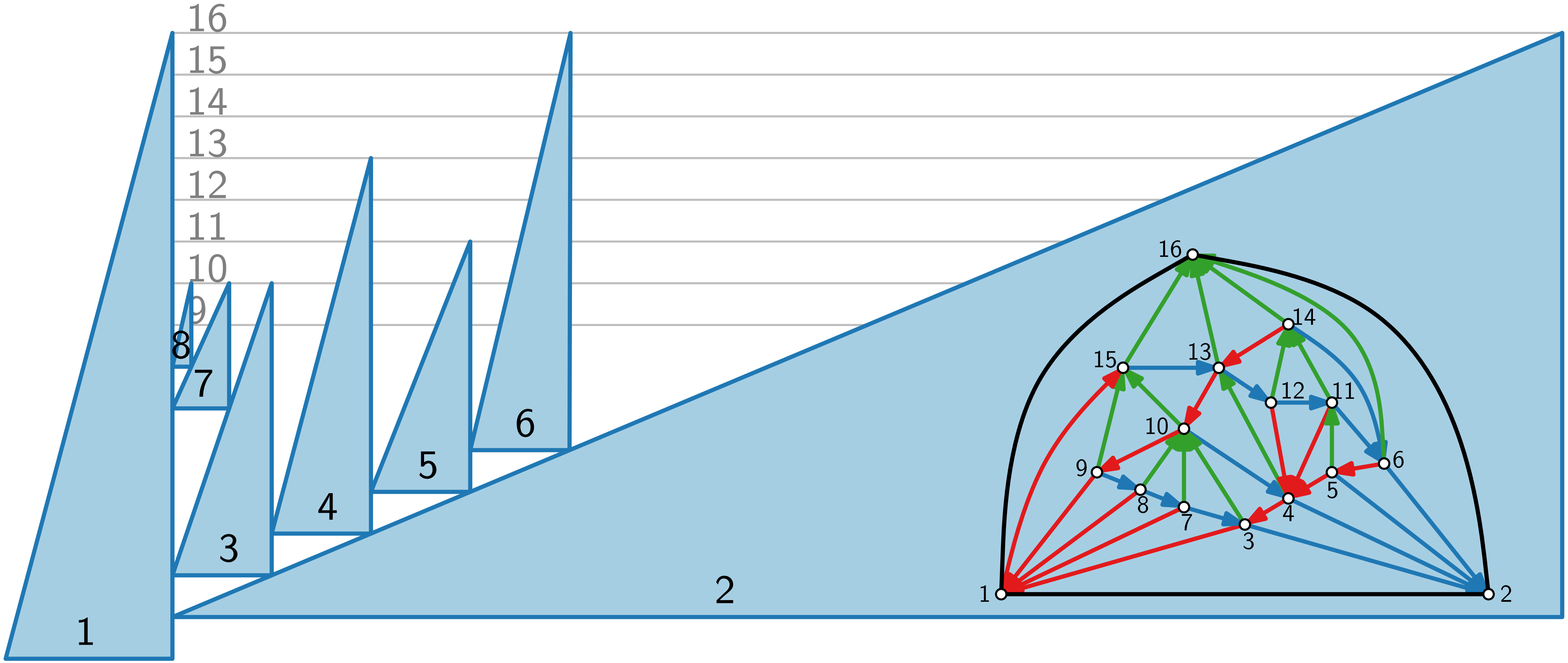

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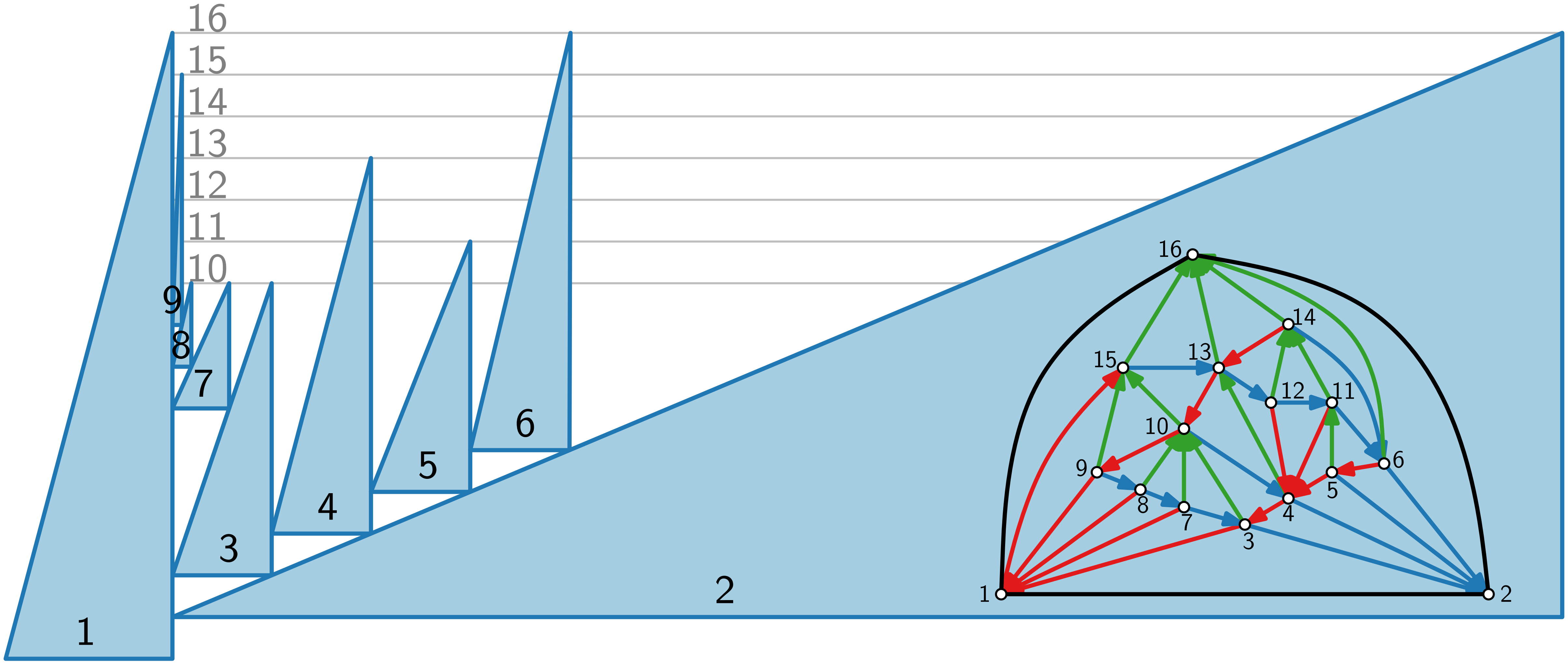

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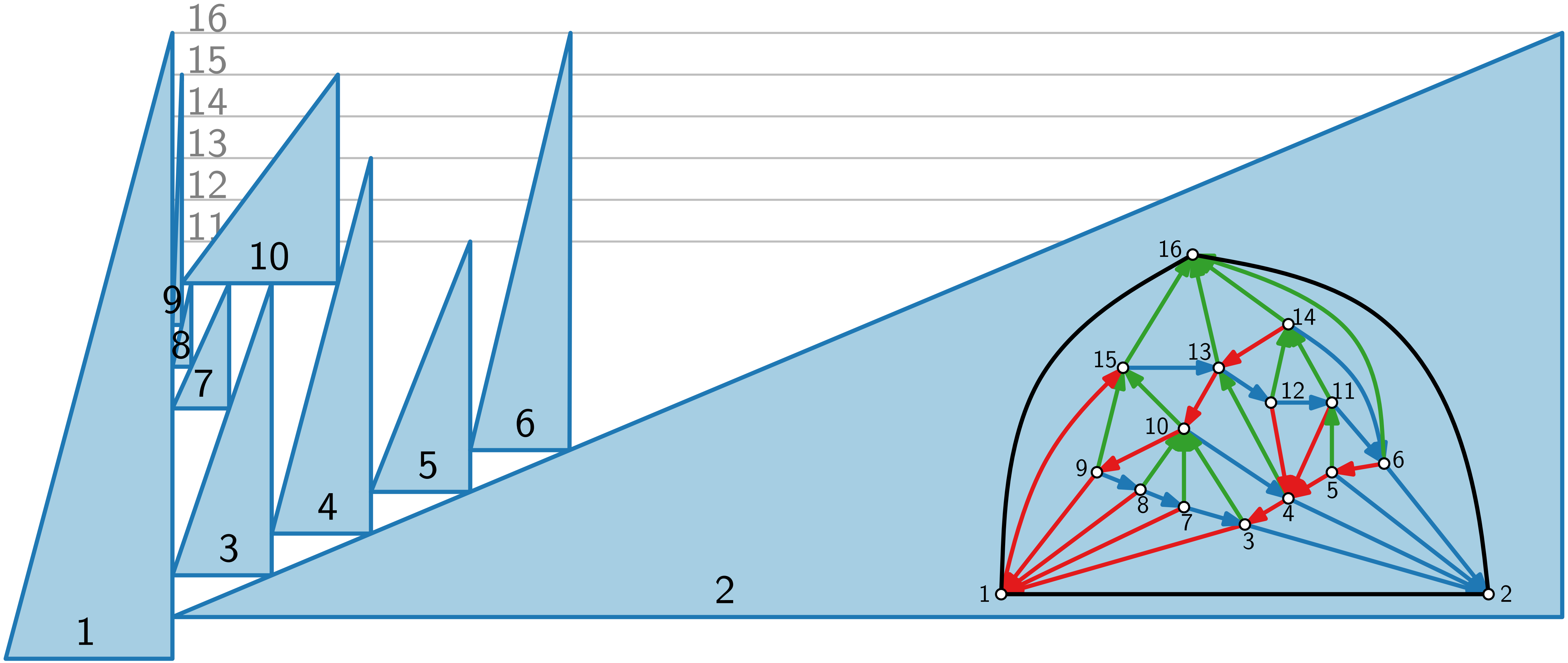

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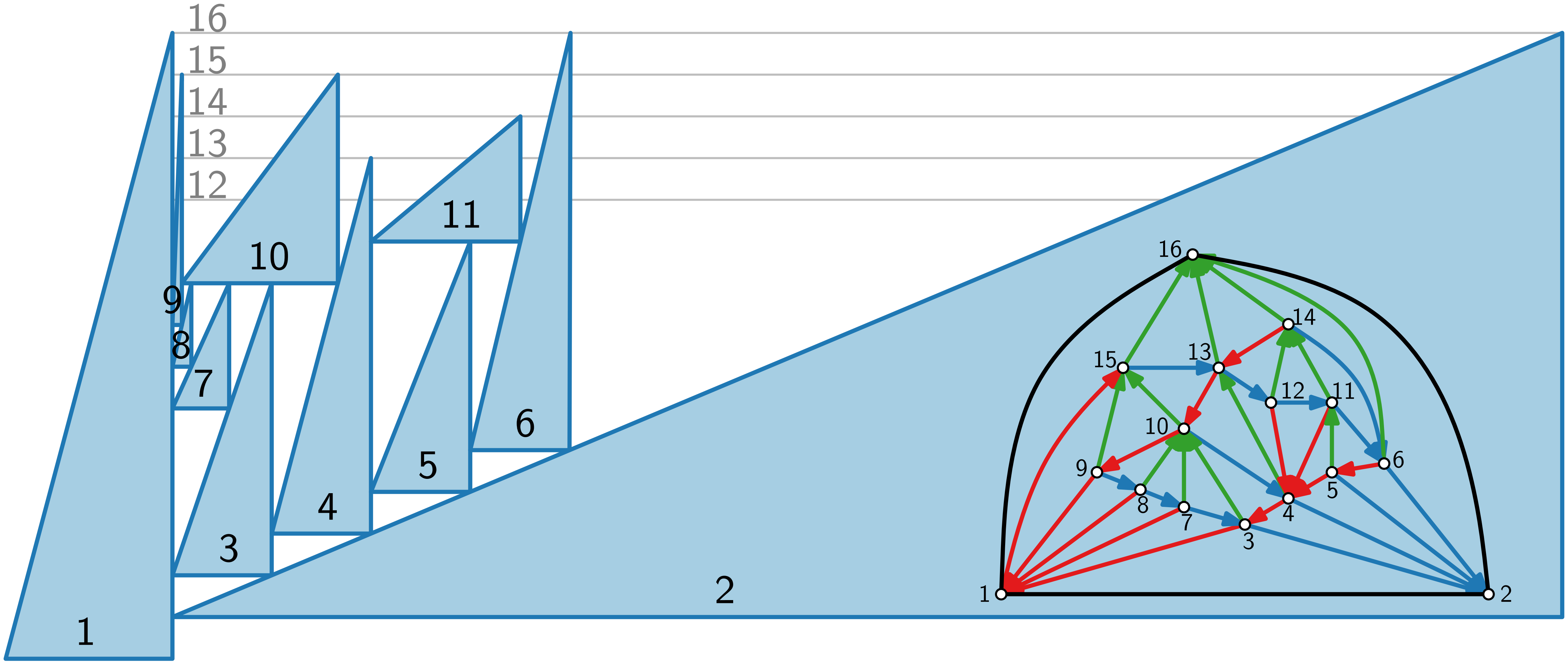

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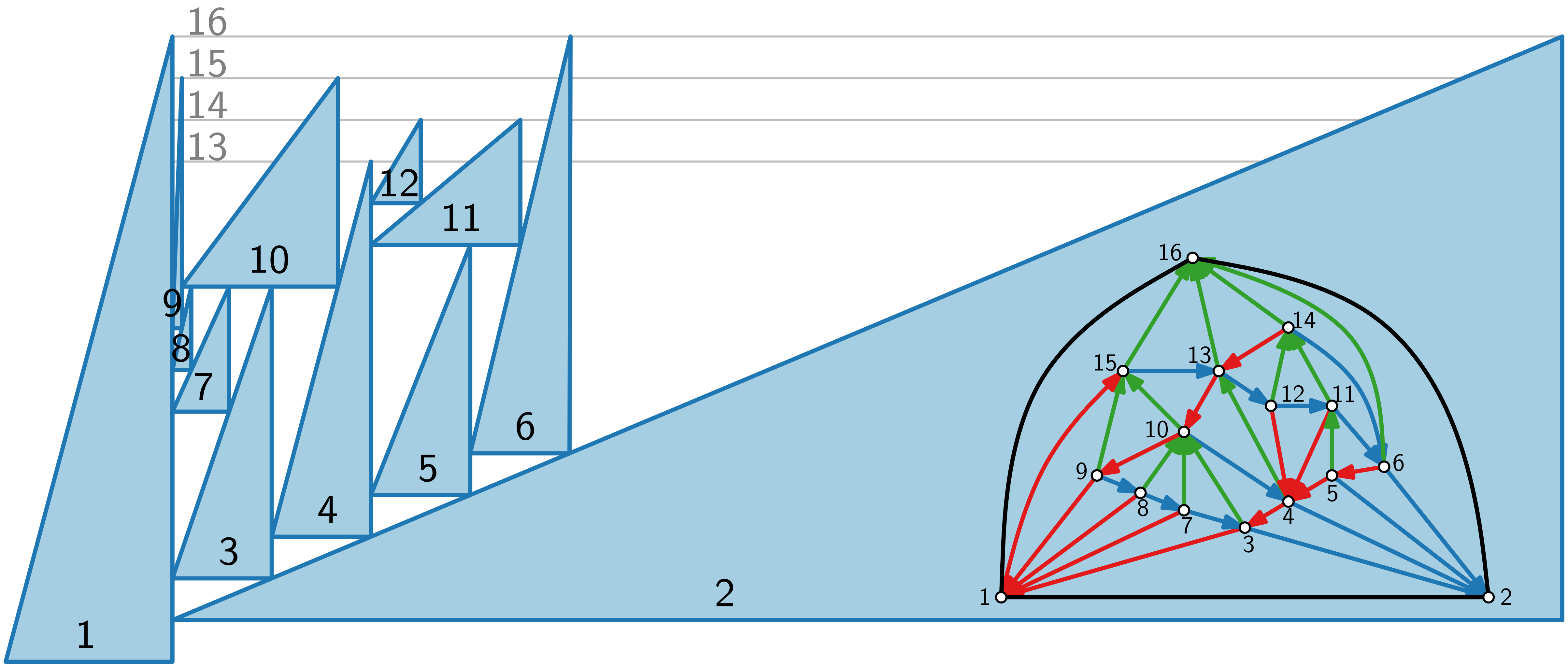

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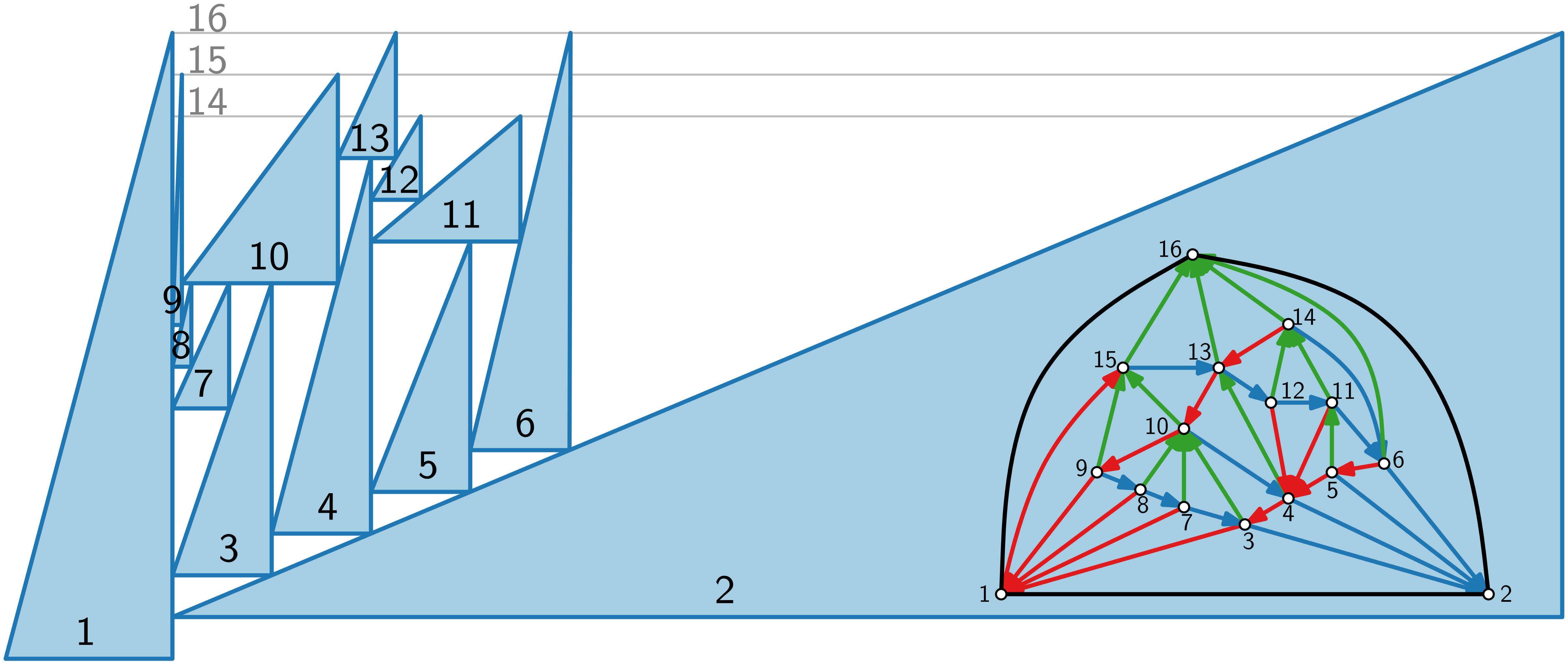

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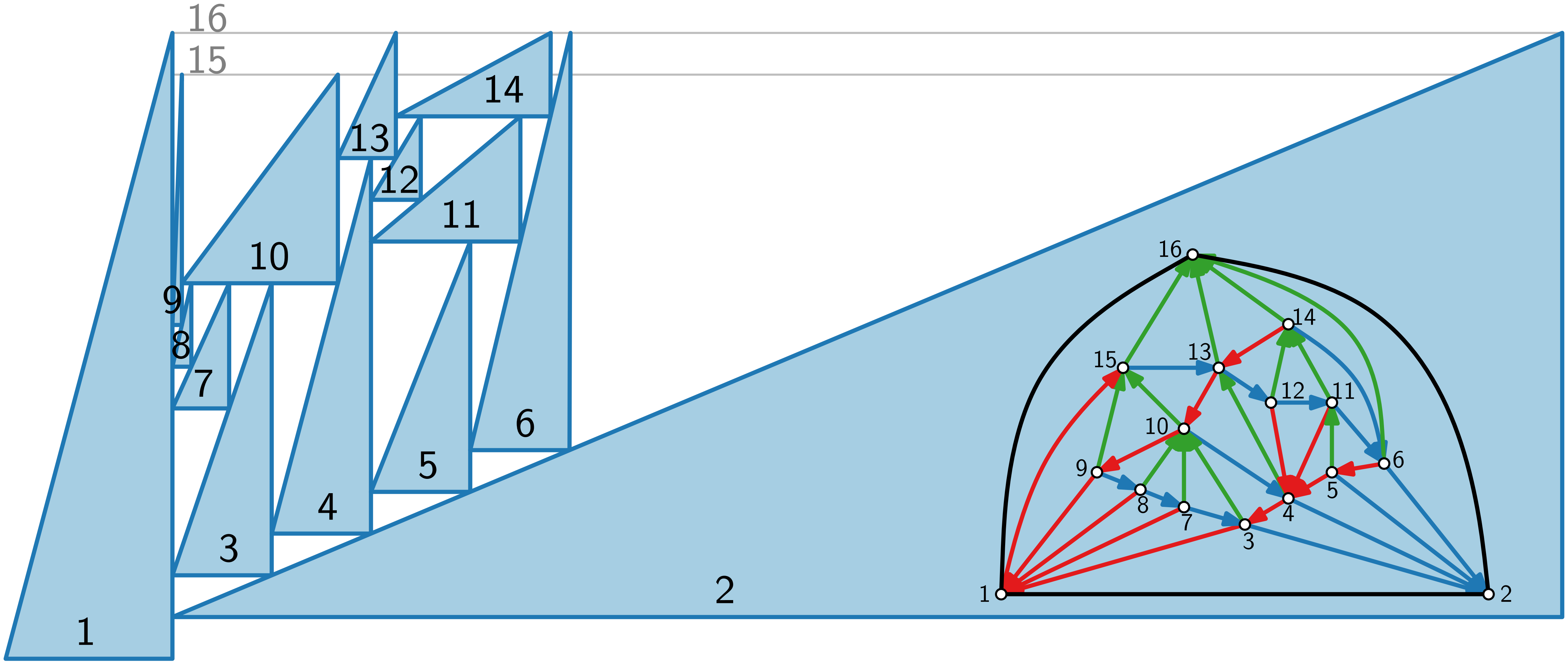

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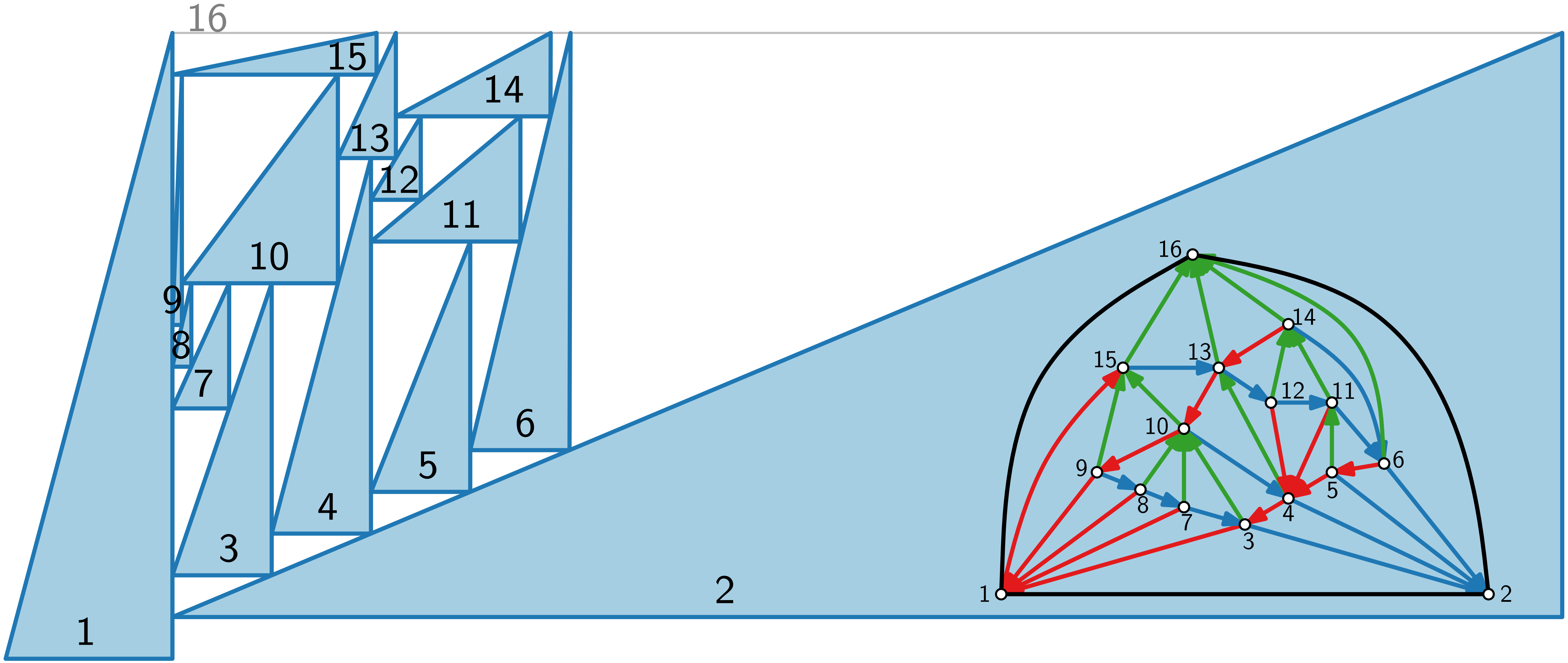

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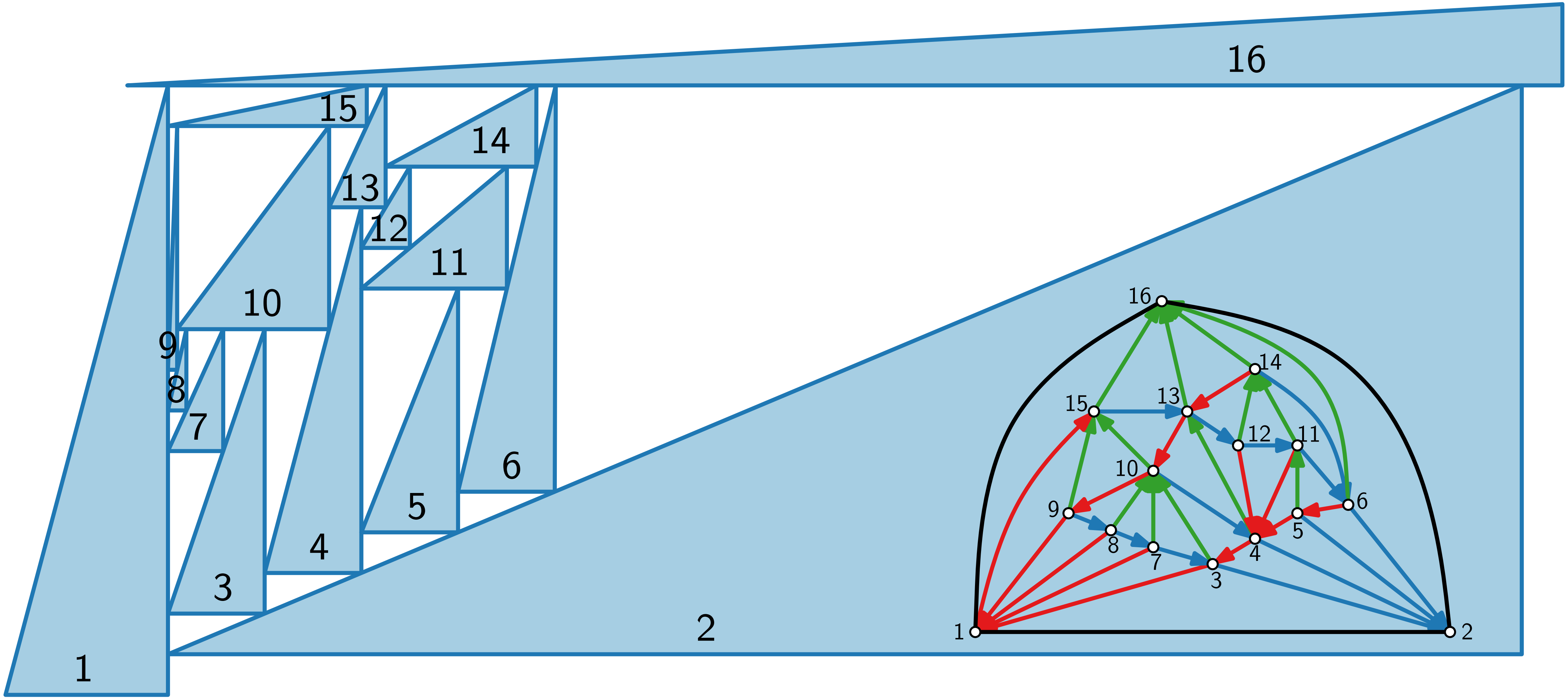
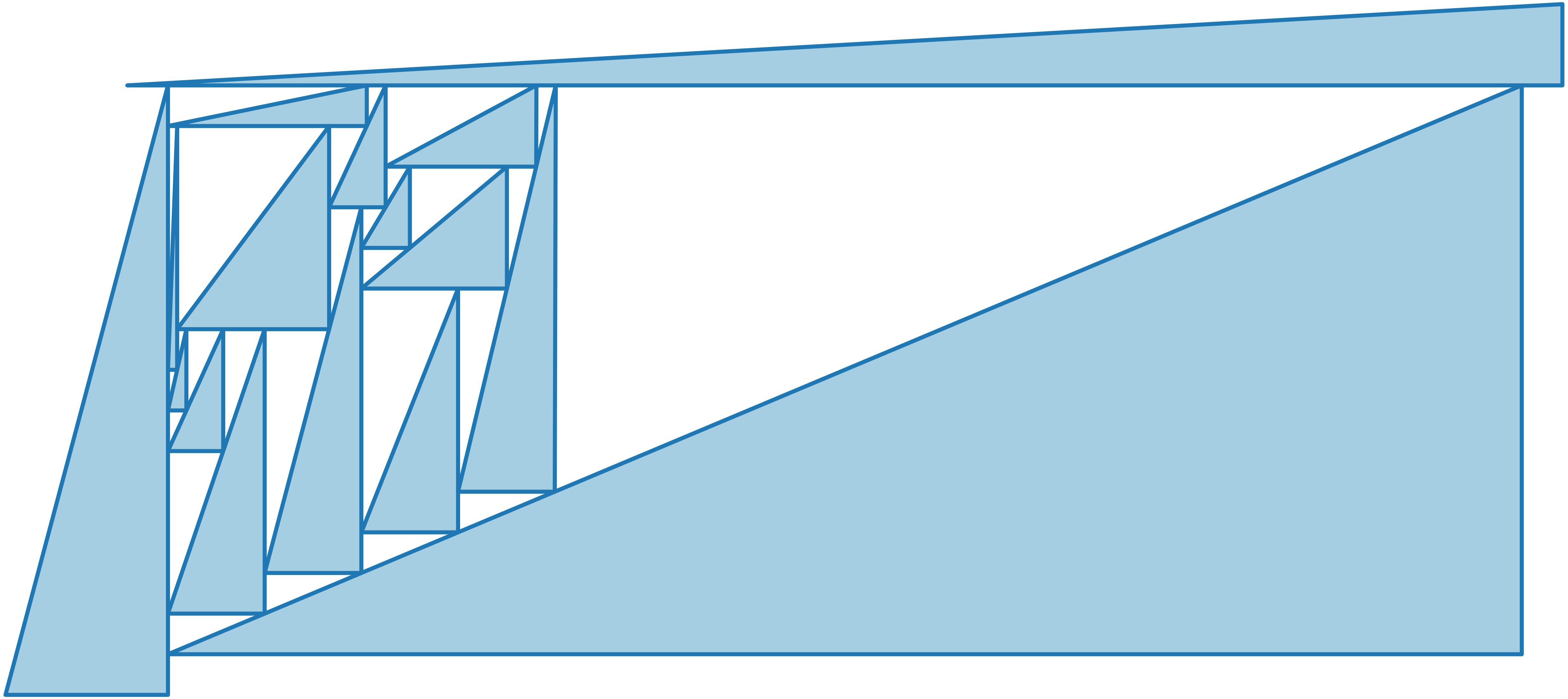


T-shape Contact Representation



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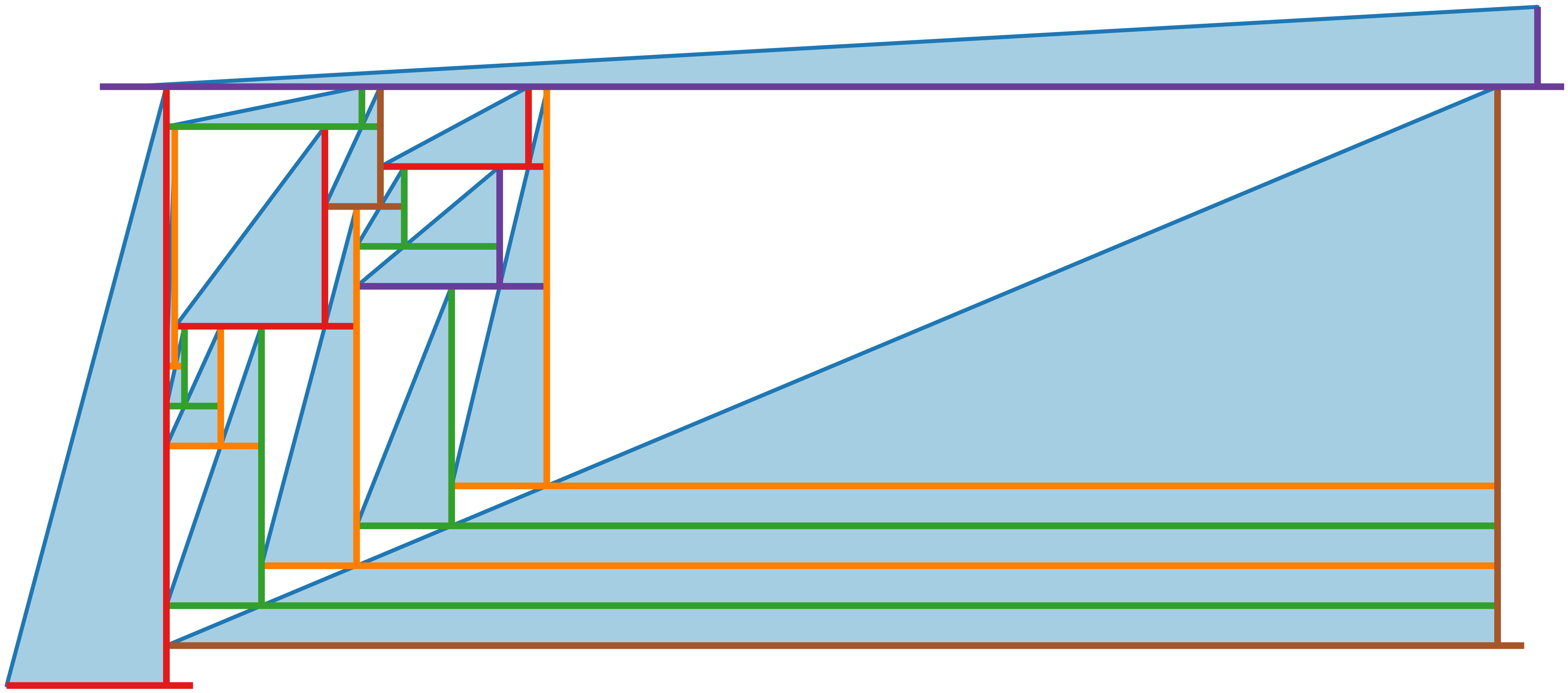

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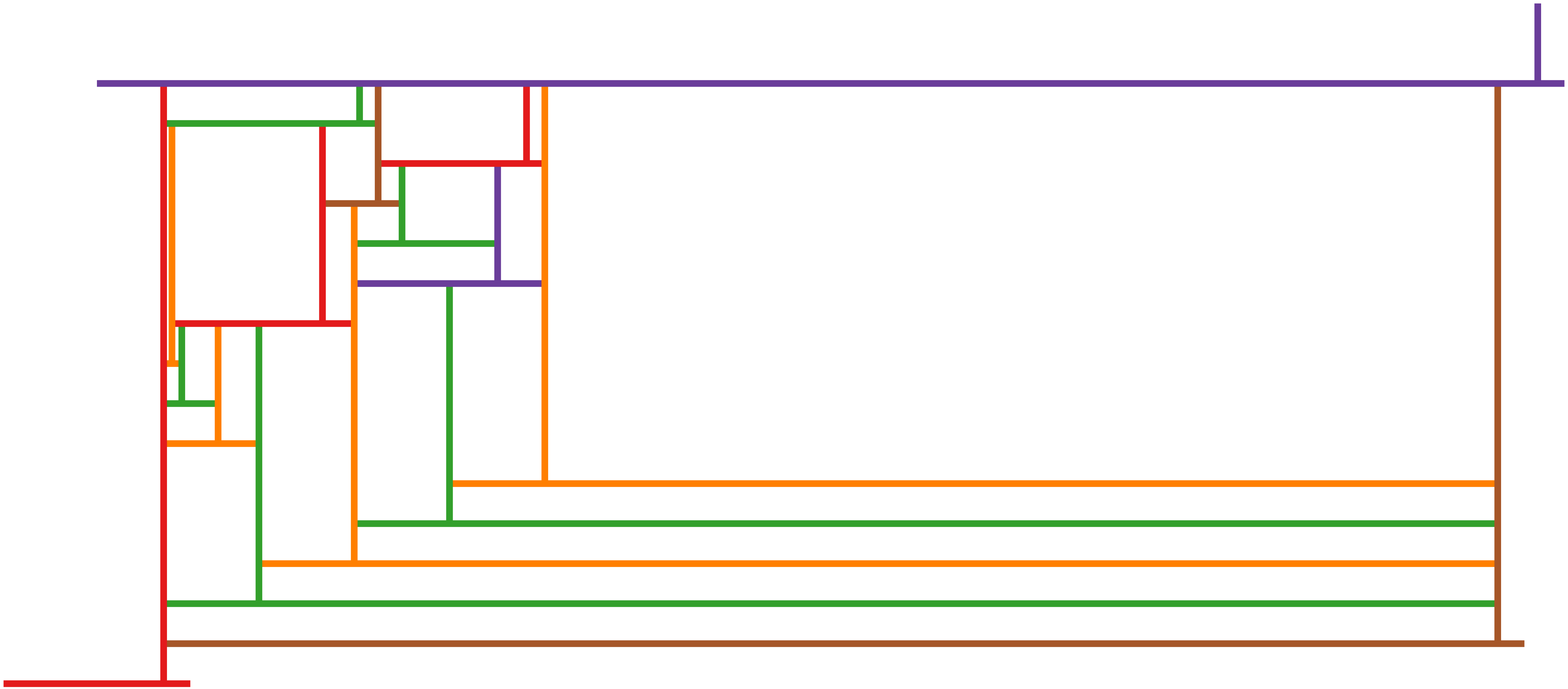

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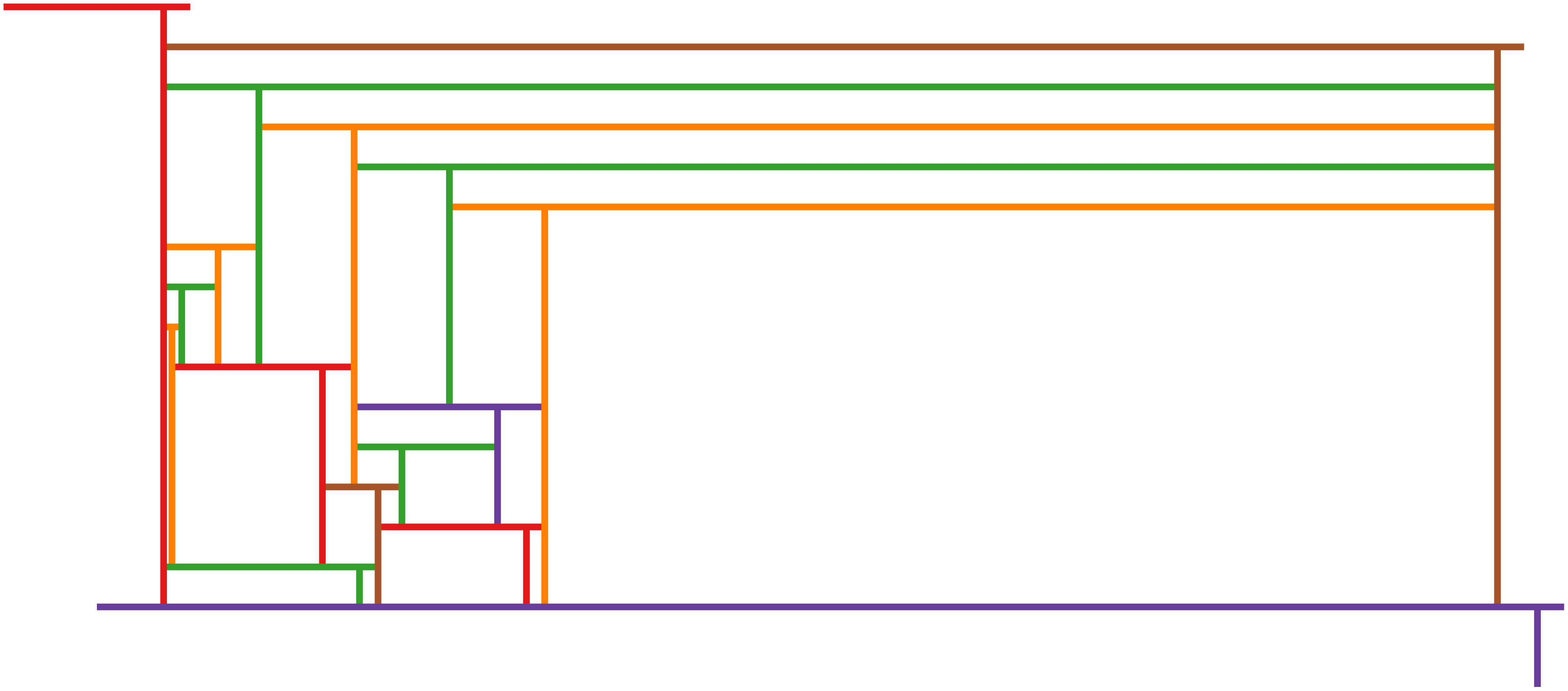

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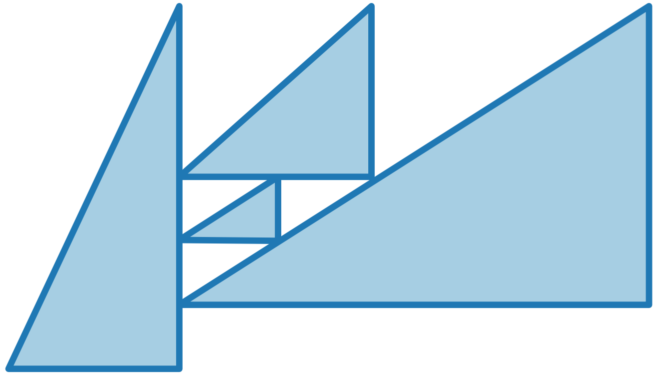
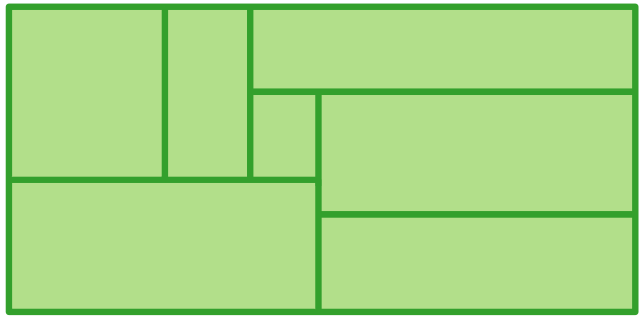
Visualization of Graphs

Lecture 7:
Contact Representations of Planar Graphs:
Triangle Contacts and Rectangular Duals



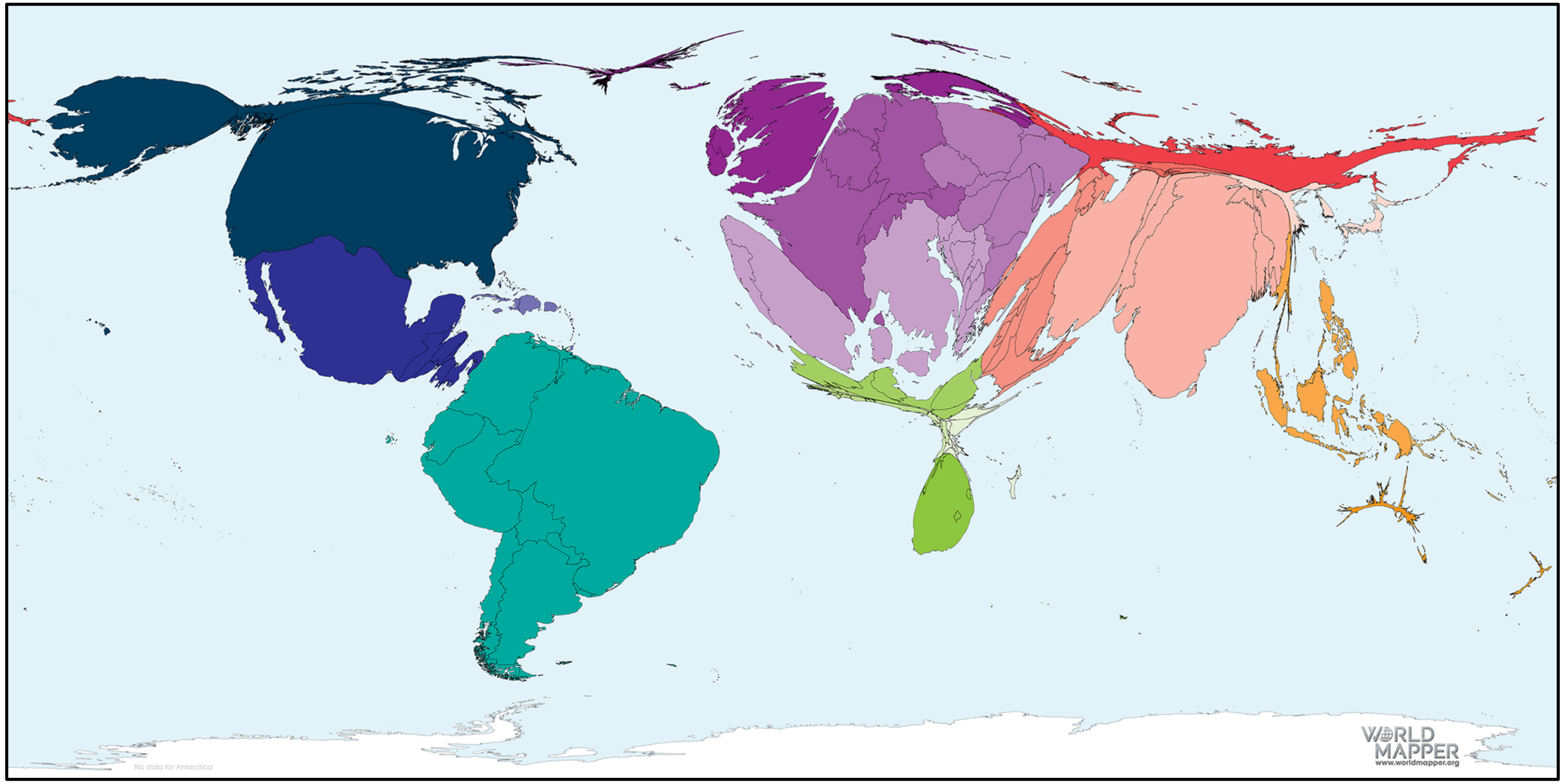
Part III:
Rectangular Duals

Alexander Wolff



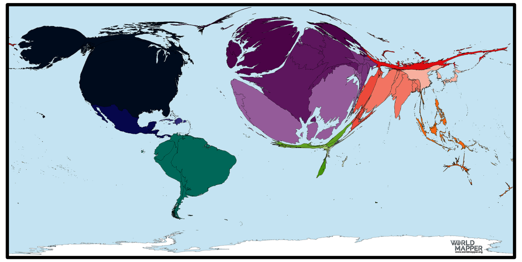
Cartograms

Cartograms

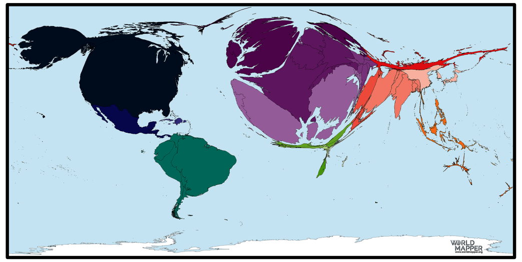


COVID19 reported deaths (January 1, 2021)

Cartograms



Cartograms



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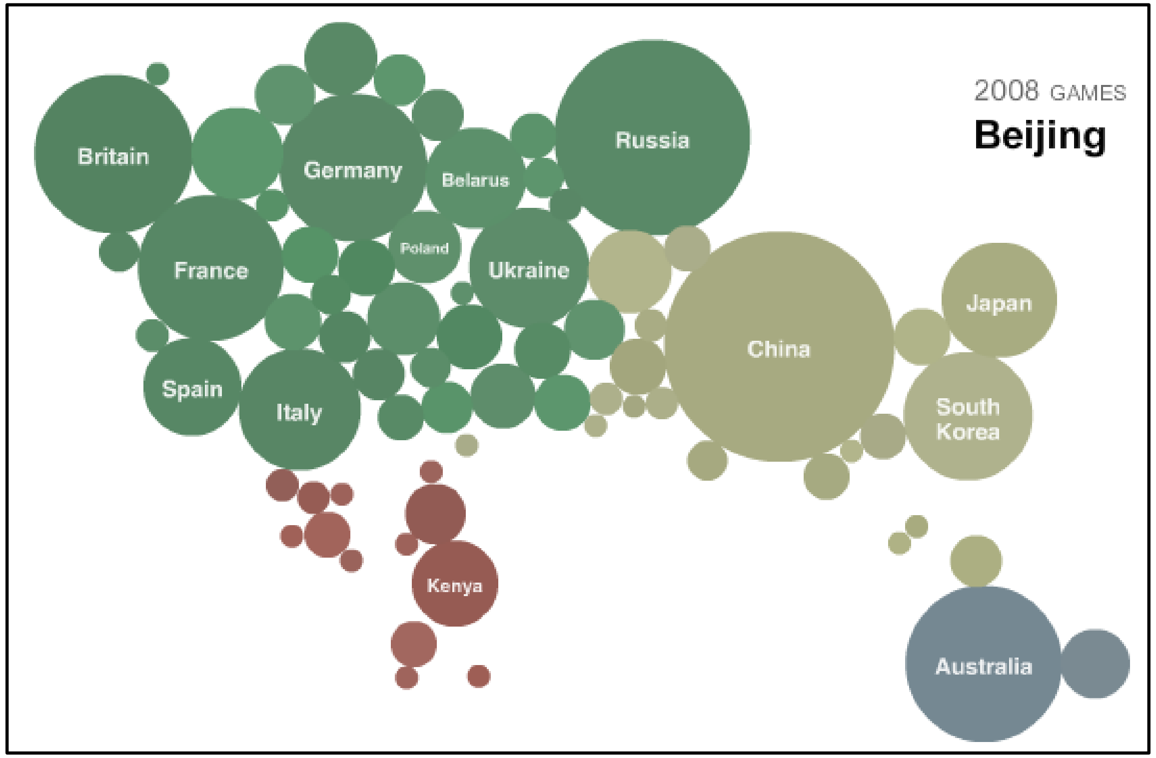

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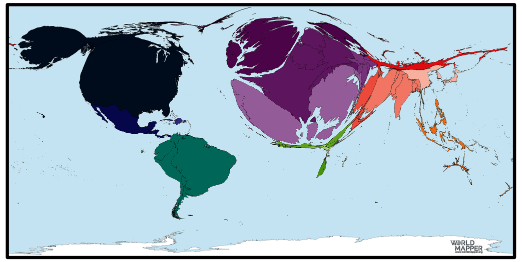

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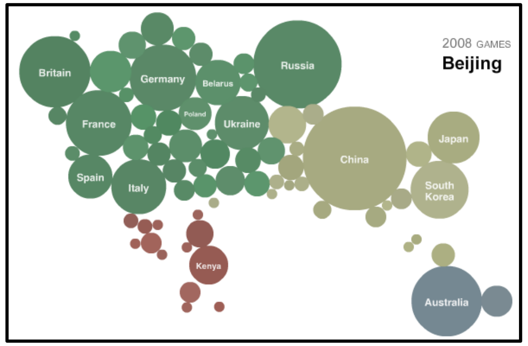

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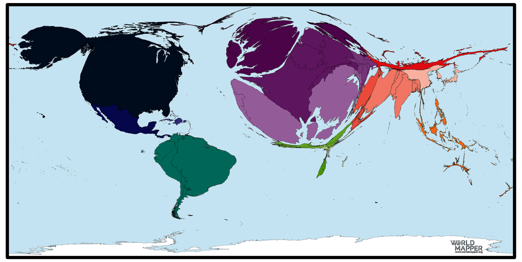

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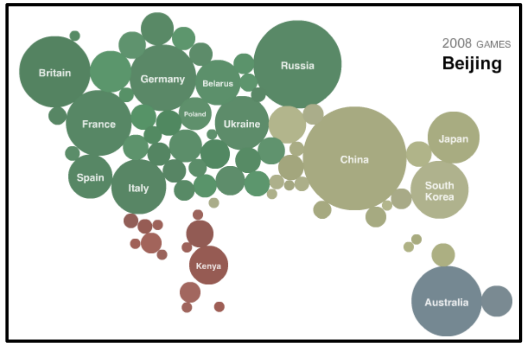

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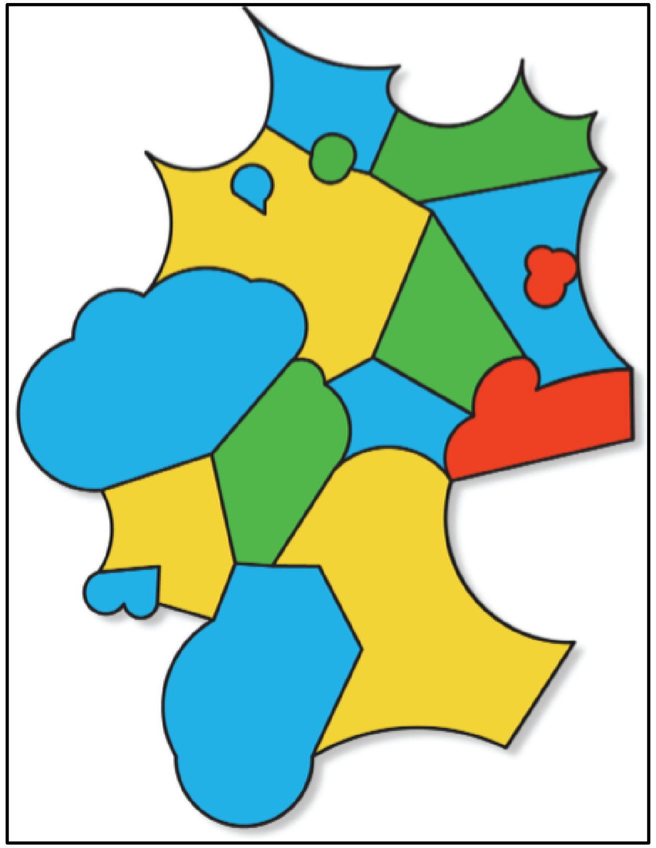

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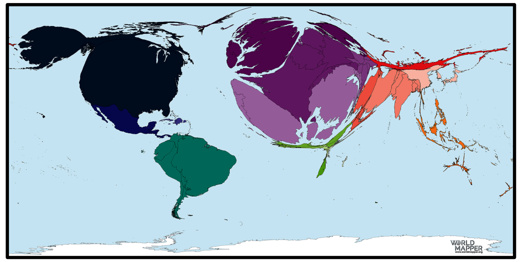

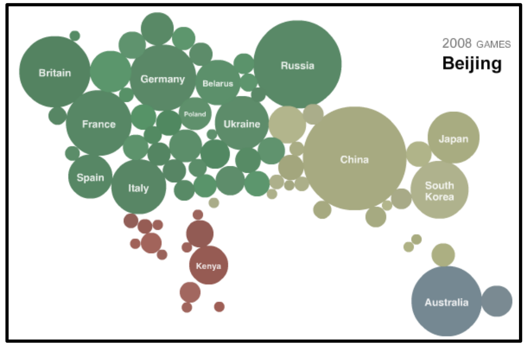
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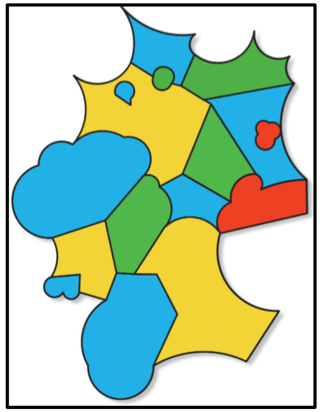

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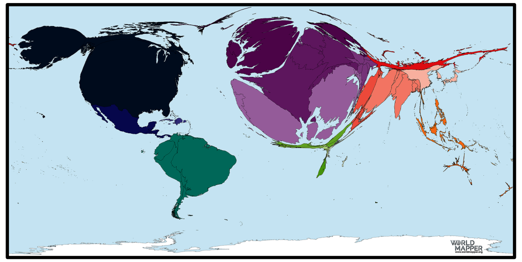

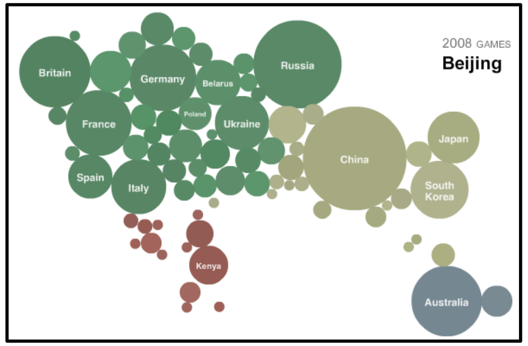
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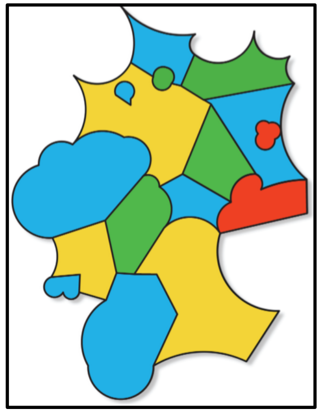

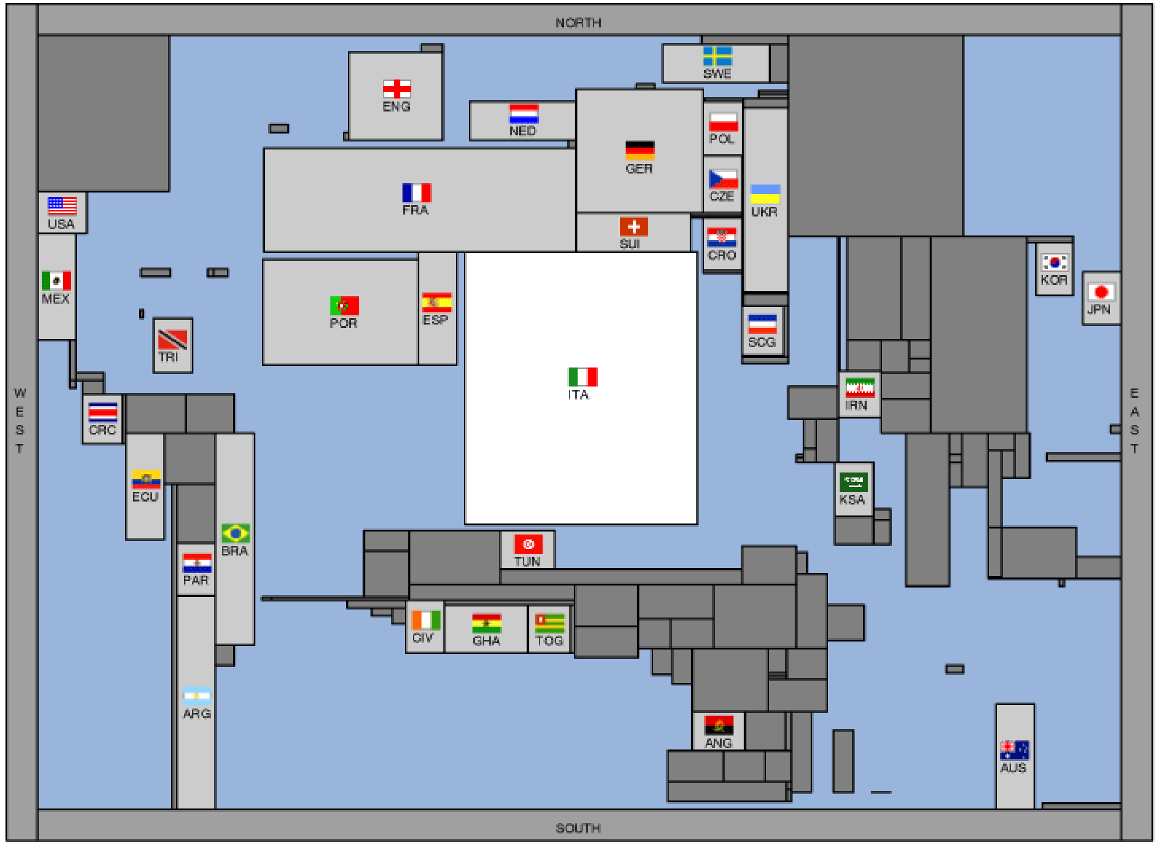
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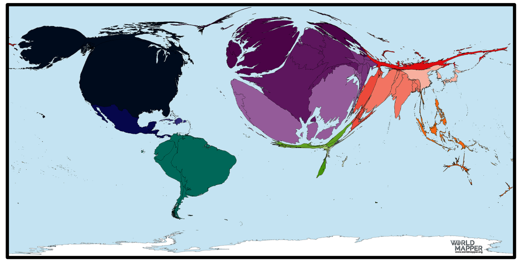

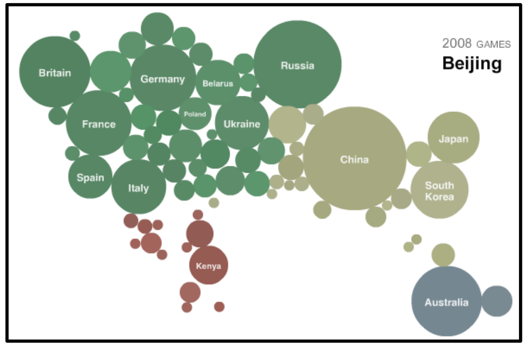
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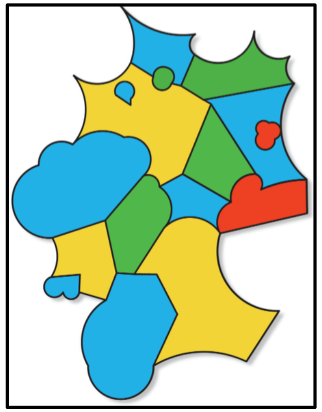

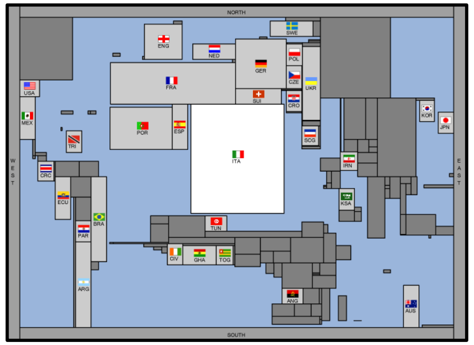

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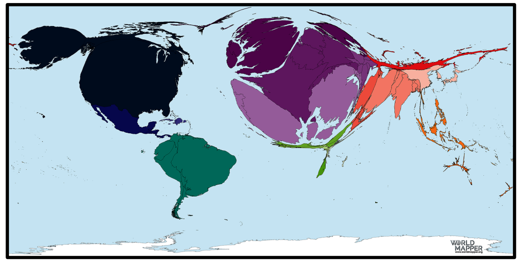

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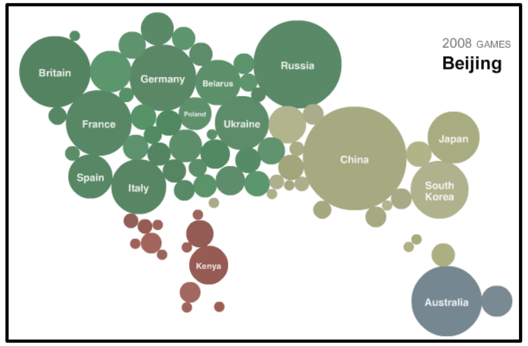

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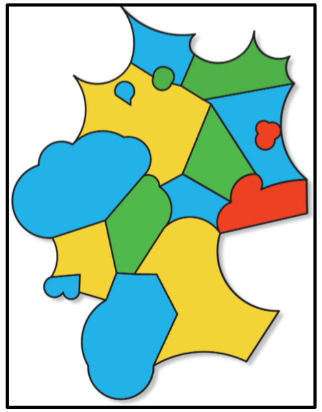

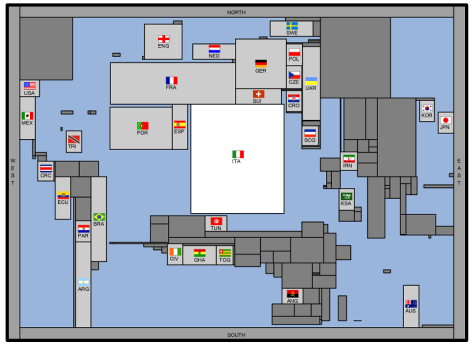

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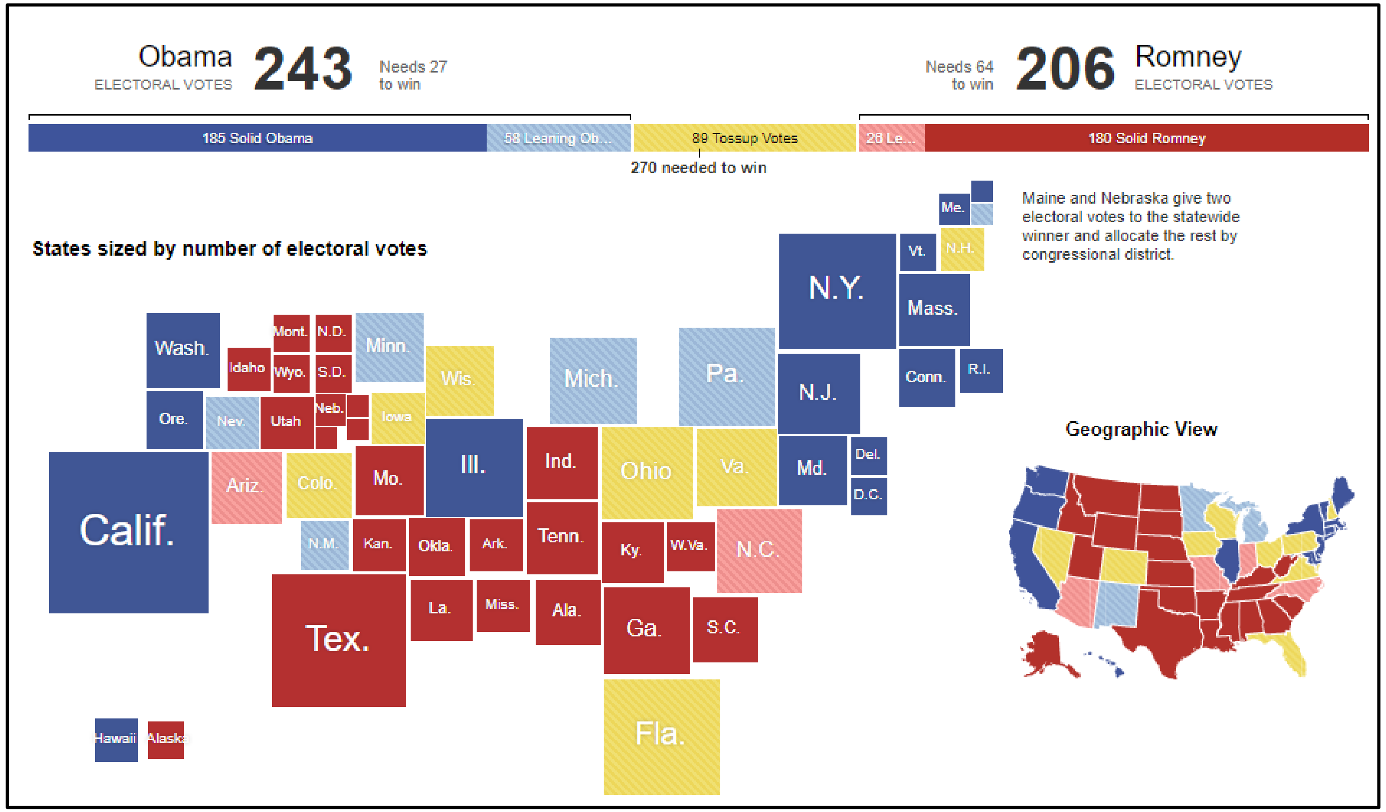

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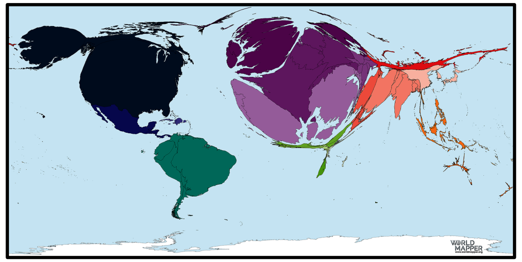

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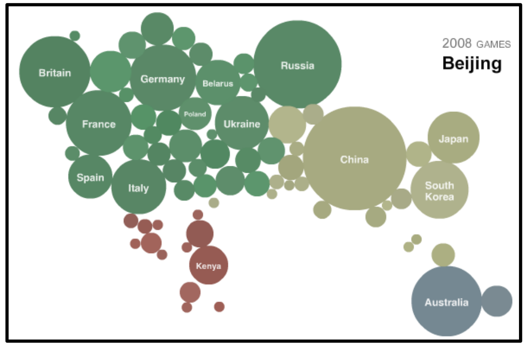

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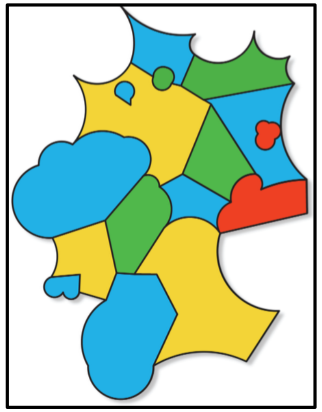

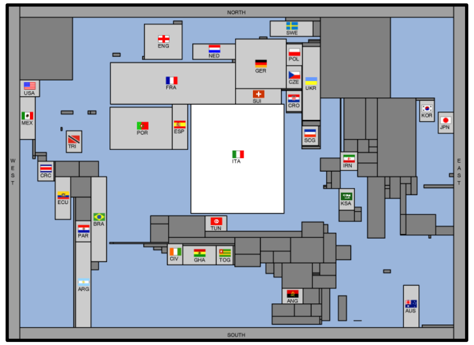

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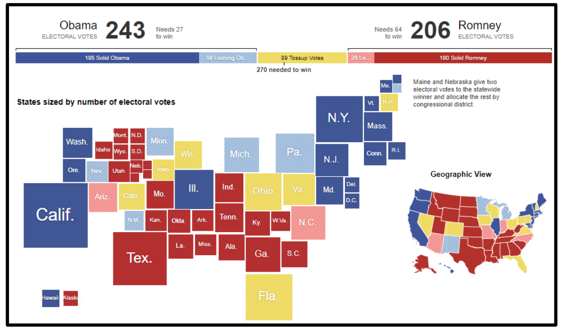

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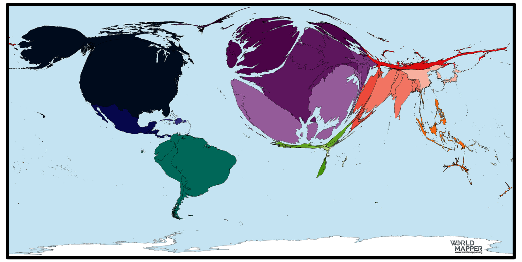

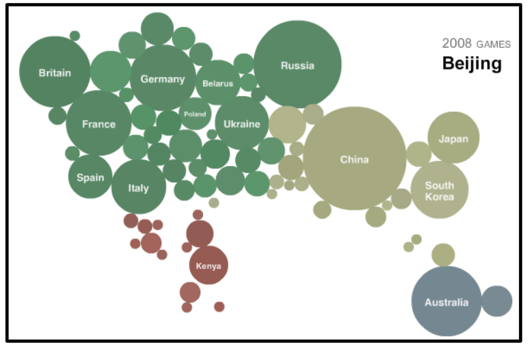
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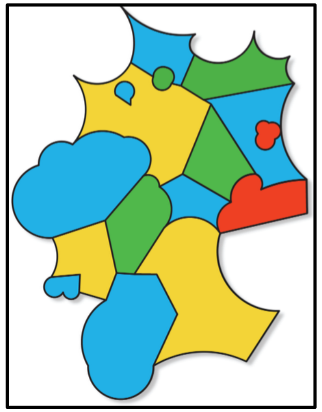

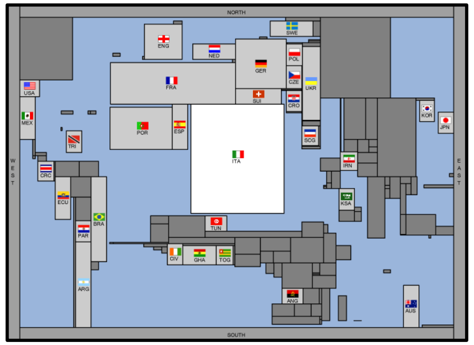

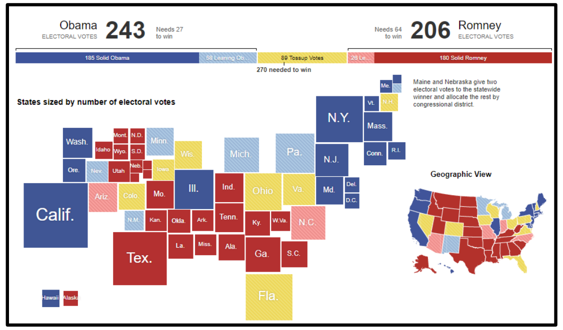
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Romney
States sized by number of electoral votes
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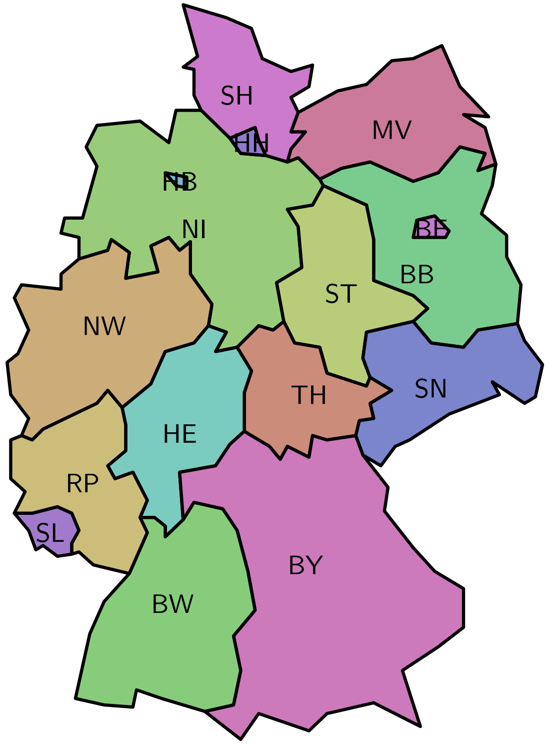
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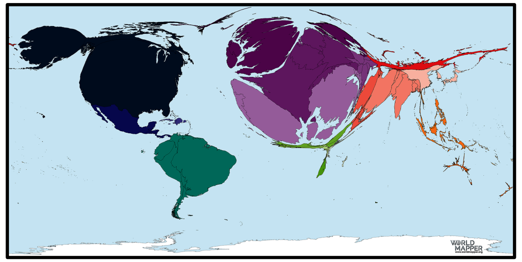

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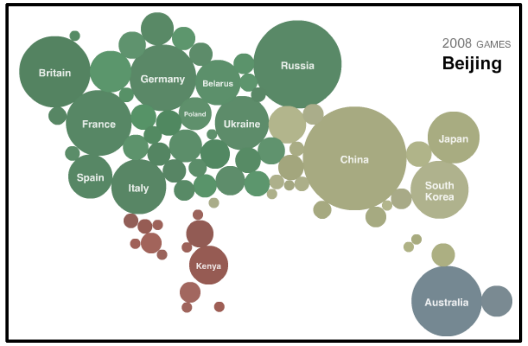

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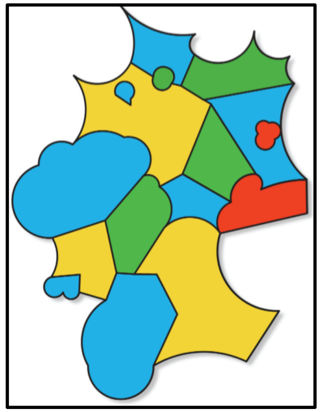

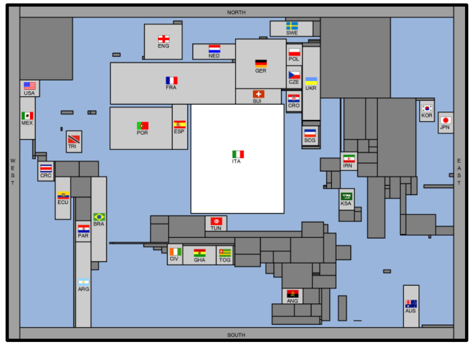

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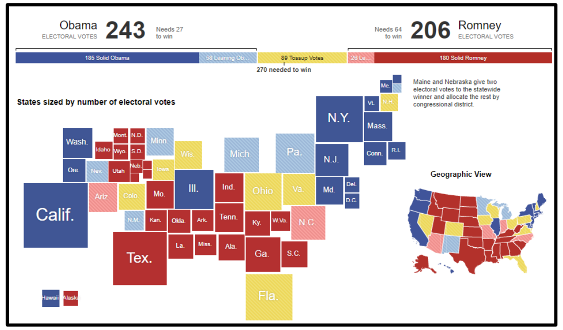

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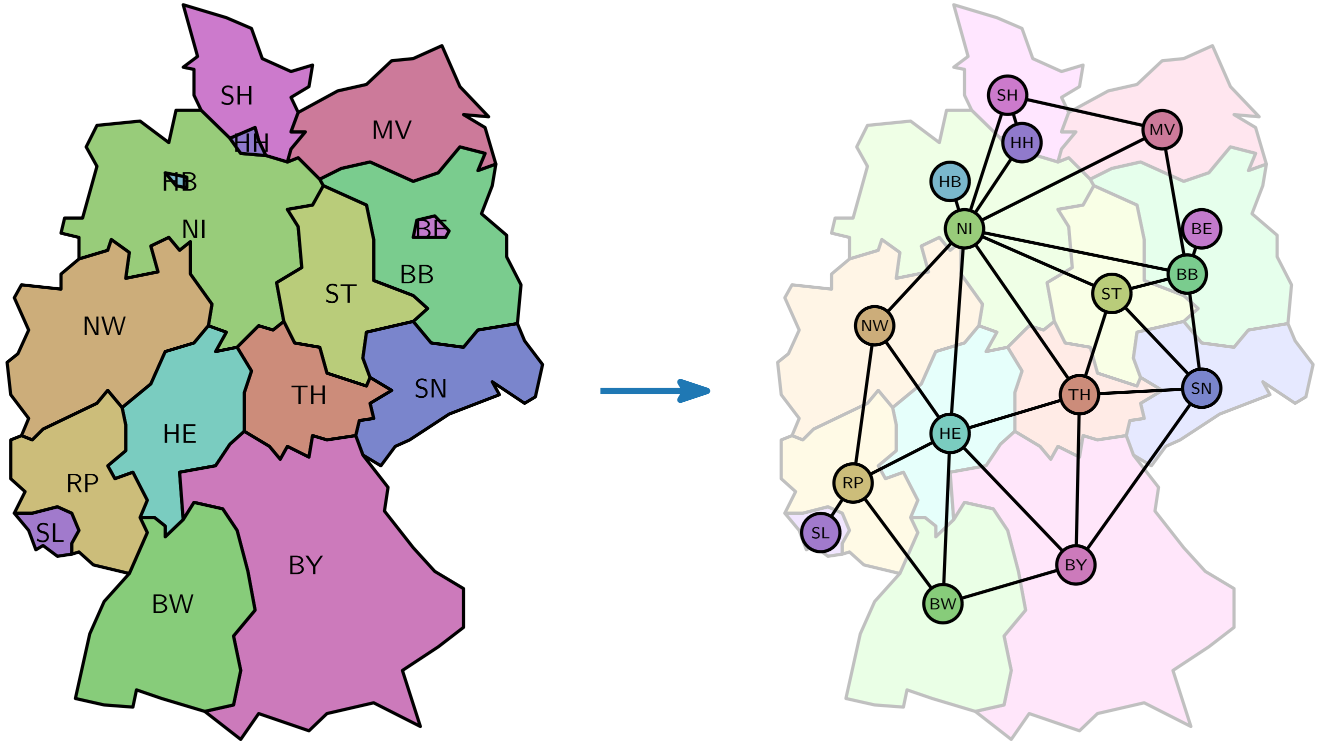

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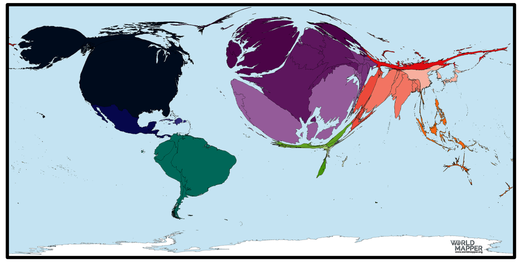

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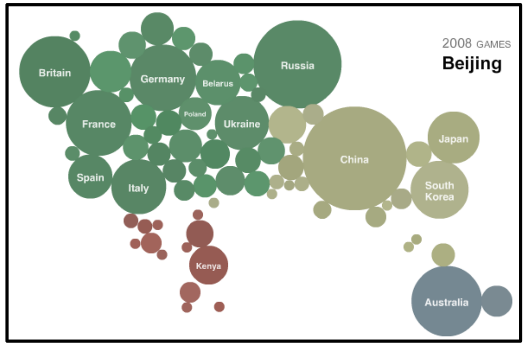

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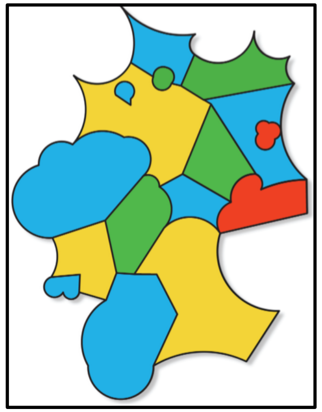

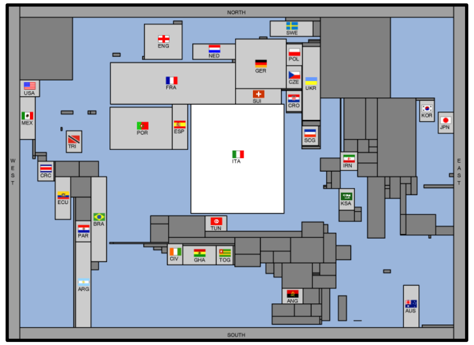

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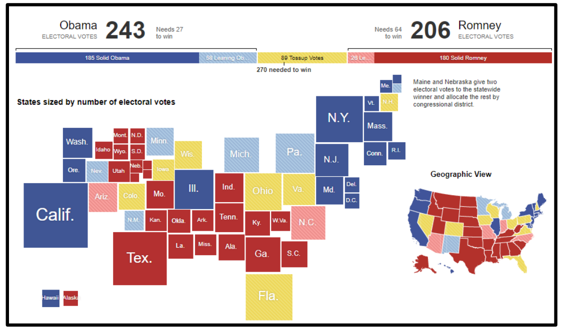

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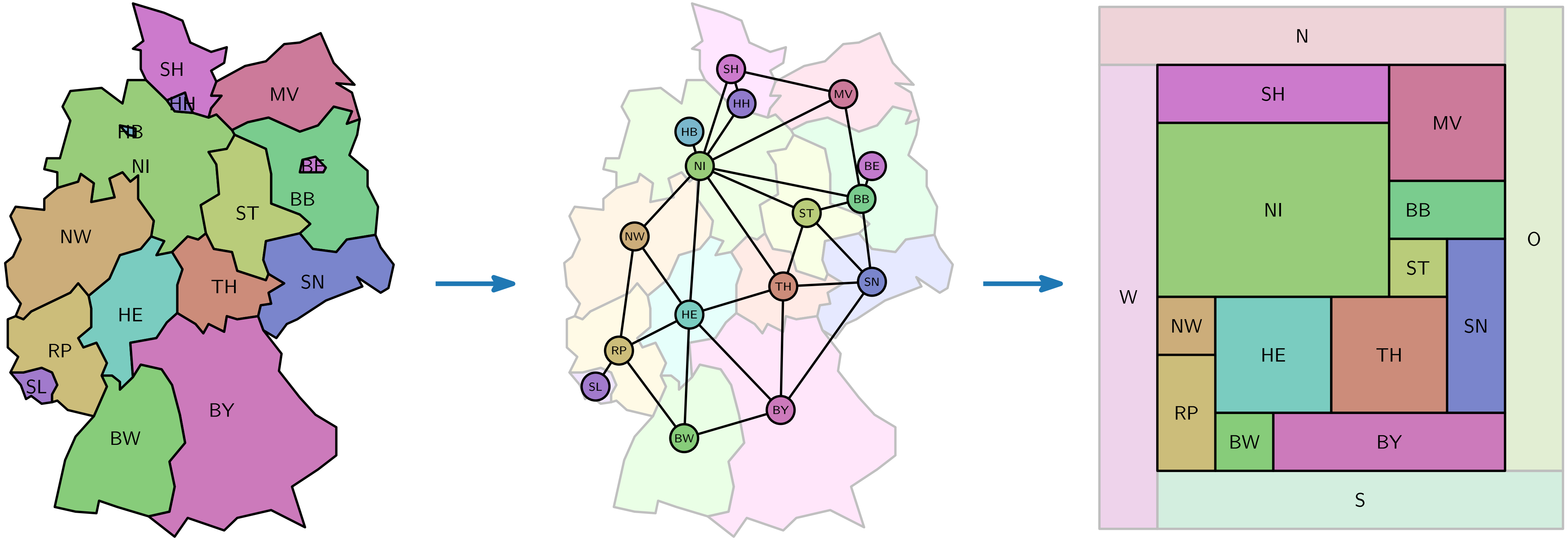

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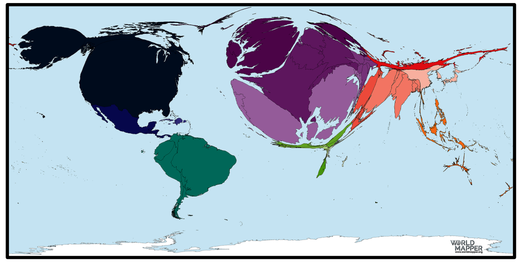

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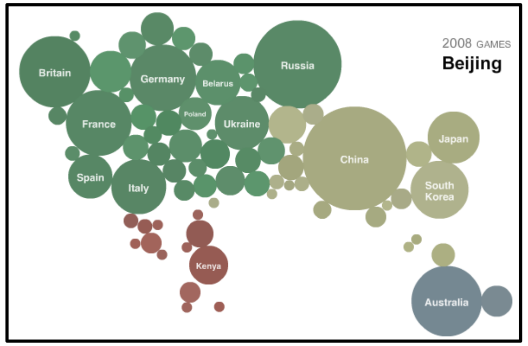

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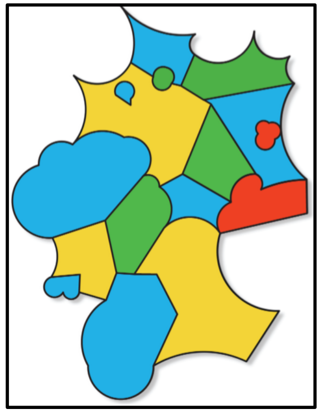

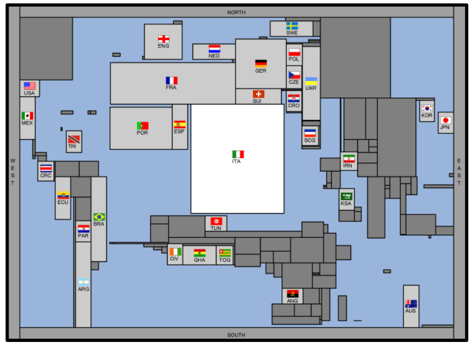

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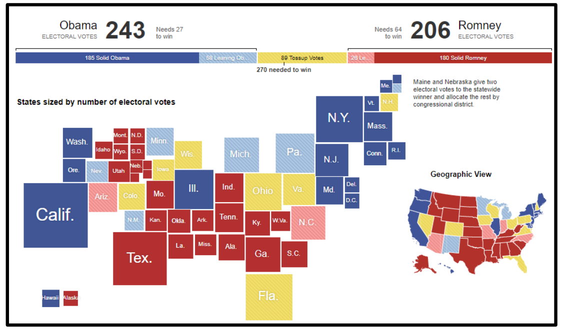

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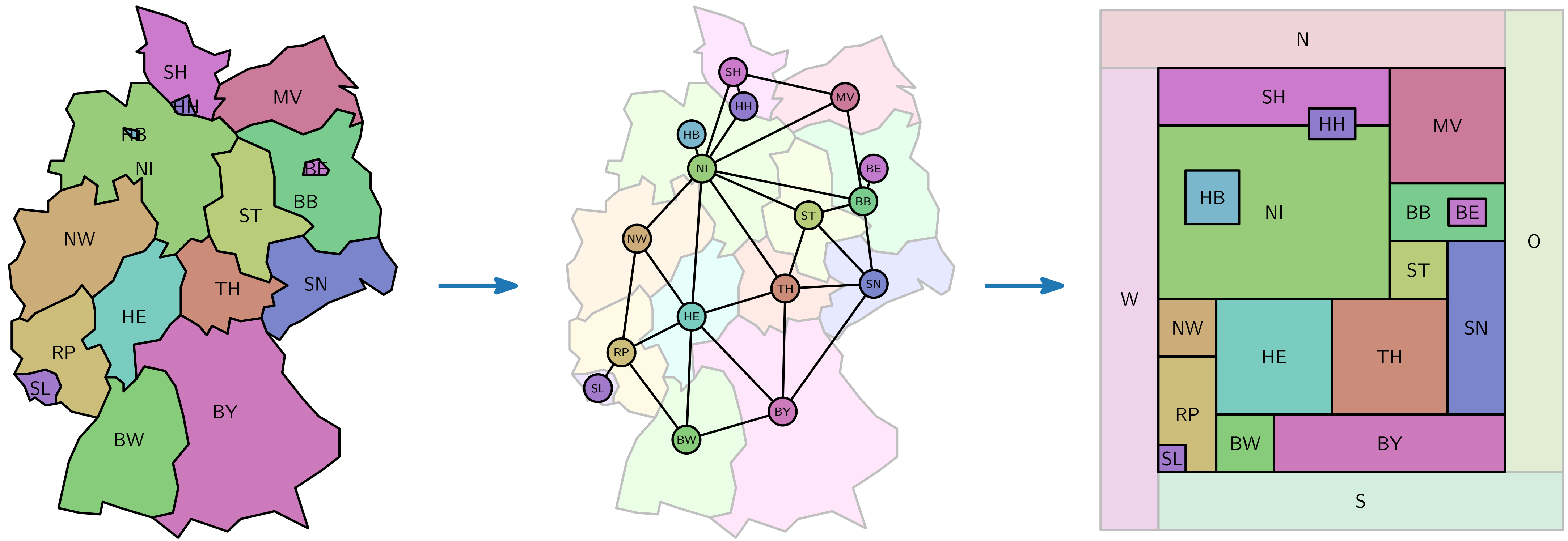
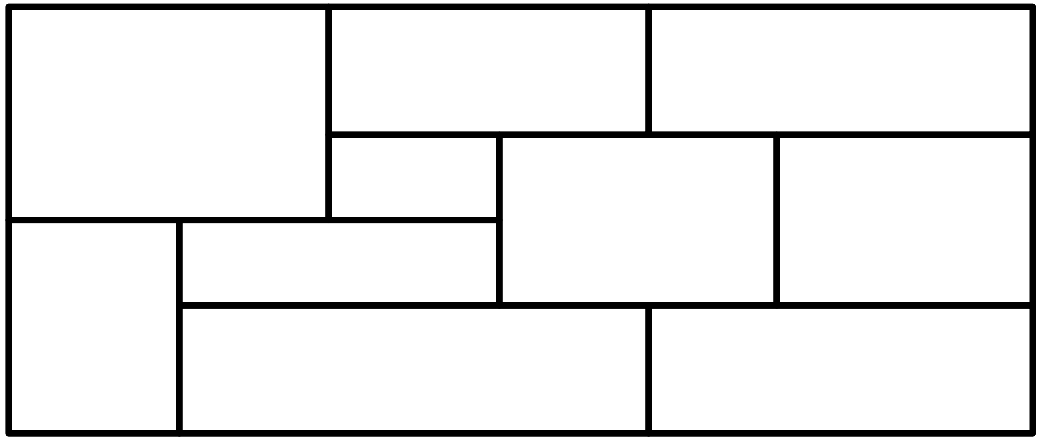


Rectangular Dual

Rectangular Dual



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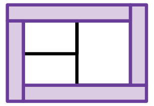
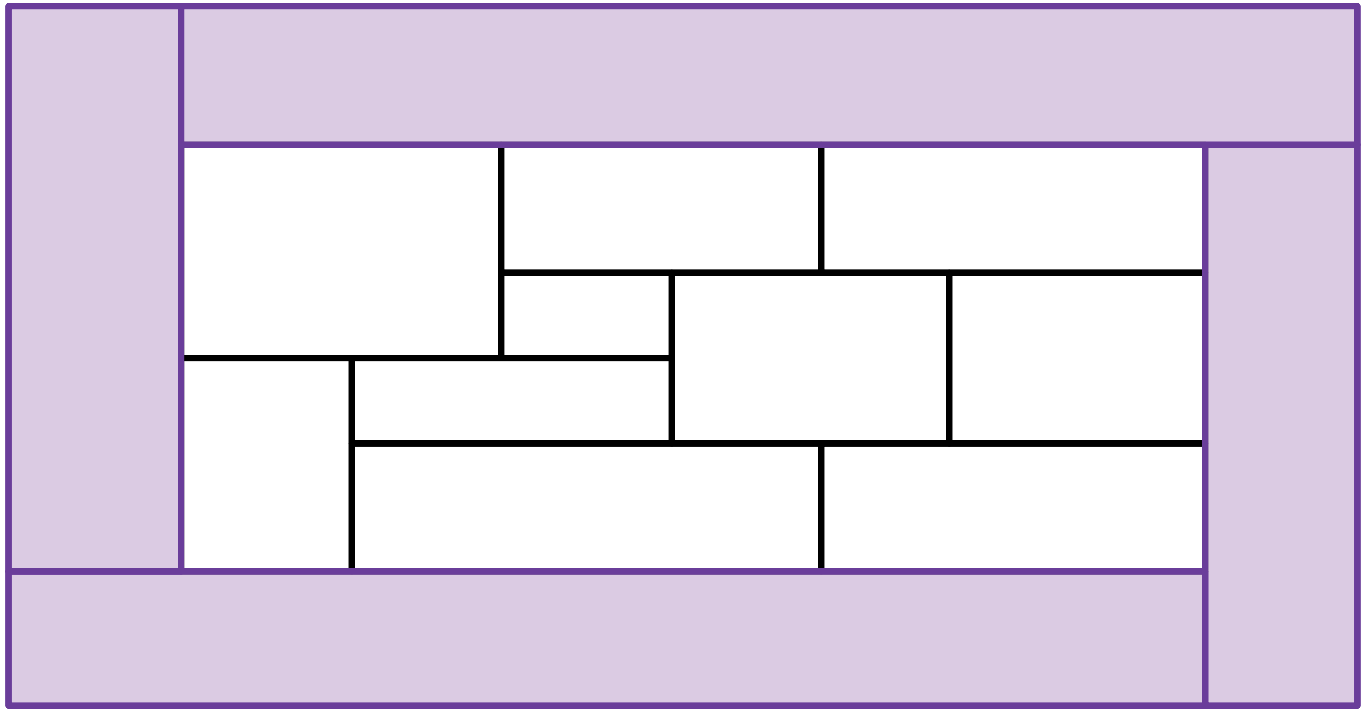
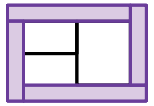


Rectangular Dual $\mathcal{R}$



Rectangular Dual



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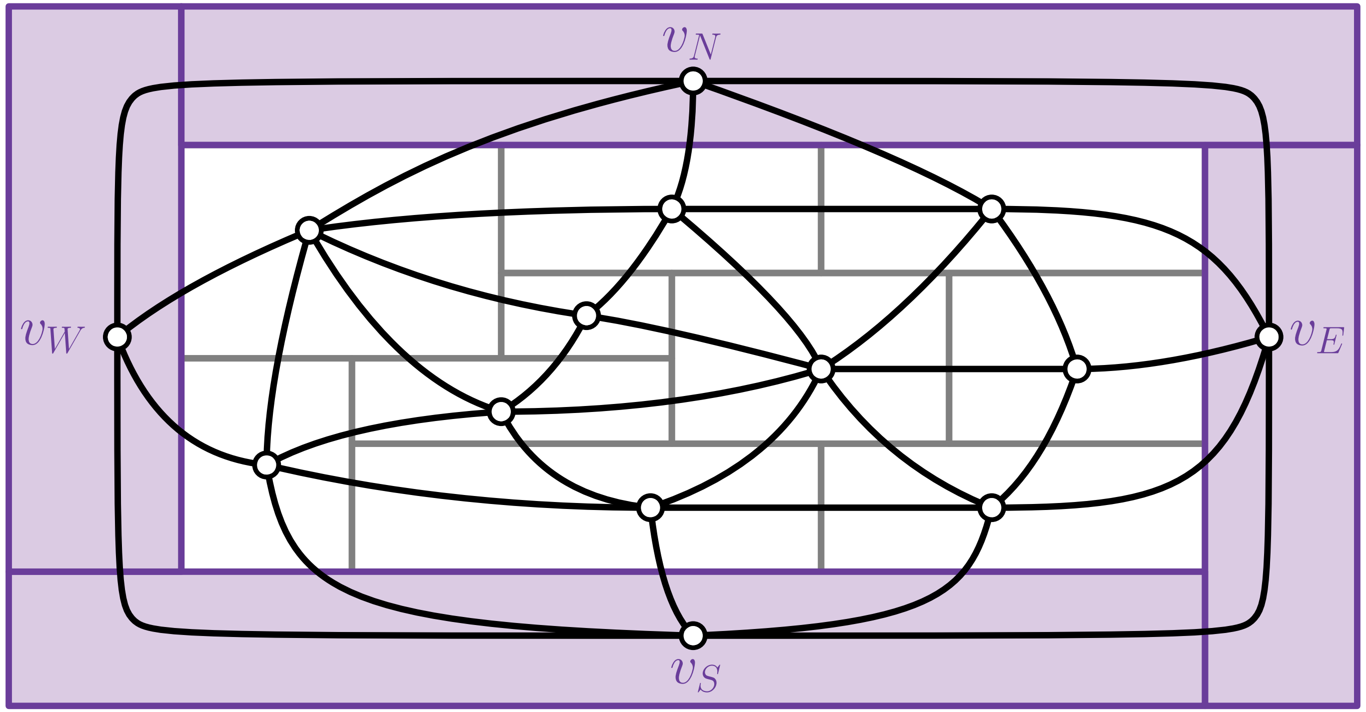

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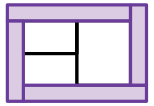

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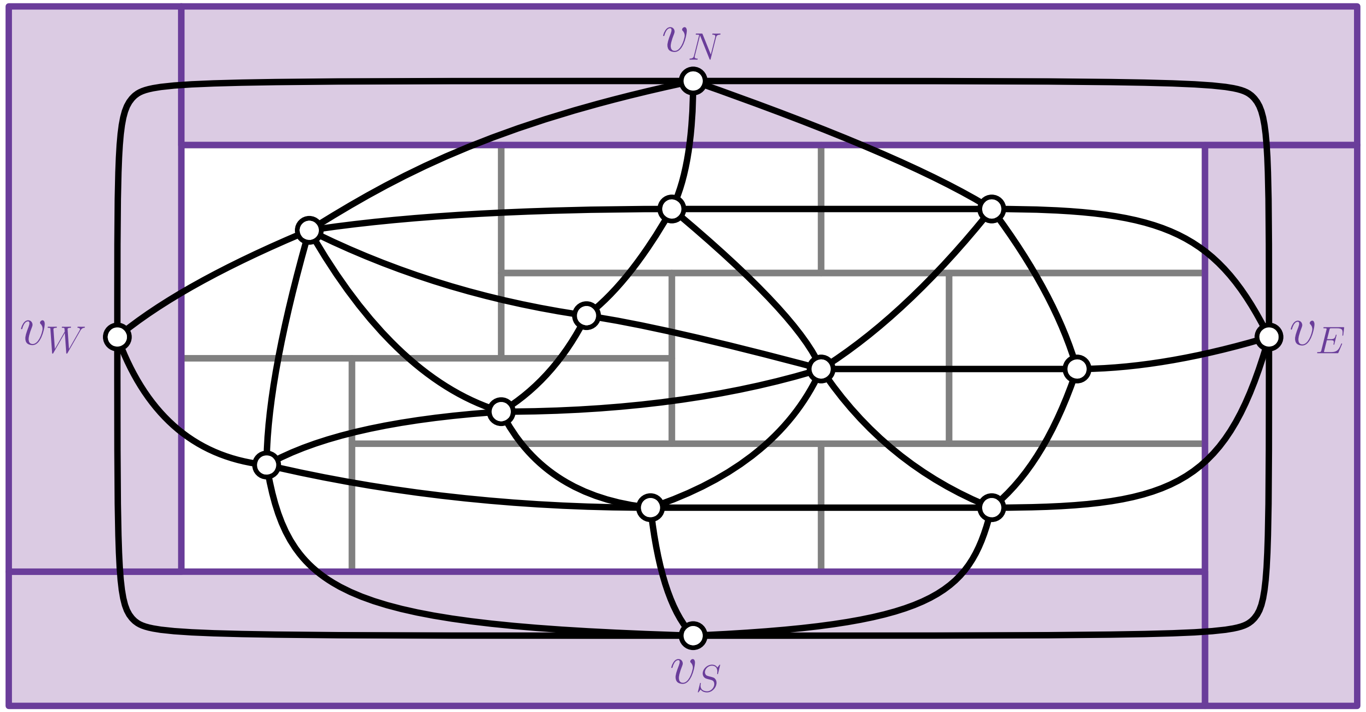
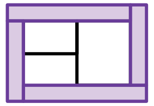


A **rectangular dual** of a graph G is a contact representation with axis-aligned rectangles s.t.

Rectangular Dual



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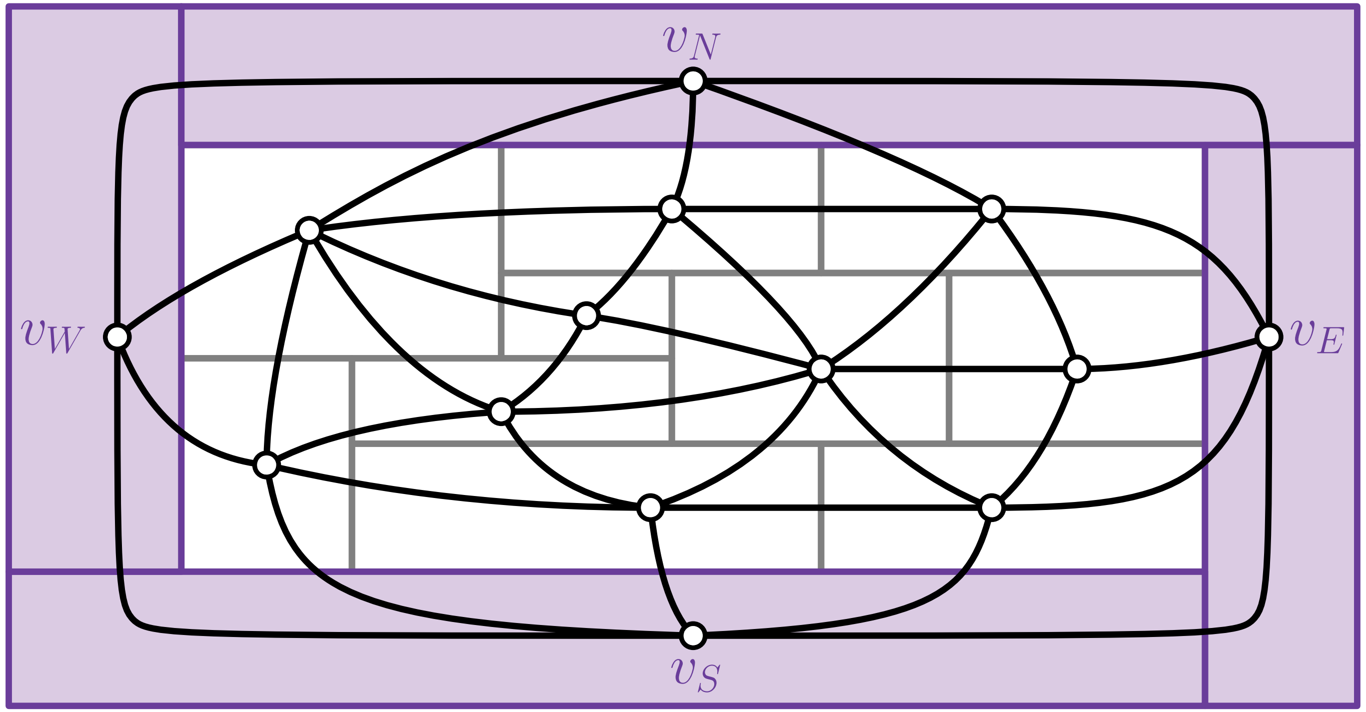

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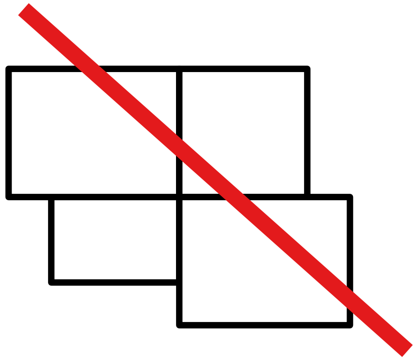

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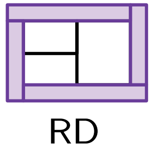

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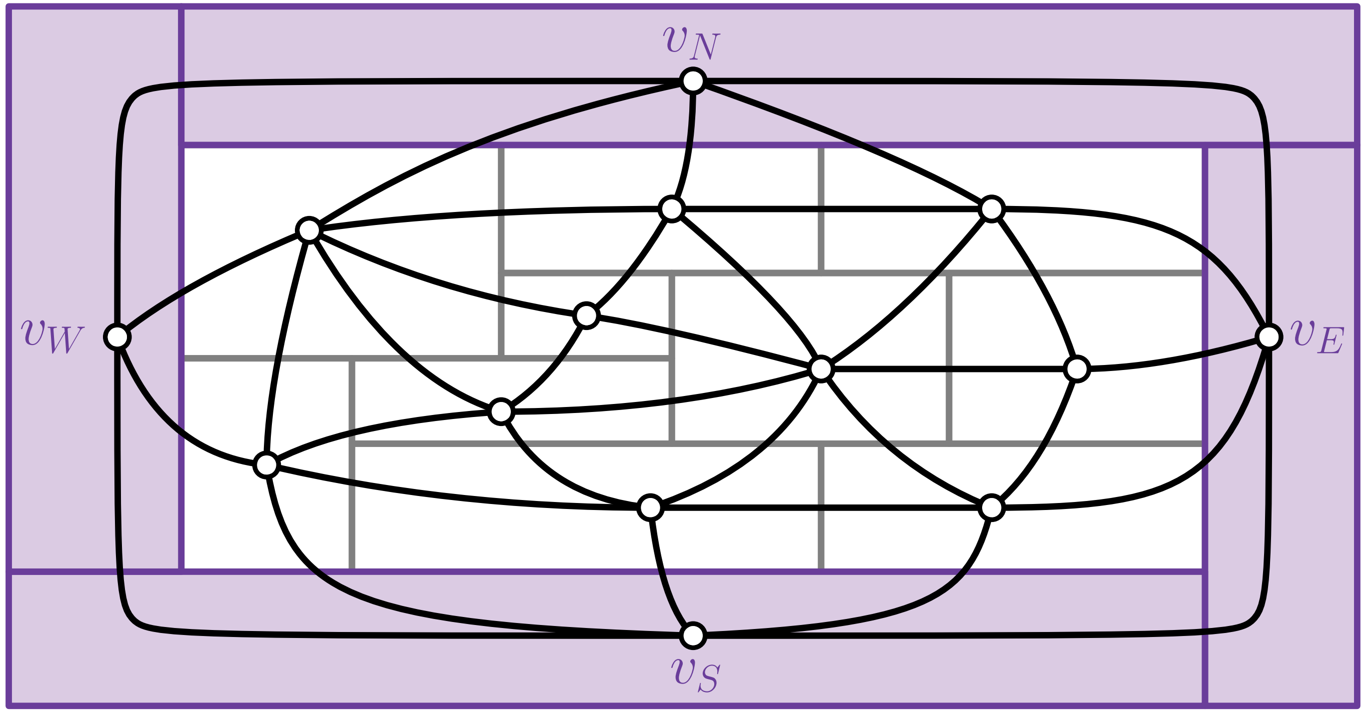

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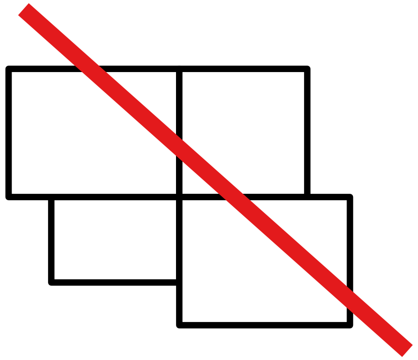

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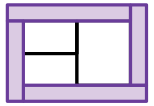

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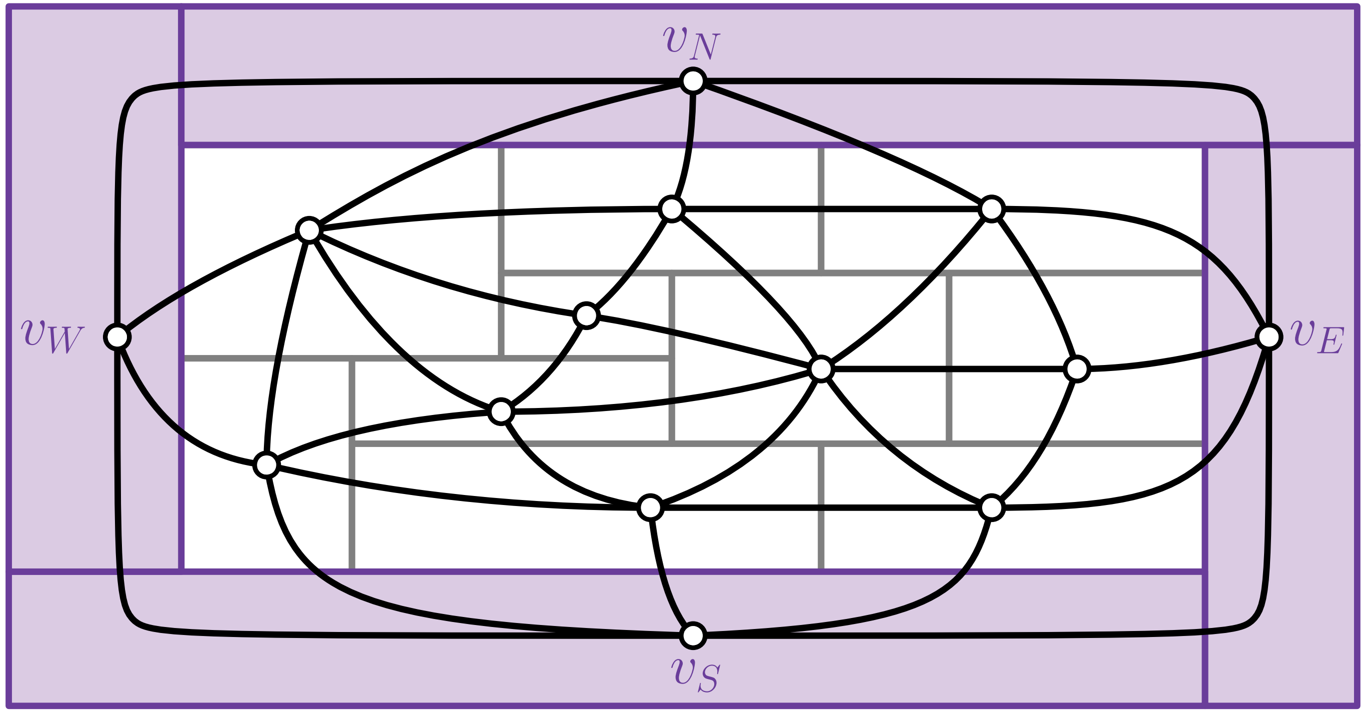

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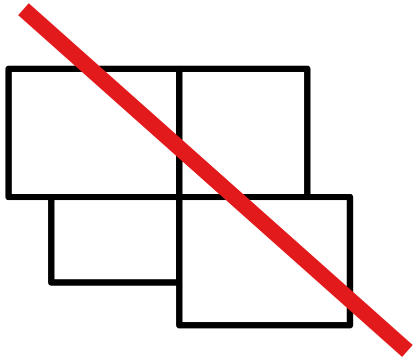

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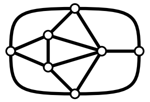
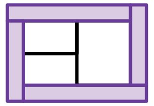
Rectangular Dual


PTP

Properly Triangulated Planar Graph G


RD

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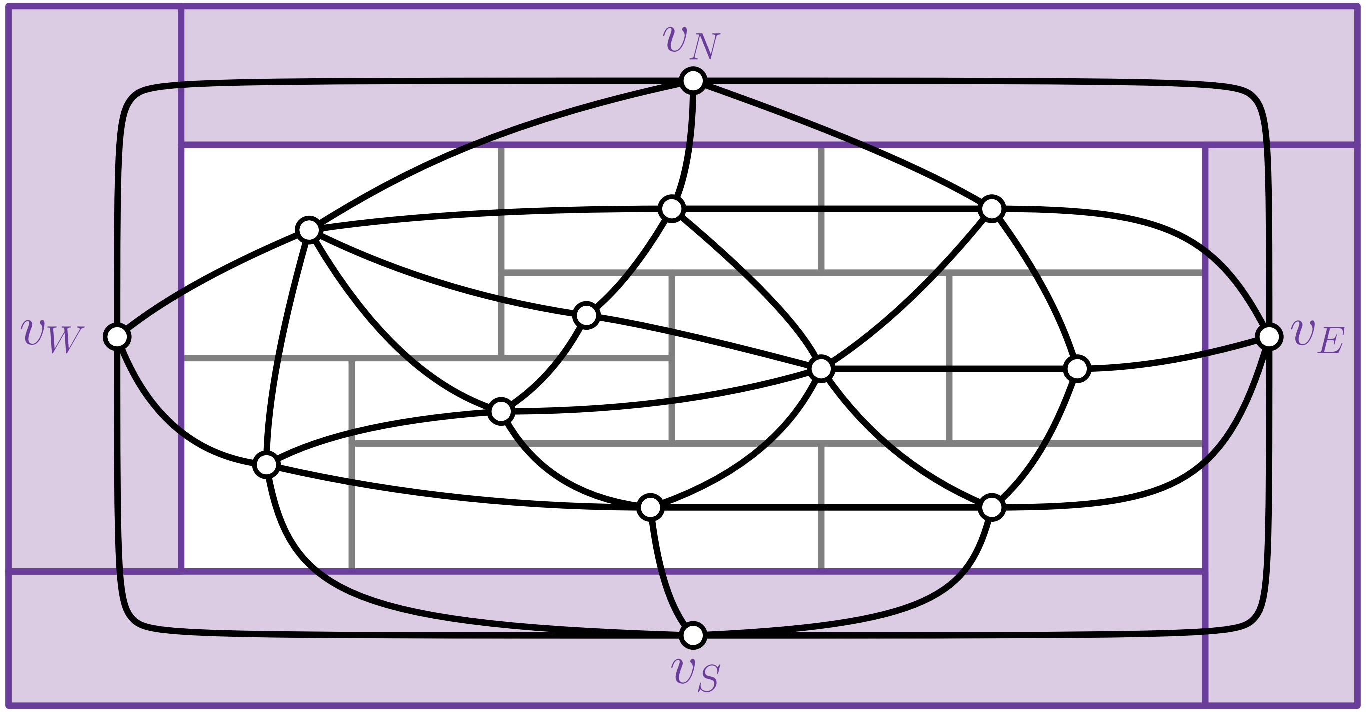

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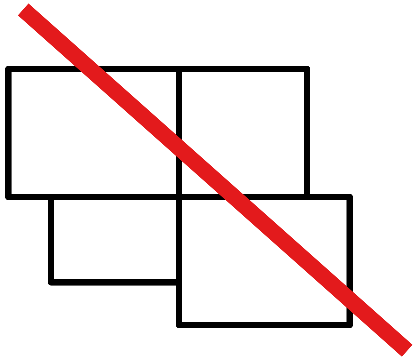

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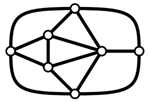

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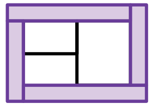

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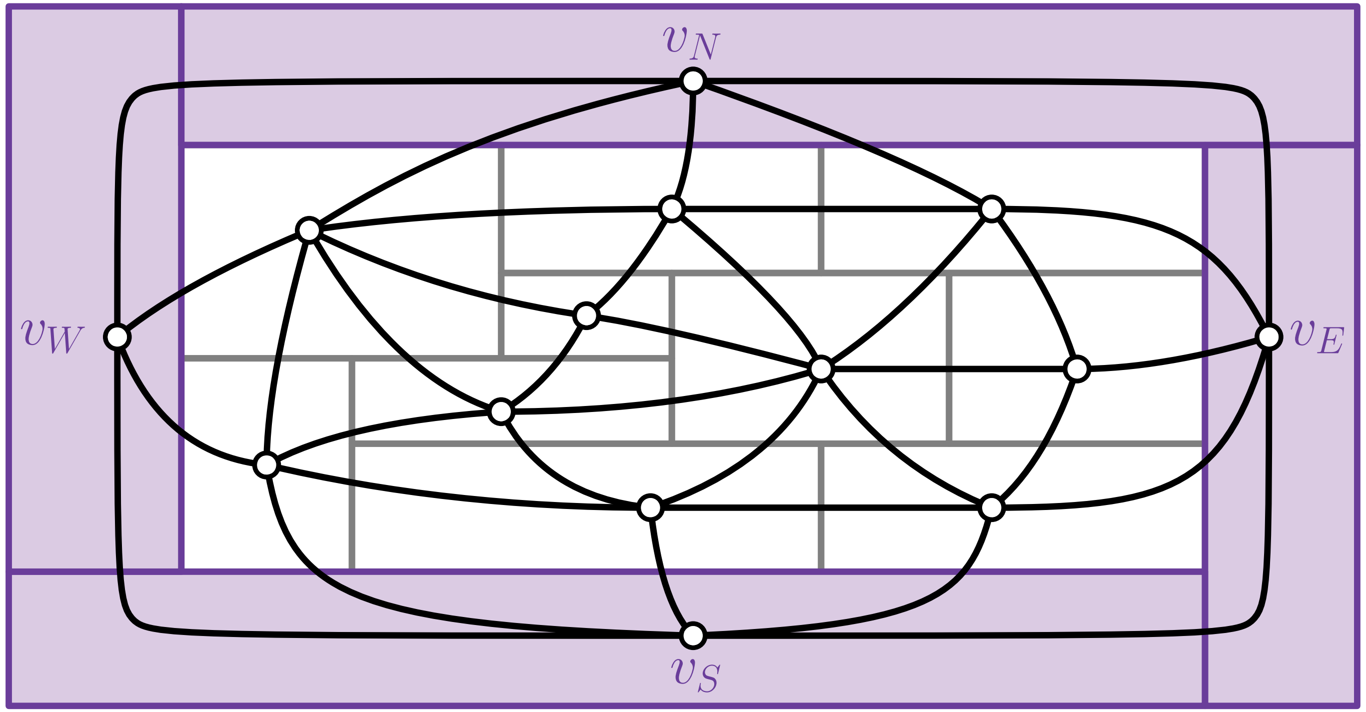

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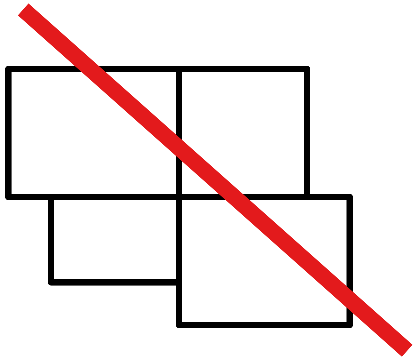

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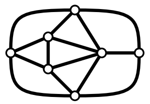
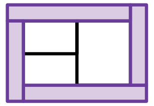


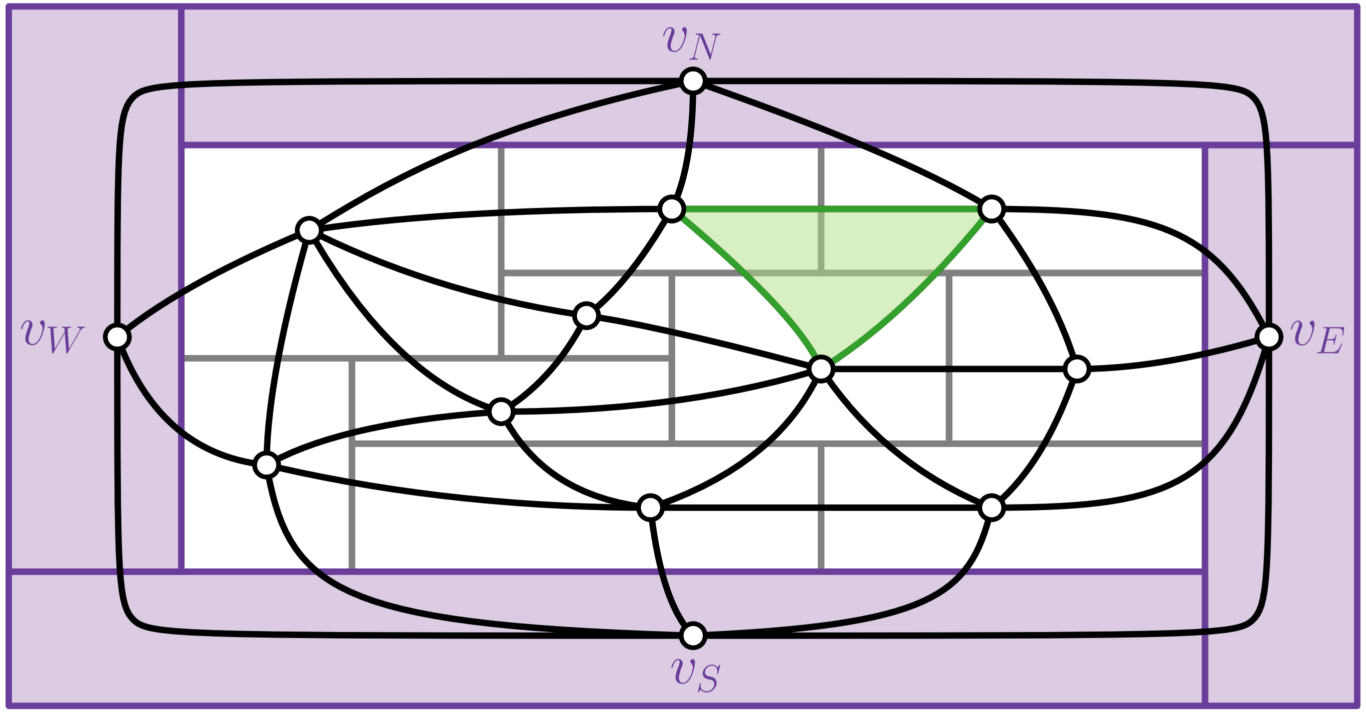
Properly Triangulated Planar Graph G

PTP



Rectangular Dual $\mathcal{R}$

RD



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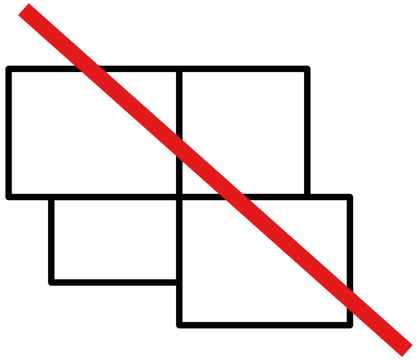

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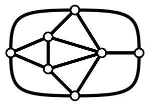

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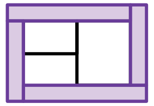

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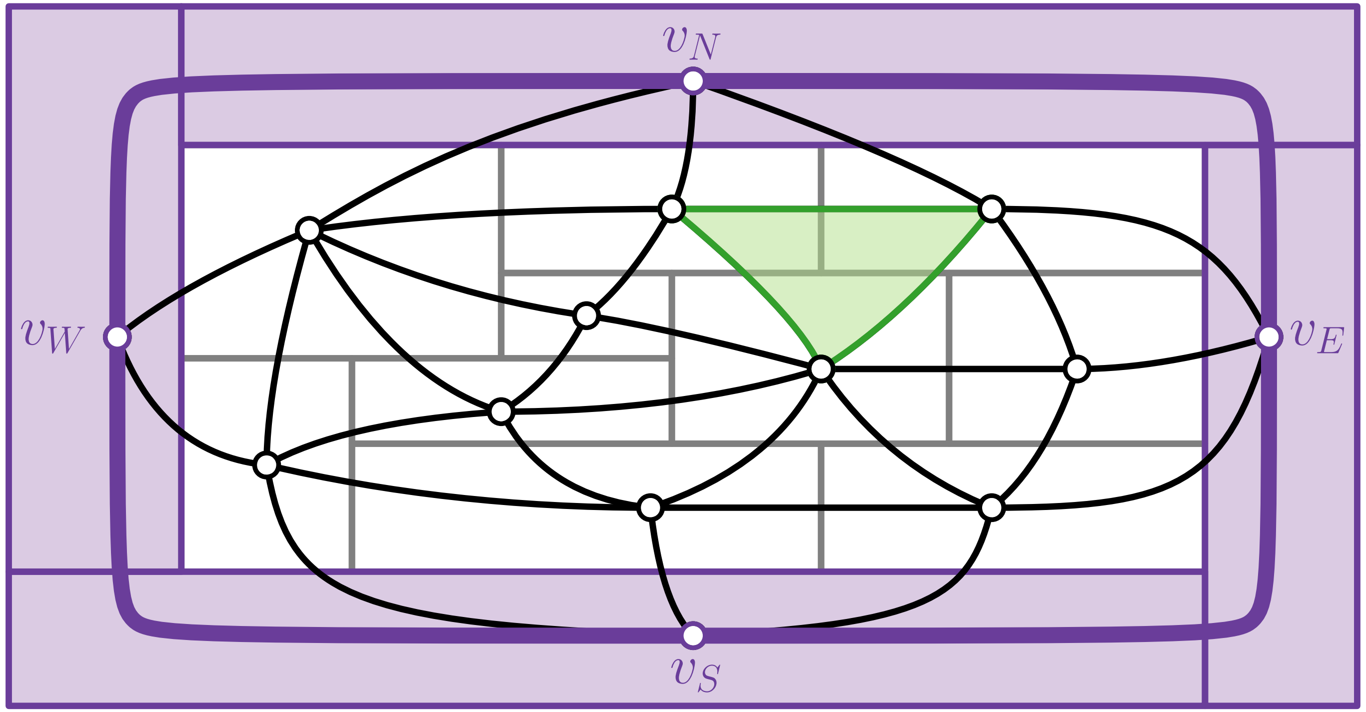

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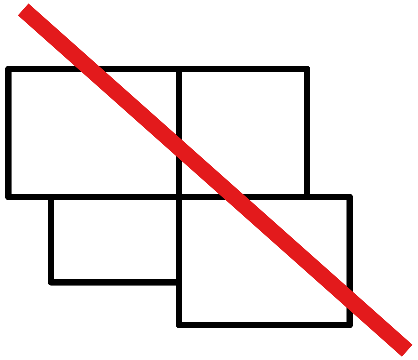

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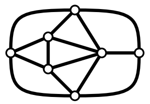
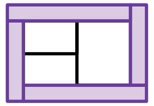
Rectangular Dual

Exactly 4 vertices on outer face



Properly Triangulated Planar Graph G



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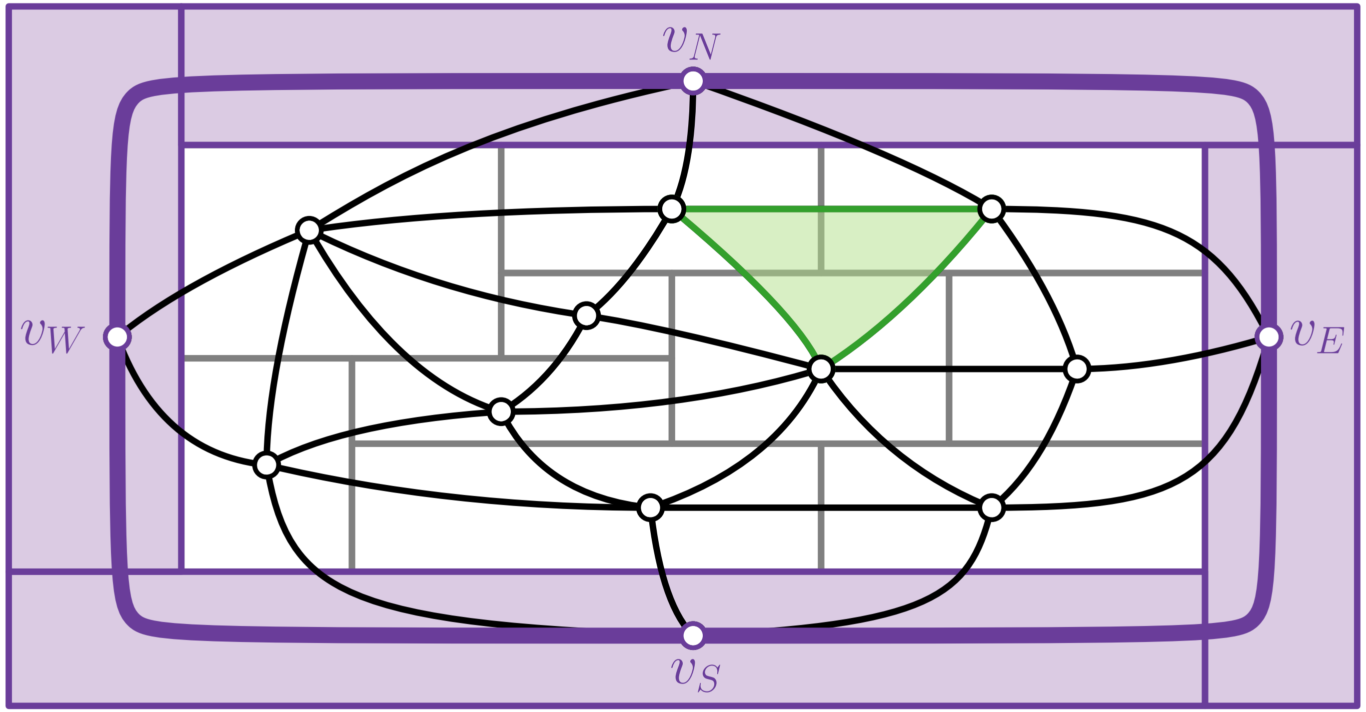

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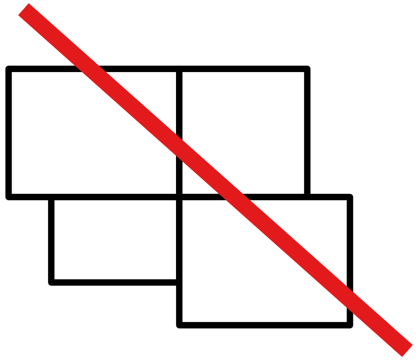

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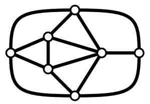

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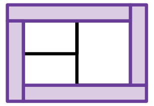

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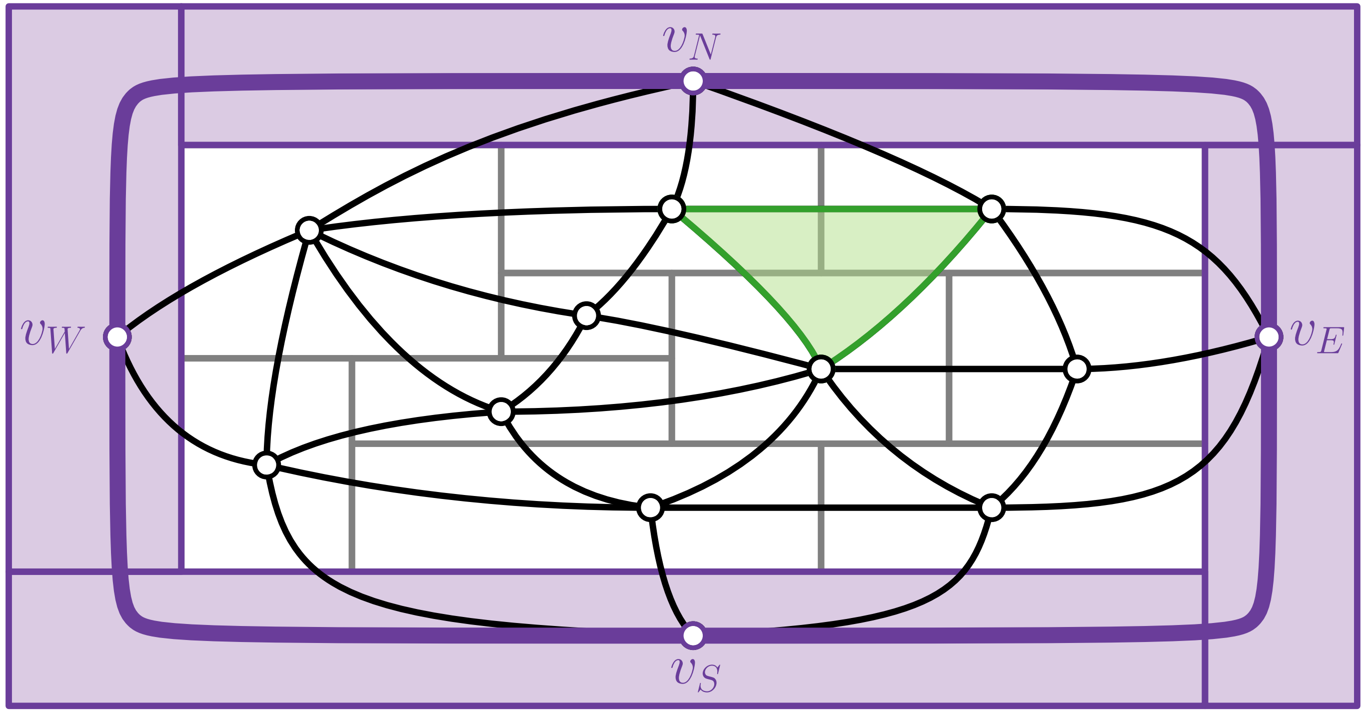

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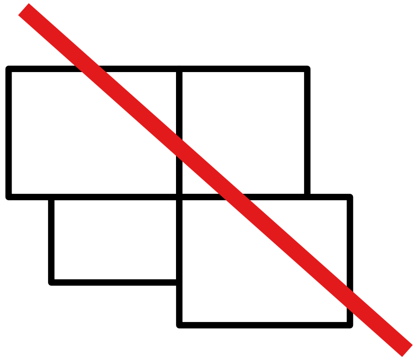

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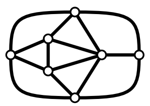

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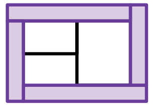

Rectangular Dual $\mathcal{R}$

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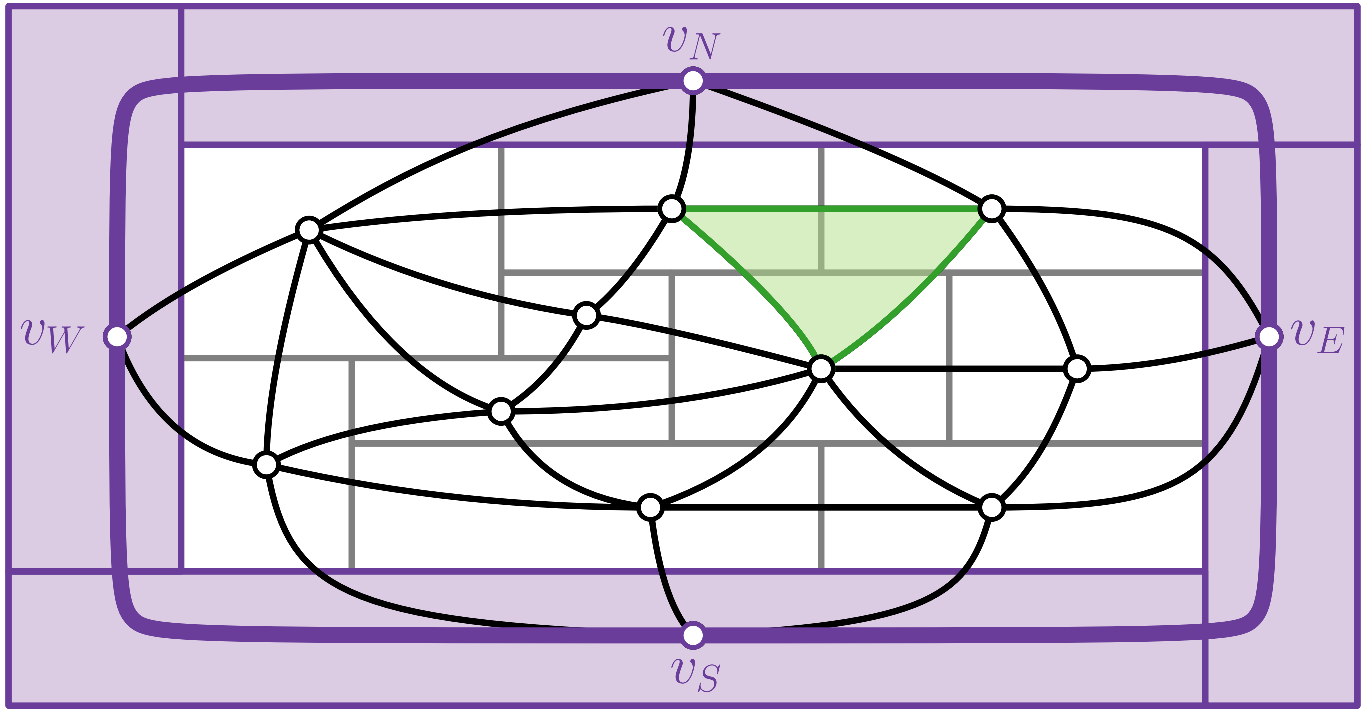

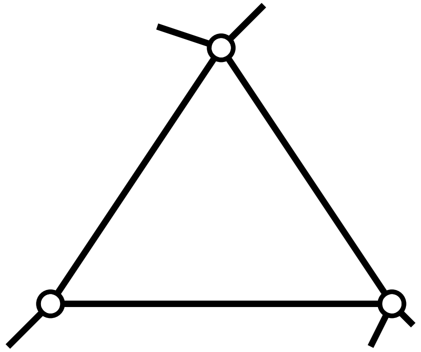

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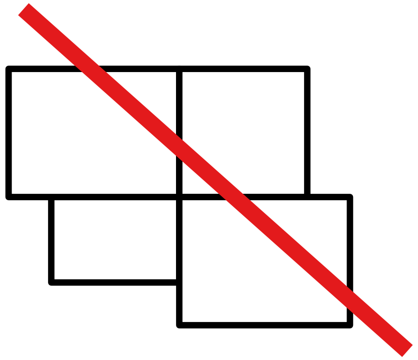

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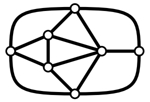
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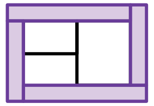
RD

Rectangular Dual $\mathcal{R}$

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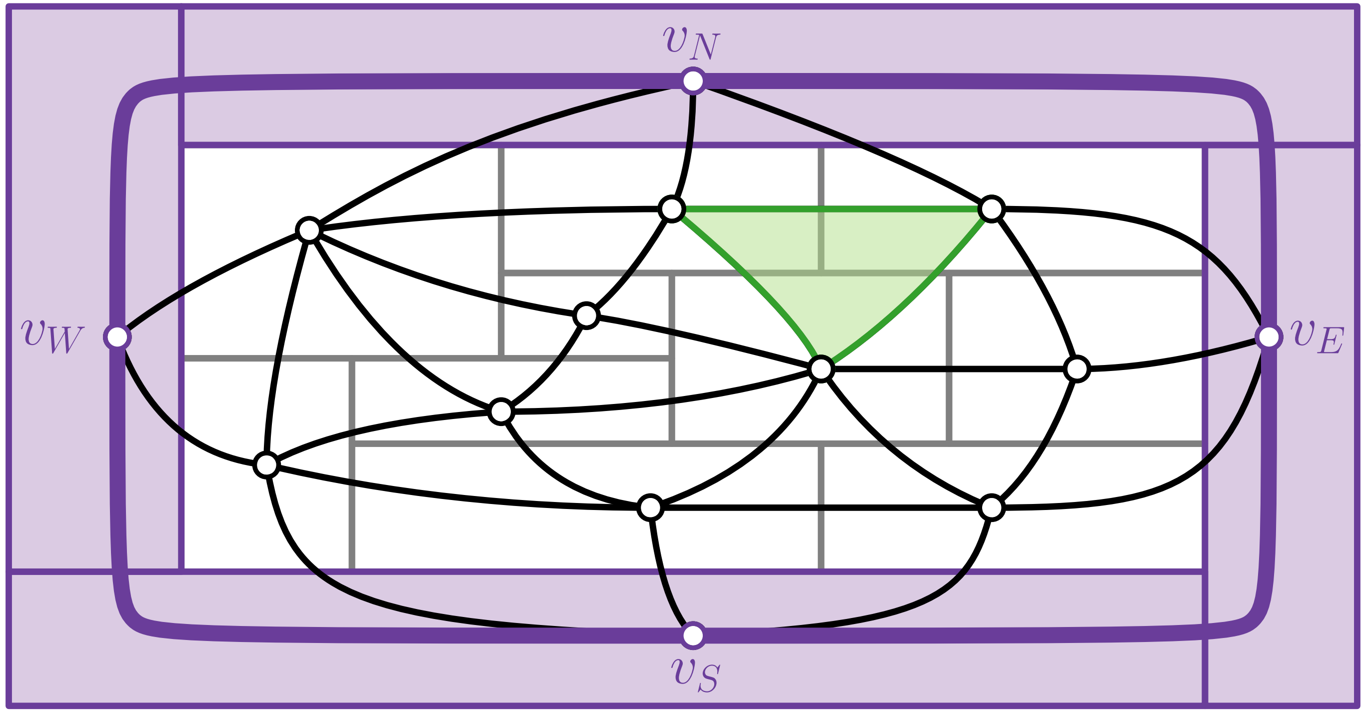

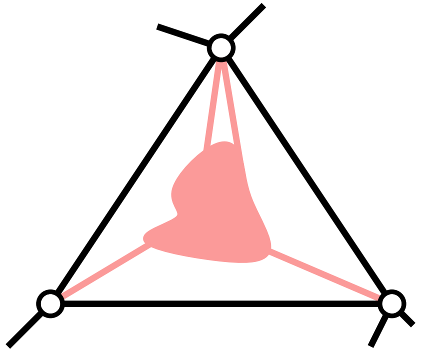

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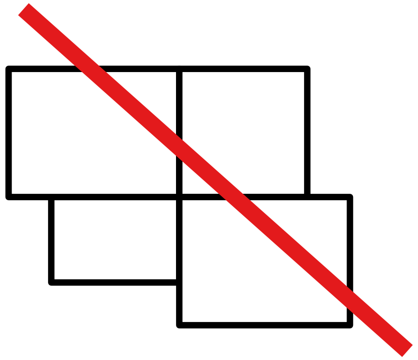

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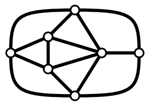
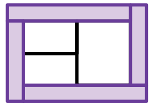


Properly Triangulated Planar Graph G



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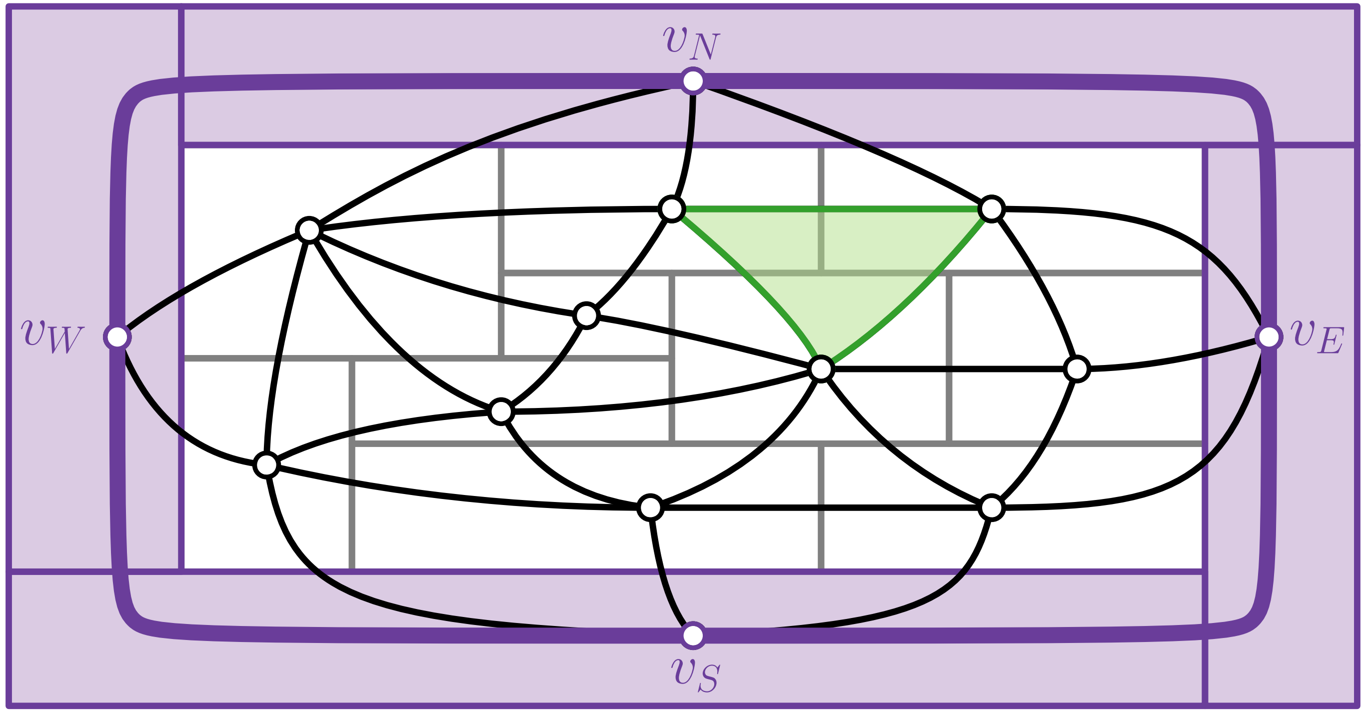

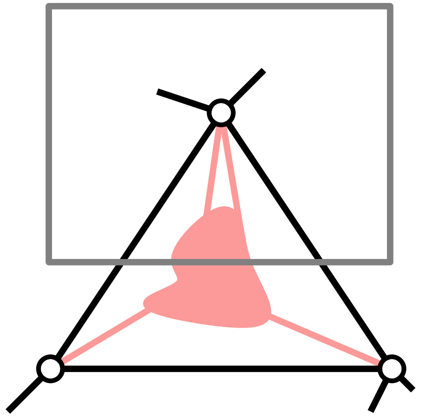

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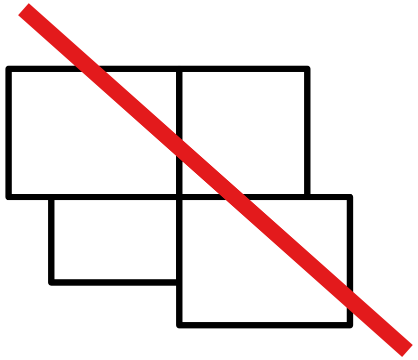

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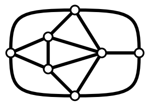

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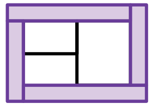

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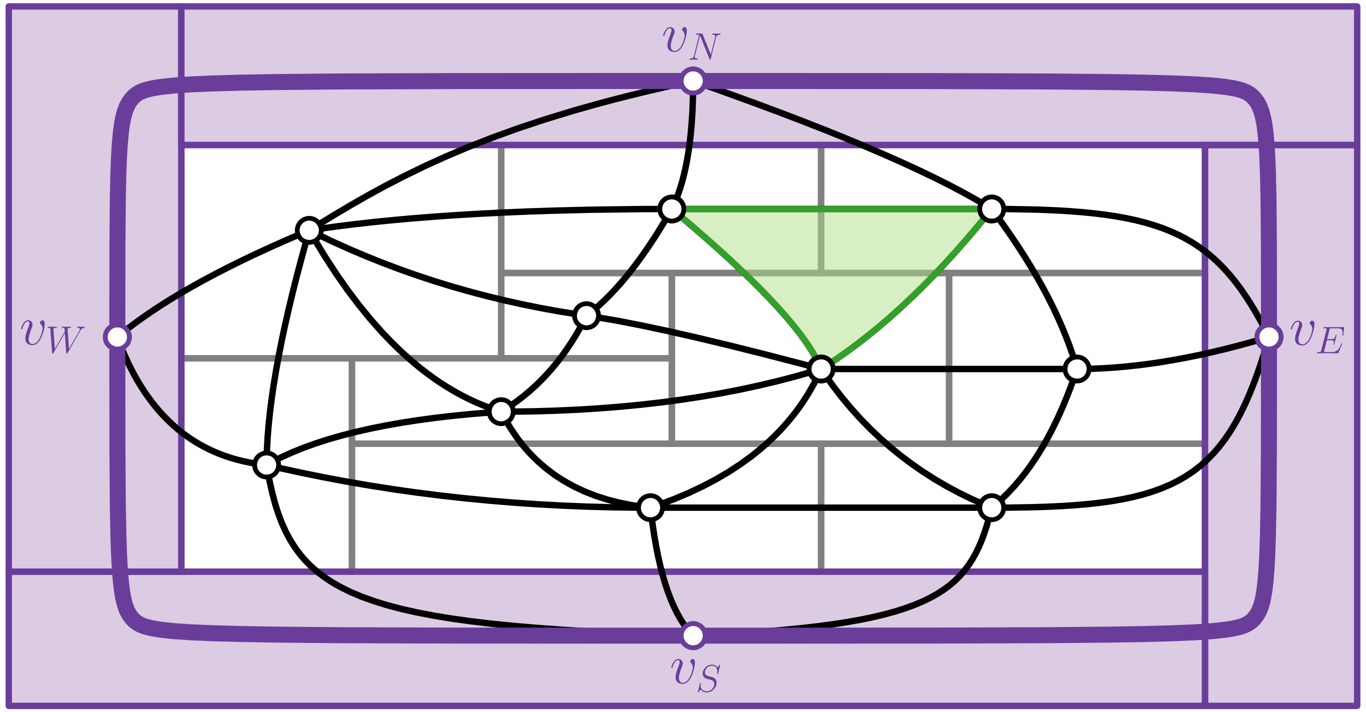

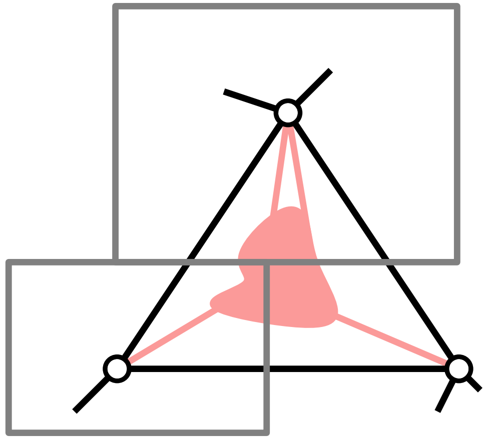

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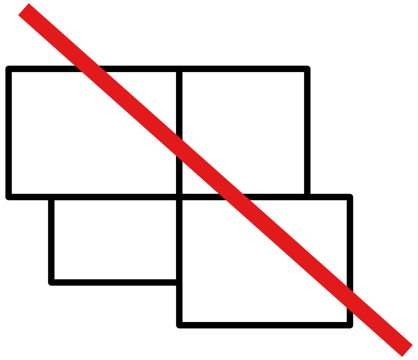

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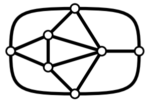
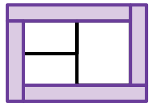
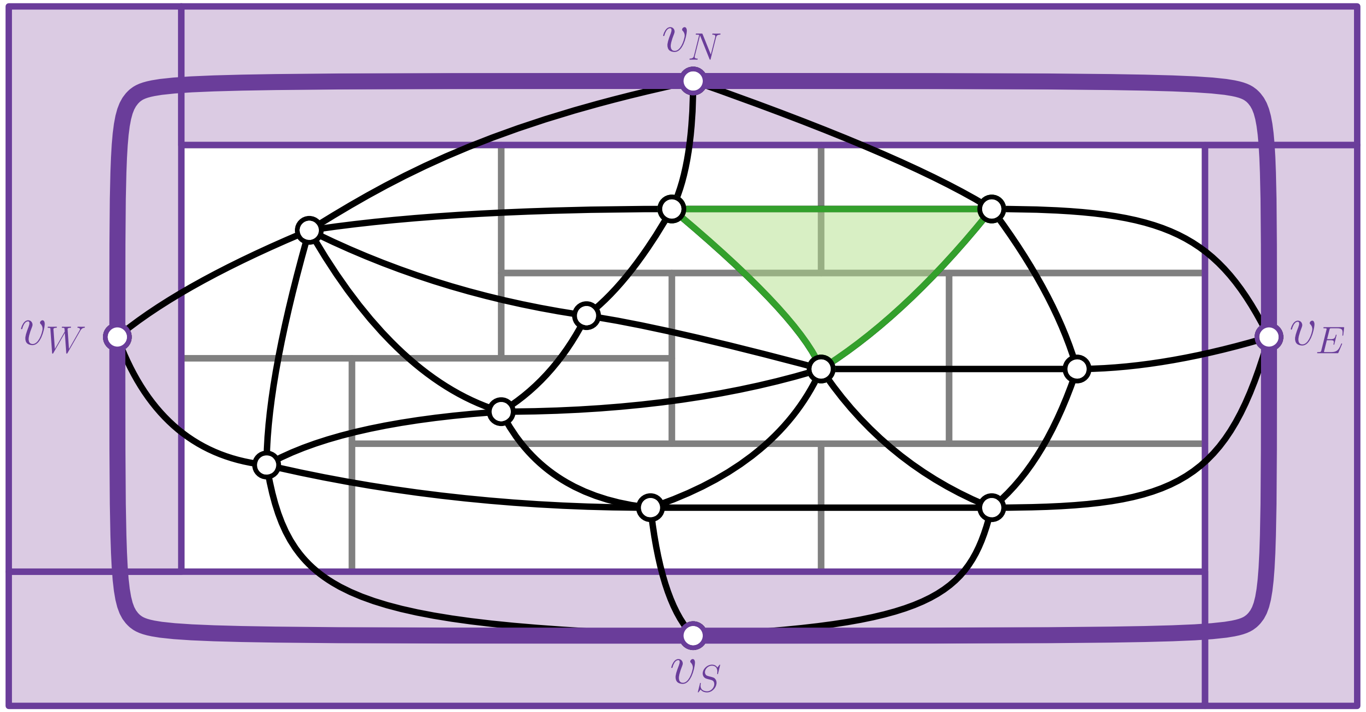
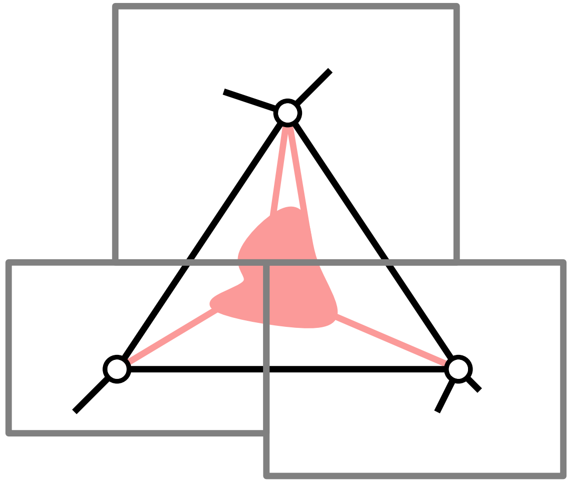
Rectangular Dual





Exactly 4 vertices on outer face

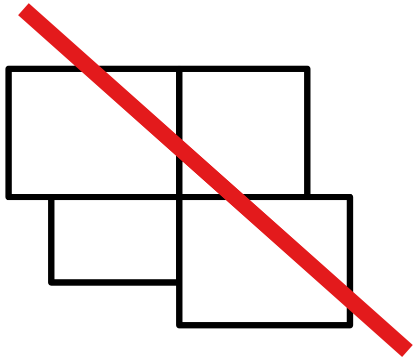




no separating triangle

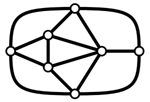
A **rectangular dual** of a graph G is a contact representation with axis-aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle

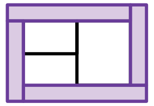


Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

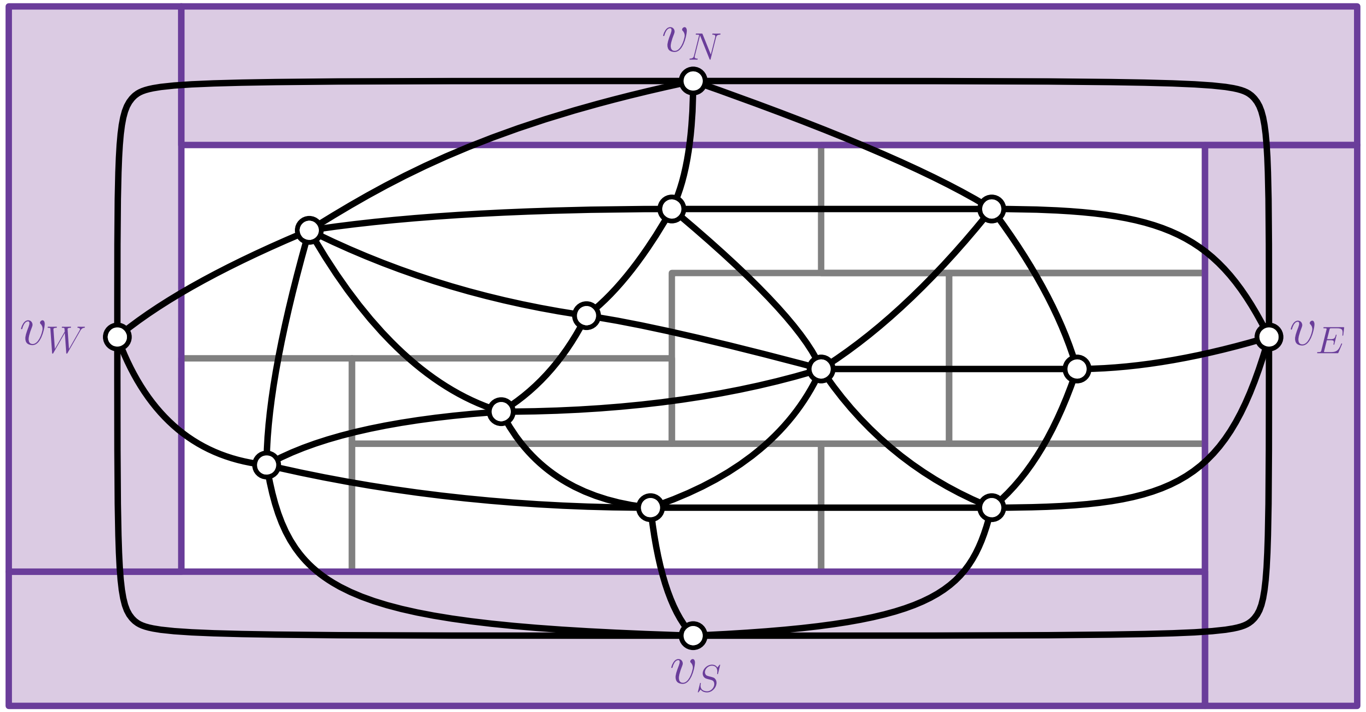
Regular Edge Labeling



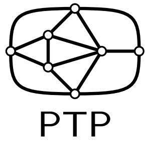
Properly Triangulated Planar Graph G



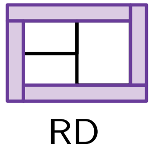
Rectangular Dual $\mathcal{R}$



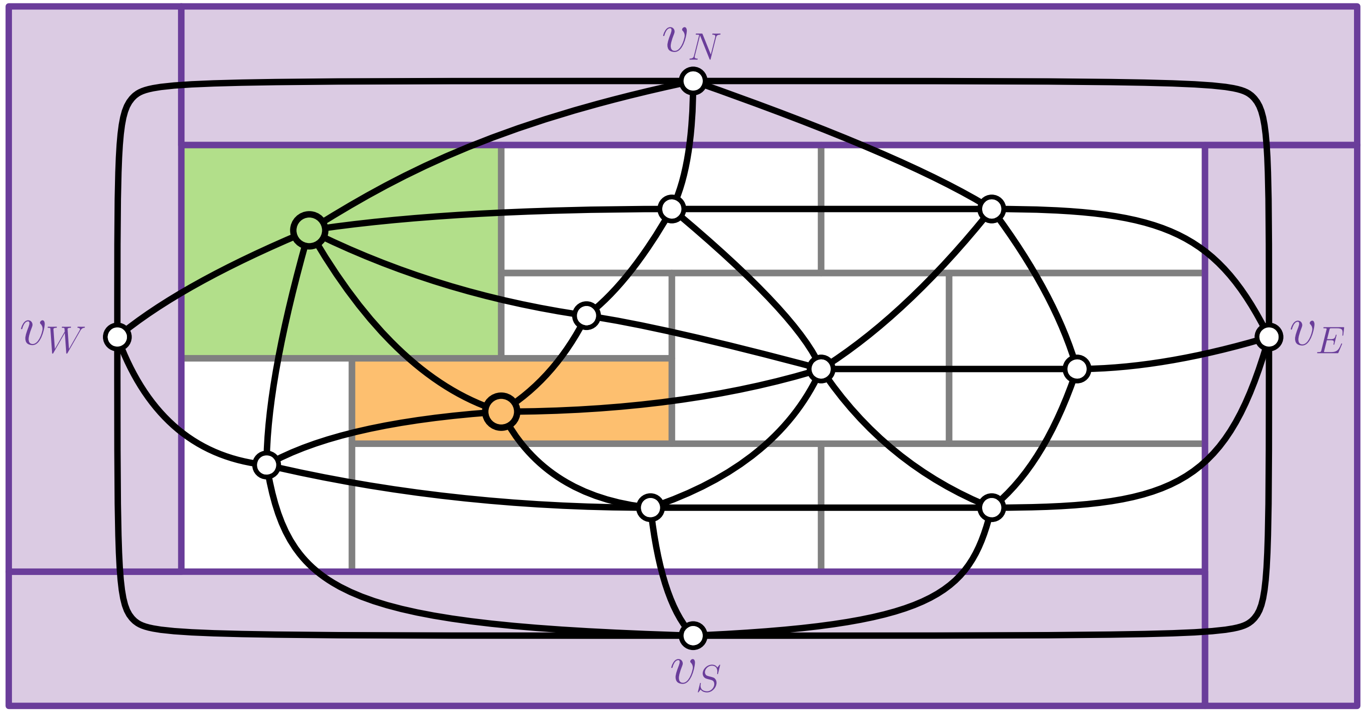
Regular Edge Labeling



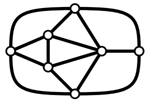
Properly Triangulated Planar Graph G



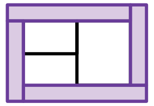
Rectangular Dual $\mathcal{R}$



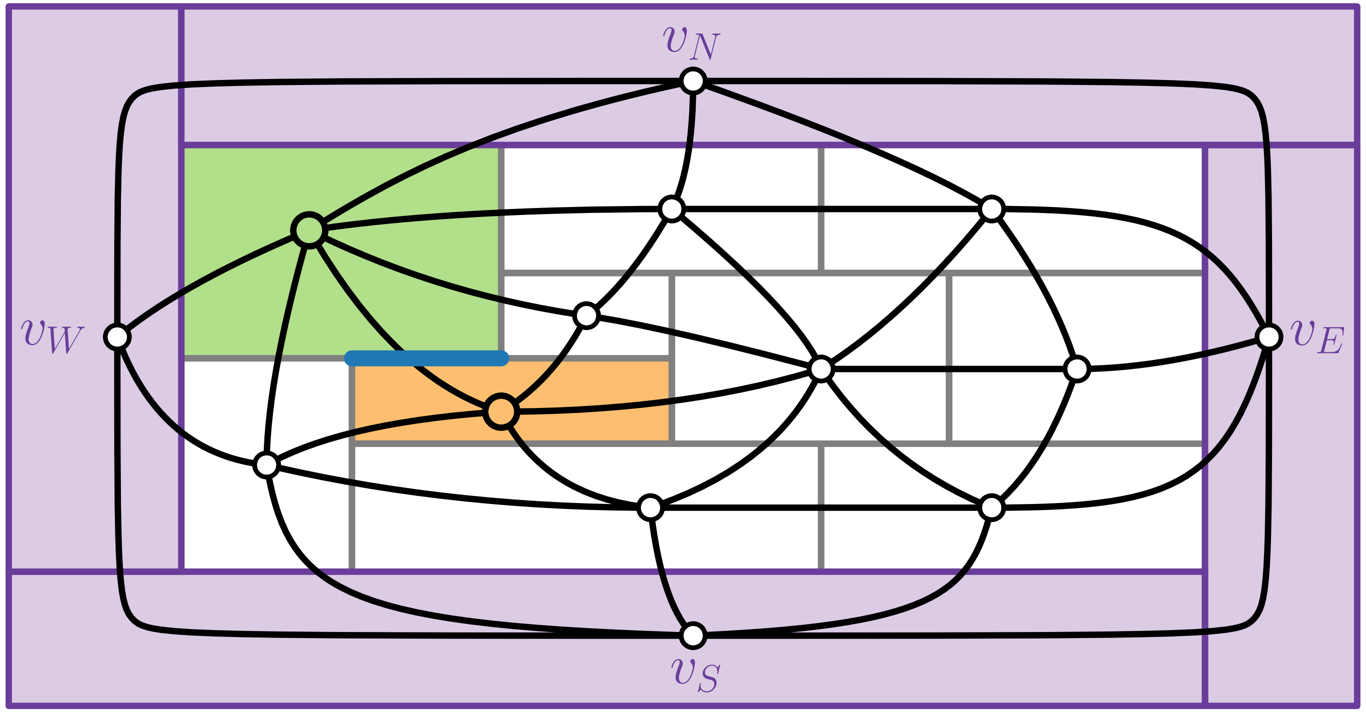
Regular Edge Labeling



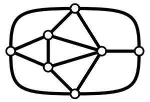
Properly Triangulated Planar Graph G



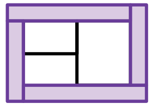
Rectangular Dual $\mathcal{R}$



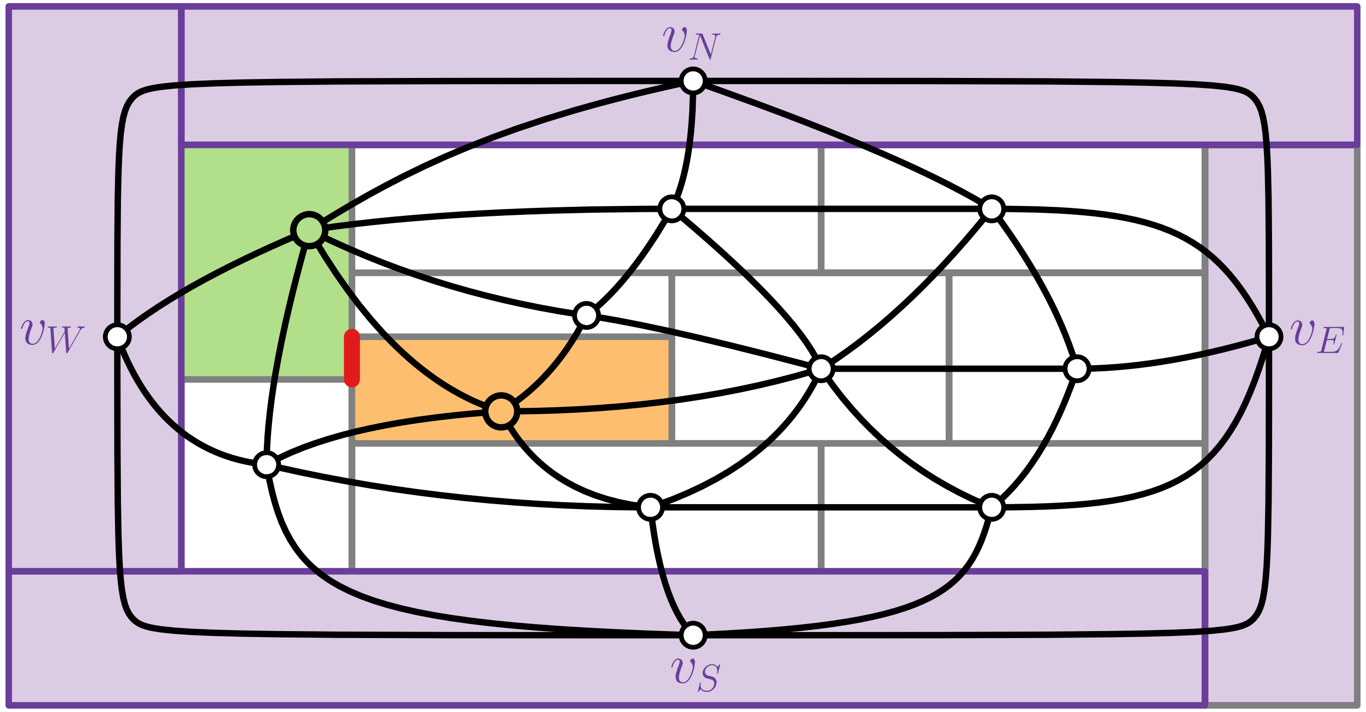
Regular Edge Labeling



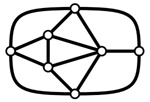
Properly Triangulated Planar Graph G



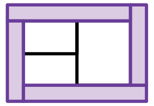
Rectangular Dual $\mathcal{R}$



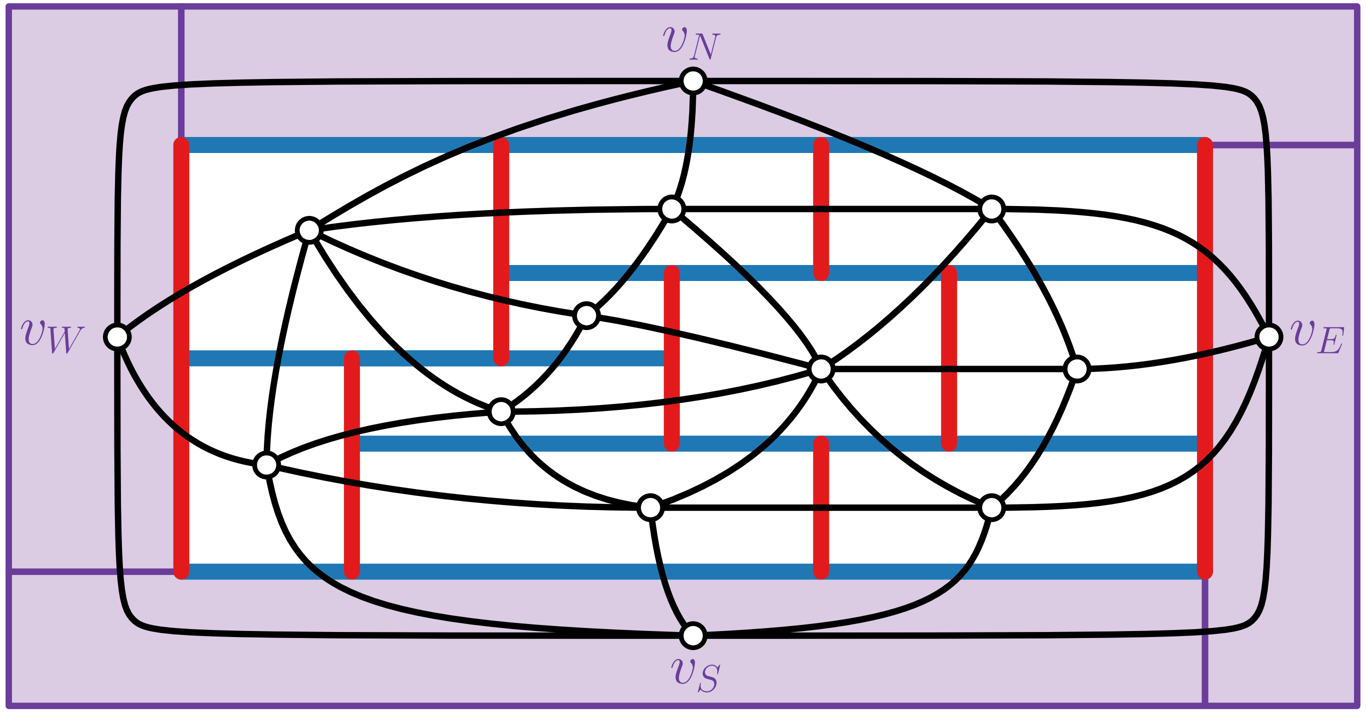
Regular Edge Labeling



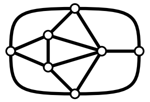
Properly Triangulated Planar Graph G



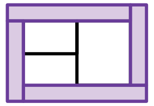
Rectangular Dual $\mathcal{R}$



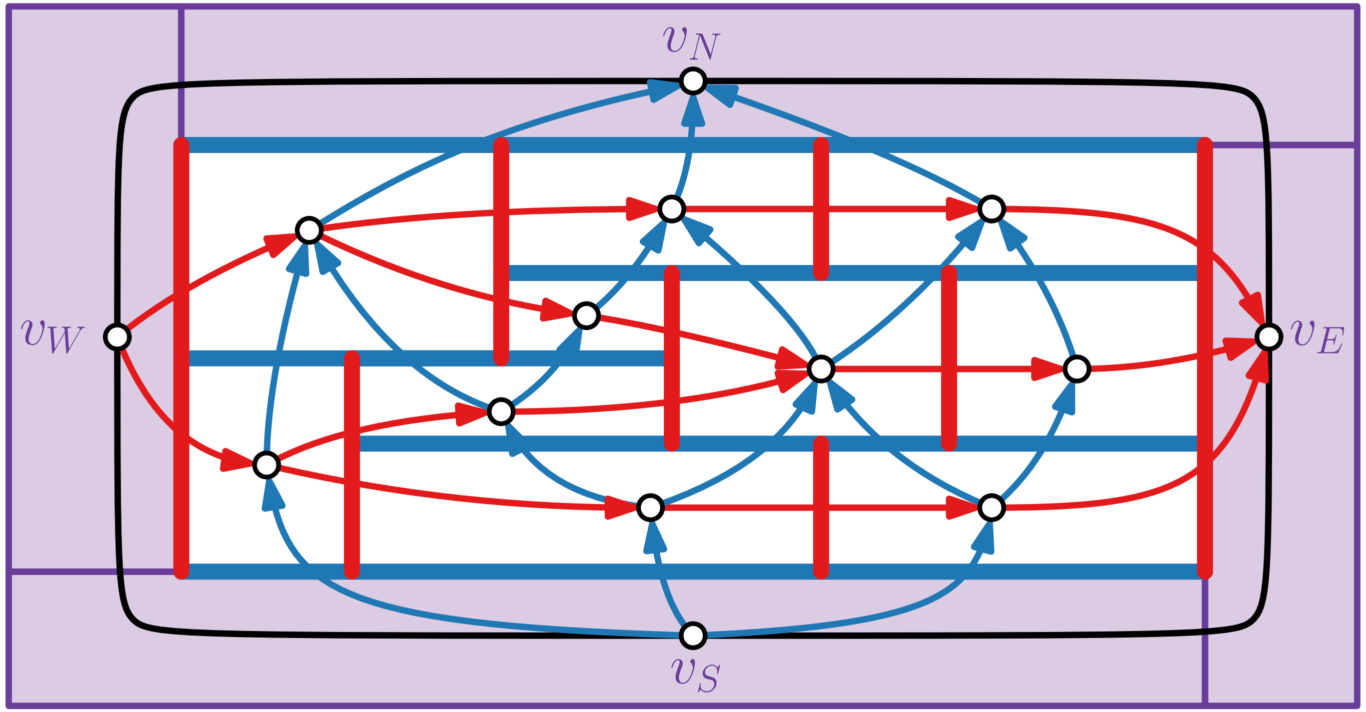
Regular Edge Labeling



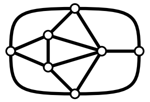
Properly Triangulated Planar Graph G



Rectangular Dual $\mathcal{R}$

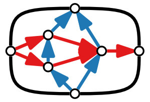


Regular Edge Labeling



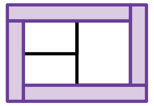
PTP

Properly Triangulated Planar Graph G



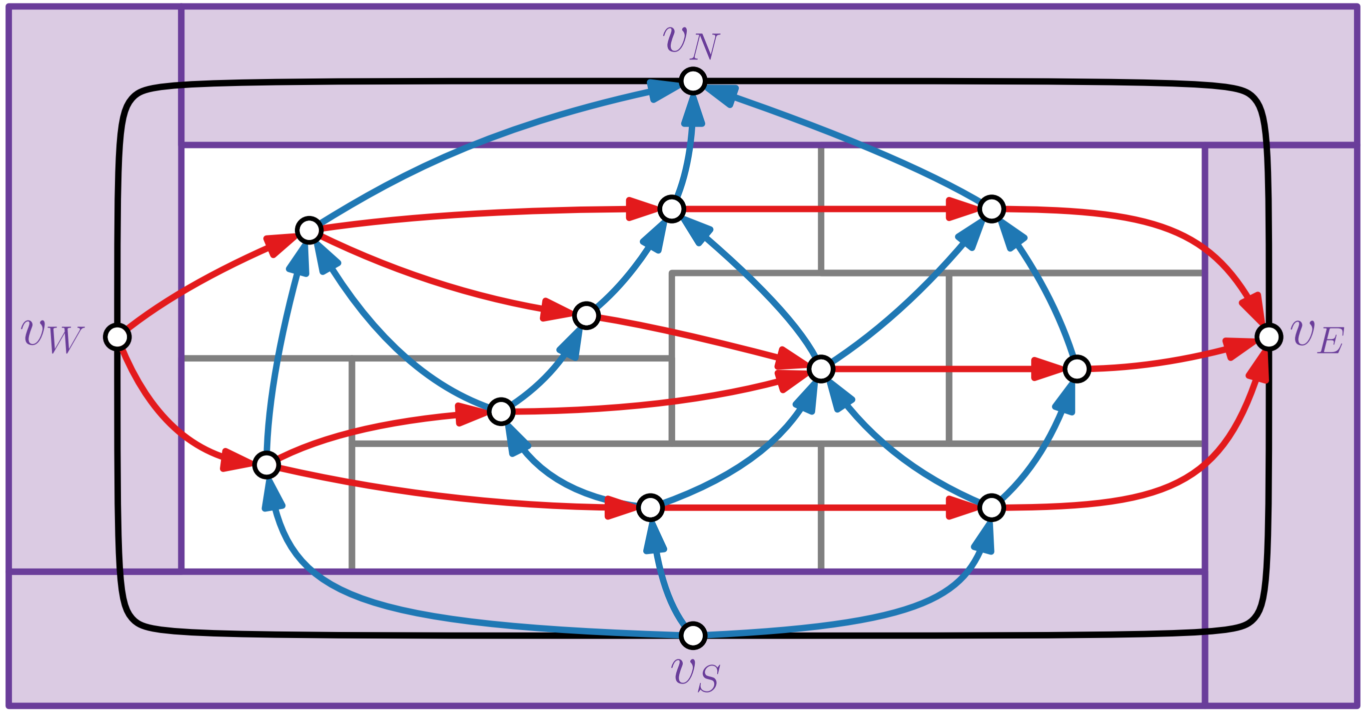
REL

Regular Edge Labeling

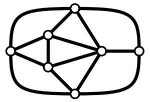


RD

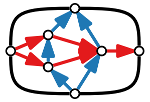
Rectangular Dual $\mathcal{R}$



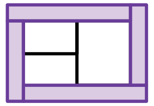
Regular Edge Labeling



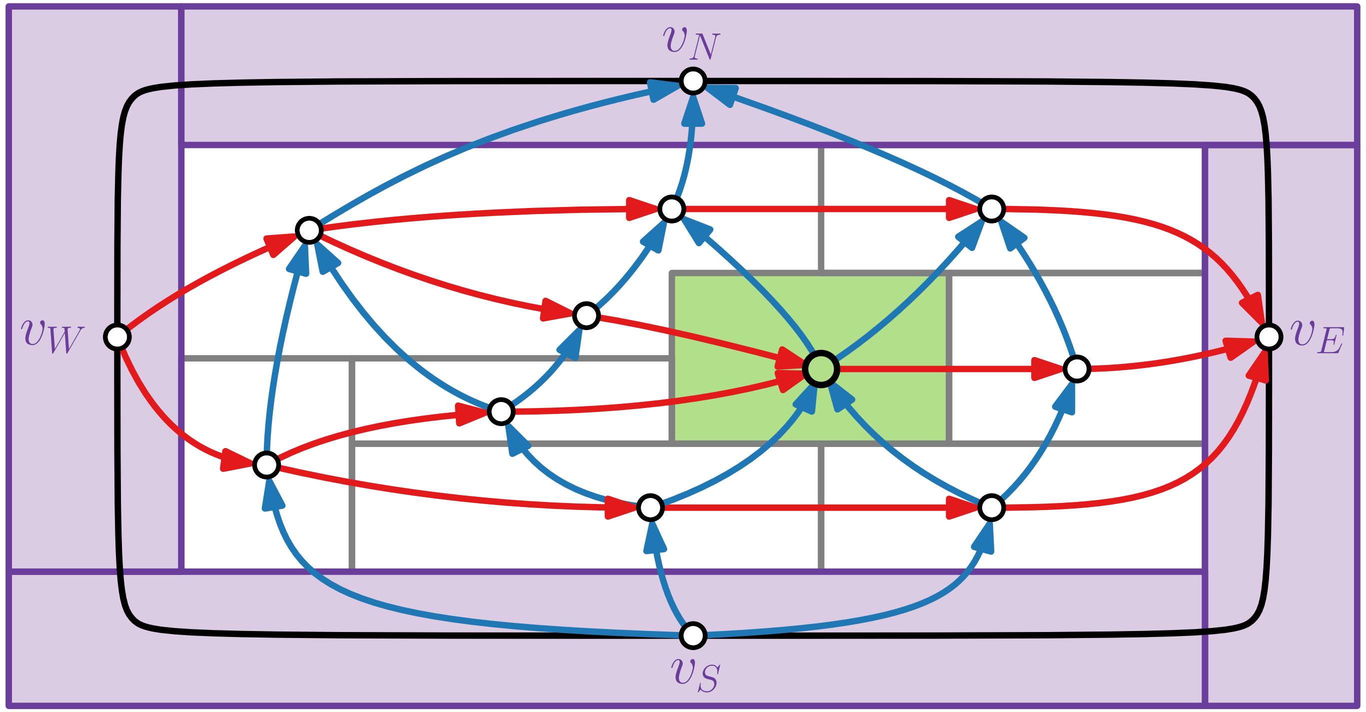
Properly Triangulated Planar Graph G



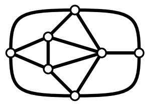
Regular Edge Labeling



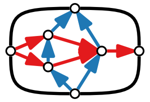
Rectangular Dual $\mathcal{R}$



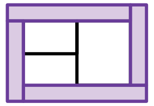
Regular Edge Labeling



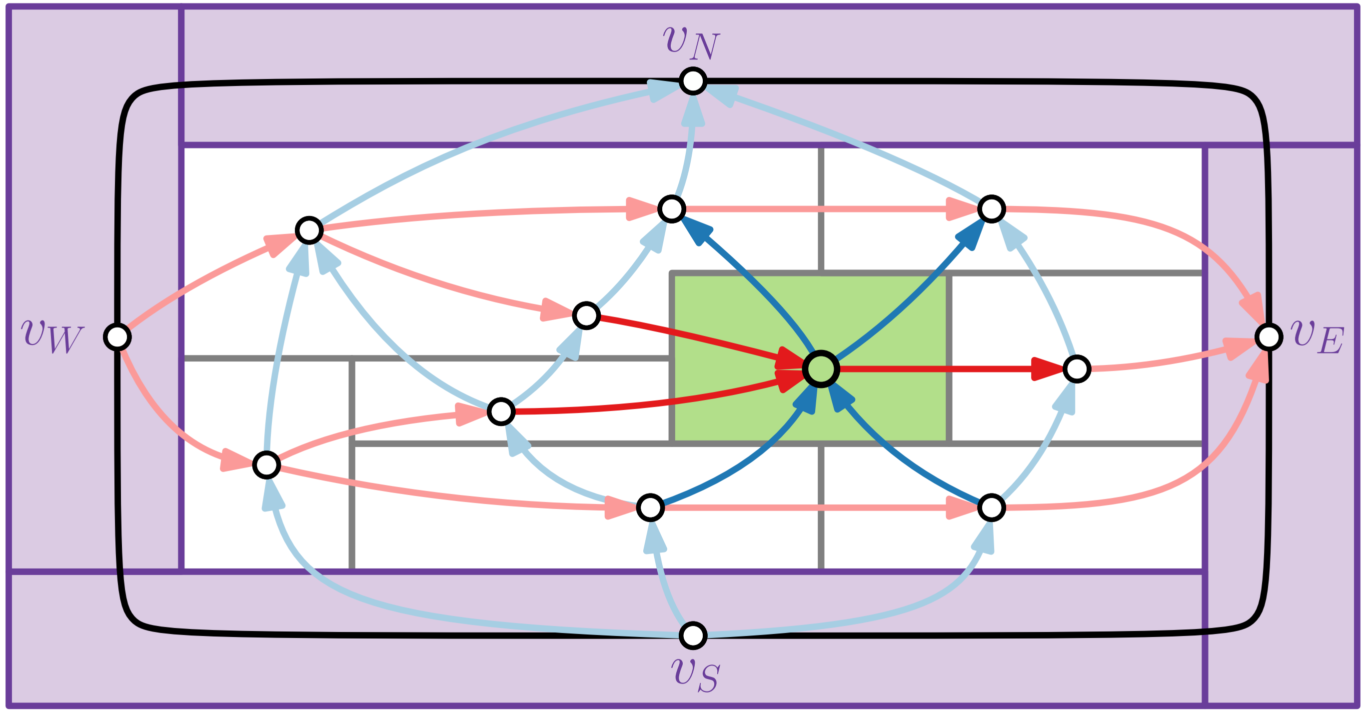
Properly Triangulated Planar Graph G



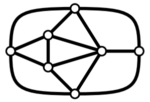
Regular Edge Labeling



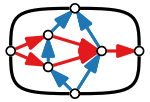
Rectangular Dual $\mathcal{R}$



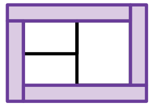
Regular Edge Labeling



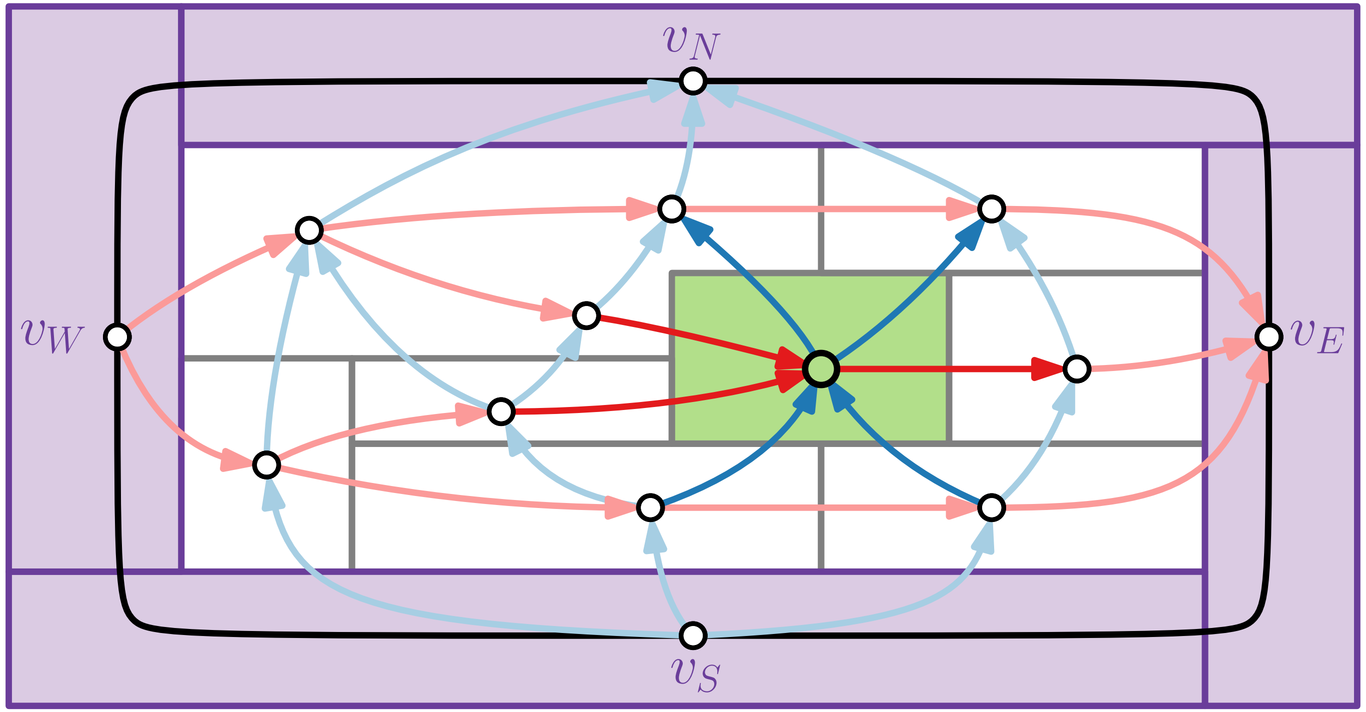
Properly Triangulated Planar Graph G

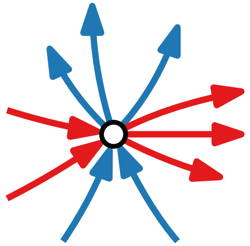


Regular Edge Labeling



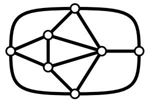
Rectangular Dual $\mathcal{R}$



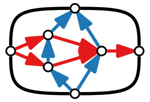


for every inner vertex

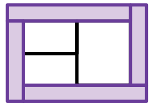
Regular Edge Labeling



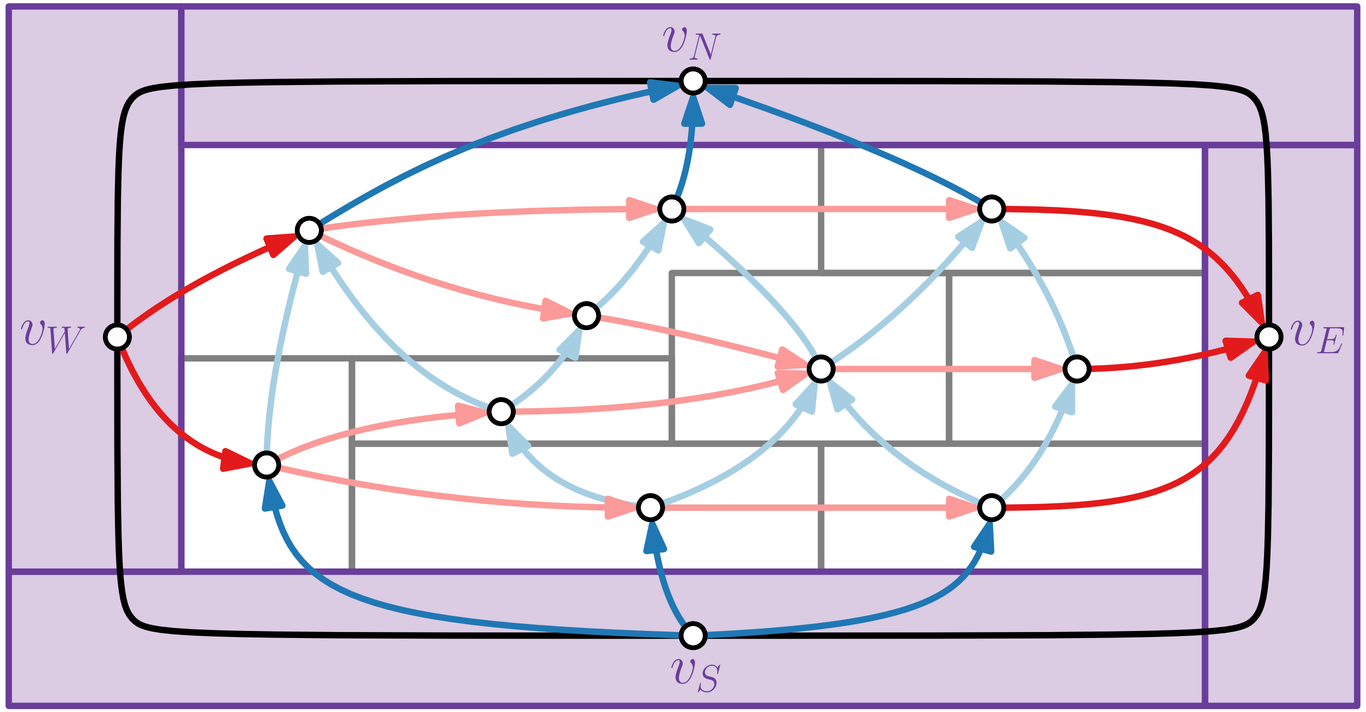
Properly Triangulated Planar Graph G

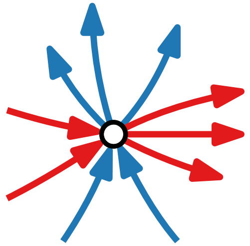


Regular Edge Labeling



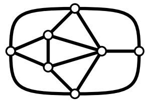
Rectangular Dual $\mathcal{R}$



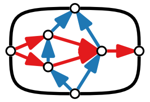


for every inner vertex

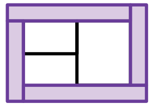
Regular Edge Labeling



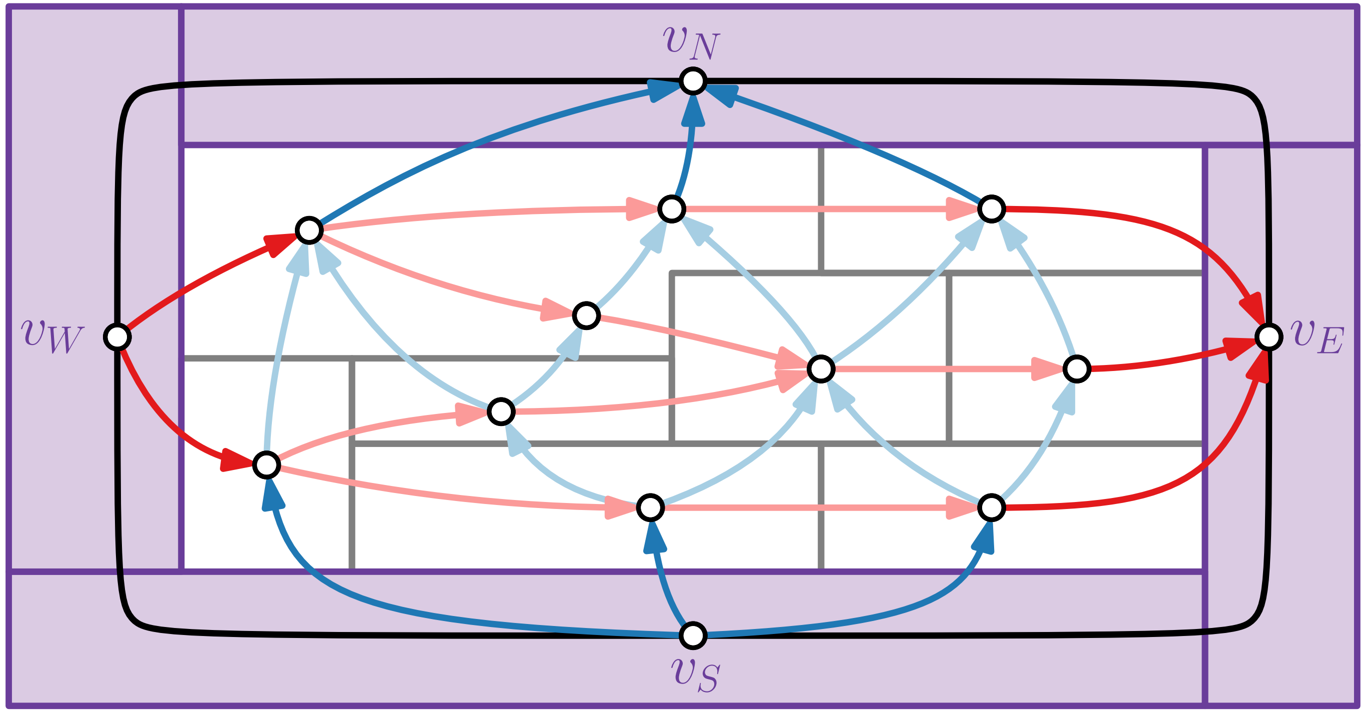
Properly Triangulated Planar Graph G

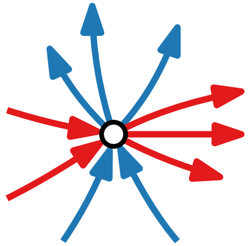


Regular Edge Labeling

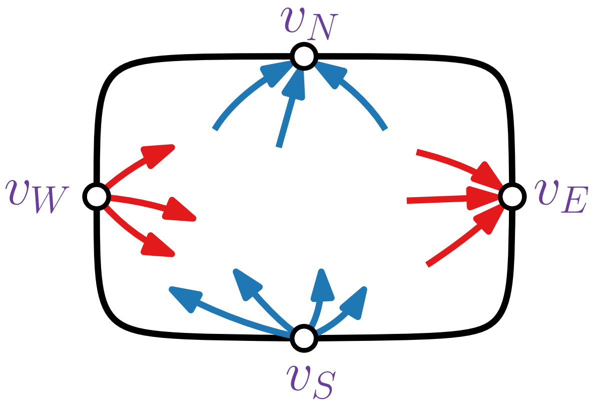


Rectangular Dual $\mathcal{R}$



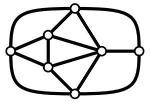


for every inner vertex

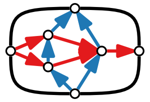


for four outer vertices

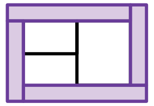
Regular Edge Labeling



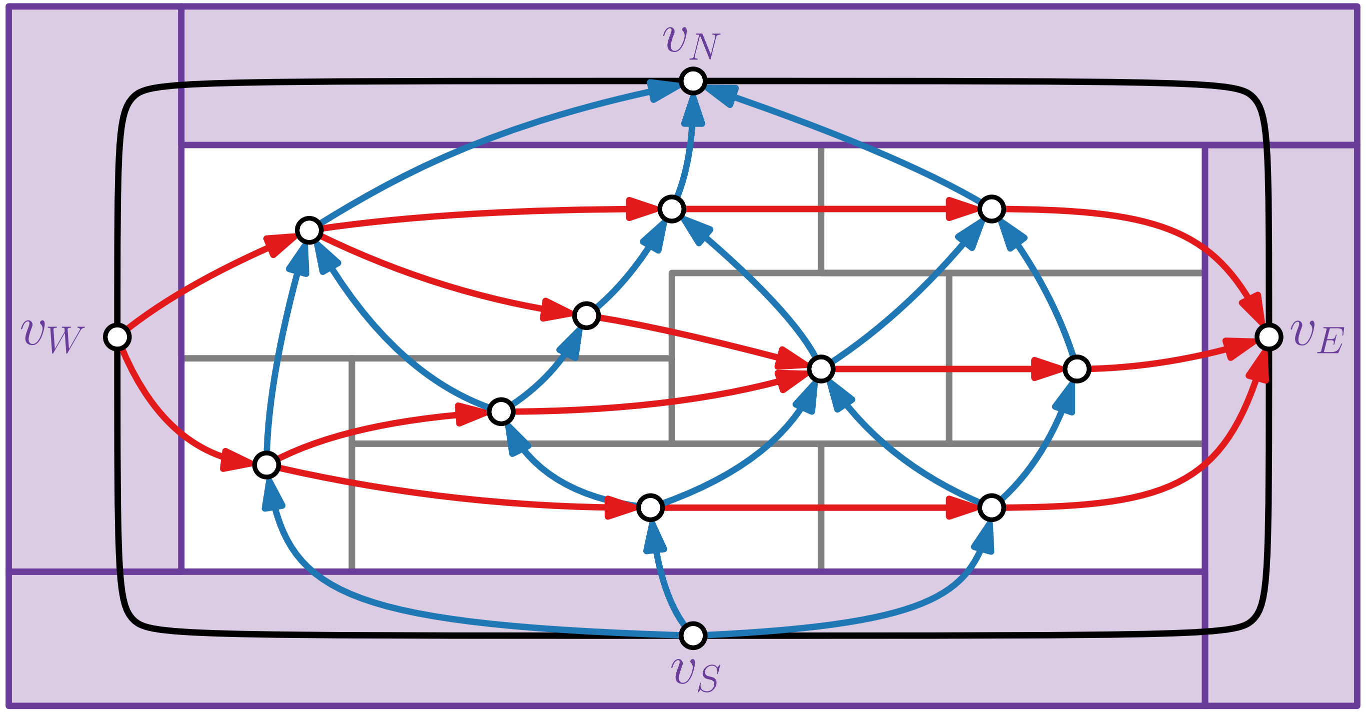
Properly Triangulated Planar Graph G

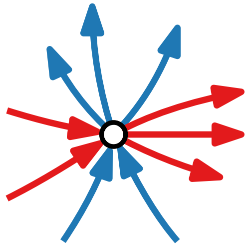


Regular Edge Labeling

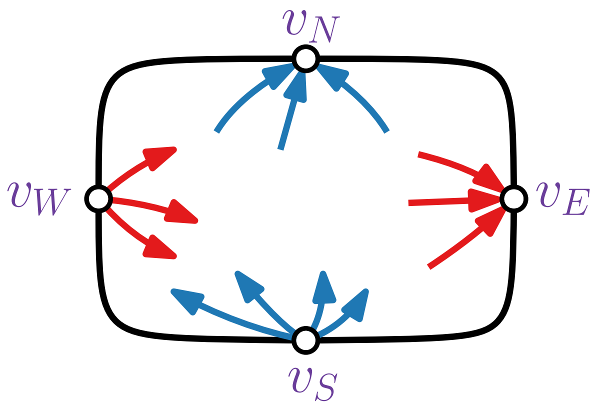


Rectangular Dual $\mathcal{R}$



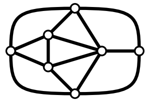


for every inner vertex



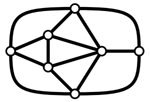
for four outer vertices

[Kant, He '94]: In linear time

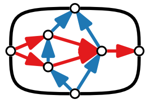


PTP

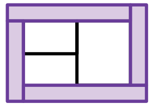
Regular Edge Labeling


PTP

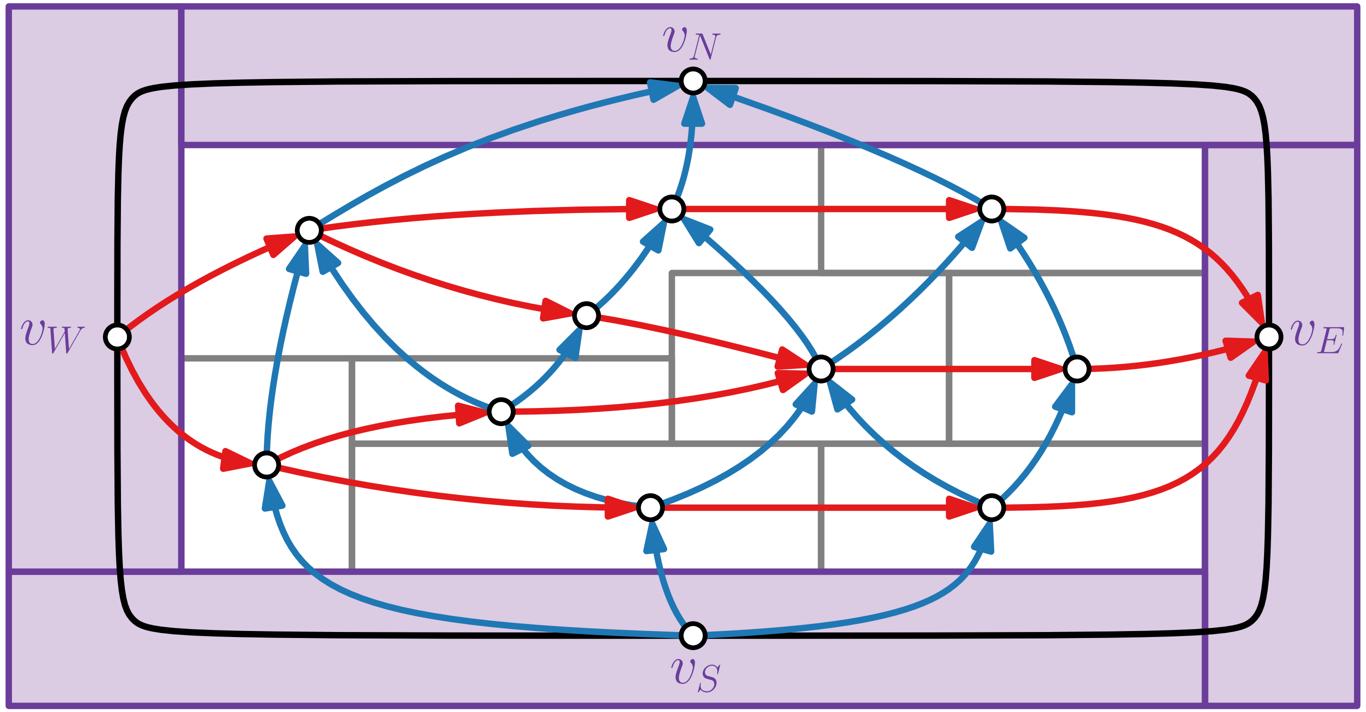
Properly Triangulated Planar Graph G

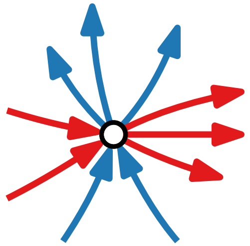

REL

Regular Edge Labeling

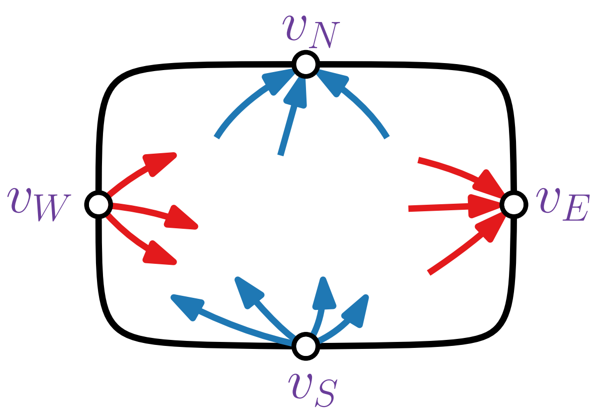

RD

Rectangular Dual $\mathcal{R}$



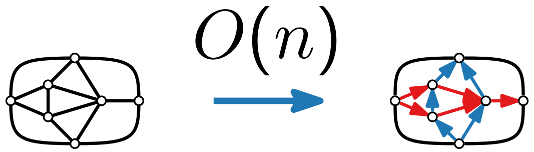


for every inner vertex



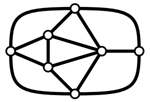
for four outer vertices

[Kant, He '94]: In linear time

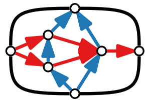


PTP REL

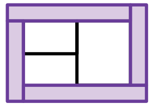
Regular Edge Labeling


PTP

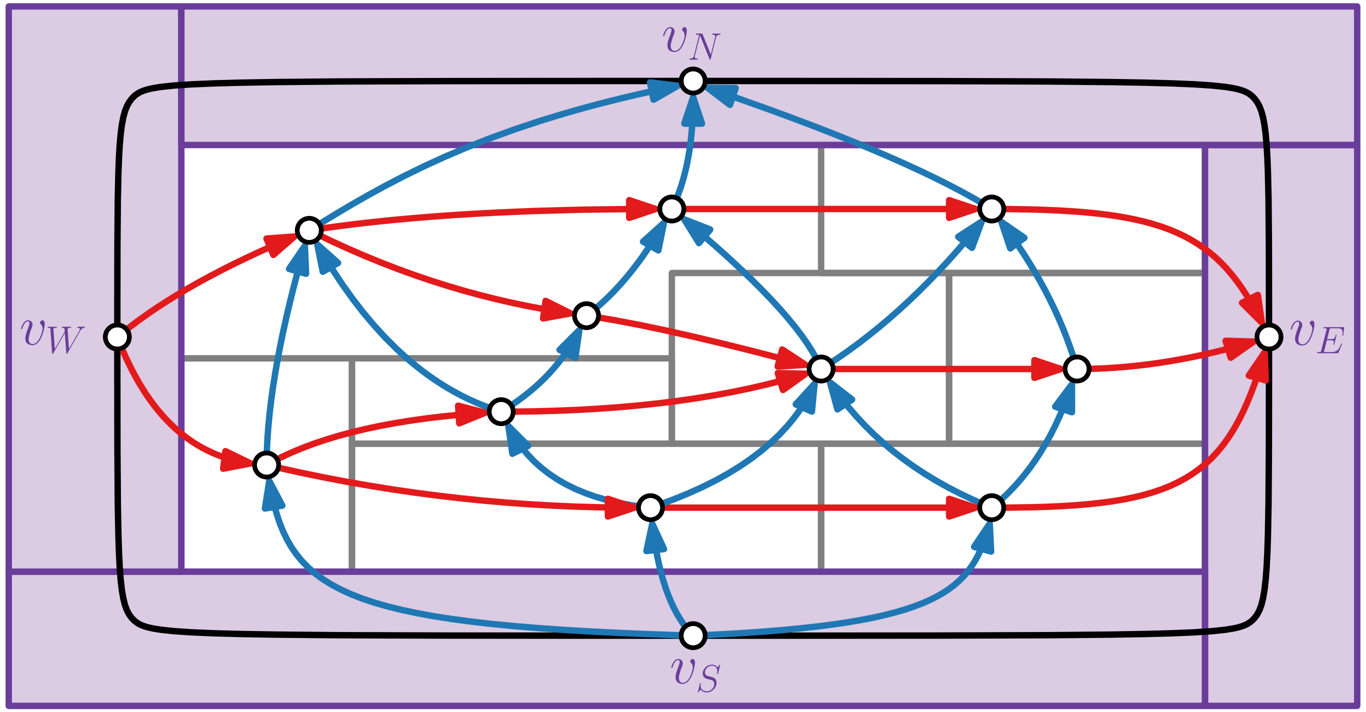
Properly Triangulated Planar Graph G

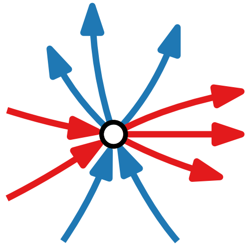

REL

Regular Edge Labeling

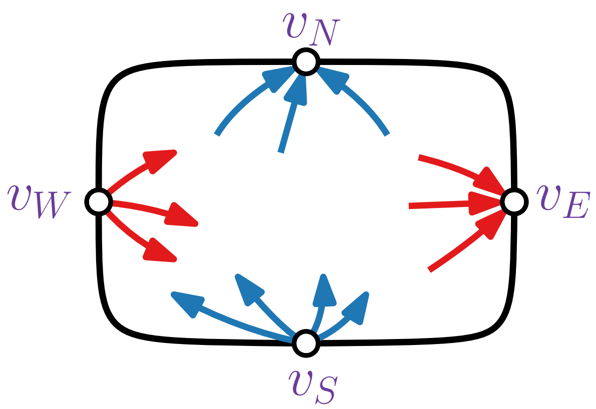

RD

Rectangular Dual $\mathcal{R}$



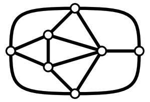


for every inner vertex



for four outer vertices

[Kant, He '94]: In linear time


PTP

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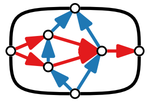
REL

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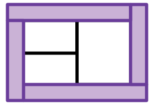
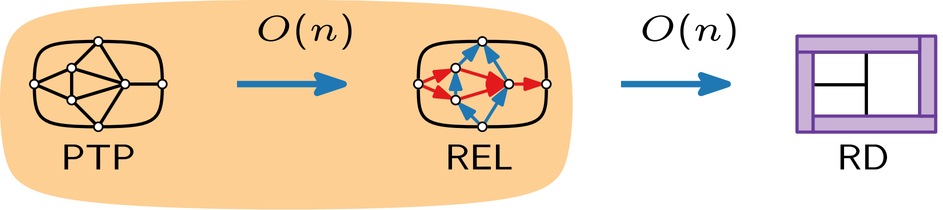
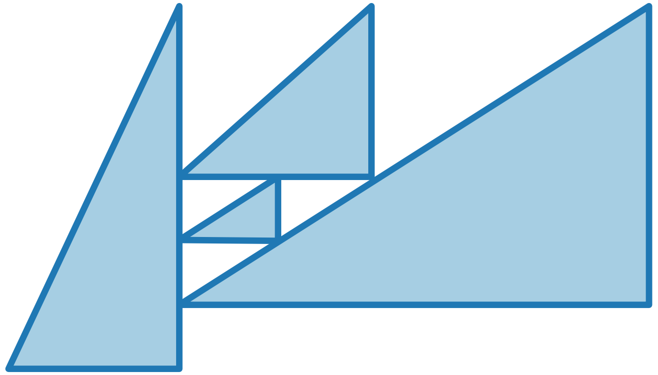

RD

[Kant, He '94]:

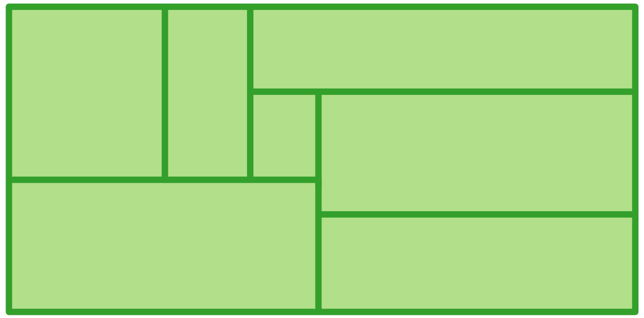


Visualization of Graphs

Lecture 7:
Contact Representations of Planar Graphs: Triangle Contacts and Rectangular Duals



Part IV:
Computing a REL



Alexander Wolff

Refined Canonical Order

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Refined Canonical Order

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$v_1 = v_S, v_2 = v_W, v_3, \ldots, v_n = v_N$ of the vertices of G

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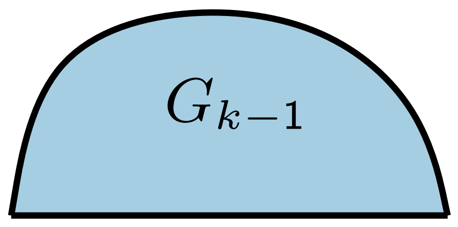

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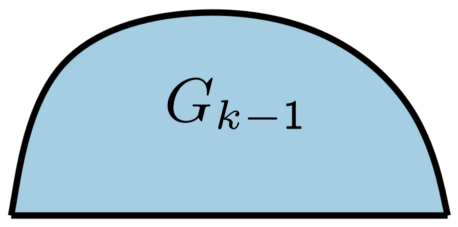

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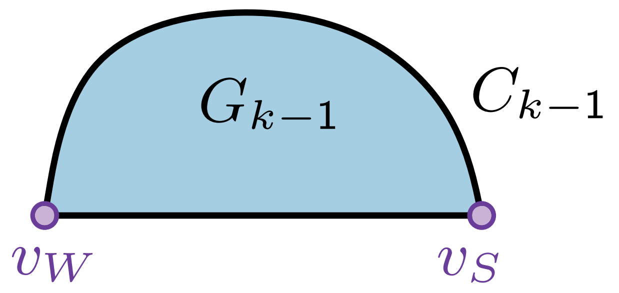

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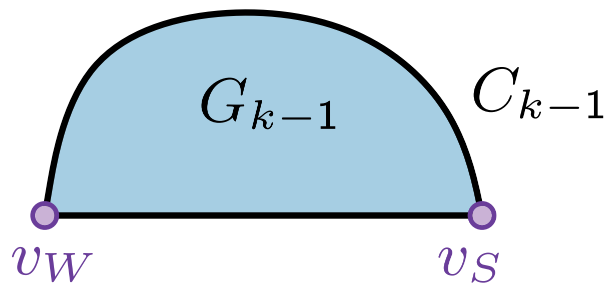

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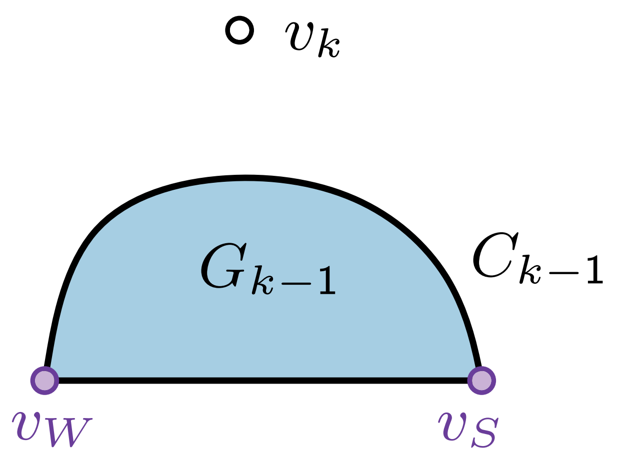

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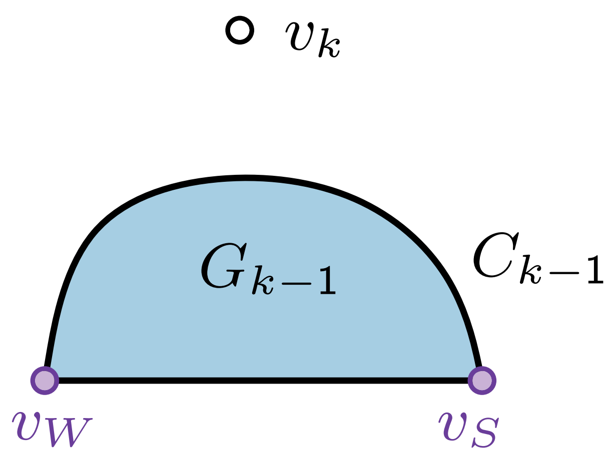

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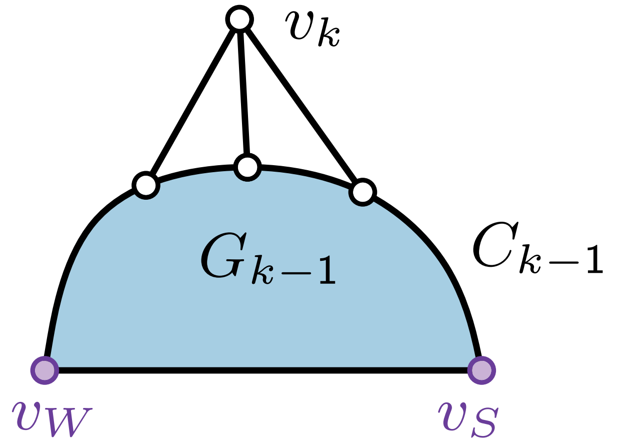

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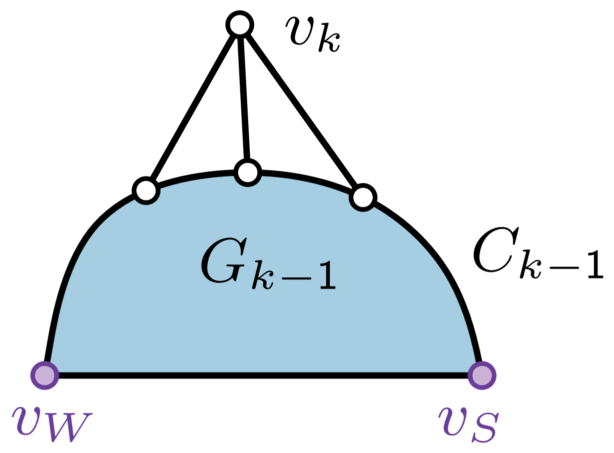

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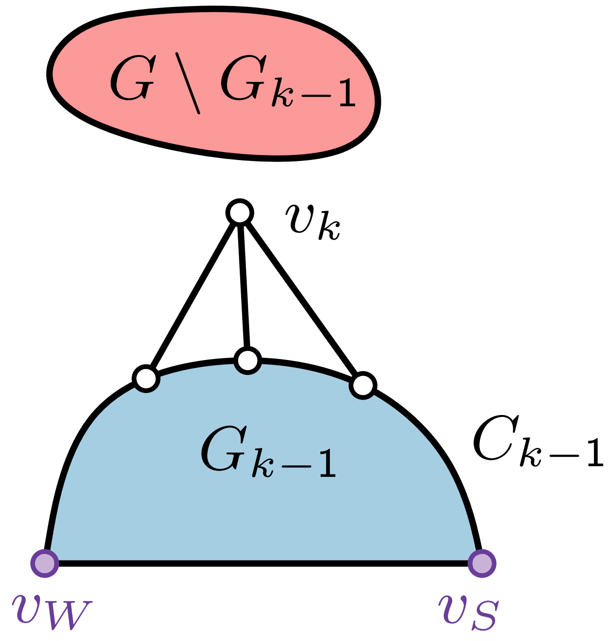

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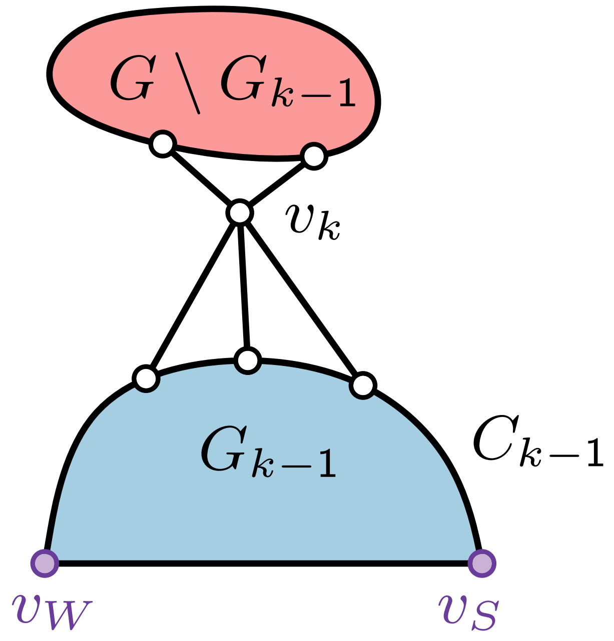

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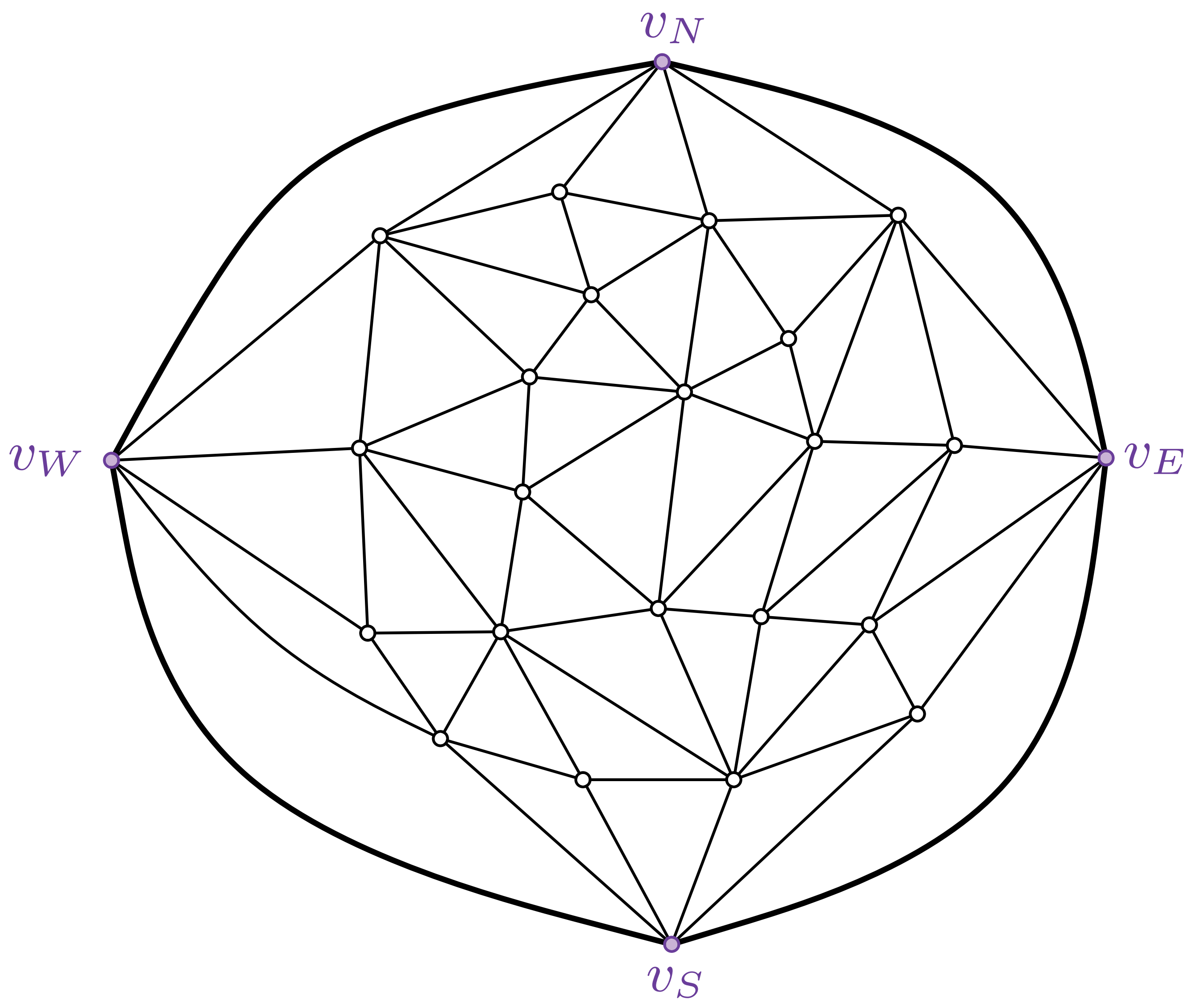

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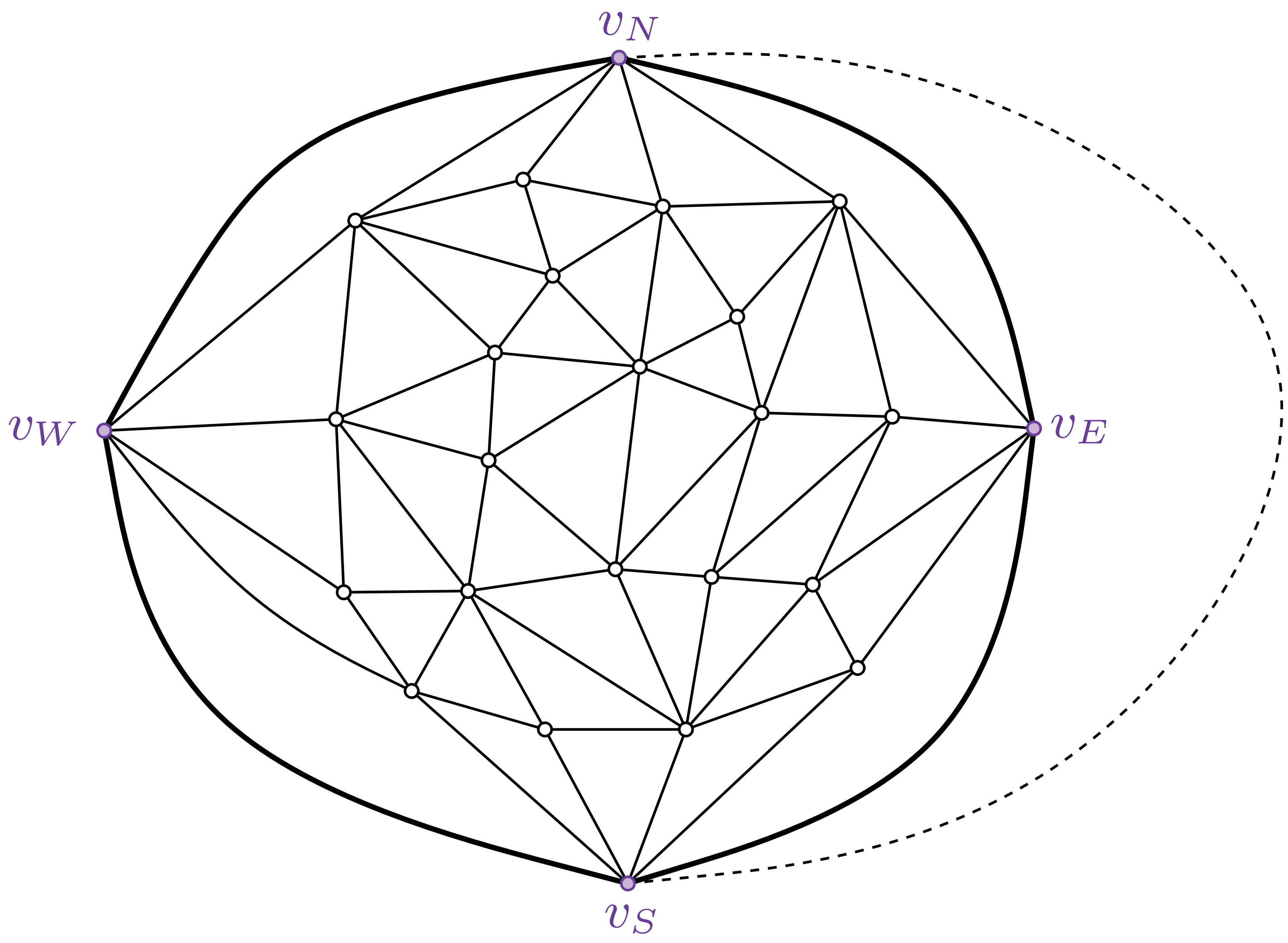

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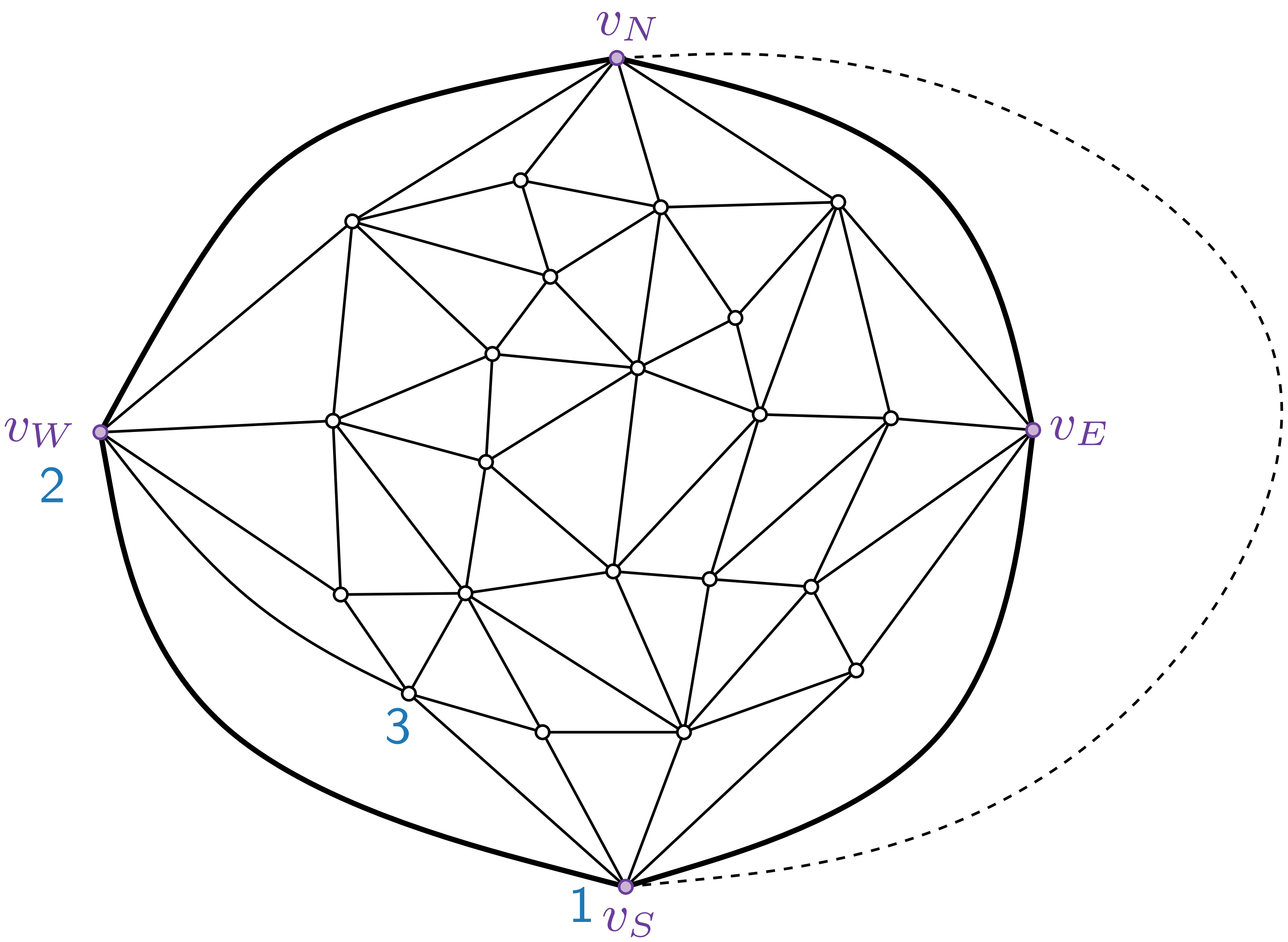

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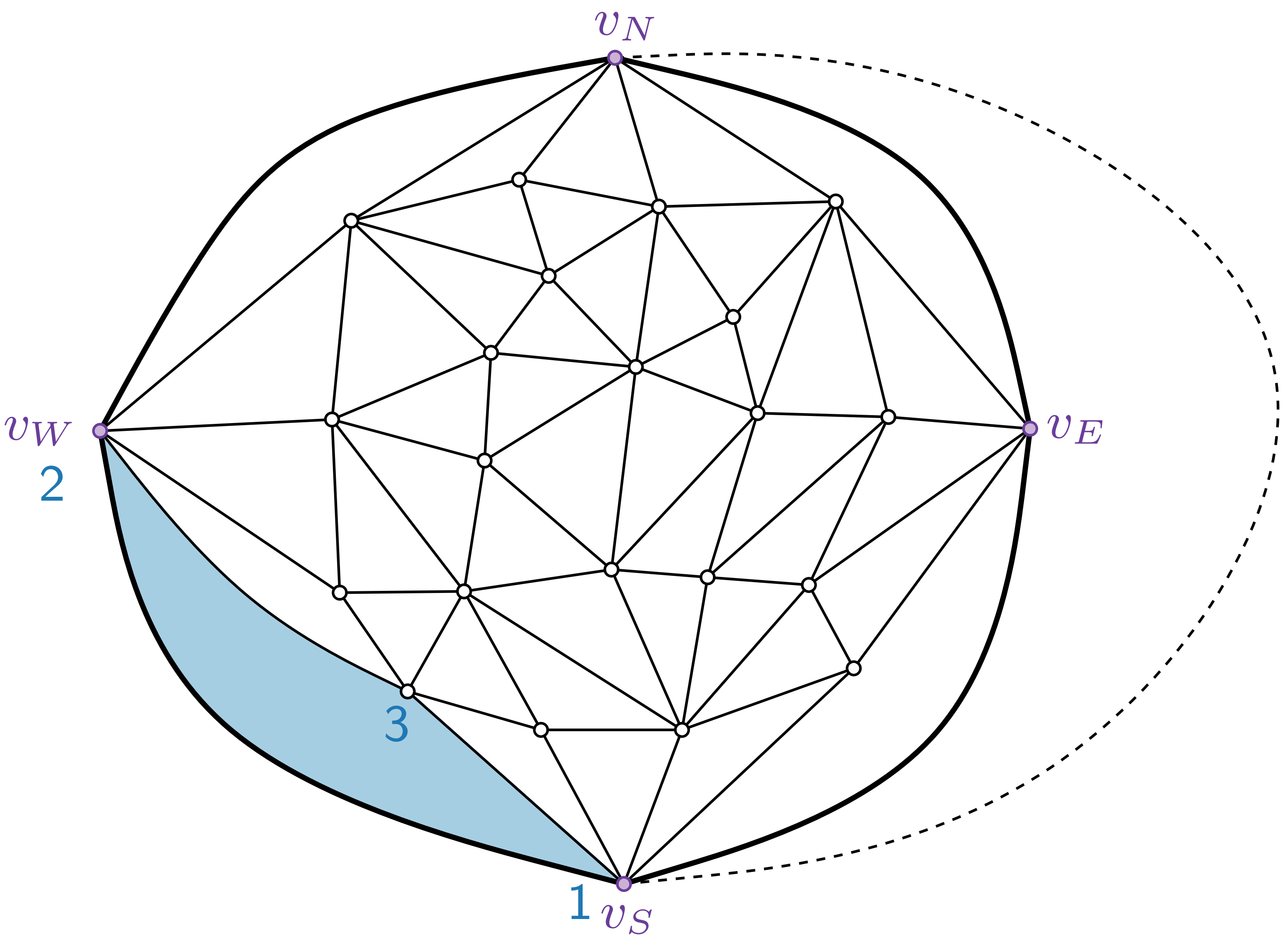

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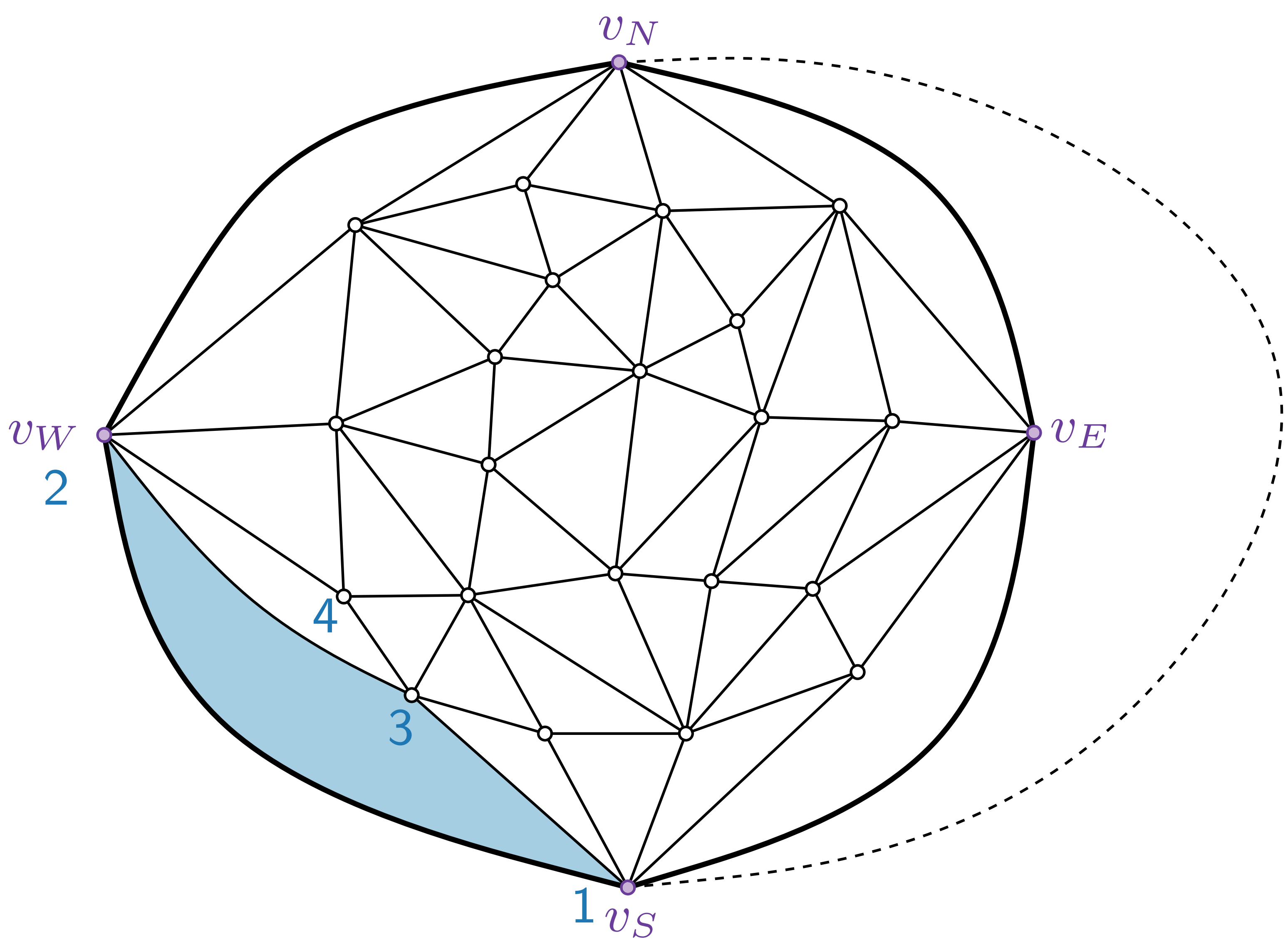

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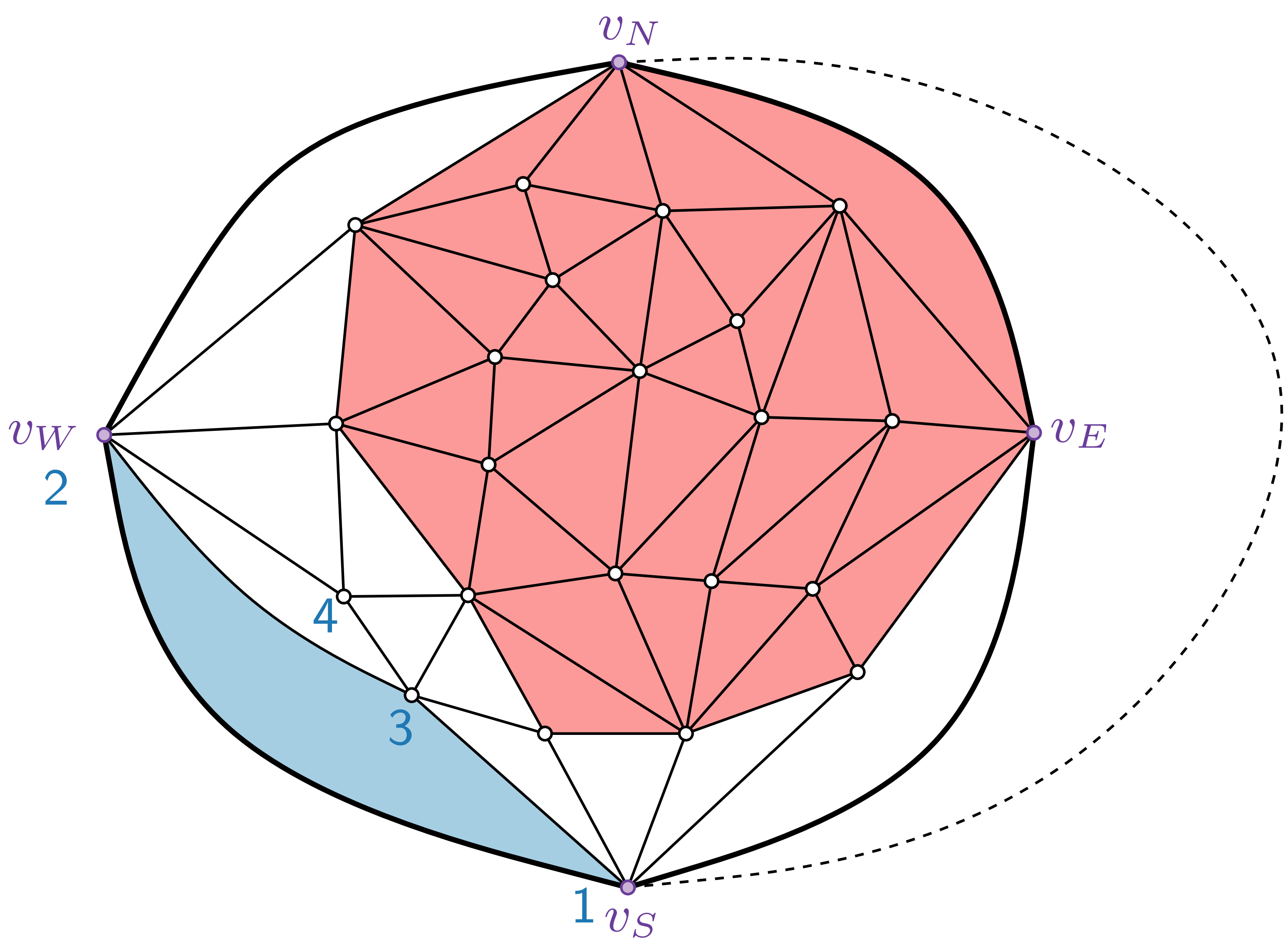

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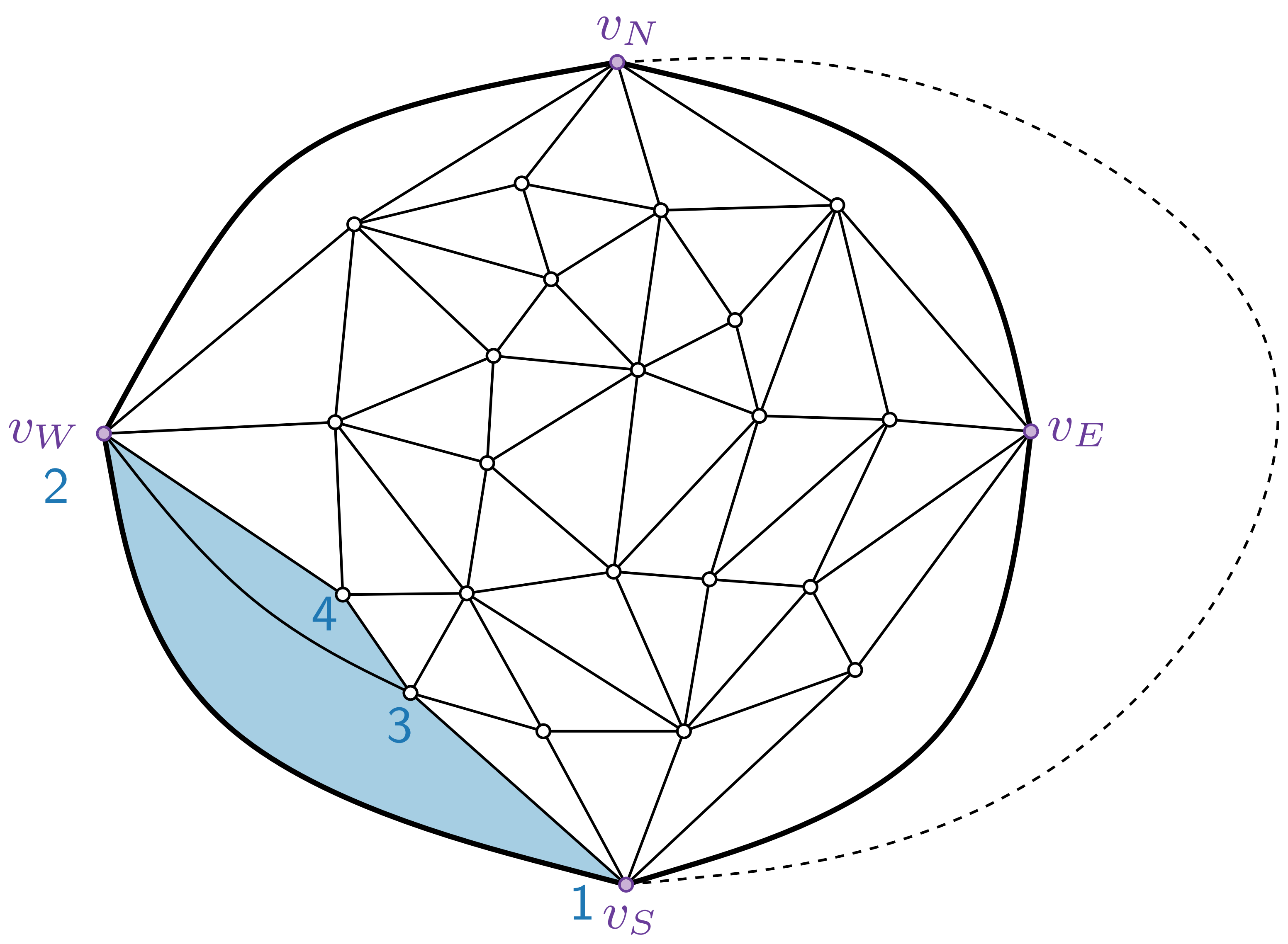

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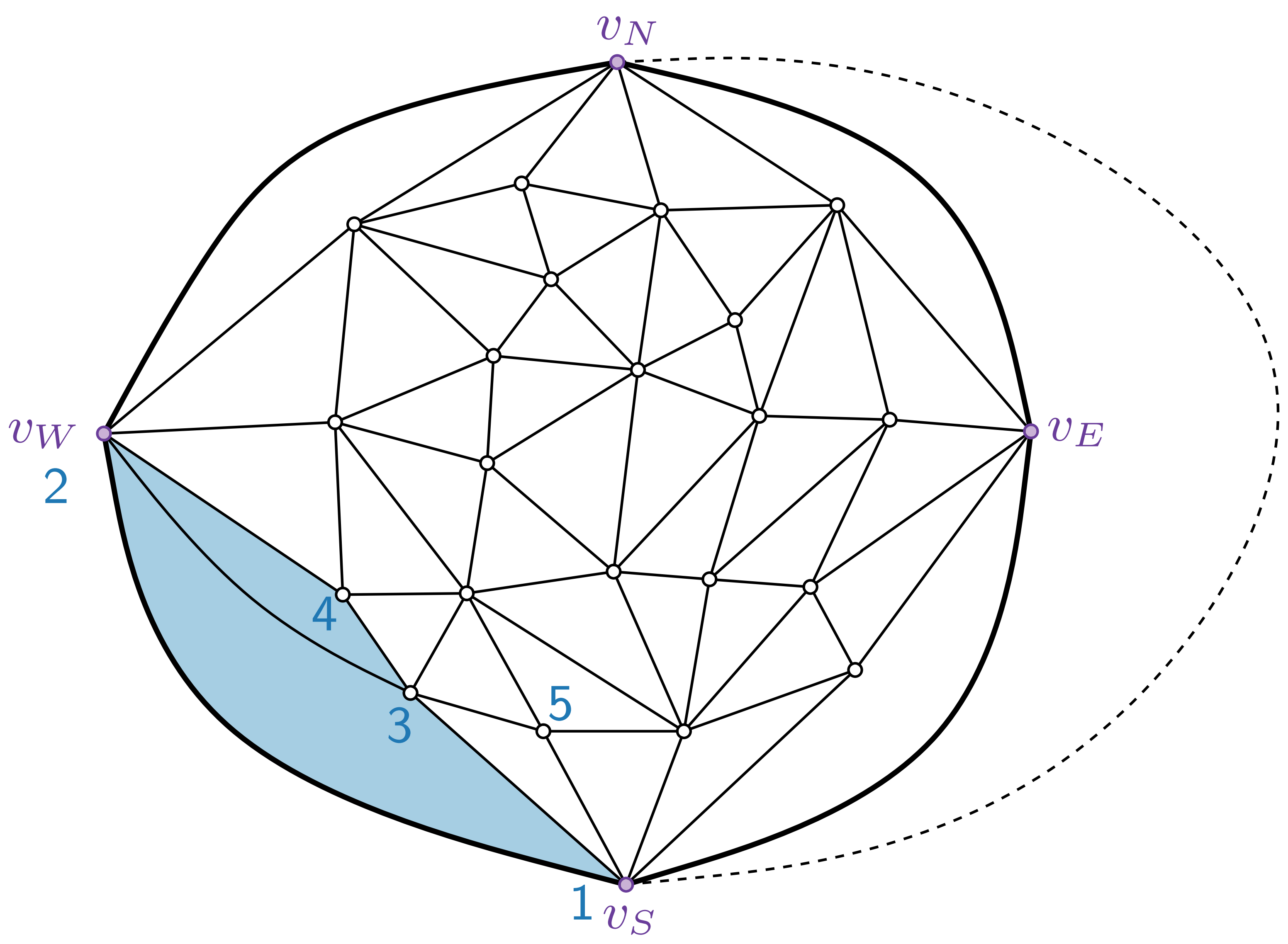

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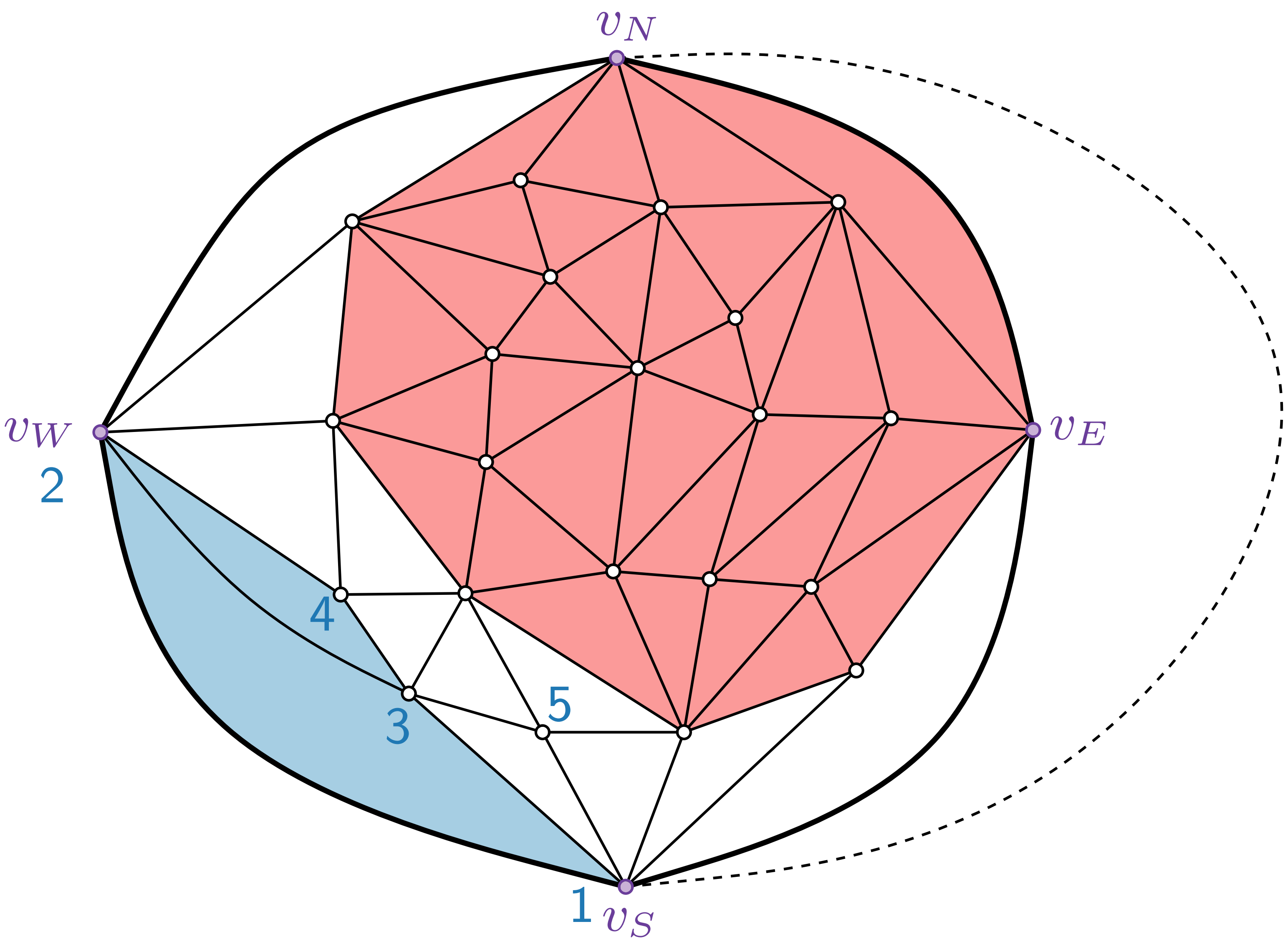

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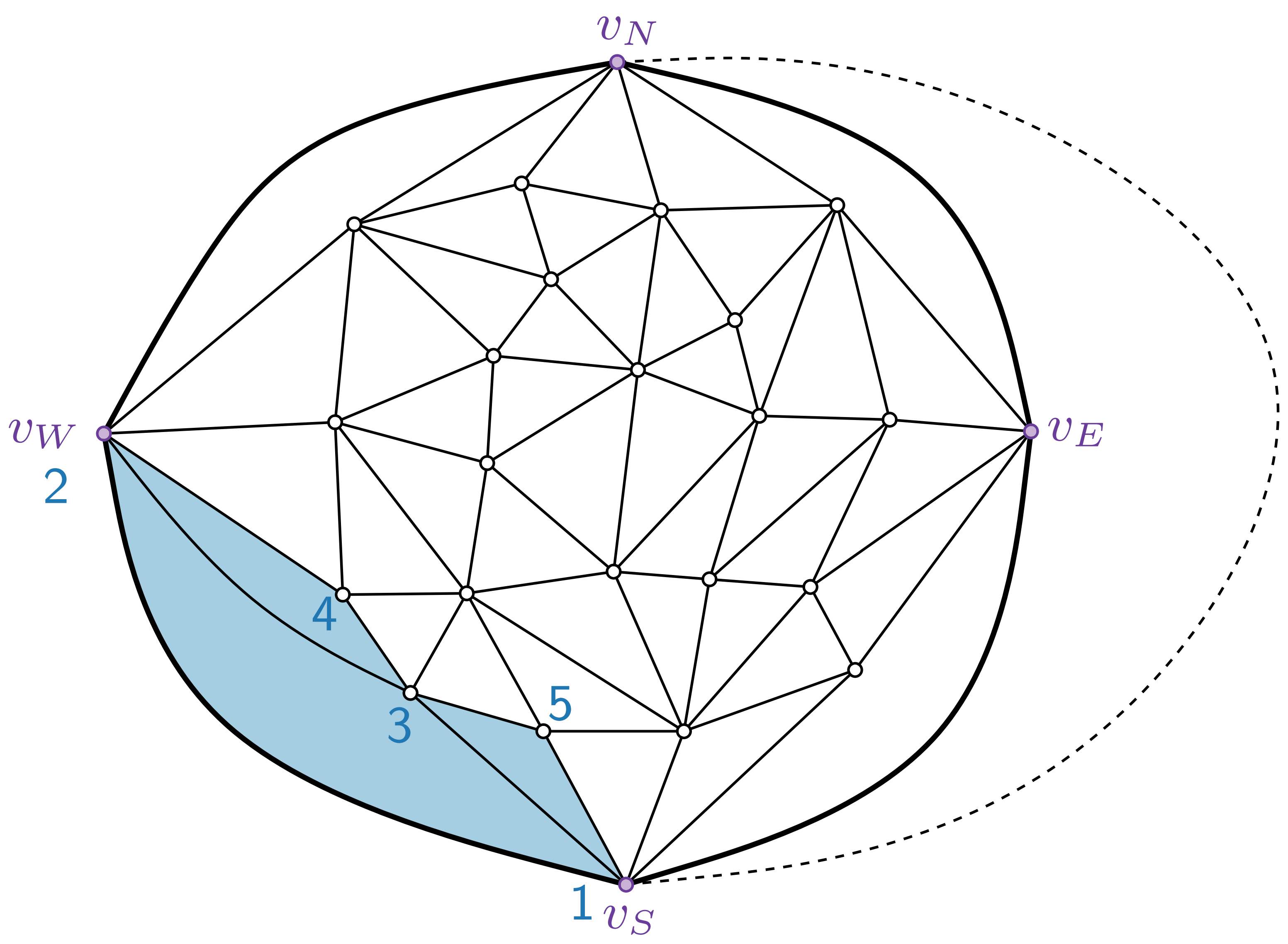

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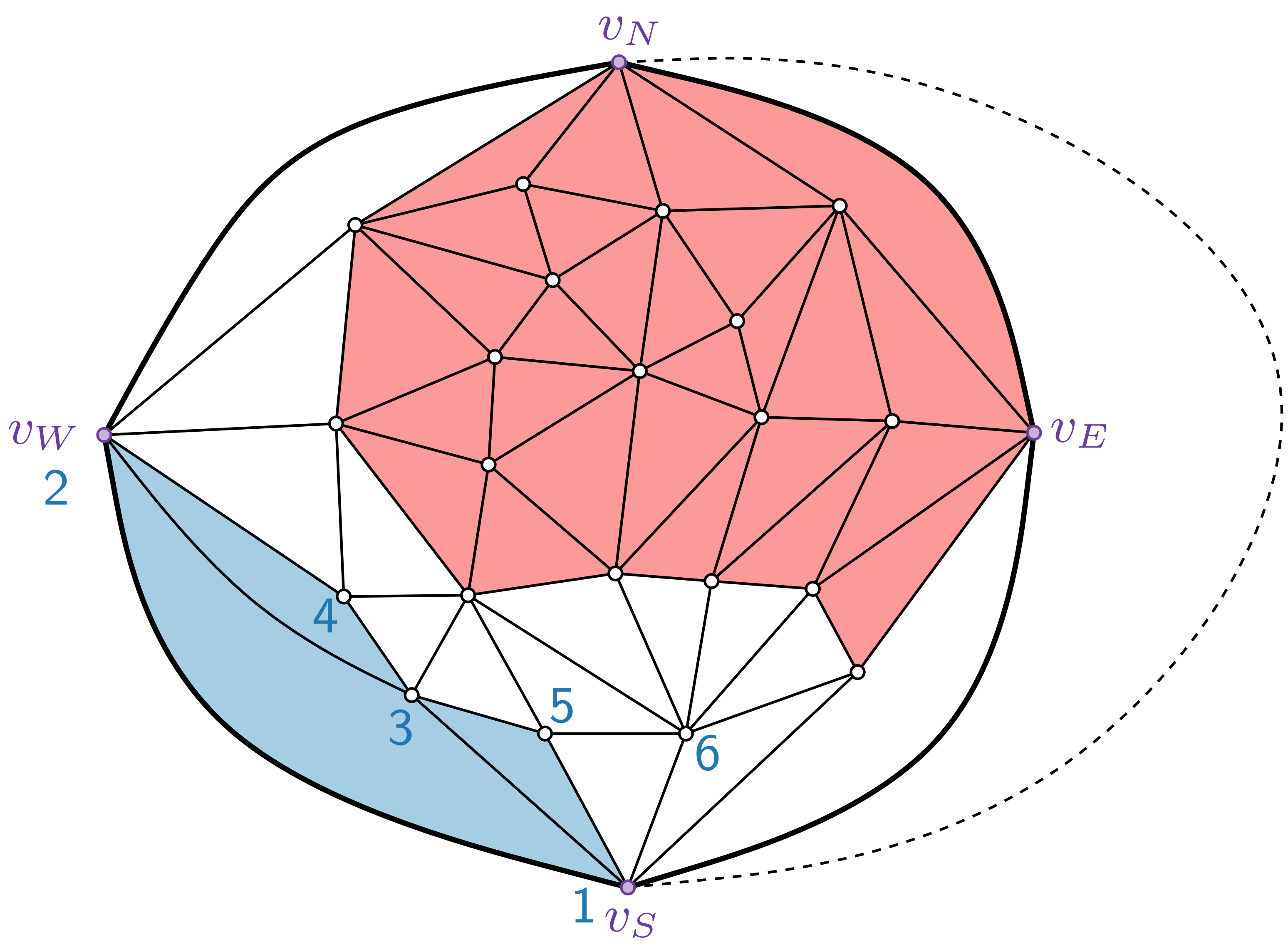

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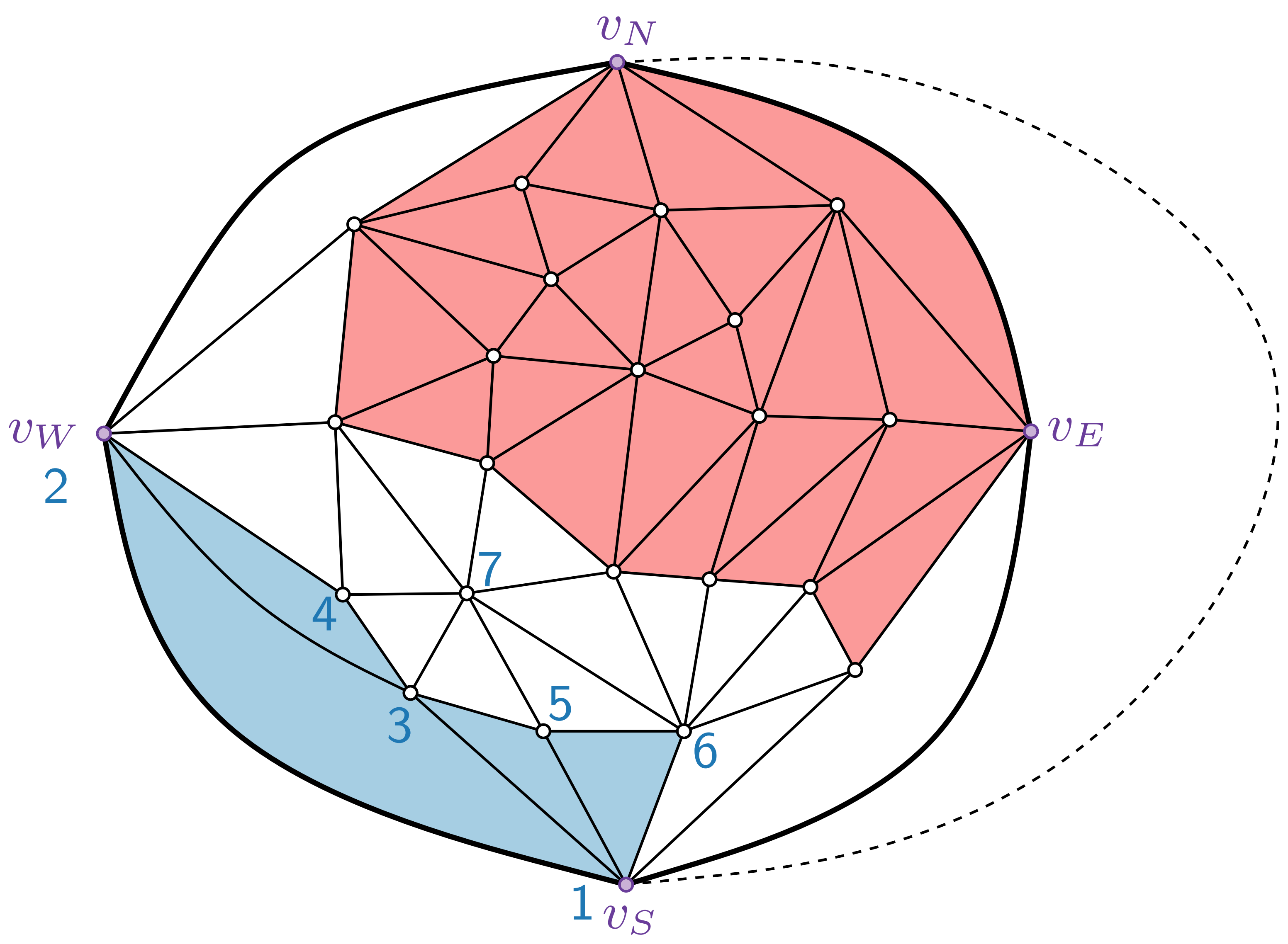

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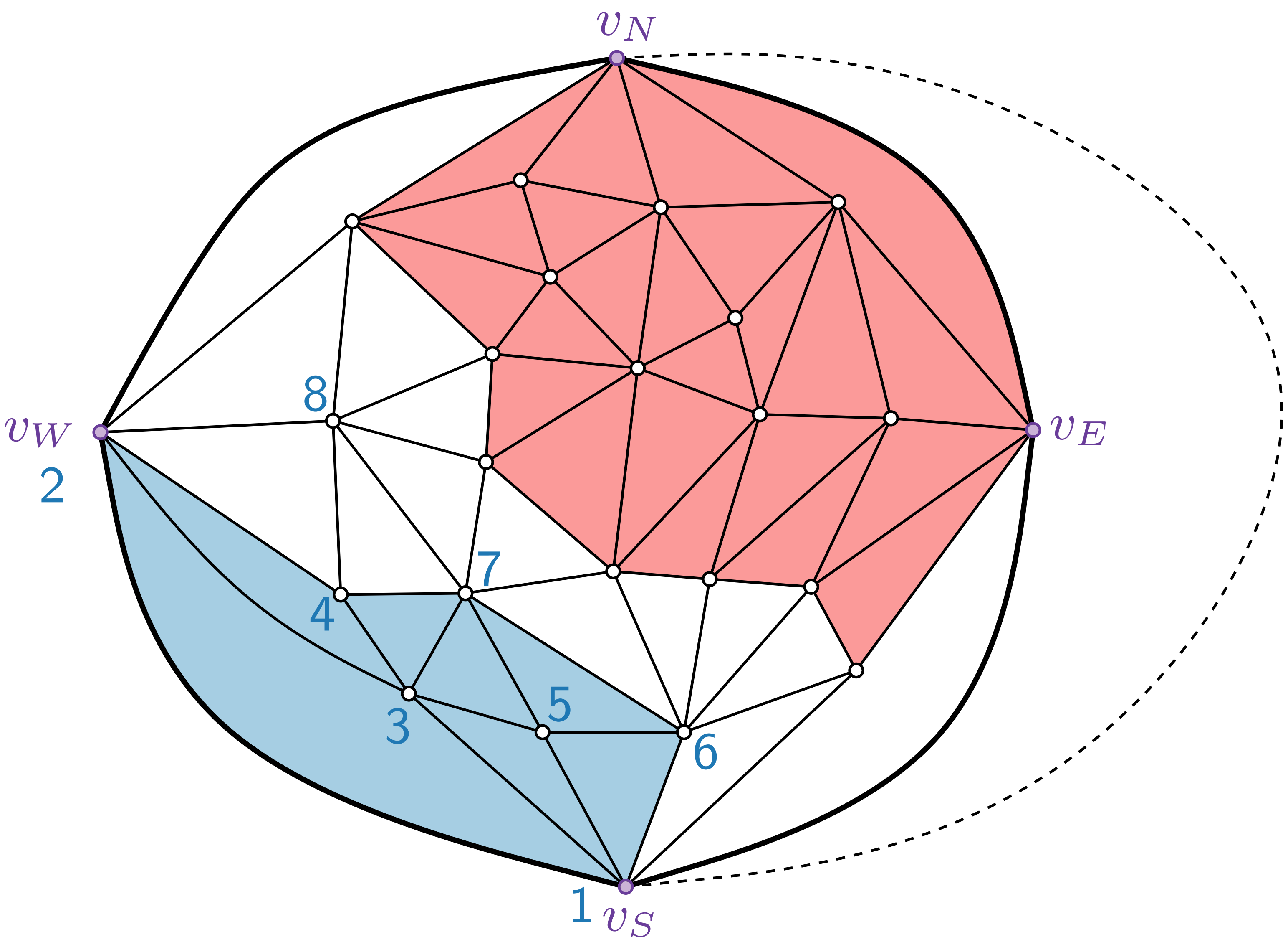

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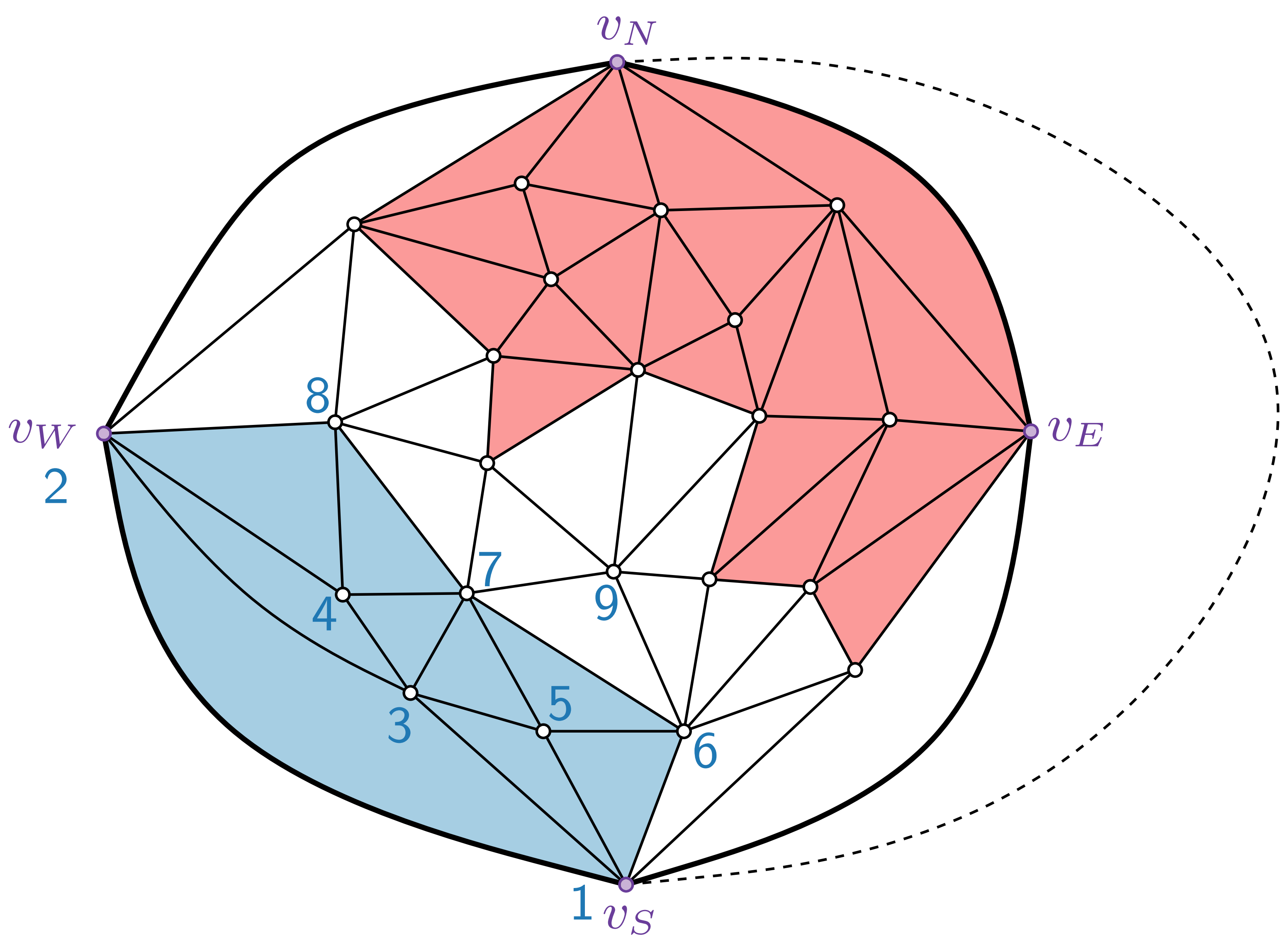

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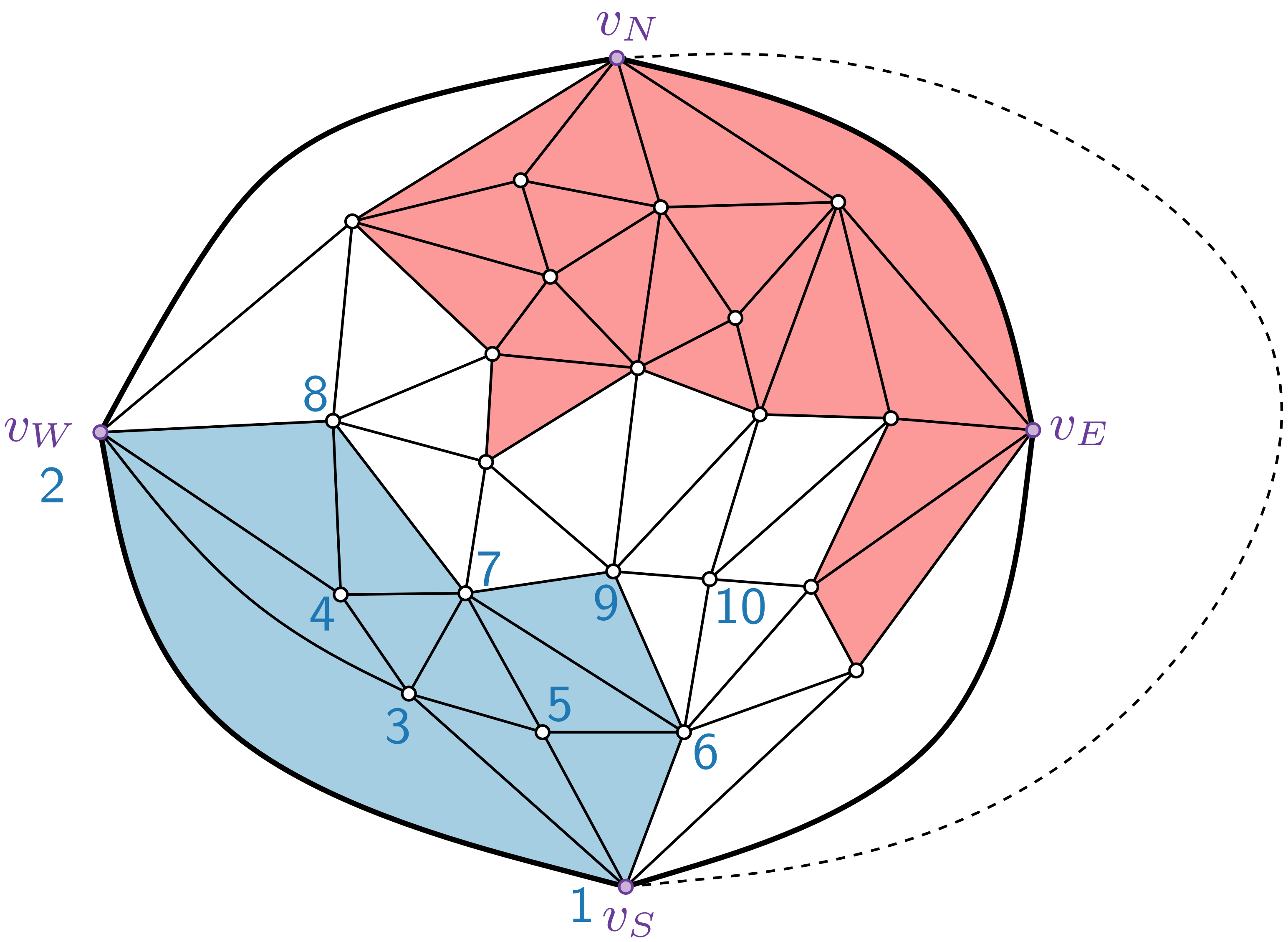

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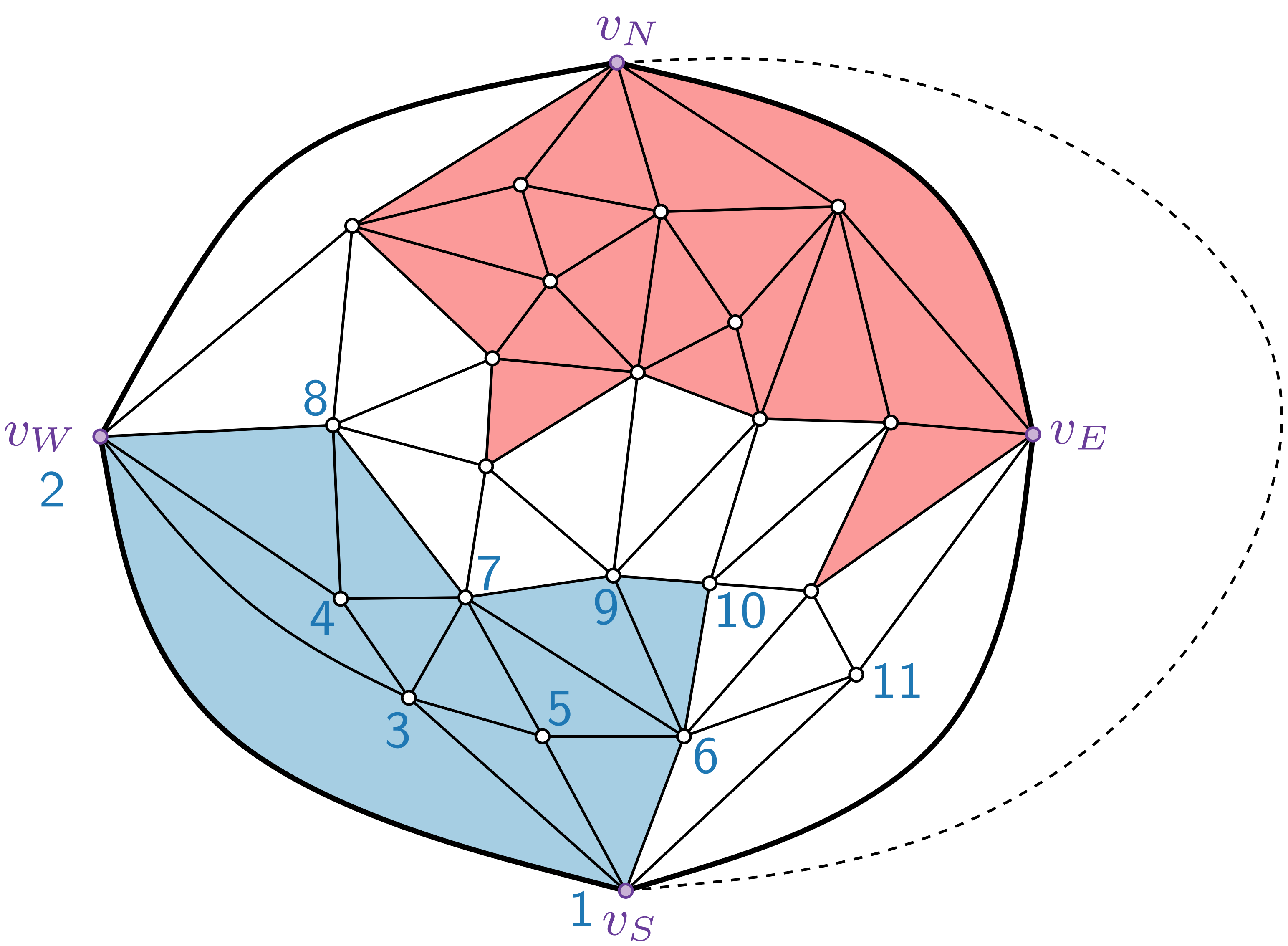

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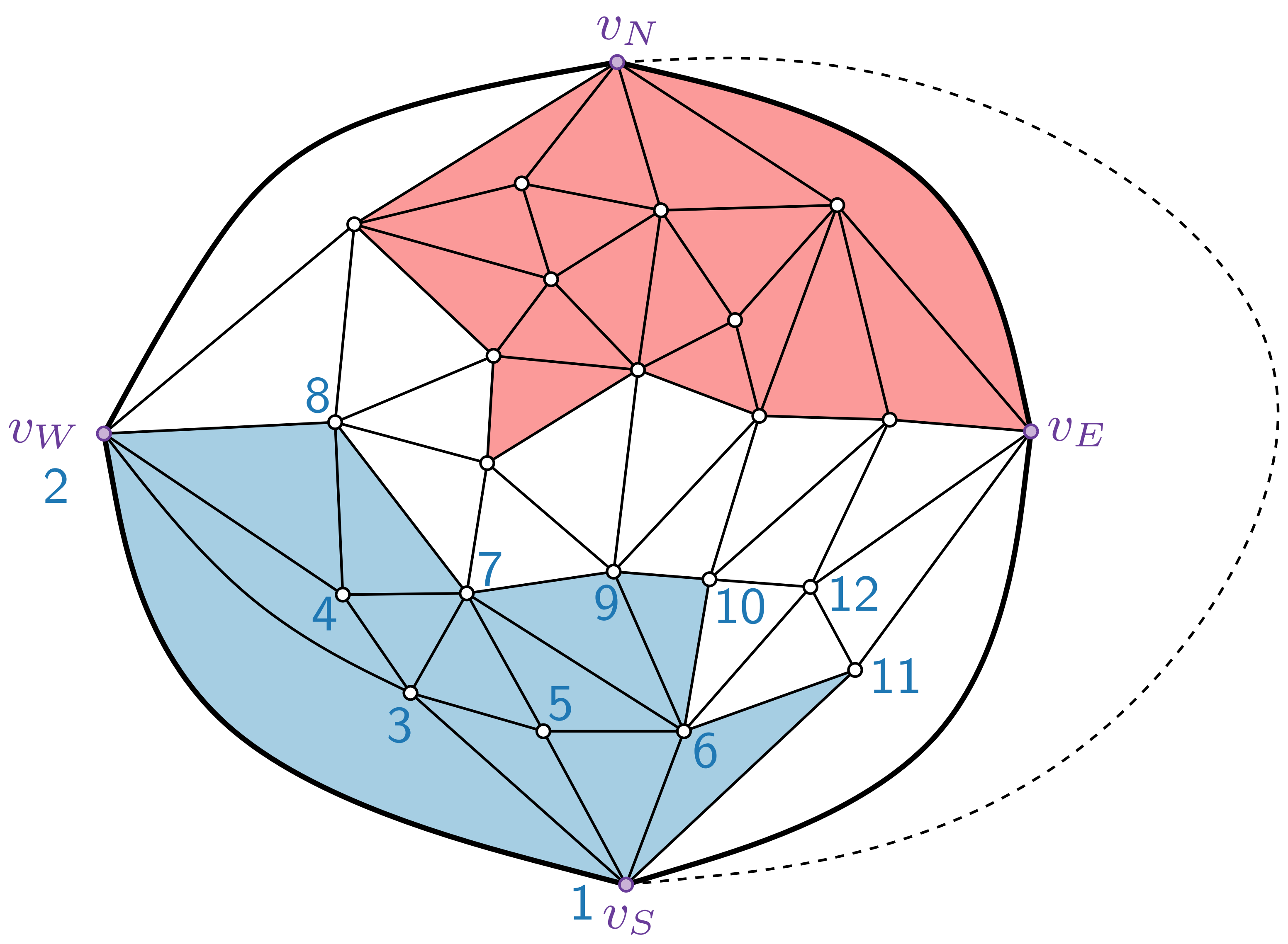

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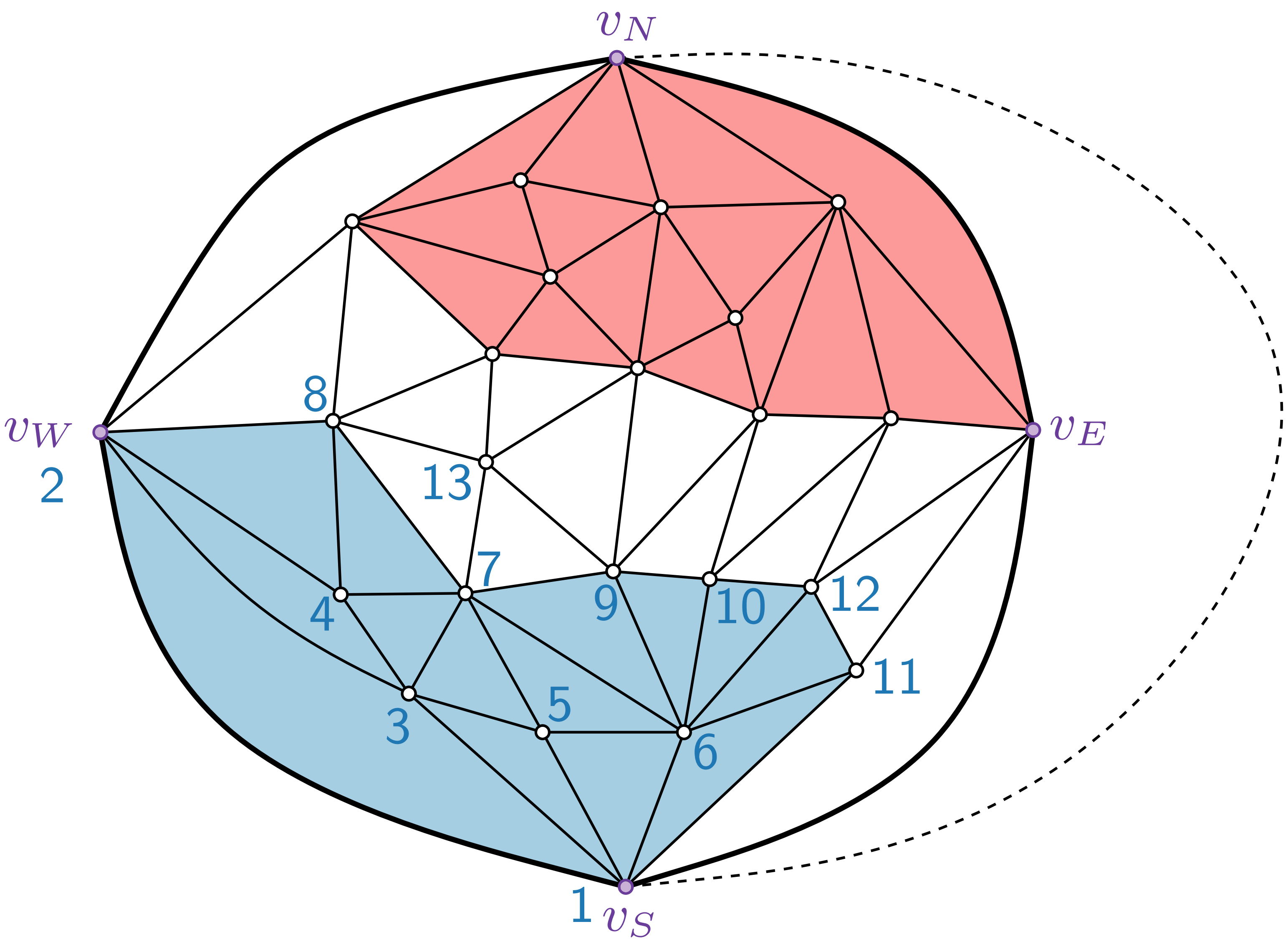

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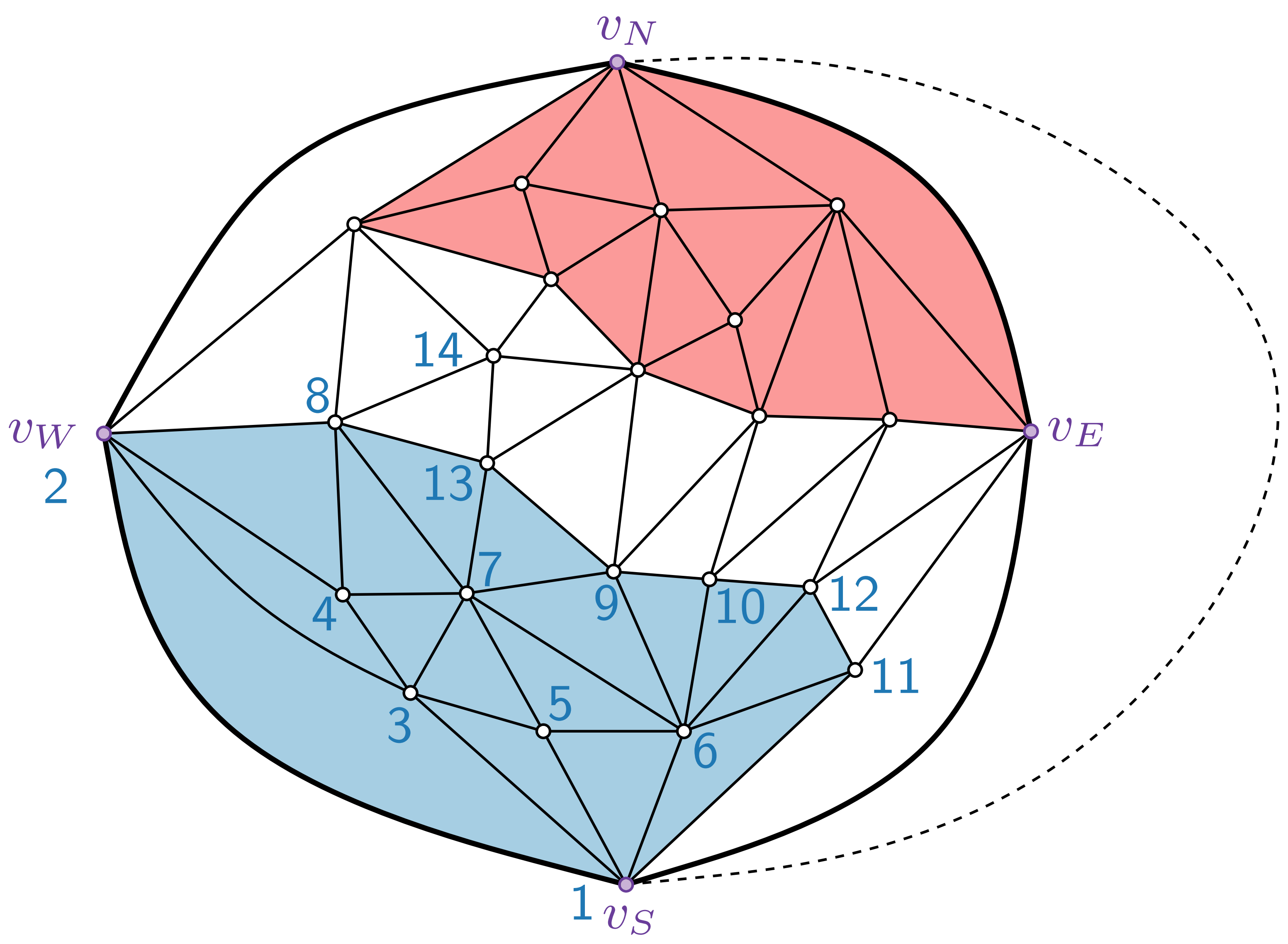

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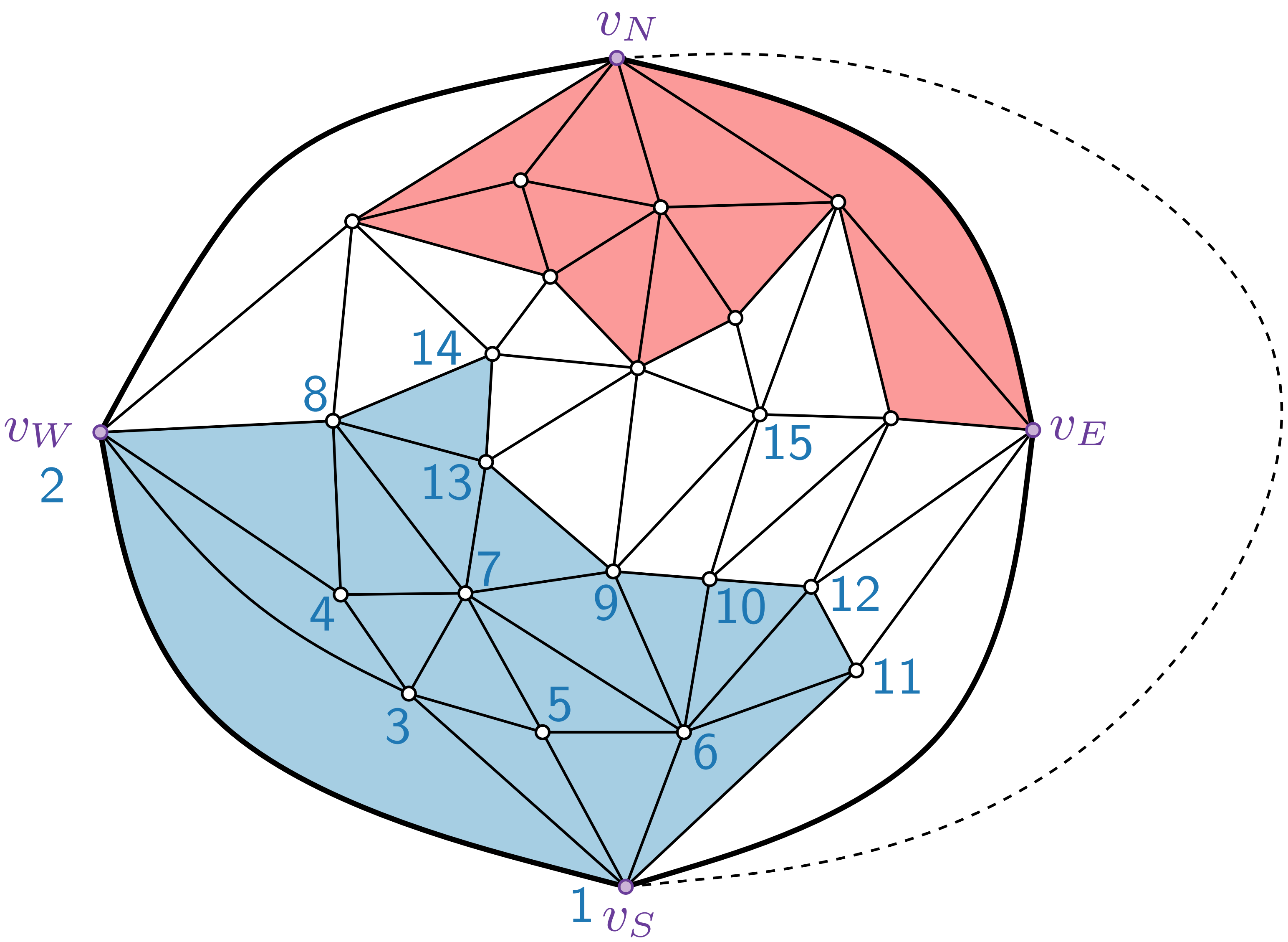

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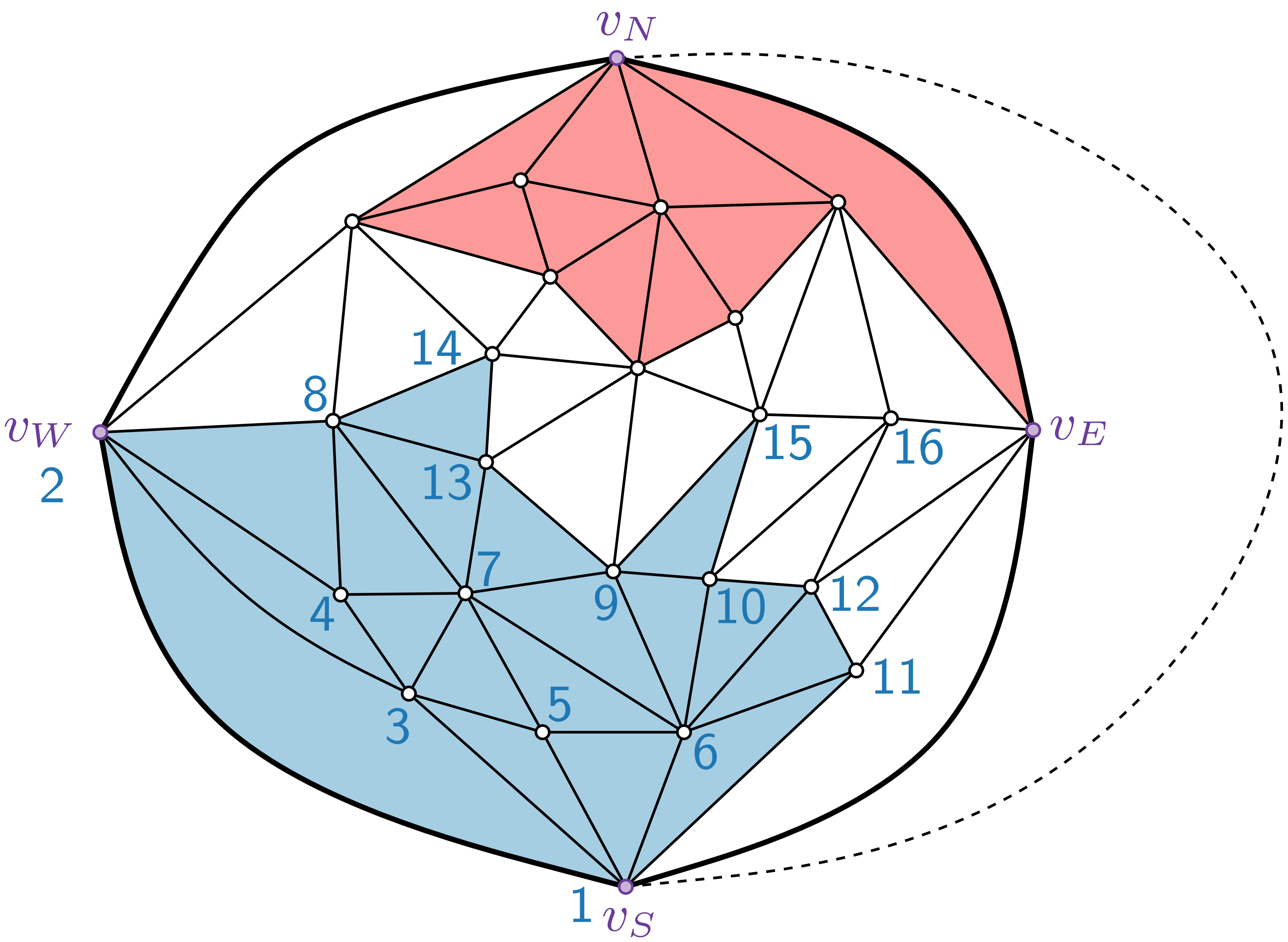

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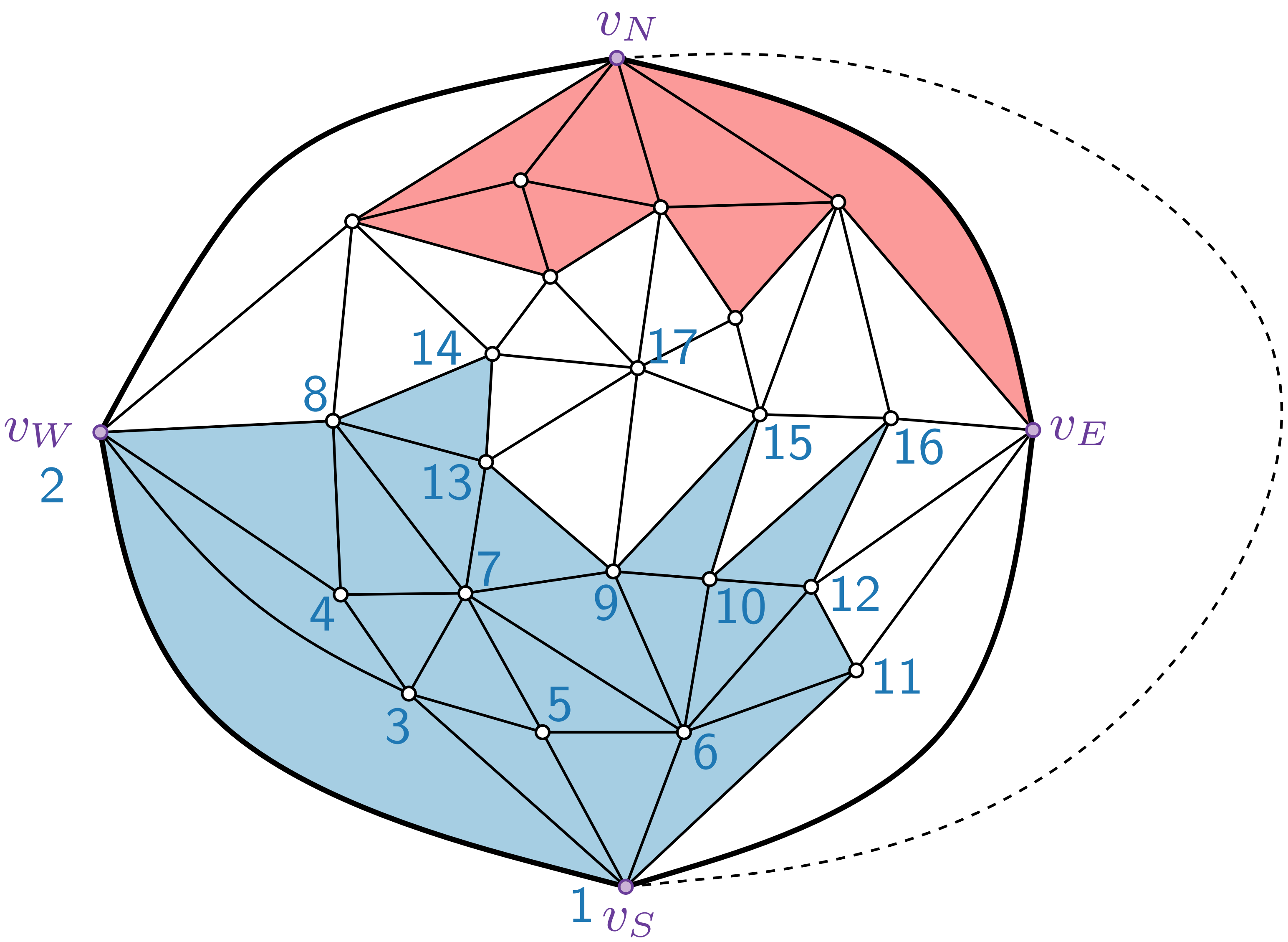

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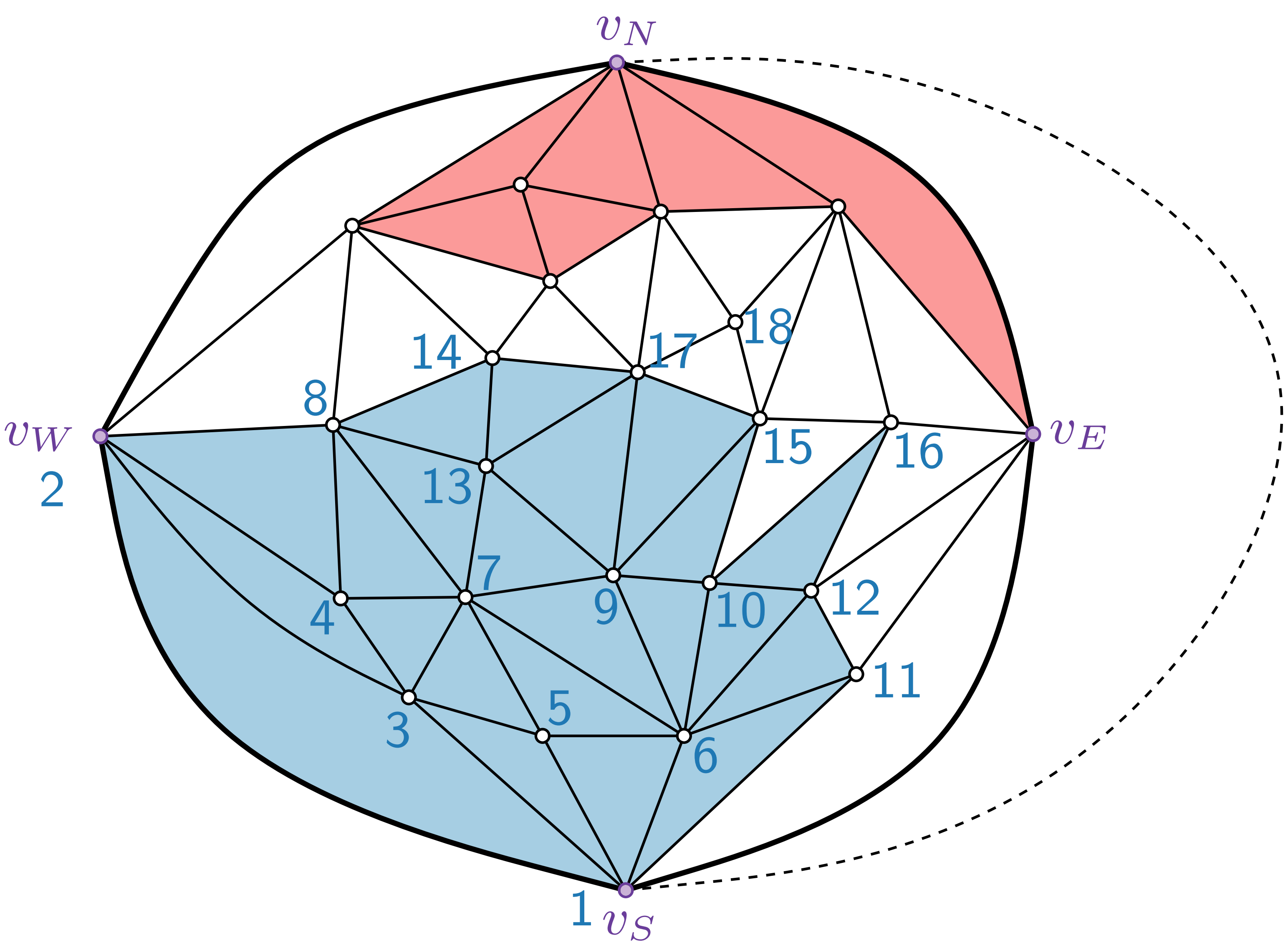

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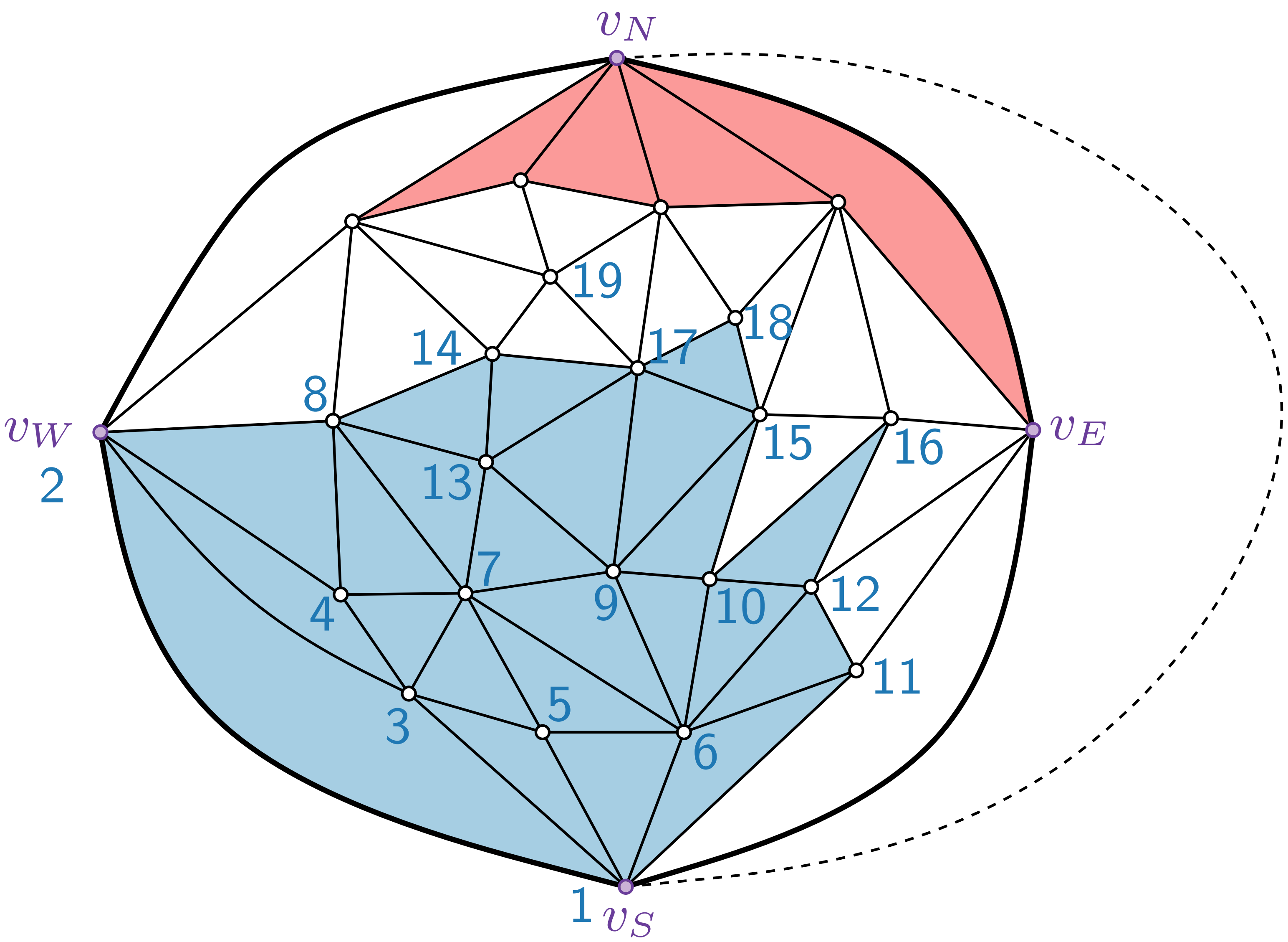

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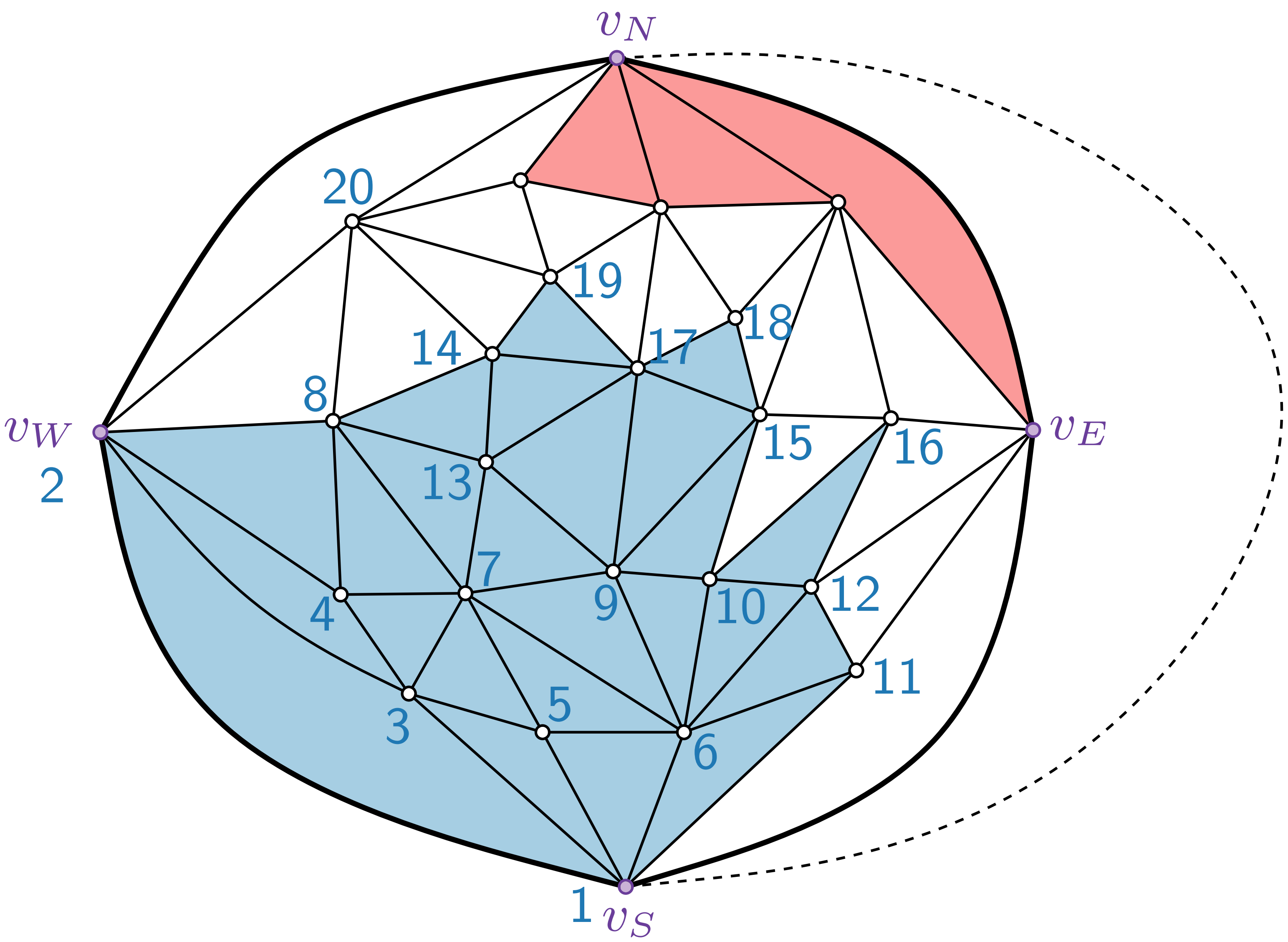

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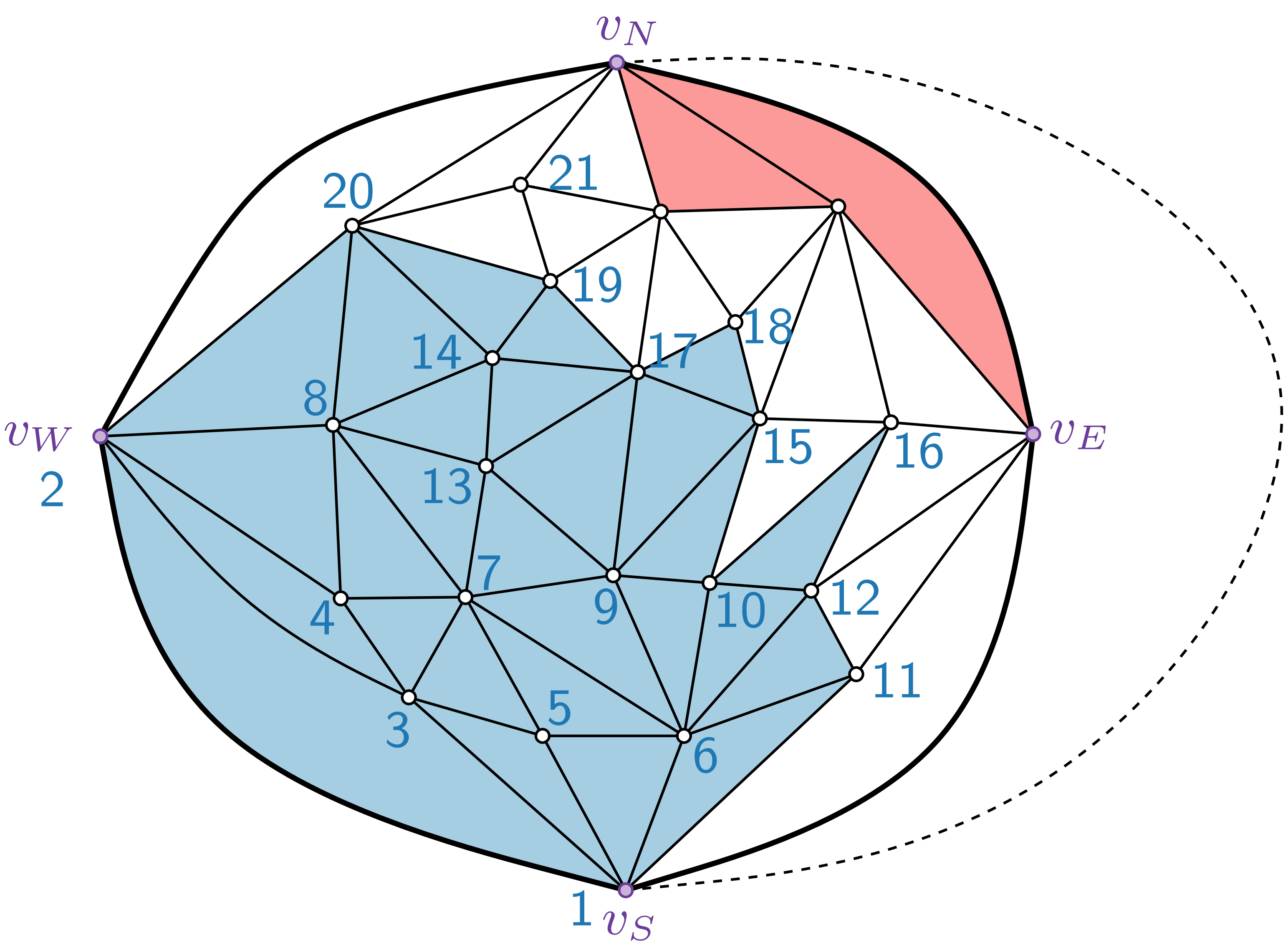

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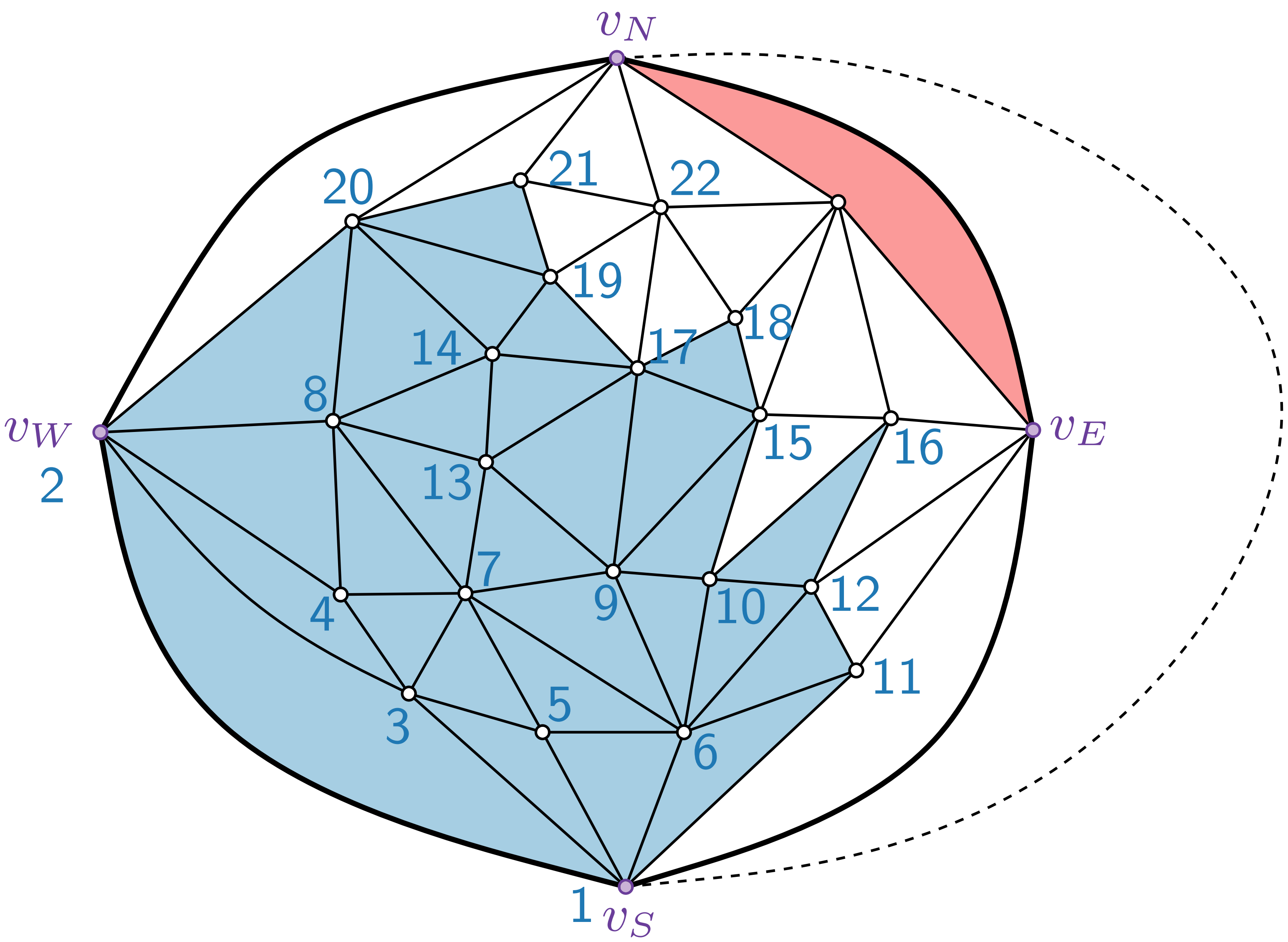

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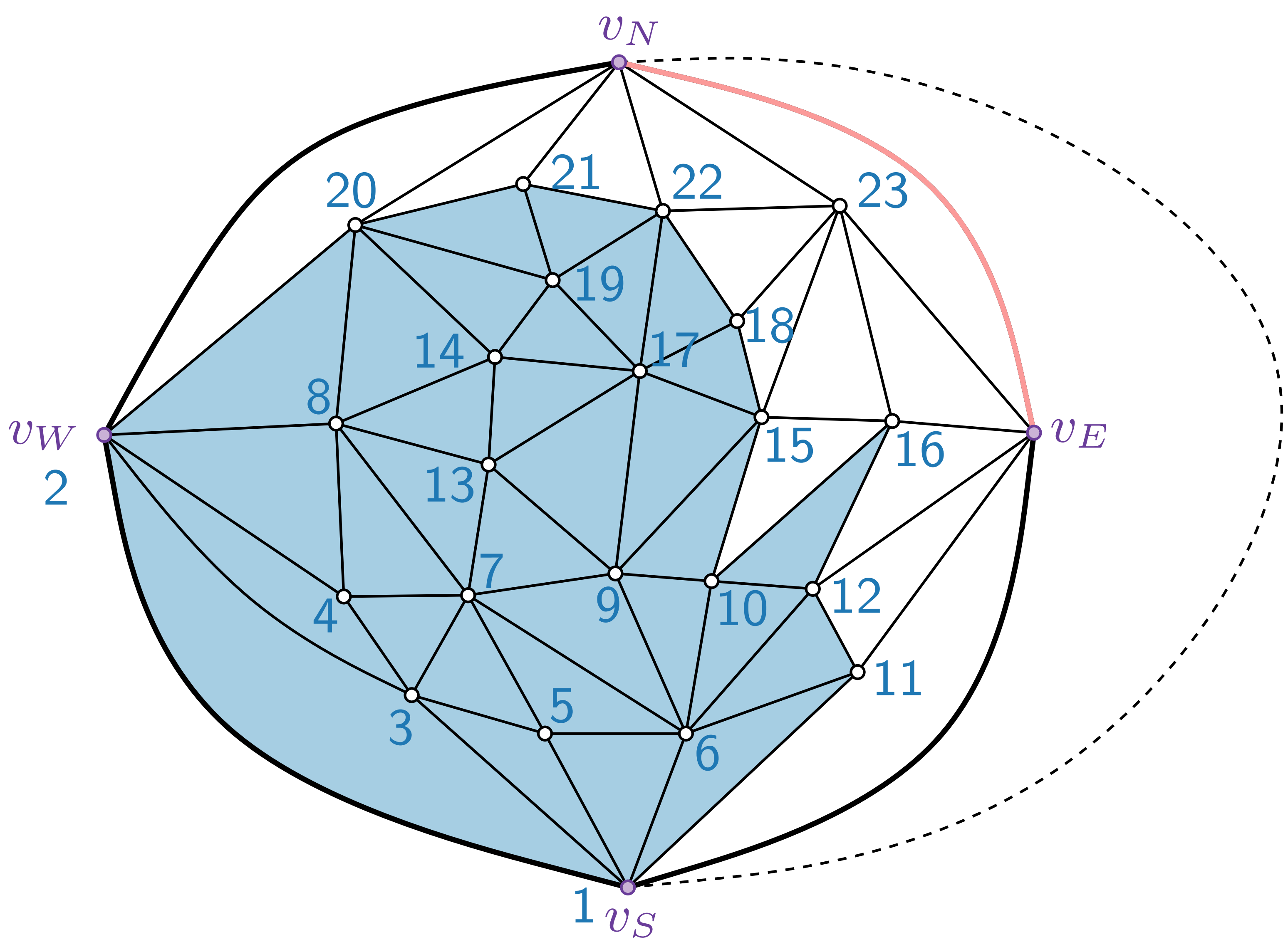

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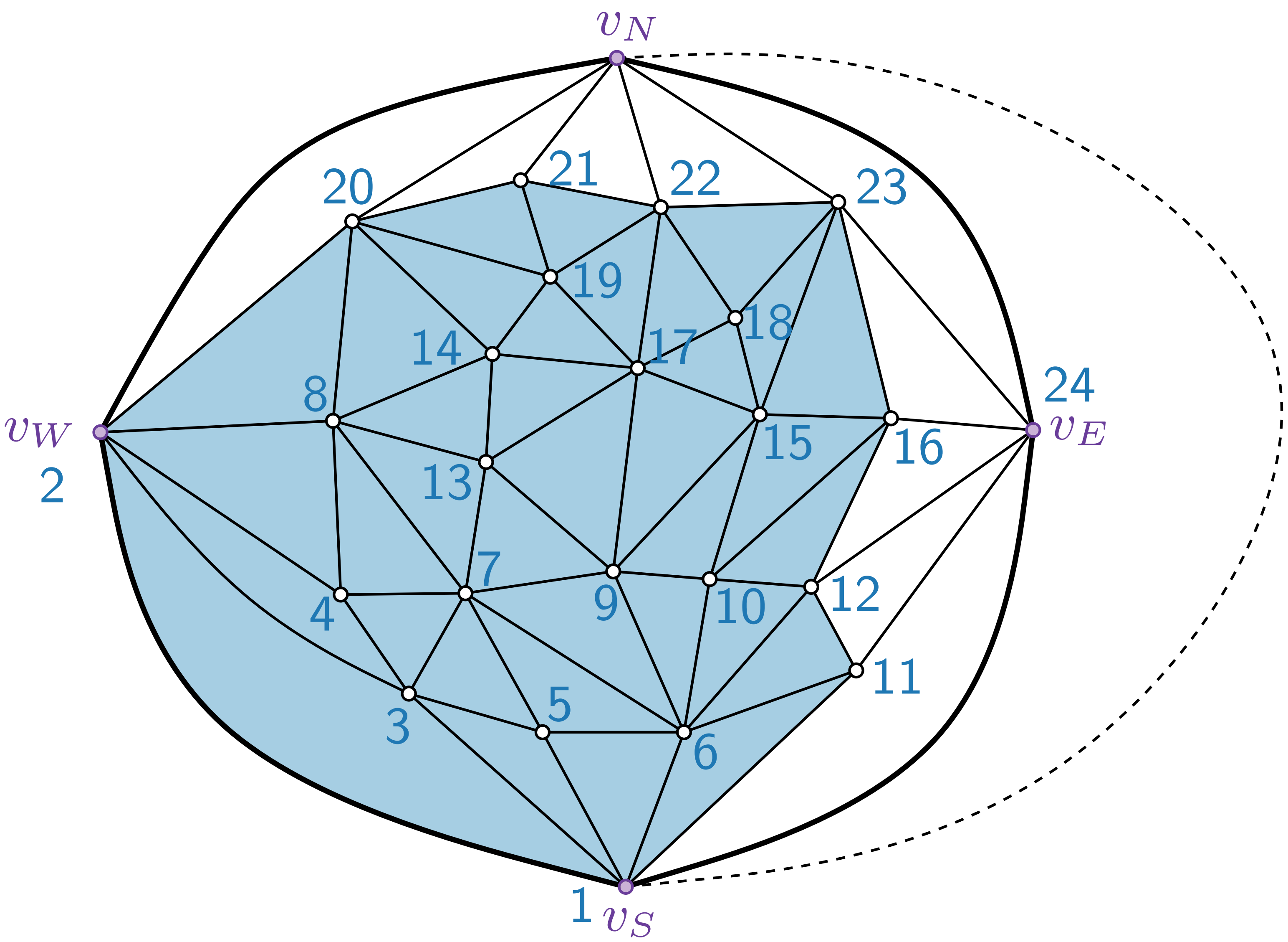

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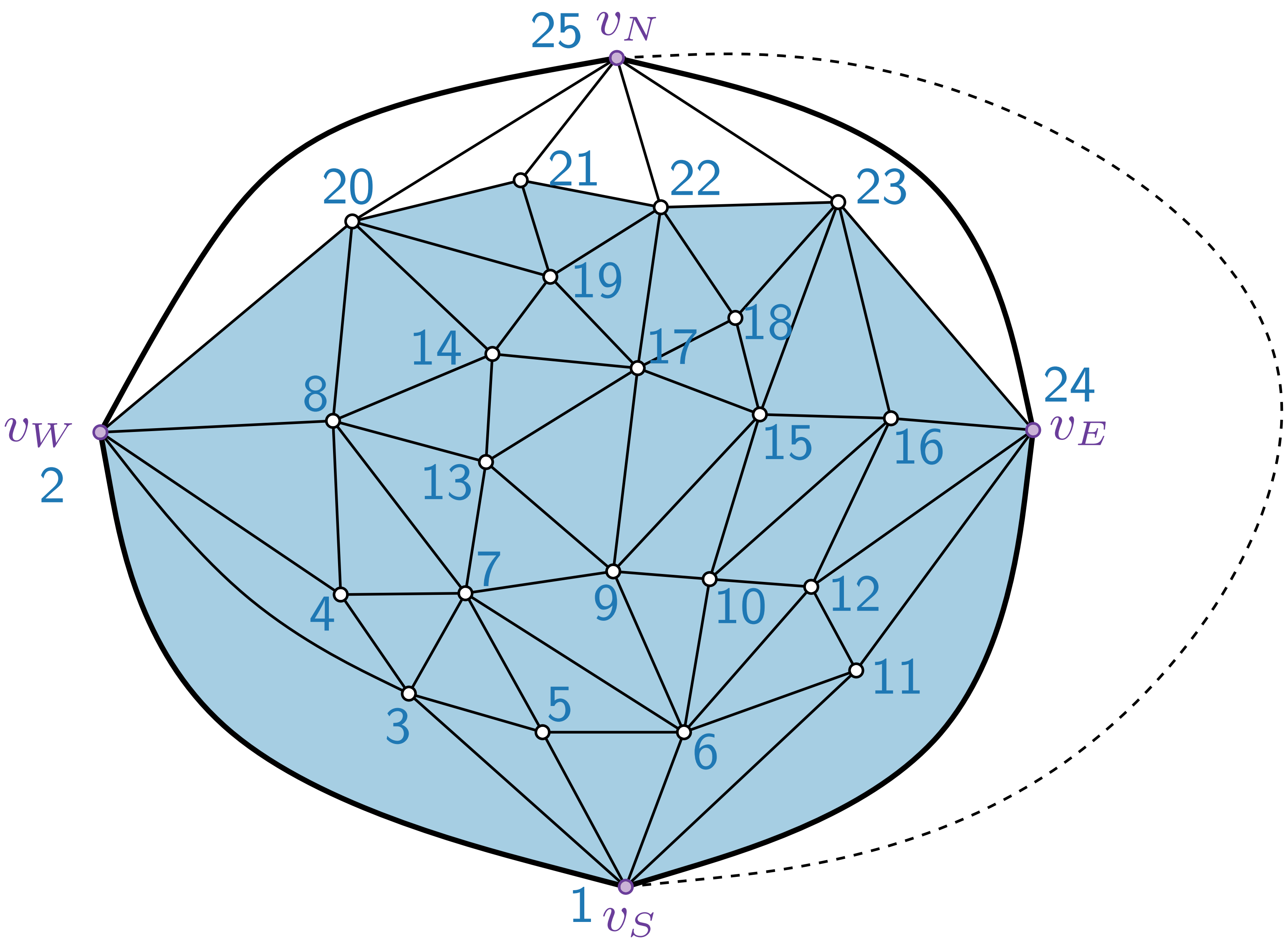

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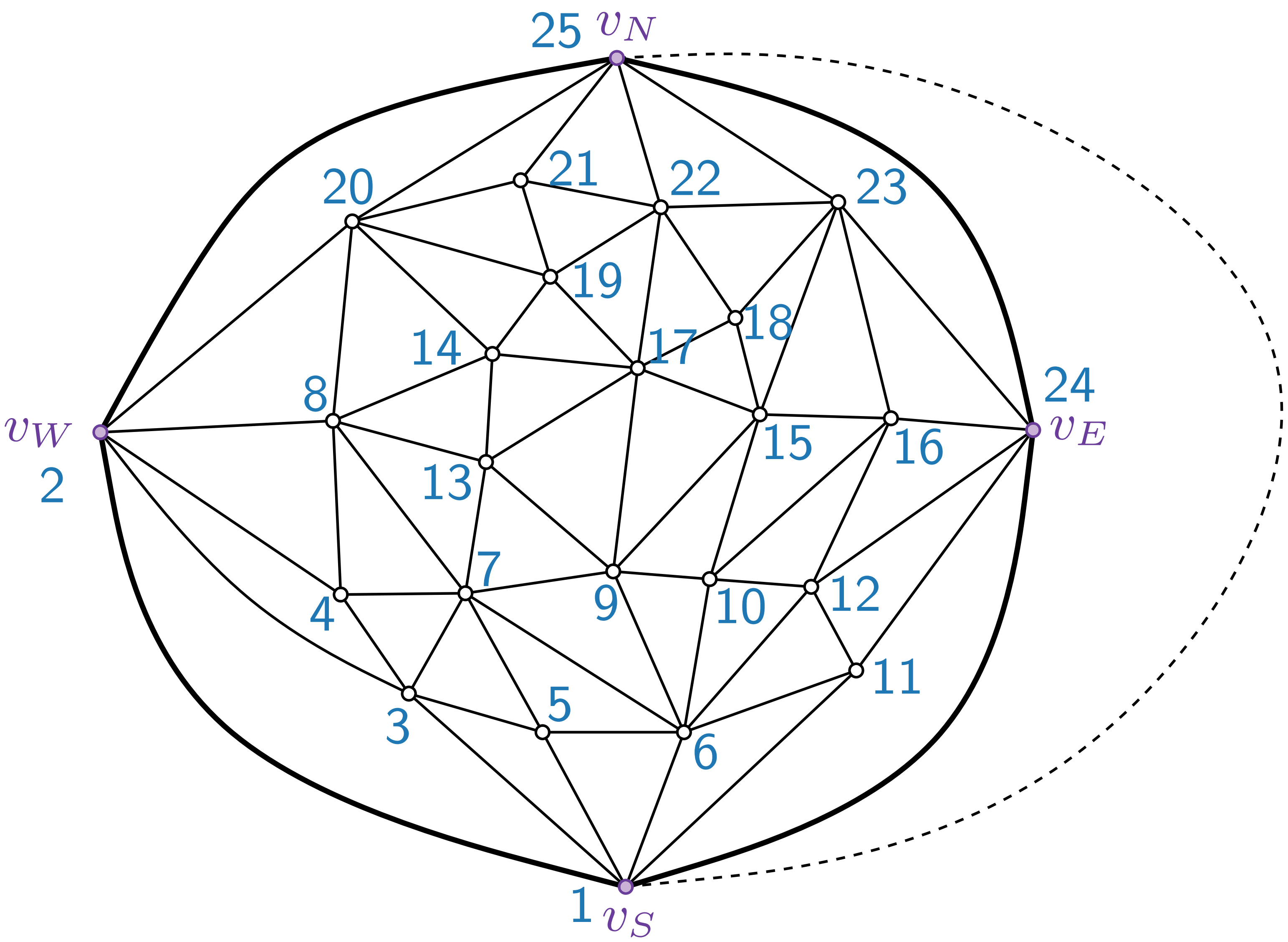

Refined Canonical Order → REL

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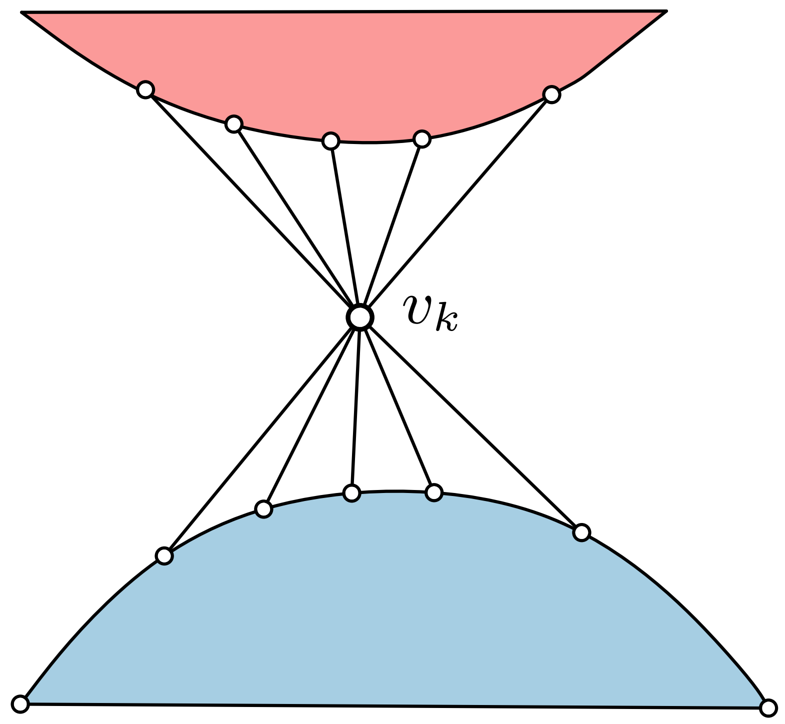

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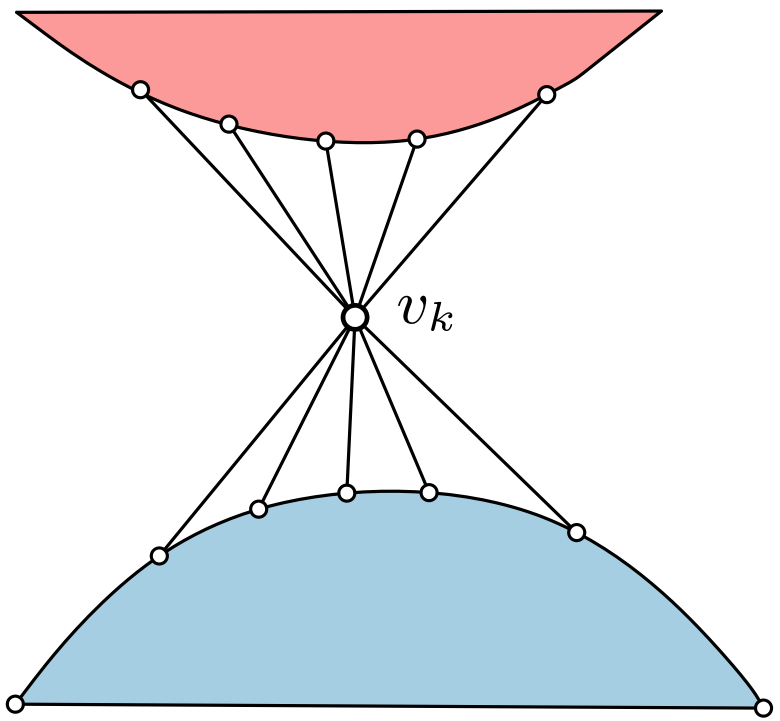

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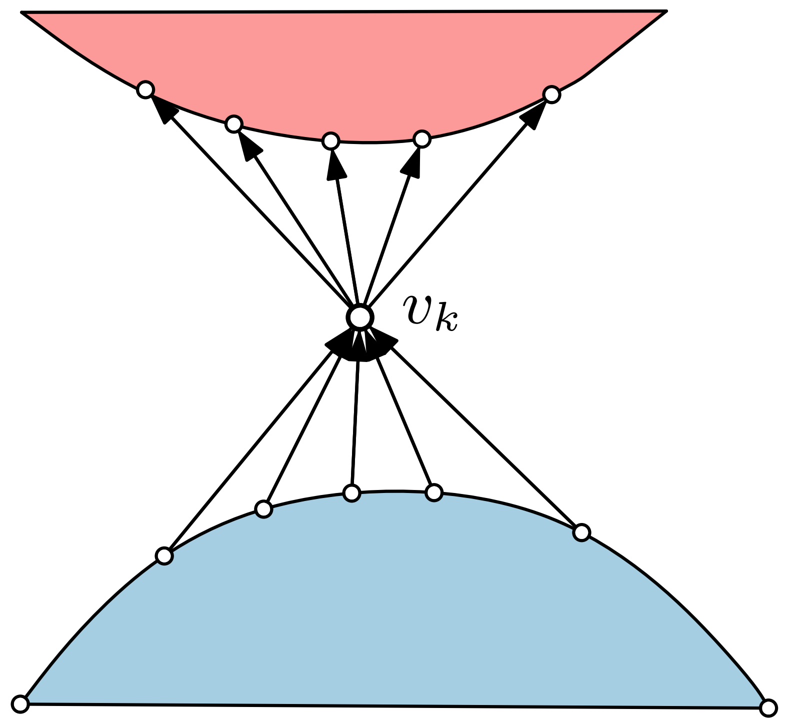

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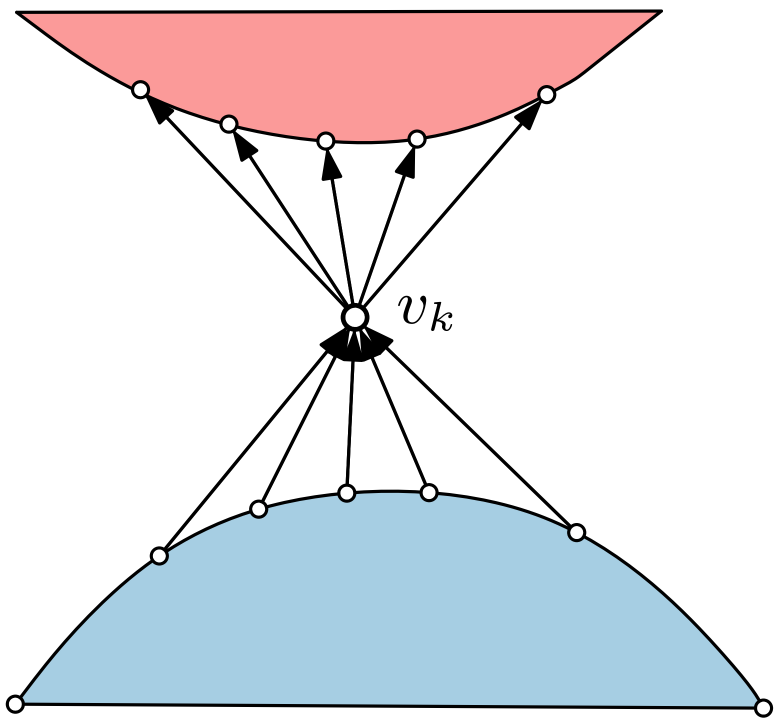

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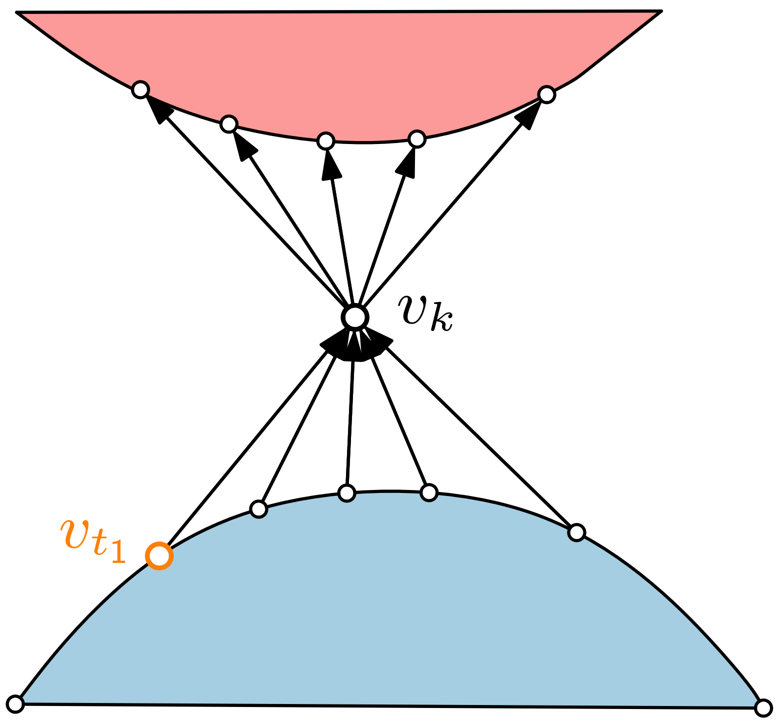

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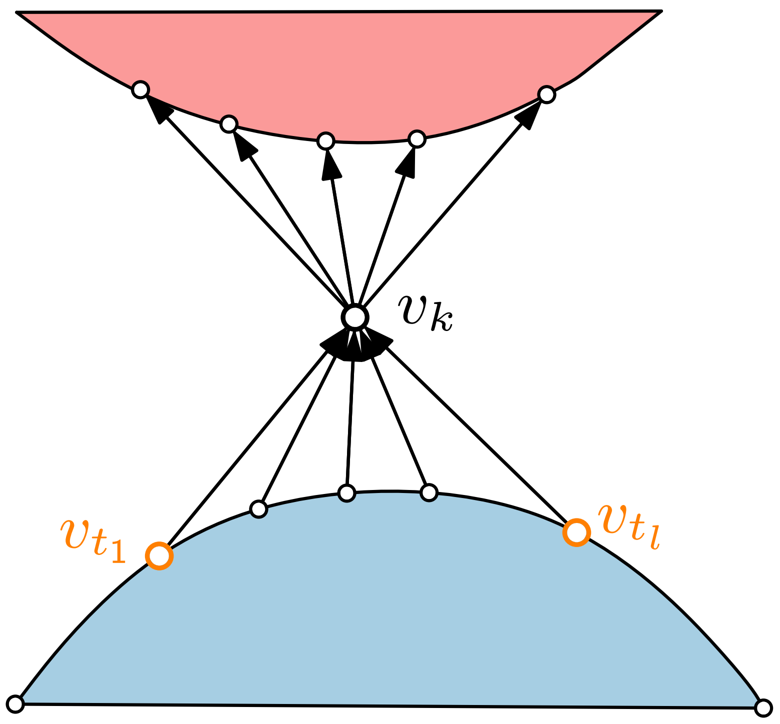

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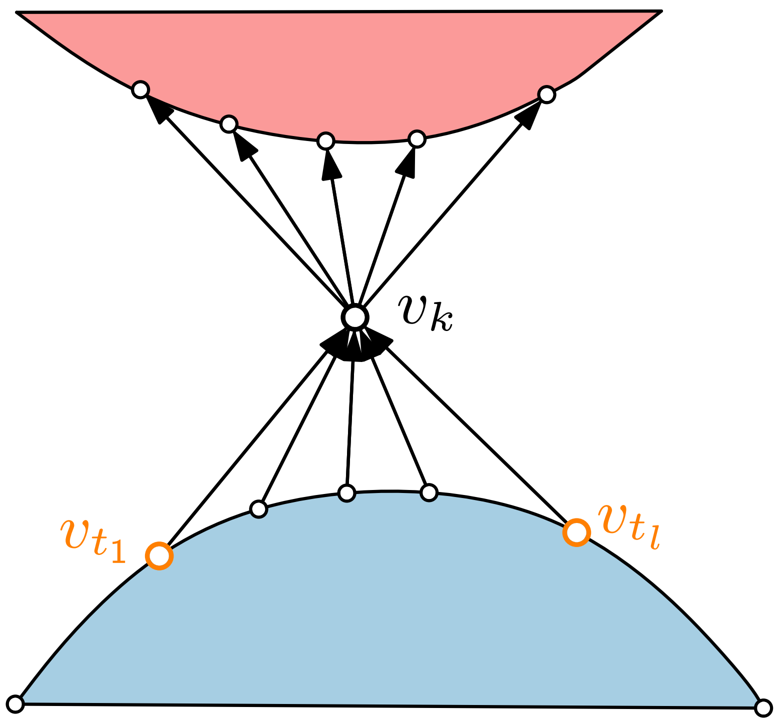

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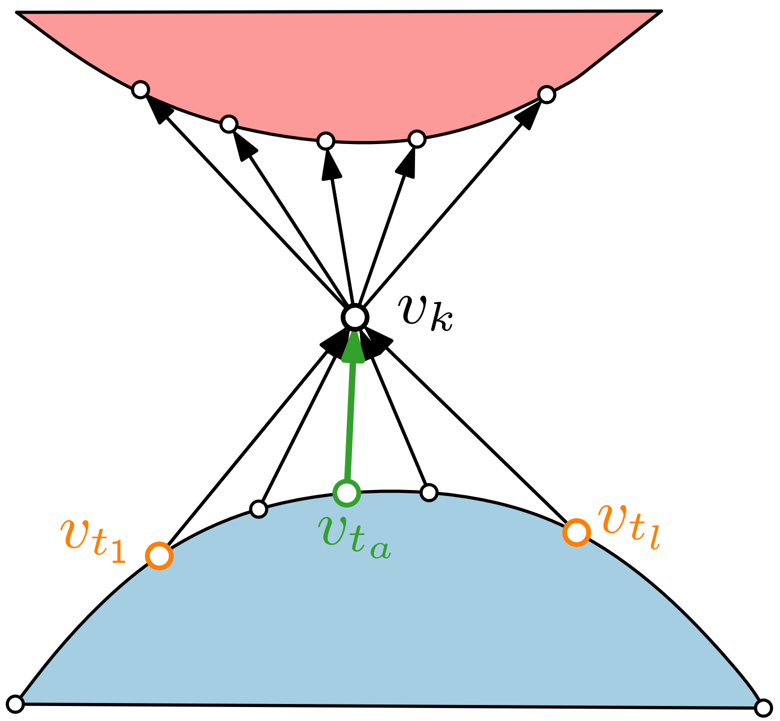

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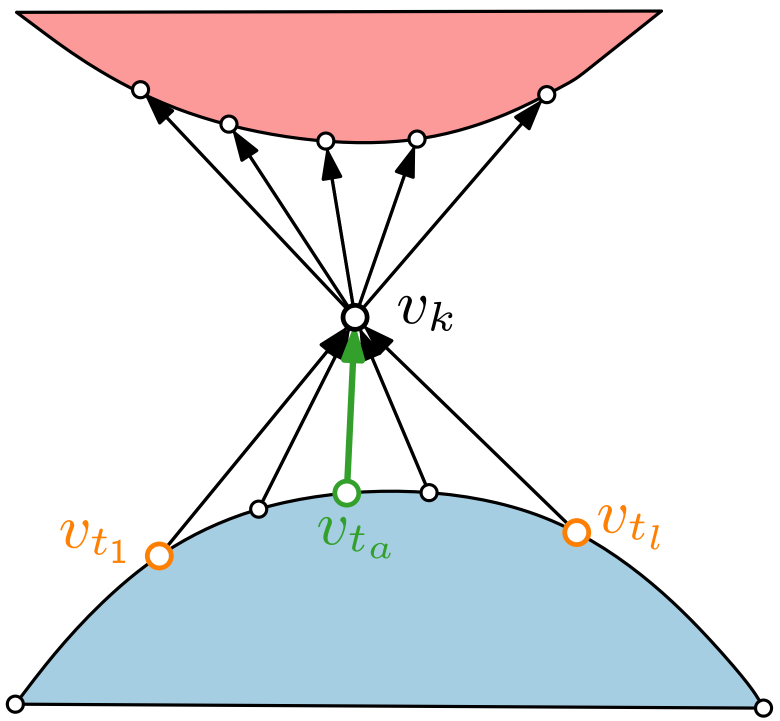

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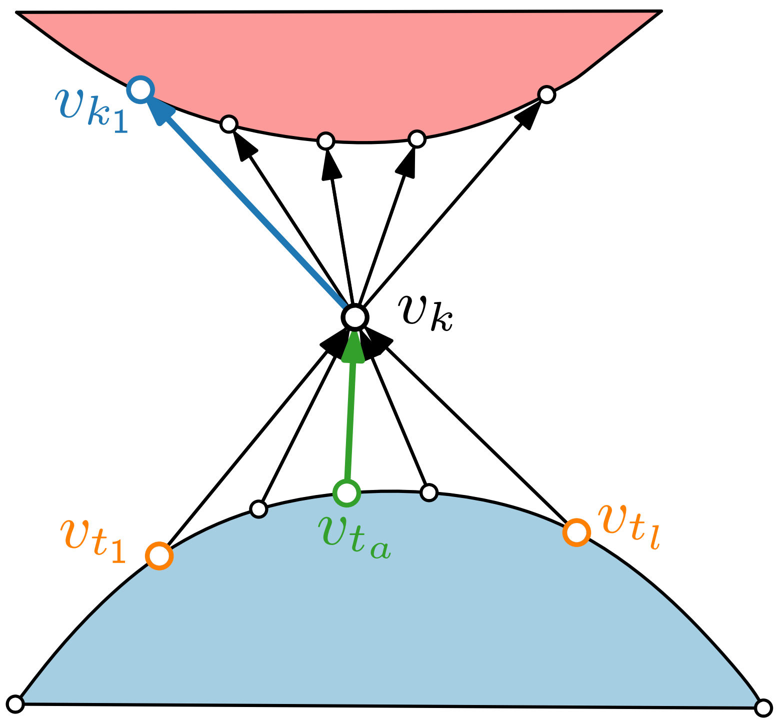

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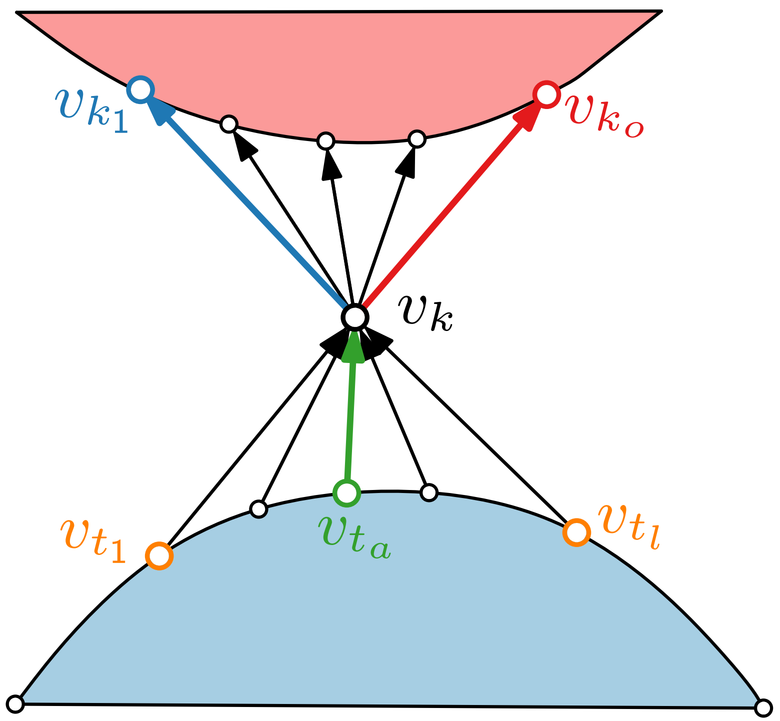

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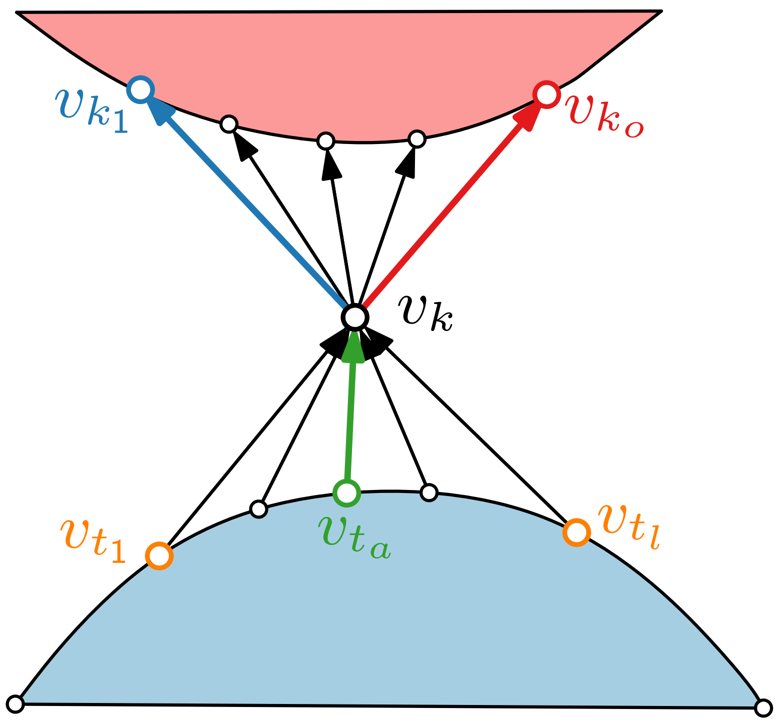

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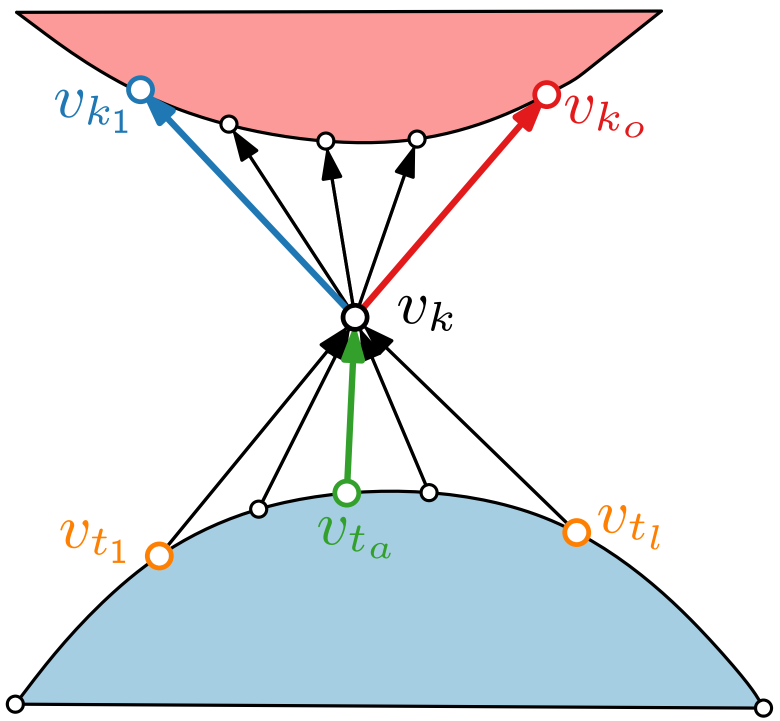

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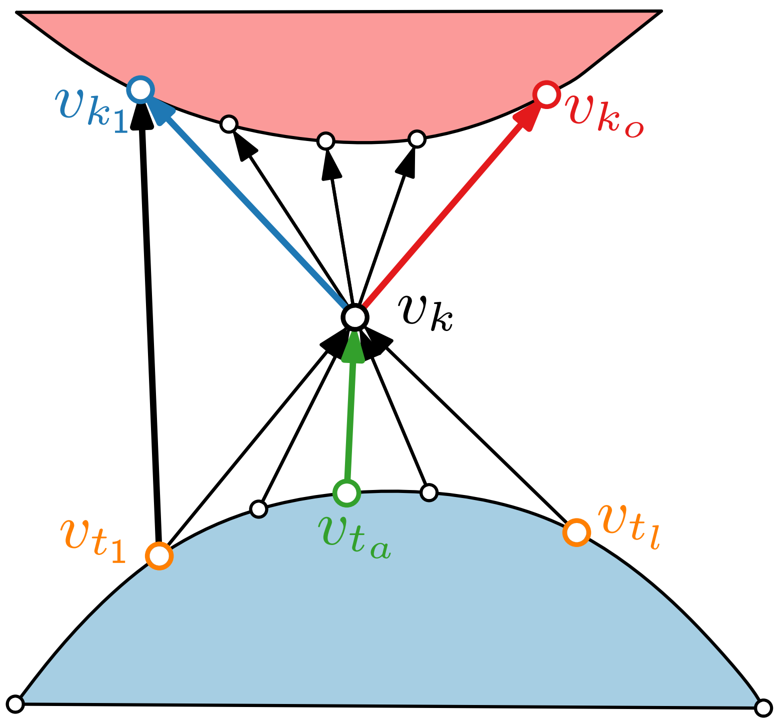

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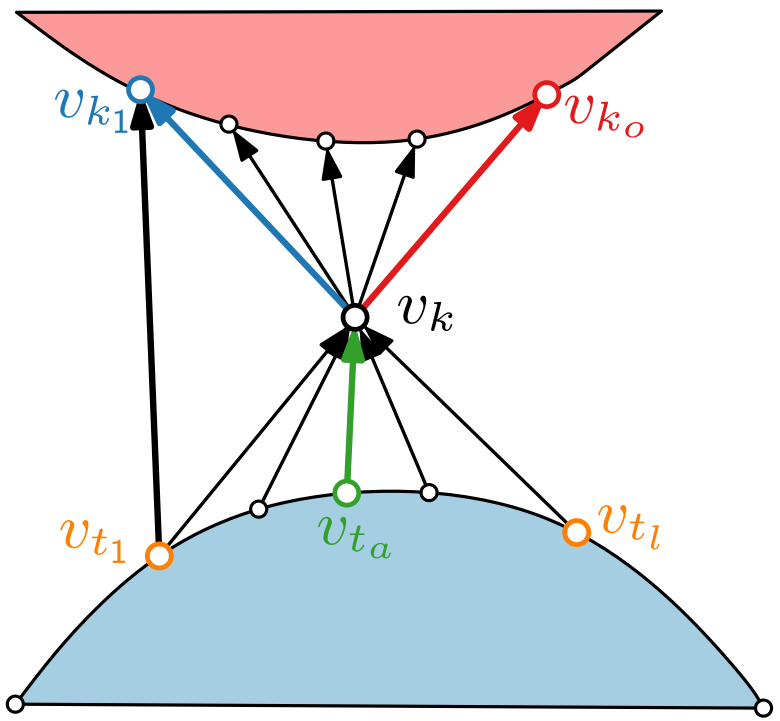

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Contradiction since $k > t_1$.

Refined Canonical Order → REL

Lemma 2.
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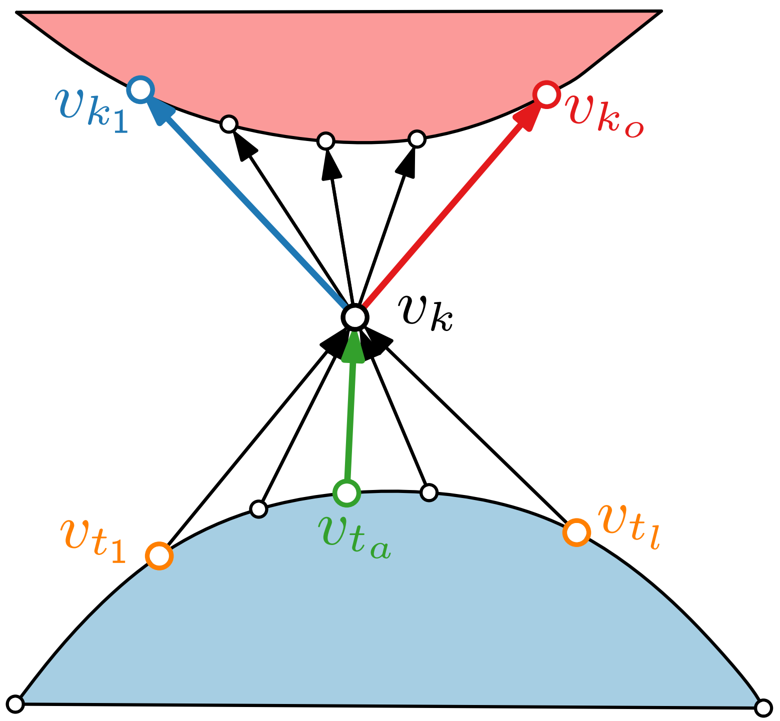

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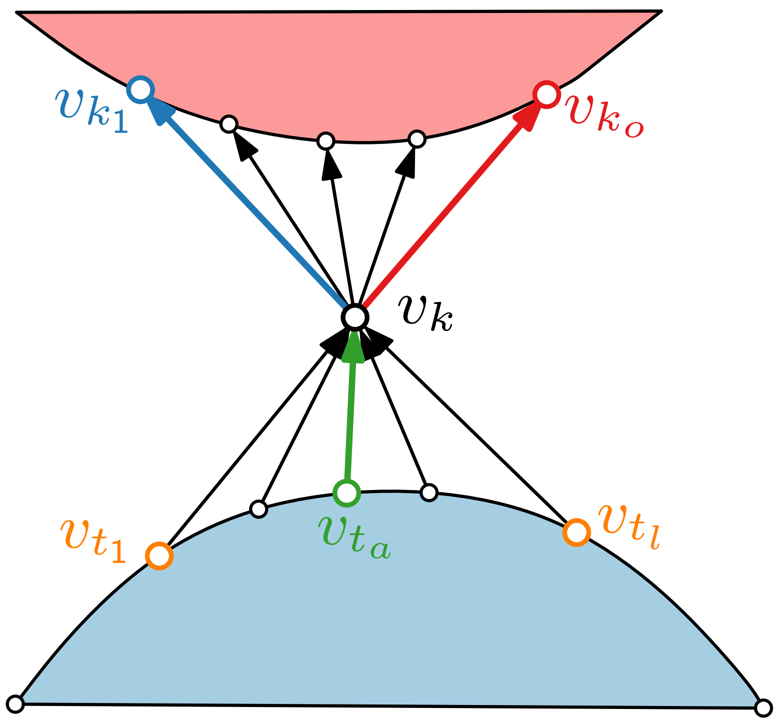

Refined Canonical Order $\to$ REL

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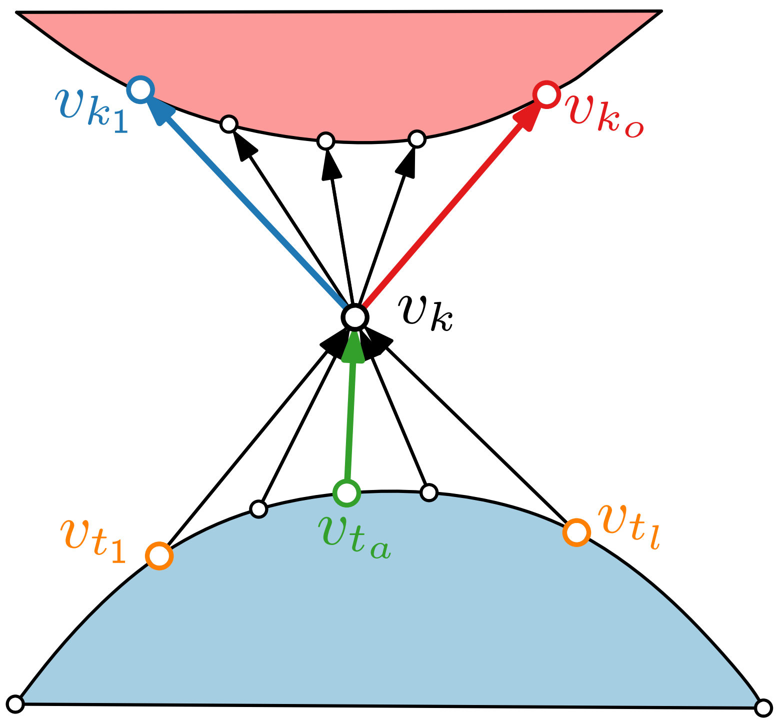

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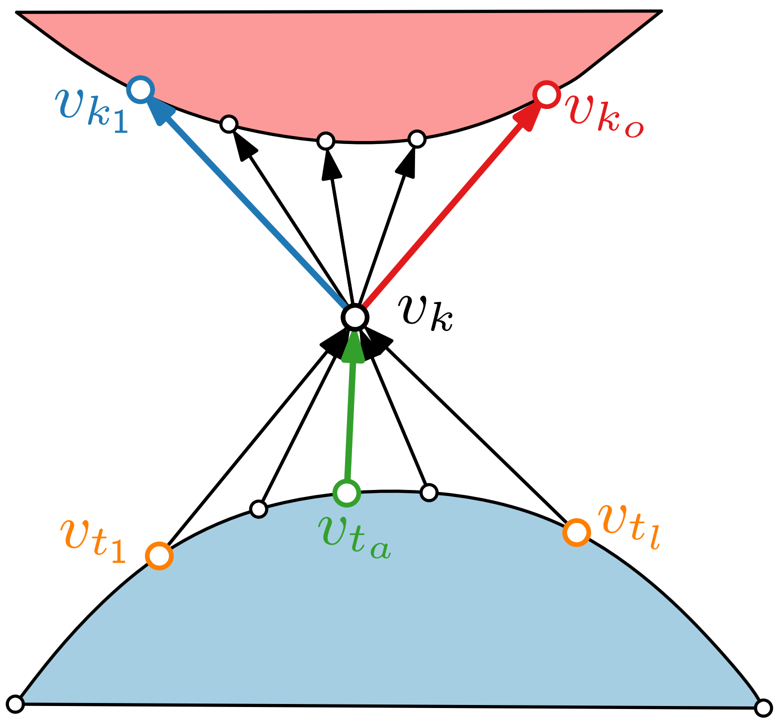

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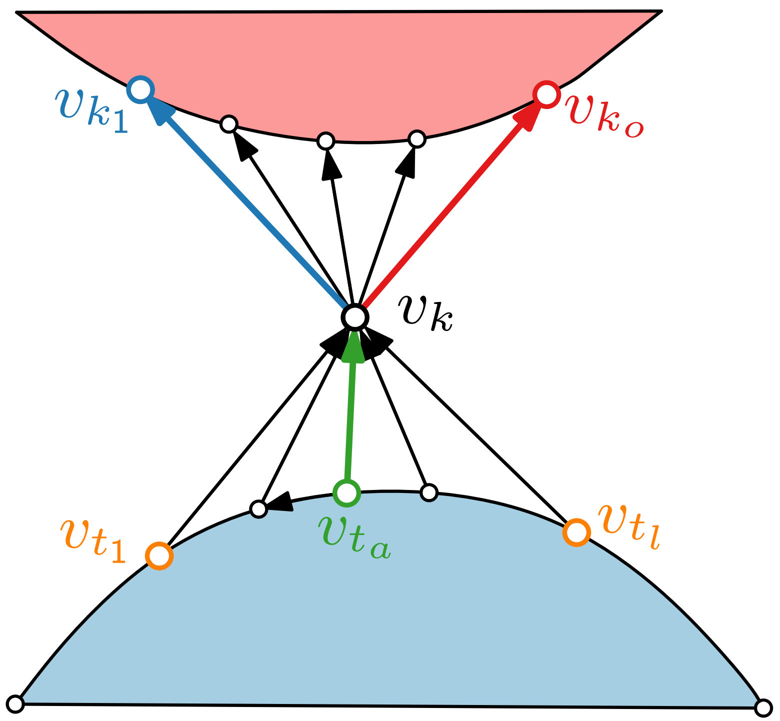

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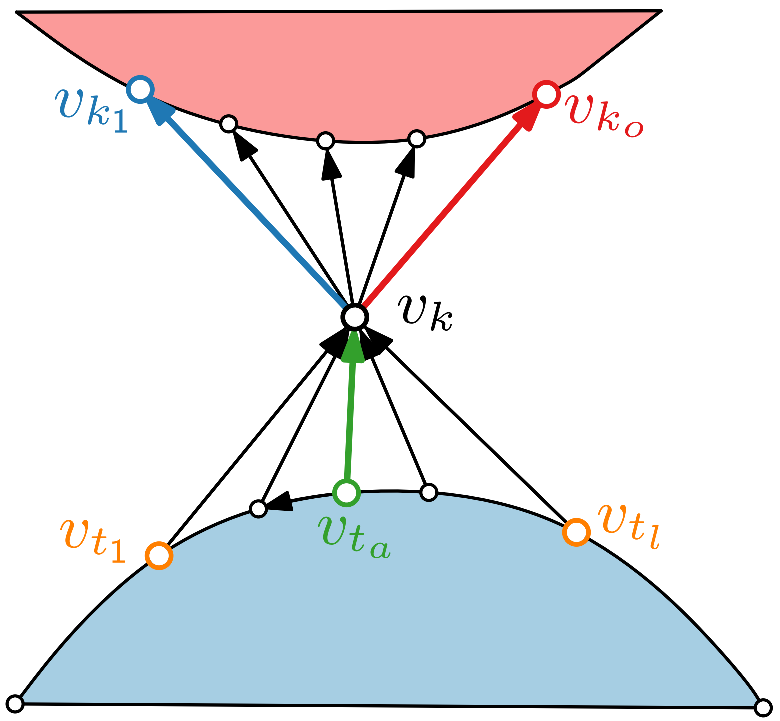

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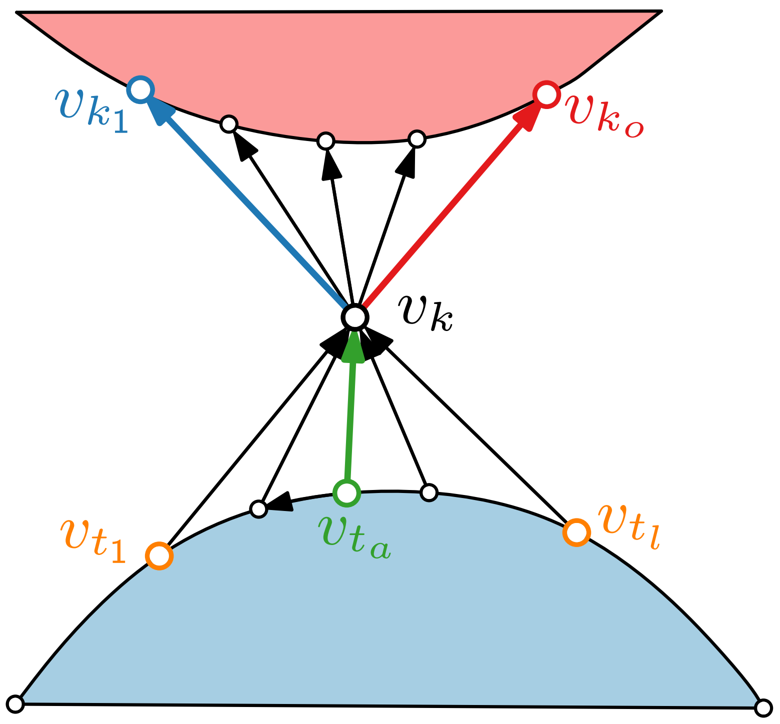

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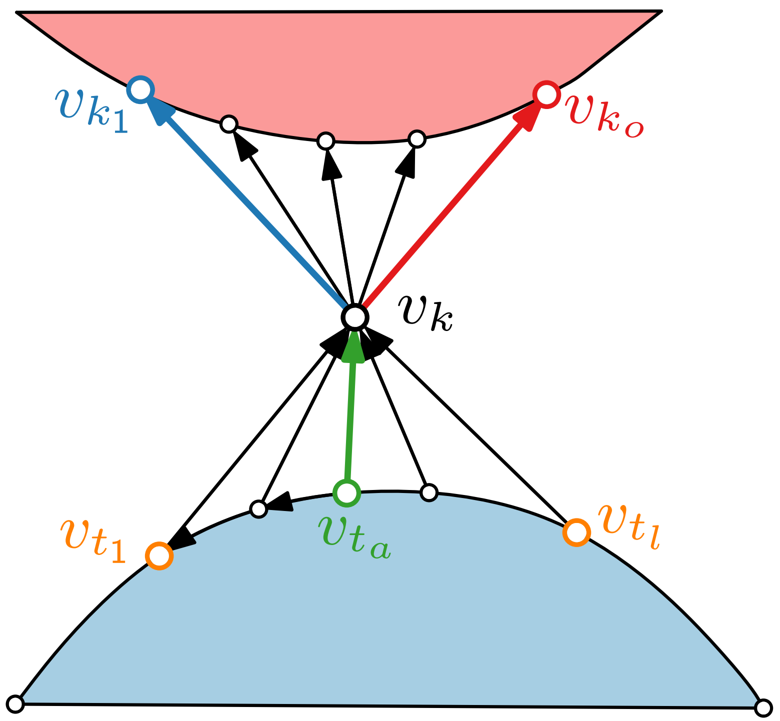

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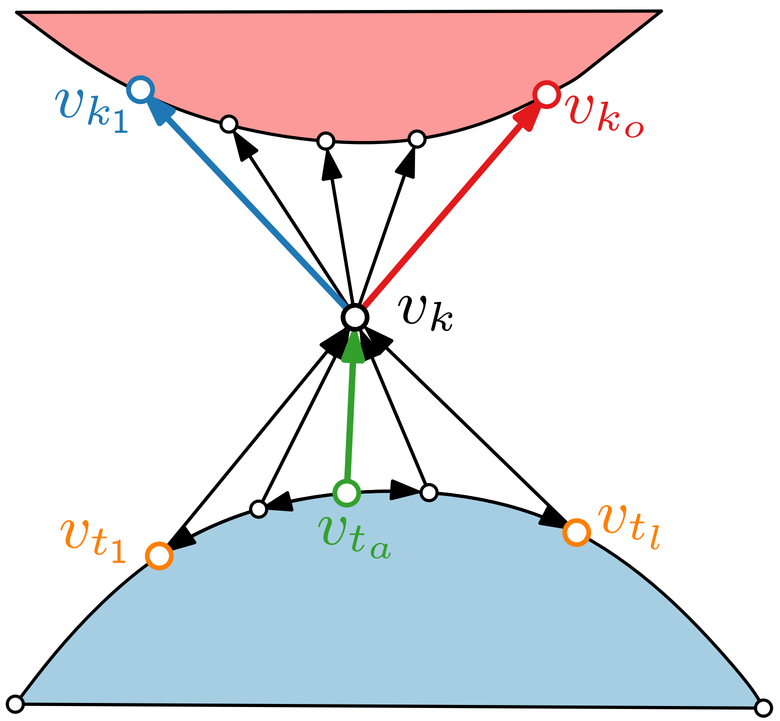

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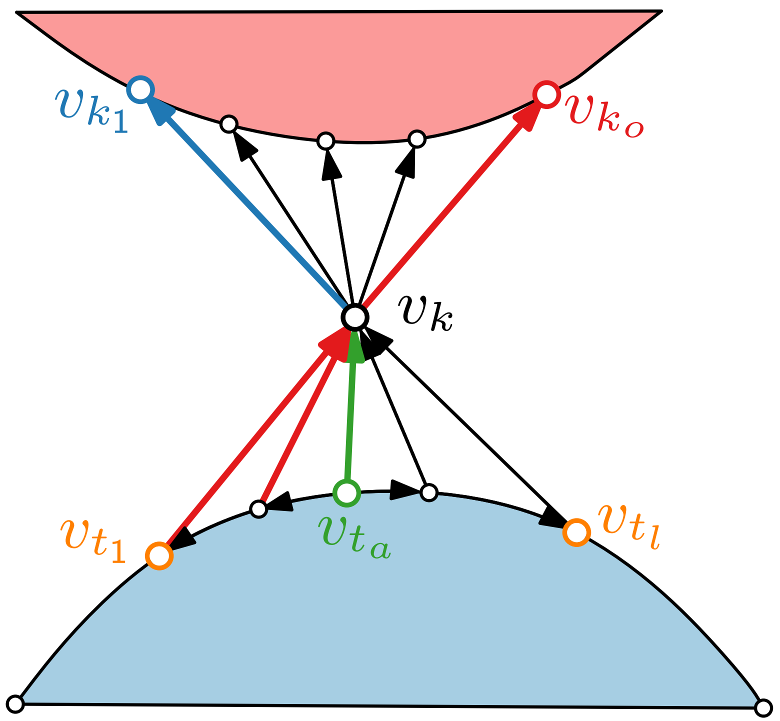

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- Edges (v_{t_i}, v_k), $1 \leq i < a - 1$, are right edges.

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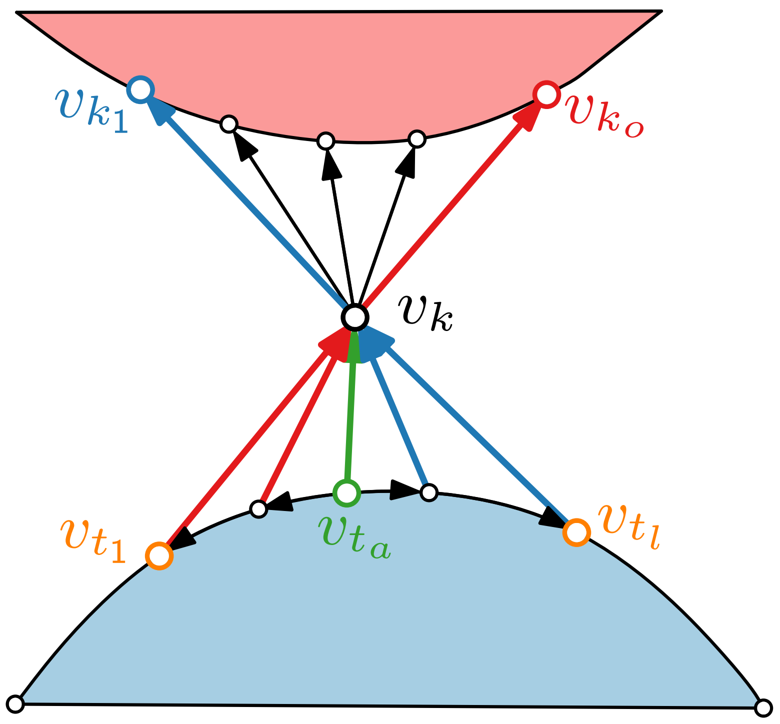

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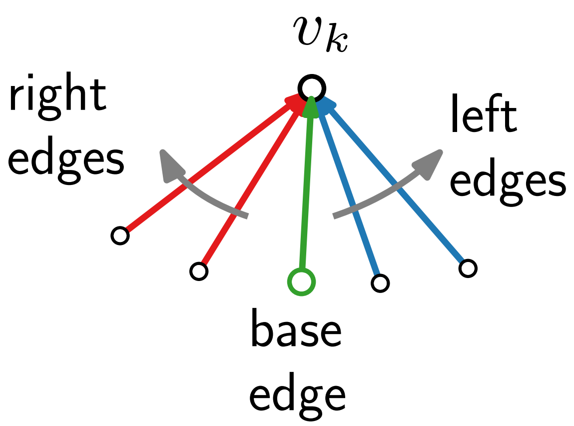
- Edges (v_{t_i}, v_k), $1 \leq i < a-1$, are right edges.
- Similarly, (v_{t_i}, v_k), for $a+1 \leq i \leq l$, are left edges.

Refined Canonical Order → REL



Refined Canonical Order $\rightarrow$ REL

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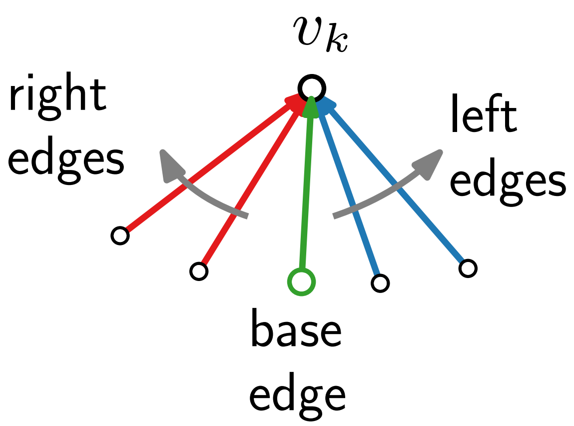

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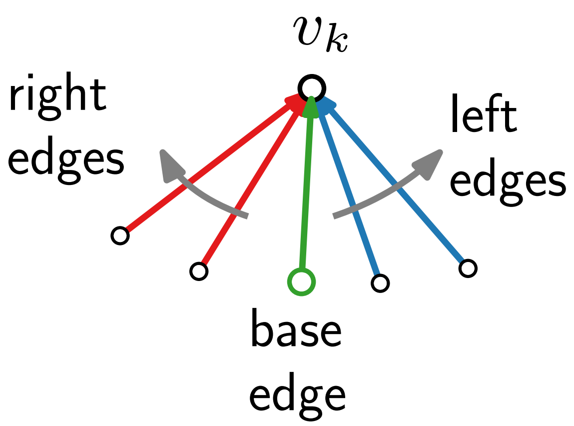

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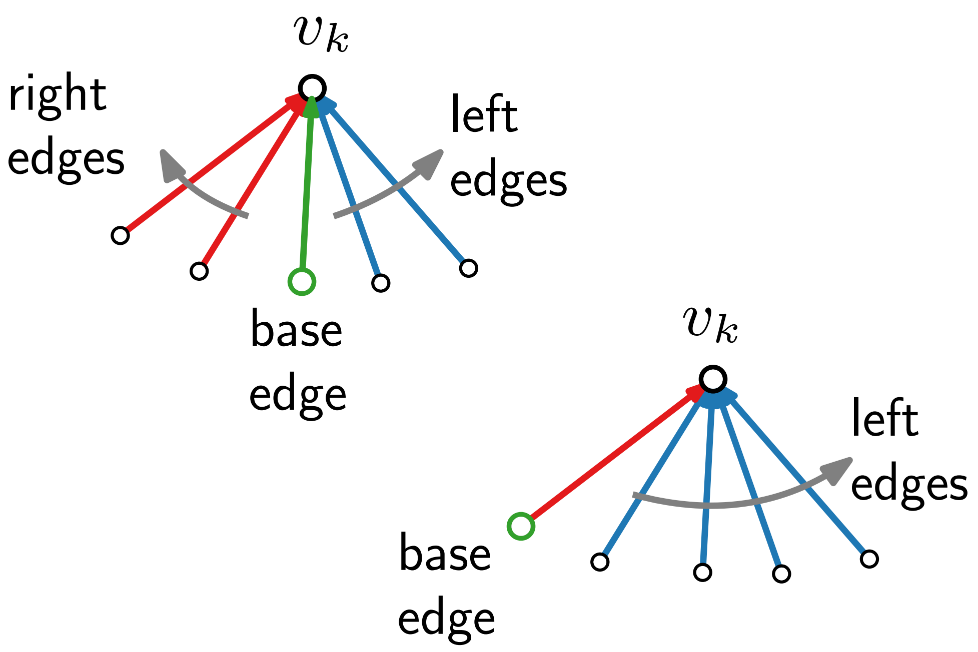

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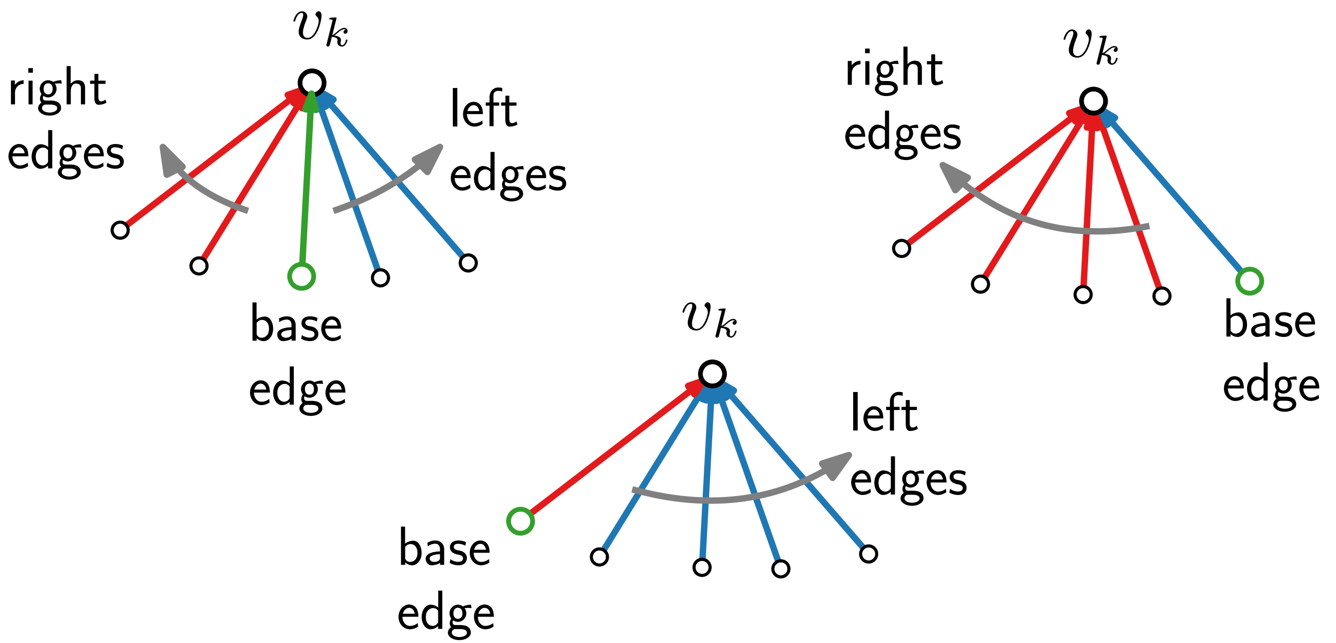

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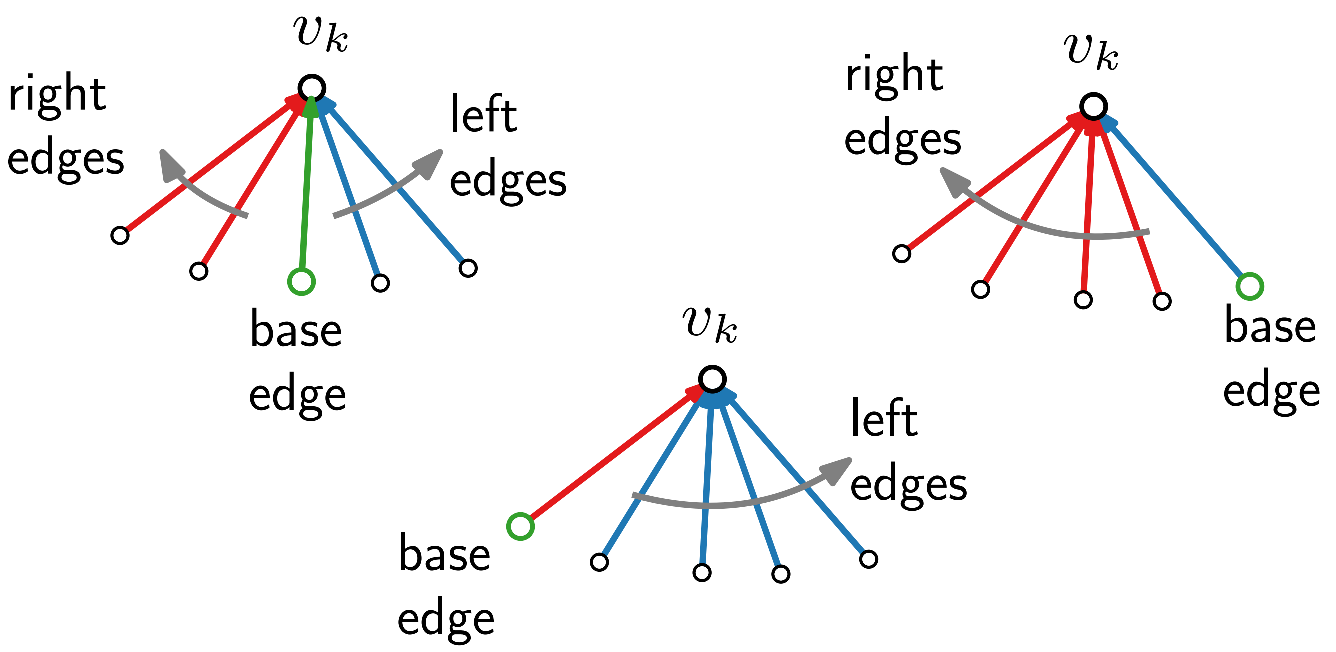

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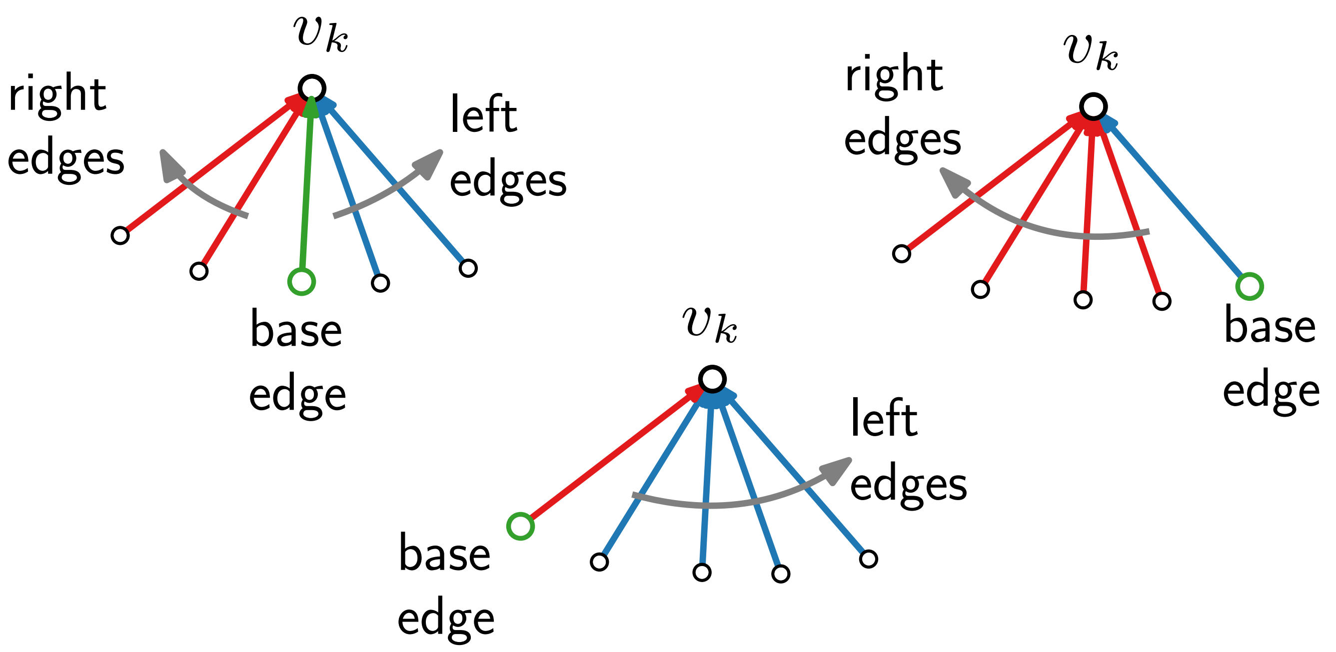
$\{T_r, T_b\}$ is a regular edge labeling.



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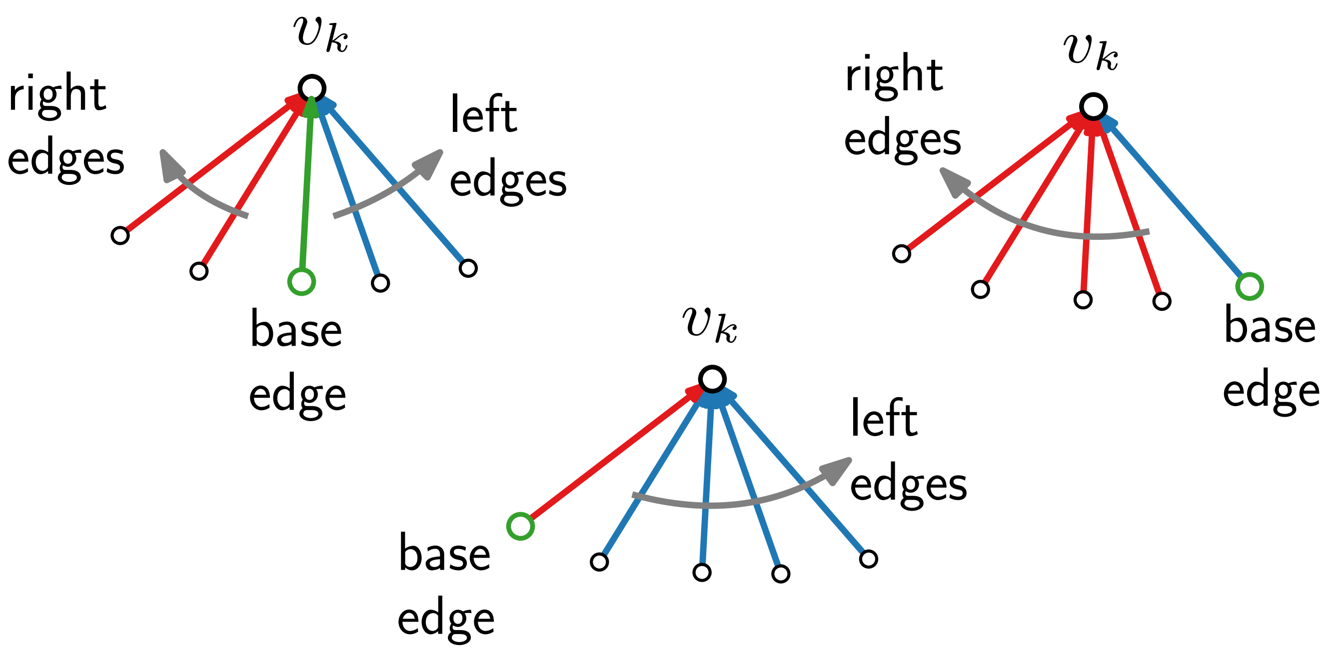

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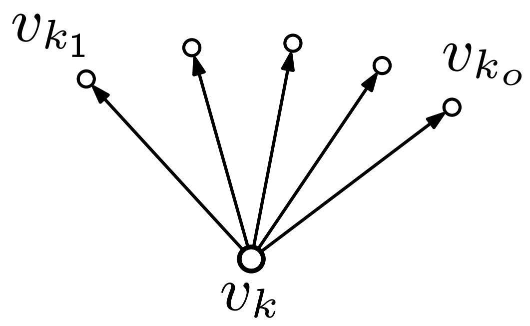

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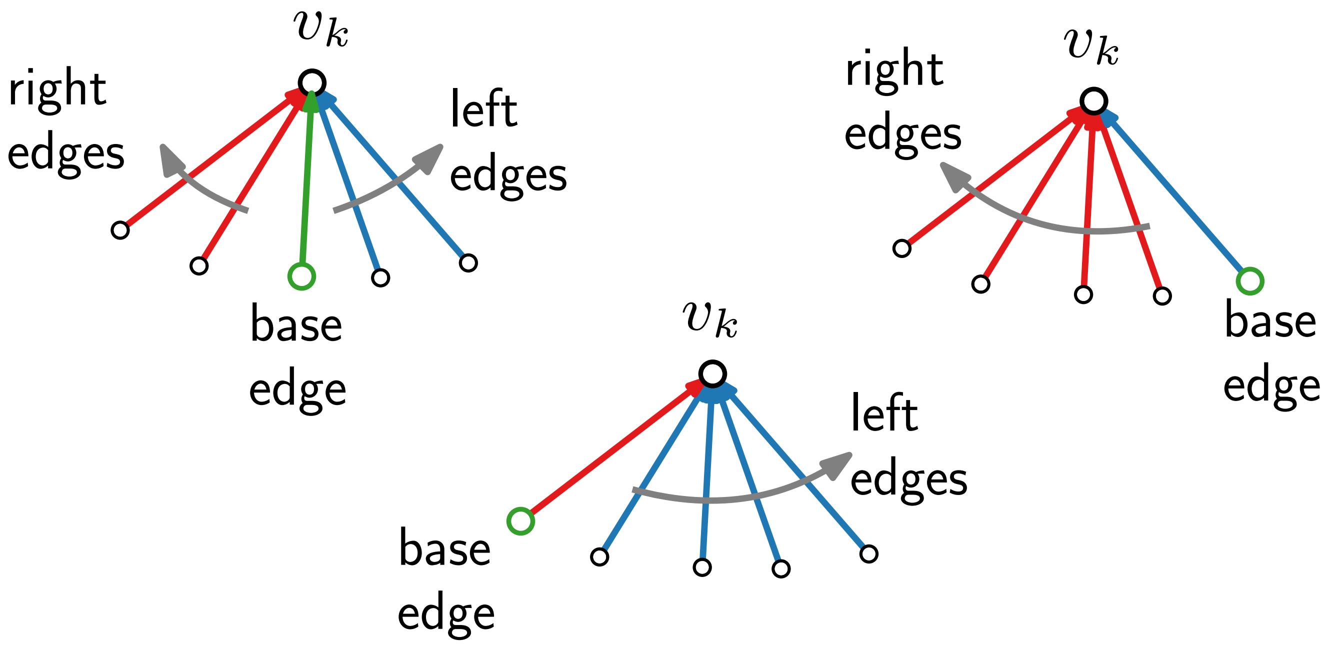

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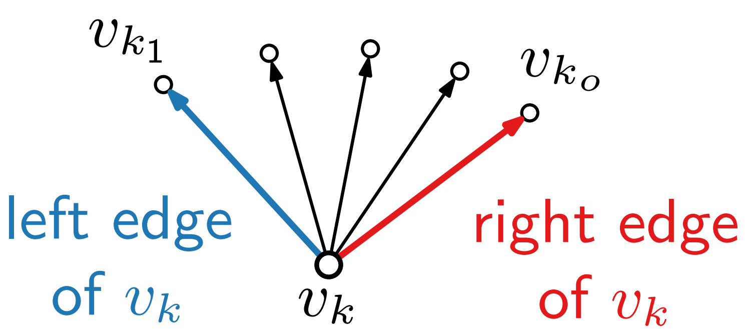

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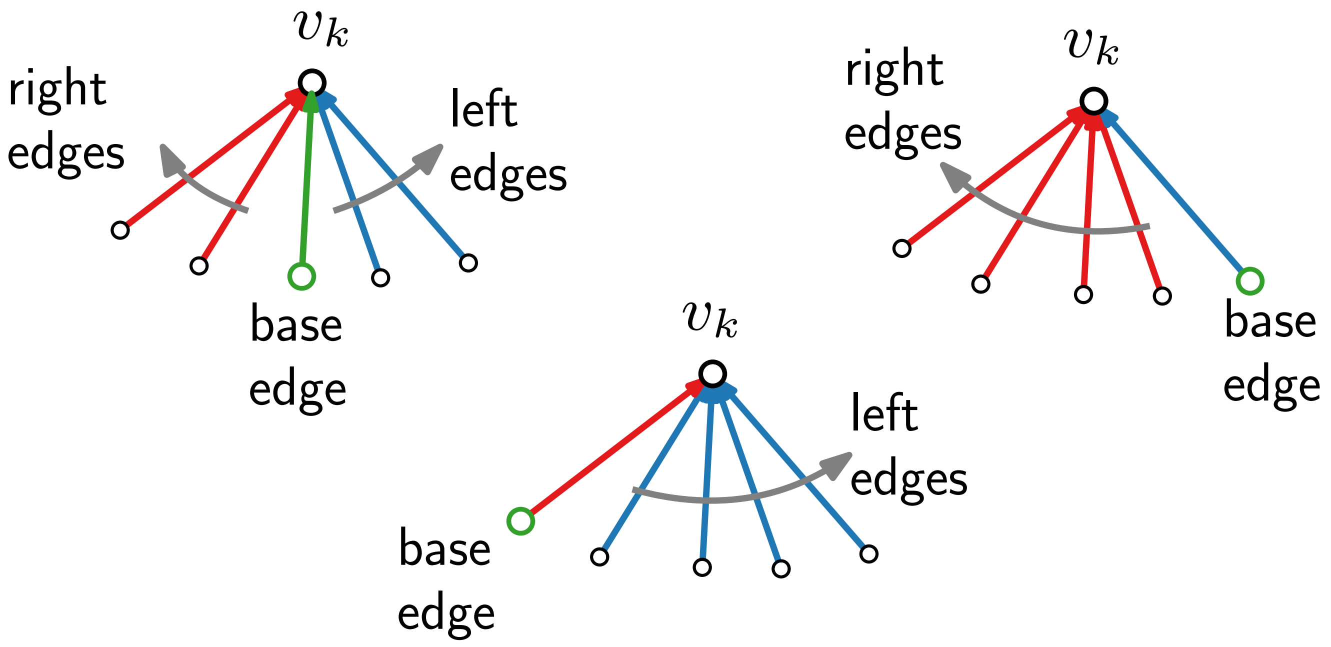

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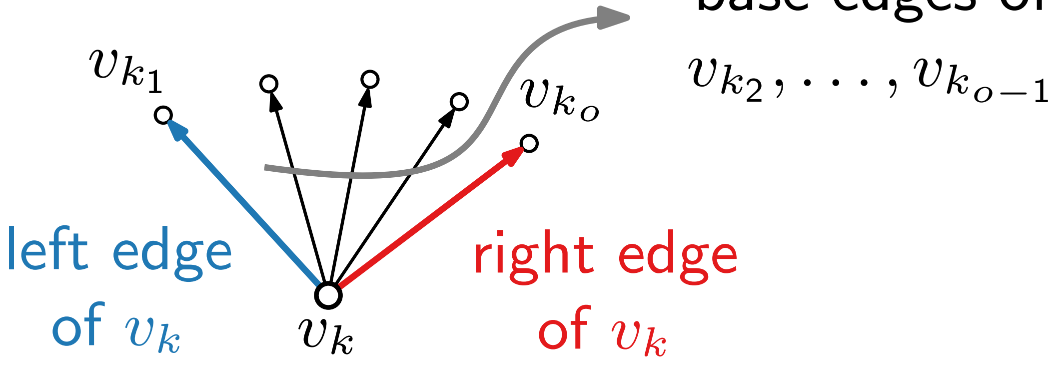

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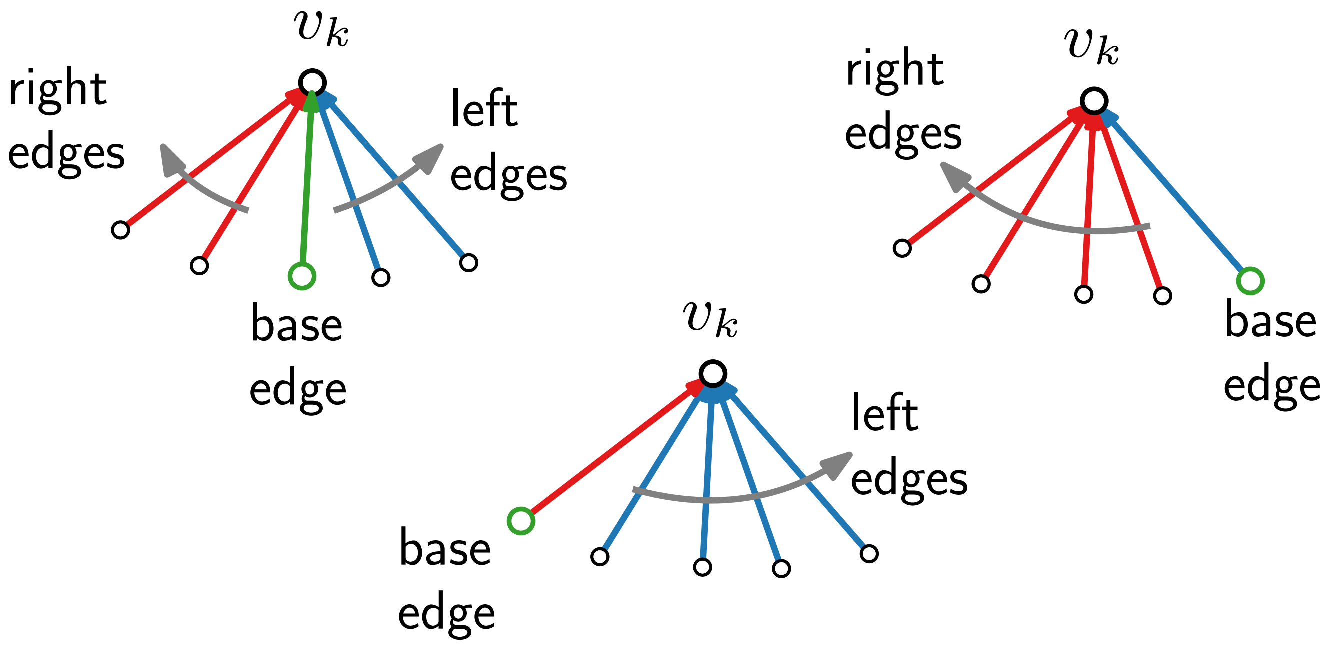

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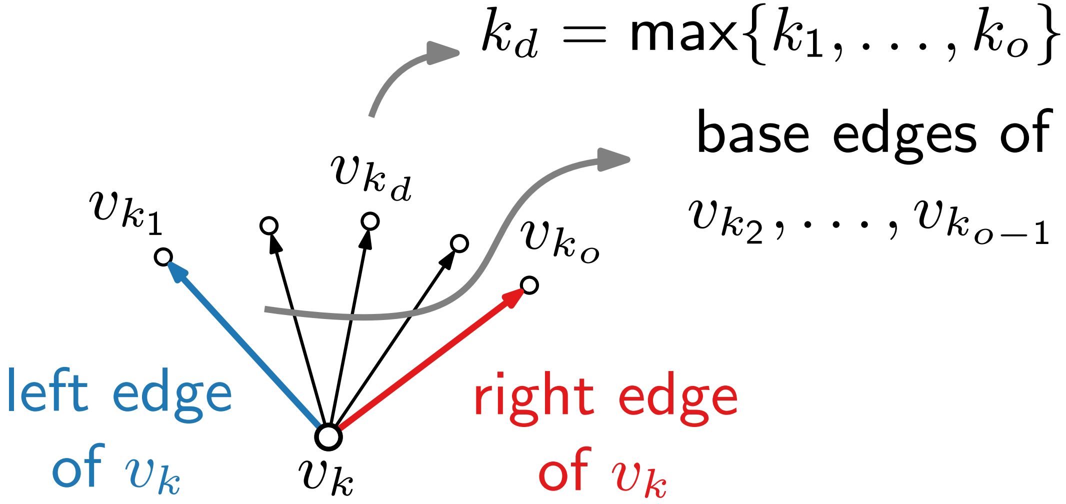

Refined Canonical Order → REL

Coloring.

- Color right (left) edges in red (blue).
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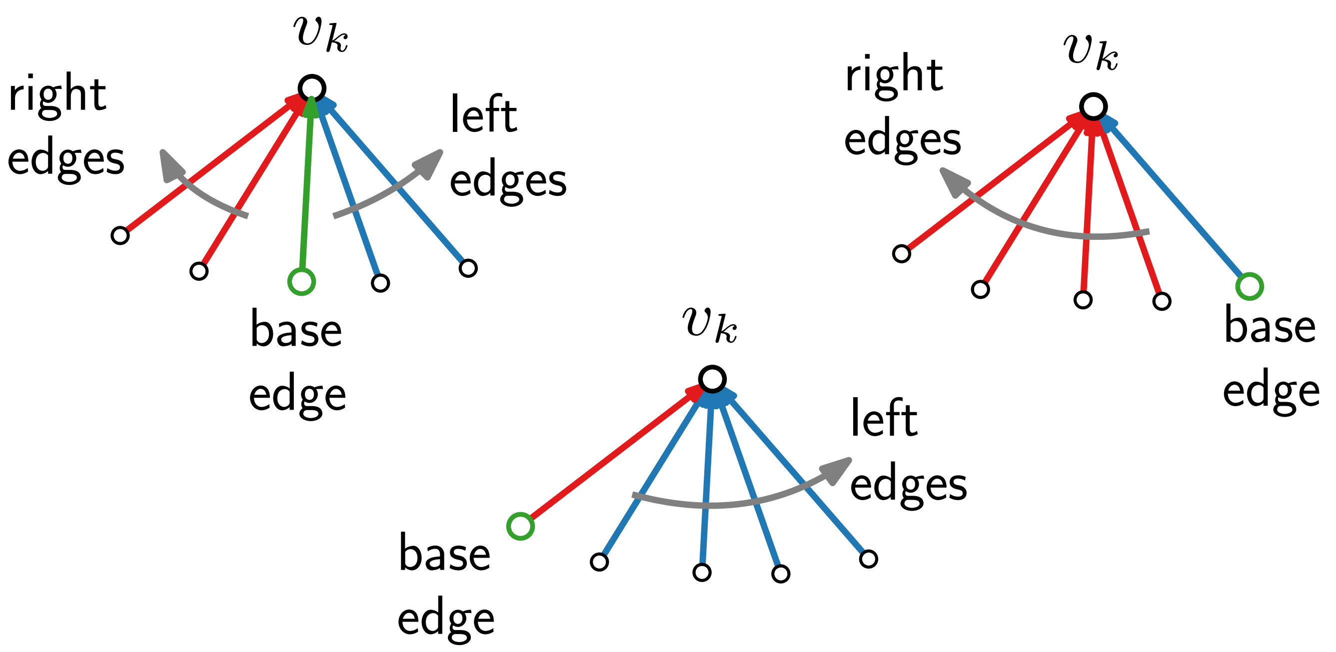
Let T_r be the red edges and T_b the blue edges.



Lemma 3.
$\{T_r, T_b\}$ is a regular edge labeling.

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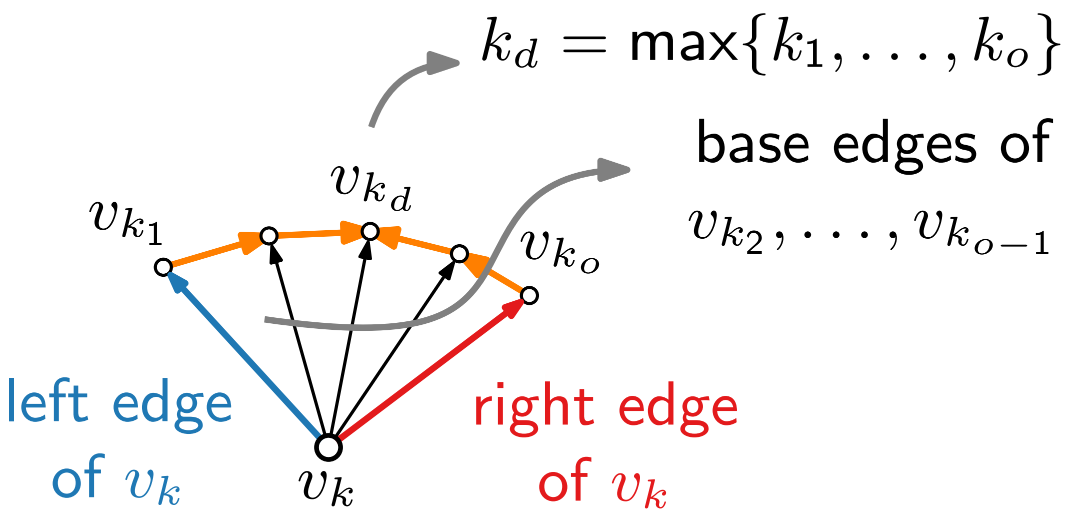
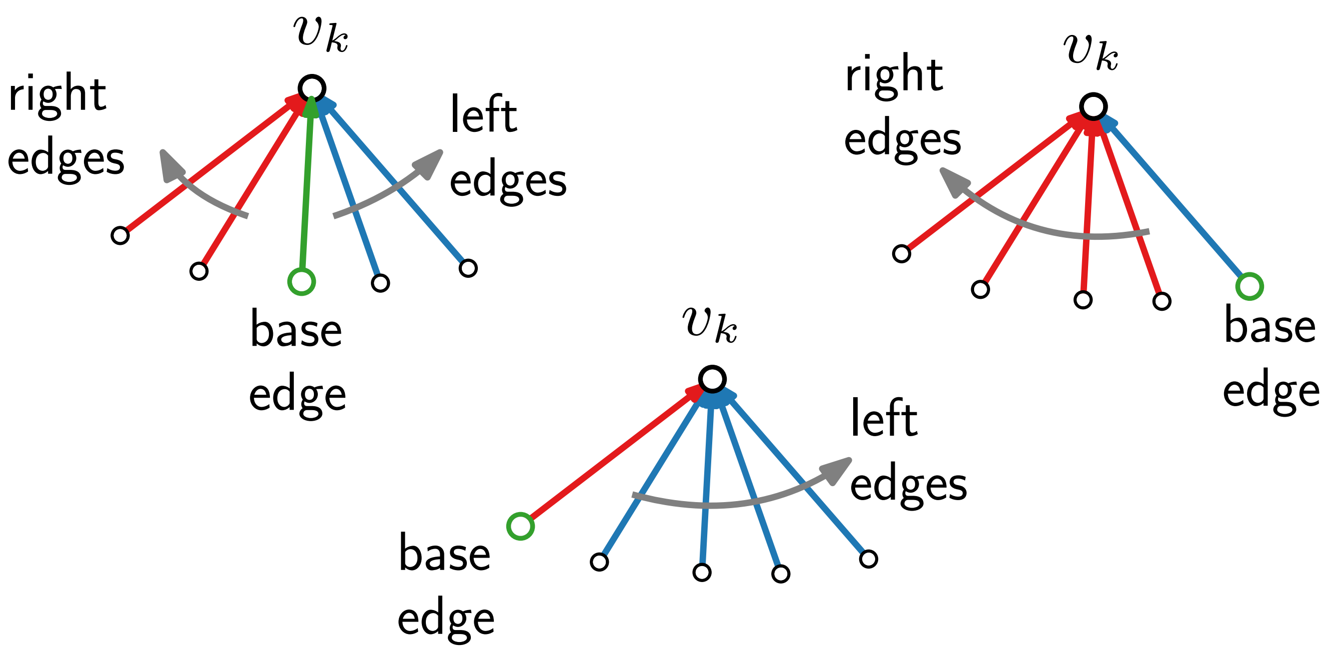


- $k_1 < k_2 < \ldots < k_d$ and $k_d > k_{d+1} > \ldots > k_o$

Refined Canonical Order → REL



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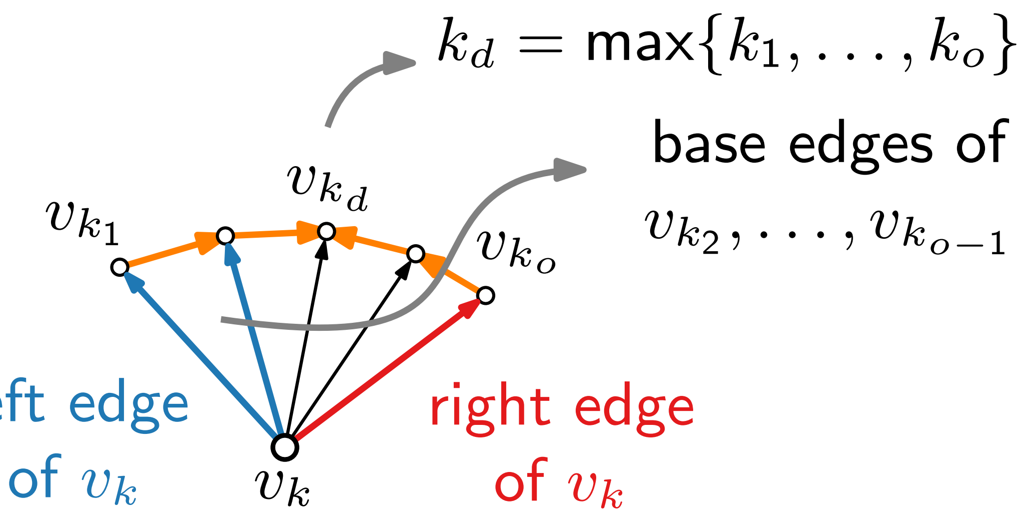

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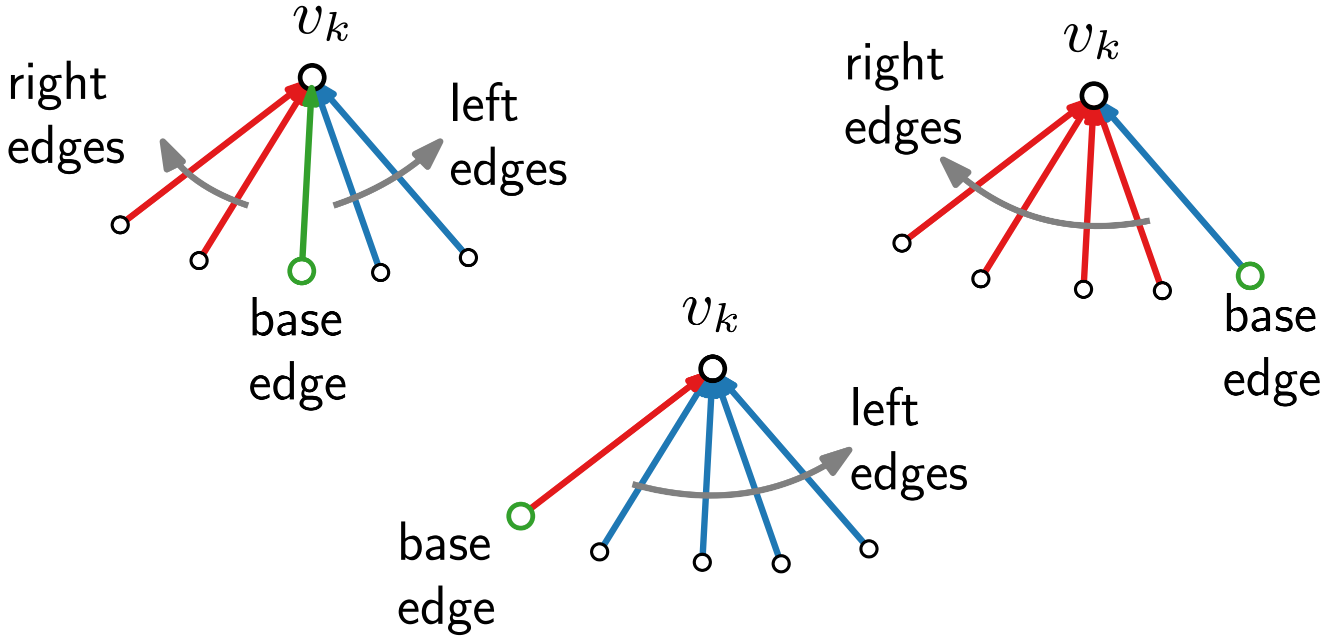

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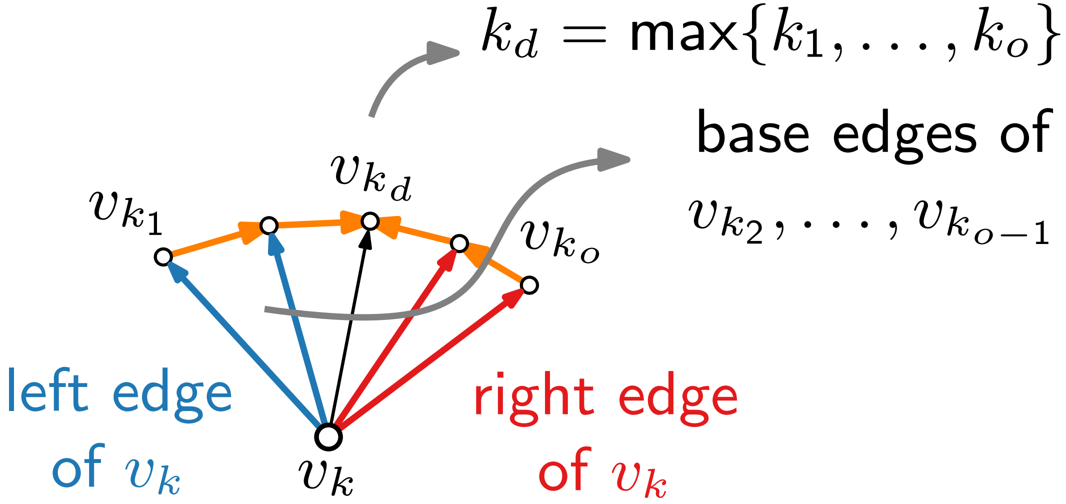

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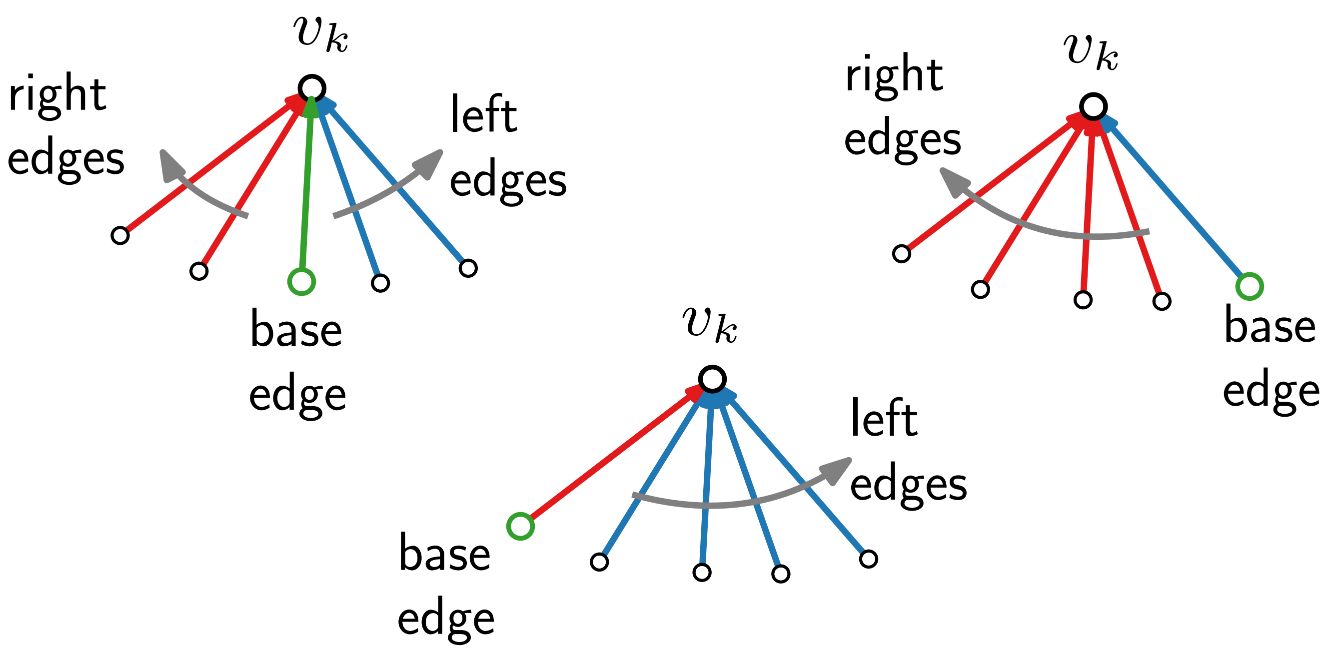

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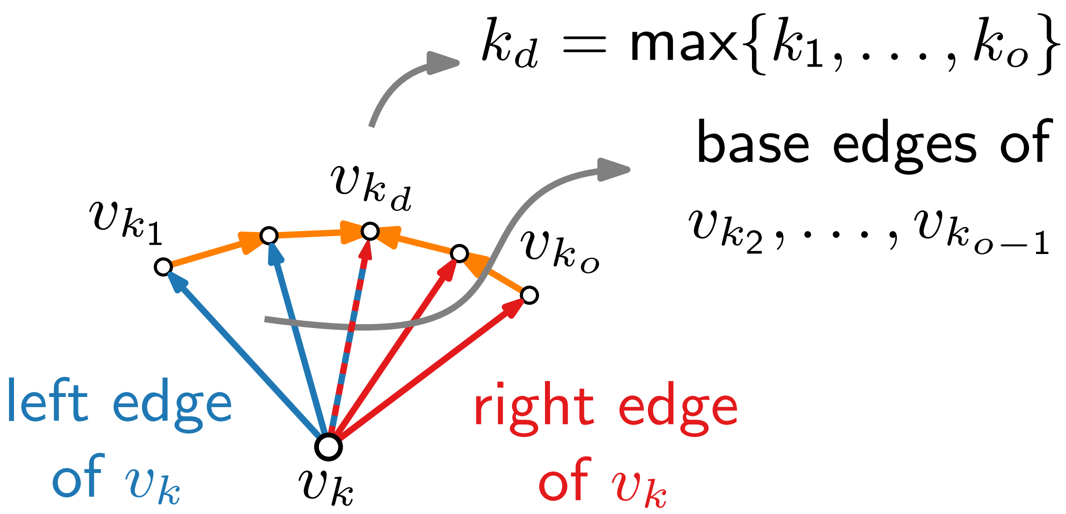

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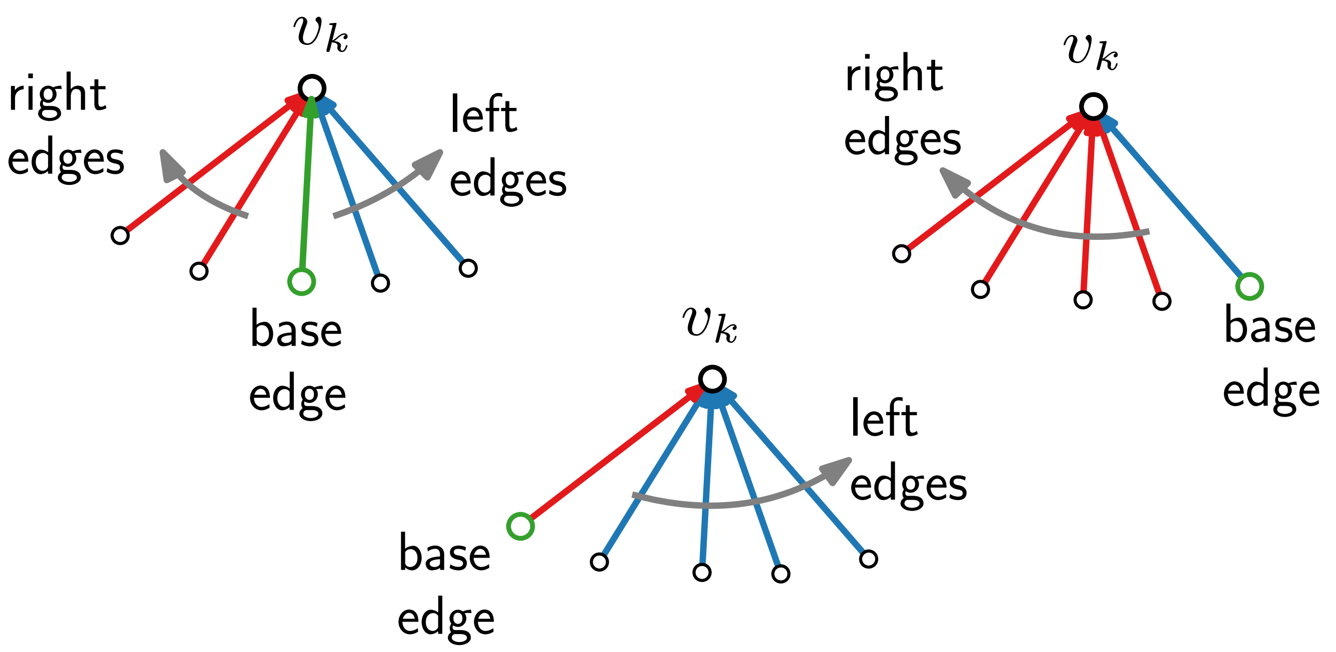

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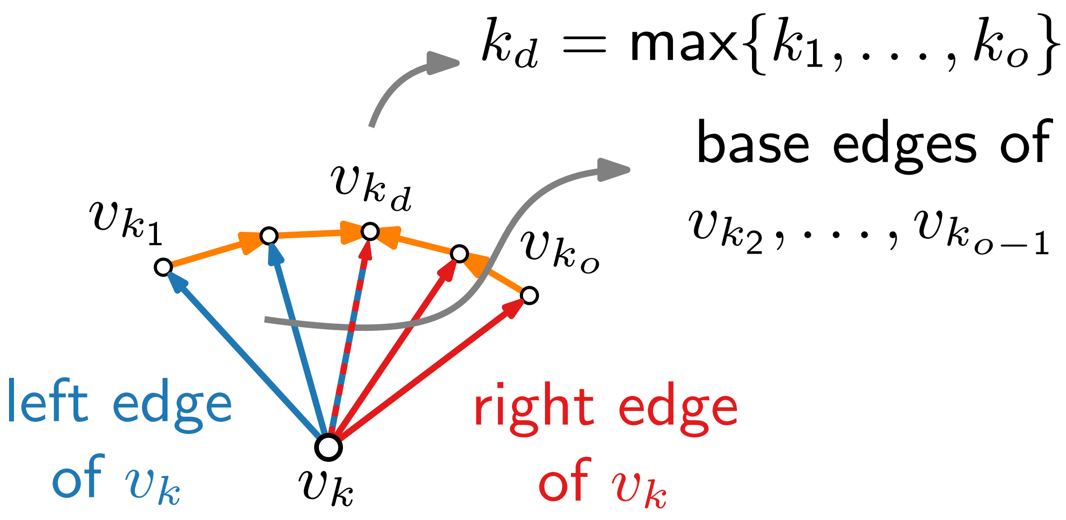

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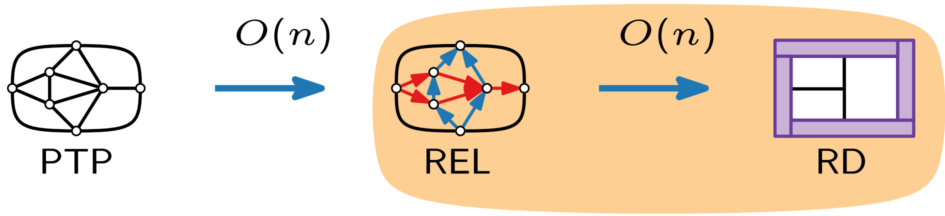
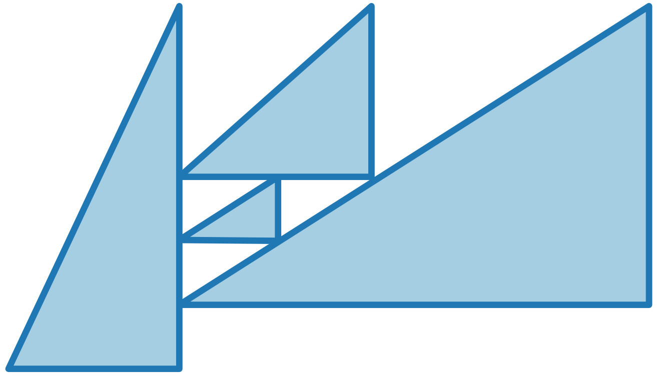
$\Rightarrow$ Circular order of outgoing edges at v_k correct.

[Kant, He '94]:



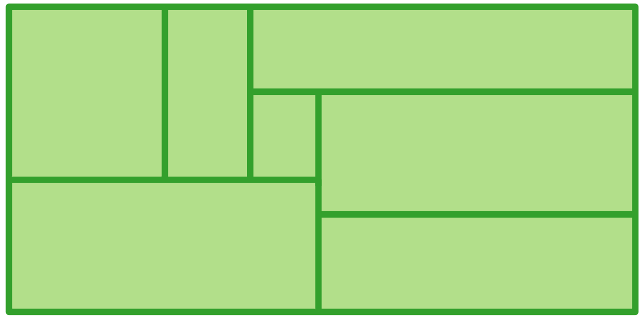
Visualization of Graphs

Lecture 8:
Contact Representations of Planar Graphs:
Triangle Contacts and Rectangular Duals

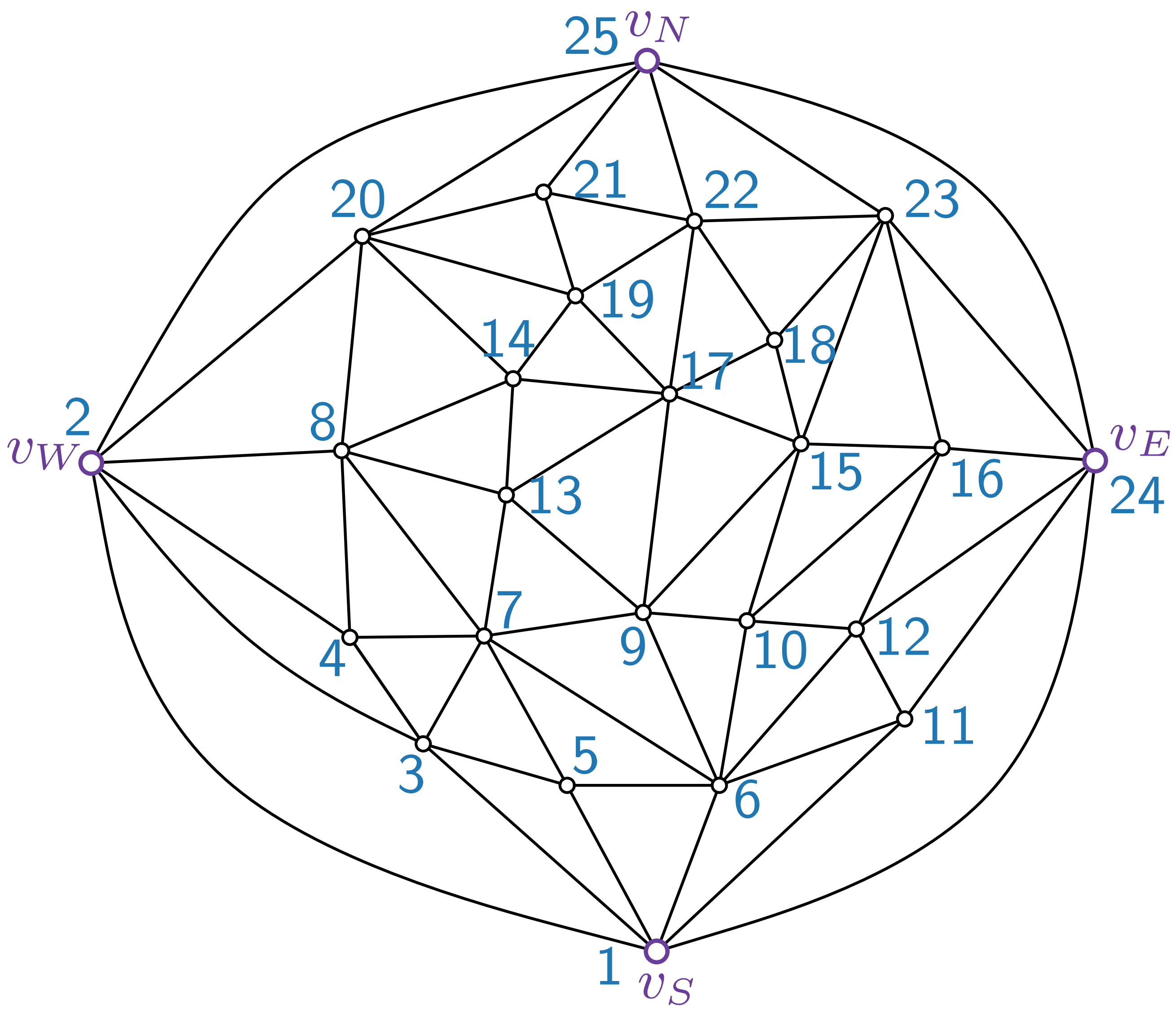


Part V:
Computing the Coordinates

Alexander Wolff



From REL to st-Digraphs to Coordinates



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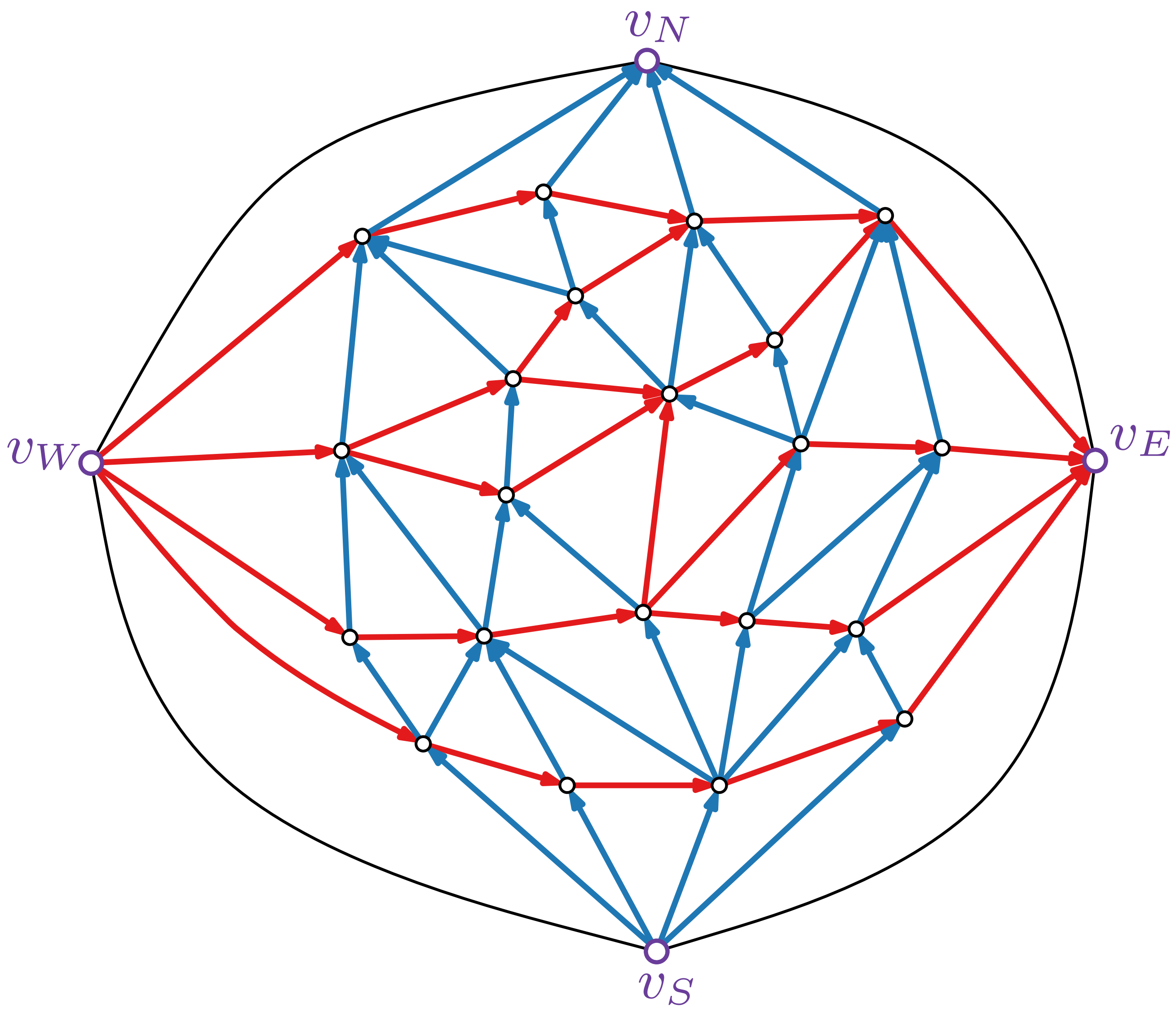

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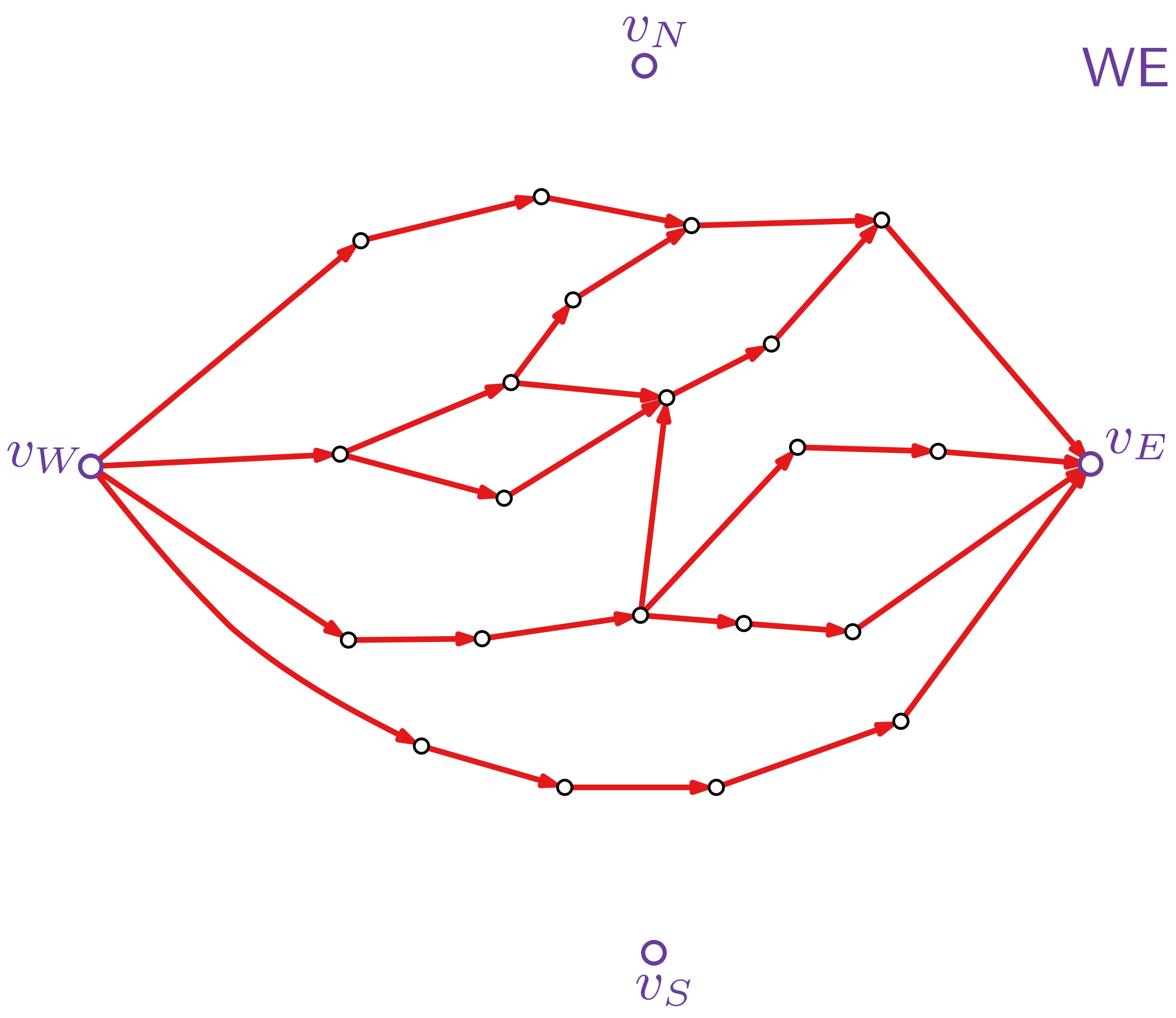
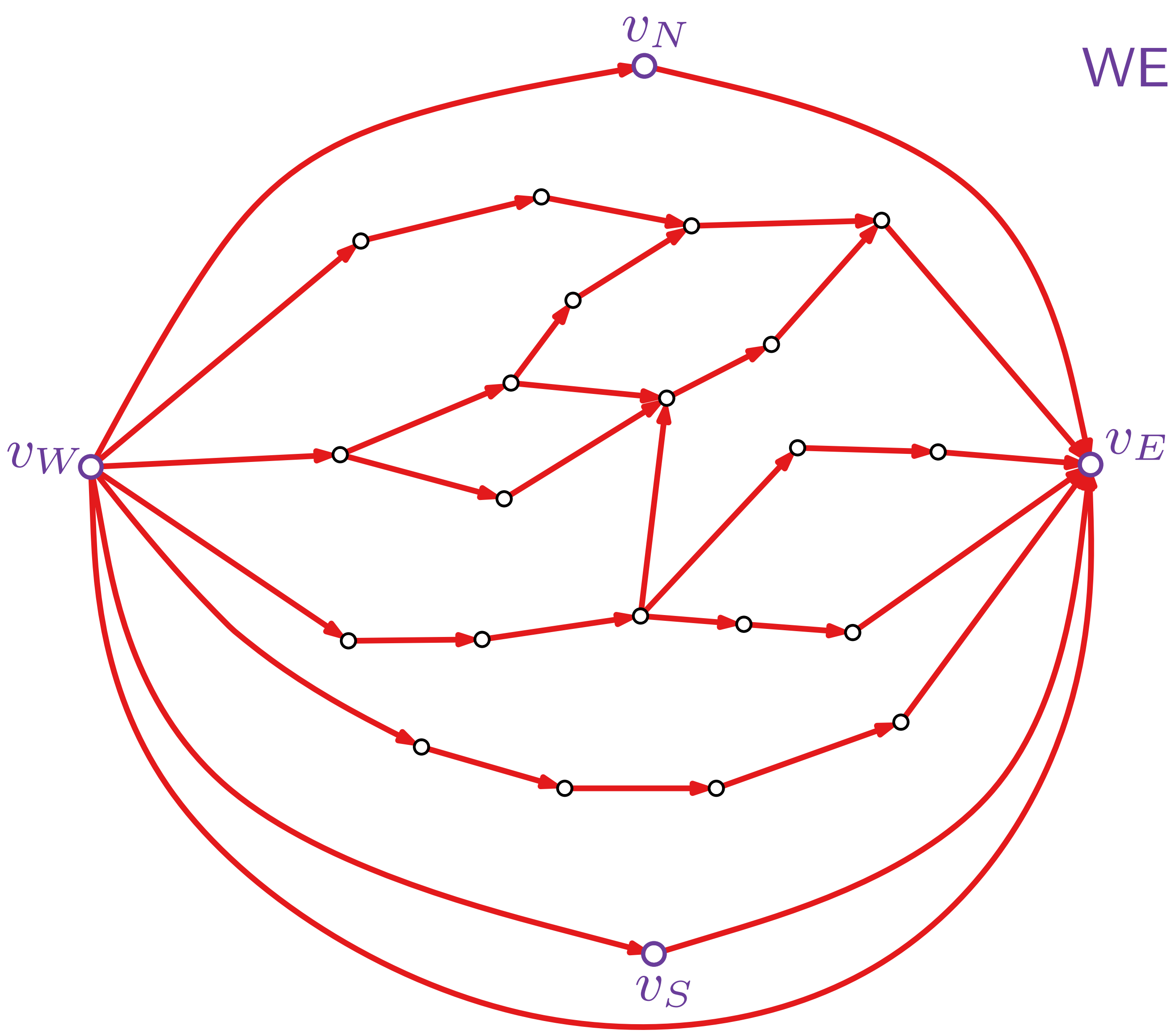


WE network G_{hor}

From REL to st-Digraphs to Coordinates



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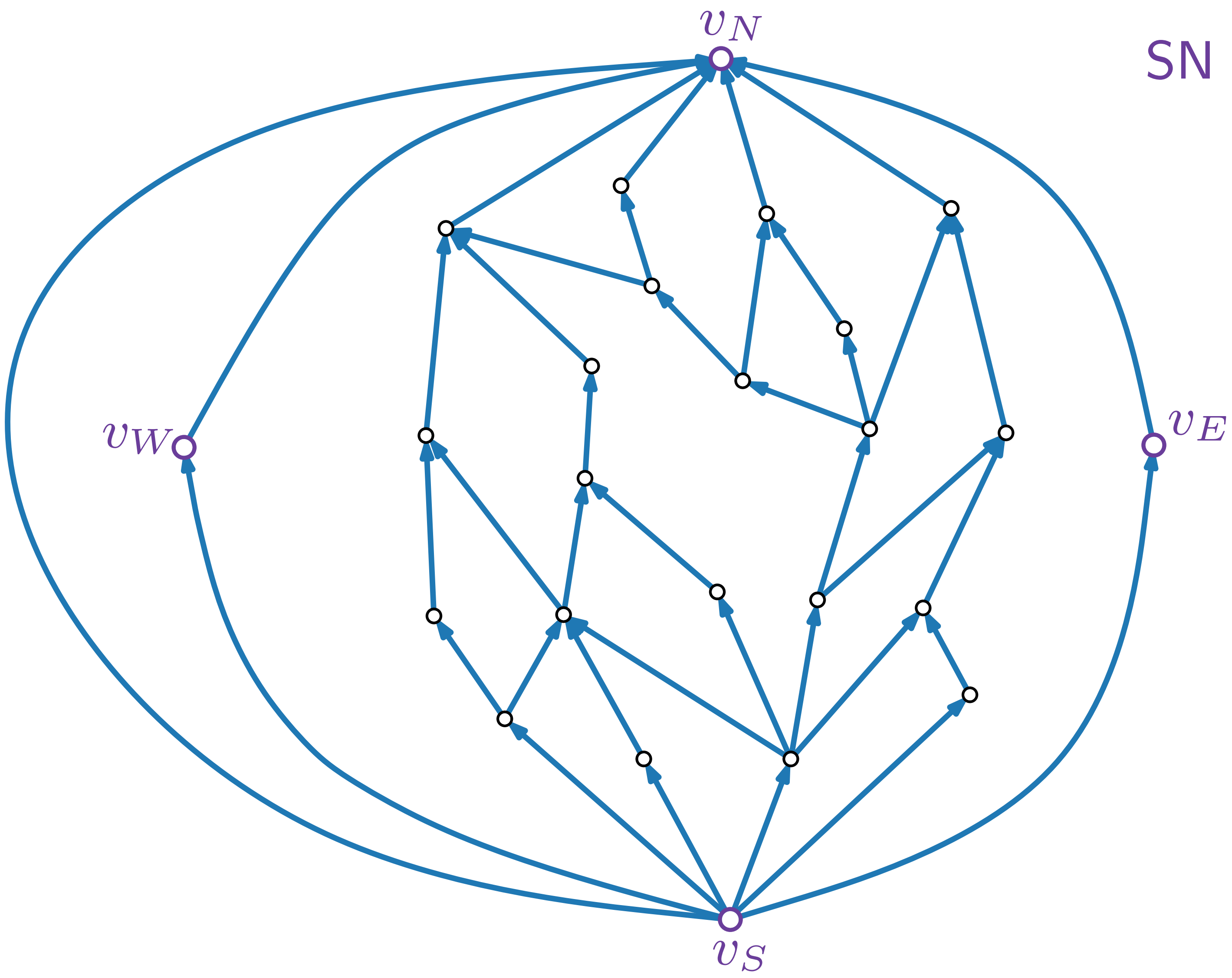
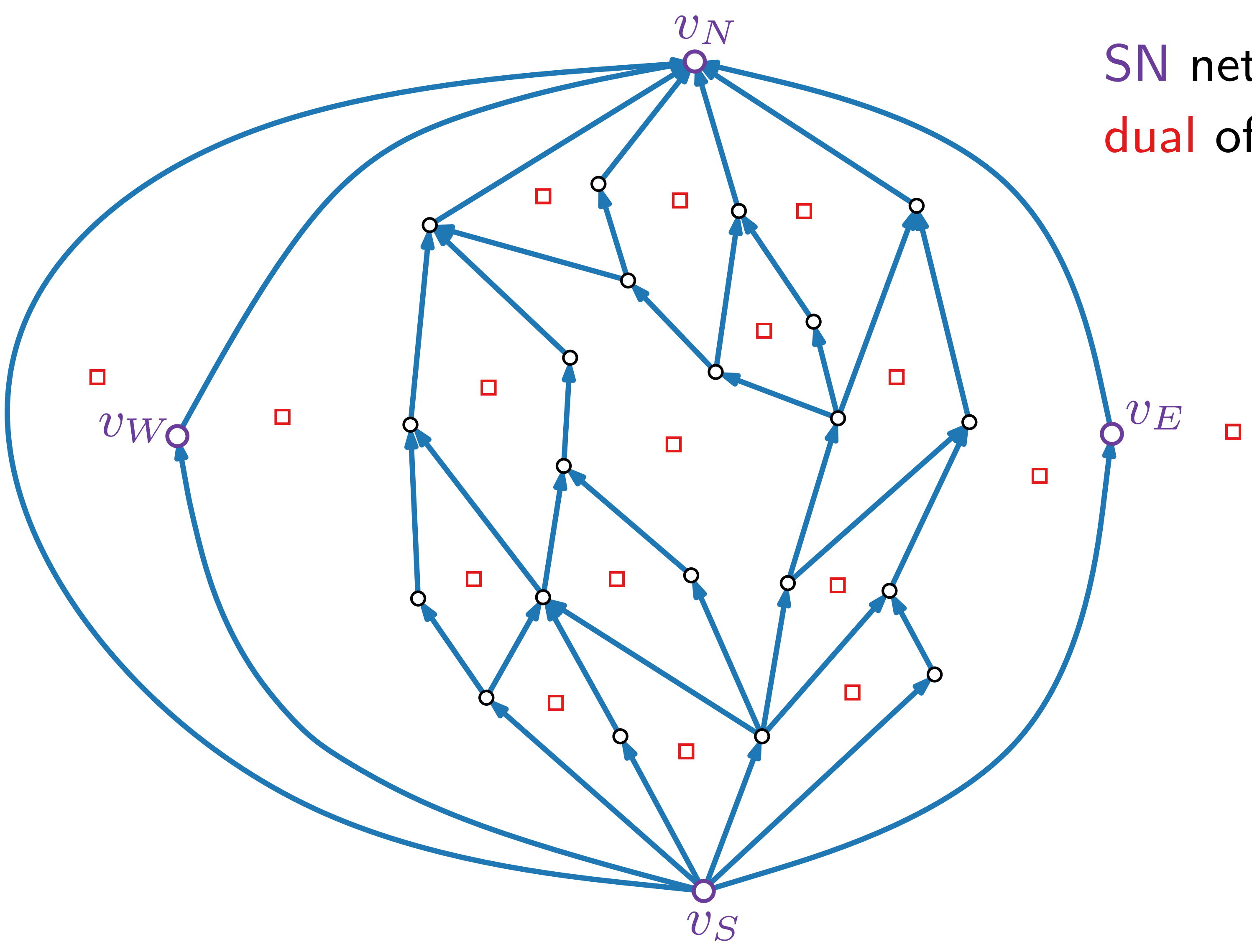


SN network G_{ver}

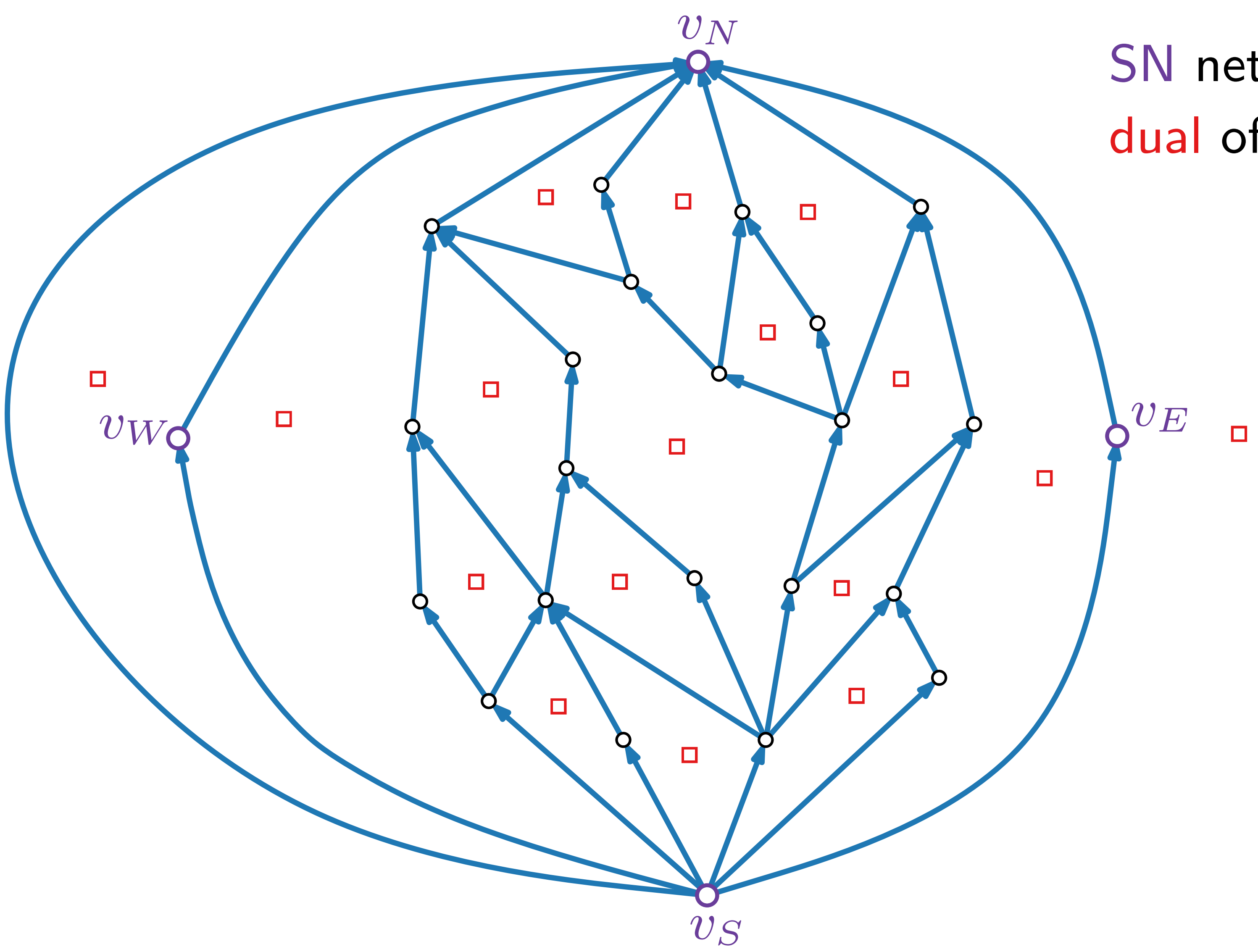
From REL to st-Digraphs to Coordinates



SN network G_{ver}

dual of G_{ver}

From REL to st-Digraphs to Coordinates



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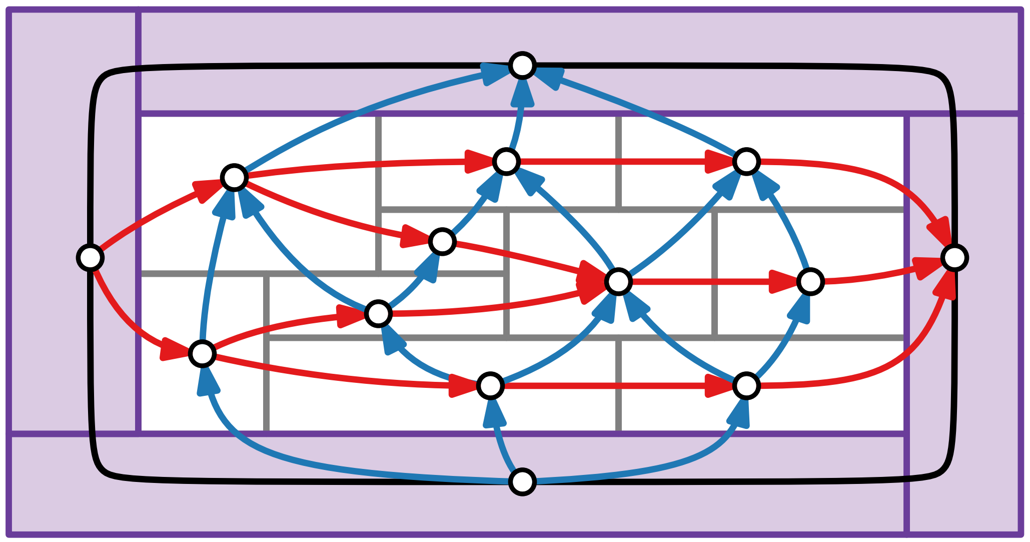

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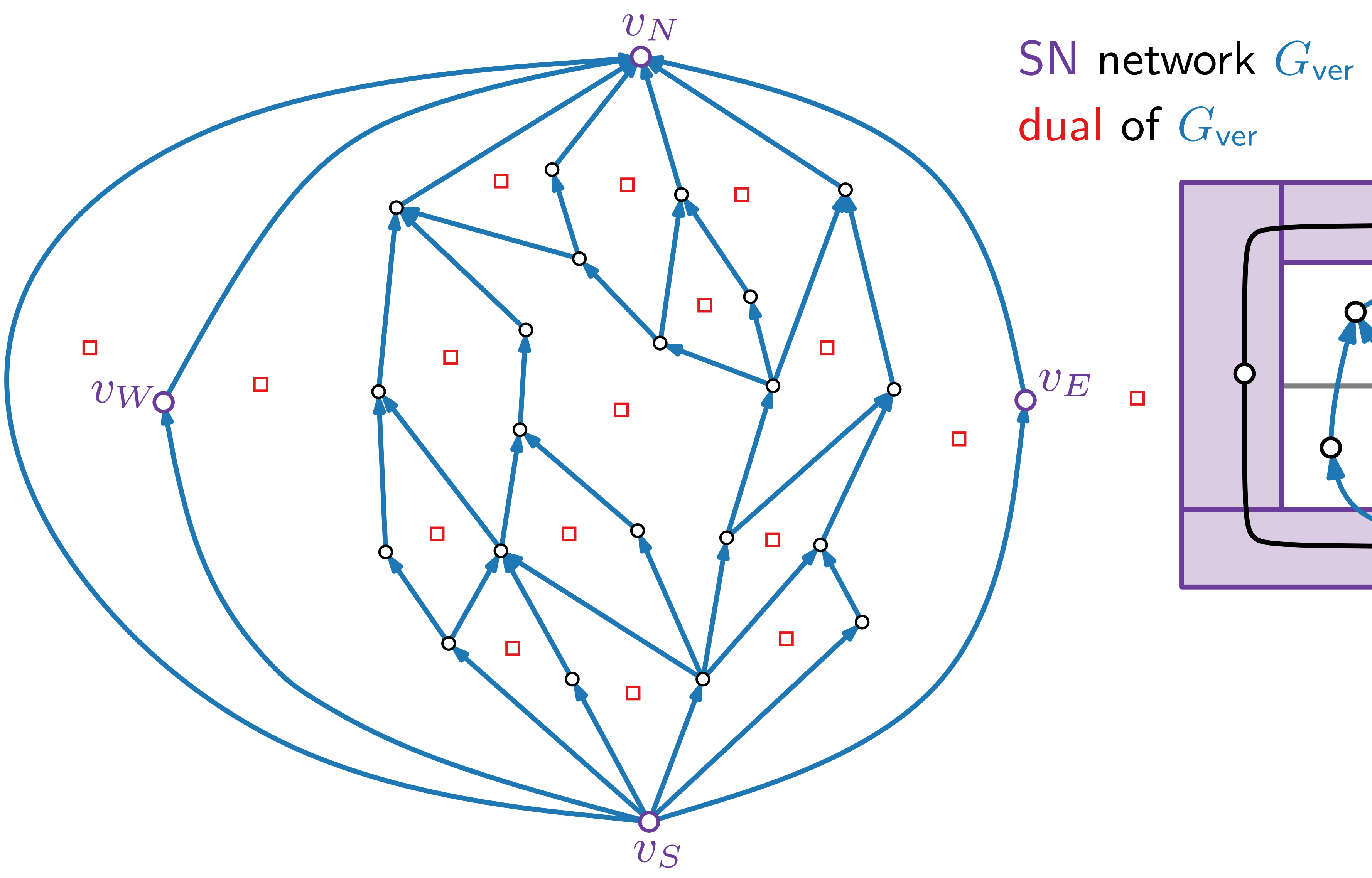

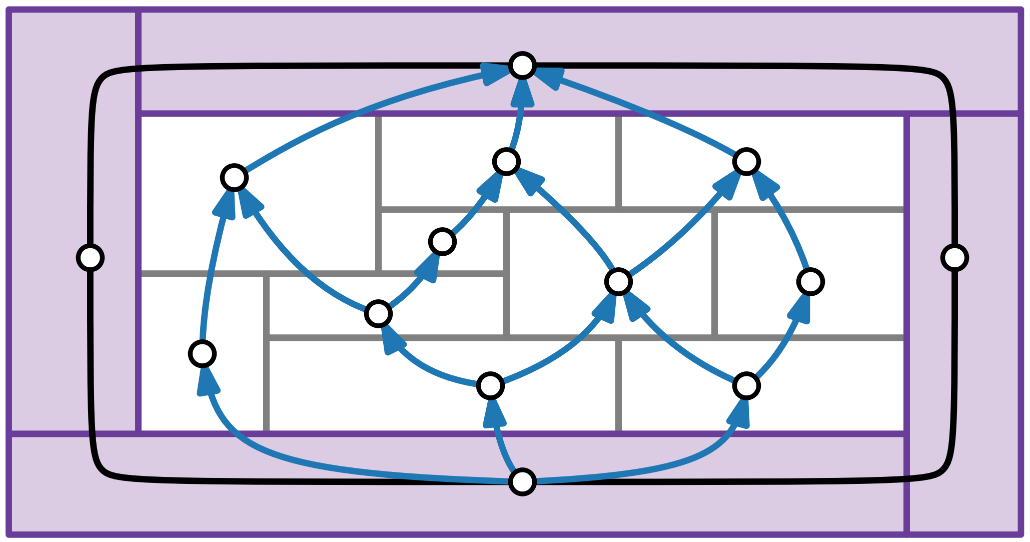

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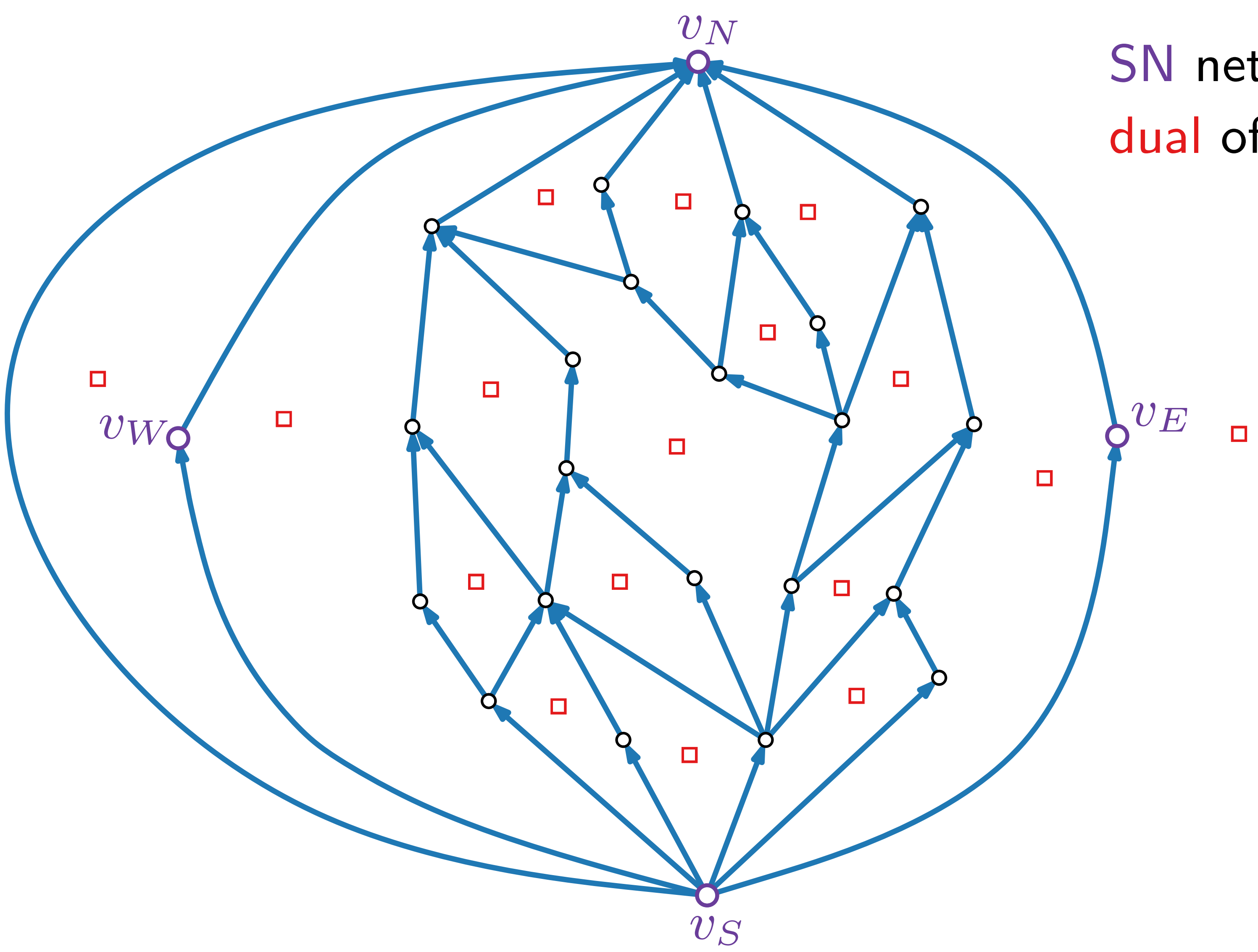

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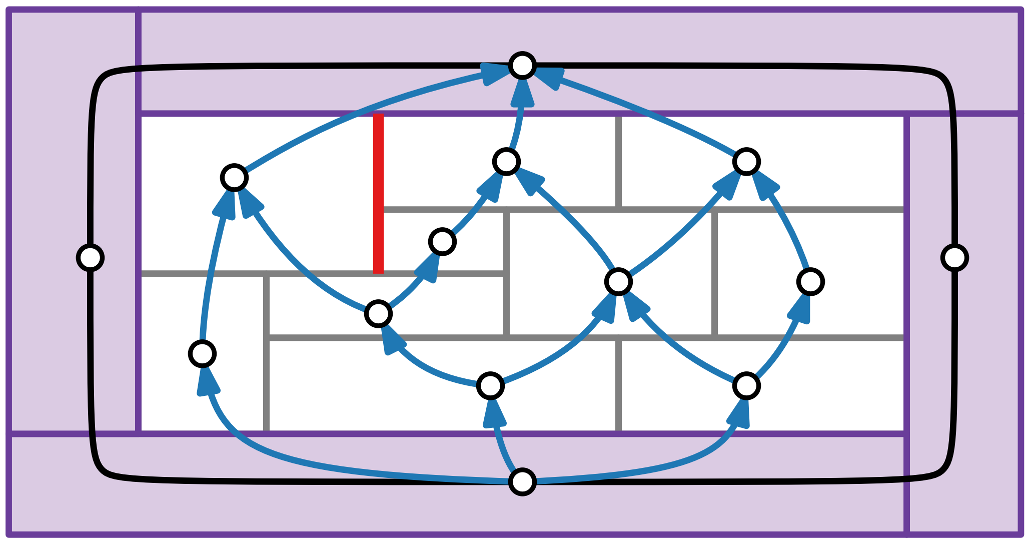

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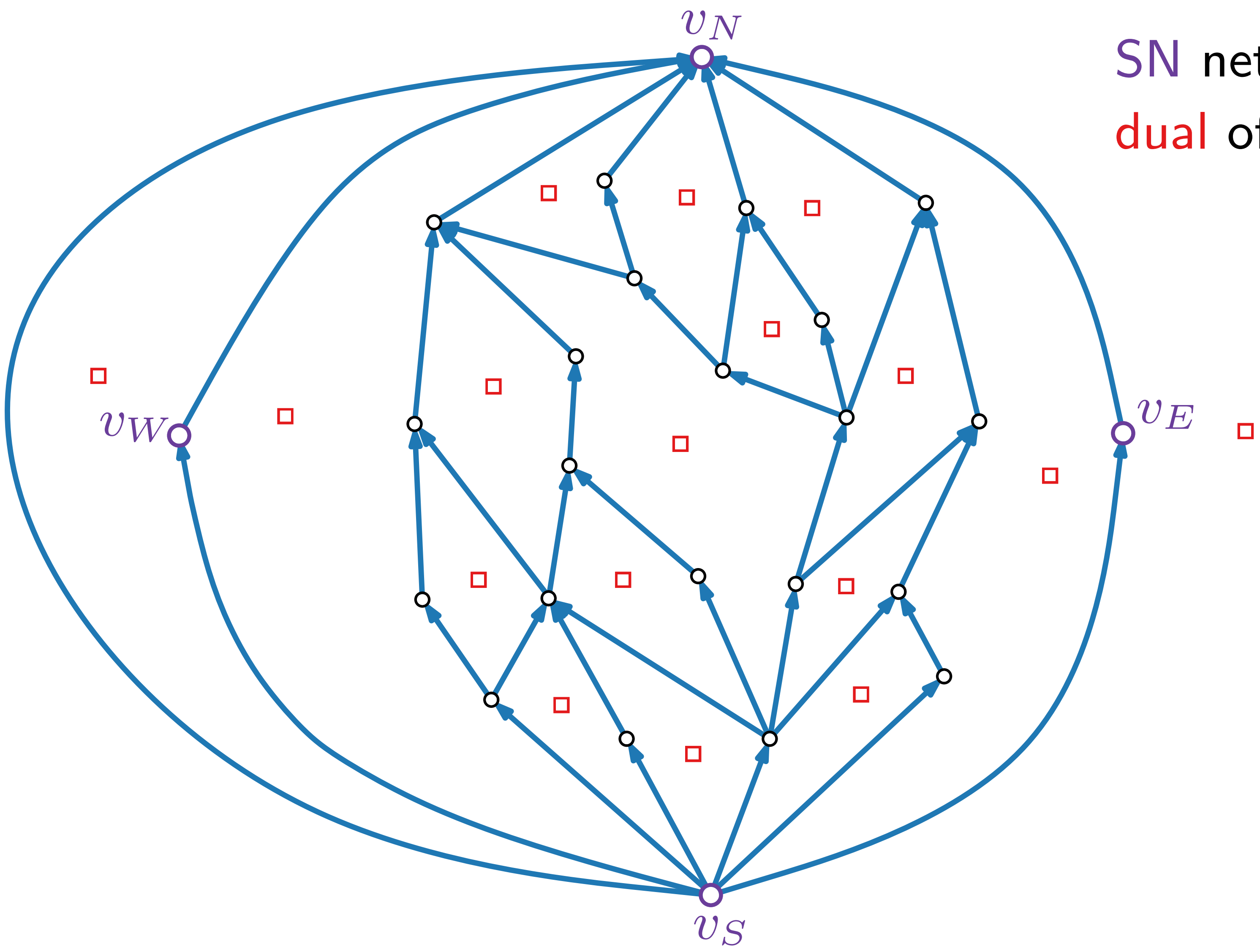

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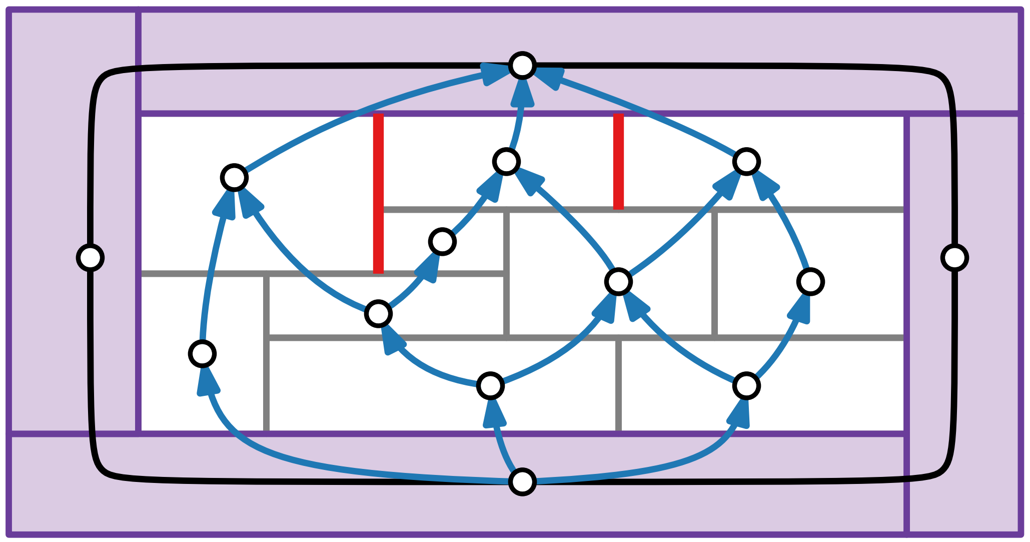

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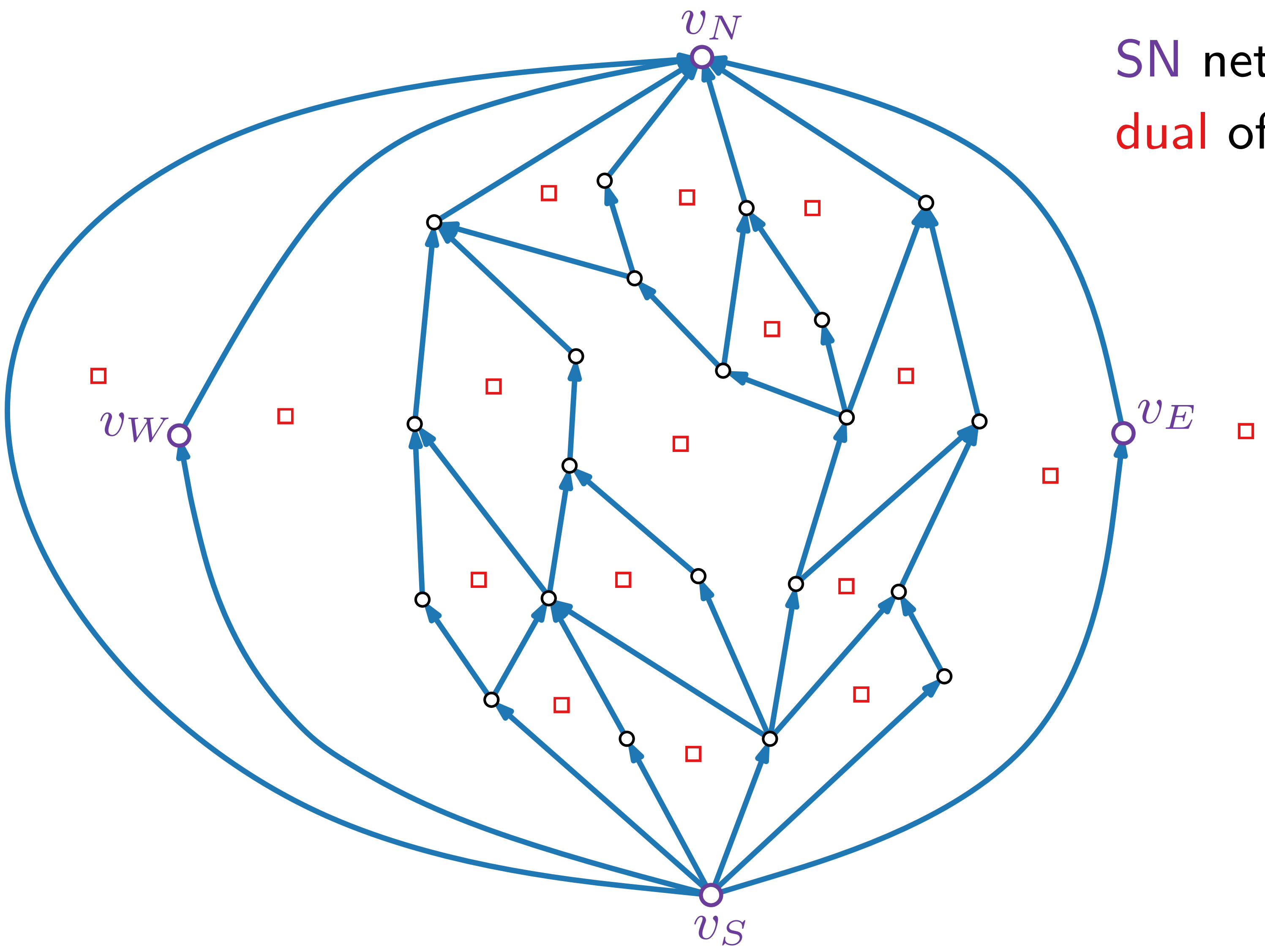

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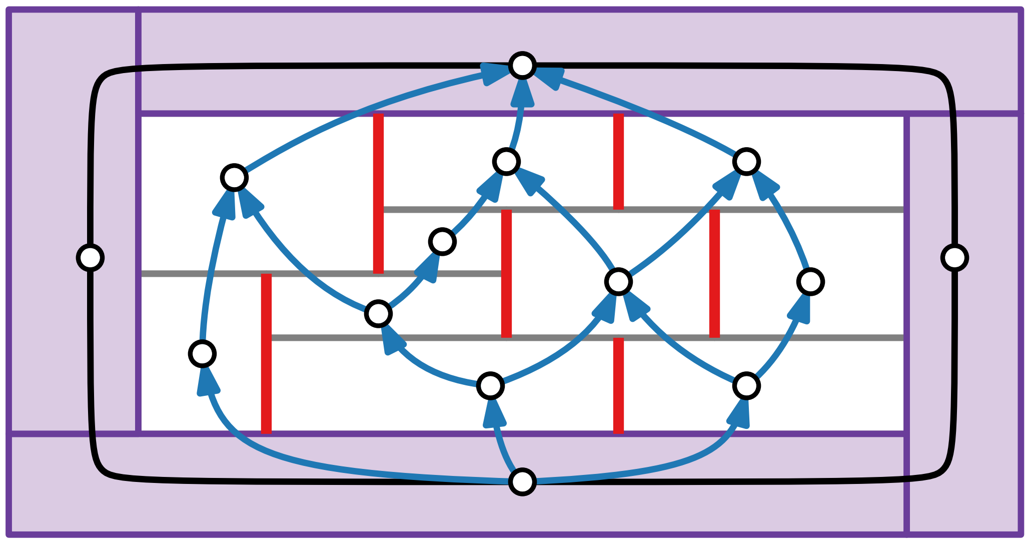

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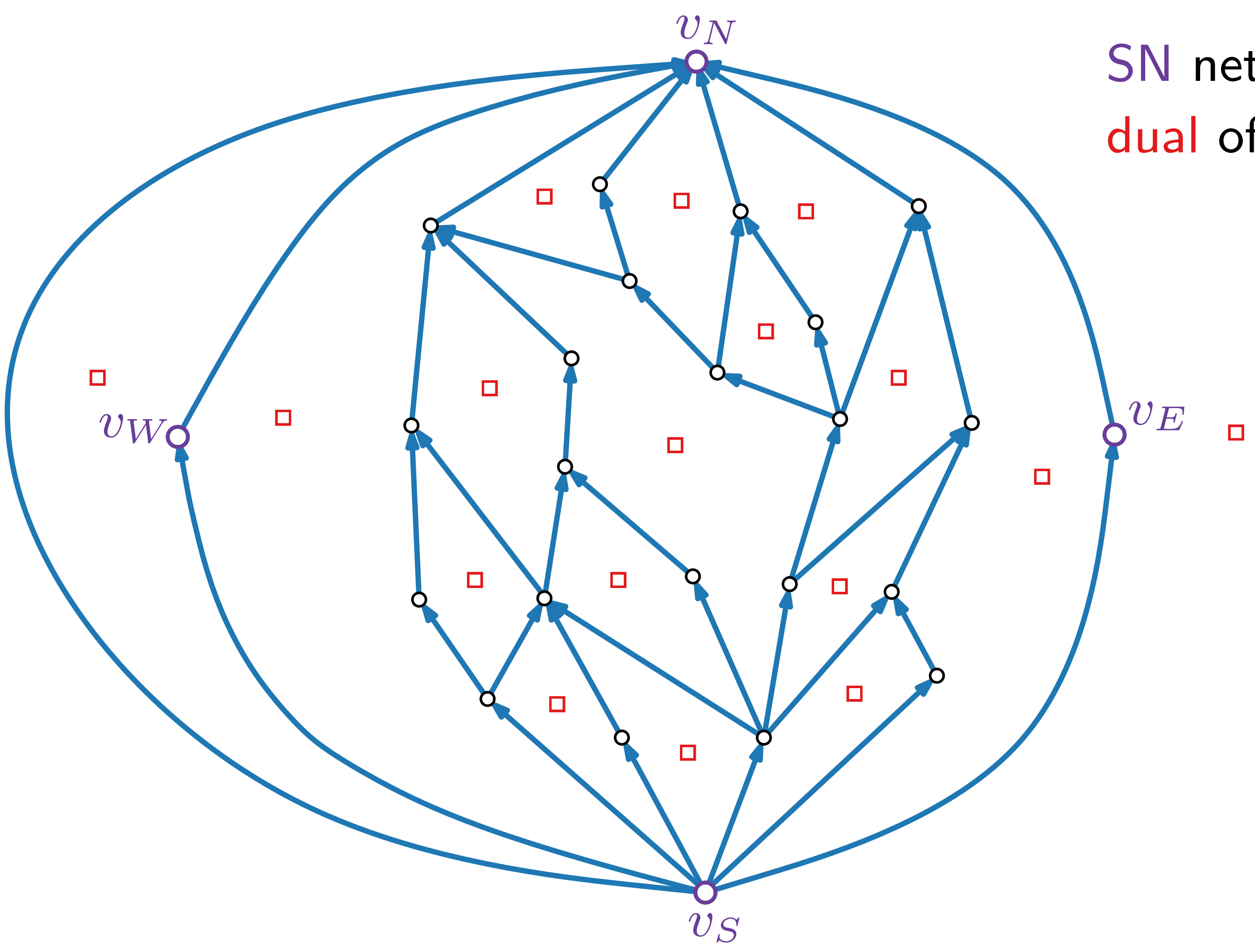

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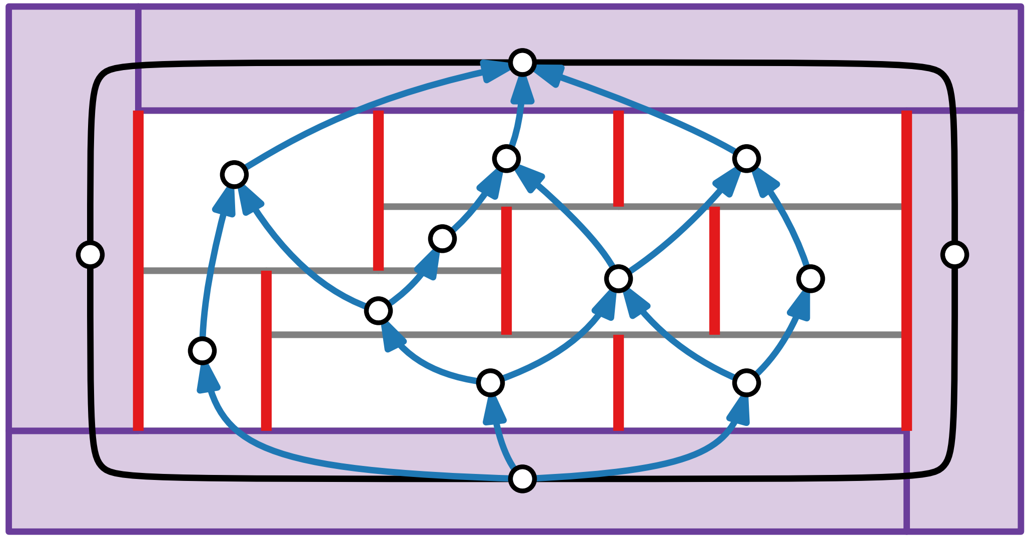

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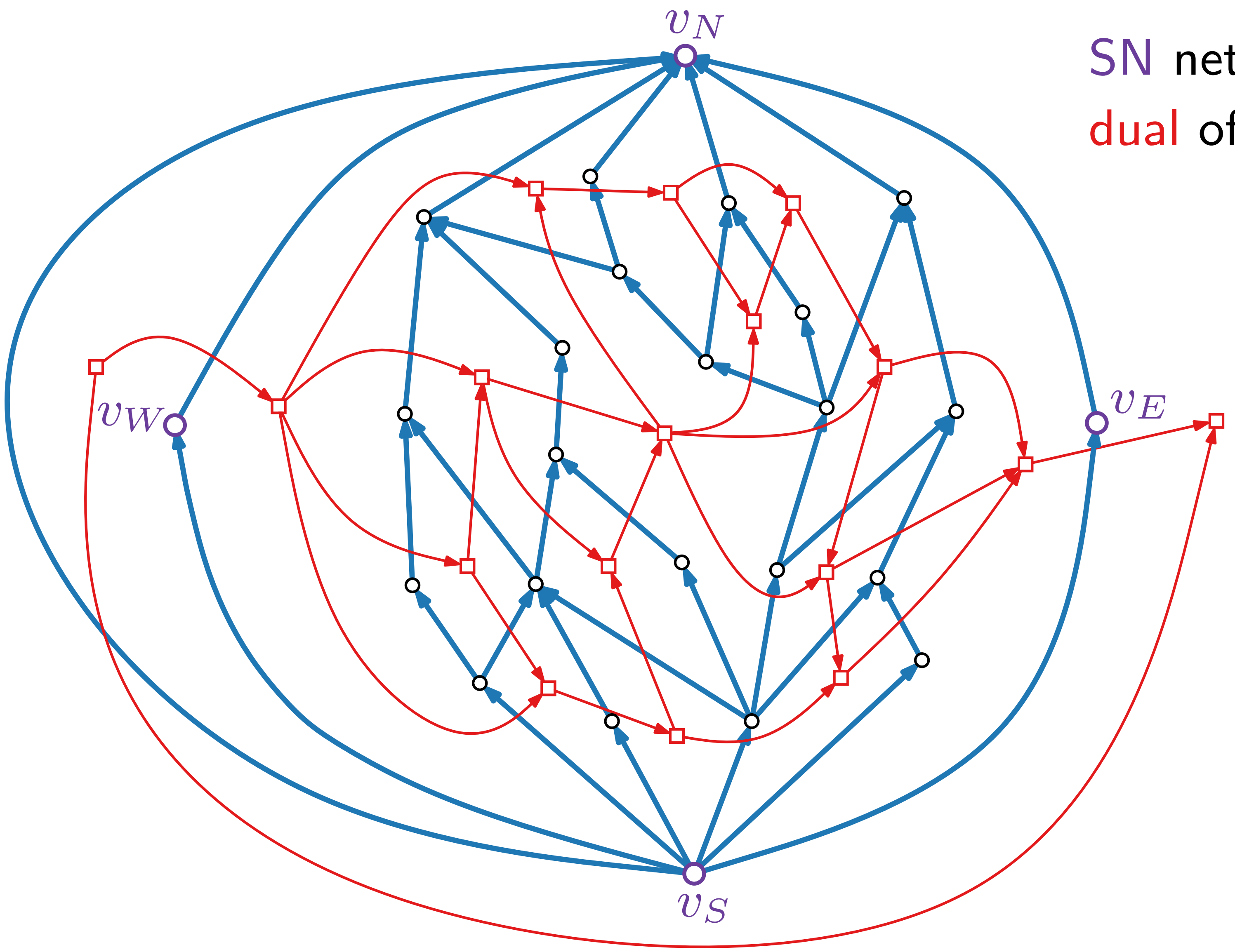

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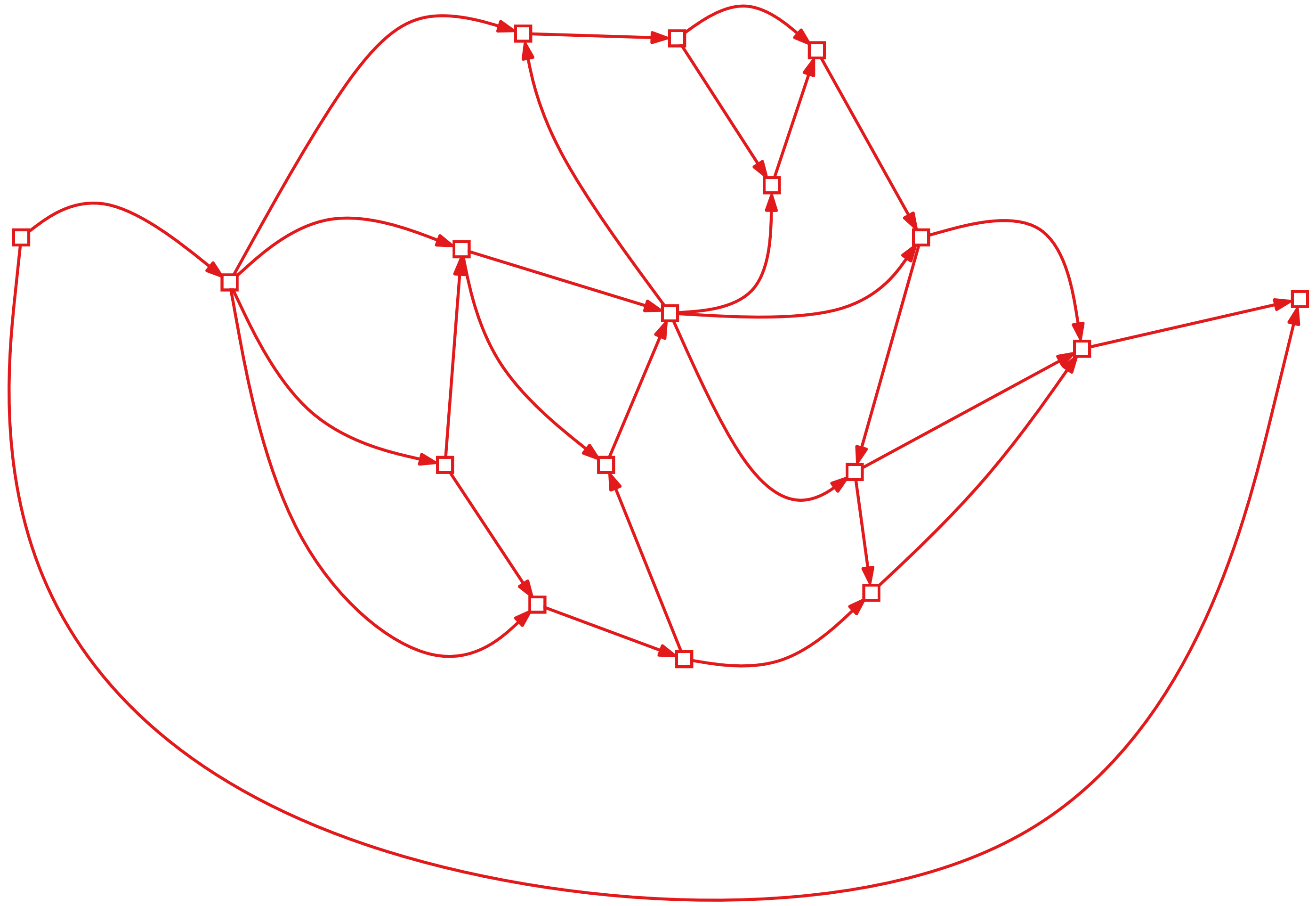

dual of G_{ver}

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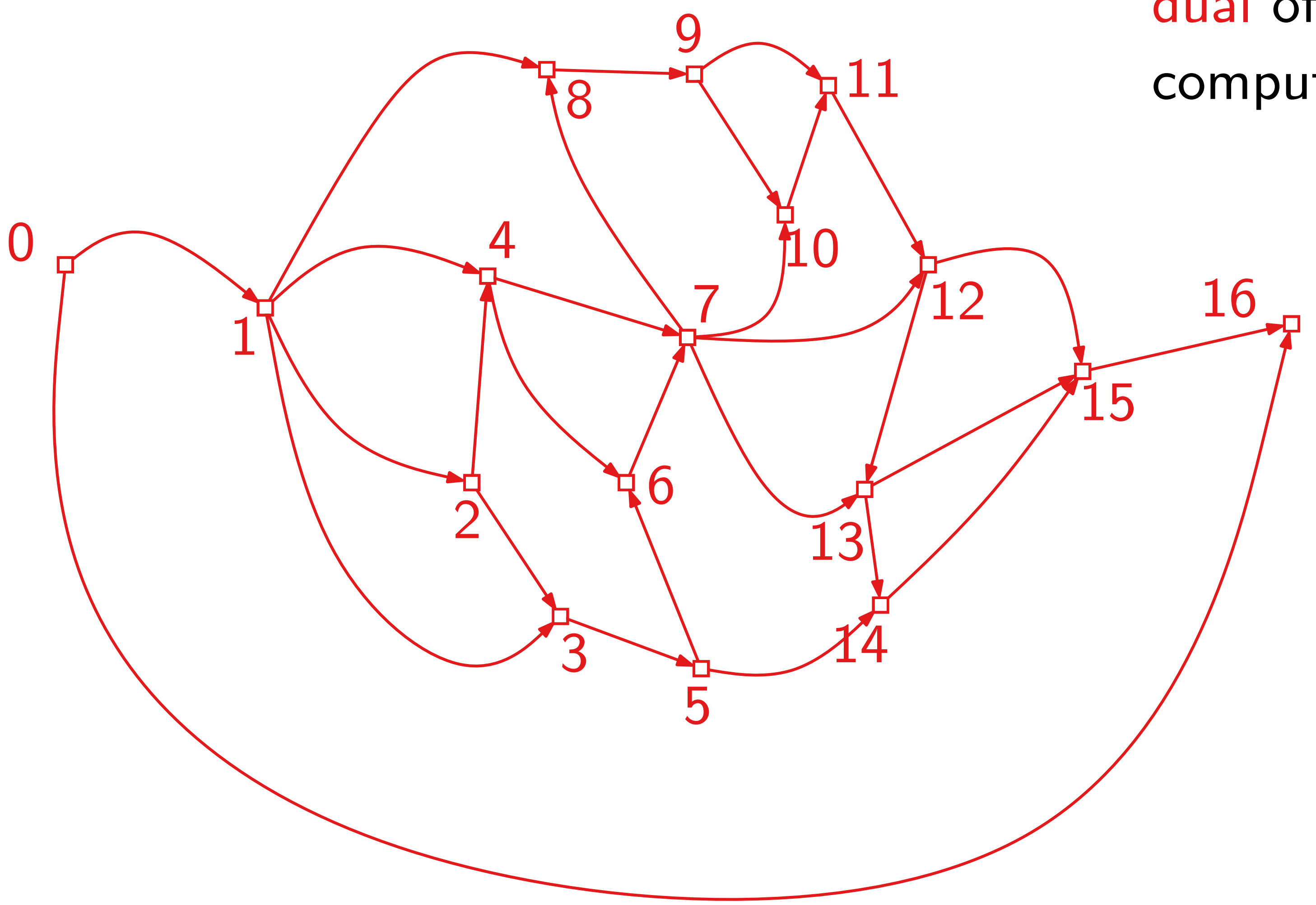
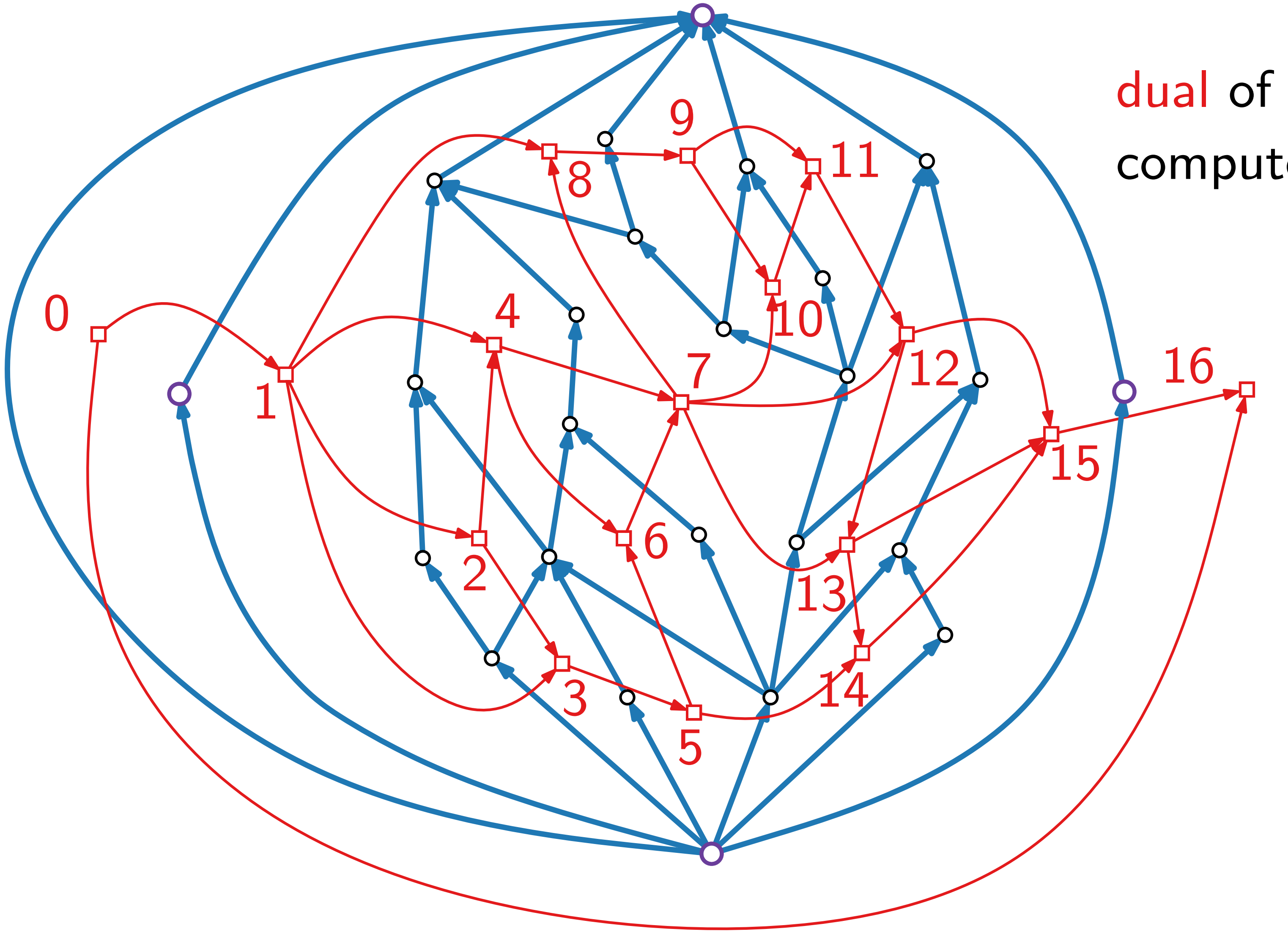


dual of G_{ver}

compute topological order

From REL to st-Digraphs to Coordinates



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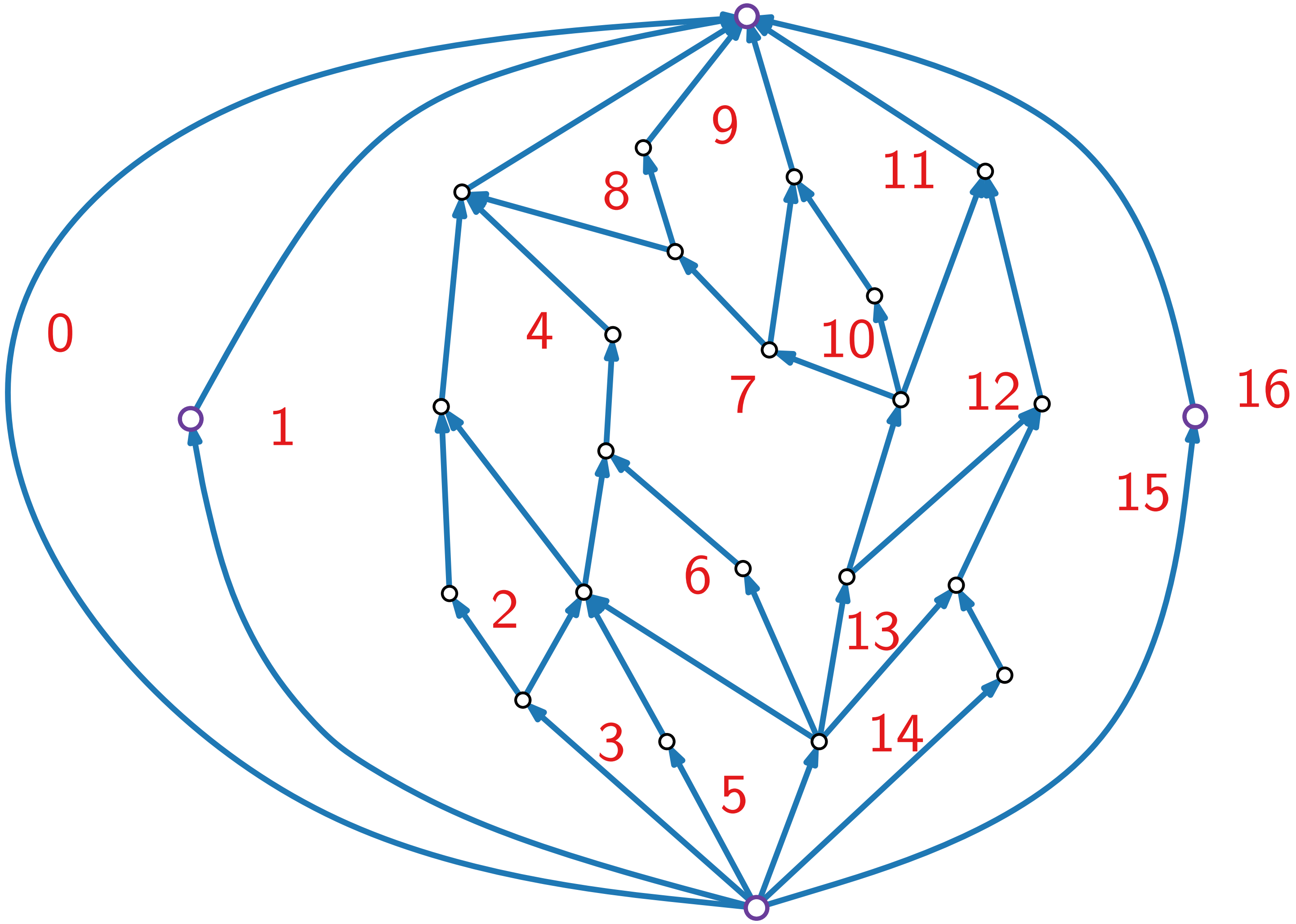

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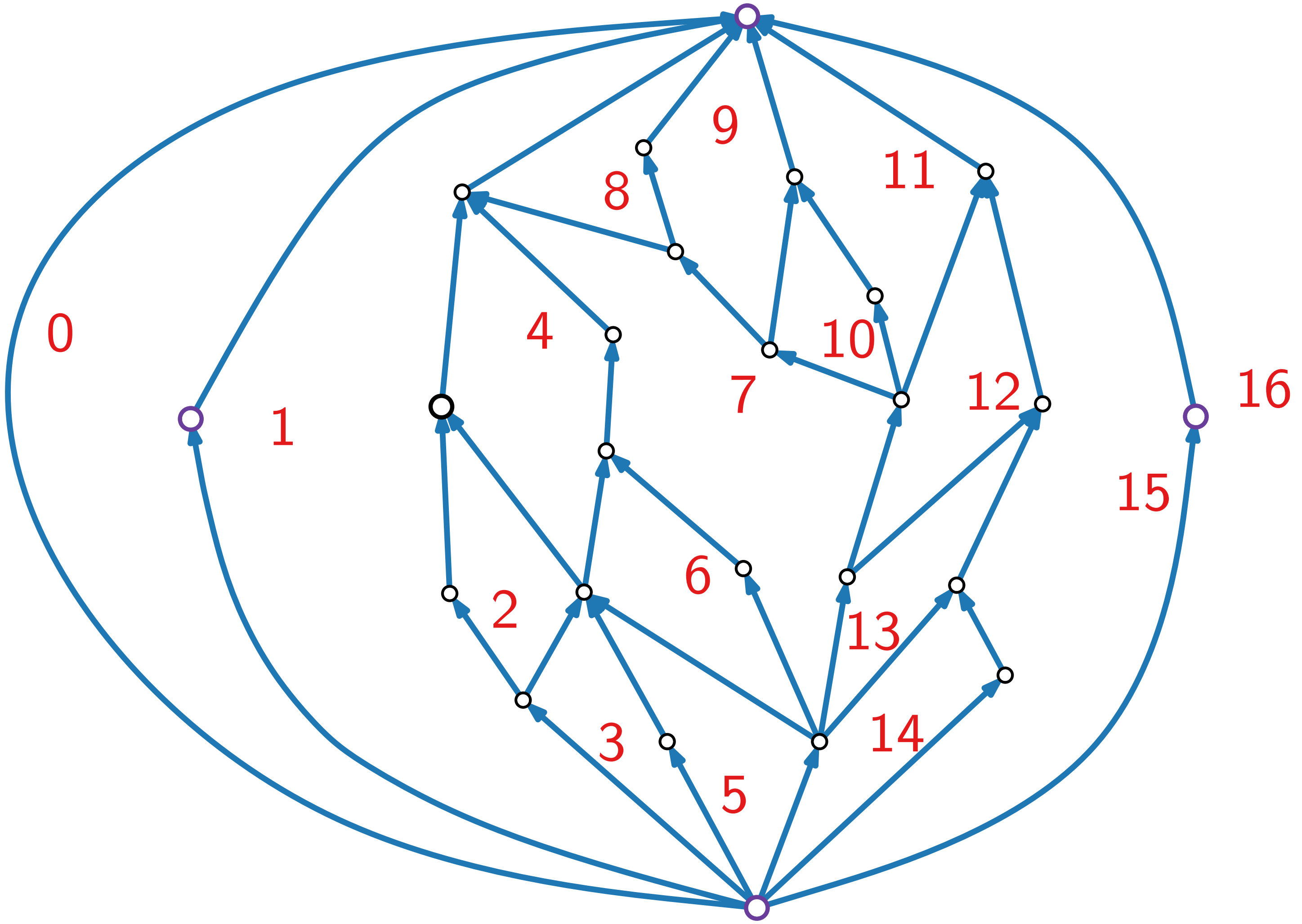

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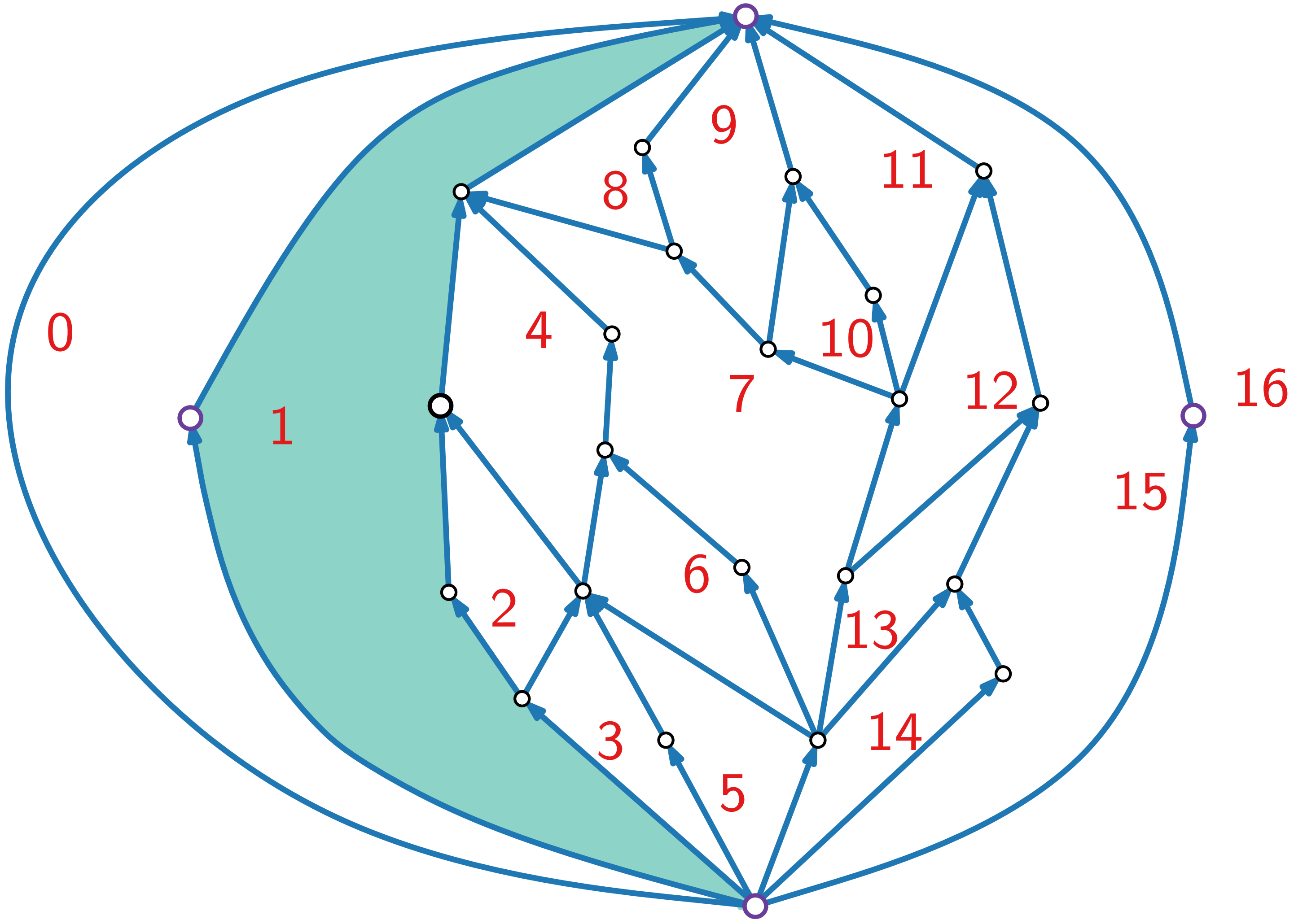

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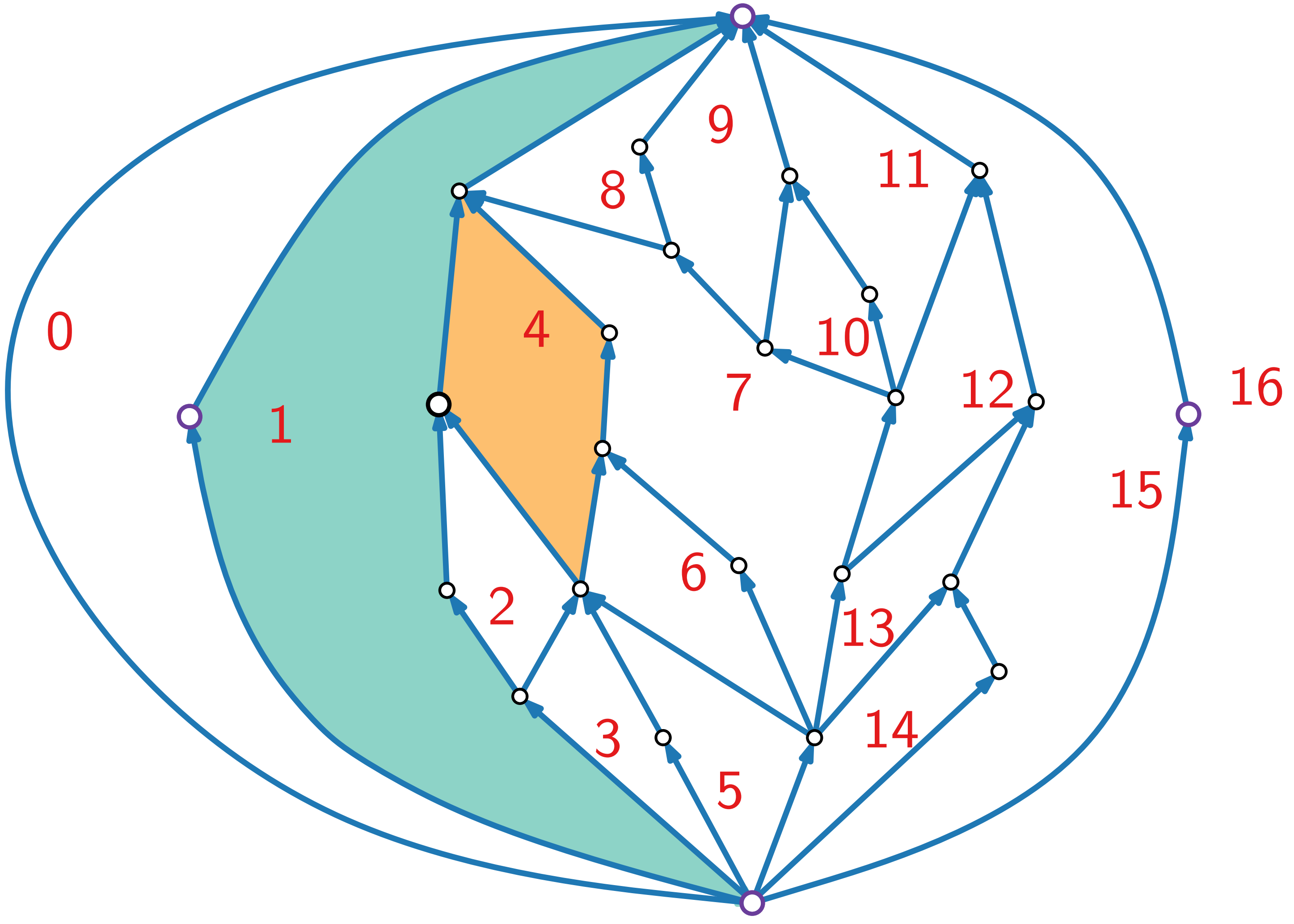

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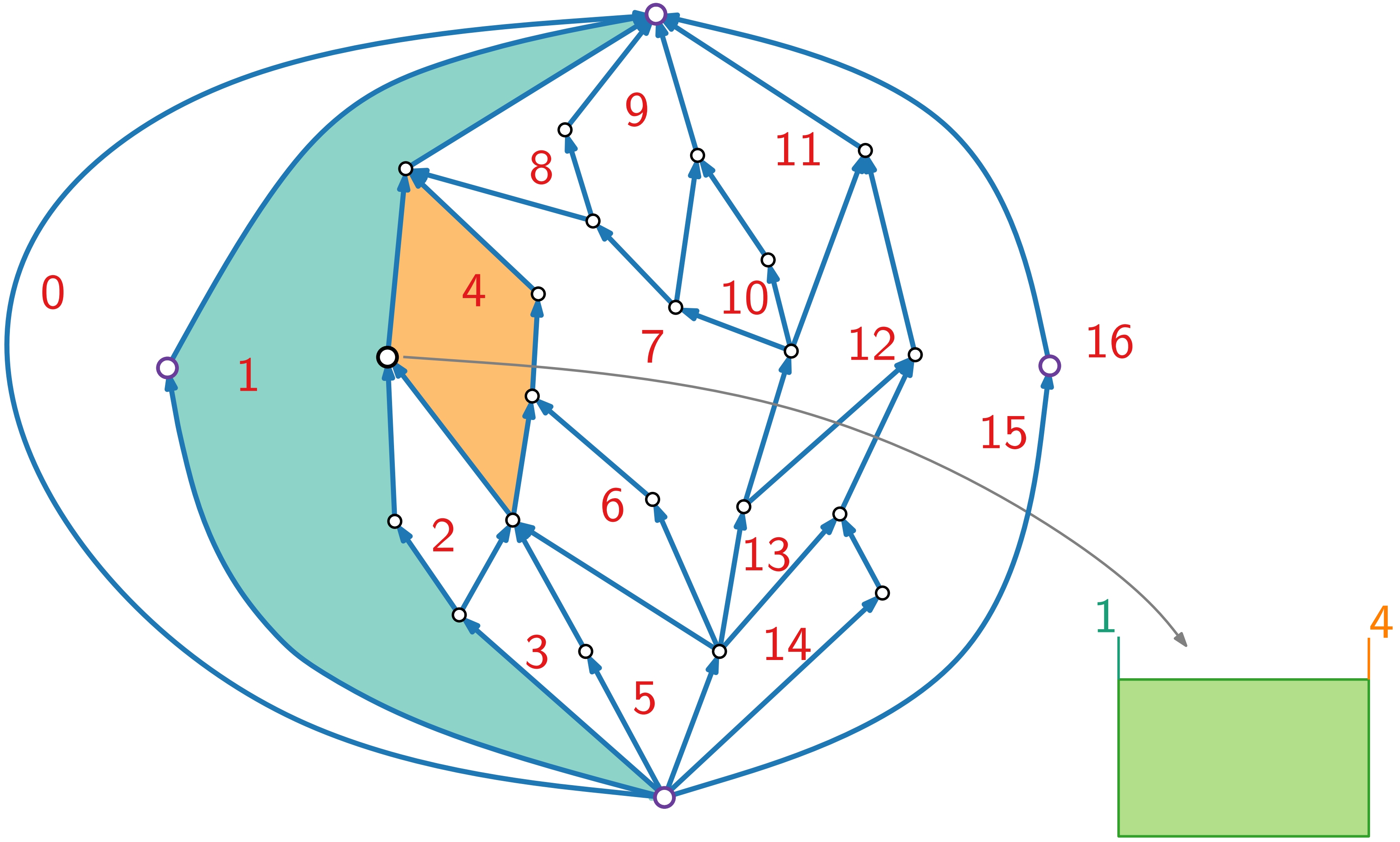

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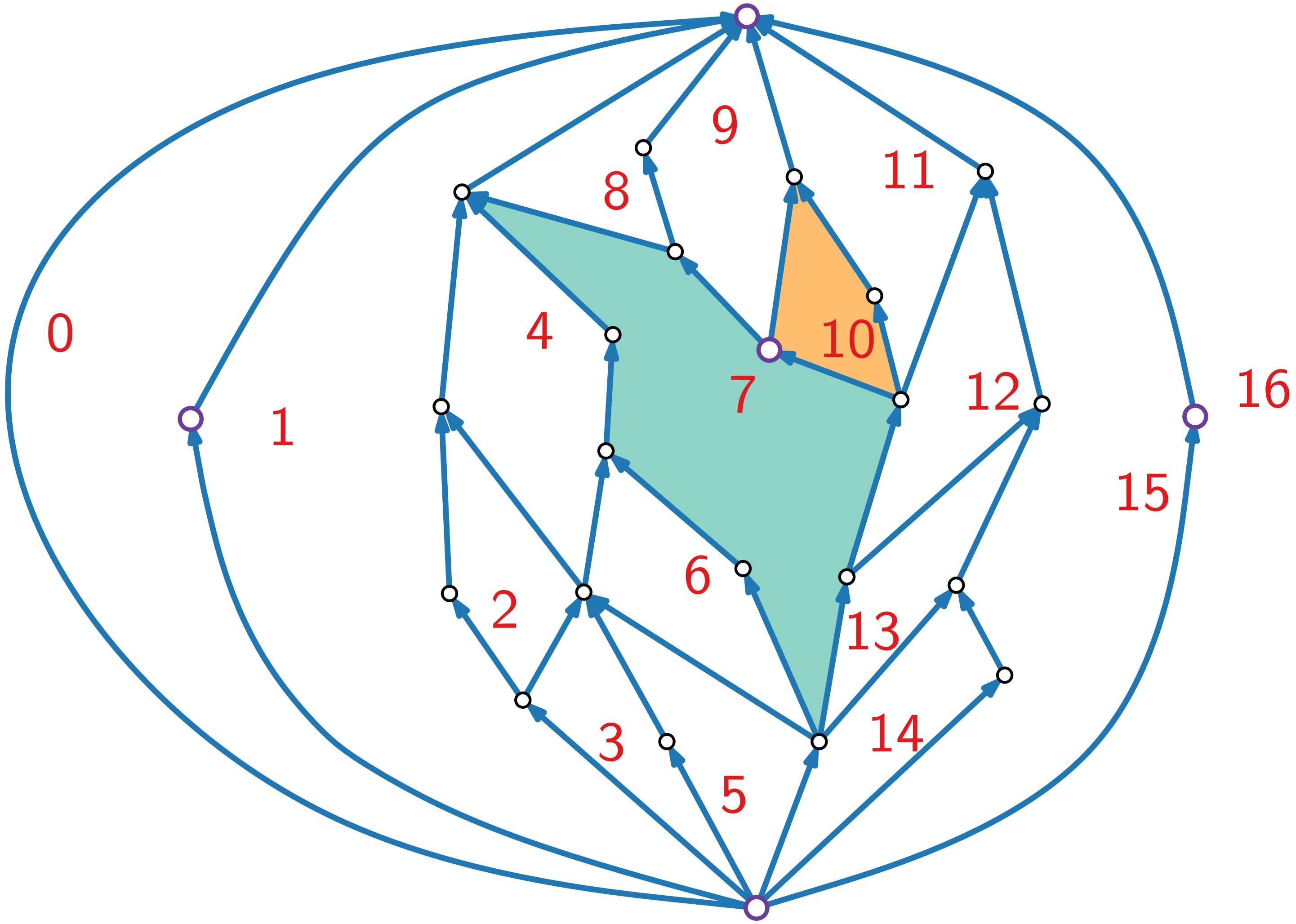

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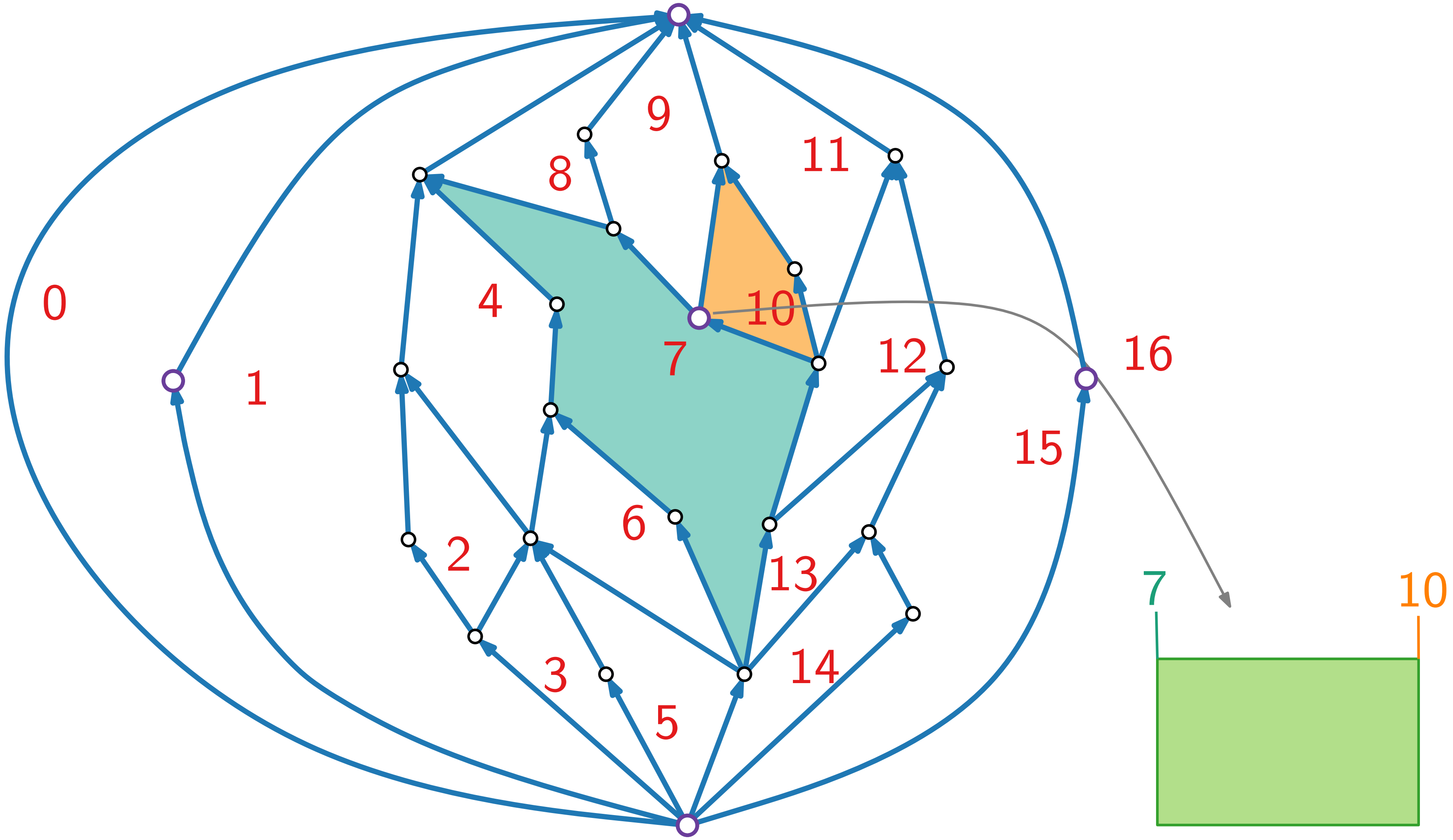

Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

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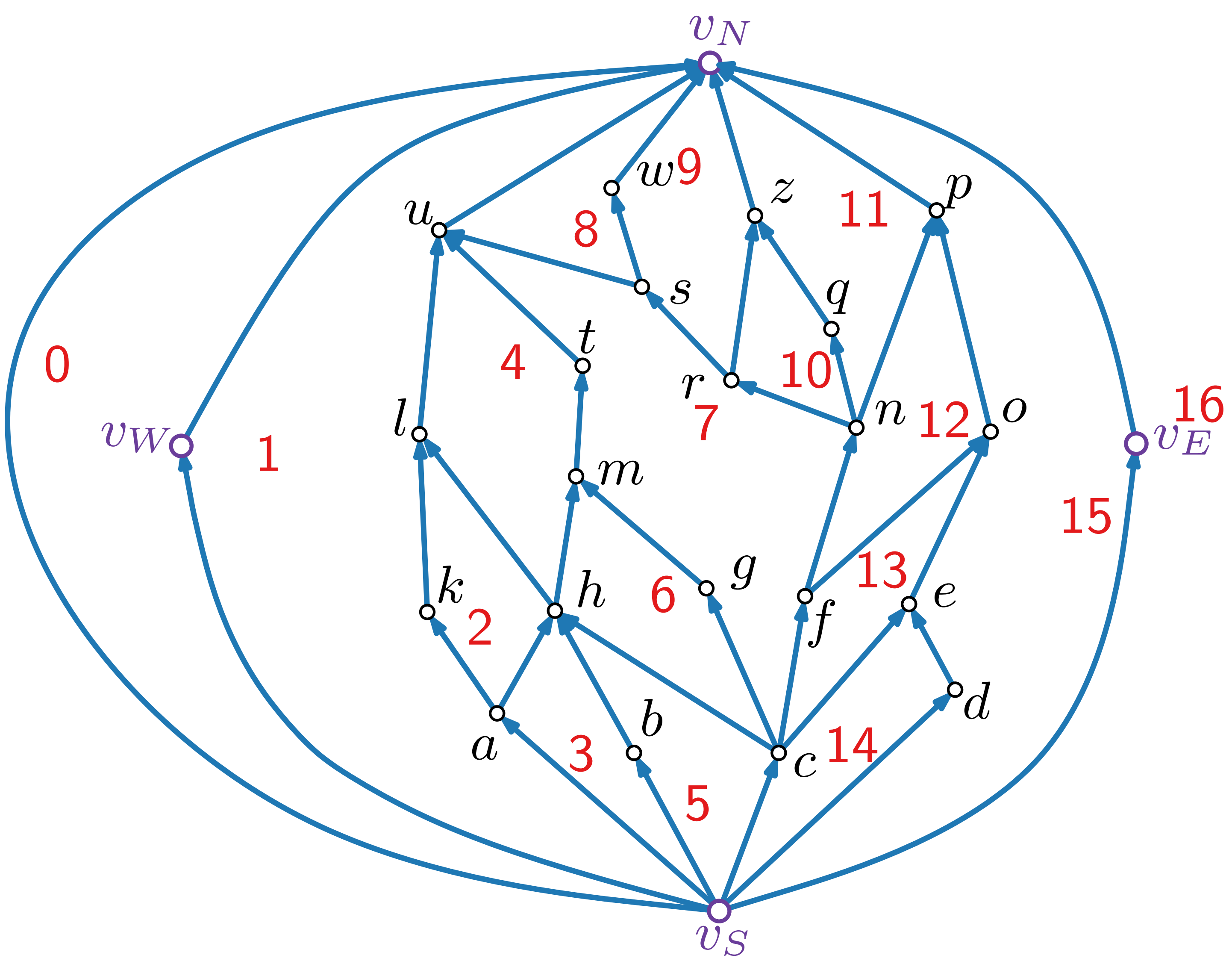
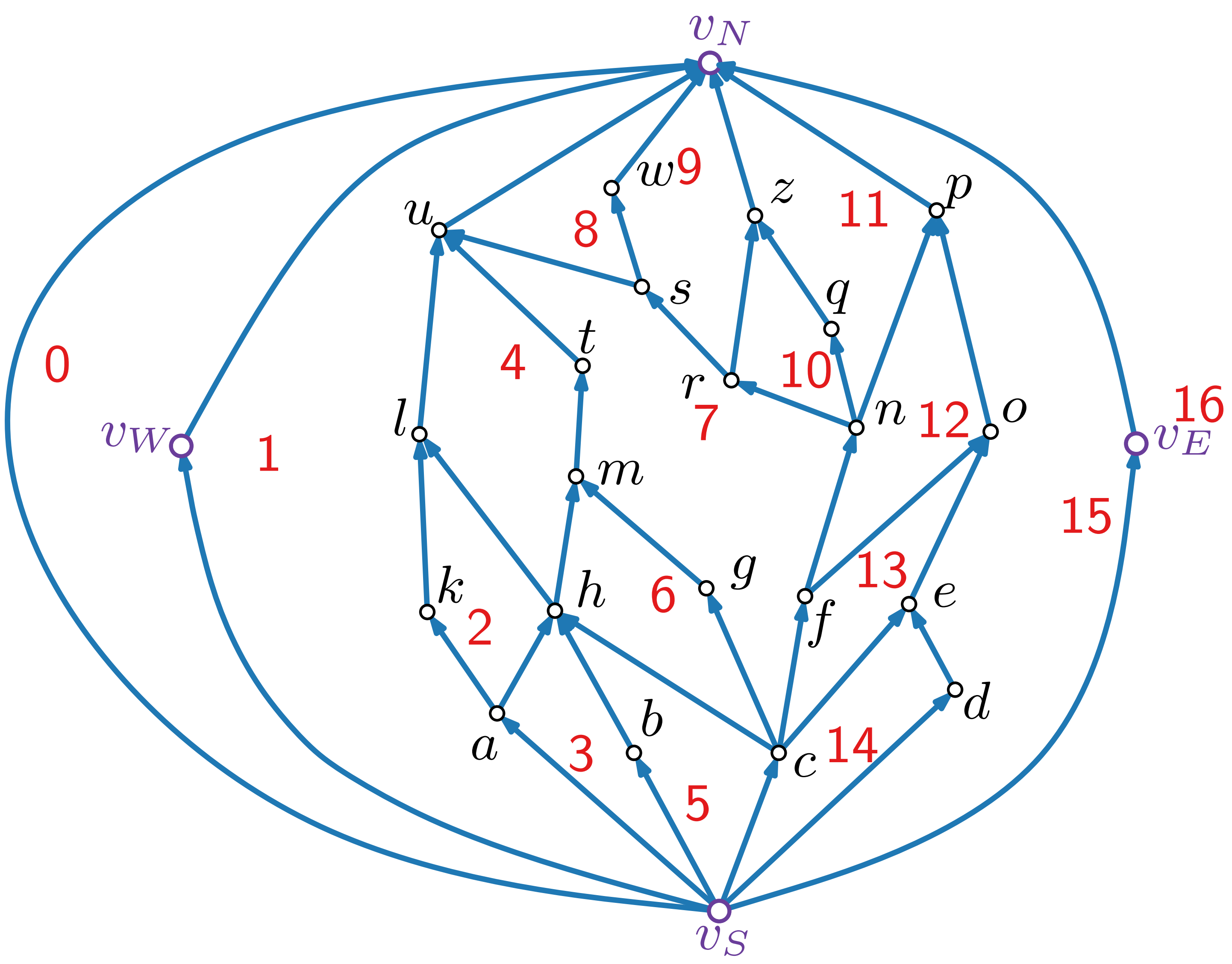
- For each $v \in V$, let $R(v) = [x_1(v), x_2(v)] \times [y_1(v), y_2(v)]$.

Reading off Coordinates to Get Rectangular Dual

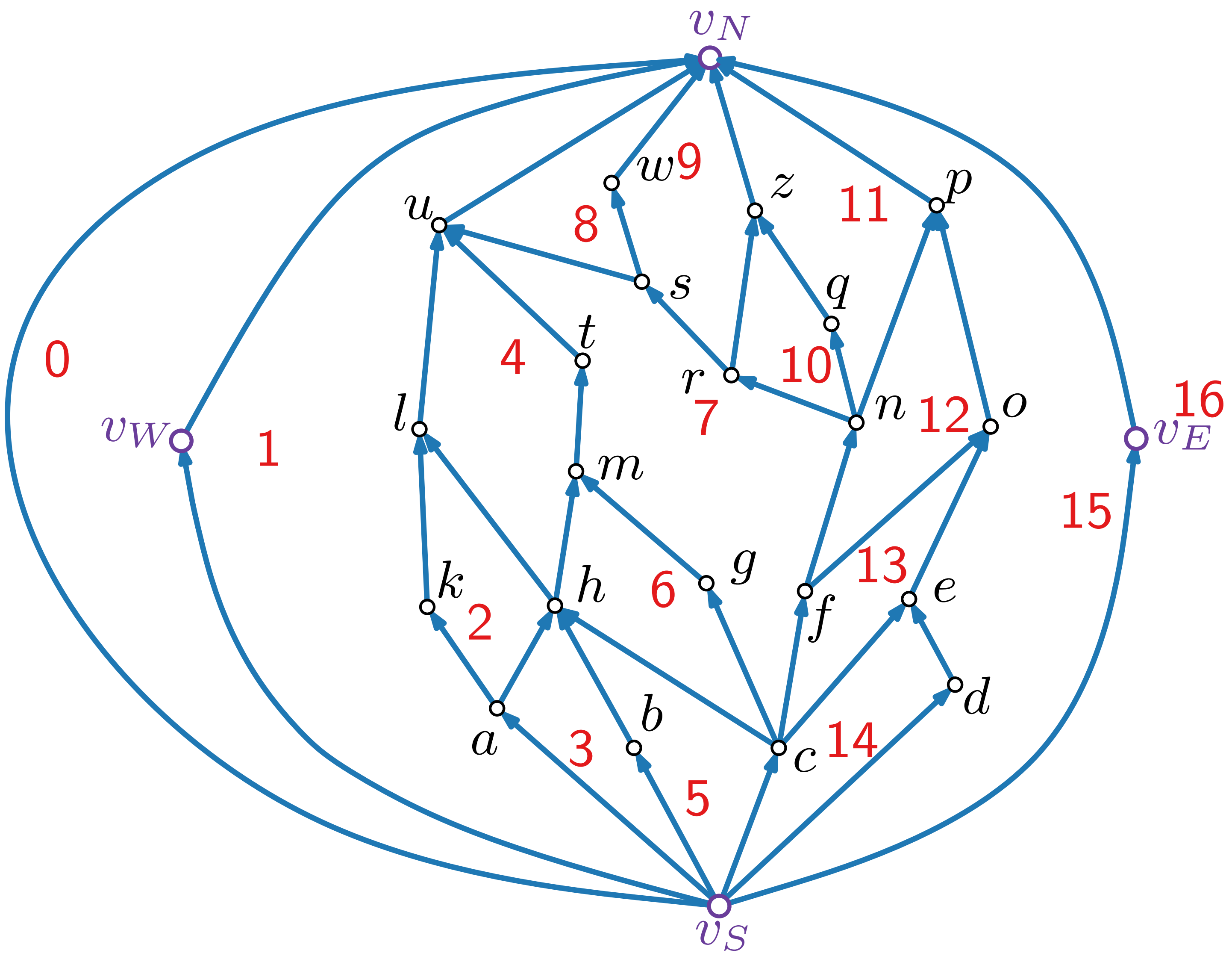


Reading off Coordinates to Get Rectangular Dual

$x_1(v_N) = 1,\ x_2(v_N) = 15$

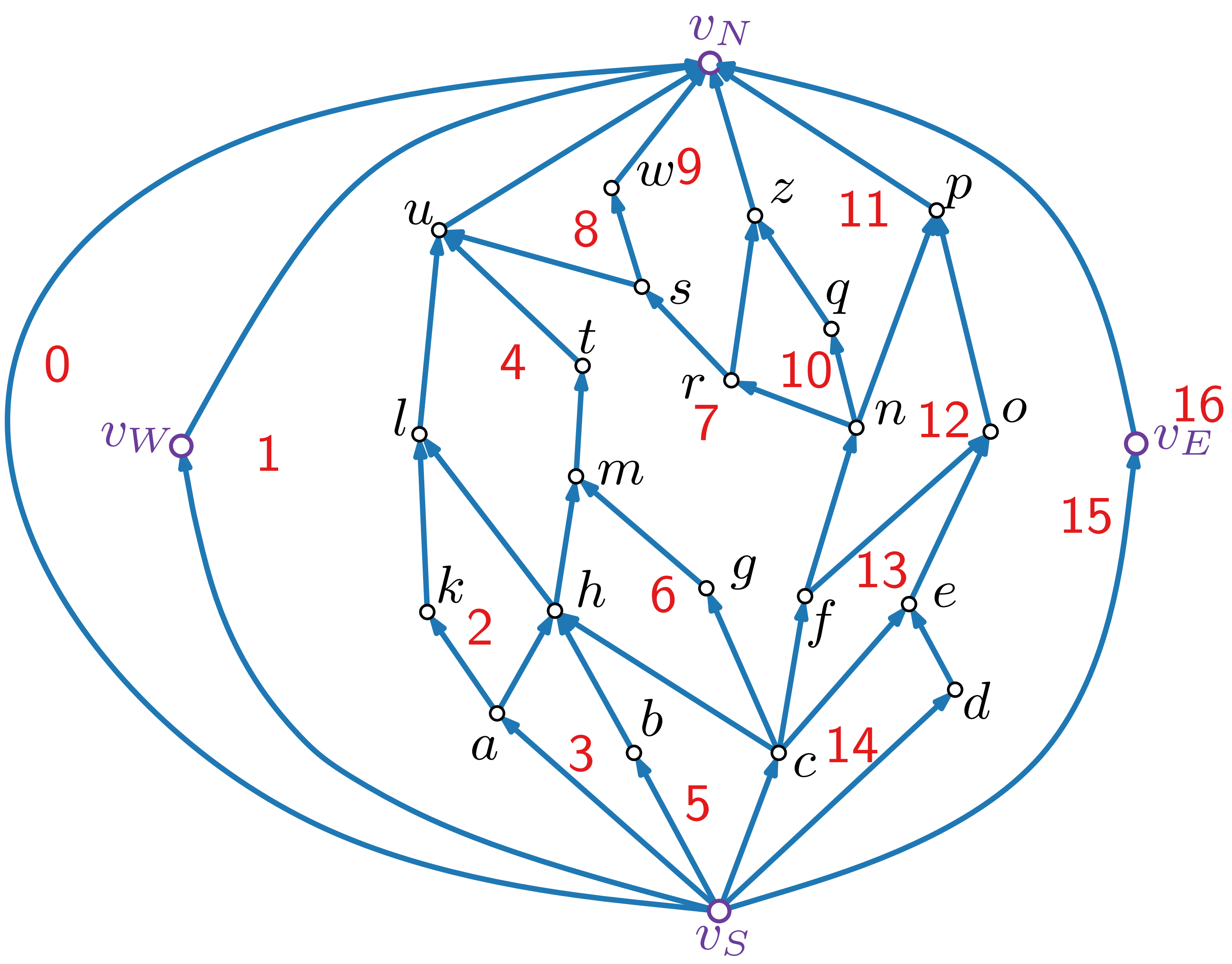


Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$

Reading off Coordinates to Get Rectangular Dual



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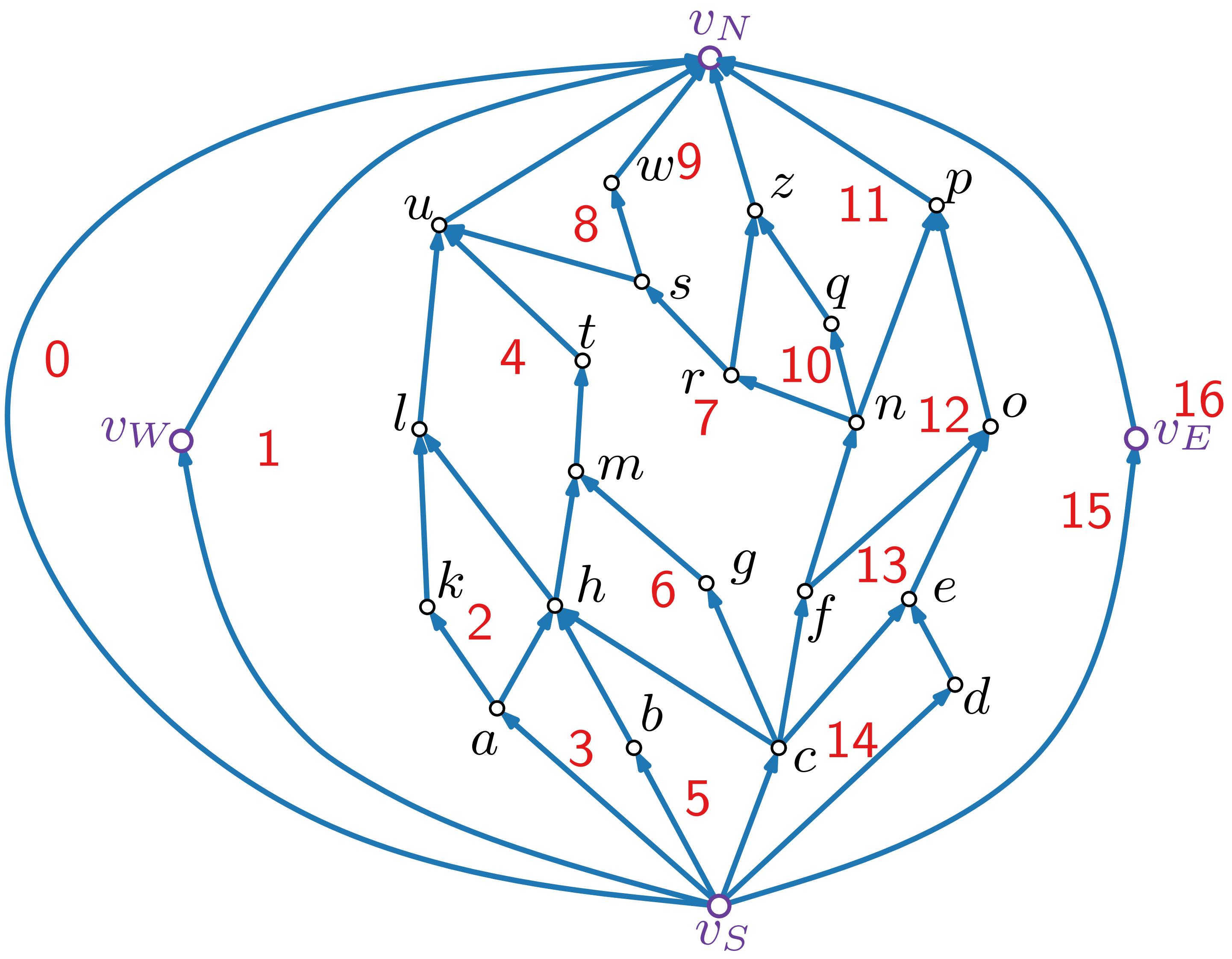
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$

Reading off Coordinates to Get Rectangular Dual



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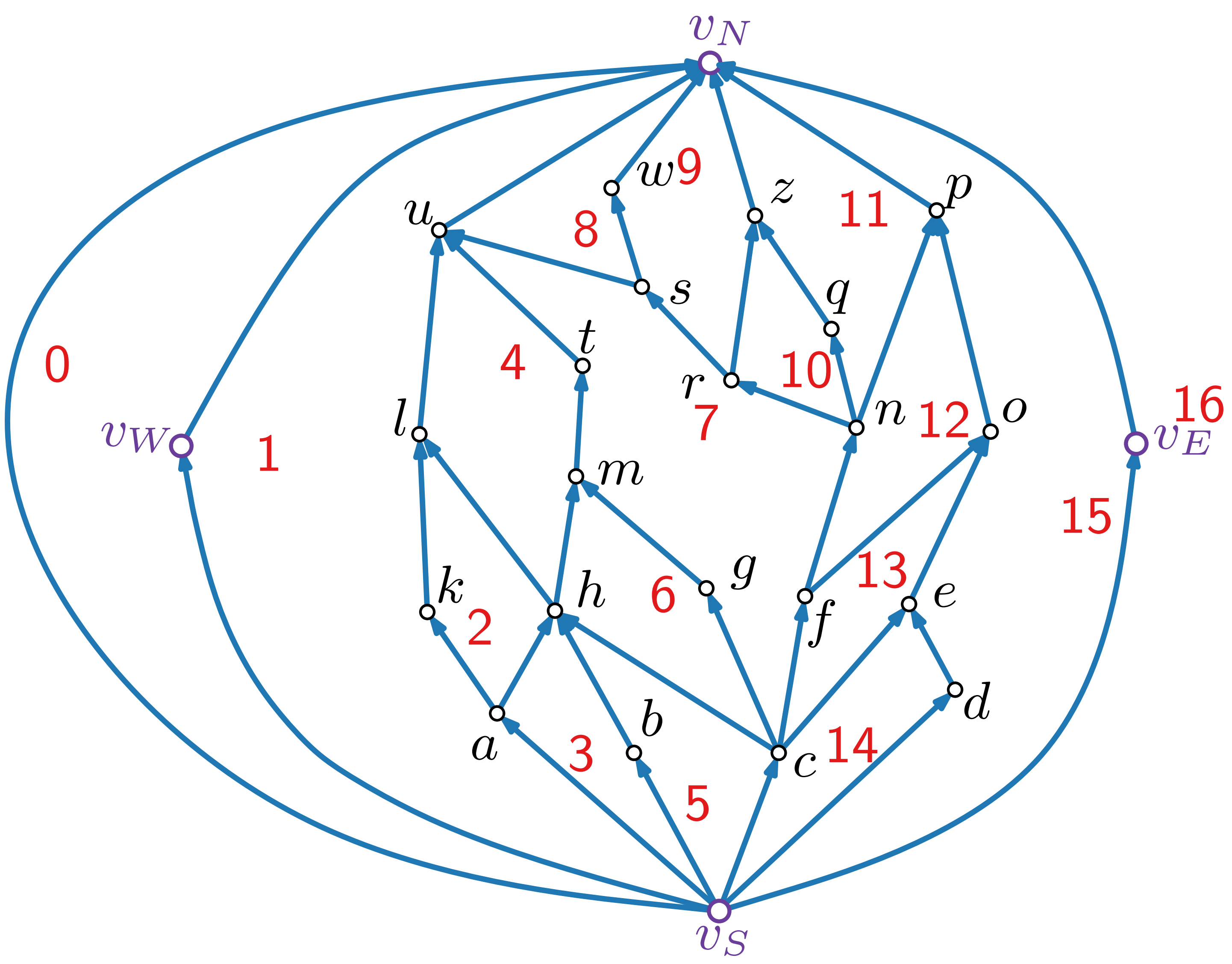
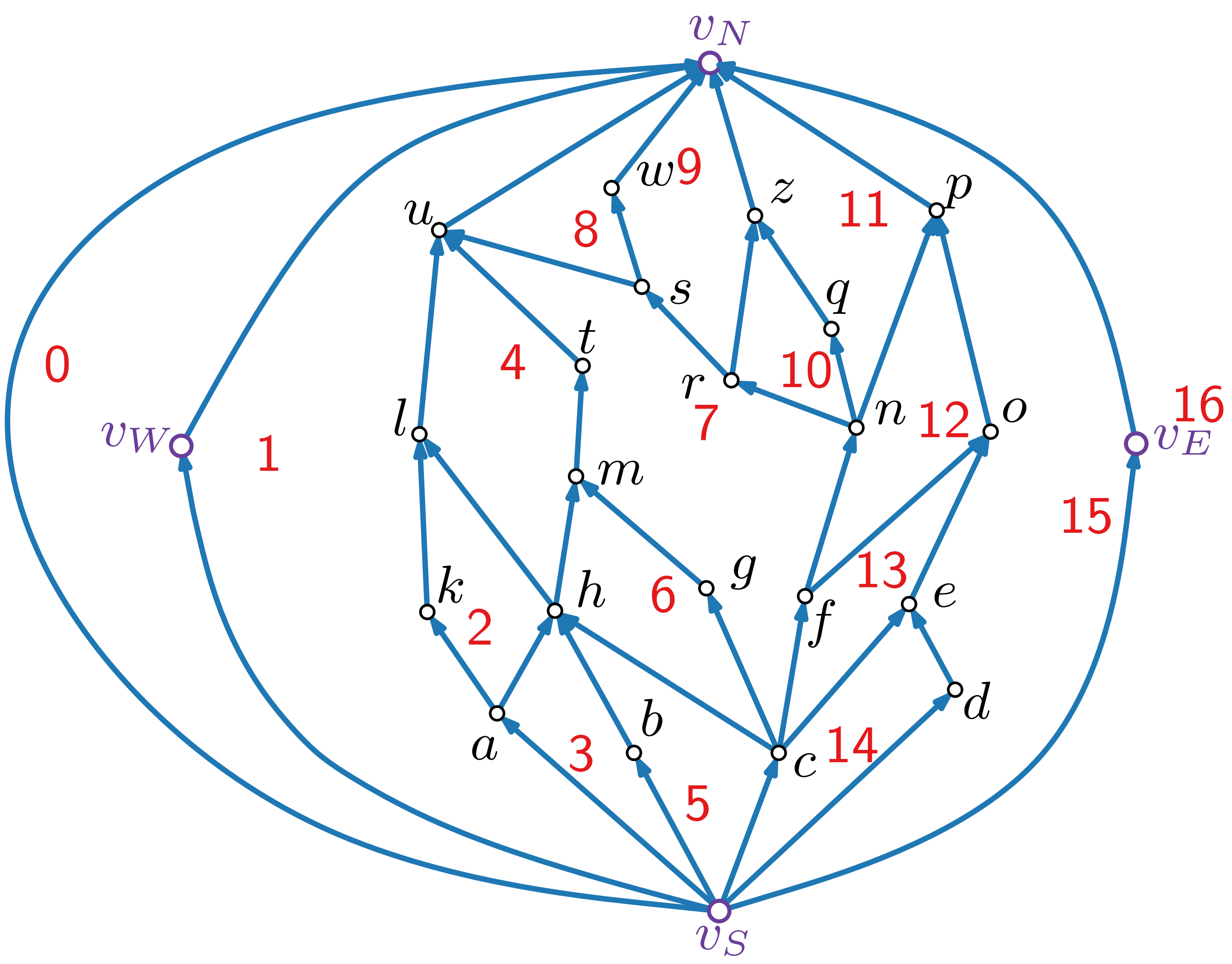
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$

Reading off Coordinates to Get Rectangular Dual



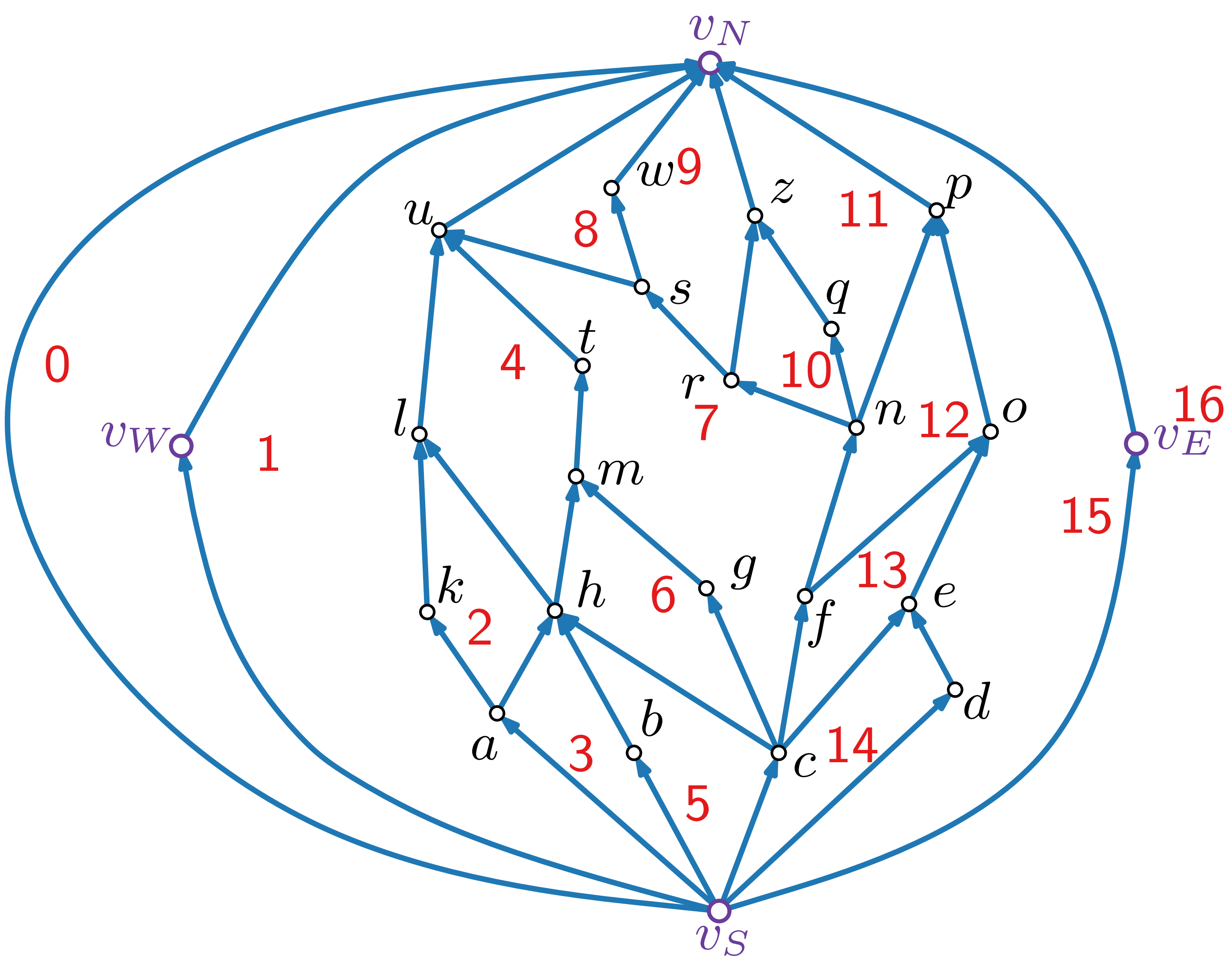
$x_1(v_N) = 1, \; x_2(v_N) = 15$
$x_1(v_S) = 2, \; x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15, \; x_2(v_E) = 16$
$x_1(a) = 1, \; x_2(a) = 3$

Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$

Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$

$x_1(v_S) = 2,\ x_2(v_S) = 16$

$x_1(v_W) = 0, x_2(v_W) = 1$

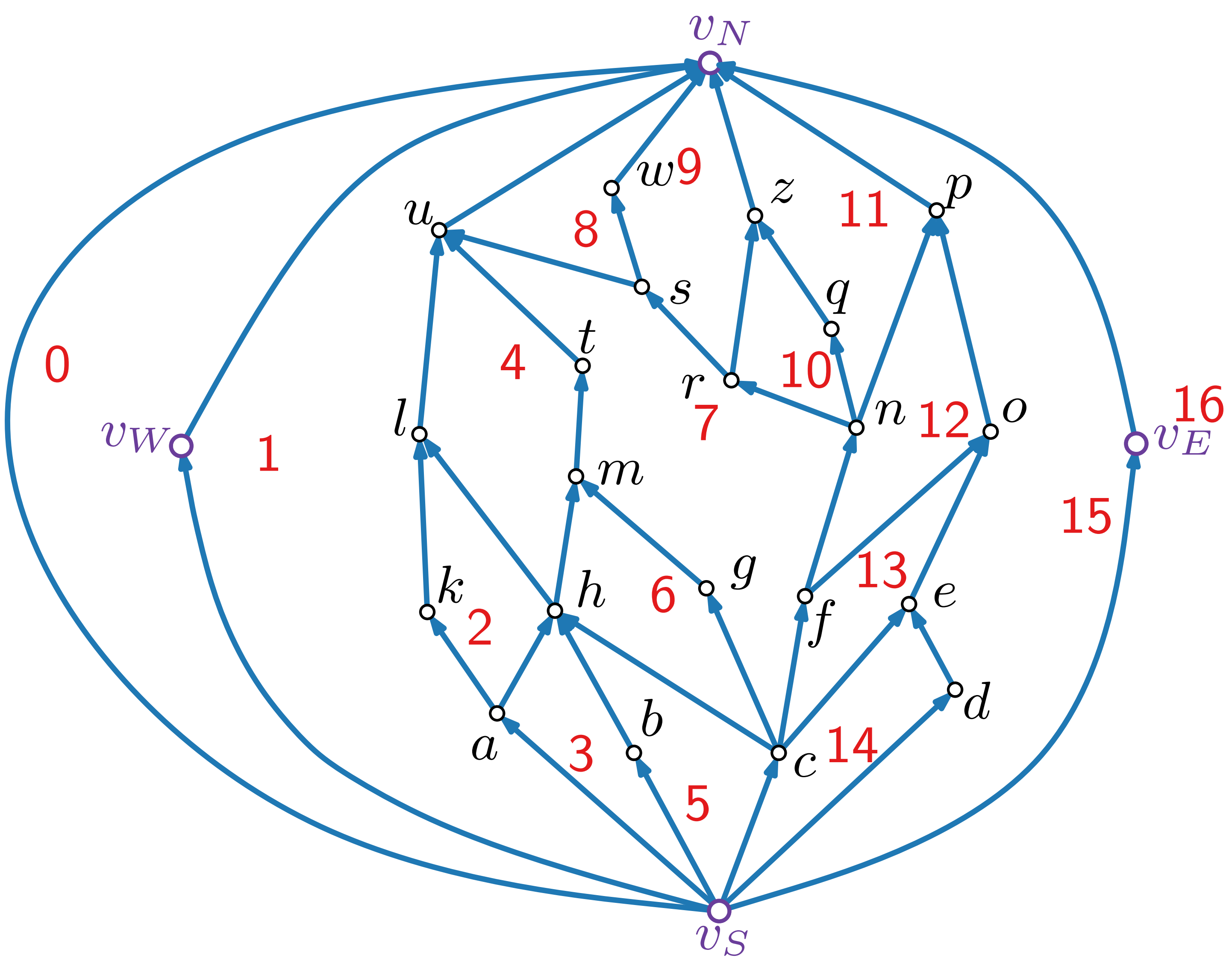
$x_1(v_E) = 15,\ x_2(v_E) = 16$

$x_1(a) = 1,\ x_2(a) = 3$

$x_1(b) = 3,\ x_2(b) = 5$

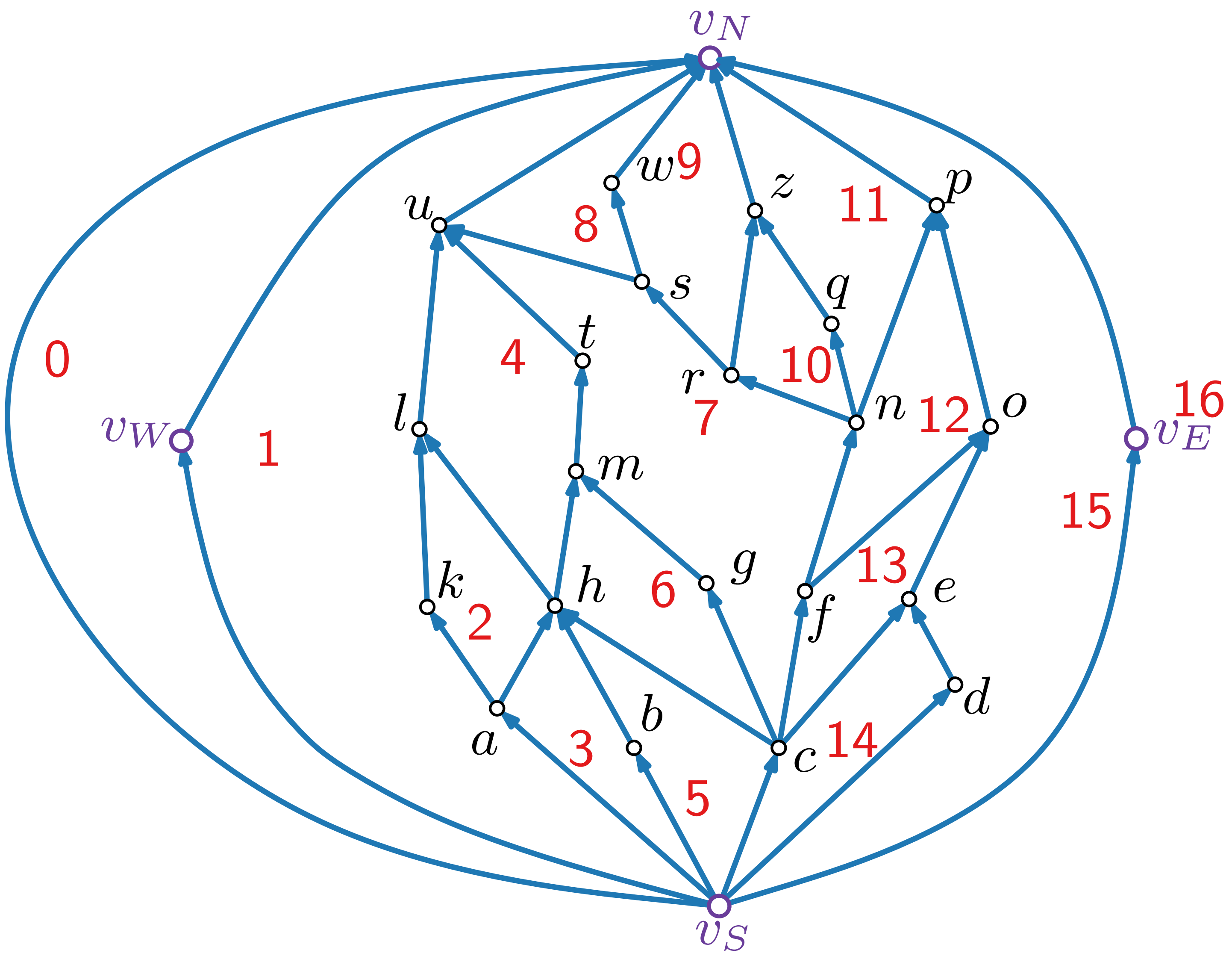
$x_1(c) = 5,\ x_2(c) = 14$

Reading off Coordinates to Get Rectangular Dual



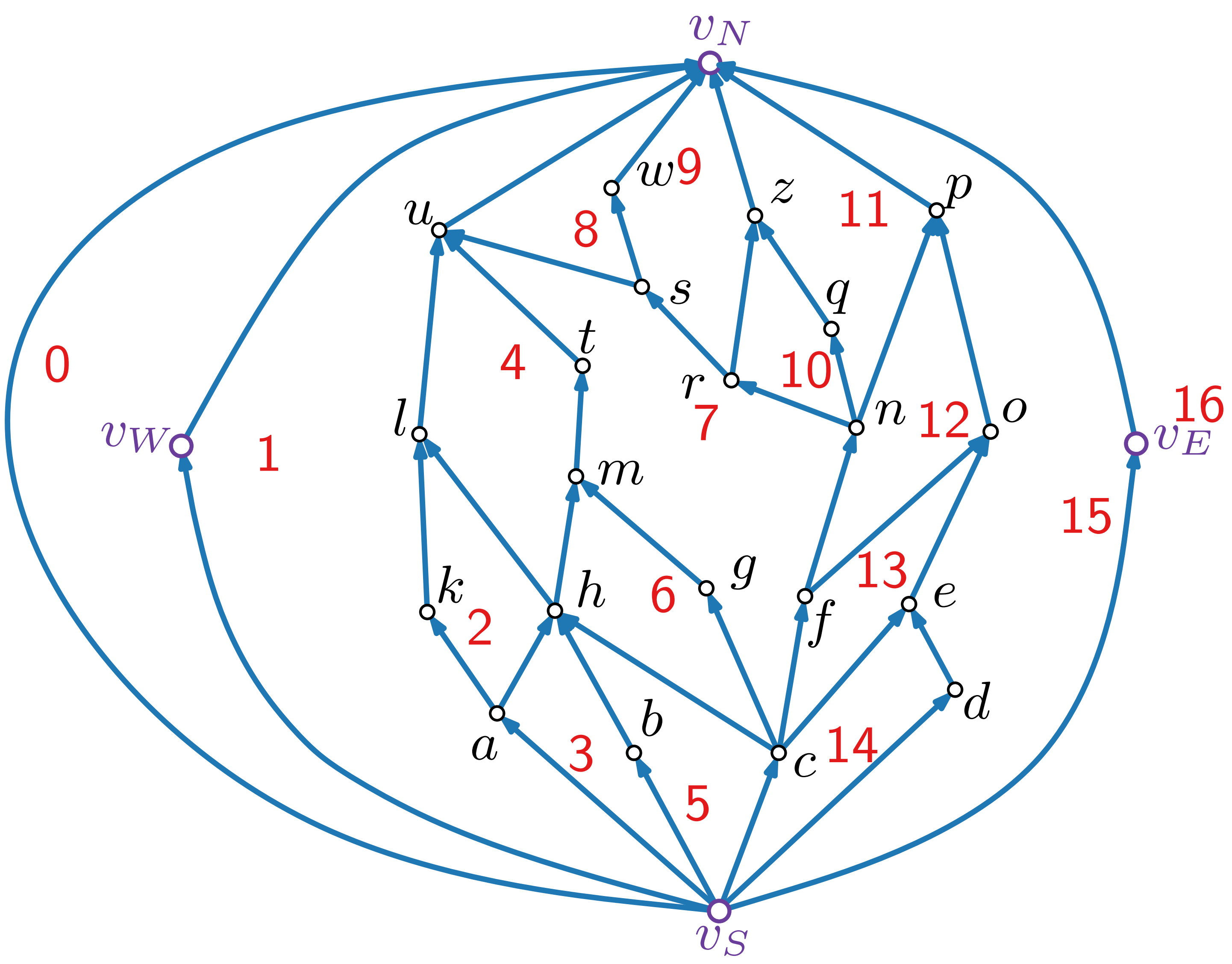
$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$

Reading off Coordinates to Get Rectangular Dual



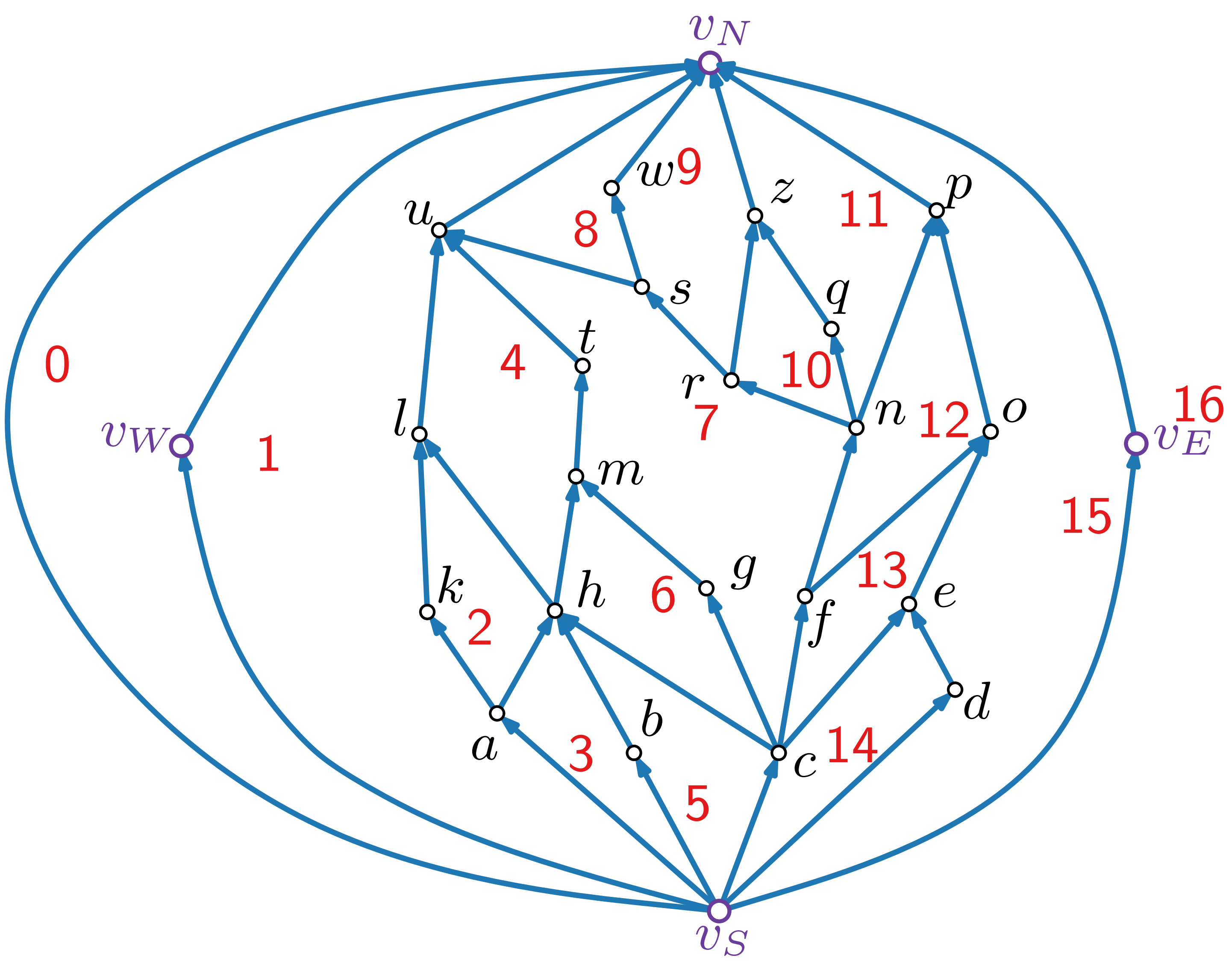
$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$

Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

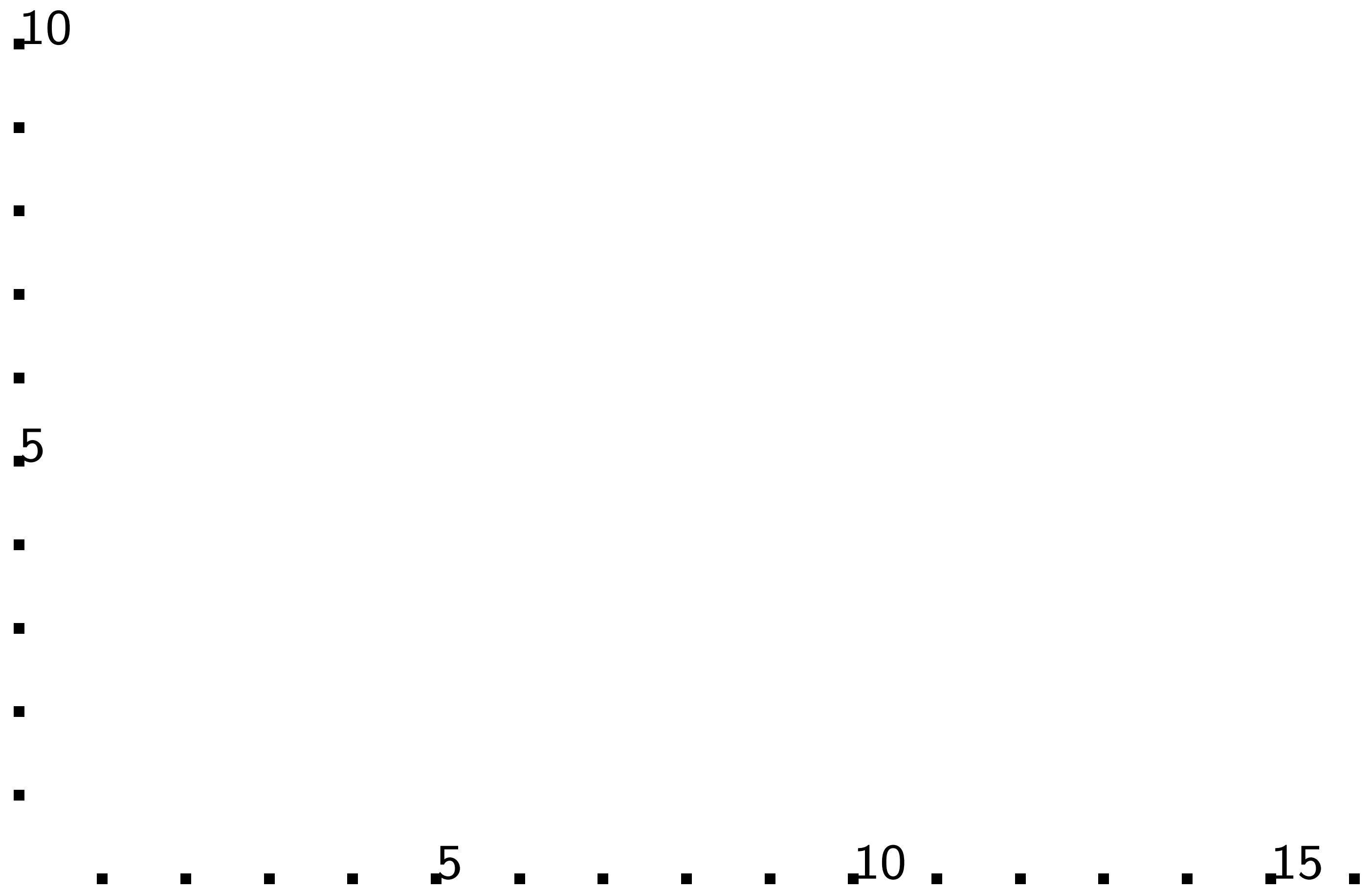
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

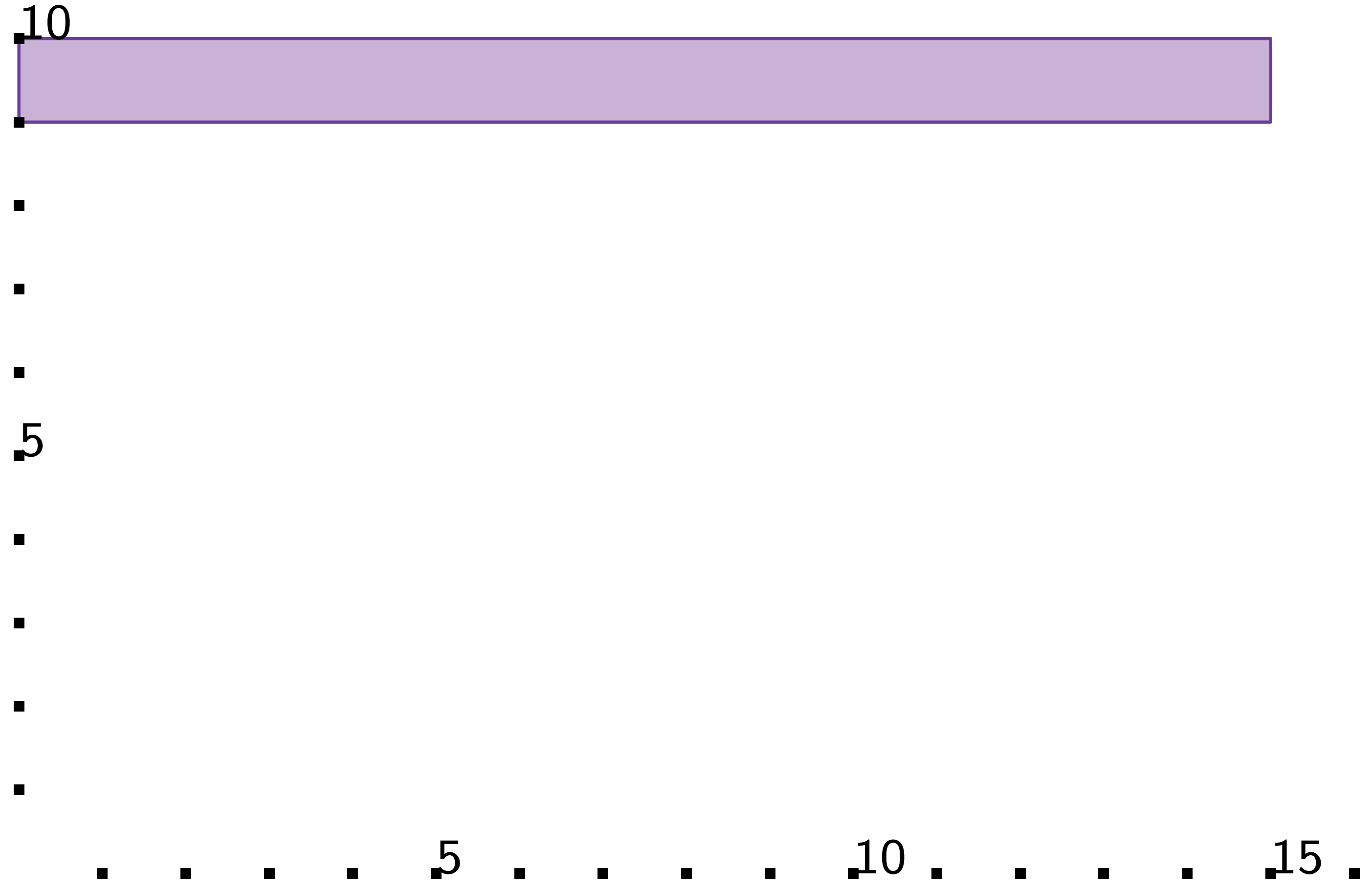
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

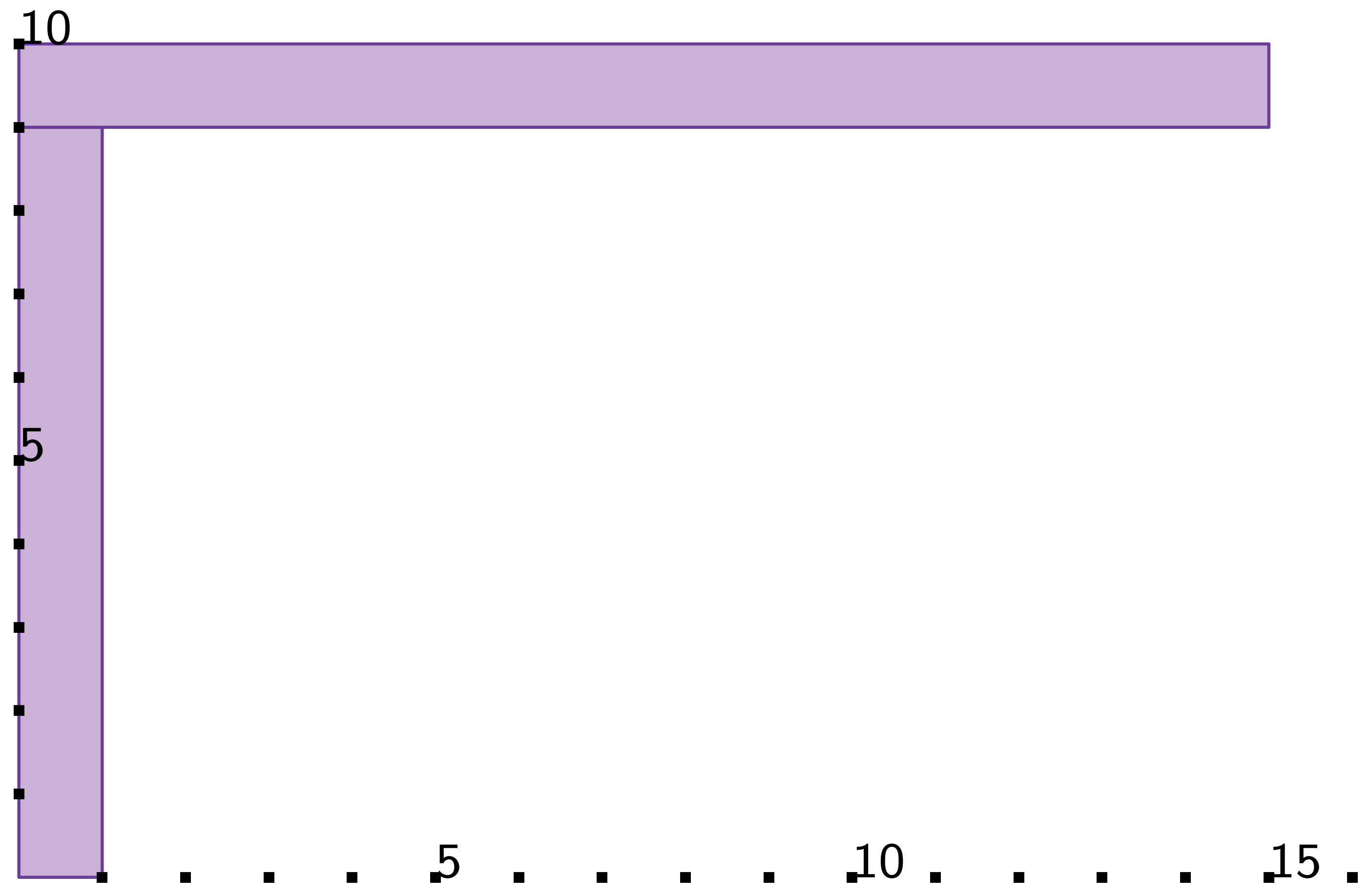
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

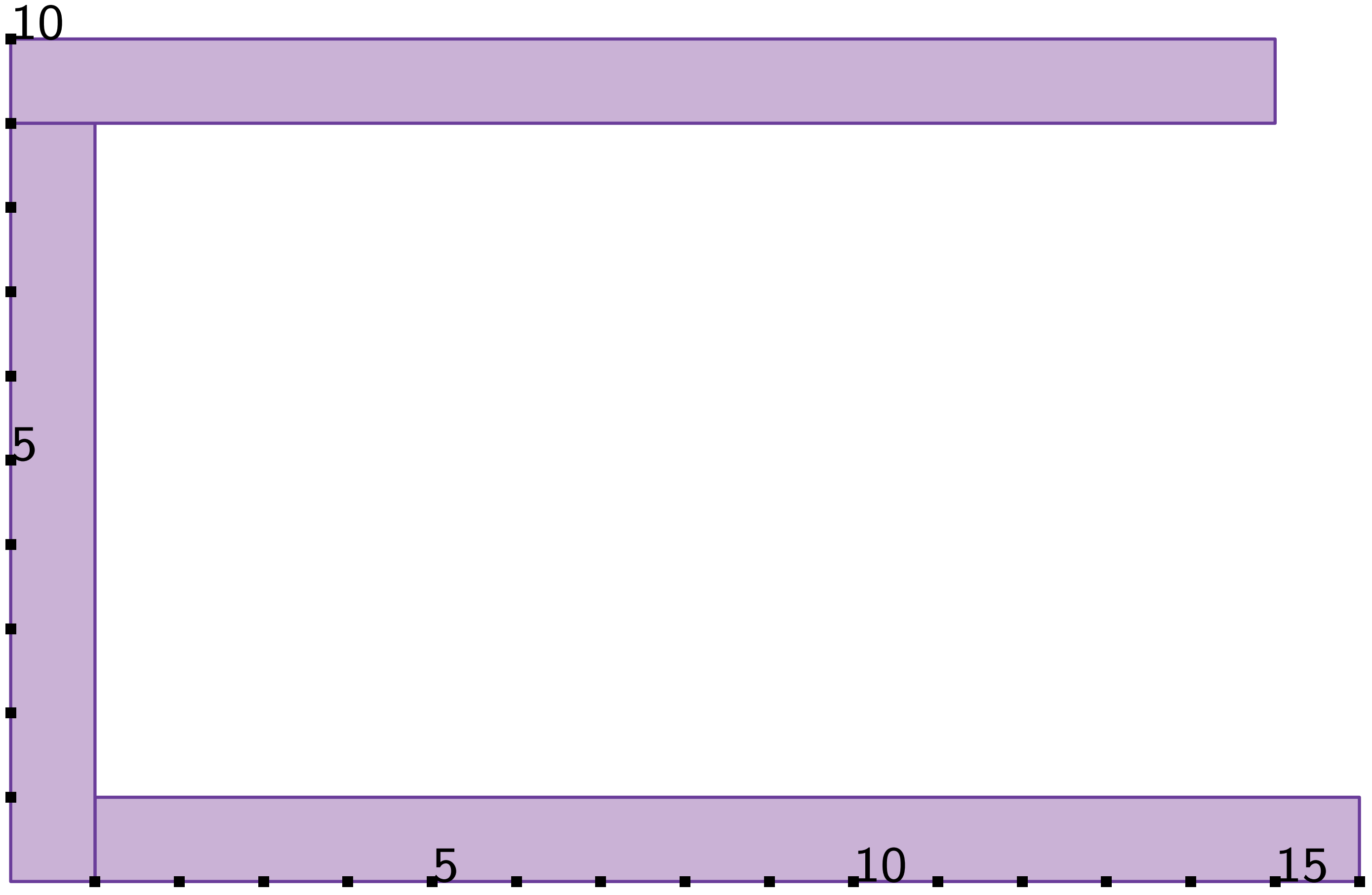
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

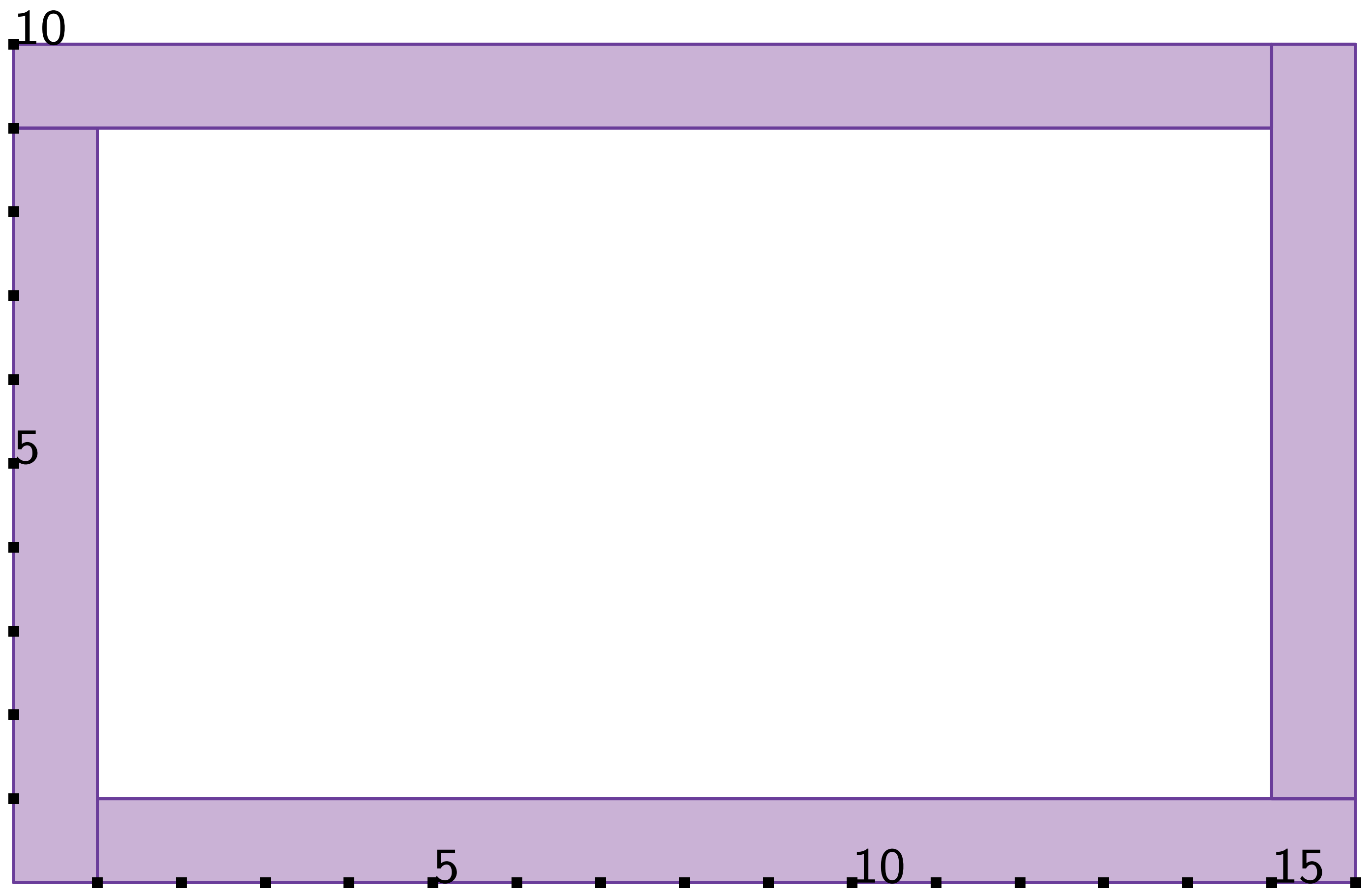
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

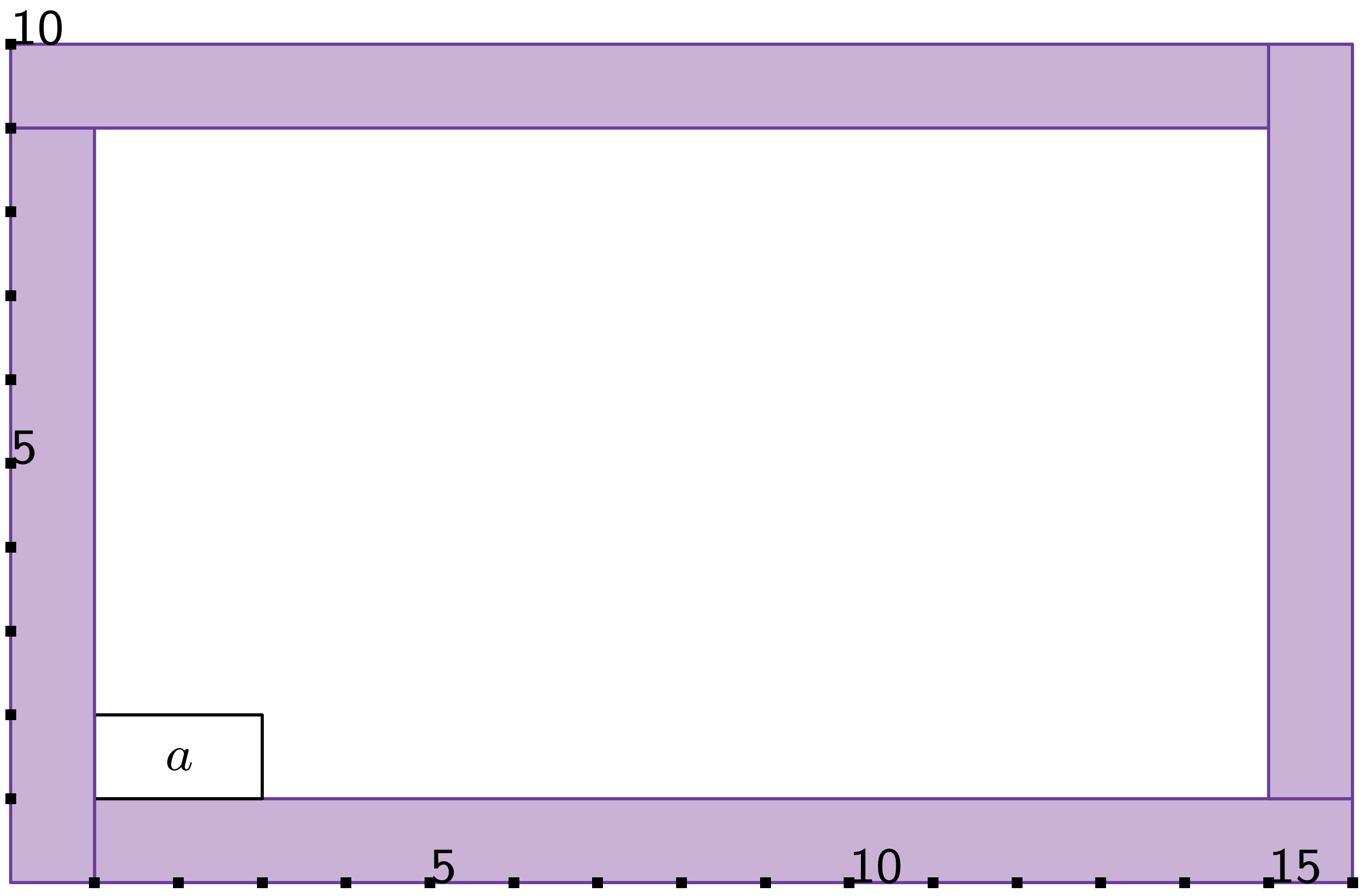
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

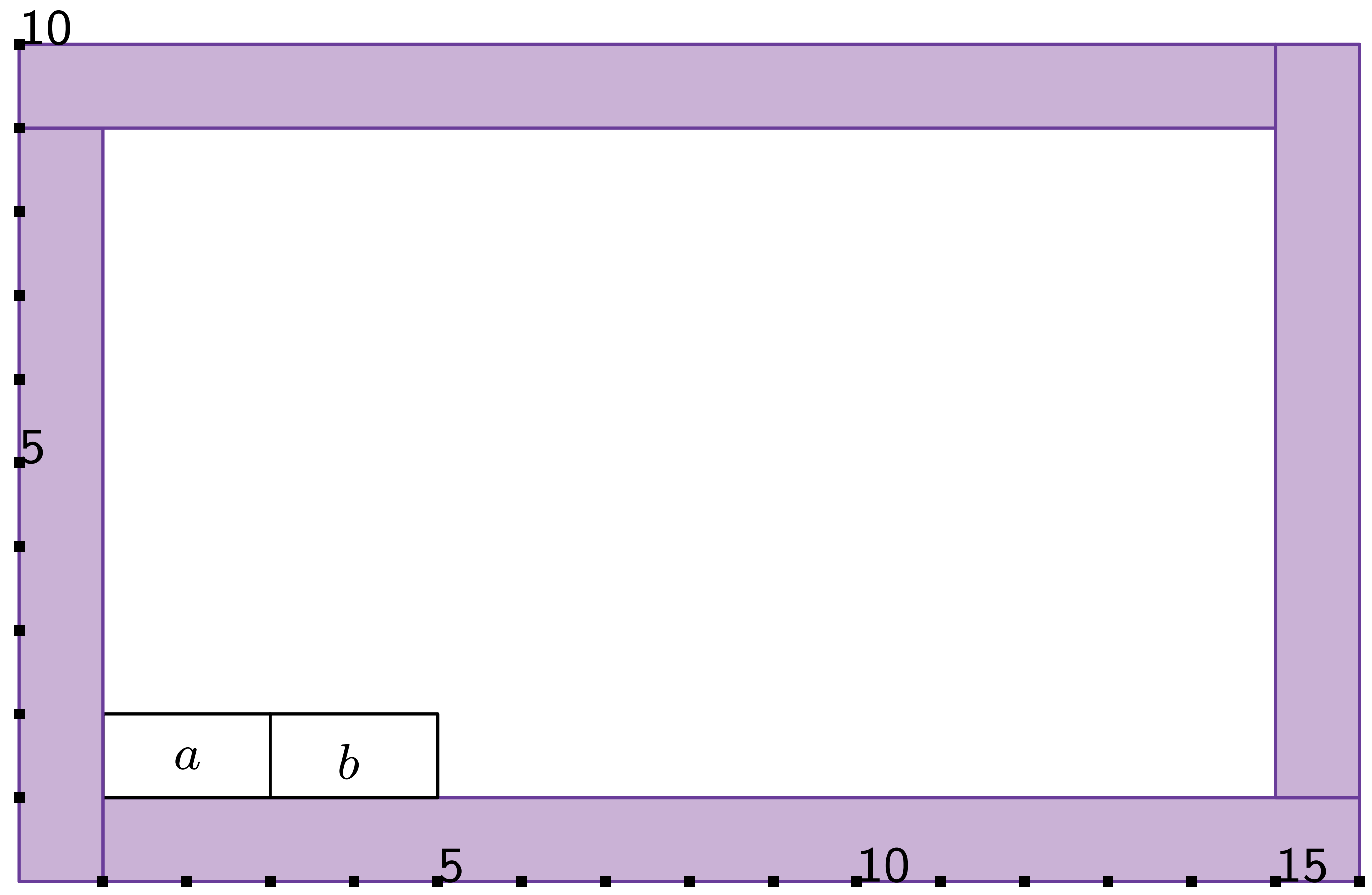
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

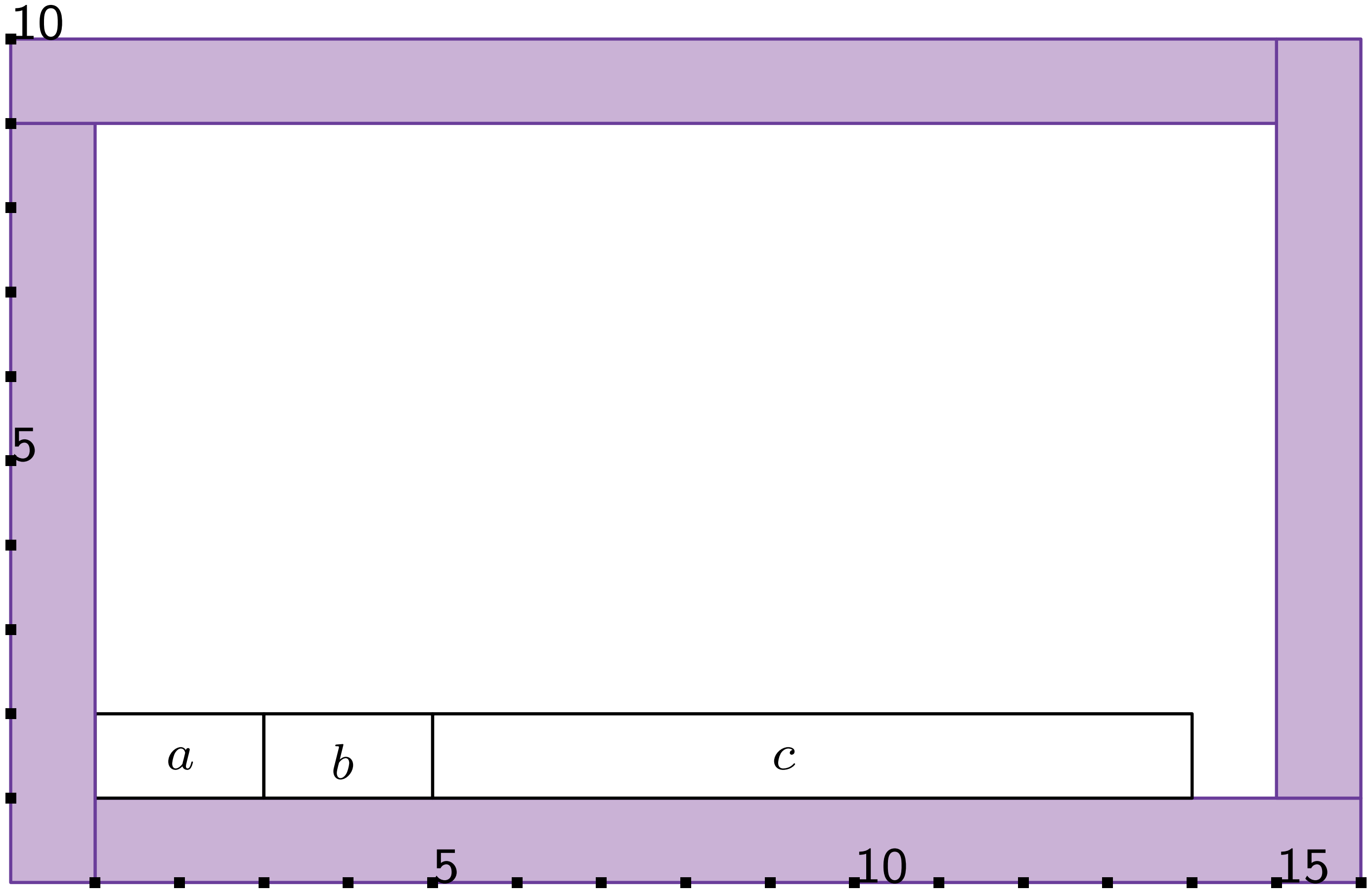
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

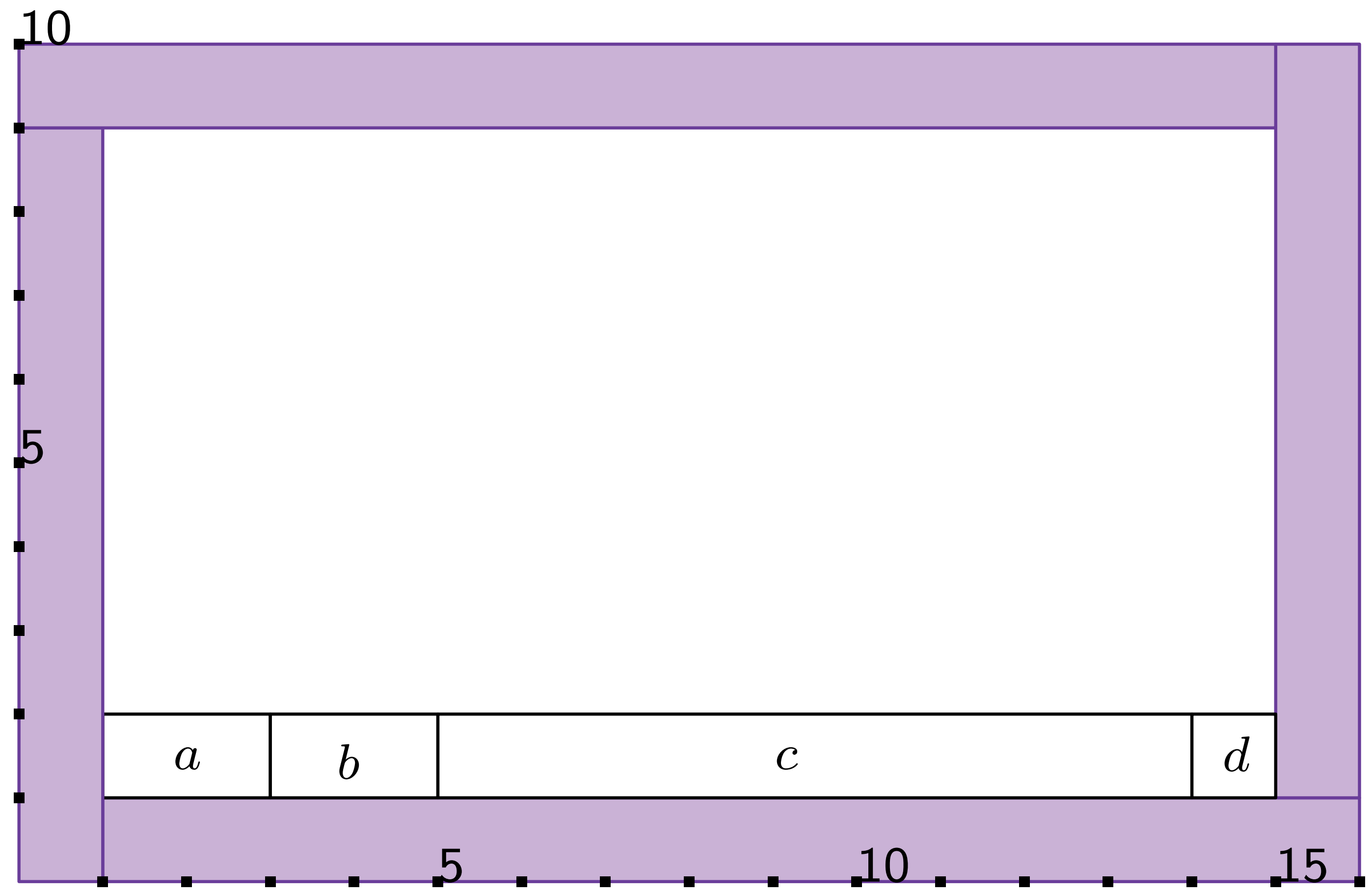
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

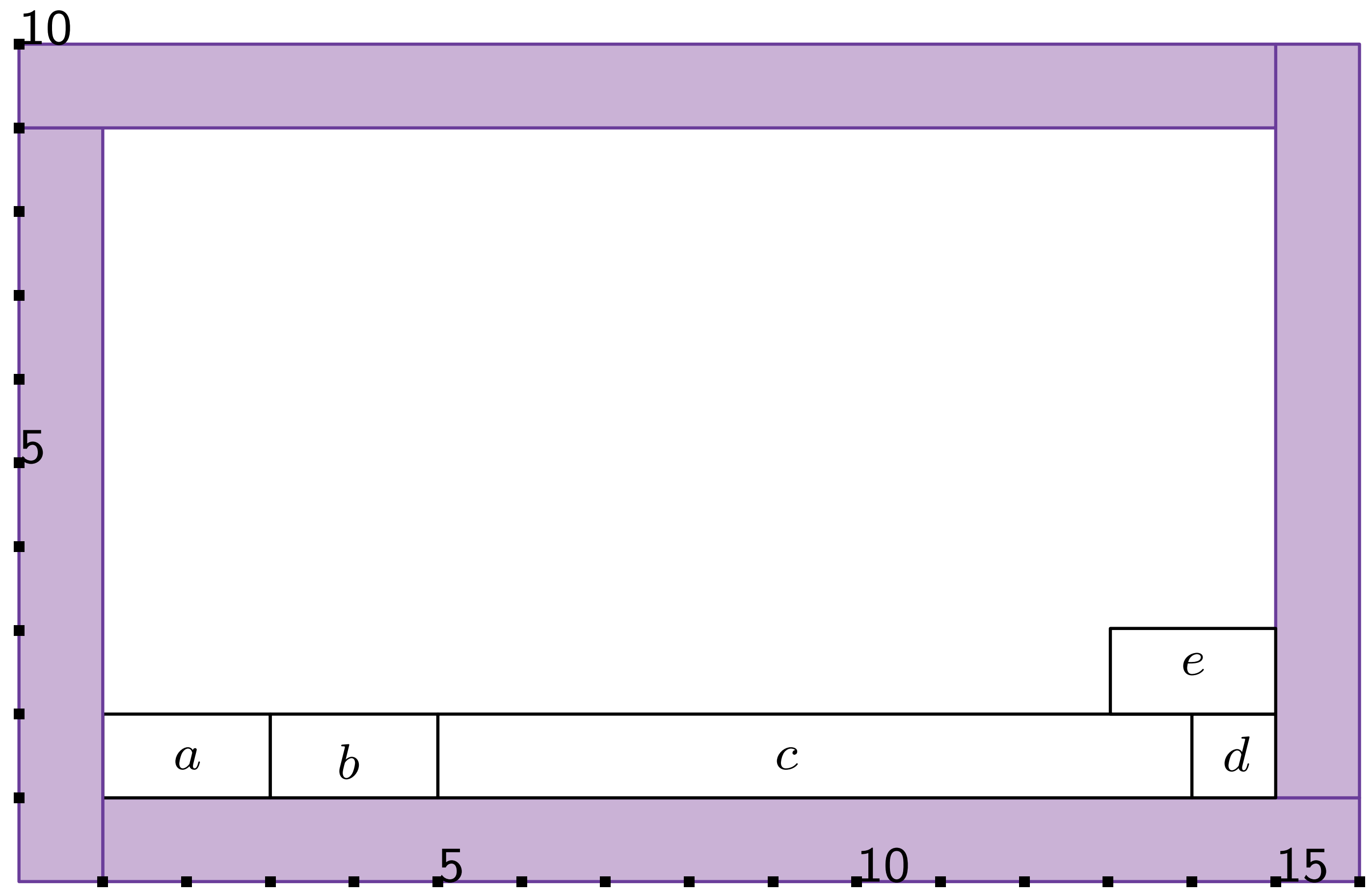
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

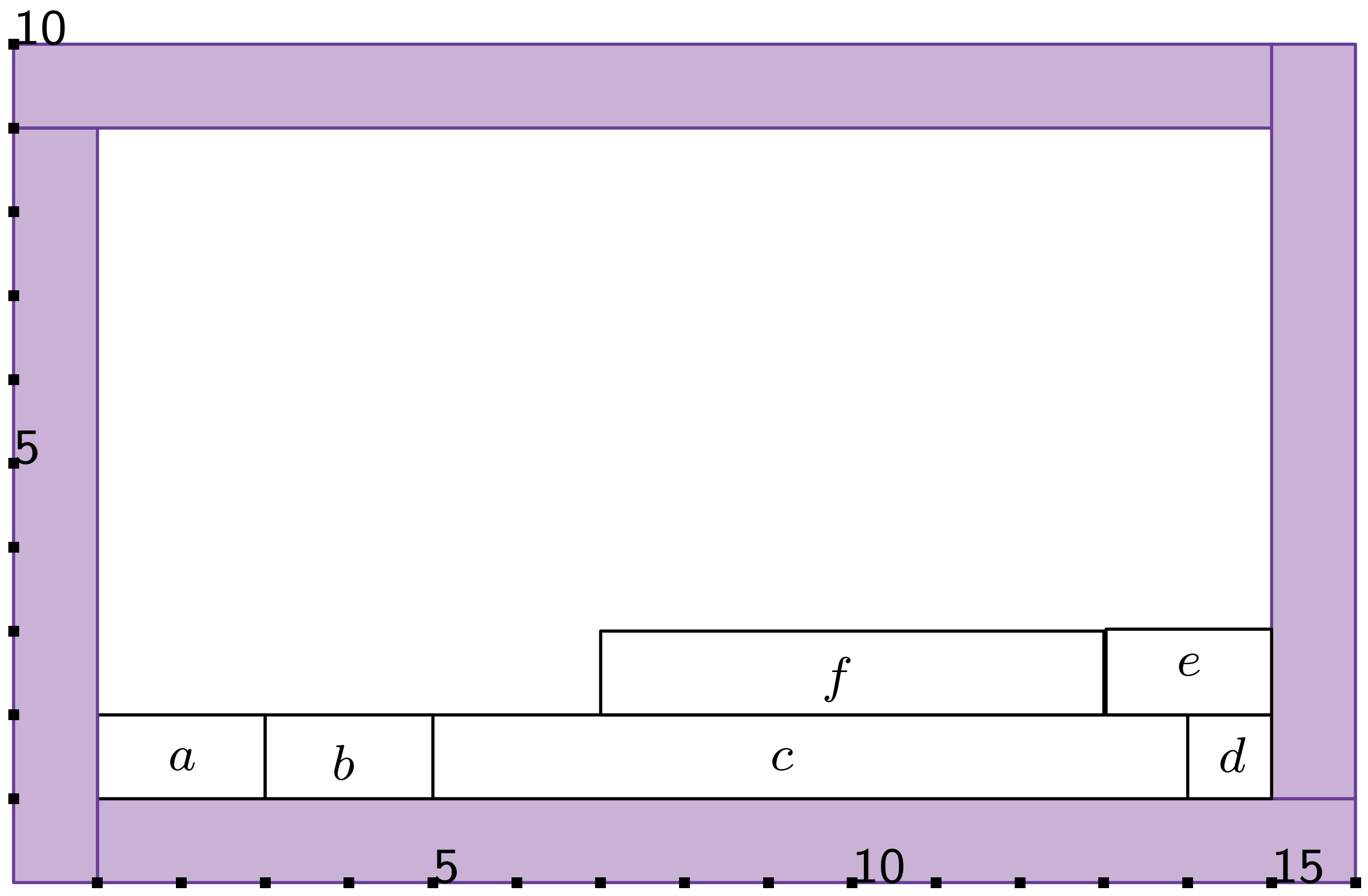
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

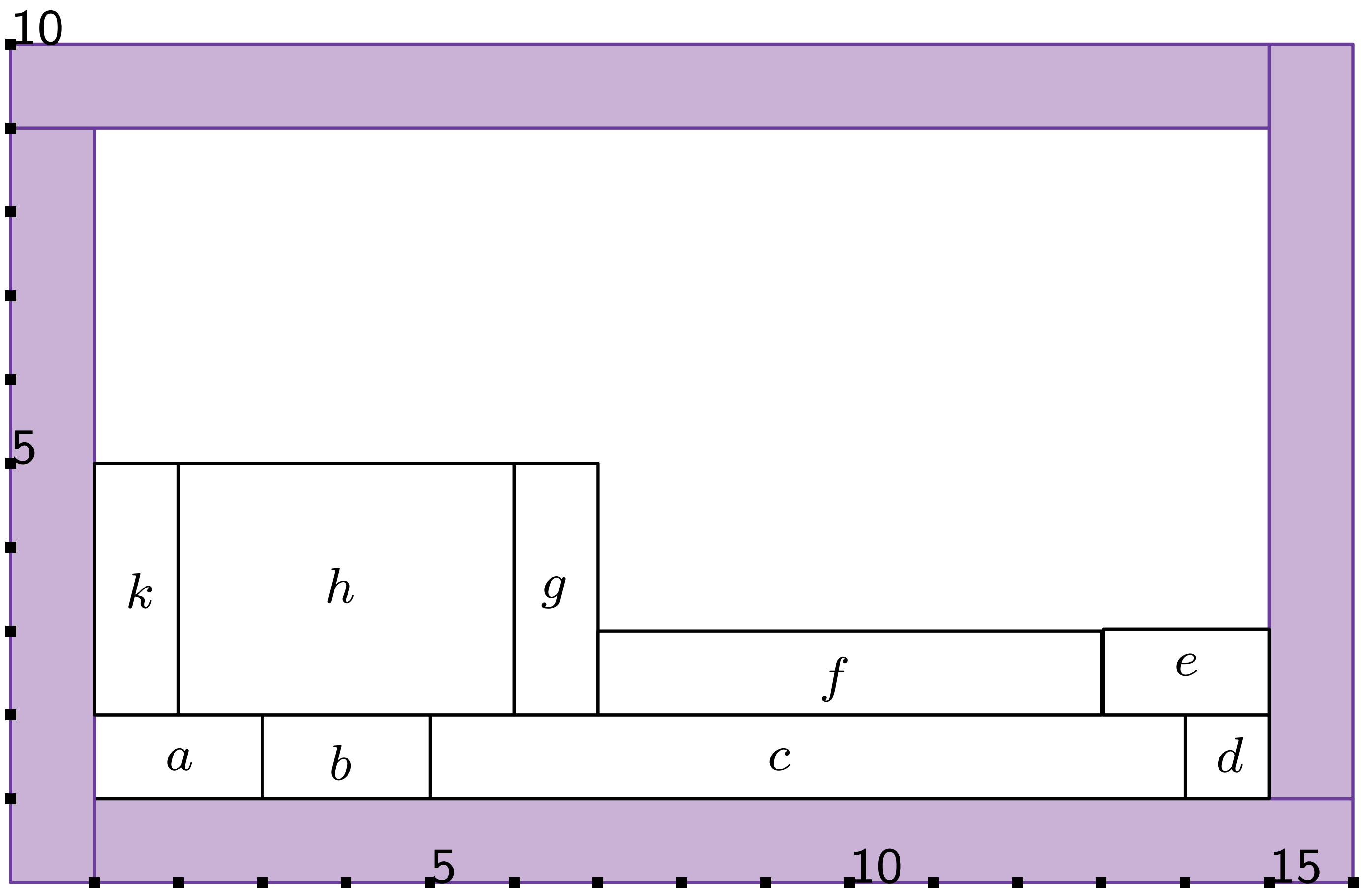
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

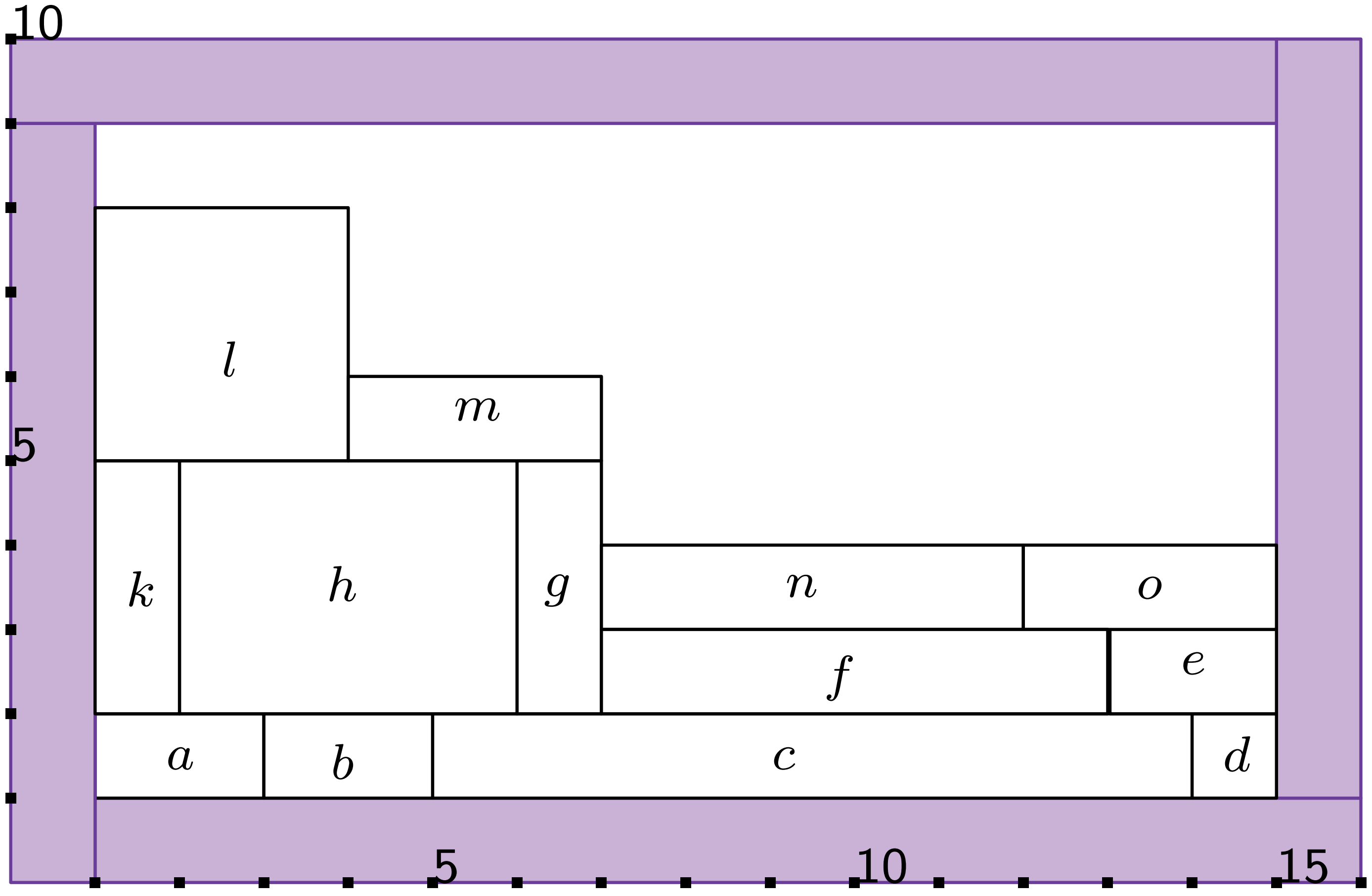
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
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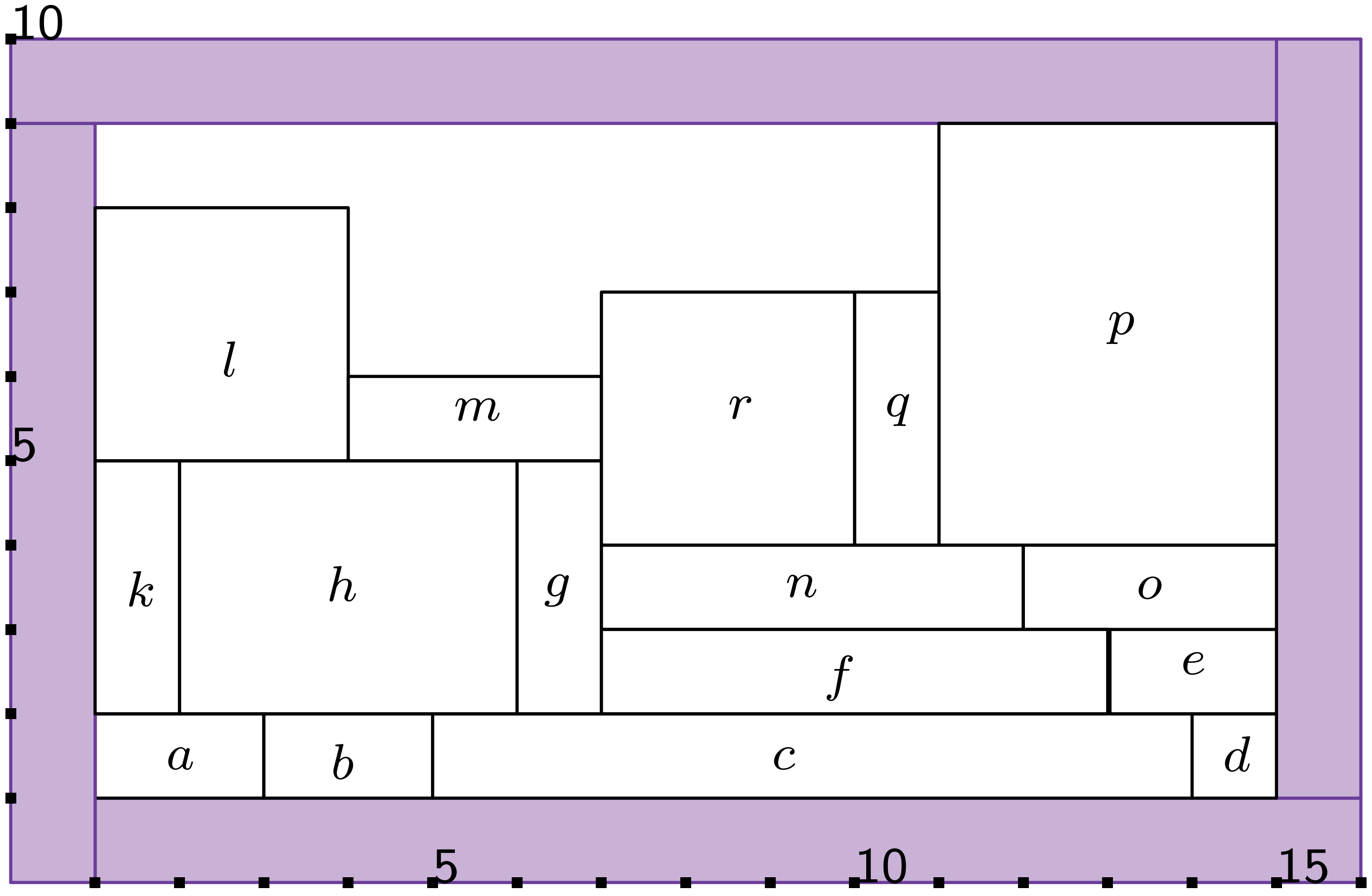
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
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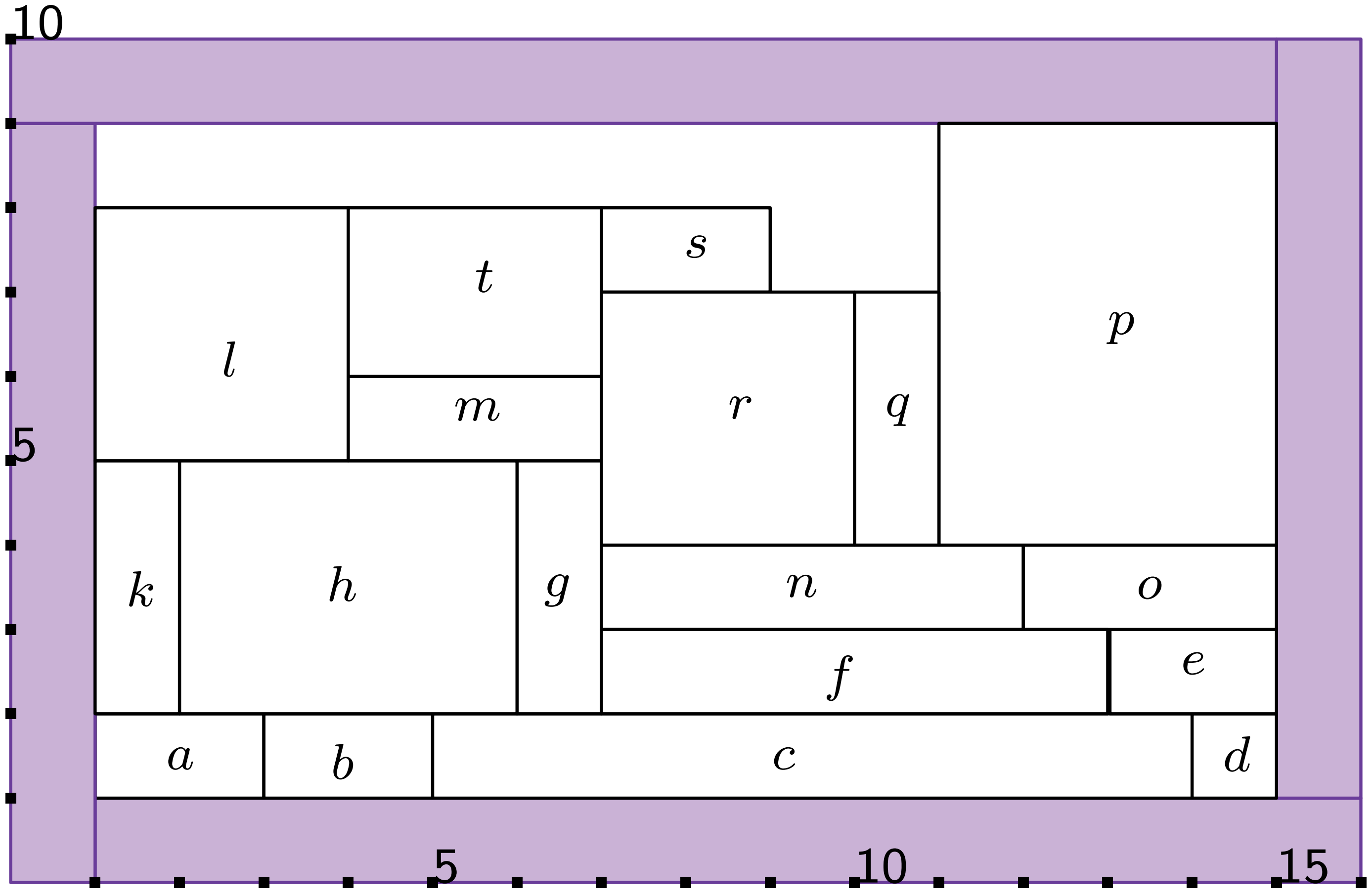

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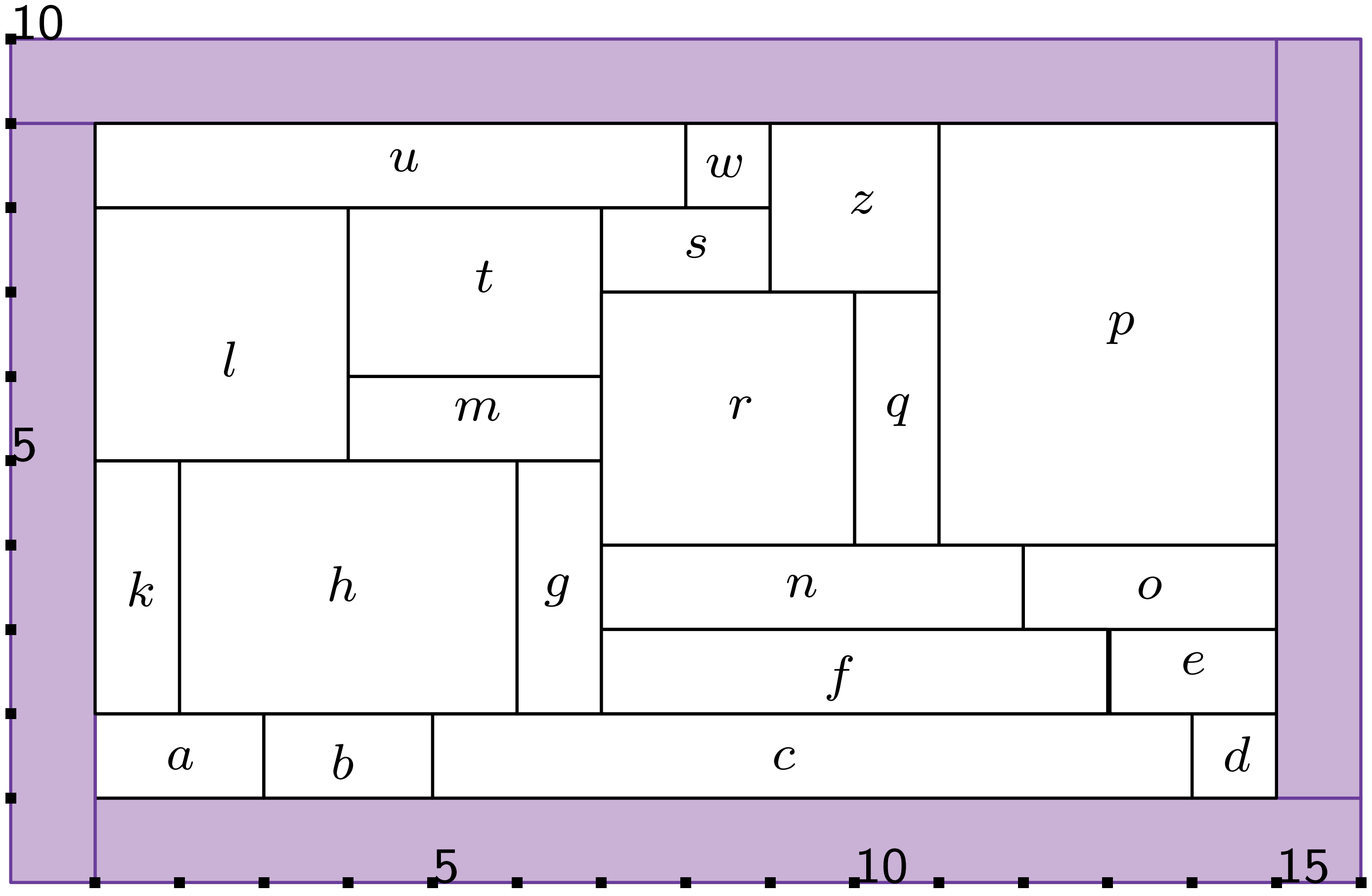

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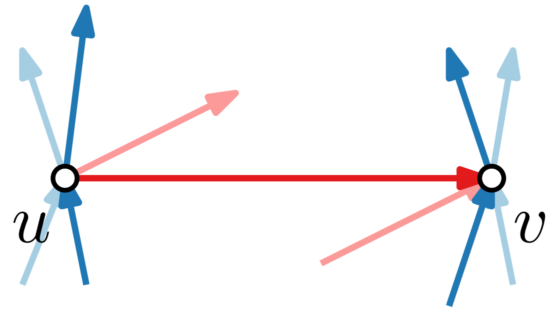

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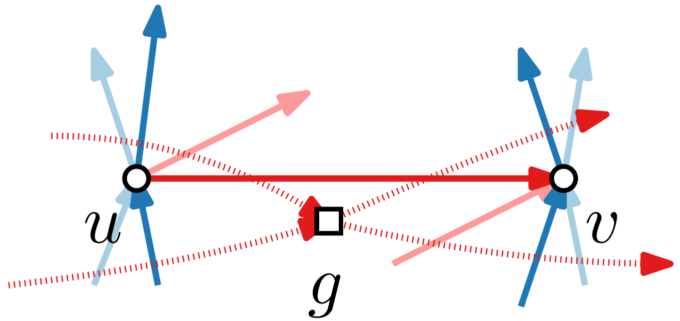

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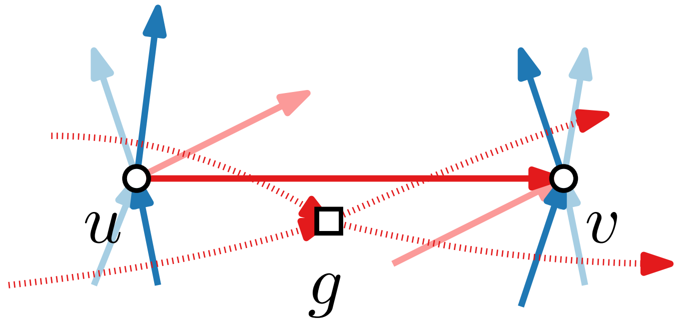

$$x_2(u) = f_{\text{ver}}(g) = x_1(v)$$

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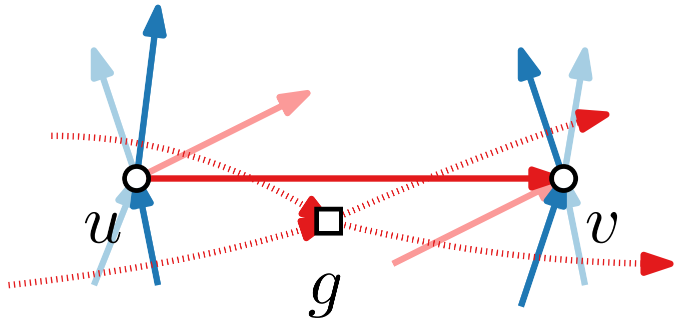

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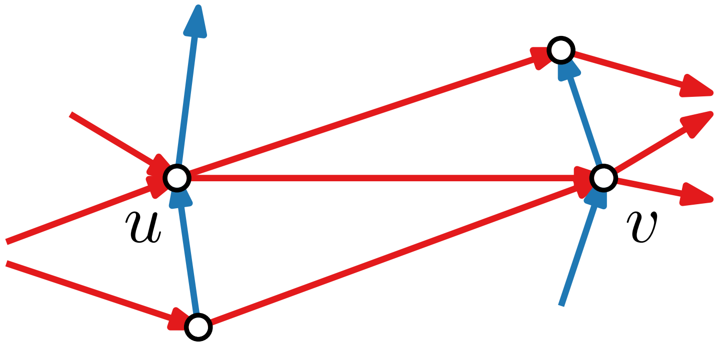

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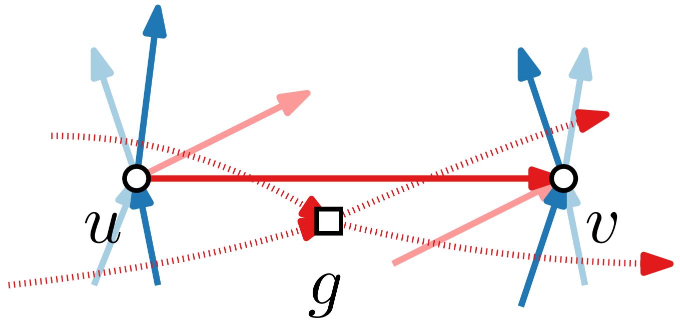

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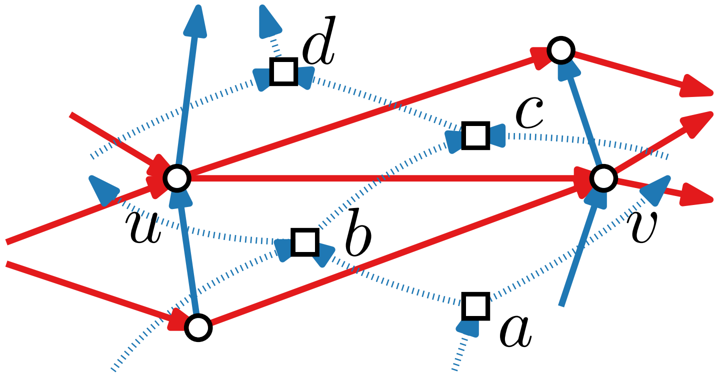

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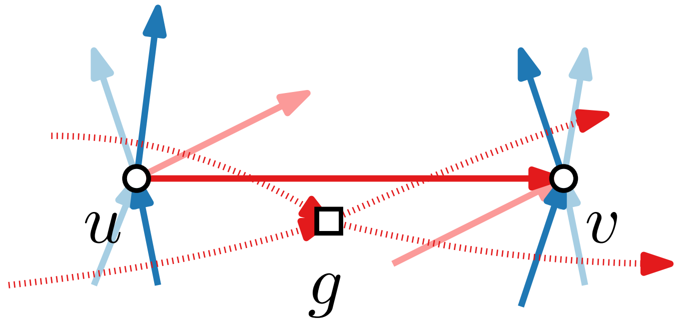

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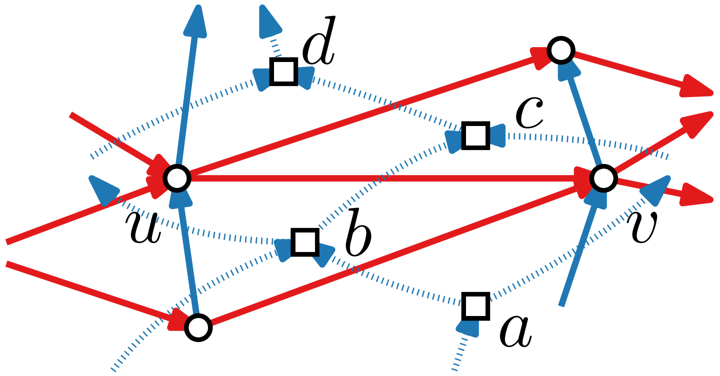

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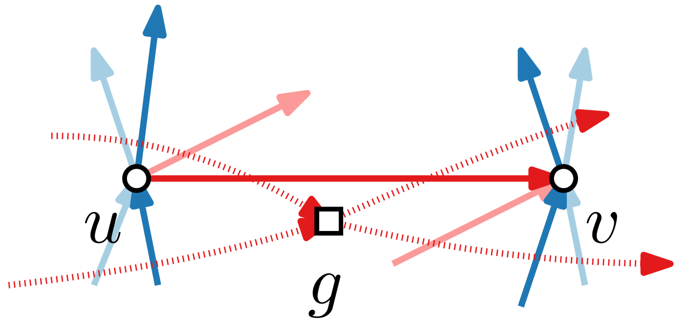

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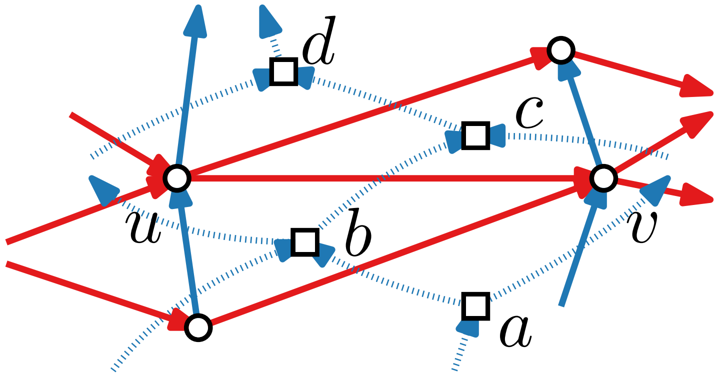

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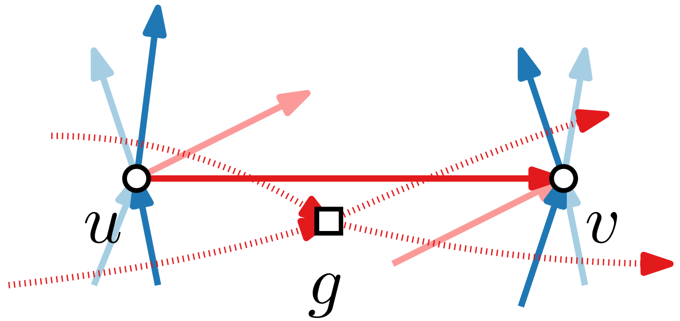

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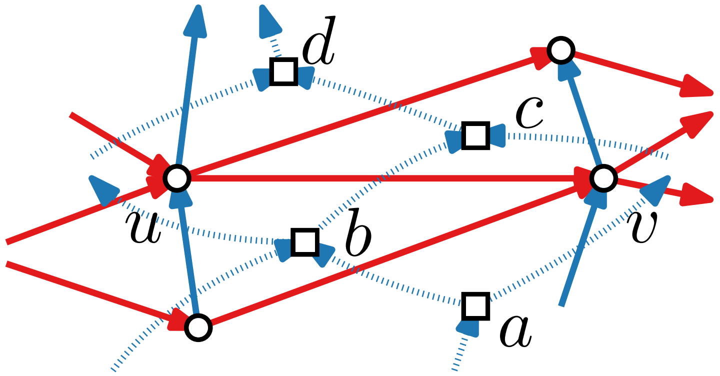

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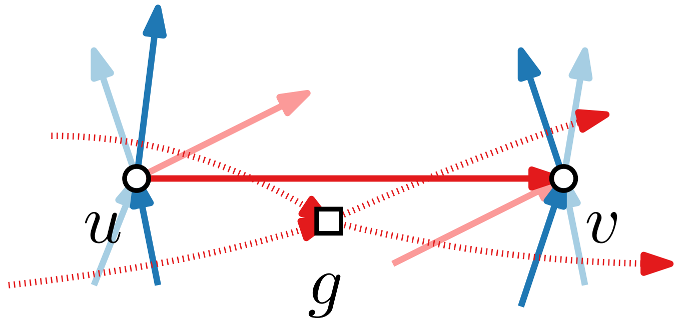

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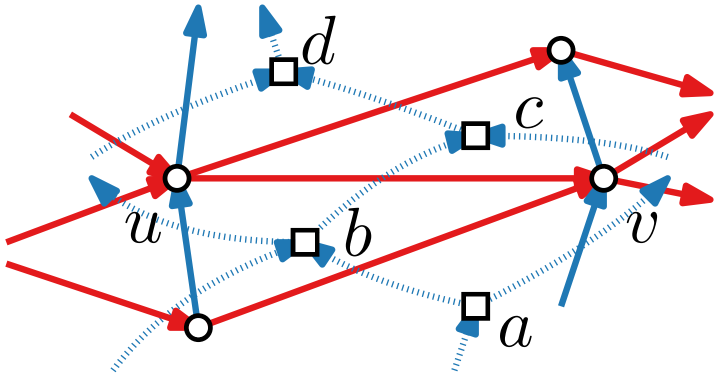

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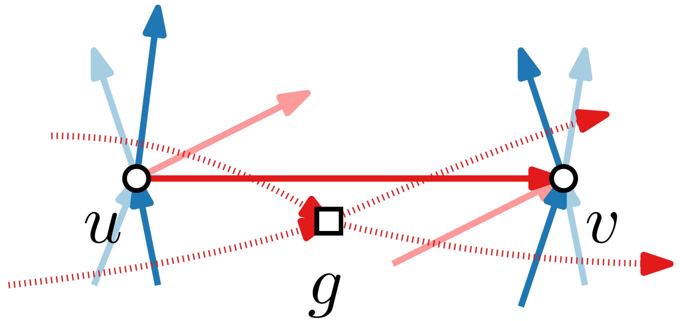

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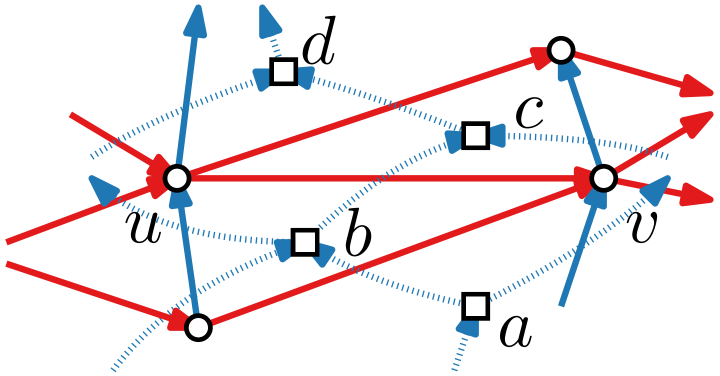

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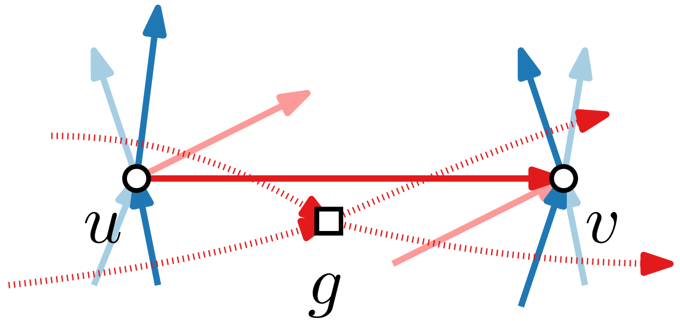

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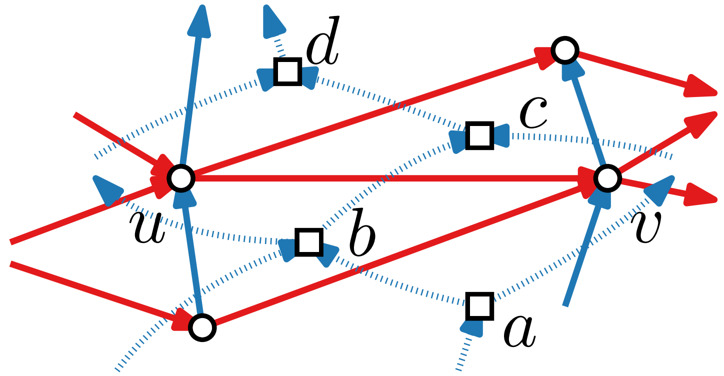

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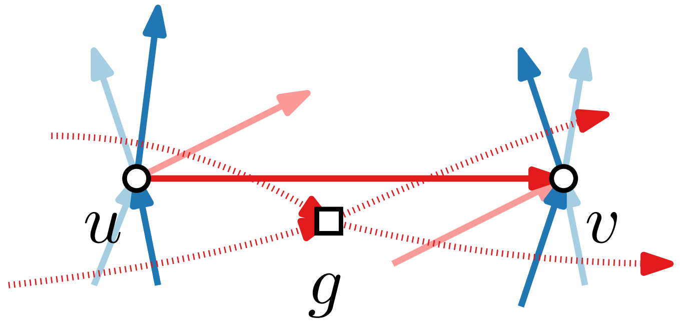

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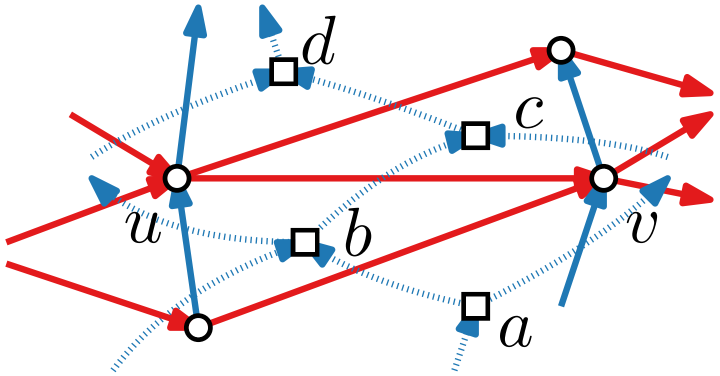

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- For details, see He's paper [He '93].

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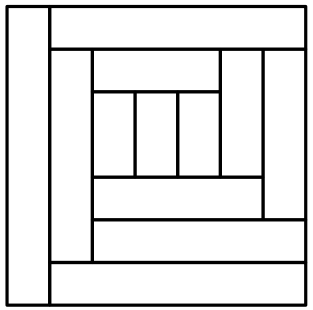

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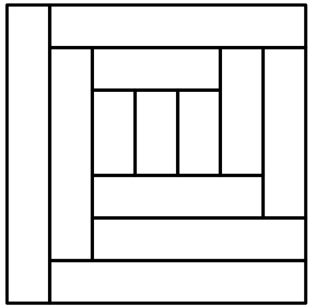

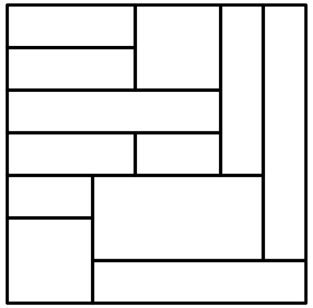

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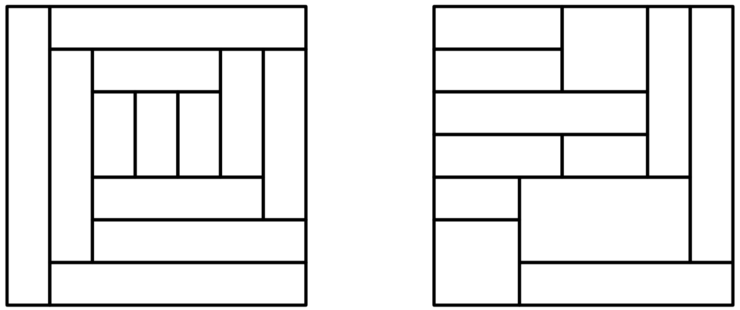

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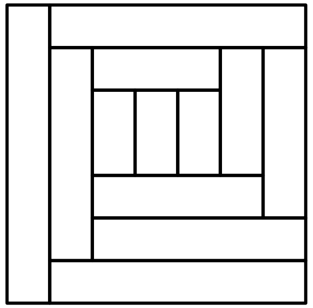

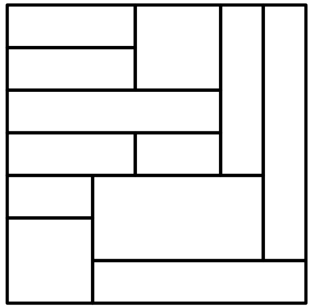

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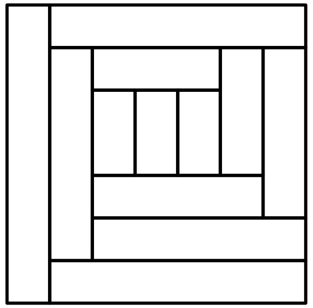

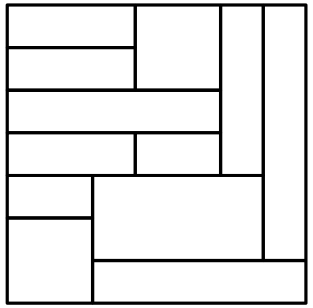

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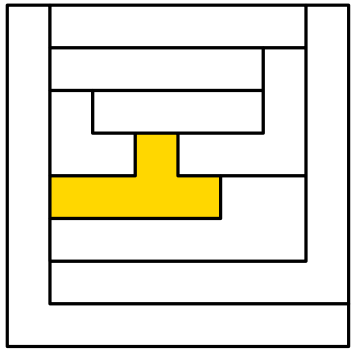

Literature

Construction of triangle contact representations based on

- [de Fraysseix, Ossona de Mendez, Rosenstiehl '94] On Triangle Contact Graphs

Construction of rectangular dual based on

- [He '93] On Finding the Rectangular Duals of Planar Triangulated Graphs
- [Kant, He '94] Two algorithms for finding rectangular duals of planar graphs

and originally from

- [Koźmiński, Kinnen '85] Rectangular Duals of Planar Graphs