Business Cycles

- Exercise 5 -

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The Neoclassical Model

Question:
This exercise will ask you to work through the derivation of the IS curve under various different scenarios.

a) Graphically derive the IS curve for a generic specification of the consumption function and the investment demand function.

Demand side:

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$$
\begin{aligned}
Y_t &= C_t + I_t + G_t \\
C_t &= C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \\
I_t &= I^d(r_t, A_{t+1}, K_t)
\end{aligned}
$$

Demand side:

$$Y_t = C_t + I_t + G_t$$

$$C_t = C^d(\underbrace{Y_t - G_t}_{+}, \underbrace{Y_{t+1} - G_{t+1}}_{+}, \underbrace{r_t}_{-})$$

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Aggregated desired expenditures:

Demand side:

$$Y_t = C_t + I_t + G_t$$

$$C_t = C^d(\underbrace{Y_t - G_t}_{+}, \underbrace{Y_{t+1} - G_{t+1}}_{+}, \underbrace{r_t}_{-})$$

$$I_t = I^d(\underbrace{r_t}_{-}, \underbrace{A_{t+1}}_{+}, \underbrace{K_t}_{-})$$

Aggregated desired expenditures:

$$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) + I^d(r_t, A_{t+1}, K_t) + G_t$$

Demand side:

$$Y_t = C_t + I_t + G_t$$
$$C_t = C^d(\underbrace{Y_t - G_t}_{+}, \underbrace{Y_{t+1} - G_{t+1}}_{+}, \underbrace{r_t}_{-})$$
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Aggregated desired expenditures:

$$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) + I^d(r_t, A_{t+1}, K_t) + G_t$$

Autonomous expenditures:

Demand side:

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C_t &= C^d(\underbrace{Y_t - G_t}_{+}, \underbrace{Y_{t+1} - G_{t+1}}_{+}, \underbrace{r_t}_{-}) \\
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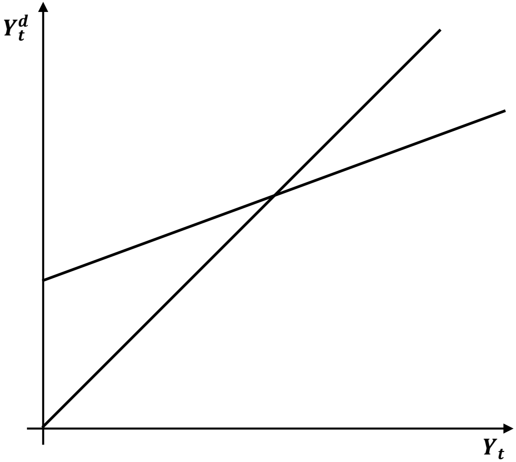
\end{aligned}
$$

Aggregated desired expenditures:

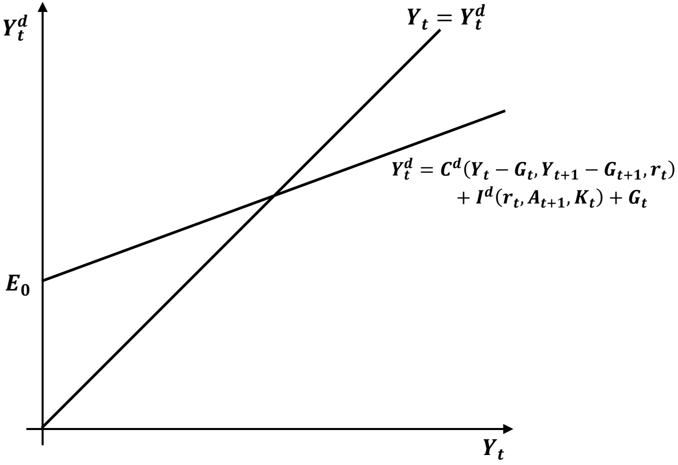
$$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) + I^d(r_t, A_{t+1}, K_t) + G_t$$

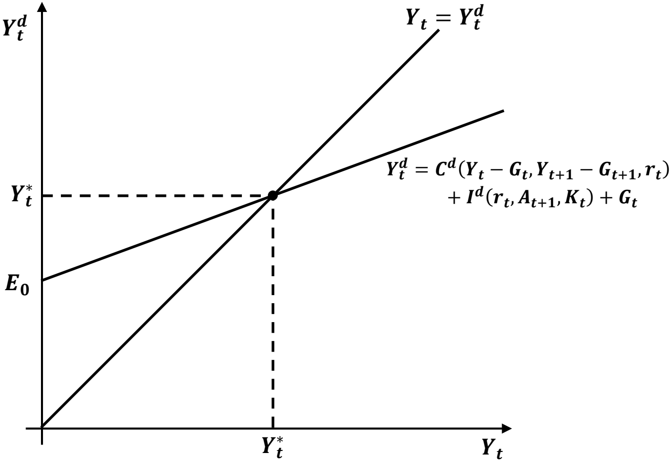
Autonomous expenditures:

$$E_0 = C^d(-G_t, Y_{t+1} - G_{t+1}, r_t) + I^d(r_t, A_{t+1}, K_t) + G_t$$



Y_t^d
Y_t


Y_t^d
$Y_t = Y_t^d$
$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$
$+ I^d(r_t, A_{t+1}, K_t) + G_t$
E_0
Y_t



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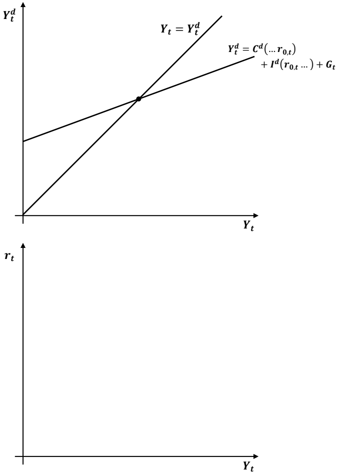
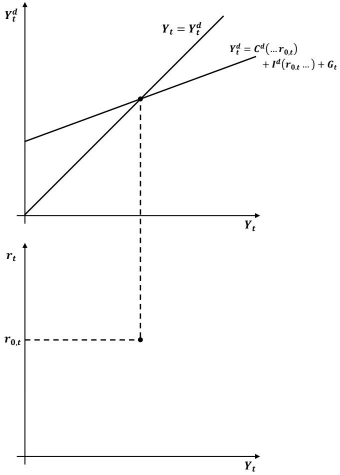
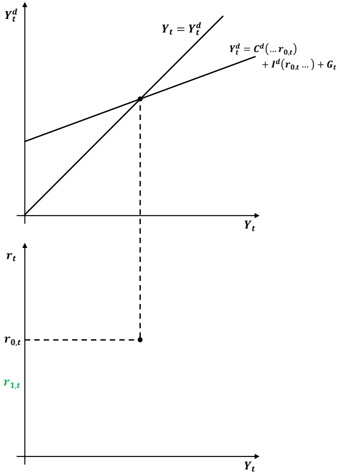
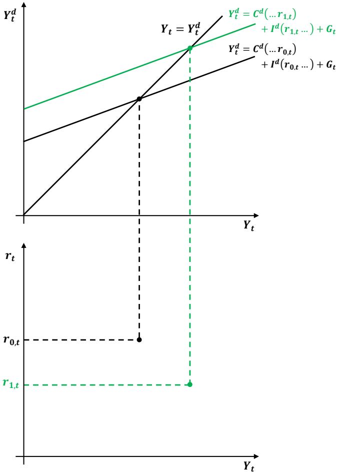
$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$
$+ I^d(r_t, A_{t+1}, K_t) + G_t$
Y_t^*
E_0
Y_t^*
Y_t


Y_t^d
$Y_t = Y_t^d$
$Y_t^d = C^d(\dots r_{0,t})$
$+ I^d(r_{0,t} \dots) + G_t$
Y_t
r_t
Y_t


Y_t^d
$Y_t = Y_t^d$
$Y_t^d = C^d(\dots r_{0,t})$
$+ I^d(r_{0,t} \dots) + G_t$
Y_t
r_t
$r_{0,t}$
Y_t

1. IS curve





Y_t^d
$Y_t = Y_t^d$
$Y_t^d = C^d(\dots r_{1,t})$
$+ I^d(r_{1,t} \dots) + G_t$
$Y_t^d = C^d(\dots r_{0,t})$
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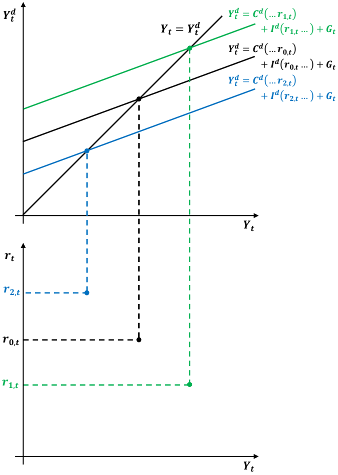
Y_t
r_t
$r_{0,t}$
$r_{1,t}$
Y_t



Y_t^d
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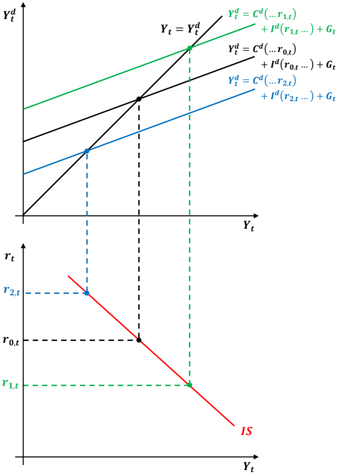
$Y_t^d = C^d(\dots r_{2,t}) + I^d(r_{2,t} \dots) + G_t$
Y_t
r_t
$r_{2,t}$
$r_{0,t}$
$r_{1,t}$
Y_t


Y_t^d
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$Y_t^d = C^d(\ldots r_{2,t})$
$+ I^d(r_{2,t} \ldots) + G_t$
Y_t
r_t
$r_{2,t}$
$r_{0,t}$
$r_{1,t}$
IS
Y_t

Question:
This exercise will ask you to work through the derivation of the IS curve under various different scenarios.

b) Suppose that investment demand is relatively more sensitive to the real interest rate than in (a). Relative to a), how will this impact the shape of the IS curve?

c) Suppose that the MPC is larger than in a) but still smaller than one. How will this affect the shape of the IS curve?

b:

Investment demand becomes more sensitive to changes in the real interest rate

- For every value of r_t, desire for investment I^d decreases $\Rightarrow$ autonomous expenditures E_0 decreases $\Rightarrow$ desired expenditure line Y_t^d shifts down

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b:

Investment demand becomes more sensitive to changes in the real interest rate

- For every value of r_t, desire for investment I^d decreases $\Rightarrow$ autonomous expenditures E_0 decreases $\Rightarrow$ desired expenditure line Y_t^d shifts down
- The higher r_t, the stronger is the effect on I^d
- IS curve becomes flatter than in a) $\Rightarrow$ changes in r_t lead to more pronounced changes in Y

1. IS curve

c:

MPC increases

- Less consumption smoothing

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- Slope of desired expenditure line Y_t^d becomes steeper

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- Slope of desired expenditure line Y_t^d becomes steeper
- And autonomous expenditures E_0 decreases slightly

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MPC increases

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- Slope of desired expenditure line Y_t^d becomes steeper
- And autonomous expenditures E_0 decreases slightly
- IS curve is flatter than in a)

Question:
In this question, you are asked to derive the Y^s curve again.

a) Graphically derive the Y^s curve for a generic specification of the aggregated production function, the labor supply curve, and the labor demand curve.

Supply side:

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$$N_t = N^s(w_t, \theta_t)$$
$$N_t = N^d(w_t, A_t, K_t)$$
$$Y_t = A_t F(K_t, N_t)$$

Supply side:

$$N_t = N^s(\underbrace{w_t}_{+}, \underbrace{\theta_t}_{-})$$
$$N_t = N^d(w_t, A_t, K_t)$$
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Supply side:

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- Both labor supply and demand function determine N_t together

Supply side:

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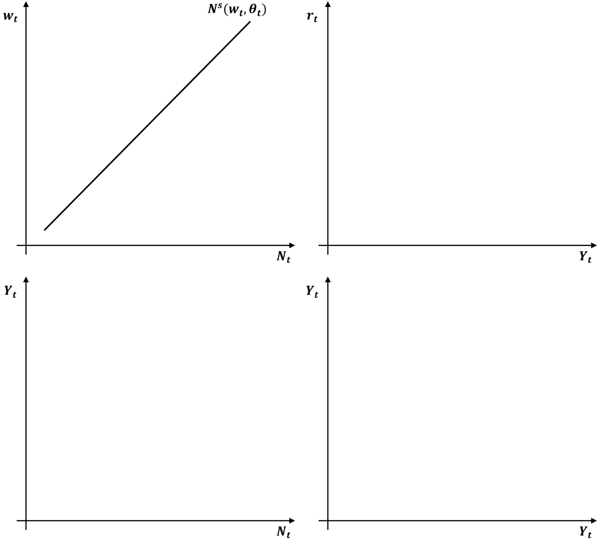
$$N_t = N^d(\underbrace{w_t}_{-}, \underbrace{A_t}_{+}, \underbrace{K_t}_{+})$$

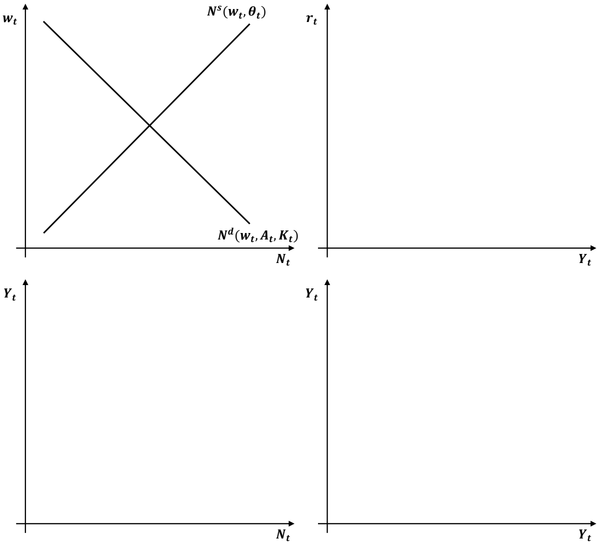
$$Y_t = \underbrace{A_t}_{+} F(\underbrace{K_t}_{+}, \underbrace{N_t}_{+})$$

- Both labor supply and demand function determine N_t together
- Given N_t, the production function determines Y_t

2. Y^s curve



2. Y^s curve



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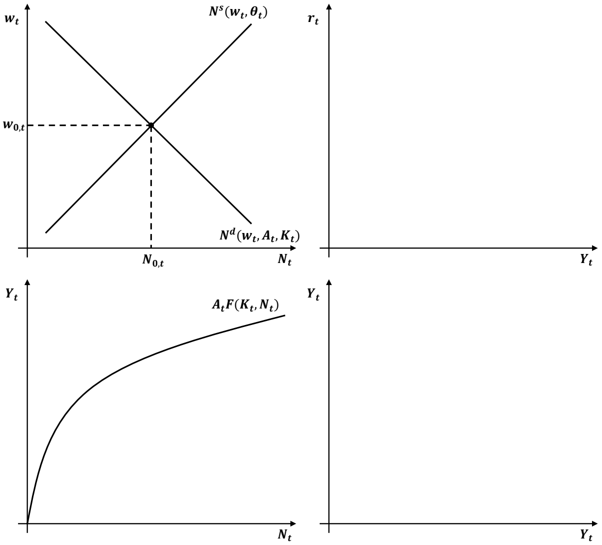
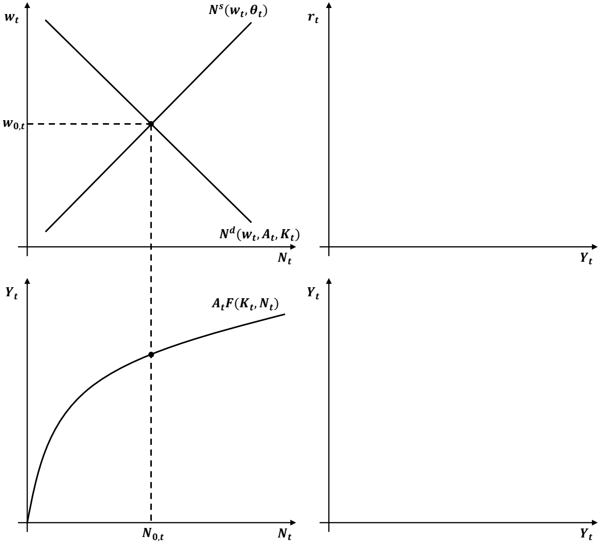
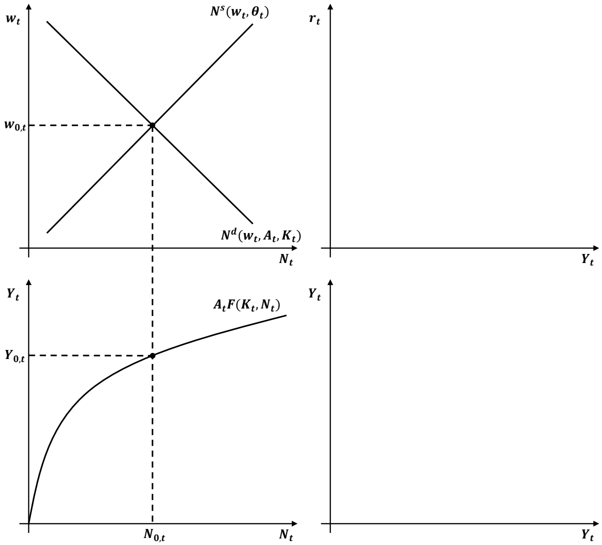




w_t
$N^s(w_t, \theta_t)$
$w_{0,t}$
$N^d(w_t, A_t, K_t)$
N_t
r_t
Y_t
Y_t
$A_tF(K_t, N_t)$
$N_{0,t}$
N_t
Y_t
Y_t


w_t
$N^s(w_t, \theta_t)$
$w_{0,t}$
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r_t
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$Y_{0,t}$
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N_t
Y_t
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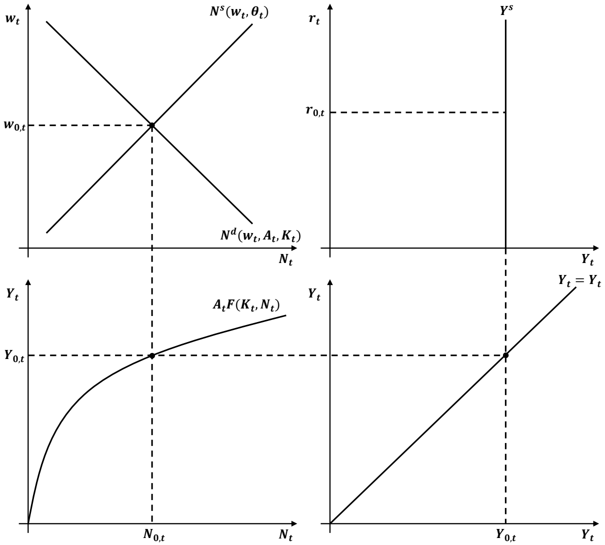
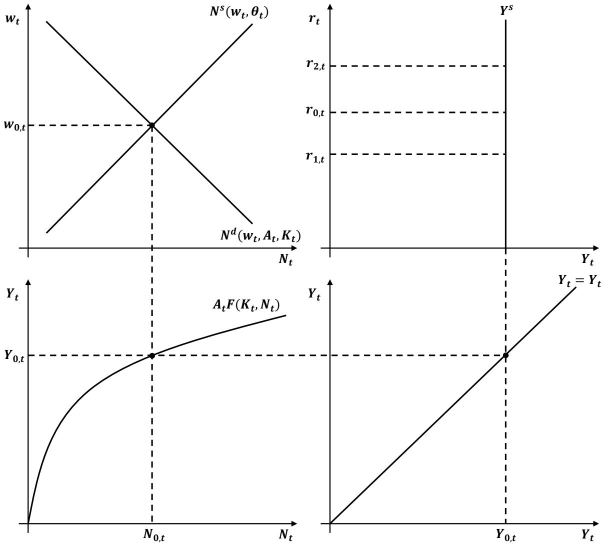

w_t
$N^s(w_t, \theta_t)$
$w_{0,t}$
$N^d(w_t, A_t, K_t)$
N_t
r_t
Y^s
$r_{0,t}$
Y_t
Y_t
$A_tF(K_t, N_t)$
$Y_{0,t}$
$N_{0,t}$
N_t
Y_t
$Y_t = Y_t$
$Y_{0,t}$
Y_t

2. Y^s curve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N_t
Y_t
$Y_t = Y_t$
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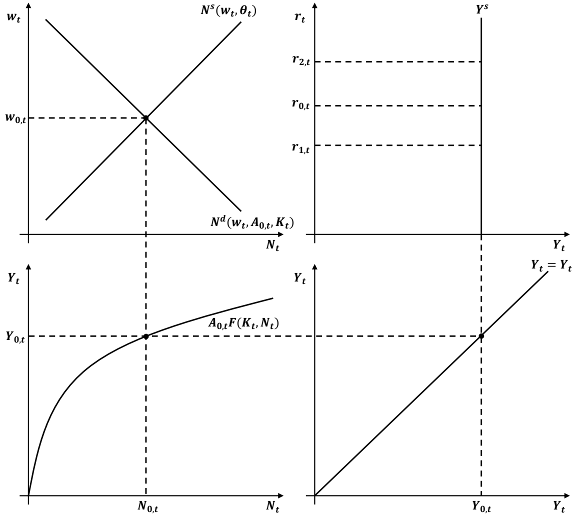
Y_t

Question:
In this question, you are asked to derive the Y^s curve again.

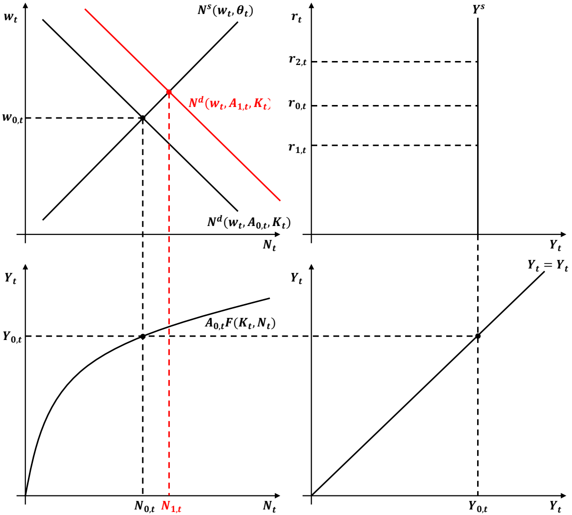
b) Show graphically and explain how an increase in the current productivity A_t affects the Y^s curve.

c) Show graphically and explain how an increase in the money supply M_t^s affects the Y^s curve.

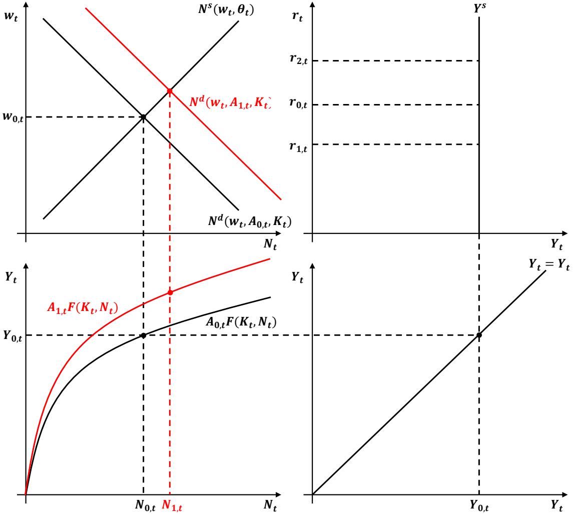
Increase in A_t: supply side shock



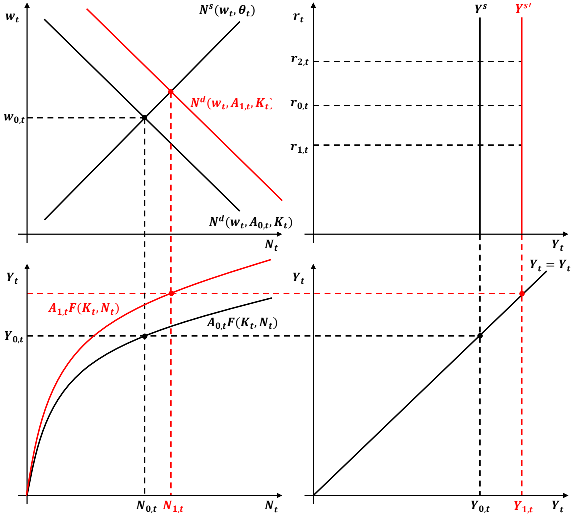
Increase in A_t: supply side shock



Increase in A_t: supply side shock



Increase in A_t: supply side shock



Increase in A_t: supply side shock

- Labor demand increases $\Rightarrow N^d$ curve shifts to the right

Increase in A_t: supply side shock

- Labor demand increases $\Rightarrow$ N^d curve shifts to the right
- N_t and w_t increase

Increase in A_t: supply side shock

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- Production function shifts up (for given K_t and N_t, Y_t increases)

Increase in A_t: supply side shock

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- Y^s curve goes to the right

Increase in A_t: supply side shock

- Labor demand increases $\Rightarrow N^d$ curve shifts to the right
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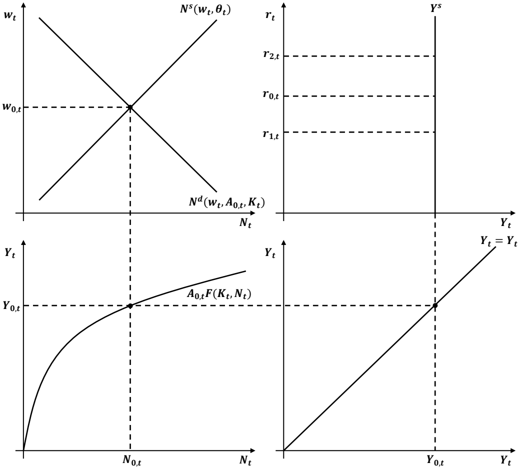
Increase in M_t:



Increase in M_t:

- No affect at all on Y^s or on any other real variable

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- Money neutrality in the mid- to long-run

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Question:
The neoclassical model is characterized by eight equations all simultaneously holding. In class you derived a graphical apparatus to characterize the equilibrium. Re-derive the equilibrium determined by the "real block" and by the "nominal block" graphically and explain the decision rules of each actor!

Real block

Real block

$$Y_t = C_t + I_t + G_t$$
$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$
$$I_t = I^d(r_t, A_{t+1}, K_t)$$

- IS curve: summary of (r_t, Y_t) for which the aggregate resource constraint holds where the households and firms choose C_t and I_t optimally.

Real block

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Y_t &= C_t + I_t + G_t \\
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I_t &= I^d(r_t, A_{t+1}, K_t) \\
N_t &= N^s(w_t, \theta_t) \\
N_t &= N^d(w_t, A_t, K_t) \\
Y_t &= A_t F(K_t, N_t)
\end{aligned}$$

- IS curve: summary of (r_t, Y_t) for which the aggregate resource constraint holds where the households and firms choose C_t and I_t optimally.
- Y^s curve: summary of (r_t, Y_t) for where labor demand and supply are optimally determined, consistent with the production technology

Government

- The government consumes some private output (G_t, G_{t+1}) and finances its spending with a mix of taxes (T_t, T_{t+1}) and by issuing debt (all exogenous to the model)

Government

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- *Ricardian Equivalence* holds in our model $\Rightarrow$ all that matters for the equilibrium behavior are G_t and G_{t+1}; the timings and amounts of T_t and T_{t+1} are irrelevant for decision making of agents (as is the level of debt issued by the government)

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- Given that households care only about the present value of its *net income*, and given that the government budget constraint has to be fulfilled, we can act as though the government balances its budget each period $(G_t = T_t$ and $G_{t+1} = T_{t+1})$

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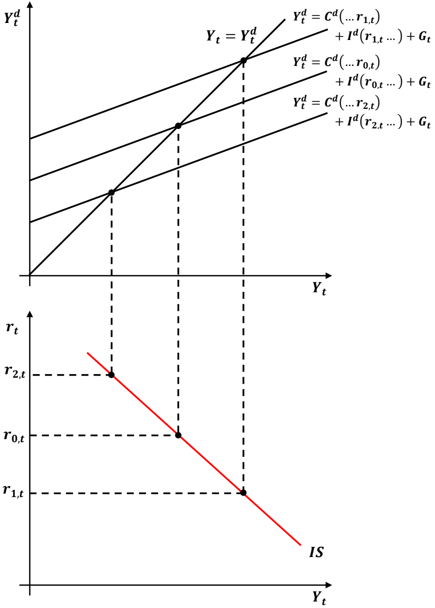
- Given that households care only about the present value of its *net income*, and given that the government budget constraint has to be fulfilled, we can act as though the government balances its budget each period ($G_t = T_t$ and $G_{t+1} = T_{t+1}$)



Y_t^d
$Y_t = Y_t^d$
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$Y_t^d = C^d(\dots r_{0,t}) + I^d(r_{0,t} \dots) + G_t$
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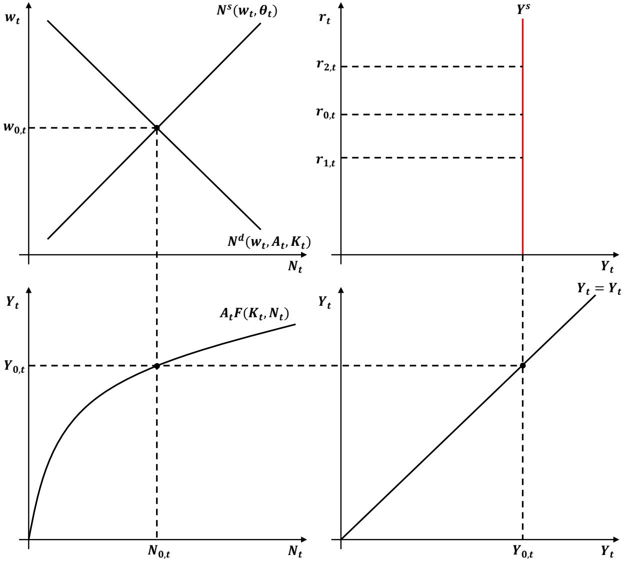
Y_t
r_t
$r_{2,t}$
$r_{0,t}$
$r_{1,t}$
IS
Y_t



w_t
$N^s(w_t, \theta_t)$
$w_{0,t}$
$N^d(w_t, A_t, K_t)$
N_t
r_t
Y^s
$r_{2,t}$
$r_{0,t}$
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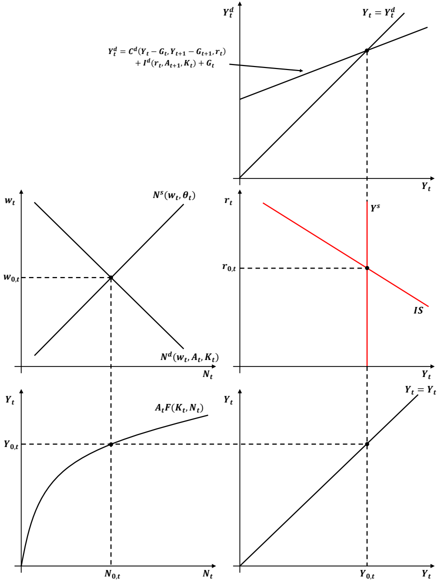
Y_t
Y_t
$A_t F(K_t, N_t)$
$Y_{0,t}$
$N_{0,t}$
N_t
Y_t
$Y_t = Y_t$
$Y_{0,t}$
Y_t



Y_t^d
$Y_t = Y_t^d$
$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) + I^d(r_t, A_{t+1}, K_t) + G_t$
Y_t
w_t
$N^s(w_t, \theta_t)$
$w_{0,t}$
$N^d(w_t, A_t, K_t)$
N_t
r_t
Y^s
$r_{0,t}$
IS
Y_t
Y_t
$A_t F(K_t, N_t)$
$Y_{0,t}$
$N_{0,t}$
N_t
Y_t
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Y_t

Nominal block

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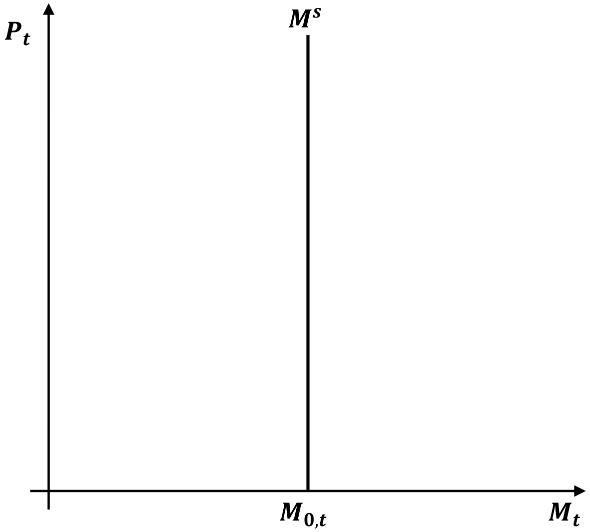
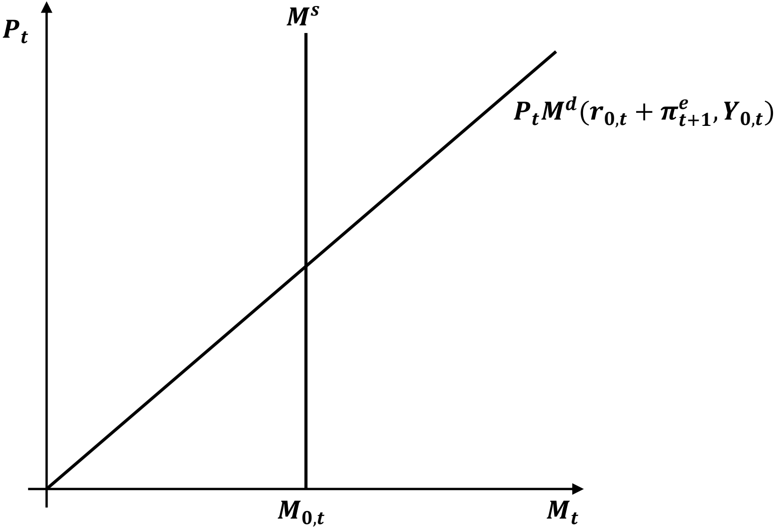
- Using the Fisher equation $r_t = i_t - \pi^e_{t+1}$ we get

$$M_t = P_t M^d(r_t + \pi^e_{t+1}, Y_t)$$


P_t
M^s
$M_{0,t}$
M_t


P_t
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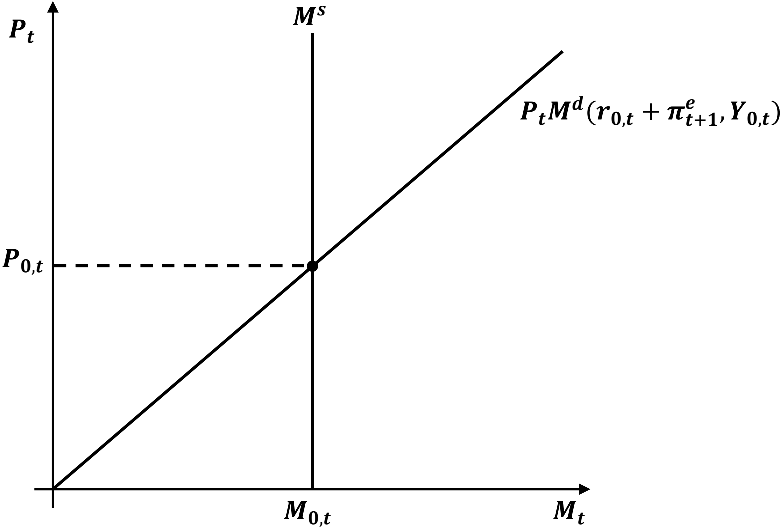

P_t
M^s
$P_t M^d(r_{0,t} + \pi^e_{t+1}, Y_{0,t})$
$P_{0,t}$
$M_{0,t}$
M_t

Question:
Suppose that we assume specific functional forms for the consumption function and the investment demand function. These are:

$$C_t = c_1(Y_t - G_t) + c_2(Y_{t+1} - G_{t+1}) - c_3 r_t$$
$$I_t = -d_1 r_t + d_2 A_{t+1} + d_3 K_t$$

Here, c_1 through c_4 and d_1 through d_3 are fixed parameters governing the sensitivity of consumption and investment to different factors relevant for those decisions.

a) We must have $Y_t = C_t + I_t + G_t$. Use the given functional forms for the consumption and investment with the resource constraint to derive an algebraic expression for the IS curve.

b) Use this to derive an expression for the slope of the IS curve (i.e. $\frac{\partial Y_t}{\partial r_t}$).

IS curve

IS curve

$$\begin{aligned} Y_t &= C_t + I_t + G_t \\ &= c_1(Y_t - G_t) + c_2(Y_{t+1} - G_{t+1}) - c_3 r_t \dots \\ &\qquad - d_1 r_t + d_2 A_{t+1} + d_3 K_t + G_t \\ Y_t(1 - c_1) &= (1 - c_1)G_t + c_2(Y_{t+1} - G_{t+1}) \dots \\ &\qquad + d_2 A_{t+1} + d_3 K_t - (c_3 + d_1) r_t \\ Y_t &= \frac{1}{1 - c_1}\Big((1 - c_1)G_t + c_2(Y_{t+1} - G_{t+1}) \dots \\ &\qquad + d_2 A_{t+1} + d_3 K_t - (c_3 + d_1) r_t\Big) \end{aligned}$$

IS curve

$$\begin{aligned} Y_t &= C_t + I_t + G_t \\ &= c_1(Y_t - G_t) + c_2(Y_{t+1} - G_{t+1}) - c_3 r_t \dots \\ &\qquad - d_1 r_t + d_2 A_{t+1} + d_3 K_t + G_t \end{aligned}$$

$$\begin{aligned} Y_t(1 - c_1) &= (1 - c_1)G_t + c_2(Y_{t+1} - G_{t+1}) \dots \\ &\qquad + d_2 A_{t+1} + d_3 K_t - (c_3 + d_1) r_t \end{aligned}$$

$$Y_t = \frac{1}{1 - c_1}\Big((1 - c_1)G_t + c_2(Y_{t+1} - G_{t+1}) \dots + d_2 A_{t+1} + d_3 K_t - (c_3 + d_1) r_t\Big)$$

Slope of the IS curve

$$\frac{\partial Y_t}{\partial r_t} = -\frac{c_3 + d_1}{1 - c_1}$$