Lehrstuhl für Informatik I
Algorithmen und
Komplexität
Universität Würzburg

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Exercise sheet 9

Visualization of Graphs

Exercise 1 – Visibility Representations

Let $R = [0, x] \times [0, y] \subset \mathbb{R}^2$ be an axis-parallel rectangle in the plane that contains a set S of pairwise disjoint horizontal line segments. Let the line segments $[0, x] \times \{0\}$ and $[0, x] \times \{y\}$ be contained in S.

We consider the ε-bar visibility graph $G = (S, E)$ with the set of directed edges

$$E = \{(u, v) \mid v \text{ is vertically upwards visible from } u\}.$$

a) Show that G is an st-graph. **2 Points**

b) Describe how we could derive an upward planar drawing of G from the given visibility representation. **5 Points**

Hint: A first step could be to transform the line segments into axis-parallel rectangles with small height.

Exercise 2 – Computing Coordinates for a Visibility Representation

We want to compute an ε-bar visibility representation ψ of an st-graph $G = (V, E)$. In addition to G, we are also given minimal (vertical) distances between pairs of bars corresponding to two adjacent vertices and a minimal width for every bar. More precisely, for a given edge-weight function $h\colon E \to \mathbb{R}_{\geq 0}$ and edge $(u, v) \in E$, the vertical distance of the bars $\psi(u)$ and $\psi(v)$ has to be at least $h((u, v))$. The function $w\colon V \to \mathbb{R}_{\geq 0}$ gives the minimal width for each bar $\psi(v)$ for $v \in V$.

a) Describe a linear-time algorithm that calculates the y-coordinates for the bars in the visibility representation with a minimum maximal height. Argue why your algorithm achieves this asymptotic runtime. **7 Points**

b) Show how we can use the algorithm for y-coordinates to compute the x-coordinates of each bar in the visibility representation with respect to w and such that we achieve a minimum maximal width for the whole representation. **6 Points**

   Hint: Consider the st-dual of G.

This assignment is due on July 14th at 10 am. Please submit your solutions via WueCampus. The exercises will be discussed in the tutorial session on July 18th at 16:15.