Lehrstuhl für Informatik I
Algorithmen und
Komplexität
Universität Würzburg

Würzburg, 30 June 2022

Prof. Dr. Alexander Wolff
Tim Hegemann
Diana Sieper

Exercise sheet 8
Visualisation of graphs

Exercise 1 – Minimum Feedback (Arc) Set

Let $G = (V, E)$ be a directed graph. For a set of edges $E' \subseteq E$, let $E'_r := \{vu \mid uv \in E'\}$ be the set of reversed edges. A set of edges $E^\star \subseteq E$ with minimal cardinality

- such that $D - E^\star$ is acyclic is called a Minimum Feedback Arc Set, and
- such that $D - E^\star + E^\star_r$ is acyclic is called a Minimum Feedback Set.

Show that $E^\star \subseteq E$ is a Minimum Feedback Set if and only if $E^\star$ is a Minimum Feedback Arc Set. **6 Points**

Exercise 2 – Optimal one-sided crossing minimization

We consider the problem of one-sided crossing minimization, i.e., we are given a bipartite graph $G = (L_1 \cup L_2, E)$ with a permutation π_1 of L_1 and we search for a permutation π_2 of L_2 that minimizes the number of crossings.

Suppose there exists a permutation $\pi_2^\star$ of L_2 such that no edges cross.

a) Show that in this case the *barycenter heuristic* also yields a permutation π'_2 that results in no crossings. **3 Points**

b) Show that in this case the *median heuristic* also yields a permutation π''_2 that results in no crossings. **3 Points**

Hint: Show how the barycenter heuristic and the median heuristic can handle the case when multiple vertices have the same barycenter or median, respectively. Think about when this can happen under the assumption that there exists a permutation such that no edges cross.

Exercise 3 – Planar Drawings

By iteratively applying the heuristics from the lecture, does one always find a crossing-free drawing of a graph (with more than two layers, where necessary) if the graph is upward planar? Justify your answer. **2 Points**

Exercise 4 – Precedence-Constrained Multi-Processor Scheduling

Give an infinite class of instances for which the $(2 - 1/W)$-approximation algorithm for the scheduling problem PRECEDENCE-CONSTRAINED MULTI-PROCESSOR SCHEDULING from the lecture yields schedules of length $(2 - 1/W)\,\mathrm{OPT}$. It suffices to consider a fixed value for W. **6 Points**

This assignment is due on July 7th at 10 am. Please submit your solutions via WueCampus. The exercises will be discussed in the tutorial session on July 11th at 16:15.