Lehrstuhl für Informatik I
Algorithmen und
Komplexität
Universität Würzburg

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Exercise sheet 5

Visualisation of graphs

Exercise 1 – Simple upward planar graphs

Prove or disprove that the following graph classes are upward planar:

a) directed acyclic graphs, whose underlying undirected graph is a simple cycle; **3 Points**

b) directed acyclic graphs, whose underlying undirected graph is a tree. **3 Points**

Exercise 2 – False friends

In the lecture we introduced three necessary conditions for a digraph G to be upward planar (planar, acylcic, bimodal). Show that these conditions are not sufficient. To do so find a directed graph that adheres to all three conditions and proove that it is *not* upward planar. **2 Points**

Exercise 3 – Refinement of the outerface

Let $G = (V, E)$ be a directed acylcic graph with given embedding, set of faces F, and outer face f_0. Let $\Phi \colon \mathcal{S} \cup \mathcal{T} \to F$ be a consistent assignment of the large angles of the sinks and sources to the incident faces. We consider the situation in which the inner faces have been refined already, i.e., there exist no large angles on the inner faces anymore.

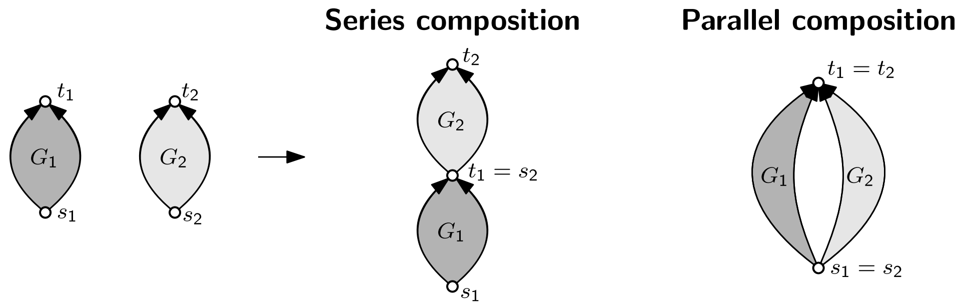
Show how you can complete the transformation into a planar st-graph by refining f_0. **4 Points**

Hint: A possible intermediate step could be to first refine f_0 such that the sources and sinks of the graph form two disjoint intervals on the boundary of the outer face.

Exercise 4 – Upwardplanar drawings of series-parallel graphs

A graph G with a source vertex s and a sink vertex t is series-parallel, if

- it contains a single directed edge (s, t), or
- it consists of two series-parallel graphs G_1, G_2 with sources s_1, s_2 and sinks t_1, t_2 that are combined such that
  - $t = t_2$, $t_1 = s_2$, and $s = s_1$ (*series composition*), or
  - $t = t_1 = t_2$, and $s = s_1 = s_2$ (*parallel composition*)



In contrast to general acyclic graphs, all series-parallel graphs have an upward planar drawing. Describe in words an algorithm that generates such drawings. Assume that the series-parallel graph to be drawn is given with its decomposition tree. In the output drawing, edges are allowed to have bends, but all bends and vertices must have integer coordinates.

a) Describe your algorithm. **4 Points**

b) Argue why your algorithm generates an upward planar drawing. **4 Points**

c) Estimate the area requirement of the drawings generated by your algorithm (subject to n, the number of vertices). **1 Bonus point**

d) What is the maximum number of bends one edge in the output drawing can have? Justify your answer. **1 Bonus point**

This assignment is due at the beginning of the next lecture, that is, on June 9th at 10 am. Please submit your solutions via WueCampus. The exercises will be discussed in the tutorial session on June 13th at 16:15.