Business Cycles
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Problem Set

Production and Labor Supply

1. Suppose that the household only lives for one period. The household's optimization problem is:

$$\max_{C_t,N_t} U = \ln C_t + \theta_t \frac{(1-N_t)^{1-\gamma}}{1-\gamma}$$
$$\text{s.t.} \quad C_t = w_t N_t$$

In this problem, the household receives no dividend from the firm.

a) Solve for the optimality condition characterizing the household problem.

b) From this optimality condition, what can you say about the effect of w_t on N_t? What is your explanation for this finding?

2. Suppose that you have a firm with a Cobb-Douglas production function for production in period t:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

The only twist relative to our setup in the main text is that the firm does not use labor to produce output in period $t+1$. The production function in that period is:

$$Y_{t+1} = A_{t+1} K_{t+1}^{\alpha}$$

a) Write down the optimization problem for the firm in this setup. It has to pay labor in period t w_t, and it discounts future dividends by $\frac{1}{1+r_t}$. It must borrow to finance its investment at r_t. The capital accumulation equation is standard.

b) What is the terminal condition for the firm? Explain the economic logic.

c) Using this specification of production, derive the first order optimality conditions for the optimal choices of N_t and K_{t+1}.

d) Re-arrange the first order optimality conditions to derive explicit expression for the demand for N_t and the demand for K_{t+1}.

e) Why is investment increasing in future productivity but is not affected by current productivity?

f) Re-arrange your answer from the previous part, making use of the capital accumulation equation, to solve for an expression for I_t.

3. Consider a representative agent with the utility function

$$U = \ln C_t + \theta \ln(1-N_t)$$

The budget constraint is

$$C_t + w_t(1-N_t) = w_t + \Pi_t$$

where w_t is the wage and Π_t is non-wage income.

a) Solve for the optimal quantities of consumption and labor.

b) Suppose that the government implements a lump sum subsidy to all workers. The budget constraint is now

$$C_t + w_t(1 - N_t) = w_t + \Pi_t + T_t$$

How are the optimal quantities of C_t and N_t affected by the introduction of the subsidy? Specifically, do people consume more or less leisure? What is the economic intuition for this?

c) Instead of a lump sum subsidy, suppose the government subsidies work. With the subsidy, the workers receive an effective wage rate of $w_t(1+\tau_t)$. The budget constraint is

$$C_t + w_t(1+\tau_t)(1 - N_t) = w_t(1+\tau_t) + \Pi_t$$

How are the optimal quantities of C and N affected by the introduction of the subsidy? Specifically, do people consume more or less leisure? What is the economic intuition for this?

4. Ricardian Equivalence

a) Explain the extent you agree with this statement: Ricardian Equivalence shows that government deficits do not matter.

b) List the assumptions of the Ricardian Equivalence theorem.

5. Further (voluntary) homework:
Suppose the economy described in question 2 and create an Excel file. Assume the following values for exogenous parameters: $\alpha = 1/3$, $\delta = 0.1$, $A_t = 1$, $A_{t+1} = 1$, and $K_t = 2$.

a) Create a column of possible values of w_t, ranging from a low of 1 to a high of 1.5, with a step of 0.01 between entries (i.e. create a column going from 1 to 1.01 to 1.02 all the way to 1.5). For each possible value of w_t, solve for a numeric value of N_t. Plot w_t against the optimal value of N_t. Does the resulting demand curve for labor qualitatively look like Figure 12.2 in the book?

b) Suppose that A_t increases to 1.1. Re-calculate the optimal value of N_t for each value of w_t. Plot the resulting N_t values against w_t in the same plot as what you did on the previous part. What does the increase in A_t do to the position of the labor demand curve?

c) Go back to assuming the parameter and exogenous values we started with. Create a grid of values of r_t ranging from a low 0.02 to a high of 0.1, with a space of 0.001 between (i.e. create a column going from 0.020, to 0.021, to 0.022, and so on). For each value of r_t, solve for the optimal level of I_t. Create a graph with r_t on the vertical axis and I_t on the horizontal axis. Plot this graph. Does it qualitatively look like Figure 12.3 in the book?

d) Suppose that A_{t+1} increases to 1.1. For each value of r_t, solve for the optimal I_t. Plot this in the same figure as on the previous part. What does the increase in A_{t+1} do to the position of the investment demand curve?