Business Cycles

- Exercise 3 -

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Equilibrium in an Endowment Economy

Question:
Suppose the economy is populated by many identical agents. These agents act as price takers and take current and future income as given. They live for two periods: t and $t+1$. They solve a standard consumption-savings problem which yields a consumption function

$$C_t = C(Y_t, Y_{t+1}, r_t).$$

a) What are the signs of the partial derivative of the consumption function? Explain the economic intuition.

b) Suppose there is an increase in Y_t holding Y_{t+1} and r_t fixed. How does the consumer want to adjust its consumption and saving? Explain the economic intuition.

c) Suppose there is an increase in Y_{t+1} holding Y_t and r_t fixed. How does the consumer want to adjust its consumption and saving? Explain the economic intuition.

a:

Consumption Function

$$C_t = C(Y_t, Y_{t+1}, r_t)$$

$$\frac{\partial C_t}{\partial Y_t} = ?$$
$$\frac{\partial C_t}{\partial Y_{t+1}} = ?$$
$$\frac{\partial C_t}{\partial r_t} = ?$$

a:

Consumption Function

$$C_t = C(Y_t, Y_{t+1}, r_t)$$

$$\frac{\partial C_t}{\partial Y_t} > 0 \Rightarrow \textbf{M}\text{arginal } \textbf{P}\text{ropensity to } \textbf{C}\text{onsum}$$

$$\frac{\partial C_t}{\partial Y_{t+1}} > 0 \Rightarrow \text{consumption smoothing}$$

$$\frac{\partial C_t}{\partial r_t} < 0 \Rightarrow \text{intertemporal price of goods}$$

b:

ceteris paribus: $Y_t \uparrow$

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- households would like to increase current consumption

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ceteris paribus: $Y_t \uparrow$

- households would like to increase current consumption
- but MPC< 1 (consumption smoothing: $\frac{\partial C_{t+1}}{\partial Y_t} > 0$)

c:

ceteris paribus: $Y_{t+1} \uparrow$

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ceteris paribus: $Y_{t+1} \uparrow$

- households would like to smooth consumption stream
- $\dfrac{\partial C_t}{\partial Y_{t+1}} > 0$

Question:
Suppose the economy is populated by many identical agents. These agents act as price takers and take current and future income as given. They live for two periods: t and $t+1$. They solve a standard consumption-savings problem which yields a consumption function

$$C_t = C(Y_t, Y_{t+1}, r_t).$$

d) Now let's go to equilibrium. What is the generic definition of a competitive equilibrium?

e) Define the IS curve and graphically derive it.

d:

Competitive Equilibrium

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- here: r_t is the intertemporal price of goods and households choose the optimal quantities $C_t(j)$ and $C_{t+1}(j)$

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Competitive Equilibrium

- set of prices and quantities for which all agents are **behaving optimally** and all markets **simultaneously clear**
- here: r_t is the intertemporal price of goods and households choose the optimal quantities $C_t(j)$ and $C_{t+1}(j)$
- endowment economy: Y_t and Y_{t+1} are given

d:

Competitive Equilibrium $\Rightarrow$ endowment economy

- Each household decides optimally upon consumption

$$C_t(j) = C^d(Y_t(j), Y_{t+1}(j), r_t)$$

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Competitive Equilibrium $\Rightarrow$ endowment economy

- Each household decides optimally upon consumption

$$C_t(j) = C^d(Y_t(j), Y_{t+1}(j), r_t)$$

- In the aggregate: $C_t = Y_t$

$\Rightarrow$ r_t has to adjust accordingly

e:
IS-Curve

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- set of (r_t, Y_t) pairs where income equals expenditure and the households behave optimally

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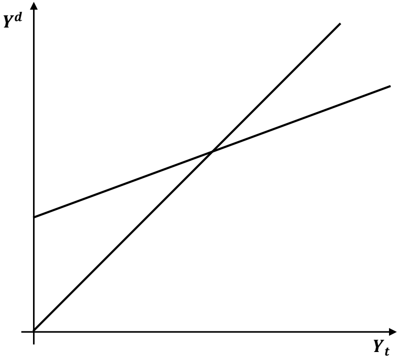

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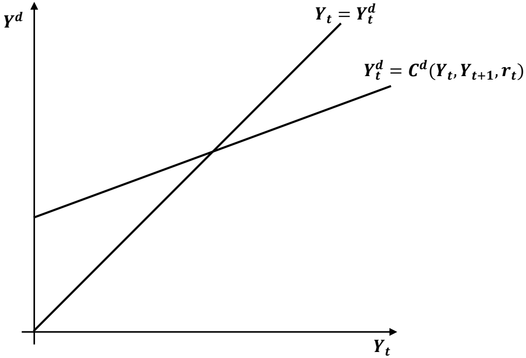

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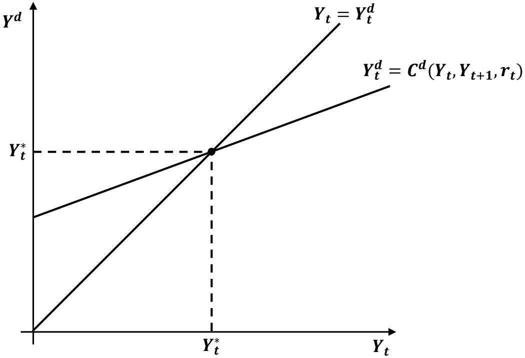

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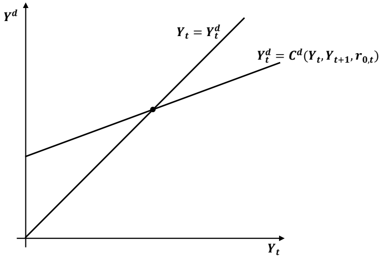

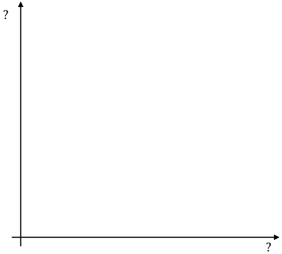

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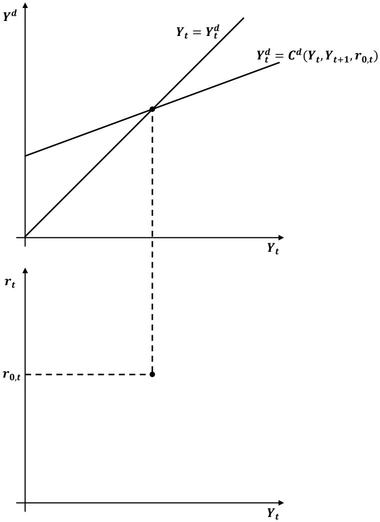

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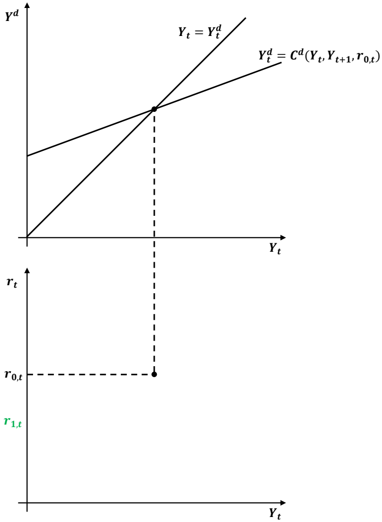

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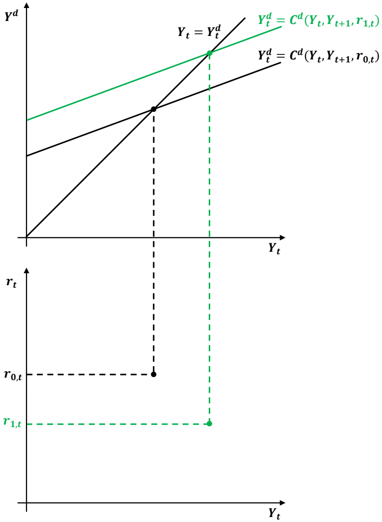

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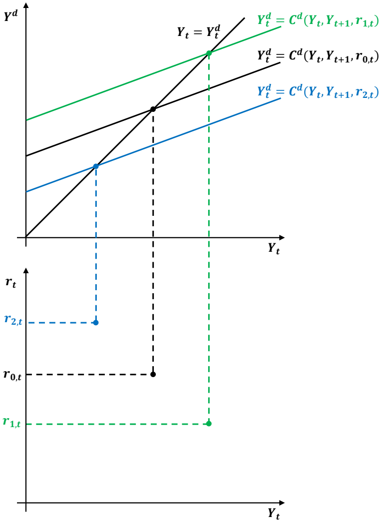
Y^d
$Y_t = Y_t^d$
$Y_t^d = C^d(Y_t, Y_{t+1}, r_{1,t})$
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Y_t
r_t
$r_{2,t}$
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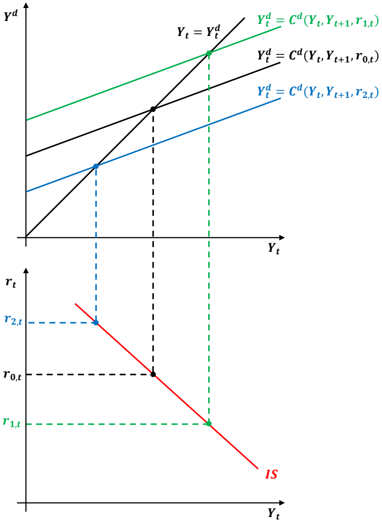
$r_{1,t}$
Y_t

e: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Y_t
r_t
$r_{2,t}$
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IS
Y_t

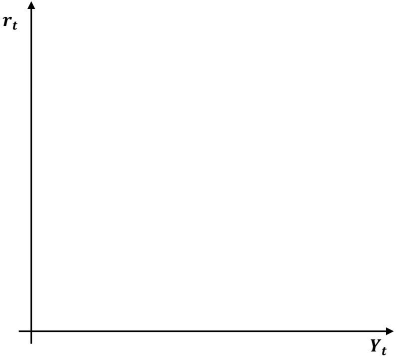
Question:
Suppose the economy is populated by many identical agents. These agents act as price takers and take current and future income as given. They live for two periods: t and $t+1$. They solve a standard consumption-savings problem which yields a consumption function

$$C_t = C(Y_t, Y_{t+1}, r_t).$$

f) Graph the Y^s curve with the IS curve and show how you determine the real interest rate.

g) Suppose there is an increase in Y_t. Show how this affects the equilibrium real interest rate. Explain the economic intuition for this.

f:



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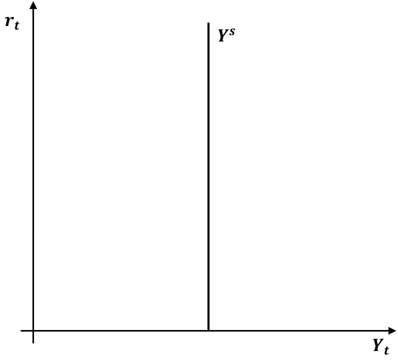

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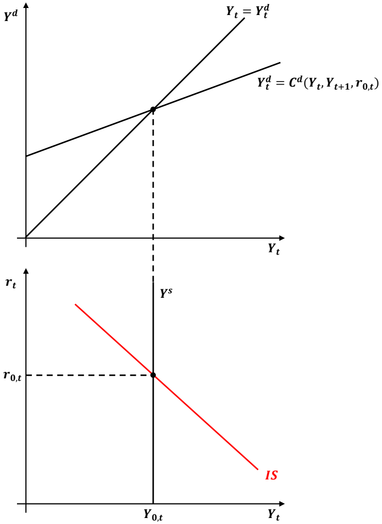
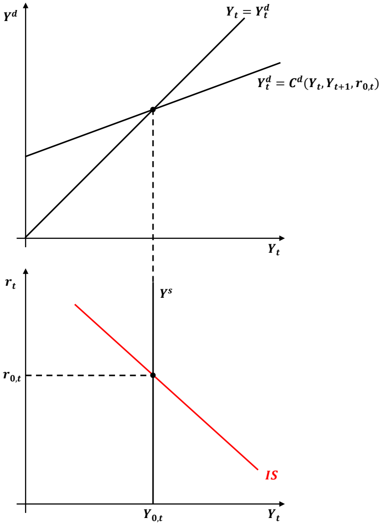


g:



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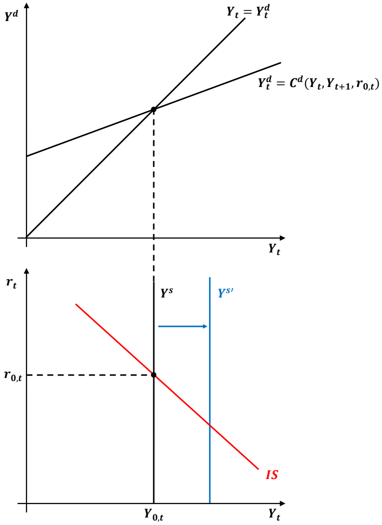

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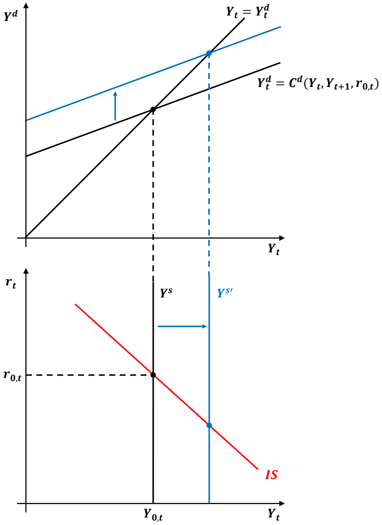

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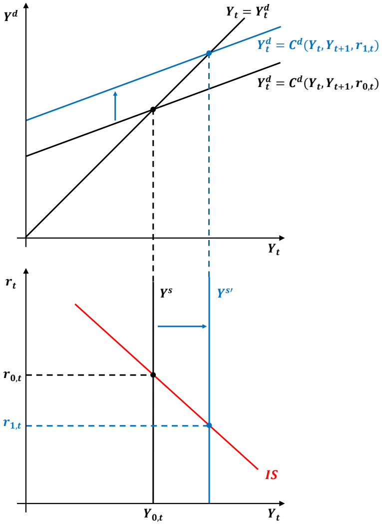

Y^d
$Y_t = Y_t^d$
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Y_t
r_t
Y^s
$Y^{s'}$
$r_{0,t}$
$r_{1,t}$
IS
$Y_{0,t}$
Y_t

Question:
Suppose that there exist many identical households in an economy. The representative household has the following lifetime utility function:

$$U = \ln C_t + \beta \ln C_{t+1}$$

It faces a sequence of period budget constraints which can be combined into one intertemporal budget constraint:

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

The endowment, Y_t and Y_{t+1}, is exogenous, and the household takes the real interest rate as given.

a) Derive the consumption function for the representative household.
b) Solve for expressions for the equilibrium values of r_t.
c) How does r_t react to changes in Y_t and Y_{t+1}. What is the economic intuition for this?

a:
Consumption function:

2. General Equilibrium - Analytically

a:
Consumption function:

1. Derive Euler equation.

a:
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$$\frac{C_{t+1}}{C_t} = \beta(1 + r_t)$$

2. General Equilibrium - Analytically

a:
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2. Plug Euler equation in budget constraint and solve for C_t.

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2. Plug Euler equation in budget constraint and solve for C_t.

$$C_t + \frac{\beta(1 + r_t)C_t}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}$$

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2. Plug Euler equation in budget constraint and solve for C_t.

$$C_t + \frac{\beta(1 + r_t)C_t}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}$$

$$C_t = \frac{1}{1 + \beta} Y_t + \frac{1}{1 + \beta} \frac{Y_{t+1}}{1 + r_t}$$

b:
Total desired expenditures:

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$$Y_t^d = C_t$$
$$Y_t^d = \frac{1}{1+\beta} Y_t + \frac{1}{1+\beta} \frac{Y_{t+1}}{1+r_t}$$

Equilibrium:

b:
Total desired expenditures:

$$Y_t^d = C_t$$
$$Y_t^d = \frac{1}{1+\beta} Y_t + \frac{1}{1+\beta} \frac{Y_{t+1}}{1+r_t}$$

Equilibrium: $Y_t = Y_t^d$

$$Y_t = Y_t^d$$
$$Y_t = \frac{1}{1+\beta} Y_t + \frac{1}{1+\beta} \frac{Y_{t+1}}{1+r_t}$$
$$1 + r_t = \frac{1}{\beta} \frac{Y_{t+1}}{Y_t}$$

Question:
Suppose that we have an economy with many identical households. There is a government that exogenously consumes some output and pays for it with lump sum taxes. Lifetime utility for a household is:

$$U = \ln C_t + \beta \ln C_{t+1}$$

The household faces two within period budget constraints given by:

$$C_t + S_t = Y_t - T_t$$
$$C_{t+1} = Y_{t+1} - T_{t+1} + (1 + r_t)S_t$$

a) Derive the intertemporal budget constraint and the Euler equation. Is the Euler equation at all affected by the presence of taxes, T_t and T_{t+1}? Use your results to derive an expression for the consumption function.

Budget constraint:

$$C_t + S_t = Y_t - T_t$$

$$C_{t+1} = Y_{t+1} - T_{t+1} + (1 + r_t)S_t$$

Budget constraint:

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$$S_t = \frac{C_{t+1}}{1 + r_t} - \frac{Y_{t+1}}{1 + r_t} + \frac{T_{t+1}}{1 + r_t}$$

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$$C_t + \frac{C_{t+1}}{1 + r_t} - \frac{Y_{t+1}}{1 + r_t} + \frac{T_{t+1}}{1 + r_t} = Y_t - T_t$$

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t - T_t + \frac{Y_{t+1} - T_{t+1}}{1 + r_t}$$

Budget constraint:

$$C_t + S_t = Y_t - T_t$$

$$C_{t+1} = Y_{t+1} - T_{t+1} + (1 + r_t)S_t$$

$$S_t = \frac{C_{t+1}}{1 + r_t} - \frac{Y_{t+1}}{1 + r_t} + \frac{T_{t+1}}{1 + r_t}$$

$$C_t + \frac{C_{t+1}}{1 + r_t} - \frac{Y_{t+1}}{1 + r_t} + \frac{T_{t+1}}{1 + r_t} = Y_t - T_t$$

$$C_t + \frac{C_{t+1}}{1 + r_t} = \underbrace{Y_t - T_t + \frac{Y_{t+1} - T_{t+1}}{1 + r_t}}_{\text{PV of the stream of net income}}$$

Euler equation:

$$\max_{C_t, C_{t+1}} U = \ln C_t + \beta \ln C_{t+1}$$

$$\text{s.t.} \quad C_t + \frac{C_{t+1}}{1+r_t} = Y_t - T_t + \frac{Y_{t+1} - T_{t+1}}{1+r_t}$$

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$$\text{s.t.} \quad C_t + \frac{C_{t+1}}{1+r_t} = Y_t - T_t + \frac{Y_{t+1} - T_{t+1}}{1+r_t}$$

$$\frac{C_{t+1}}{C_t} = \beta(1+r_t)$$

Consumption function:

Consumption function:

$$C_t = Y_t - T_t + \frac{Y_{t+1} - T_{t+1}}{1 + r_t} - \frac{C_{t+1}}{1 + r_t}$$

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Consumption function:

$$C_t = Y_t - T_t + \frac{Y_{t+1} - T_{t+1}}{1 + r_t} - \frac{C_{t+1}}{1 + r_t}$$

$$= Y_t - T_t + \frac{Y_{t+1} - T_{t+1}}{1 + r_t} - \frac{\beta(1 + r_t)C_t}{1 + r_t}$$

$$C_t = \frac{1}{1 + \beta}(Y_t - T_t) + \frac{1}{1 + \beta}\left(\frac{Y_{t+1} - T_{t+1}}{1 + r_t}\right)$$

Question:
Suppose that we have an economy with many identical households. There is a government that exogenously consumes some output and pays for it with lump sum taxes.

...

The government faces two within period budget constraints:

$$G_t + S_t^G = T_t$$
$$G_{t+1} = T_{t+1} + (1 + r_t)S_t^G$$

b) Combine the two period budget constraints for the government into one intertemporal budget constraint.

c) Suppose that the representative household knows that the government's intertemporal budget constraint must hold. Combine this information with the household's consumption function you derived above. What happens to T_t and T_{t+1}? What is your intuition for this?

b:

Government budget:

$$G_t + S_t^G = T_t$$

$$G_{t+1} = T_{t+1} + (1 + r_t)S_t^G$$

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Government budget:

$$G_t + S_t^G = T_t$$

$$G_{t+1} = T_{t+1} + (1 + r_t) S_t^G$$

$$S_t^G = \frac{G_{t+1}}{1 + r_t} - \frac{T_{t+1}}{1 + r_t}$$

b:

Government budget:

$$G_t + S_t^G = T_t$$

$$G_{t+1} = T_{t+1} + (1 + r_t)S_t^G$$

$$S_t^G = \frac{G_{t+1}}{1 + r_t} - \frac{T_{t+1}}{1 + r_t}$$

$$G_t + \frac{G_{t+1}}{1 + r_t} = T_t + \frac{T_{t+1}}{1 + r_t}$$

c:

Consumption function:

$$C_t = \frac{1}{1+\beta}(Y_t - T_t) + \frac{1}{1+\beta}\left(\frac{Y_{t+1} - T_{t+1}}{1+r_t}\right)$$

c:

Consumption function:

$$C_t = \frac{1}{1+\beta}(Y_t - T_t) + \frac{1}{1+\beta}\left(\frac{Y_{t+1} - T_{t+1}}{1+r_t}\right)$$

$\Rightarrow$ Households know that the present discounted value of tax payments must equal the present discounted value of government spending

c:

Consumption function:

$$C_t = \frac{1}{1+\beta}(Y_t - T_t) + \frac{1}{1+\beta}\left(\frac{Y_{t+1} - T_{t+1}}{1+r_t}\right)$$

⇒ Households know that the present discounted value of tax payments must equal the present discounted value of government spending

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$$C_t = \frac{1}{1+\beta}(Y_t - G_t) + \frac{1}{1+\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right)$$

⇒ Households' perspective: government balances its budget each period

Question:
Suppose that we have an economy with many identical households. There is a government that exogenously consumes some output and pays for it with lump sum taxes.

...

The government faces two within period budget constraints:

$$G_t + S_t^G = T_t$$
$$G_{t+1} = T_{t+1} + (1 + r_t)S_t^G$$

d) Equilibrium requires that $Y_t = C_t + G_t$. Plug in your expression for the consumption function (assuming that the household knows the government's intertemporal budget constraint must hold) to derive an expression for Y_t.

e) Assuming that Y_t is exogenous, what must happen to r_t after an increase in G_t?

d:

Equilibrium:

$$Y_t = C_t + G_t$$

d:

Equilibrium:

$$Y_t = C_t + G_t$$

$$Y_t = \frac{1}{1+\beta}(Y_t - G_t) + \frac{1}{1+\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right) + G_t$$

d:

Equilibrium:

$$Y_t = C_t + G_t$$

$$Y_t = \frac{1}{1+\beta}(Y_t - G_t) + \frac{1}{1+\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right) + G_t$$

$$Y_t = G_t + \frac{1}{\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right)$$

d:

Equilibrium:

$$Y_t = C_t + G_t$$

$$Y_t = \frac{1}{1+\beta}(Y_t - G_t) + \frac{1}{1+\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right) + G_t$$

$$Y_t = G_t + \frac{1}{\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right)$$

Real interest rate:

d:

Equilibrium:

$$Y_t = C_t + G_t$$
$$Y_t = \frac{1}{1+\beta}(Y_t - G_t) + \frac{1}{1+\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right) + G_t$$
$$Y_t = G_t + \frac{1}{\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right)$$

Real interest rate:

$$1 + r_t = \frac{1}{\beta}\left(\frac{Y_{t+1} - G_{t+1}}{Y_t - G_t}\right)$$

d:

Equilibrium:

$$Y_t = C_t + G_t$$

$$Y_t = \frac{1}{1+\beta}(Y_t - G_t) + \frac{1}{1+\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right) + G_t$$

$$Y_t = G_t + \frac{1}{\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right)$$

Real interest rate:

$$1 + r_t = \frac{1}{\beta}\left(\frac{Y_{t+1} - G_{t+1}}{Y_t - G_t}\right)$$

e: $\frac{\partial(1+r_t)}{\partial G_t}$

d:

Equilibrium:

$$Y_t = C_t + G_t$$

$$Y_t = \frac{1}{1+\beta}(Y_t - G_t) + \frac{1}{1+\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right) + G_t$$

$$Y_t = G_t + \frac{1}{\beta}\left(\frac{Y_{t+1} - G_{t+1}}{1+r_t}\right)$$

Real interest rate:

$$1 + r_t = \frac{1}{\beta}\left(\frac{Y_{t+1} - G_{t+1}}{Y_t - G_t}\right)$$

e: $\frac{\partial(1+r_t)}{\partial G_t} > 0$

Question:
Suppose that we have an economy with many identical households. There is a government that exogenously consumes some output and pays for it with lump sum taxes.

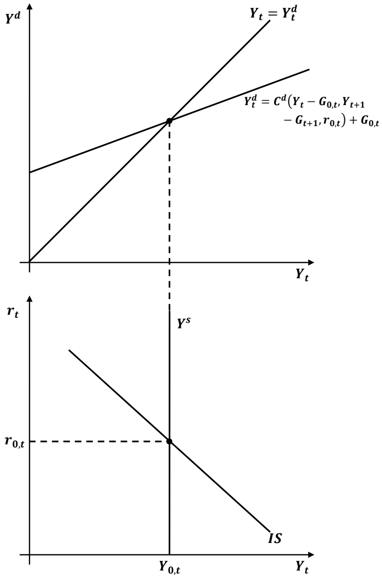
...

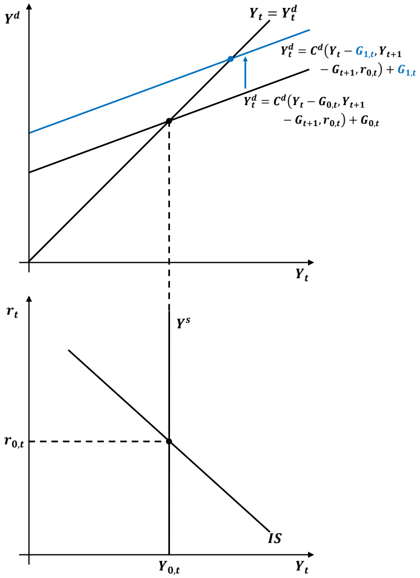
The government faces two within period budget constraints:

$$G_t + S_t^G = T_t$$
$$G_{t+1} = T_{t+1} + (1 + r_t)S_t^G$$

f) Graphically analyze an increase in G_t in an endowment economy. Clearly explain the economic intuition.

3. Fiscal Policy: Equilibrium




Y^d
$Y_t = Y_t^d$
$Y_t^d = C^d(Y_t - G_{1,t}, Y_{t+1} - G_{t+1}, r_{0,t}) + G_{1,t}$
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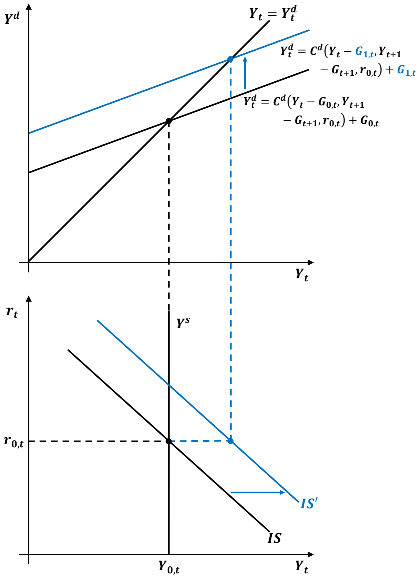
Y_t
r_t
Y^s
$r_{0,t}$
IS
$Y_{0,t}$
Y_t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Y_t
r_t
Y^s
$r_{0,t}$
IS'
IS
$Y_{0,t}$
Y_t

3. Fiscal Policy: Equilibrium