Business Cycles

- Exercise 2 -

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The Consumption-Saving Model

Question:
Write down the Euler equation in general terms and describe its economic intuition.

Consumption Euler equation

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1})$$
$$\frac{u'(C_t)}{u'(C_{t+1})} = \beta(1 + r_t)$$

Derivation:

$$\max_{C_t, C_{t+1}} U = u(C_t) + \beta u(C_{t+1})$$
$$\text{s.t.} \quad C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}$$

Question:
Write down the Euler equation in general terms and describe its economic intuition.

Consumption Euler equation

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1})$$
$$\frac{u'(C_t)}{u'(C_{t+1})} = \beta(1 + r_t)$$

Intuition:

> At an optimum, the household picks C_t and C_{t+1} so that the marginal utility of period t consumption, $u(C_t)$, equals the marginal utility of period $t + 1$ consumption, $u(C_{t+1})$, multiplied by the gross real interest rate

Question:
Consider a consumer with a lifetime utility function

$$U = \ln C_t + \beta \ln C_{t+1}$$

The period t and $t+1$ budget constraints are

$$C_t + S_t = Y_t$$
$$C_{t+1} + S_{t+1} = Y_{t+1} + (1+r)S_t$$

a) What is the optimal value of S_{t+1}? Impose this optimal value and derive the lifetime budget constraint.

b) Derive the Euler equation.

2. Consumption smoothing: budget constraint

General budget constraint:

$$C_t + S_t = Y_t \tag{1}$$

$$C_{t+1} + S_{t+1} = Y_{t+1} + (1+r)S_t \tag{2}$$

Rearrange (2):

$$S_t = \frac{C_{t+1}}{1+r} + \frac{S_{t+1}}{1+r} - \frac{Y_{t+1}}{1+r} \tag{3}$$

$S_{t+1} = 0$ **[a) Recall why!)]**

Plugging (3) into (1) and rearrange yields:

$$C_t + \frac{C_{t+1}}{1+r} = Y_t + \frac{Y_{t+1}}{1+r}$$

$$\underbrace{C_t + \frac{C_{t+1}}{1+r}}_{\text{PV of consumption}} = \underbrace{Y_t + \frac{Y_{t+1}}{1+r}}_{\text{PV of income}}$$

b:

$$\max_{C_t, C_{t+1}} U = \ln C_t + \beta \ln C_{t+1}$$

$$\text{s.t.} \quad C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

Lagrange: $\mathcal{L} =$

b:

$$\max_{C_t, C_{t+1}} U = \ln C_t + \beta \ln C_{t+1}$$
$$\text{s.t.} \quad C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

Lagrange:
$$\mathcal{L} = \ln C_t + \beta \ln C_{t+1} + \lambda \left(Y_t + \frac{Y_{t+1}}{1+r_t} - C_t - \frac{C_{t+1}}{1+r_t} \right)$$

$$\frac{\partial \mathcal{L}}{\partial C_t} =$$

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$$\frac{\partial \mathcal{L}}{\partial C_t} = \frac{1}{C_t} - \lambda = 0 \Rightarrow \lambda = \frac{1}{C_t}$$

$$\frac{\partial \mathcal{L}}{\partial C_{t+1}} =$$

b:

$$\max_{C_t, C_{t+1}} U = \ln C_t + \beta \ln C_{t+1}$$
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$$\frac{\partial \mathcal{L}}{\partial C_t} = \frac{1}{C_t} - \lambda = 0 \Rightarrow \lambda = \frac{1}{C_t}$$

$$\frac{\partial \mathcal{L}}{\partial C_{t+1}} = \frac{\beta}{C_{t+1}} - \lambda \frac{1}{1+r} = 0 \Rightarrow \lambda = \frac{\beta(1+r)}{C_{t+1}}$$

b:

$$\max_{C_t, C_{t+1}} U = \ln C_t + \beta \ln C_{t+1}$$

$$\text{s.t.} \quad C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

Lagrange:

$$\mathcal{L} = \ln C_t + \beta \ln C_{t+1} + \lambda \left(Y_t + \frac{Y_{t+1}}{1+r_t} - C_t - \frac{C_{t+1}}{1+r_t} \right)$$

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$$\frac{\partial \mathcal{L}}{\partial C_{t+1}} = \frac{\beta}{C_{t+1}} - \lambda \frac{1}{1+r} = 0 \Rightarrow \lambda = \frac{\beta(1+r)}{C_{t+1}}$$

$$\frac{1}{(1+r)C_t} = \frac{\beta}{C_{t+1}} \Rightarrow$$

b:

$$\max_{C_t, C_{t+1}} U = \ln C_t + \beta \ln C_{t+1}$$

$$\text{s.t.} \quad C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

Lagrange:

$$\mathcal{L} = \ln C_t + \beta \ln C_{t+1} + \lambda \left(Y_t + \frac{Y_{t+1}}{1+r_t} - C_t - \frac{C_{t+1}}{1+r_t} \right)$$

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$$\frac{\partial \mathcal{L}}{\partial C_{t+1}} = \frac{\beta}{C_{t+1}} - \lambda \frac{1}{1+r} = 0 \Rightarrow \lambda = \frac{\beta(1+r)}{C_{t+1}}$$

$$\frac{1}{C_t} = \frac{\beta(1+r)}{C_{t+1}} \Rightarrow \frac{C_{t+1}}{\beta C_t} = (1+r)$$

Question:
Consider a consumer with a lifetime utility function

$$U = \ln C_t + \beta \ln C_{t+1}$$

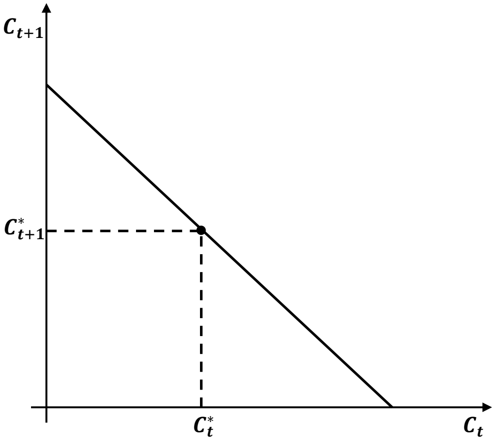
The period t and $t+1$ budget constraints are

$$C_t + S_t = Y_t$$
$$C_{t+1} + S_{t+1} = Y_{t+1} + (1+r)S_t$$

c) Graphically depict the optimality condition. Carefully label the intercepts of the budget constraint. What is the slope of the indifference curve at the optimal consumption basket,(C_t^*, C_{t+1}^*)?

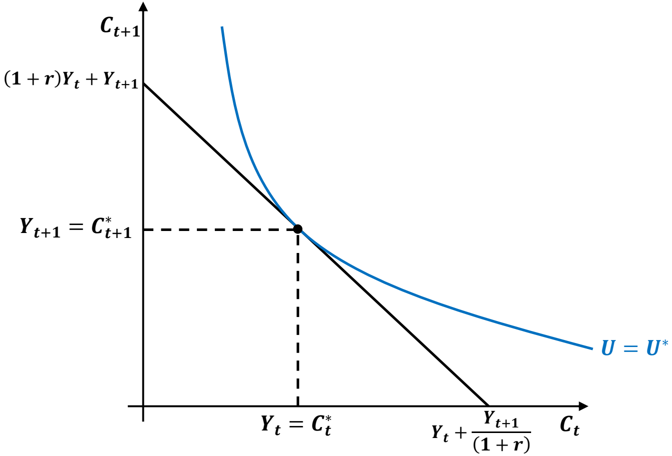
d) Graphically depict the effects of an increase in Y_{t+1}. Carefully label the intercepts of the budget constraint. Is the slope of the indifference curve at the optimal consumption basket, (C_t^*, C_{t+1}^*), different than in part c?

c:

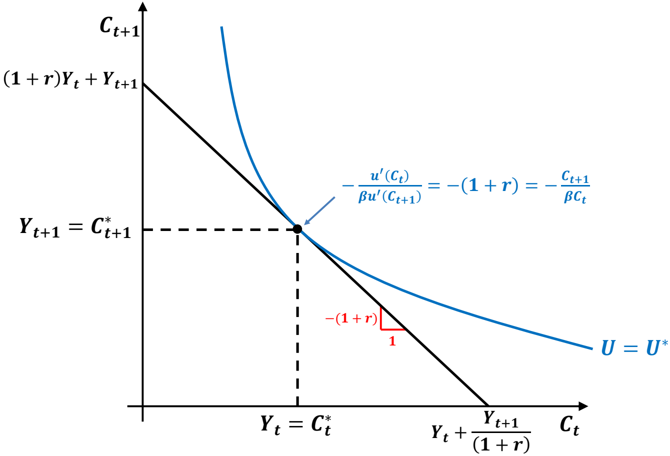


2. Consumption smoothing: indiff. curve

c:



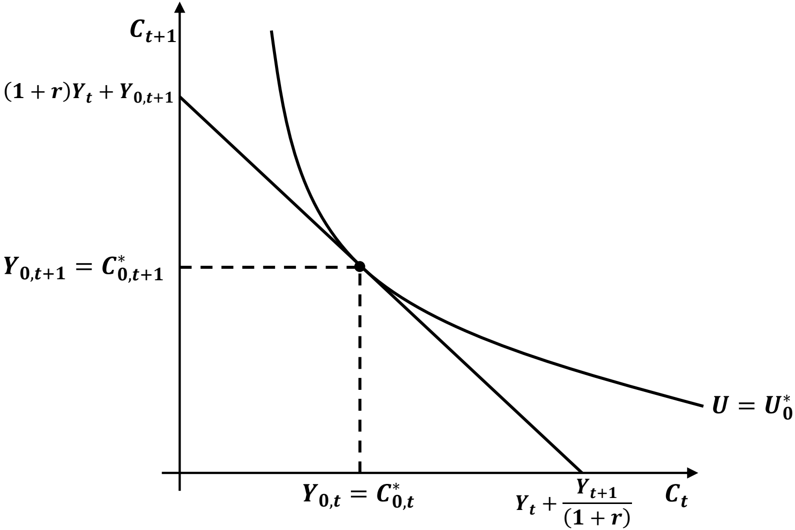
c:



C_{t+1}
$(1+r)Y_t + Y_{t+1}$
$-\frac{u'(C_t)}{\beta u'(C_{t+1})} = -(1+r) = -\frac{C_{t+1}}{\beta C_t}$
$Y_{t+1} = C^*_{t+1}$
$-(1+r)$
1
$U = U^*$
$Y_t = C^*_t$
$Y_t + \frac{Y_{t+1}}{(1+r)}$
C_t

2. Consumption smoothing: increase in Y_{t+1}

d:



2. Consumption smoothing: increase in Y_{t+1}

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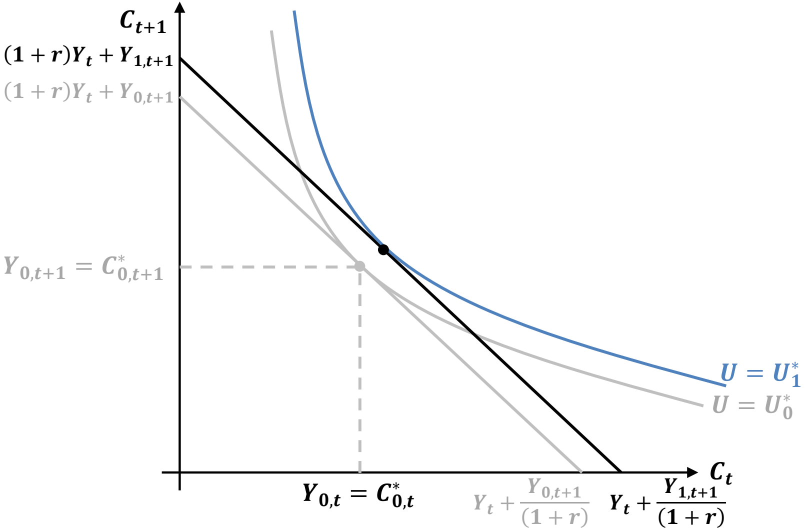
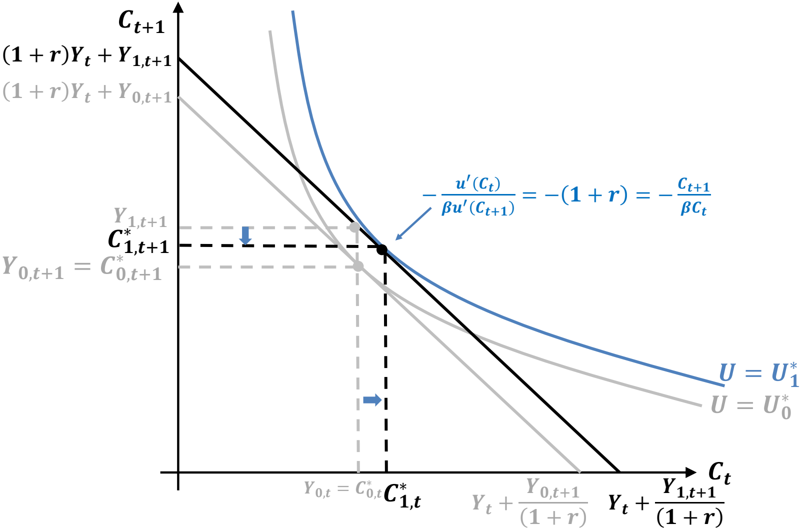


2. Consumption smoothing: increase in Y_{t+1}

d:



Question:
Consider a consumer with a lifetime utility function

$$U = \ln C_t + \beta \ln C_{t+1}$$

The period t and $t+1$ budget constraints are

$$\begin{aligned} C_t + S_t &= Y_t \\ C_{t+1} + S_{t+1} &= Y_{t+1} + (1+r)S_t \end{aligned}$$

e) Now suppose C_t is taxed at rate τ so consumers pay $1+\tau$ for one unit of period t consumption. Redo parts a-c under these new assumptions.

Budget constraint:

$$C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1+r)S_t$$

2. Consumption smoothing: taxes

Budget constraint:

$$(1+\tau)C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1+r)S_t$$

2. Consumption smoothing: taxes

Budget constraint:

$$(1+\tau) C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1+r) S_t$$

$$\Rightarrow S_t = \frac{1}{1+r} C_{t+1} - \frac{1}{1+r} Y_{t+1}$$

2. Consumption smoothing: taxes

Budget constraint:

$$(1+\tau)C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1+r)S_t$$

$$\Rightarrow S_t = \frac{1}{1+r}C_{t+1} - \frac{1}{1+r}Y_{t+1}$$

$$(1+\tau)C_t + \frac{1}{1+r}C_{t+1} = Y_t + \frac{1}{1+r}Y_{t+1}$$

Euler equation:

$$\max_{C_t, C_{t+1}} U = \ln C_t + \beta \ln C_{t+1}$$

$$\text{s.t.} \quad (1+\tau) C_t + \frac{C_{t+1}}{1+r} = Y_t + \frac{Y_{t+1}}{1+r}$$

$$\max_{C_t, C_{t+1}} U = \ln C_t + \beta \ln C_{t+1} + \lambda \left[Y_t + \frac{Y_{t+1}}{1+r} - (1+\tau) C_t - \frac{C_{t+1}}{1+r} \right]$$

2. Consumption smoothing: taxes

$$\max_{C_t, C_{t+1}} U = \ln C_t + \beta \ln C_{t+1} + \lambda \left[Y_t + \frac{Y_{t+1}}{1+r} - (1+\tau) C_t - \frac{C_{t+1}}{1+r} \right]$$

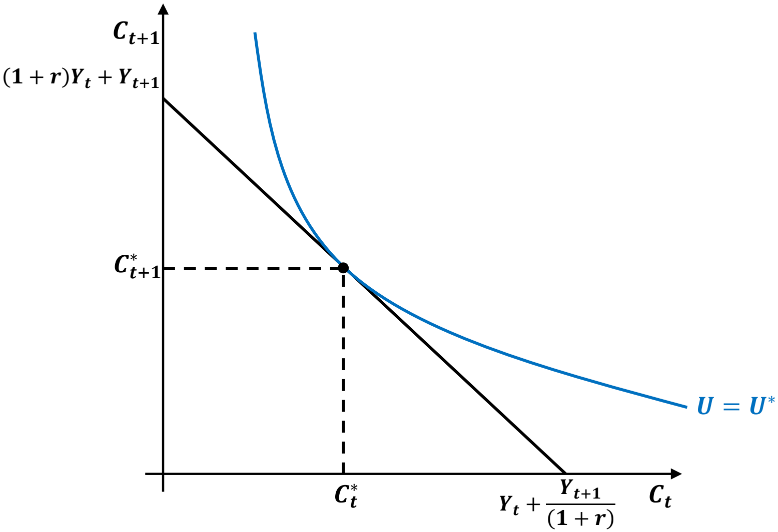
FOCs:

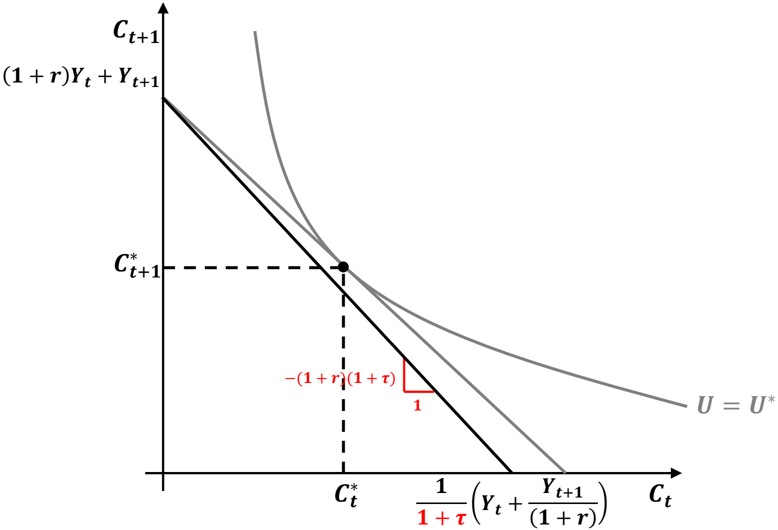
$$\frac{\partial L}{\partial C_t} = \frac{1}{C_t} - \lambda(1+\tau) = 0 \Rightarrow \lambda = \frac{1}{C_t(1+\tau)}$$

$$\frac{\partial L}{\partial C_{t+1}} = \frac{\beta}{C_{t+1}} - \lambda \frac{1}{1+r} = 0 \Rightarrow \lambda = \frac{\beta(1+r)}{C_{t+1}}$$

Euler-Equation: $$\underbrace{\frac{C_{t+1}}{\beta C_t}}_{Slope\ Ind-Kurve} = \underbrace{(1+r)(1+\tau)}_{Slope\ budget\ constraint}$$

2. Consumption smoothing: taxes




C_{t+1}
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$-(1+r)(1+\tau)$
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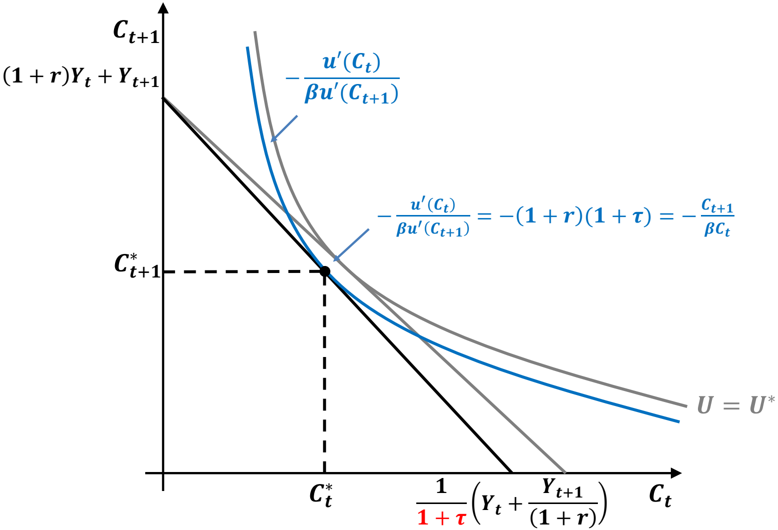

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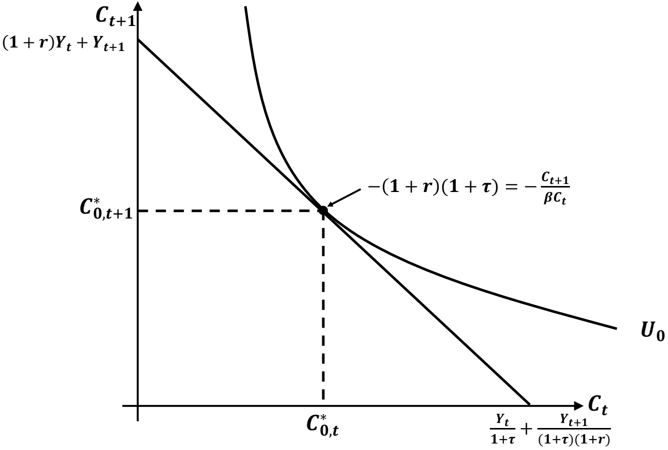
The period t and $t+1$ budget constraints are

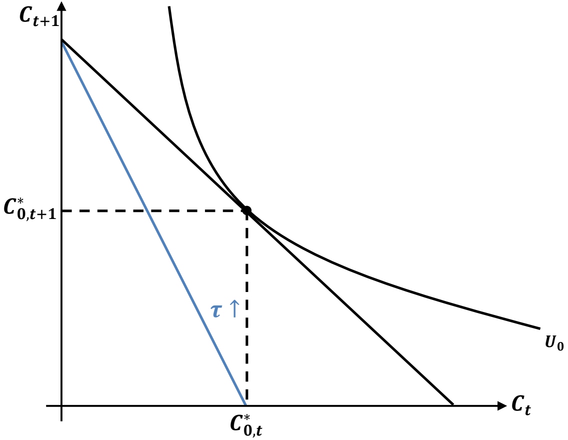
$$C_t + S_t = Y_t$$
$$C_{t+1} + S_{t+1} = Y_{t+1} + (1+r)S_t$$

f) Suppose the tax rate increases from τ to τ'. Graphically depict this. Carefully label the intercepts of the budget constraint. Is the slope of the indifference curve at the optimal consumption basket, (C_t^*, C_{t+1}^*), different than in part e? Intuitively describe the roles played by the substitution and income effects. Using this intuition, can you definitively prove the sign of $\frac{\partial C_t^*}{\partial \tau}$ and $\frac{\partial C_{t+1}^*}{\partial \tau}$? It is not necessary to use math for this. Describing it in words is fine.

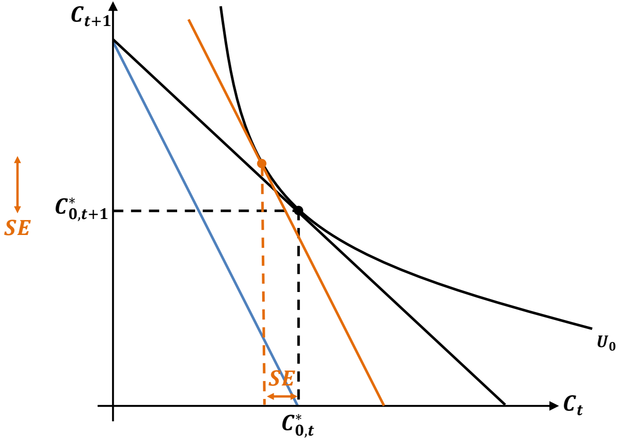
2. Consumption smoothing: taxes



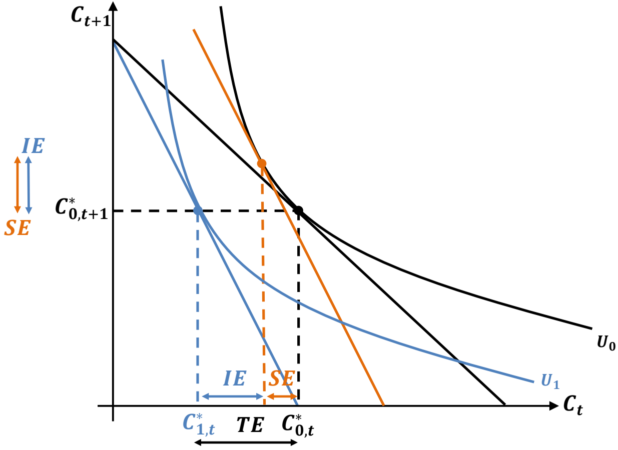
2. Consumption smoothing: taxes



2. Consumption smoothing: taxes



2. Consumption smoothing: taxes



Substitution effect:

The representative household substitutes away from the relatively more expensive good and into the relatively cheaper good

It shows how the consumption bundle would change after a change in the relative prices, where the household is compensated with sufficient income so as to leave lifetime utility unchanged

Income effect:

The income effect is the movement from the hypothetical bundle with a higher relative price for C_t but unchanged lifetime utility to a new indifference curve tangent to the new budget line

The household reduces, relative to the **hypothetical** consumption bundle, consumption in both periods

3. Borrowing constraint: budget and Euler equation

Question:
Consider the following consumption-savings problem. The consumer maximizes

$$\max_{C_t, C_{t+1}, S_t} \ln C_t + \beta \ln C_{t+1}$$

subject to the lifetime budget constraint

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

and the borrowing constraint

$$C_t \leq Y_t.$$

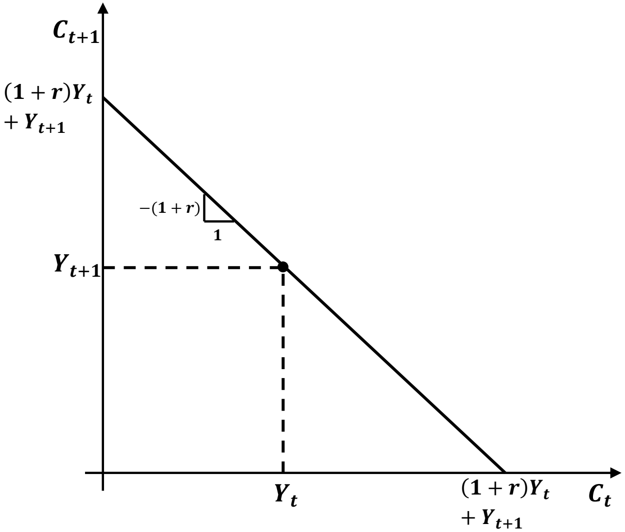
This last constraint says that savings cannot be negative in the first period. Equivalently, this is saying consumers cannot borrow in the first period.

a) Draw the budget constraint.

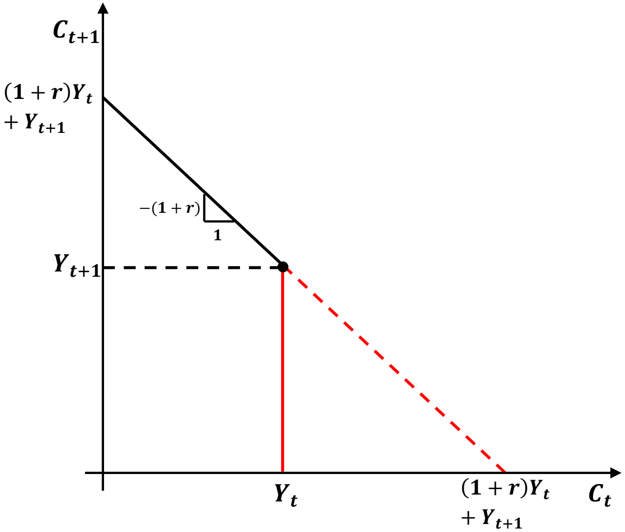
b) Assuming the constraint does not bind, what is the Euler equation?

3. Borrowing constraint: budget and Euler equation

a:



a:



3. Borrowing constraint: budget and Euler equation

b:

Euler equation: $S_t \geq 0$

If constraint does not bind $\Rightarrow$ households prefer to save in t ($S_t > 0$)

The fact that it can not borrow is irrelevant

$$-\frac{u'(C_t)}{\beta u'(C_{t+1})} = -(1+r)$$
$$\frac{C_{t+1}}{\beta C_t} = (1+r)$$
$$\frac{C_{t+1}}{C_t} = \beta(1+r)$$

3. Borrowing constraint: binding of the constraint

Question:
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and the borrowing constraint

$$C_t \leq Y_t.$$

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c) Using the Euler equation, lifetime budget constraint and borrowing constraint, solve for the period t consumption function. Clearly state under what circumstances the borrowing constraint binds.

3. Borrowing constraint: binding of the constraint

Consumption function:

3. Borrowing constraint: binding of the constraint

Consumption function:

1. Euler equation:

Consumption function:

1. Euler equation: $C_{t+1} = \beta(1 + r_t)C_t$

3. Borrowing constraint: binding of the constraint

Consumption function:

1. Euler equation: $C_{t+1} = \beta(1 + r_t)C_t$
2. Budget constraint:

3. Borrowing constraint: binding of the constraint

Consumption function:

1. Euler equation: $C_{t+1} = \beta(1 + r_t)C_t$
2. Budget constraint: $C_{t+1} = (1 + r_t)Y_t - (1 + r_t)C_t + Y_{t+1}$

1. $=$ 2.

3. Borrowing constraint: binding of the constraint

Consumption function:

1. Euler equation: $C_{t+1} = \beta(1 + r_t)C_t$
2. Budget constraint: $C_{t+1} = (1 + r_t)Y_t - (1 + r_t)C_t + Y_{t+1}$

1\. = 2.

$$\beta(1 + r_t)C_t = (1 + r_t)Y_t - (1 + r_t)C_t + Y_{t+1}$$

3. Borrowing constraint: binding of the constraint

Consumption function:

1. Euler equation: $C_{t+1} = \beta(1 + r_t)C_t$
2. Budget constraint: $C_{t+1} = (1 + r_t)Y_t - (1 + r_t)C_t + Y_{t+1}$

1. = 2.

$$\beta(1 + r_t)C_t = (1 + r_t)Y_t - (1 + r_t)C_t + Y_{t+1}$$
$$C_t = \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

3. Borrowing constraint: binding of the constraint

Borrowing constraint binds if: $C_t > Y_t$

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$$Y_t < C_t = \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

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Borrowing constraint binds if: $C_t > Y_t$

$$Y_t < C_t = \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

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Borrowing constraint binds if: $C_t > Y_t$

$$Y_t < C_t = \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

$$Y_t < \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

$$\frac{Y_{t+1}}{Y_t} > \beta(1+r_t)$$

3. Borrowing constraint: binding of the constraint

Borrowing constraint binds if: $C_t > Y_t$

$$Y_t < C_t = \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

$$Y_t < \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

$$\frac{Y_{t+1}}{Y_t} > \beta(1+r_t) = \frac{C_{t+1}}{C_t}$$

3. Borrowing constraint: binding of the constraint

Borrowing constraint binds if: $C_t > Y_t$

$$Y_t < C_t = \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

$$Y_t < \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

$$\frac{Y_{t+1}}{Y_t} > \beta(1+r_t) = \frac{C_{t+1}}{C_t}$$

$$\frac{Y_{t+1}}{Y_t} > \frac{C_{t+1}}{C_t}$$

3. Borrowing constraint: binding of the constraint

Borrowing constraint binds if: $C_t > Y_t$

$$Y_t < C_t = \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

$$Y_t < \frac{1}{1+\beta}\left(Y_t + \frac{Y_{t+1}}{1+r_t}\right)$$

$$\frac{Y_{t+1}}{Y_t} > \beta(1+r_t) = \frac{C_{t+1}}{C_t}$$

$$\frac{Y_{t+1}}{Y_t} > \frac{C_{t+1}}{C_t}$$

$$1 + g_Y > 1 + g_C$$

Question:
Consider the following consumption-savings problem. The consumer maximizes

$$\max_{C_t, C_{t+1}, S_t} \ln C_t + \beta \ln C_{t+1}$$

subject to the lifetime budget constraint

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

and the borrowing constraint

$$C_t \leq Y_t.$$

This last constraint says that savings cannot be negative in the first period. Equivalently, this is saying consumers cannot borrow in the first period.

d) Suppose $Y_t = 3$, $Y_{t+1} = 10$, $\beta = 0.95$ and $r = 0.1$. Show that the borrowing constraint binds.

3. Borrowing constraint: example

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$$\beta(1 + r_t) = \frac{C_{t+1}}{C_t}$$

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$$0.95(1 + 0.1) = 1.045$$

3. Borrowing constraint: example

$$\beta(1 + r_t) = \frac{C_{t+1}}{C_t}$$

$$0.95(1 + 0.1) = 1.045$$

$$\frac{Y_{t+1}}{Y_t} = \frac{10}{3} = 3.\bar{3}$$

3. Borrowing constraint: example

$$\beta(1 + r_t) = \frac{C_{t+1}}{C_t}$$

$$0.95(1 + 0.1) = 1.045$$

$$\frac{Y_{t+1}}{Y_t} = \frac{10}{3} = 3.\bar{3}$$

$$\frac{Y_{t+1}}{Y_t} = 3.\bar{3} > 1.045 = \frac{C_{t+1}}{C_t}$$

3. Borrowing constraint: example

$$\beta(1 + r_t) = \frac{C_{t+1}}{C_t}$$

$$0.95(1 + 0.1) = 1.045$$

$$\frac{Y_{t+1}}{Y_t} = \frac{10}{3} = 3.\bar{3}$$

$$\frac{Y_{t+1}}{Y_t} = 3.\bar{3} > 1.045 = \frac{C_{t+1}}{C_t}$$

Income increases substantially $\Rightarrow$ desire to smooth consumption but this is not possible due to the borrowing constraint

Question:
Consider the following consumption-savings problem. The consumer maximizes

$$\max_{C_t, C_{t+1}, S_t} \ln C_t + \beta \ln C_{t+1}$$

subject to the lifetime budget constraint

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}$$

and the borrowing constraint

$$C_t \leq Y_t.$$

This last constraint says that savings cannot be negative in the first period. Equivalently, this is saying consumers cannot borrow in the first period.

e) Suppose there is a one time tax rebate that increases Y_t to 4. Leave $Y_{t+1} = 10$, $\beta = 0.95$ and $r = 0.1$. What is the marginal propensity to consume out of this tax rebate?

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$$\frac{1}{1+0.95} \approx 0.51$$