Business Cycles

Part 2: Microeconomic Foundations
Lecture 4: Production and Labor Supply

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Outline

Part 1: Introduction

Part 2: Microeconomic Foundations

- Lecture 2: Consumption-Savings Problem
- Lecture 3: Equilibrium in an endownment economy, fiscal policy
- Lecture 4: Production and labor supply

Part 3: The Real Business Cycle Model

Part 4: The New Keynesian Model

Part 5: Financial Crises

Learning Objective of Todays Lecture

1. Understanding the microeconomic foundations of the production side.
2. Expand the household decisions to labor supply.
3. Completes the real business cycle (RBC) / neoclassical model.

Literature

Required reading:

- Textbook chapter 12

Optional reading:

- Textbook chapter 14 (microfoundations of money)
- Textbook chapter 15 (efficiency)

Production and Labor Supply

- We continue working with a two period, optimizing, equilibrium model of the economy
- We augment the model with which we have been working along the following two dimensions:
    1. We model production and capital accumulation
    2. We model endogenous labor supply

Firm

- There exists a representative firm. The firm produces output using capital, K_t, and labor, N_t, according to the following production function:

$$Y_t = A_t F(K_t, N_t)$$

- Think about output as units of fruit. Capital is stock of fruit trees. Labor is time spent picking from the trees.
- A_t is exogenous productivity variable. Abstract from trend growth.

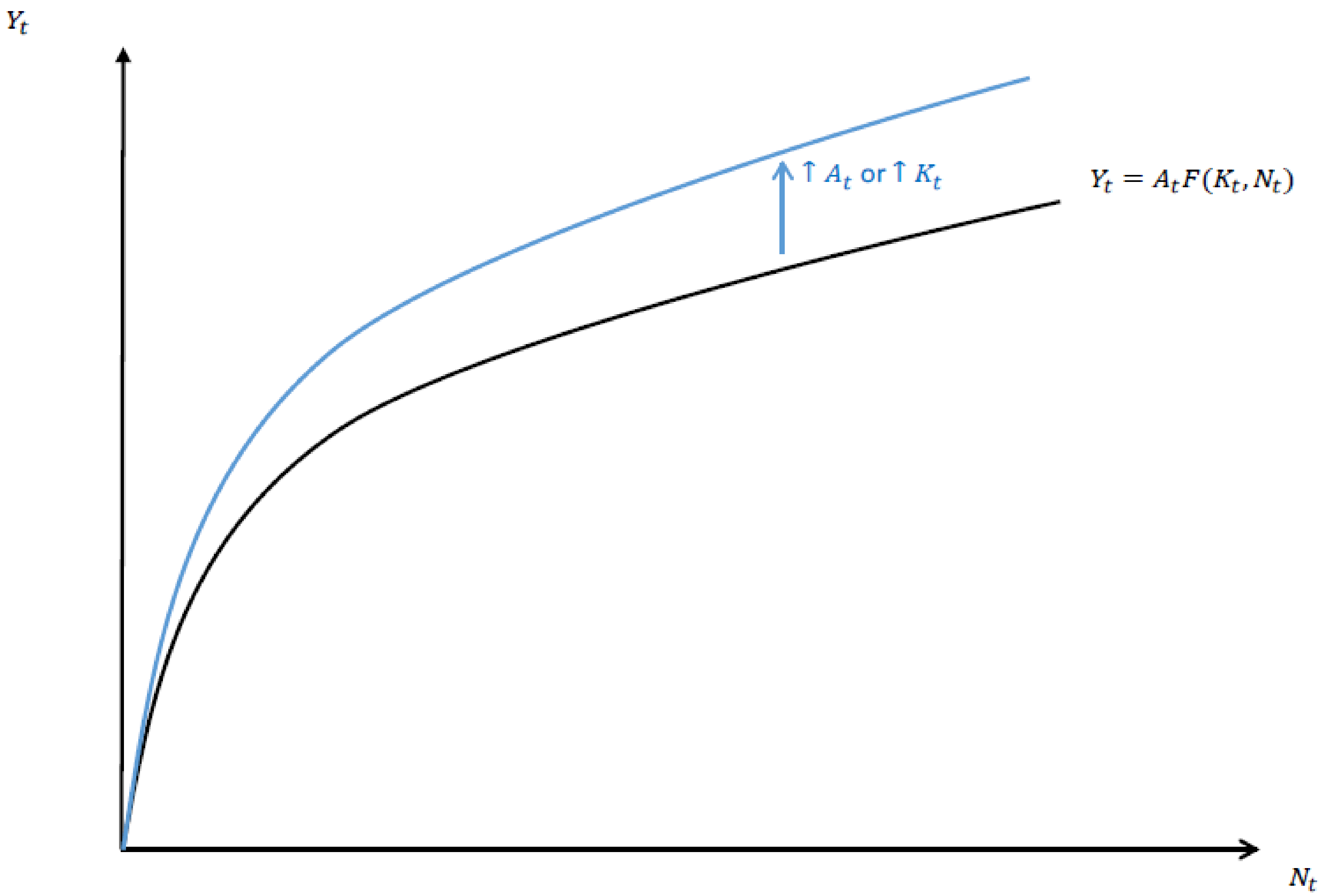
Properties of Production Function

- Both inputs necessary: $F(0, N_t) = F(K_t, 0) = 0$
- Increasing in both inputs: $F_K(K_t, N_t) > 0$ and $F_N(K_t, N_t) > 0$
- Concave in both inputs: $F_{KK}(K_t, N_t) < 0$ and $F_{NN}(K_t, N_t) < 0$
- Constant returns to scale: $F(qK_t, qN_t) = qF(K_t, N_t)$

- Example production function: Cobb-Douglas:

$$F(K_t, N_t) = K_t^{\alpha} N_t^{1-\alpha}, \qquad 0 < \alpha < 1$$

The Production Function



Capital Accumulation

- Differently than the standard Solow model, we assume that the firm owns its capital and makes the capital accumulation decisions
- Actually, doesn't matter whether household or firm makes capital accumulation decisions
- Current capital, K_t, is predetermined and hence exogenous. Capital accumulates according to:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

- Firms must borrow funds from financial intermediaries to fund investments. The cost of borrowing is r_t.
- Assumption: all of investment expenditures are financed by borrowing

$$B_t^I = I_t$$

Dividends and Firm Valuation

- The representative household owns the firm. The firm returns any difference between revenue and cost to the household each period in the form of a dividend

- Dividend is output less payments to labor less expenditure on new capital, which is investment:

$$D_t = Y_t - w_t N_t$$

$$D_{t+1} = Y_{t+1} - w_{t+1} N_{t+1} - I_{t+1} - (1 + r_t) B_t^I$$

- Paying off the loan take out in t: $(1 + r_t) B_t^I$

- "Liquidation sale" of remaining capital in $t + 1$ as there is no $t + 2$: $I_{t+1} = -(1 - \delta) K_{t+1}$

- Value of the firm is present discounted value of stream of dividends:

$$V_t = D_t + \frac{D_{t+1}}{1 + r_t}$$

Firm Problem

- From period t perspective, the firm needs to maximize its value $V_t = D_t + \frac{D_{t+1}}{1+r_t}$ by choosing N_t and I_t subject to the production function, capital accumulation restriction and the requirement that investment be financed by borrowing.
- From period $t+1$ perspective the same maximization occurs, but only N_{t+1} has to be chosen as I_{t+1} is determined by the terminal condition $K_{t+2} = 0$.
- One can write down a Lagrange optimization problem or substitute in all constraints. We do the latter here.

Firm Problem

$$\max_{N_t, N_{t+1}, I_t} V_t = D_t + \frac{D_{t+1}}{1+r_t} = Y_t - w_t N_t$$

$$+\frac{1}{1+r_t}\left[Y_{t+1} - w_{t+1}N_{t+1} - I_{t+1} - (1+r_t)B_t^I\ \right]$$

Plug in the terminal condition

$$K_{t+2} = I_{t+1} + (1-\delta)K_{t+1} = 0 \quad \Leftrightarrow I_{t+1} = -(1-\delta)K_{t+1}$$

$$\max_{N_t, N_{t+1}, I_t} Y_t - w_t N_t + \frac{1}{1+r_t}\left[Y_{t+1} - w_{t+1}N_{t+1} + (1-\delta)K_{t+1} - (1+r_t)B_t^I\ \right]$$

Plug in production function: $\mathrm{Y_t} = A_t F(K_t, N_t)$

$$\max_{N_t, N_{t+1}, I_t} A_t F(K_t, N_t) - w_t N_t$$

$$+\frac{1}{1+r_t}\left[A_{t+1}F(K_{t+1}, N_{t+1}) - w_{t+1}N_{t+1} + (1-\delta)K_{t+1} - (1+r_t)B_t^I\ \right]$$

Firm Problem

Combine the two constraints $K_{t+1} = I_t + (1-\delta)K_t$ and $B_t^I = I_t$ to one:

$$B_t^I = K_{t+1} - (1-\delta)K_t$$

Substitute in $B_t^I = K_{t+1} - (1-\delta)K_t$ to eliminate B_t^I

$$\max_{N_t, N_{t+1}, I_t} A_t F(K_t, N_t) - w_t N_t + \frac{1}{1+r_t}[A_{t+1}F(K_{t+1}, N_{t+1}) - w_{t+1}N_{t+1} + (1-\delta)K_{t+1} - (1+r_t)(K_{t+1} - (1-\delta)K_t)]$$

Firm Problem

- Pick N_t, N_{t+1}, and K_{t+1} (implies I_t) to maximize the firm's value

$$\max_{N_t, N_{t+1}, I_t} A_t F(K_t, N_t) - w_t N_t$$
$$+\frac{1}{1+r_t}[A_{t+1}F(K_{t+1}, N_{t+1}) - w_{t+1}N_{t+1} + (1-\delta)K_{t+1} - (1+r_t)(K_{t+1}-(1-\delta)K_t)]$$

- First order conditions are:

$$w_t = A_t F_N(K_t, N_t)$$

$$w_{t+1} = A_{t+1} F_N(K_{t+1}, N_{t+1})$$

$$1 + r_t = A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1-\delta)$$

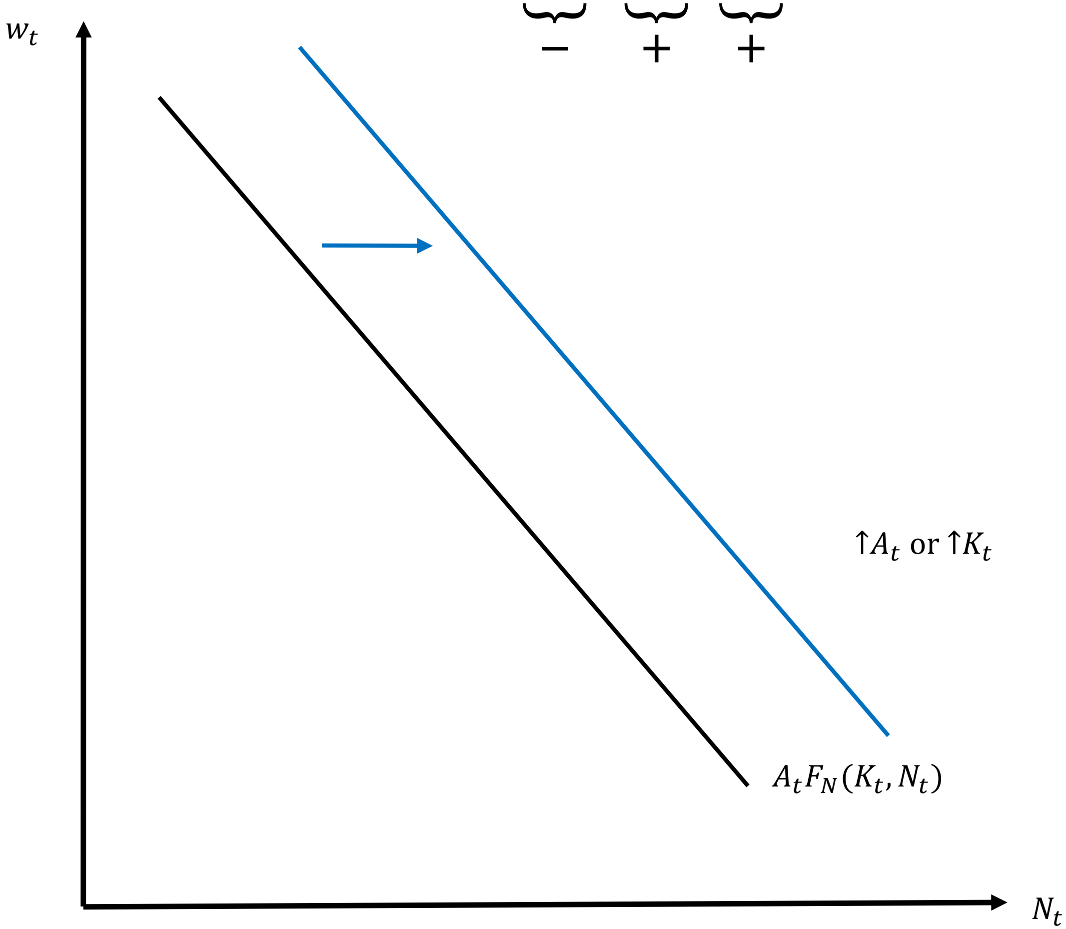
- Period t and period $t+1$ implicit labor demand functions ($w_t = MPL$).

- Capital demand function implying investment demand

Labor Demand

First two conditions are "static" (same in each period) and implicitly characterize a downward-sloping labor demand curve:

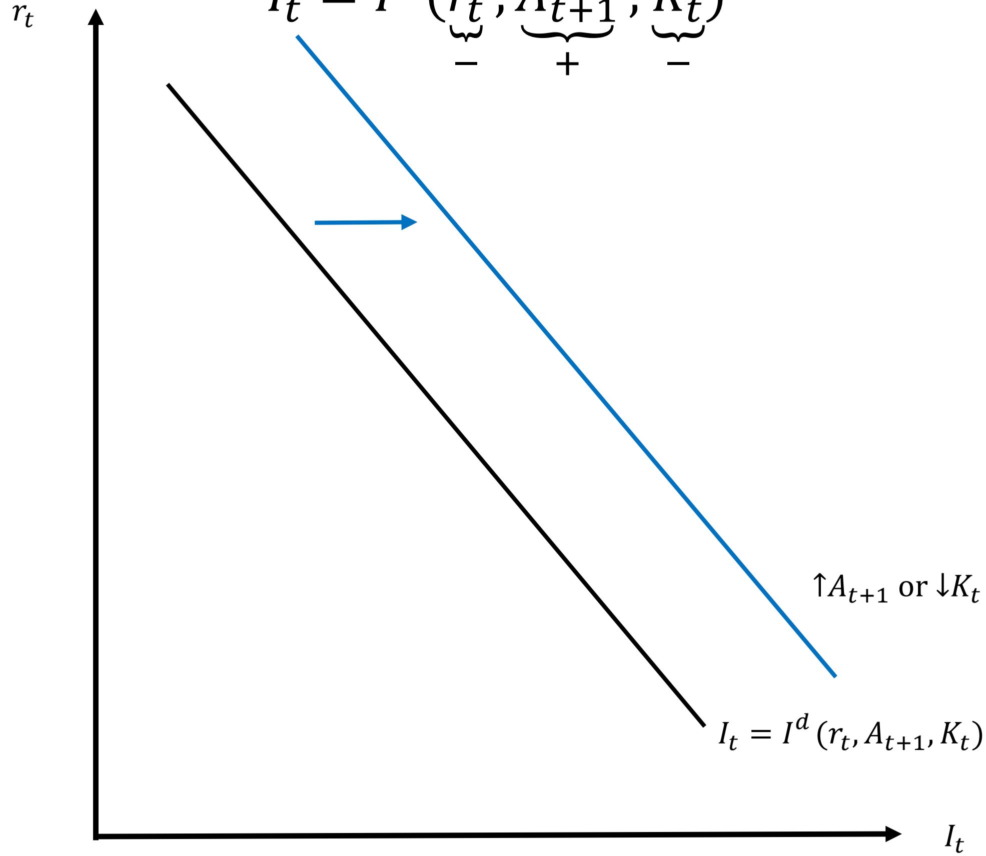
$$N_t = N^d(\underbrace{w_t}_{-}, \underbrace{A_t}_{+}, \underbrace{K_t}_{+})$$



Investment Demand

- Second first order condition implicitly defines a demand for K_{t+1}, which can be used in conjunction with the accumulation equation to get an investment demand curve:

$$I_t = I^d(\underbrace{r_t}_{-}, \underbrace{A_{t+1}}_{+}, \underbrace{K_t}_{-})$$



Households

- There exists a representative household. Households gets utility from consumption and leisure, where leisure is $L_t = 1 - N_t$, with N_t labor and available time normalized to 1
- Lifetime utility:

$$U = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1})$$

- Example flow utility function:

$$u(C_t, 1 - N_t) = \ln C_t + \theta_t \ln(1 - N_t)$$

- Here, θ_t is an exogenous “labor supply shock” governing utility from leisure (equivalently, disutility from labor)
- Notation: u_C denotes marginal utility of consumption, u_L marginal utility of leisure (marginal utility of labor is $-u_L$)

Budget Constraints

- Household faces two flow budget constraints, conceptually the same as before, but now income is partly endogenous:

$$C_t + S_t \leq w_t N_t + D_t$$

$$C_{t+1} + S_{t+1} \leq w_{t+1} N_{t+1} + D_{t+1} + D_{t+1}^I + (1 + r_t) S_t$$

- Household takes D_t and D_{t+1} as given (ownership different than management)
- D_{t+1}^I denotes divident payout from the financial intermediary. This is also taken as given.
- Terminal condition: $S_{t+1} = 0$. Gives rise to IBC:

$$C_t + \frac{C_{t+1}}{1 + r_t} = w_t N_t + D_t + \frac{w_{t+1} N_{t+1} + D_{t+1} + D_{t+1}^I}{1 + r_t}$$

First Order Conditions

- Do the optimization in the usual way. The following first order conditions emerge:

$$u_C(C_t, 1 - N_t) = \beta(1 + r_t)\, u_C(C_{t+1}, 1 - N_{t+1})$$

- This is the usual Euler equation, only looks different to accommodate utility from leisure.

- FOC with respect to N_t and N_{t+1}:

$$u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t)$$

$$u_L(C_{t+1}, 1 - N_{t+1}) = w_{t+1} u_C(C_{t+1}, 1 - N_{t+1})$$

Optimal Decision Rules

- Can go from first order conditions to optimal decision rules
- Cutting a few corners, we get the same consumption function as before:

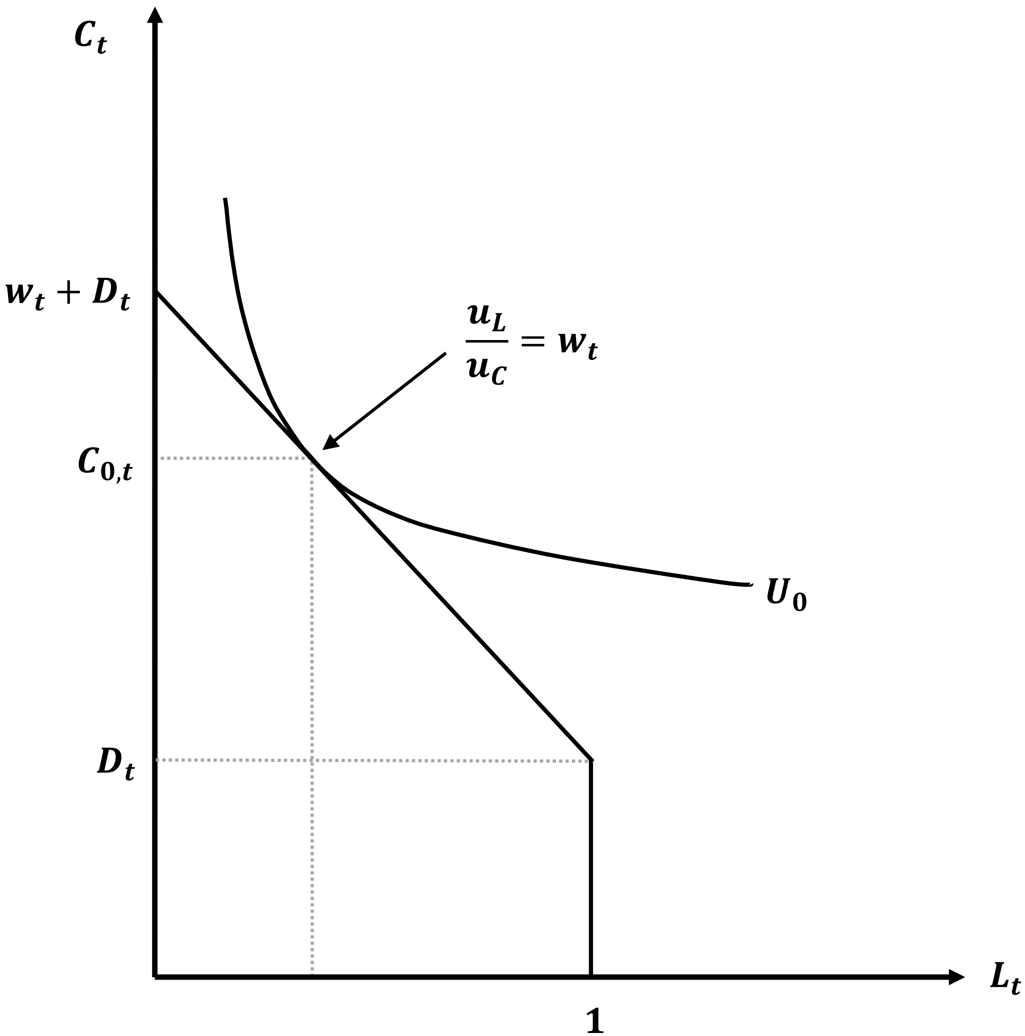
$$C_t = C^d(\underbrace{Y_t}_{+}, \underbrace{Y_{t+1}}_{+}, \underbrace{r_t}_{-})$$

- Or, if there were government spending, with Ricardian Equivalence we'd have:

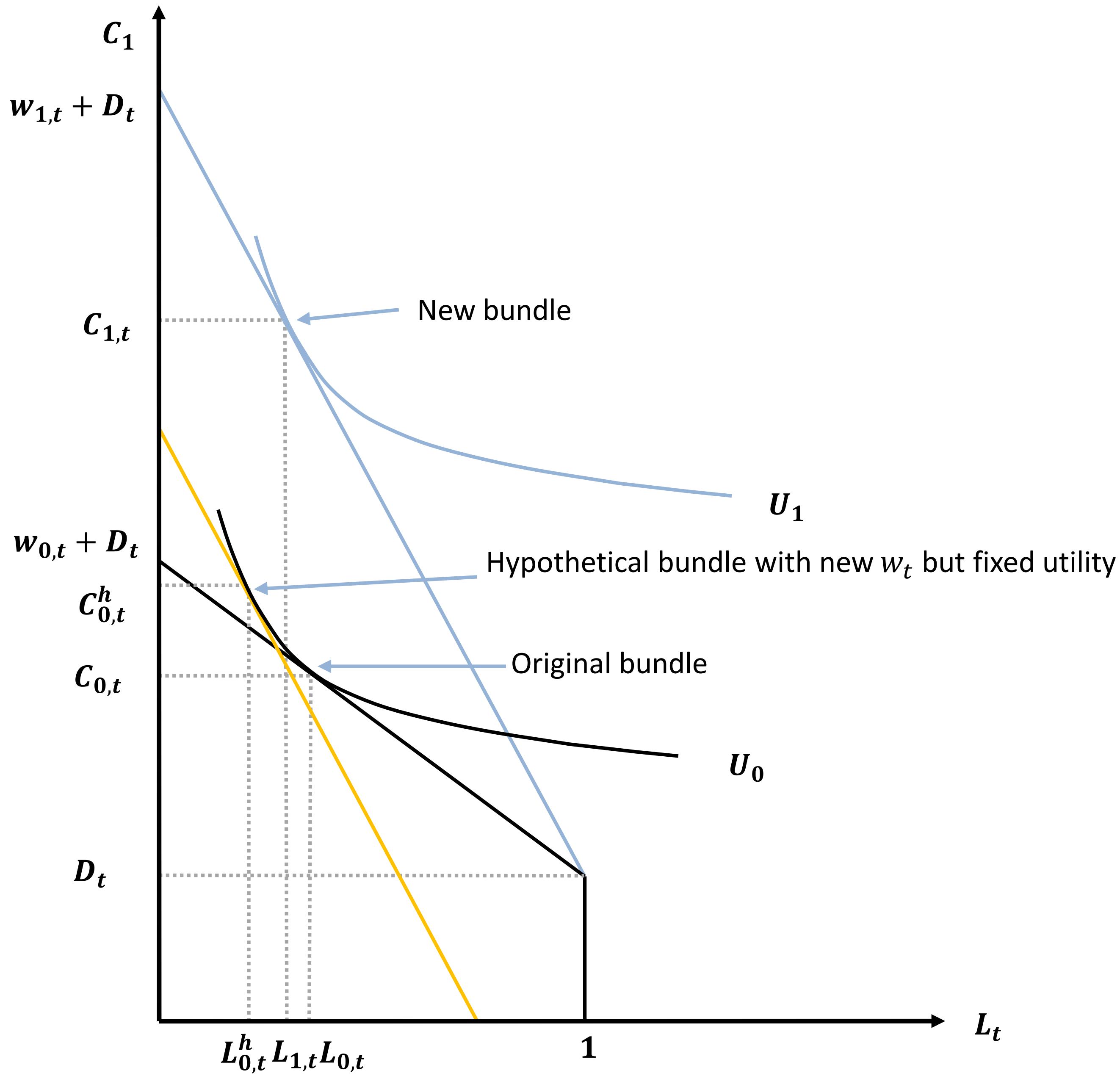
$$C_t = C^d(\underbrace{Y_t - G_t}_{+}, \underbrace{Y_{t+1} - G_{t+1}}_{+}, \underbrace{r_t}_{-})$$

Labor Supply

First order condition for N_t can be characterized by an indifference curve / budget line diagram similar to the two-period consumption case:



Income and Substitution Effects of Higher Wage

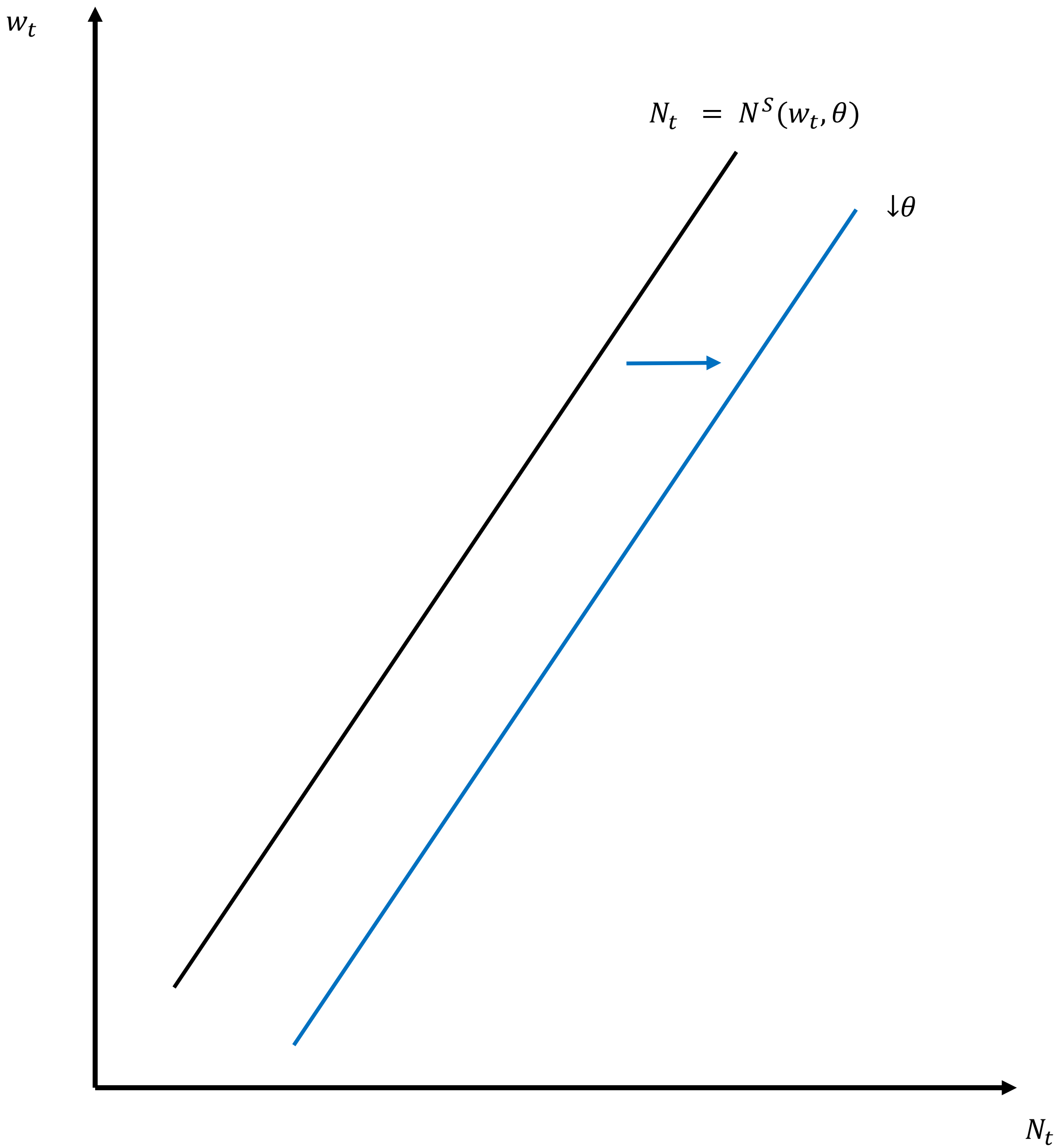


Labor Supply Function

- We assume that the substitution effect of a higher wage dominates the income effect
- This means that labor supply is increasing in w_t
- In principle, labor supply would also be affected by non-wage income and the real interest rate (anything which would impact C_t)
- We will abstract from this. We assume that labor is an increasing function of w_t and a decreasing function of θ_t, an exogenous variable which we take to be a measure of preferences for leisure (or more generally anything other than w_t which affects labor supply):

$$N_t = N^s(\underbrace{w_t}_{+}, \underbrace{\theta_t}_{-})$$

Labor Supply Graphically



Financial Intermediary

- Financial intermediary, e.g. a bank, is operating in the background
- Households borrow/save via S_t, firms fund their capital investments
- The financial intermediary "takes" funds from the households, S_t, and lends them to the firm, B_t^I, charging the same interest rate on borrowing and saving, r_t.
- Dividends earned by the financial intermediary:

$$D_{t+1}^I = r_t B_t^I - r_t S_t$$

- In this model, the financial intermediary is passive. In equilibrium, its dividends are zero.

Equilibrium

- Four optimal decision rules:

$$C_t = C^d(Y_t, Y_{t+1}, r_t)$$
$$N_t = N^s(w_t, \theta_t)$$
$$N_t = N^d(w_t, A_t, K_t)$$
$$I_t = I^d(r_t, A_{t+1}, K_t)$$

- Market-clearing condition plus production function:

$$Y_t = C_t + I_t$$
$$Y_t = A_t F(K_t, N_t)$$

- Endogenous variables: Y_t, N_t, C_t, I_t (quantities), w_t and r_t (prices)
- Exogenous variables: A_t, A_{t+1}, K_t, θ_t

Adding a Government

- Doesn't change much. Ricaridian Equivalence still holds:

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$
$$N_t = N^s(w_t, \theta_t)$$
$$N_t = N^d(w_t, A_t, K_t)$$
$$I_t = I^d(r_t, A_{t+1}, q_t, K_t)$$
$$Y_t = C_t + I_t + G_t$$
$$Y_t = A_t F(K_t, N_t)$$

- Now G_t and G_{t+1} are exogenous, and T_t and T_{t+1} are irrelevant

The real business cycle model / neoclassical model

- We have developed a completely microfounded macroeconomic model.
- This model is known as the real business cycle model or the neoclassical model.
- Note that there are only real variables. We have completely ignored money. We can analyze real variables separately from nominal variables as long as there are no nominal frictions (for example price and wage rigidities) in the economy: this is known as the classical dichotomy.
- Nominal variables: We will introduce money, inflation, and the nominal interest rate in the complete model in the next lecture without deriving microeconomic foundations (see bookchapter 14 for microfoundations of money).

Efficiency

- The competitive equilibrium is efficient in the sense that the allocations correspond to the allocations a social planner would decide to maximize the representative household‘s present discounted value of lifetime utility.
- The competitive equilibrium is Pareto efficient. This is an example of the First Welfare Theorem: under conditions of perfect competition and no distortionary taxation, all competitive equilibrium are efficient.
- An activist policy (monetary or fiscal or redistribution by social planner) can only reduce welfare.
- The model is therefore an important benchmark case of an efficient allocation rather than a very realistic model.

Summary

- Firms choose labor and capital to maximize the present discounted value of dividends. These dividends are rebated to households.
- The household chooses leisure and consumption to maximize utility. Labor supply may increase or decrease after a change in the real wage as there are offsetting income and substitution effects. Unless otherwise stated, we assume that the substitution effect dominates, and that labor supply is therefore increasing in the real wage.
- Complete real business cycle model. The allocation is efficient. Important benchmark case.
- Next lecture: Analysing the model graphically and adding nominal variables (we don’t discuss their microeconomic foundations).
- Afterwards, we will go to a more realistic model that adds price and wage rigidities, i.e. an inefficiency. Policy will try to drive the allocation as closely to the one implied by the real business cycle, i.e. the efficient allocation, as possible.