Lehrstuhl für Informatik I
Algorithmen und
Komplexität
Universität Würzburg

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Exercise sheet 4

Visualisation of graphs

Exercise 1 – Fast construction of Schnyder realiser

In the lecture we proved that every triangulated plane graph $G = (V, E)$ has a Schnyder realiser or a respective Schnyder labelling. The proof yields a recursive algorithm: contract an edge $\{a, x\}$, find recursively a Schnyder forest in the resulting graph and then add x consistently back. A naive implementation of this algorithm yields a runtime of $O(n^2)$, in particular, because we need to find the contracted edge. Explain how the algorithm can be improved to get linear runtime. **7 Points**

Hint: Think about the candidate-edges for contraction. How can they be updated quickly during the algorithm?

Exercise 2 – Fast calculation of barycentric coordinates

Let $G = (V, E)$ be a triangulated plane graph with a Schnyder realiser T_1, T_2, T_3. As in the lecture, for each inner vertex v, let $|V(R_i(v))|$ be the number of vertices in the region R_i with respect to v (including two outside vertices and v itself). Let furthermore $|P_i(v)|$ be the number of vertices on the path from v to a_i in T_i (including v). Let $v'_i = |V(R_i(v))| - |P_{i-1}(v)|$.

Show that the values v' can be calculated for all inner vertices v at once with a total runtime of $O(n)$. **7 Points**

Exercise 3 – Weak barycentric representations

Let $G = (V, E)$ be an embedded, triangulated graph with a weak barycentric representation $v \in V \mapsto (v_1, v_2, v_3) \in \mathbb{R}^3$. Let $A, B, C \in \mathbb{R}^2$ be points in general position.

Show that the function $f\colon v \in V \mapsto v_1 A + v_2 B + v_3 C$ yields a crossing-free drawing.

6 Points

This assignment is due at the beginning of the next lecture, that is, on May 26th at 10 am. Please submit your solutions via WueCampus. The exercises will be discussed in the tutorial session on May 30th at 16:15.