Problem Set
The Endowment Economy

1. Suppose the economy is populated by many identical agents. These agents act as price takers and take current and future income as given. They live for two periods: t and $t+1$. They solve a standard consumption-savings problem which yields a consumption function

$$C_t = C(Y_t, Y_{t+1}, r_t).$$

   a) What are the signs of the partial derivative of the consumption function? Explain the economic intuition.

   b) Suppose there is an increase in Y_t holding Y_{t+1} and r_t fixed. How does the consumer want to adjust its consumption and saving? Explain the economic intuition.

   c) Suppose there is an increase in Y_{t+1} holding Y_t and r_t fixed. How does the consumer want to adjust its consumption and saving? Explain the economic intuition.

   d) Now let's go to equilibrium. What is the generic definition of a competitive equilibrium?

   e) Define the IS curve and graphically derive it.

   f) Graph the Y^s curve with the IS curve and show how you determine the real interest rate.

   g) Suppose there is an increase in Y_t. Show how this affects the equilibrium real interest rate. Explain the economic intuition for this.

2. Suppose that there exist many identical households in an economy. The representative household has the following lifetime utility function:

$$U = \ln C_t + \beta \ln C_{t+1}$$

   It faces a sequence of period budget constraints which can be combined into one intertemporal budget constraint:

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

   The endowment, Y_t and Y_{t+1}, is exogenous, and the household takes the real interest rate as given.

   a) Derive the consumption function for the representative household.

   b) Solve for expressions for the equilibrium values of r_t.

   c) How does r_t react to changes in Y_t and Y_{t+1}. What is the economic intuition for this?

3. Suppose that we have an economy with many identical households. There is a government that exogenously consumes some output and pays for it with lump sum taxes. Lifetime utility for a household is:

$$U = \ln C_t + \beta \ln C_{t+1}$$

The household faces two within period budget constraints given by:

$$C_t + S_t = Y_t - T_t$$
$$C_{t+1} = Y_{t+1} - T_{t+1} + (1 + r_t)S_t$$

a) Derive the intertemporal budget constraint and the Euler equation. Is the Euler equation at all affected by the presence of taxes, T_t and T_{t+1}? Use your results to derive an expression for the consumption function.

The government faces two within period budget constraints:

$$G_t + S_t^G = T_t$$
$$G_{t+1} = T_{t+1} + (1 + r_t)S_t^G$$

b) Combine the two period budget constraints for the government into one intertemporal budget constraint.

c) Suppose that the representative household knows that the government's intertemporal budget constraint must hold. Combine this information with the household's consumption function you derived above. What happens to T_t and T_{t+1}? What is your intuition for this?

d) Equilibrium requires that $Y_t = C_t + G_t$. Plug in your expression for the consumption function (assuming that the household knows the government's intertemporal budget constraint must hold) to derive an expression for Y_t.

e) Assuming that Y_t is exogenous, what must happen to r_t after an increase in G_t?

f) Graphically analyze an increase in G_t in an endowment economy. Clearly explain the economic intuition.