Business Cycles
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Problem Set

The Consumption-Saving Model

1. Write down the Euler equation in general terms and describe its economic intuition.

2. Consider a consumer with a lifetime utility function

$$U = u(C_t) + \beta u(C_{t+1})$$

that satisfies all the standard assumptions discussed in the lecture. Assume further that the utility function is $u(C_t) = \ln C_t$. The period t and $t+1$ budget constraints are

$$C_t + S_t = Y_t$$
$$C_{t+1} + S_{t+1} = Y_{t+1} + (1+r)S_t$$

a) What is the optimal value of S_{t+1}? Impose this optimal value and derive the lifetime budget constraint.

b) Derive the Euler equation.

c) Graphically depict the optimality condition. Carefully label the intercepts of the budget constraint. What is the slope of the indifference curve at the optimal consumption basket,(C_t^*, C_{t+1}^*)?

d) Graphically depict the effects of an increase in Y_{t+1}. Carefully label the intercepts of the budget constraint. Is the slope of the indifference curve at the optimal consumption basket, (C_t^*, C_{t+1}^*), different than in part *c*?

e) Now suppose C_t is taxed at rate τ so consumers pay $1+\tau$ for one unit of period t consumption. Redo parts a-c under these new assumptions.

f) Suppose the tax rate increases from τ to τ'. Graphically depict this. Carefully label the intercepts of the budget constraint. Is the slope of the indifference curve at the optimal consumption basket, (C_t^*, C_{t+1}^*), different than in part e? Intuitively describe the roles played by the substitution and income effects. Using this intuition, can you definitively prove the sign of $\frac{\partial C_t^*}{\partial \tau}$ and $\frac{\partial C_{t+1}^*}{\partial \tau}$? It is not necessary to use math for this. Describing it in words is fine.

3. Consider the following consumption-savings problem. The consumer maximizes

$$\max_{C_t, C_{t+1}, S_t} \ln C_t + \beta \ln C_{t+1}$$

subject to the lifetime budget constraint

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

and the borrowing constraint

$$C_t \leq Y_t.$$

This last constraint says that savings cannot be negative in the first period. Equivalently, this is saying consumers cannot borrow in the first period.

a) Draw the budget constraint.

b) Assuming the constraint does not bind, what is the Euler equation?

c) Using the Euler equation, lifetime budget constraint and borrowing constraint, solve for the period t consumption function. Clearly state under what circumstances the borrowing constraint binds.

d) Suppose $Y_t = 3$, $Y_{t+1} = 10$, $\beta = 0.95$ and $r = 0.1$. Show that the borrowing constraint binds.

e) Suppose there is a one time tax rebate that increases Y_t to 4. Leave $Y_{t+1} = 10$, $\beta = 0.95$ and $r = 0.1$. What is the marginal propensity to consume out of this tax rebate?