Business Cycles

Part 2: Microeconomic Foundations
Lecture 2: Consumption-savings problem

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Outline

Part 1: Introduction

Part 2: Microeconomic Foundations

- Lecture 2: Consumption-Savings Problem
- Lecture 3: Equilibrium in an endownment economy, fiscal policy
- Lecture 4: Production and labor supply

Part 3: The Real Business Cycle Model

Part 4: The New Keynesian Model

Part 5: Financial Crises

Learning Objective of Todays Lecture

1. Understand what microeconomic foundation of a macroeconomic equation means.
2. Understand the consumption-savings problem of households and how we can derive a microfounded consumption function based on this.
3. Study several macroeconomic applications of the household‘s consumption optimization problem.

Literature

Required reading:

- Textbook chapter 9

Optional reading:

- Textbook chapter 10 (A Multi-Period Consumption-Saving Model)
- Chapter 8 on consumption in Romer‘s Advanced Macroeconomics
- Chapters 4.1 - 4.5 in Wickens‘ Macroeconomics Theory
- Chapter 15 in Sorensen‘s and Whitta-Jacobsen‘s Introducing Advanced Macroeconomics

Microeconomics of Macro

- Building blocks of the models in this course consist of decision rules of optimizing agents and a concept of equilibrium.
- Will be studying optimal decision rules first.
- Framework is dynamic but only two periods (t, the present, and $t + 1$, the future).
- Consider representative agents: one household and one firm.
- Unrealistic but useful abstraction.

Consumption

- Consumption the largest expenditure category in GDP (60-70 percent).
- Study problem of a representative household.
- Household receives exogenous amount of income in periods t and $t + 1$.
- Must decide how to divide its income in t between consumption and saving/borrowing.
- All variables are real, not nominal.

Basics

- Representative household earns income of Y_t and Y_{t+1}. Future income known with certainty.
- Consumes C_t and C_{t+1}.
- Begins life with no wealth and can save $S_t = Y_t - C_t$ (can be negative, which is borrowing).
- Earns/pays real interest rate r_t on saving/borrowing.
- Household a price-taker: takes r_t as given.
- Do not model a financial intermediary (i.e., bank), but assume existence of option to borrow/save through this intermediary.

Budget Constraints

Two flow budget constraints in each period:

$$C_t + S_t \leq Y_t$$

$$C_{t+1} + S_{t+1} \leq Y_{t+1} + (1 + r_t)S_t$$

- Savings S_t link both periods.
- $r_t\, S_t$: income earned on the stock of savings brought into $t\ +1$

Terminate conditions:

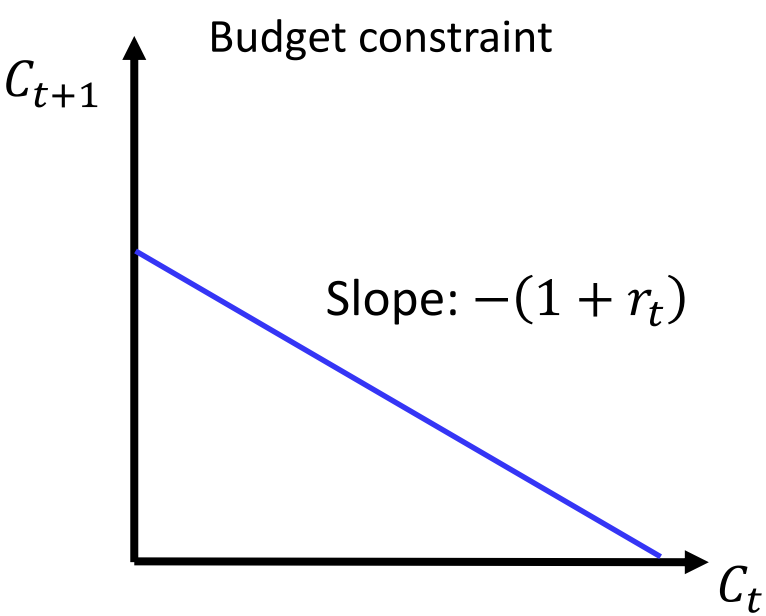
- Household would not want $S_{t+1} > 0$. Why? There is no $t + 2$. Don’t want to die with positive assets.
- Household would like $S_{t+1} < 0$ – die in debt. Lender would not allow that.
- Hence, $S_{t+1} = 0$ is a terminal condition (sometimes “no Ponzi”).

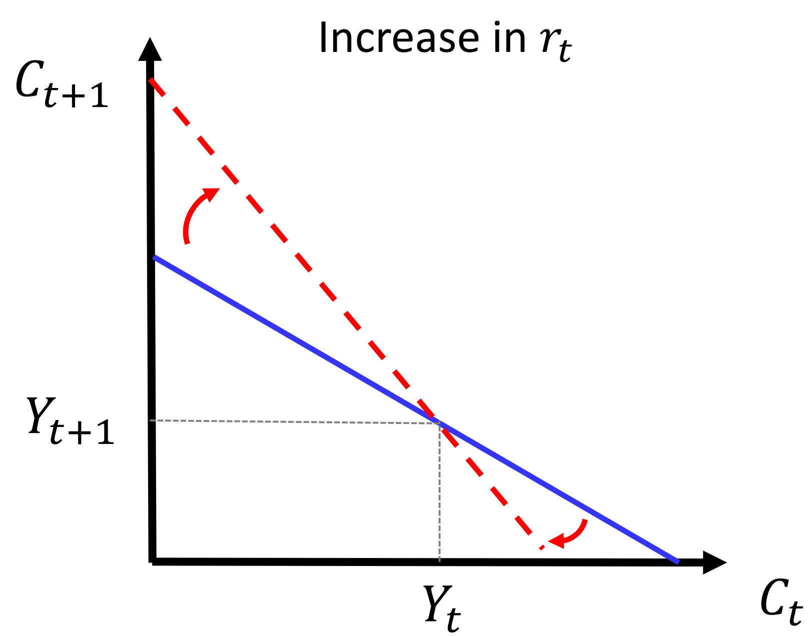
Intertemporal Budget Constraint

- Assume budget constraints hold with equality (otherwise leaving income on the table), and eliminate S_t, leaving:

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}$$

- This is called the intertemporal budget constraint (IBC).
- Says that present discounted value of stream of consumption equals present discounted value of stream of income.





Preferences

- Household gets utility from how much it consumes
- Utility function: $u(C_t)$. "Maps" consumption into utility units
- Assume:
  - $u'(C_t) > 0$ (positive marginal utility)
  - $u''(C_t) < 0$ (diminishing marginal utility)
  - "More is better, but at a decreasing rate"
- Example utility function:

$$u(C_t) = \ln(C_t)$$
$$u'(C_t) = \frac{1}{C_t} > 0$$
$$u''(C_t) = -\frac{1}{C_t^2} < 0$$

- Utility is completely ordinal – no meaning to magnitude of utility (it can be negative). Only useful to compare alternatives

Lifetime Utility

Lifetime utility is a weighted sum of utility from period t and $t+1$ consumption:

$$U = u(C_t) + \beta\, u(C_{t+1})$$

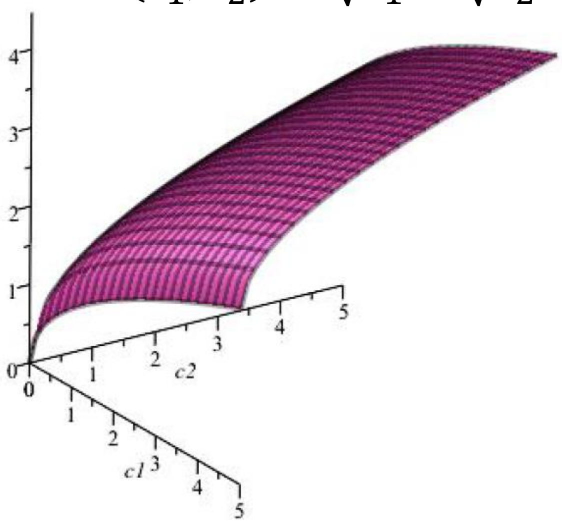
- $0 < \beta < 1$ is the discount factor – it is a measure of how impatient the household is.

Utility Functions Graphically

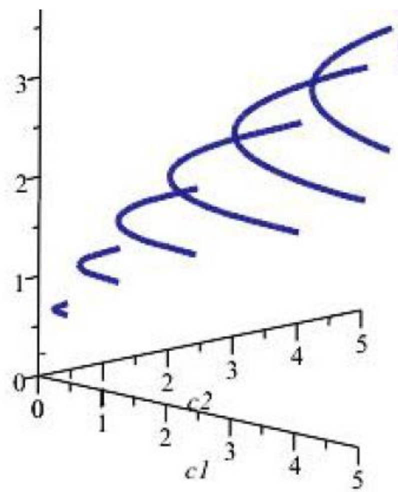
Describe how much satisfaction an individual experiences from consuming goods

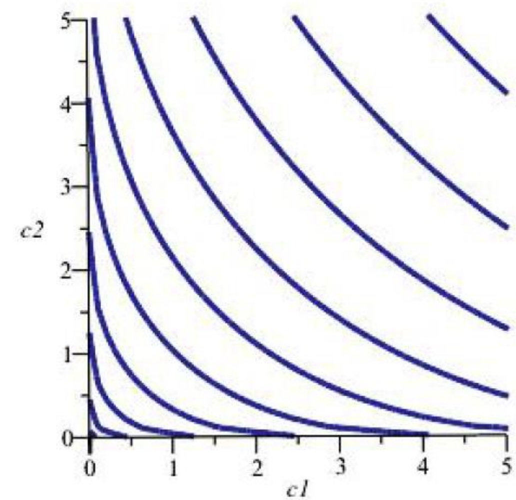
- Utility functions with two or more arguments are useful to study trade-offs
- Standard property: strictly increasing at a strictly decreasing rate (diminishing marginal utility): $u' > 0, u'' < 0$.

$$U(c_1, c_2) = \sqrt{c_1} + \sqrt{c_2}$$



Indifference curves





Source: Chugh (2016)

Indifference Curve

- Think of C_t and C_{t+1} as different goods (different in time dimension)
- Indifference curve: combinations of C_t and C_{t+1} yielding fixed overall level of lifetime utility
- Different indifference curve for each different level of lifetime utility. Direction of increasing preference is northeast
- Slope of indifference curve at a point is the negative ratio of marginal utilities:

$$slope = -\frac{u'(C_t)}{\beta\, u'(C_{t+1})}$$

- Given assumption of $u''(\cdot) < 0$, steep near origin and flat away from it
- This is known as the *marginal rate of substitution*: Maximum quantity of one good the consumer is willing to give up to obtain an extra unit of the other good.

Optimization

- Lagrange function:

$$\max L(C_t, C_{t+1}, \lambda) = u(C_t) + \beta u(C_{t+1}) + \lambda \left[Y_t + \frac{Y_{t+1}}{1+r_t} - C_t - \frac{C_{t+1}}{1+r_t} \right]$$

- FOC:

$$u'(C_t) - \lambda = 0$$

$$\beta u'(C_{t+1}) - \lambda \frac{1}{1+r_t} = 0$$

$$Y_t + \frac{Y_{t+1}}{1+r_t} - C_t - \frac{C_{t+1}}{1+r_t} = 0$$

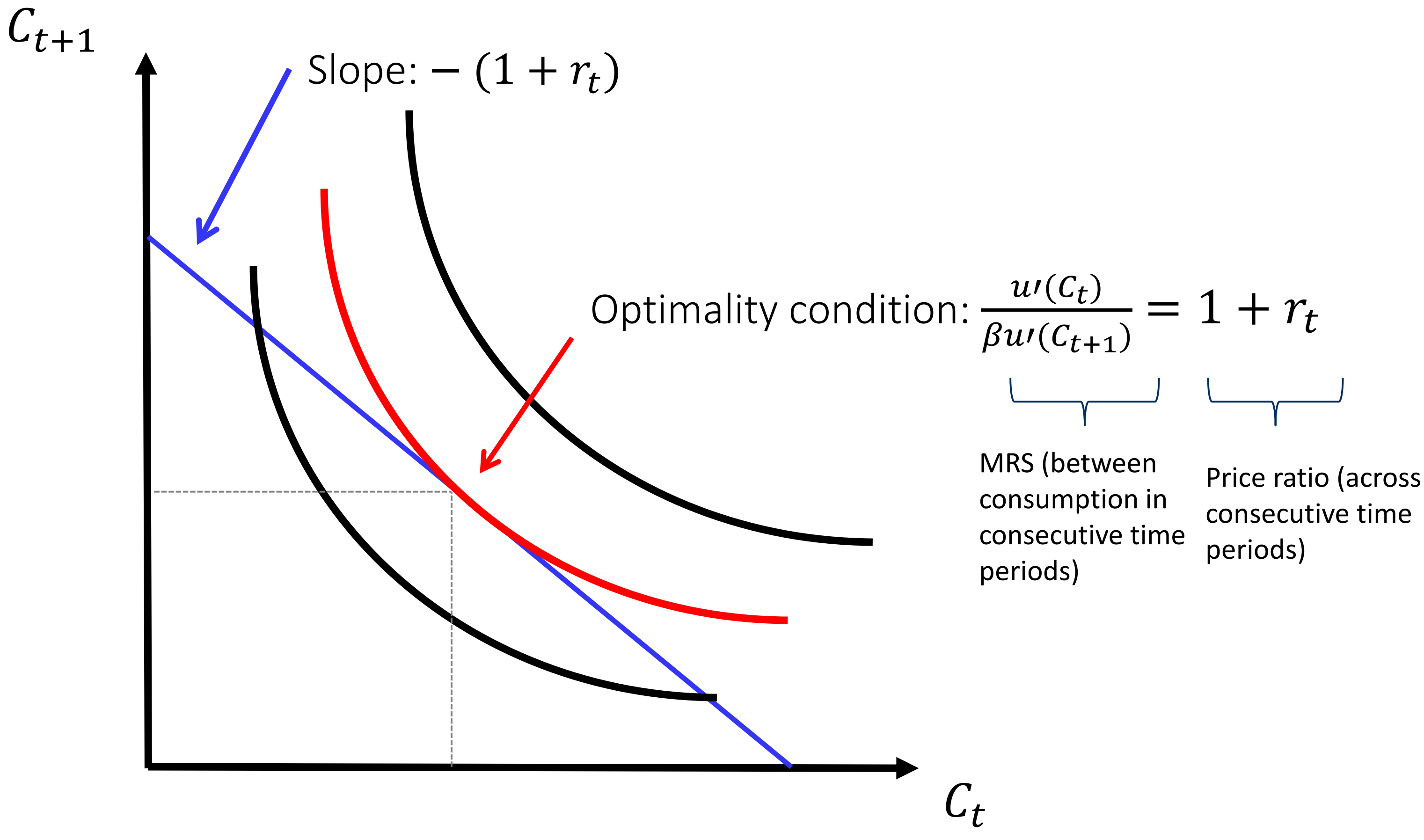
- Rearranging and eliminating λ yields the Euler equation:

$$\frac{u'(C_t)}{\beta u'(C_{t+1})} = 1 + r_t$$

MRS = - slope of budget constraint
(price ratio between period t and t+1)

Optimization Graphically

- Objective is to choose a consumption bundle on highest possible indifference curve
- At this point, indifference curve and budget line are tangent (which is same condition as Euler equation)



Consumption Function

- What we want is a decision rule that determines C_t as a function of things which the household takes as given: Y_t, Y_{t+1}, and r_t
- Consumption function:

$$C_t = C^d(Y_t, Y_{t+1}, r_t)$$

- Can use indifference curve – budget line diagram to qualitatively figure out how changes in Y_t, Y_{t+1}, and r_t affect C_t

Increases in Y_t and Y_{t+1}

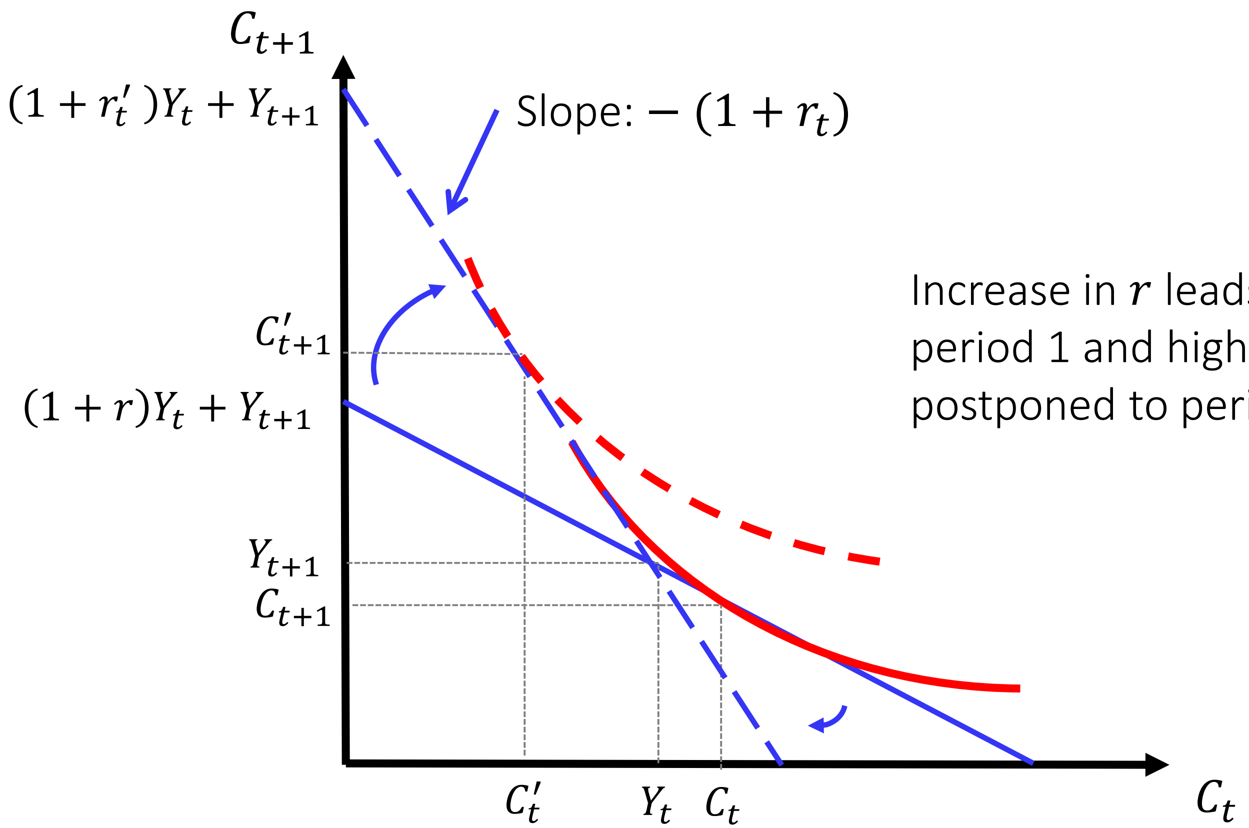
- An increase in Y_t or Y_{t+1} causes the budget line to shift out horizontally to the right
- In new optimum, household will locate on a higher indifference curve with higher C_t and C_{t+1}
- Important result: wants to increase consumption in both periods when income increases in either period
- Wants its consumption to be "smooth" relative to its income
- Achieves smoothing its consumption relative to income by adjusting saving behavior increases S_t when Y_t goes up, reduces S_t when Y_{t+1} goes up
- Can conclude that $\frac{\partial C_t}{\partial Y_t} > 0$ and $\frac{\partial C_t}{\partial Y_{t+1}} > 0$
- Further, $\frac{\partial C_t}{\partial Y_t} < 1$. Call this the marginal propensity to consume, MPC

Increase in r_t

- A little trickier
- Causes budget line to become steeper, pivoting through endowment point
- Competing income and substitution effects:
    - Substitution effect: how would consumption bundle change when r_t increases and income is adjusted so that household would locate on unchanged indifference curve?
    - Income effect: how does change in r_t allow household to locate on a higher/lower indifference curve?
- Substitution effect always to reduce C_t, increase S_t
- Income effect depends on whether initially a borrower ($C_t > Y_t$, income effect to reduce C_t) or saver ($C_t < Y_t$, income effect to increase C_t)

Increase in the real interest rate

- What happens if the central bank increases the interest rate?
- Assume that prices are fixed in the *short-term* so that a change in the nominal interest rate lead to a change in the real interest rate ($i_t = r_t + \bar{\pi}$).
- Main transmission mechanism in New Keynesian models: *intertemporal substitution of consumption*.



Increase in r leads to lower consumption in period 1 and higher savings. Consumption is postponed to period 2.

Note that we have chosen the form of the utility function and incomes Y_t, Y_{t+1} in a way that the substiution effect dominates which is in line with the empirical macro evidence.

The Consumption Function

- We will assume that the substitution effect always dominates for the interest rate
- Qualitative consumption function (with signs of partial derivatives)

$$C_t = C^d(\underbrace{Y_t}_{+}, \underbrace{Y_{t+1}}_{+}, \underbrace{r_t}_{-})$$

- Example with log utility: Suppose $u(C_t) = \ln(C_t)$
  - Euler equation is:

$$C_{t+1} = \beta(1 + r_t)C_t$$

  - Budget constraint:

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}$$

  - Combine both to get the consumption function is:

$$C_t = \frac{1}{1 + \beta}\left[Y_t + \frac{Y_{t+1}}{1 + r_t}\right]$$

  - MPC: $\frac{1}{1+\beta}$
  - Go through other partials.

Permanent Income Hypothesis (PIH)

- Our analysis is consistent with Friedman's (1957) PIH
- Consumption ought to be a function of "permanent income"
- Permanent income: present value of lifetime income
- Special case: $r_t = 0$ and $\beta = 1$: consumption equal to average lifetime income
- Implications:
  1. Consumption forward-looking. Consumption should not react to changes in income that were predictable in the past
  2. MPC less than 1.
  3. The longer you live, the lower is the MPC.
- Important empirical implications for econometric analysis. Regression of C_t on Y_t will not identify MPC (which is relevant for things like fiscal multiplier) if in historical data changes in Y_t are persistent.

Applications and Extensions

We will consider several applications / extensions:

1. Wealth
2. Permanent vs. transitory changes in income
3. Random walk / Borrowing Contraints

Wealth

- Allow household to begin life with stock of wealth H_{t-1}. Real price of this asset in t is Q_t
- Household can accumulate more of this asset or sell it.
- Think about a quantity of housing or shares of stock.
- Period t constraint:

$$C_t + S_t + Q_t(H_t - H_{t-1}) \leq Y_t$$

- Period $t + 1$ constraint:

$$C_{t+1} + S_{t+1} + Q_{t+1}(H_{t+1} - H_t) \leq Y_{t+1} + (1 + r_t)S_t$$

- Imposing terminal conditions, IBC is:

$$C_t + \frac{C_{t+1}}{1 + r_t} + Q_t H_t = Y_t + \frac{Y_{t+1}}{1 + r_t} + Q_t H_{t-1} + \frac{Q_{t+1} H_t}{1 + r_t}$$

Simplifying Assumptions and the Consumption Function

- First, assume household must choose $H_t = \mathbf{0}$. It simply sells off the asset in period t at price Q_t:

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t} + Q_t H_{t-1}$$

- Increase in Q_t then functions just like increase in Y_t

$$C_t = C^d(\underbrace{Y_t}_{+}, \underbrace{Y_{t+1}}_{+}, \underbrace{r_t}_{-}, \underbrace{Q_t}_{+})$$

- Empirical application: stock market boom (increase in Q_t)

Alternative Simplifying Assumption

- Assume $H_{t-1} = 0$, and assume that household must purchase an exogenous amount of the asset, H_t (e.g. has to buy a house)
- IBC:

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t} + H_t\left(\frac{Q_{t+1}}{1+r_t} - Q_t\right)$$

- Increase in Q_{t+1}: functions like increase in Y_{t+1}:

$$C_t = C^d(\underbrace{Y_t}_{+}, \underbrace{Y_{t+1}}_{+}, \underbrace{r_t}_{-}, \underbrace{Q_t}_{-}, \underbrace{Q_{t+1}}_{+})$$

- Empirical applications: house price boom (anticipated increase in Q_{t+1})

Permanent vs. Transitory Changes in Income

- Go back to standard consumption function:

$$C_t = C^d(\underbrace{Y_t}_{+}, \underbrace{Y_{t+1}}_{+}, \underbrace{r_t}_{-})$$

- Take total derivative (differs from partial derivative in allowing everything to change):

$$dC_t = \frac{\partial C^d(.)}{\partial Y_t} dY_t + \frac{\partial C^d(.)}{\partial Y_{t+1}} dY_{t+1} + \frac{\partial C^d(.)}{\partial r_t} dr_t$$

- If just $dY_t \neq 0$, then $\frac{dC_t}{dY_t}$ equal to partial $\frac{\partial C^d(.)}{\partial Y_t}$
- But if changes in income are persistent ($dY_t > 0 \Rightarrow dY_{t+1} > 0$), then $\frac{dC_t}{dY_t} > \frac{\partial C^d(.)}{\partial Y_t}$
- Implication: consumption reacts more to a change in income the more persistent is that change in income

Application: Tax Cuts

- Suppose household pays taxes, T_t and T_{t+1}, to government each period, so net income is $Y_t - T_t$ and $Y_{t+1} - T_{t+1}$. Consumption function is:

$$C_t = C^d(Y_t - T_t, Y_{t+1} - T_{t+1}, r_t)$$

- A cut in taxes is equivalent to an increase in income
- Implication: tax cuts will have bigger stimulative effects on consumption the more persistent the tax cuts are
- Empirical studies: Shaprio and Slemrod (2003) and Shapiro and Slemrod (2009)
  - Initial installment of Bush tax cuts in 2001 was close to permanent (ten years). Theory predicts consumption ought to react a lot. It didn't.
  - US tax rebates 2008: known to be only one time. Theory predicts consumption should react comparatively little. It did.

Random Walk Hypothesis

- Suppose that $\beta(1 + r) = 1$
- Suppose that future income is not known with certainty, so that the Euler equation includes an expectation operator:

$$u'(C_t) = E[u'(C_{t+1})]$$

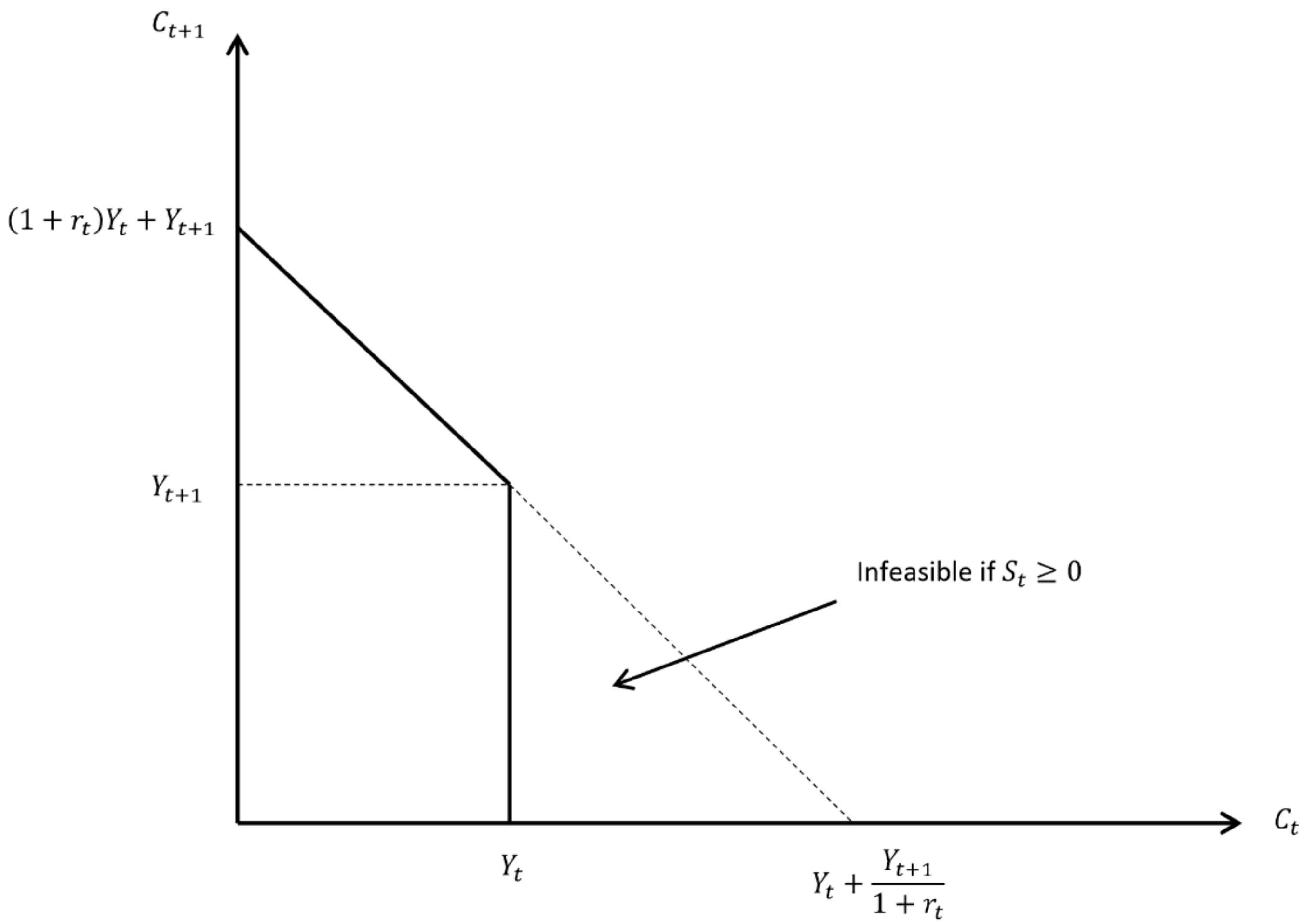
- Suppose that $u'''(\cdot) = 0$ (so no precautionary saving). Then this implies that $E[C_{t+1}] = C_t$
- In expectation, future consumption ought to equal current consumption. This is the “random walk” hypothesis
- Doesn’t mean that future consumption always equals current consumption
- But it does imply future changes in consumption ought to not be predictable, because in expectation future consumption should equal current consumption
- Random walk model due to Hall (1978)

Empirical Tests

- Random walk hypothesis one of the most tested macroeconomic theories
- Generally fails:
    - Parker (1999): exploits facts about social security withholding and predictable changes of these over course of the year. Consumption reacts to predictable changes in take home pay ⇒ inconsistent with predictions of RWH
    - Evans and Moore (2012): look at relationship between receipt of paycheck (which is predictable) and within-month mortality cycle - They argue that the receipt of a paycheck, leads to a consumption boom, which triggers higher mortality ⇒ inconsistent with predictions of RWH since consumption shouldn't react to such a predictable change in income

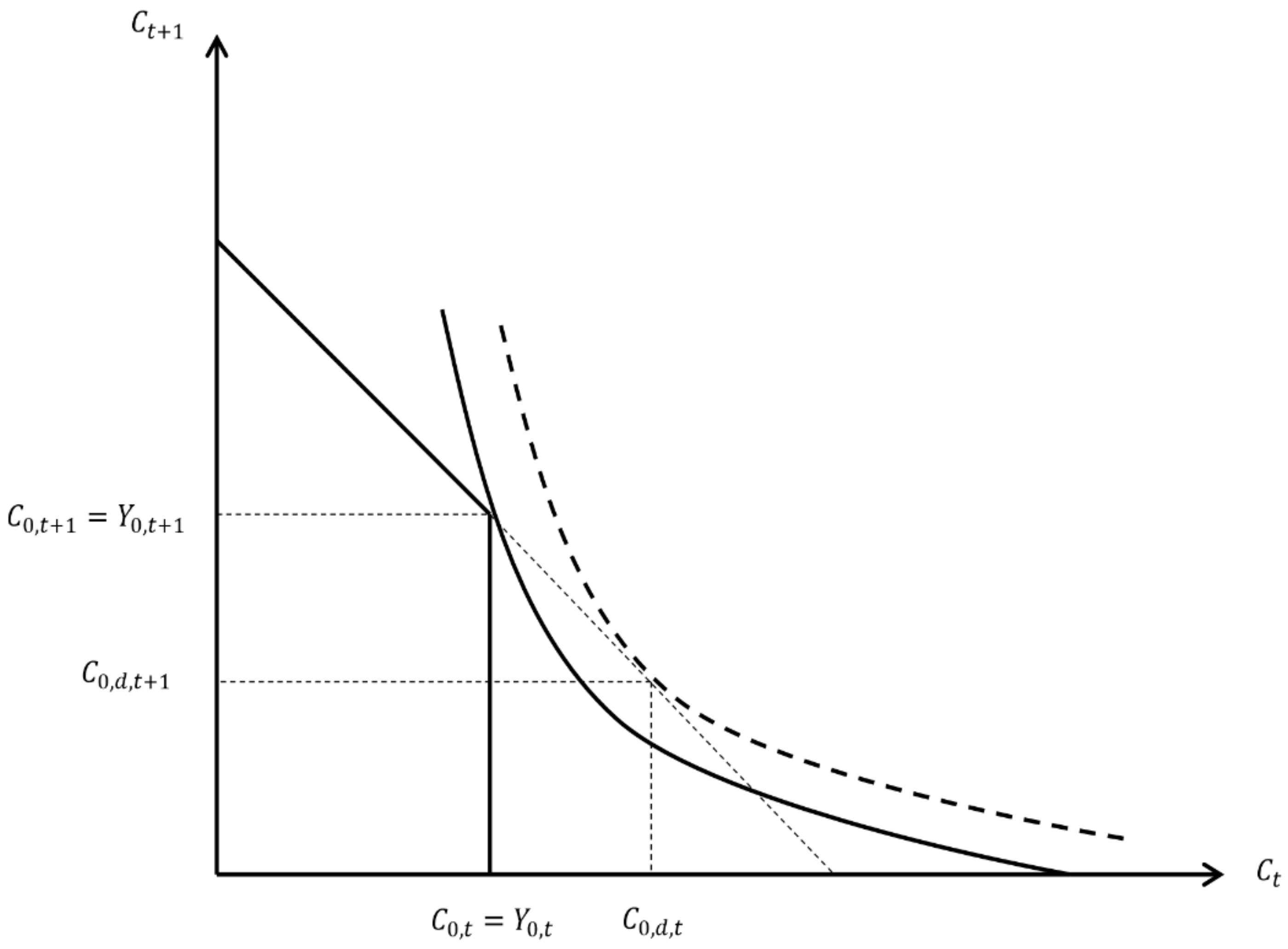
Borrowing Constraints

- Empirical failures can potentially be accounted for by borrowing constraints
- Simplest form of a borrowing constraint: you can't. $S_t \geq 0$. Introduces kink into budget line.



Binding Borrowing Constraint

If borrowing constraint "binds" you locate at the kink in the budget line (i.e. Euler equation does not hold)



Implications of a Binding Borrowing Constraint

- Current consumption equals current income
- Means that if household gets more income, will spend all of it
- Further means that if household expects more income in future, can't adjust consumption until the future – future consumption will react to anticipated change in income
- Potential resolution of some empirical failures of random walk / permanent income hypothesis (PIH) model
- Also has policy implications. Makes sense to target taxes/transfers to households likely to be borrowing constrained if objective is to stimulate consumption

Summary

- Modern macroeconomics is based on microeconomic foundations, i.e. representative agent theory.
- Two-period framework is very useful to develop intuition.
- Households maximize utility function with respect to a budget constraint.
- Households like to smooth consumption via saving.
- Optimal solution: Slope of the indifference curve (marginal rate of substitution) equals the slope of the budget constraint.
- Households are forwards looking: Consumption depends on current and future income. Additional income increases consumption. How much? Depends on persistence of income increase.
- Changes in the real interest rate lead to intertemporal substitution.
- Microfounded Consumption Function: Decision rule that relates optimal consumption to things that the household takes as given.
- Have studied a number of extensions and applications.