Lehrstuhl für Informatik I
Algorithmen und
Komplexität
Universität Würzburg

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Exercise sheet 2

Visualization of Graphs

Exercise 1 – Unit edge lengths

In a drawing of a graph G with *unit edge lengths* each edge is drawn as a line segment of length 1.

a) Prove or disprove that all trees admit a *crossing-free* drawing with unit edge lengths.
2 Points

We now go one step further and consider drawings with unit edge lengths of G where the Euclidean distance between two vertices u and v is equal to the distance of u and v in G (i.e. equal to the number of edges of a shortest path from u to v).

b) Characterize the set of connected graphs that can be drawn in this way. **3 Points**

Exercise 2 – Adapting forces for positioning

In the force-directed approach, we may add additional forces to all or some vertices. Describe functions for forces that are suitable to

a) keep a vertex v close to a specified position,

b) position a vertex u close to the x-axis,

c) align an edge $\{a, b\}$ parallel to the y-axis (approximately),

d) draw directed edges upward.

6 Points

Exercise 3 – Adapting forces for vertices with area > 0

The force-directed methods introduced in the lecture assume that all vertices are represented as points, i.e., disks with radius 0. Which modifications are necessary to represent vertices as disks? What about other convex shapes? **4 Points**

Hint: For disks, consider a physical analogy again first.

Exercise 4 – Tutte Drawings

Prove the following properties for Tutte drawings.

a) If G is connected, then a Tutte drawing can have vertex overlaps. **1 Point**

b) If G is 2-connected, then a Tutte drawing can have vertex overlaps.

   Hint: Try to find small examples. You don't need many vertices! What is the smallest example you can find? **2 Points**

c) In the literature, the Tutte forces are often described without dividing by the degree of the vertex:

$$f_{\text{attr}}(u, v) = \begin{cases} 0 & u \text{ fixed} \\ ||p_u - p_v|| & \text{else} \end{cases}$$

   Find an example where iteratively applying these forces does not find the equilibrium. Does that mean that no equilibrium exists?

   Hint: Find a situation where a vertex "shoots" too far over the optimum position.

   4 Points

This assignment is due at the beginning of the next lecture, that is, on May 12th at 10 am. Please submit your solutions via WueCampus. The exercises will be discussed in the tutorial session on May 16th at 16:00.