Business Cycles
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# Problem Set

## Math Primer and Macroeconomic Variables

1. **Logs:**

   a) Express the following equations as log-linear functions, i.e. take logs and simplify as much as possible.

      i) $Y = zK^{\alpha}N^{1-\alpha}$

      ii) $Z = ce^{rt}\beta^{K}$

   b) Suppose that the growth rate of some variable, $X$, is constant and equal to $g > 0$ from time $t_0$ to time $t_1$; drops to 0 at time $t_1$; rises gradually from 0 to $g$ from time $t_1$ to time $t_2$; and is constant and equal to $g$ after time $t_2$.

      i) Sketch a graph of the growth rate of $X$ as a function of time.

      ii) Sketch a graph of $\ln X$ as a function of time.

2. **Growth Rates:**

   a) Show that the growth rate of the ratio of two variables is approximately the difference of their growth rates. Hint: Remember the fact that the growth rate of a variable equals approximately the log first difference.

   b) The real GDP of Germany, measured in year 2010 prices, rose from EUR 2,038,505 million in 1991 to EUR 2,843,226 million in 2016. What was the average annual growth rate?

3. **Calculus:**

   Calculate all the first, second, and cross derivatives of the following functions.

   a) $F(K,N) = zK^{\alpha}N^{1-\alpha}$

   b) $F(K,N) = \ln(z) + \alpha\ln(K) + (1-\alpha)\ln(N)$

   c) $U(C,L) = \dfrac{C^{1-\gamma}-1}{1-\gamma} + L$

   d) $F(K,N) = \left[\alpha K^{\frac{\nu-1}{\nu}} + (1-\alpha)N^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}$ (first derivatives are sufficient)

4. **Optimization:**

   a) Solve the following constrained maximization problem using Lagrange multipliers!

$$\max_{x_1,x_2,x_3} U = x_1^{\alpha_1}x_2^{\alpha_2}x_3^{\alpha_3}$$
$$\text{s.t.} \quad w_0 = p_1x_1 + p_2x_2 + p_3x_3$$

   b) Consider an individual who receives utility from consumption, $c$, and leisure, $l$. The individual has $\bar{L}$ time to allocate to work, $n$, and leisure. The individual's consumption is a function of how much he works. In particular, $c = \sqrt{n}$. The individual's maximization problem is

$$\begin{aligned} &\max_{c,l,n} U = \ln(c) + \theta l \\ &\text{s.t.} \quad c = \sqrt{n} \\ &\qquad\ \ \bar{L} = n + l \end{aligned}$$

where $\theta > 0$. Solve the maximization problem!

5. **Further (voluntary) homework:**
Download data of the GDP components of your home country - or any other country you like - from **Penn World Tables** or from the homepage of the **OECD**. Try to replicate Figure 1.2 from the lecture slides. Do the stylized facts discussed in the lecture hold also for this country?