Lehrstuhl für Informatik I
Algorithmen und
Komplexität
Universität Würzburg

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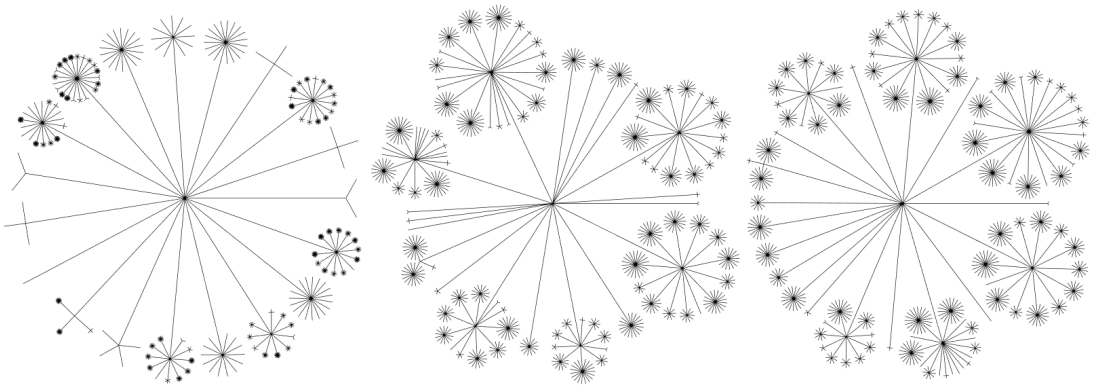
Exercise sheet 1

Visualization of Graphs

Exercise 1 – Drawing conventions & aesthetics of balloon layouts

The three drawings of the same tree below are drawn with a *balloon layout*. Try to find at least two common drawing conventions and two possible drawing aesthetics for this layout style. **2 Points**



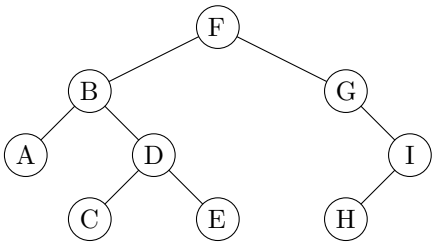
Exercise 2 – Binary trees with pre- and postorder coordinates

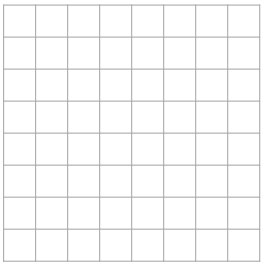
Let $T = (V, E)$ be a binary tree with root r. For each $v \in V$, let $x(v) := \mathrm{preorder}(v)$ and $y(v) := \mathrm{postorder}(v)$. Recall that $T(v)$ denotes the subtree rooted at v.

You may use the graph and grid below to try an example.

a) Prove that this coordinate assignment yields a planar drawing of T. **4 Points**

b) Give thight bounds on the area requirement of the generated drawing. **2 Points**





c) Prove that if you direct all edges of T such that they "point away" from r – that is, all vertices can be reached from r – then all arcs in the drawing point downwards. **2 Points**

Exercise 3 – Space-saving HV-drawings of complete binary trees

Let T be a *complete binary tree* of height h, that is, a binary tree where all vertices of depth $1, \dots, h-1$ have exactly 2 children and all vertices of depth h are leaves. Consider the following HV-drawing algorithm.

Algorithm 1: BalancedHVDraw(node v, depth d)

if $v ==$ nil **then return** $\emptyset$
$v_l, v_r \leftarrow$ left / right child of v
$\Gamma_1 \leftarrow$ BalancedHVDraw(v_l, $d+1$)
$\Gamma_2 \leftarrow$ BalancedHVDraw(v_r, $d+1$)
if d odd **then return** horizontal combination of Γ_1 and Γ_2
if d even **then return** vertical combination of Γ_1 and Γ_2

a) Prove that the right-heavy HV-layout algorithm from the lecture produces a drawing of T with area $\Omega(n \log n)$. **2 Points**

b) Prove that BalancedHVDraw produces a drawing of T with area $O(n)$. **6 Points**

c) What is the aspect ratio (i.e., the ratio between the width and the height) of the generated drawing in the worst case? **2 Points**

This assignment is due at the beginning of the next lecture, that is, on May 05 at 10 am. Please submit your solutions via WueCampus. The exercises will be discussed in the tutorial session on May 02 at 16:00.