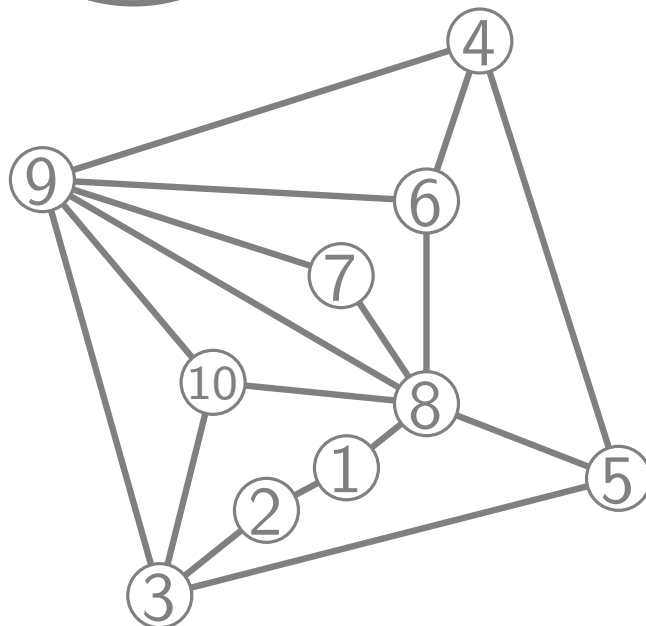
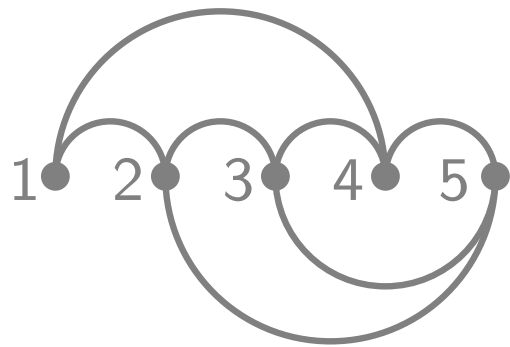


Visualization of Graphs

Lecture 1a:

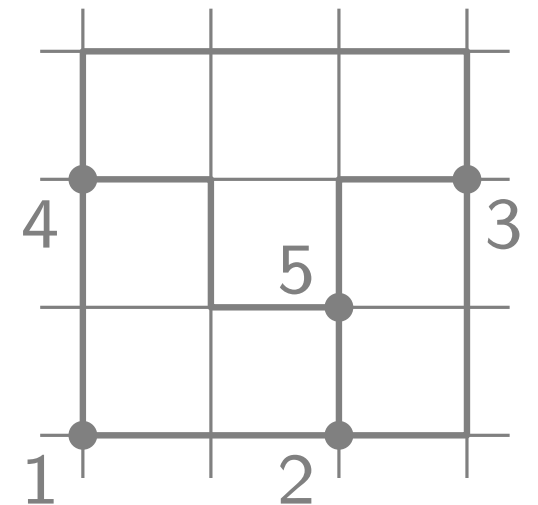
The Graph Visualization Problem



Part I:
Organizational & Overview

Alexander Wolff

Slide layout: Jonathan Klawitter



Organizational

Lectures: ■ Alexander Wolff (M4, room 01.001, alexander.wolff@uni-...)

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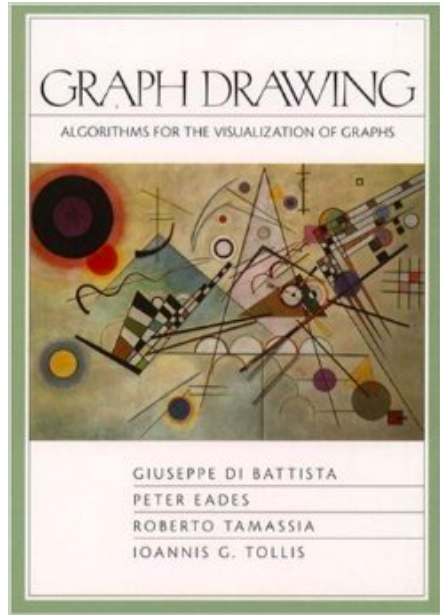
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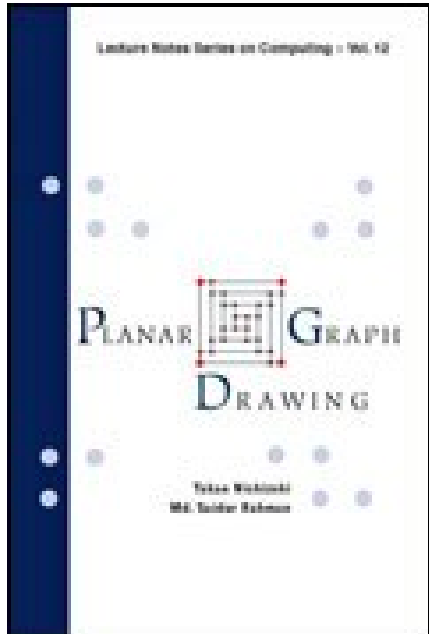
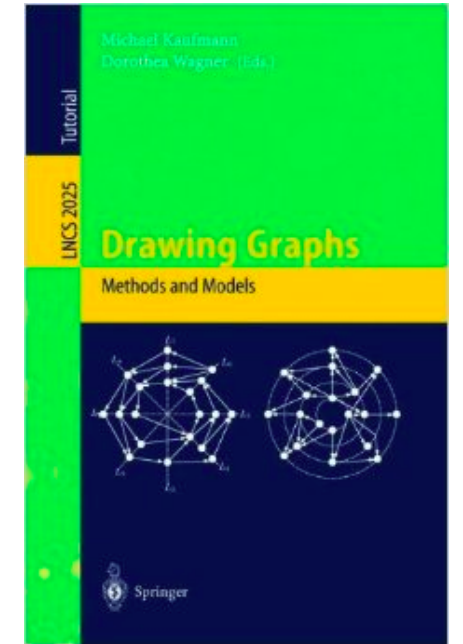
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Books



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Graph Drawing: Algorithms for the Visualization of Graphs
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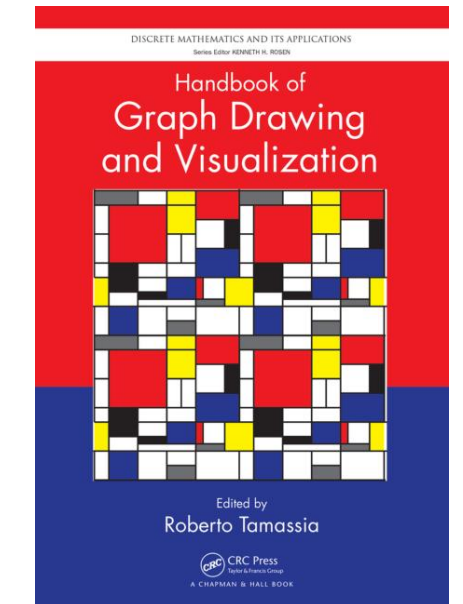
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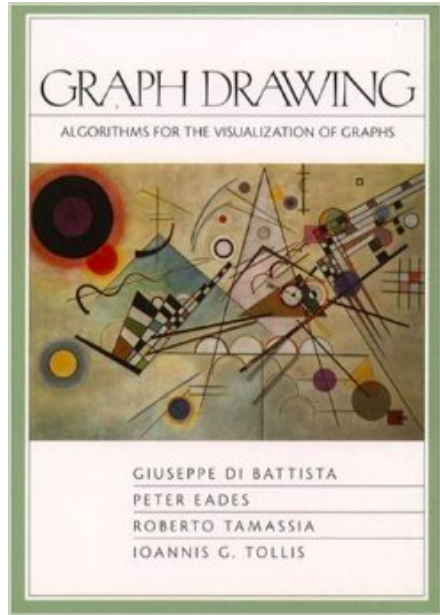
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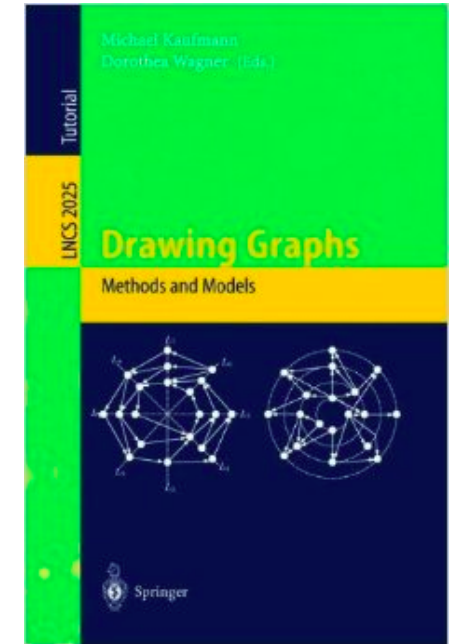


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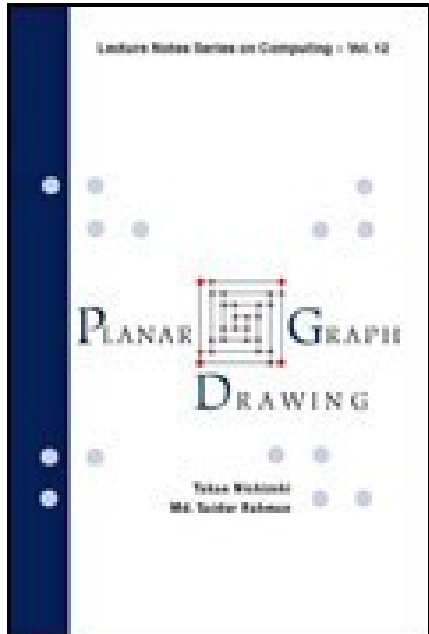
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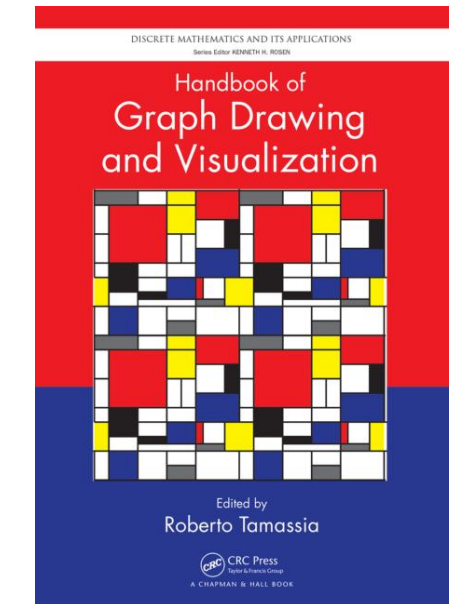


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- Overview of graph visualization

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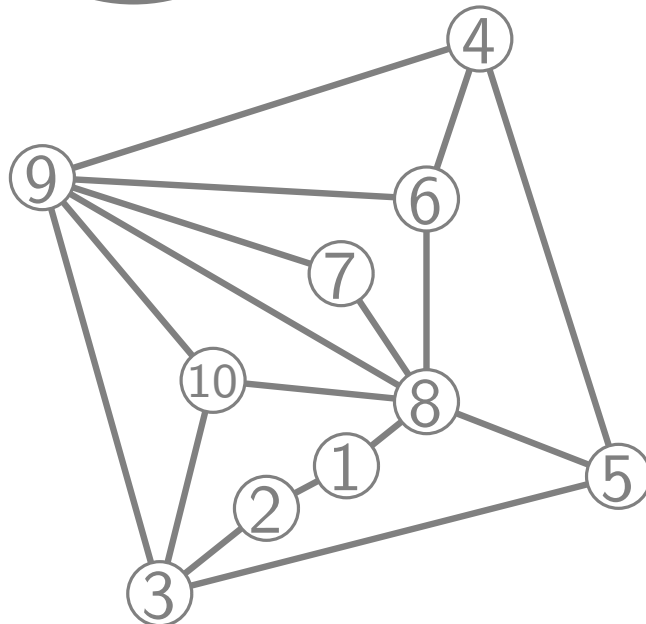
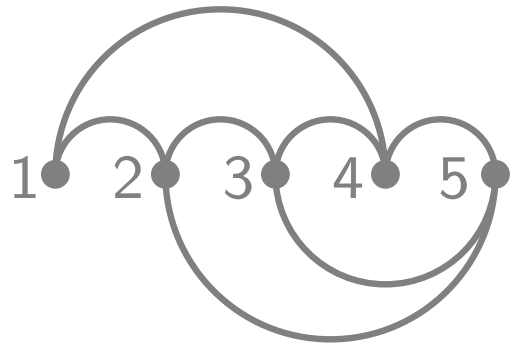
Topics

- Drawing Trees and Series-Parallel Graphs
- Tutte Embedding and Force-Based Drawing Algorithms
- Straight-Line Drawings of Planar Graphs
- Orthogonal Grid Drawings
- Octilinear Drawings for Metro Maps
- Upwards Planar Drawings
- Hierarchical Layouts of Directed Graphs
- Contact Representations
- Visibility Representations
- The Crossing Lemma
- Beyond Planarity

Visualization of Graphs

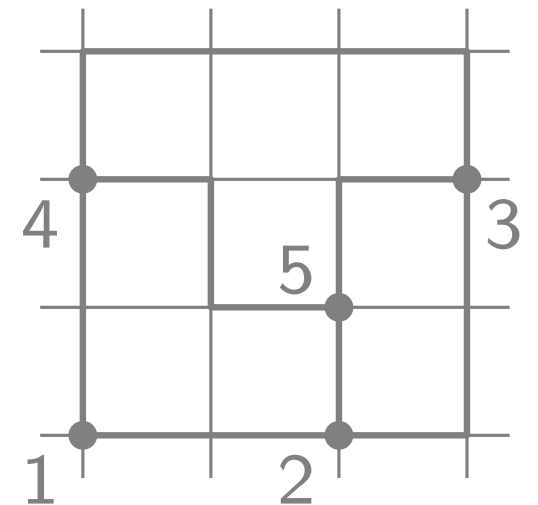
Lecture 1a:

The Graph Visualization Problem



Part II: The Layout Problem

Jonathan Klawitter



Graphs and their representations

What is a graph?

- graph $G = (V, E)$
- vertices $V = \{v_1, v_2, \dots, v_n\}$
- edge $E = \{e_1, e_2, \dots, e_m\}$

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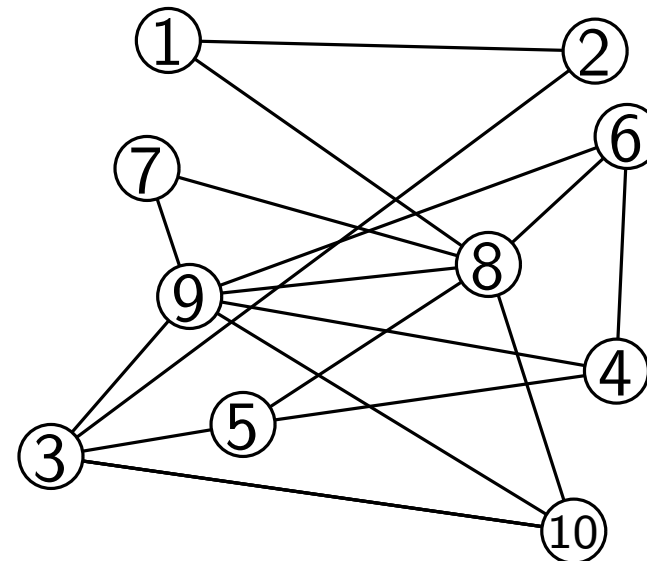
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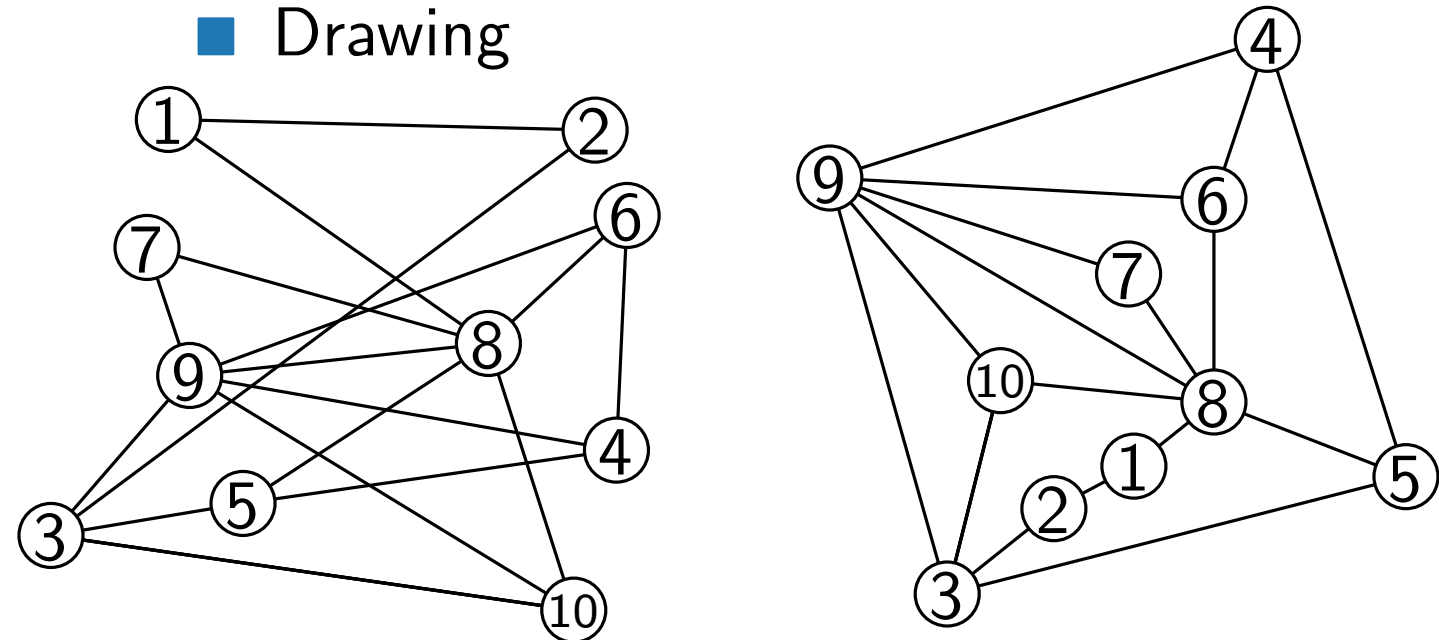
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■ Drawing



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- Metro systems
- Road networks
- Power grids
- Telecommunication networks
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Abstract networks

- Social networks
- Communication networks
- Phylogenetic networks
- Metabolic networks
- Class/Object Relation Diagrams (UML)
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We need algorithms that draw graphs automatically to make networks more accessible to humans.

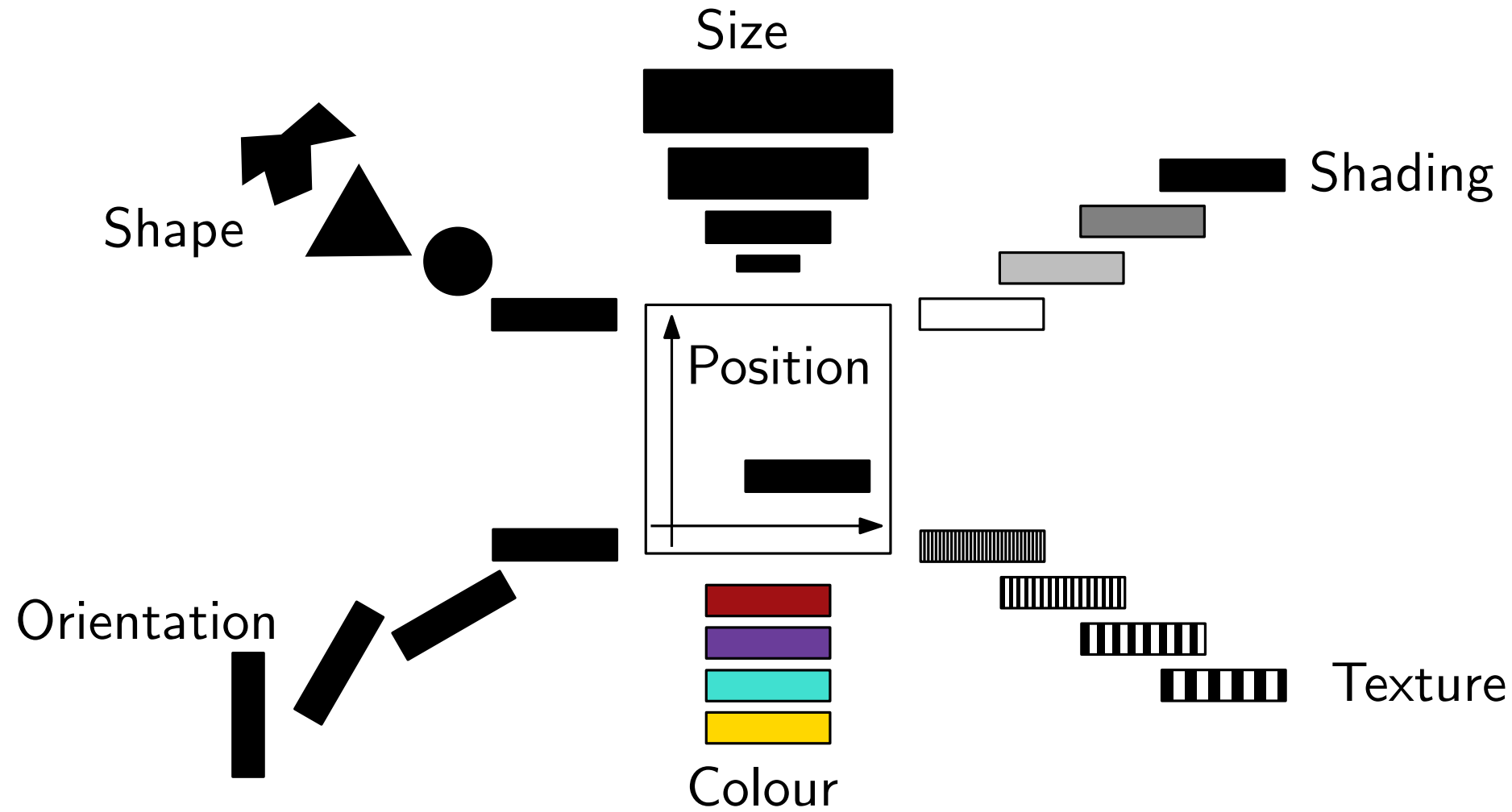
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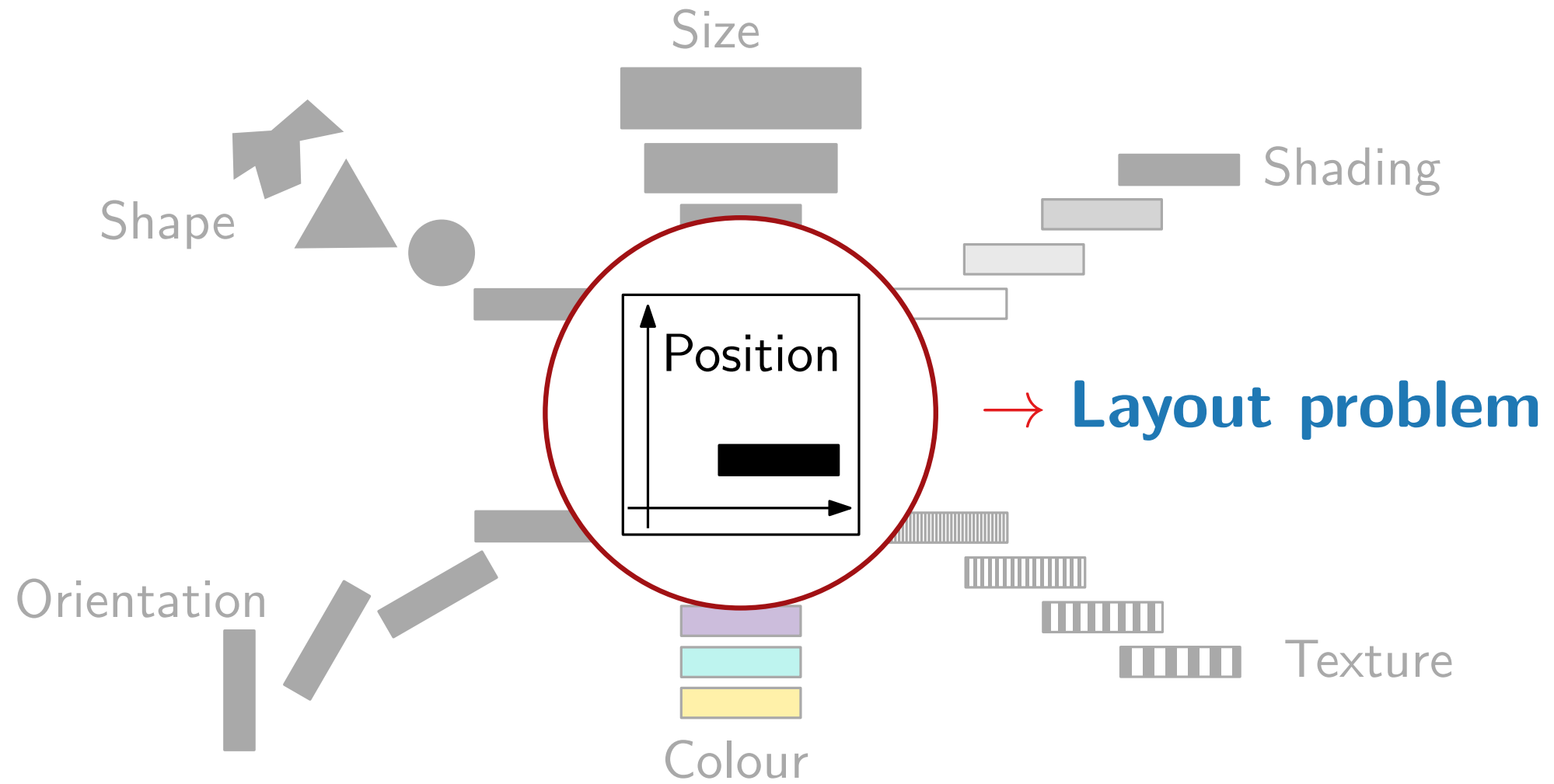
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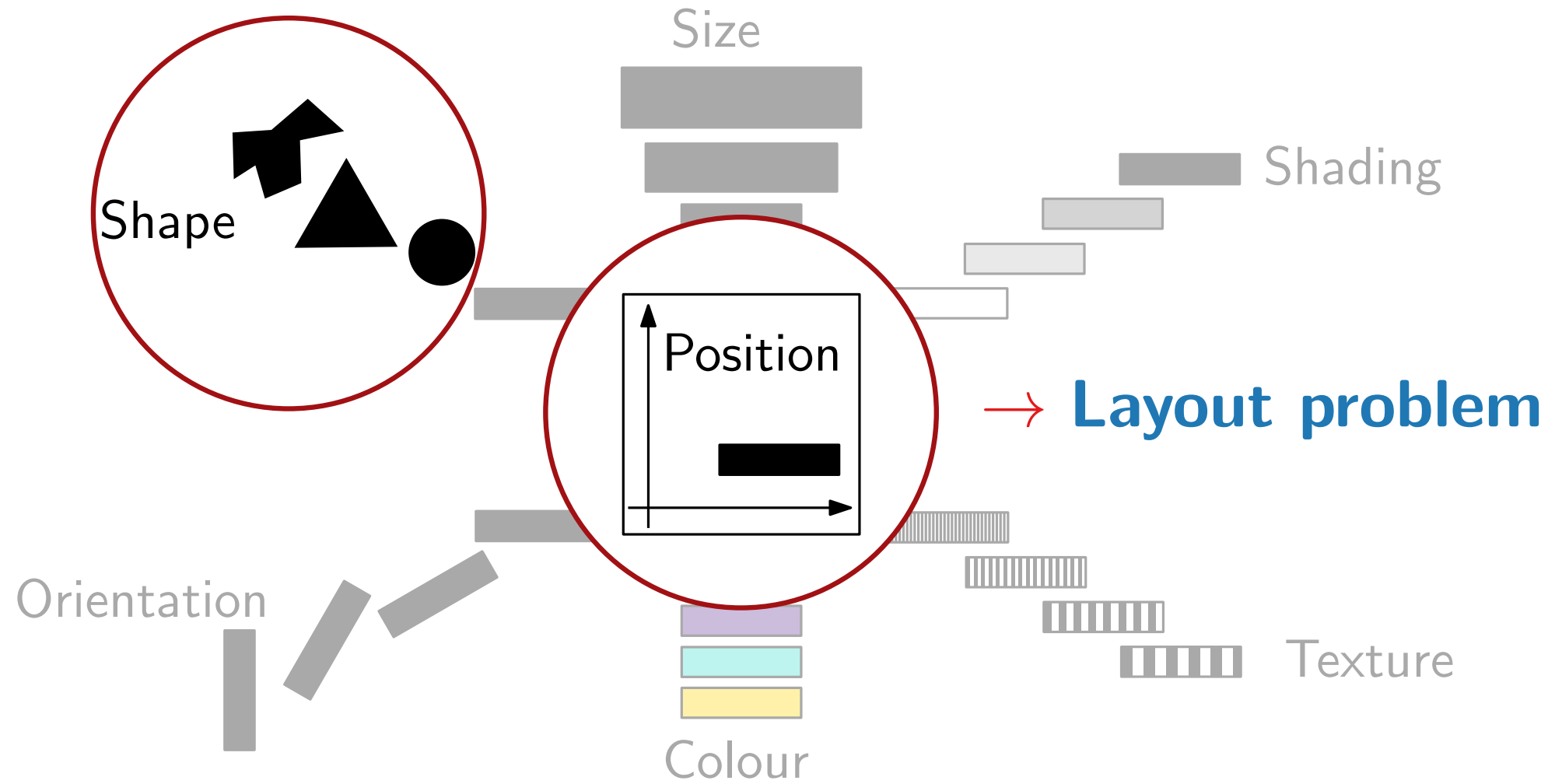
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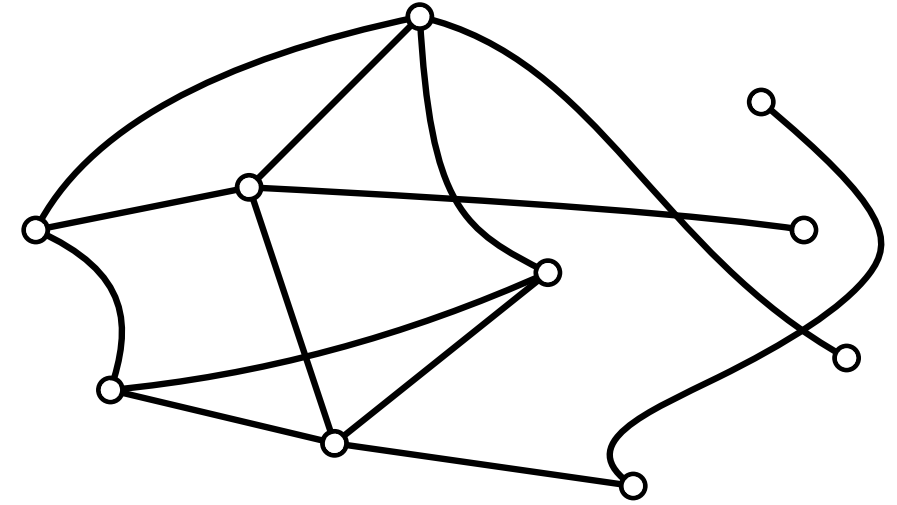
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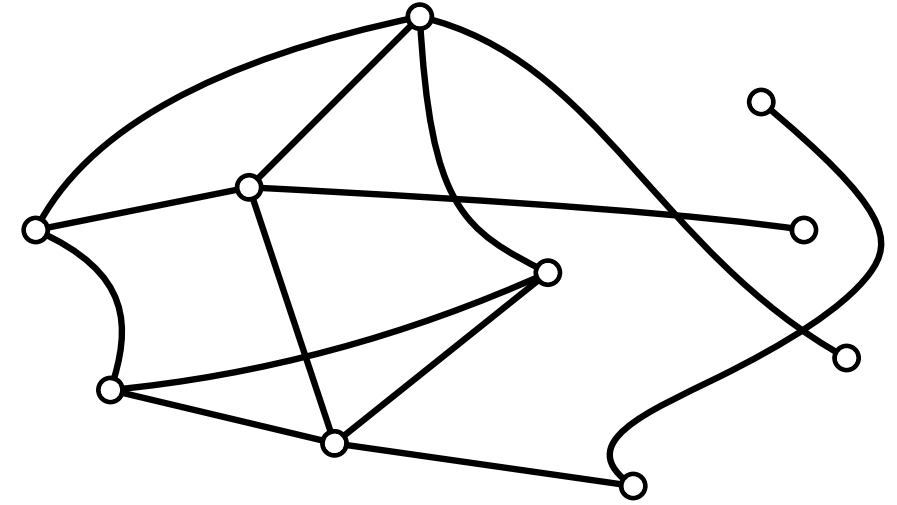
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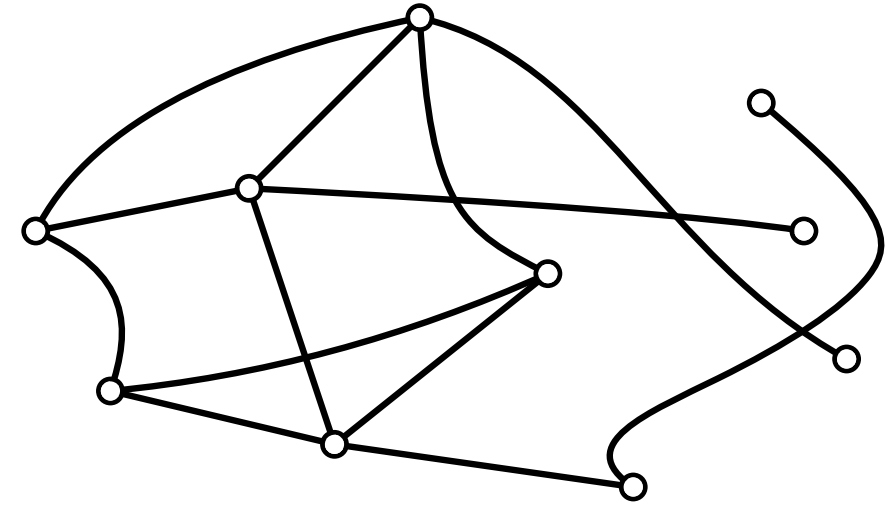
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in: graph G

out:

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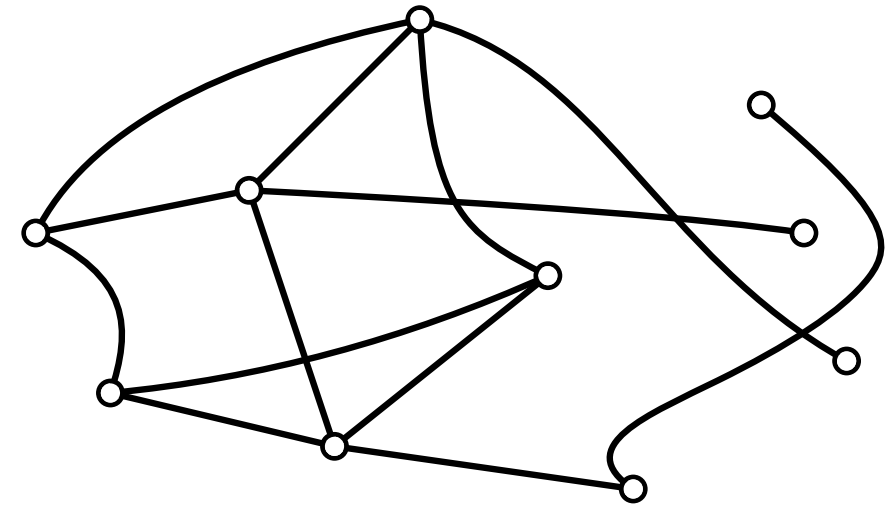
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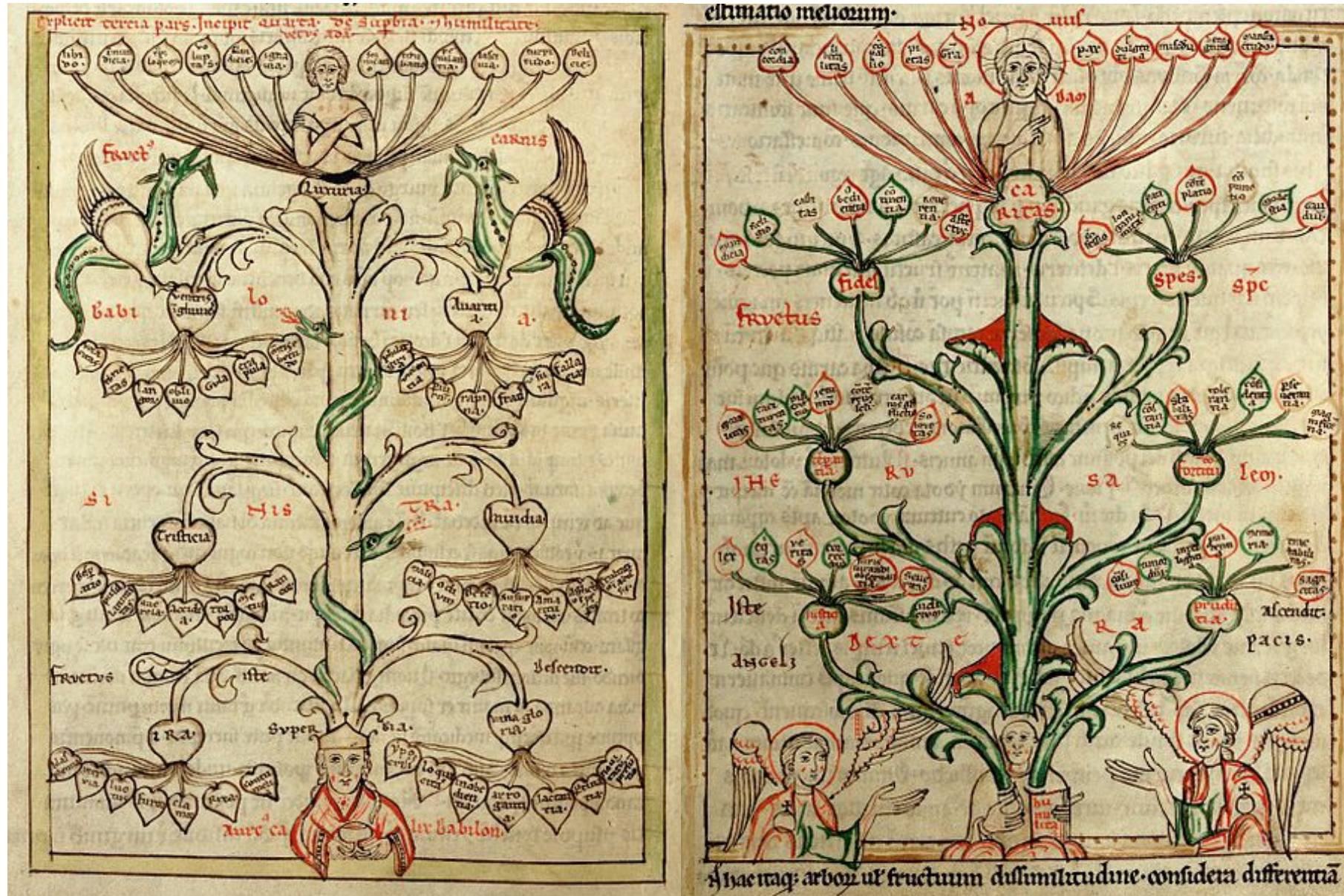
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But what is a **nice** drawing?

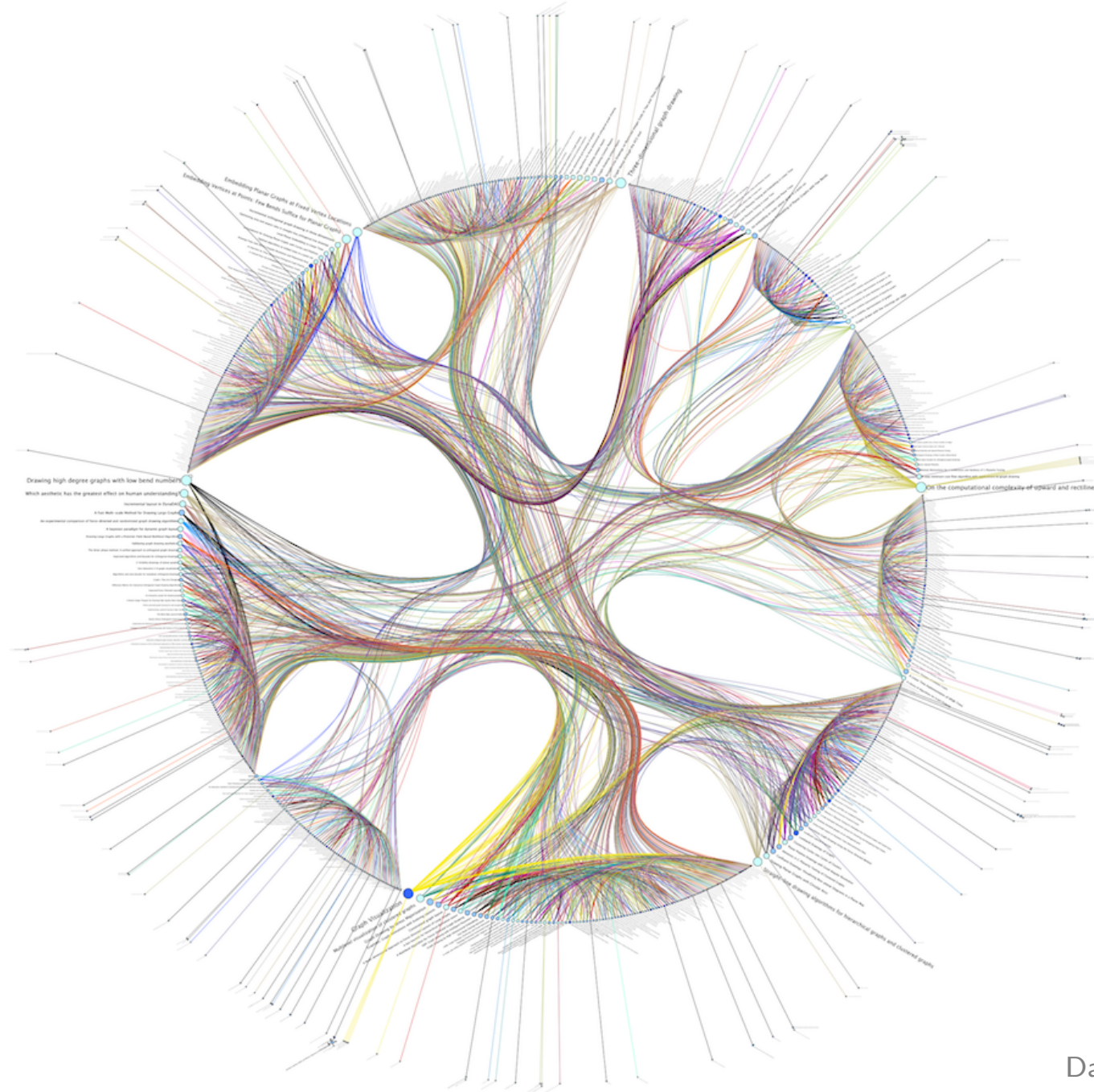
Tree of virtues and tree of vice

ca. 1200



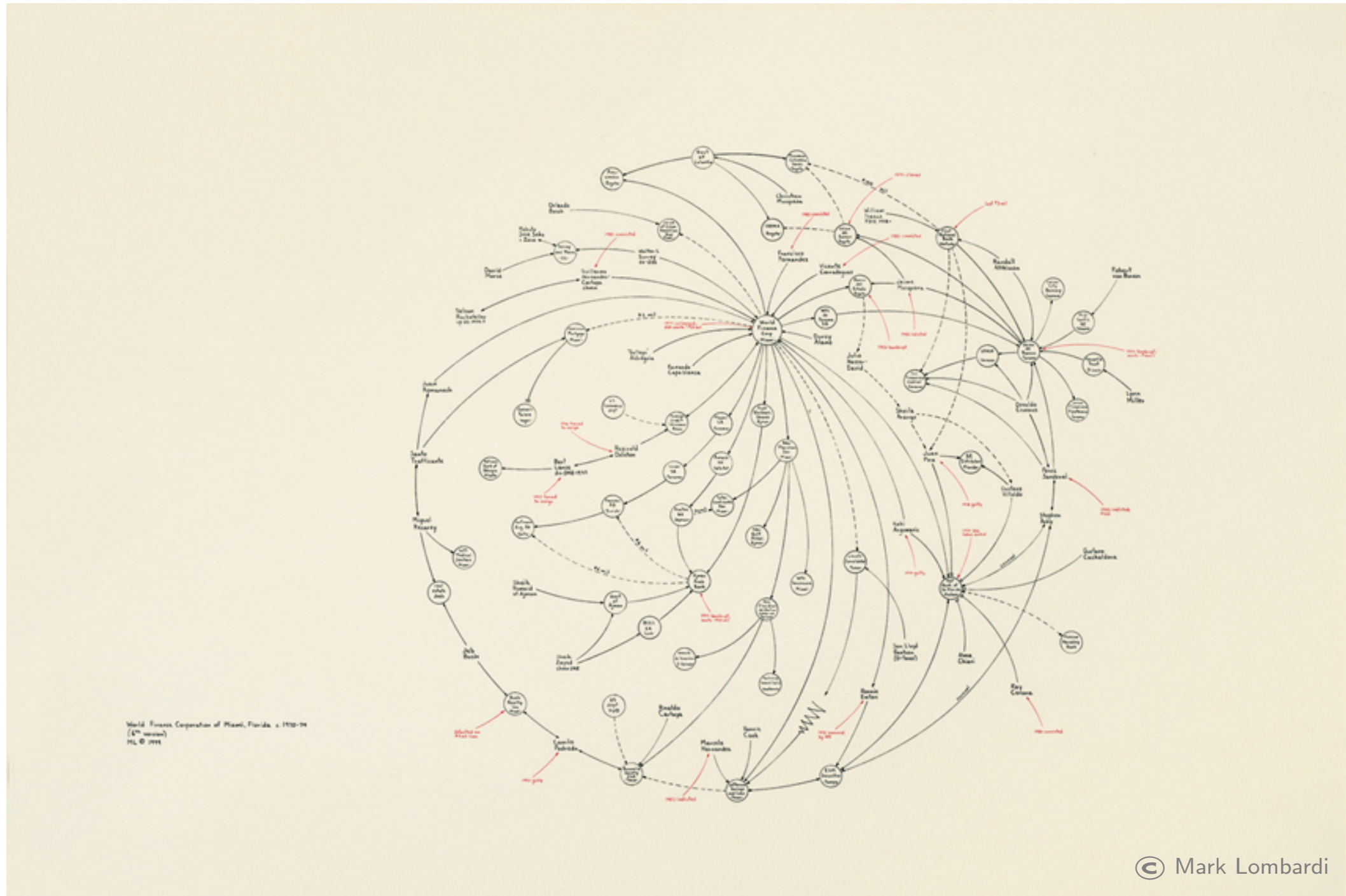


Social network – citation graph

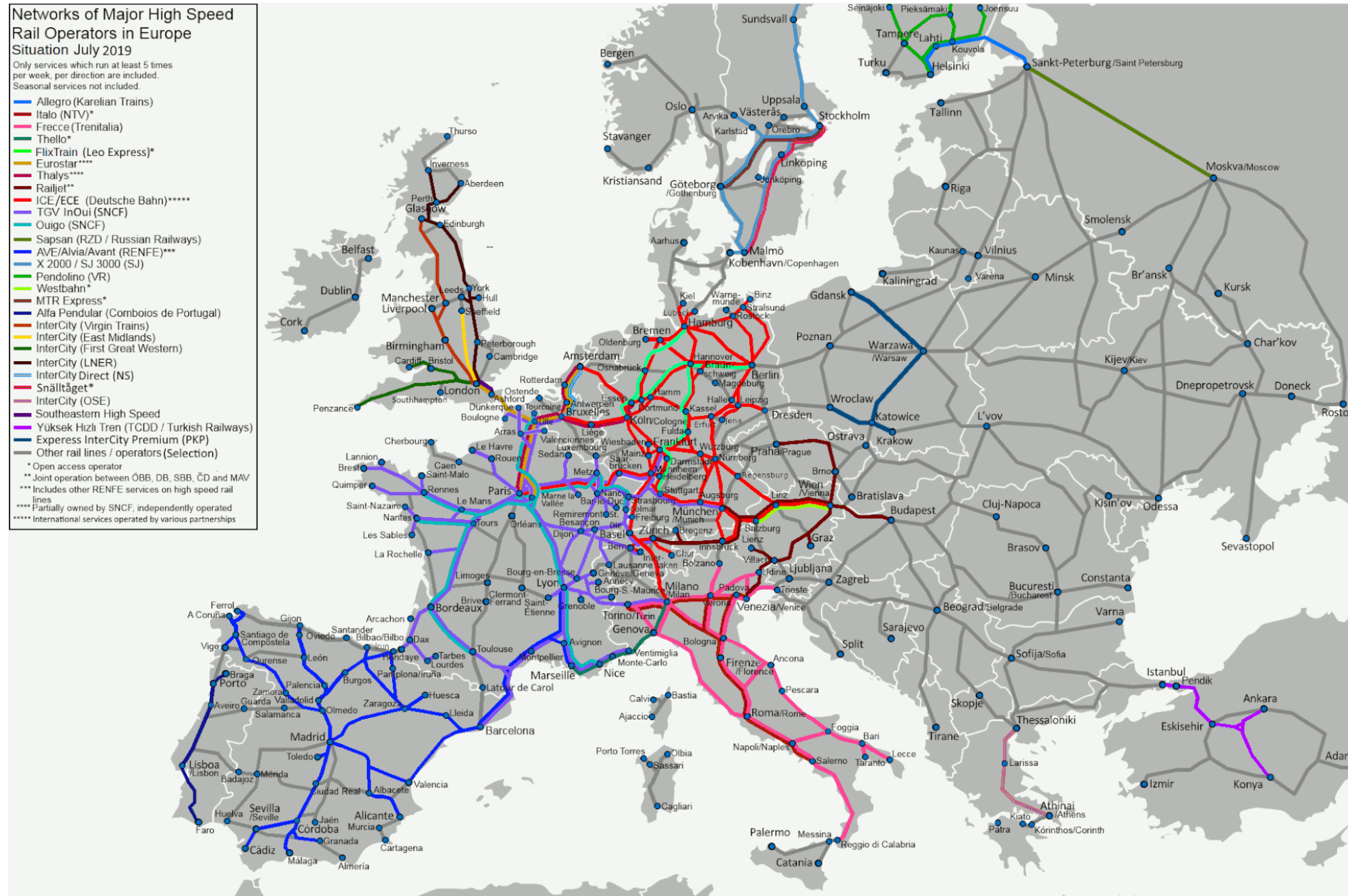




Social network - world finance corporation



Transportation network – European high speed railroads



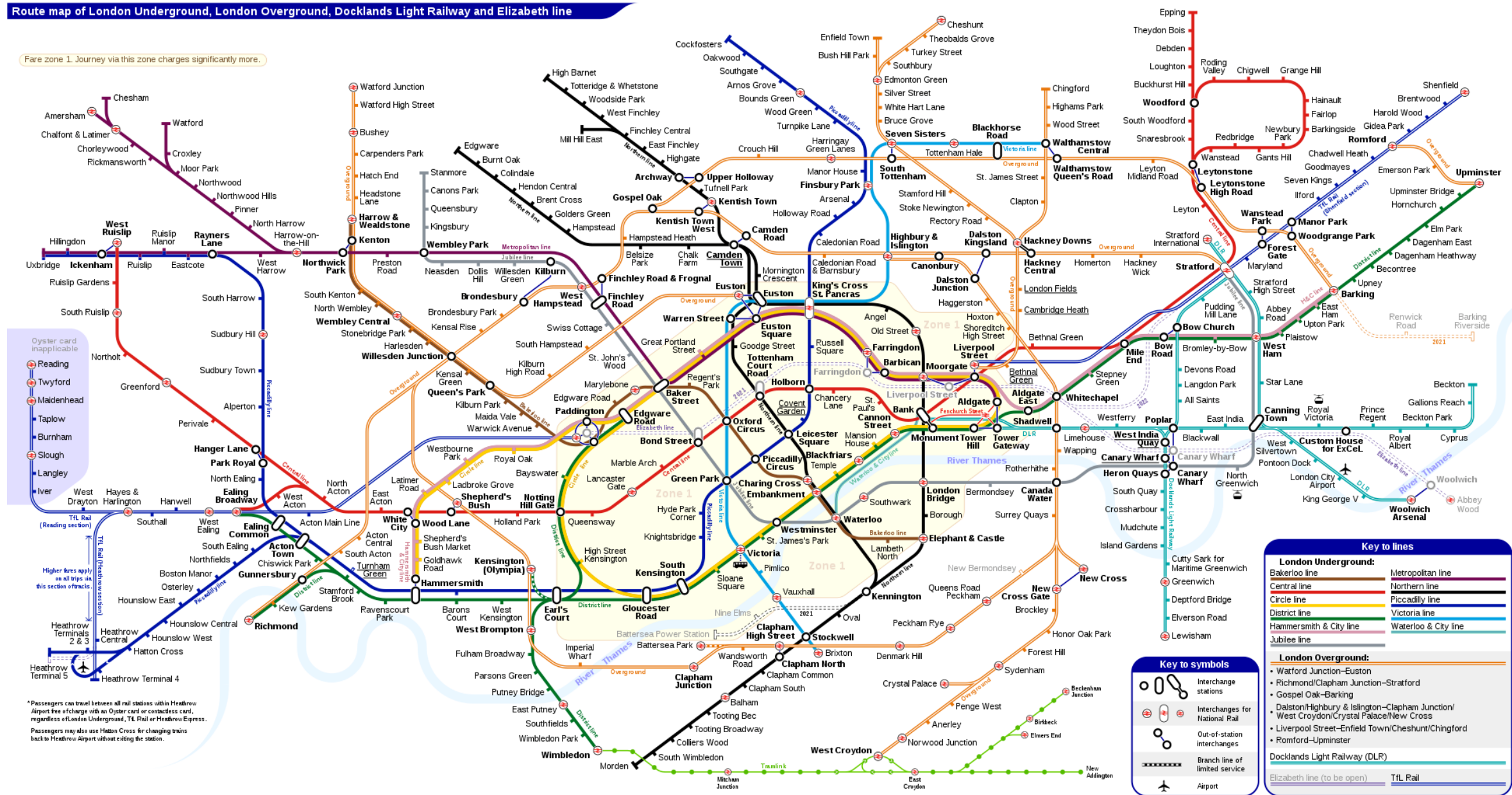
Source: Wiki Commons: Networks of Major High Speed Rail Operators in Europe - CC BY-SA 3.0

Transportation network – London Underground

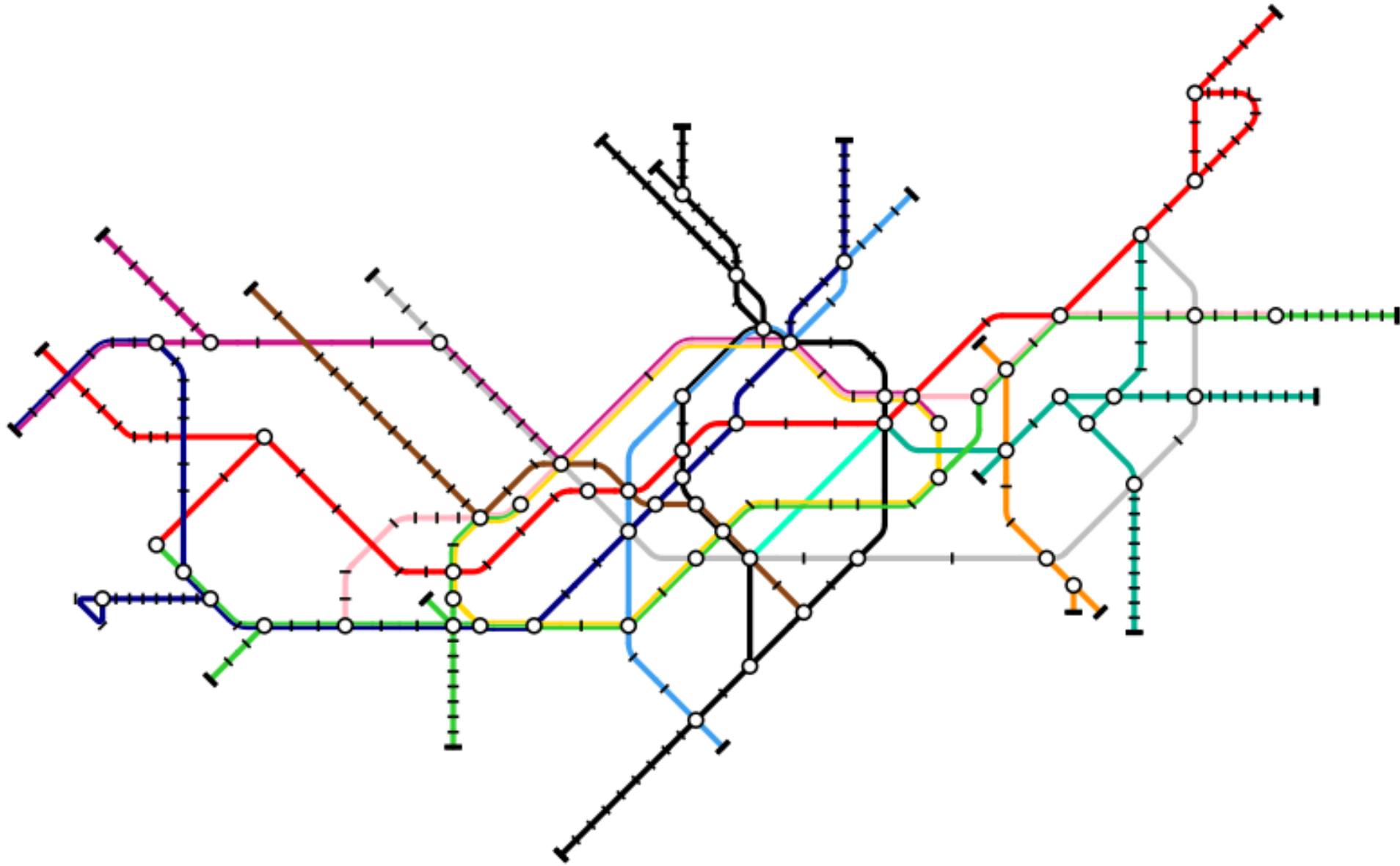


Source: Wiki Commons: London Underground full map - CC BY-SA 3.0

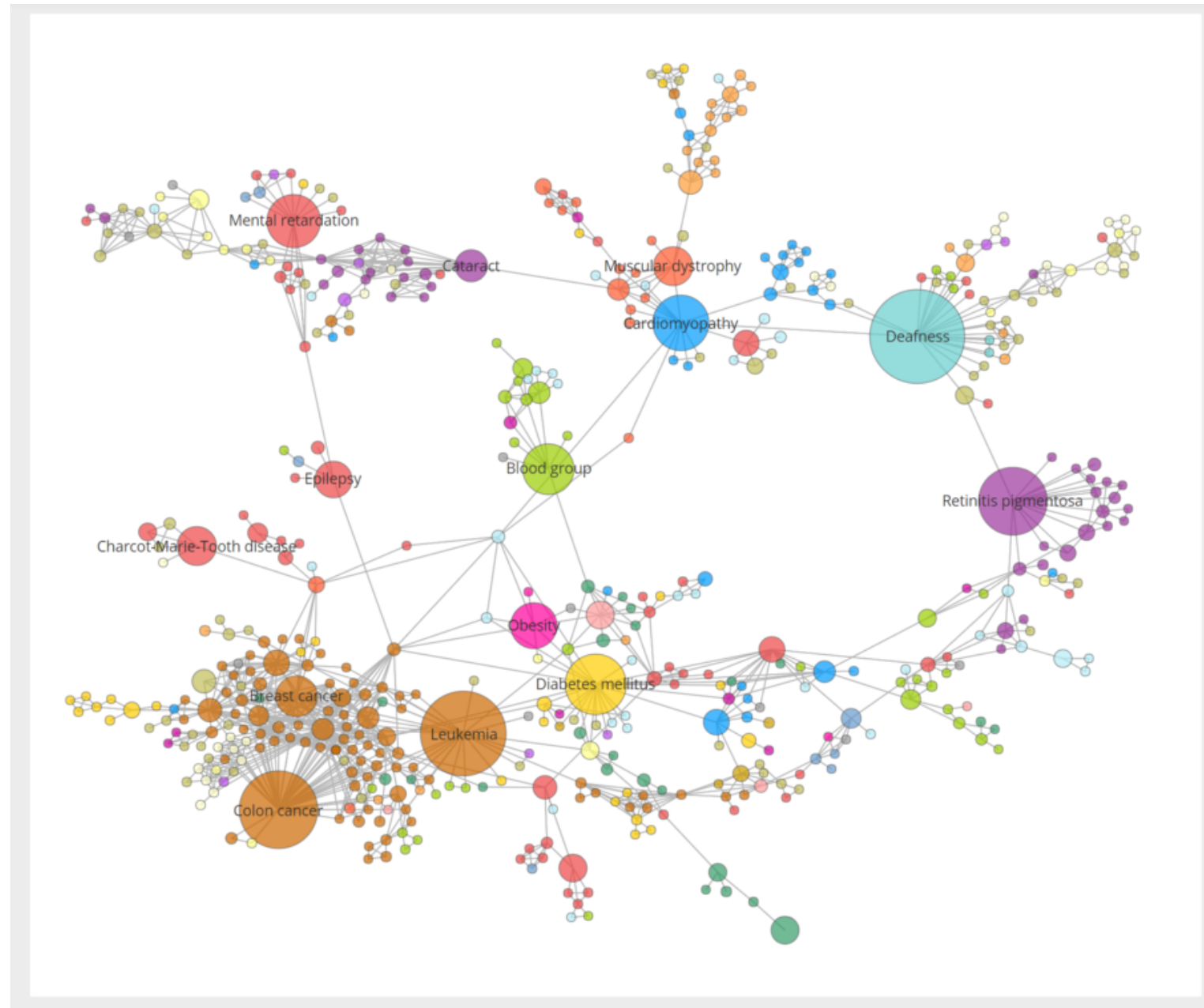
Transportation network – London Underground



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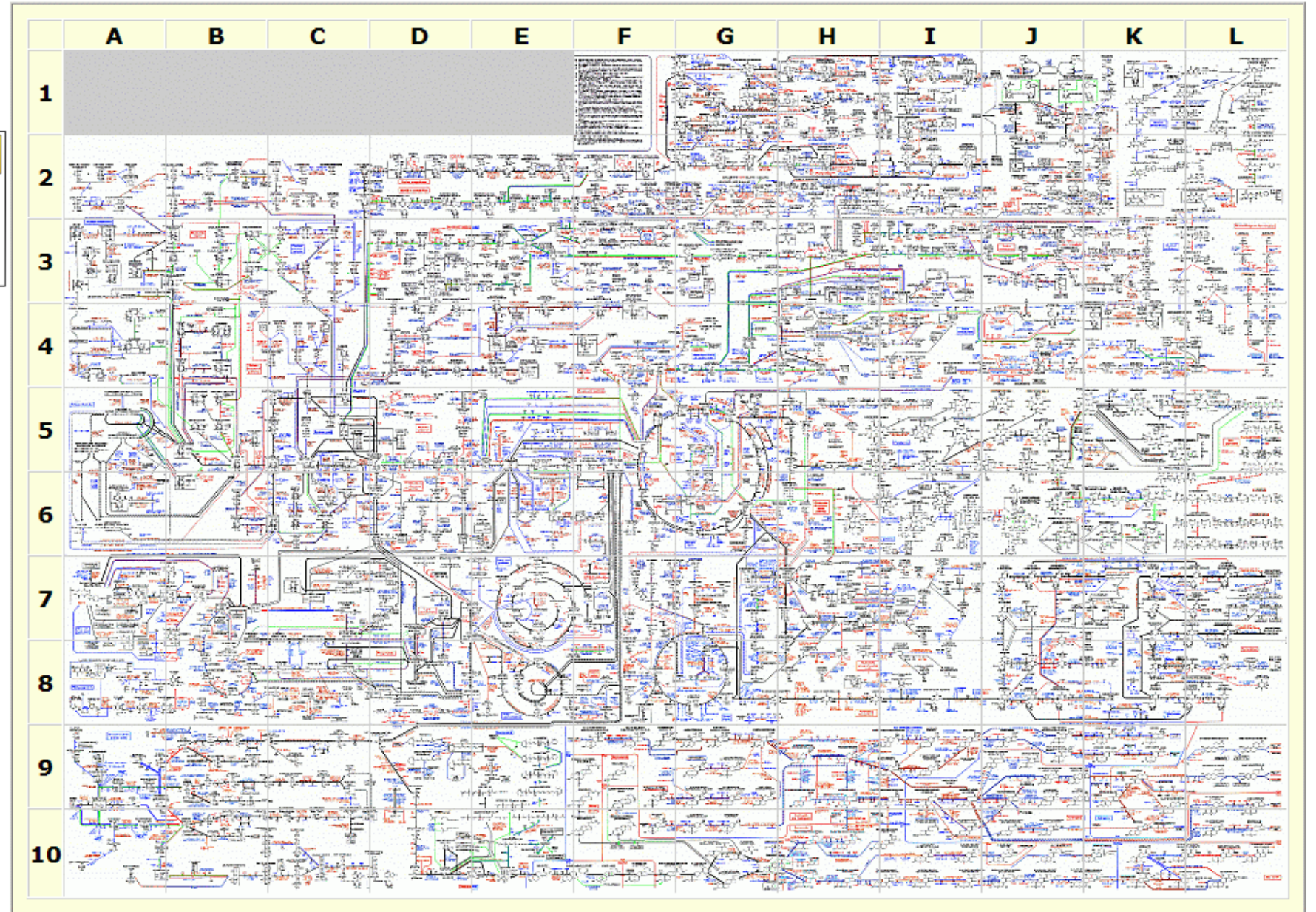
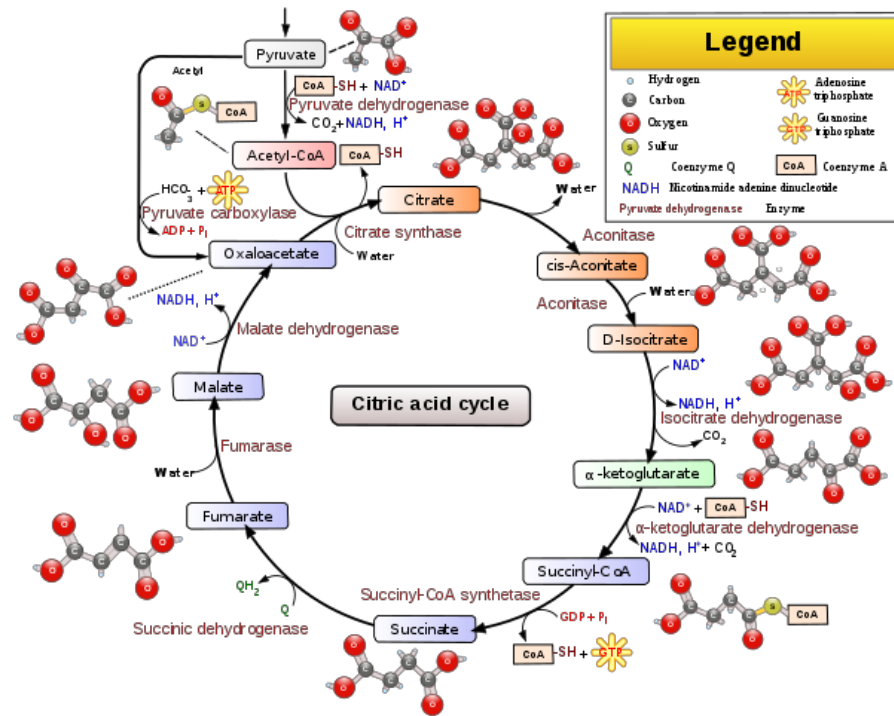


Bioinformatics – disease interaction

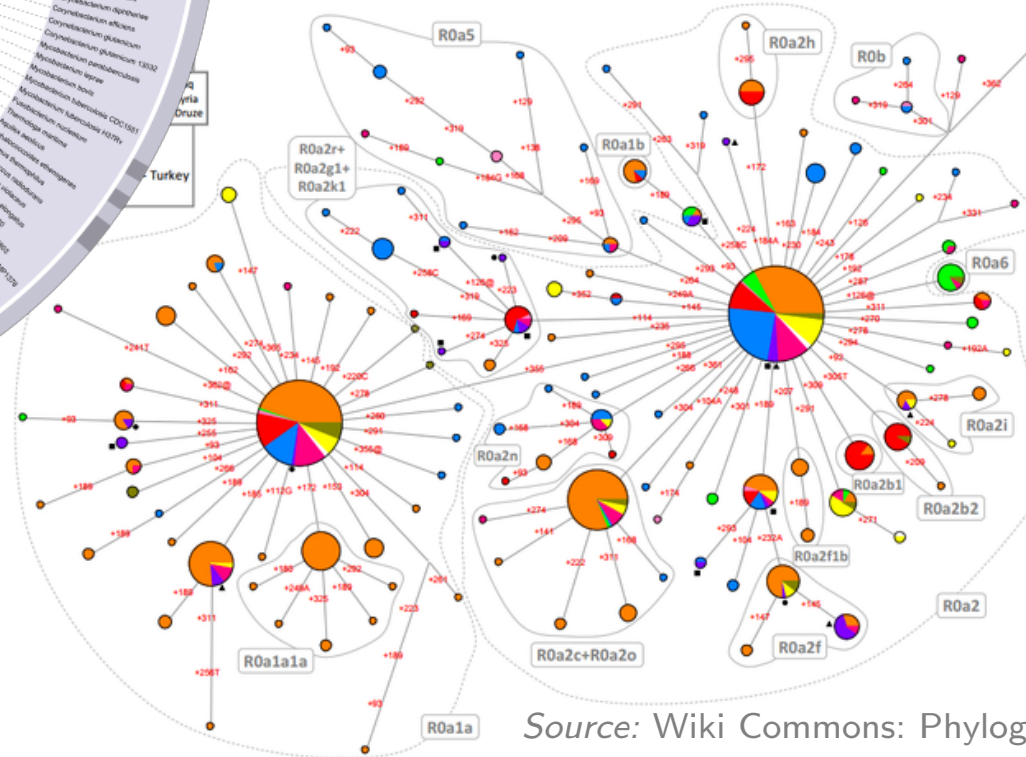
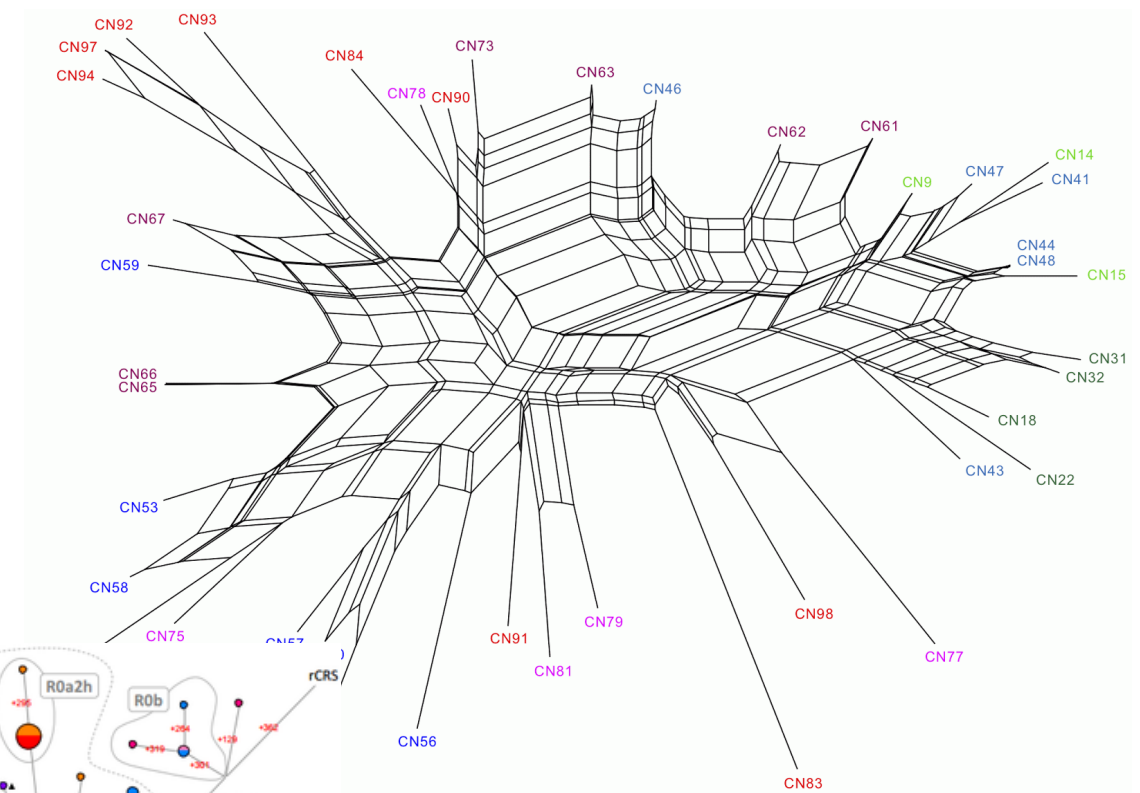
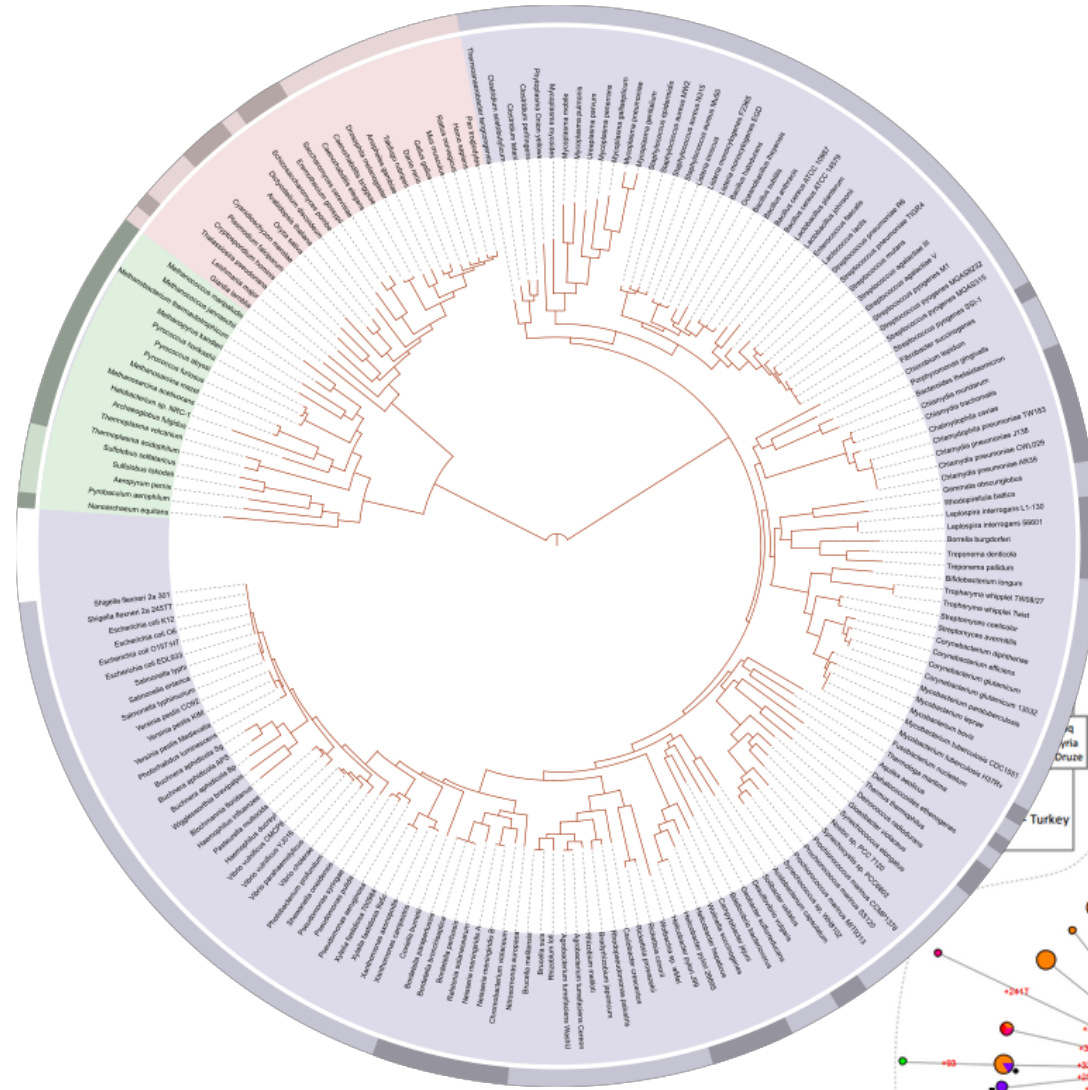


Source: Wiki Commons: Human disease network - CC BY-SA 4.0

Bioinformatics – molecular metabolic network

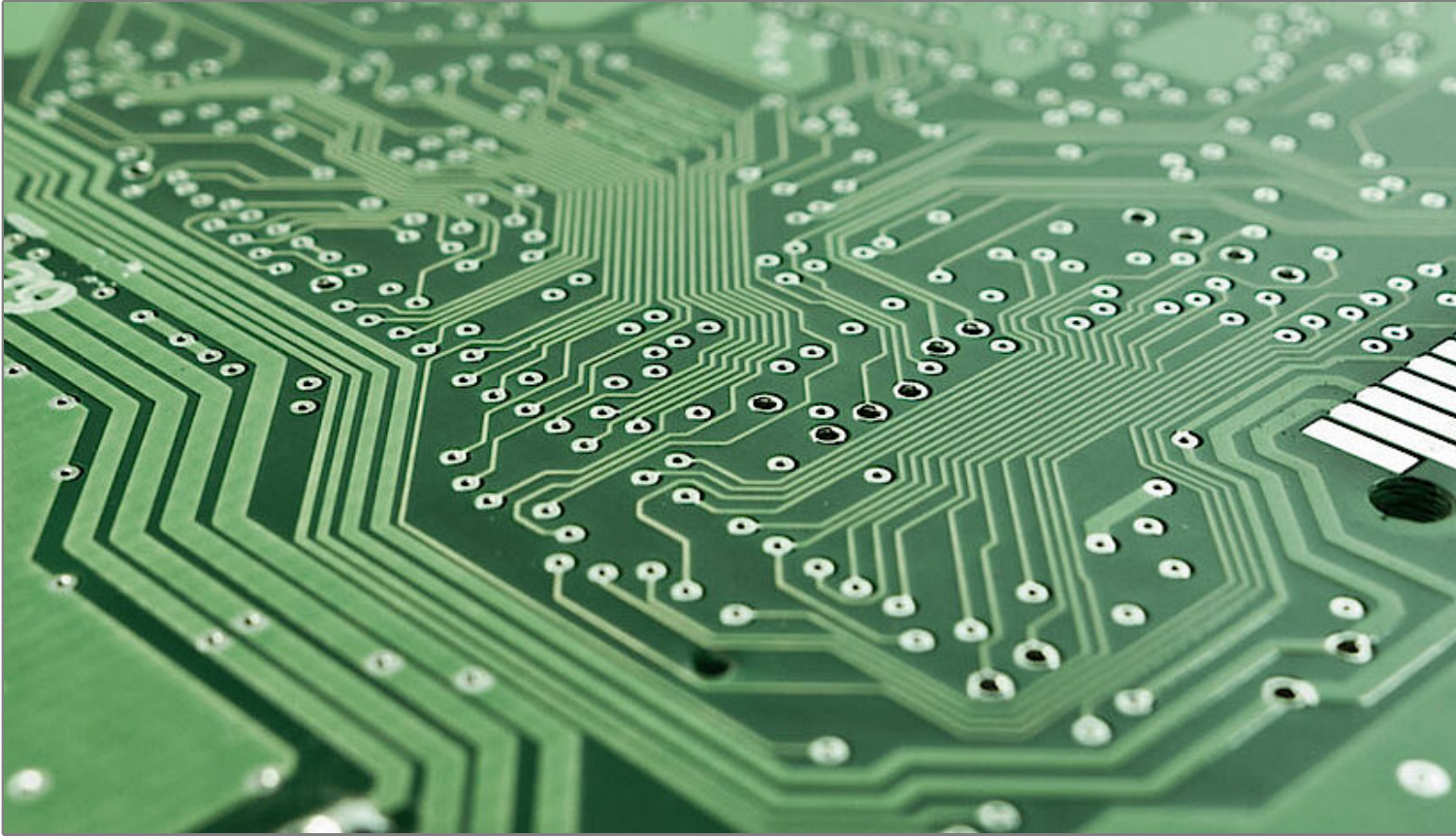


Bioinformatics – phylogenetic trees & networks



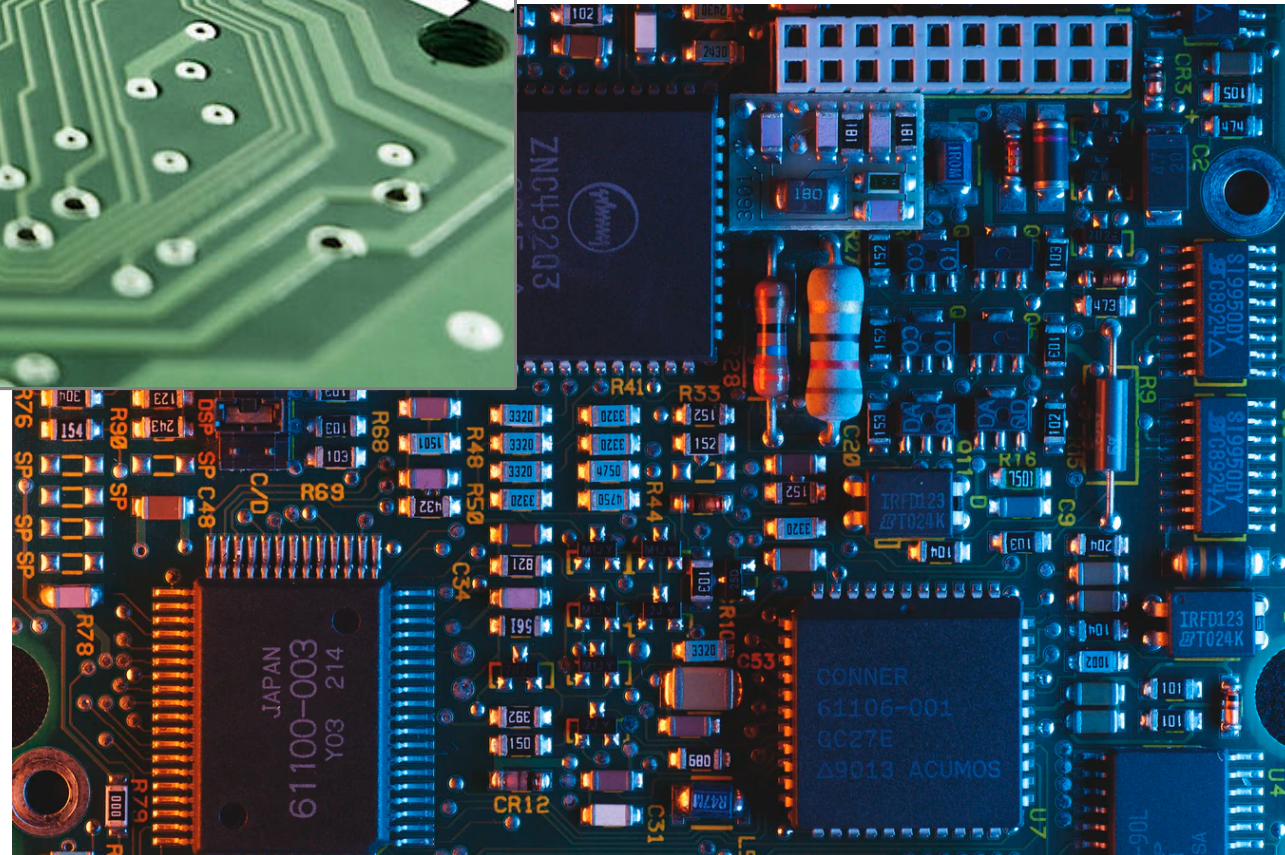
Source: Wiki Commons: Phylogenetic network of HVS-I variation - CC BY 4.0

Technical network – very large-scale integration (VLSI)

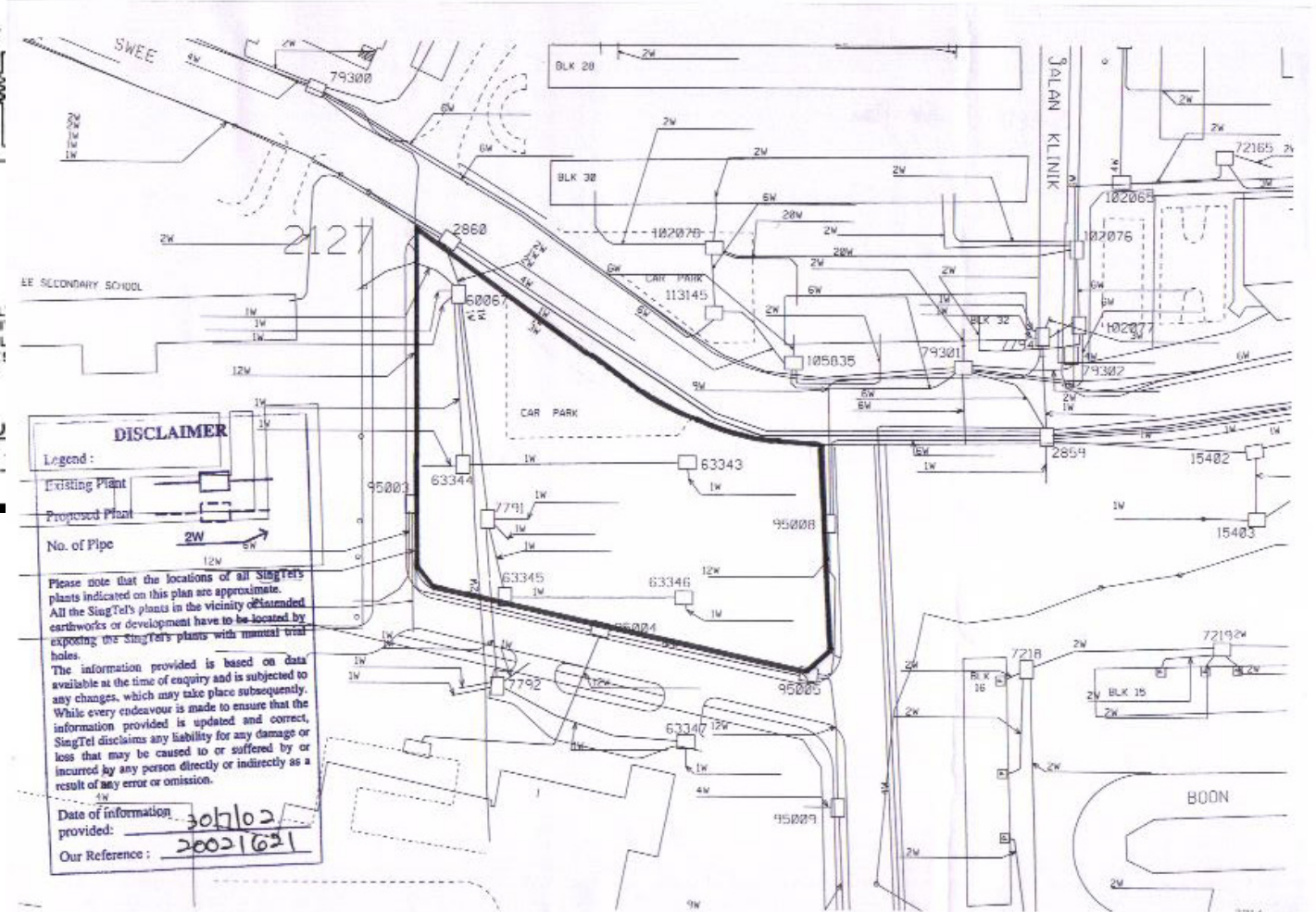
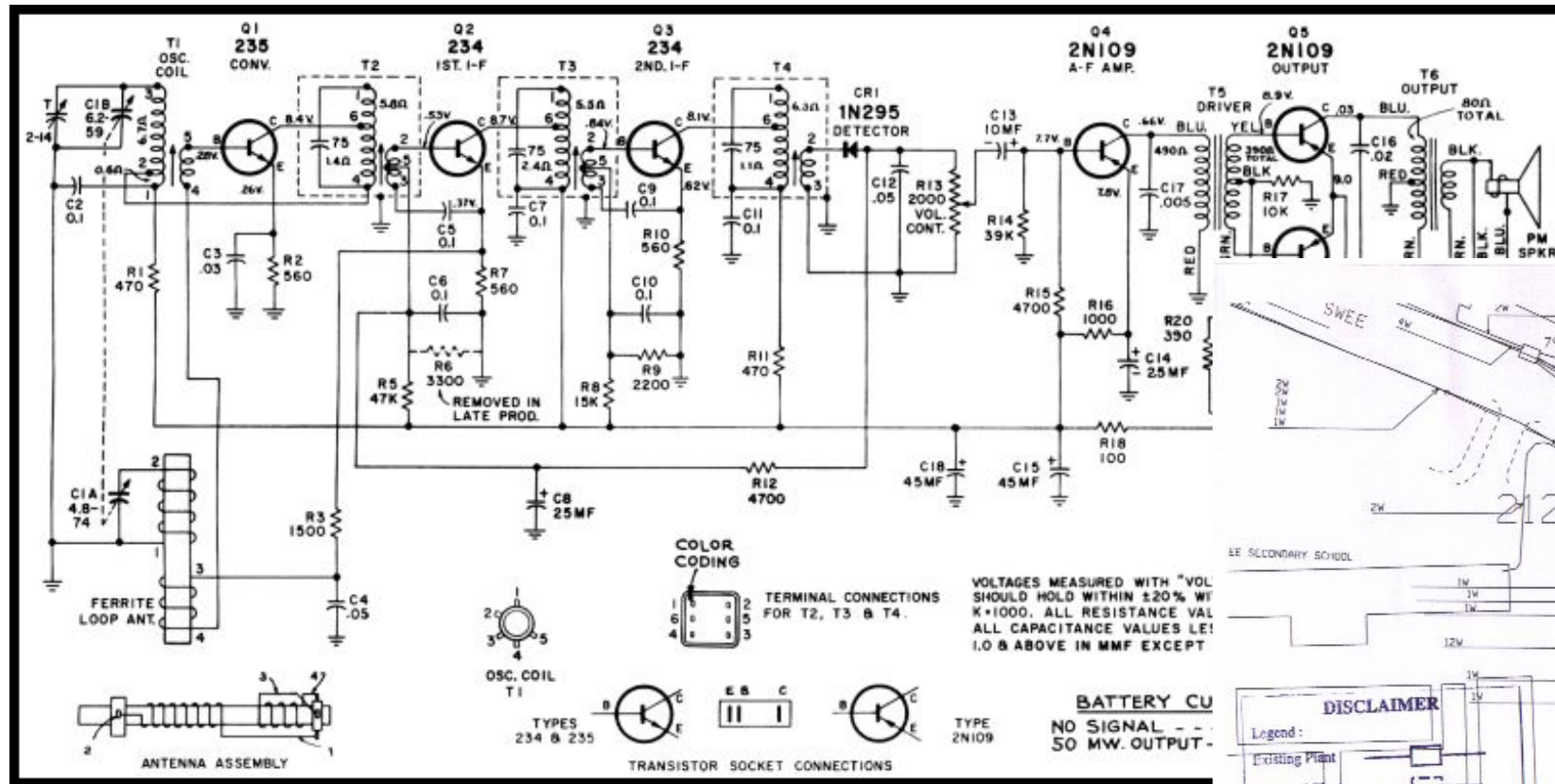


Source: Pixabay

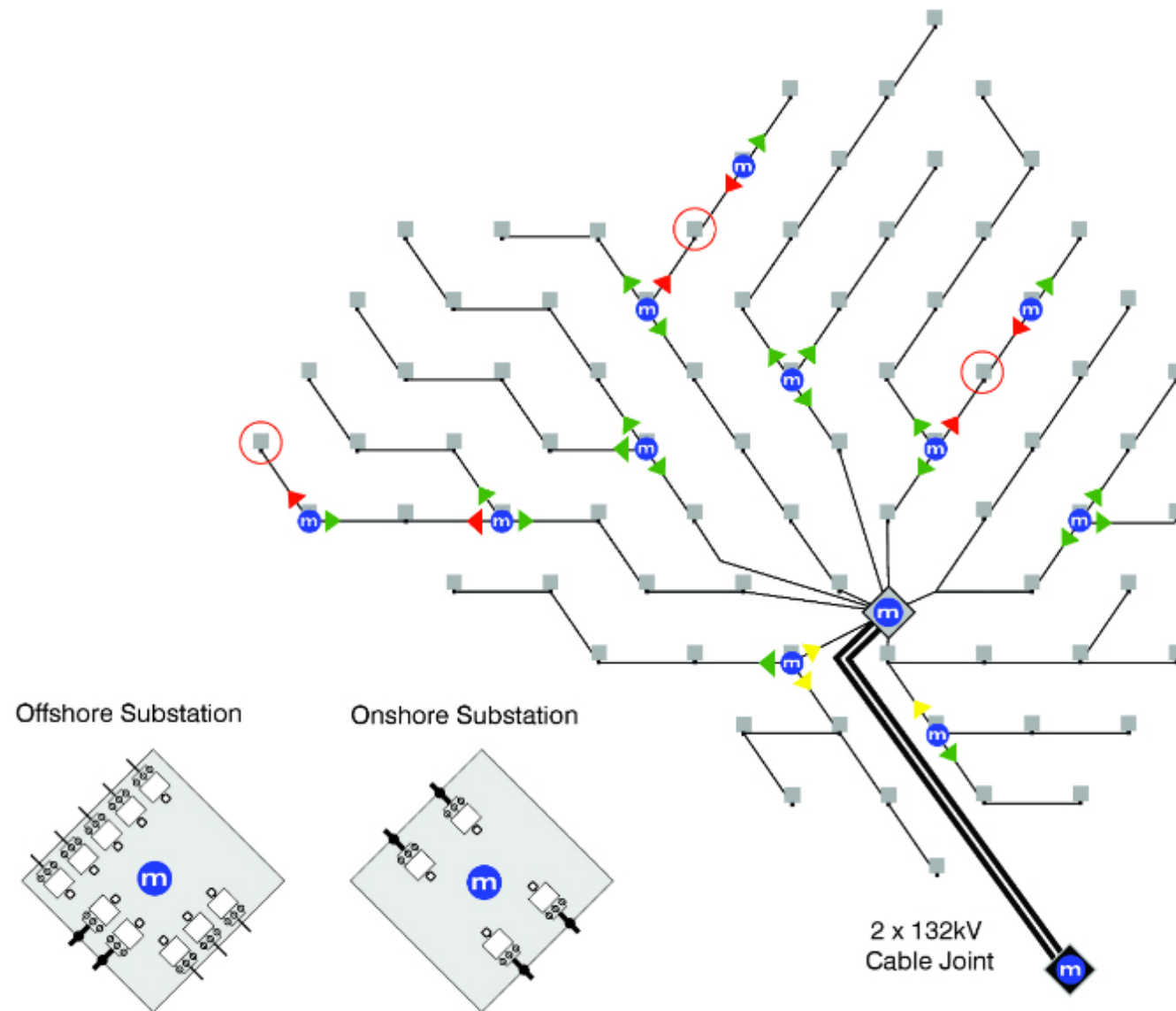
Source: Wiki Commons: Diopsis - CC BY-SA 3.0



Technical network – transistor diagram, wiring

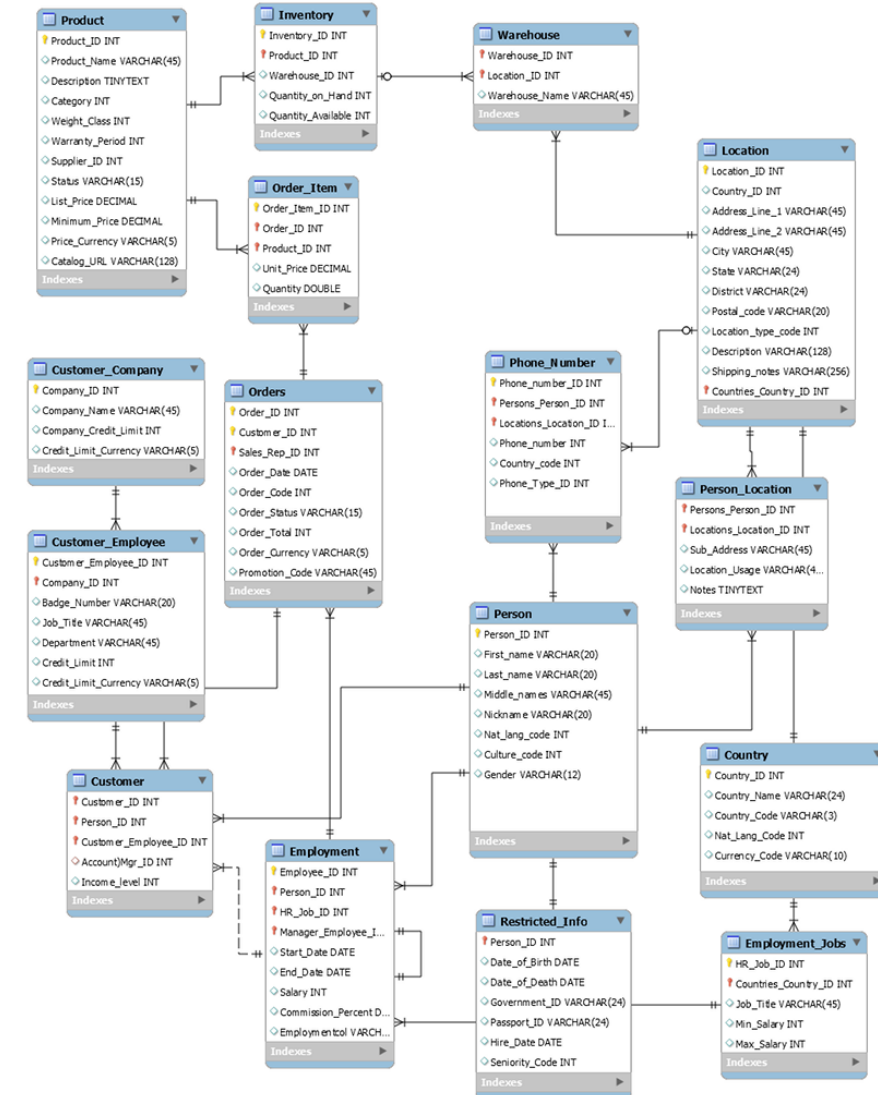
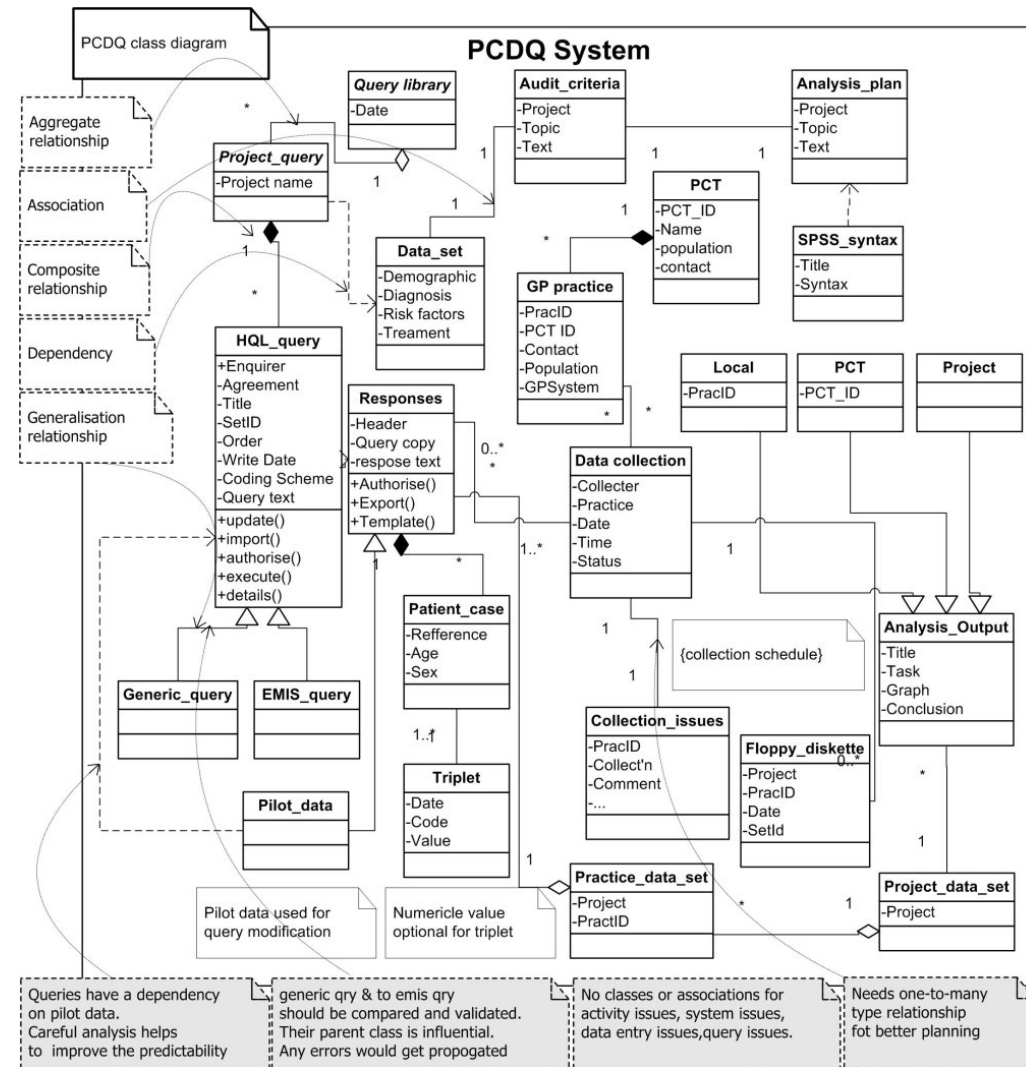


Technical networks – offshore wind farms



Source: Wiki Commons: Alpha Ventus Windmills - CC BY-SA 3.0

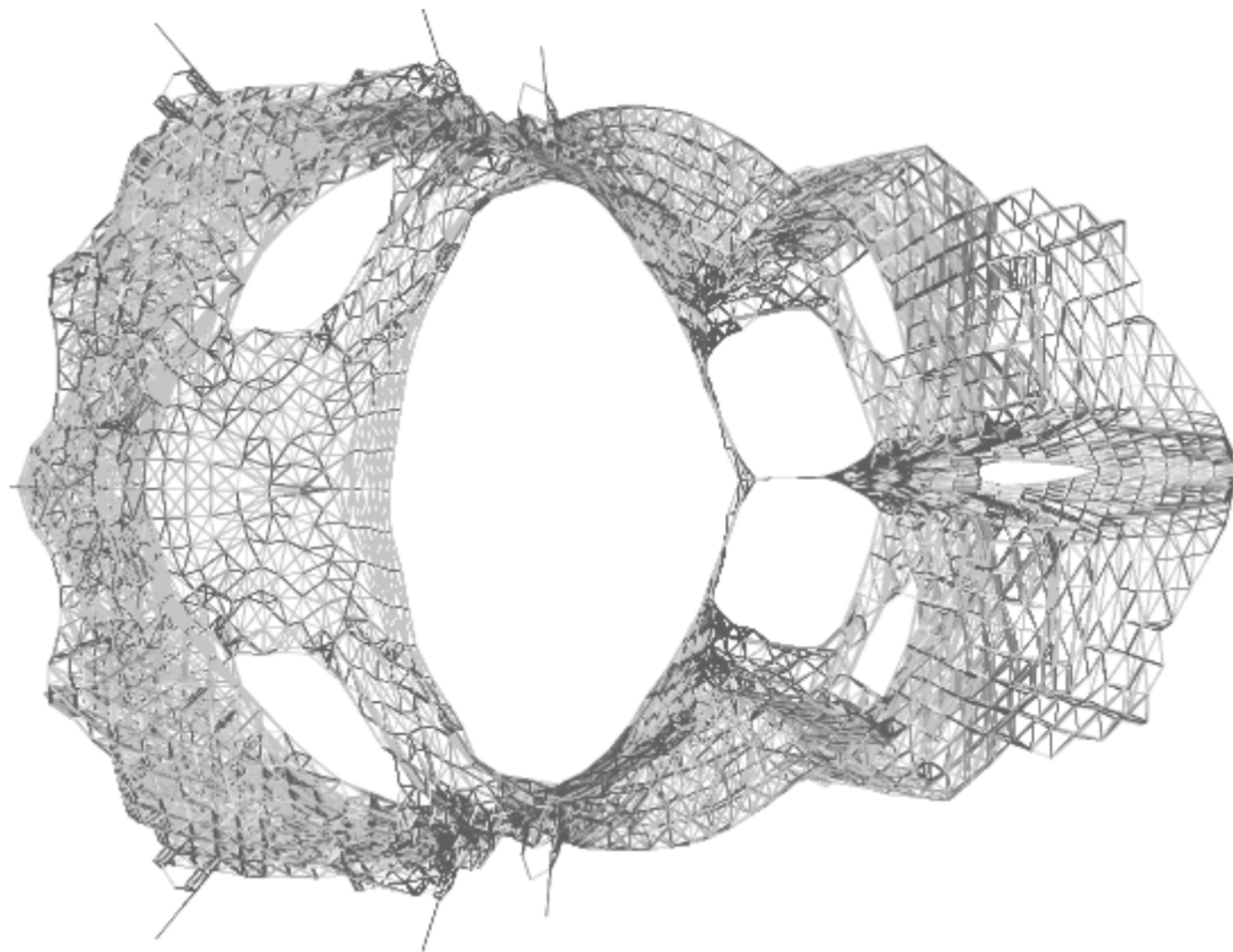
Technical network – UML diagram



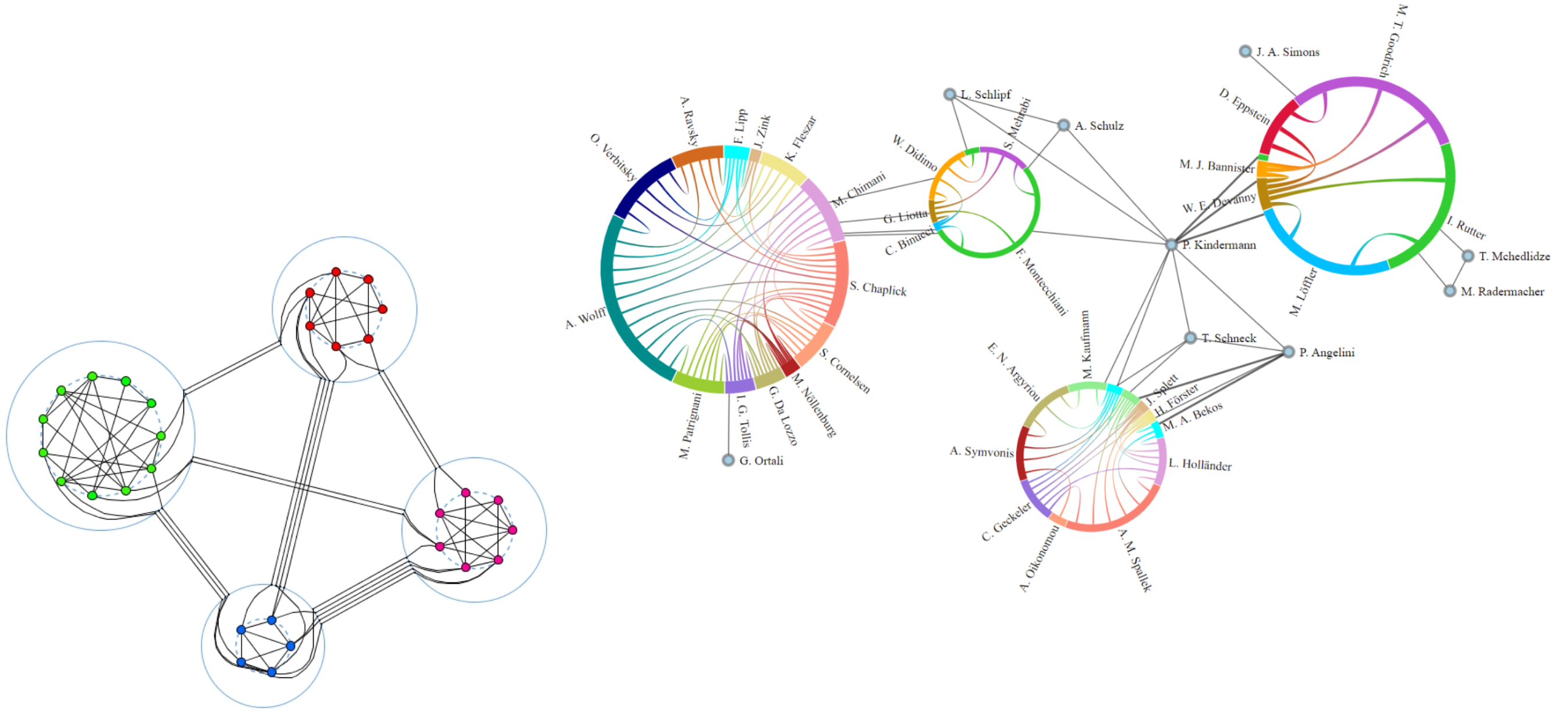
LORD OF THE RINGS



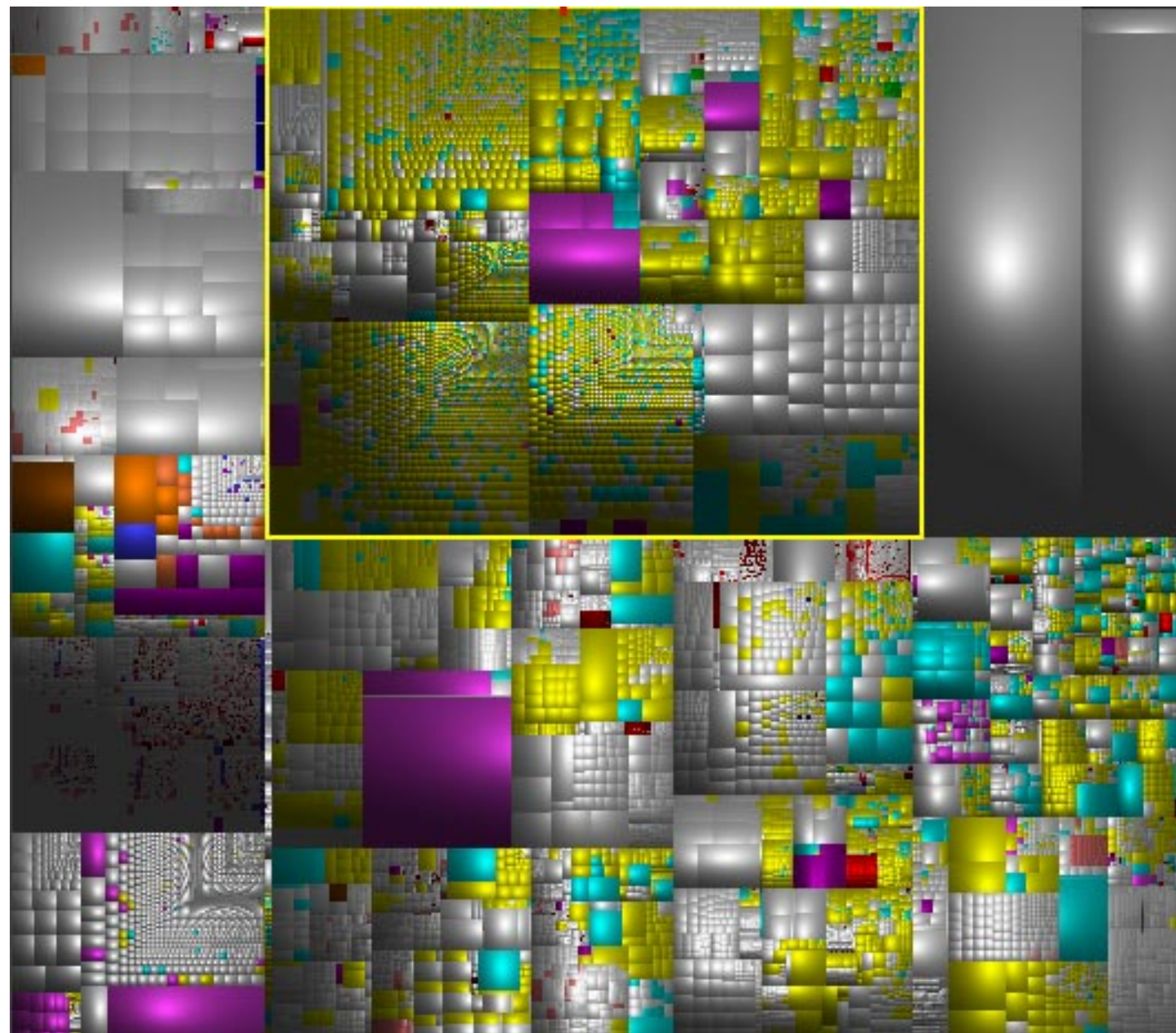
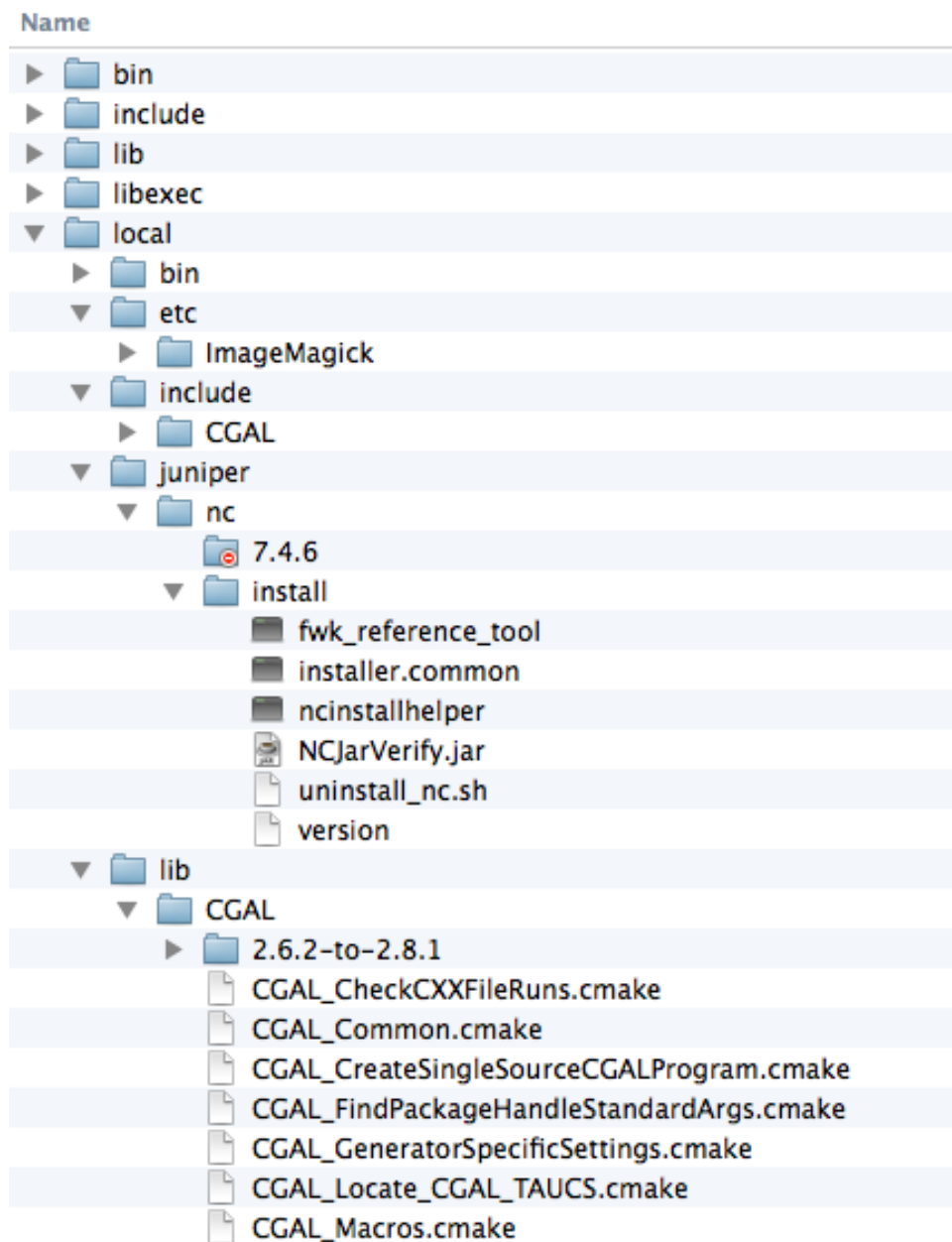
Large graphs – object mesh



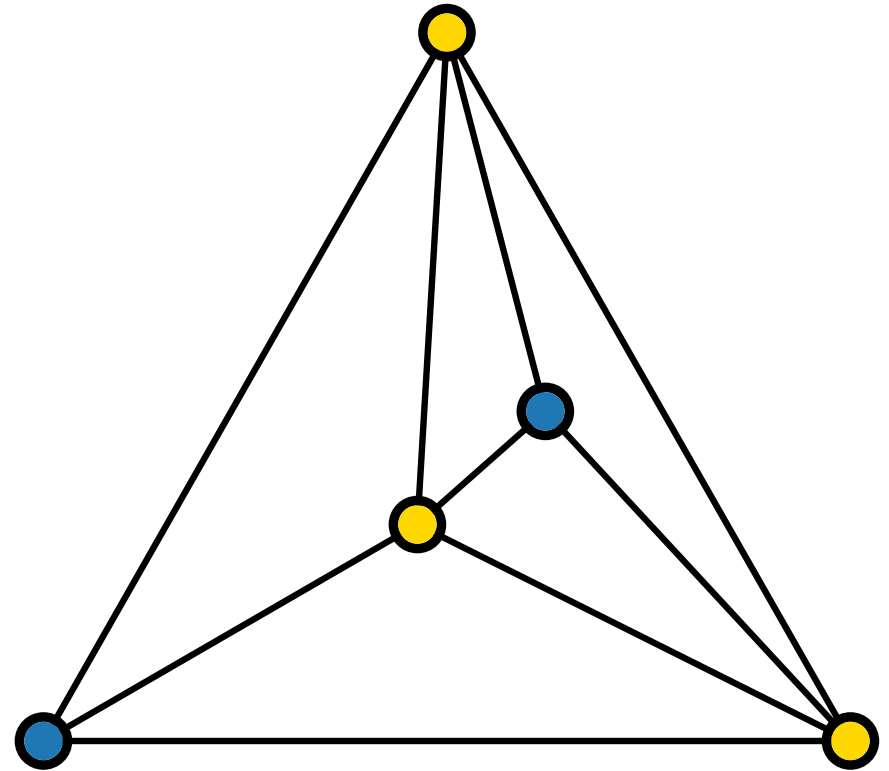
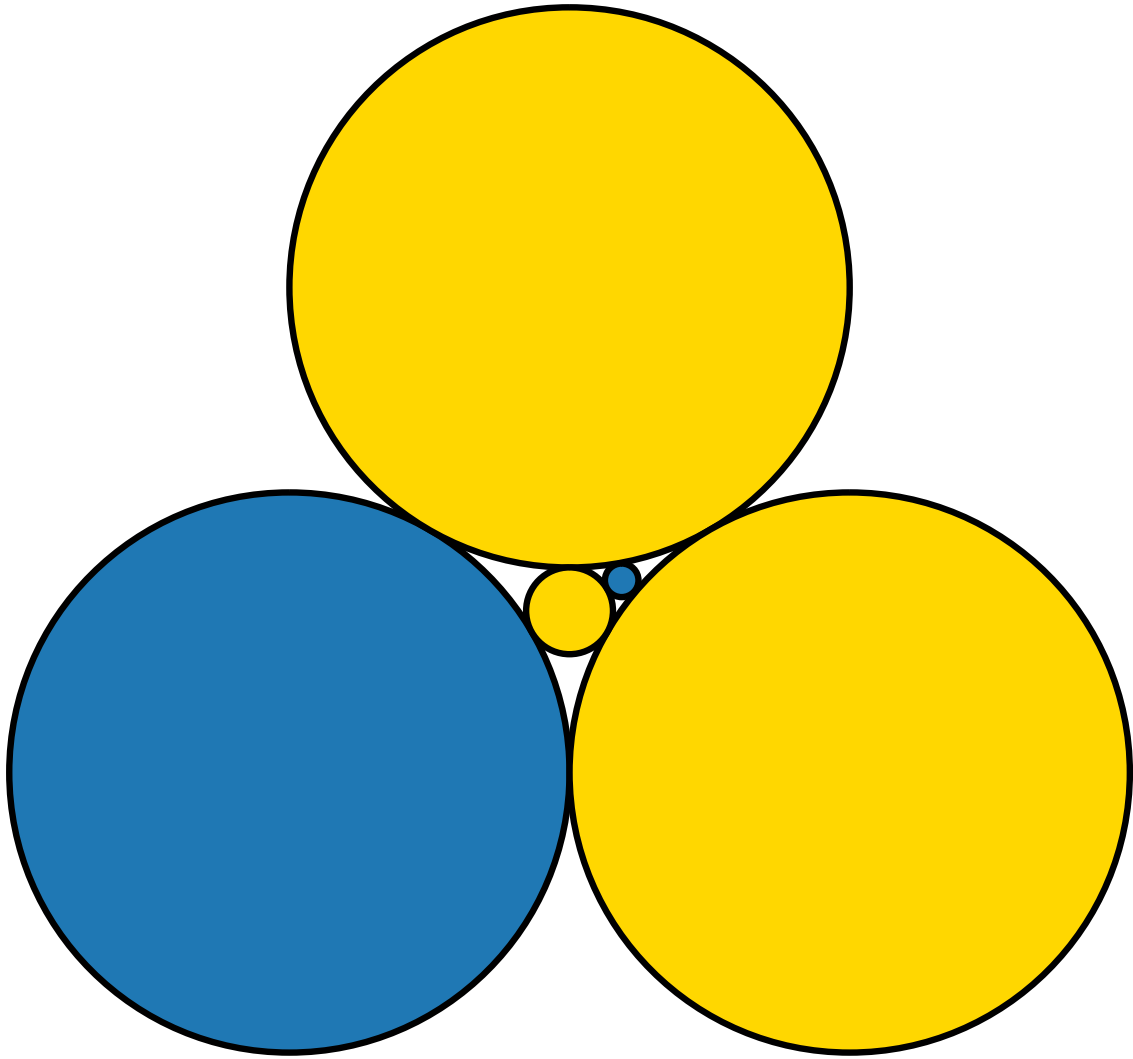
General graphs – micro-macro layout



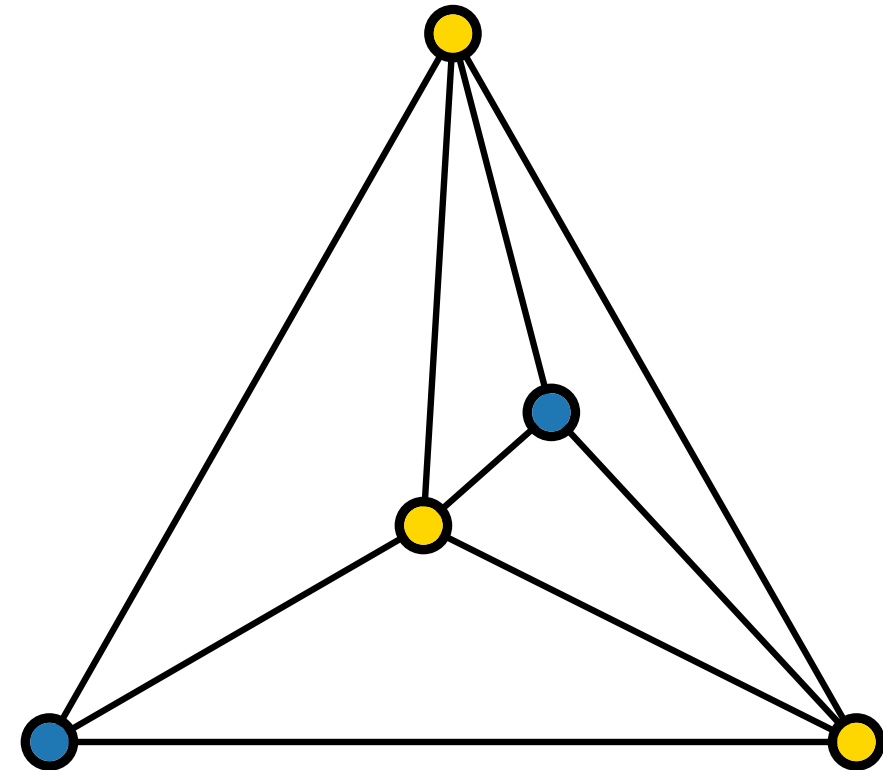
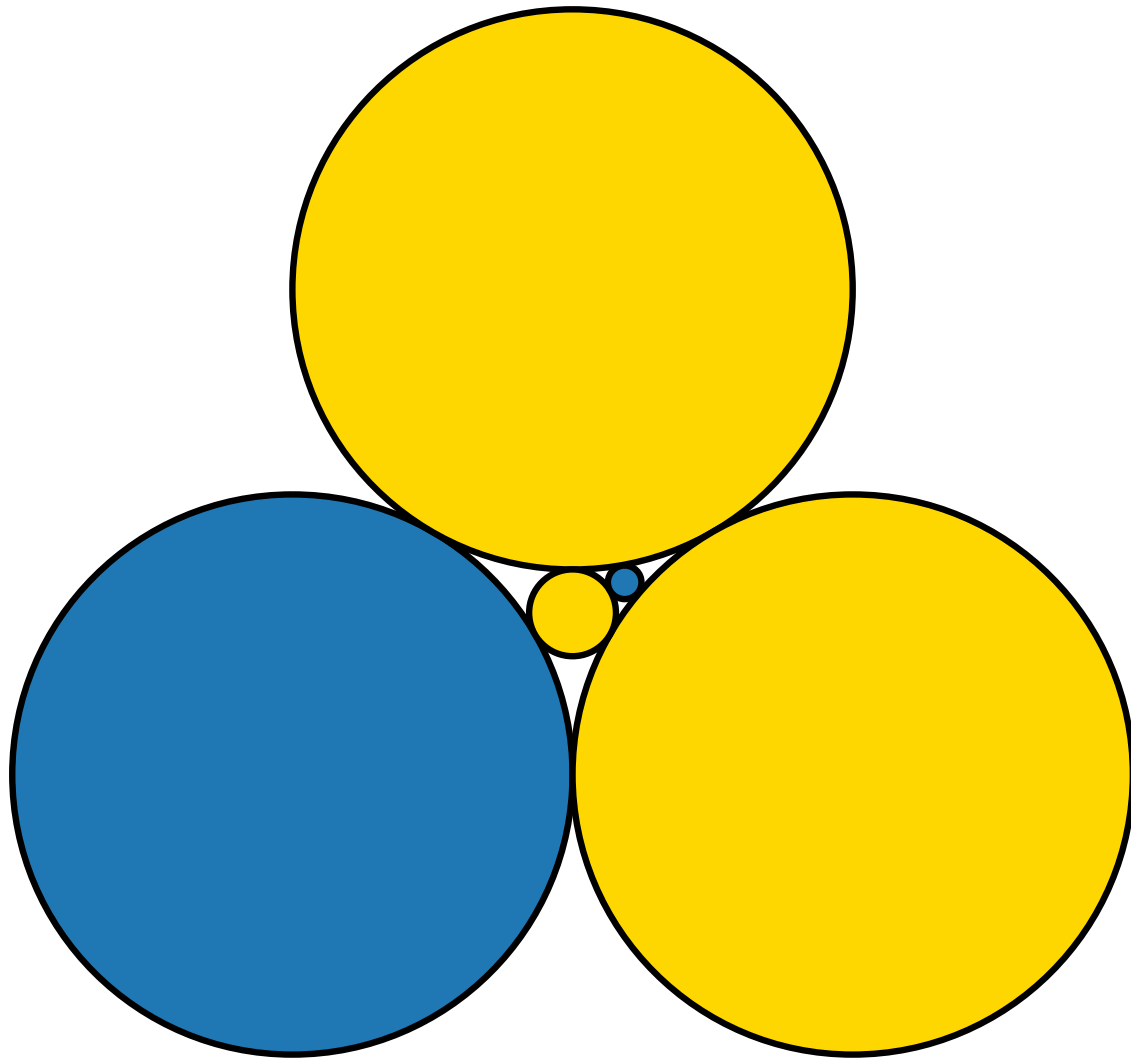
Alternative representations – treemap



Alternative representations – contact graphs



Alternative representations – contact graphs



■ For more examples see visualcomplexity.com

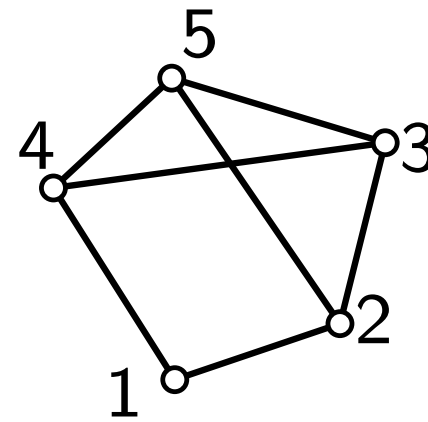
Requirements of a graph layout

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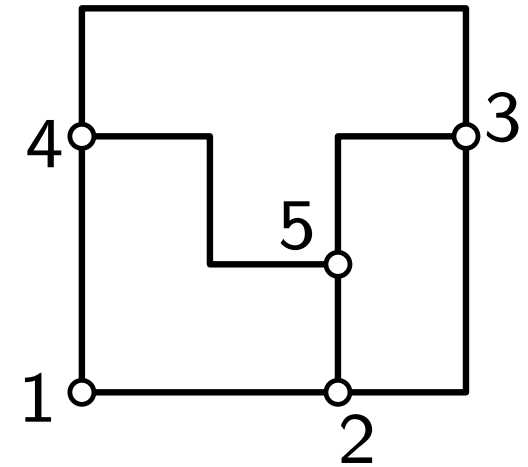
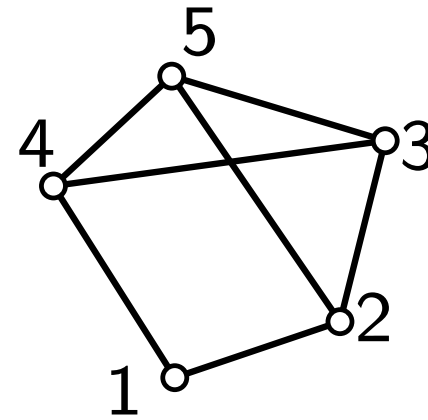
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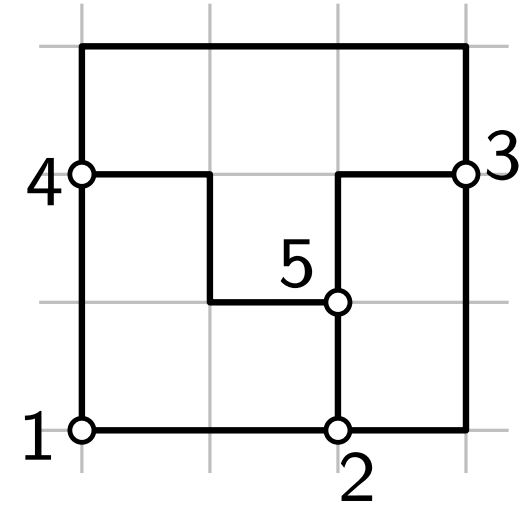
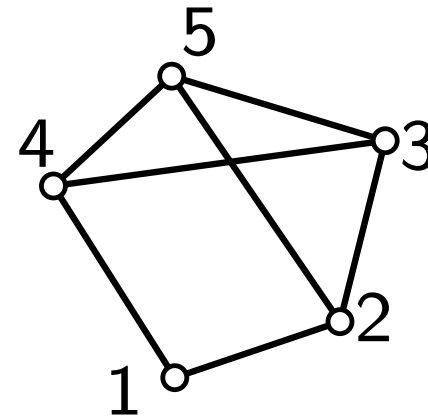
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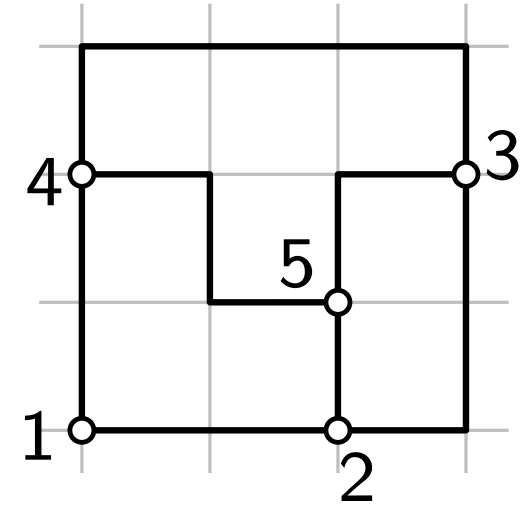
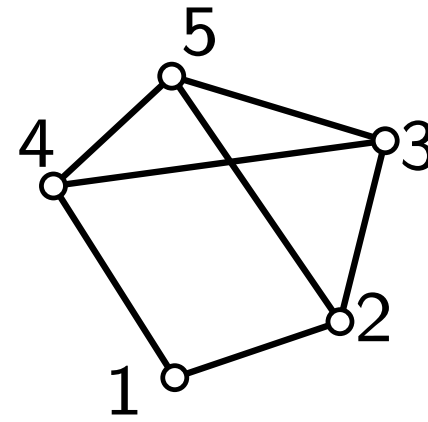
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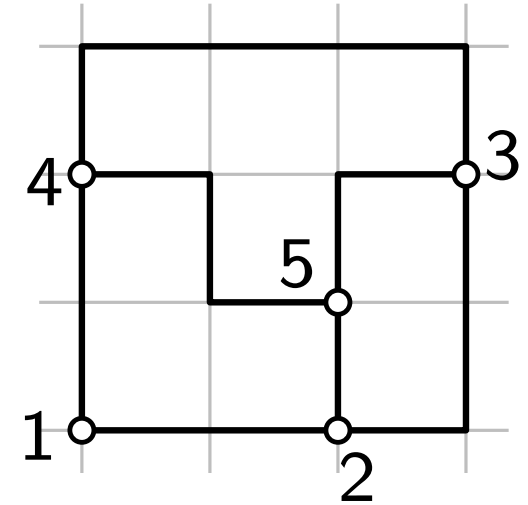
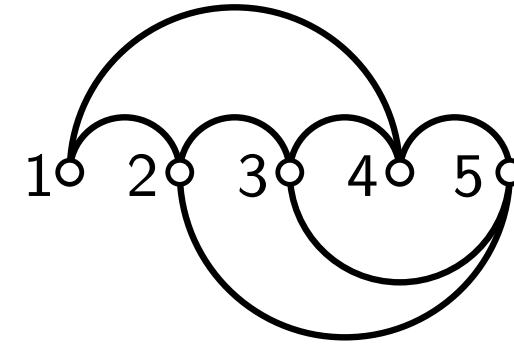
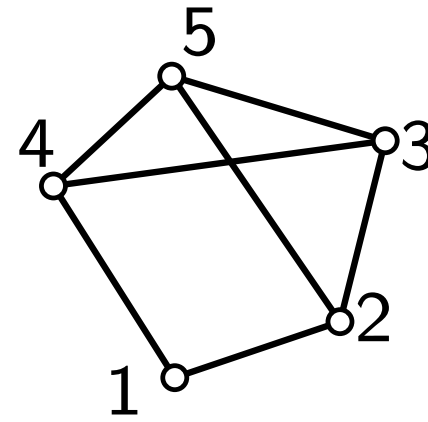
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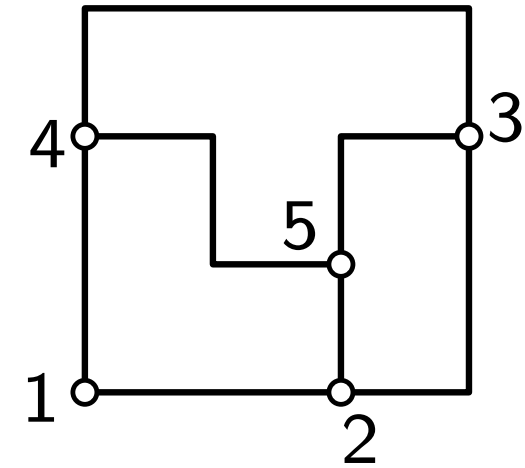
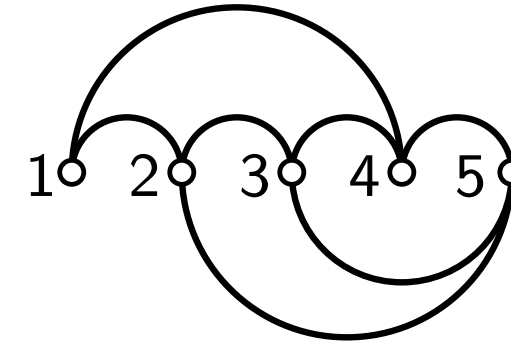
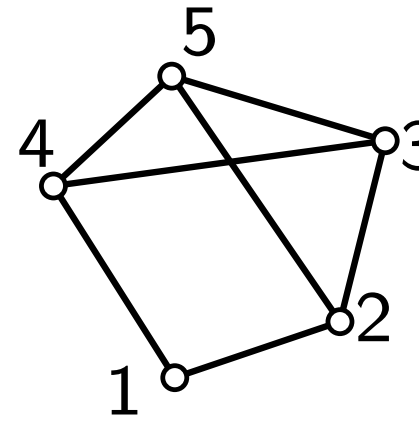


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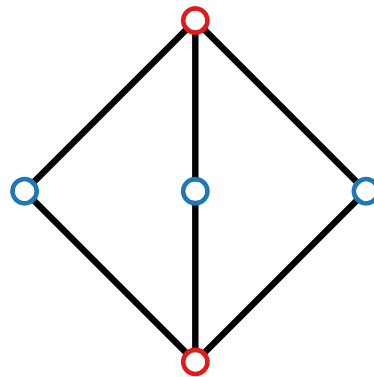
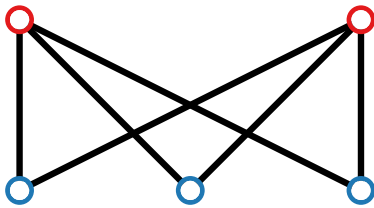
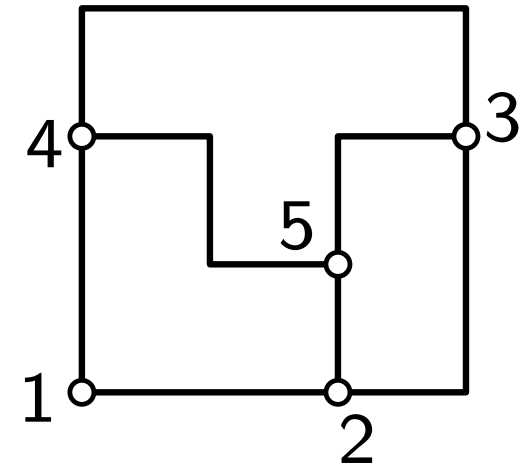
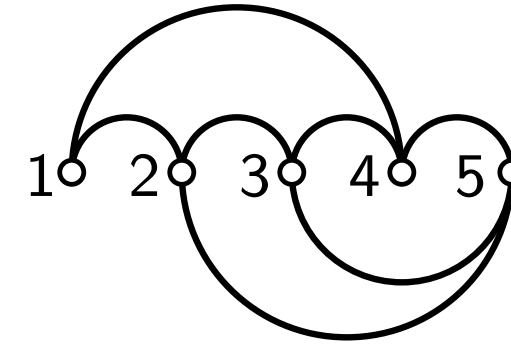
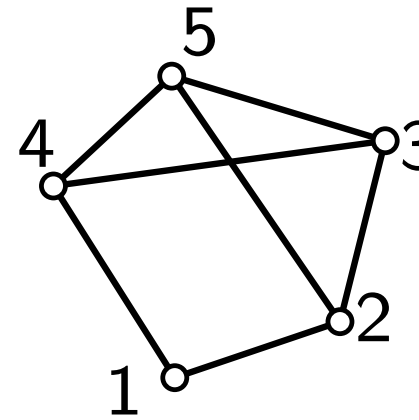
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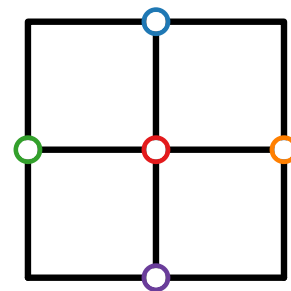
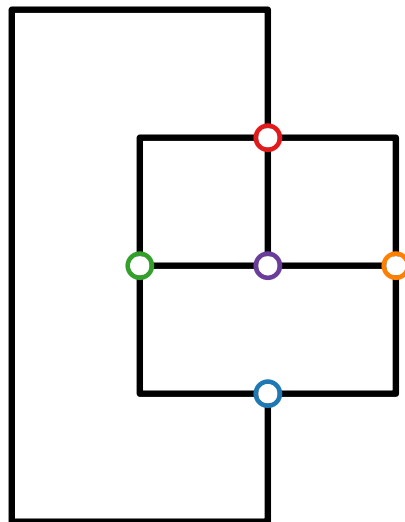
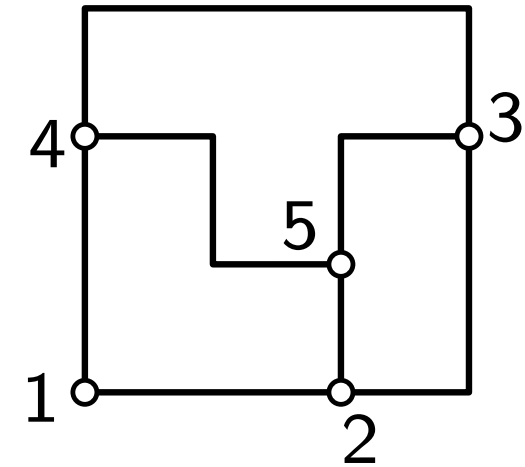
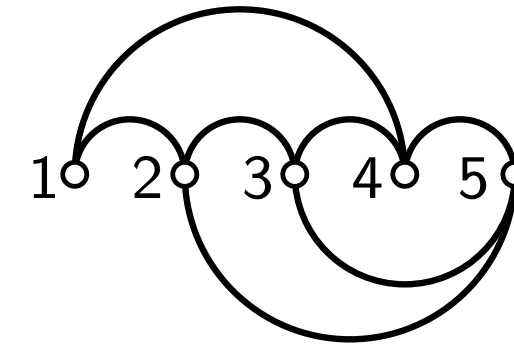
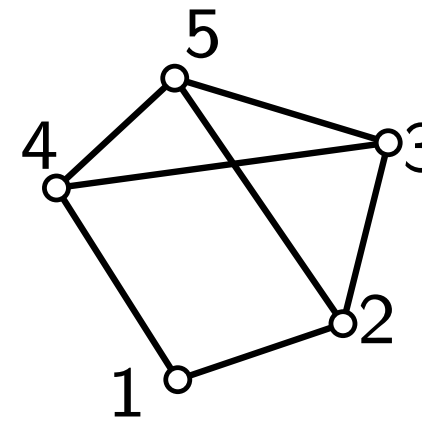
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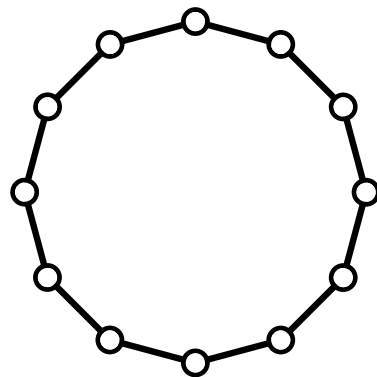
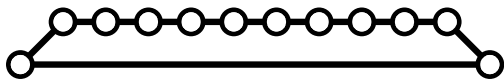
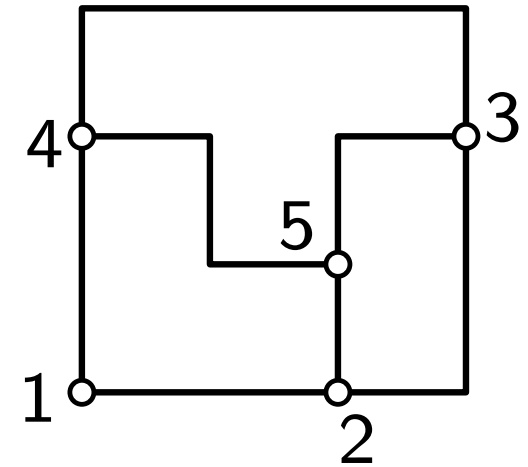
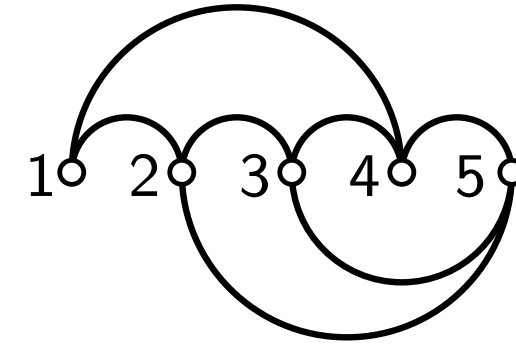
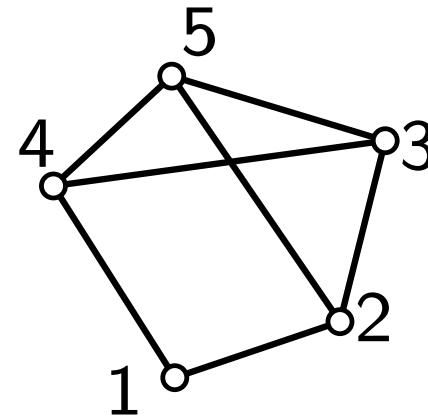
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. /



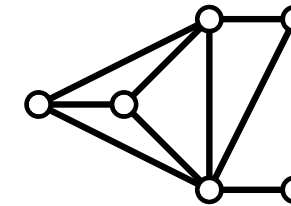
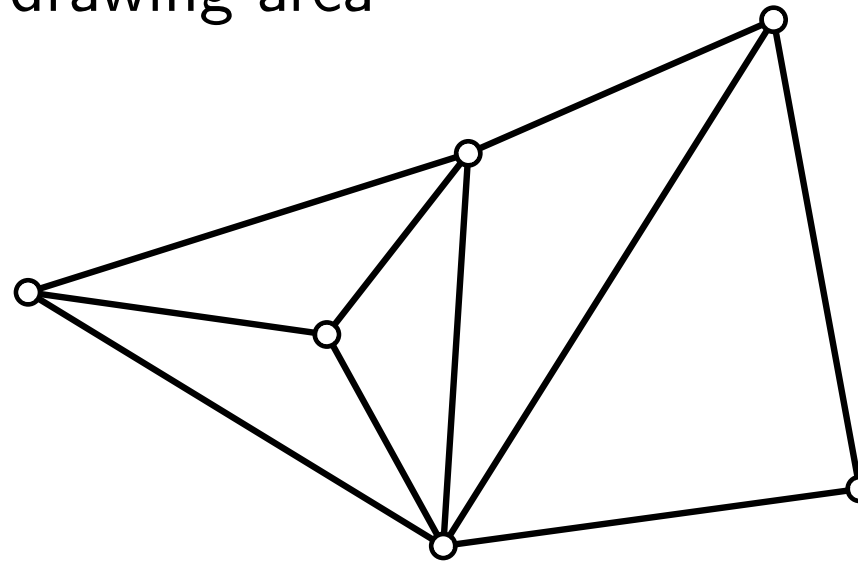
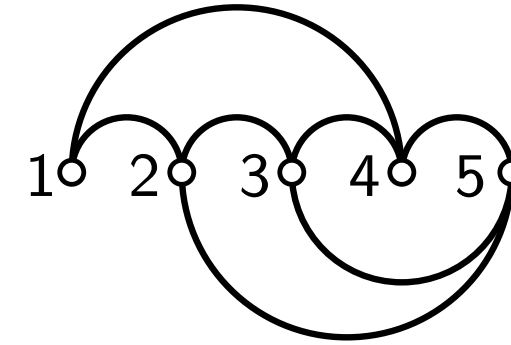
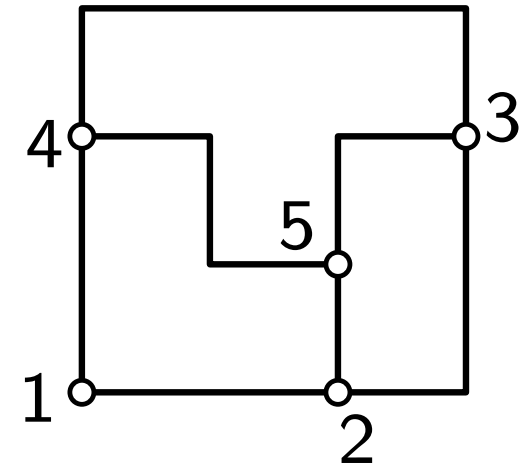
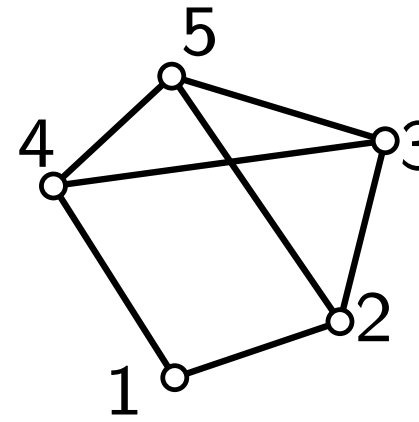
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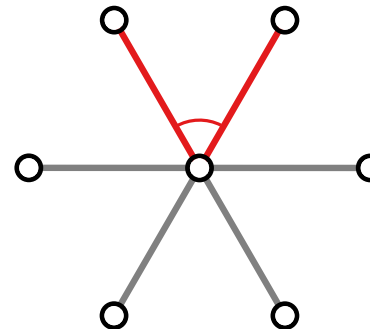
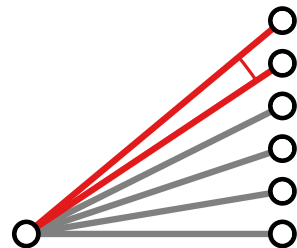
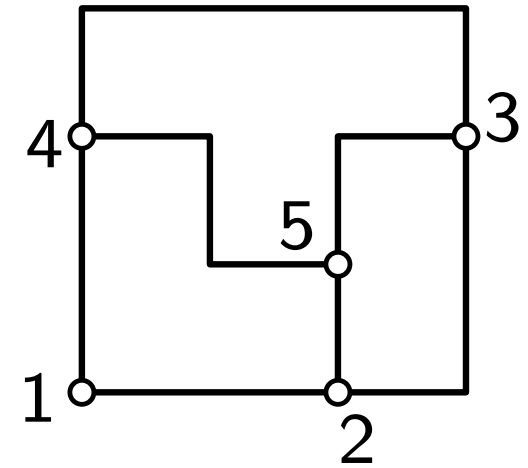
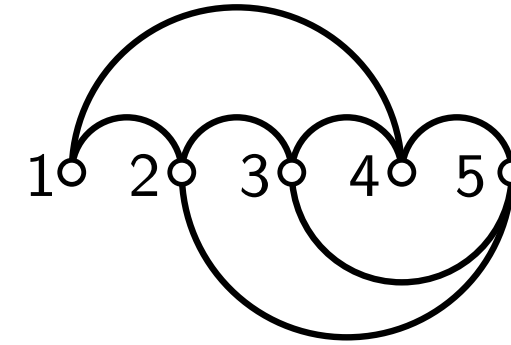
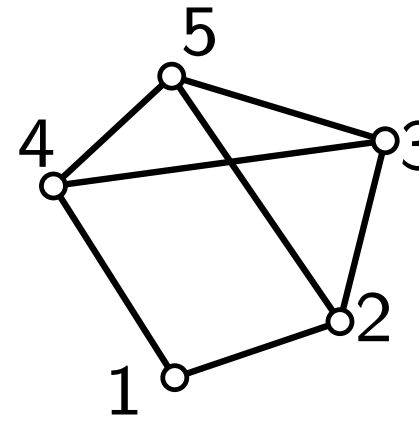
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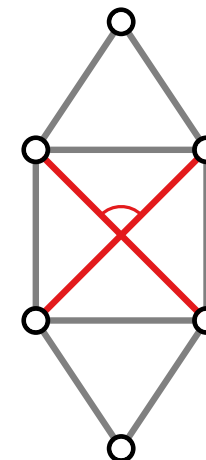
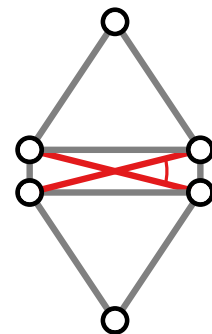
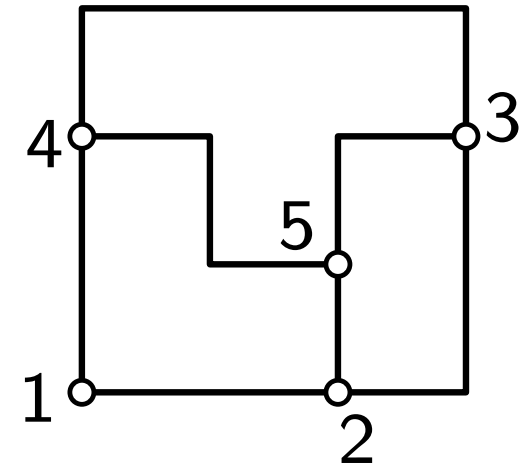
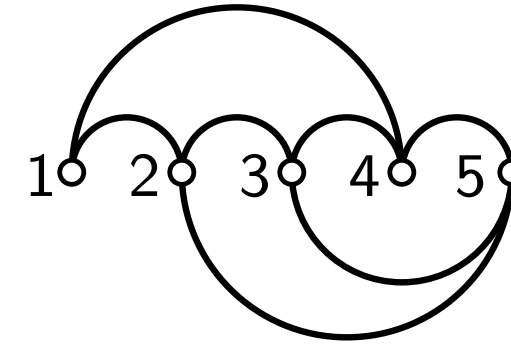
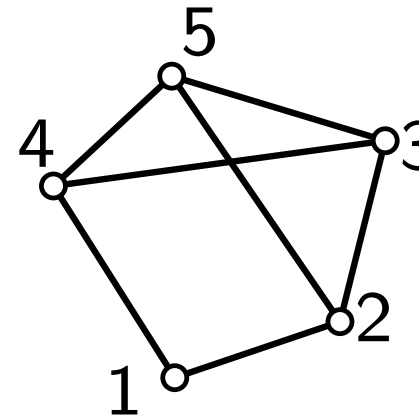
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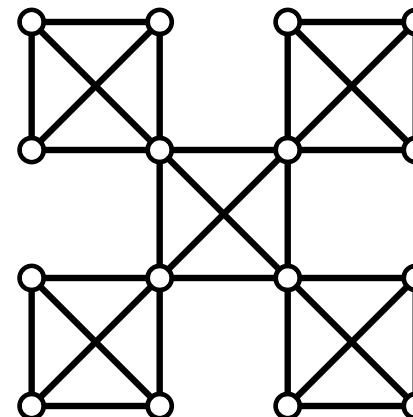
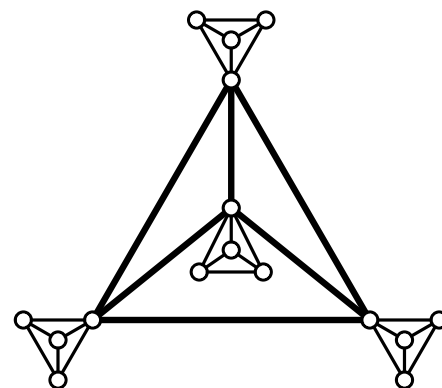
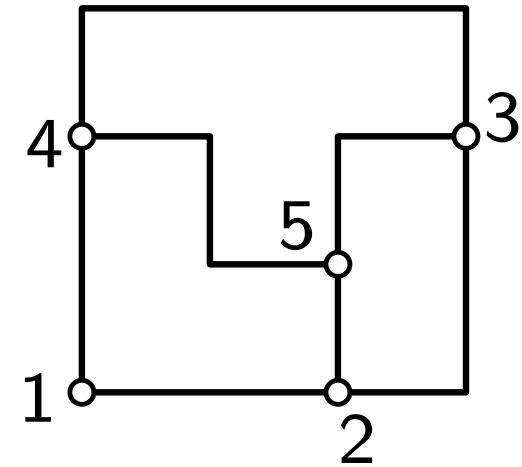
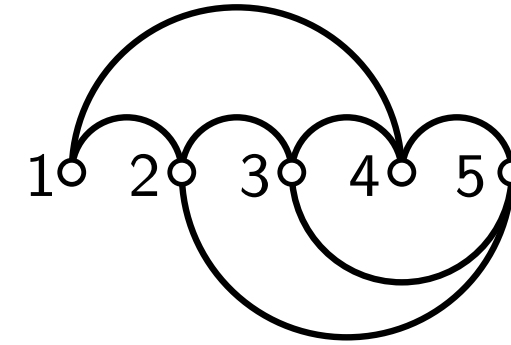
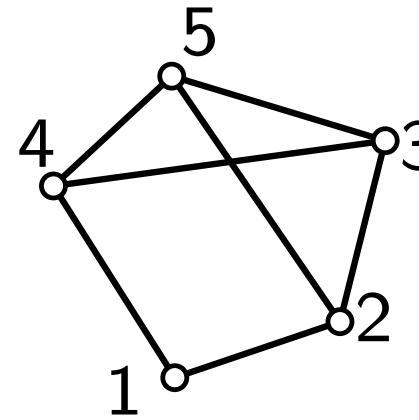
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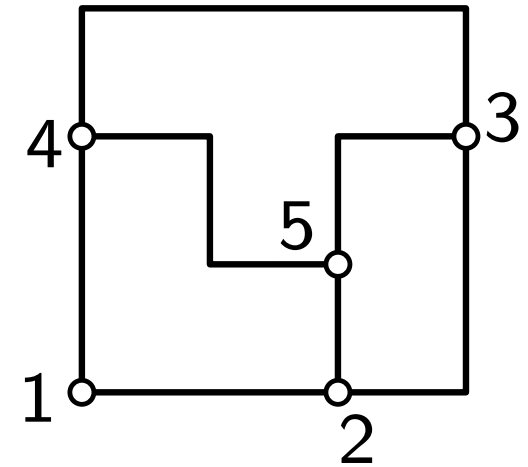
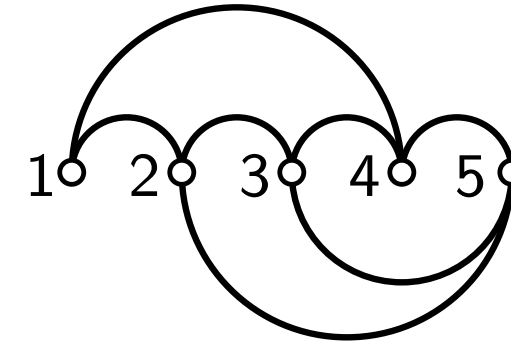
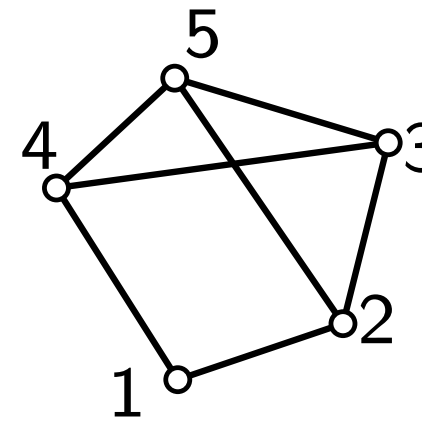
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→ such criteria are often inversely related

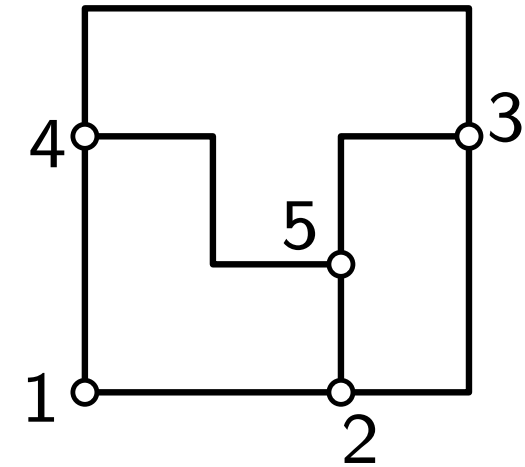
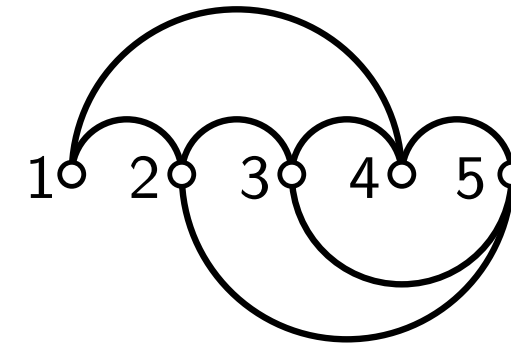
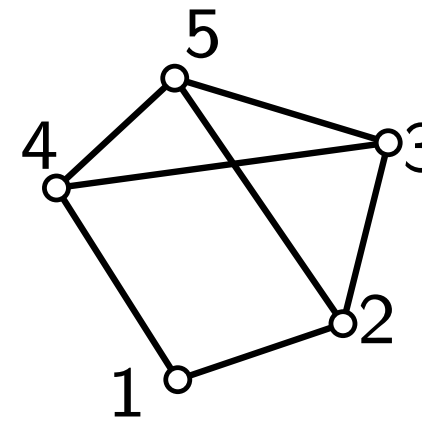
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- grid drawings
- without crossing

2. Aesthetics to be optimized, e.g.

- crossing/bend minimization
- edge length uniformity
- minimizing total edge length/drawing area
- angular resolution
- symmetry/structure



→ such criteria are often inversely related

→ lead to NP-hard optimization problems

Requirements of a graph layout

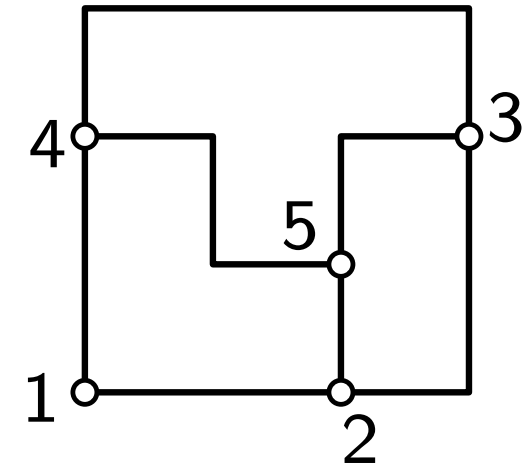
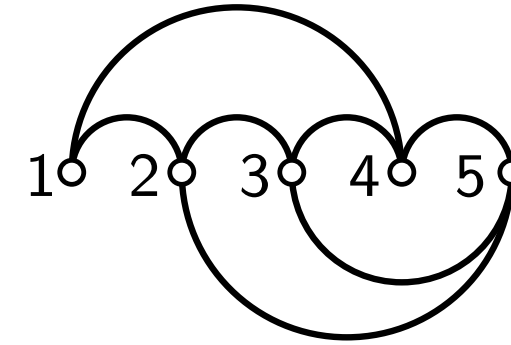
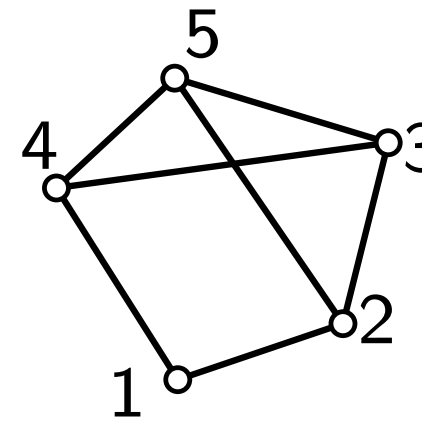
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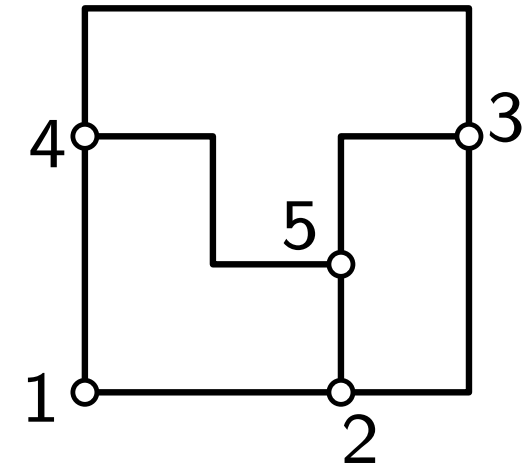
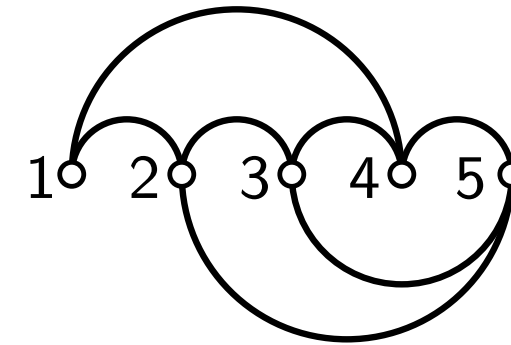
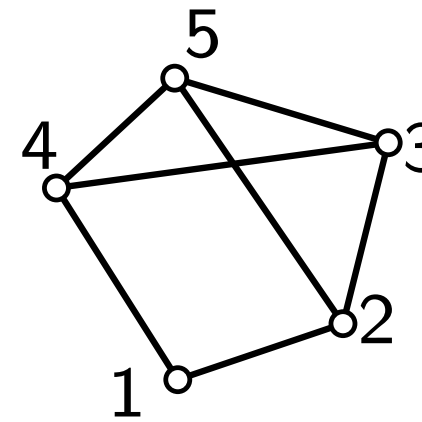
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3. Local Constraints, e.g.

- restrictions on neighboring vertices (e.g., “upward”).
- restrictions on groups of vertices/edges (e.g., “clustered”).



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The layout problem

Graph visualisation problem

in: Graph G

out: Drawing Γ of G such that

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- **drawing conventions** are met,
- **aesthetic criteria** are optimised, and
- some **additional constraints** are satisfied.