Monetary Policy

Part 2: Conventional Monetary Policy

Exercise 7: Solving the IS-MP-PC Model, Rational Expectations, Time-Inconsistency

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Task 1: Solving the IS-MP-PC Model

Task 1 (a)

Consider the two-equation model of the lecture. Which model variables are exogenously determined, and which are determined endogenously? How can we infer the nominal and real interest rate?

Task 1 (a)

The IS-MP and PC Model:

$$\pi_t = \pi_t^e + \gamma\,(y_t - y_t^*) + \varepsilon_t^\pi$$
$$y_t = y_t^* - \alpha(\beta_\pi - 1)(\pi_t - \pi^*) + \varepsilon_t^y$$

Exogenous variables:

- $y_t^*, \pi^*, \varepsilon_t^\pi, \varepsilon_t^y$

Endogenous variables:

- π_t, y_t

Task 1 (a)

Expectations term:

- Depends on the model solution method
    - Taken as given: engog. variables in terms of all other model elements
    - Solve explicitly for expectations term: assume rational expectations; solution expresses endog. variables as functions of exog. variables and the expectations process.

Nominal interest rate:

- Can be inferred via the original MP equation.

Real interest rate:

- Can be inferred via the Fisher equation.

Task 1 (b)

Solve the model for inflation and describe what determines the level of inflation fundamentally (i.e. shocks are assumed to be zero for now).

Task 1 (b)

$$\pi_t = \pi_t^e + \gamma\,(y_t - y_t^*) + \varepsilon_t^{\pi}$$

$$y_t = y_t^* - \alpha(\beta_\pi - 1)(\pi_t - \pi^*) + \varepsilon_t^{y}$$

Task 1 (b)

What determines inflation?

$$\pi_t = \theta \pi_t^e + (1-\theta)\pi^*$$

- Weighted average of expected and targeted inflation.

What is θ?

$$\theta = \frac{1}{1 + \alpha\gamma(\beta_\pi - 1)}$$

- γ: parameter stemming from the PC determining how strongly inflation reacts to changes in the output gap.
- α: parameter stemming from the IS curve determining how strongly output reacts to changes in the real rate gap.
- β_π: policy parameter describing how strongly the Central Bank reacts to deviations of inflation from target

Task 1 (c)

Based on the equation for inflation, what happens to the dynamics if the central bank becomes more active?

Task 1 (c)

What is θ?

$$\theta = \frac{1}{1 + \alpha\gamma(\beta_\pi - 1)}$$

- α: structural parameter from IS curve
- γ: structural parameter from PC curve

Central bank becomes more active:

- β_π increases, i.e. θ decreases

$$\pi_t = \theta\pi_t^e + (1 - \theta)\pi^*$$

- As a consequence, inflation driven more by central banks target and less by expected inflation.

Task 1 (d)

Explain how supply and demand shocks affect the current level of inflation.

Task 1 (d)

$$\pi_t = \theta\pi_t^e + (1-\theta)\pi^* + \theta(\gamma\varepsilon_t^y + \varepsilon_t^\pi)$$

Task 1 (d)

In general:

- Positive supply and demand shocks increase the level of inflation
- Both shocks are dampened in their effect by the parameter θ
- The demand shock is additionally distorted by the parameter γ, which is the slope of the PC

When is the effect of shocks strong?

$$\theta = \frac{1}{1 + \alpha\gamma(\beta_\pi - 1)}$$

- The higher θ, the stronger the effect of shocks.
- The higher γ, the stronger the effect of a demand shock.

Task 1 (e)

Solve the model for output and describe what determines output fundamentally (i.e. shocks are assumed to be zero for now).

Task 1 (e)

$$y_t = y_t^* - \alpha(\beta_\pi - 1)(\pi_t - \pi^*) + \varepsilon_t^y$$

Task 1 (e)

In general:

- Output is driven by the difference between expected and targeted inflation
- Assuming plausible values for the parameters, the inflation gap negatively influences output.

$$(\pi_t^e - \pi^*)$$

Increase in inflation expectations:

- Since $-\theta\alpha(\beta_\pi - 1) < 0$,
- negative effect on output.

Task 1 (f)

Explain how supply and demand shocks affect output.

Task 1 (f)

$$y_t = y_t^* - \theta\alpha(\beta_\pi - 1)(\pi_t^e - \pi^* + \varepsilon_t^\pi) + (1 - \theta\alpha\gamma(\beta_\pi - 1))\varepsilon_t^y$$

Task 1 (f)

In general:

- A positive demand shock increases output.
- A positive supply shock decreases output.
    - If supply is shocked, output increases above target.
    - The central bank increases the nominal (and real) interest rate.
    - Output decreases via the IS curve.

Pass-through of shocks:

- Depends on the structural and policy parameters.
- In case of a positive demand shock, the higher is β_π, the smaller is $(1 - \theta\alpha\gamma(\beta_\pi - 1))$ and the smaller is the pass-through of the shock.

Task 1 (g)

Calculate partial derivatives illustrating how inflation and output change with an increase in inflation expectations c.p.

Task 1 (g)

Task 1 (g)

Effect of higher inflation expectations:

- Output is affected negatively.
- Inflation is affected positively.

Pass-through:

- The effect on output is $\alpha(\beta_\pi - 1)$ times larger than the effect on inflation.
- Central bank faces trade-off in being more or less aggressive.

Task 2: Rational Expectations Solution

Task 2 (a)

Rederive the rational expectations solution of the model. Which assumptions must be made regarding the shock processes?

Task 2 (a)

$$\pi_t = \pi_t^e + \gamma\,(y_t - y_t^*) + \varepsilon_t^{\pi}$$
$$y_t = y_t^* - \alpha(\beta_{\pi} - 1)(\pi_t - \pi^*) + \varepsilon_t^{y}$$

Task 2 (a)

Task 2 (b)

Which assumptions must be made regarding the shock processes?

Task 2 (b)

Assumption regarding shocks:

- Shocks are assumed to be zero.

Calibration of shock processes:

- Mean zero
- Some standard deviation

Task 2 (c)

What is the solution’s intuition for the anchoring of inflation expectations?

Task 2 (c)

Rational expectations (RE):

- Agents in the model form their expectations according to the model‘s equations.
- RE assume no uncertainty.
- The model is assumed to be a realistic description of the reality.

Deviations from target:

- Inflation only deviates from target, when an unexpected shock occurs.
- Output only deviates from target, when an unexpected shock occurs.

Strict anchoring:

- Strong assumption.
- If possible, business cycle (volatility) substantially reduced.

Task 3: Unstable Solutions

Task 3 (a)

What does stability in case of difference equations imply?

Task 3 (a)

Stability:

- Stable solutions yield monotonical convergence to a certain value.

Instability:

- Instable solutions let the value of a variable convergence to $\pm\infty$.

Implication:

- Calibrate model parameters such that solution is stable.

Task 3 (b)

What is the intuition behind an unstable solution to our model, i.e. for values $1 < \theta < \infty$?

Task 3 (b)

Stable solution:

- Temporary demand / supply shocks let output and inflation deviate from long-run level initially.
- If shock fades out, output and inflation revert back to long-run level.

Unstable solution:

- Demand / supply shocks let output and inflation deviate from long-run level.
- System explodes, i.e. inflation / output converge to $\pm\infty$.

6.3 Time Inconsistency

Task 4 (a)

Describe your intuition for the central bank's Social Loss Function in the time in-consistency problem.

$$SL_t = (y_t - y_t^e)^2 + \kappa \pi_t^2$$

$$y_t^e = y_t^* + \omega$$

Task 4 (a)

Social loss function:

- Mathematical formula for loss in society
- Assuming central bank as a social planner

Here, two components: $SL_t = (y_t - y_t^e)^2 + \kappa \pi_t^2$

- First part: total deviation of current output from efficient output level
- Second part: squared level of inflation with preference parameter κ
- Inflation aversion of central bank

Task 4 (b)

Rederive the first-order condition under the so-called „cheating solution“.

Task 4 (b)

$$\pi_t = \pi_t^e + y_t - y_t^*$$

$$y_t = y_t^* - \alpha(r_t - r^*)$$

$$SL_t = (y_t - y_t^e)^2 + \kappa\pi_t^2$$

Task 4 (b)

Task 4 (b)

Rewrite SL in terms of output gap:

$$SL_t = (y_t - y_t^* - \omega)^2 + \kappa\pi_t^2$$

Substitute in the Phillips curve:

$$SL_t = (\pi_t - \pi_t^e - \omega)^2 + \kappa\pi_t^2$$

Take FOC:

$$\frac{\partial SL_t}{\partial \pi_t} = 2(\pi_t - \pi_t^e - \omega) + 2\kappa\pi_t = 0$$

Task 4 (b)

Assume $\pi^* = 0$, so that $\pi_t^e = 0$.

Solution:

$$\pi_t^C = \frac{\omega}{1+\kappa}, \quad y_t^C = y_t^* + \frac{\omega}{1+\kappa}$$

Task 4 (c)

Compare the social loss from the cheating solution and the rule-based solution.

Task 4 (c)

$$SL_t = (y_t - y_t^* - \omega)^2 + \kappa\pi_t^2$$

Task 4 (c)

$$SL_t^R - SL_t^C$$

Task 4 (d)

Why will the cheating solution not be implemented by the central bank? What is the consequence?

Task 4 (d)

Positive welfare gain from cheating:

- If central bank announces to follow a monetary policy rule,
- there is a benefit from deviating from the rule.

Agents are assumed to be rational:

- Per assumption: agents expect central bank to follow rule
- $\pi^e = \pi^*$

But (!) agents are aware of welfare gain from cheating:

- Agents know of the time inconsistency
- Adapt their expectations accordingly

Task 4 (d)

Consequence:

- $\pi_t^e = \frac{\omega}{\kappa}$
- Inflation bias due to time inconsistency
- Expected inflation higher than targeted inflation
- Output, however, equal to potential

Task 4 (d)

Cheating Solution:

$$\pi_t^C = \frac{\omega}{1+\kappa}\ ,\quad y_t^C = y_t^* + \frac{\omega}{1+\kappa}$$

Rule-based Solution:

$$\pi_t^R = 0, y_t^R = y_t^*$$

Discretionary Policy:

$$\pi_t^D = \frac{\omega}{\kappa}, \mathrm{y}_\mathrm{t}^\mathrm{D} = \mathrm{y}_\mathrm{t}^*$$