Exercise Session 5: Monetary Policy

Monetary Policy Framework: Optimal Rate of Inflation, Inflation Targeting

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TASK 1: Optimal Rate of Inflation

a) Briefly summarize the Pros and Cons for positive/negative/zero inflation.

Arguments for Zero Inflation

Costs of non-zero inflation:

- Redistribution effects (inflation: from creditors to debtors; deflation, vice versa)
- Hedging against those risks absorbs resources
- Unanticipated inflation: discourages savings, hampering investment & growth
- If not all prices do adjust instantaneously → distortions of price signal & inefficient use of resources
- Cost of price changes
- Inflation: tax burden might increase if not adjusted for
- High inflation: cost of managing cash holdings (‘shoe leather costs’)

TASK 1: Optimal Rate of Inflation

a) Briefly summarize the Pros and Cons for positive/negative/zero inflation.

Arguments for Positive Inflation

- Inaccurate measurement may overstate true inflation (by about 1 %) (measurement might dismiss substitution effects)
- Labor market effects: since adjusting wages downwards is difficult (also in recessions), having positive inflation decrease real wages, given that nominal wages stay constant
- Managing danger of deflation(ary) spirals (increasing real debt burdens → debt service gets more expensive → defaults on loans amplifying the downturn)
- ZLB on interest rates is reached faster & more often after an adverse shock the lower the inflation target

Arguments for Deflation

- Opportunity cost of holding money faced by private agents, i.e. the nominal interest rate, should equal the social cost of creating additional fiat money
- If the marginal cost of creating additional money is zero or approximately zero, the nominal interest rate should be zero; together with a positive real interest rate (e.g. 2 %), this implies deflation via the Fisher equation:

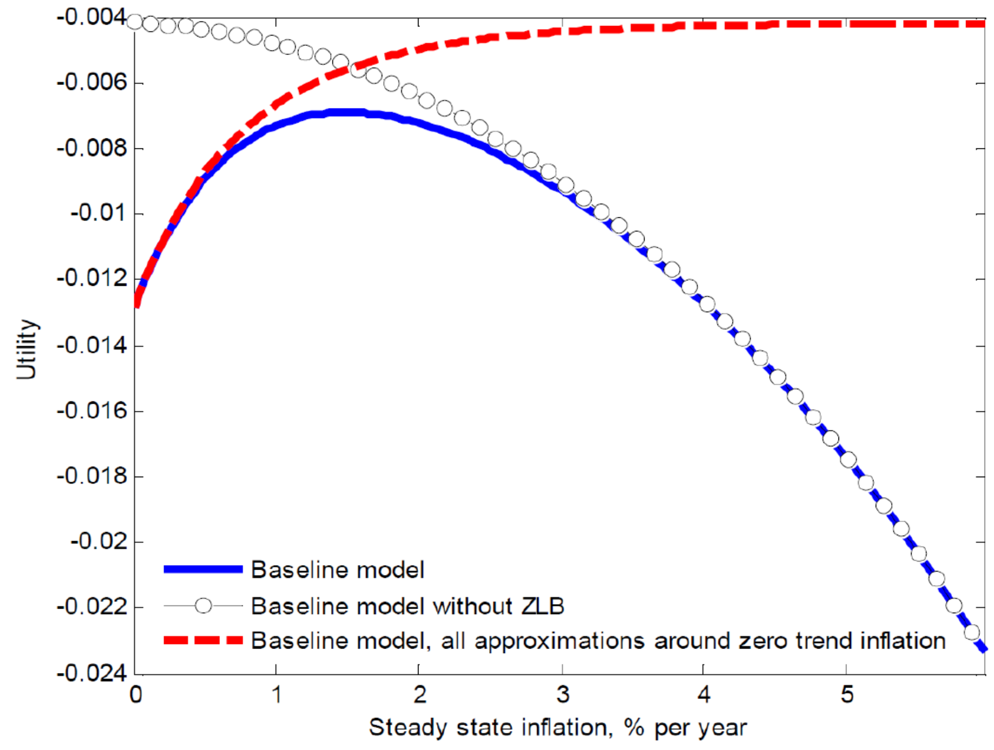
$$\underbrace{i}_{0} = \underbrace{r}_{2} - \underbrace{\pi}_{-2}$$

Known as the Friedman Rule (optimal inflation rate = minus the long-run real interest rate). This result holds in frictionless (flex price) models without zero lower bound constraint.

NKMs (including nominal rigidities) indicate optimal inflation rate of zero (larger than Friedman Rule).

TASK 1: Optimal Rate of Inflation

b) While the optimal inflation rate in the baseline New Keynesian model is zero, this optimal solution changes if we take into account that monetary policy may be constrained by a ZLB on nominal interest rates. Discuss how the solution changes based on the graph provided below.



Source: Coibion, Gorodnichenko and Wieland (2011)

Circles: without ZLB, but with relative price distortions (cost of inflation) [standard NKM], optimal rate of inflation=zero (so that rel. price distortions are minimized) → only costs of inflation & no benefits considered.

Red line: includes ZLB, but without relative price distortion effects, i.e. no costs of inflation; so utility strictly increases as steady-state inflation increases

Blue line: includes both frictions (ZLB & rel. price distortions) so that costs & benefits of inflation are considered and trade-off becomes apparent, utility is increasing at very low levels of inflation so that zero inflation is not optimal when the ZLB is present; peak occurs ~1.5% at annualized rate

TASK 2: Inflation Targeting – Svensson's Model (1997)

a) After how many periods do the output gap and the inflation react to a change in the interest rate i_t?

- $\Delta i_t \curvearrowright x_{t+1} \curvearrowright \pi_{t+1}$
- Hence, output gap reacts after 1 period
- While inflation rate reacts after 2 periods

TASK 2: Inflation Targeting – Svensson's Model (1997)

Assume further that the economy's central bank follows a strategy of strict inflation targeting, i.e. it seeks to minimize the following period loss function:

$$L(\pi_{t+2}) = E_t\left[\frac{1}{2}(\pi_{t+2} - \pi^*)^2\right]$$

b) What is the policymaker's control variable?

- control variable: $i_t \rightarrow$ set i_t to minimize loss function

c) What is/are the state variable(s) that the policymaker takes as given when deciding upon policy?

- state variables: π_t, x_t (policymaker takes these as given when s(he) decides upon policy)

TASK 2: Inflation Targeting – Svensson's Model (1997)

d) Explain the difference between strict and flexible inflation targeting.

Strict inflation targeting:

- Announce target for inflation and use policy instrument to achieve this target
- Raise/cut interest rate if (forecasted) inflation is above/below target
- Highly transparent, but very restrictive

Flexible inflation targeting:

- Give some weight to output stabilization in addition to a pure inflation target
- Formal or informal versions of flexible inflation targeting have been adopted by most central banks

TASK 2: Inflation Targeting – Svensson's Model (1997)

e) Derive the central bank's optimal targeting rule under strict inflation targeting. Interpret your result economically.

$$L(\pi_{t+2}) = E_t\left[\frac{1}{2}(\pi_{t+2} - \pi^*)^2\right]$$

→ to get to the **targeting rule**, derive FOC

First step: iterate PC-curve forward to get expression for π_{t+2}:

$\pi_{t+2} =$

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First step: iterate PC-curve forward to get expression for π_{t+2}:

$$\pi_{t+2} = \underbrace{\pi_{t+1}}_{repl.\,with\,PC-curve} + \alpha_1 \underbrace{x_{t+1}}_{repl.\,with\,IS-curve} + \varepsilon_{t+2}$$

$$\pi_{t+2} = (\pi_t + \alpha_1 x_t + \varepsilon_{t+1}) + \alpha_1(\beta_1 x_t - \beta_2(i_t - \pi_t) + \eta_{t+1}) + \varepsilon_{t+2}$$

Factor out:

$$\pi_{t+2} = (1 + \alpha_1\beta_2)\pi_t + \alpha_1(1 + \beta_1)x_t - \alpha_1\beta_2 i_t + (\varepsilon_{t+1} + \alpha_1\eta_{t+1} + \varepsilon_{t+2})$$

TASK 2: Inflation Targeting – Svensson's Model (1997)

e) Derive the central bank's optimal targeting rule under strict inflation targeting. Interpret your result economically.

Redefine the notation to make your life easier:

Define: $a_1 = (1 + \alpha_1\beta_2) \wedge a_2 = \alpha_1(1 + \beta_1) \wedge a_3 = \alpha_1\beta_2$

Rewrite: $\pi_{t+2} = a_1\pi_t + a_2 x_t - a_3 i_t + (\varepsilon_{t+1} + \alpha_1\eta_{t+1} + \varepsilon_{t+2})$

TASK 2: Inflation Targeting – Svensson's Model (1997)

e) Derive the central bank's optimal targeting rule under strict inflation targeting. Interpret your result economically.

$$\pi_{t+2} = a_1\pi_t + a_2 x_t - a_3 i_t + (\varepsilon_{t+1} + \alpha_1 \eta_{t+1} + \varepsilon_{t+2})$$

Second step: compute FOC

$$\begin{aligned}\frac{\partial L(\pi_{t+2})}{\partial i_t} &= E_t\left[(\pi_{t+2} - \pi^*) \cdot \frac{\partial \pi_{t+2}}{\partial i_t}\right] \\ &= -a_3\left(\pi_{t+2|t} - \pi^*\right) = 0 \\ \Leftrightarrow \pi_{t+2|t} &= \pi^*\end{aligned}$$

- Optimal target rule under strict inflation targeting → Inflation forecast targeting
- Intermediate target: set $i = f(\pi, x)$ such that $E_t(\pi_{t+2}) = \pi^*$ (target rate of below, but close to 2%)

TASK 2: Inflation Targeting – Svensson's Model (1997)

f) Derive the central bank's optimal instrument rule, which states i_t in term of current inflation, the current output gap, and the model parameters under strict inflation targeting. Interpret your result economically.

To get from central bank's optimal targeting to optimal instrument rule, determine $\pi_{t+2|t}$

$\Longrightarrow \pi_{t+2|t}$: conditional on the information about π_{t+2} available in period t

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$\Longrightarrow \pi_{t+2|t}$: conditional on the information about π_{t+2} available in period t

$$\pi_{t+2|t} = a_1\pi_t + a_2x_t - a_3i_t \mid no\ knowledge\ about\ future\ shocks$$

$$a_3i_t = a_1\pi_t + a_2x_t - \pi^* \quad \mid rearrange\ for\ instrument\ i$$

$$i_t = \frac{1}{a_3}(a_1\pi_t - \pi^* + a_2x_t)$$

TASK 2: Inflation Targeting – Svensson's Model (1997)

f) Derive the central bank's optimal instrument rule, which states i_t in term of current inflation, the current output gap, and the model parameters under strict inflation targeting. Interpret your result economically.

Rewrite the monetary policy rule in terms of the model parameters:

$$i_t = \frac{1}{a_3}(a_1\pi_t - \pi^* + a_2 x_t) \quad |\, multiply\ out$$

$$i_t = \frac{a_1}{a_3}\pi_t - \frac{1}{a_3}\pi^* + \frac{a_2}{a_3}x_t \quad |\, replace\ with\ model\ parameters$$

$$i_t = \frac{1+\alpha_1\beta_2}{\alpha_1\beta_2}\pi_t - \frac{1}{\alpha_1\beta_2}\pi^* + \frac{\alpha_1(1+\beta_1)}{\alpha_1\beta_2}x_t \quad |\, cancel\ out\ \&\ rearrange$$

$$= \frac{\alpha_1\beta_2}{\alpha_1\beta_2}\pi_t + \frac{1}{\alpha_1\beta_2}\pi_t - \frac{1}{\alpha_1\beta_2}\pi^* + \frac{1+\beta_1}{\beta_2}x_t \,|\, simplify$$

$$i_t = \pi_t + \frac{1}{\alpha_1\beta_2}(\pi_t - \pi^*) + \frac{1+\beta_1}{\beta_2}x_t$$

TASK 2: Inflation Targeting – Svensson's Model (1997)

g) Explain the difference between a targeting and an instrument rule verbally.

FOCs are targeting rules:

- General condition that the central bank aims to fulfill
- Can be implemented using different models and one can use more information to forecast inflation than reflected in one specific model
- <u>No</u> explicit recipe how to adjust the interest rate
- Possibility in making mistakes in interest rate setting as no precise formula is given
- Commitment to target rule in reality plausible

TASK 2: Inflation Targeting – Svensson's Model (1997)

g) Explain the difference between a targeting and an instrument rule verbally.

Instrument rules:

- Precise formula on how to set the interest rate given a specific model
- Less flexible and possibly less robust than targeting rule
- Interest rate setting might be inefficient if the model is misspecified
- <u>No</u> central bank would commit to a narrowly defined instrument rule

TASK 2: Inflation Targeting – Svensson's Model (1997)

h) Now consider the case of *flexible* inflation targeting. Explain how the loss function would change. Further, explain how the loss function for *strict* inflation targeting is nested in the loss function for *flexible* inflation targeting.

$$L(\pi_t) \quad = E_t\left[\frac{1}{2}(\pi_t - \pi^*)^2 + \lambda x_t^2\right] \quad \rightarrow \text{flexible inflation targeting}$$

- $\lambda > 0$: captures relative weight on output stabilization
- $\lambda = 0$: strict inflation targeting $\Longrightarrow$ hence, nested in loss function of flexible inflation targeting

TASK 2: Inflation Targeting – Svensson's Model (1997)

i) The central bank's optimal instrument rule under flexible inflation targeting is given by $i_t = \pi_t + \frac{1-c}{\alpha_1\beta_2}(\pi_t - \pi^*) + \frac{1-c+\beta_1}{\beta_2}x_t$. Compare this rule with the one under strict inflation targeting and explain how the latter is nested in the former.

(1) Strict inflation targeting: $i_t = \pi_t + \frac{1}{\alpha_1\beta_2}(\pi_t - \pi^*) + \frac{1+\beta_1}{\beta_2}x_t$

(2) Flexible inflation targeting: $i_t = \pi_t + \frac{1-c}{\alpha_1\beta_2}(\pi_t - \pi^*) + \frac{1-c+\beta_1}{\beta_2}x_t$

- If c = 0, (2) simplifies to (1) → optimal instrument rule under strict inflation targeting

TASK 2: Inflation Targeting – Svensson's Model (1997)

j) Now consider a positive demand shock $\eta_0 > 0$.

Calculate the initial response of the interest rate under strict and flexible targeting. Compare them. Explain why the central bank reacts to a demand shock at all under strict inflation targeting.

Start from long-run equilibrium in $t = 0$ (shock period):

IS-curve: $x_0 = \beta_1 x^* - \beta_2(i^* - \pi^*) + \eta_0$

Initial interest rate response: $i_0 = \pi_0 + \frac{1}{\alpha_1 \beta_2}(\pi_0 - \pi^*) + \frac{1+\beta_1}{\beta_2} x_0$

Assuming that we start in steady state: $i^* = \pi^* \wedge x^* = 0$

For a positive demand shock, it then follows that:

$x_0 = \beta_1 0 - \beta_2(0) + \eta_0 \;\rightarrow\; x_0 = \eta_0 > 0$

TASK 2: Inflation Targeting – Svensson's Model (1997)

j) Now consider a positive demand shock $\eta_0 > 0$.

Calculate the initial response of the interest rate under strict and flexible targeting. Compare them. Explain why the central bank reacts to a demand shock at all under strict inflation targeting.

Positive demand shock: $x_0 = \eta_0 > 0$

Initial interest rate response under *strict* inflation targeting:

$$i_0 = \pi_0 + \frac{1}{\alpha_1 \beta_2}(\pi_0 - \pi^*) + \frac{1 + \beta_1}{\beta_2}\eta_0$$

Initial interest rate response under *flexible* inflation targeting:

$$i_0 = \pi_0 + \frac{1 - c}{\alpha_1 \beta_2}(\pi_0 - \pi^*) + \frac{1 - c + \beta_1}{\beta_2}\eta_0$$

TASK 2: Inflation Targeting – Svensson's Model (1997)

j) Now consider a positive demand shock $\eta_0 > 0$.

Calculate the initial response of the interest rate under strict and flexible targeting. Compare them. Explain why the central bank reacts to a demand shock at all under strict inflation targeting.

With $0 < c < 1$, $i^{flex} < i^{strict}$.

- Initial response of central bank smaller to $\eta_0 > 0$ in flexible than in strict inflation targeting regime, due to greater weight on business cycle smoothing
- Implication: future inflation will be higher since $i^{flex} < i^{strict}$ ($\curvearrowright x_{t+1} \nearrow \curvearrowright \pi_{t+1} \nearrow$)
- Central bank reacts to a demand shock under strict inflation targeting since the output gap impacts future inflation via the PC curve

TASK 2: Inflation Targeting – Svensson's Model (1997)

k) Now consider a positive demand shock $\eta_0 > 0$.

Under which of the strategies do you expect the demand shock to be more persistent?

- Demand shocks will prevail longer under flexible inflation targeting.

- Under strict inflation targeting, the central bank's reaction is stronger with $i^{strict} > i^{flex}$ ⇒ greater weight to return back to inflation target: therefore strong response and quick return

- Flexible inflation targeting: central bank also considers the path of the output gap and responds less strongly to smooth the transition & dampen business cycle volatility