Exercise Session 3
The Long Run 2: Natural Interest Rate, Estimating Unobservable Variables, and Inflation in the Long Run

Task 1: Repetition. Recall the solution for the long run interest rate derived from the Solow Growth model. How is each parameter driving r^*? Provide economic intuition.

$$r^* = \alpha\left(\frac{\delta + z + n}{s}\right).$$

Task 2: Argue with the help of the Fisher equation why a persistent decrease in r^* yields the zero lower bound to bind more often.

Task 3: Summarize the possible reasons of a decline in r^* discussed in class concisely.

Task 4: Read the required reading and discuss the following questions concisely based on the article by Jordà and Taylor (2019):

(a) What drives a country's rate of interest?
(b) What explains the variation in interest rates?
(c) Discuss the insights from the Solow Model in context of the article. Which aspects are reflected in the model, which are not?

TASK 5: Explain how the HP filter and the bandpass filter decompose a time series into trend and cycle in context of GDP.

TASK 6: Estimating Trends/Cycles (Potential output/output gaps) in EXCEL

(a) Deterministic detrending: Subtract a linear time trend from real GDP. Write down the algebraic expression.
(b) Regress real GDP on a linear, quadratic, and cubic time trend. Write down the OLS regression equation in matrix notation and show how one can obtain an output gap estimate from this regression.

I will provide solutions for the following excel-specific parts in a video, uploaded before the next exercise.

(c) Open the EXCEL sheet *TrendAndCycles.xlsx* and fill in the blanks.
   a. First, transform real GDP into logs.
   b. Build the regressor matrix X buy adding a constant, a linear, quadratic, and cubic time trend.
   c. Compute the matrix X'X for the three specifications: linear, quadratic, and cubic trend, using the MMUL-command.
   d. Compute the variance-covariance X'Y matrix, using the MMUL-command.
   e. Compute the beta coefficients, using the OLS estimator and the MMUL- and MINV-commands.
   f. Compute the trend components as X*beta.
   g. Compute the respective output gaps as the difference between actual real GDP and your trend estimates. Transform your result into full percentage points.
   h. Discuss the plots. What would the results imply in context of business cycle stabilization policies?

i. Repeat the exercise for the Hamilton Filter by filling in the blanks: transform real GDP into logs, build the regressor matrix, compute the respective matrices, and the beta coefficients. Compute the trend measure and the resulting output gap measure.
j. Discuss the plots also in terms of one- and two-sided filtering.

Task 7: Compare the approaches used to estimate the natural interest rate in contexts of their characteristics, but also their limitations.

Task 8: Quantity Theory

(a) Use the Quantity Theory to show how long run inflation is controlled by the central bank.

(b) Use the Quantity Theory to explain why inflation has been persistently low over the last decade.