

# Monetary Policy

## Part 1: Basic Macroeconomic Concepts

### Lecture 2: The Long Run: Solow Model, Equilibrium Unemployment, Quantity Theory

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# Outline

## Part 1: Basic Macroeconomic Concepts

- Lecture 1: GDP Measurement, Growth and Business Cycles
- **Lecture 2: The Long Run: Solow Model and Equilibrium Unemployment**
- Lecture 3: The Long Run: Natural Interest Rate and Quantity Theory
- Lecture 4: The Short Run: The IS-MP-PC Model

## Part 2: Conventional Monetary Policy

## Part 3: Monetary Policy at the Zero Lower Bound on Nominal Interest Rate

## Part 4: Monetary and Fiscal Interactions

## Part 5: Financial Stability (if time permits)

## Mock Exam

## Learning Objective of Today's Lecture

We will review the concept of aggregate supply and unemployment in the long run

1. What are important drivers of production, labor, capital, wages and interest rates in the long run?
2. Understanding unemployment in the long run

## Required reading

Unless you are fully familiar with the Solow Growth Model, I suggest that you read about it in an undergraduate textbook

- Very short version of the Solow Model:  
The Global Economy, Chapter 3 “Production Function” and 4 “Solow Model”, NYU Stern,  
[https://www.stern.nyu.edu/sites/default/files/assets/documents/The\\_Global\\_Economy\\_Amazon\\_Digital%20%283%29.pdf](https://www.stern.nyu.edu/sites/default/files/assets/documents/The_Global_Economy_Amazon_Digital%20%283%29.pdf)
- A more detailed, easy to understand version is in Blanchard “Macroeconomics” in the chapters on long run growth (or the German version by Blanchard/Illing)
- An excellent very detailed treatment is in Sørensen and Whitta-Jacobsen “Introducing Advanced Macroeconomics” (more technical than needed here)

## 2.1 Aggregate Supply in the Long Run

# Why Should We Study Long-Term Developments?

## Growth-oriented policies

- Understanding long-term developments is important for designing growth-oriented policies. We will look at some examples today, though this is not the main focus of this course.

## Business-cycle oriented stabilization policies

- Understanding long-term developments is crucial for being able to define measures of cyclical fluctuations, i.e. deviations from these long-term developments.
- In modern business cycle analysis, variables are analyzed in deviations from natural rates, i.e. long-run values of variables, and these deviations are what matter for monetary and fiscal stabilization policy (and not the variables per se):
  - Output gap: deviation of GDP from potential GDP
  - Interest rate gap: deviation of the interest rate from the natural interest rate
  - Unemployment gap: deviation of unemployment from its natural rate (NAIRU or NAWRU)

# Changes in long-term components

- Many recent discussions on short-term stabilization policy focus on whether the medium- to long-term components have changed. Examples:
  - Potential GDP: Effect of retirement wave of baby boomer cohort
  - NAIRU: Slow wage growth might have decreased the NAIRU in the US. Possible reasons: fall in labor share due to superstar firms (google etc.) and/or declining worker power
  - Natural interest rate: It has most likely decreased since the Global Financial Crisis.
- Throughout this lecture we focus on aggregate supply in the long-run and the equilibrium unemployment rate.
- Next lecture we will study the natural interest rate and inflation in the long run.

# Aggregate Supply

- You have probably studied the Solow growth model in previous classes. This is the standard model to think about long-run growth. If you don't know this model anymore, I suggest to go through it again at home (see readings).
- In this model technology, capital accumulation and labor supply determine long-run GDP.
- Technology refers generally to the efficiency of combining capital and labor into output. Except for actual production technology, a country's institutional environment is important for this.
- As we will see, technology (or productivity) is key for long-run growth.
- Fluctuations in demand generate temporary deviations from potential output.



# A Simple Aggregate Supply Framework

An aggregate production function characterizes a country's average production process. Often a Cobb-Douglas function is used:

$$Y = AF(K, L) = AK^\alpha L^{1-\alpha}$$

$Y$ : aggregate real output per year

$K$ : capital input per year

$L$ : hours worked per year

$A$ : efficiency with which capital and labor are combined. It is also referred to as total factor productivity (TFP)

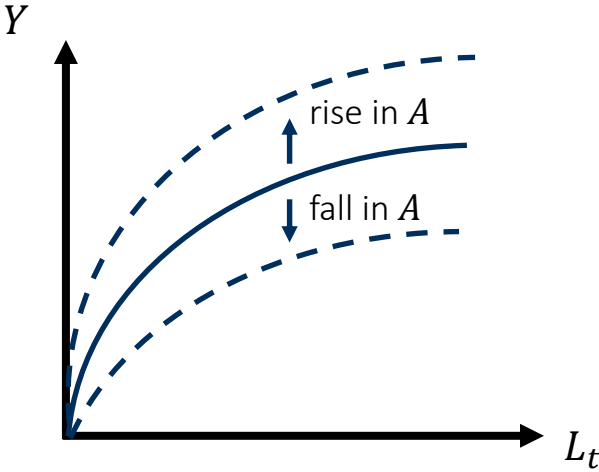
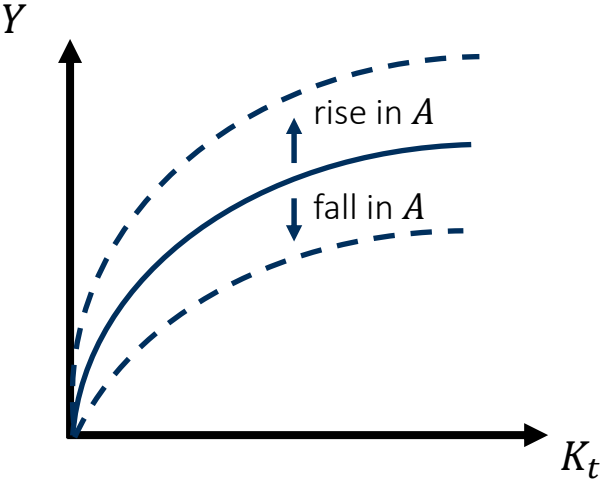
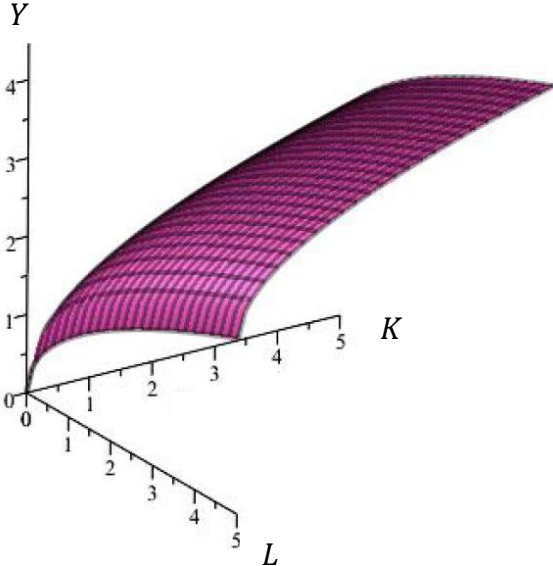
$\alpha$ : Proportionate contribution of  $K$  to total inputs. Empirically around **0.3** in advanced economies, lower in developing economies ( $\approx 0.15$ )

Three important features of Cobb-Douglas production functions:

1. Constant returns to scale
2. Marginal products of  $K$  and  $L$  are positive: if  $K$  or  $L$  increase, output increases
3. Diminishing returns of  $K$  and  $L$ : The marginal product of each input decreases as more of that input is added with an unchanged amount of the other input.

# Cobb-Douglas Production Function

Concave in labor and capital



## Solow Growth Model – Main equations

- Production function:  $Y = AF(K, L) = AK^\alpha L^{1-\alpha}$
- Resource constraint:  $Y_t = C_t + I_t$
- Exogenous savings rate  $s$ :  $I_t = s Y_t$
- Capital accumulation equation:  $K_{t+1} = I_t + (1 - \delta)K_t$
- Equations can be written in efficiency units  $k = \frac{K}{ZL}$  (with  $Z = A^{1/(1-\alpha)}$ )
- Aggregate production in efficiency units yields  $y = \frac{Y}{ZL} = k^\alpha$
- Labor force (which equals population) and technology grow at the constant rates  $n$  and  $z$ , respectively
- There are only real variables in the Solow growth model. In general, it is assumed that in the long-run money does not matter for real economic variables, so monetary policy has no effect on output in the long run.

# Solow Growth Model – Solution

- After some calculations, we obtain the central capital accumulation equation:

$$k_{t+1} - k_t = sk_t^\alpha - (\delta + z + n)k_t$$

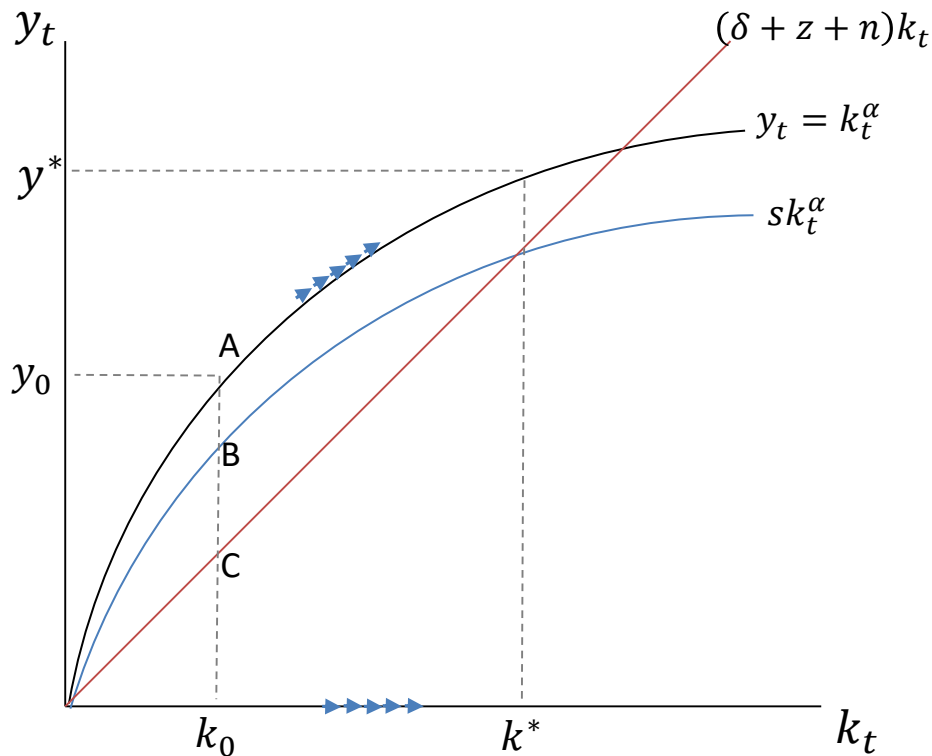
- Thus, the change in the capital stock in efficiency units equals the proportion of the product that is saved, discounted by the depreciation and the growth rates of population and technology.
- Capital (in efficiency units) reaches its steady state when its change over time equals zero:

$$sk_t^\alpha = (\delta + z + n)k_t$$

- The dynamics of the model bring the economy to the steady state, where:
  - Balanced growth path:  $k = k^*$
  - GDP grows at the rate  $z + n$
  - Per capita GDP grows with technology growth rate  $z$
  - Per capita growth can only be permanently increased if technology growth  $z$  increases.

# Solow Growth Model – dynamics and steady state

- Graphically, the dynamics of the model take the economy to the steady state as follows:



Phase 1: Convergence to steady state  $k_0$  to  $k^*$

- Growth rates are higher than in steady state as the returns to capital are higher
- Returns to capital decrease due to the concavity of the production function, so that growth decreases over time until steady state  $k^*$  is reached

Phase 2: Balanced growth path:  $k^*$

- Steady state is reached
- Per capita growth can only be permanently increased if technology growth  $z$  increases.
- An increase in  $s$  can increase growth temporary until a higher steady state is reached (can take several decades)

# What Accounts for Long-Run Growth?

In a mechanical sense, can only be two things

- Growth in productivity ( $A_t$ ): we produce more output given the same inputs
- Factor accumulation ( $L_t, K_t$ ): more factors of production help us produce more stuff

However, sustained growth over hundreds of years cannot be explained by input factor accumulation:

- Production increases in capital, but at a decreasing rate.
- To explain permanent growth, investment would need to increase more and more which would require a continuous increase in the savings rate. This is, however, bounded at 100%.
- Increases in capital can, however, lead to temporary higher growth rates that can last several decades.
- Labor input per capita is roughly trendless – empirically not a substantial source of growth in per capita output.

# Productivity growth as the source of permanent per capita growth

Continuous per capita growth over hundreds of years is explained by productivity growth

- For given factor inputs, better technology increases the output. Productivity (often measured as output/hour) increases.
- Technology  $A_t$  is exogenous in the Solow growth model.
- In endogenous growth models, it can be explained for example by modeling the R&D sector. Then, an increase in  $K_t$  might increase  $A_t$  because more resources are spent on R&D.

## What is TFP ( $A$ )?

- Technology
- Infrastructure: transportation, telecommunications, energy/power, information technology, finance
- Human capital unless it is modelled separately: education, training, health of workers
- Business environment and governance: obstacles to starting or running a business, e.g. getting permits, getting credit, enforcing contracts, ....
- Political and economic stability: security issues as well as macroeconomic stability

Policies to affect potential output, i.e. the supply side, are usually structural policies that are meant to improve the institutional and technological environment to increase productivity.

Monetary policy has no effect on output in the long run.



# How to measure TFP

- Measuring TFP as the Solow residual:

$$A_t = Y / (K_t^\alpha L_t^{1-\alpha})$$

$$\log(A) = \log(Y) - \alpha \log(K) - (1 - \alpha)\log(L)$$

- Problem: not the whole capital stock and labor might be used all the time. Might need to account for varying capacity utilization:

$$Y = A(u_K K)^\alpha (u_L L)^{1-\alpha}$$

# How are Input Factors Determined?

Labor and capital (investment) demand by firms can be determined by a profit maximization problem.

The solution shows that capital and labor are paid their marginal products (under the assumption of perfect competition, otherwise a mark-up needs to be added):

$$w_t = A_t F_L(K_t, L_t) = (1 - \alpha) A_t \left( \frac{K_t}{L_t} \right)^\alpha = MPL$$

$$r_t = A_t F_K(K_t, L_t) = \alpha A_t \left( \frac{L_t}{K_t} \right)^{1-\alpha} = MPK$$

Note:  $w_t$  denotes the real (not the nominal) wage,  $r_t$  denotes the real return to capital

Intuition:

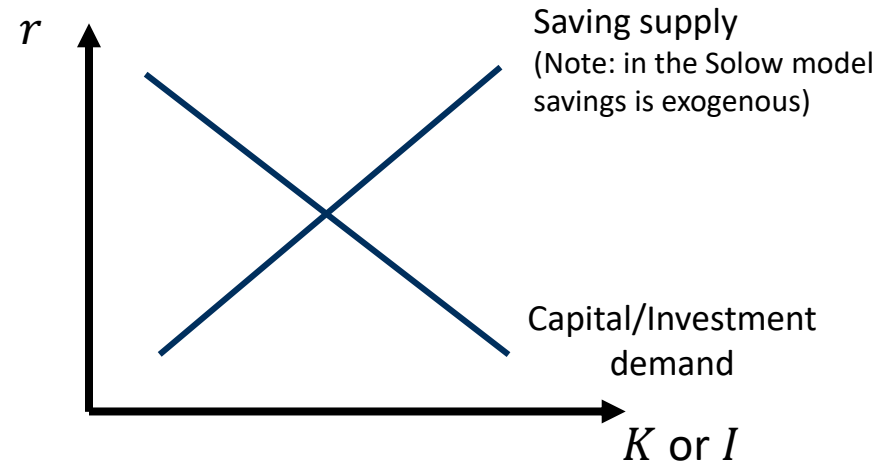
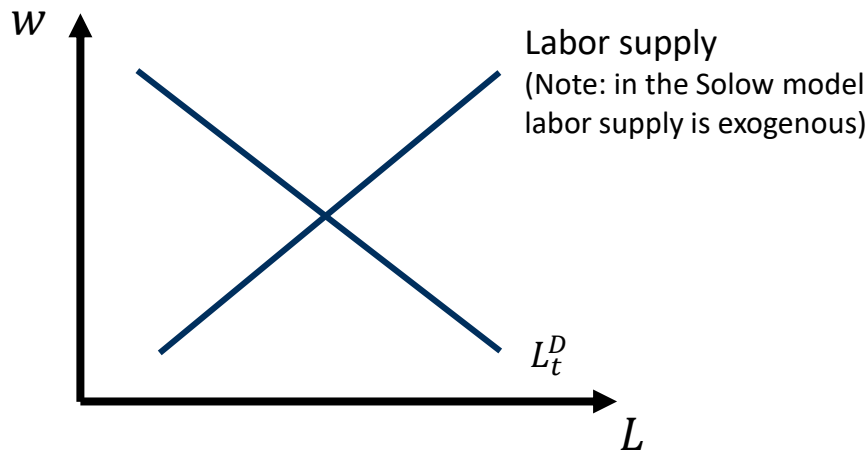
- Producers hire labor and accumulate capital up to the point where they can make profits given the cost (real wage, real cost of capital).

# How Do the Different Factors ( $A$ , $K$ , $L$ ) Affect Each Other?

Solving the first order conditions for  $L$  and  $K$  yields labor demand and capital demand (implies investment demand) functions:

$$L_t^D = \left( (1 - \alpha) \frac{A_t}{w_t} \right)^{1/\alpha} K_t$$

$$K_t^D = \left( \alpha \frac{A_t}{r_t} \right)^{1/(1-\alpha)} L_t$$



- Increase in  $A_t$  increase the marginal product of labor and capital and therefore the demand for labor and capital.
- An increase in one input factor (ex.  $K_t$ ) has a positive effect on the marginal product of the other one (ex.  $L_t$ ) and increases the demand for this one as well.

# How are Wages and Interest Rates Determined in the Long Run?

Solving the Solow model for a steady state with capital in efficiency units yields\*

$$k^* \approx \left( \frac{s}{n + z + \delta} \right)^{\frac{1}{1-\alpha}}$$

Inserting  $k^*$  in the optimality conditions for labor and capital yields the long-run wage rate and interest rate.

$$w^* \approx (1 - \alpha)A^{1-\alpha} \left( \frac{s}{n + z + \delta} \right)^{\frac{\alpha}{1-\alpha}}$$

$$r^* \approx \alpha \left( \frac{n + z + \delta}{s} \right)$$

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\* Derivations are based on the augmented Solow model with technology and population growth with  $k = K/(ZL)$  as on slide 11.

# How are Wages and Interest Rates Determined in the Long Run?

$$\text{Long-run wage: } w^* \approx (1 - \alpha)A^{1-\alpha} \left( \frac{s}{n+z+\delta} \right)^{\frac{\alpha}{1-\alpha}}$$

- Wages grow with technology
- Intuition: Labor is rather constant in the long run. If  $A$  grows and therefore the marginal product of labor grows, then wages increase.

$$\text{Long-run interest rate: } r^* \approx \alpha \left( \frac{n+z+\delta}{s} \right)$$

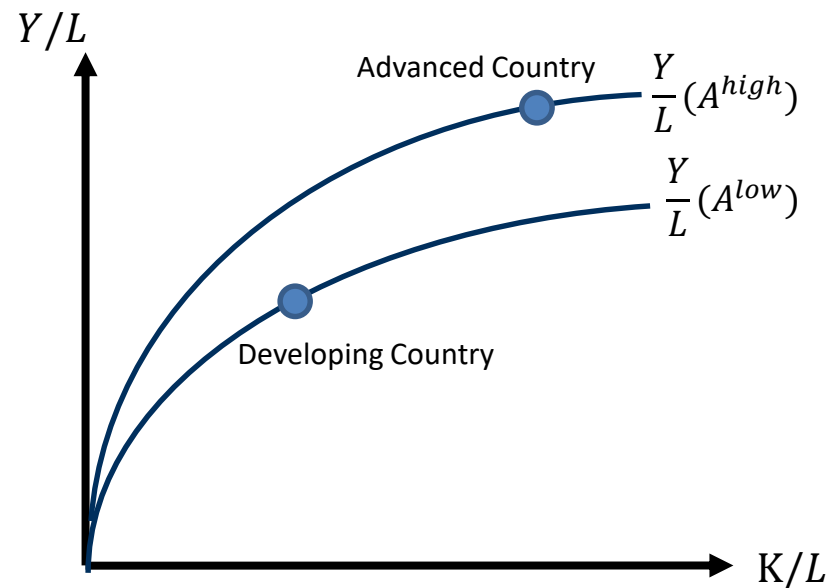
- Return to capital is constant in the long run. Capital is increasing over time with productivity growth. Hence, the marginal product of capital does not grow, so that the return to capital remains constant.
- $r^*$  increases with factors that increase the demand for investment: technology growth  $z$ , depreciation  $\delta$ , population growth  $n$ , and the capital share  $\alpha$ .
- It decreases with an increase in the savings rate as this would increase the supply of capital/investment.
- $r^*$  is closely related to the natural interest rate, which is the long-run equilibrium rate that brings savings and investment in equilibrium.

# Short Application: Developing Countries vs. Advanced Countries

How can countries increase per capita output?

$$Y = AK^\alpha L^{1-\alpha} \Rightarrow \frac{Y}{L} = A \left( \frac{K}{L} \right)^\alpha$$

1. Increase capital, i.e.  $K/L$
  2. Increase productivity  $A$
- Advanced countries: mainly via increasing  $A$ , because capital is already high. Limited additional returns from increasing capital (steady state has been reached in terms of Solow growth model)
  - Developing countries: catching up via both, increasing  $K$  and  $A$



## 2.2 Unemployment in the Long Run

# Unemployment in the Long Run

So far, we have assumed perfect competition

- In such a model, there can be only frictional unemployment (someone found a job, but has not started yet) and search unemployment (time is needed to find a good offer). This would reduce labor input, but should not exert downward pressure on wages.
- The focus so far was on technology growth. In the very long run, growth and the unemployment rate are not interlinked, so that this was a useful simplification.

Empirically, there is in the medium to long run involuntary unemployment beyond frictional and search unemployment due to deviations from perfect competition.

This means that the labor market does not clear even in the medium to long run, i.e. wages do not adjust to ensure that labor demand equals labor supply.



# Unemployment in the Long Run

Including unemployment in the analysis is important to define potential output more precisely:

$$Y = AF(K, L) = AK^\alpha L^{1-\alpha}$$

with  $L = (1 - u^*)NH$

$L$ : actual labor input

$u^*$ : natural unemployment rate

$N$ : total labor force

$H$ : average hours worked

# Reasons for Unemployment in the Medium to Long Run

What are the mechanisms that impede real wages to fully adjust to equilibrium, resulting in a demand for labor that is lower than the supply?

One important reason on the labor demand side: efficiency wages.

There are different versions either focusing on unobservable labor effort or on minimizing turn-over of employees:

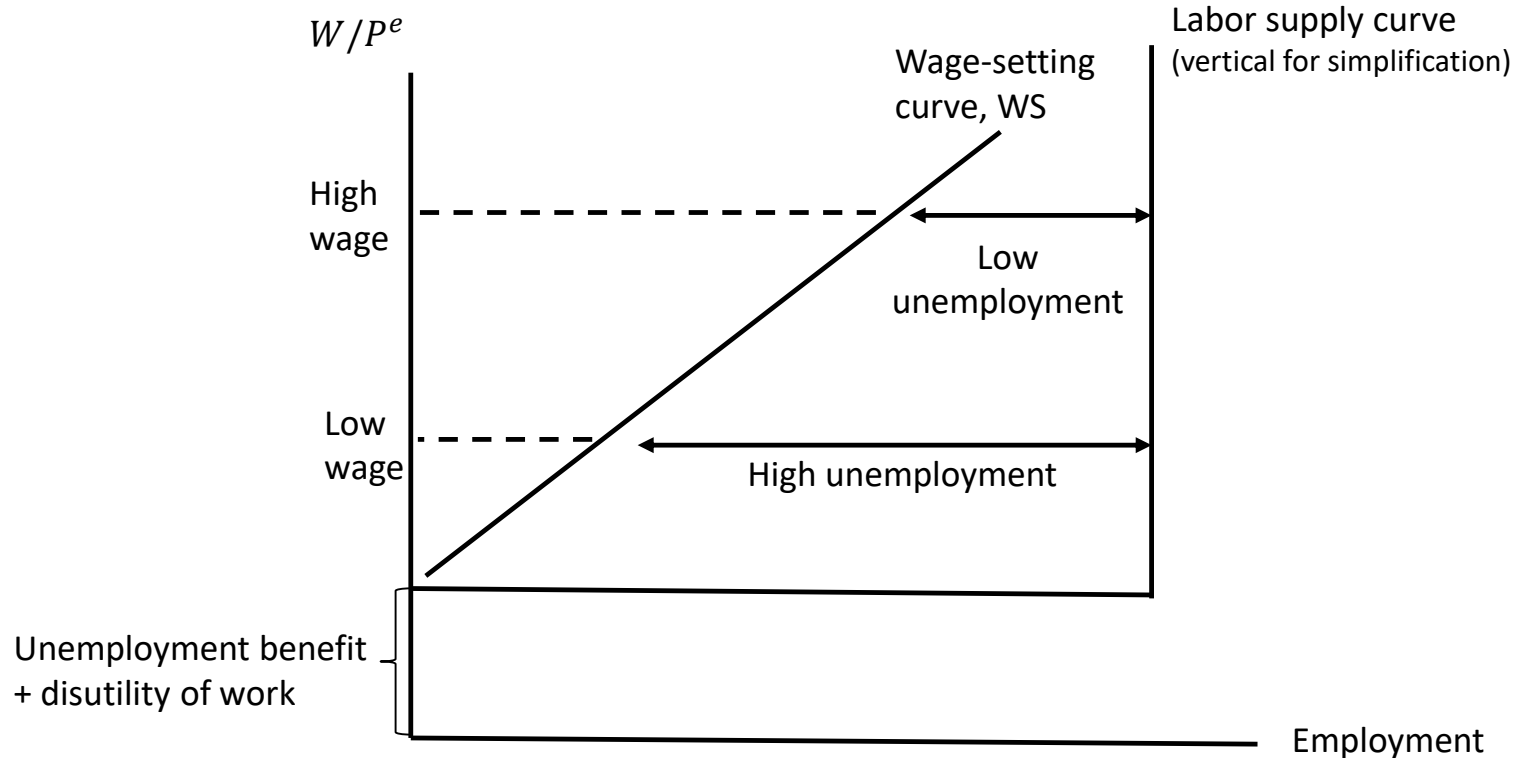
- Effort: Employers pay a higher wage than employees require to take on a job to create a cost to the worker of losing the job. An incentive to work conscientiously is created in this way.
- Turnover: Via paying wages above the market-clearing level, firms can prevent a high turnover of workers.

In both versions, employers have to offer higher wages when the probability of the worker getting another job is higher – this creates wages that move with the business cycle and the probability of becoming unemployed.

# Wage-Setting Equation

Efficiency wages imply a wage setting (WS) equation in which the wage increases with employment (the bargaining function  $B(E)$  is increasing in employment  $E$ ):

$$W = P^e B(E)$$



- Wages are set above what workers get when being unemployed + compensation for disutility of working
- Wages rise as employment rises

# Price Setting

Imperfect competition leads to price-setting power of firms.

- Firms set prices as a mark-up over marginal cost ( $MC$ ). Focus here is on the labor markets, so that we focus on the mark-up over (nominal) wages divided by the  $MPL$ :

$$P = (1 + \mu)MC$$

$$P = (1 + \mu) \frac{W}{MPL}$$

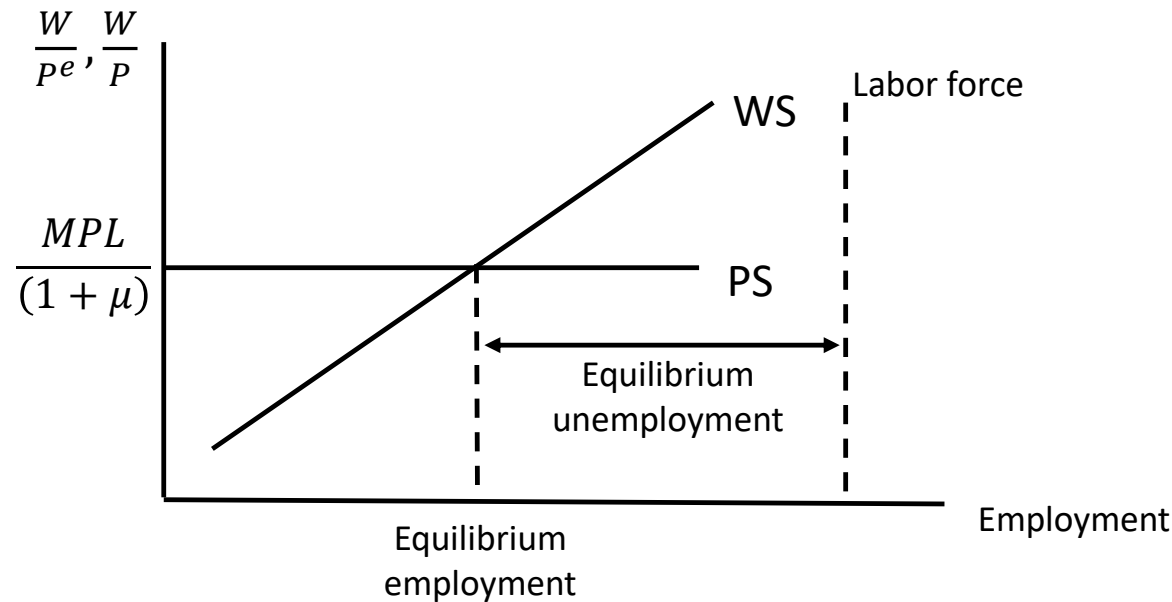
- Implies a flat price-setting (PS) curve in terms of the real wage:

$$\frac{W}{P} = \frac{MPL}{(1+\mu)}$$

- Intuition: if all firms increase the mark-up, then the price level increases, which decreases the real wage

# Supply-Side Equilibrium of the Labor Market

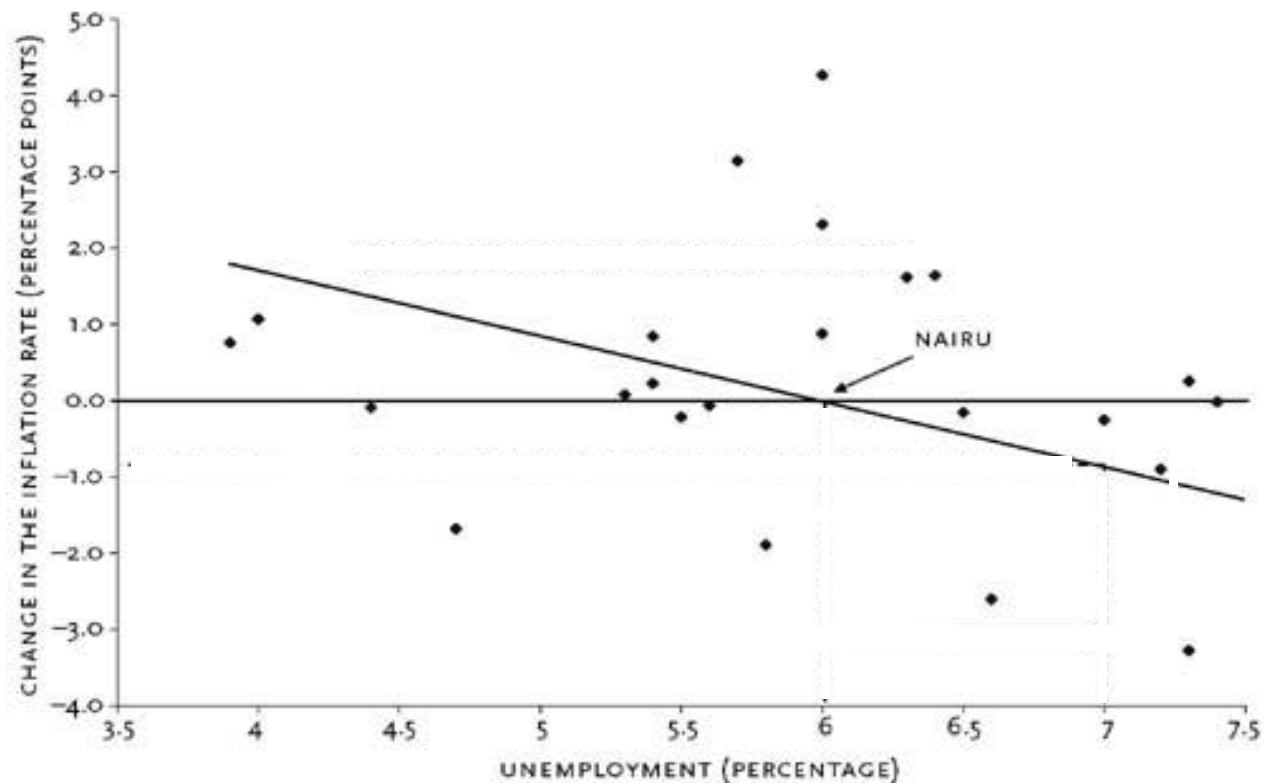
- The intersection of the WS and PS curve determines unemployment in the medium run – the natural rate of unemployment. One could also analyze cyclical unemployment in this model.



- An upward shift of the WS curve would increase unemployment, while an upward shift of the PS would lower it.
- WS-curve shift upwards if wage-setting power of workers (unions, insider-outsider theory) increases (implies wages above efficiency wages), if unemployment benefits increase, if minimum wages increase, ...
- PS could shift upwards due to an increase in competition that lowers  $\mu$
- Note that monetary policy plays no role in determining unemployment in the medium to long run.

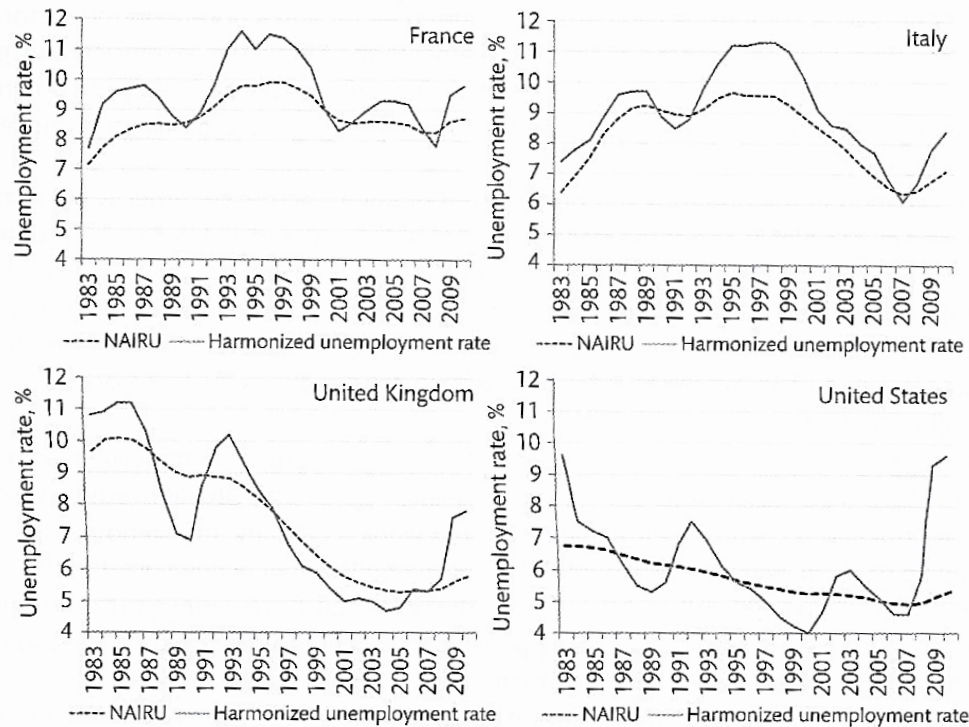
# NAIRU

- NAIRU (Non-Accelerating Inflation Rate of Unemployment) measures medium-term equilibrium unemployment as measured by the intersection of the WS and PS curve.
- A simple way to determine the NAIRU is to regress the change in inflation on the unemployment rate.
- A closely related concept is the NAWRU (Non-Accelerating Wage Inflation Rate of Unemployment) in which the focus is on stable wages.



# NAIRU Estimates

- The NAIRU can vary over time due to movements in the WS and PS curve (including factors we have not discussed like the age and educational profile of the population, hysteresis effects etc.)
- It is much less volatile than the actual unemployment rate that also includes cyclical fluctuations
- Estimates are surrounded by high uncertainty (similar to potential output estimates)



**Figure 2.8** Non-Accelerating Inflation Rate of Unemployment (NAIRU) and harmonized unemployment rates in France, Italy, the United Kingdom and the United States: 1983–2010.

Source: OECD Economic Outlook (accessed December 2011).

# Application: Production Function Approach of Measuring Potential Output

Start from a Cobb-Douglas production function:

$$Y = AK^\alpha L^{1-\alpha}$$

with  $L = (1 - u^*)NH$

- The problem of extracting a trend is shifted from  $Y$  to  $A$ ,  $L$  and  $K$ , all of which are subject to additional measurement problems.
- Nevertheless, this approach is often used in practice as it enables structural analysis of possible changes of different influencing factors of potential output (changes in demographics, labor force participation, savings rate, recent investment, depreciation, technological innovations, ...).



# Application: Production Function Approach of Measuring Potential Output

Example: potential output concept of the European Commission

- Capital is computed by adding up investment via a capital accumulation equation. Trend capital is usually assumed to equal actual capital, as capital is quite smooth  $\rightarrow K^*$
- Cyclically adjusted labor:
  1. Get an estimate of the trend labor force,  $N^*$ , via computing the trend of the participation rate with the HP filter.
  2. Trend employment,  $(1 - u^*)N^*$ , consistent with non-accelerating wage inflation (NAWRU) is computed. The NAWRU is an estimate on its own.
  3. Trend labor input is computed by multiplying trend employment with the trend of average hours worked.  $\rightarrow L^* = (1 - u^*)N^*H^*$
- Computation of cyclically-adjusted TFP: HP filtered Solow Residual  $\rightarrow A^*$
- Potential output:  $Y^* = A^*K^{*\alpha}L^{*1-\alpha}$

# Summary

## Long-run growth can be explained with the Solow model

- Long-run growth explained by productivity growth, temporary higher growth can be caused by capital accumulation
- To catch up developing countries can foster both, capital accumulation and productivity growth
- Wages increase with technology growth
- The long run real interest rate is called the natural interest rate and it varies with different factors including the savings rate, productivity growth and the capital share in production.

## Market imperfections prevent the labor market from clearing even in the long run

- Firms have incentives to set wages higher above the market clearing wage due to unobserved labor effort and for avoiding a higher worker turnover (efficiency wage theory)
- Empirically, the equilibrium unemployment rate can be measured based on the NAIRU (or NAWRU)

## Production function approach of measuring potential output

- Combines insights from the Solow model with measures of equilibrium unemployment. It is widely used for medium-term projections of output