Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Domiyes

Algorithmen für Programmierwettbewerbe

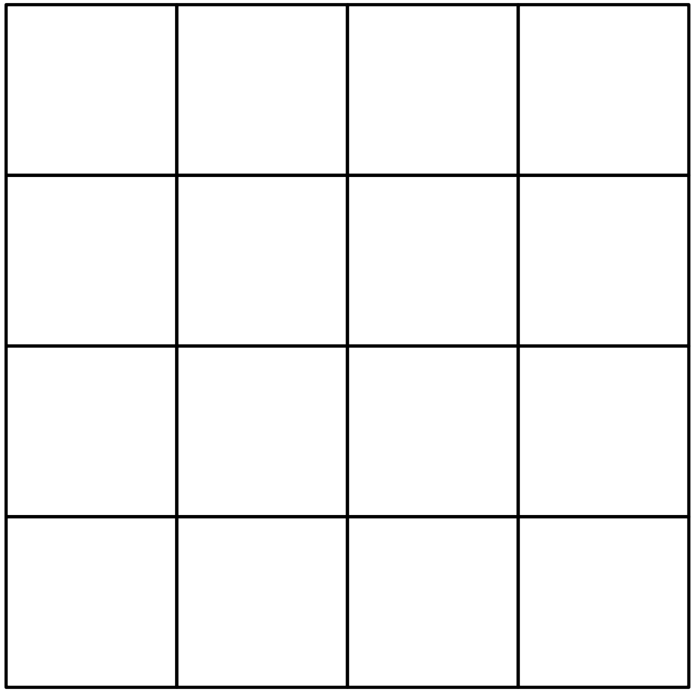
Sommersemester 2021

Sarah Bäurich

Florian Strunz

The Problem

Input: A set of dominoes positioned on a board.



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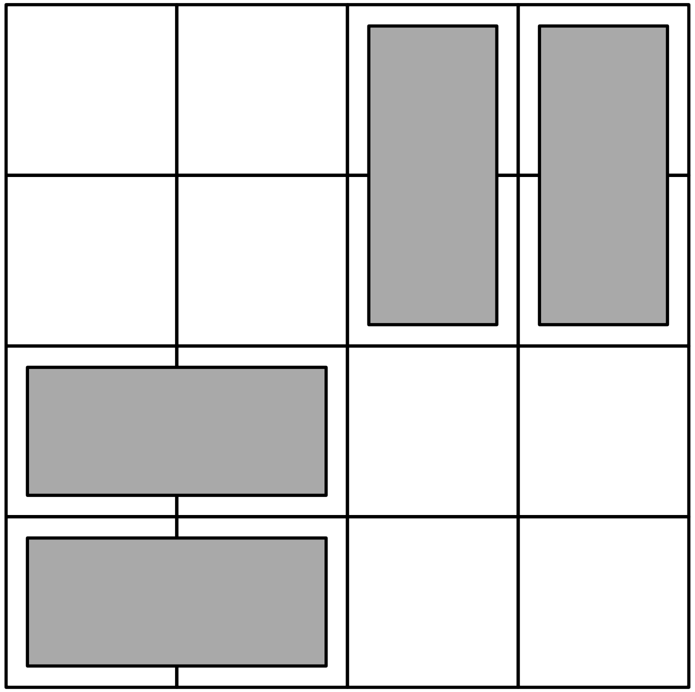

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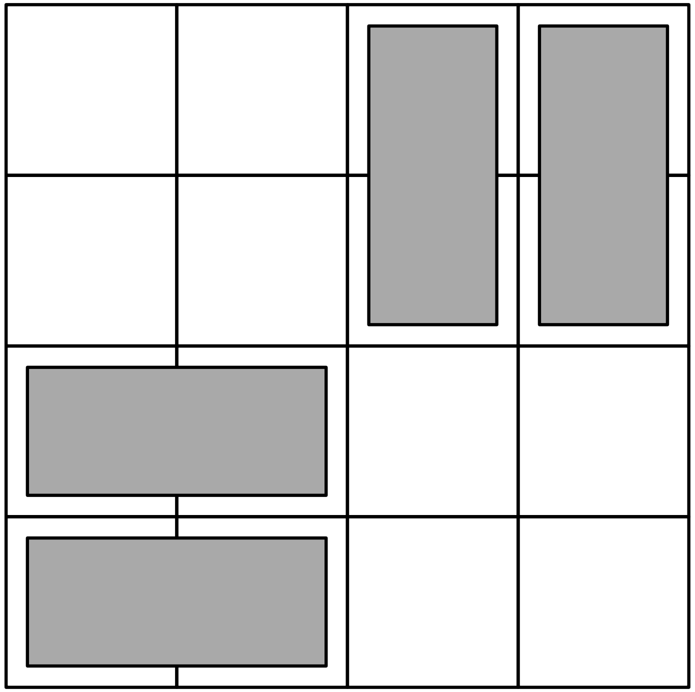

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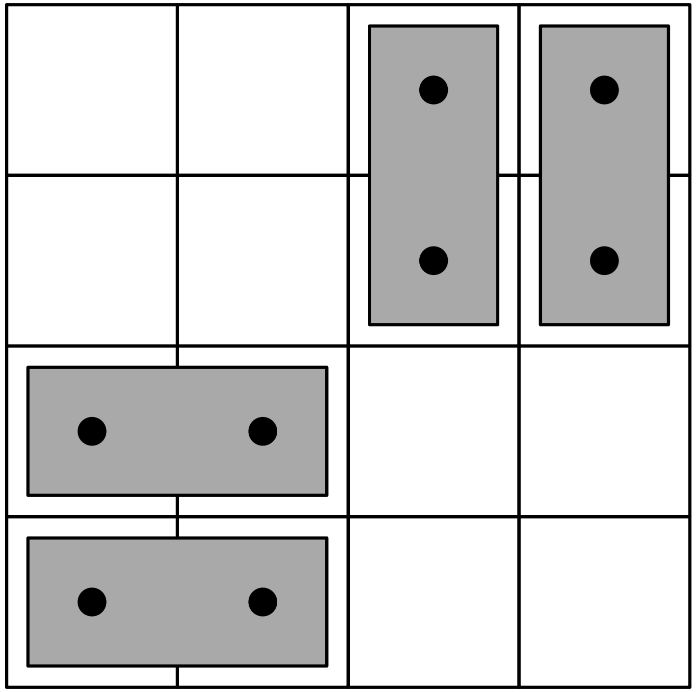

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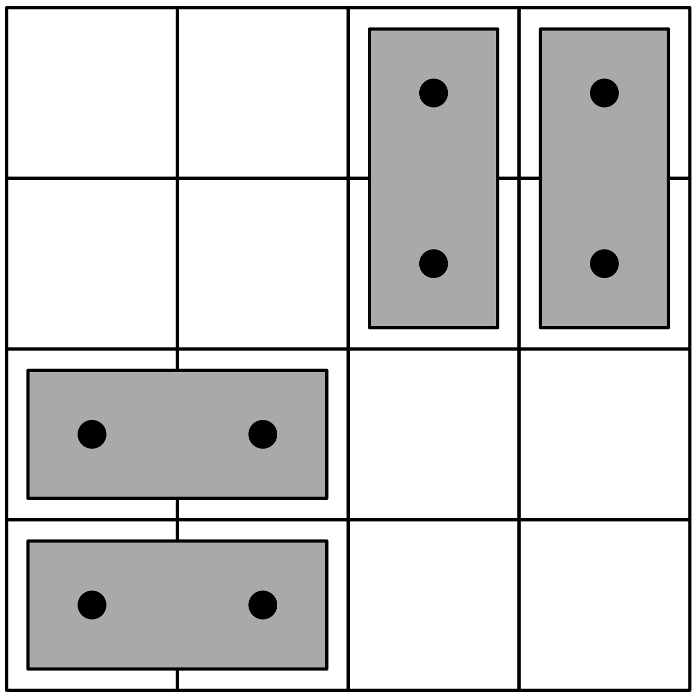

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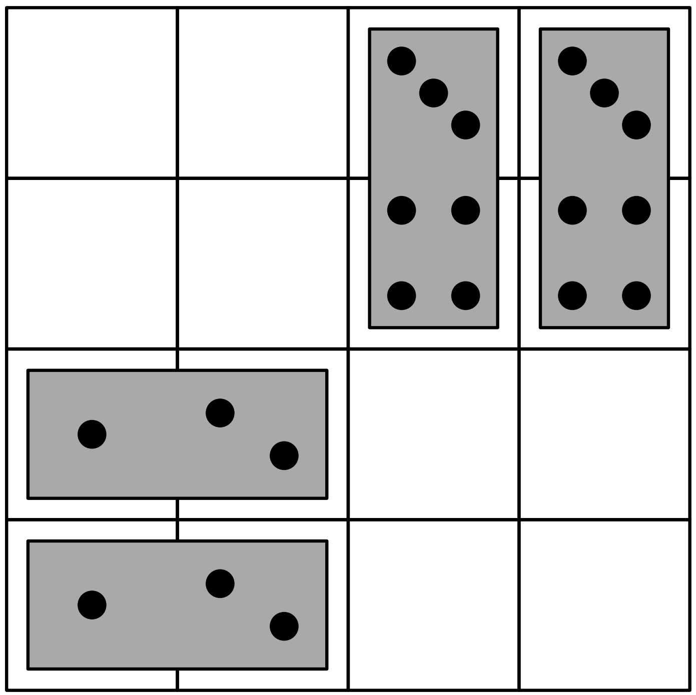

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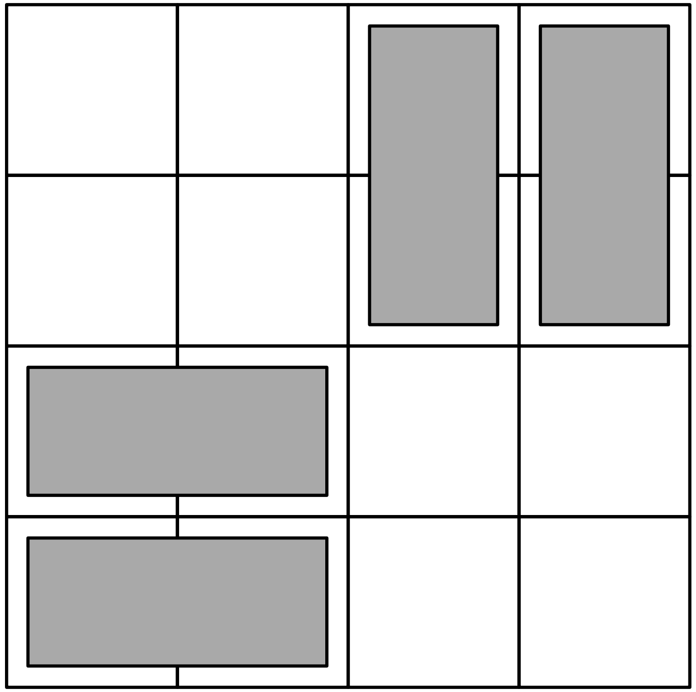

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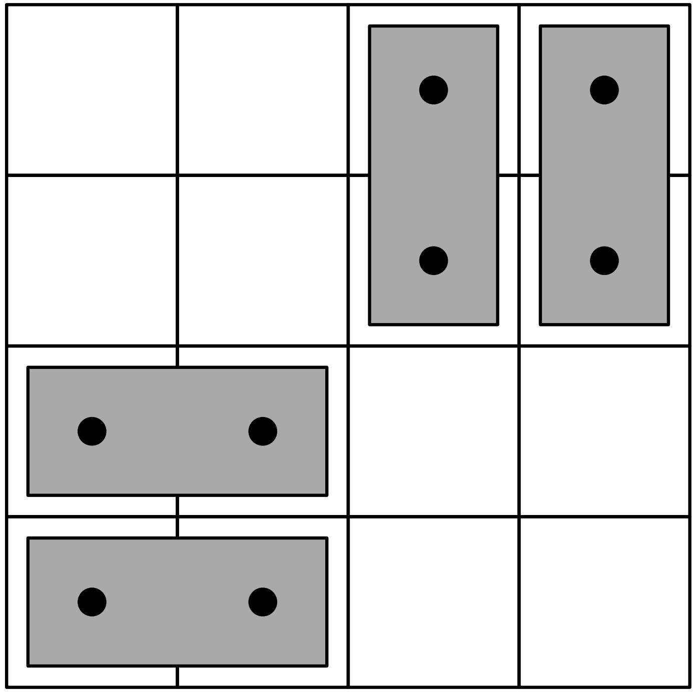

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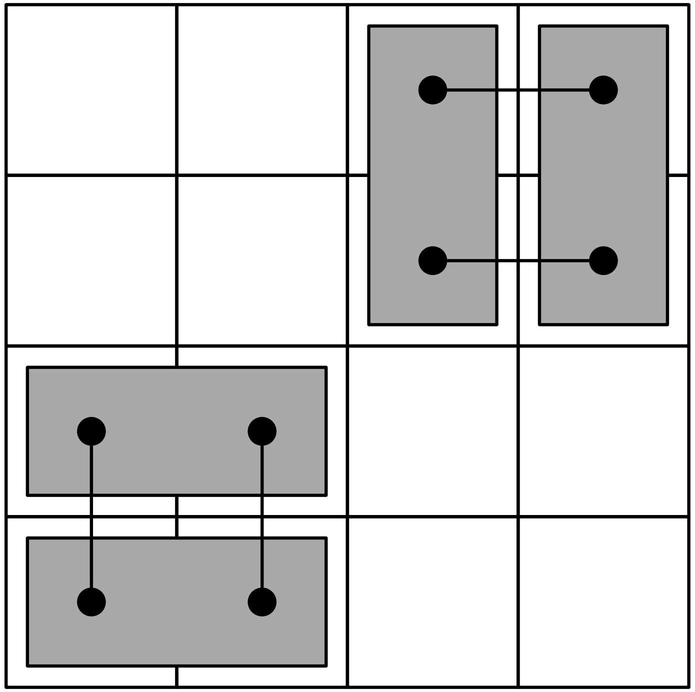

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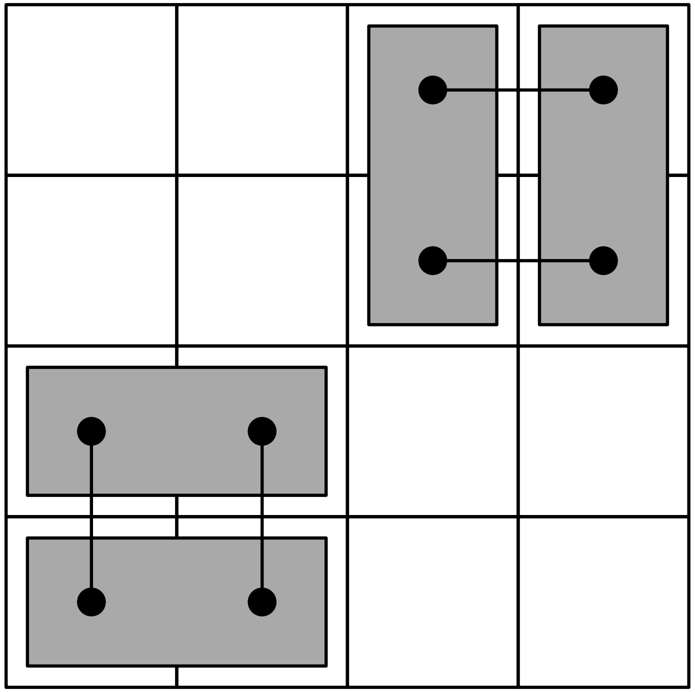

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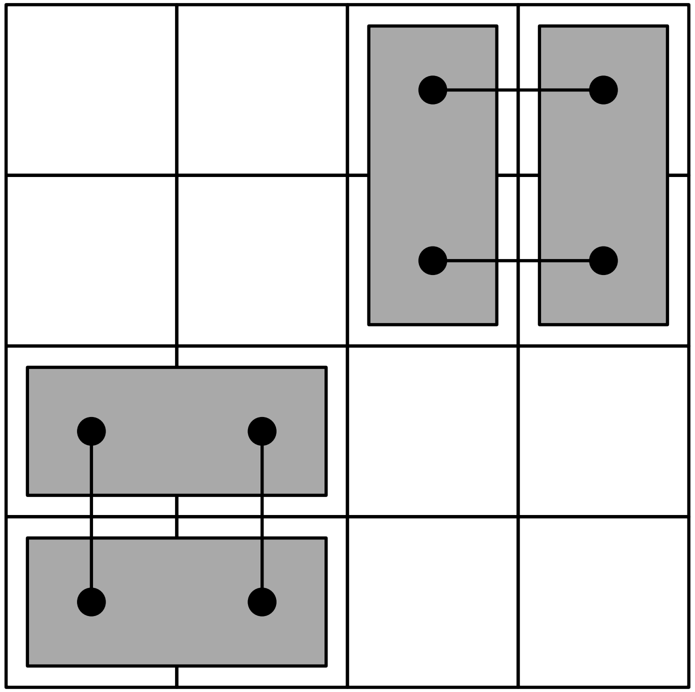

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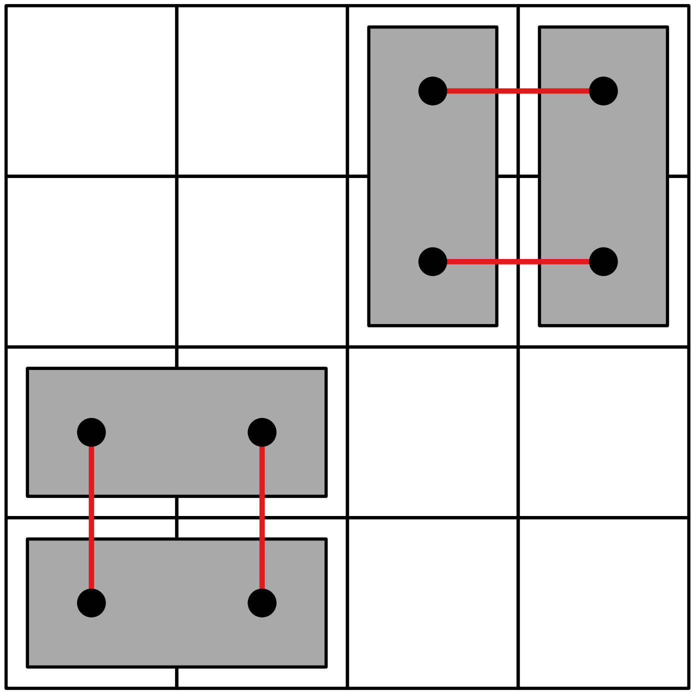

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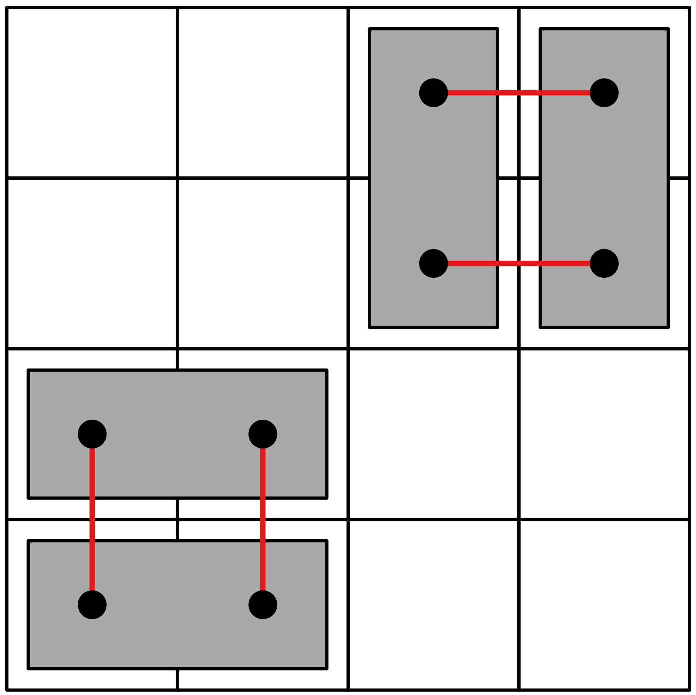

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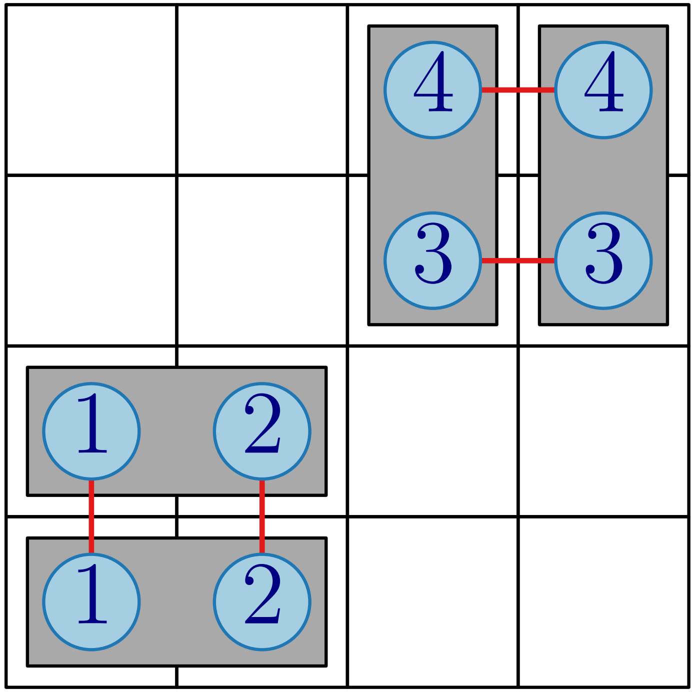

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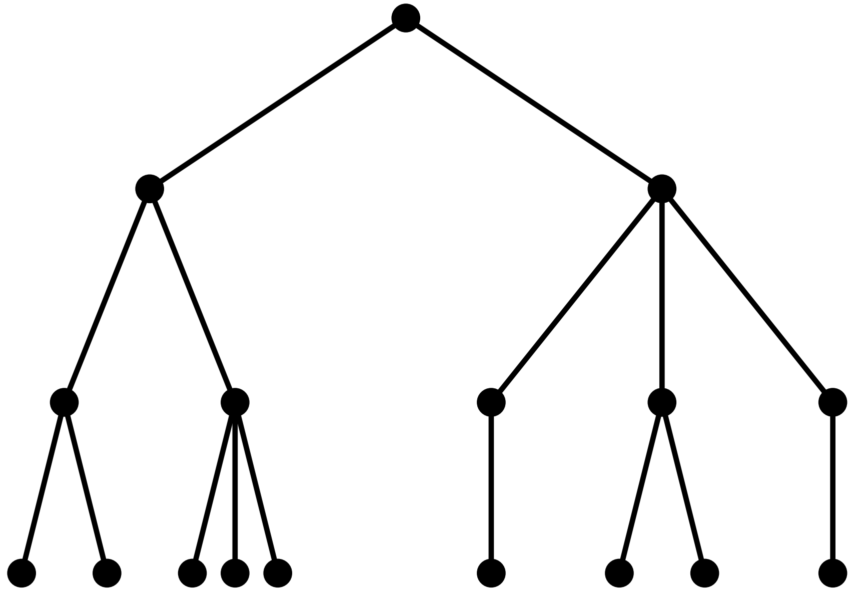

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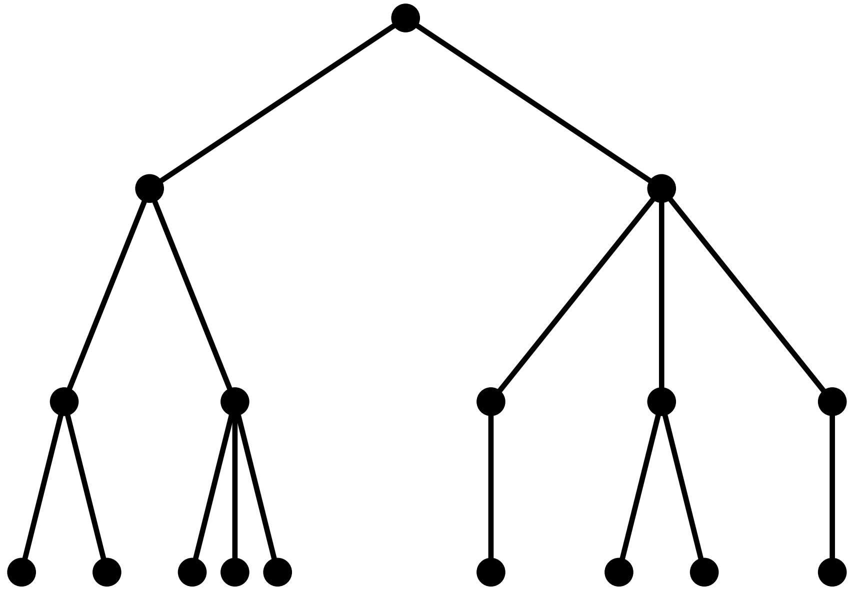

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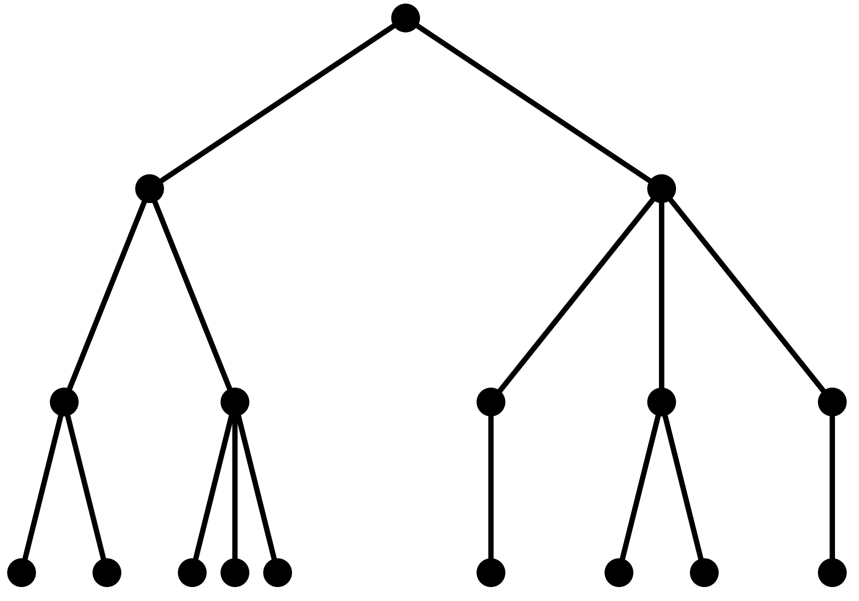

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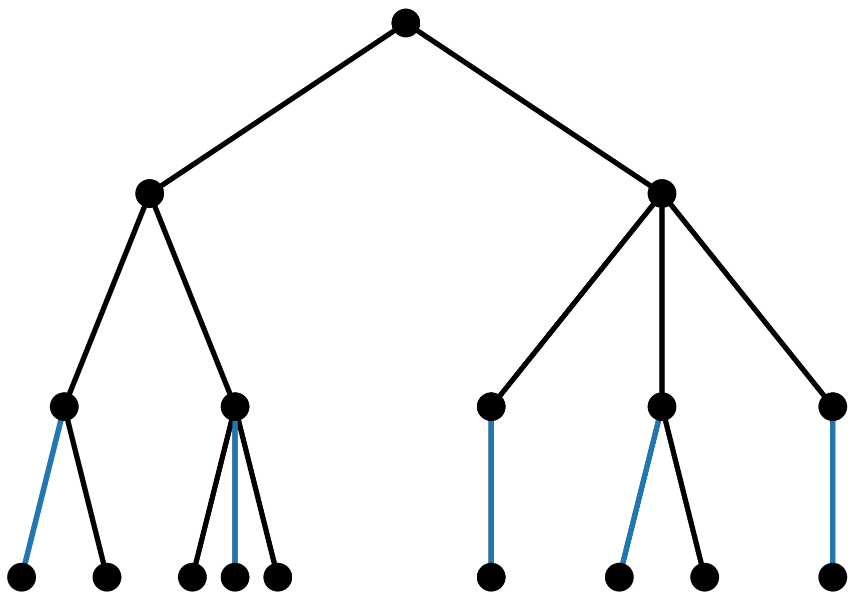

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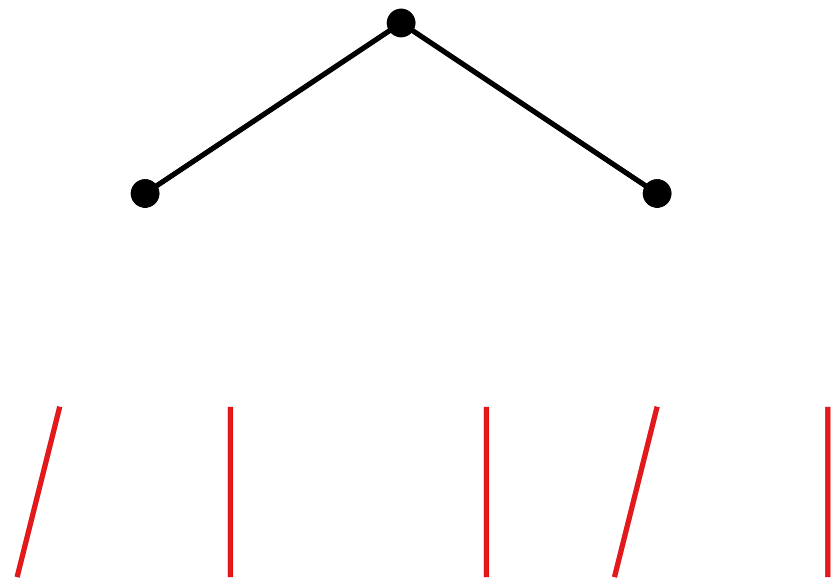

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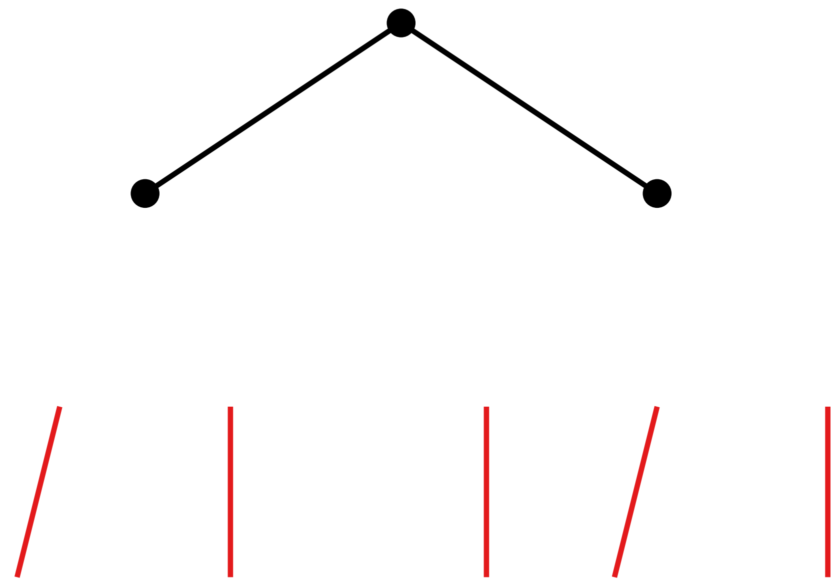

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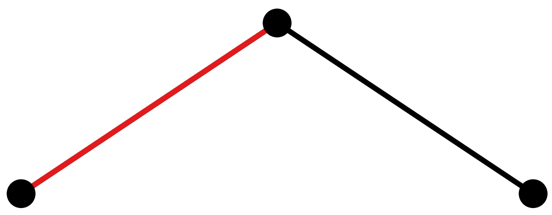

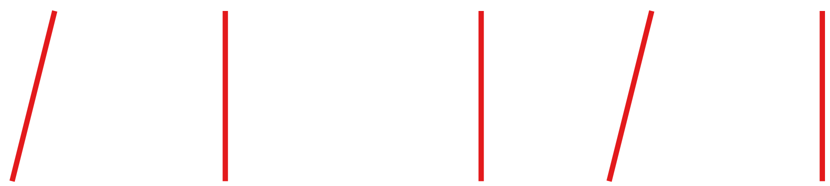

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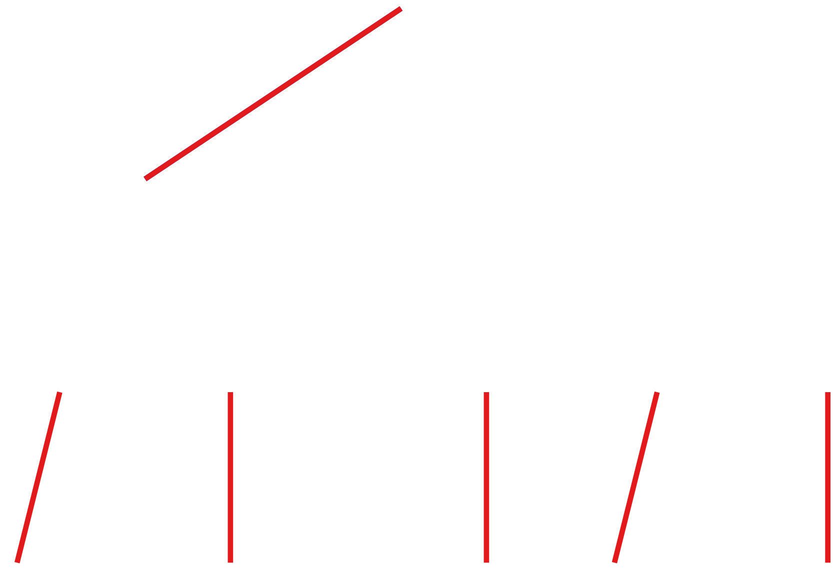

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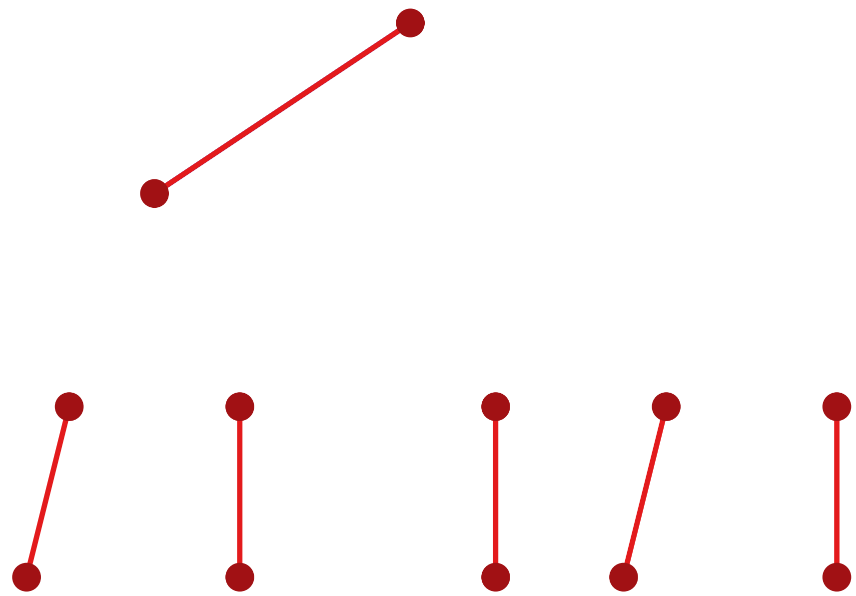

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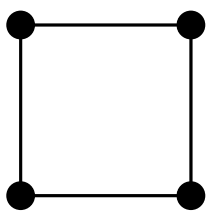 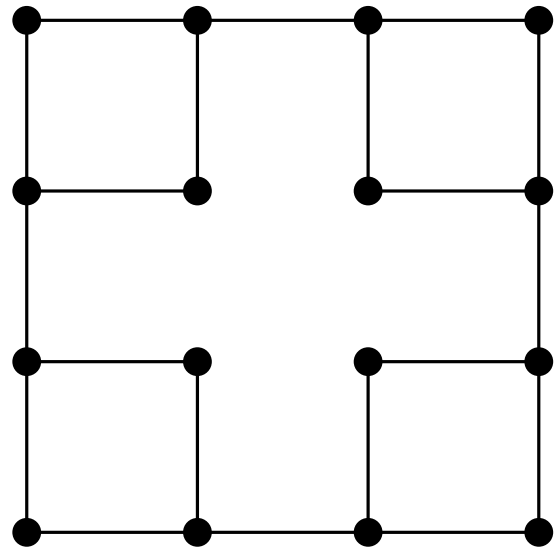

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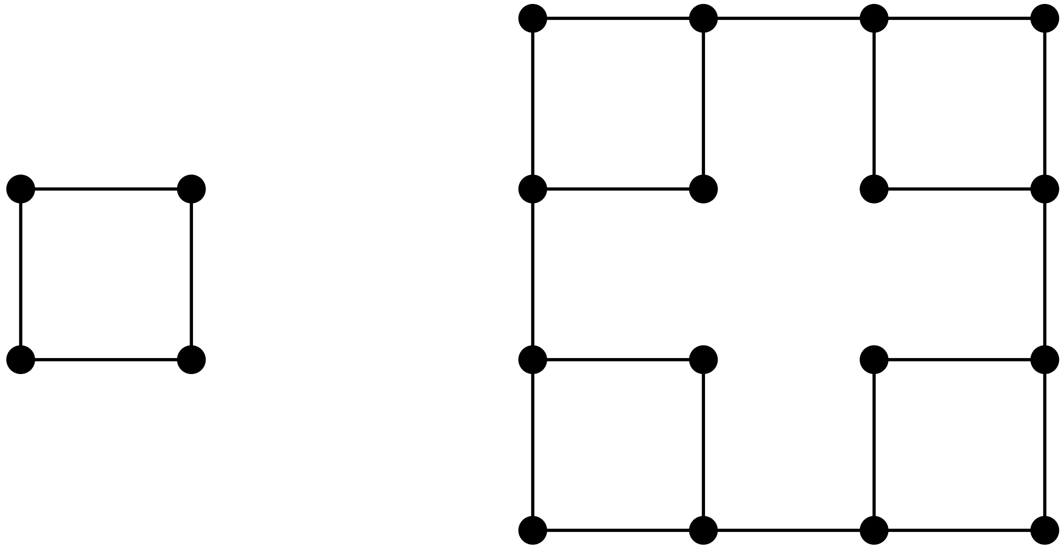

Domino graphs can have cycles! $\Rightarrow$ They are not trees. $\Rightarrow$ *Our* $\mathcal{O}(V)$ *algorithm will not work here*.

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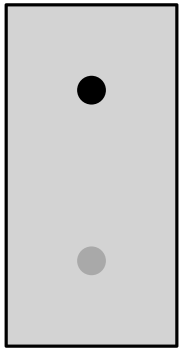

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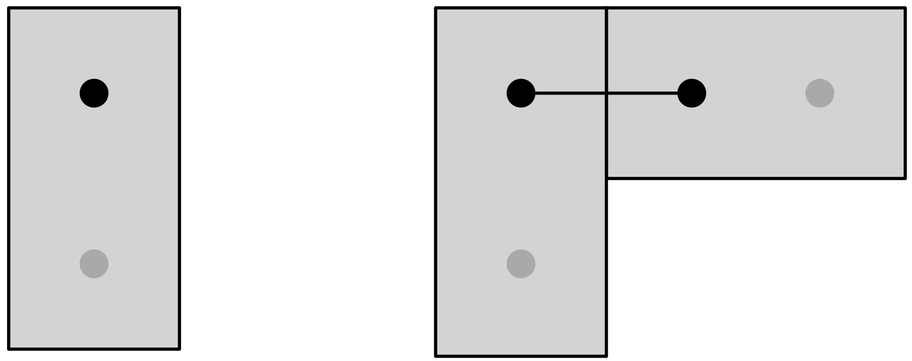

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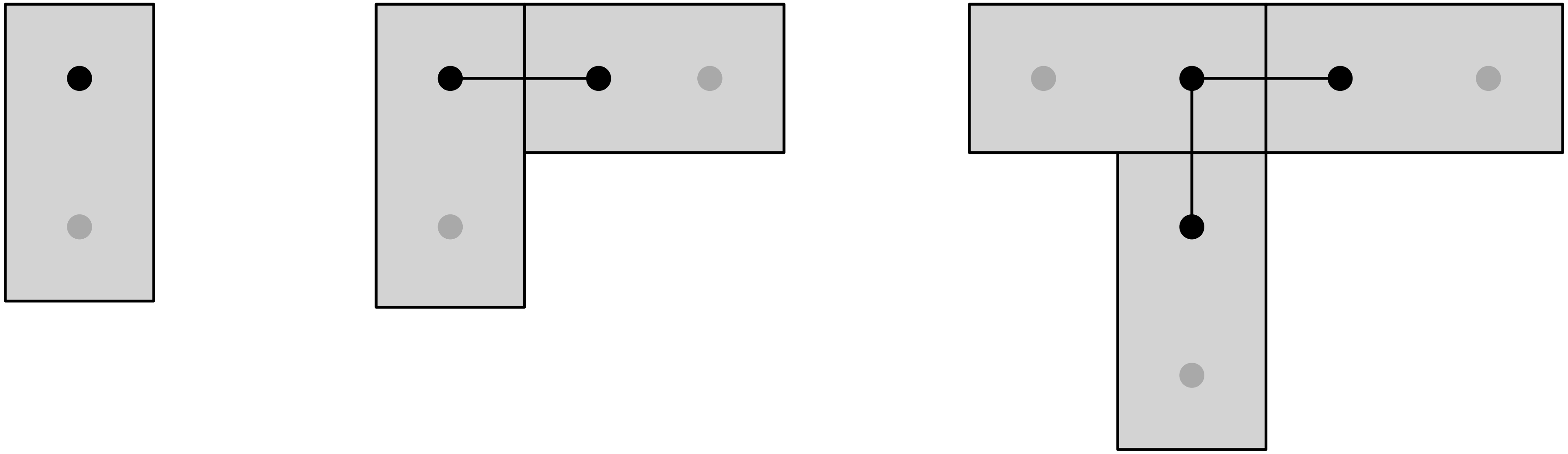

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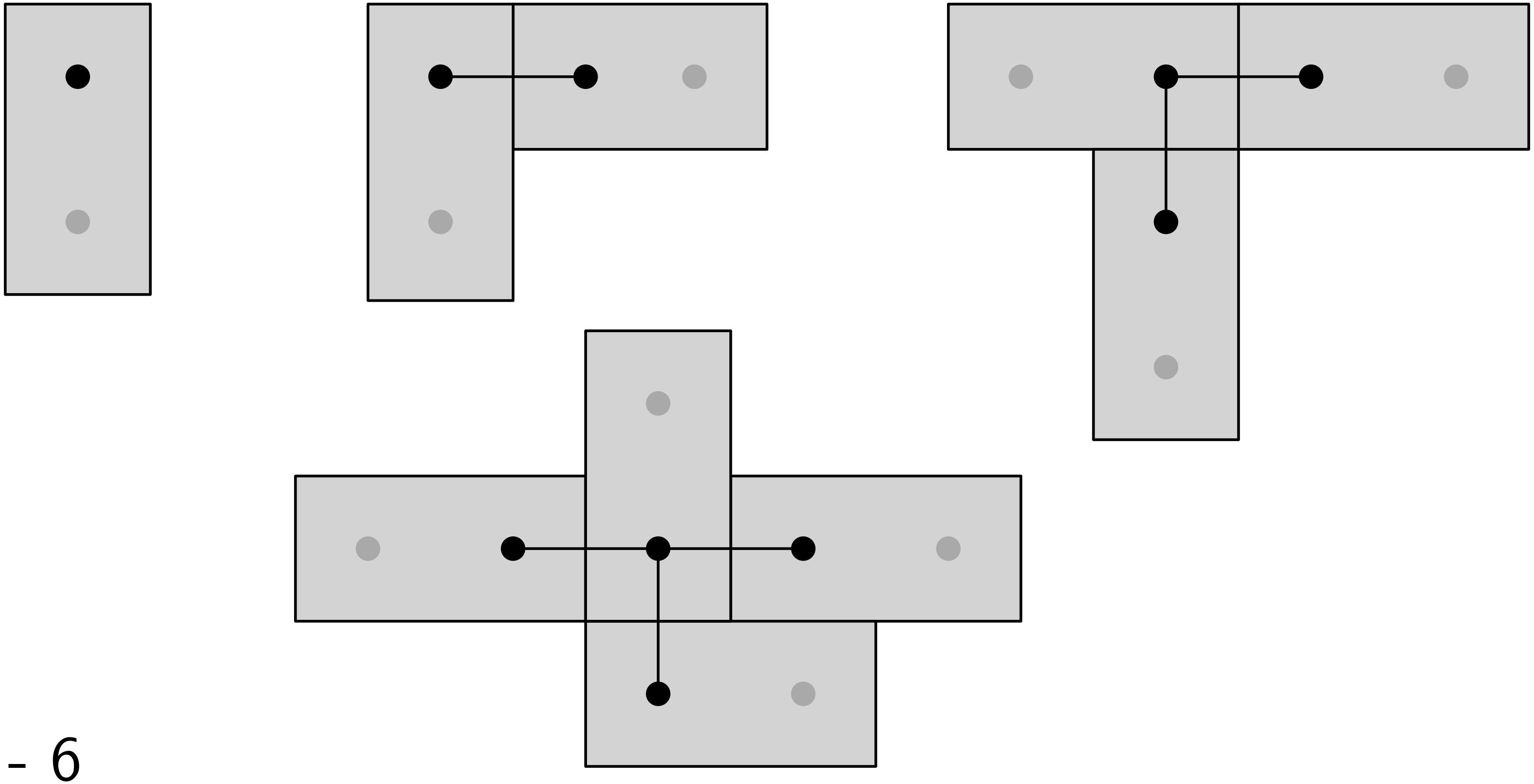

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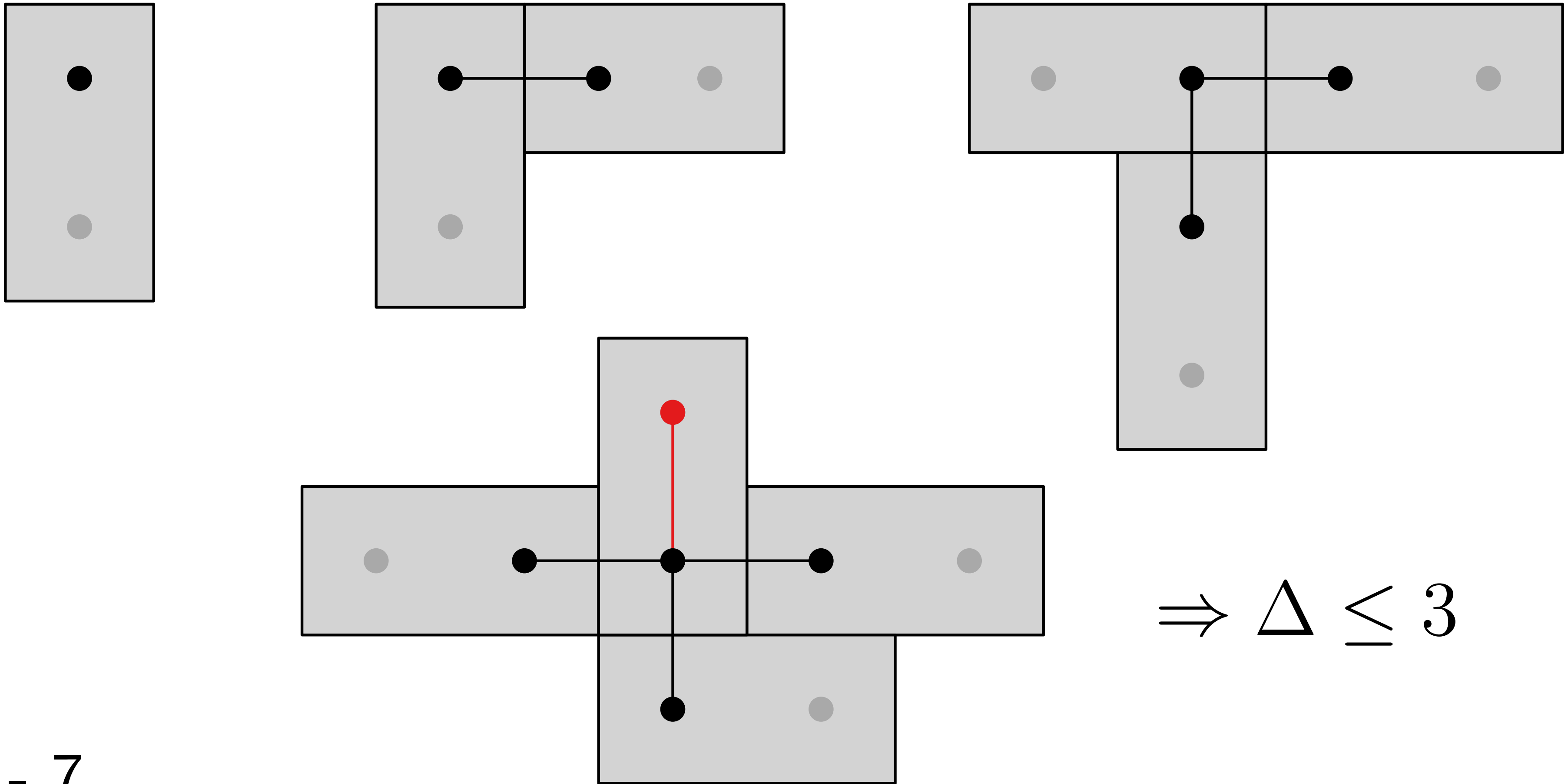

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- Hopefully, such a specialisation will give us faster and/or simpler algorithms!

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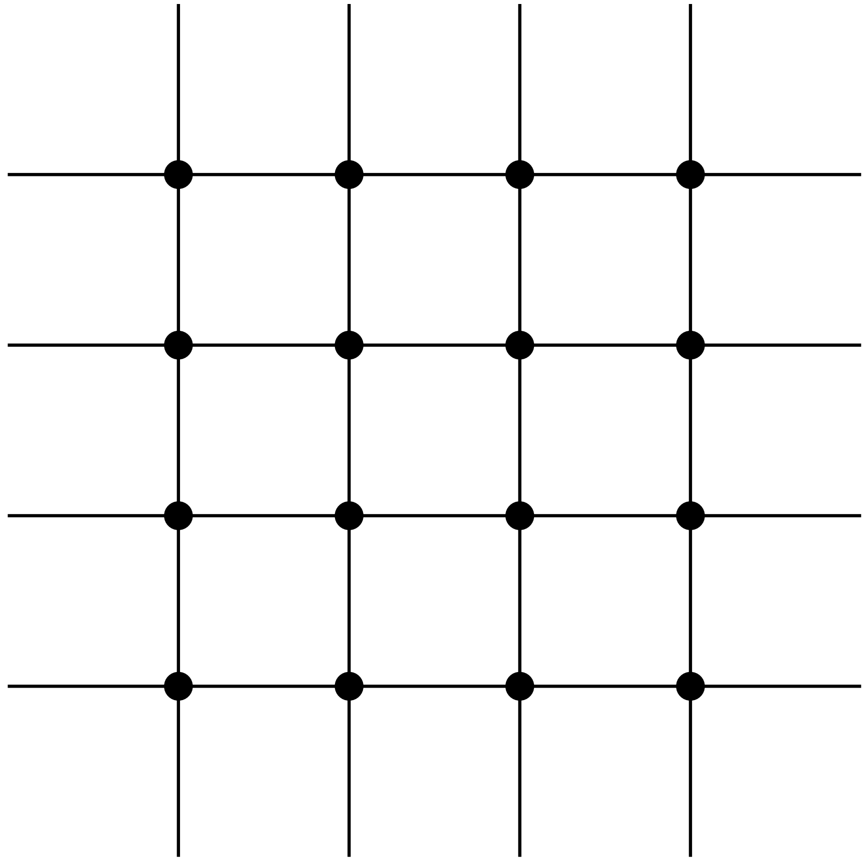

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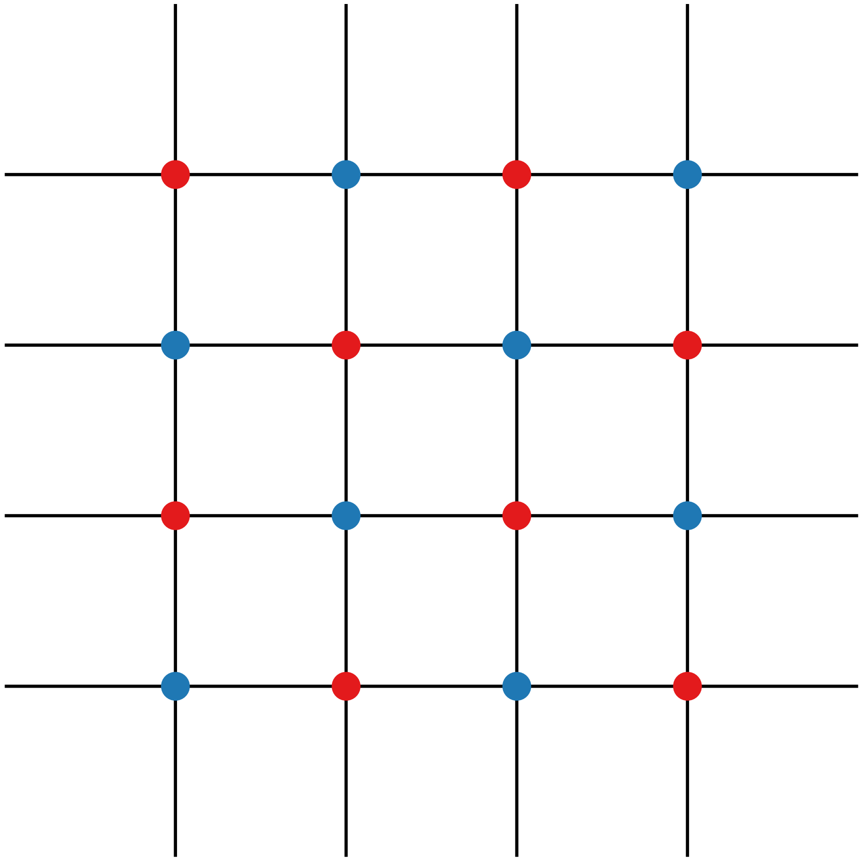

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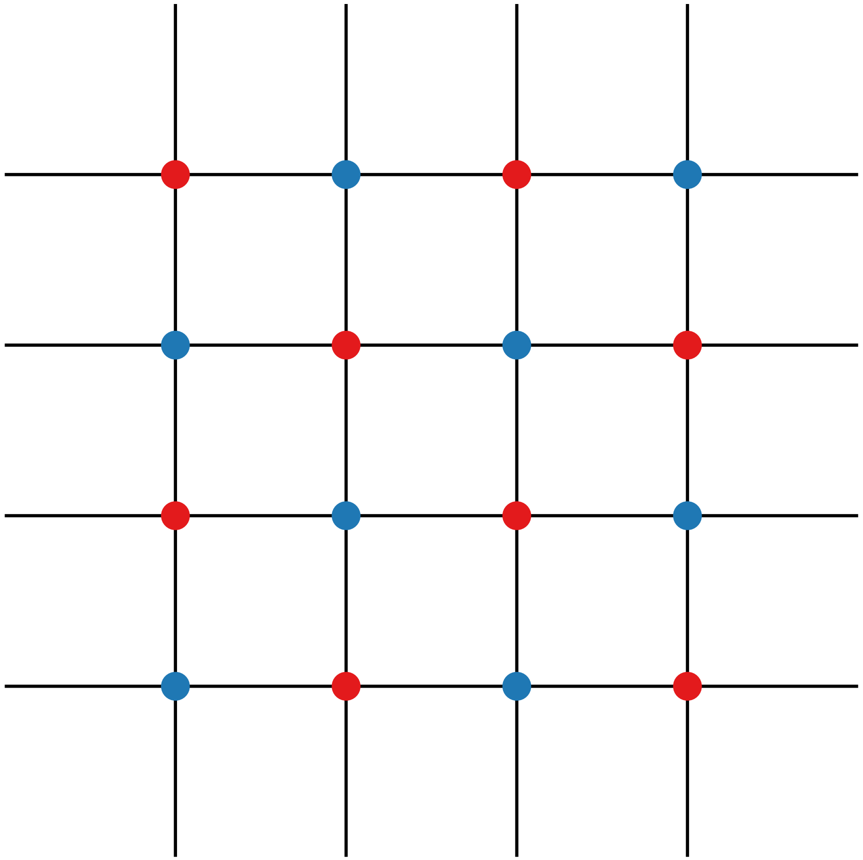
The infinite grid graph can be two-coloured. Thus, we can divide V into two edge-disjoint sets A and B.



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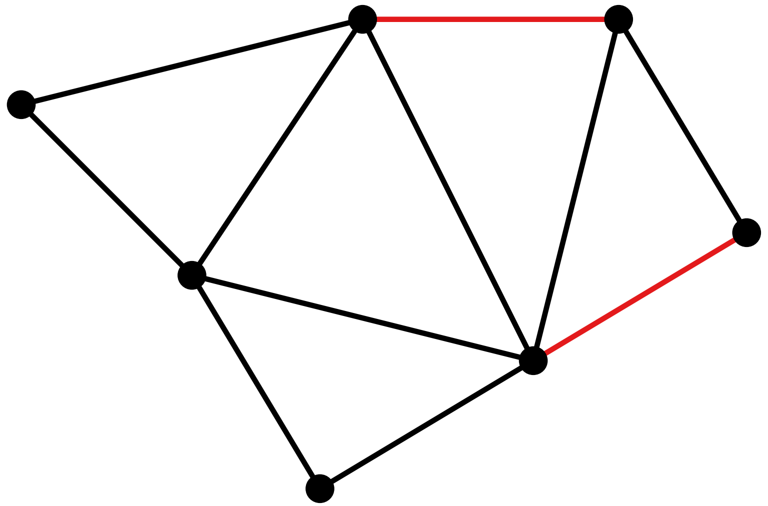

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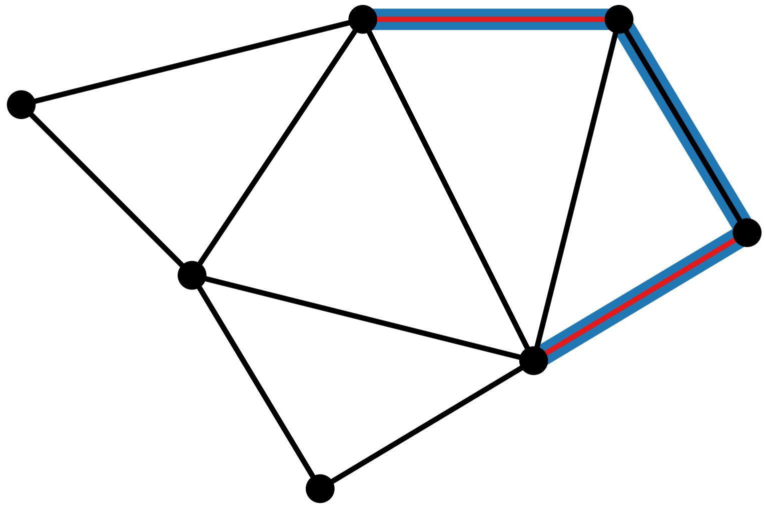

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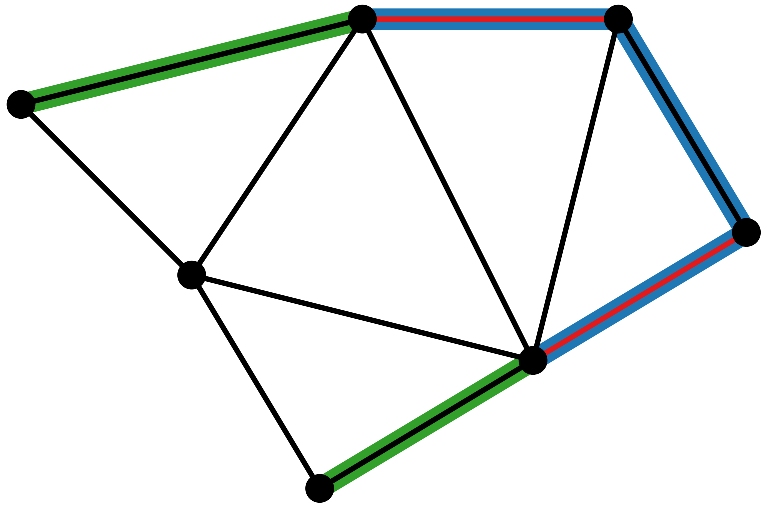

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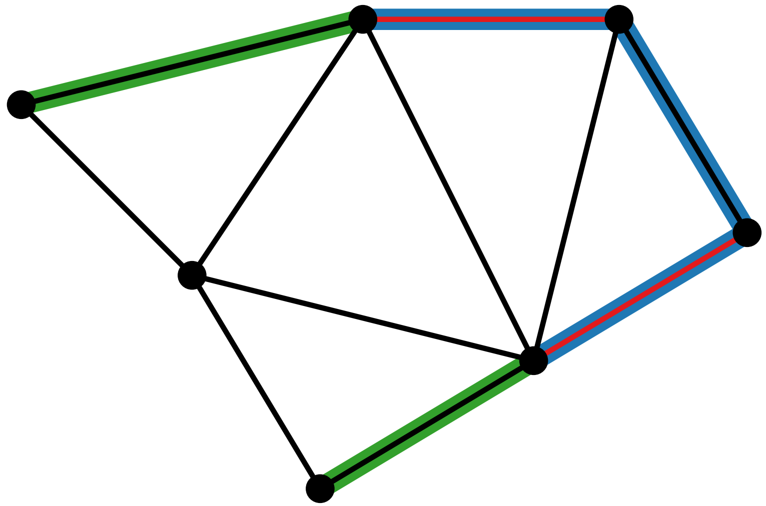

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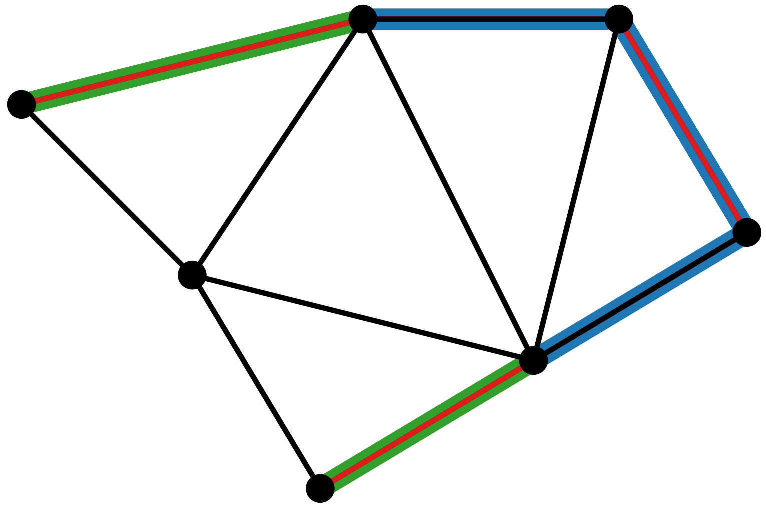

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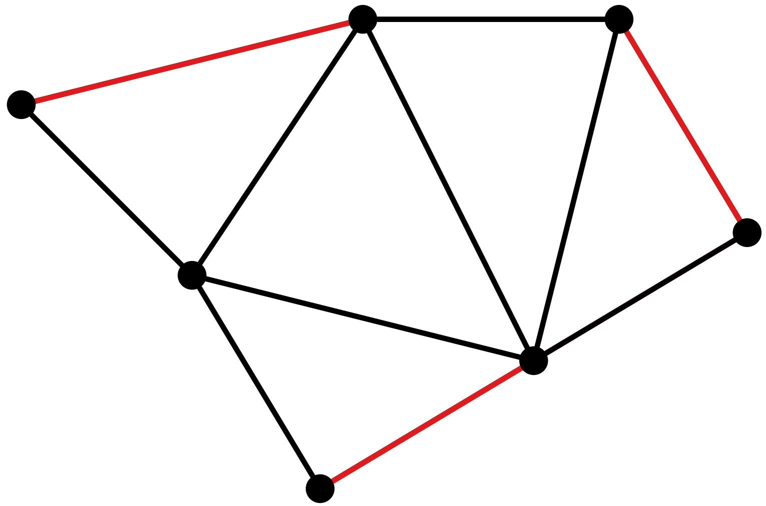

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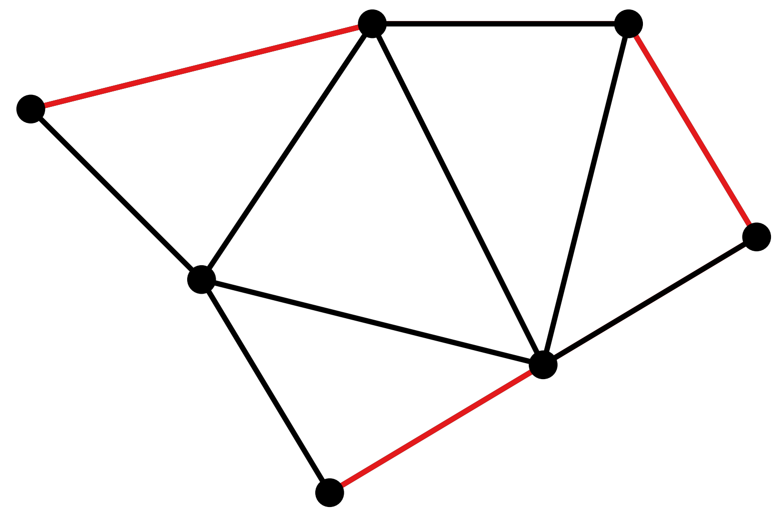

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Theorem. *(Berge)*

$$M \text{ is maximum matching} \Leftrightarrow \nexists \text{Augmenting path}$$

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- Solution: Specialise the algorithm for bipartite graphs.

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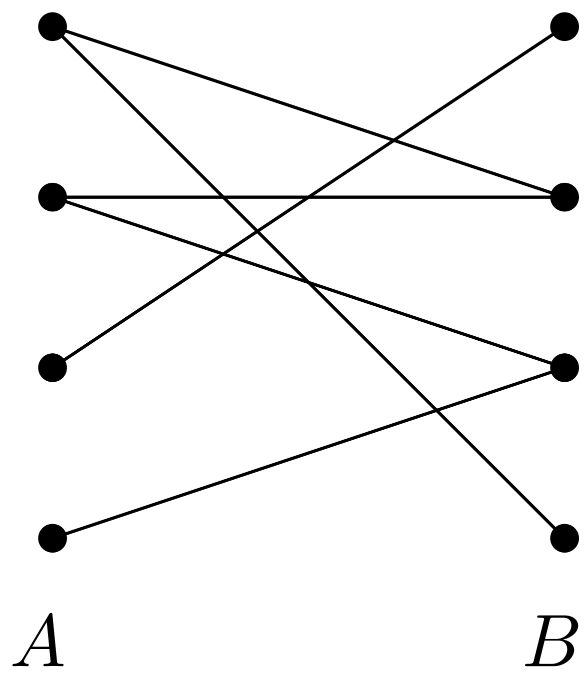

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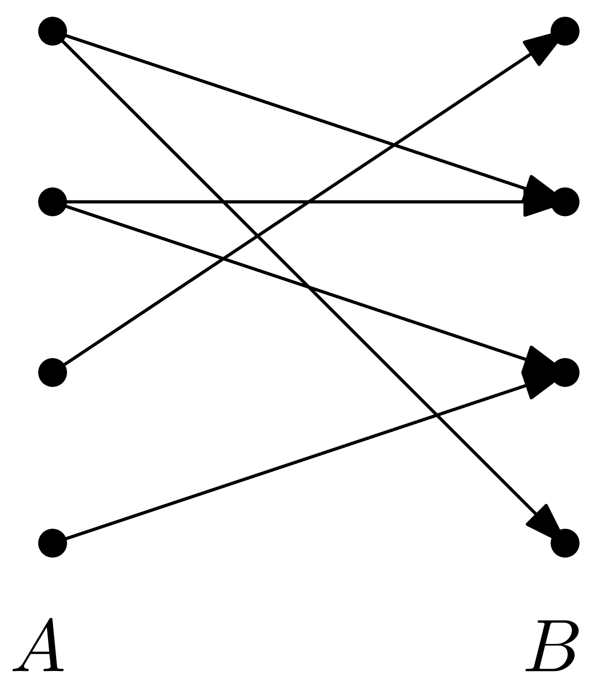

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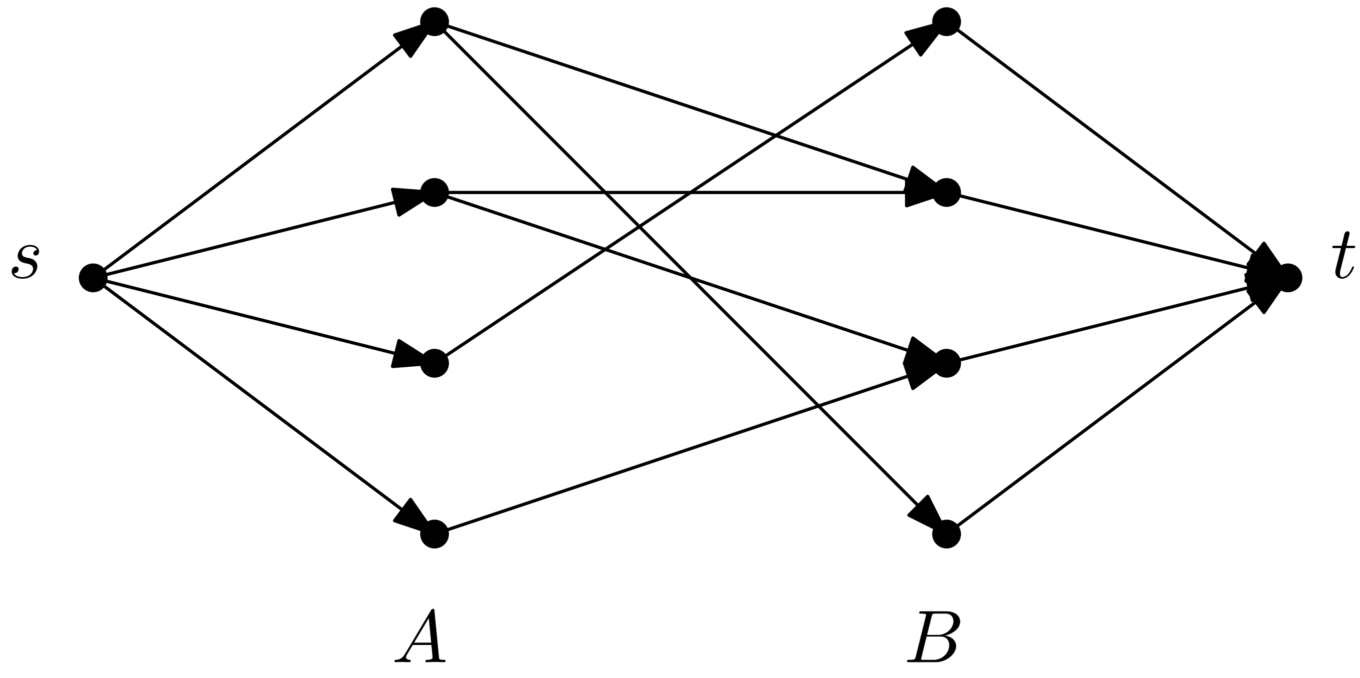

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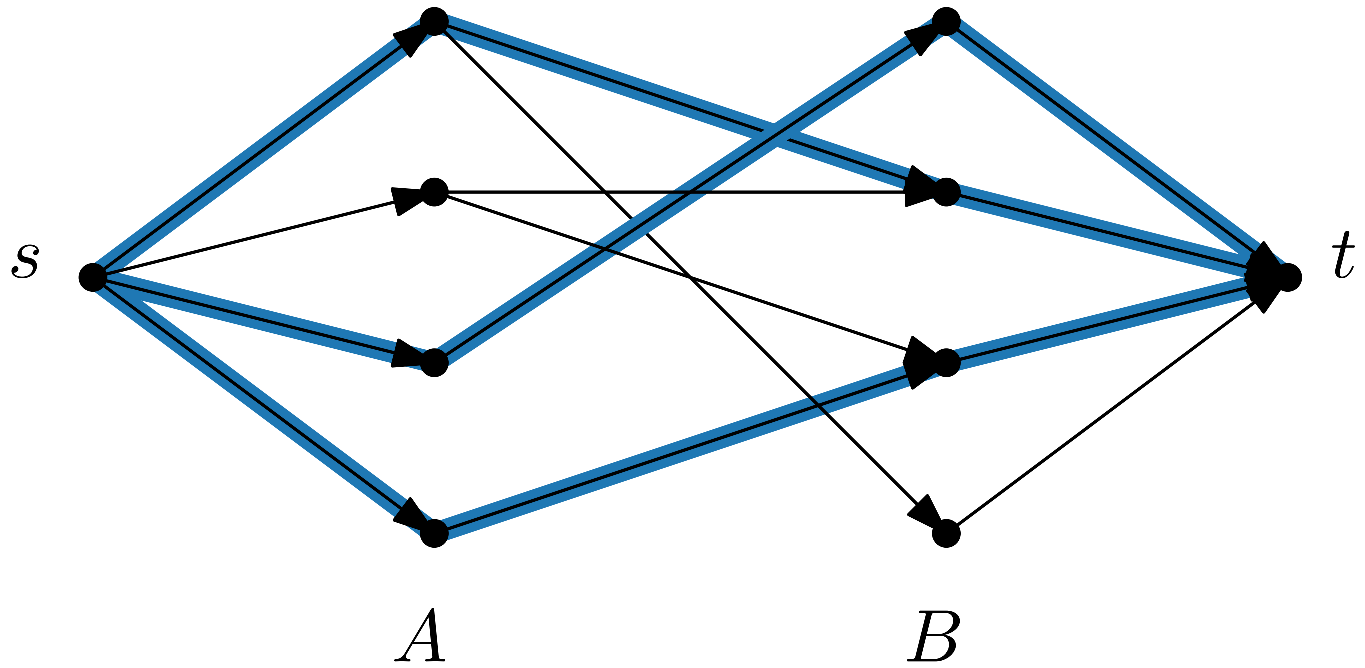

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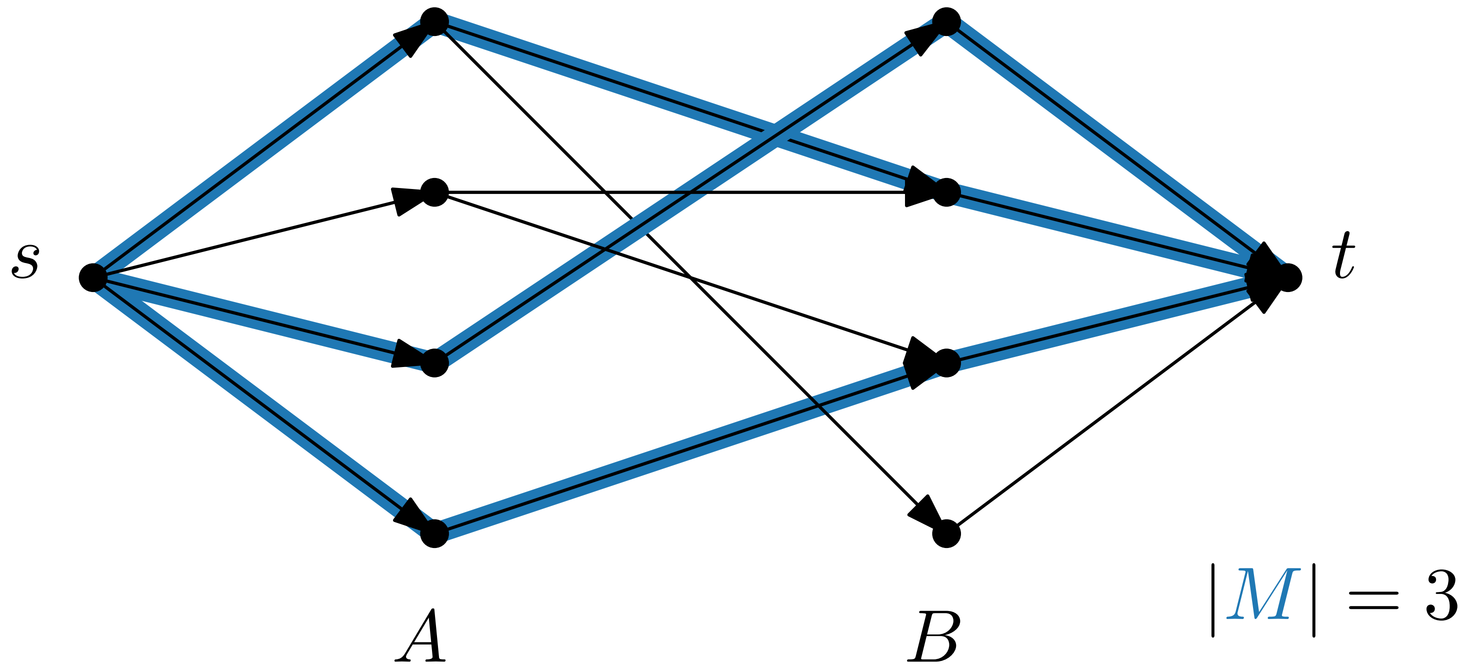

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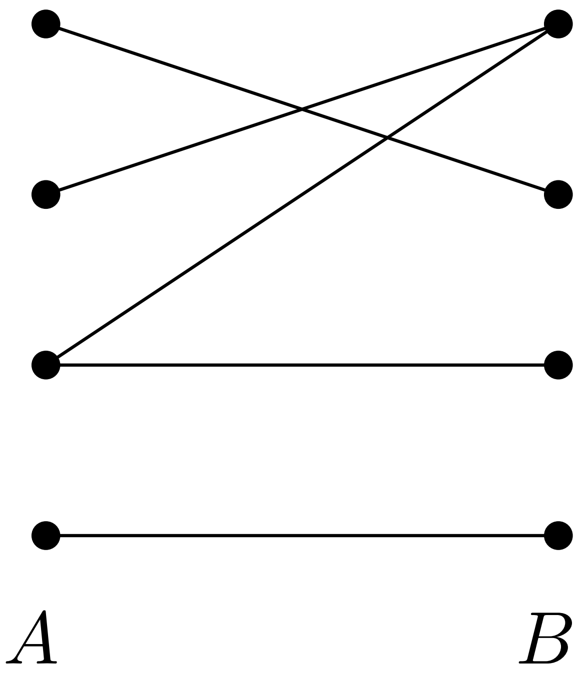

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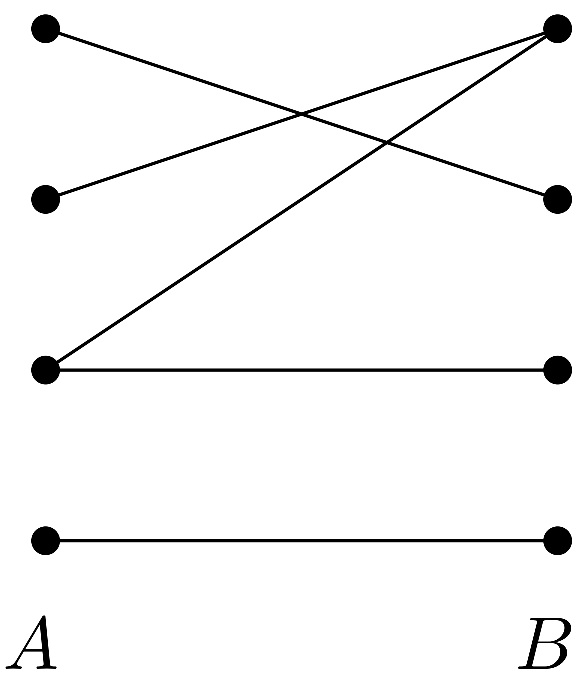

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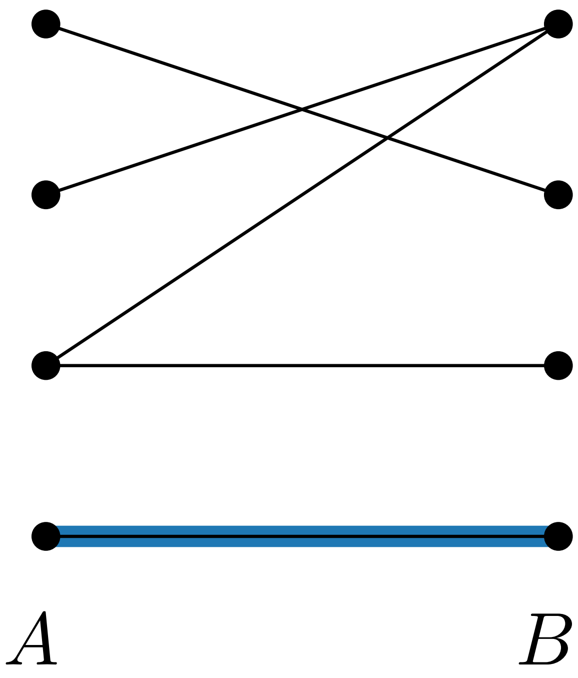

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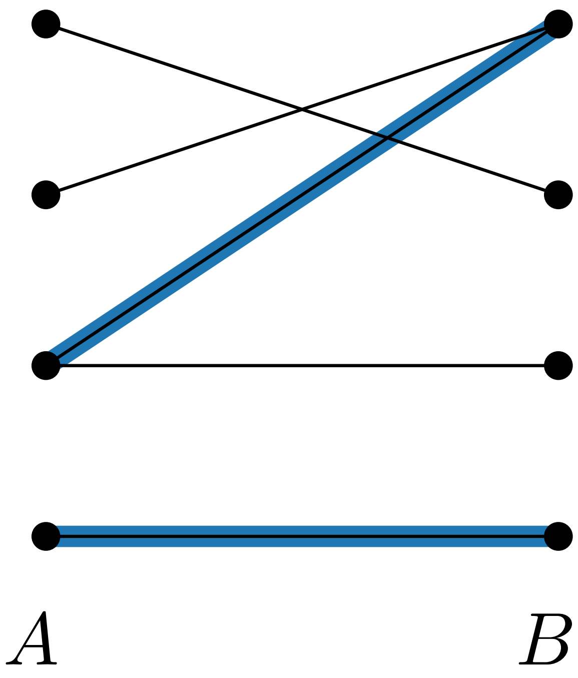

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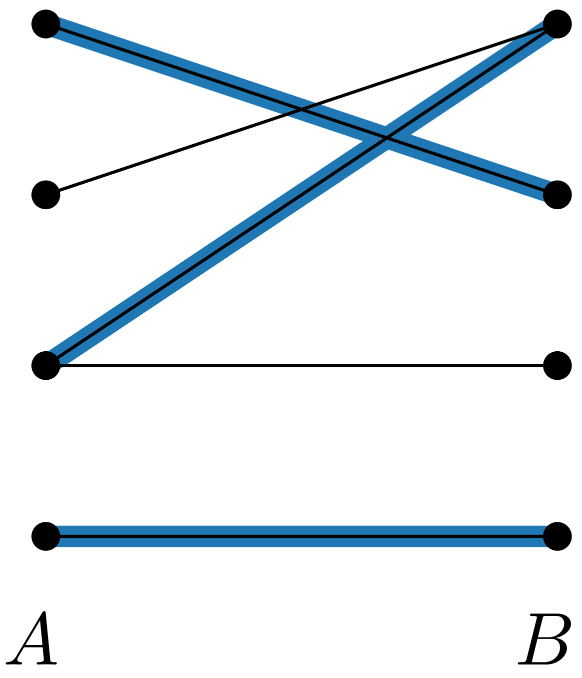

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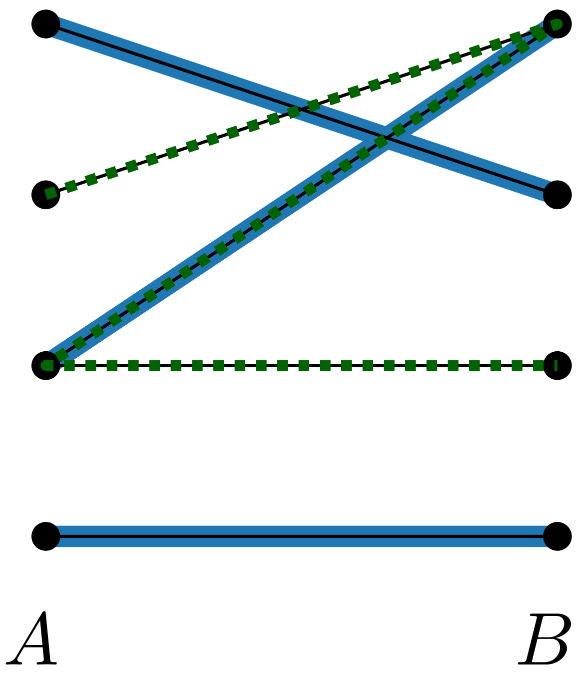

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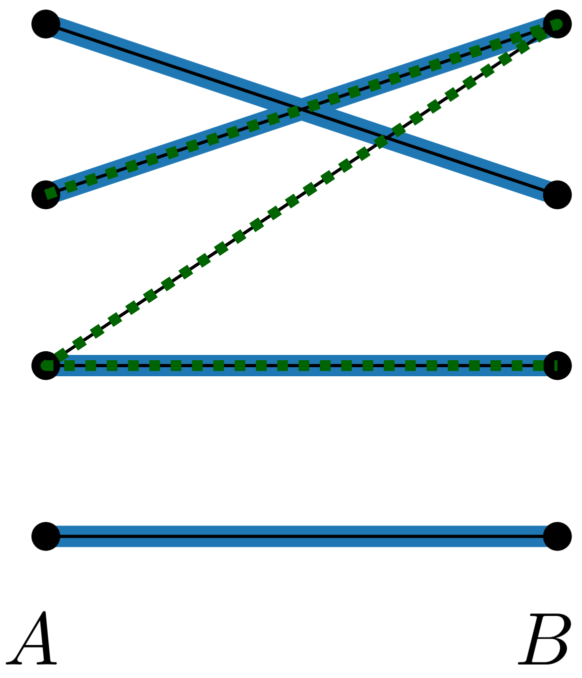

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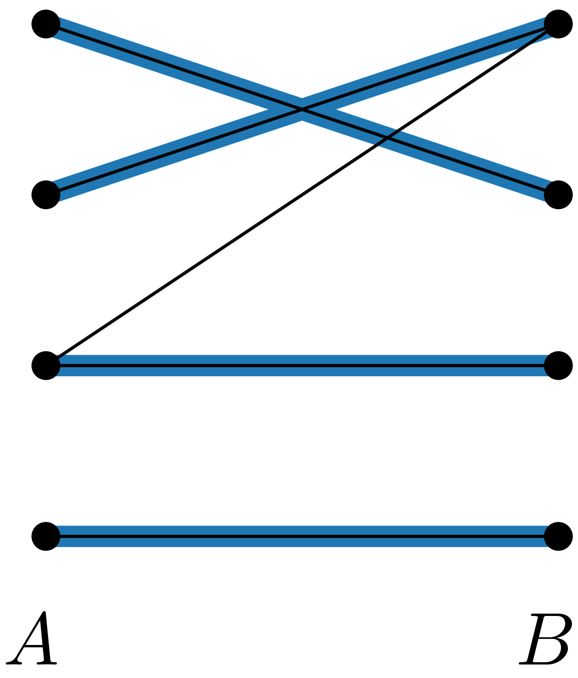

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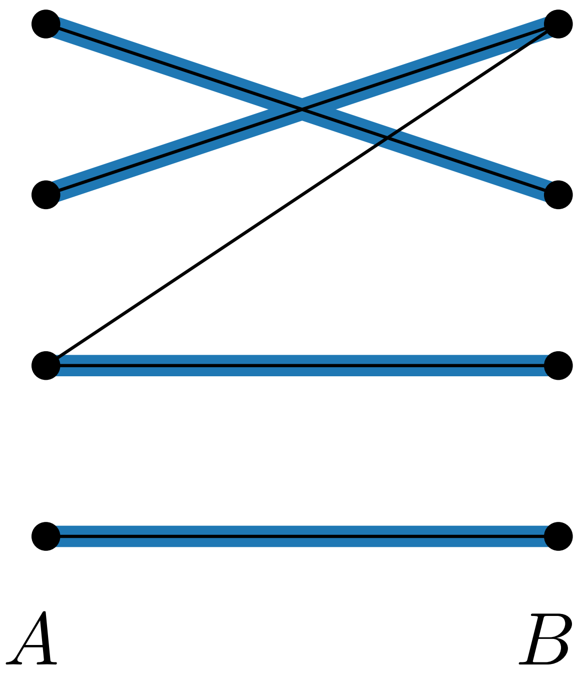

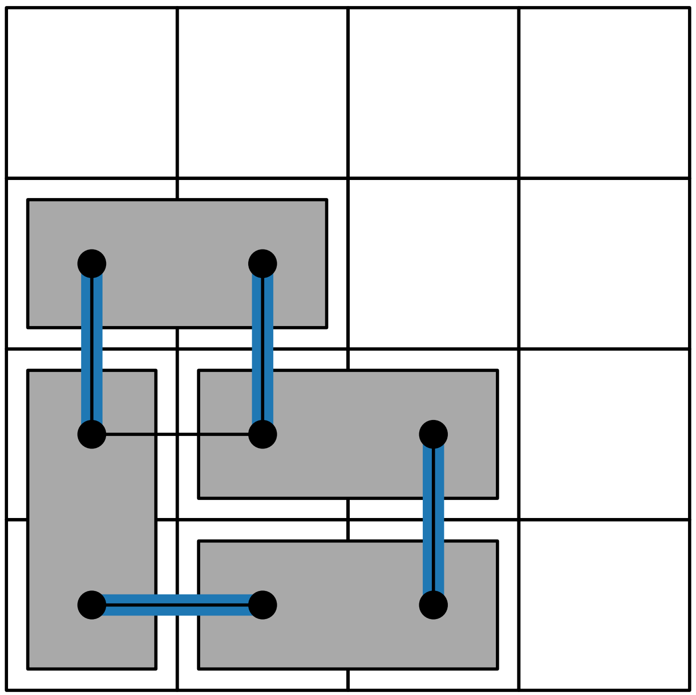

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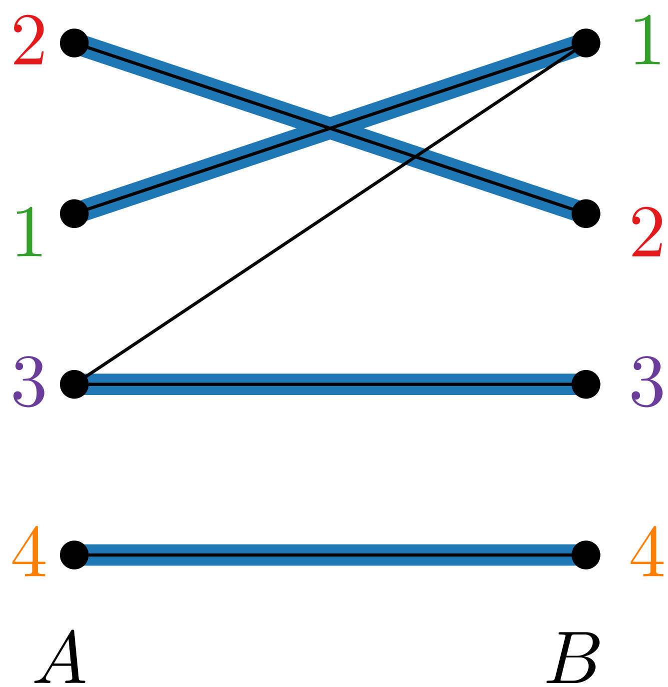

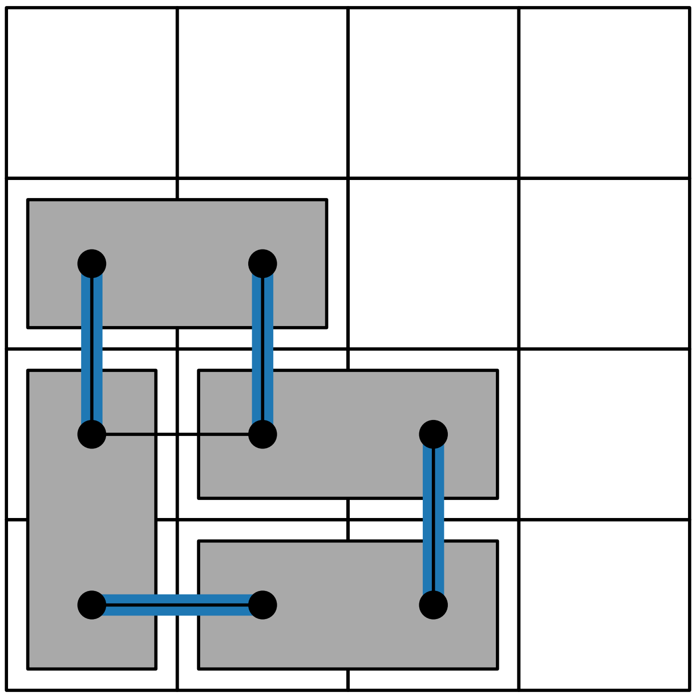

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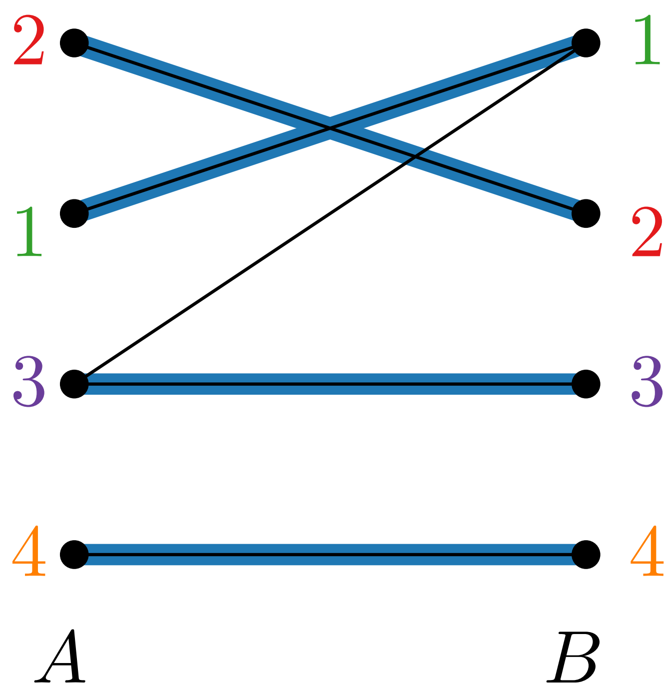

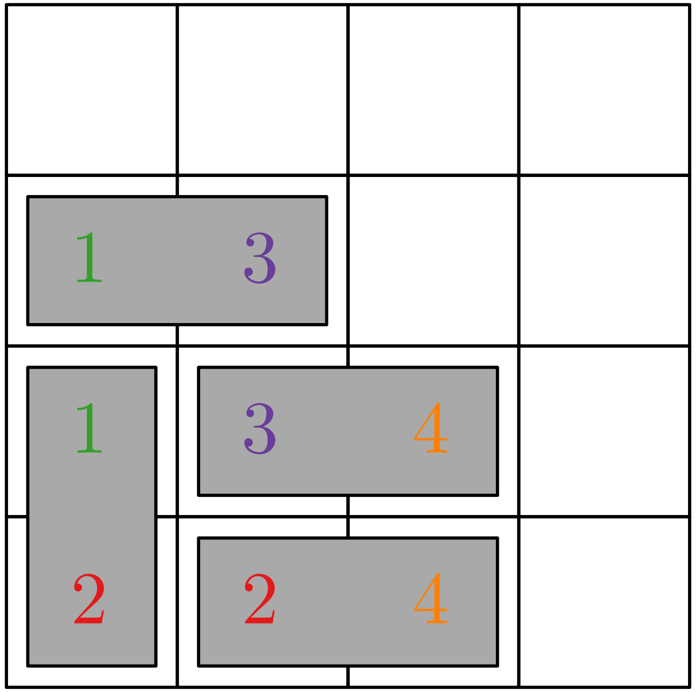

The Domiyes Algorithm

DOMIYES(Domino[] D)

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$P = \{p_1 \mid (p_1, p_2) \in D\} \cup \{p_2 \mid (p_1, p_2) \in D\}$

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- $M = \textsc{MaxBipartiteMatching}(A, B, E)$
- $k = 0$
- **foreach** $\{a, b\} \in M$ **do**
  - $f^{-1}(a)$.number $= k$; $f^{-1}(b)$.number $= k$
  - $k = k + 1$

The Domiyes Algorithm

$n := D$.length

DOMIYES(Domino[] D)

$P = \{p_1 \mid (p_1, p_2) \in D\} \cup \{p_2 \mid (p_1, p_2) \in D\}$ — $\mathcal{O}(n)$

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$\overline{\mathcal{O}(n^2 + VE)}$

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MaxBipartiteMatching$(A, B, E \subseteq \binom{A}{2} \cup \binom{B}{2})$

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$\textsc{MaxBipartiteMatching}(A,\ B,\ E \subseteq \binom{A}{2} \cup \binom{B}{2})$

$M = \emptyset$

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MaxBipartiteMatching

```
MaxBipartiteMatching(A, B, E ⊆ (A choose 2) ∪ (B choose 2))
    M = ∅
    foreach M-free a ∈ A do
        if ∃ aug. path P from a to M-free b ∈ B then
            foreach uv ∈ P do
                if {u, v} ∈ M then
                    M = M \ {{u, v}}
                else
                    M = M ∪ {{u, v}}
    return M
```

MaxBipartiteMatching

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This still runs in $\mathcal{O}(VE)$ time.
However...

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$\Delta \leq 3 \Rightarrow |E| \leq 3V$

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$\mathcal{O}(VE) =$
$\mathcal{O}(V \cdot 3V) = \mathcal{O}(V^2)$.