Lehrstuhl für Informatik I
Algorithmen und Komplexität
Universität Würzburg

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Vorlesung Algorithmen für Geographische Informationssysteme SS 2021

Clustering Equivalent Destinations

Abgabe: in 2er Gruppen bis Mittwoch, den 9.6.2021, 10:15 Uhr, in analoger Form oder über die E-Learning-Seite der Lehrveranstaltung. Bitte geben Sie auf Ihrer Ausarbeitung die Namen beider Gruppenteilnehmer an.

Equivalent destinations in trees

Let G be a graph and $\mathcal{T} = (V, E)$ the tree as in the lecture slides, rooted at $s \in V$. Let P_{ab} be the (unique) path in $\mathcal{T}$ from a to b, and let $w(\cdot)$ be the weight of a path.

Definition 1 (Directed similarity) *Let $u, v \in V$ be two vertices, $u \neq s$, and let x be their lowest common ancestor in $\mathcal{T}$. The directed similarity of u to v is defines as*

$$\sigma(u, v) = w(P_{sx})/w(P_{su}).$$

Definition 2 (α-Compatible, $\oplus$) *Two vertices $u, v \in V$ are called α-compatible if and only if both $\sigma(u, v) \geq \alpha$ and $\sigma(v, u) \geq \alpha$. When α is clear from context, we write $u \oplus v$ to assert that u and v are α-compatible.*

(a) Prove or disprove these three claims: $\oplus$ is reflexive, symmetric and transitive. *(3 Punkte)*

(b) Prove the following lemma. *(3 Punkte)*

Lemma 1 *Let $u, v \in V$ be vertices and let x be their lowest common ancestor. Then u and v are compatible if and only if they are each compatible to x, that is, $u \oplus v \iff u \oplus x \wedge v \oplus x$.*

(c) Prove the following lemma. *(3 Punkte)*

Lemma 2 *Let $x \in V$ and let $a, b \in V$ be descendants of x. If $w(P_{sa}) \leq w(P_{sb})$ and $b \oplus x$, then $a \oplus x$. That is, if a and b are both descendants of x, and b is father away from the root, then $b \oplus x \Longrightarrow a \oplus x$.*

(d) Prove the following lemma. *(1 Punkt)*

Lemma 3 *Let $x \in V$ be a vertex and let $S \subseteq V$ be a set of vertices such that for any pair of vertices in S their lowest common ancestor is x. Then $(\oplus)$ is an equivalence relation on S. In particular, let $S_\oplus = \{v \in S \colon v \oplus x\}$. All pairs of vertices in $S_\oplus$ are compatible, and any vertex in $S \setminus S_\oplus$ is not compatible to any other vertex in S.*

(e) The algorithm from the lecture merges cells bottom-up, starting with the leafs, and it is proven that this results in an optimal clique cover. Consider doing the same merging algorithm, but top-down, starting from the root and merging down. Give an instance on which this finds a clique cover with $\Theta(n)$ cells even though a clique cover with $O(1)$ cells exists. *(4 Punkte)*