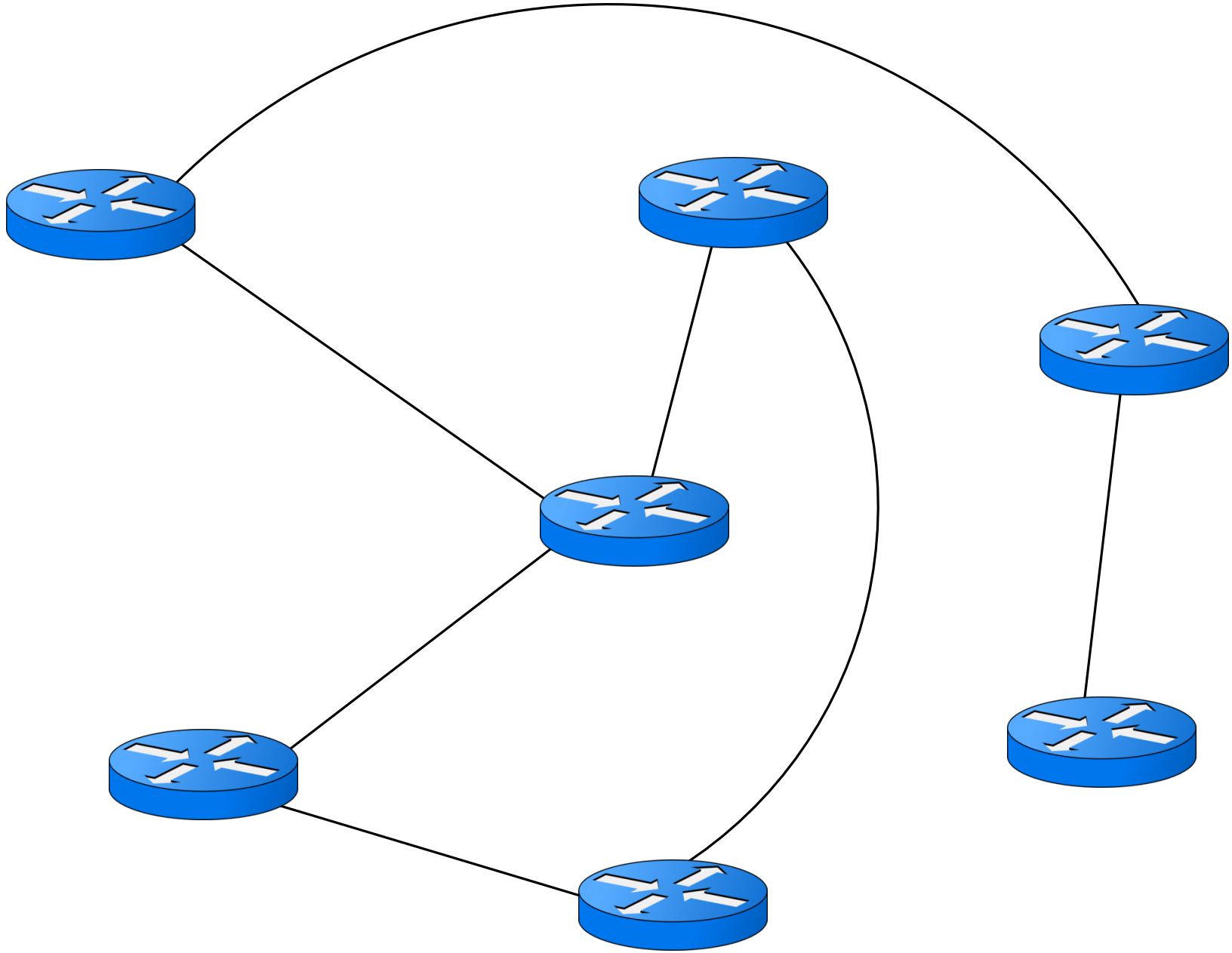
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Seminar
Algorithmen für Programmierwettbewerbe

Problem C
A Series of Tubes

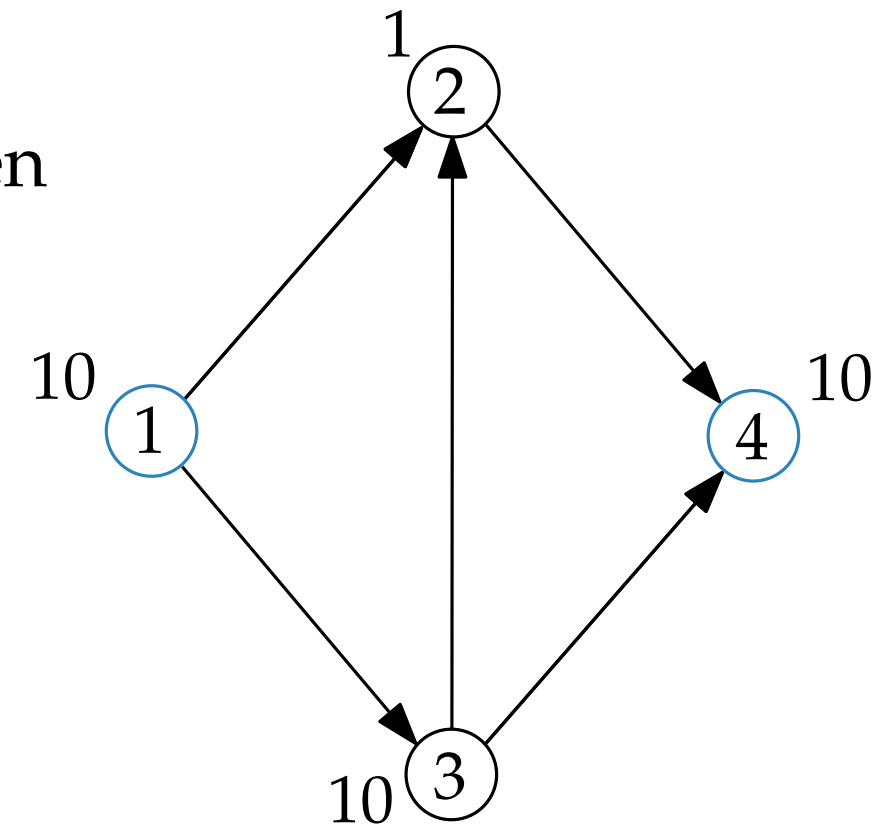
Martin Klüpfel

Ferdinand Zink



Problemstellung

Gegeben:
- Routernetzwerk
- dazugehörige Bearbeitungszeiten
- Start- und Endknoten



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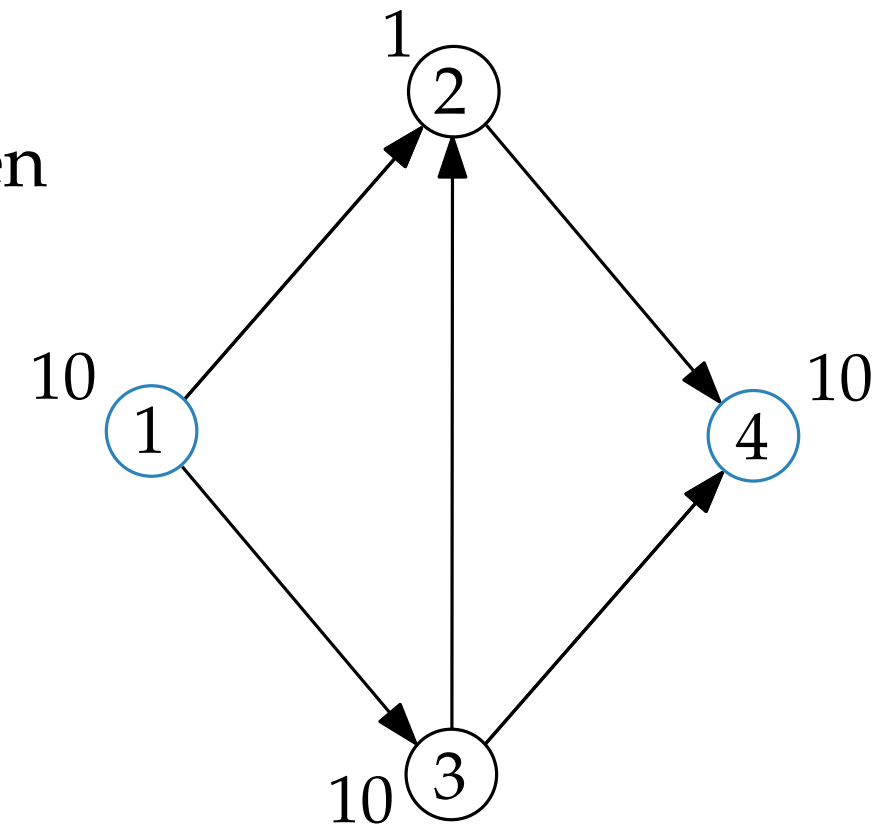

Ziel:
Bestimmung der Länge eines kürzesten Pfades

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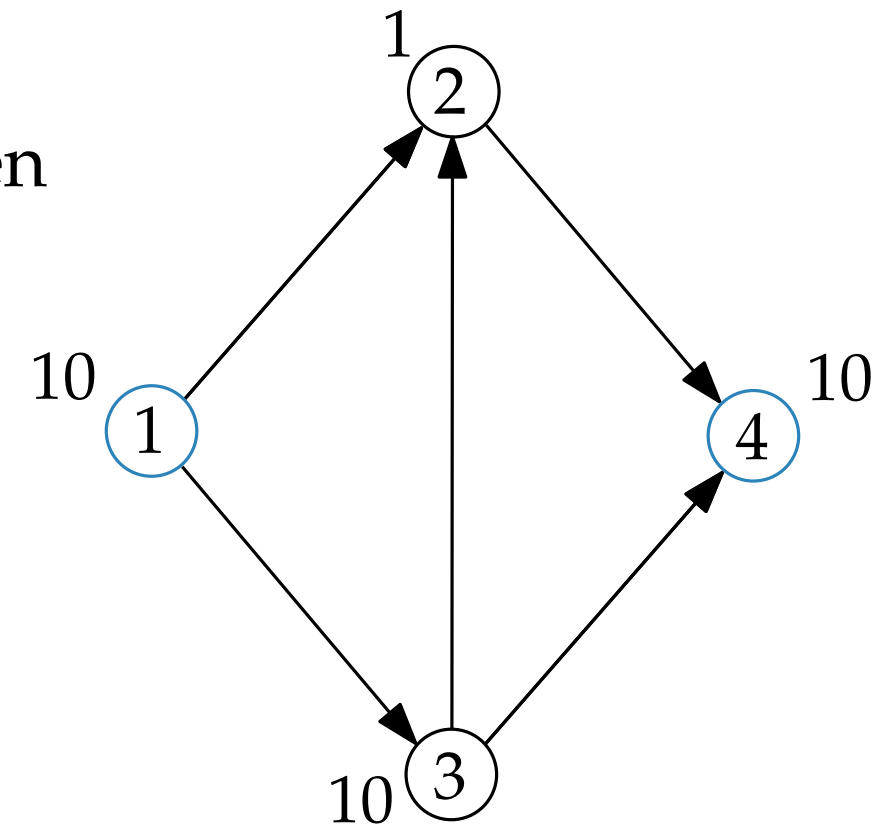

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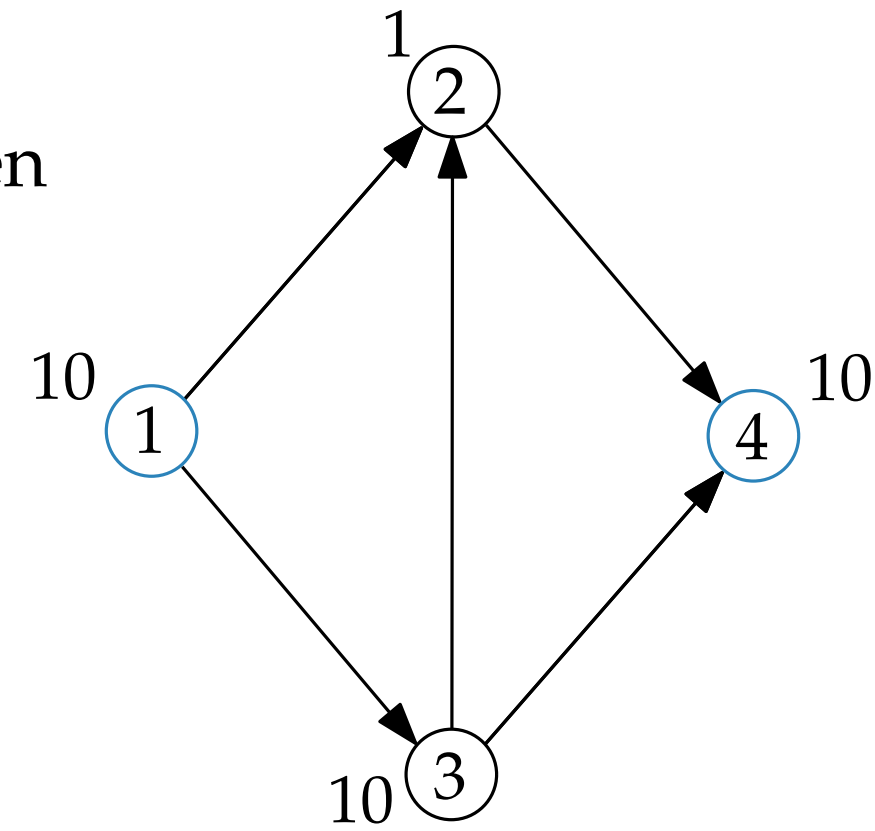

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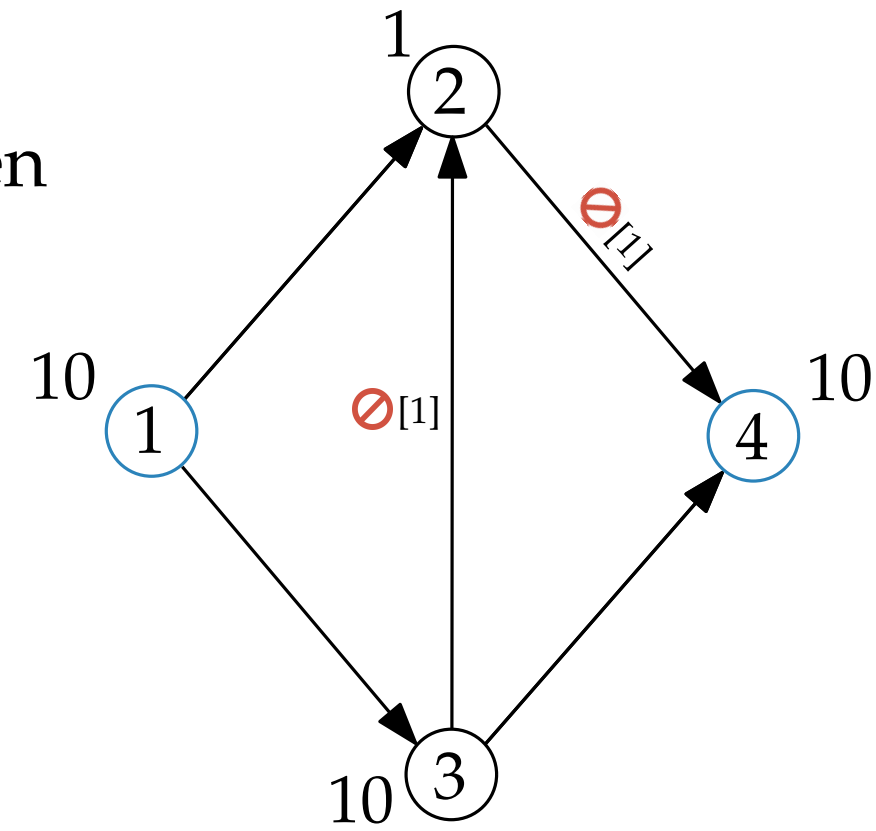

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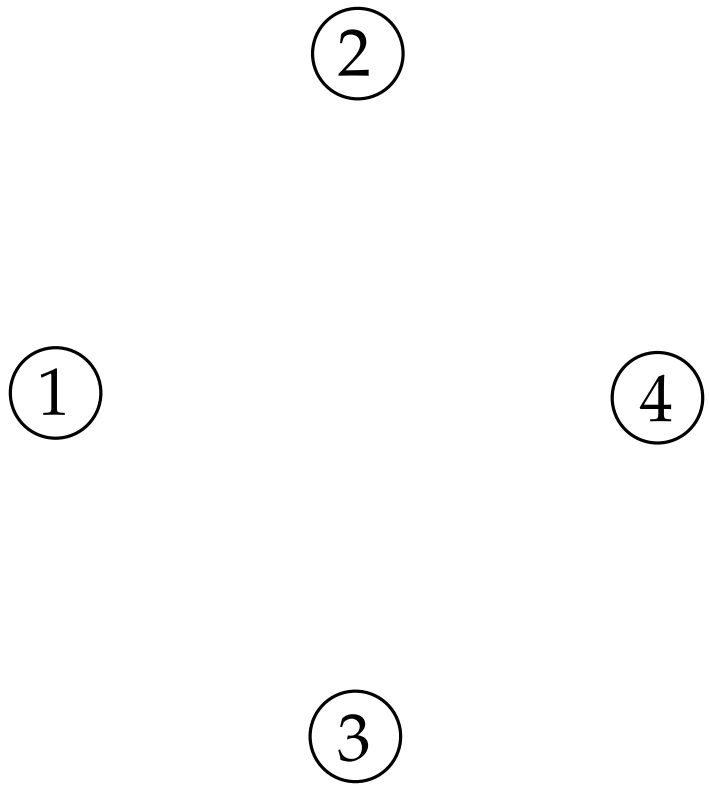
Problemstellung

Eingabe:

Problemstellung

Eingabe:

4

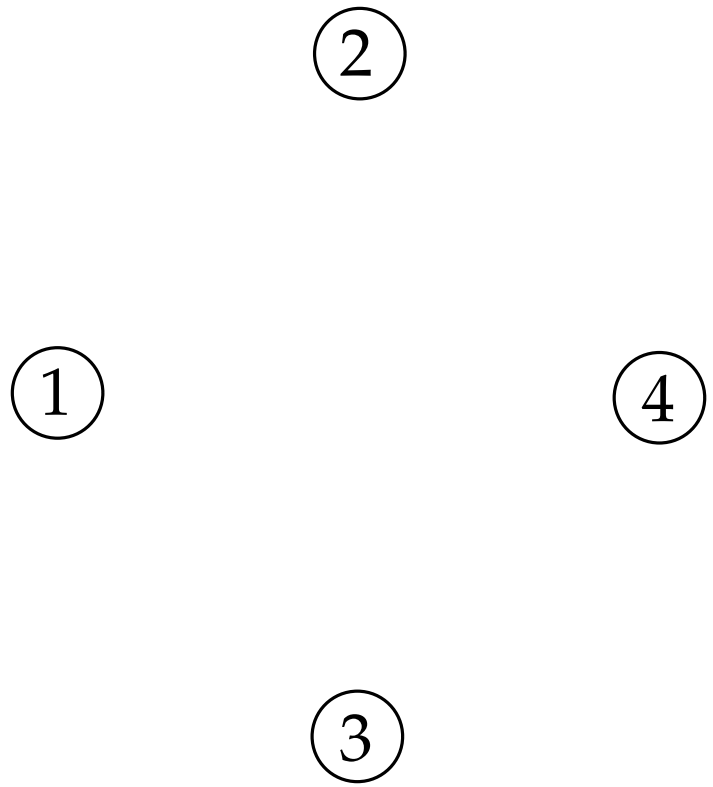


Problemstellung

Eingabe:

4

2 10



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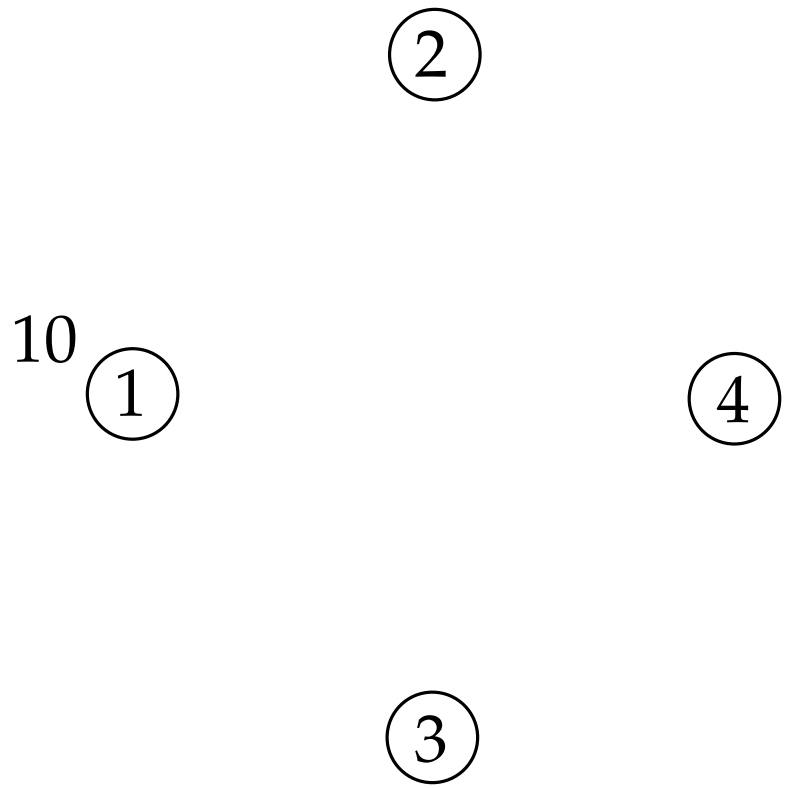
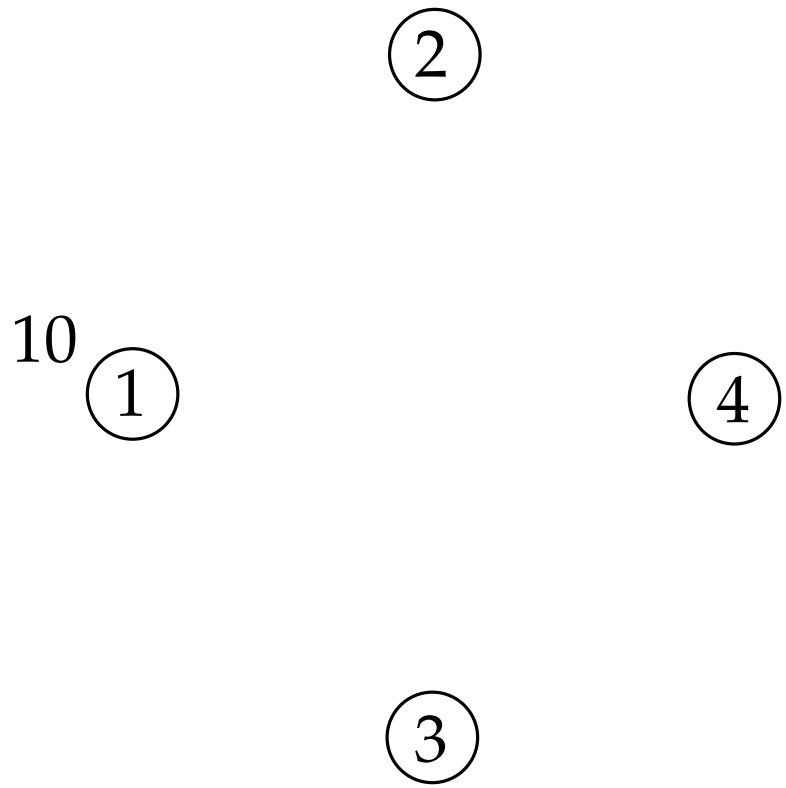


Problemstellung

Eingabe:

4

```
2 10
0 2
0 3
```



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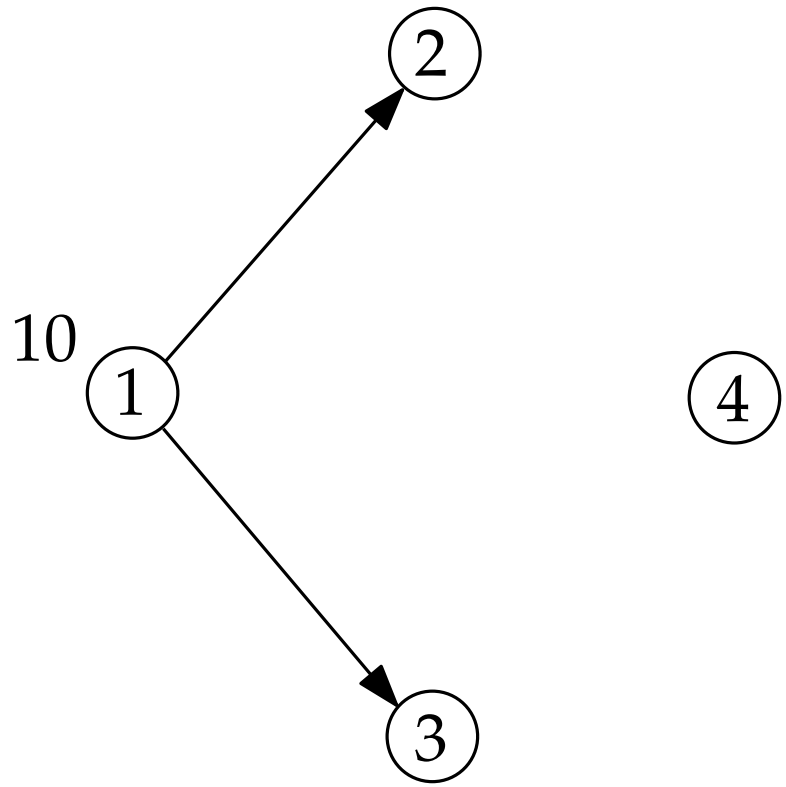

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2 10
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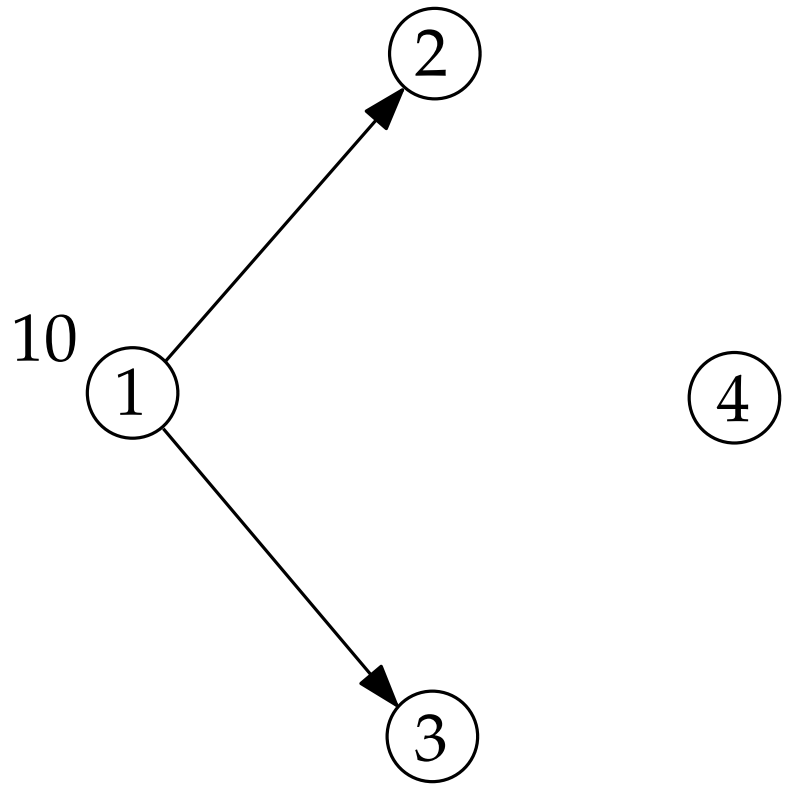
1 1



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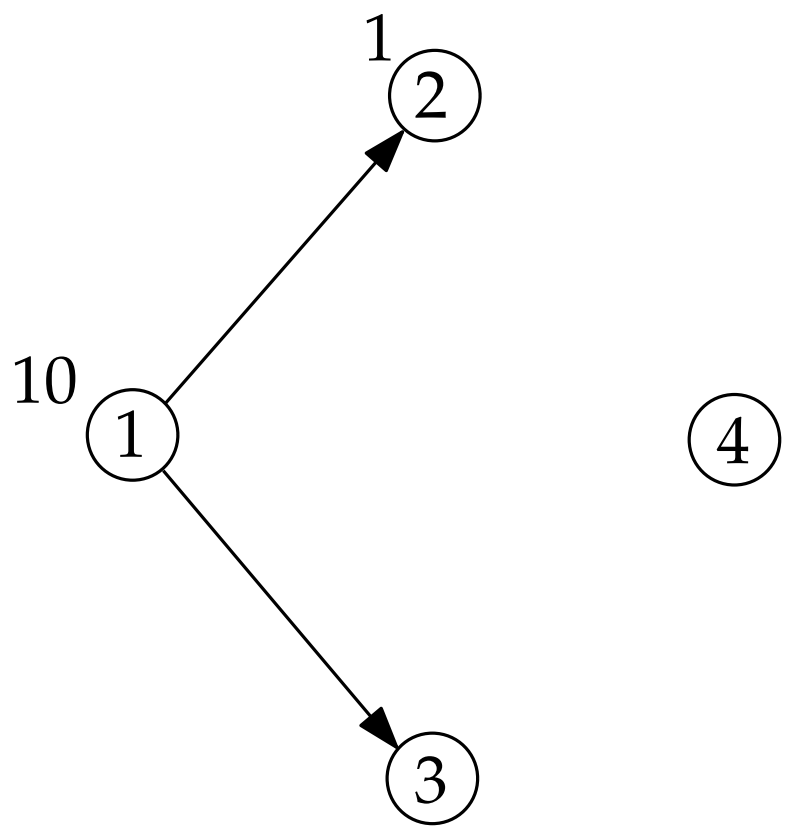

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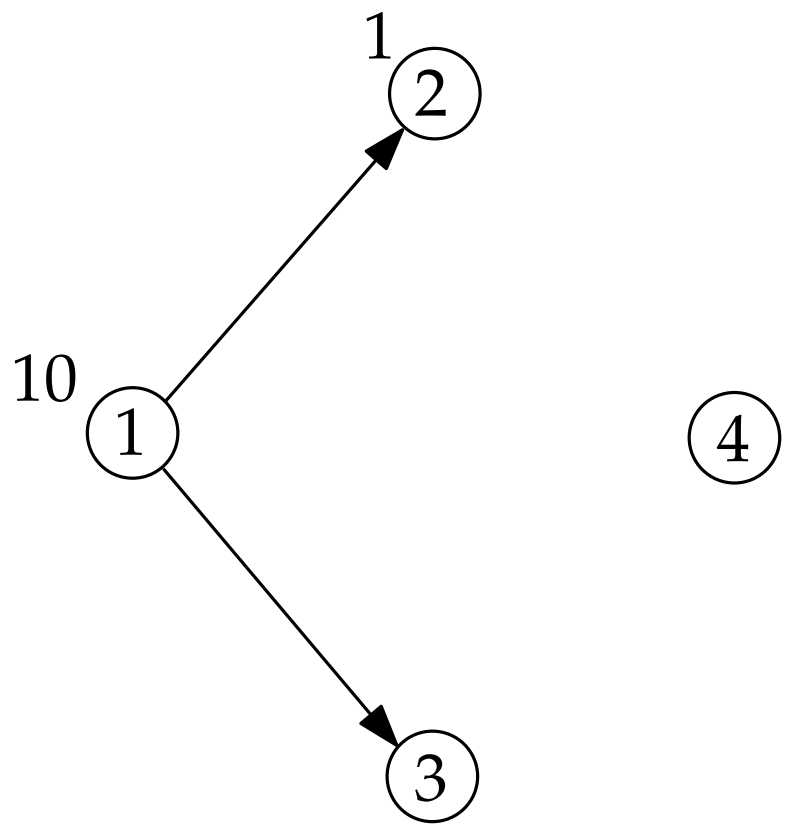

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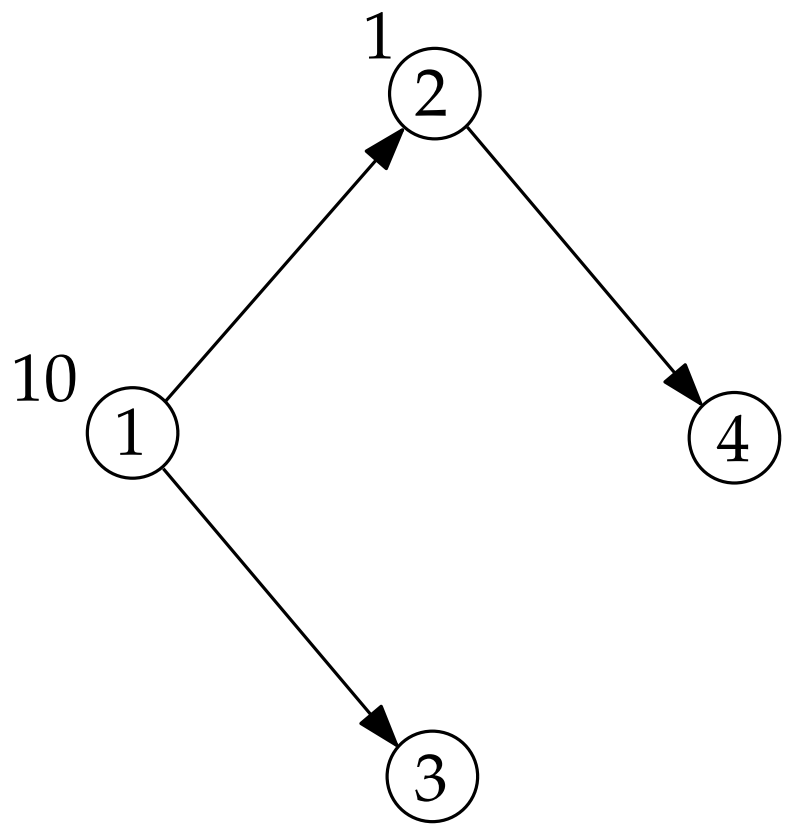

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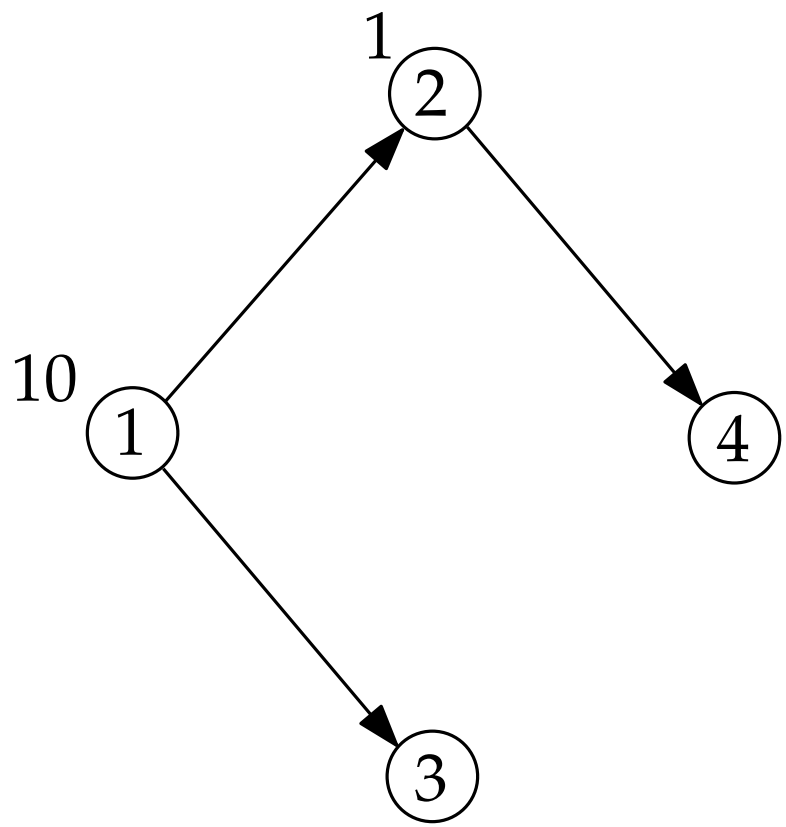

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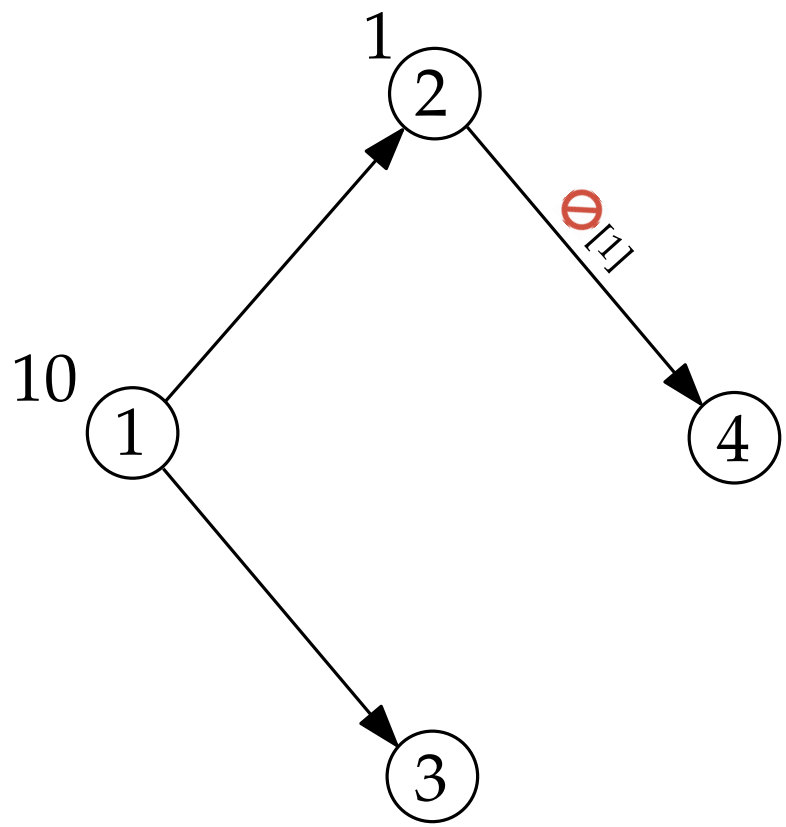

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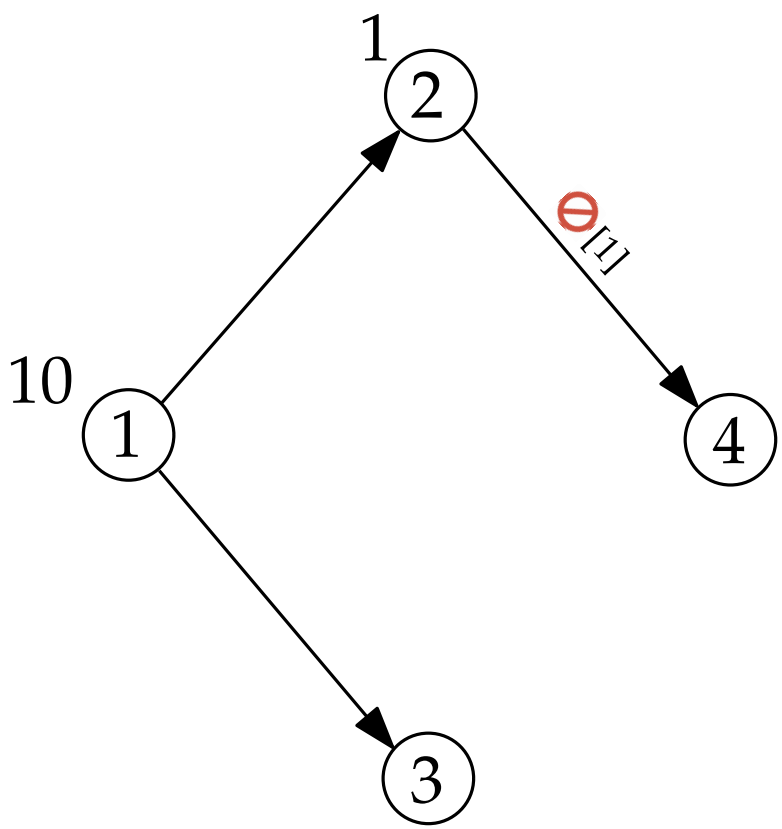

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0 3
1 1
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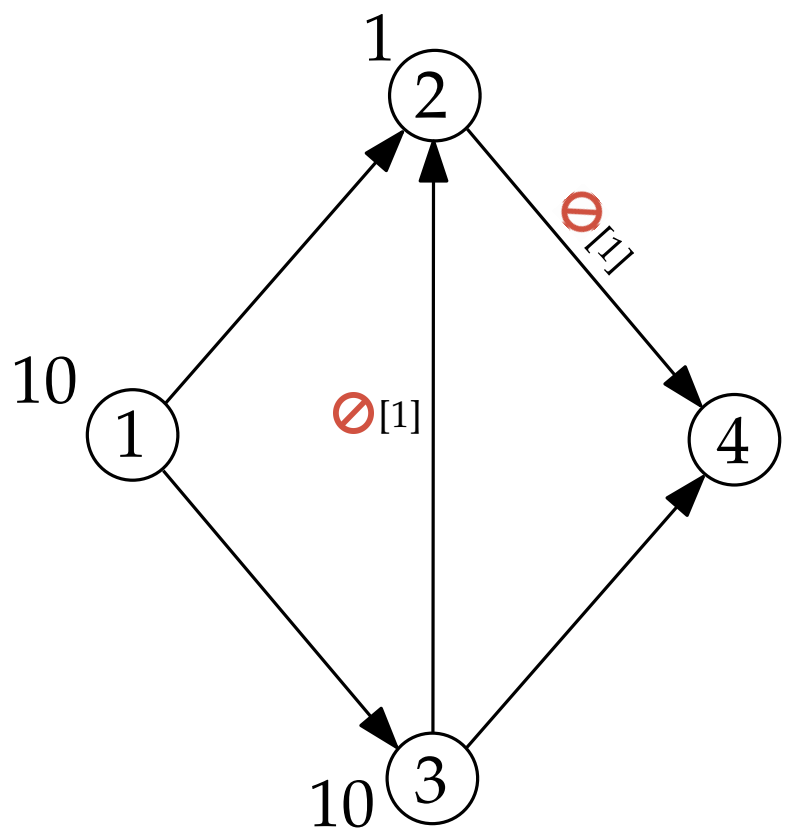
```



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1 2 1
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```
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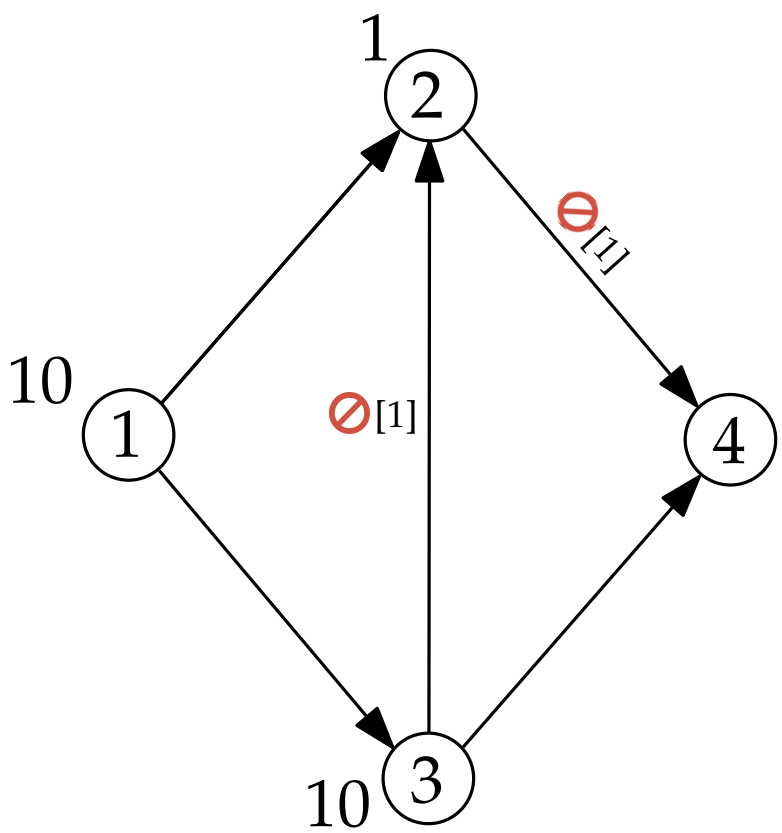

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```
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```
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```
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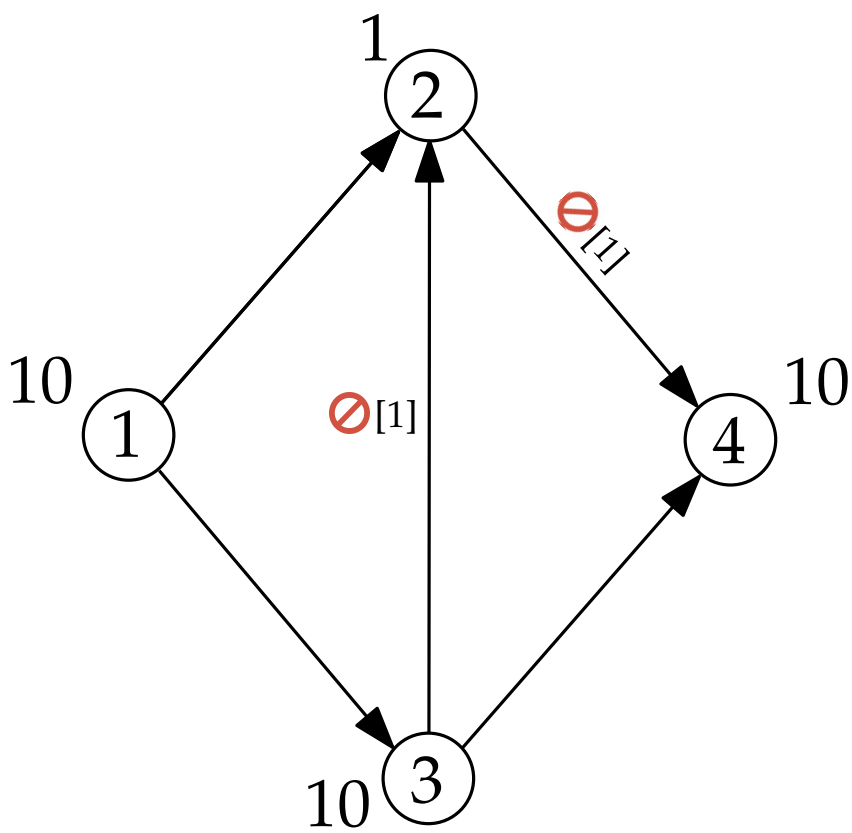

Beispiel

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```
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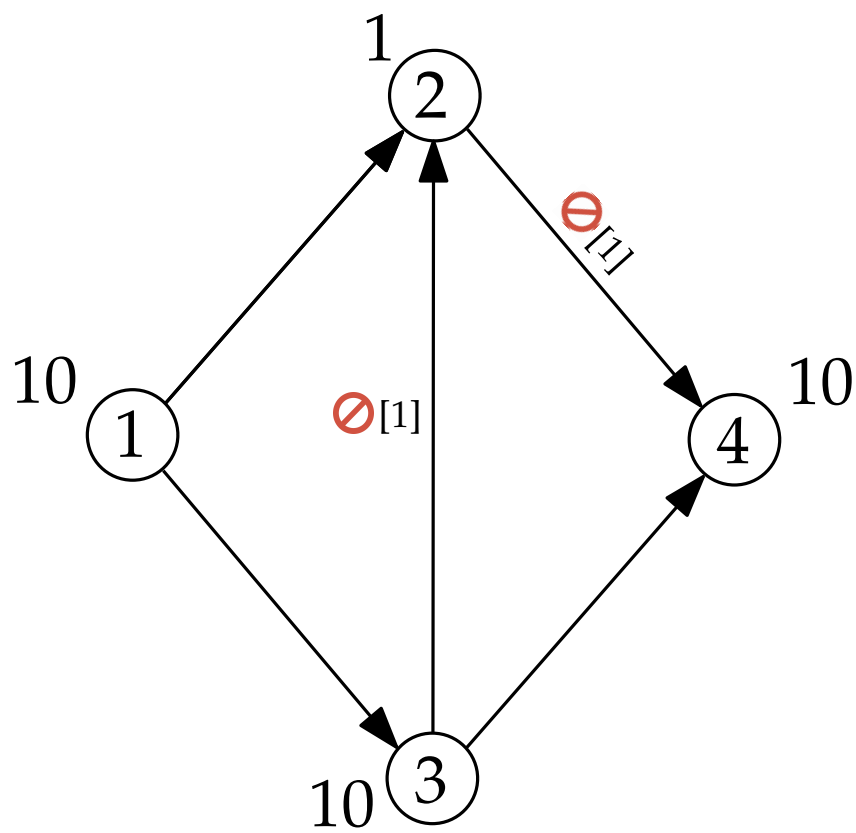
2 10
0 2
0 3
1 1
1 4 1
2 10
1 2 1
0 4
0 10
```



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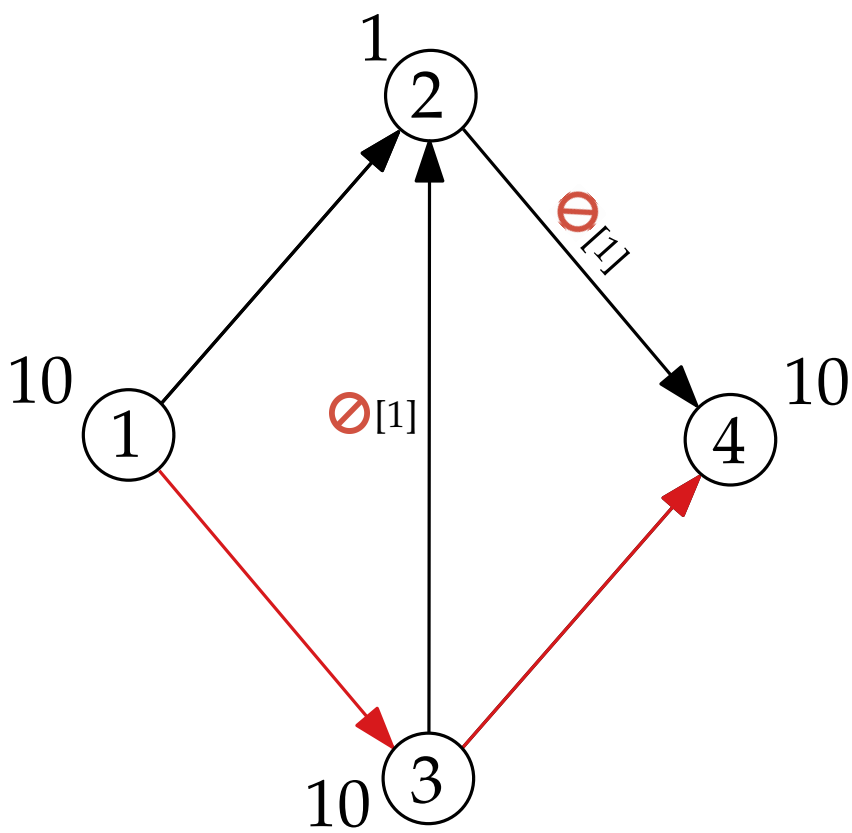

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```
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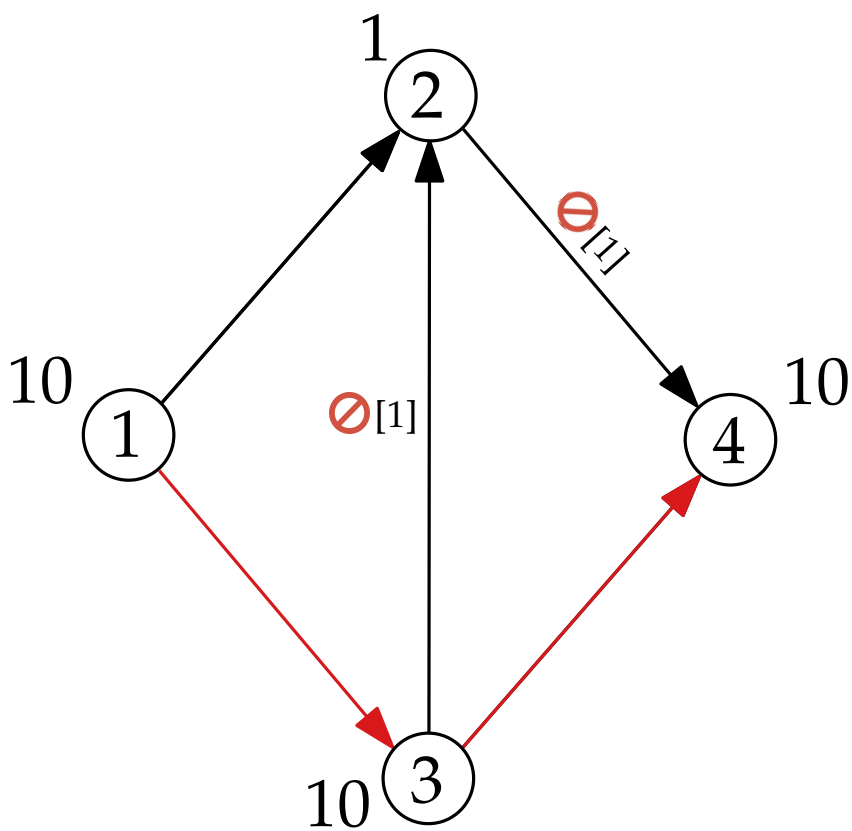
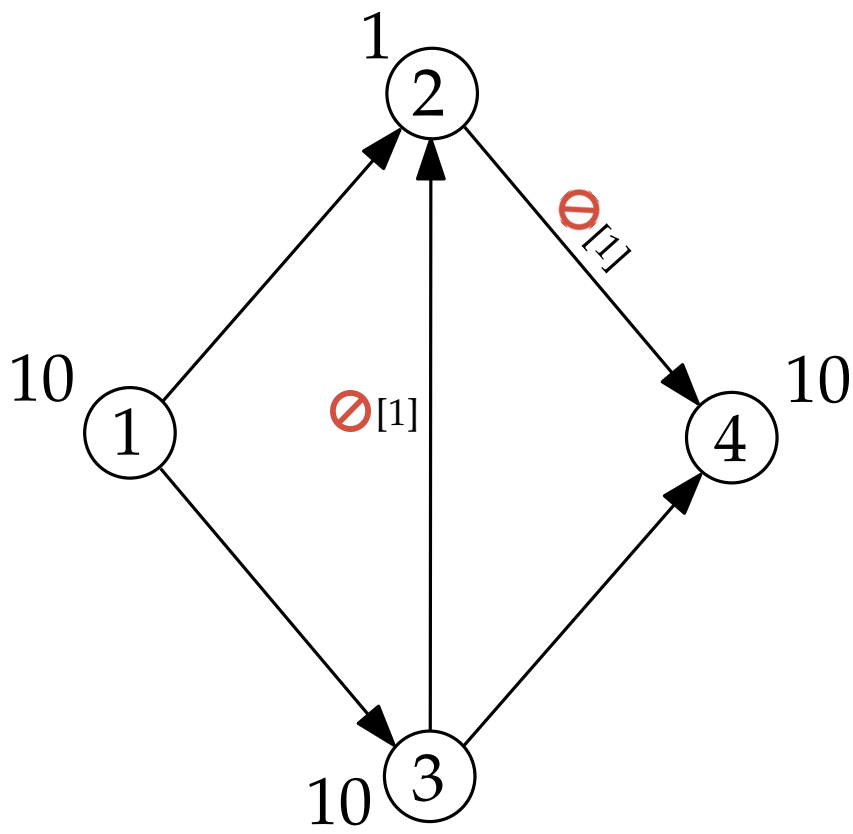
```

Ausgabe:

30

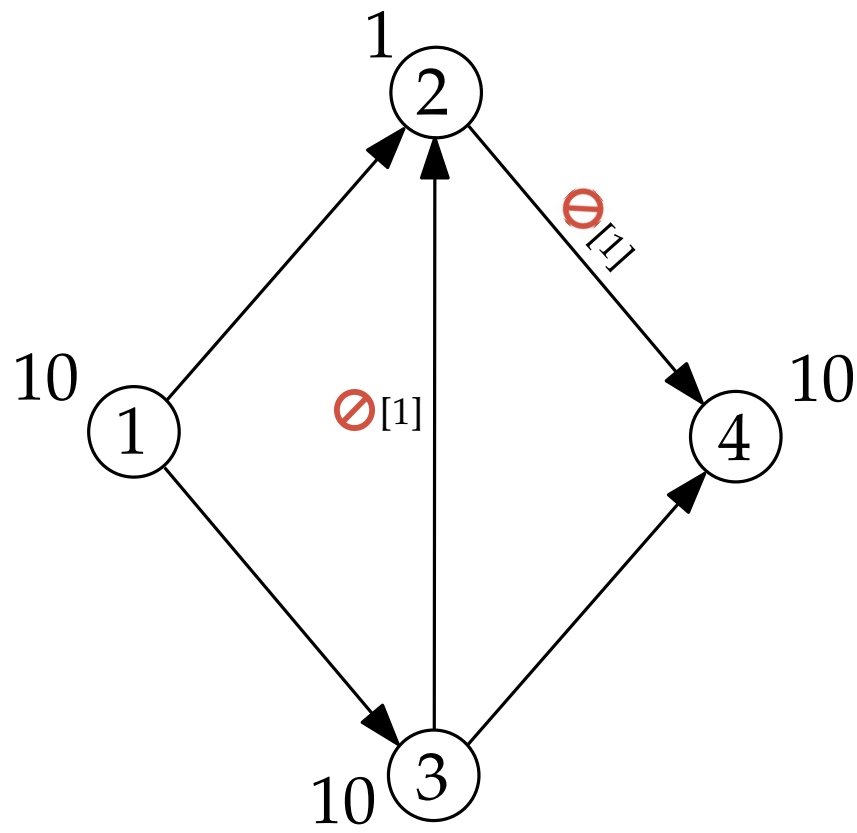


Brute-Force?



Brute-Force?

Rekursives Durchlaufen aller Pfade unter Berücksichtigung der Regeln



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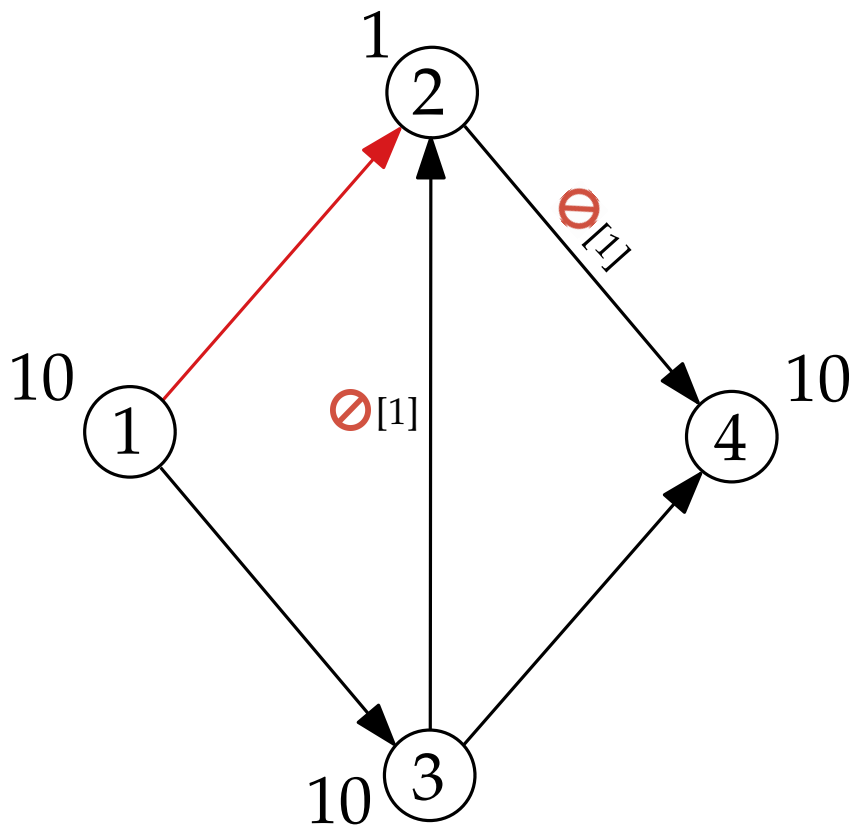

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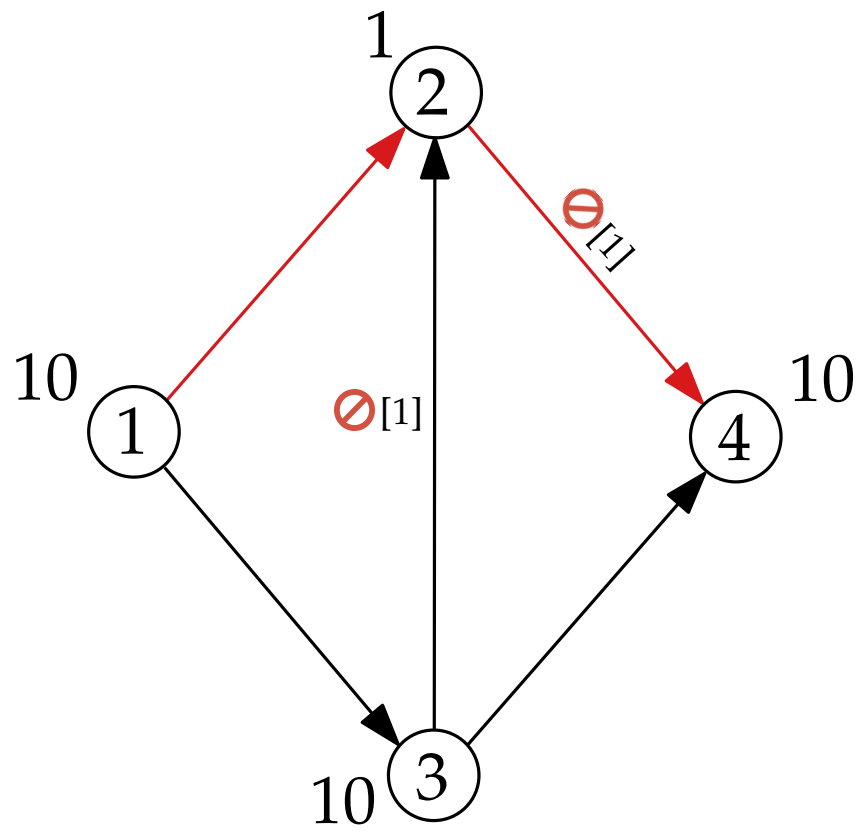

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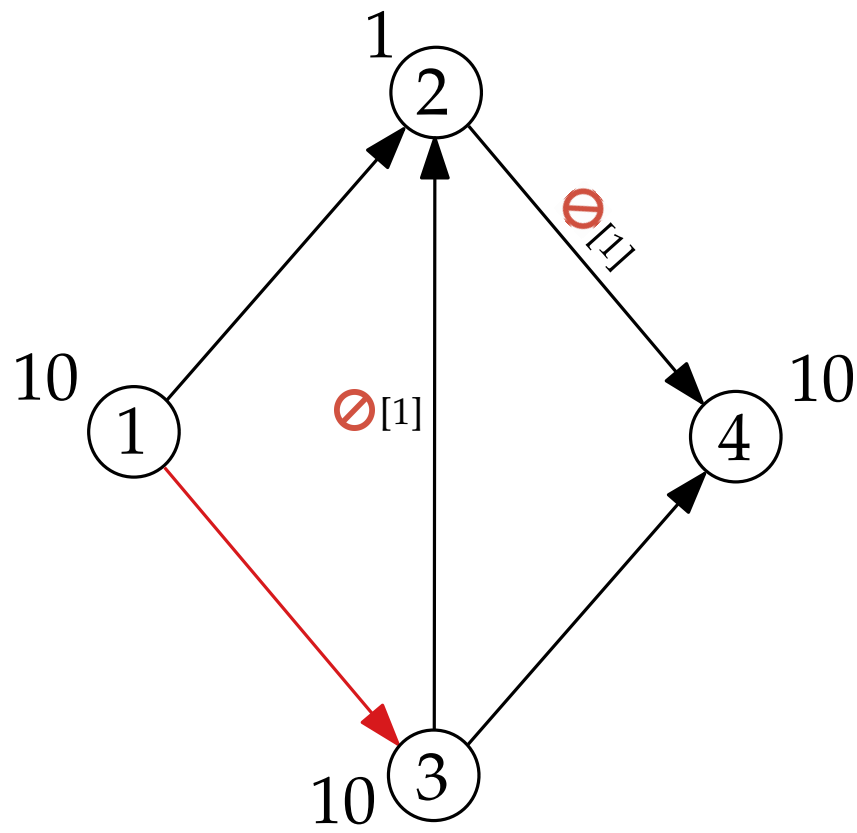

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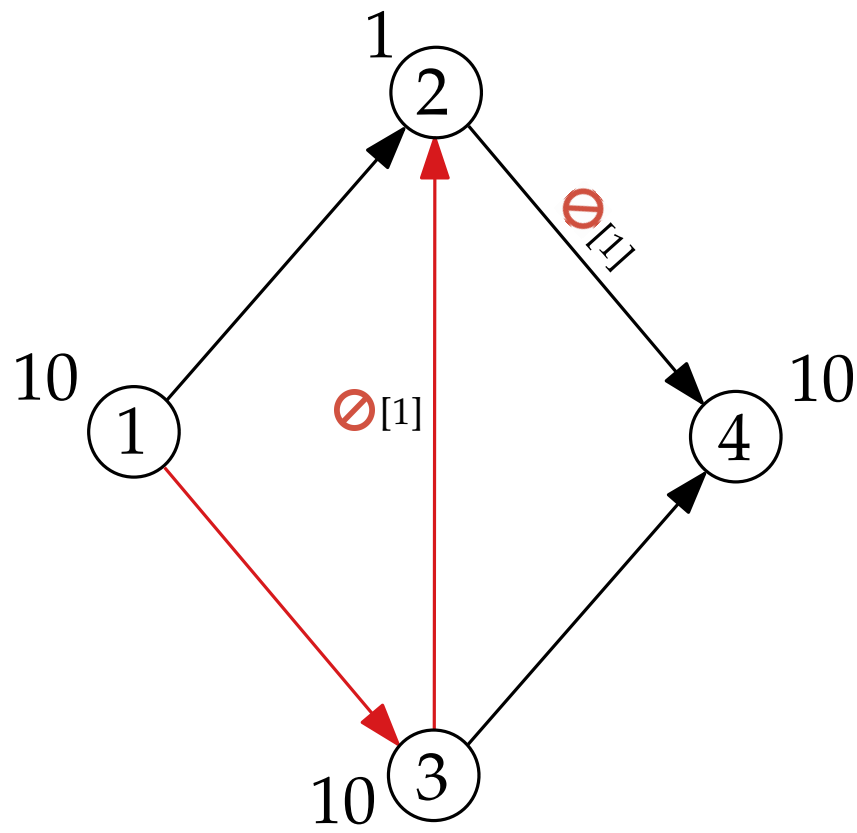

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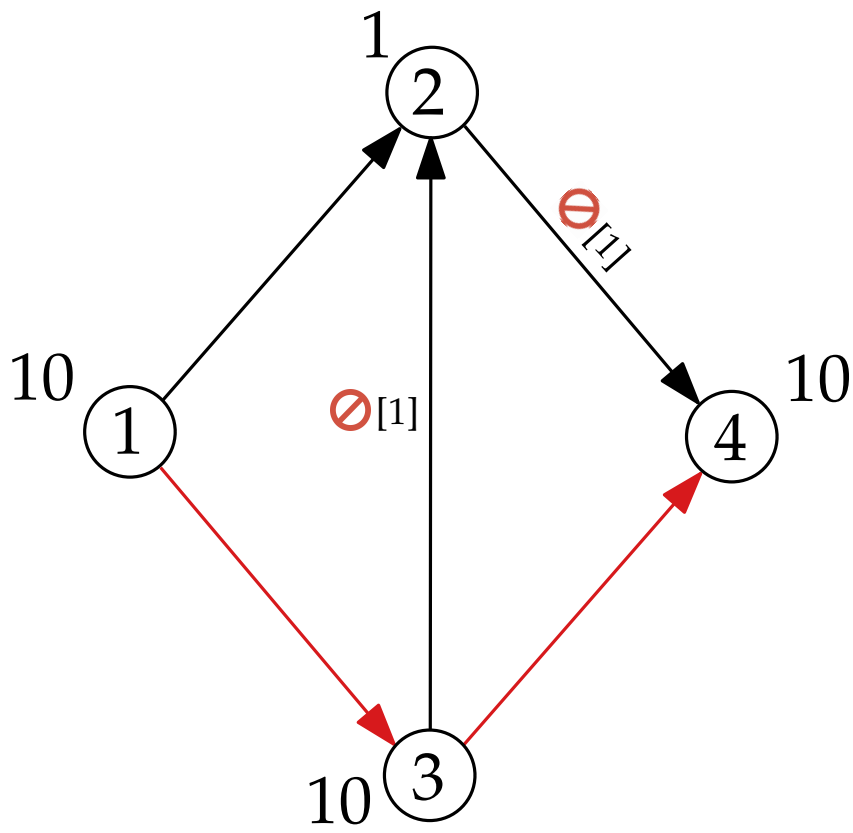

Brute-Force?

Rekursives Durchlaufen aller Pfade unter Berücksichtigung der Regeln
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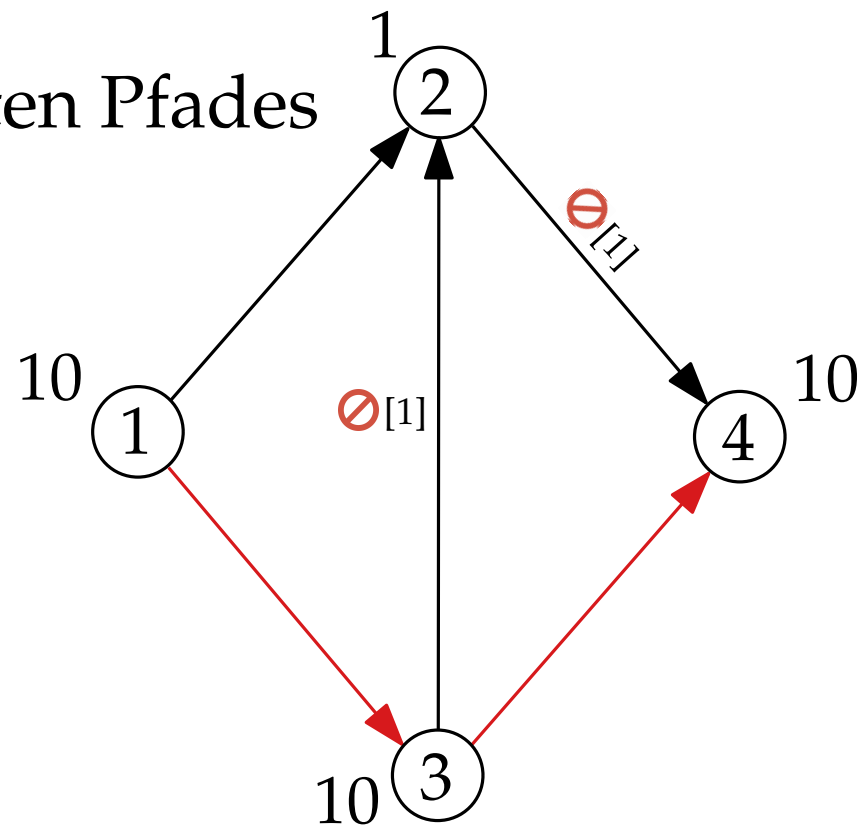
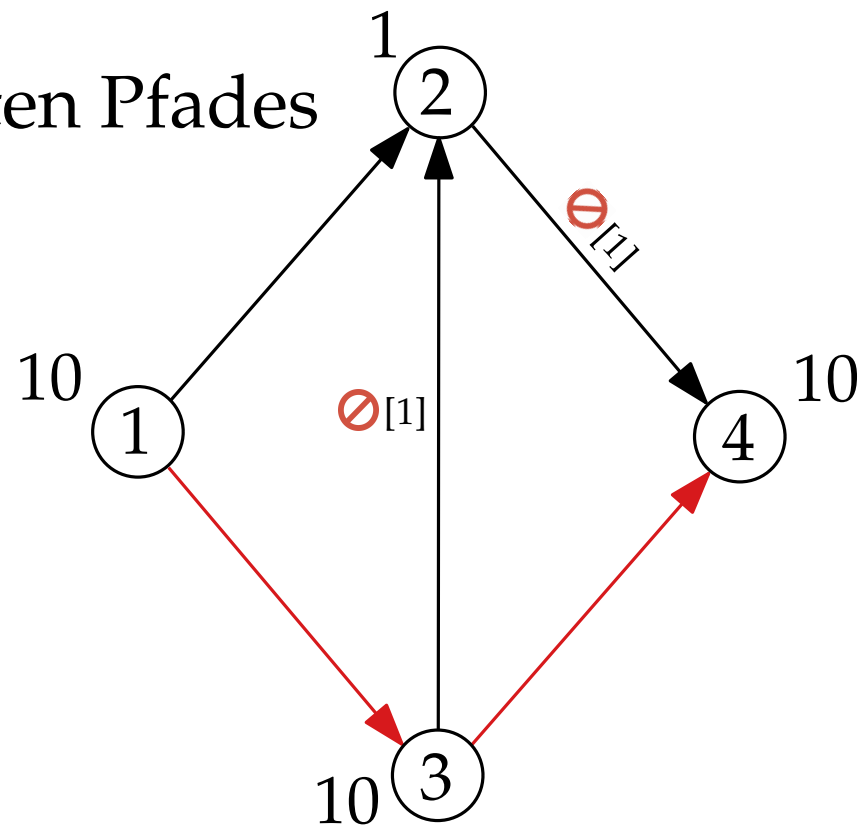


Brute-Force?

Rekursives Durchlaufen aller Pfade unter Berücksichtigung der Regeln

→ Zurückgeben des kürzesten Pfades

→ Laufzeit: $O(n!) = O(n^n)$



Ansatz: Kantengraph

Konstruieren des Kantengraphen G' aus dem gegebenen Graphen G.

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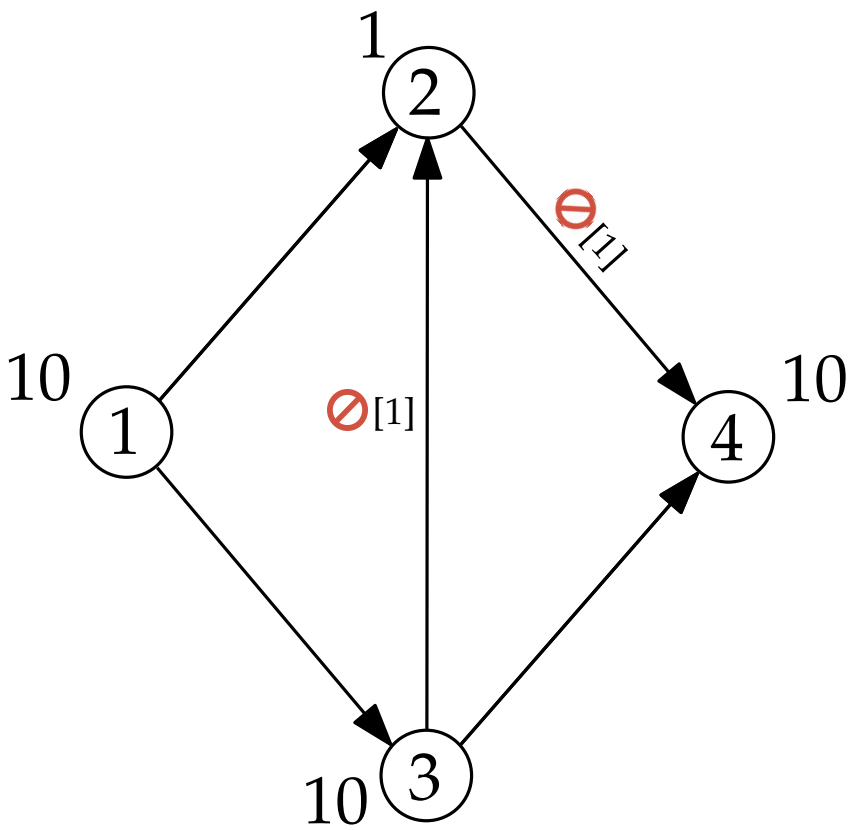

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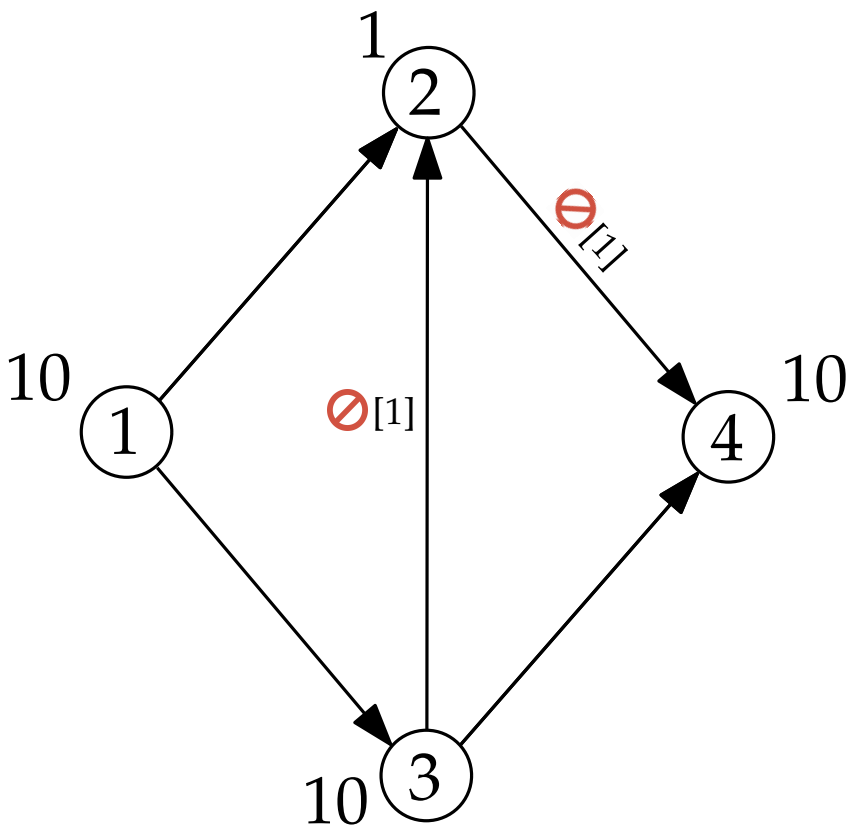

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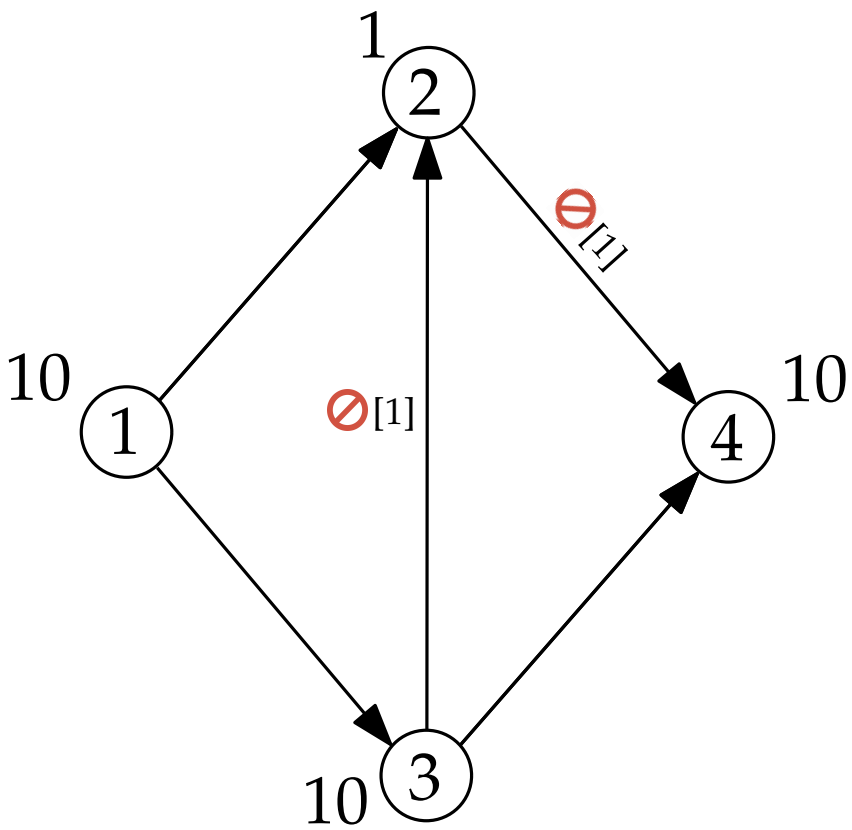

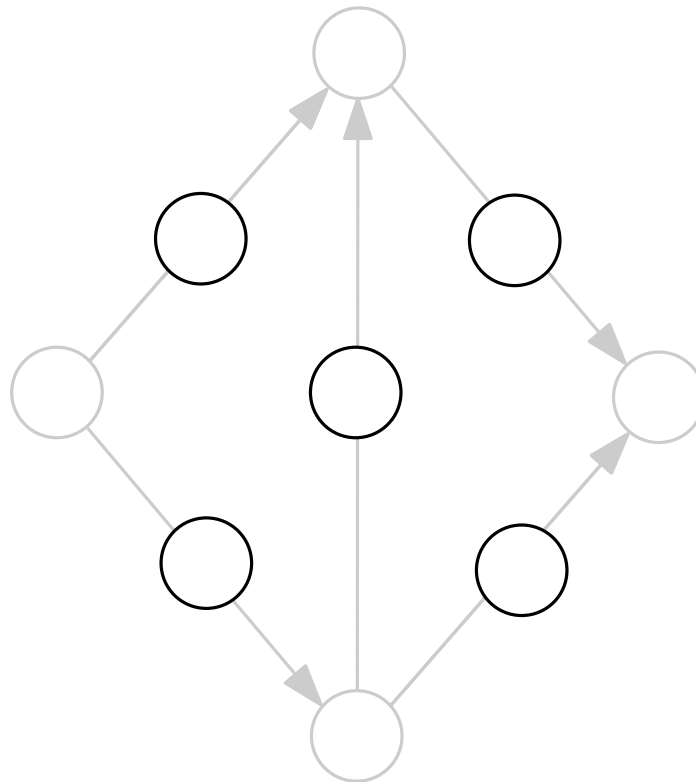

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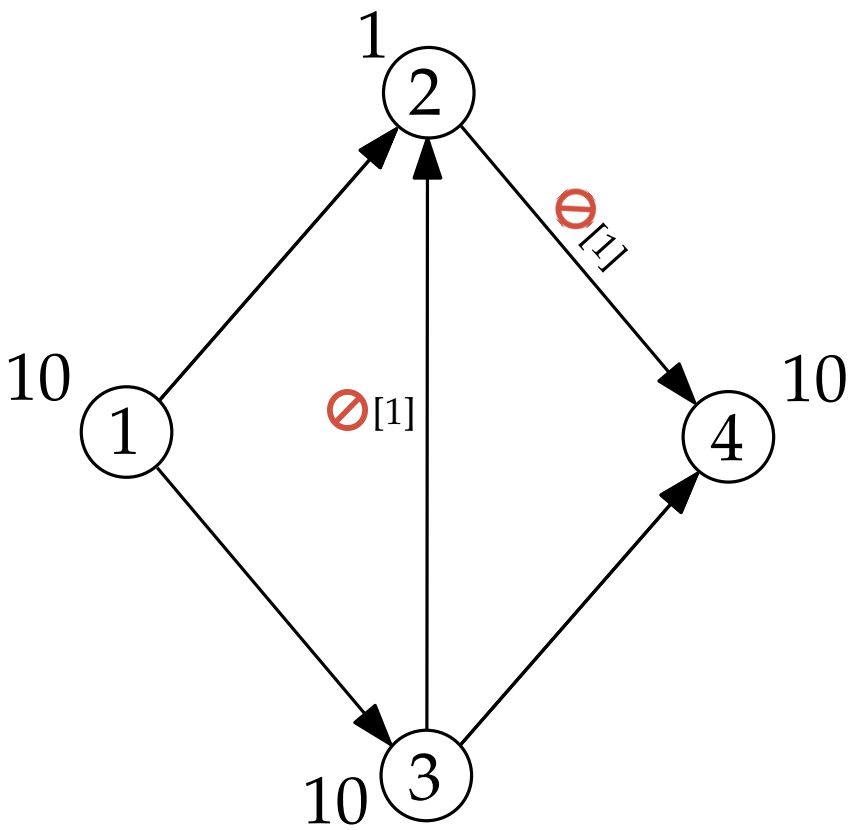

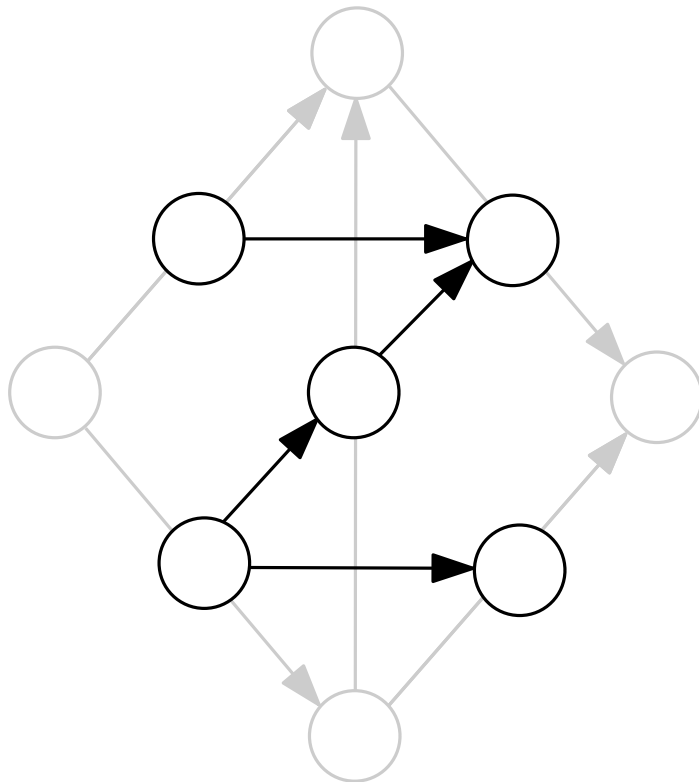

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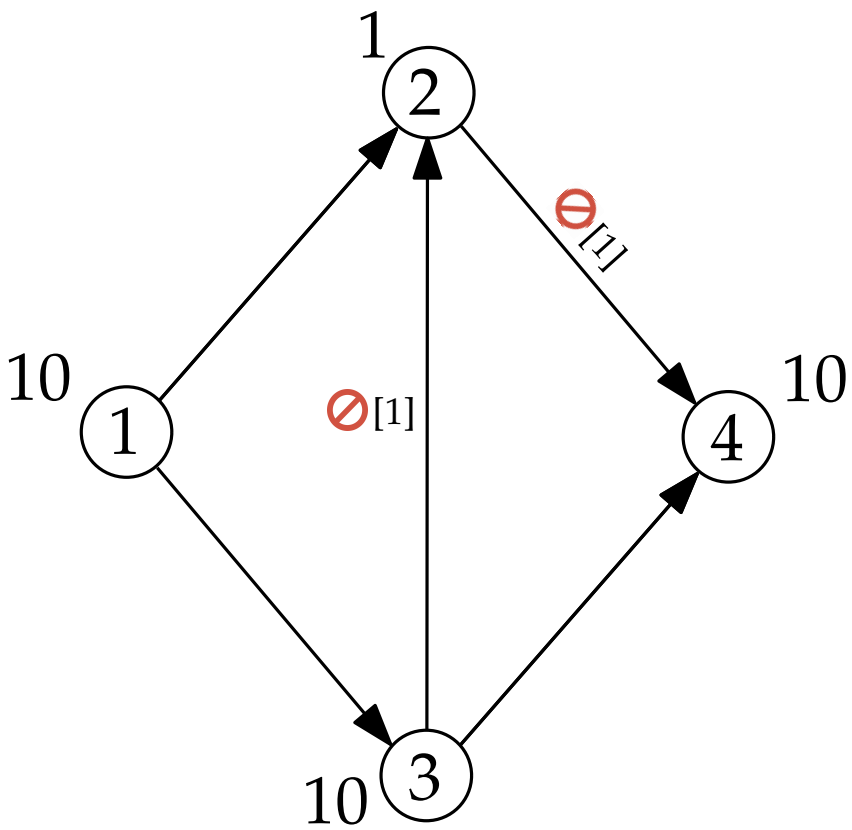

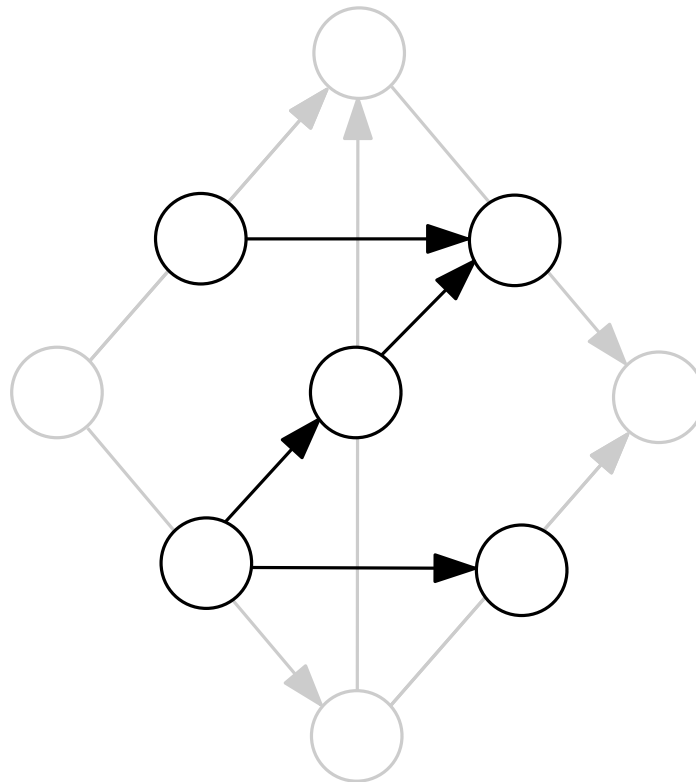

G' muss noch weiter modifiziert werden...

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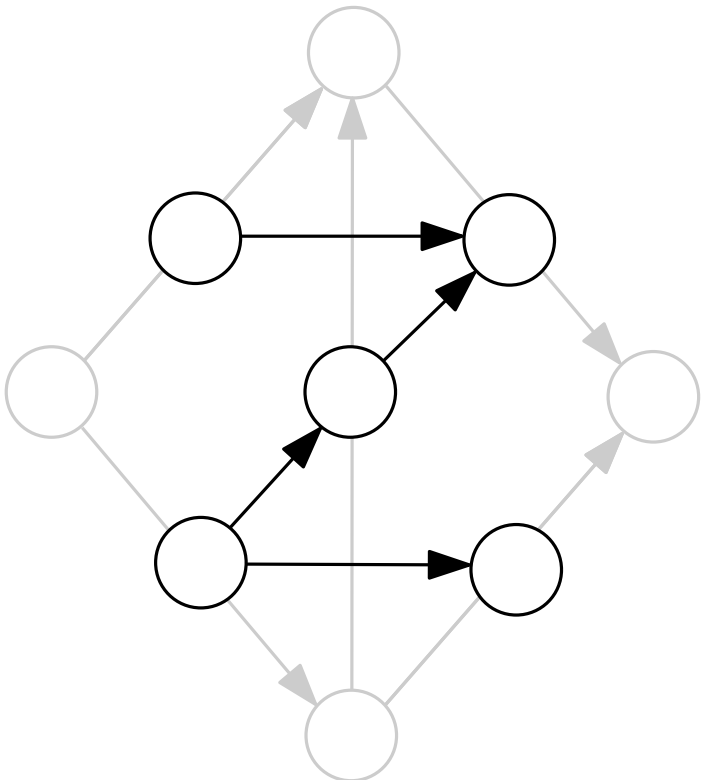
Modifikation 1: Wiederherstellung von Start- und Endknoten



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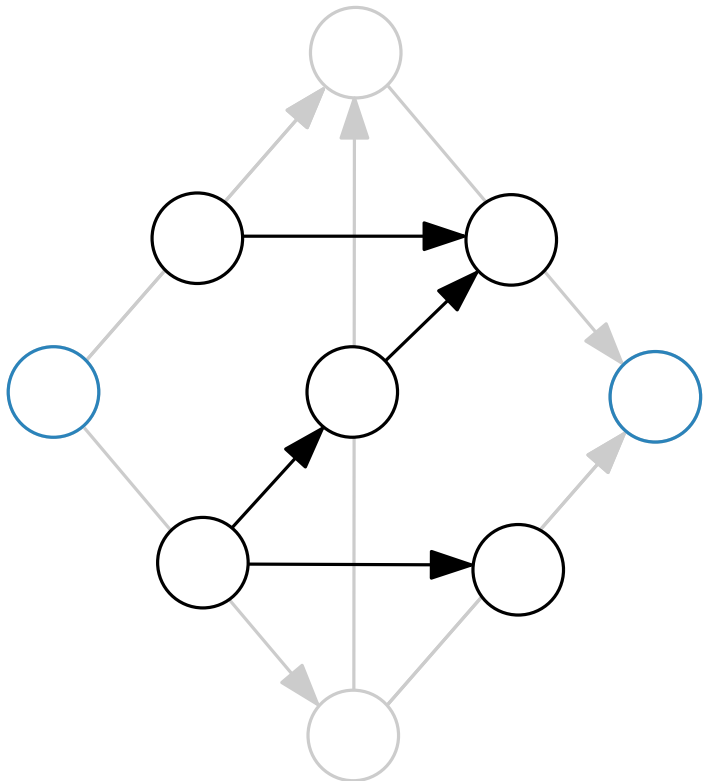

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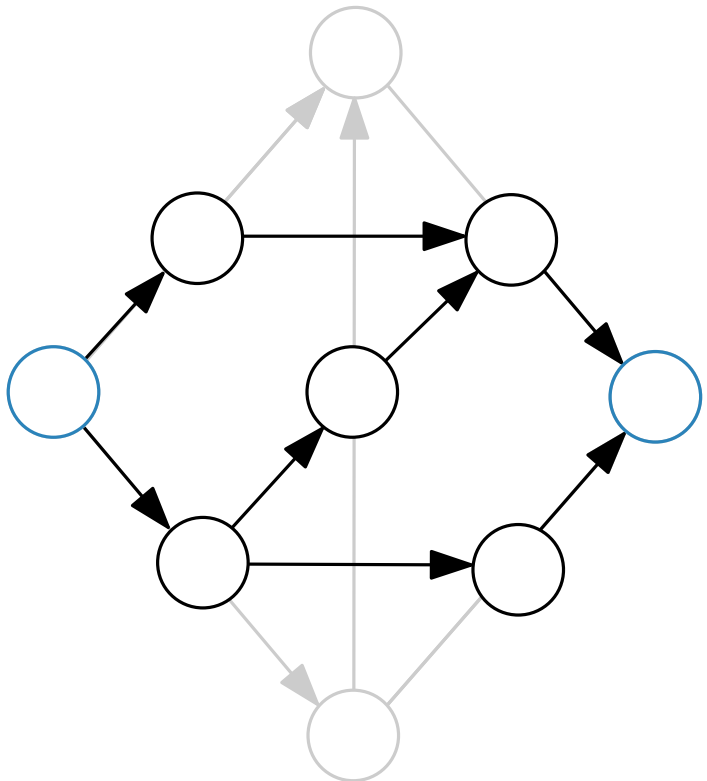

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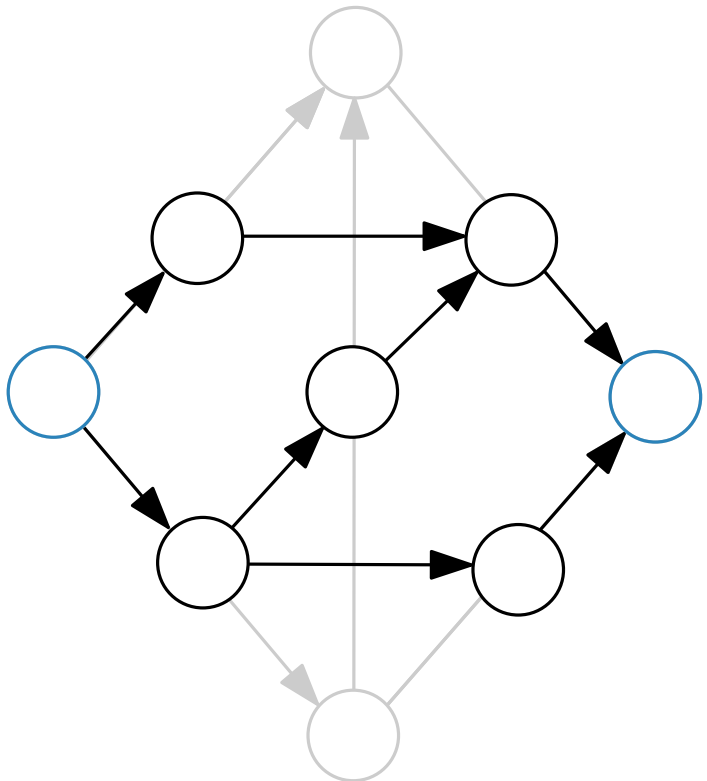
Modifikation 1: Wiederherstellung von Start- und Endknoten

Modifikation 2: Entfernen der durch Regeln verbotenen Kanten



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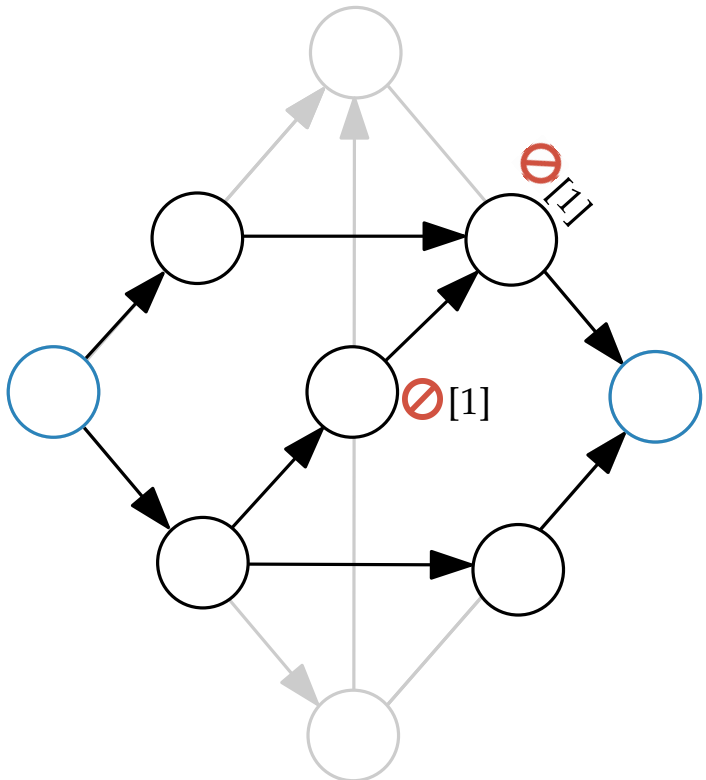

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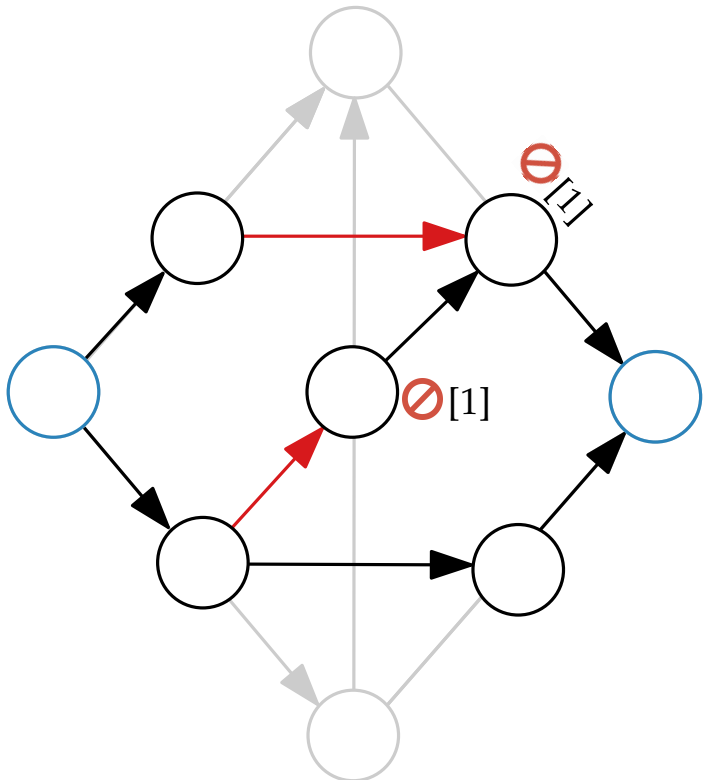

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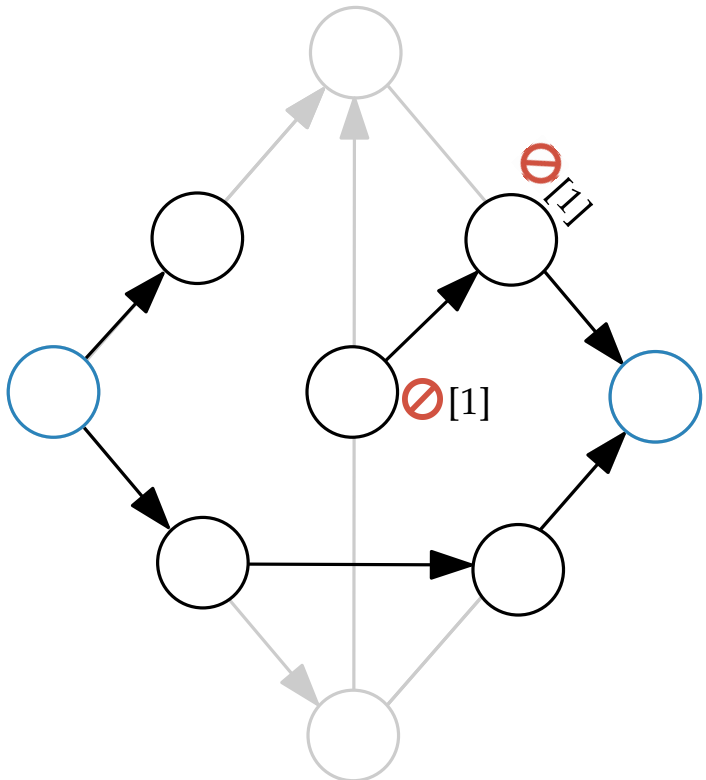

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Modifikation 1: Wiederherstellung von Start- und Endknoten

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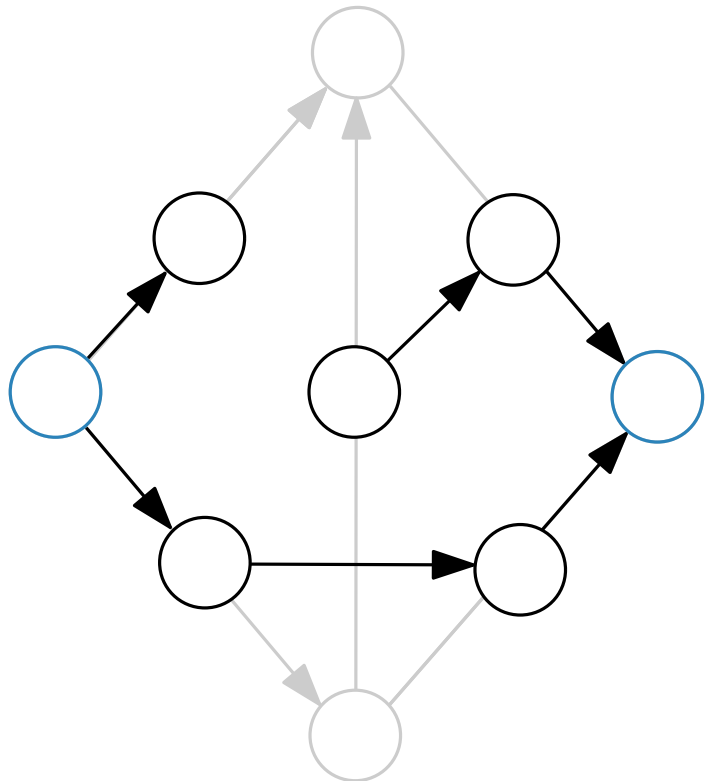
Modifikation 3: Setzen der Kantengewichte entsprechend ihrer ehemaligen Knoten



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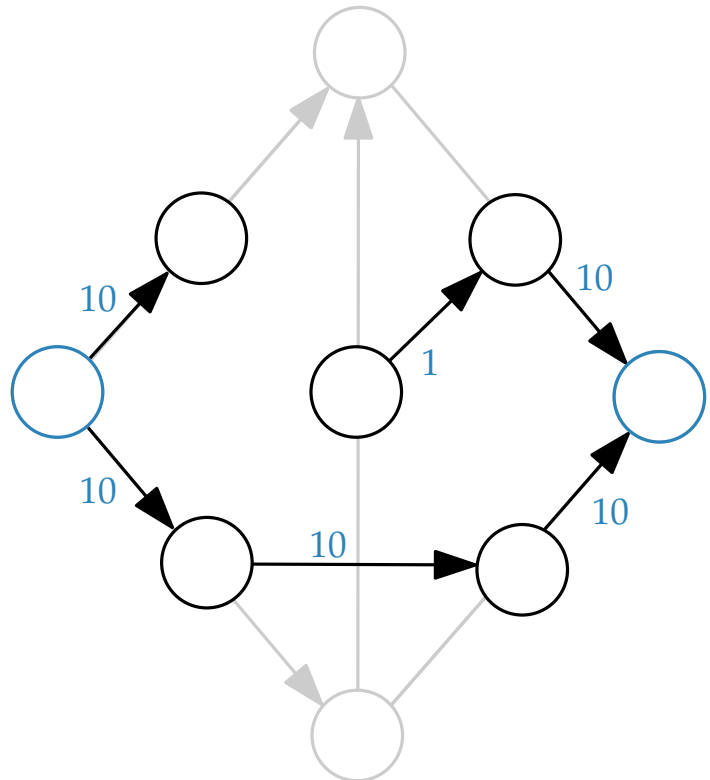

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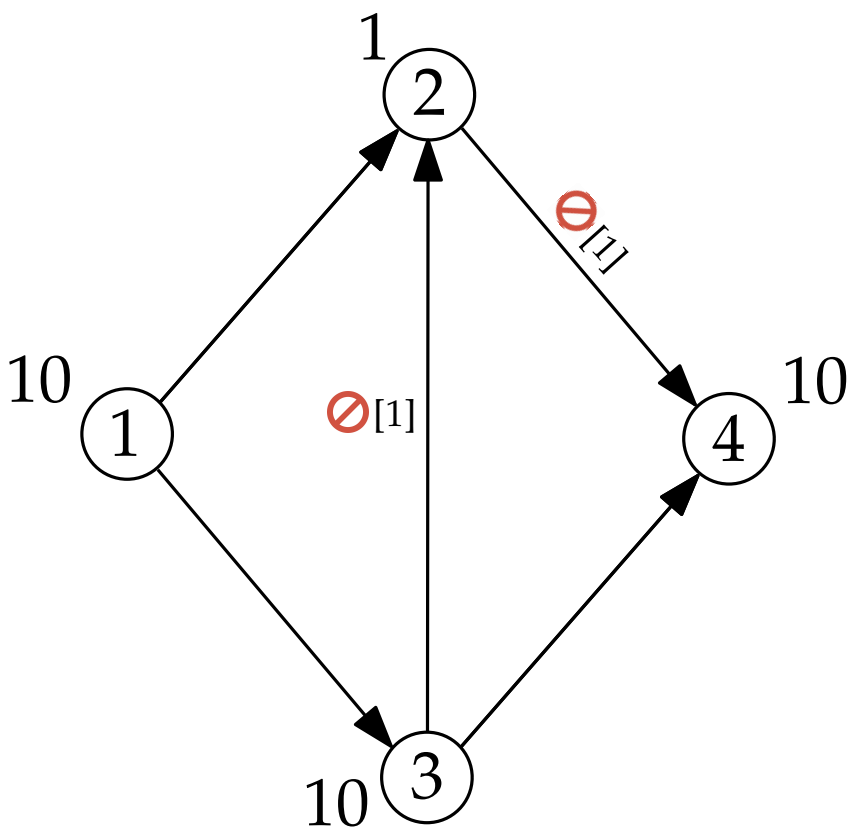

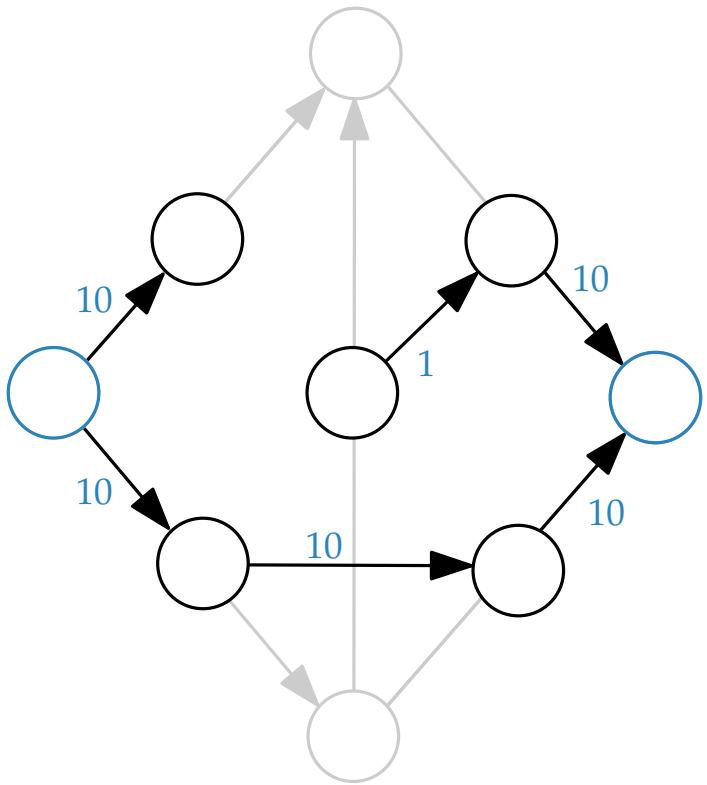

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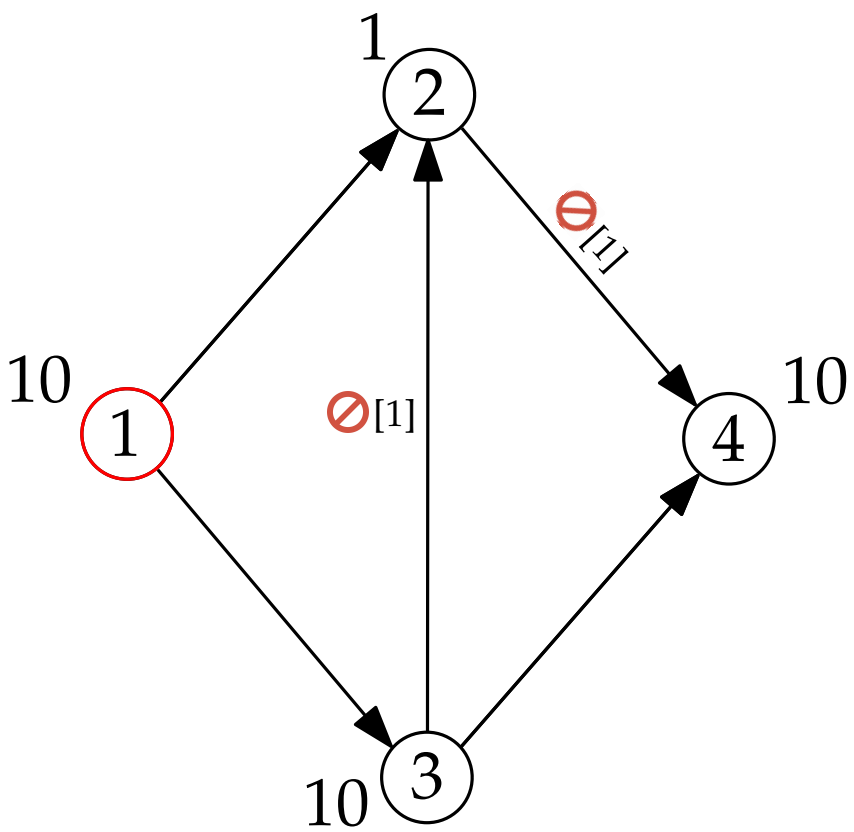

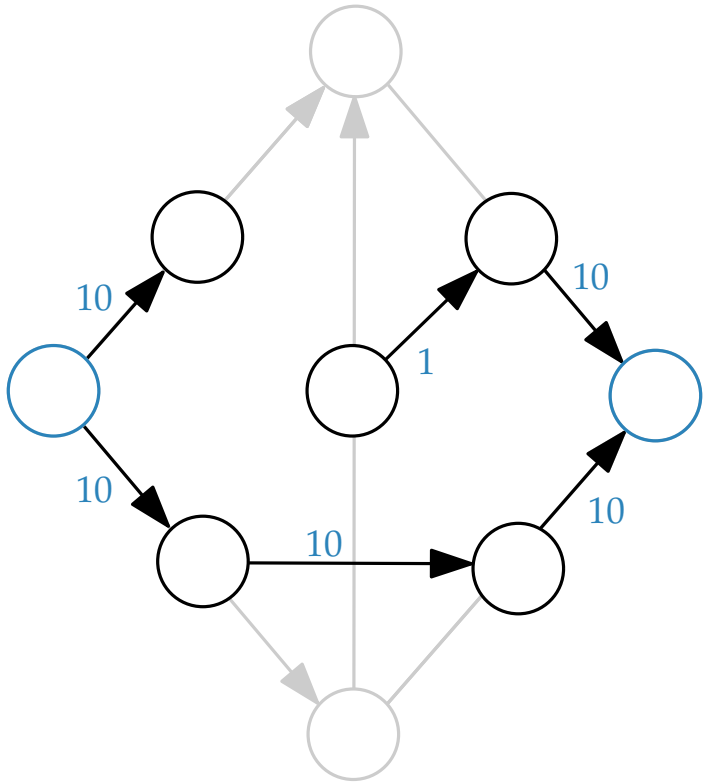

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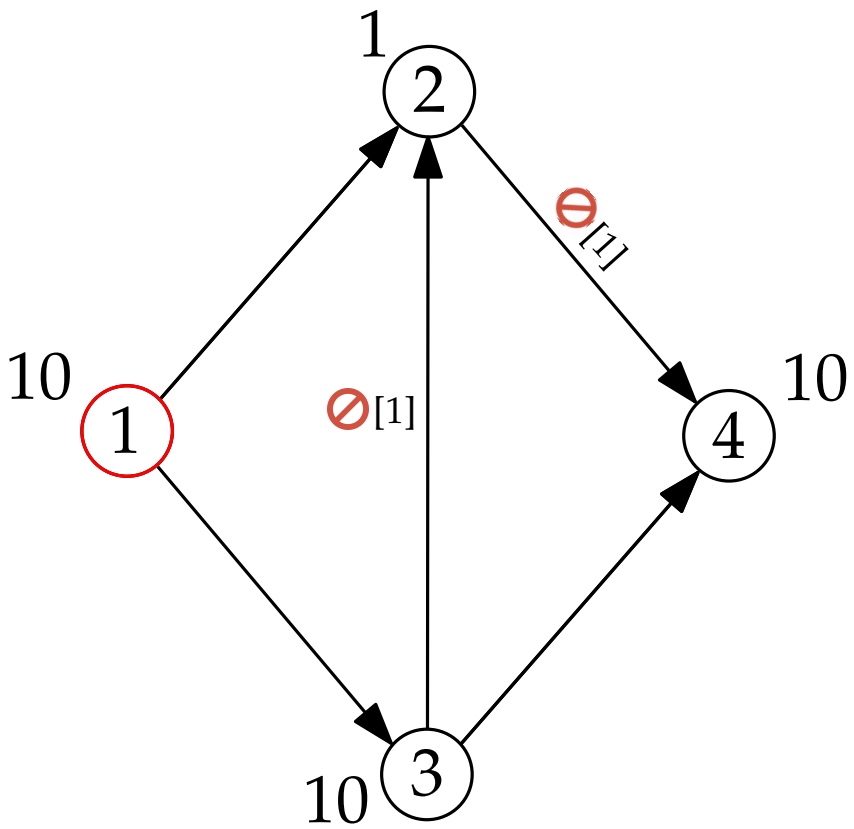

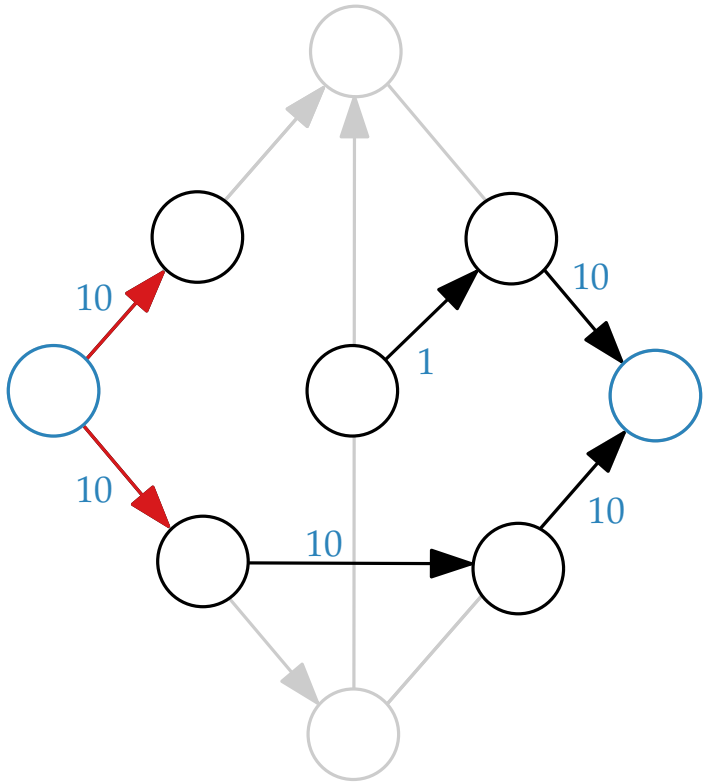

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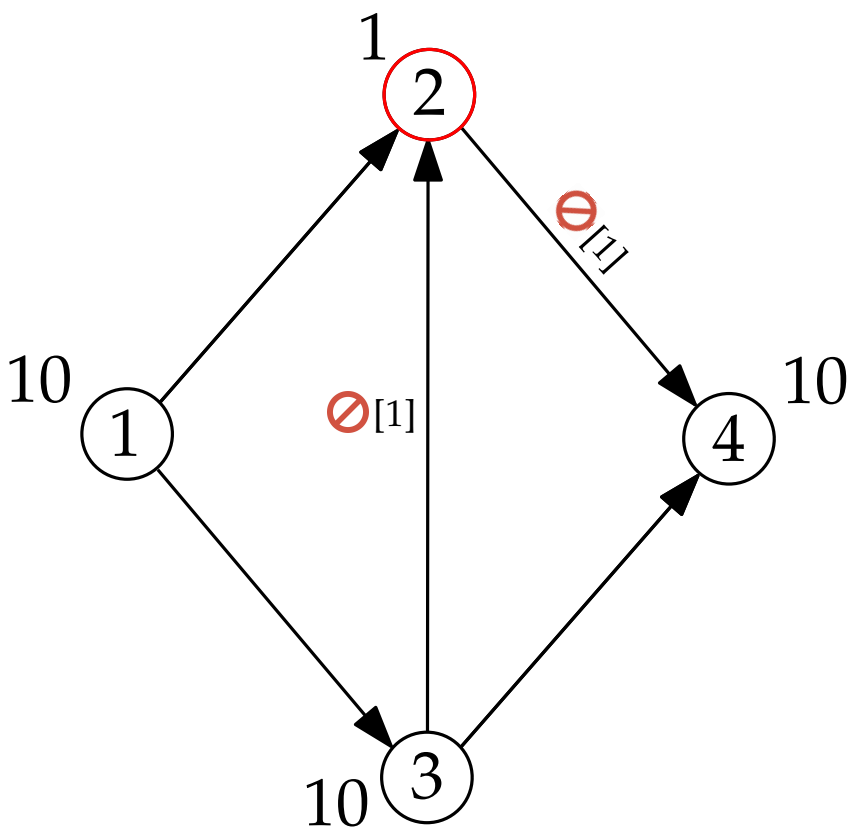

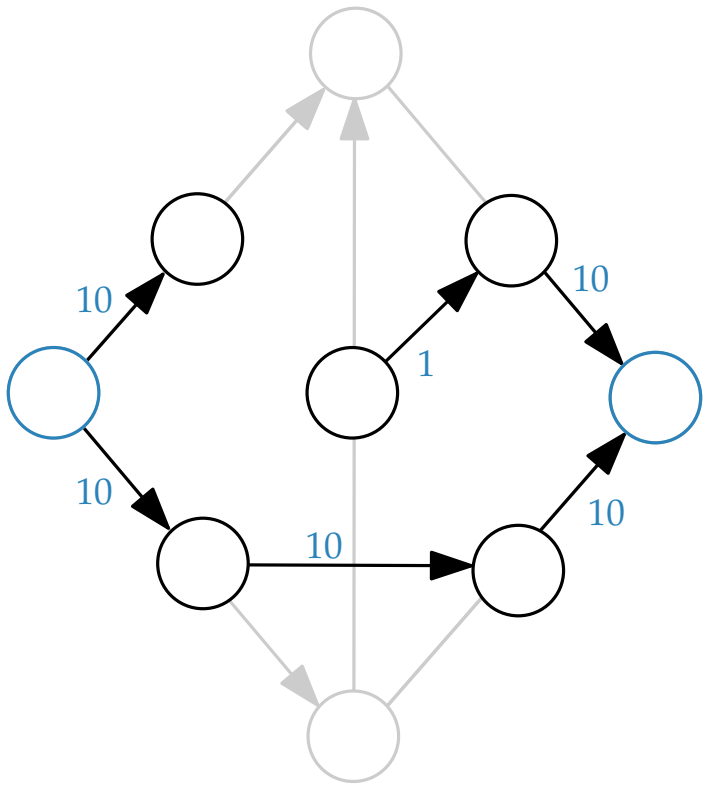

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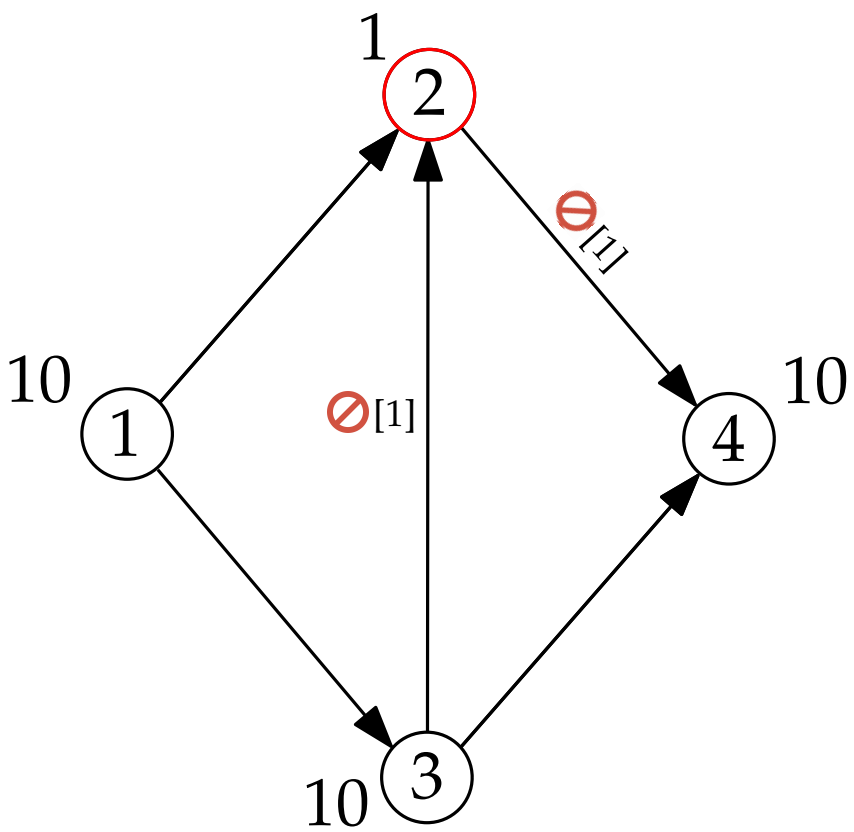

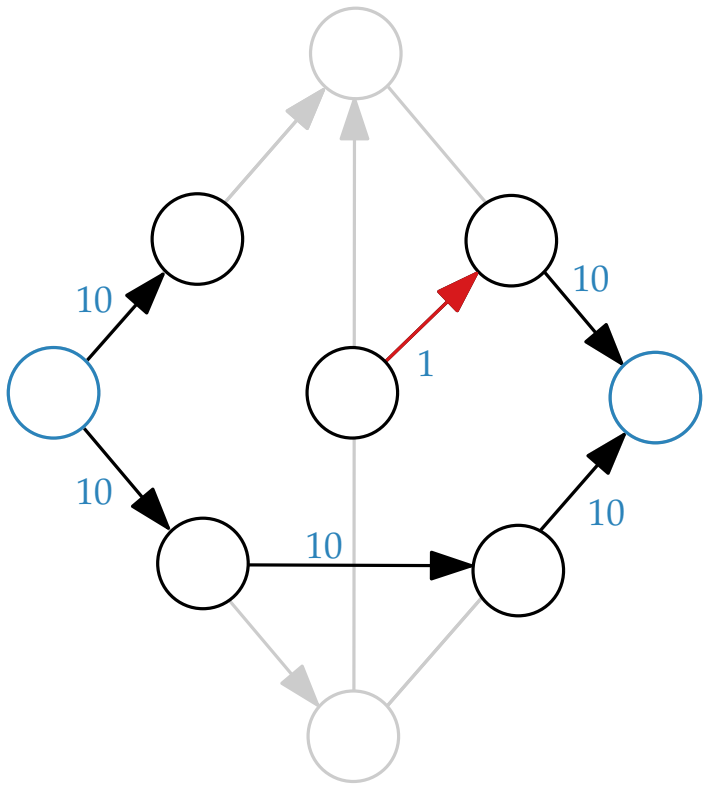

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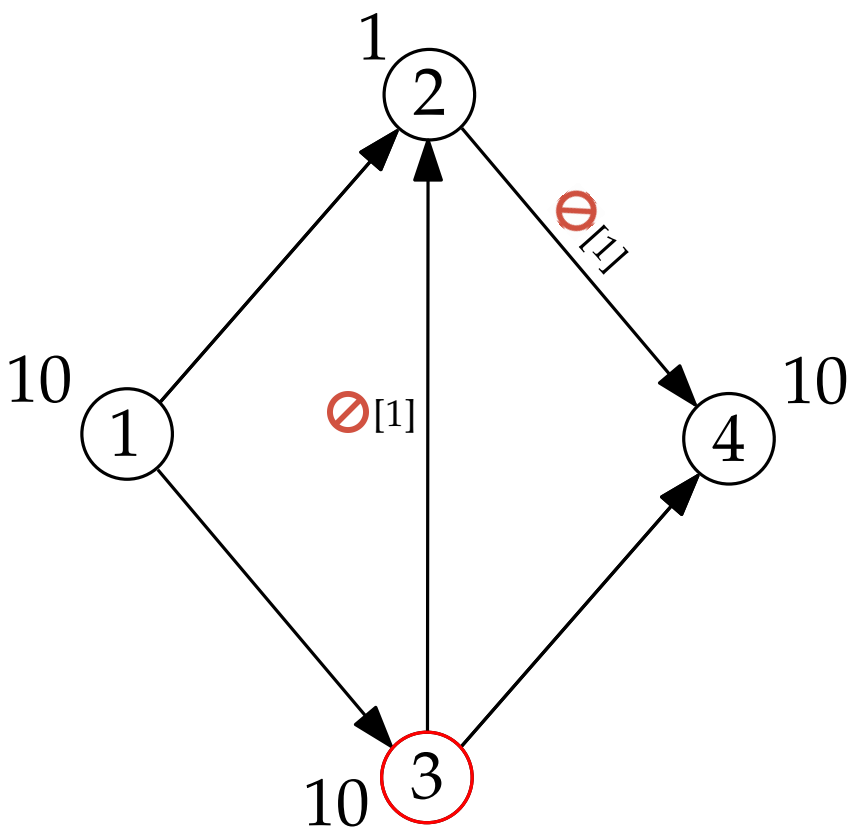

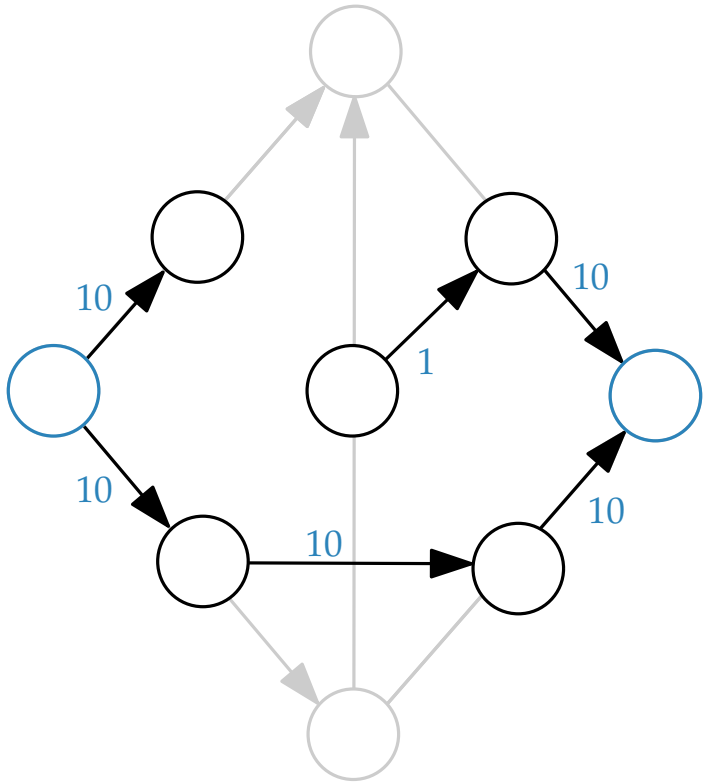

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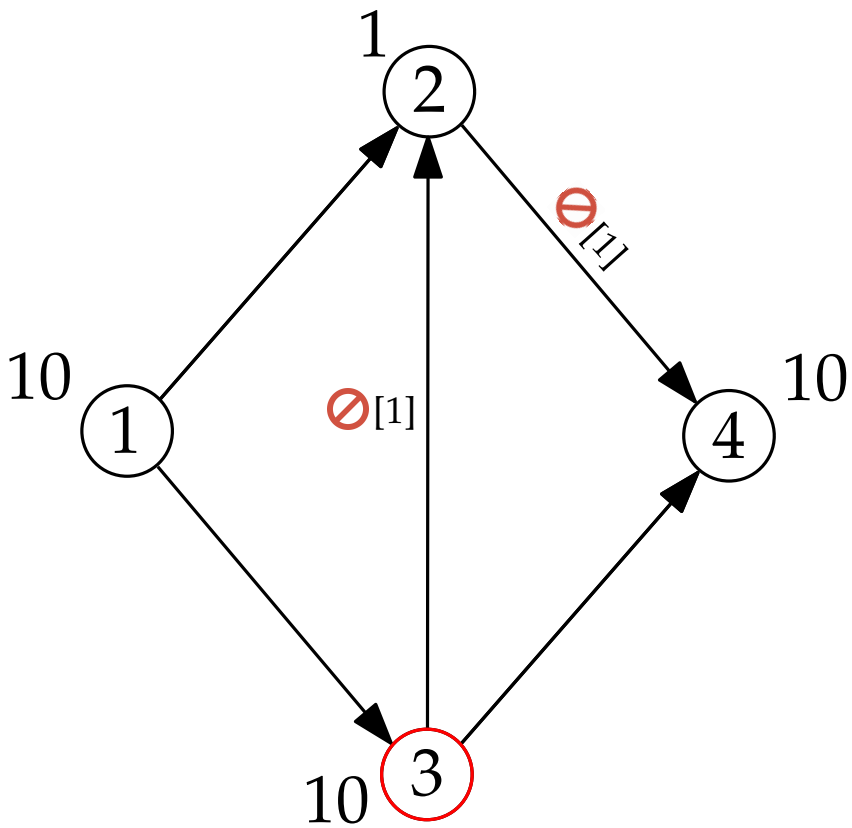

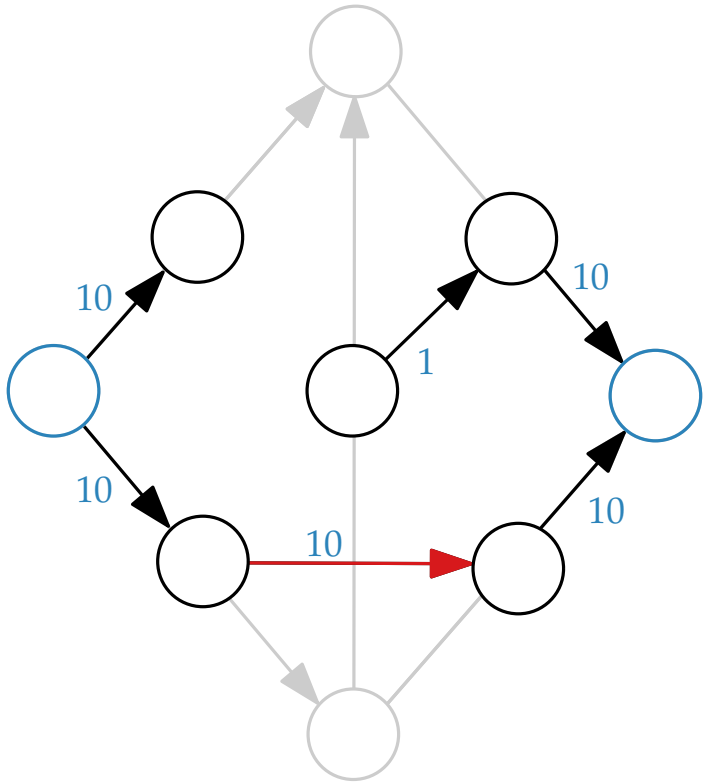

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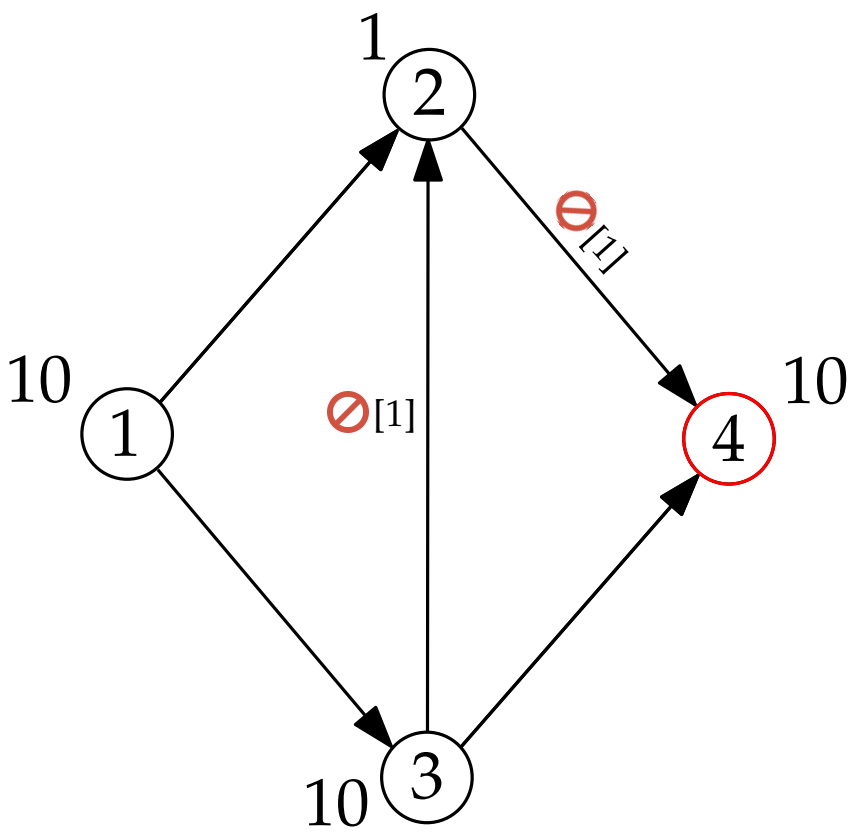

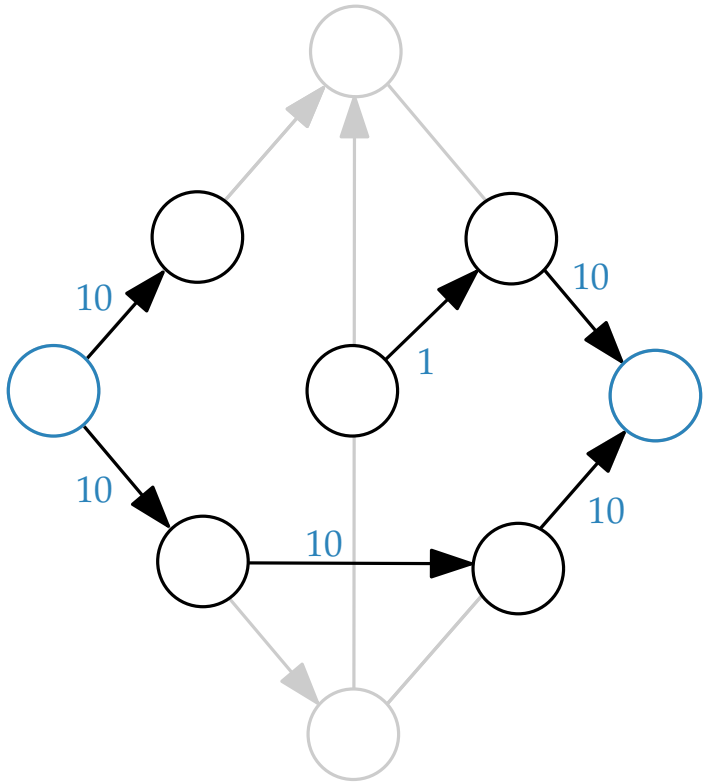

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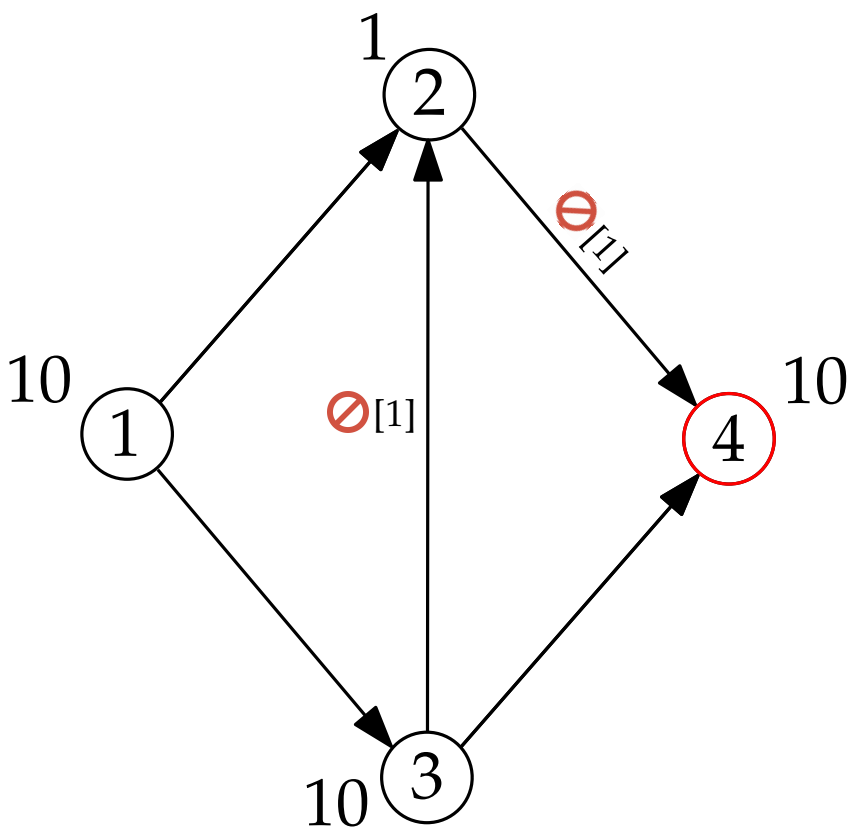

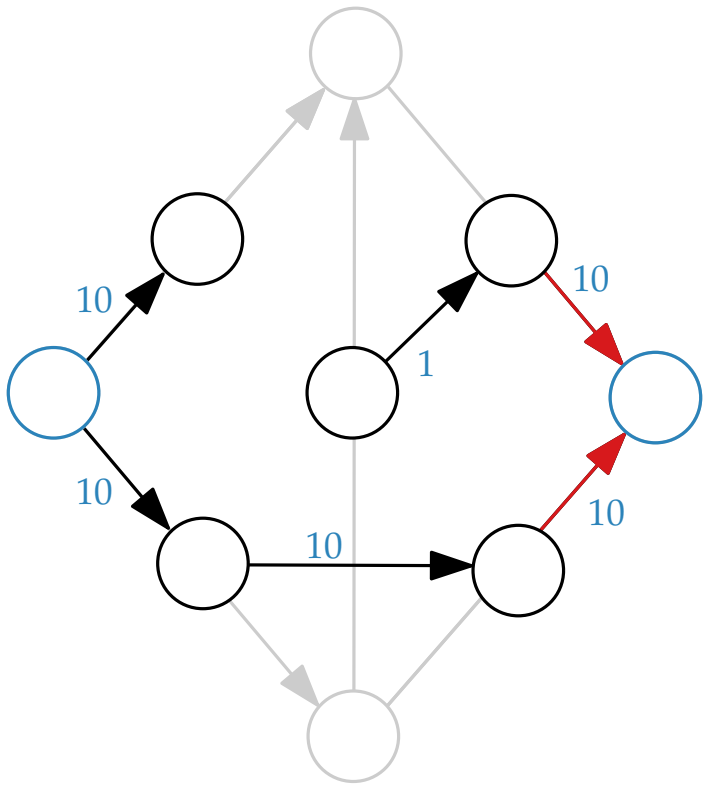

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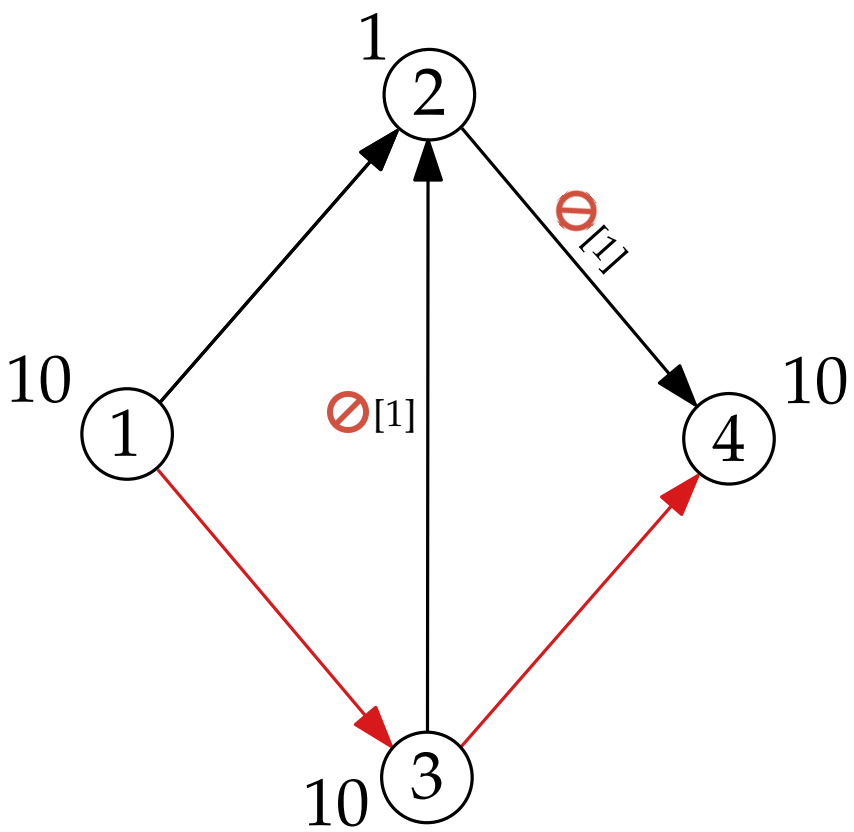
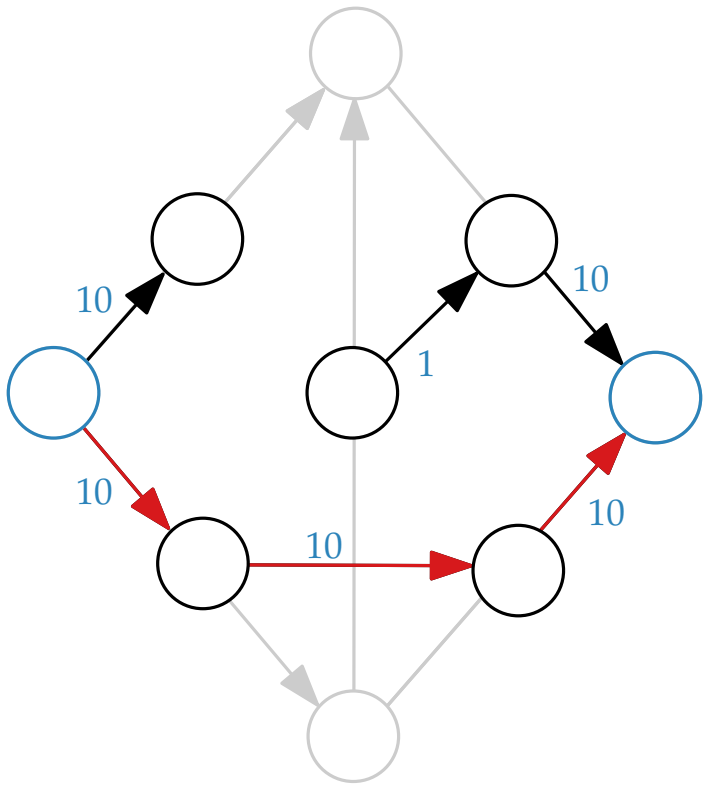
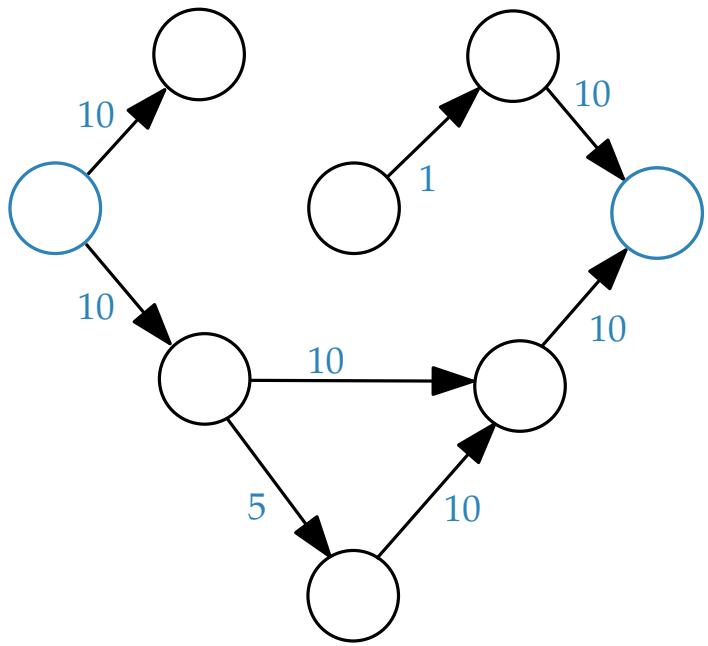




Ansatz: Kürzester Weg (Dijkstra)

→ Anwendung des Dijkstra-Algorithmus zur Bestimmung des kürzesten Weges



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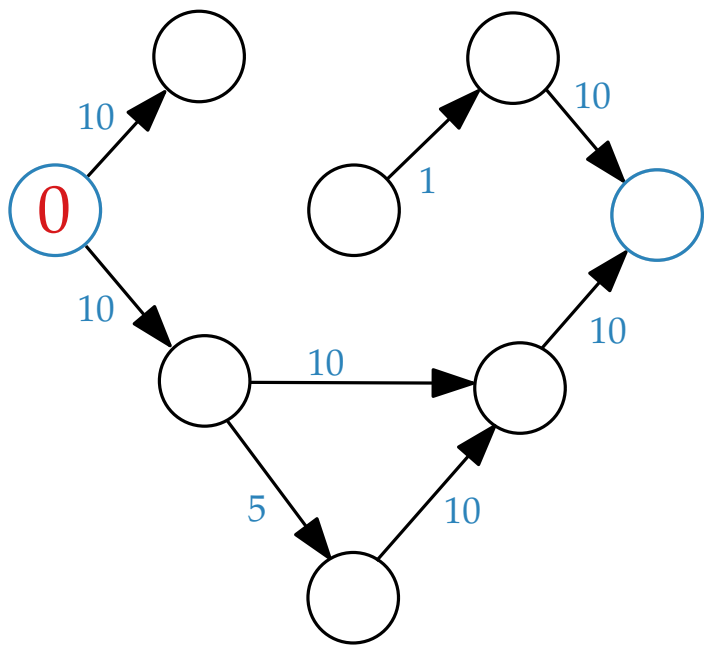

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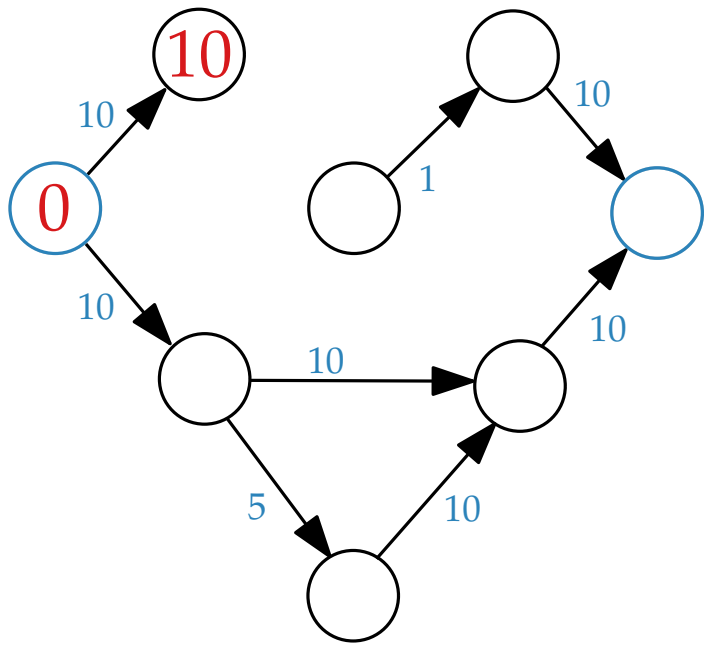

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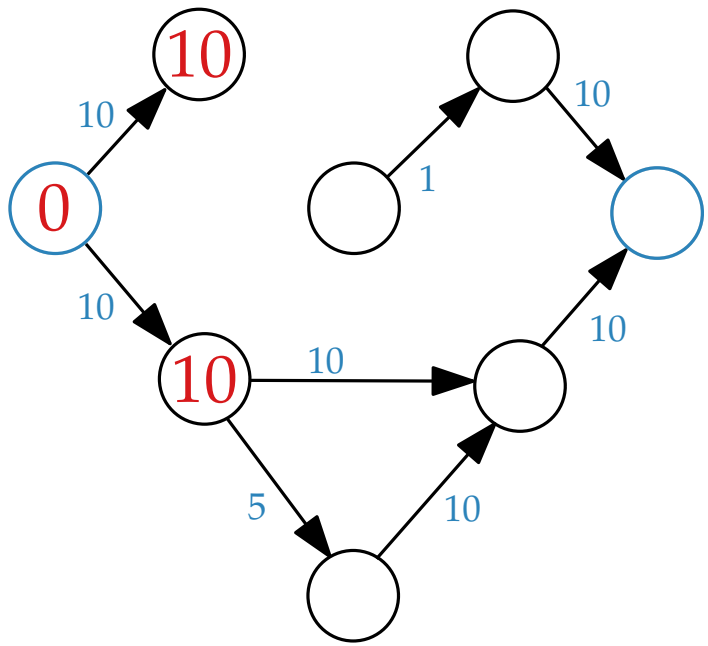

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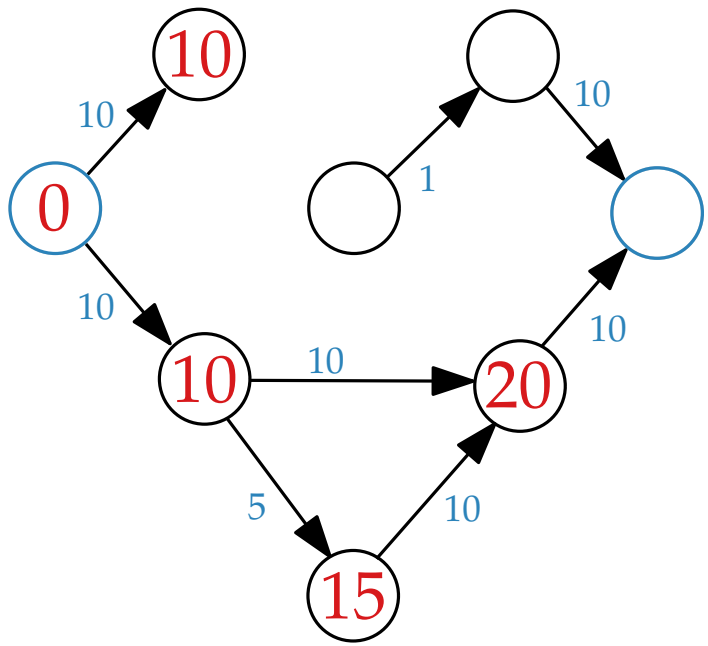

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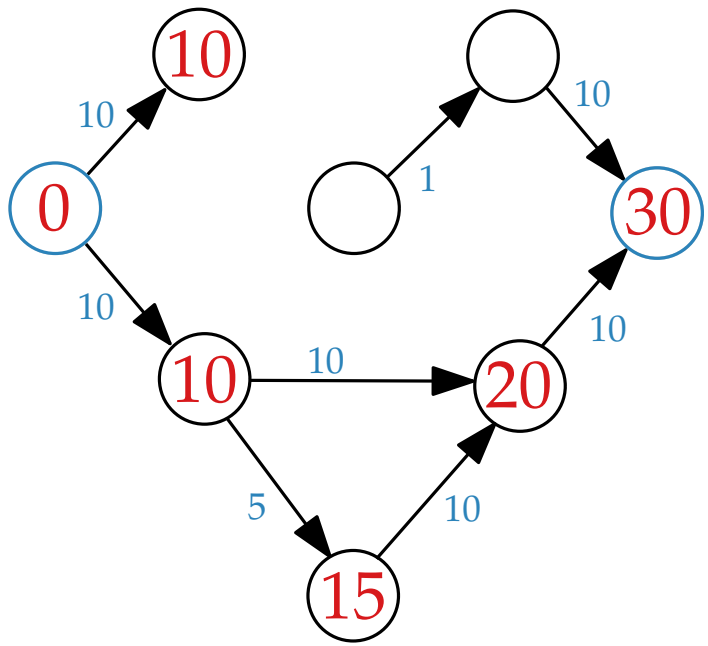

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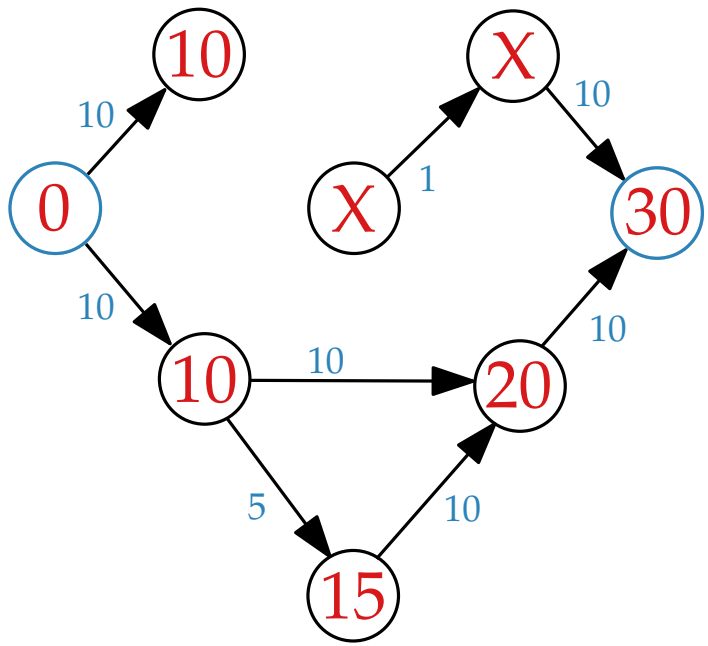

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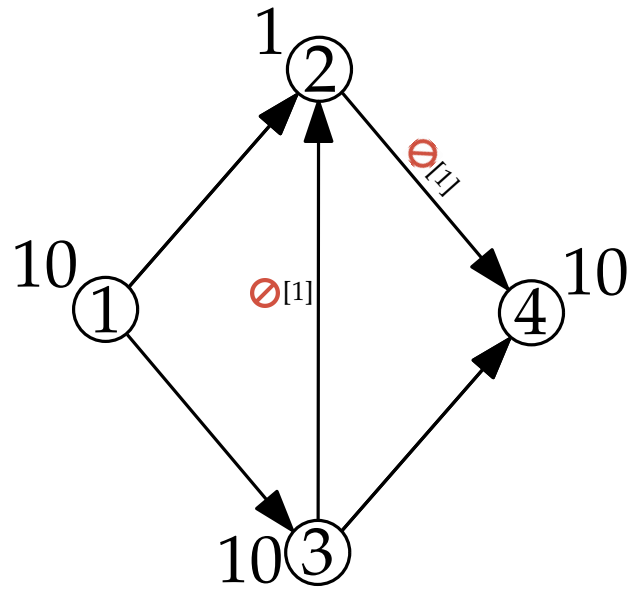
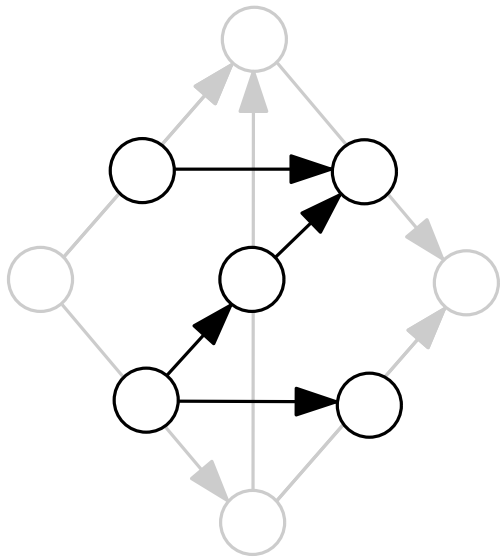
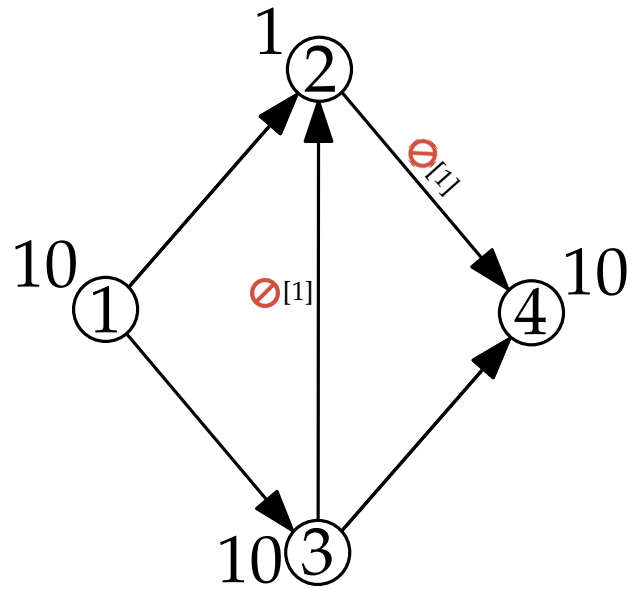
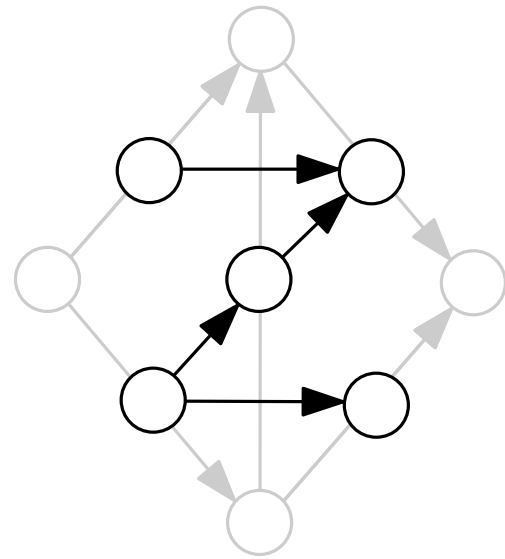




Eingaben: Graph $G = (V, E)$, Startknoten s, Zielknoten t

Ansatz: Algorithmus





Eingaben: Graph $G = (V, E)$, Startknoten s, Zielknoten t

$G' = (V', E')$
for e in E **do**
  $V' = V' \cup \{v_e\}$
for (v,u) in E **do**
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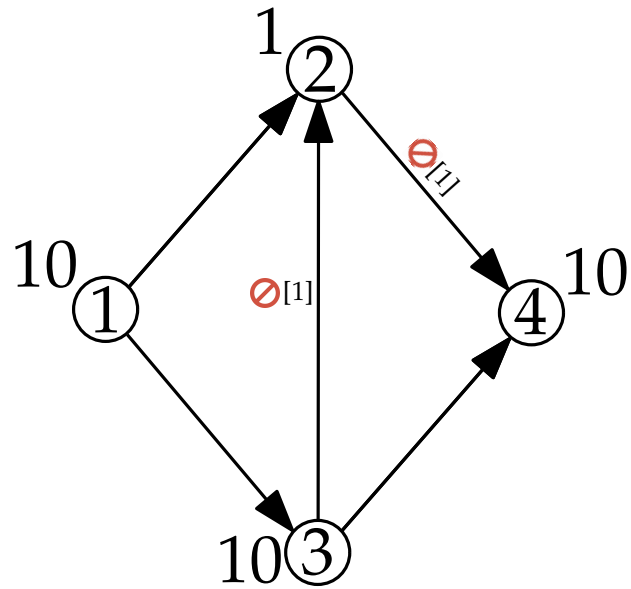
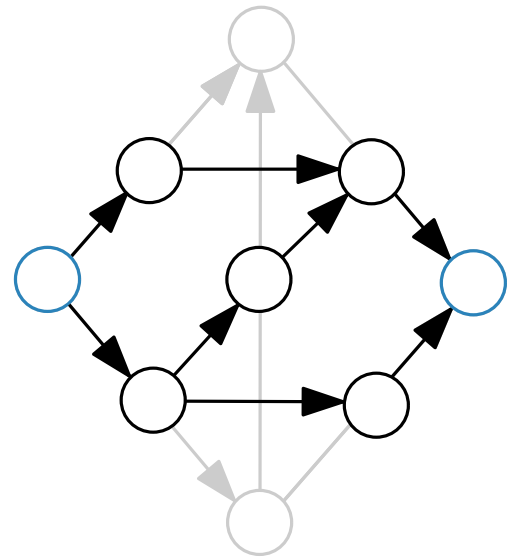
Ansatz: Algorithmus





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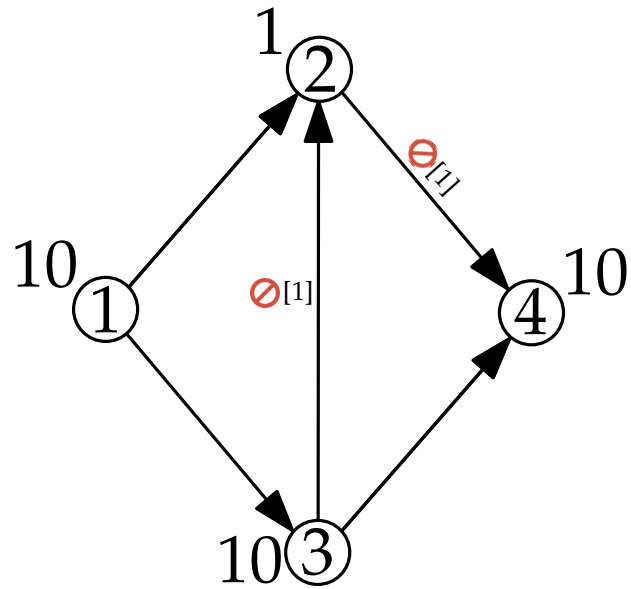

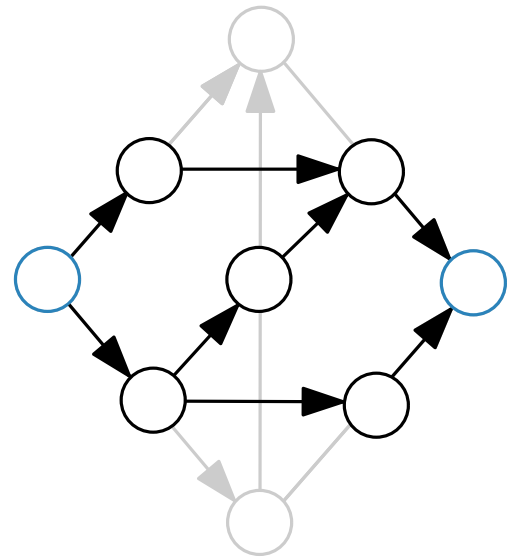

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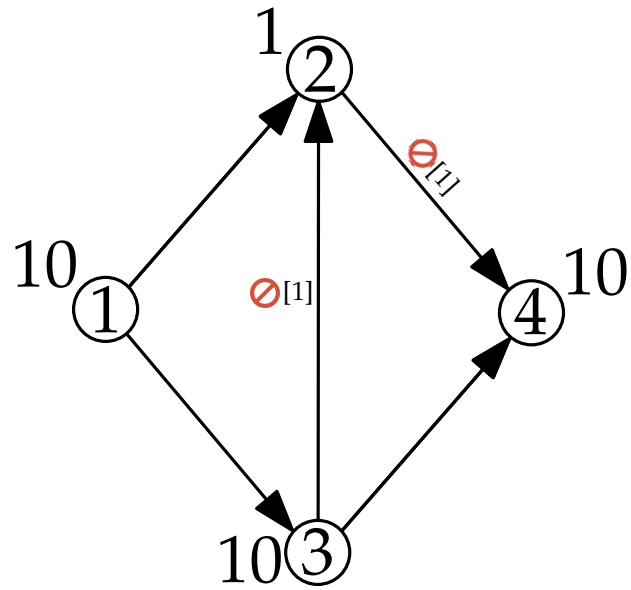
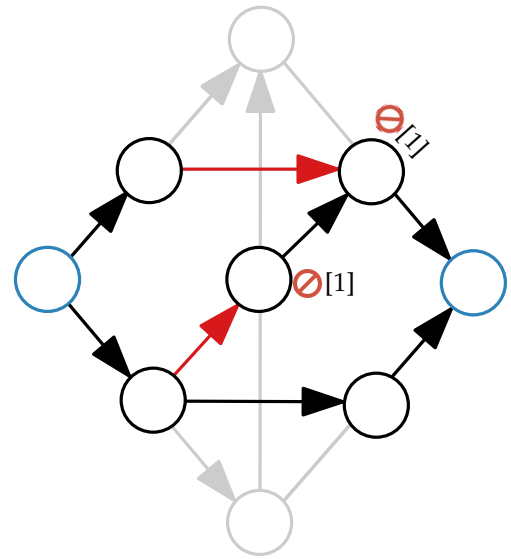
    $E' = E' \cup \{(v_{(v,u)}, v_{(u,x)})\}$

$V' = V' \cup \{s', t'\}$
for v in Adj[s] **do**
  $E' = E' \cup \{(s', v_{(s,v)})\}$
for (v, t) in E **do**
  $E' = E' \cup \{(v_{(v,t)}, t')\}$

Ansatz: Algorithmus





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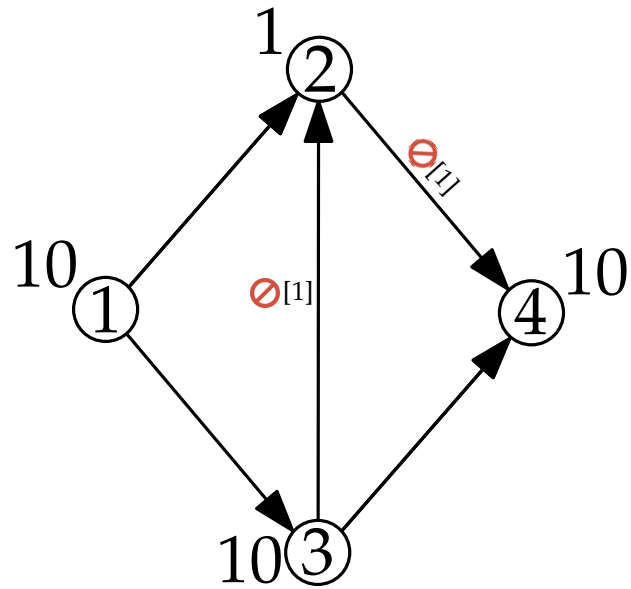
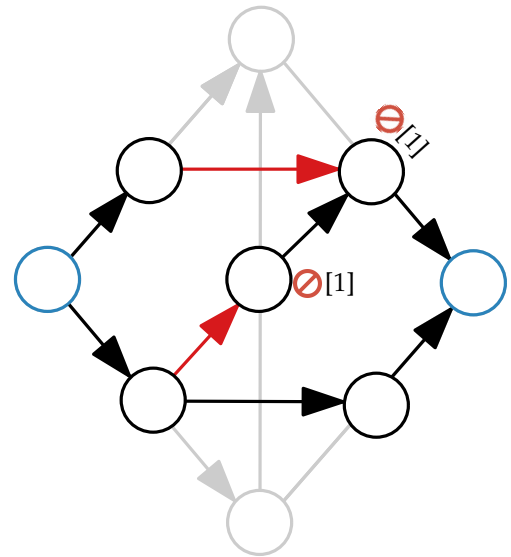
  $E' = E' \cup \{(s', v_{(s,v)})\}$
for (v, t) in E **do**
  $E' = E' \cup \{(v_{(v,t)}, t')\}$

Ansatz: Algorithmus





Eingaben: Graph $G = (V, E)$, Startknoten s, Zielknoten t

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for (v,u) in E **do**
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      $E' = E' \cup \{(v_{(v,u)}, v_{(u,x)})\}$

$V' = V' \cup \{s', t'\}$
for v in Adj[s] **do**
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for (v, t) in E **do**
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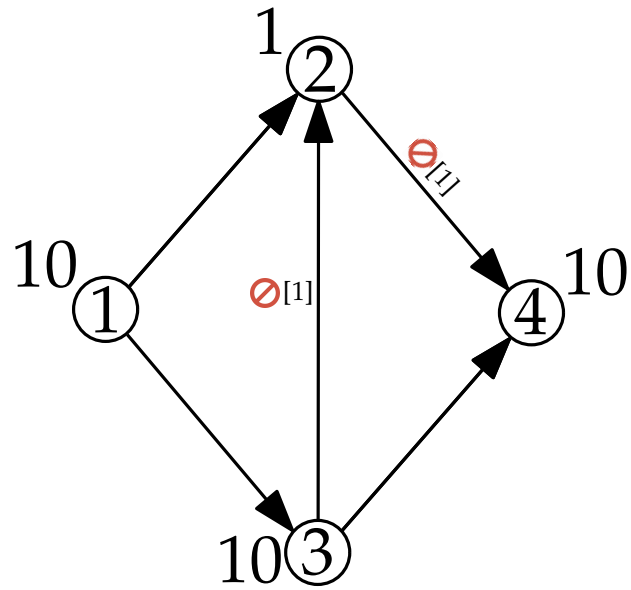
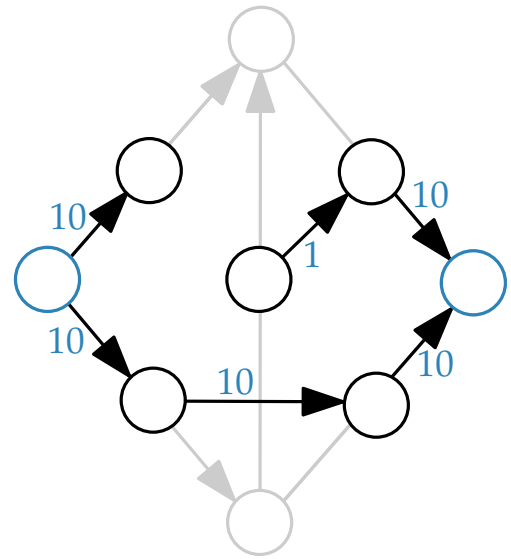
Ansatz: Algorithmus





Eingaben: Graph $G = (V, E)$, Startknoten s, Zielknoten t

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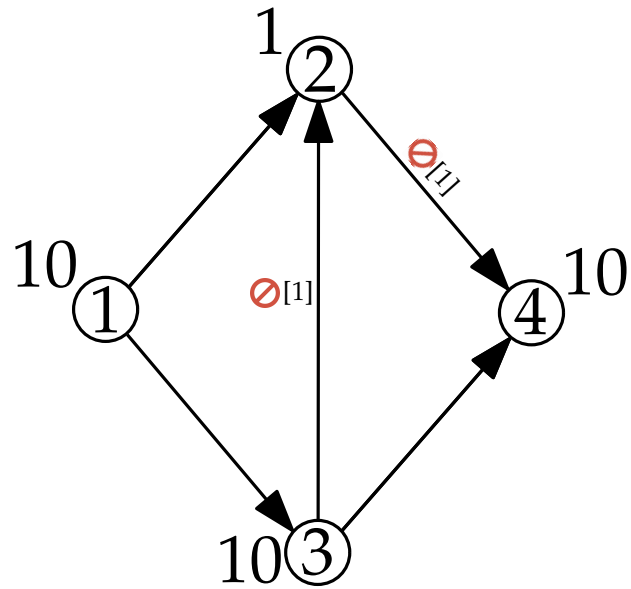
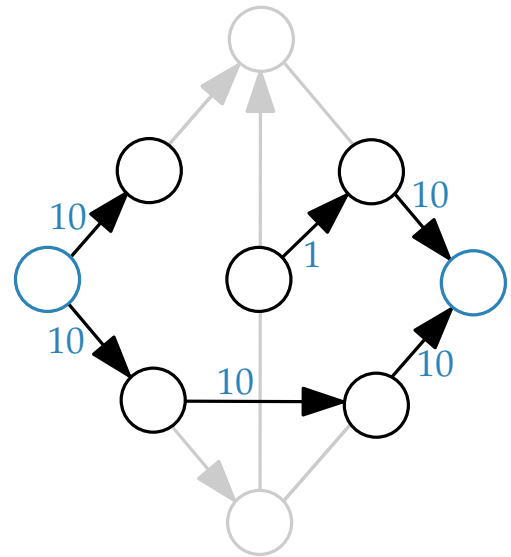
Ansatz: Algorithmus





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      $E' = E' \cup \{(v_{(v,u)}, v_{(u,x)})\}$
      $(v_{(v,u)}, v_{(u,x)}).weight := u.p$

$V' = V' \cup \{s', t'\}$
for v in Adj[s] **do**
  $E' = E' \cup \{(s', v_{(s,v)})\}$
  $(s', v_{(s,v)}).weight := s.p$
for (v, t) in E **do**
  $E' = E' \cup \{(v_{(v,t)}, t')\}$
  $(v_{(v,t)}, t').weight := t.p$

Ansatz: Laufzeit

Eingabelänge: $O(|E| \cdot |V|)$

Ansatz: Laufzeit

Eingabelänge: $O(|E| \cdot |V|)$

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      $(v_{(v,u)}, v_{(u,x)}).weight := u.processingTime$
$V' = V' \cup \{s', t'\}$
for v in Adj[s] **do**
  $E' = E' \cup \{(s', v_{s,v})\}$
for (v, t) in E **do**
  $E' = E' \cup \{(v_{(v,t)}, t')\}$

Ansatz: Laufzeit

Eingabelänge: $O(|E| \cdot |V|)$

$G' = (V', E')$
for e in E **do**
  $V' = V' \cup \{v_e\}$ $O(|E|)$
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  for x in Adj[u] **do**
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Ansatz: Laufzeit

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Ansatz: Laufzeit

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Dijkstra: $O(|E|^2)$ (eigentlich: $O(|V'|^2)$)

Ansatz: Laufzeit

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for e in E **do**
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for (v,u) in E **do** $O(|E| \cdot |V|)$
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Dijkstra: $O(|E|^2)$ (eigentlich: $O(|V'|^2)$)

Gesamtlaufzeit: $O(|E|^2) \cup O(|E| \cdot |V|)$