



# Location-dependent generalization of road networks based on equivalent destinations

T. C. van Dijk<sup>1</sup>, J.-H. Haunert<sup>2</sup>, J. Oehrlein<sup>2</sup>

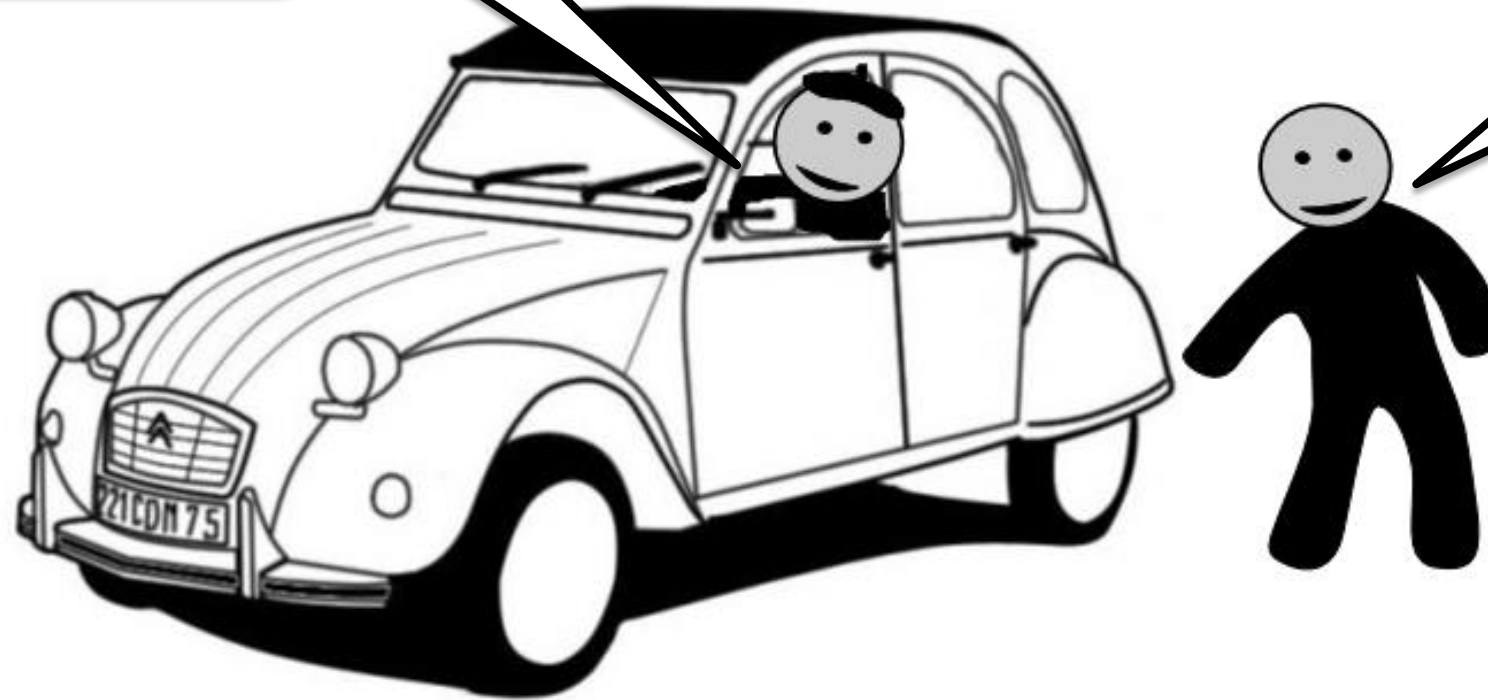
<sup>1</sup>University of Würzburg

<sup>2</sup>University of Osnabrück

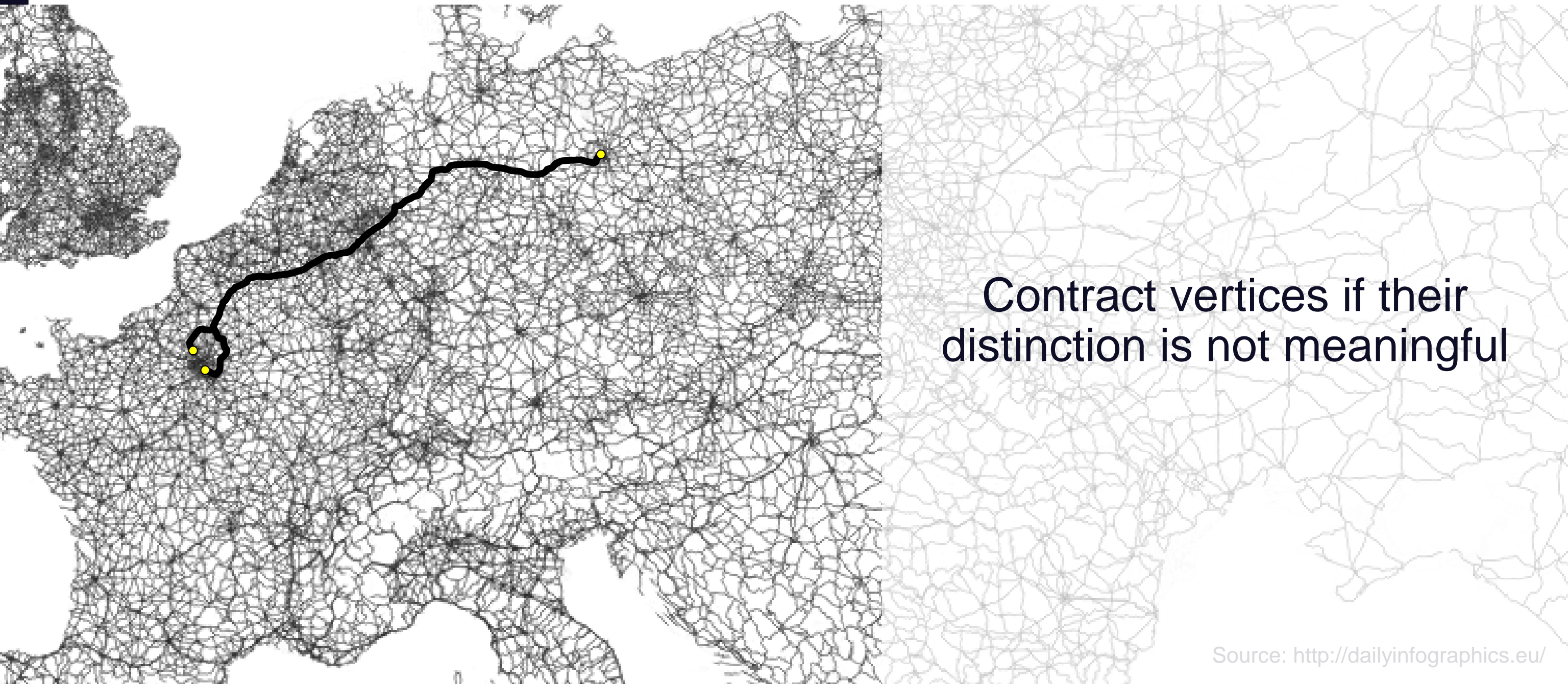
# Motivation

Hey there! Could you please give me directions to Paris?

Of course!  
What's your exact destination address?



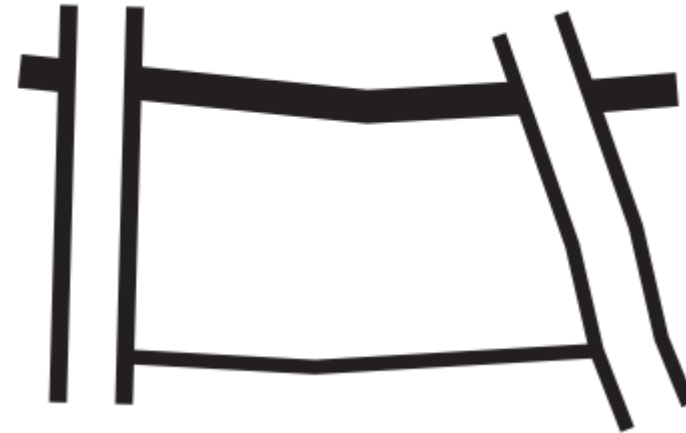
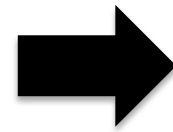
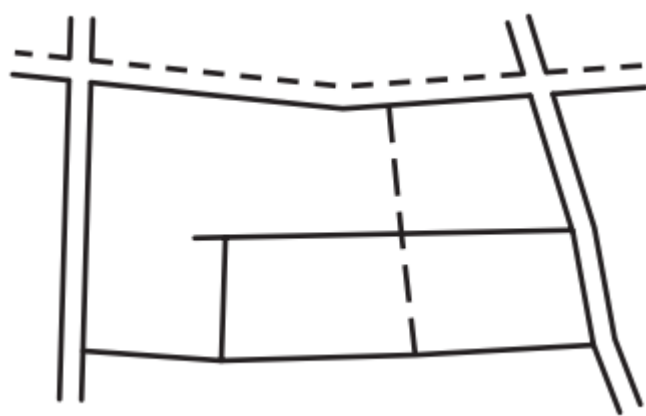
# Idea



Contract vertices if their distinction is not meaningful

Source: <http://dailyinfographics.eu/>

# Related Work: Map Generalization

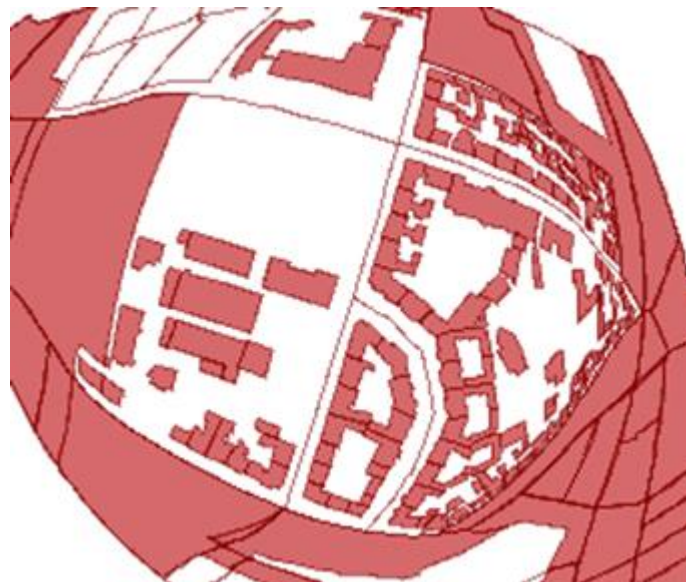
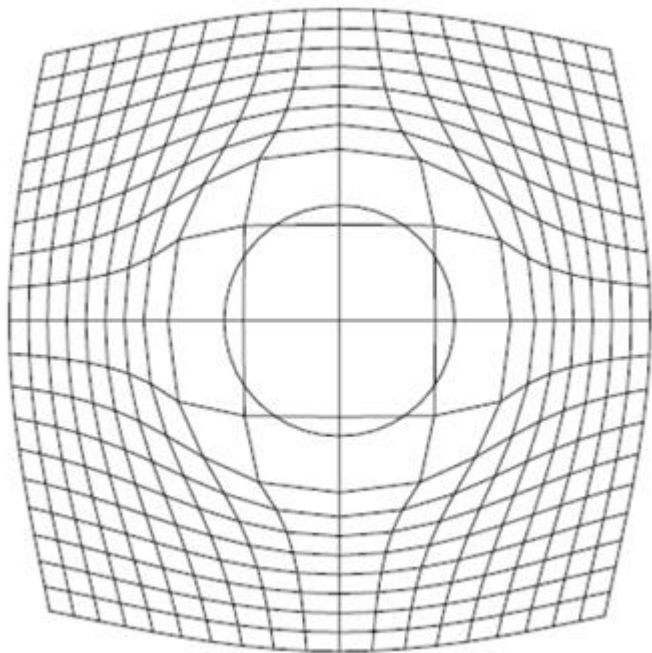


(Hake et al., 1994)

- Much work on road selection for static single-scale maps (e.g., Jiang & Claramunt, 2004; Thomson & Brooks, 2002)
- Selection is NP-hard even for rather simple models of the problem (Brunel et al., 2014)

# Related Work: Variable-Scale Maps

- Fish-eye projections for user-centered navigation maps
- Level of detail (LoD) follows scale

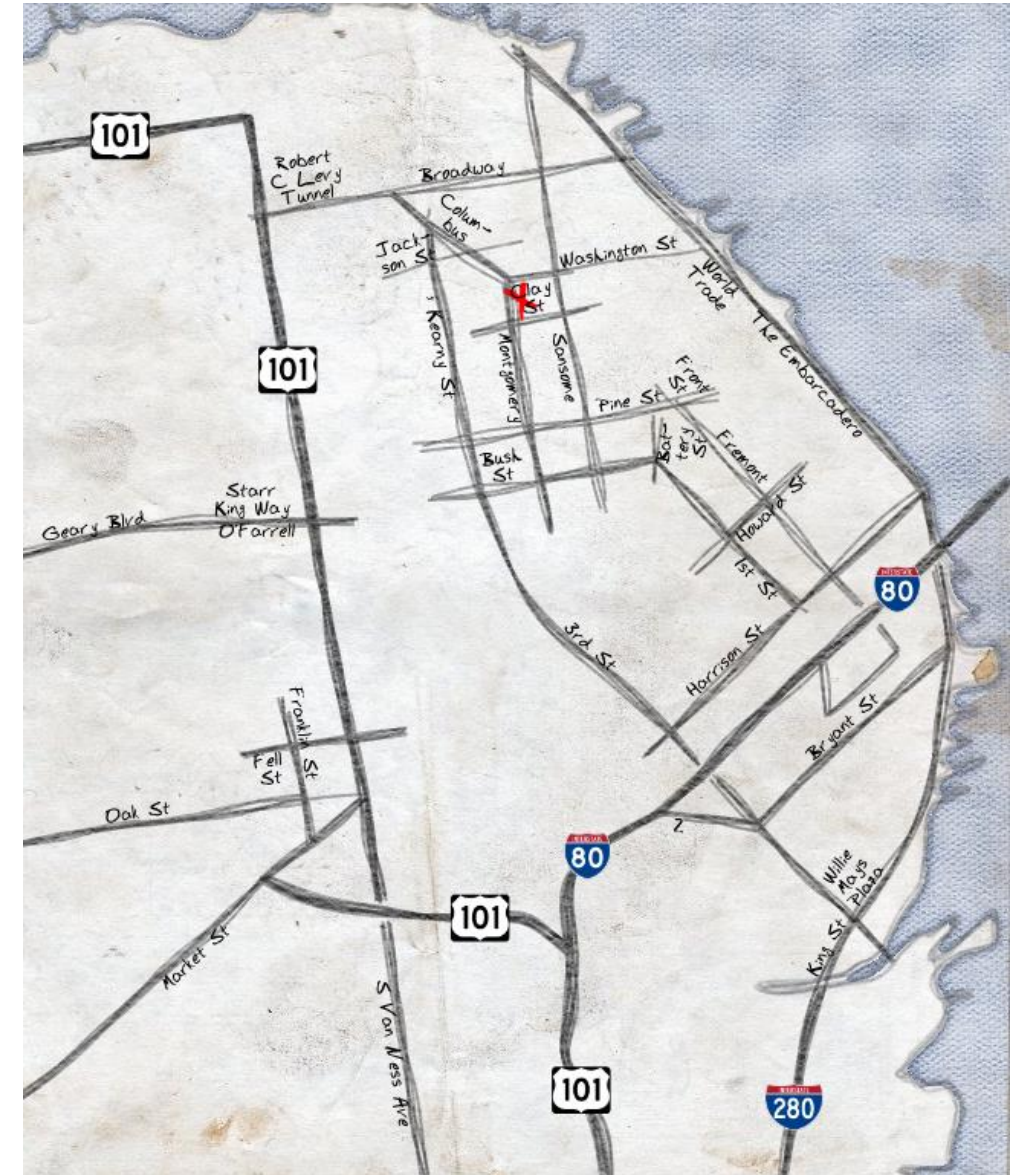


(Hampe et al., 2004)

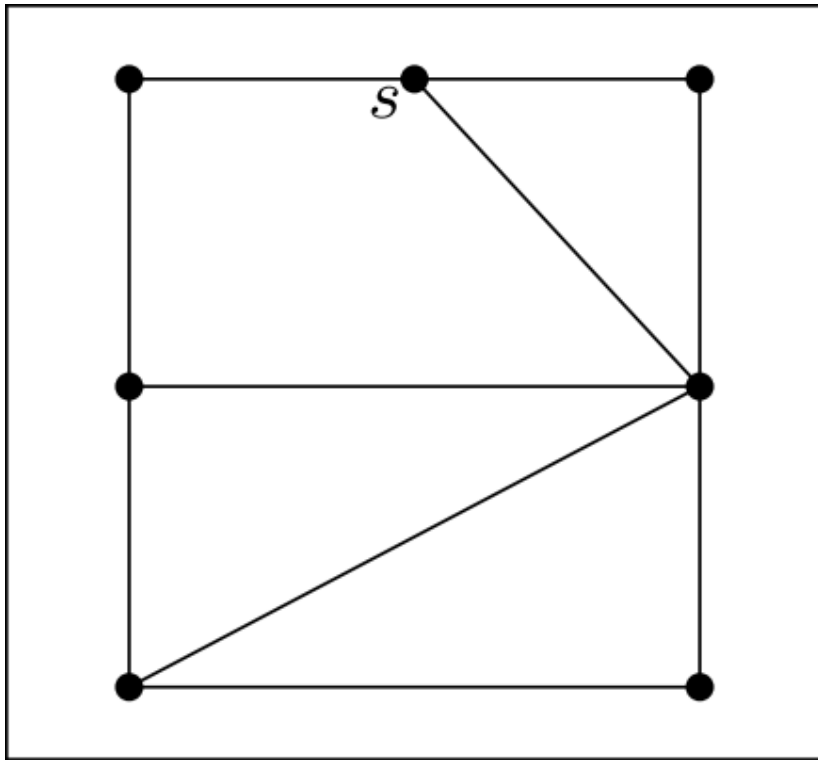
# Related Work: Destination Maps

- Road selection based on shortest paths
- Scale follows LoD
- No guarantee of optimality with respect to a well-defined objective

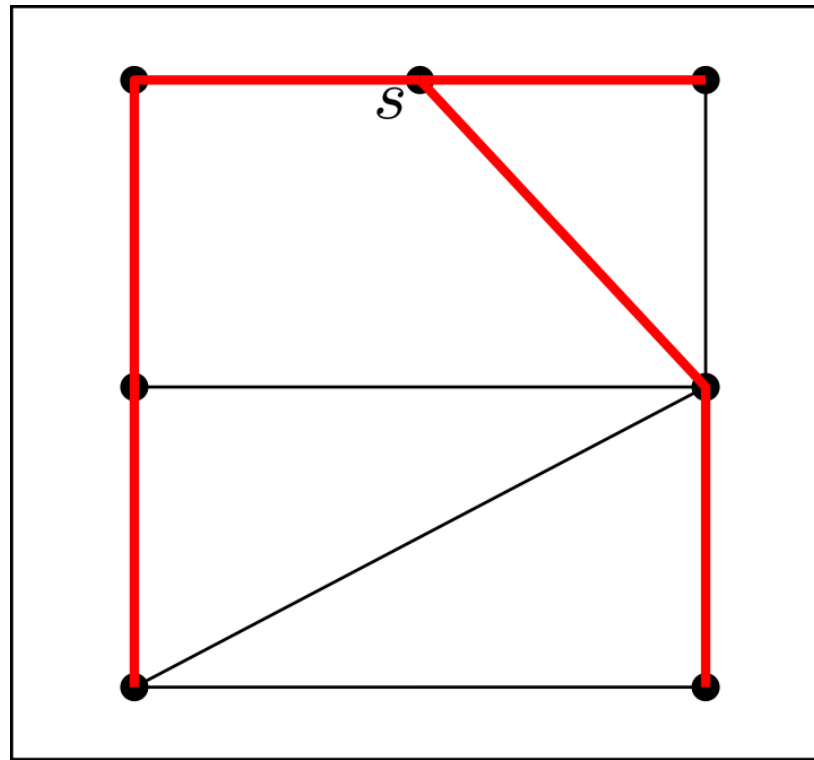
(Kopf et al., 2010)



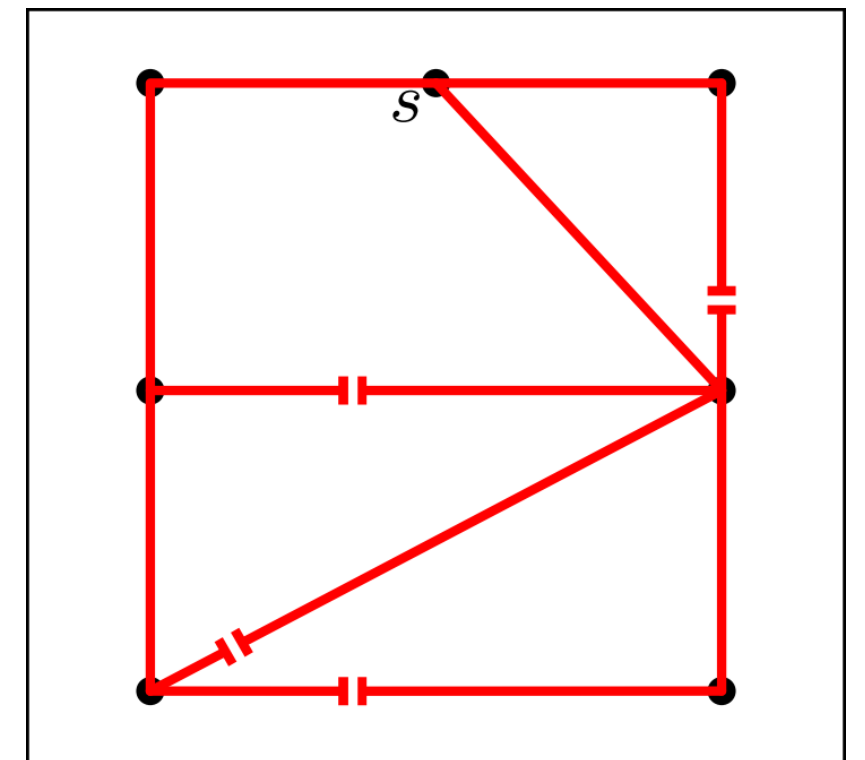
# Prerequisites



road network: graph  $G$   
user location: vertex  $s$



shortest paths from  $s$   
to all other vertices



shortest paths from  $s$   
to all points in  $G$

$\mathcal{T}$

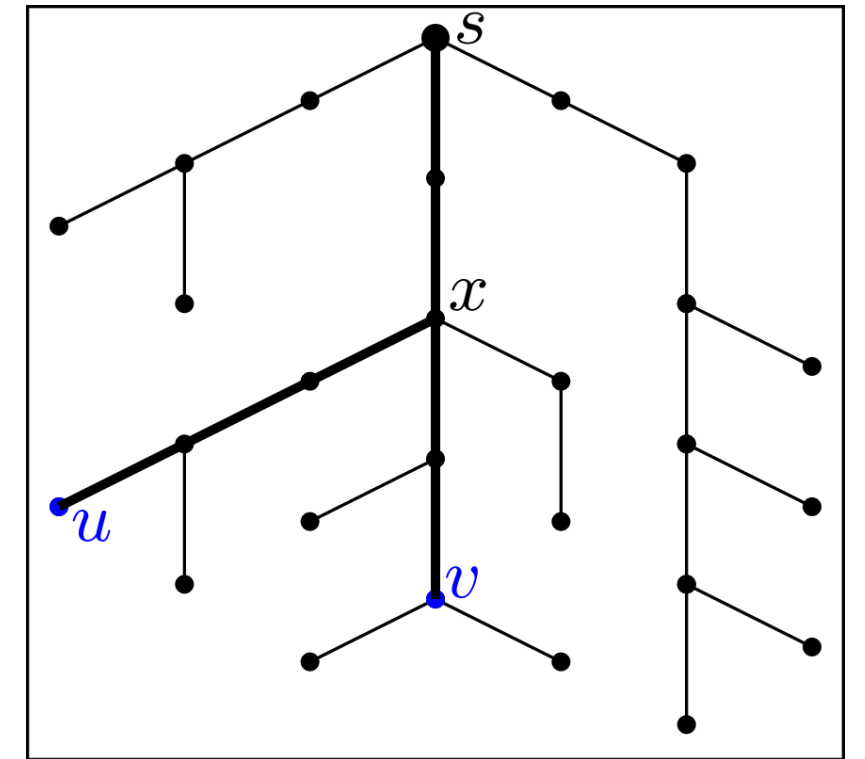
# Problem definition

- Given a tree  $\mathcal{T} = (V, E_{\mathcal{T}})$  rooted at  $s \in V$
- For any  $u, v \in V, u \neq s$  define **directed similarity** as

$$\sigma(u, v) = \frac{w(P_{sx})}{w(P_{su})}$$

where  $x \in V$  is the lowest common ancestor of  $u$  and  $v$  in  $\mathcal{T}$ .

- Any  $u, v \in V$  are called  **$\alpha$ -compatible** if  $\sigma(u, v) \geq \alpha$  and  $\sigma(v, u) \geq \alpha$  with  $\alpha \in [0, 1]$
- For fixed  $\alpha$  this is expressed as  $u \oplus v$



$$\sigma(u, v) = 2/5$$

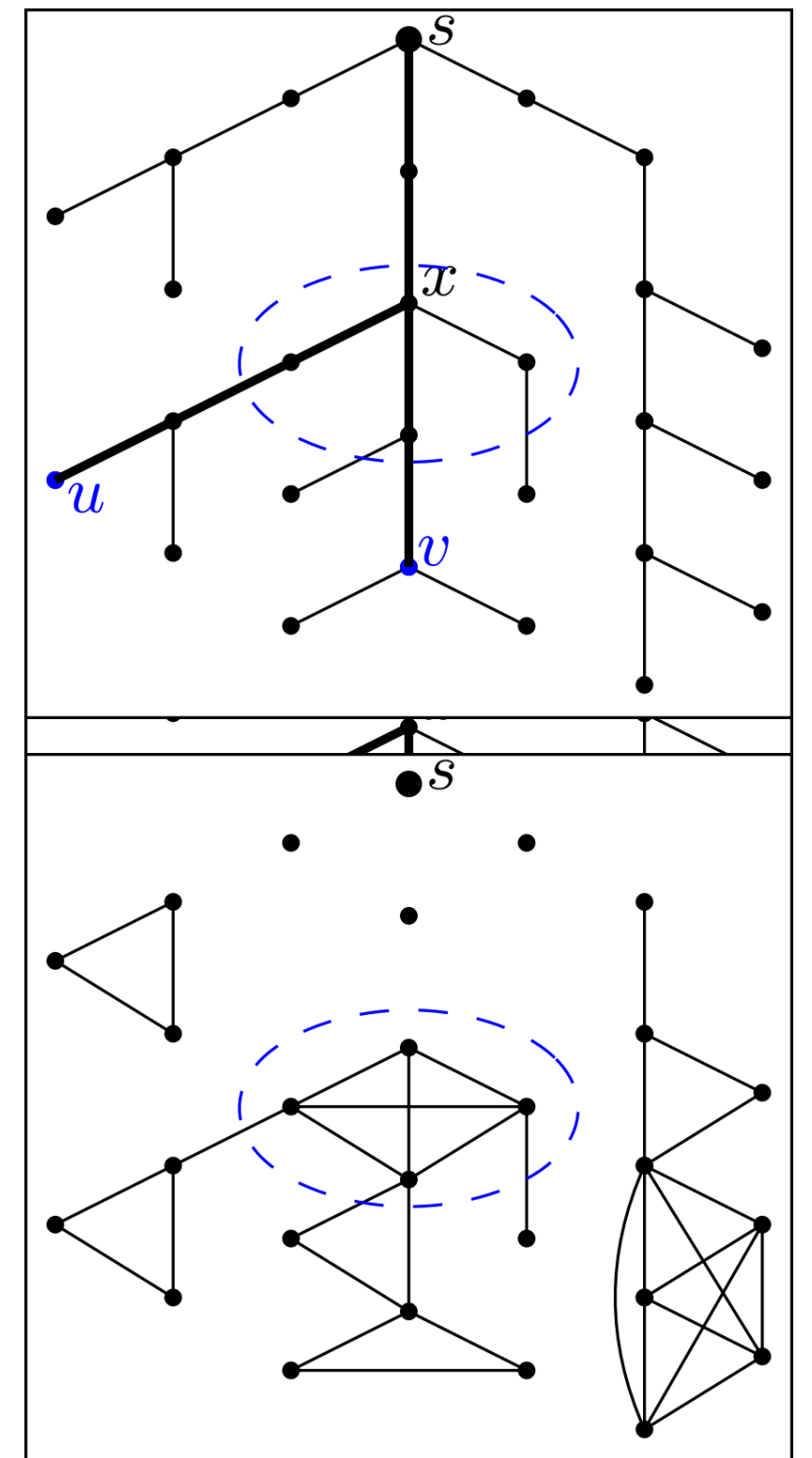
$$\sigma(v, u) = 1/2$$



# Problem definition

- Define a **compatibility graph**  $G_{\oplus} = (V, E_{\oplus})$  with  $E_{\oplus} = \{(u, v) \in V \times V : u \oplus v, u \neq v\}$
- Contracting  $S \subseteq V$  is **allowed** if and only if
  - $S$  is connected in  $\mathcal{T}$
  - $S$  is a clique in  $G_{\oplus}$

(here:  $\alpha = 2/3$ )



# Problem: TreeSummary

- Instance:
  - A tree  $\mathcal{T} = (V, E_{\mathcal{T}})$ , rooted at  $s \in V$
  - Weights on the edges  $w: E_{\mathcal{T}} \rightarrow \mathbb{R}_{\geq 0}$
  - A compatibility threshold  $\alpha \in [0,1]$
- Problem:

Find a partition of  $V$  into as few cells as possible, such that each cell is an allowed contraction!

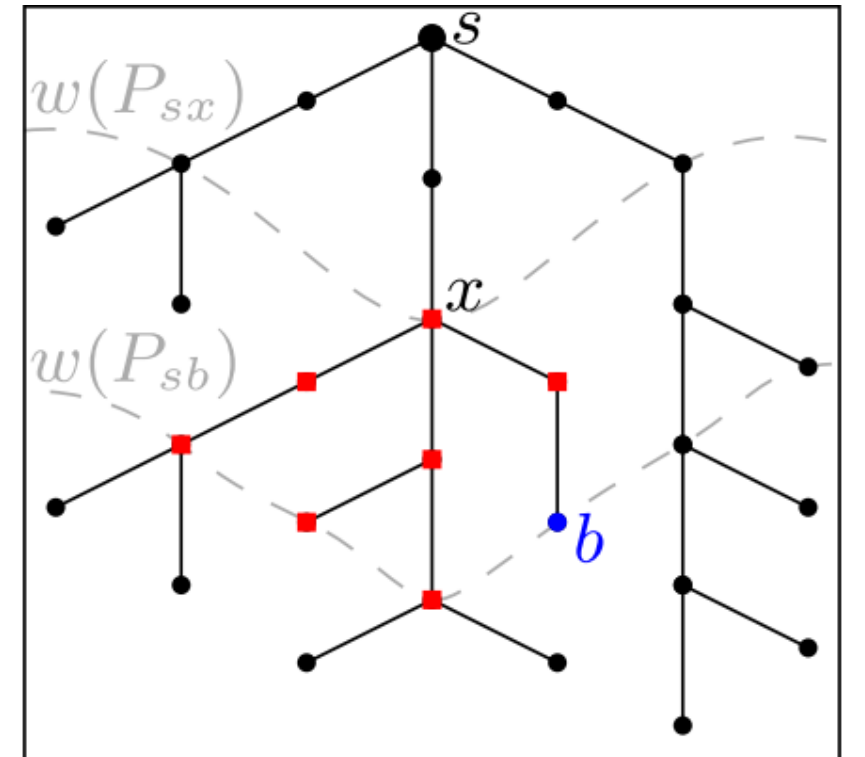
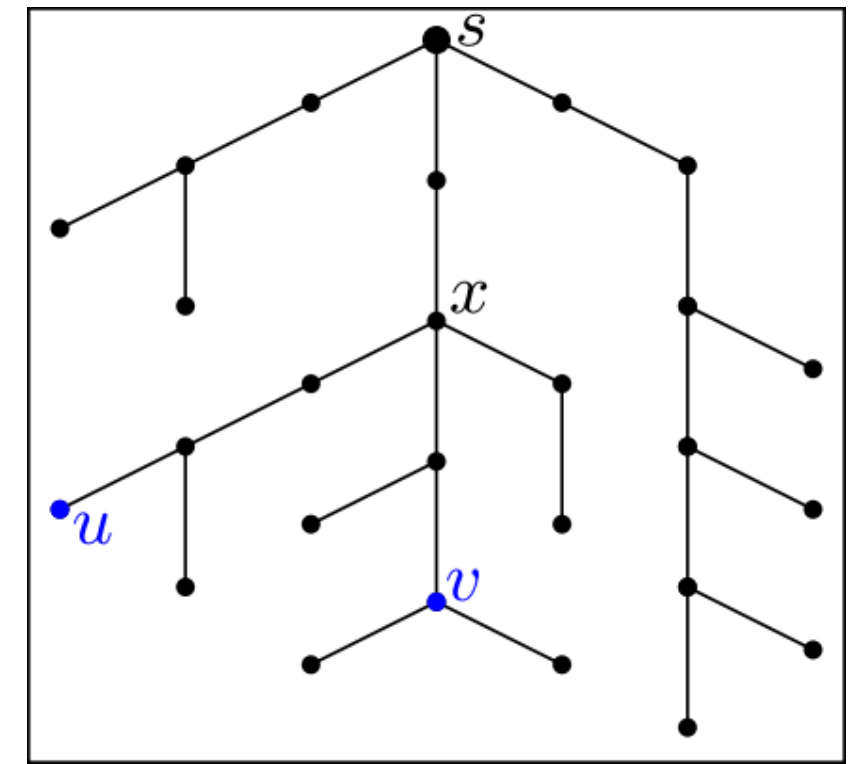
# Lemmata

- Let  $u, v \in V$  be vertices and let  $x$  be their lowest common ancestor. Then:

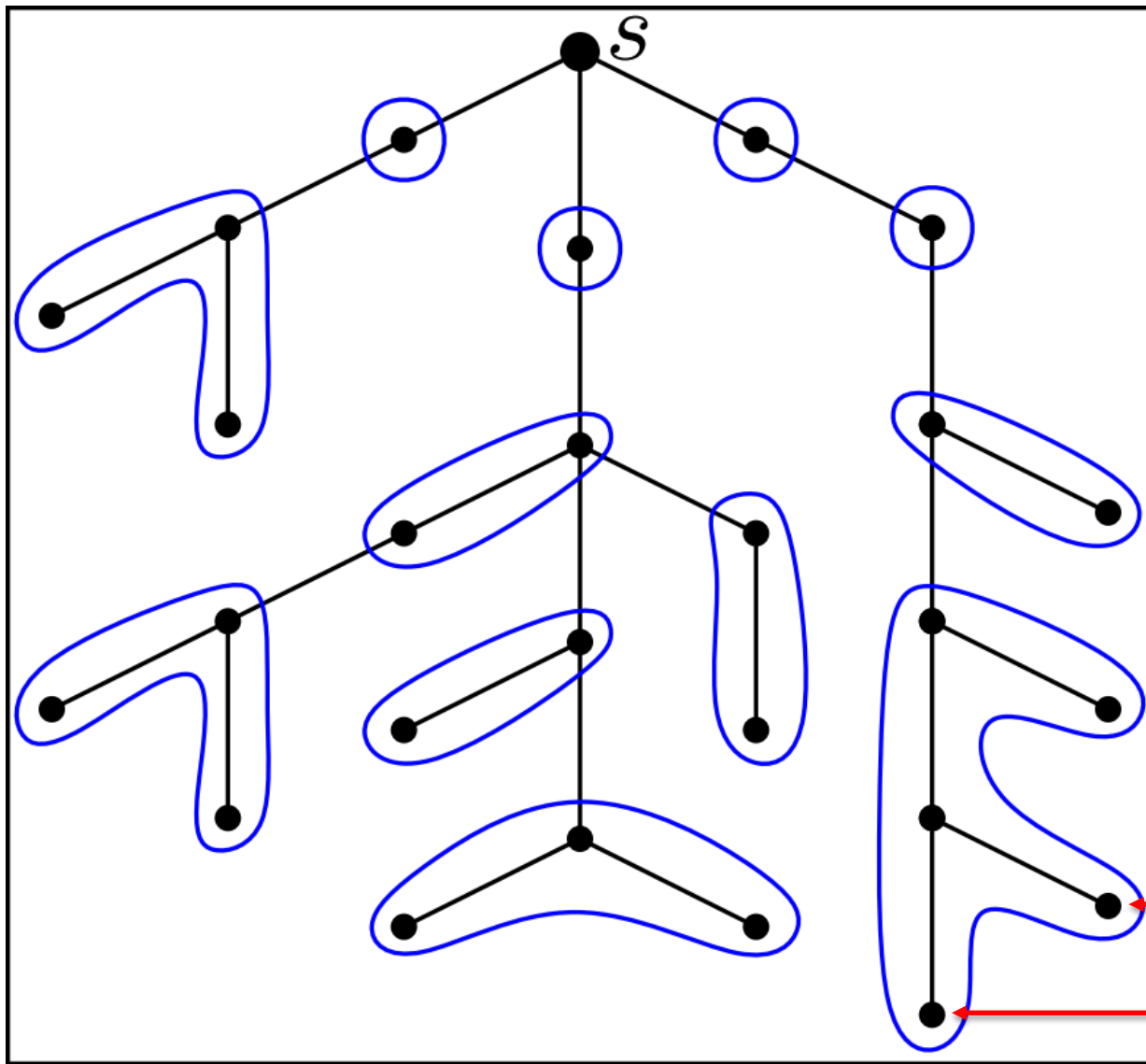
$$u \oplus v \iff u \oplus x \wedge v \oplus x$$

- Let  $x \in V$  and let  $a, b \in V$  be descendants of  $x$ . Then:

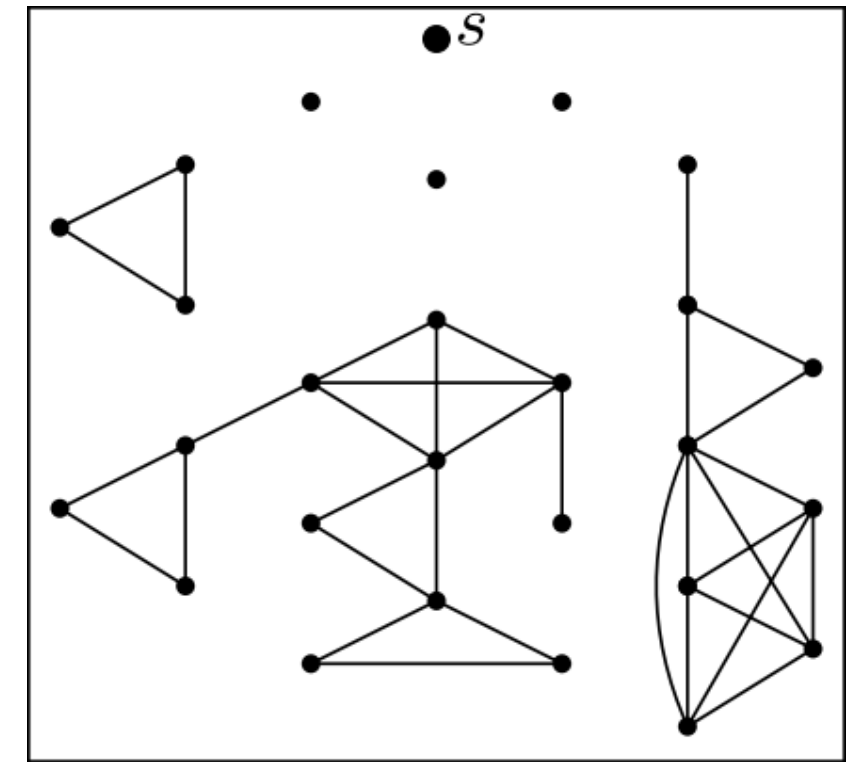
$$w(P_{sa}) \leq w(P_{sb}) \wedge b \oplus x \implies a \oplus x .$$



# Algorithm: ContractTree



(here:  $\alpha = 2/3$ )



- Following invariants of a cell  $C$  hold:
- Vertices of  $C$  are connected in  $\mathcal{T}$
  - **deep vertices** of their cell
  - $C$  is clique in  $G_{\oplus}$

# Algorithm: ContractTree


**Data:** Rooted Tree  $\mathcal{T}$  with edge weights

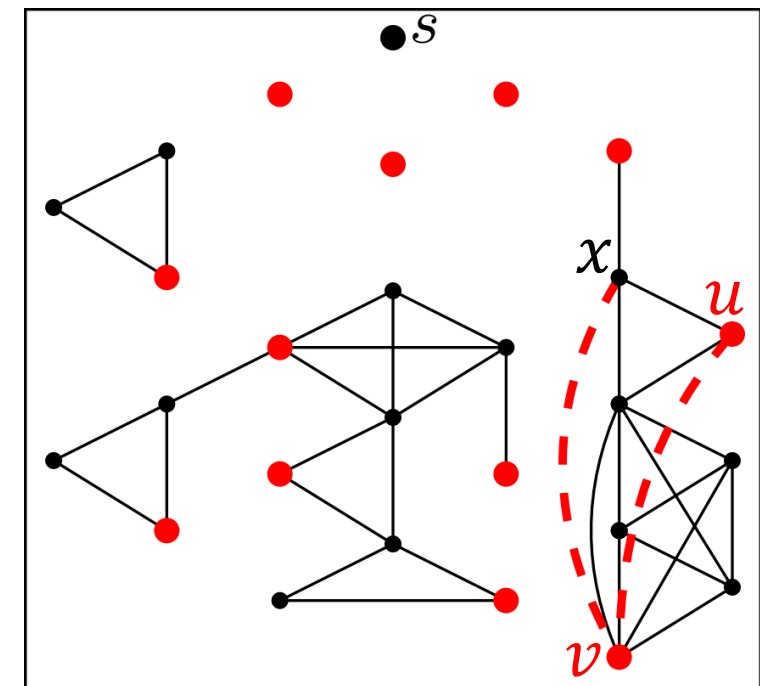
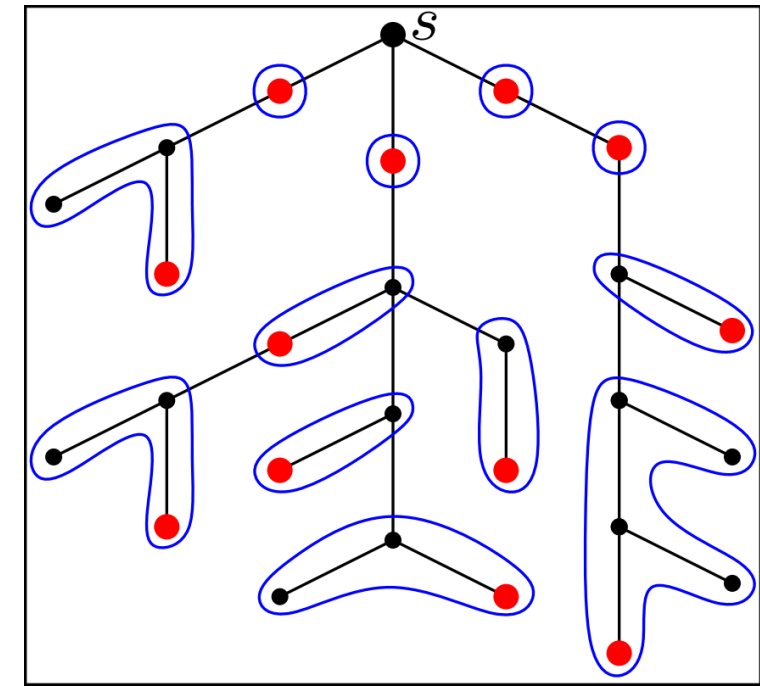
**Result:** Minimum-cardinality allowed contraction  $\mathcal{C}$

1.  $\mathcal{C} \leftarrow$  the set with a singleton cell for each vertex of  $\mathcal{T}$ ;
2. **For all** the vertices  $x \in \mathcal{T}$ , in post-order **do**
3.      $S_{\oplus} \leftarrow \{v \in \text{Children}(x) : x \oplus \text{Deep}(\text{Cell}(v))\}$ ;
4.     Merge  $\text{Cell}(x)$  and all  $\text{Cell}(v)$  for  $v \in S_{\oplus}$
5. **End**
6. **Return**  $\mathcal{C}$ ;

run-time:  $\mathcal{O}(|V|)$

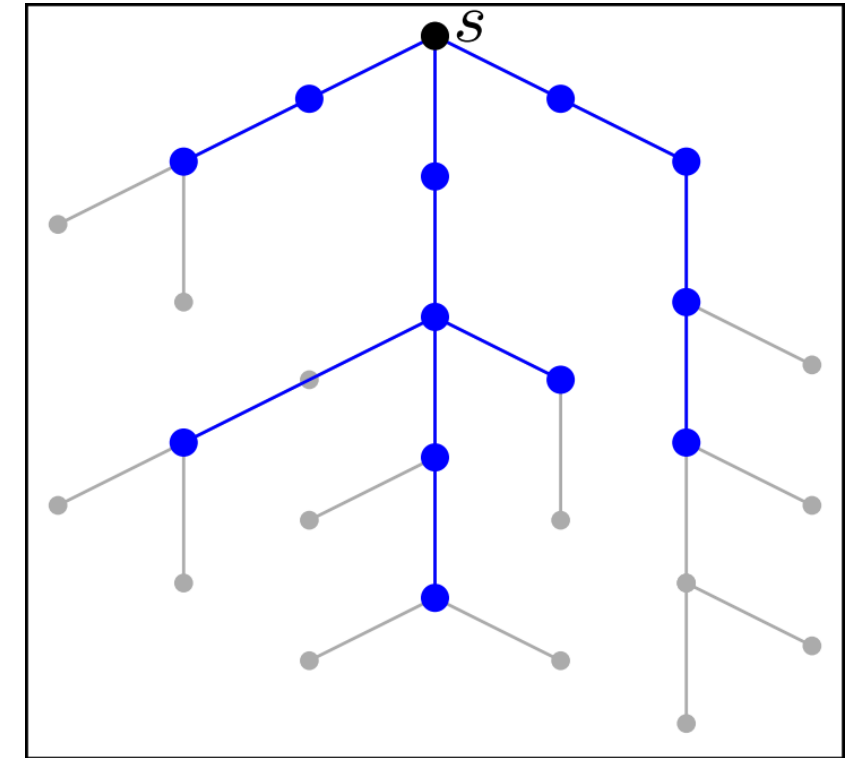
# Algorithm: Result is optimal

- Let  $\mathcal{C}$  be the set of cells of  $\mathcal{T}$  computed with the algorithm
- $\mathcal{C}$  is a clique cover of  $G_{\oplus}$
- Consider  $\mathcal{J} := \{Deep(\mathcal{C}) : \mathcal{C} \in \mathcal{C}\}$
- $\mathcal{J}$  is an independent set of  $G_{\oplus}$ :
  - Assume  $(u, v) \in E_{\oplus}$  for any  $u, v \in \mathcal{J}$
  - Let  $x$  be the lowest common ancestor of  $u$  and  $v$
  - $u \oplus v \implies x \oplus u$  and  $x \oplus v$
  - $Cell(u)$  and  $Cell(v)$  would have been merged 
- $|\mathcal{J}| = |\mathcal{C}|$
- $\mathcal{C}$  is a minimal clique cover of  $G_{\oplus}$



# Visualization

- Represent each cell by its root vertex.
- The relation between the cells is a tree structure induced by  $\mathcal{T}$ .



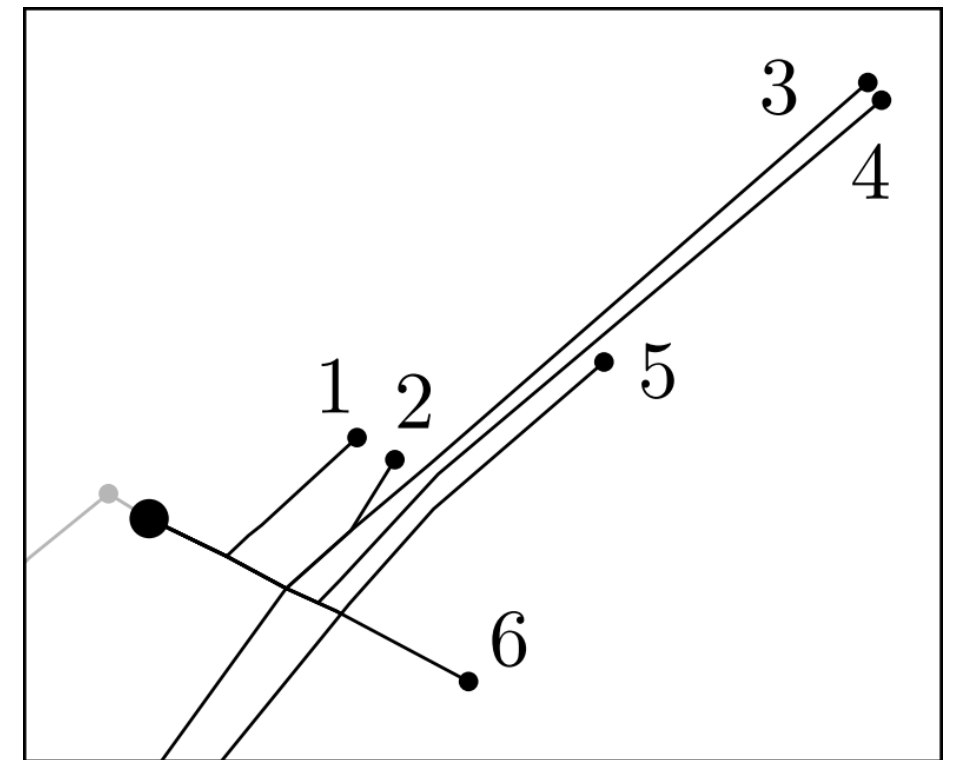
- How to visualize this tree?

# Visualization: Three proposals

## 1. Detailed drawing

For every cell  $c$  and its parent  $p$  draw  $P_{pc}$

- + topologically correct
- + same level of detail as input
- internal branches





# Visualization: Three proposals

1. Detailed drawing

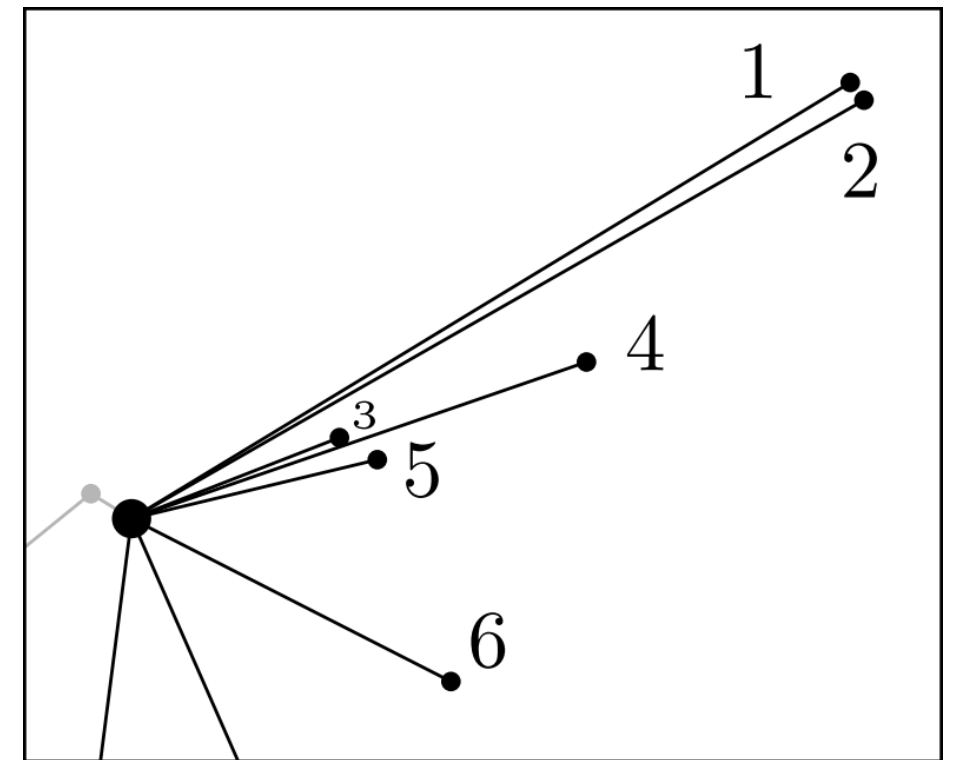
2. Direct drawing

For every cell  $c$  and its parent  $p$  draw a direct line between  $p$  and  $c$

+ simple and highly generalized

+ shows the actual clustering

— not topologically safe

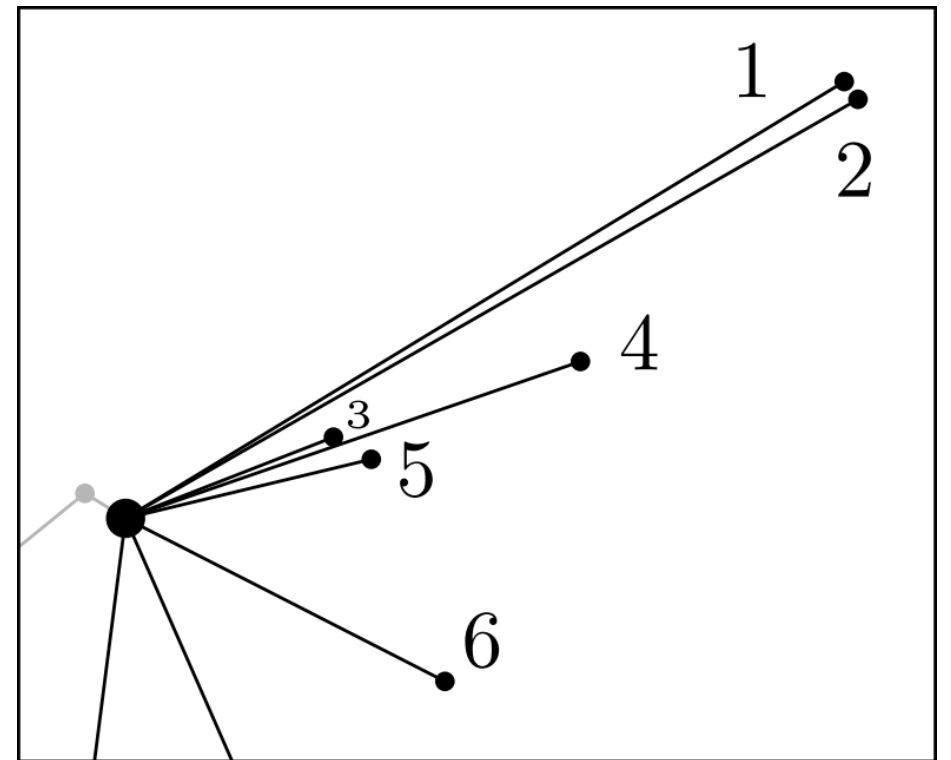
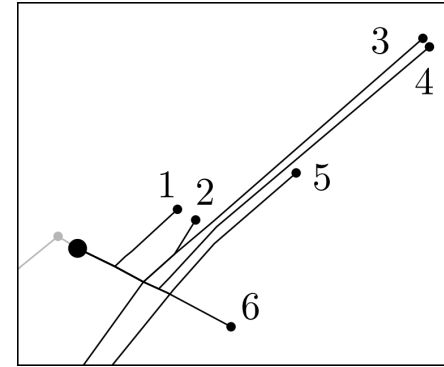


# Visualization: Three proposals

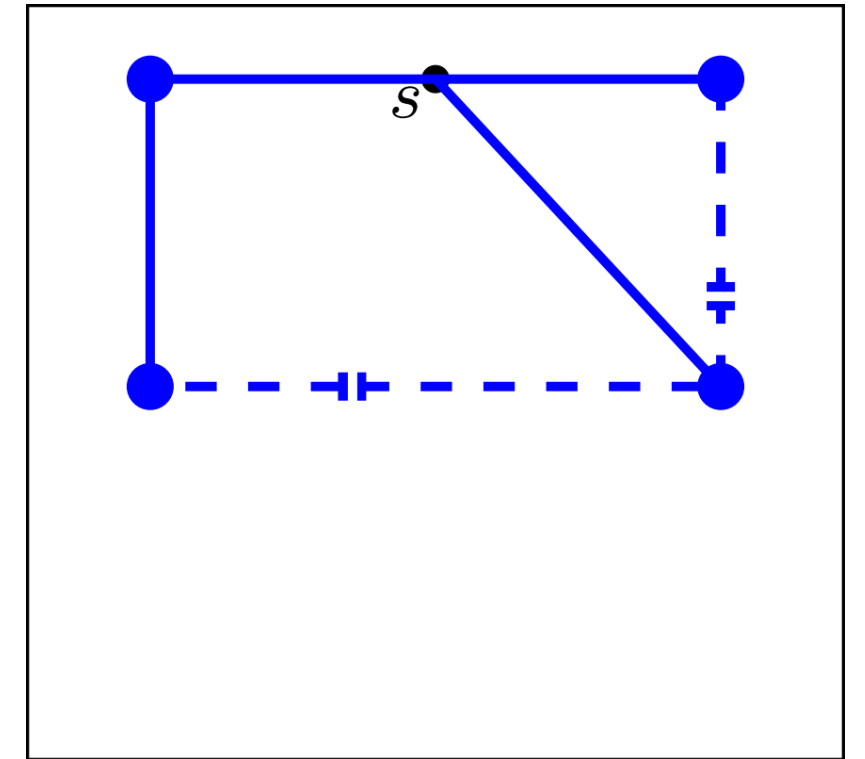
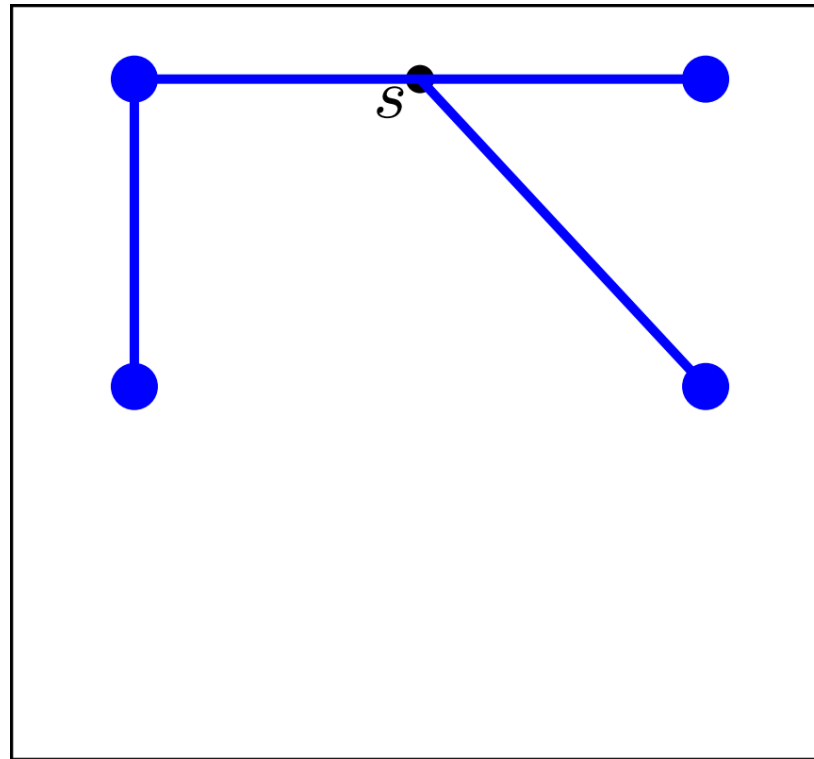
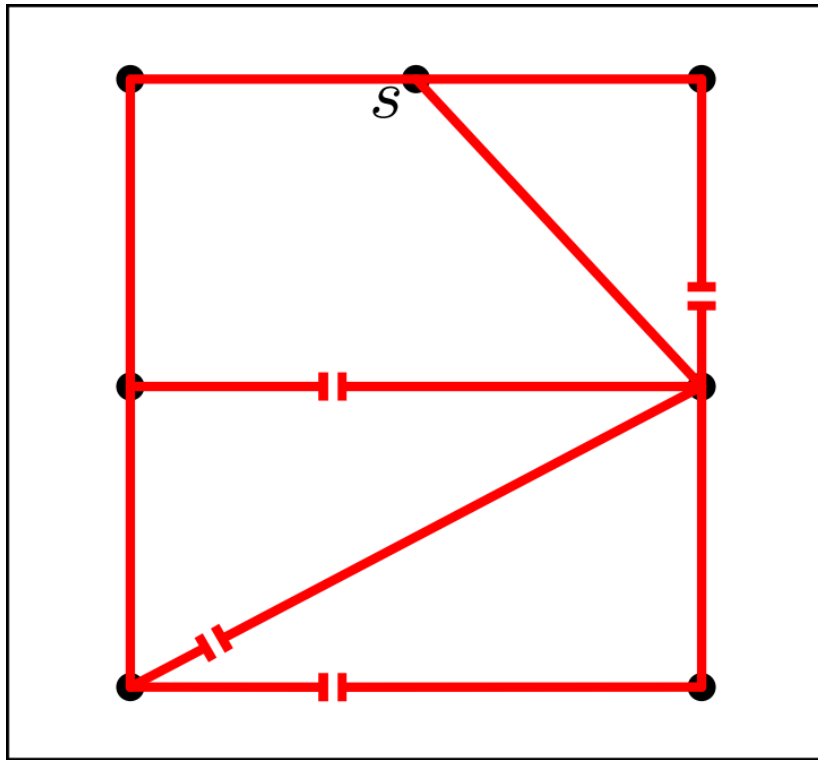
1. Detailed drawing
2. Direct drawing
3. Simplified drawing

Construct detailed drawing and apply topologically-safe simplification algorithm (Dyken et al., 2009)

- + topologically safe
- + less internal branches
- heuristic



# Visualization: Cross connections



- Resulting tree gives no information about cell adjacency
- Draw cross connections between neighboring cells

# Example results



$\alpha = 90\%$



# Example results

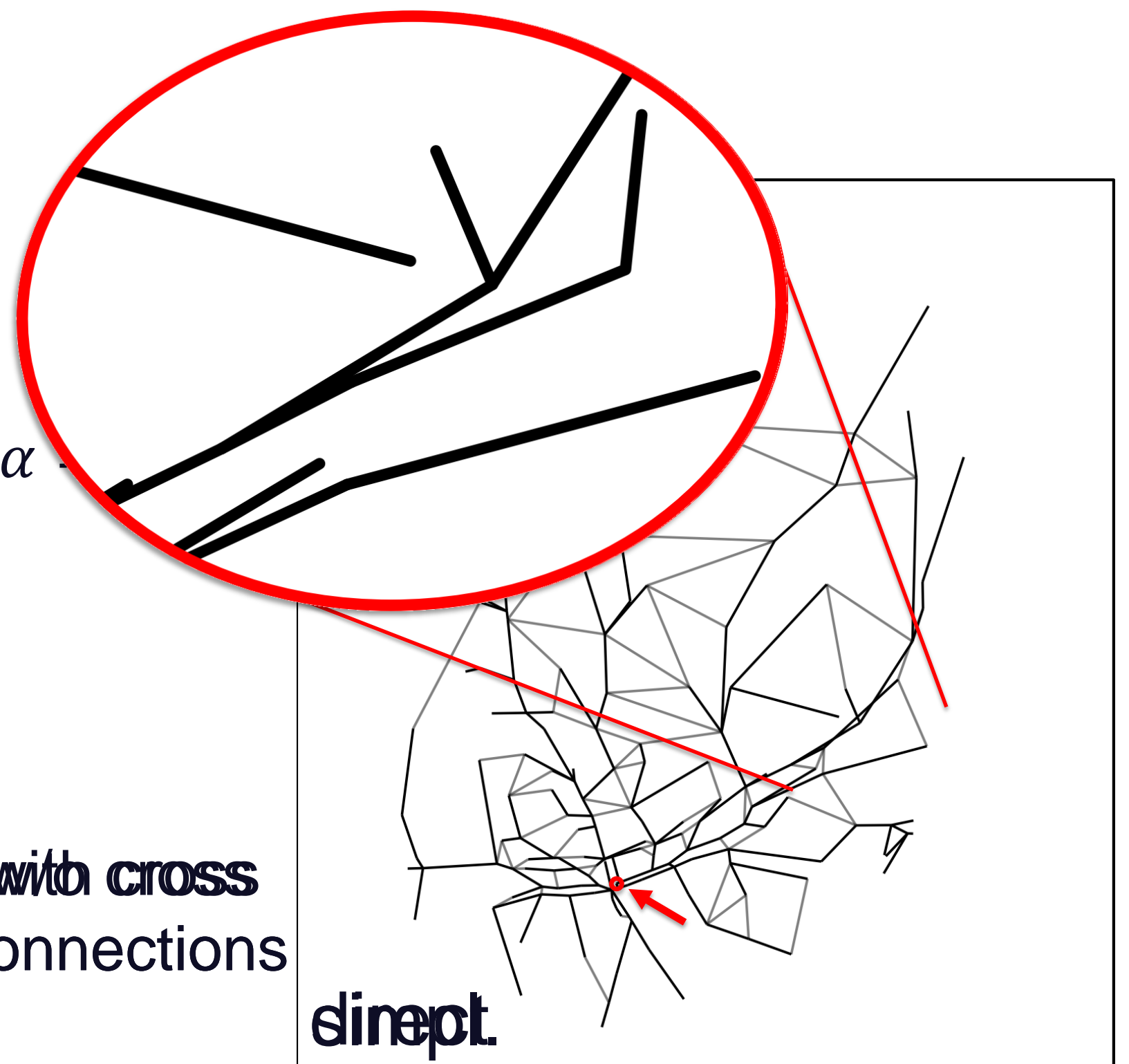
detailed



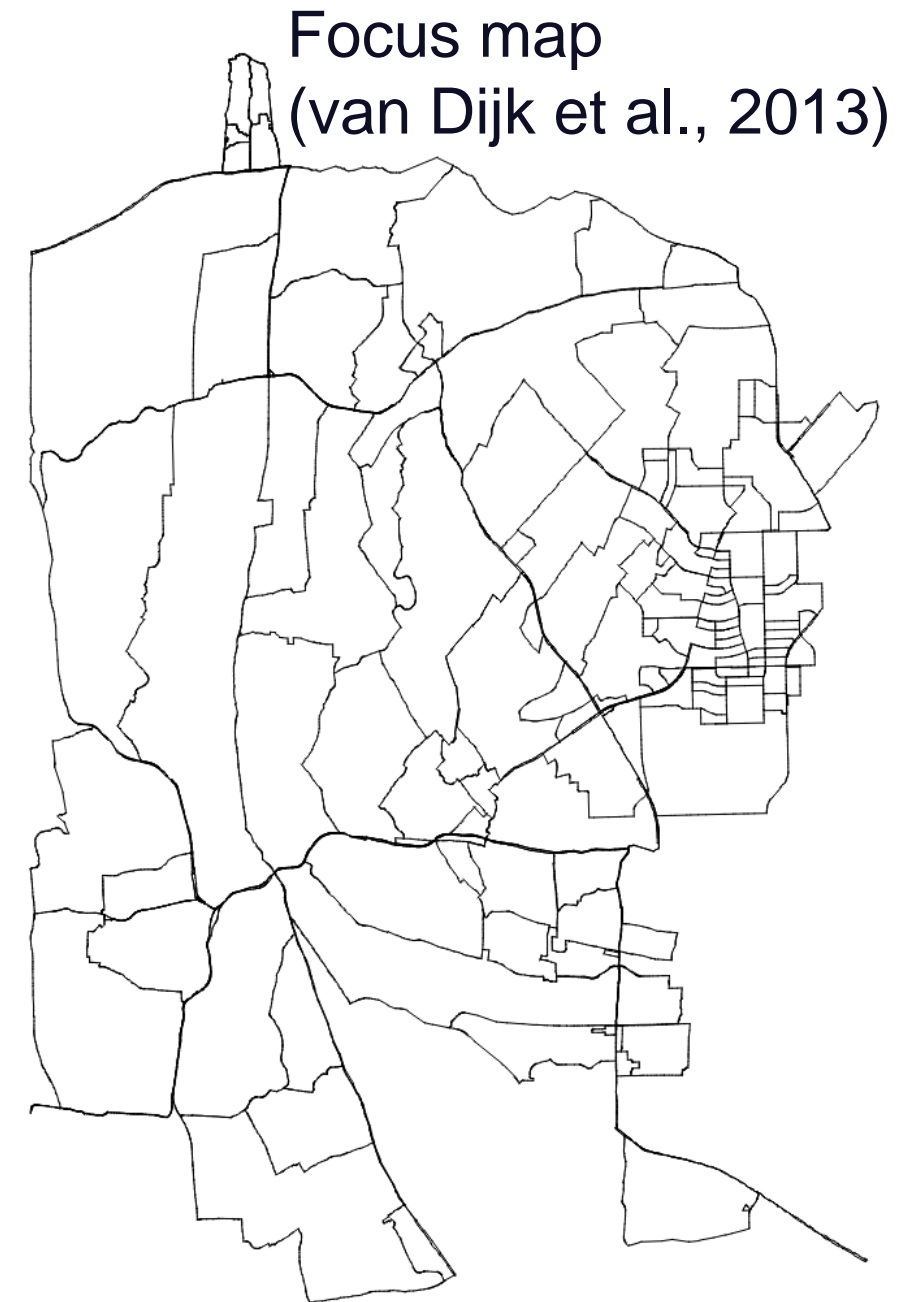
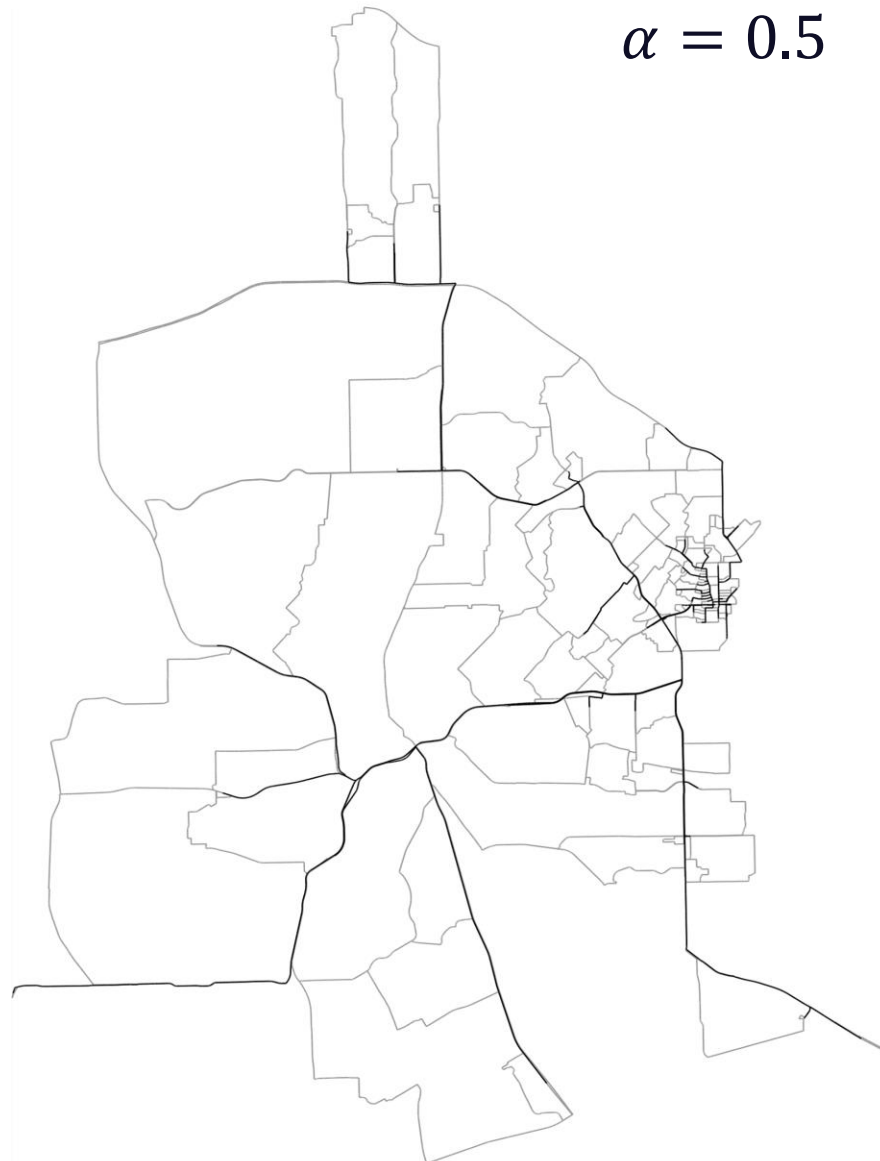
$\alpha$

with cross  
connections

dinept.



# Example results



# Summary

- Problem TreeSummary: Contraction of compatible destination
- Optimal linear-time algorithm (traversing tree in post-order)
- Three kinds of drawings:
  - Detailed
  - Direct
  - Simplified
- Cross connections