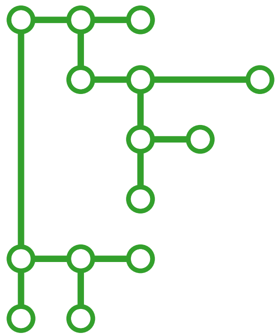
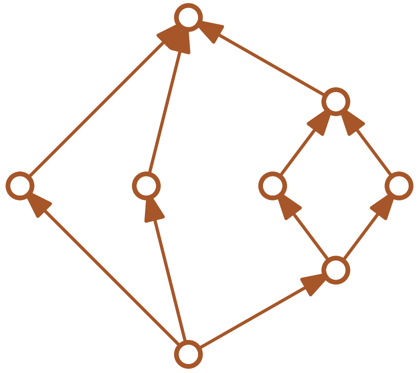
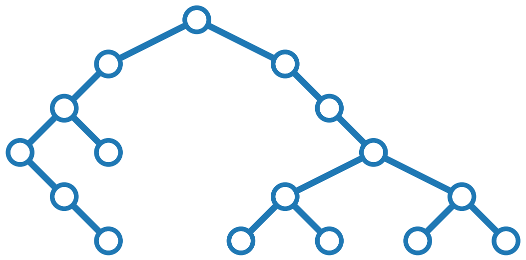
Visualization of Graphs

Lecture 1b:
Drawing Trees and Series-Parallel Graphs

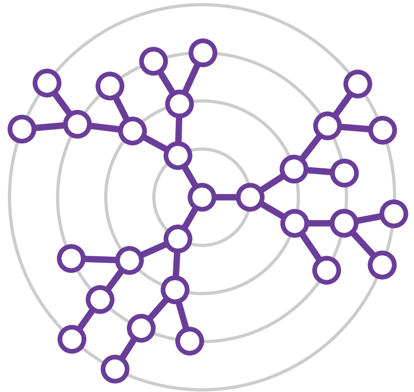




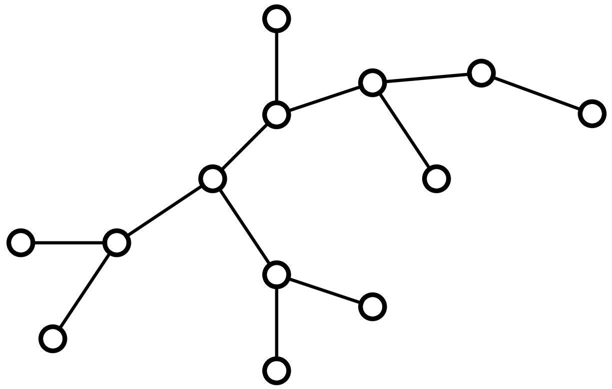
Part I:
Layered Drawings



Jonathan Klawitter

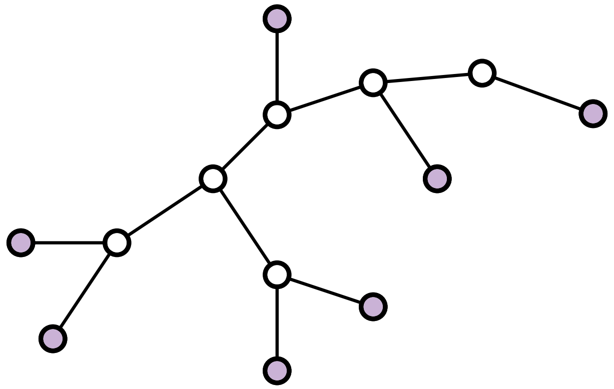


(Rooted) Trees



(Rooted) Trees

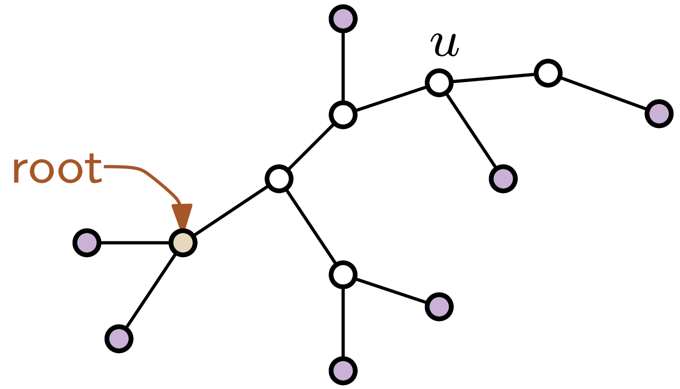
Leaf: Vertex of degree 1



(Rooted) Trees

Leaf: Vertex of degree 1

Rooted tree: tree with designated **root**

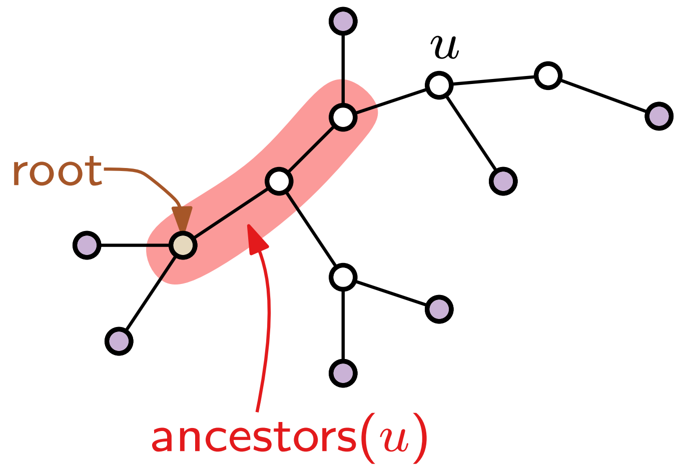


(Rooted) Trees

Leaf: Vertex of degree 1

Rooted tree: tree with designated **root**

Ancestor: Vertex on path to root



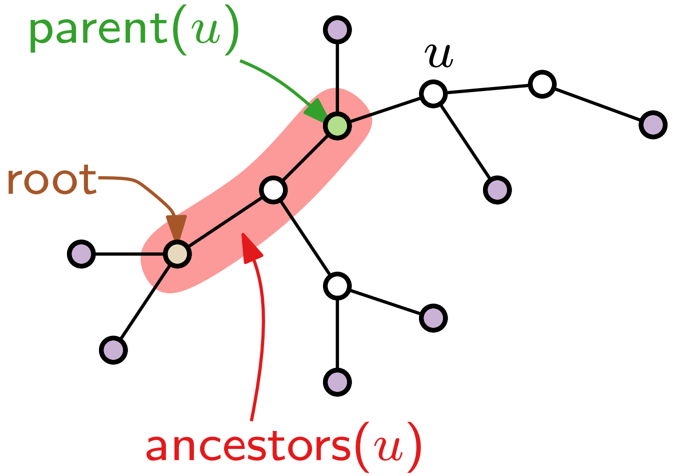
(Rooted) Trees

Leaf: Vertex of degree 1

Rooted tree: tree with designated **root**

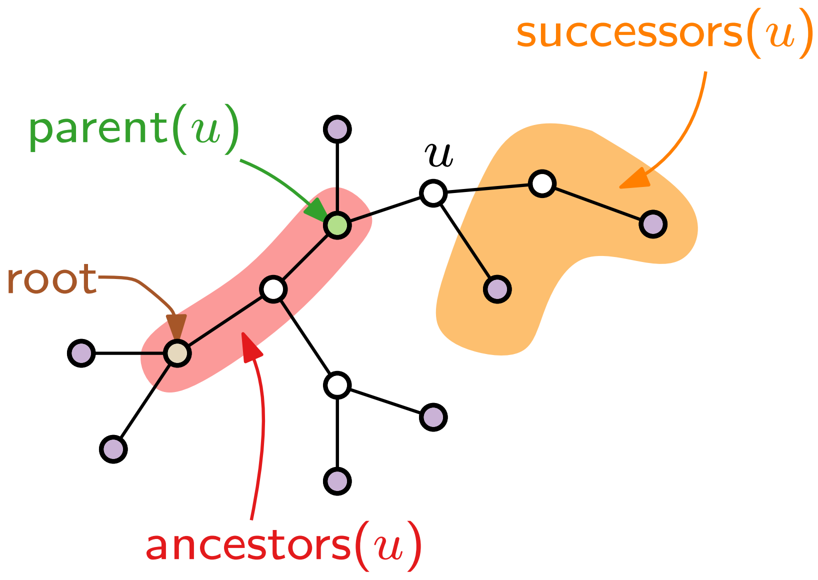
Ancestor: Vertex on path to root

Parent: Neighbor on path to root



(Rooted) Trees

Leaf: Vertex of degree 1
Rooted tree: tree with designated **root**
Ancestor: Vertex on path to root
Parent: Neighbor on path to root
Successor: Vertex on path away from root



(Rooted) Trees

Leaf: Vertex of degree 1

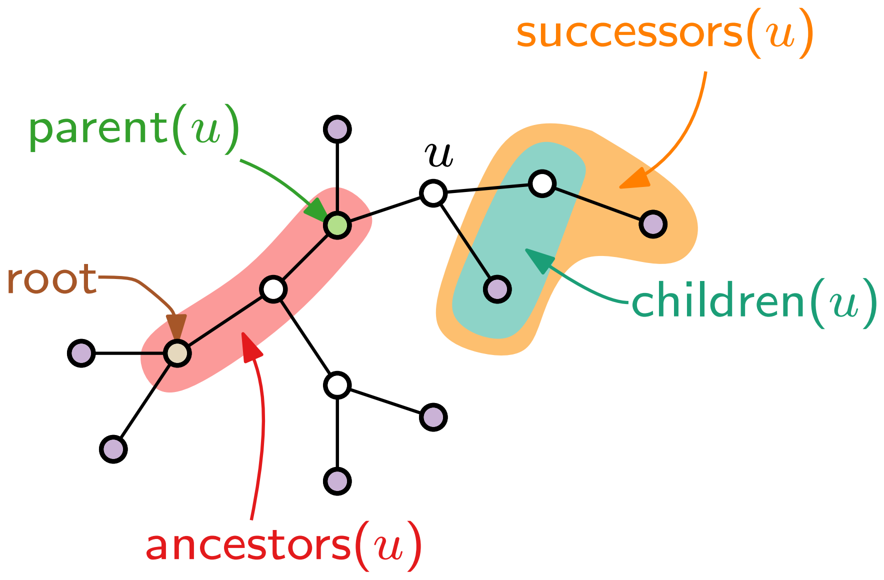
Rooted tree: tree with designated **root**

Ancestor: Vertex on path to root

Parent: Neighbor on path to root

Successor: Vertex on path away from root

Child: Neighbor not on path to root



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Parent: Neighbor on path to root

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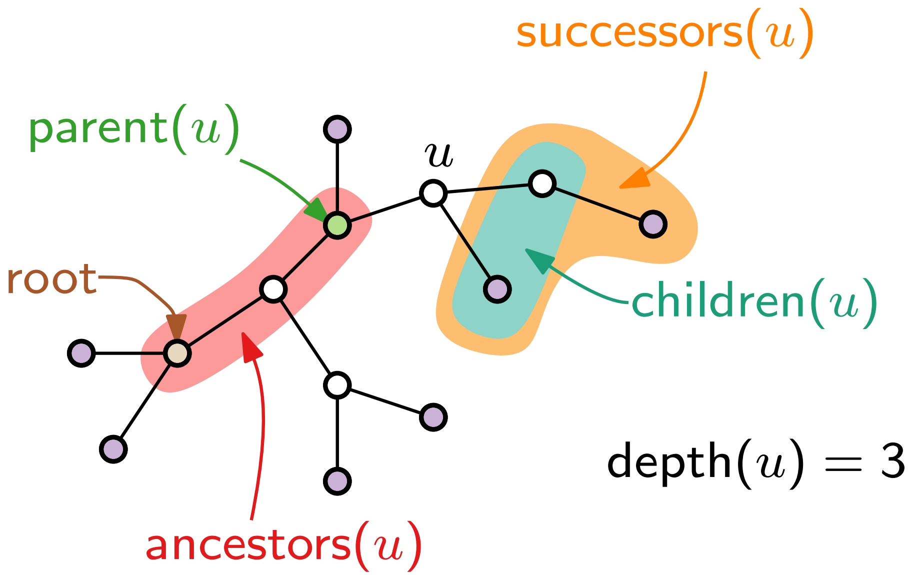
Depth: Length of path to root



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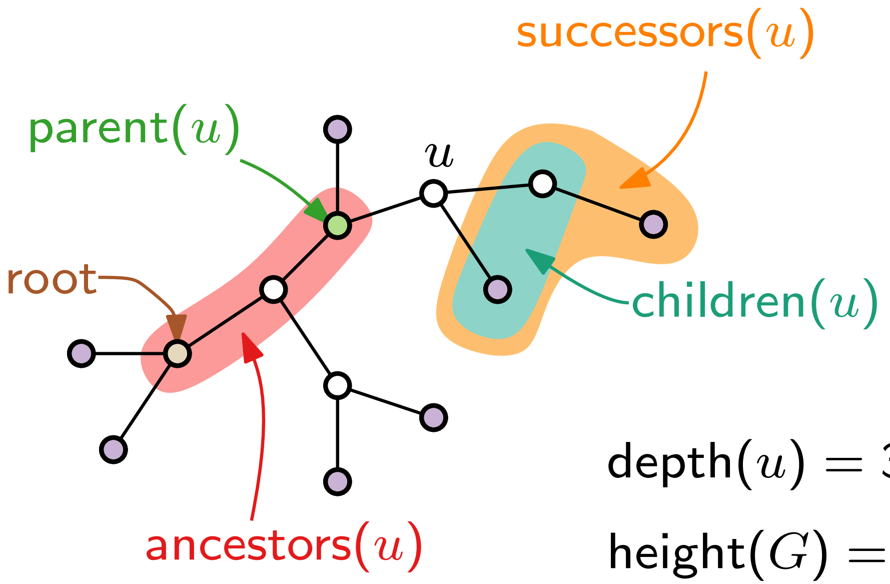

$\text{depth}(u) = 3$

$\text{height}(G) = 5$

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Leaf: Vertex of degree 1

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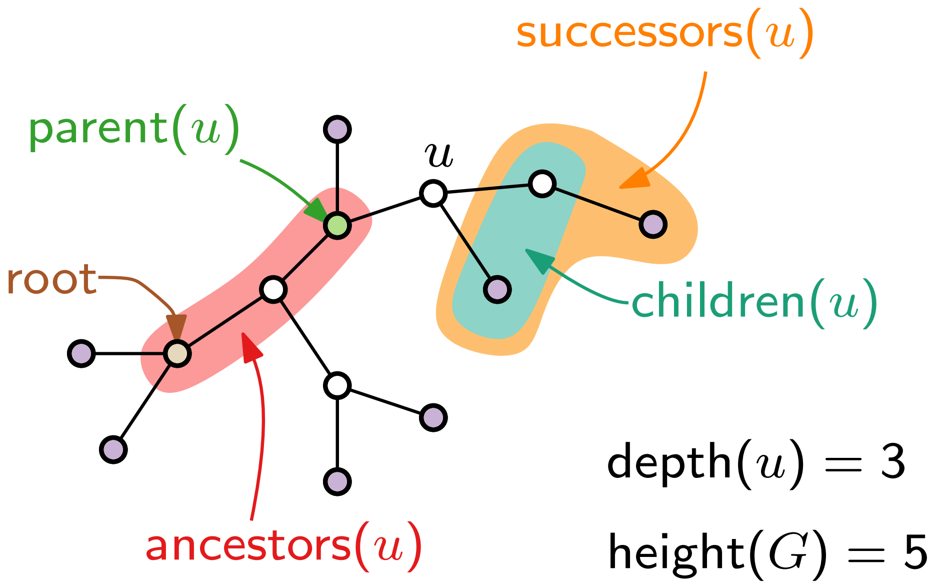
Depth: Length of path to root

Height: Maximum depth of a leaf

Binary Tree: At most two children per vertex (left / right child)



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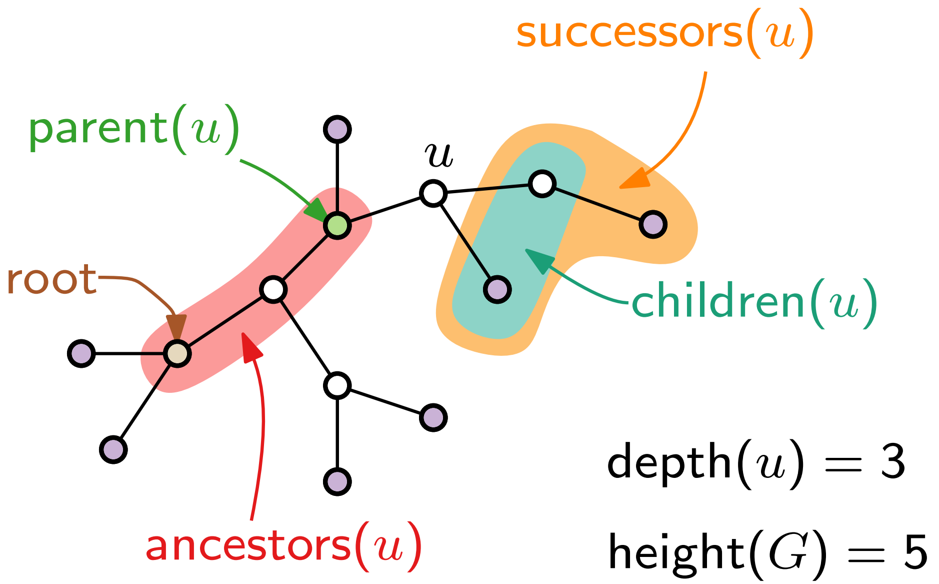

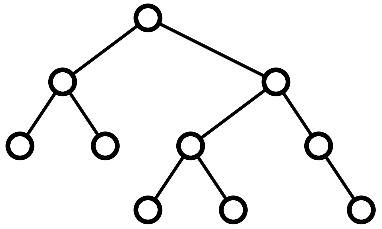

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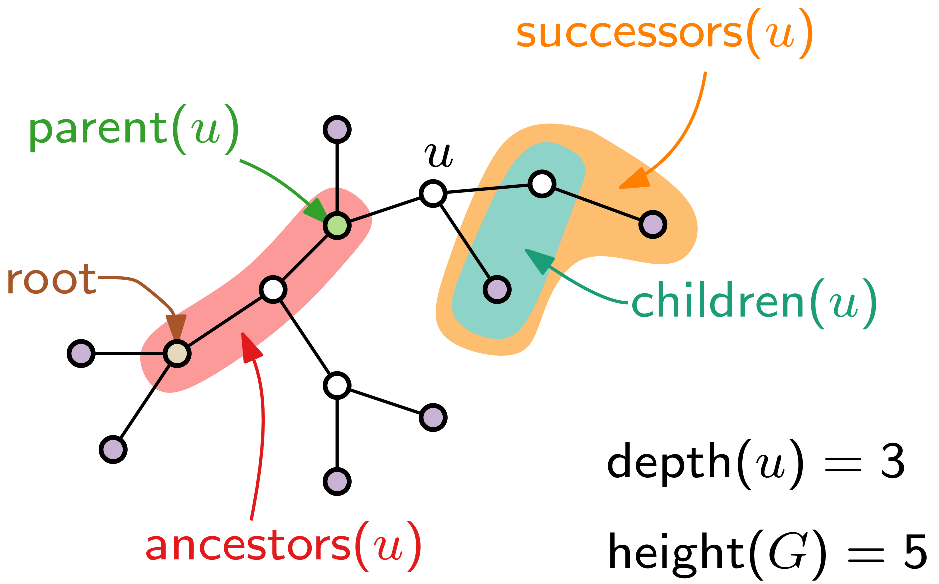
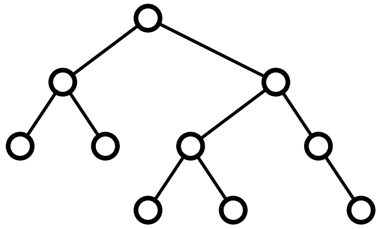


3 traversals:



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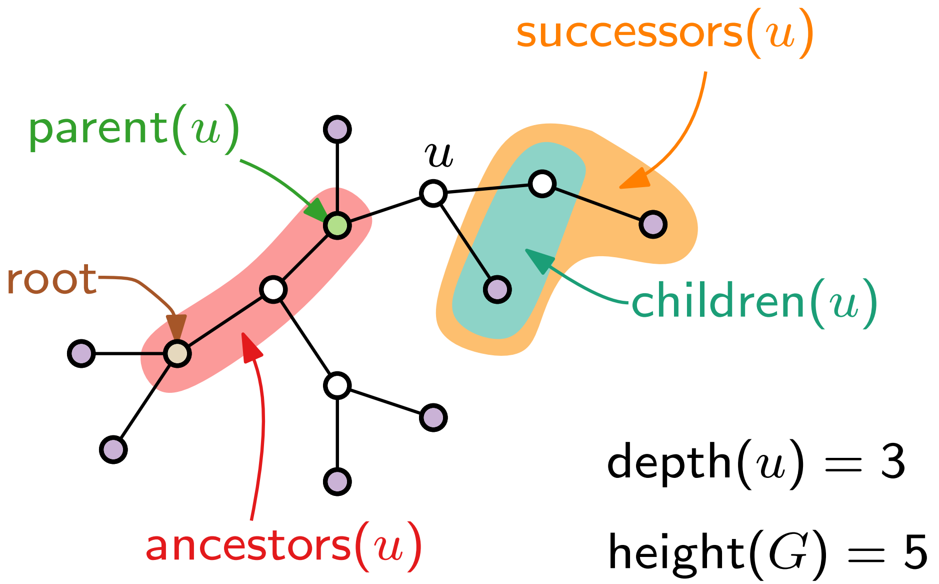

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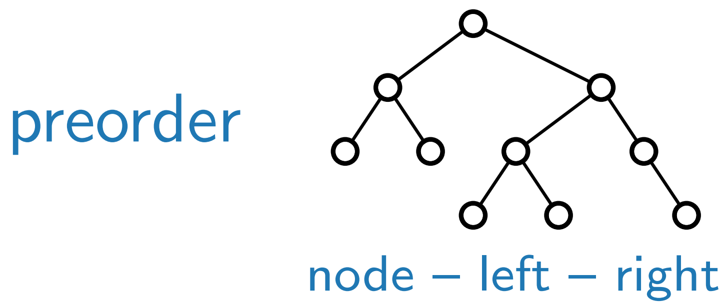

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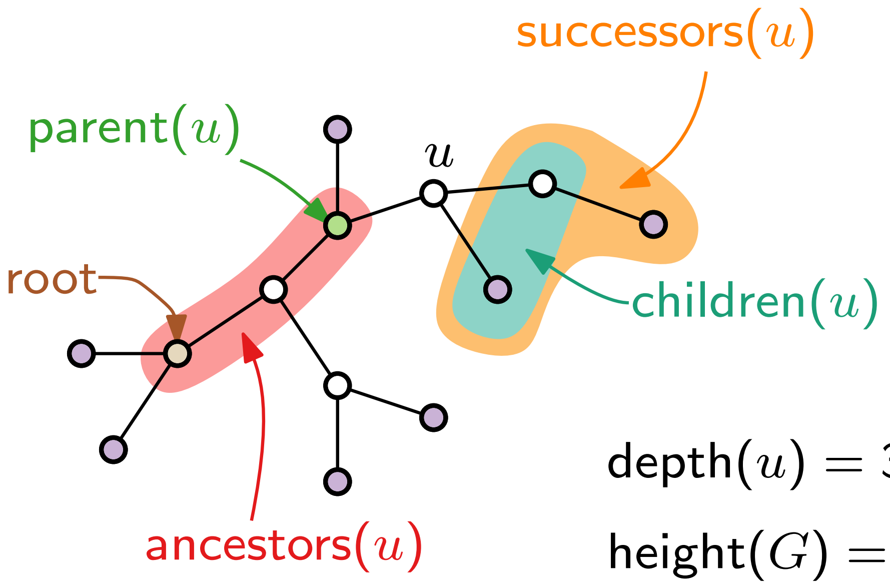
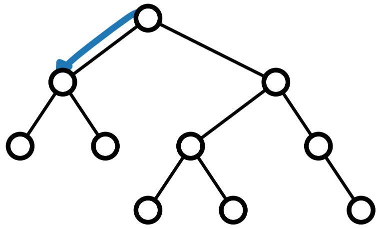


depth(u) $= 3$

height(G) $= 5$

3 traversals:

preorder



node – left – right

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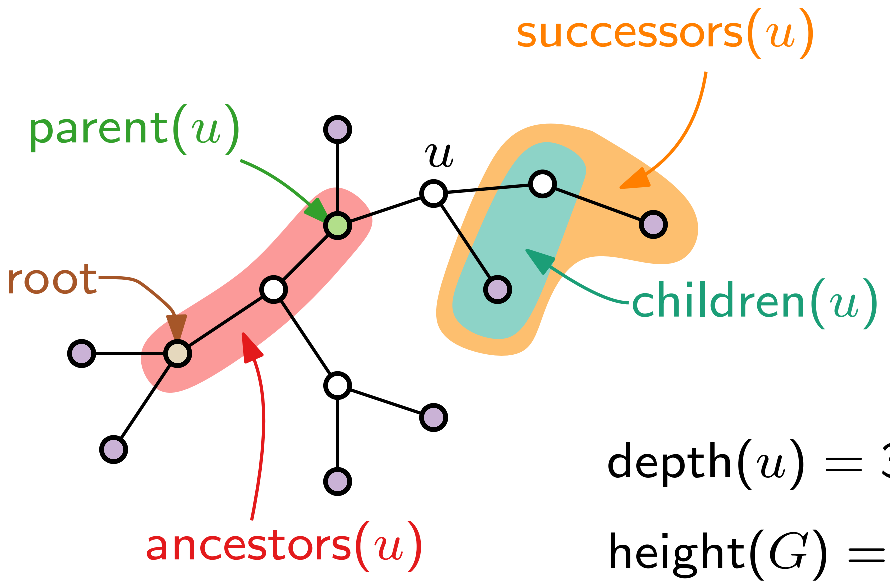

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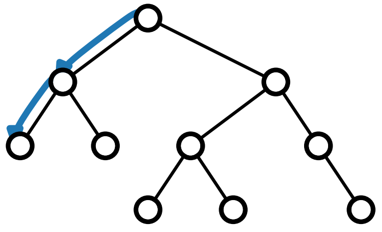

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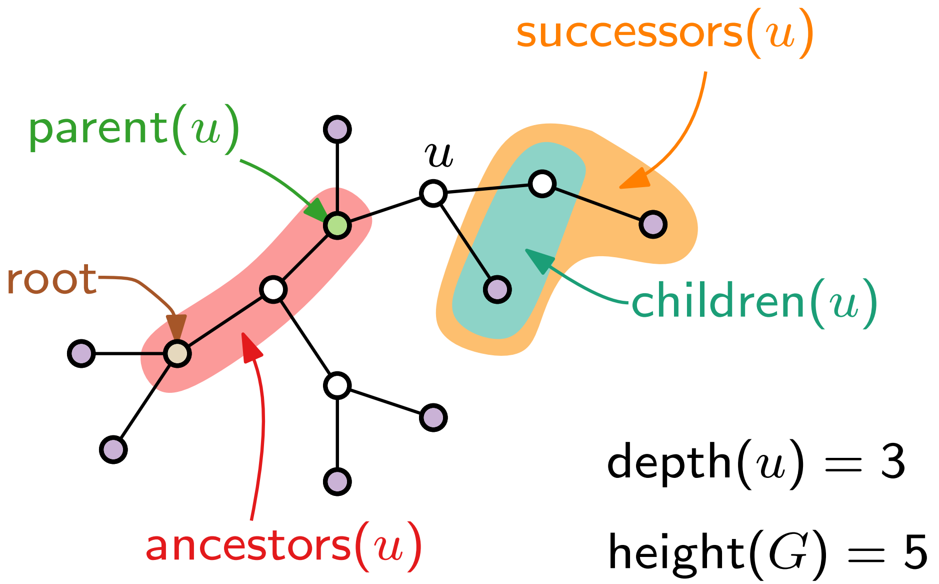

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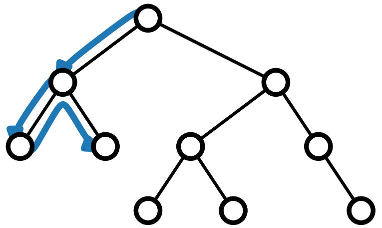
preorder



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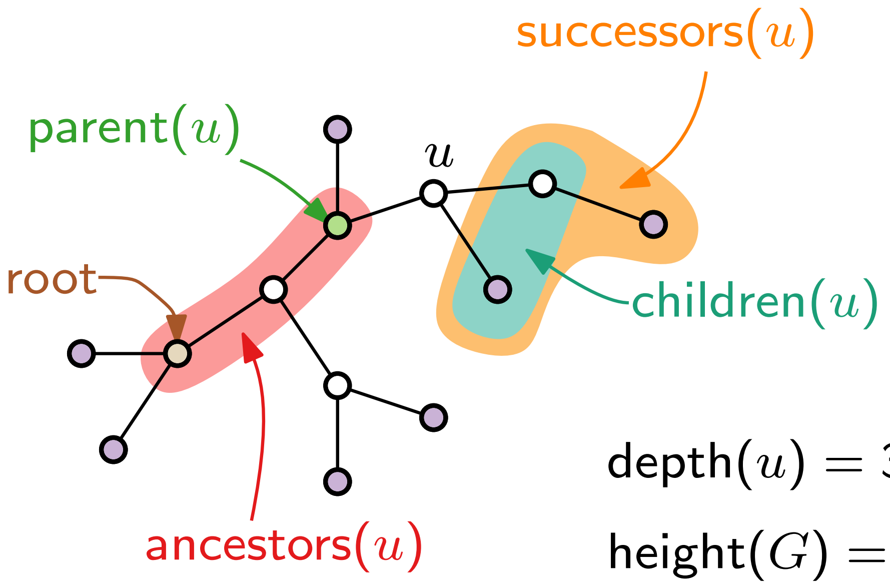

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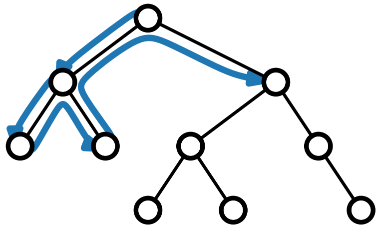

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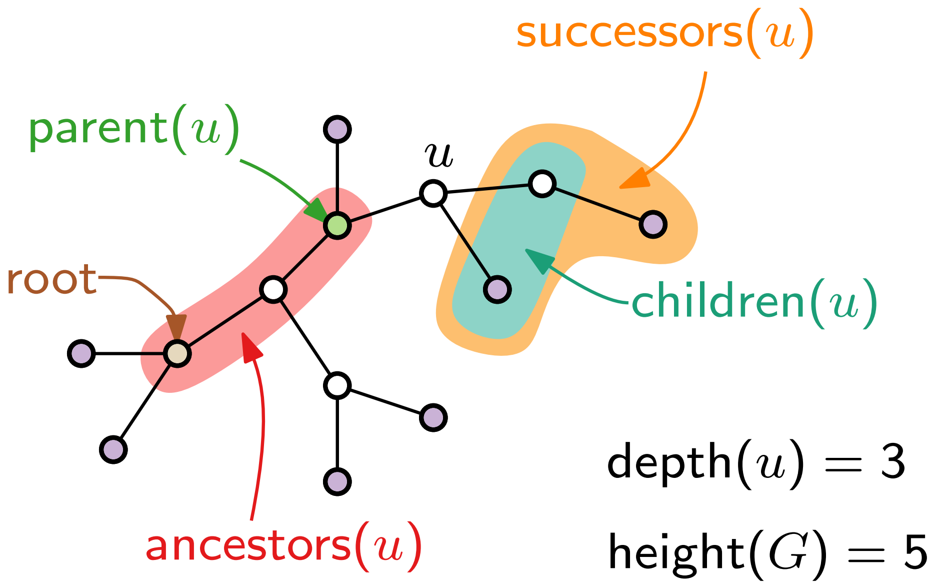

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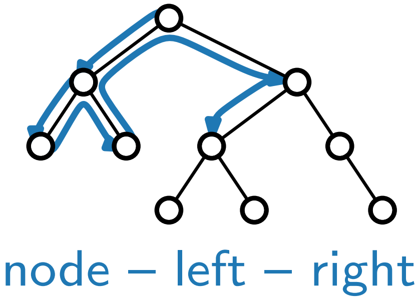
preorder



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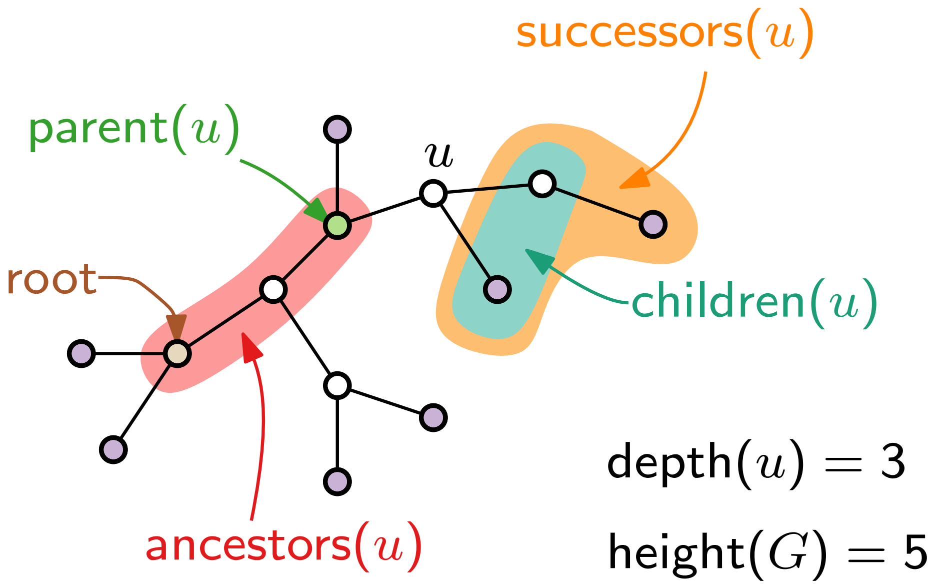

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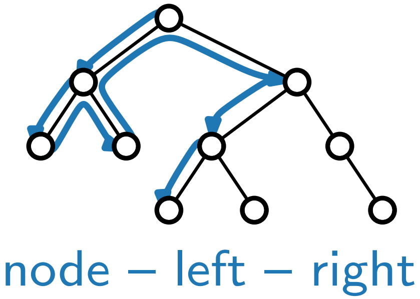
preorder



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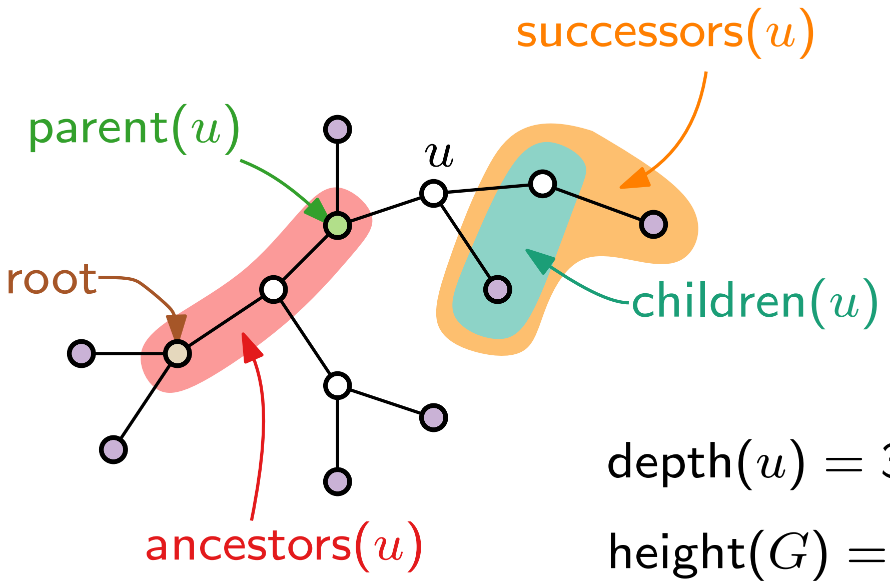

depth(u) $= 3$

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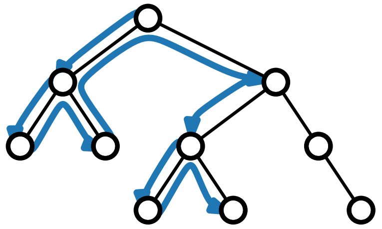
3 traversals:

preorder



node – left – right

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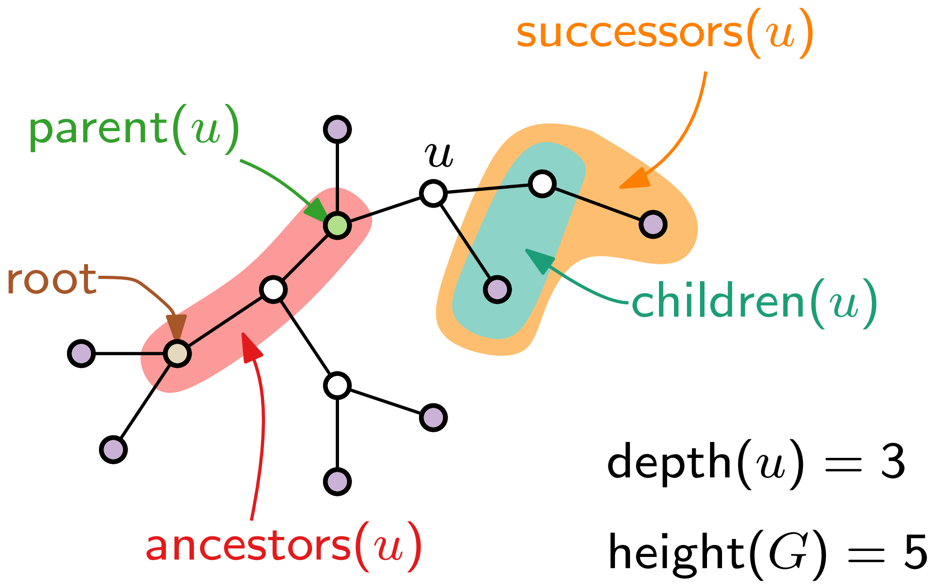
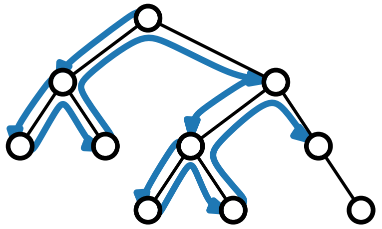


3 traversals:

preorder



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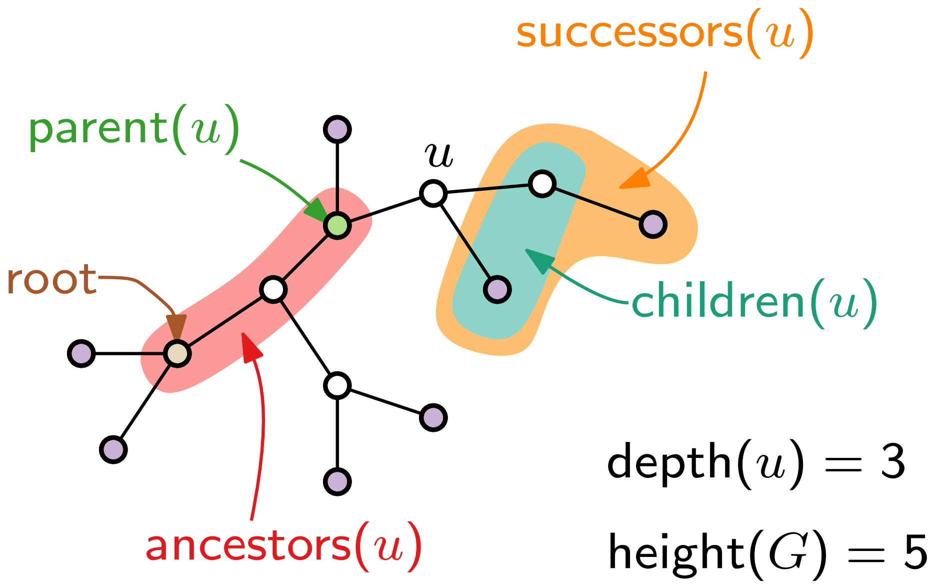

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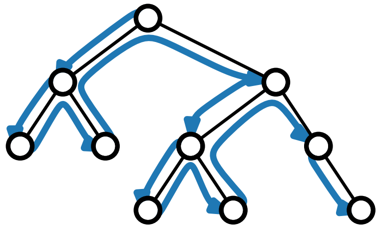
preorder



node – left – right

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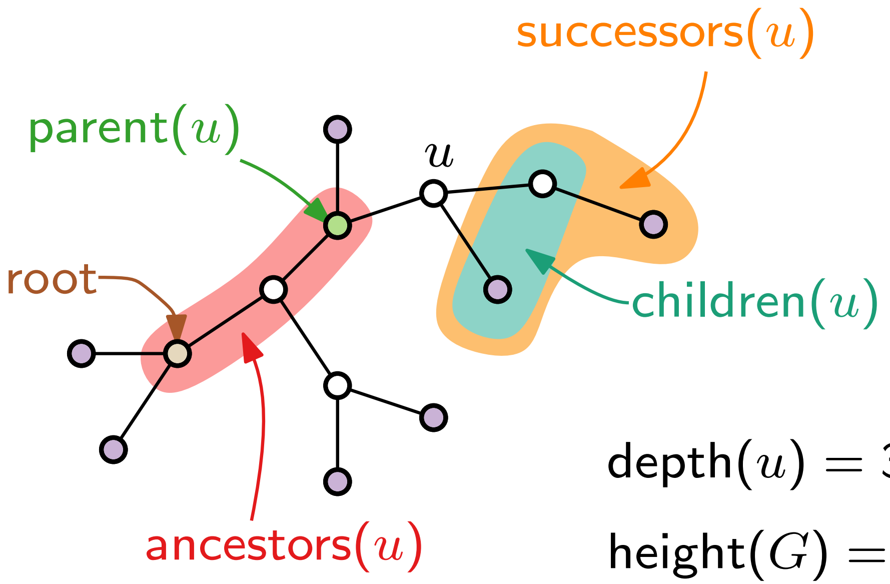

depth(u) = 3

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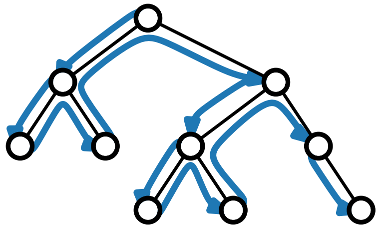
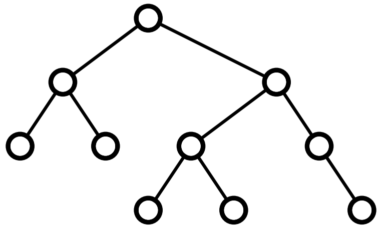
3 traversals:

preorder



node – left – right

inorder



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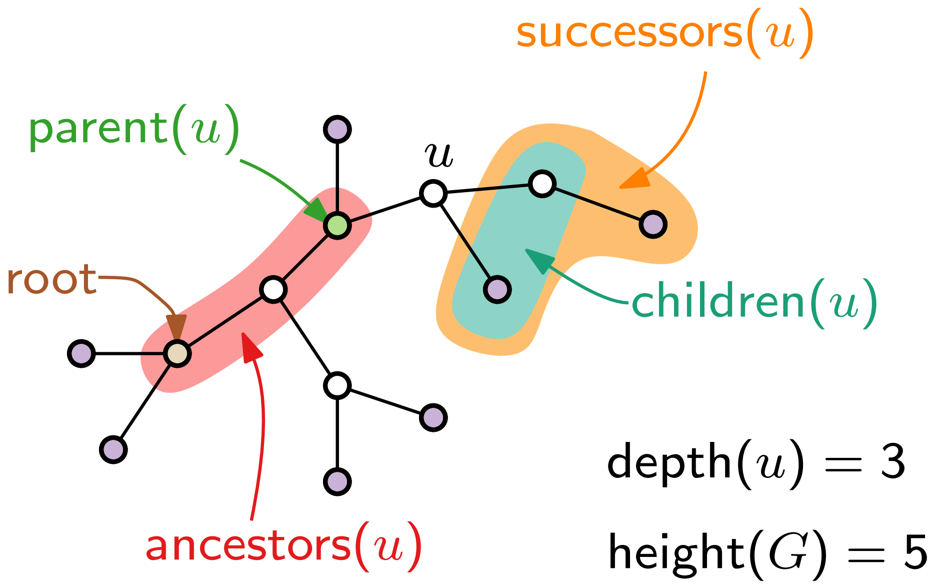
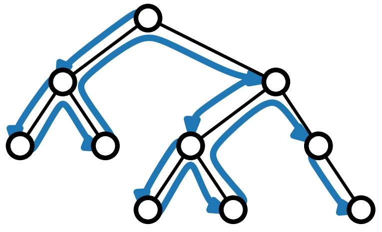


Binary Tree: At most two children per vertex (left / right child)

3 traversals:

preorder



node – left – right

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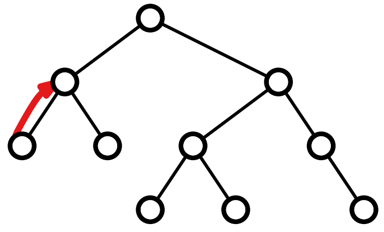

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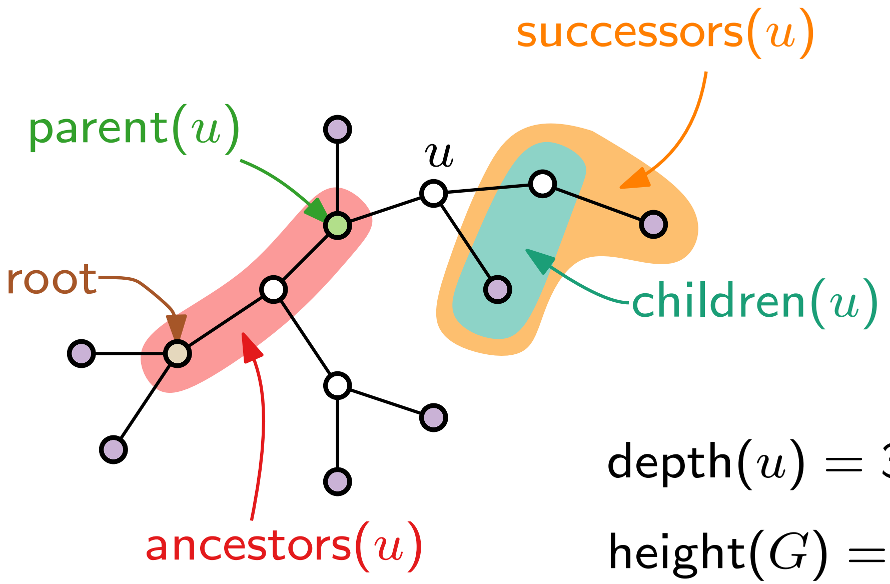

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3 traversals:

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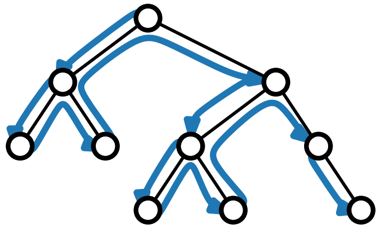
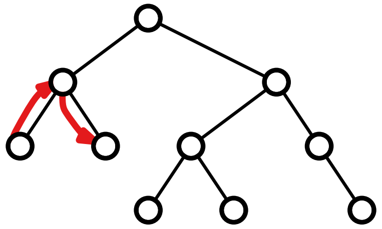


node – left – right

inorder



left – node – right

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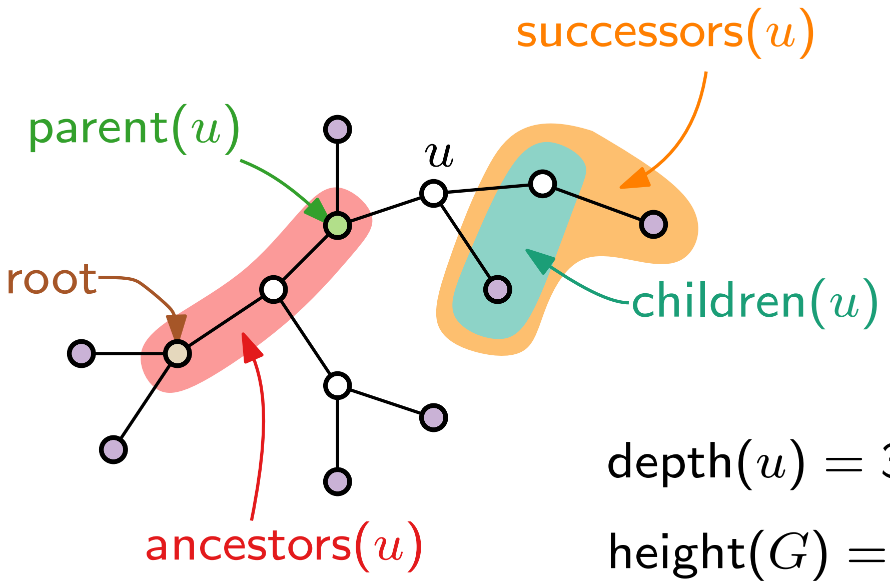

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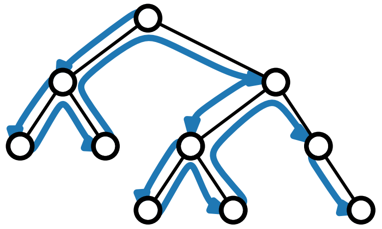

node – left – right

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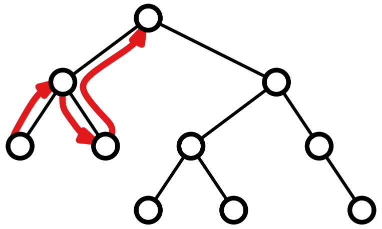

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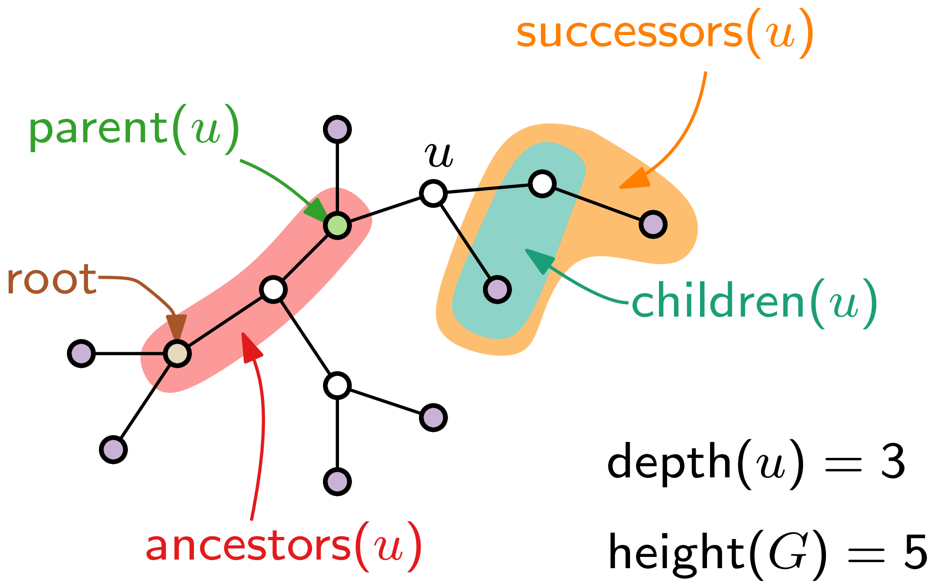
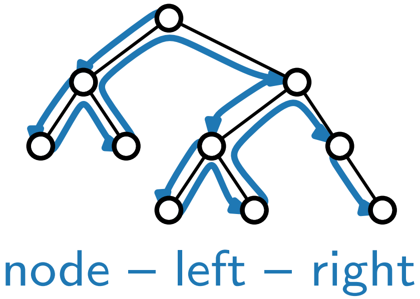


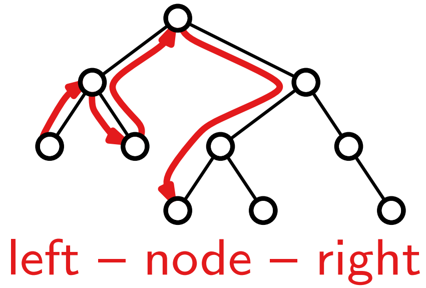
Binary Tree: At most two children per vertex (left / right child)

3 traversals:

preorder



inorder



(Rooted) Trees

Leaf: Vertex of degree 1

Rooted tree: tree with designated **root**

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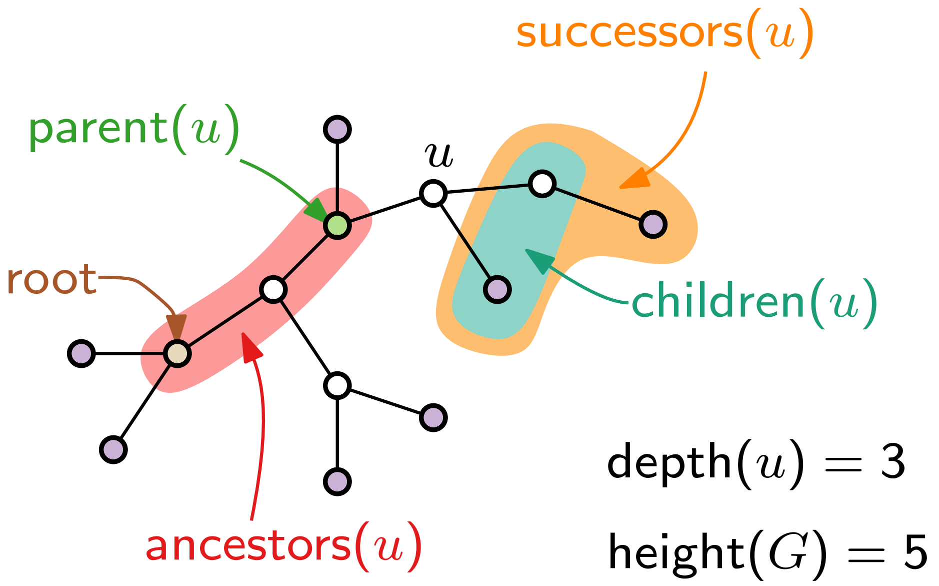

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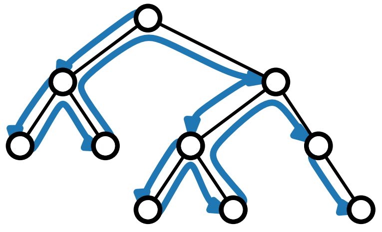
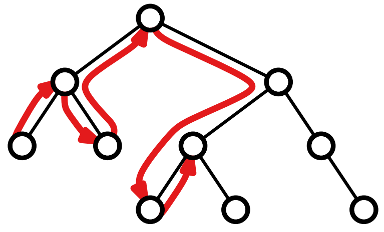


node – left – right

inorder



left – node – right

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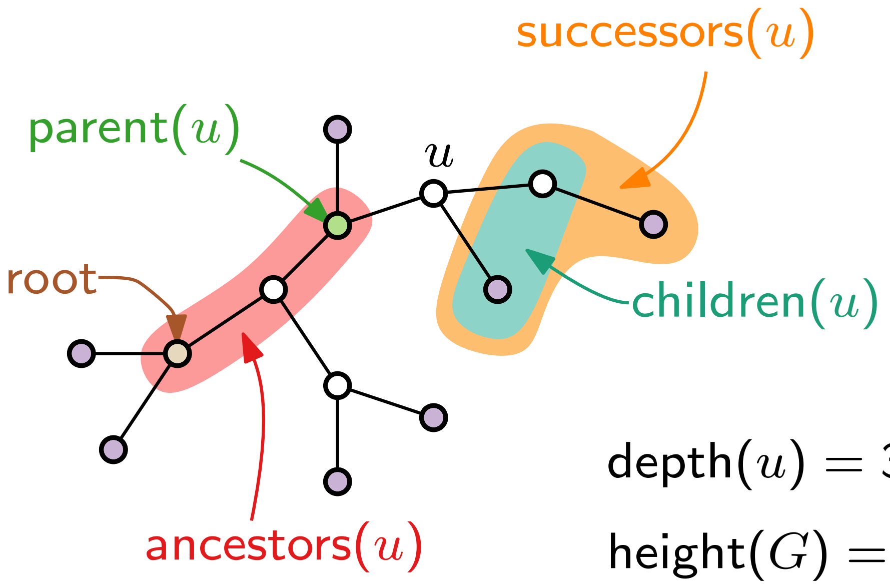


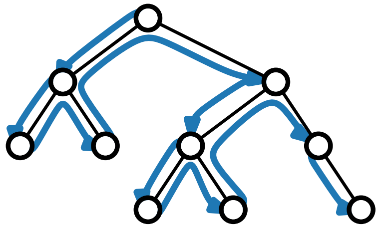
depth(u) $= 3$

height(G) $= 5$

Binary Tree: At most two children per vertex (left / right child)

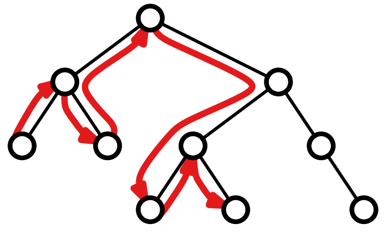
3 traversals:

preorder



node – left – right

inorder



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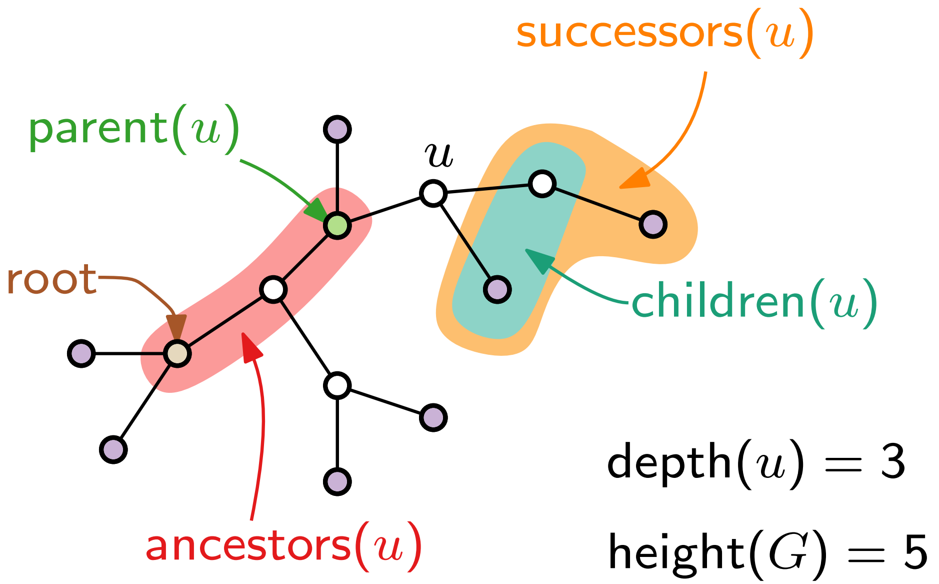
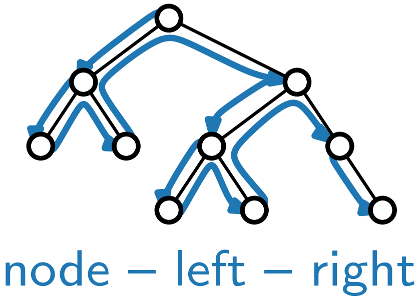
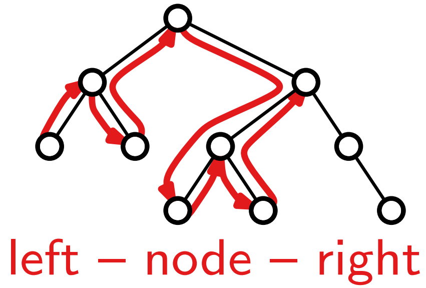


3 traversals:

preorder



inorder



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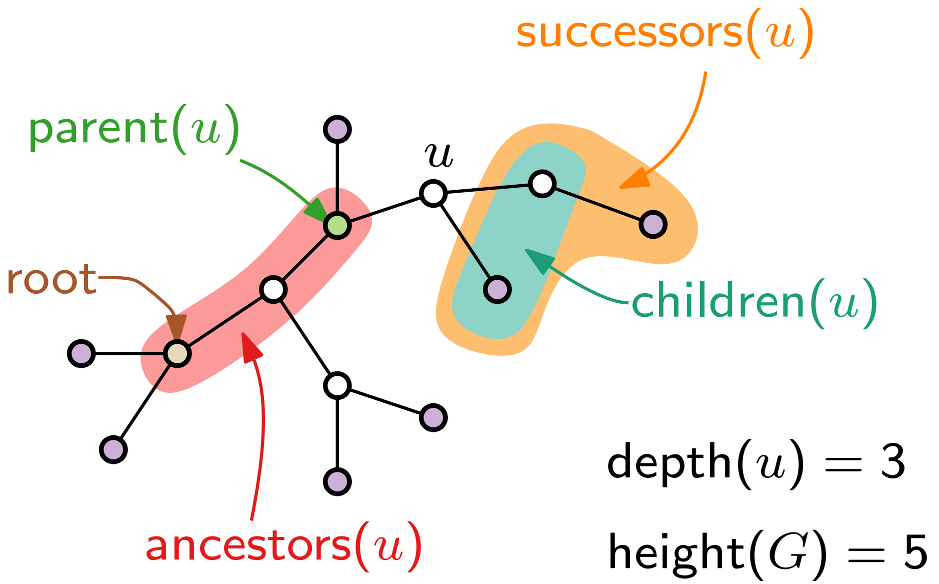

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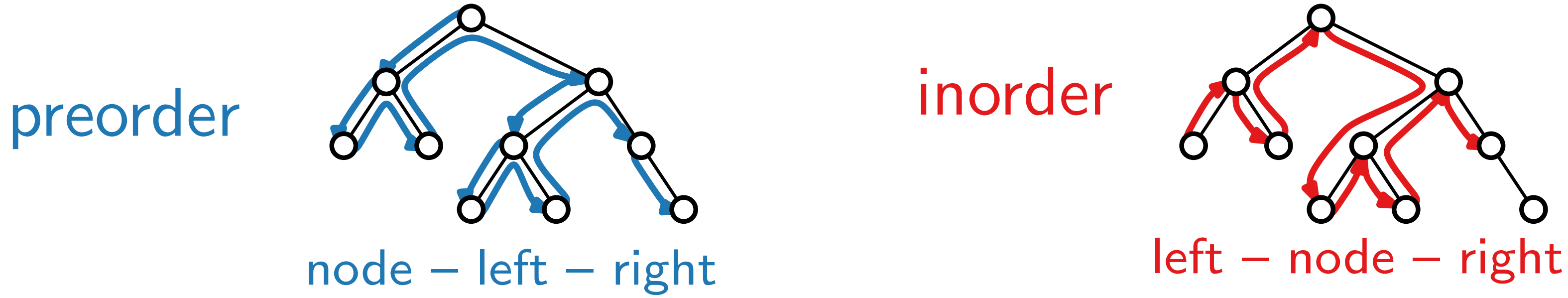

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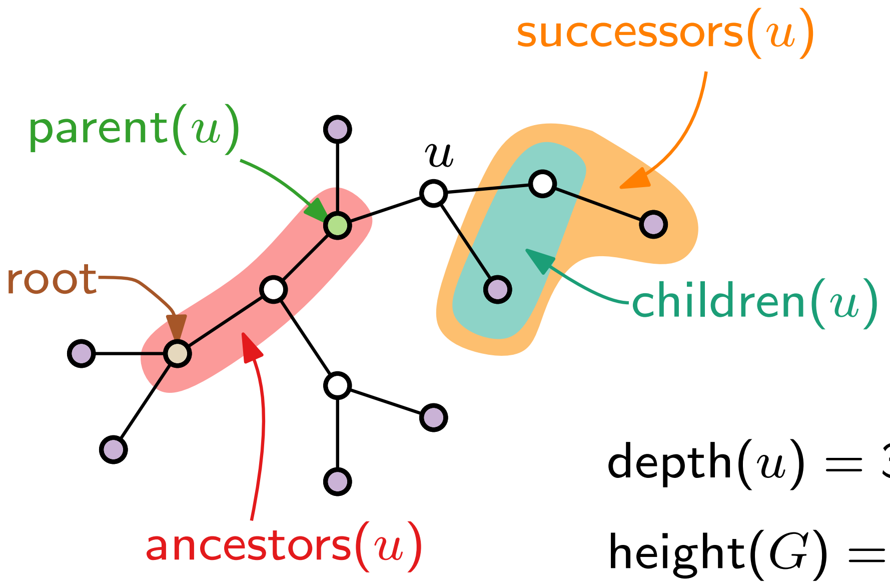

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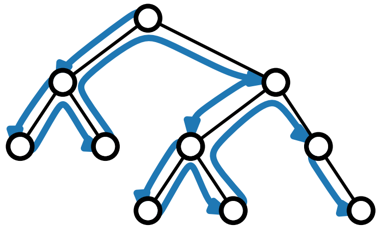
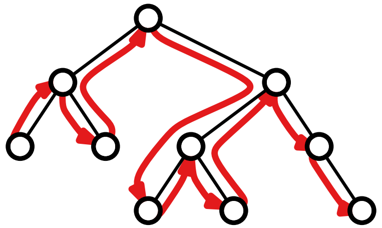


node – left – right

inorder



left – node – right

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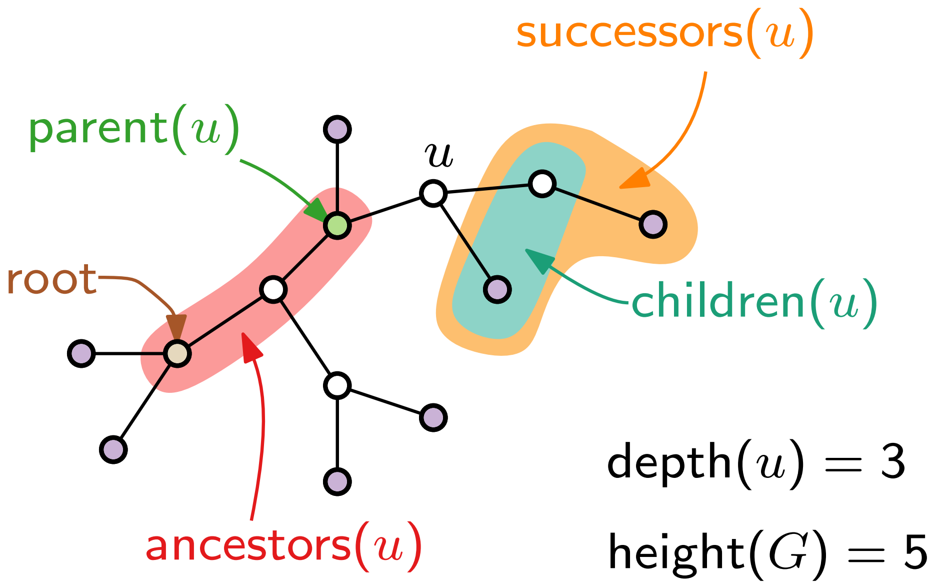
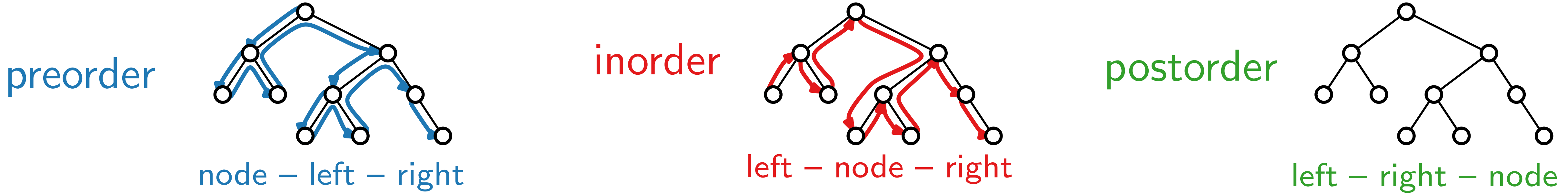


3 traversals:



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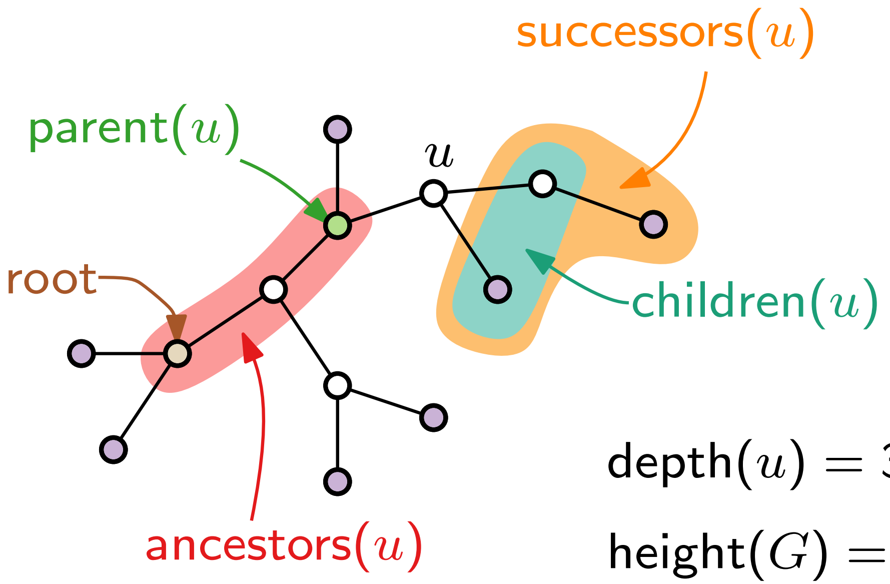

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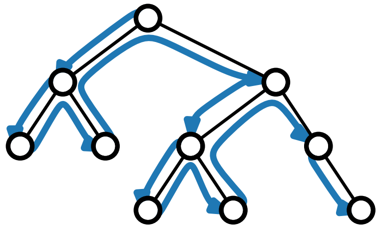
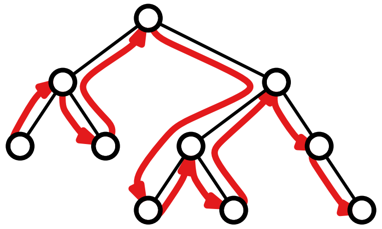


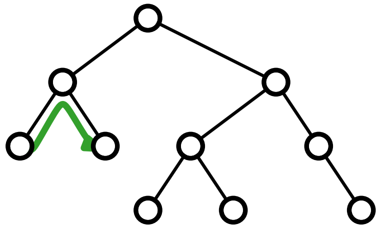
node – left – right

inorder



left – node – right

postorder



left – right – node

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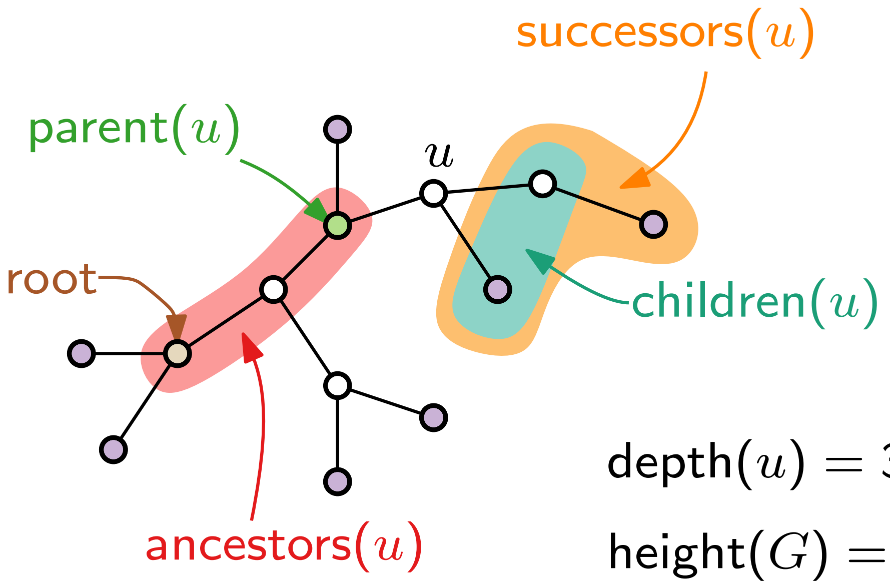

depth(u) = 3

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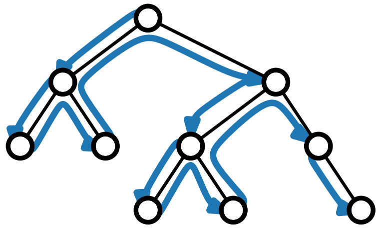
3 traversals:

preorder



node – left – right

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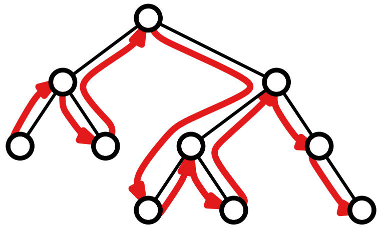
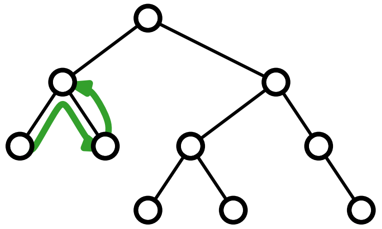


left – node – right

postorder



left – right – node

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Leaf: Vertex of degree 1
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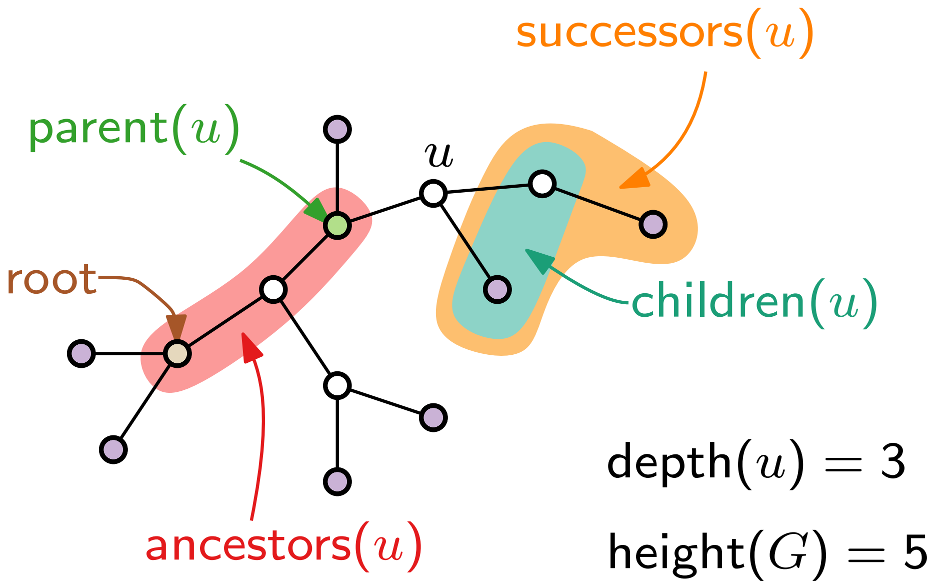

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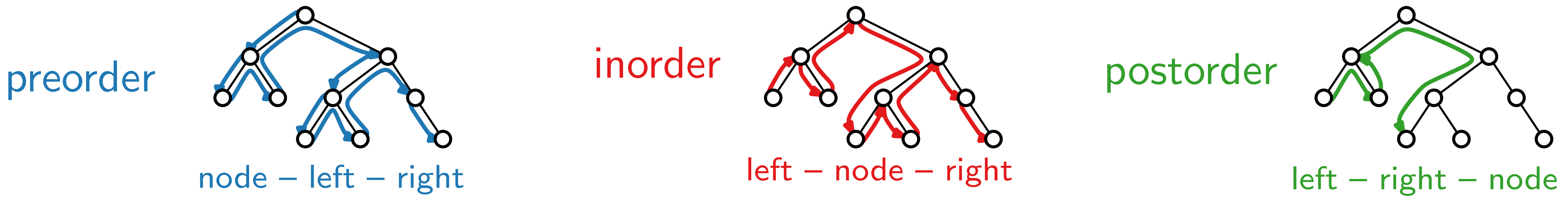

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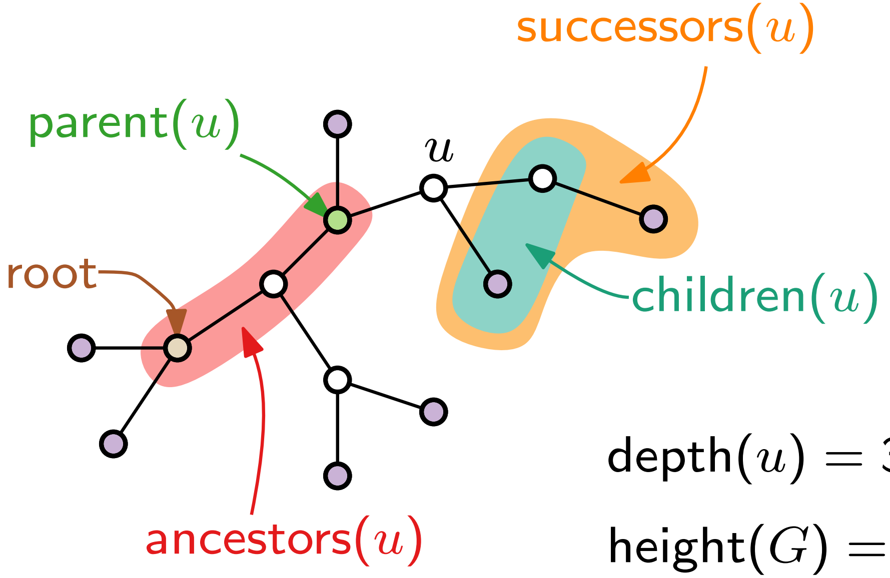

depth(u) $= 3$

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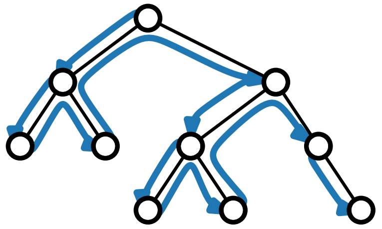
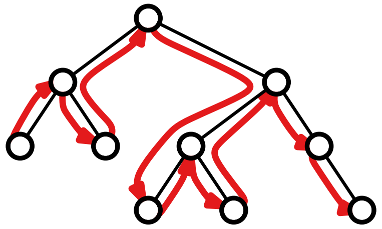
3 traversals:

preorder



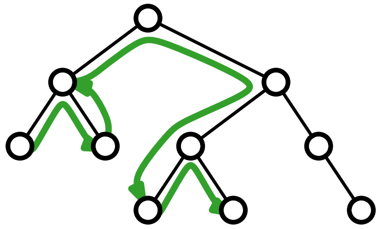
node – left – right

inorder



left – node – right

postorder



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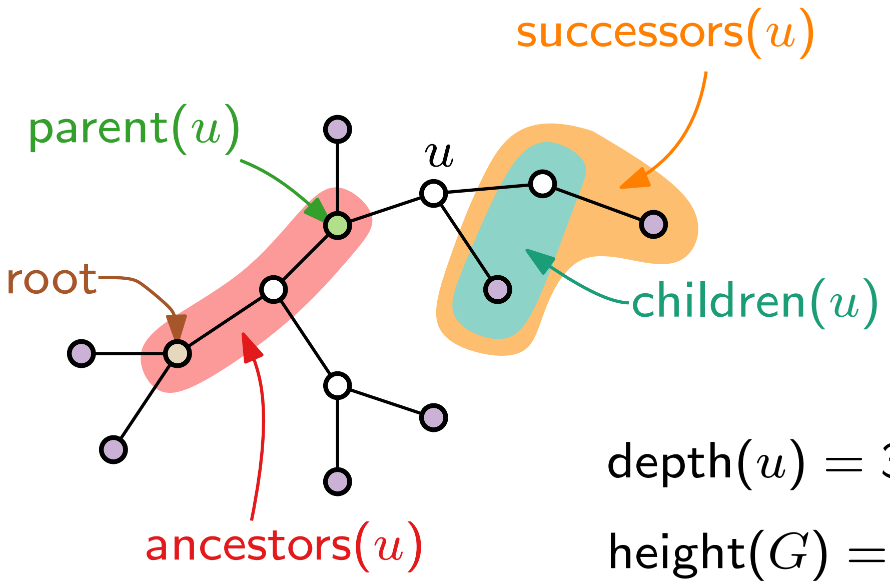

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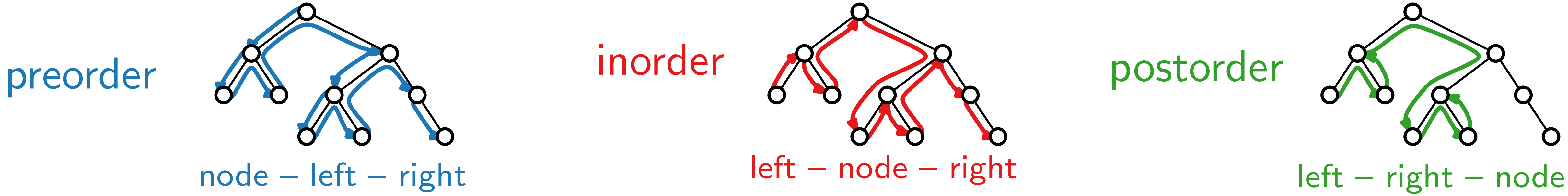

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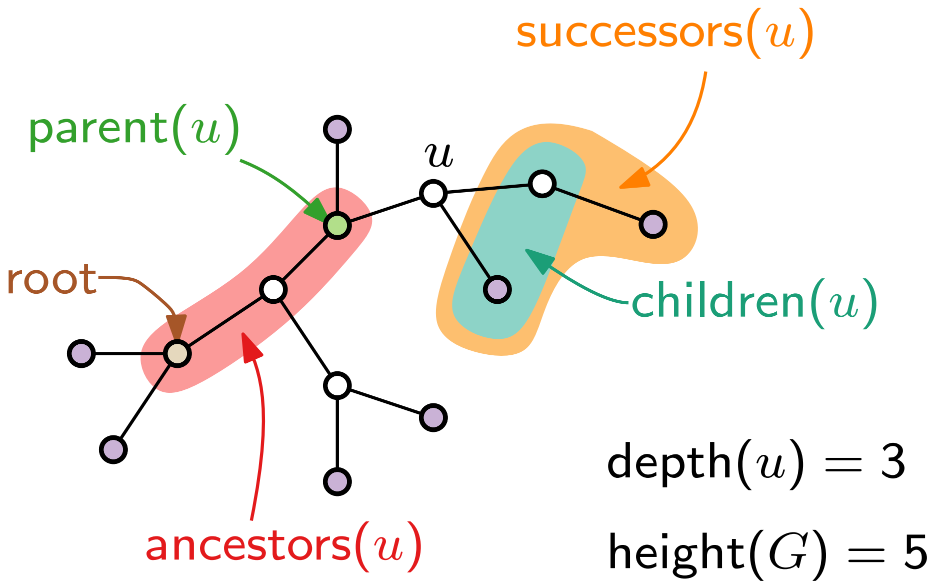

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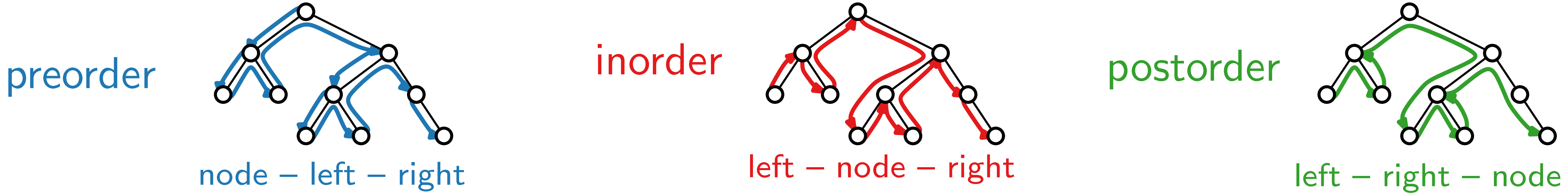

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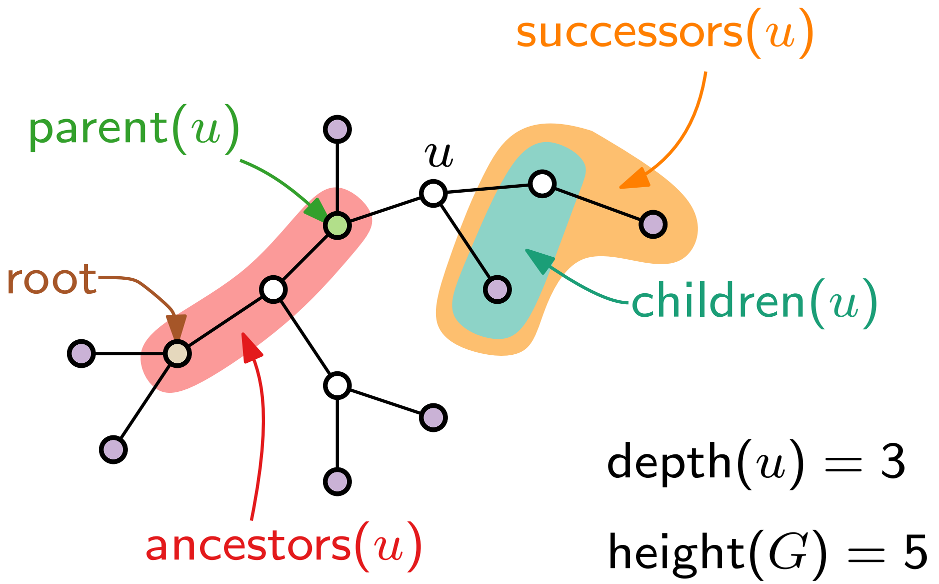

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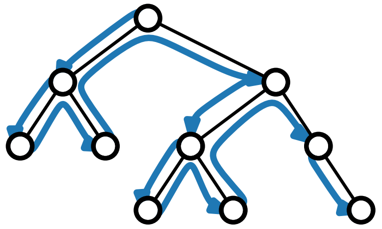
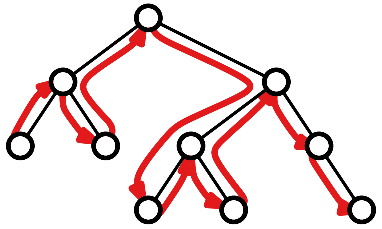


node – left – right

inorder



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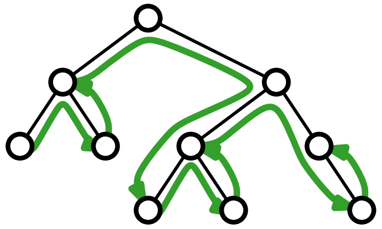

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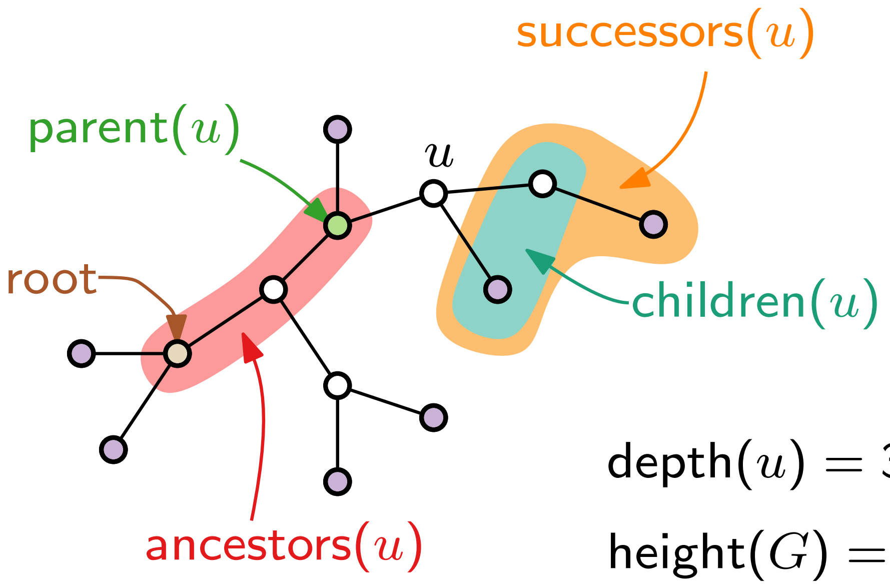

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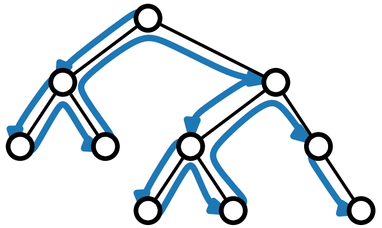
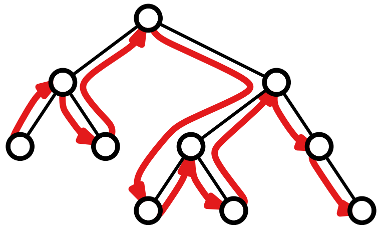
preorder

node – left – right

inorder

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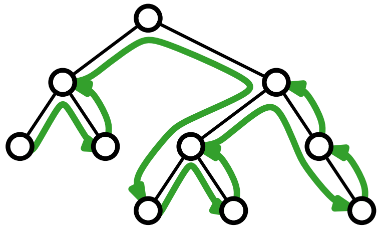
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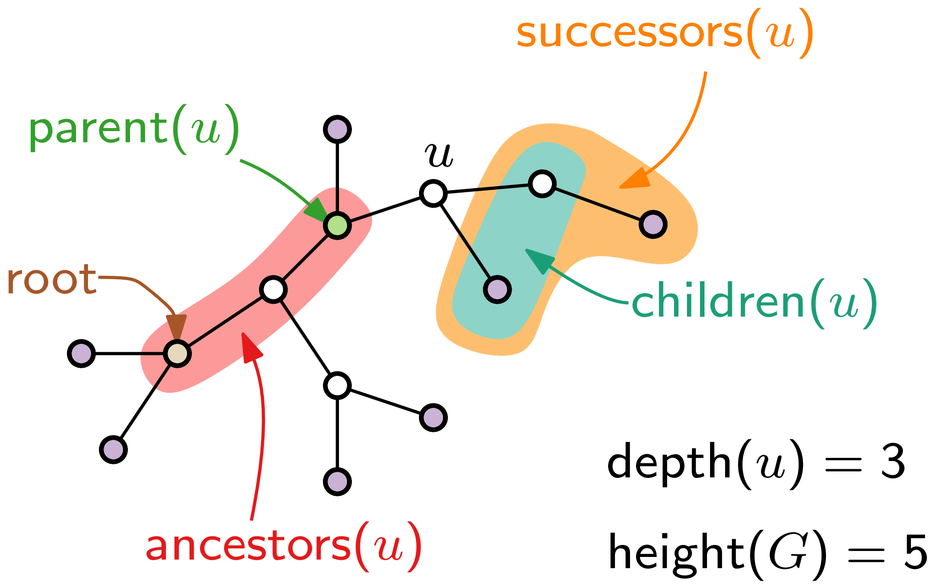

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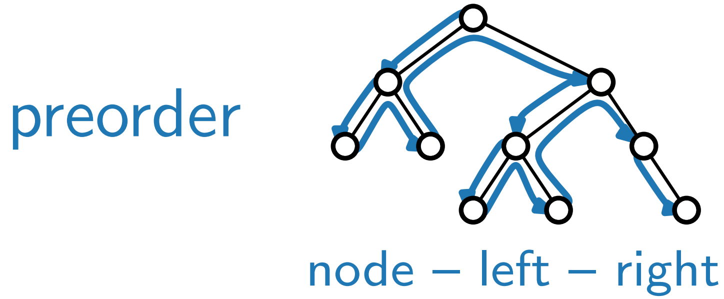

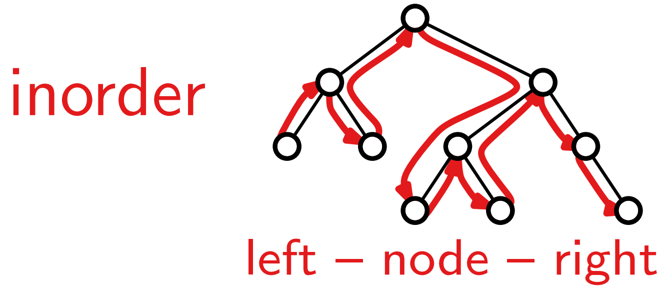

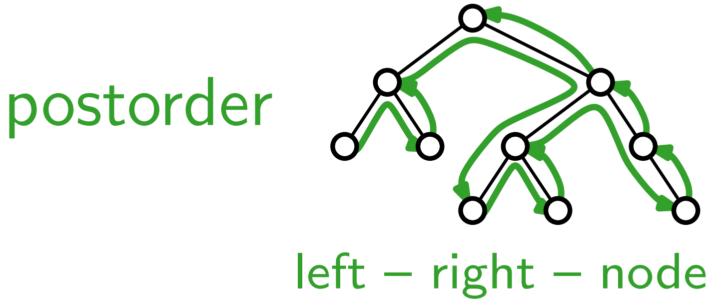

First Grid Layout of Binary Trees

1. Choose y-coordinates:

First Grid Layout of Binary Trees

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2. Choose x-coordinates:

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1. Choose y-coordinates: $y(u) =$depth(u)

2. Choose x-coordinates:

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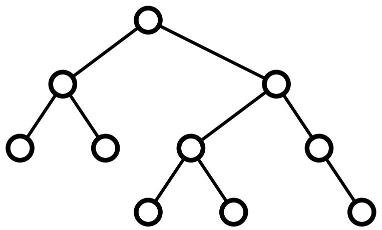

2. Choose x-coordinates:

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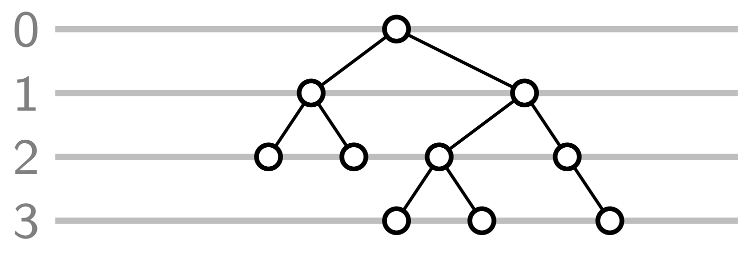

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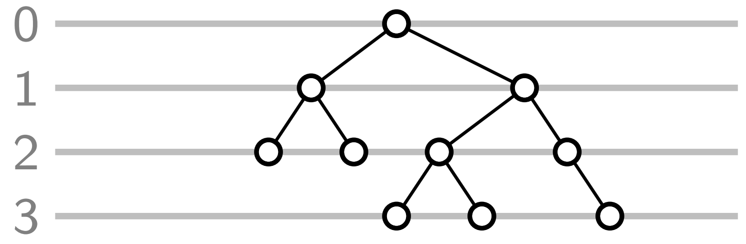

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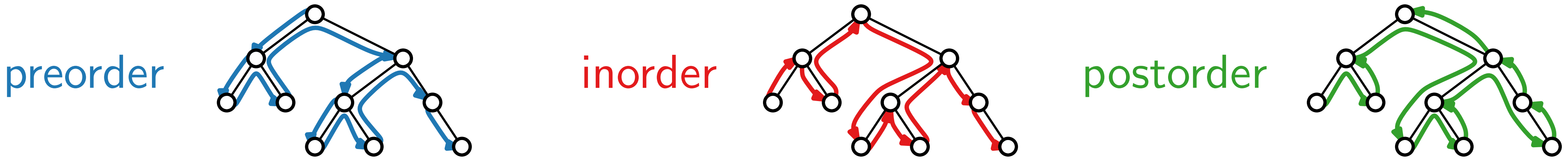

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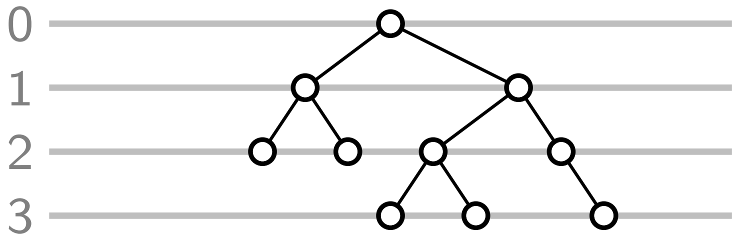
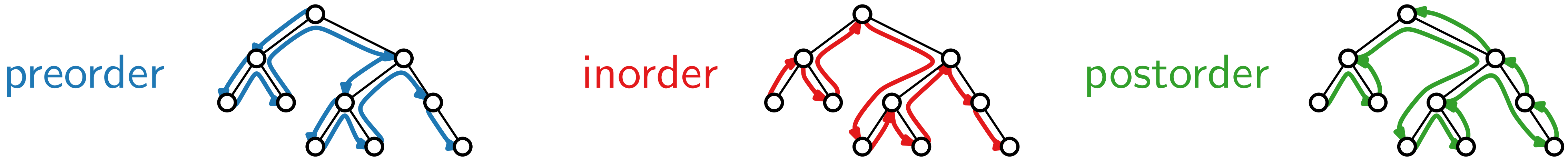
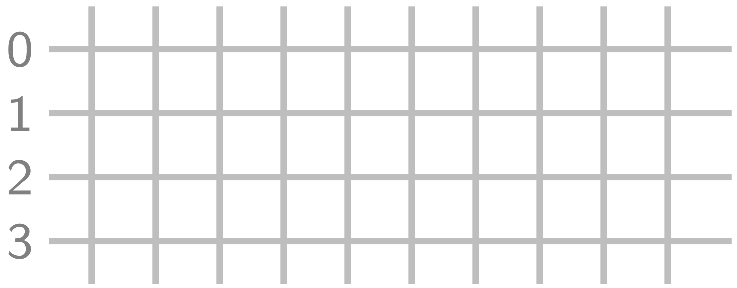


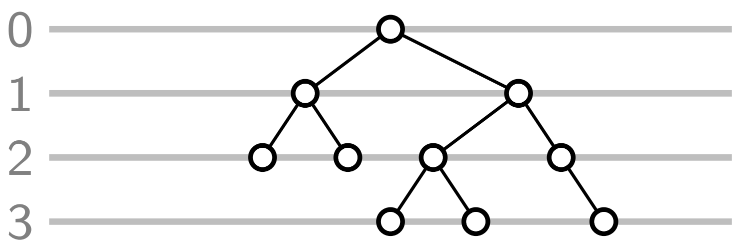
2. Choose x-coordinates:





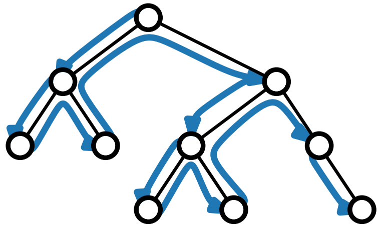
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) = \text{depth}(u)$

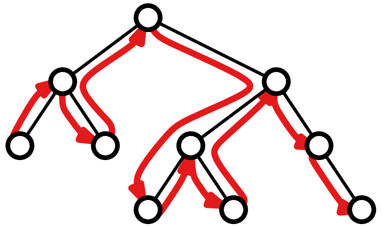


2. Choose x-coordinates:

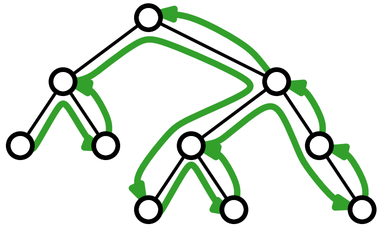
preorder

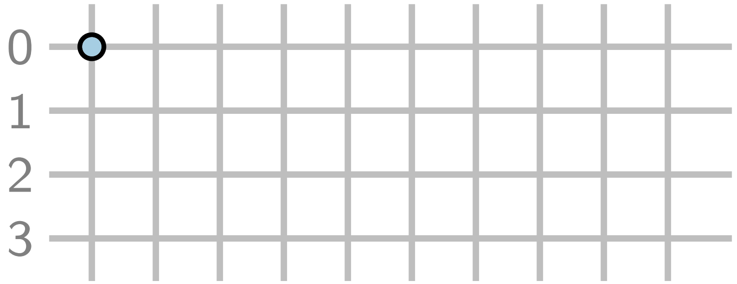


inorder



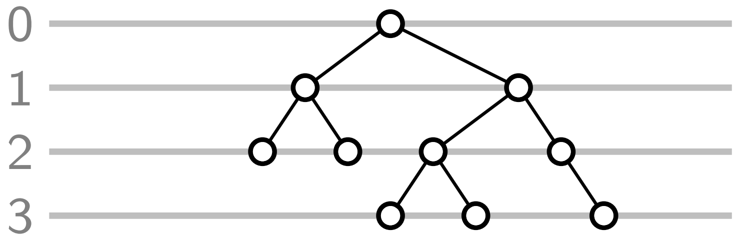
postorder





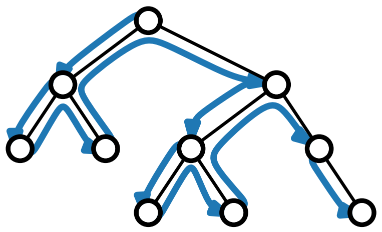
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) = $depth$(u)$

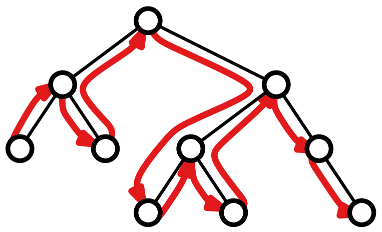


2. Choose x-coordinates:

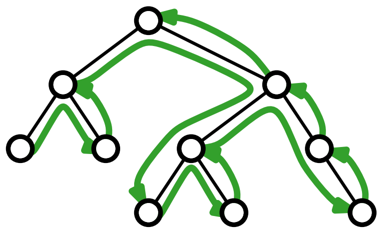
preorder

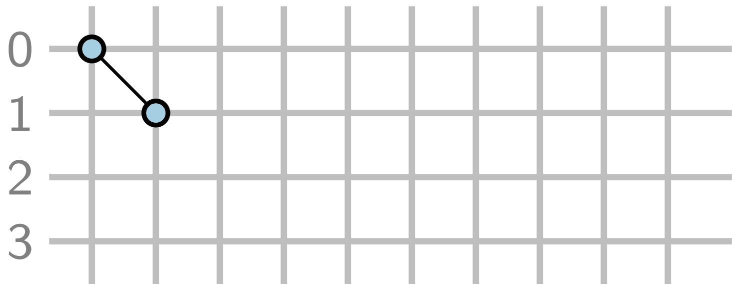


inorder



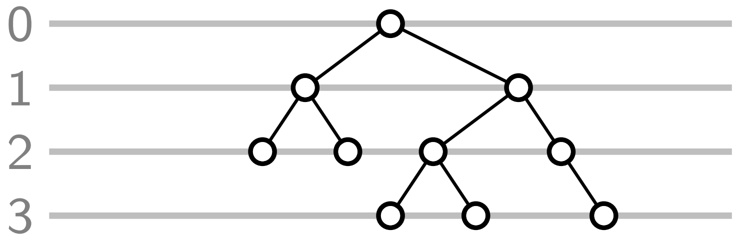
postorder



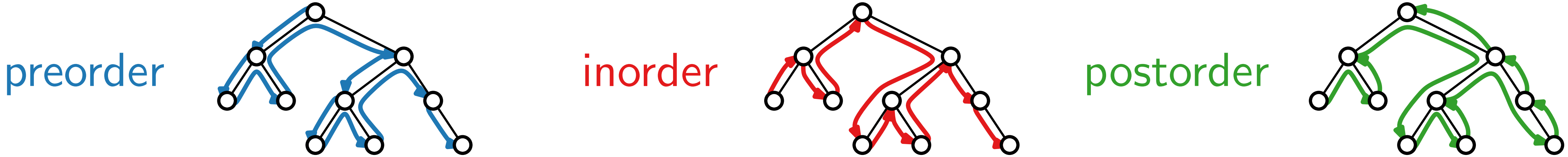


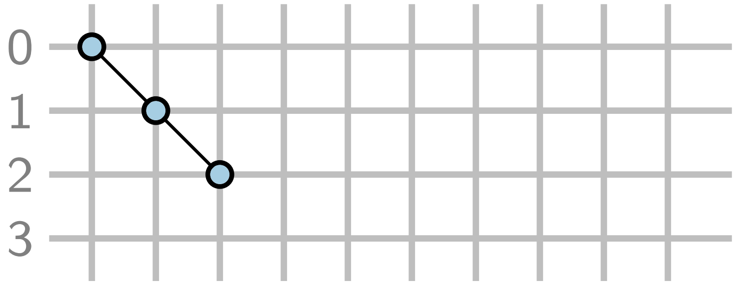
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)



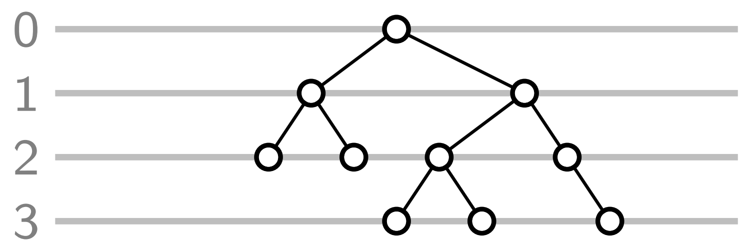
2. Choose x-coordinates:



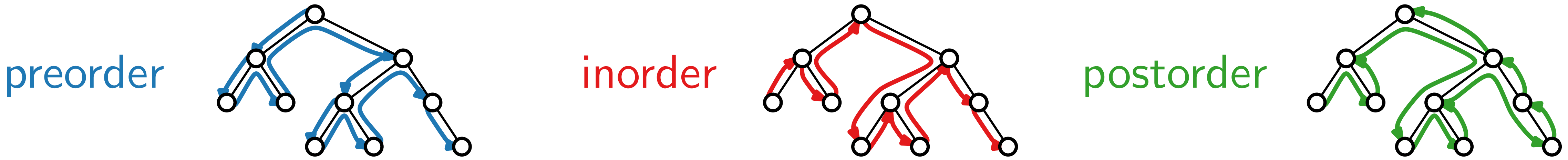


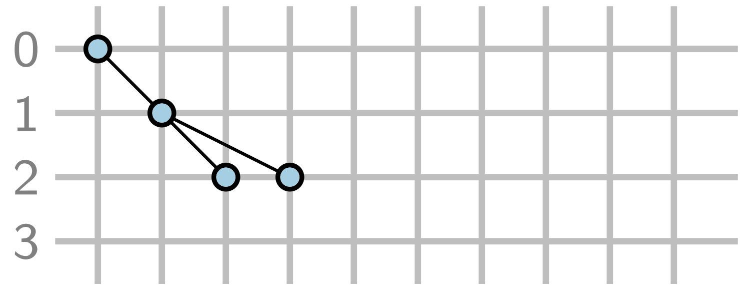
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)



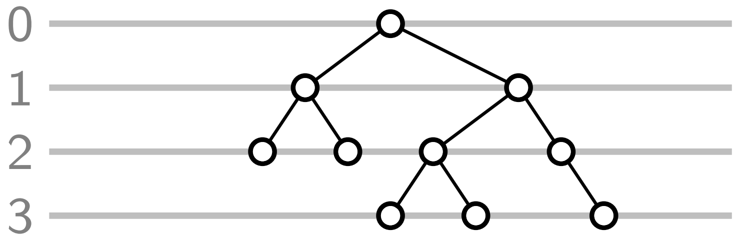
2. Choose x-coordinates:



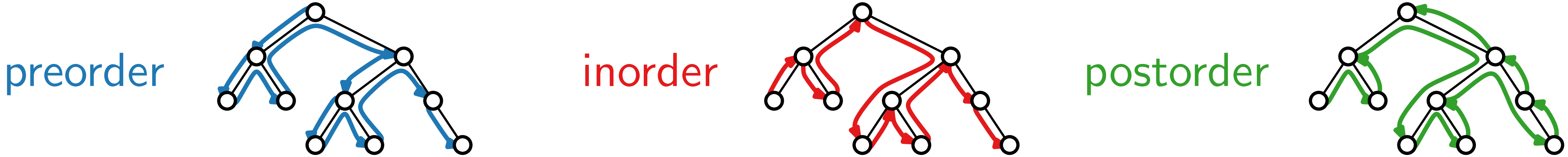


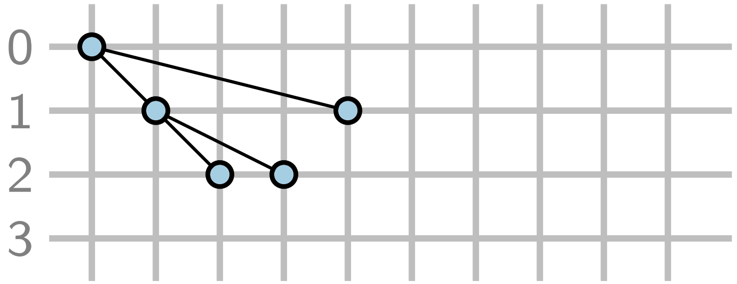
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)



2. Choose x-coordinates:





First Grid Layout of Binary Trees

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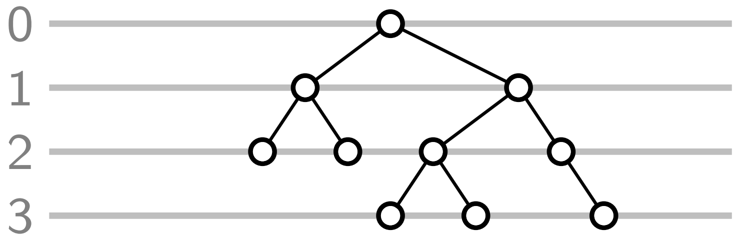
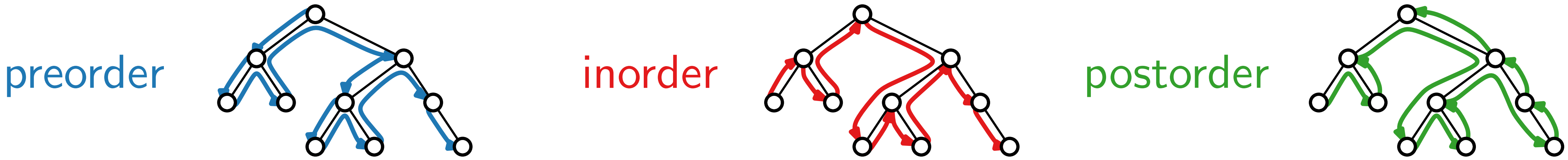
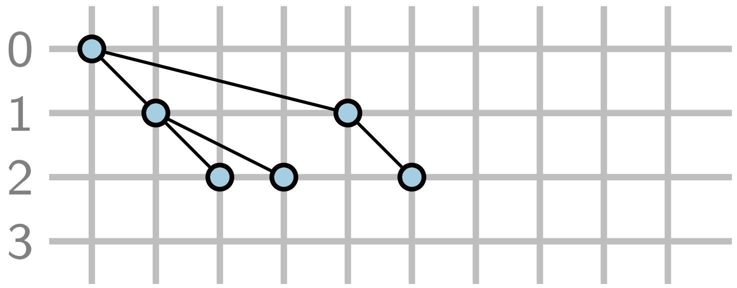


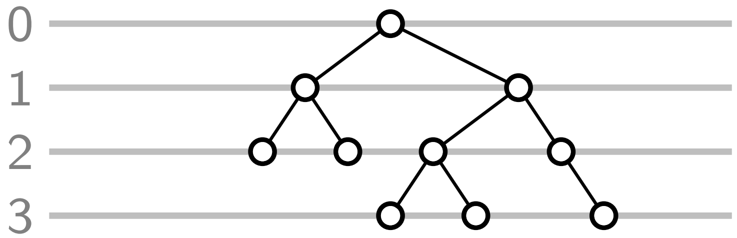
2. Choose x-coordinates:





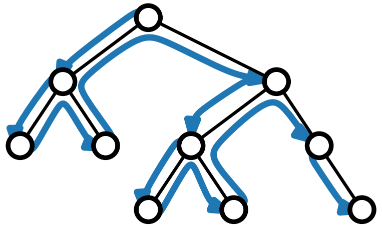
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)

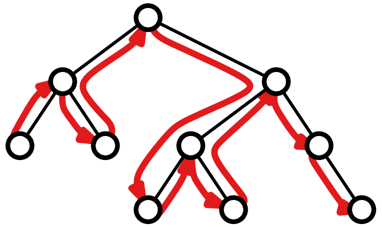


2. Choose x-coordinates:

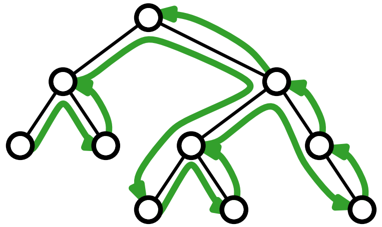
preorder

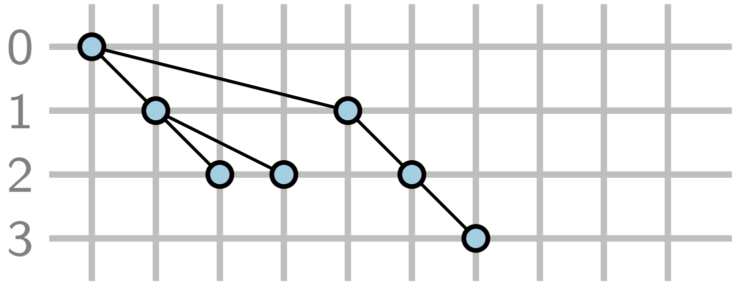


inorder



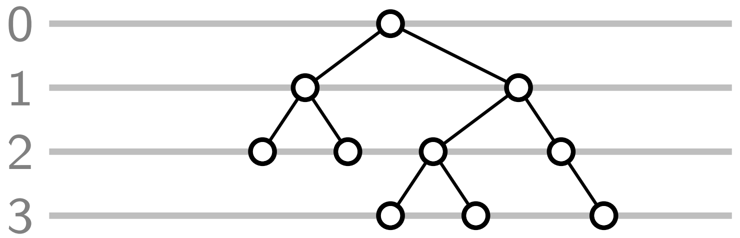
postorder



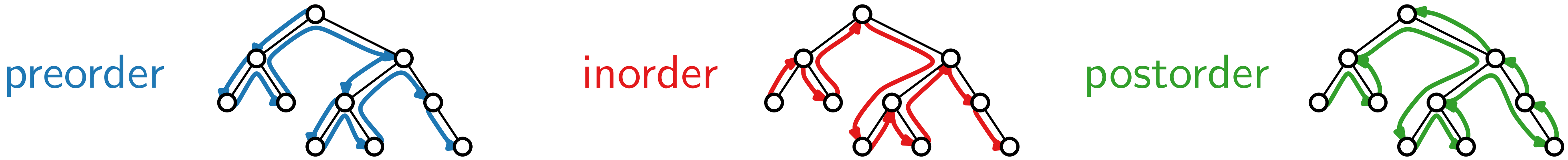


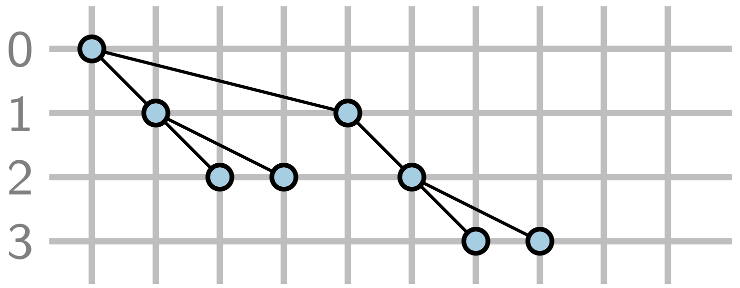
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)



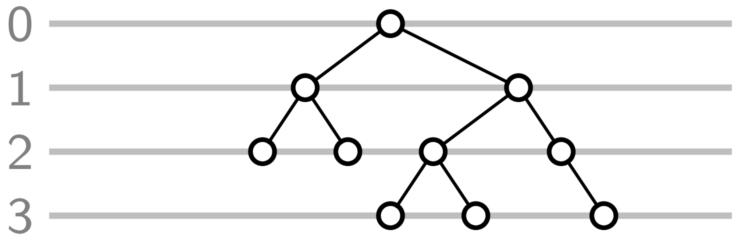
2. Choose x-coordinates:



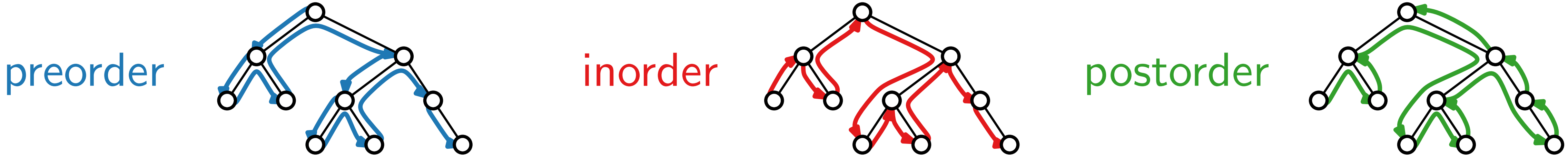


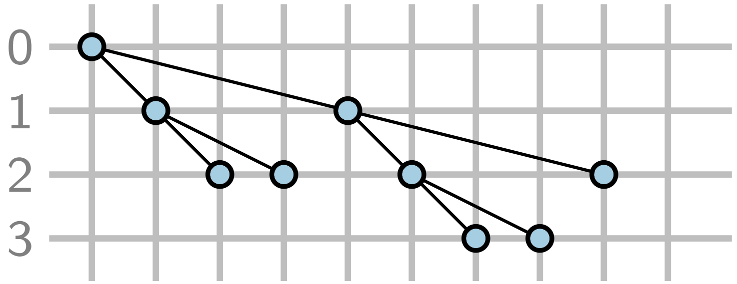
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)



2. Choose x-coordinates:





First Grid Layout of Binary Trees

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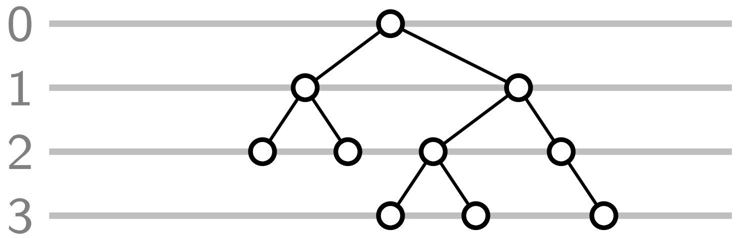
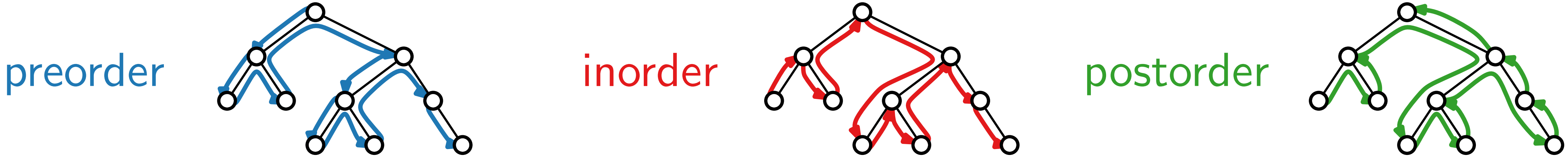
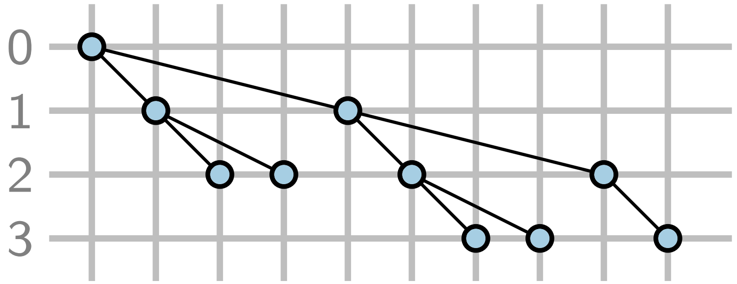


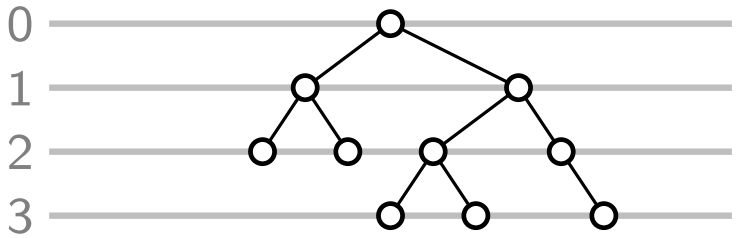
2. Choose x-coordinates:



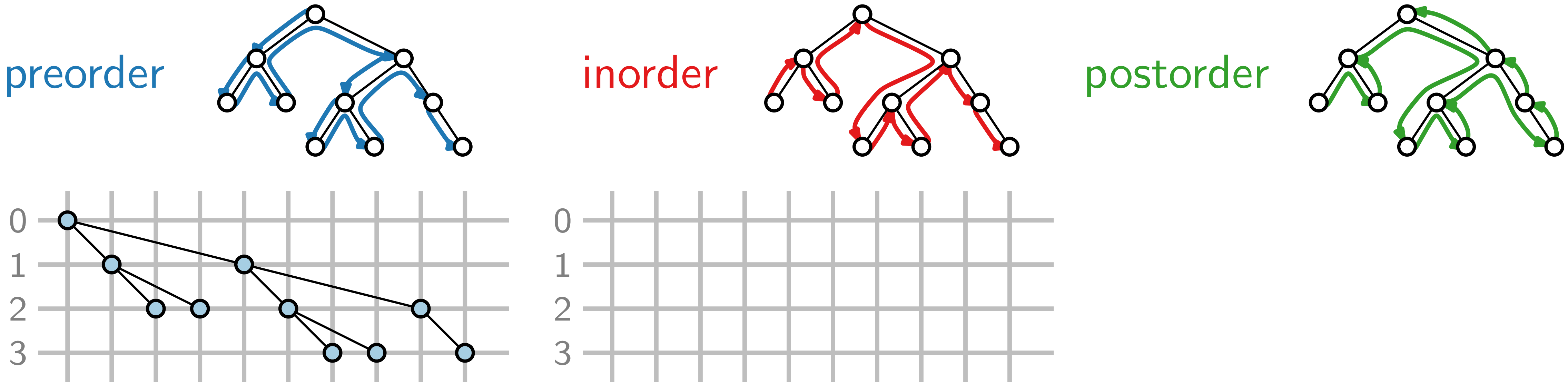


First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)



2. Choose x-coordinates:



First Grid Layout of Binary Trees

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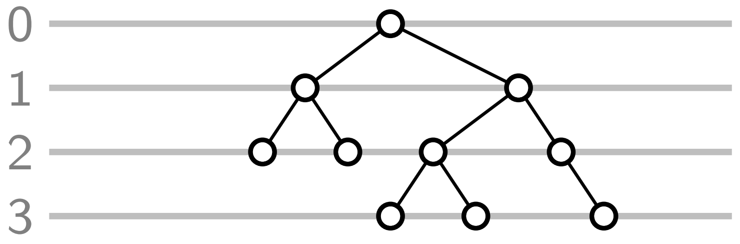
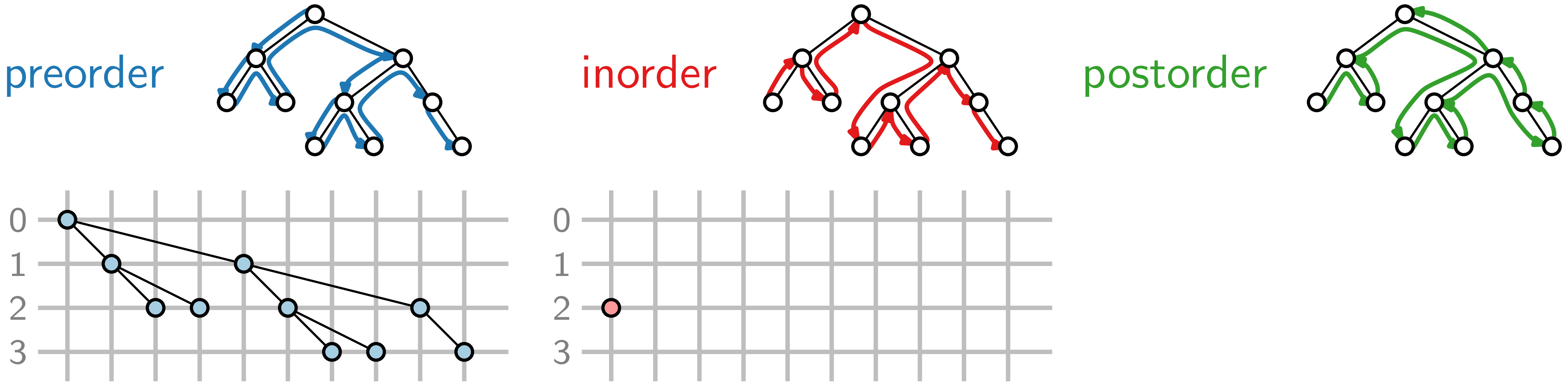
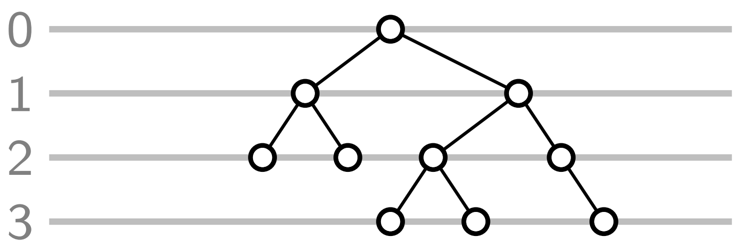


2. Choose x-coordinates:



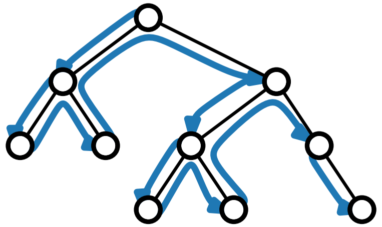
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)

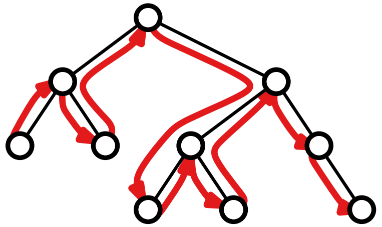


2. Choose x-coordinates:

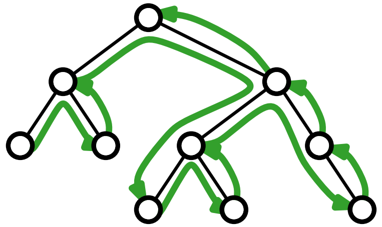
preorder

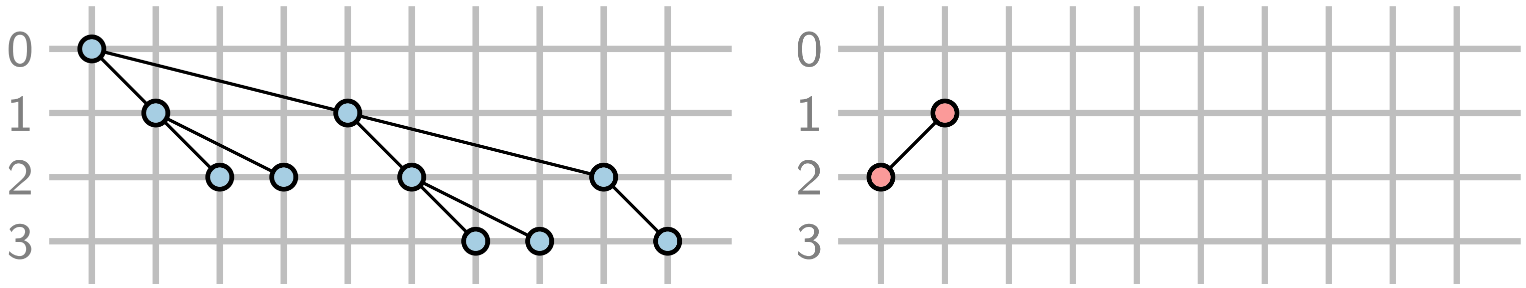


inorder



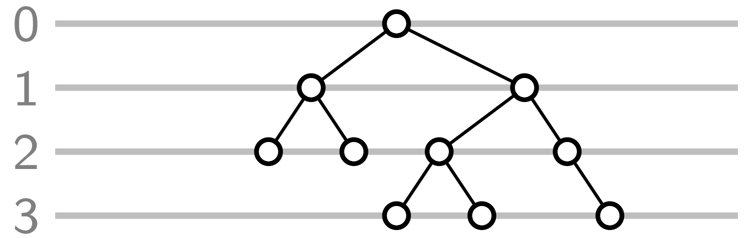
postorder





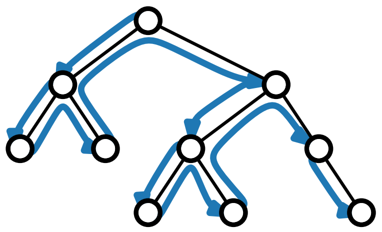
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)

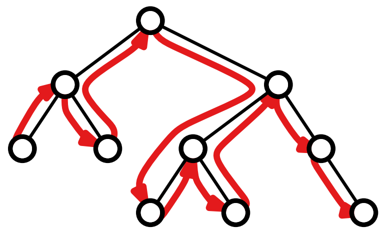


2. Choose x-coordinates:

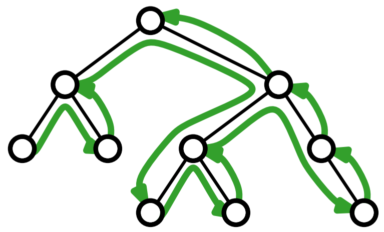
preorder

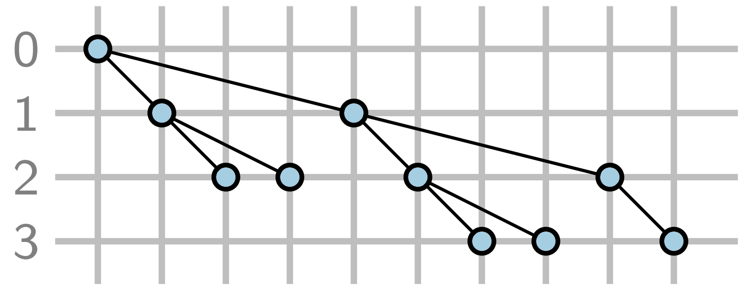


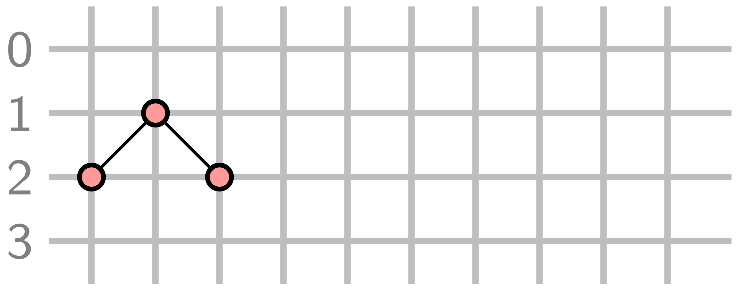
inorder



postorder

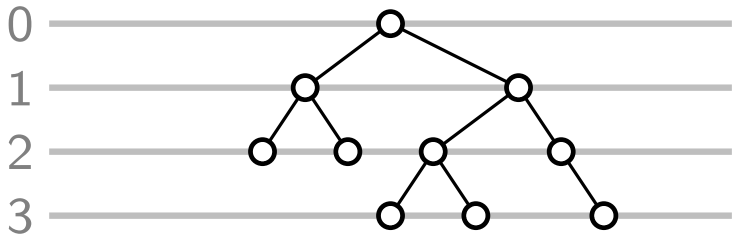




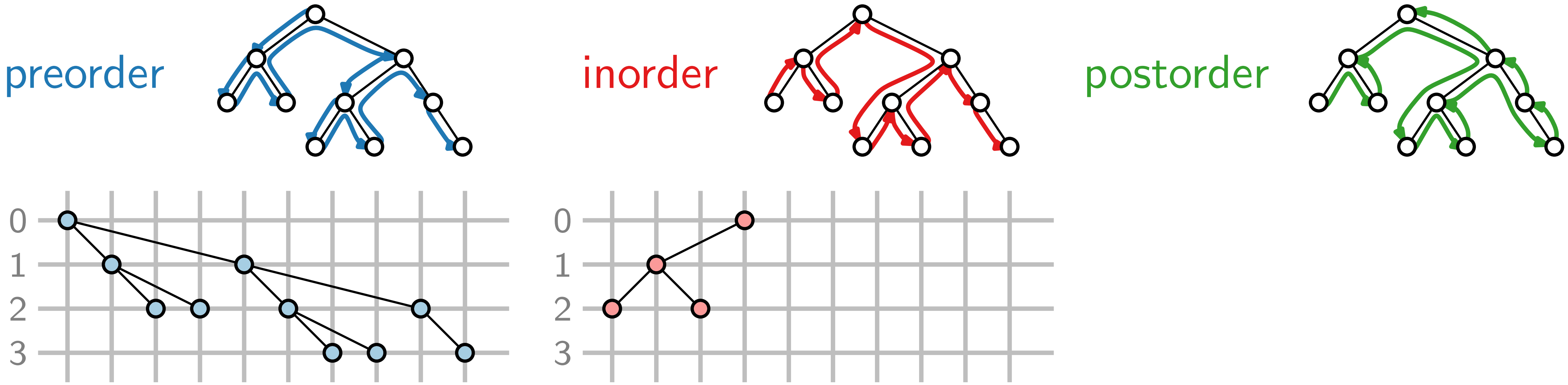


First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)



2. Choose x-coordinates:



First Grid Layout of Binary Trees

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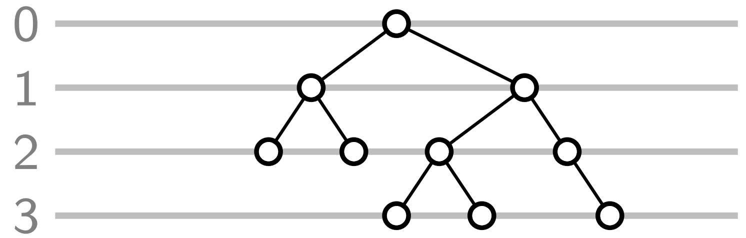
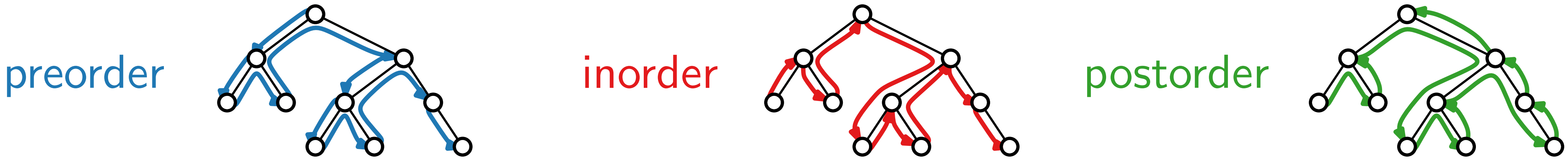
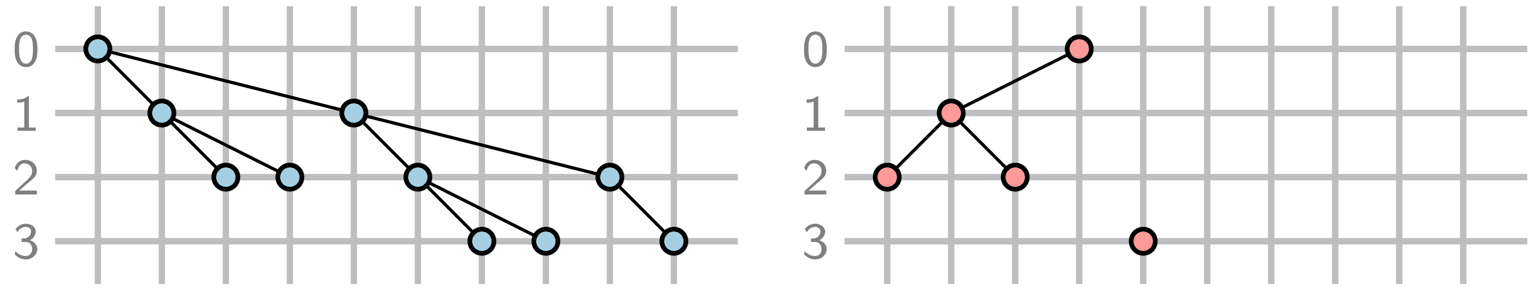


2. Choose x-coordinates:





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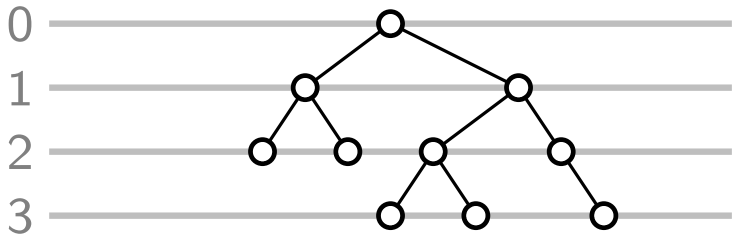

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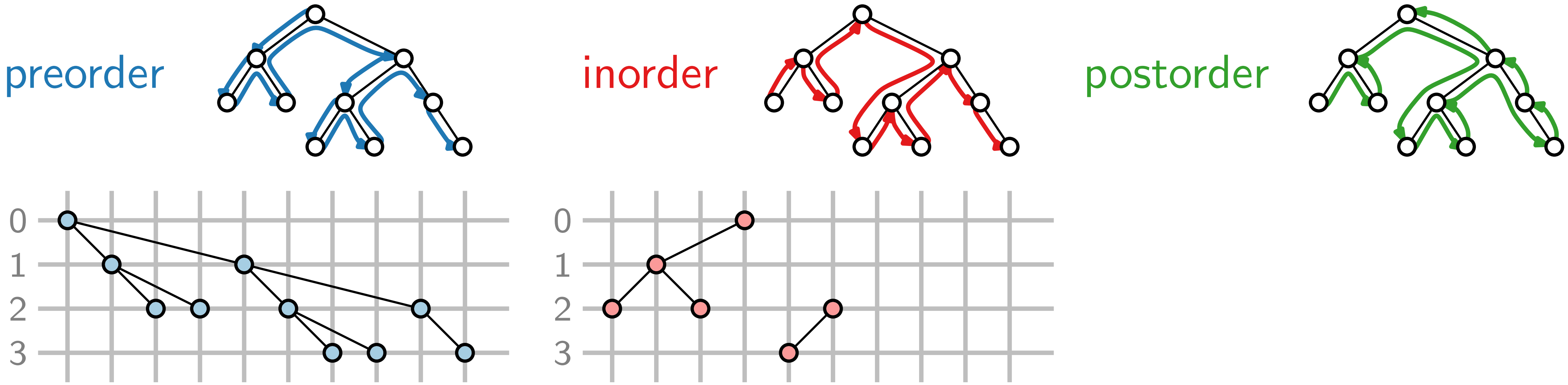

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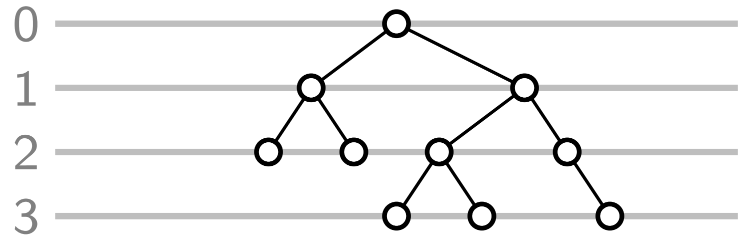

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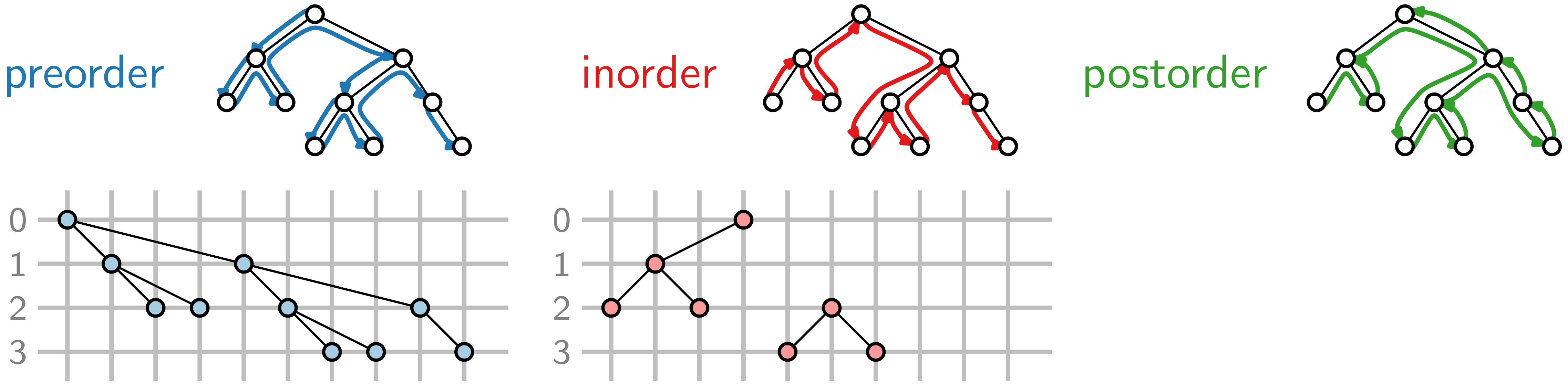

First Grid Layout of Binary Trees

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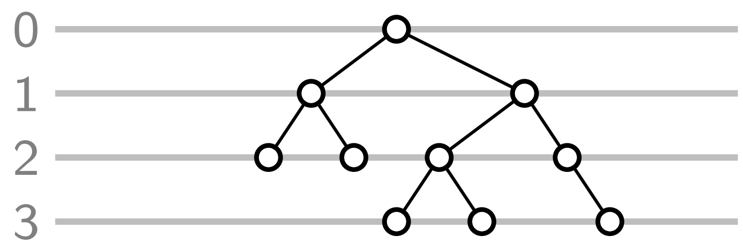
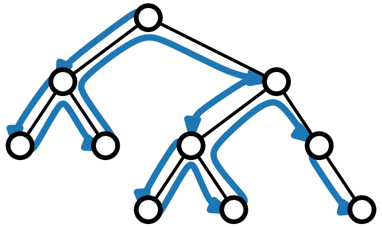
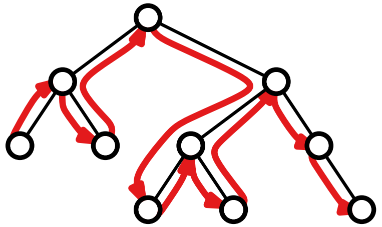


2. Choose x-coordinates:

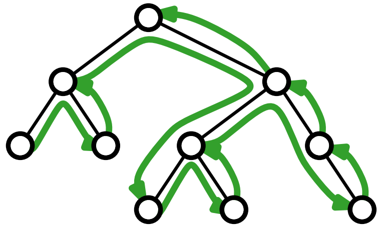
preorder

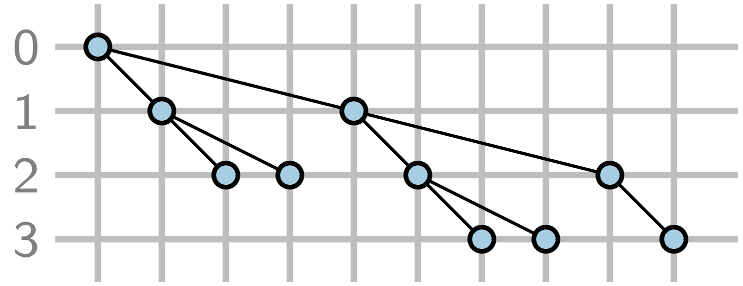


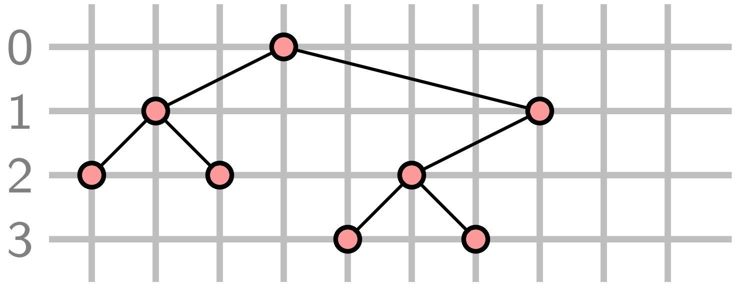
inorder



postorder







First Grid Layout of Binary Trees

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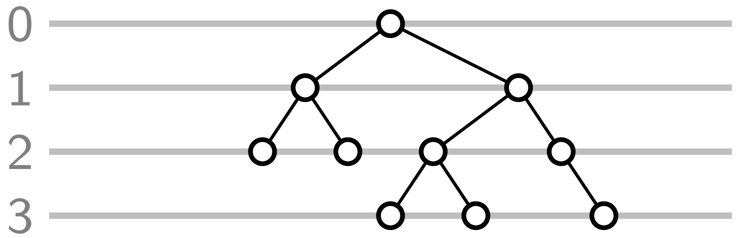
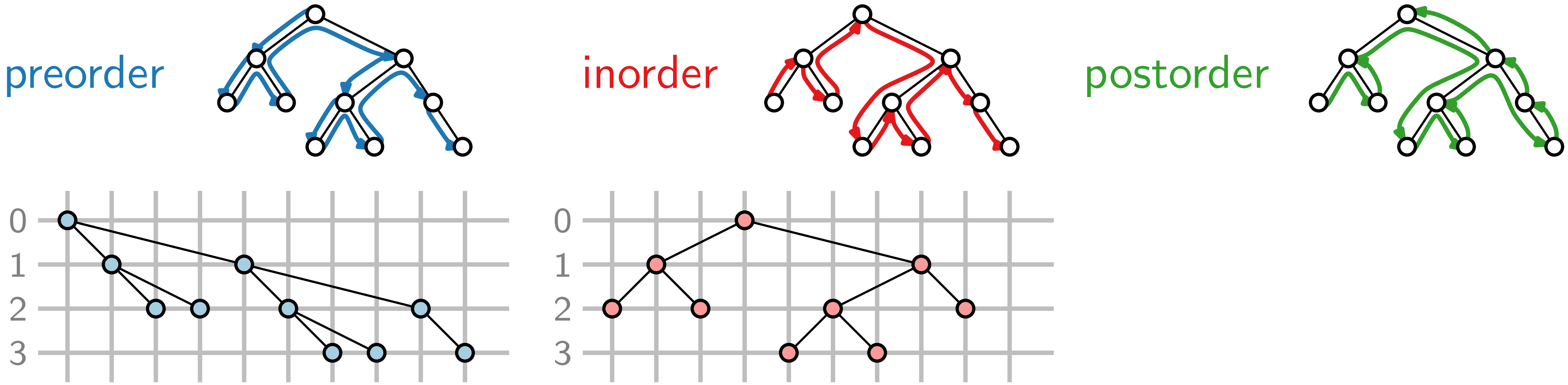
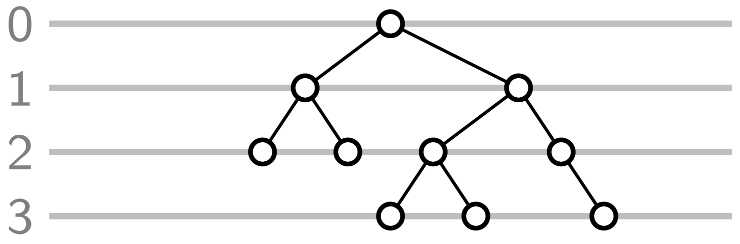


2. Choose x-coordinates:



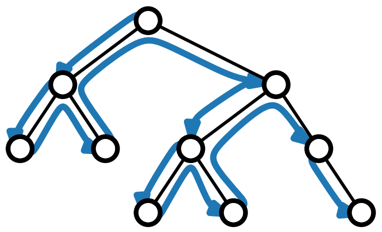
First Grid Layout of Binary Trees

1. Choose y-coordinates: $y(u) =$depth(u)

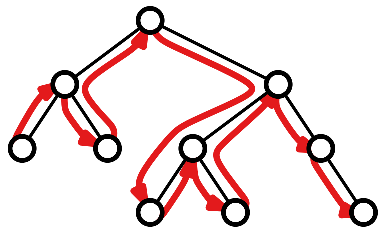


2. Choose x-coordinates:

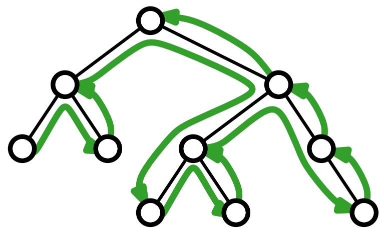
preorder

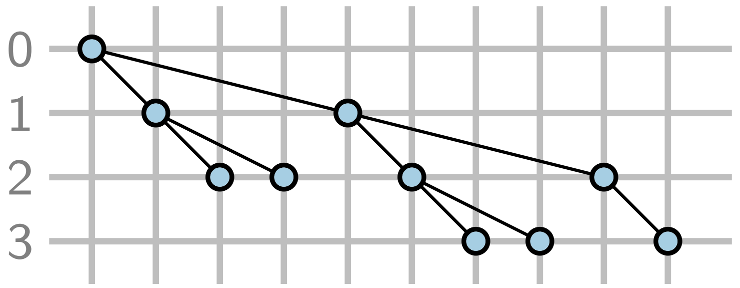


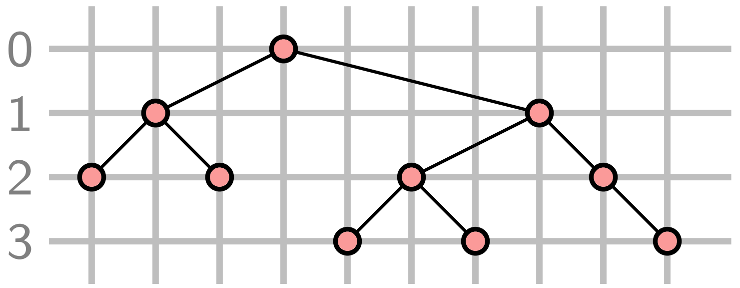
inorder



postorder







First Grid Layout of Binary Trees

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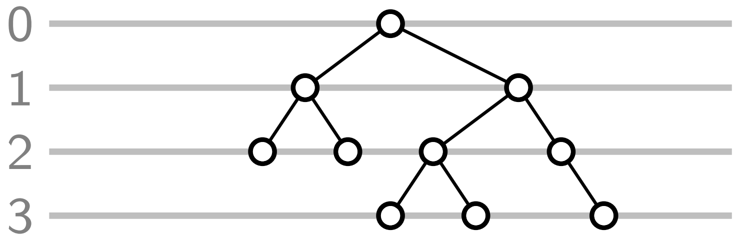
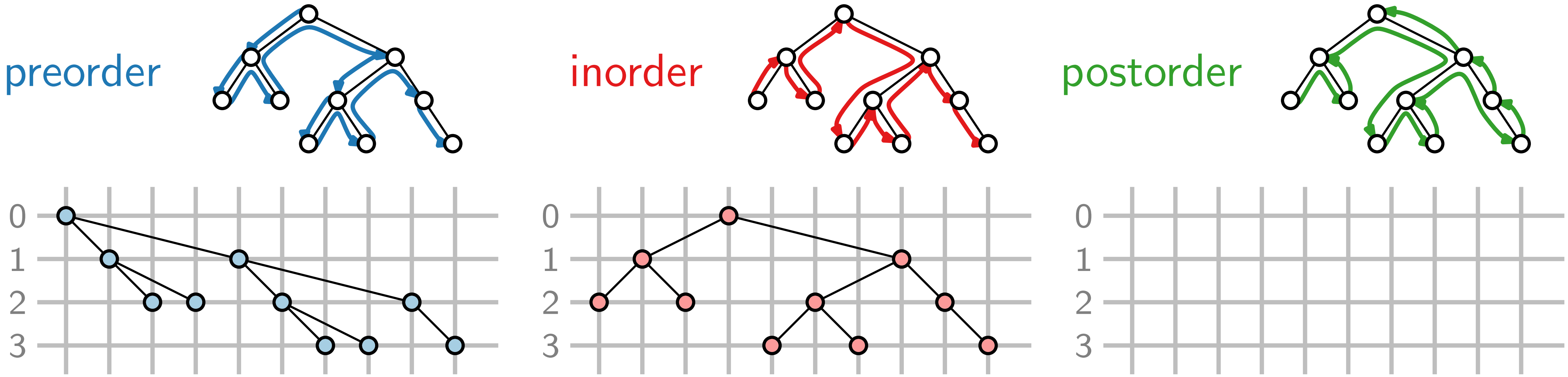


2. Choose x-coordinates:



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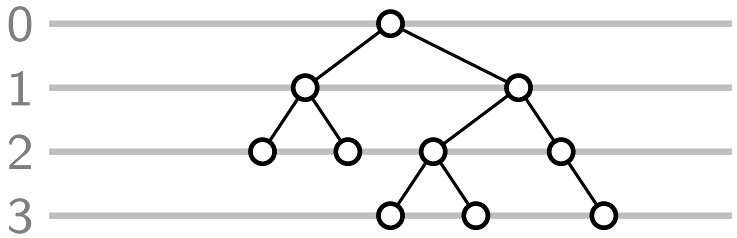

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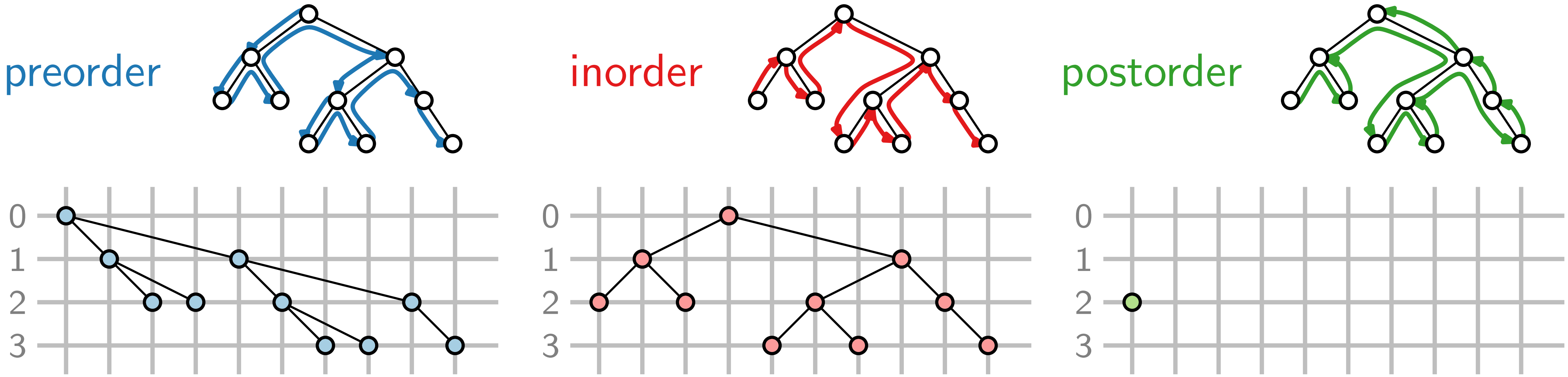

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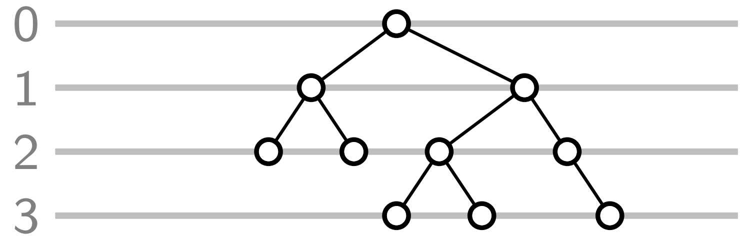

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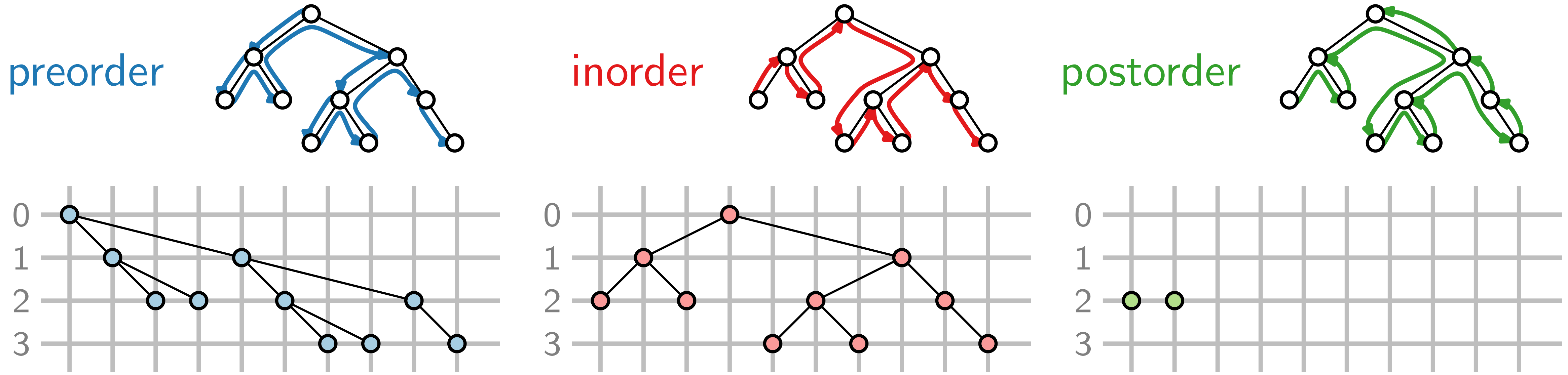

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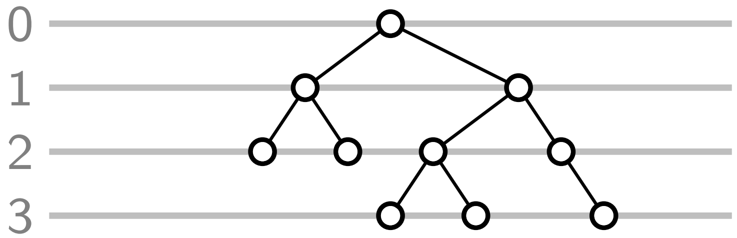

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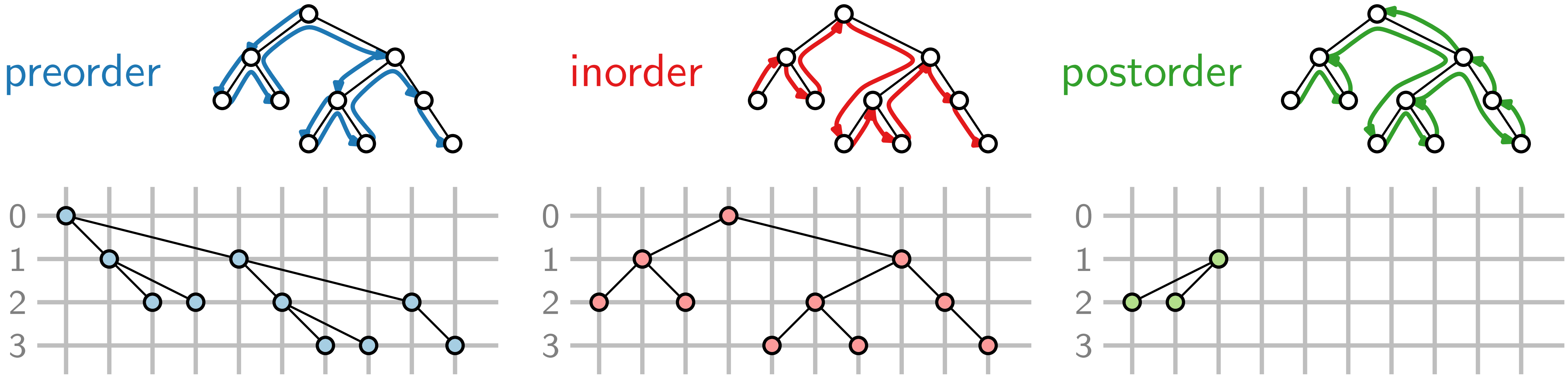

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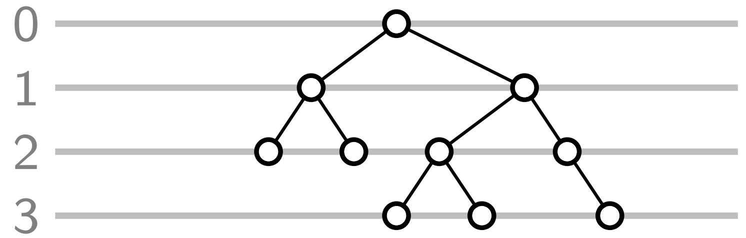

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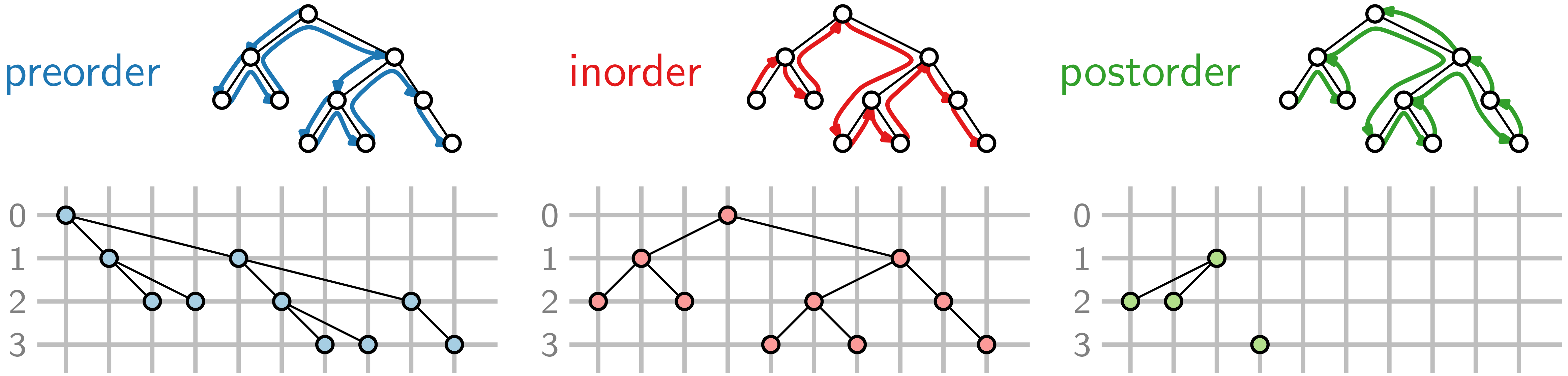

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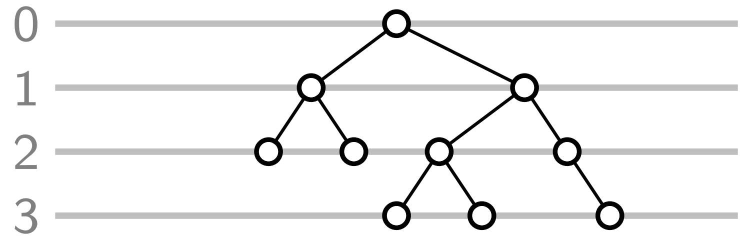

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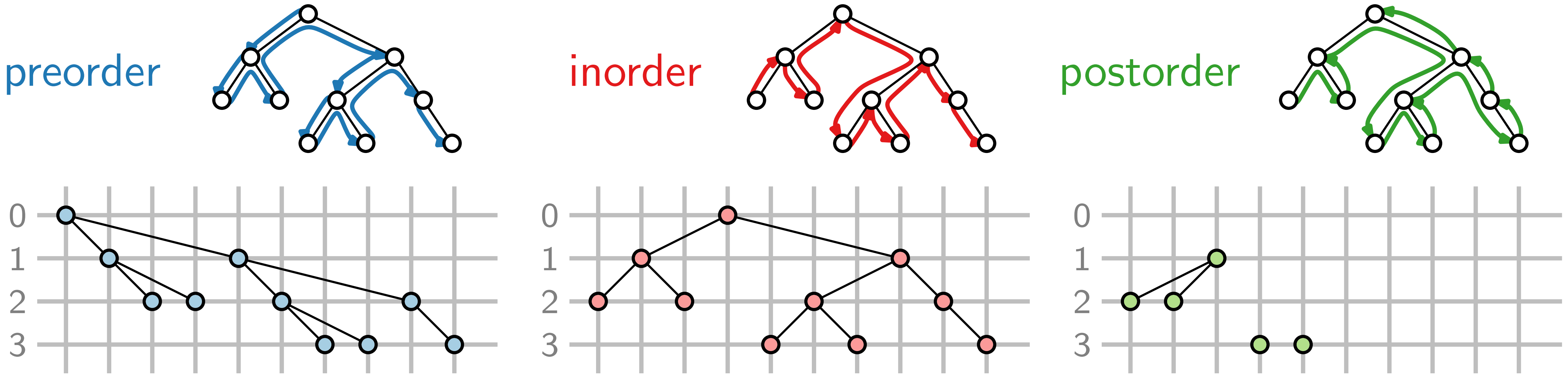

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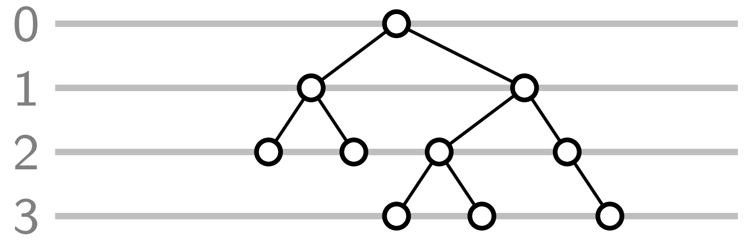
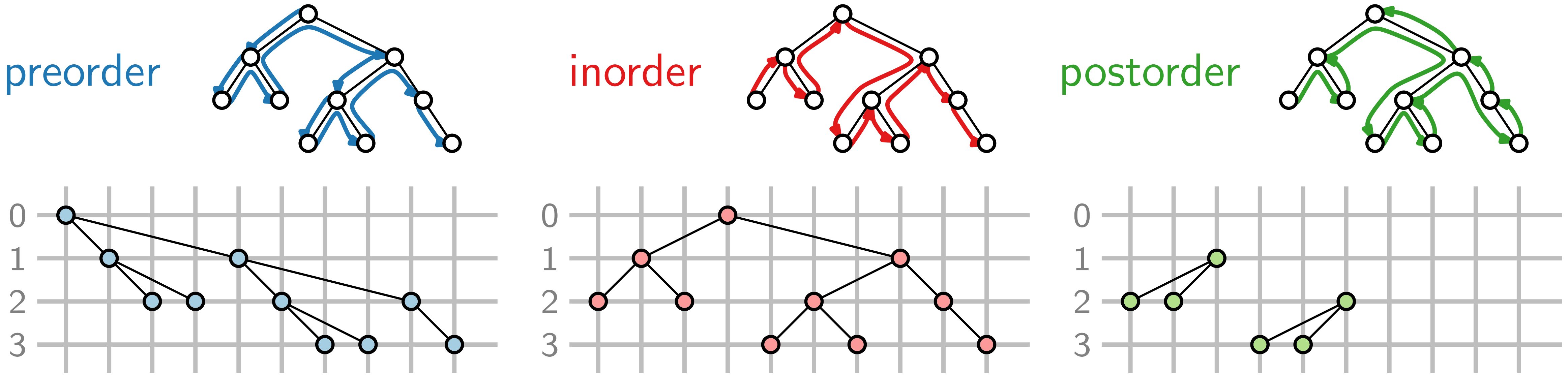


2. Choose x-coordinates:



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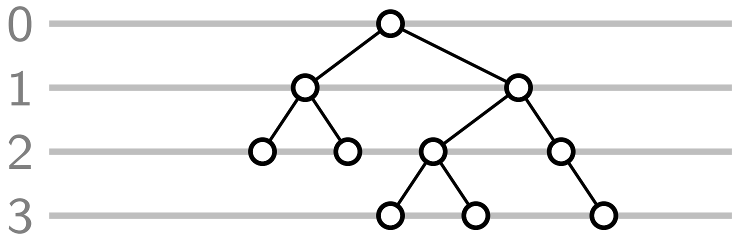

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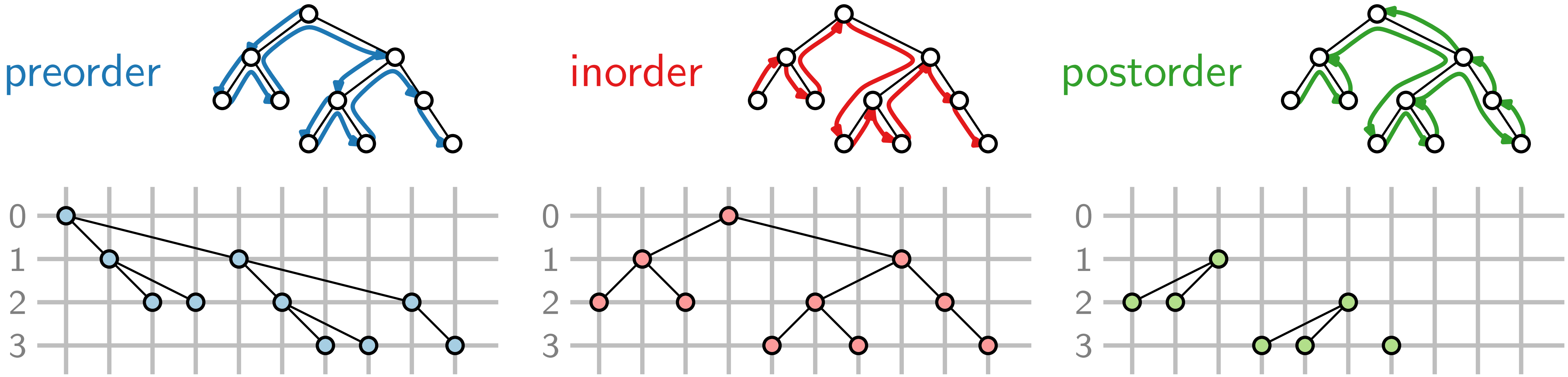

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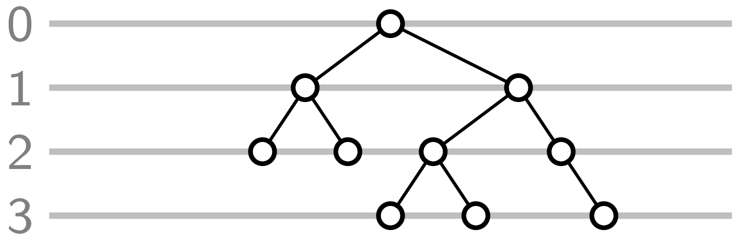
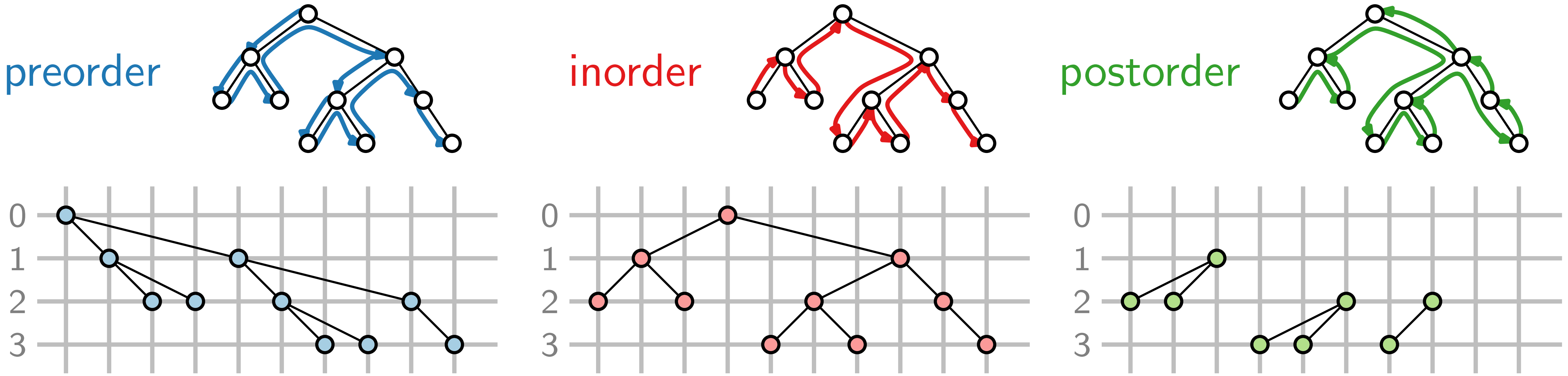


2. Choose x-coordinates:



First Grid Layout of Binary Trees

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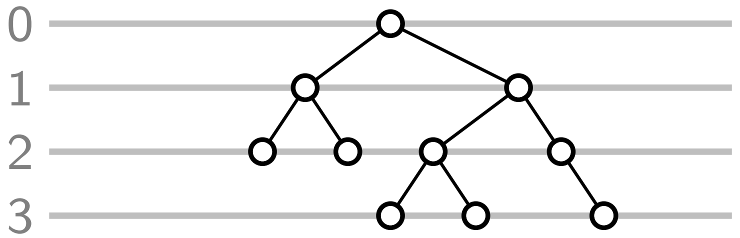
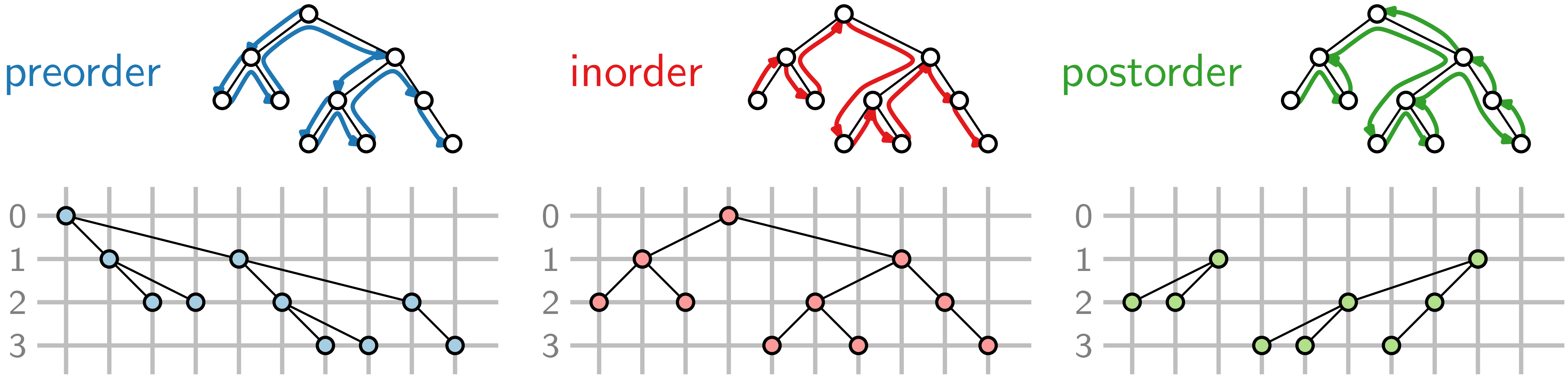


2. Choose x-coordinates:



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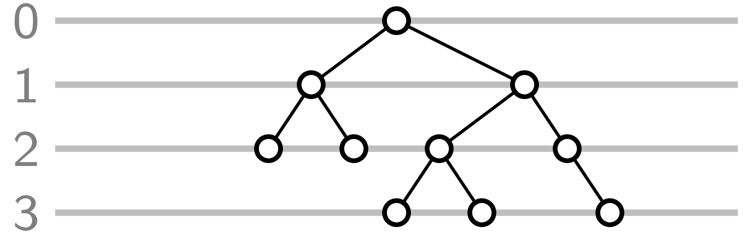

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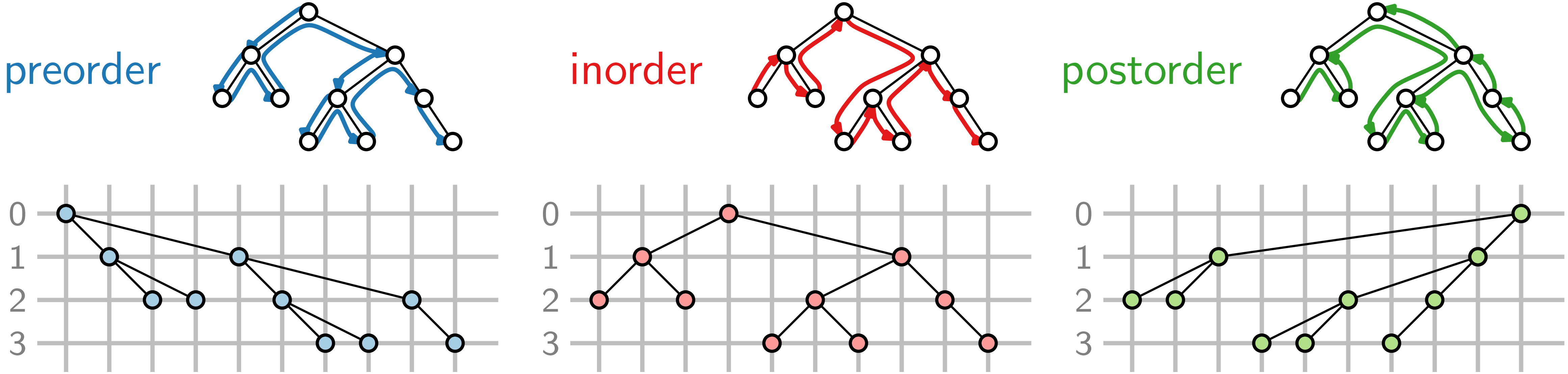
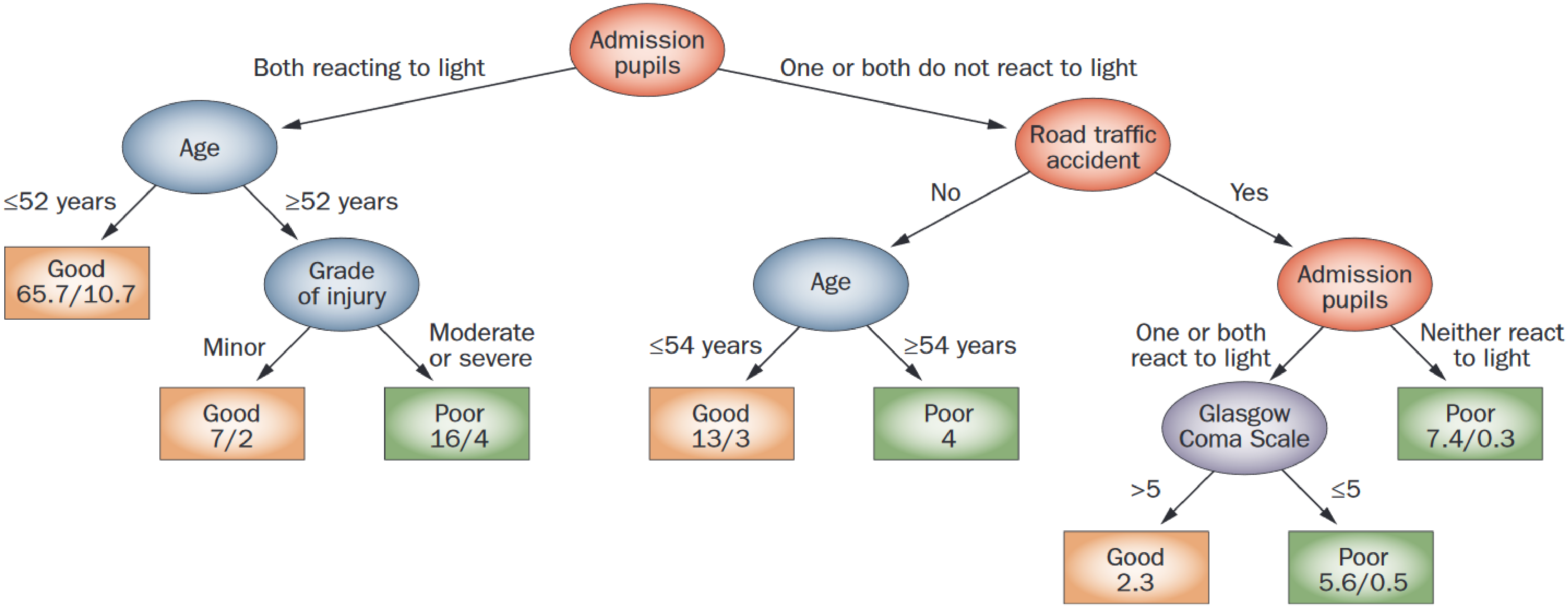


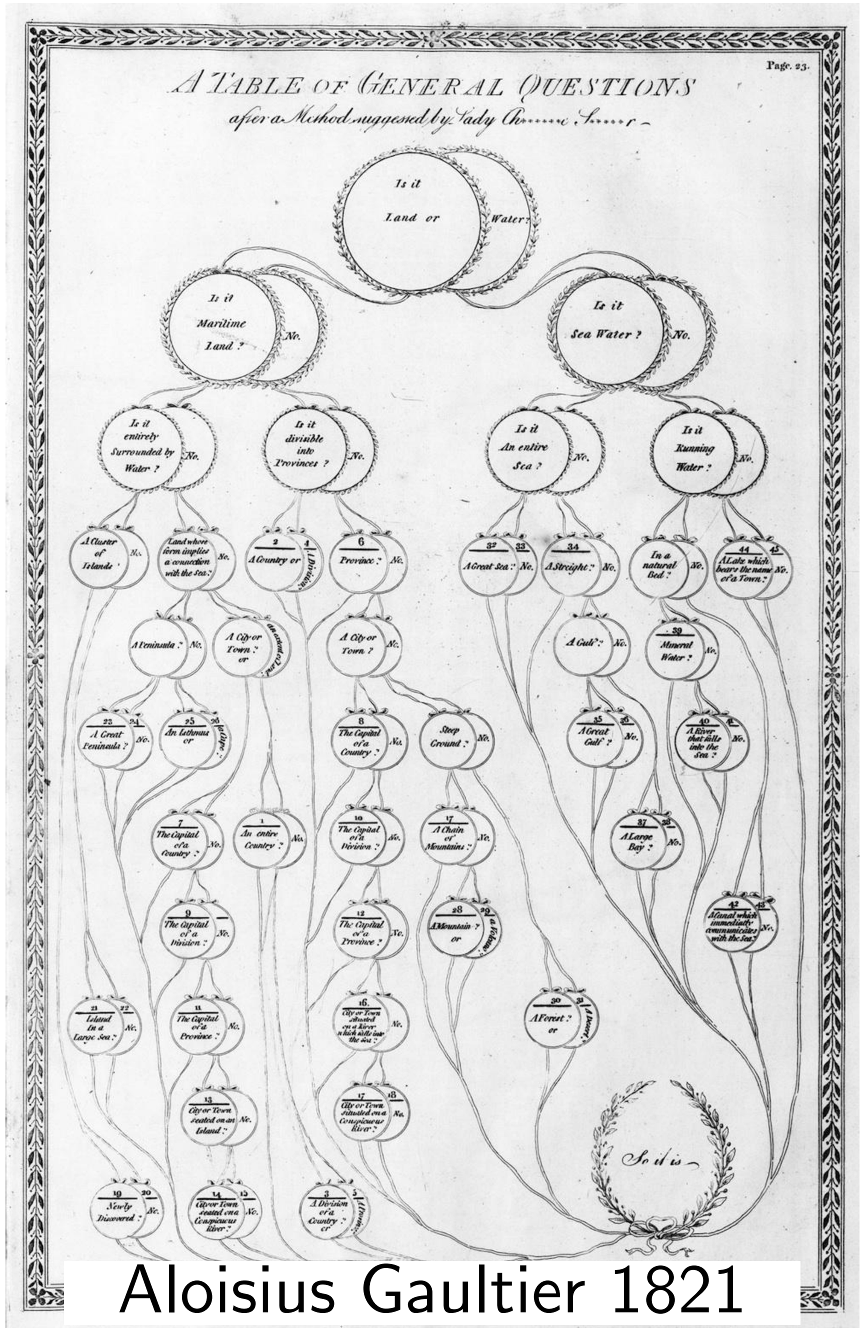
Layered Drawings – Applications



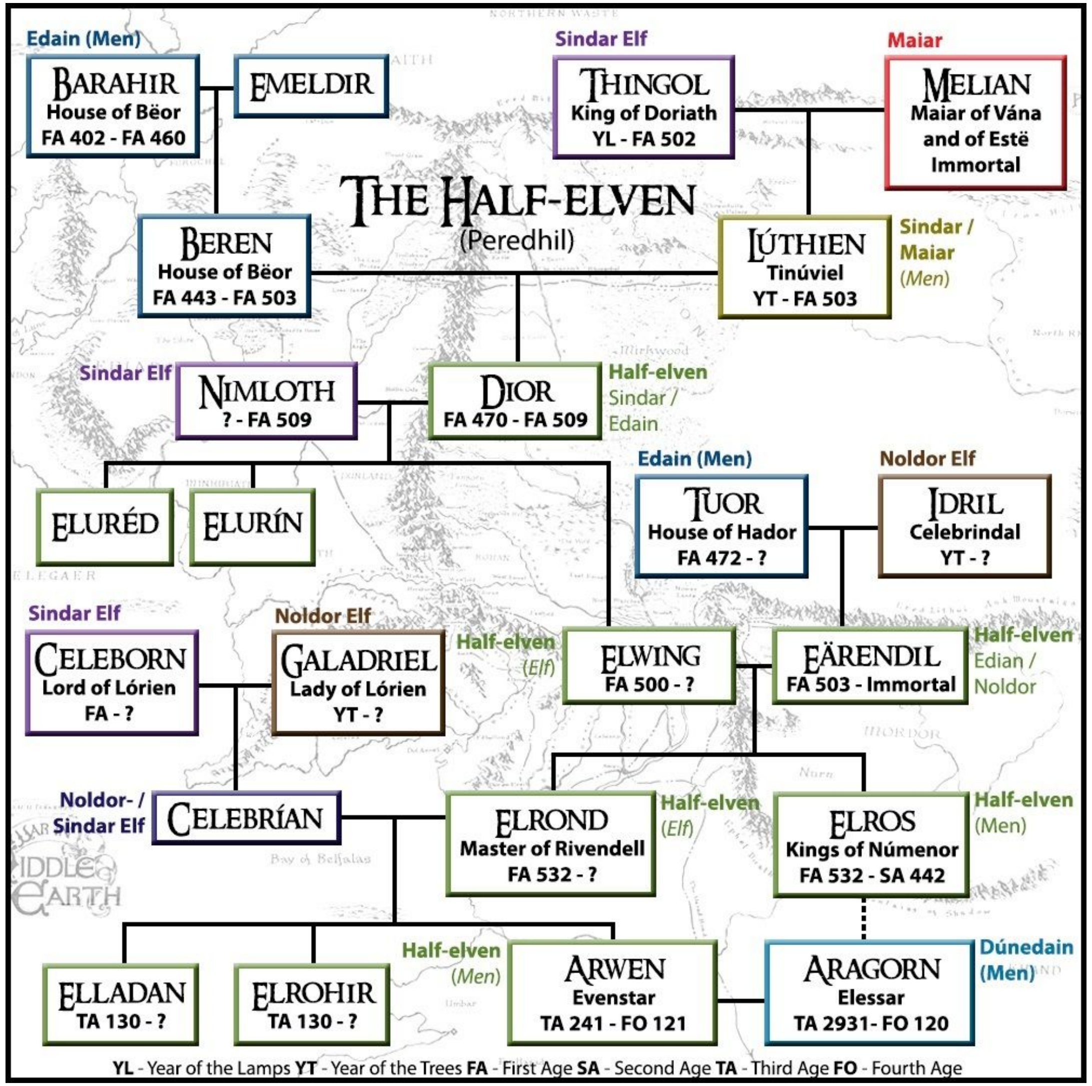
Decision tree for outcome prediction after traumatic brain injury

Source: Nature Reviews Neurology

Layered Drawings – Applications

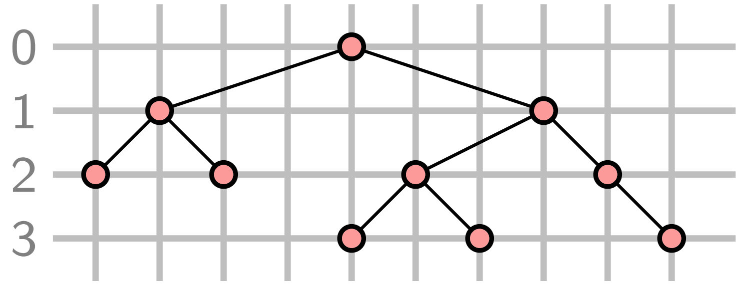


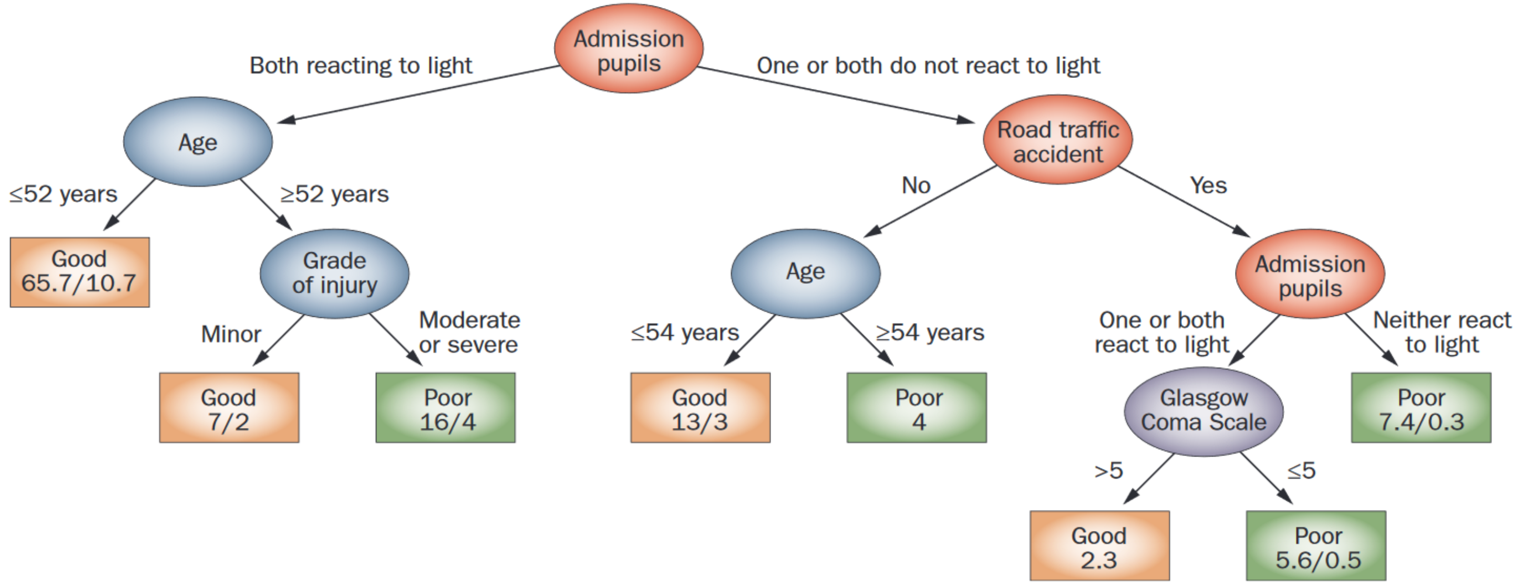
Aloisius Gaultier 1821



Family tree of LOTR elves
and half-elves

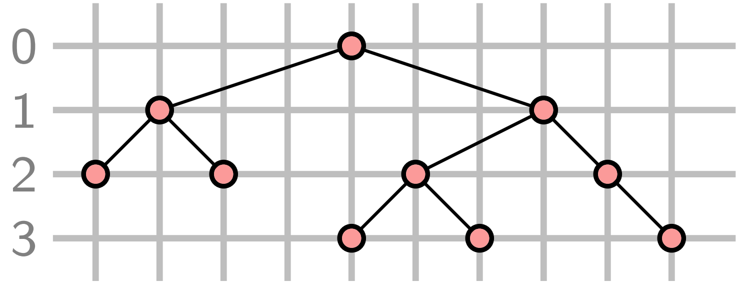
Layered Drawings – Drawing Style

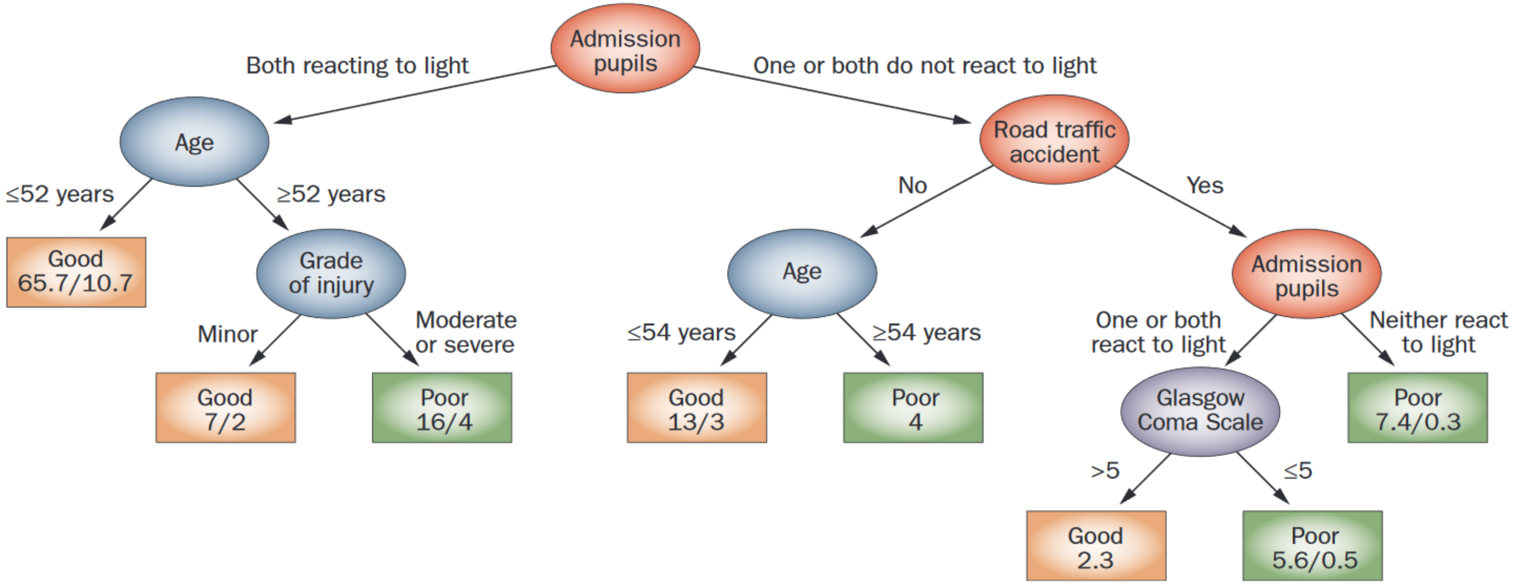




- What are properties of the layout?

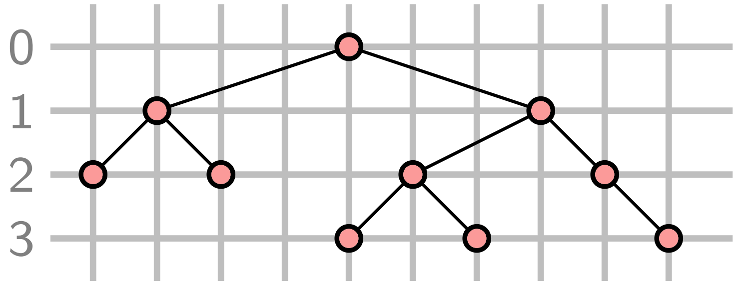
Layered Drawings – Drawing Style

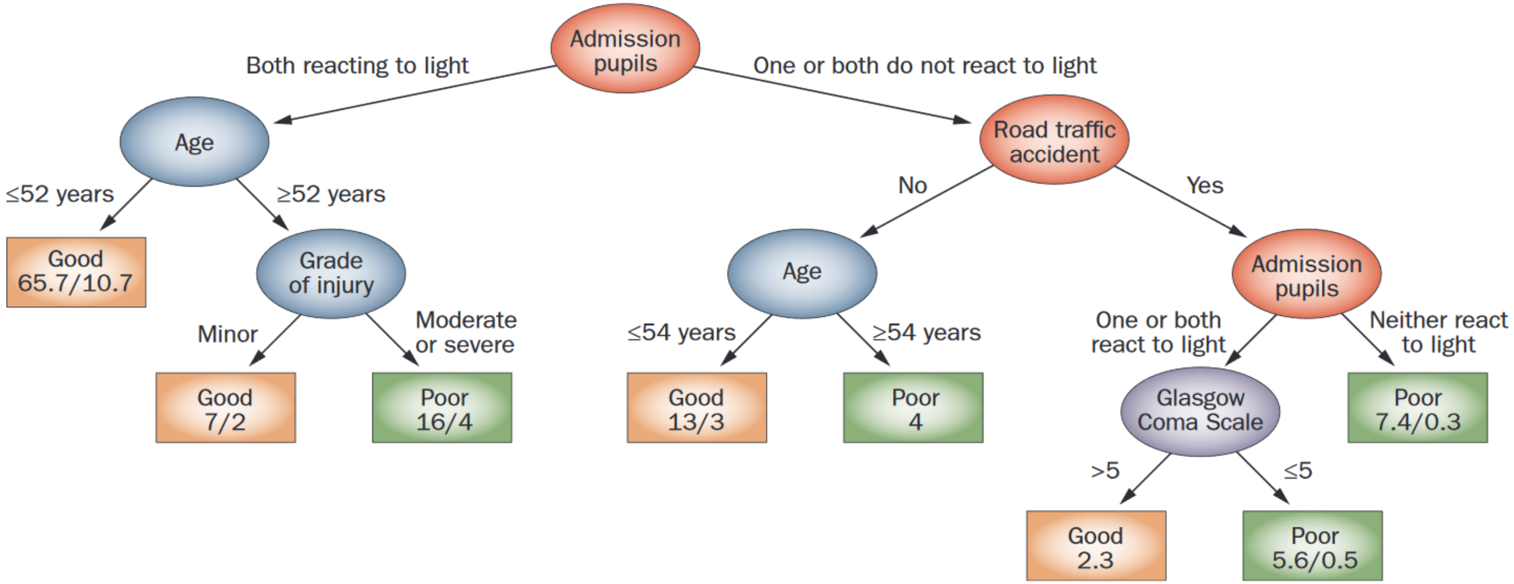




- What are properties of the layout?
- What are the drawing conventions?

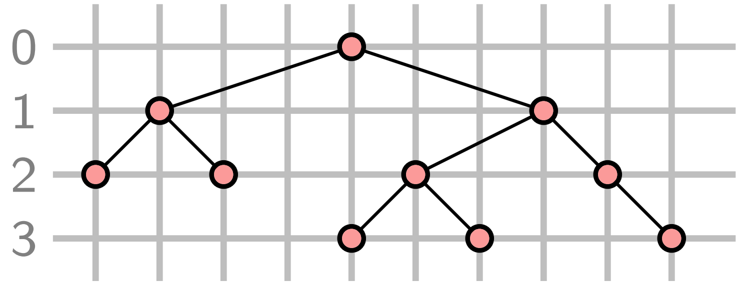
Layered Drawings – Drawing Style

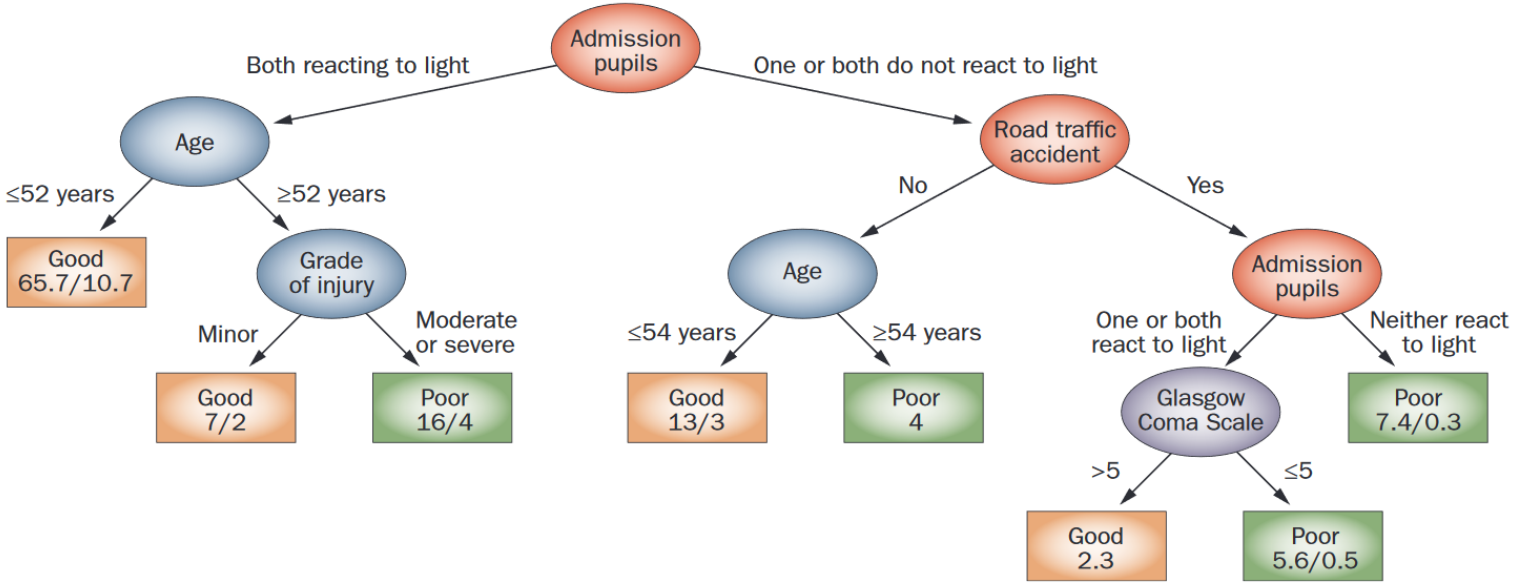




- What are properties of the layout?
- What are the drawing conventions?
- What are aesthetics to optimize?

Layered Drawings – Drawing Style





Drawing conventions

- What are properties of the layout?
- What are the drawing conventions?
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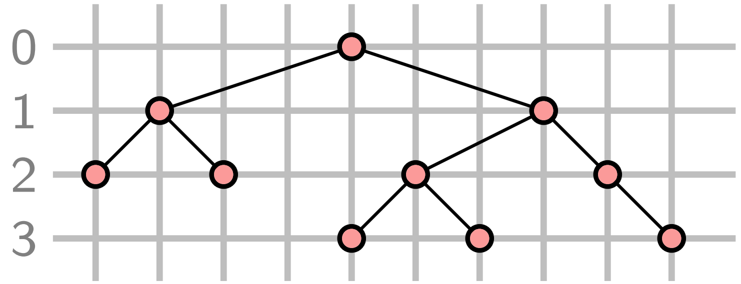
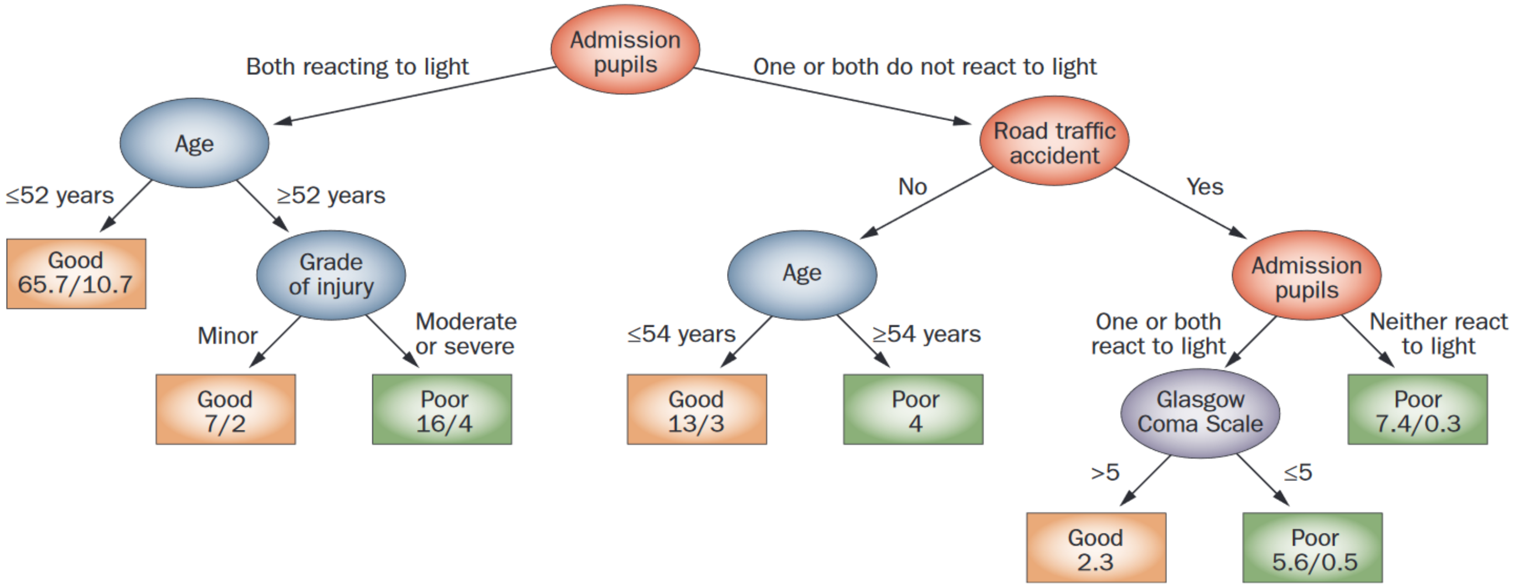
Layered Drawings – Drawing Style



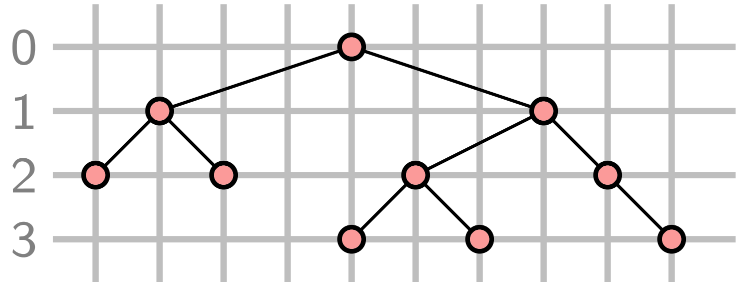


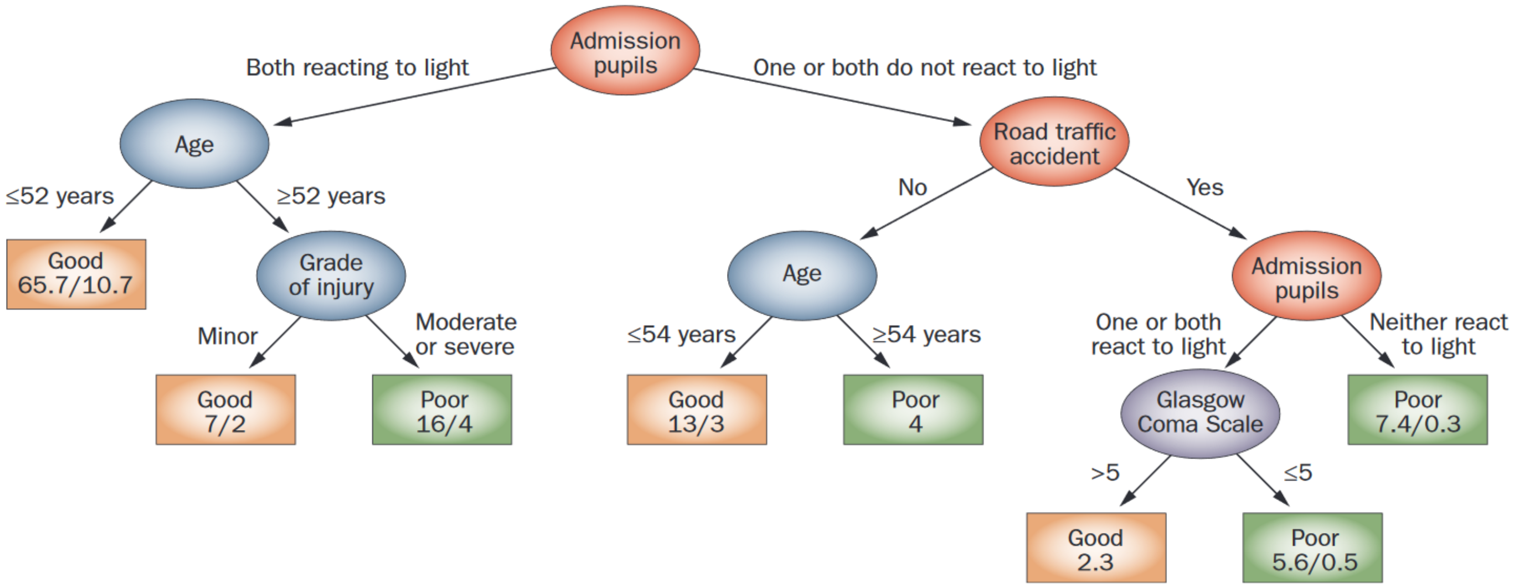
Drawing conventions

- Vertices lie on layers and have integer coordinates

- What are properties of the layout?
- What are the drawing conventions?
- What are aesthetics to optimize?

Layered Drawings – Drawing Style



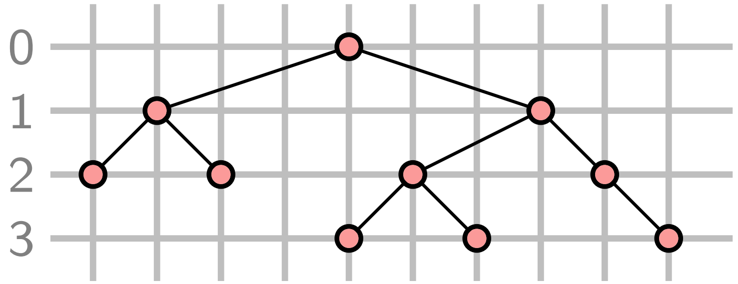


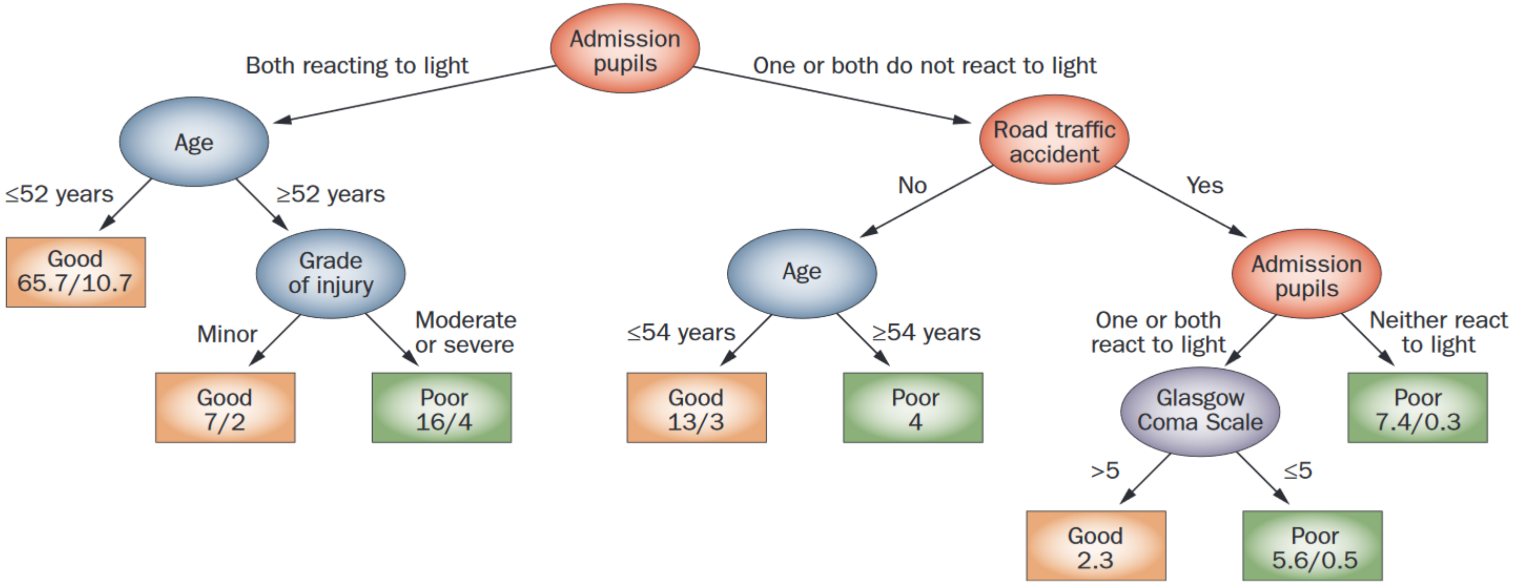
Drawing conventions

- Vertices lie on layers and have integer coordinates
- Parent centered above children

- What are properties of the layout?
- What are the drawing conventions?
- What are aesthetics to optimize?

Layered Drawings – Drawing Style





Drawing conventions

- Vertices lie on layers and have integer coordinates
- Parent centered above children
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- What are properties of the layout?
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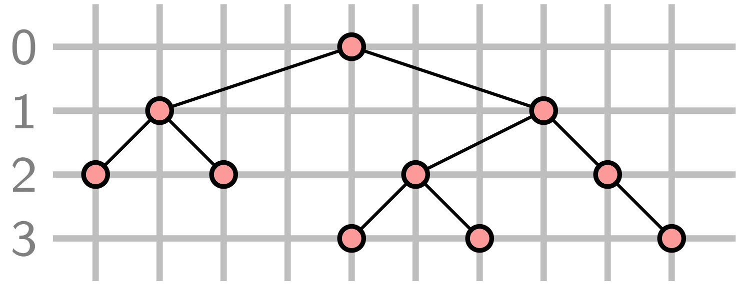
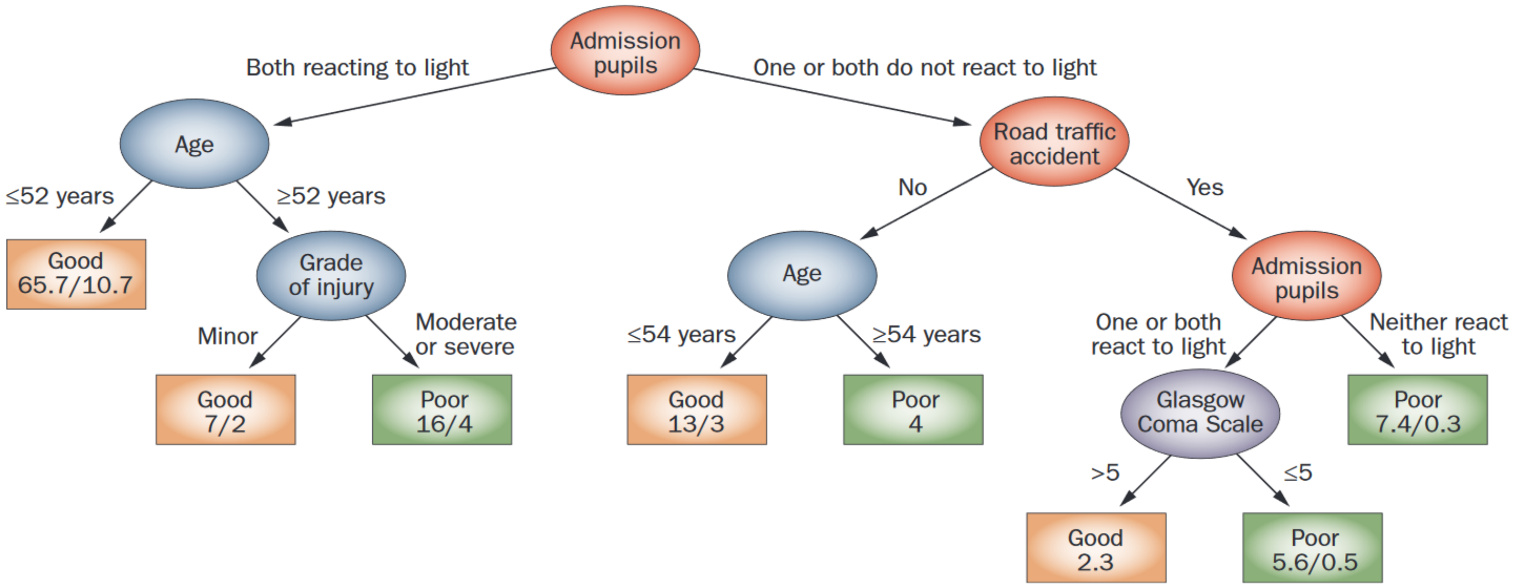
Layered Drawings – Drawing Style





- What are properties of the layout?
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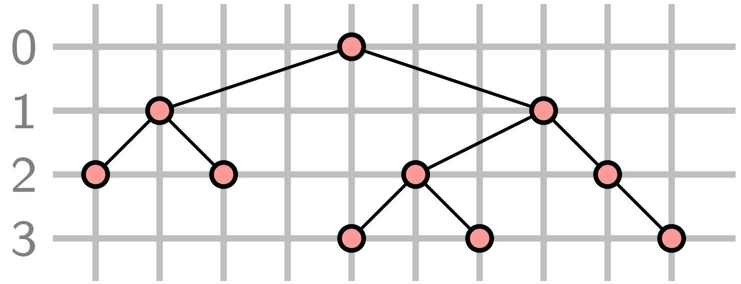
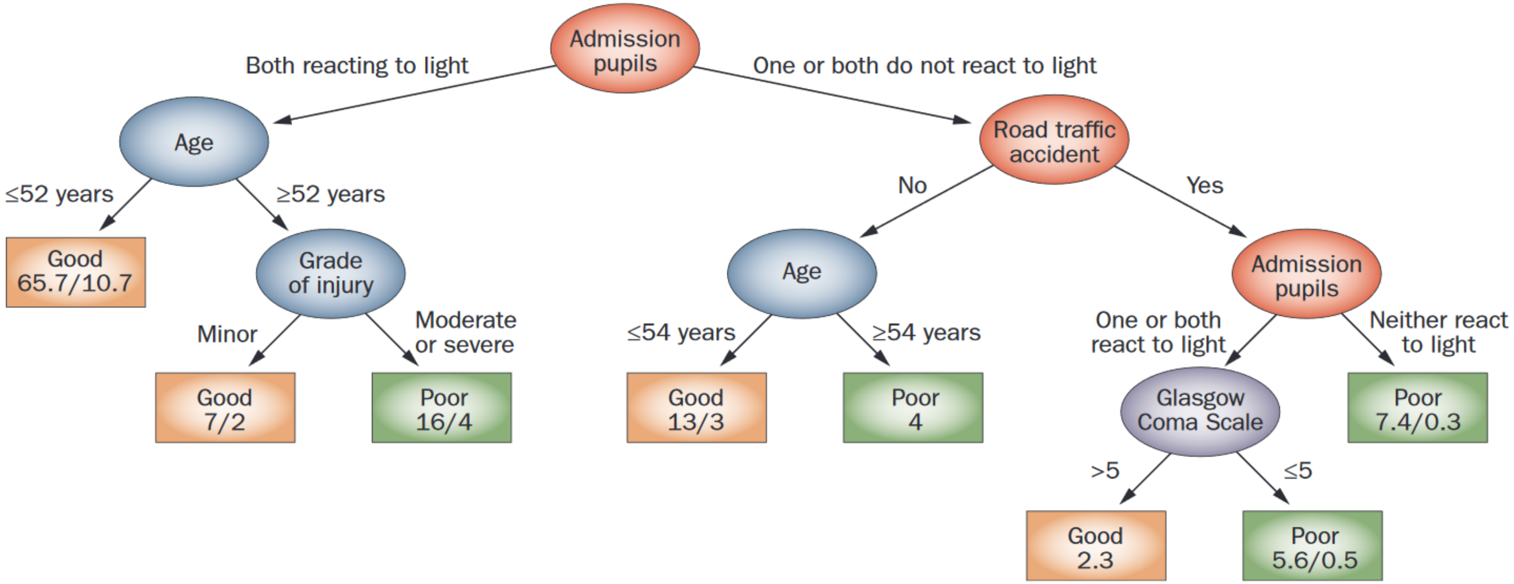
Drawing conventions

- Vertices lie on layers and have integer coordinates
- Parent centered above children
- Edges are straight-line segments
- Isomorphic subtrees have identical drawings

Layered Drawings – Drawing Style





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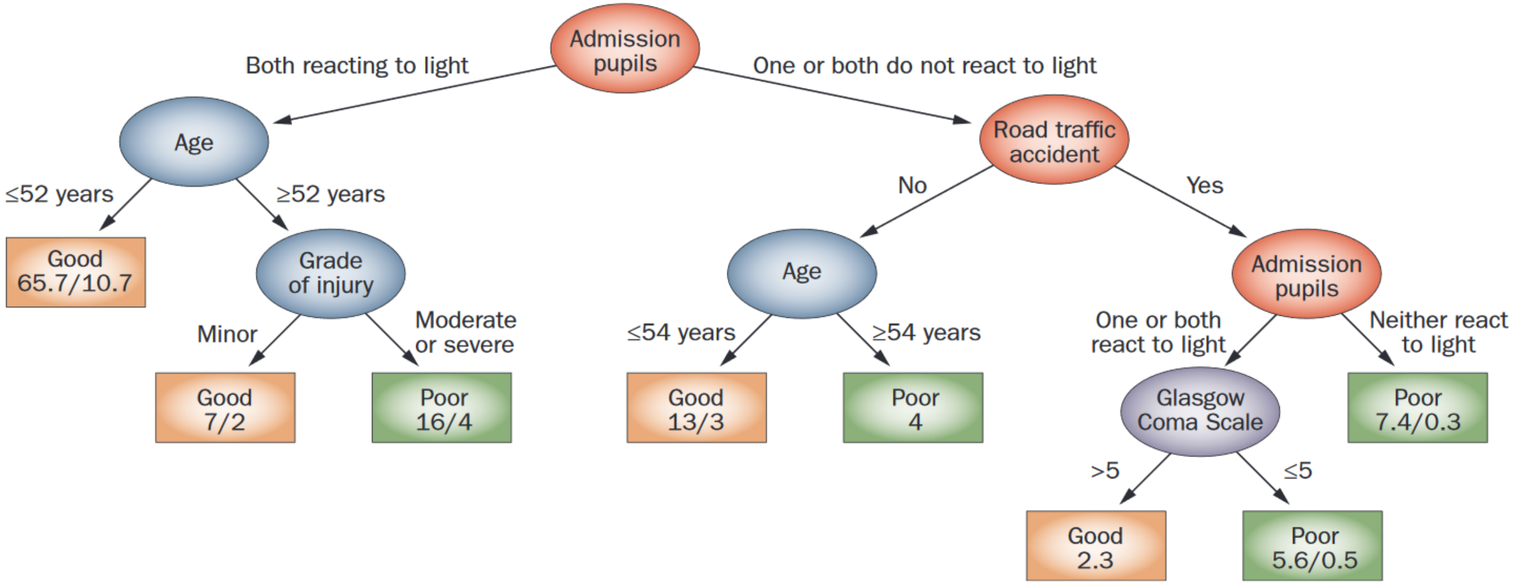

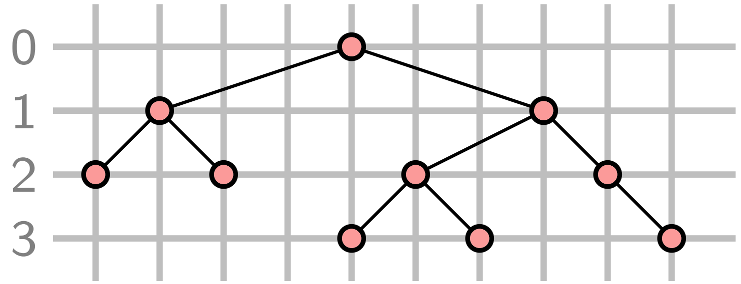

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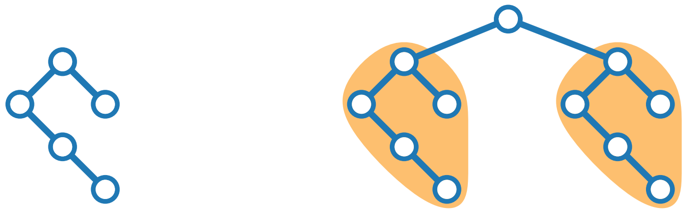
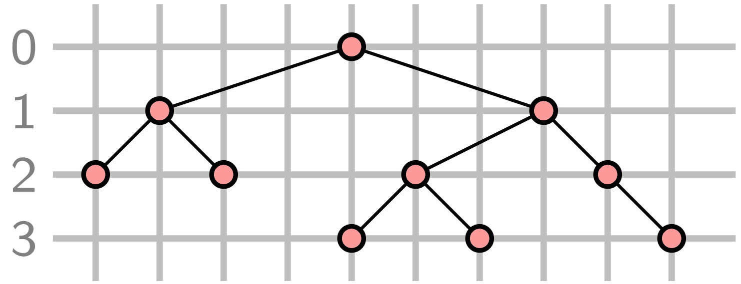
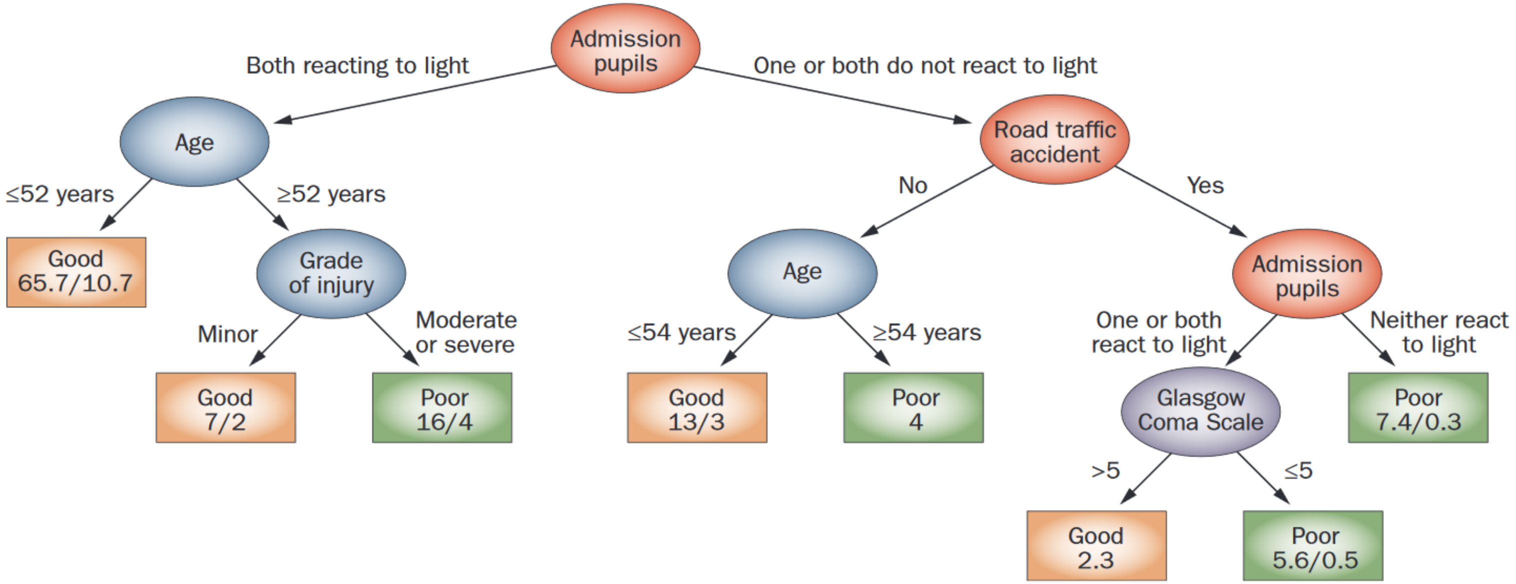


Layered Drawings – Drawing Style





Drawing conventions

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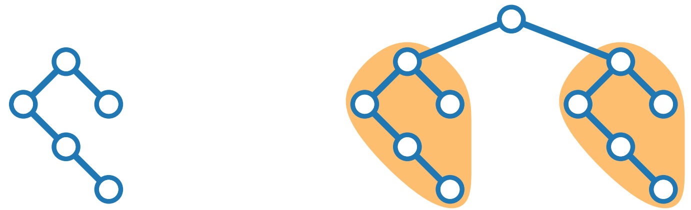
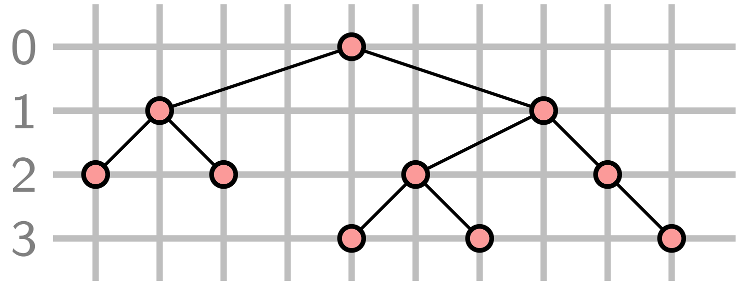
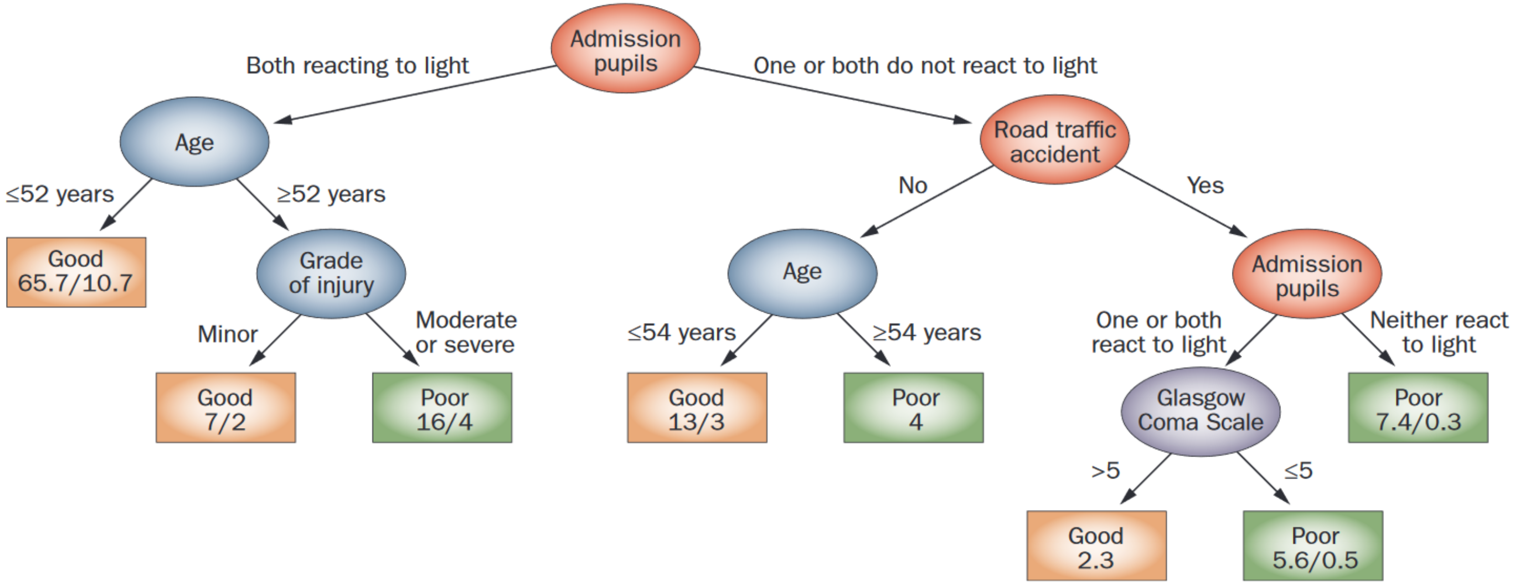
Drawing aesthetics

- What are properties of the layout?
- What are the drawing conventions?
- What are aesthetics to optimize?



Layered Drawings – Drawing Style





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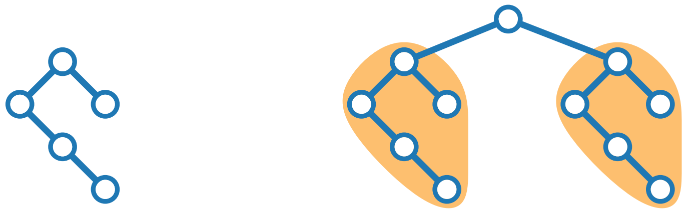


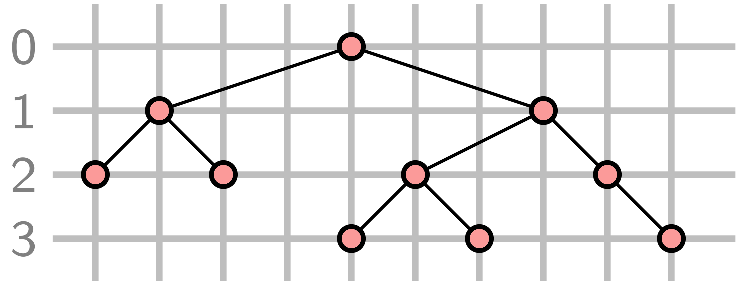
Drawing conventions

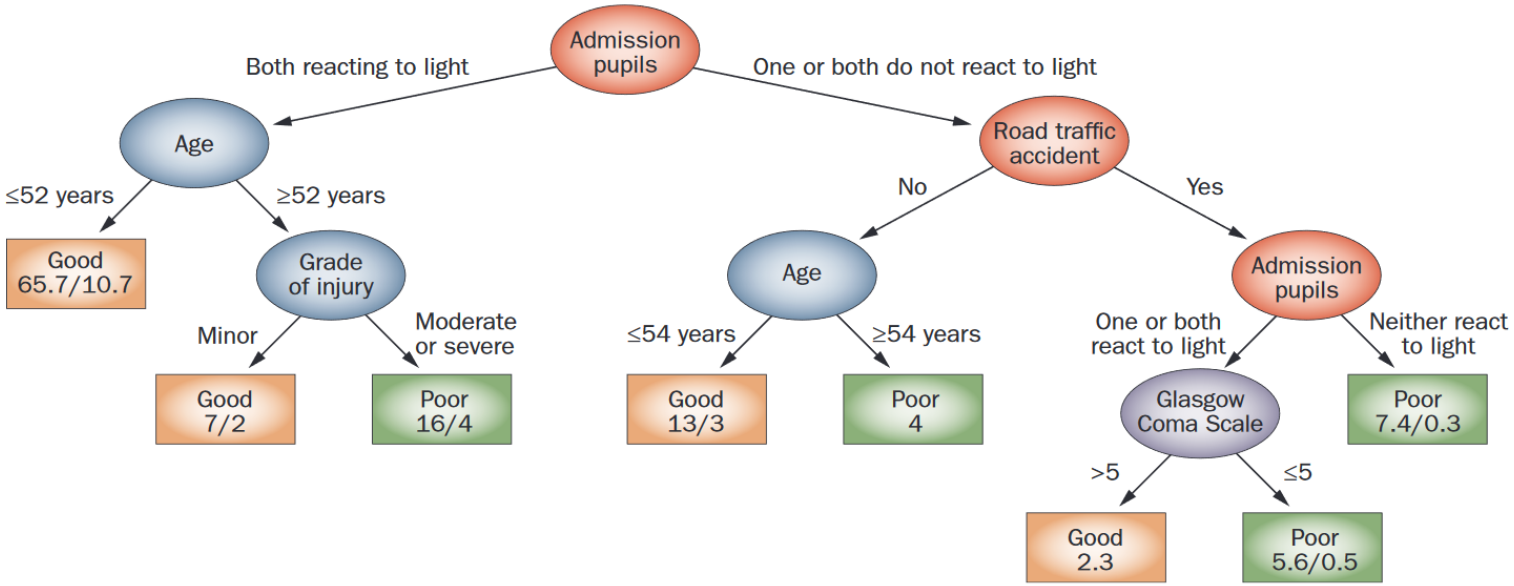
- Vertices lie on layers and have integer coordinates
- Parent centered above children
- Edges are straight-line segments
- Isomorphic subtrees have identical drawings

Drawing aesthetics

- Area

Layered Drawings – Drawing Style





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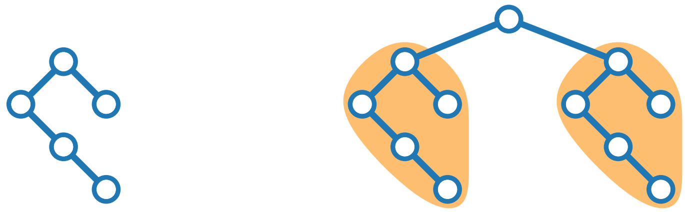

Drawing conventions

- Vertices lie on layers and have integer coordinates
- Parent centered above children
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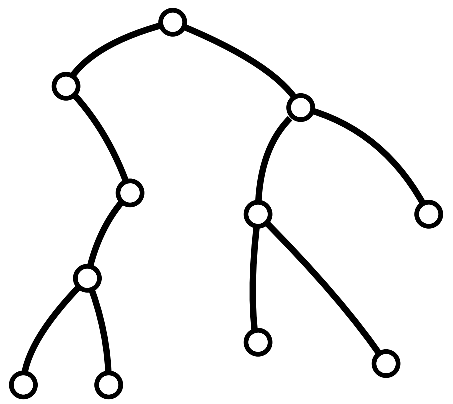
- Isomorphic subtrees have identical drawings

Drawing aesthetics

- Area
- Symmetries

Layered Drawings – Algorithm

Input: A binary tree T
Output: A layered drawing of T



Layered Drawings – Algorithm

Input: A binary tree T
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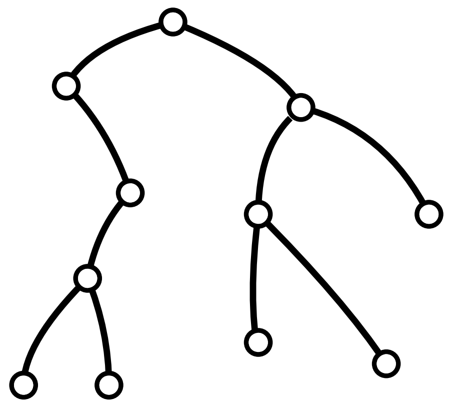
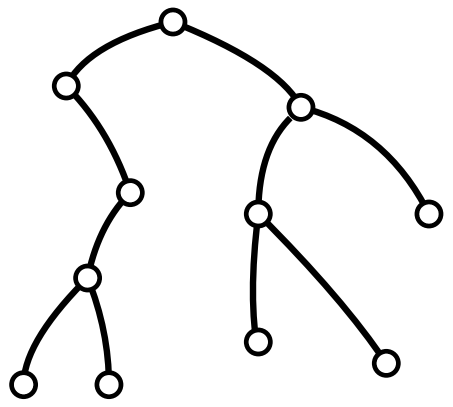


Base case:
Divide:

Conquer:

Layered Drawings – Algorithm

Input: A binary tree T
Output: A layered drawing of T

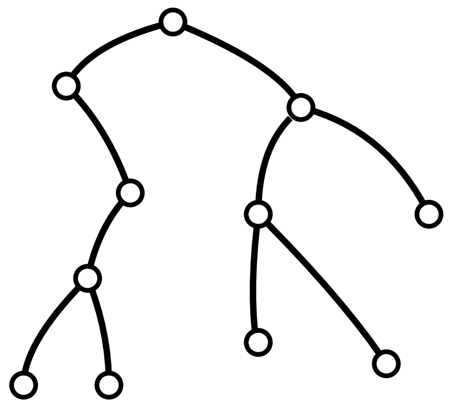


Base case: A single vertex 
Divide:

Conquer:

Layered Drawings – Algorithm

Input: A binary tree T
Output: A layered drawing of T



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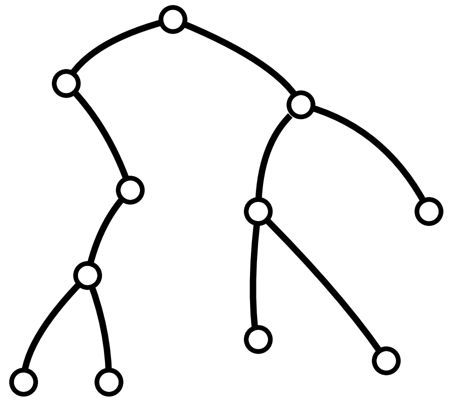

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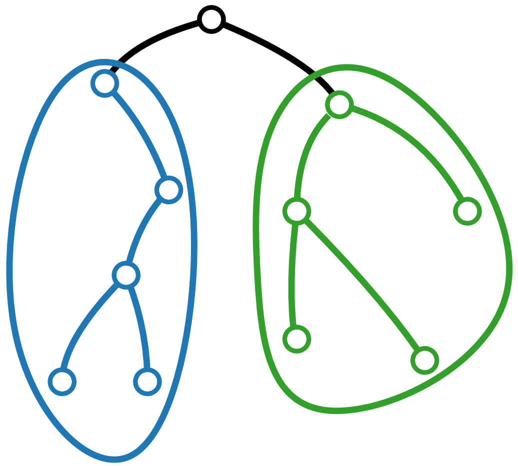

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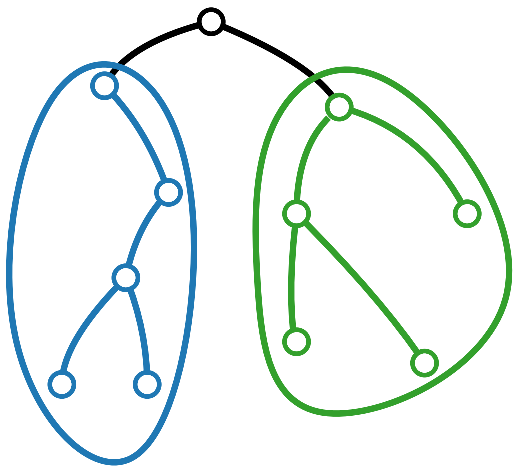

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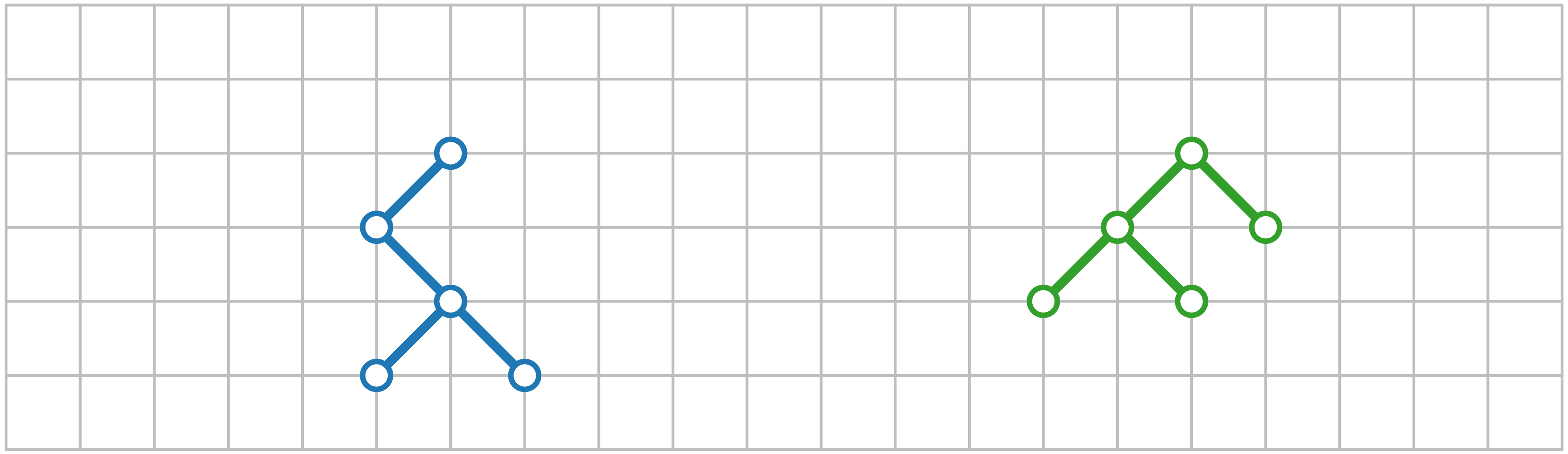

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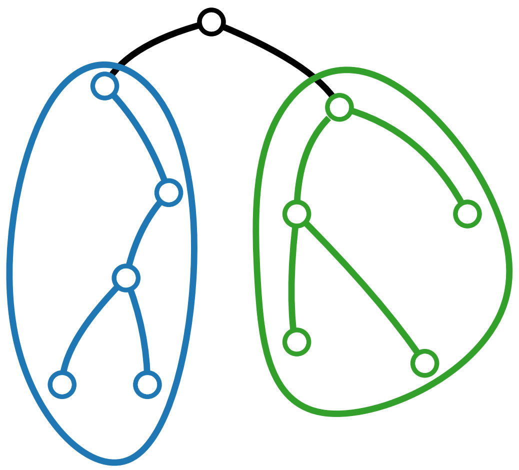

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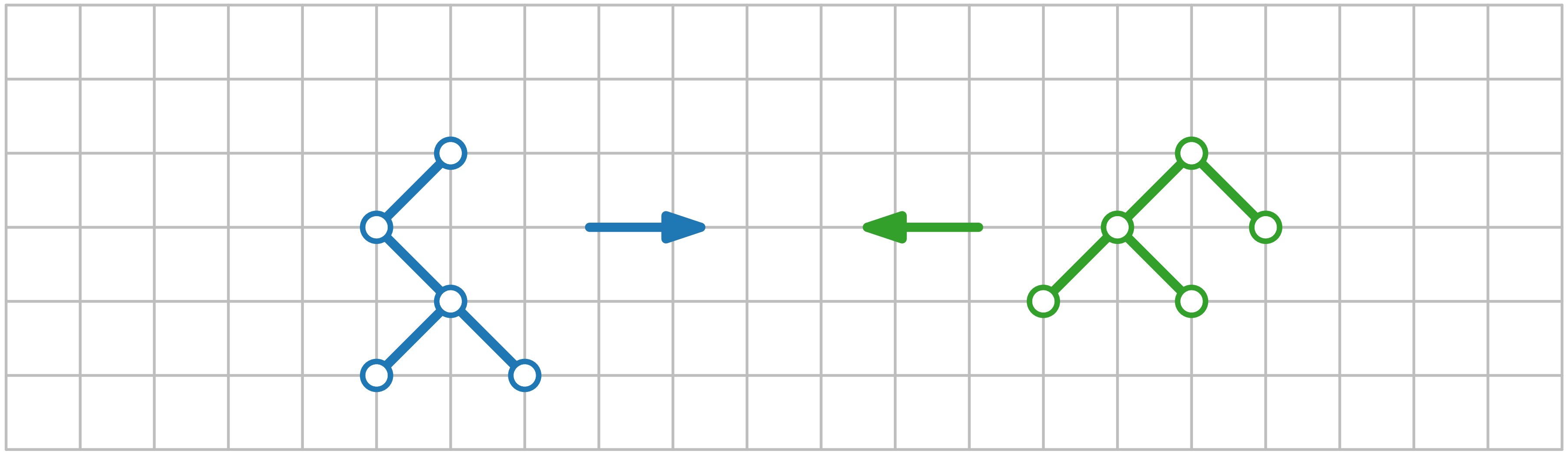

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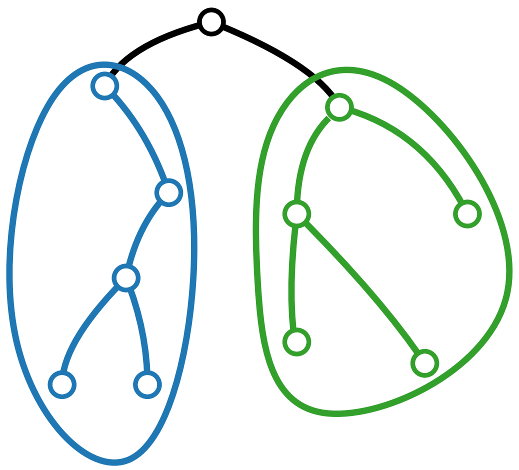

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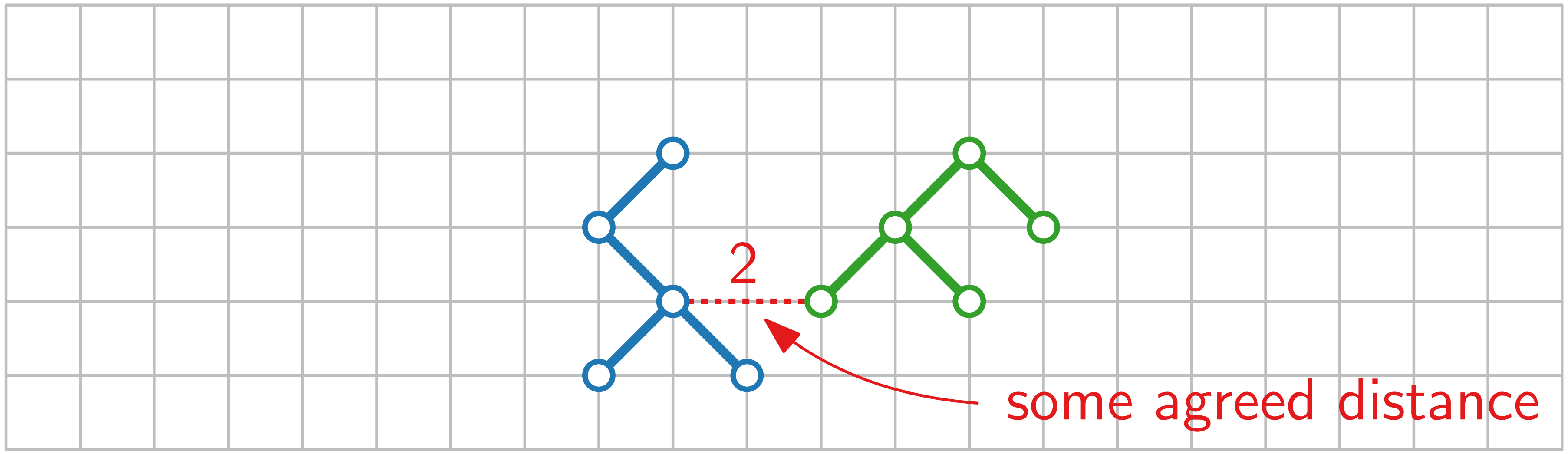

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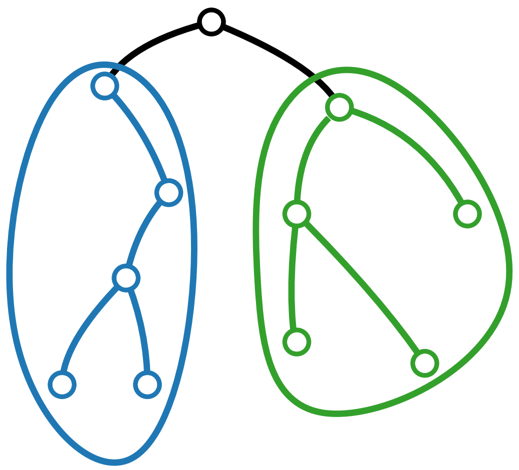

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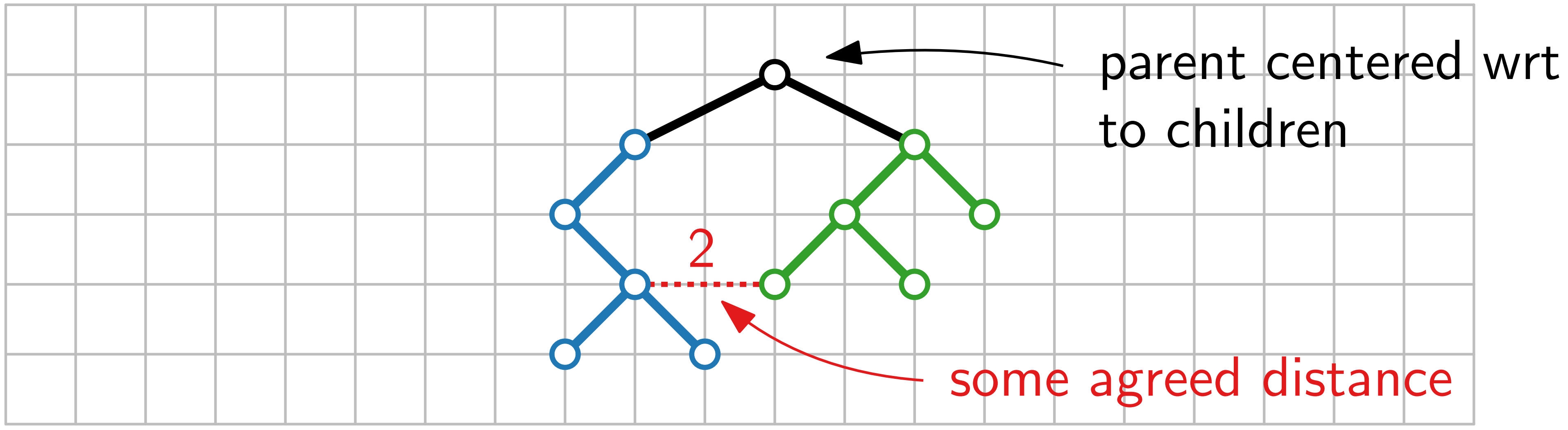

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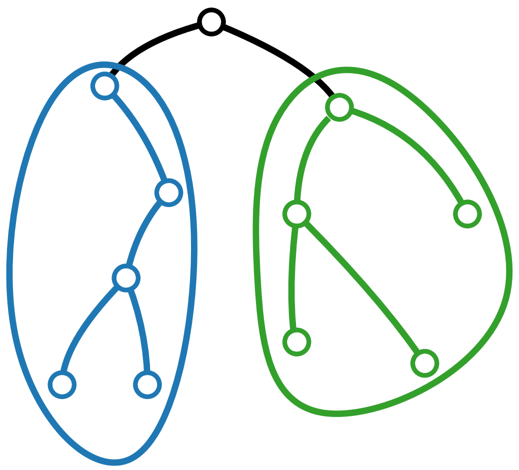

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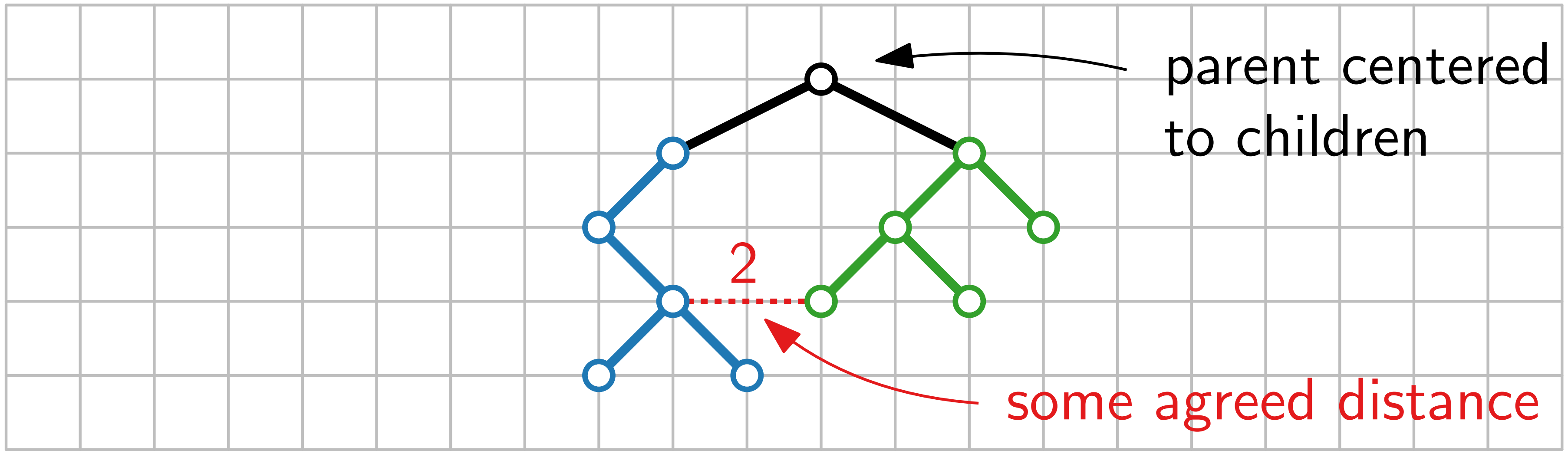

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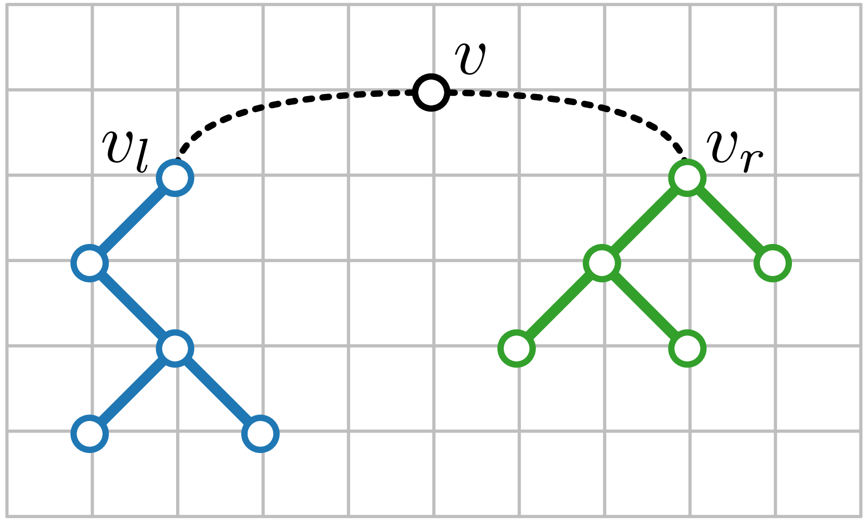

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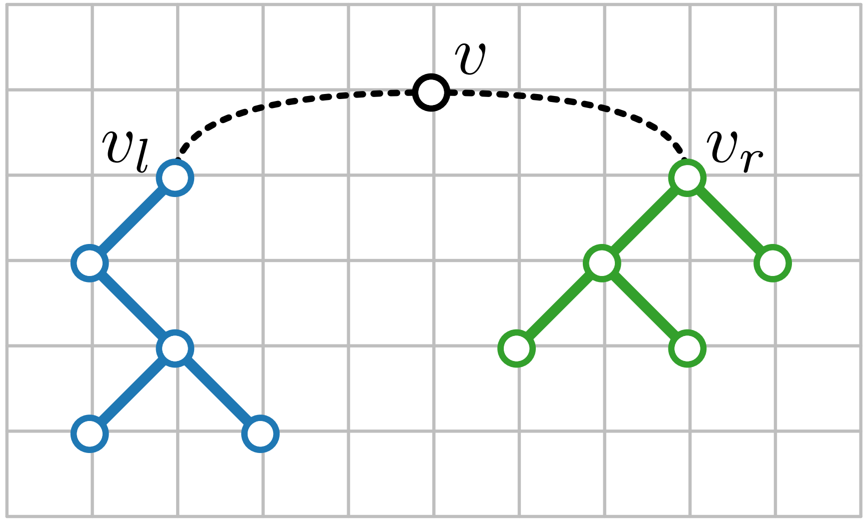

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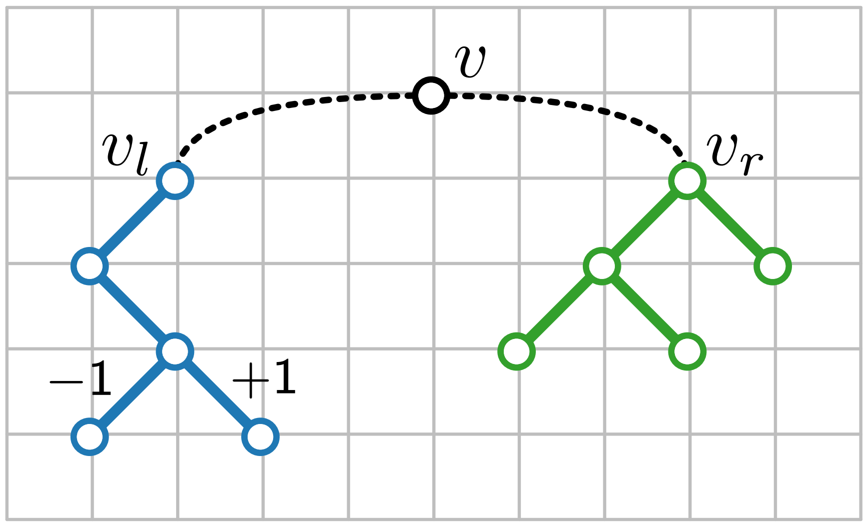

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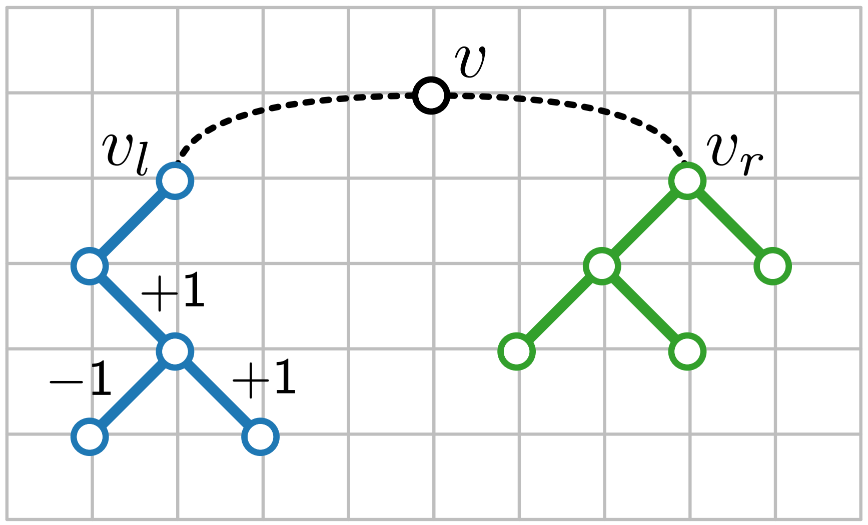

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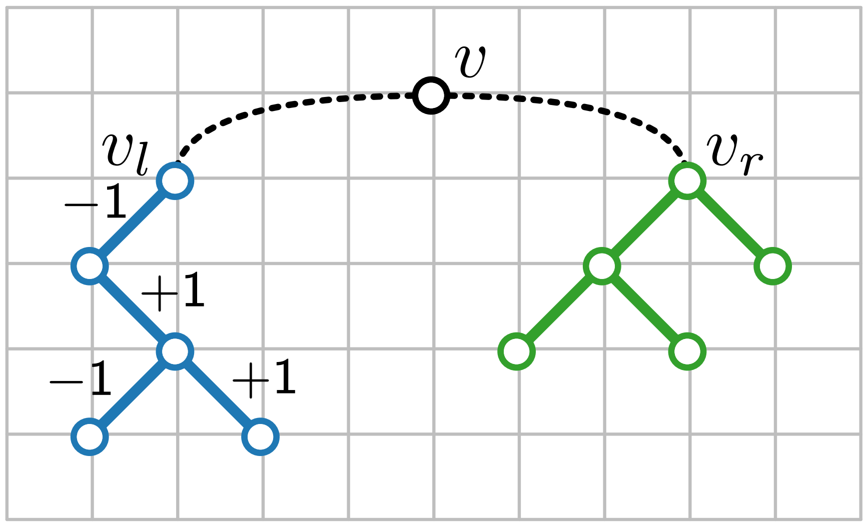

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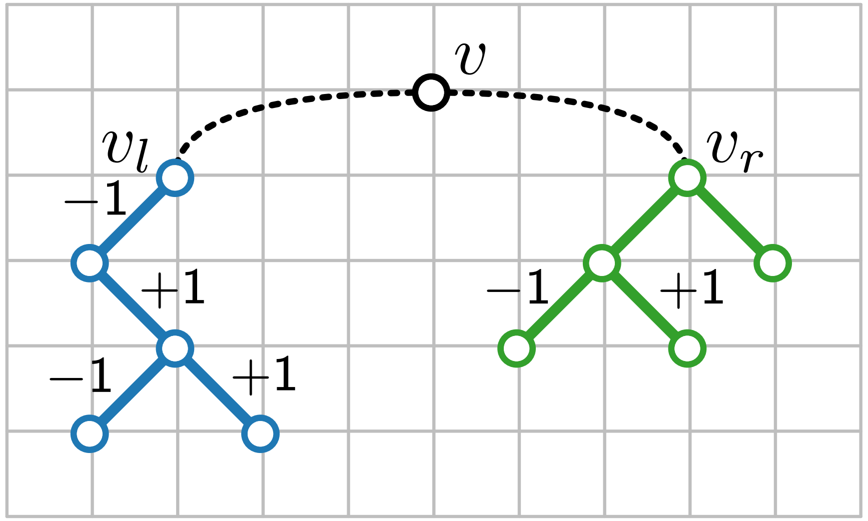

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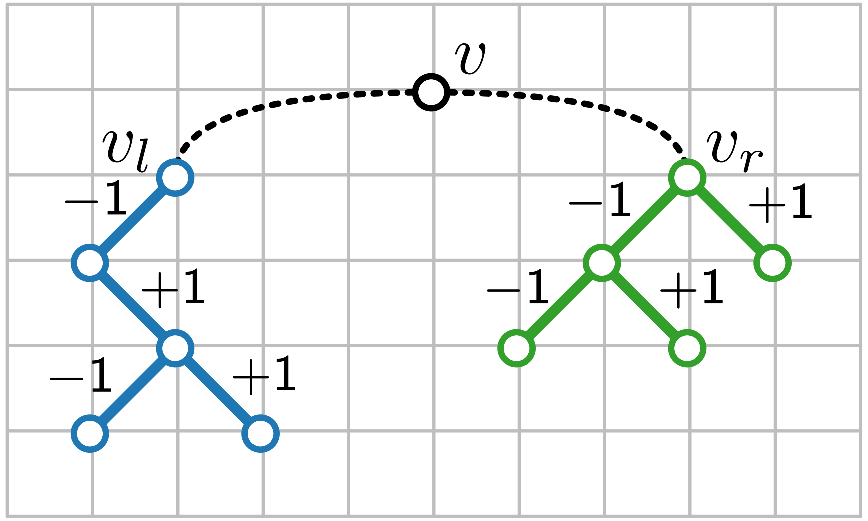

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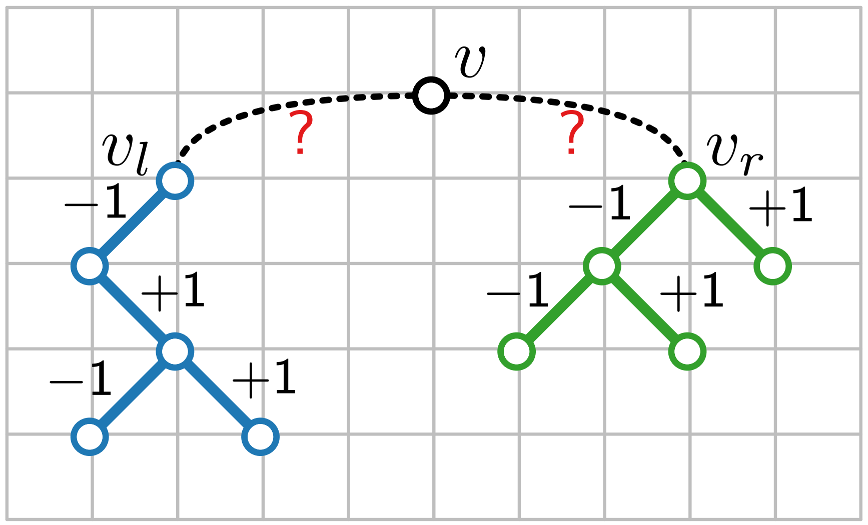

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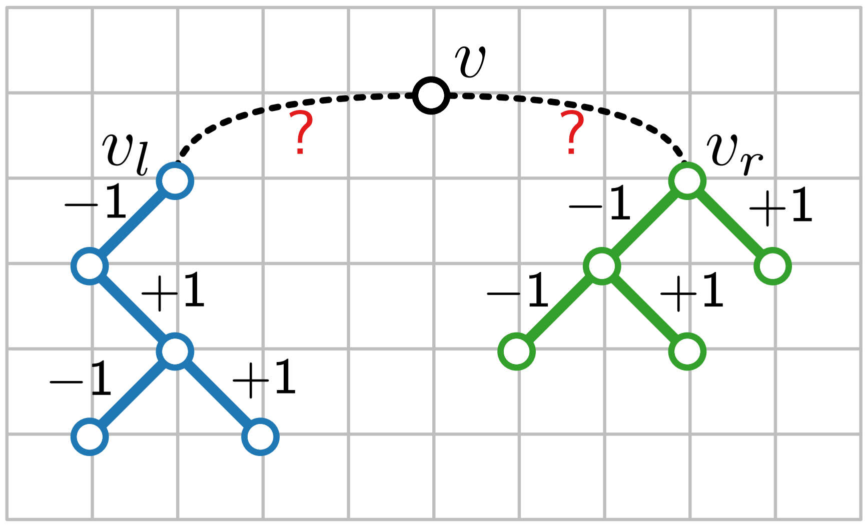

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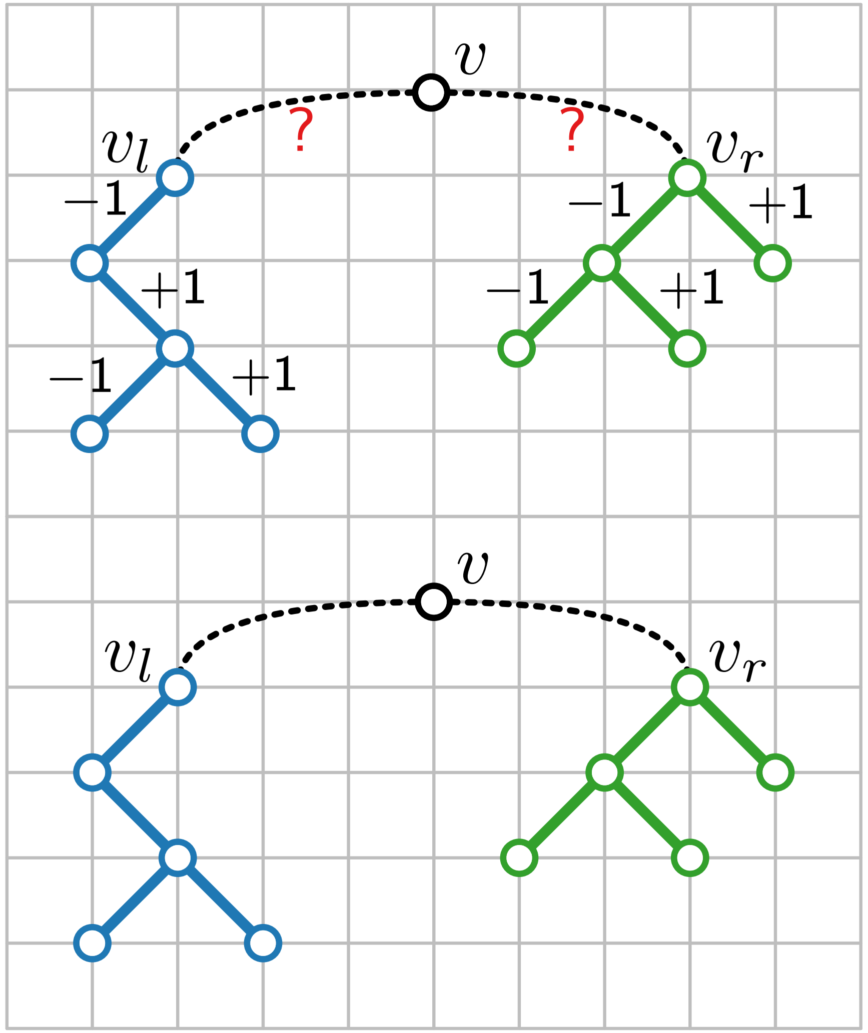

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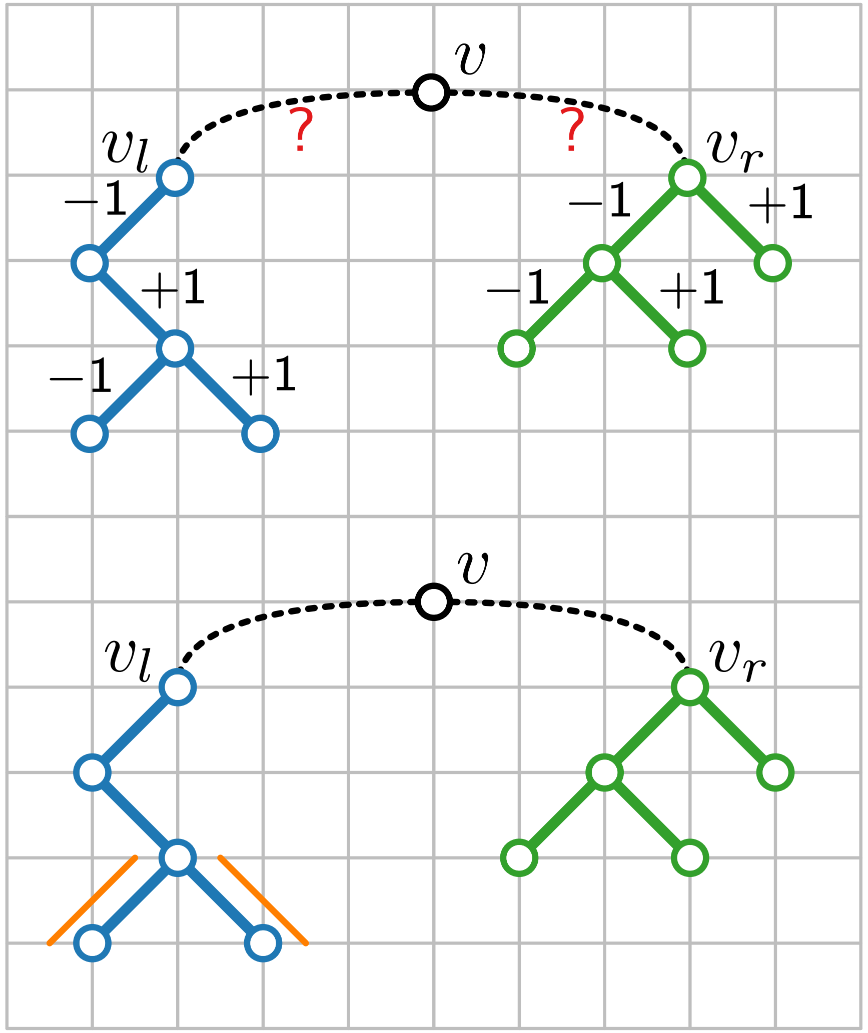

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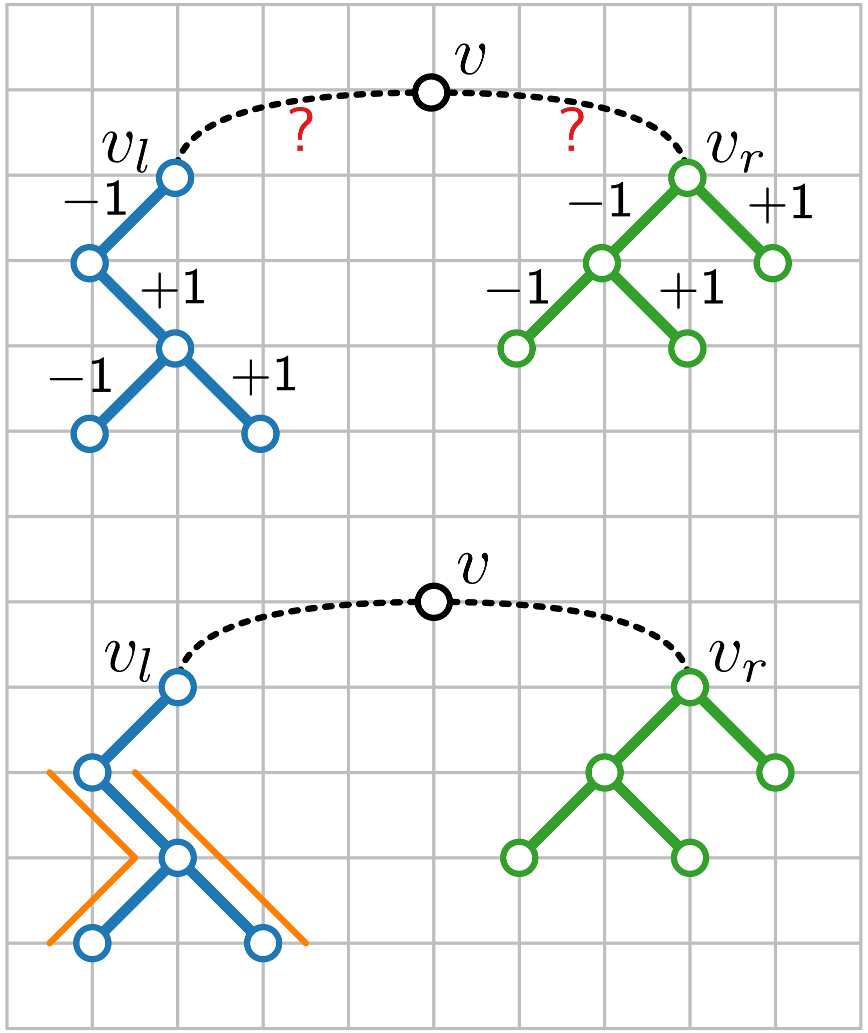

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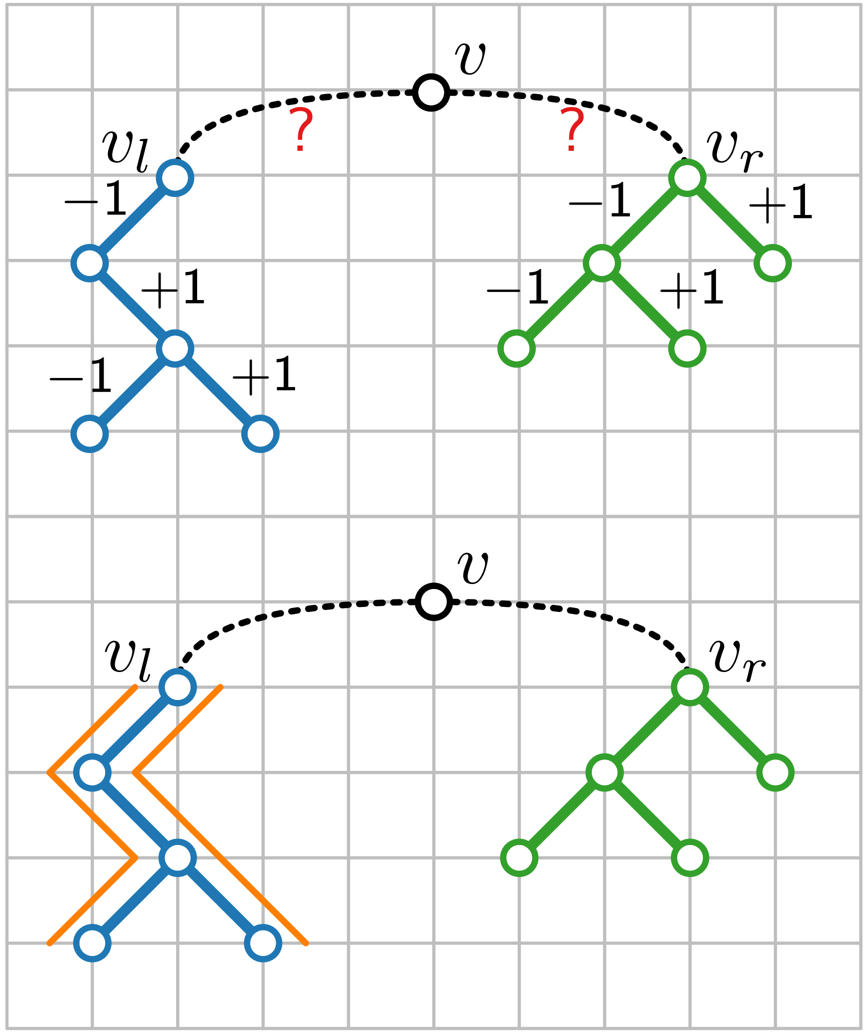

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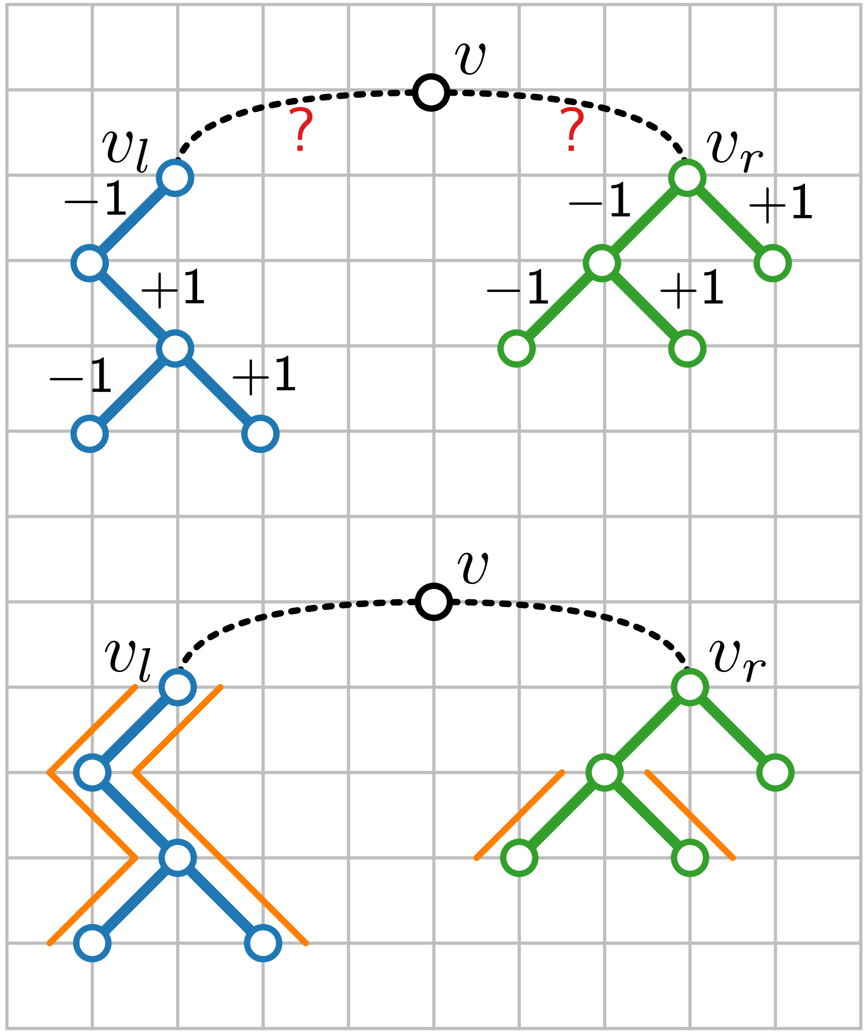

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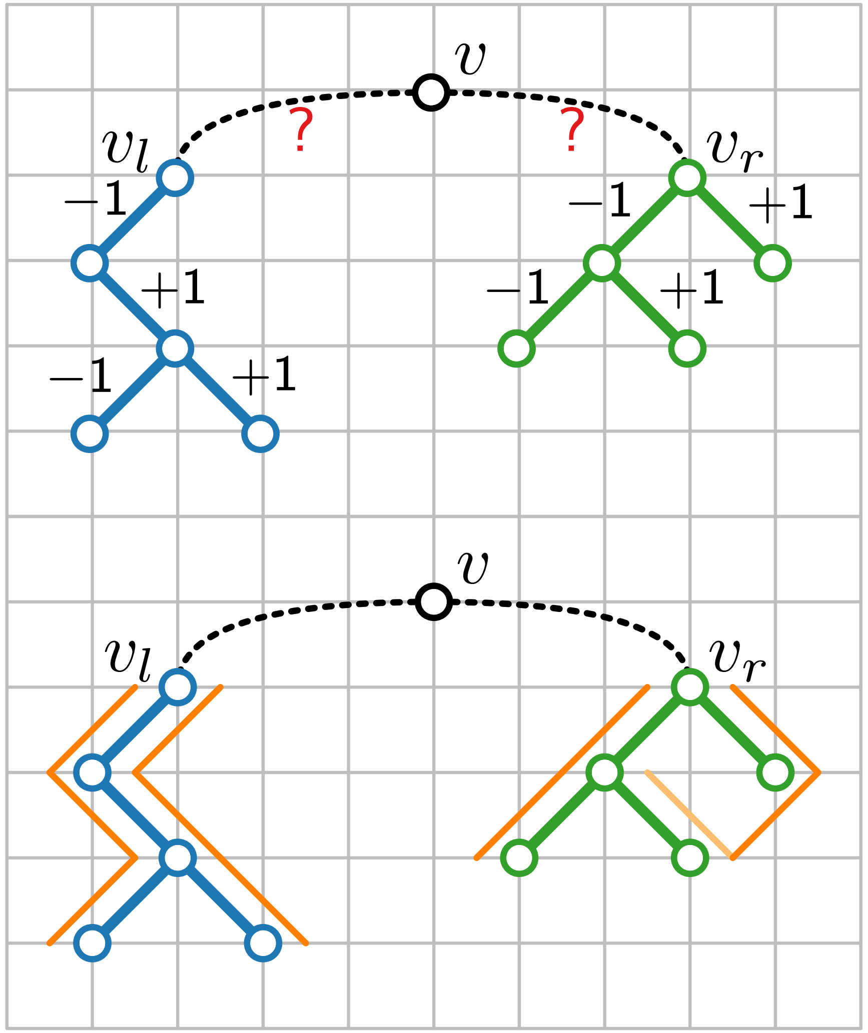

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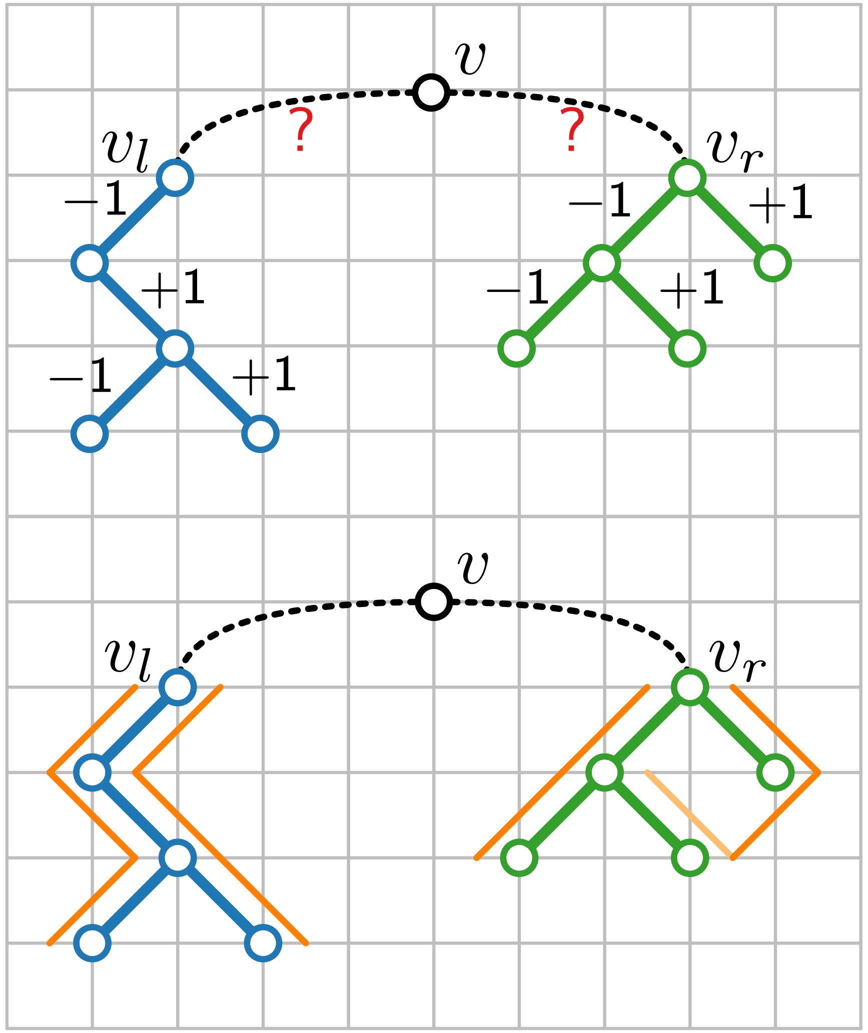

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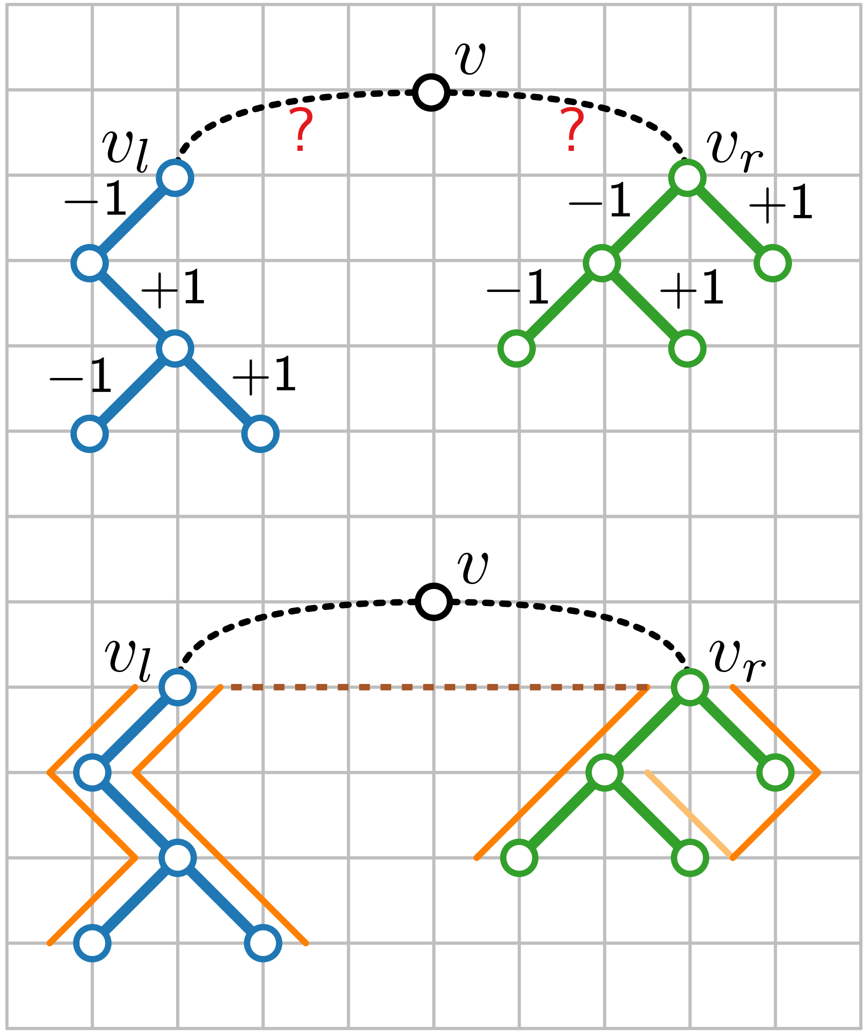

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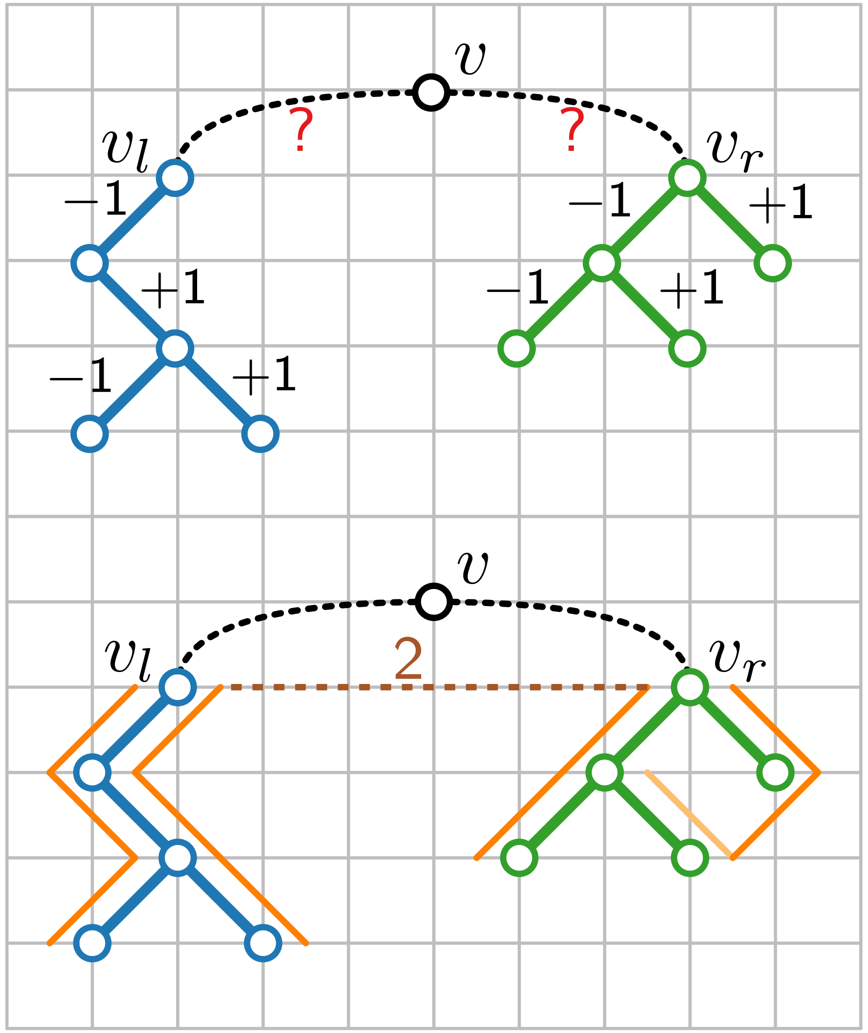

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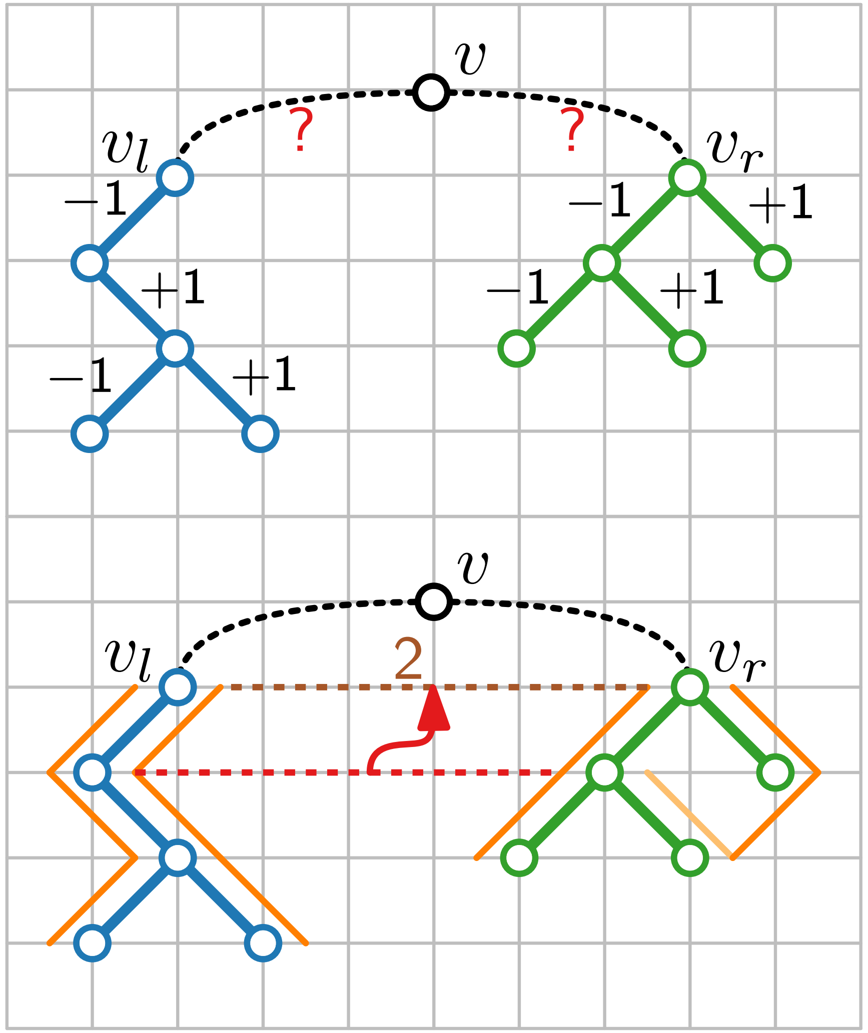

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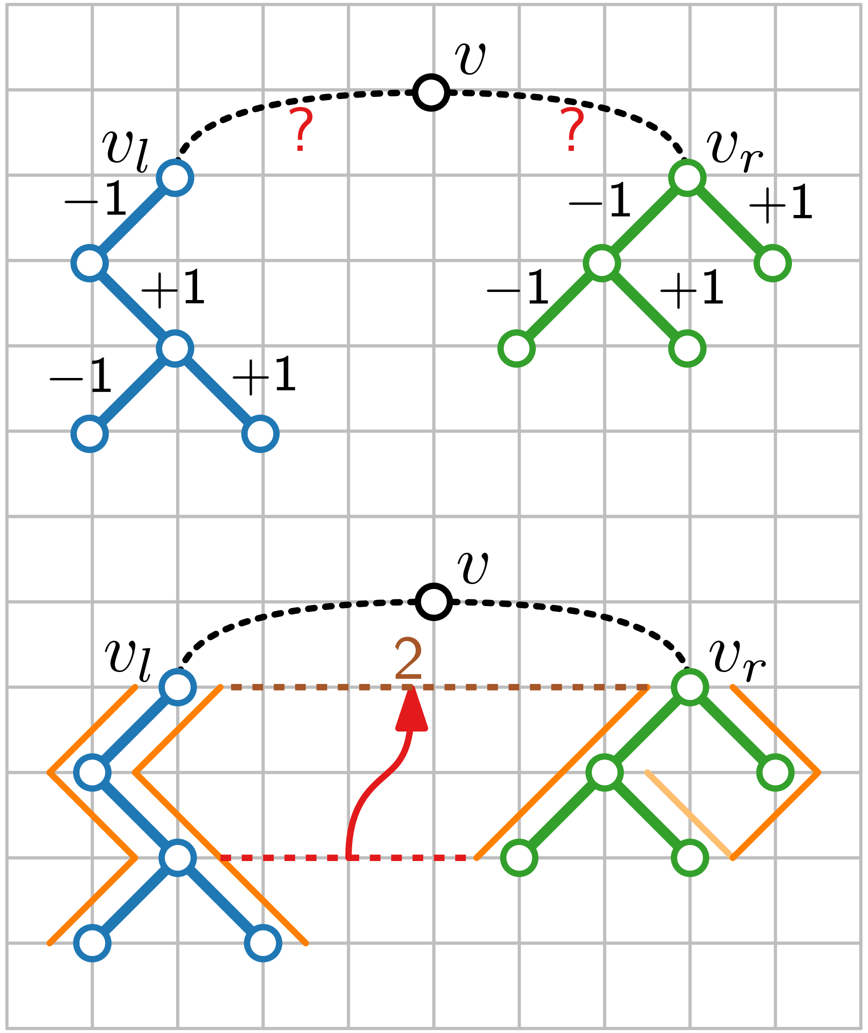

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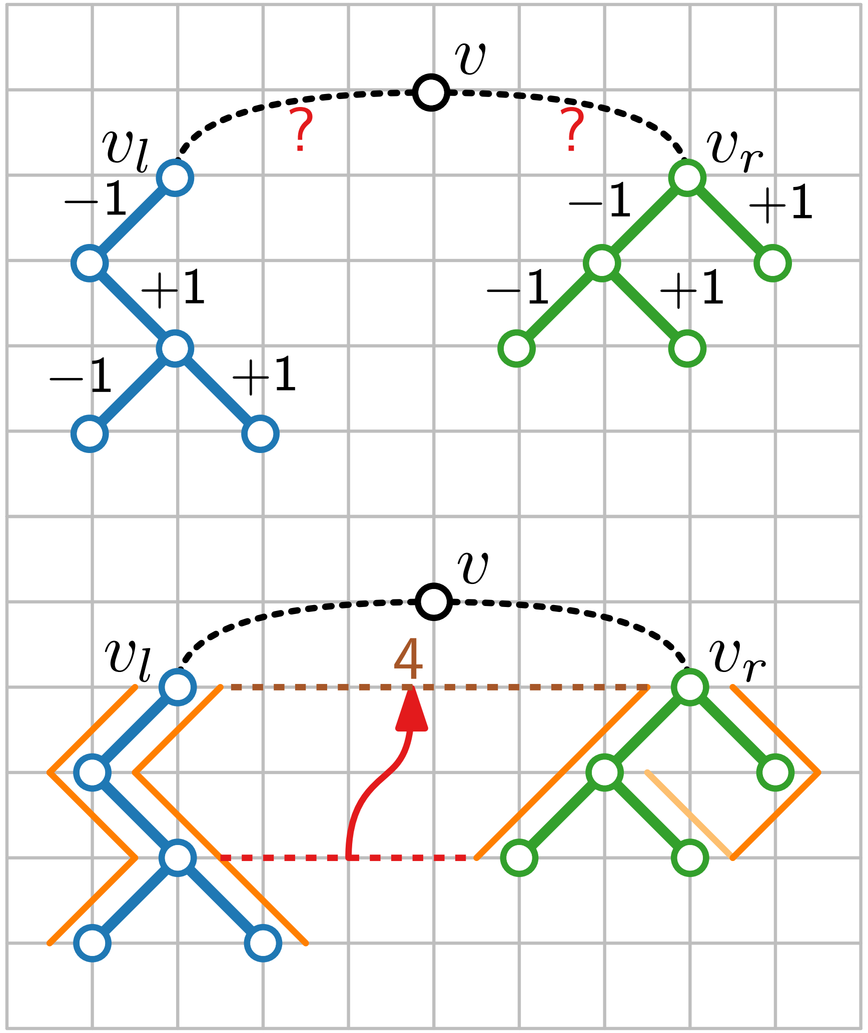

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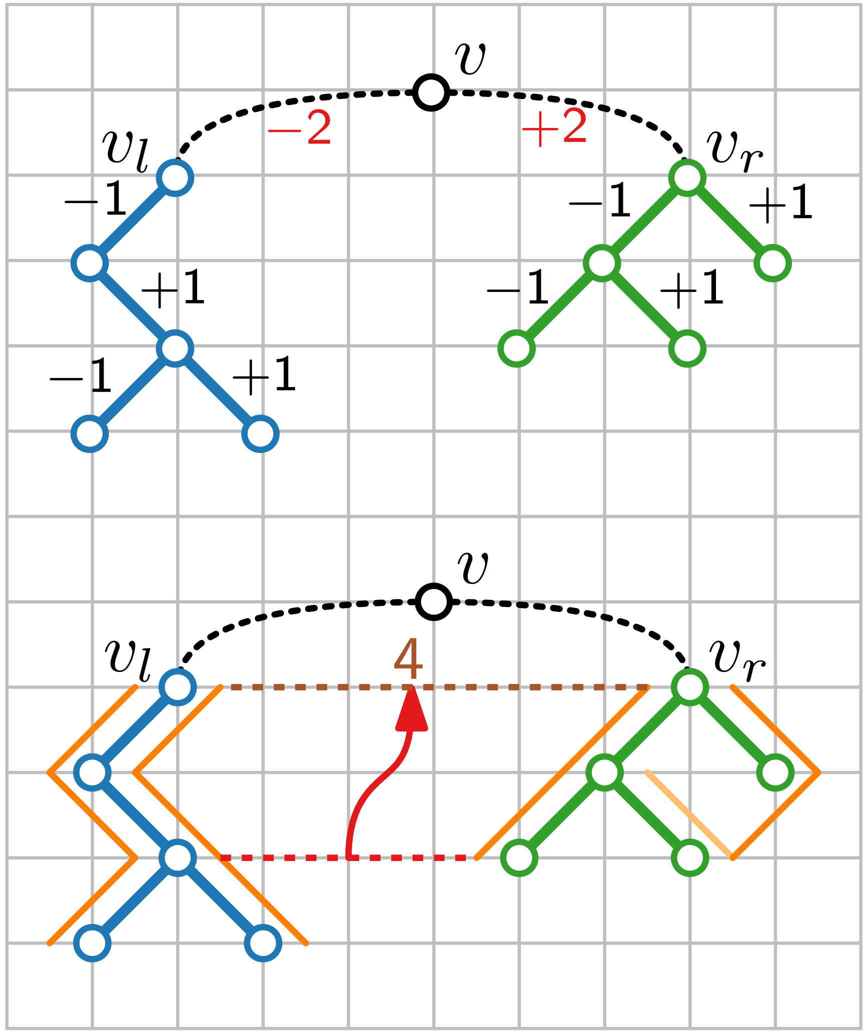

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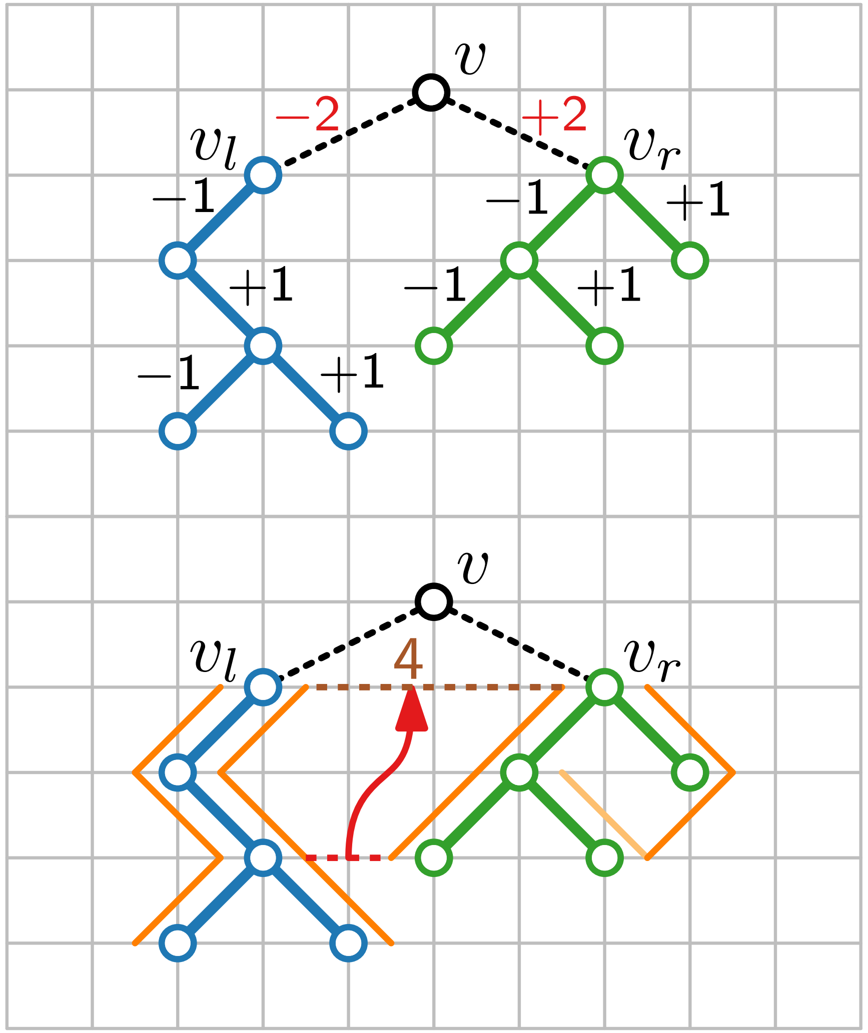

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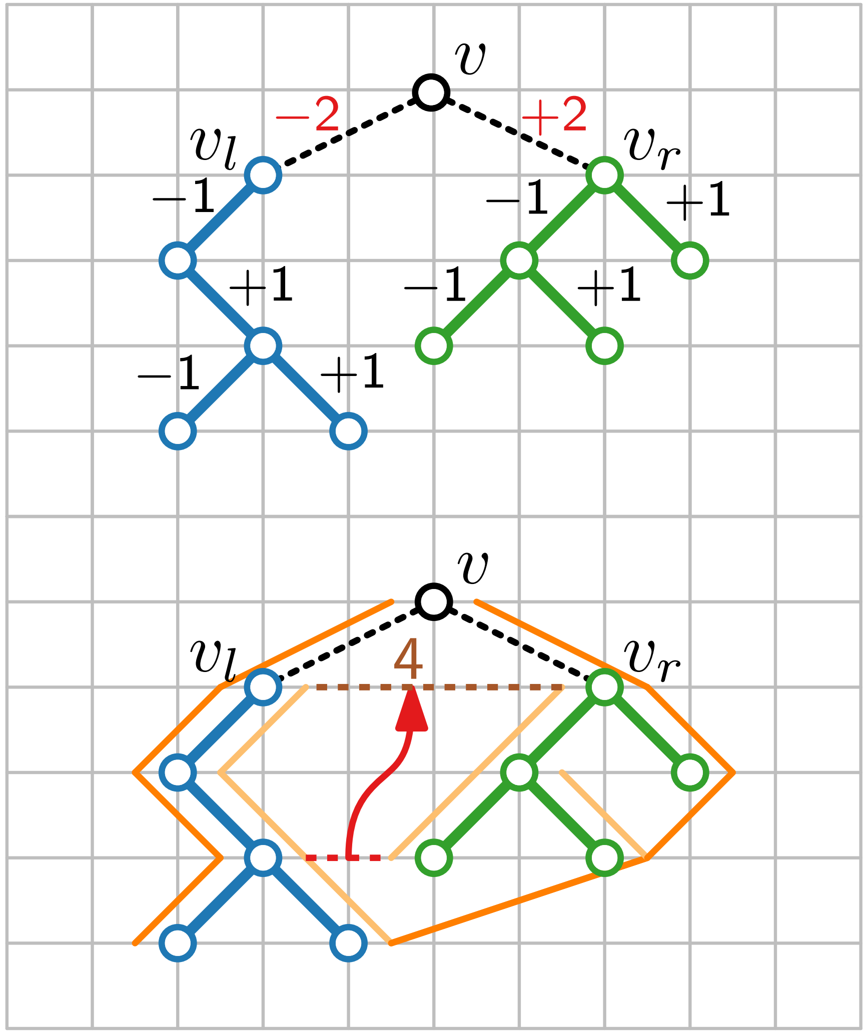

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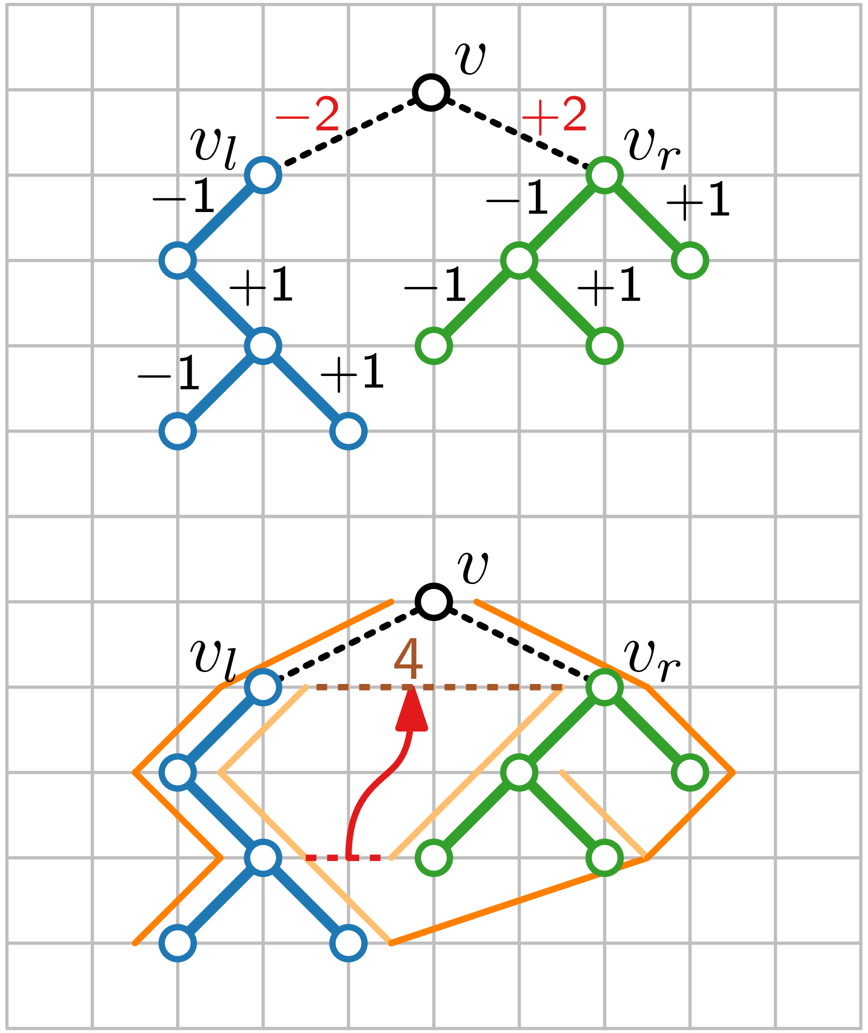

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Runtime?

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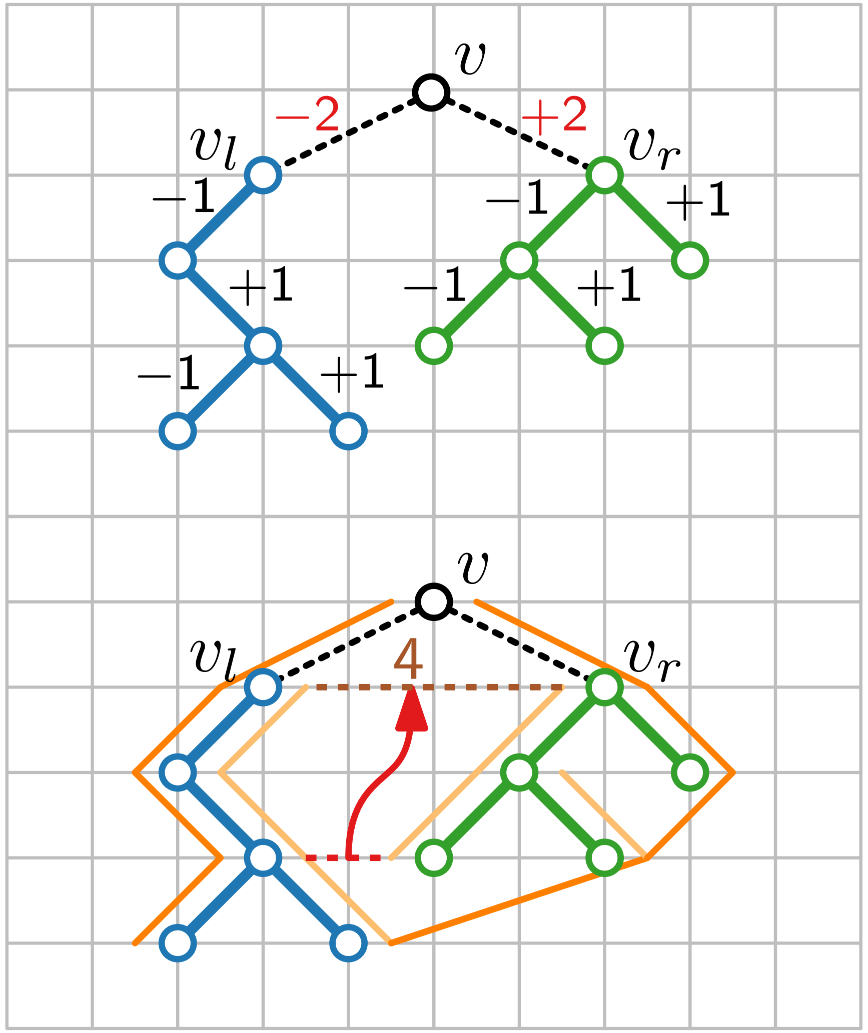

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- How often do we have to walk along a contour?

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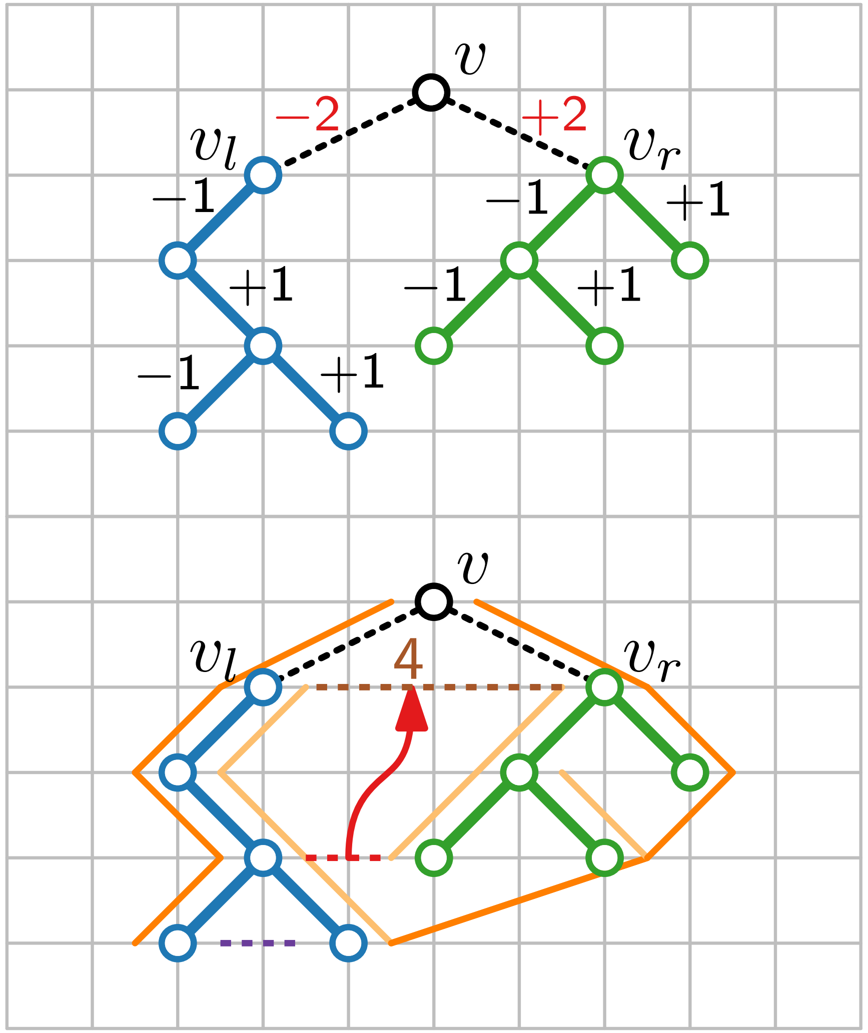

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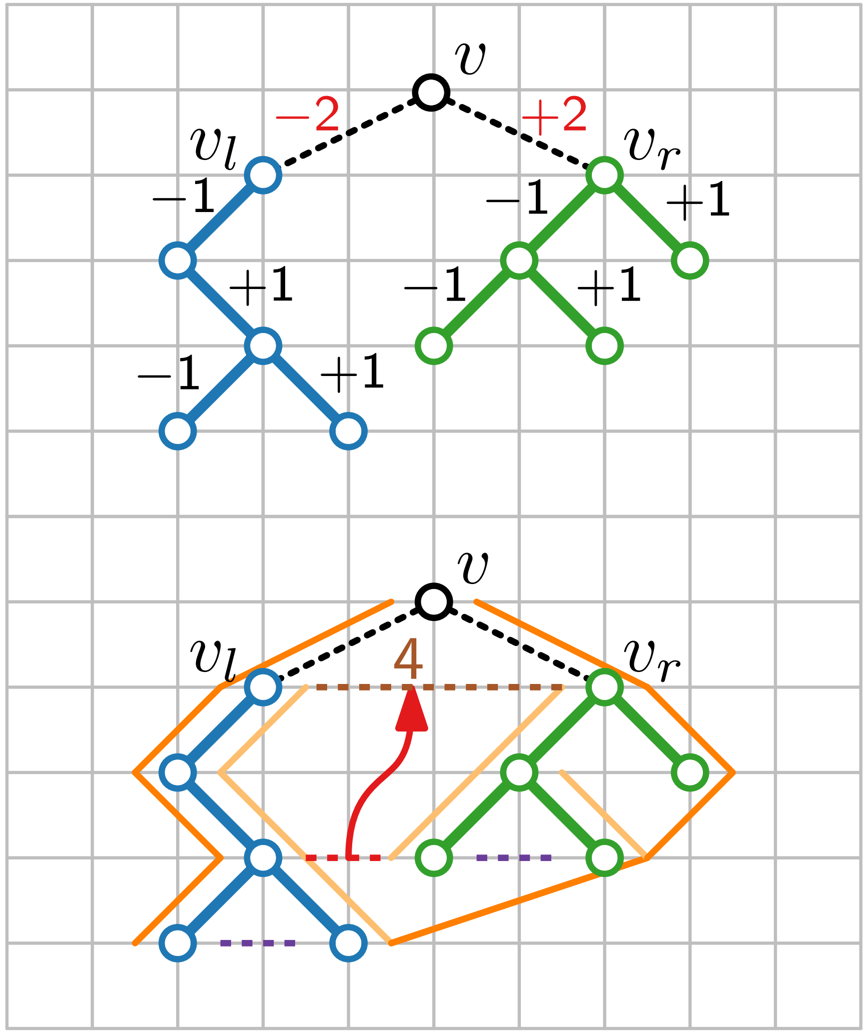

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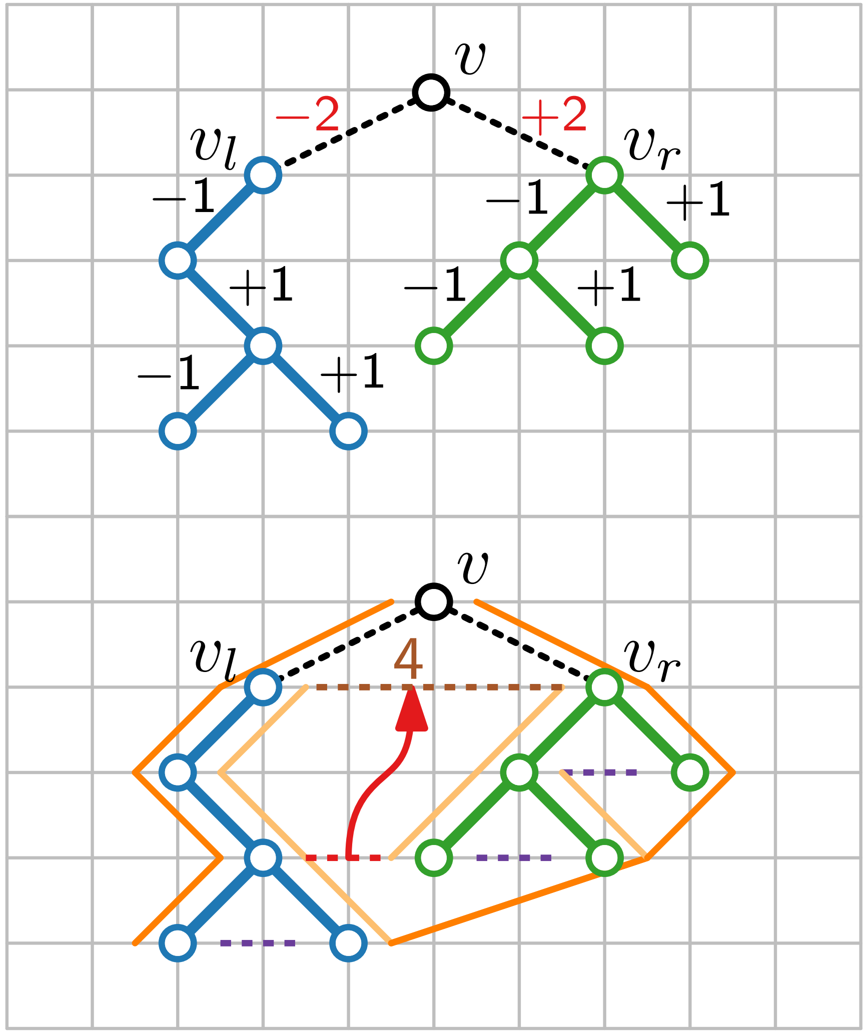

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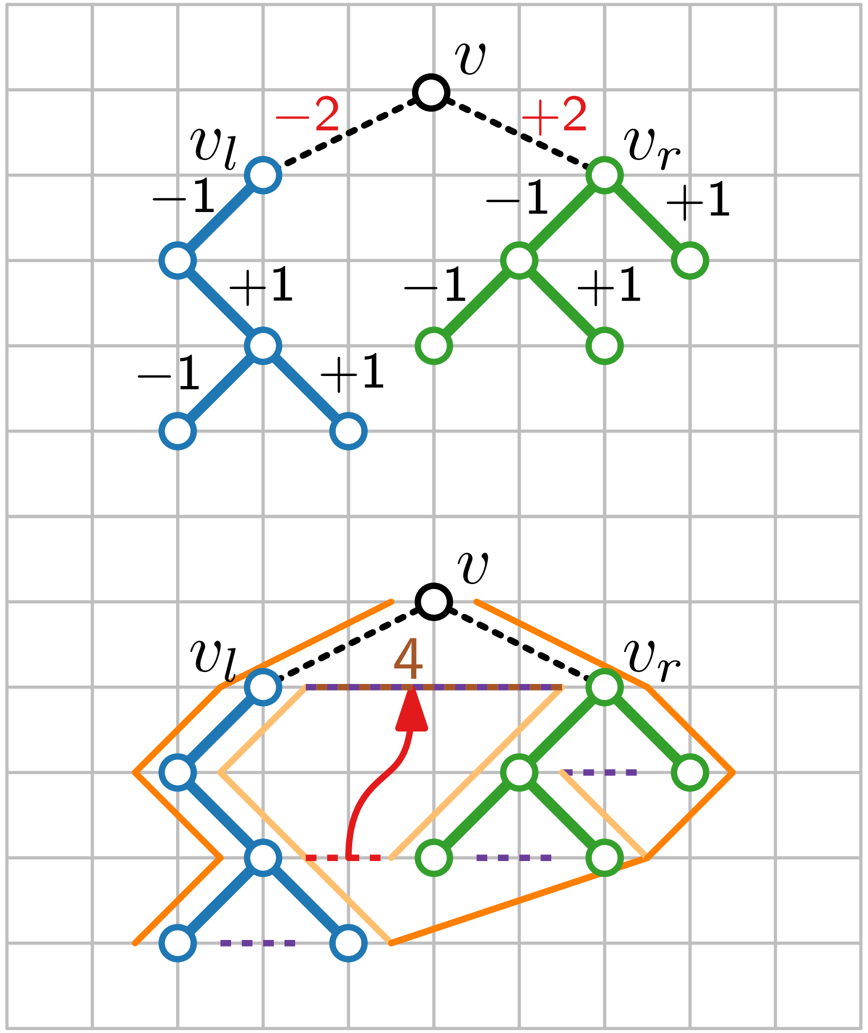

Phase 2 – preorder traversal:

- Compute x- and y-coordinates

Runtime?

- How often do we have to walk along a contour?

Layered Drawings – Algorithm Details

Phase 1 – postorder traversal:

- For each vertex compute horizontal displacement of left and right child
- x-offset$(v_l) = -\lceil \frac{d_v}{2} \rceil$, x-offset$(v_r) = \lceil \frac{d_v}{2} \rceil$
- At vertex u (below v) store left and right contour of subtree $T(u)$
- Contour is linked list of vertex coordinates/offsets
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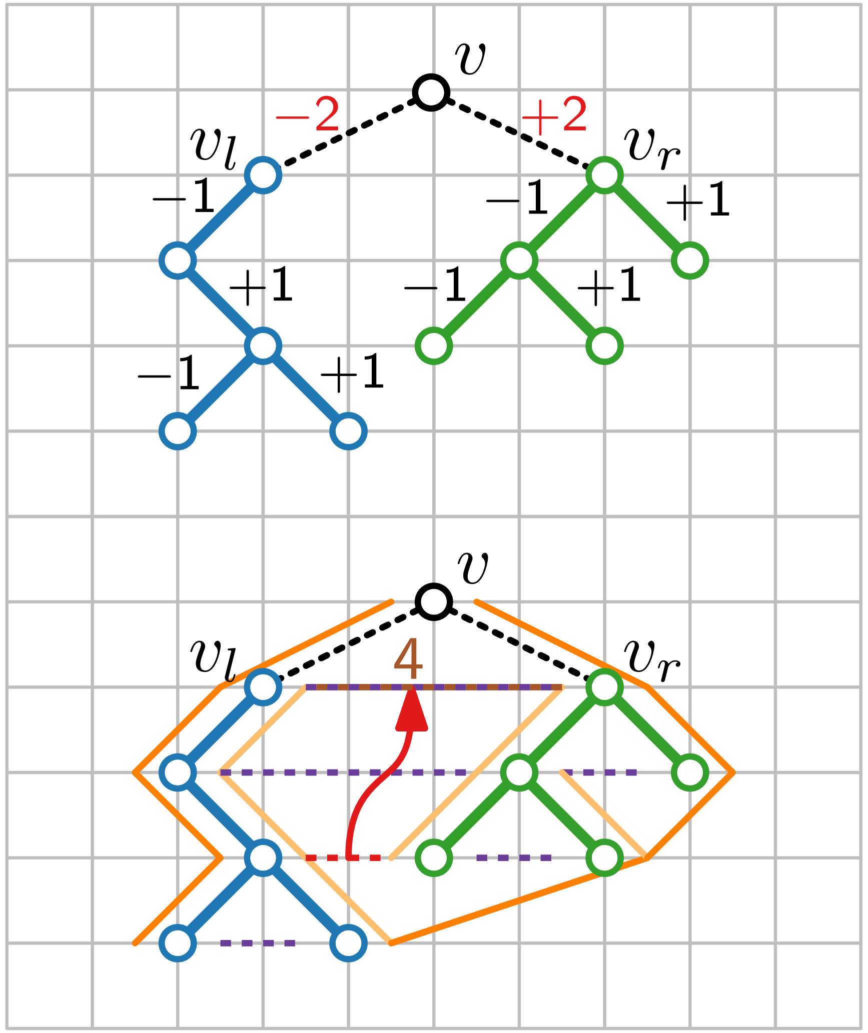

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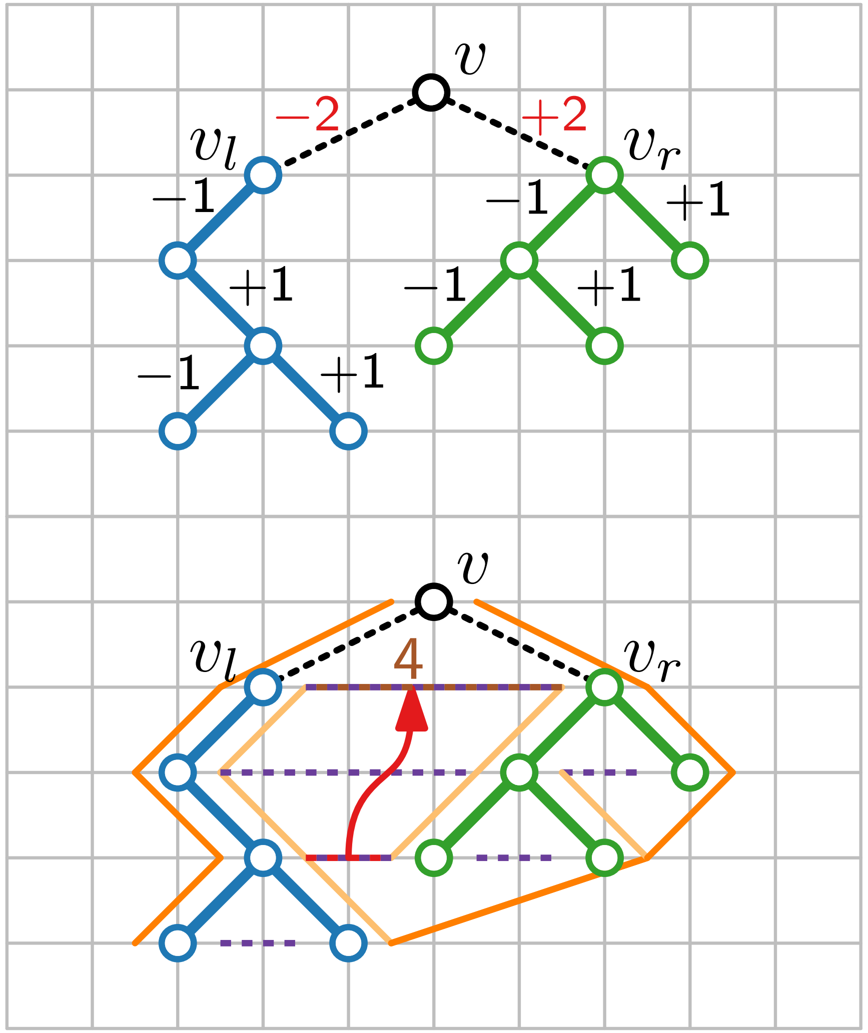

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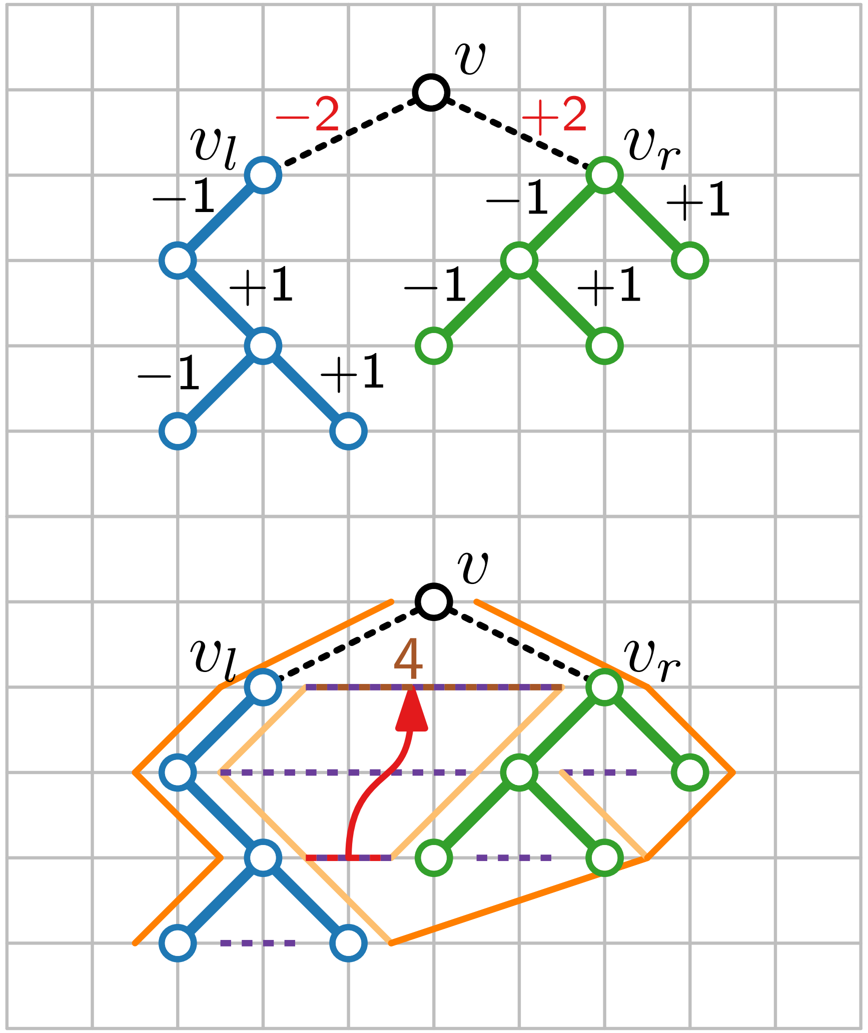

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$\Rightarrow \mathcal{O}(n)$

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Let T be a binary tree with n vertices. We can construct a drawing Γ of T in $\mathcal{O}(n)$ time, such that:

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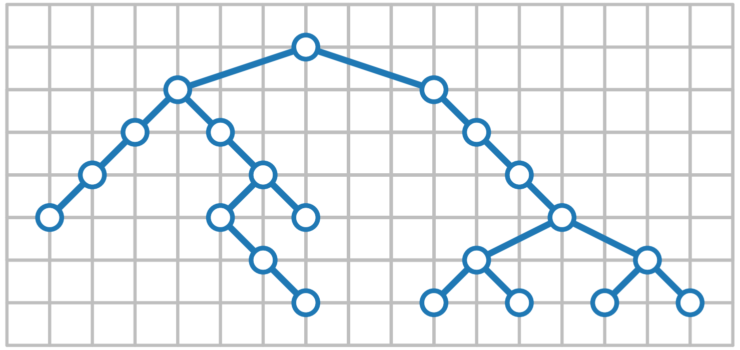

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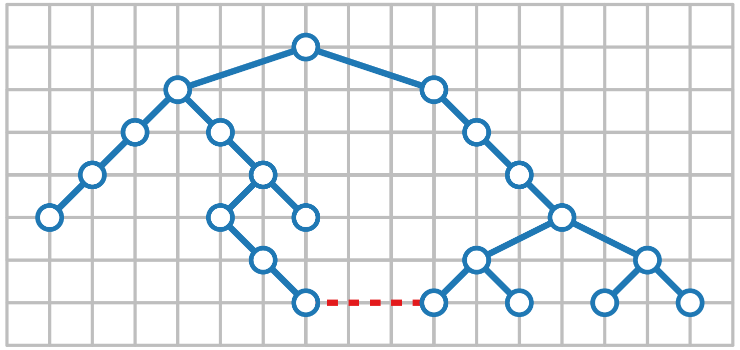

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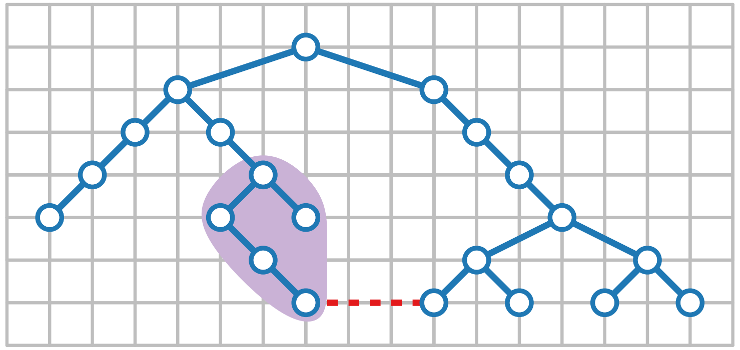

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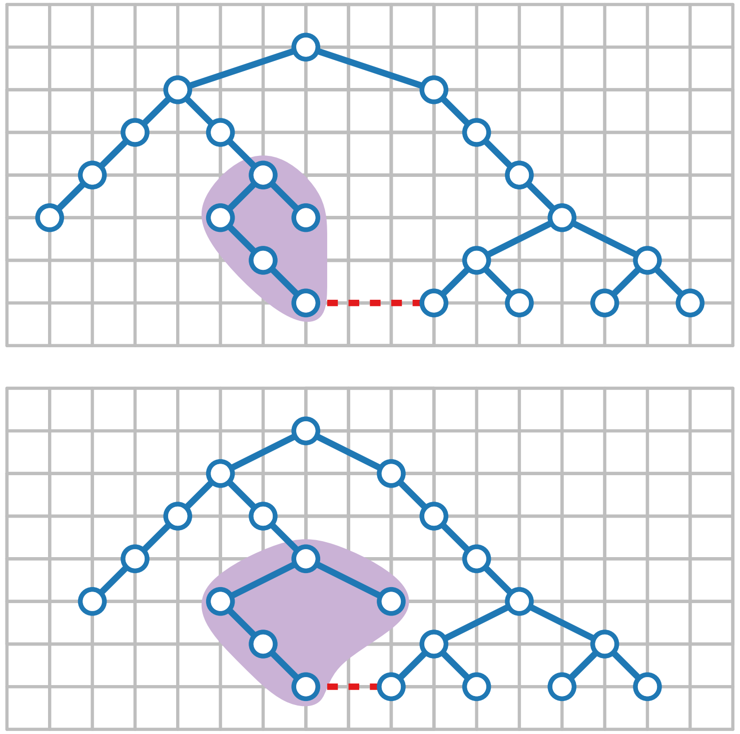

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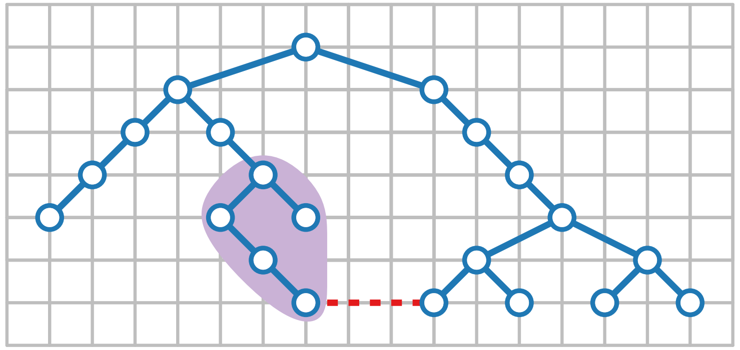

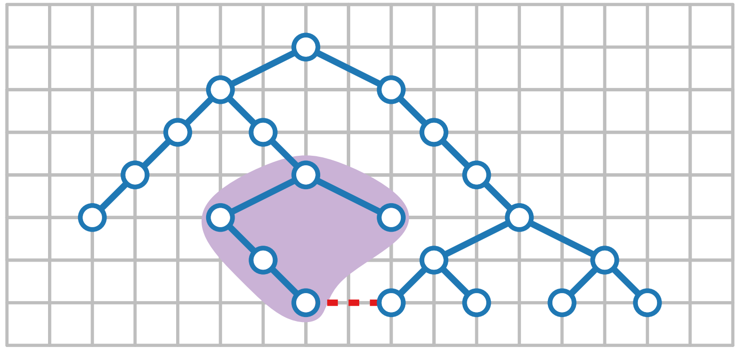

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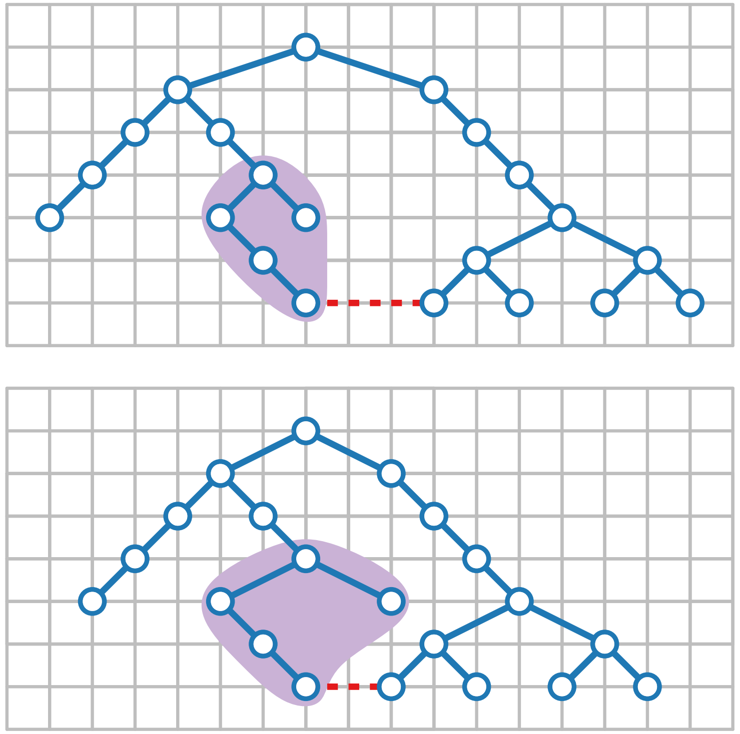

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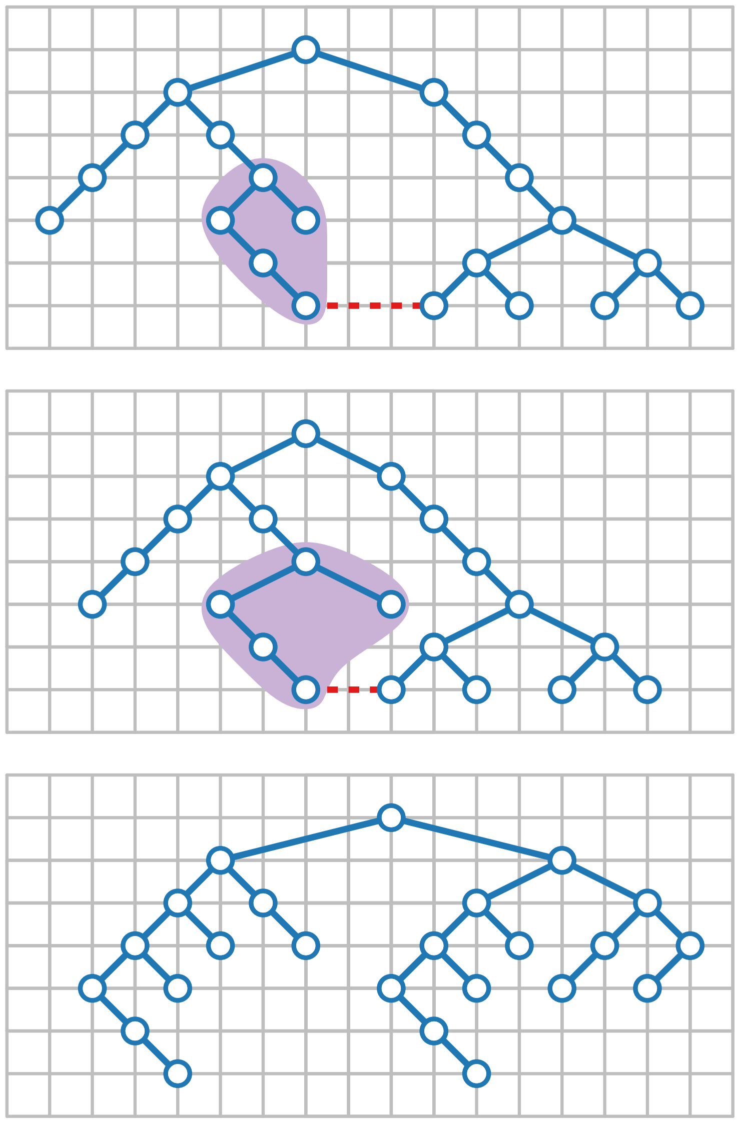

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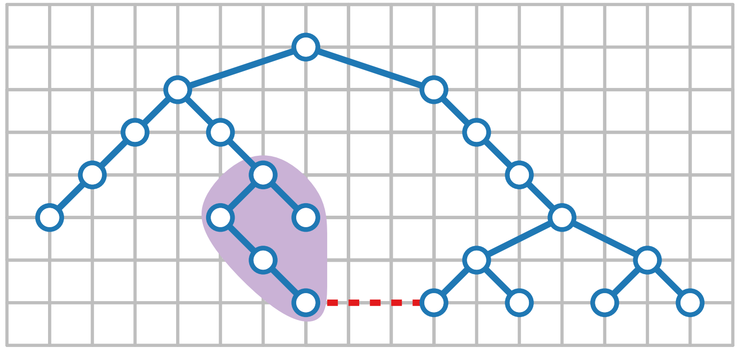

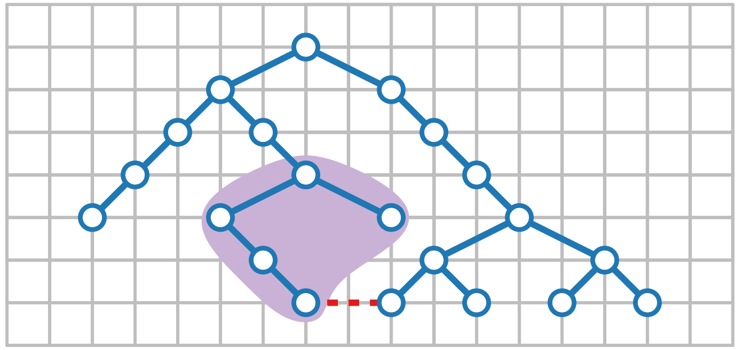

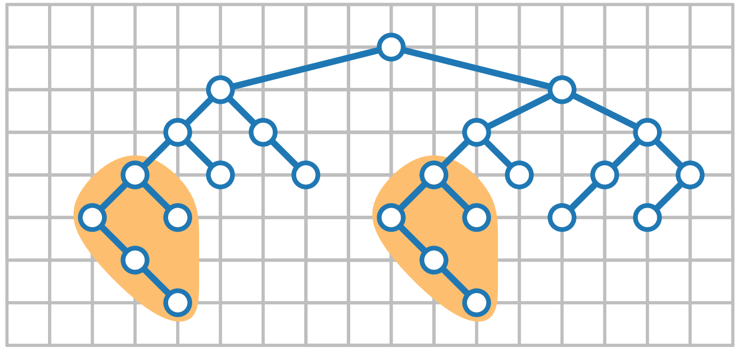

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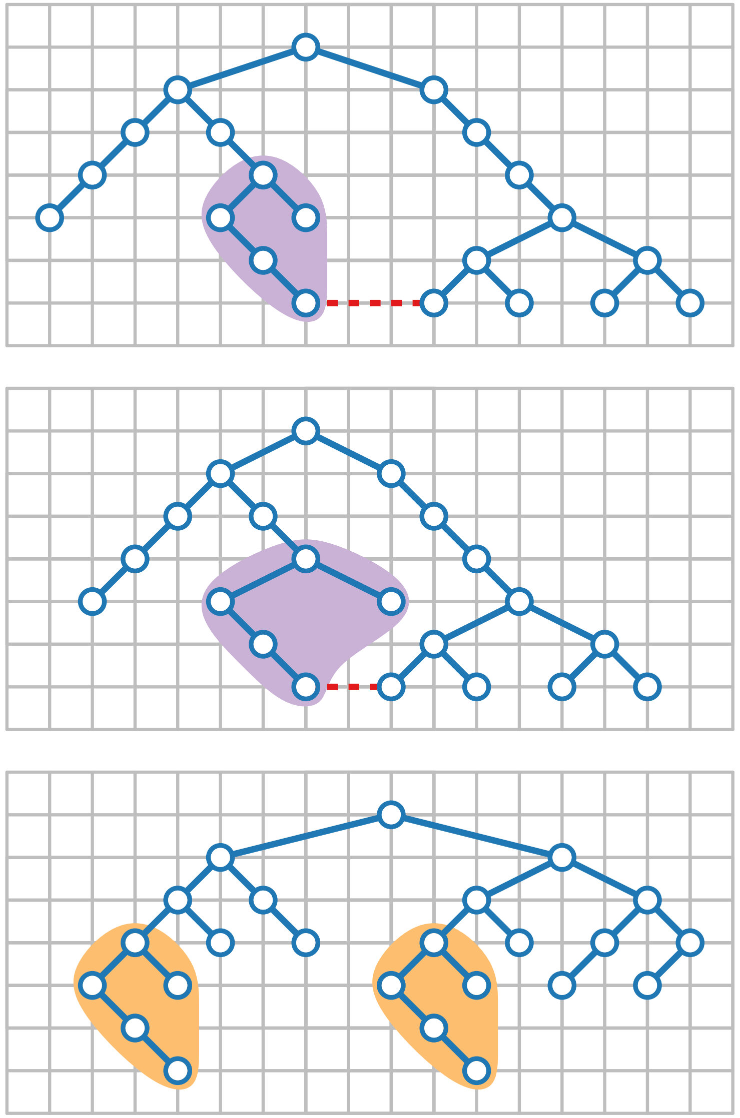

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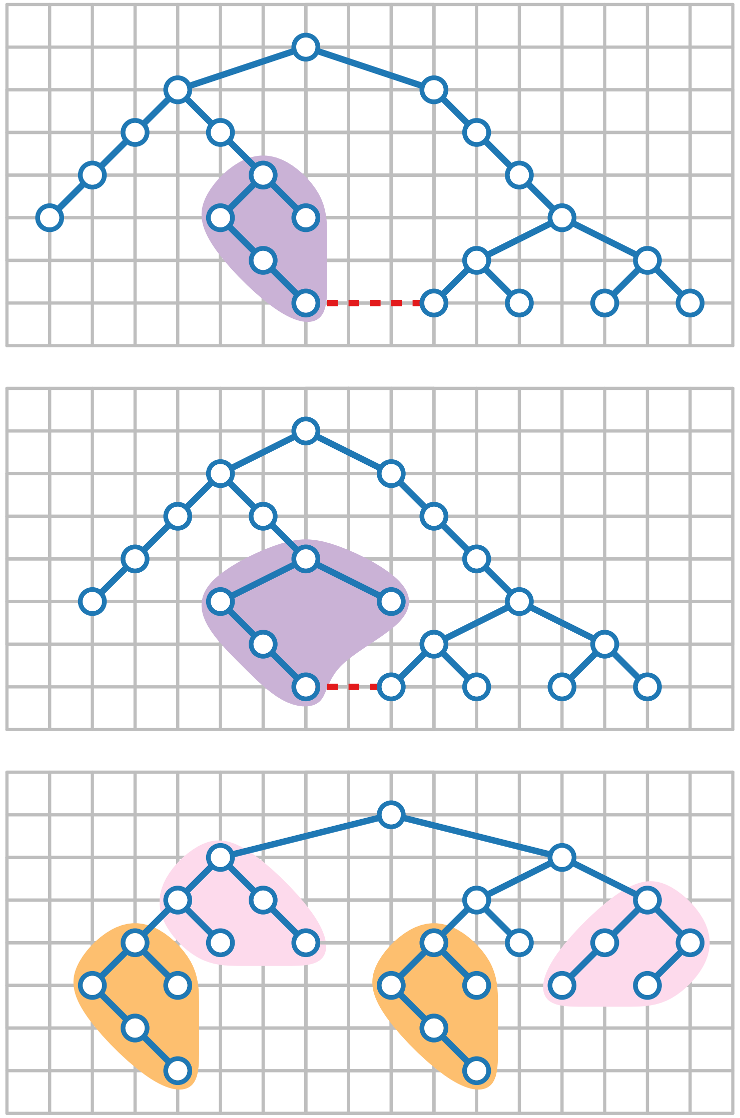

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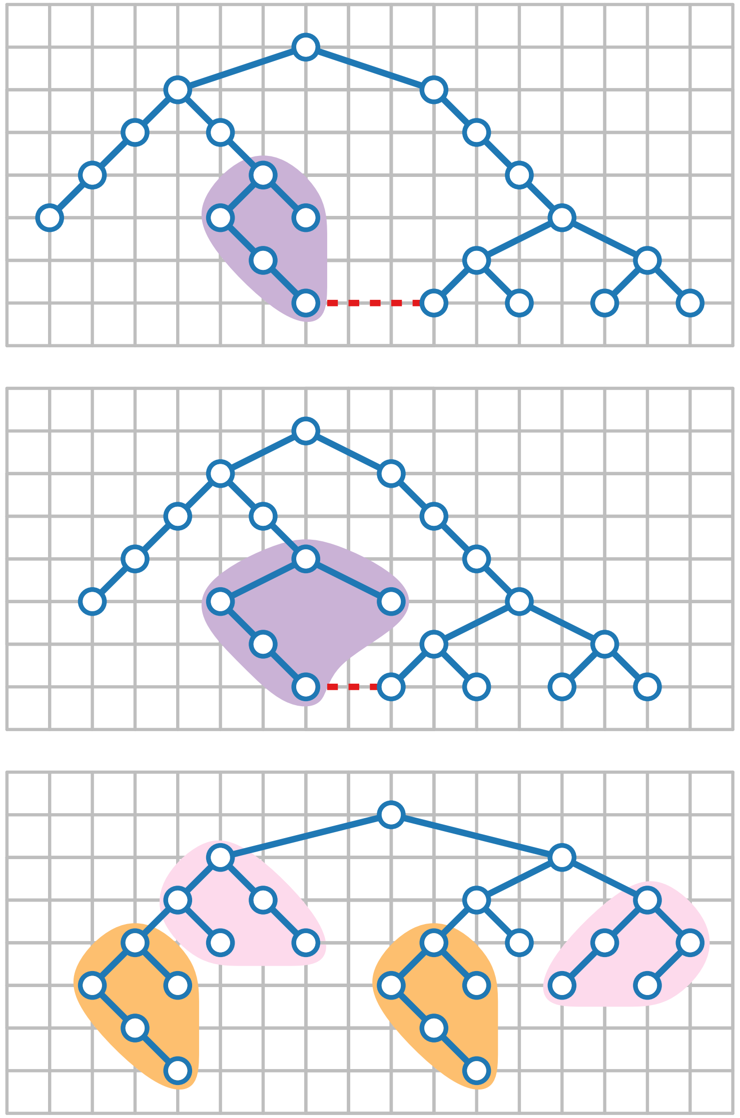

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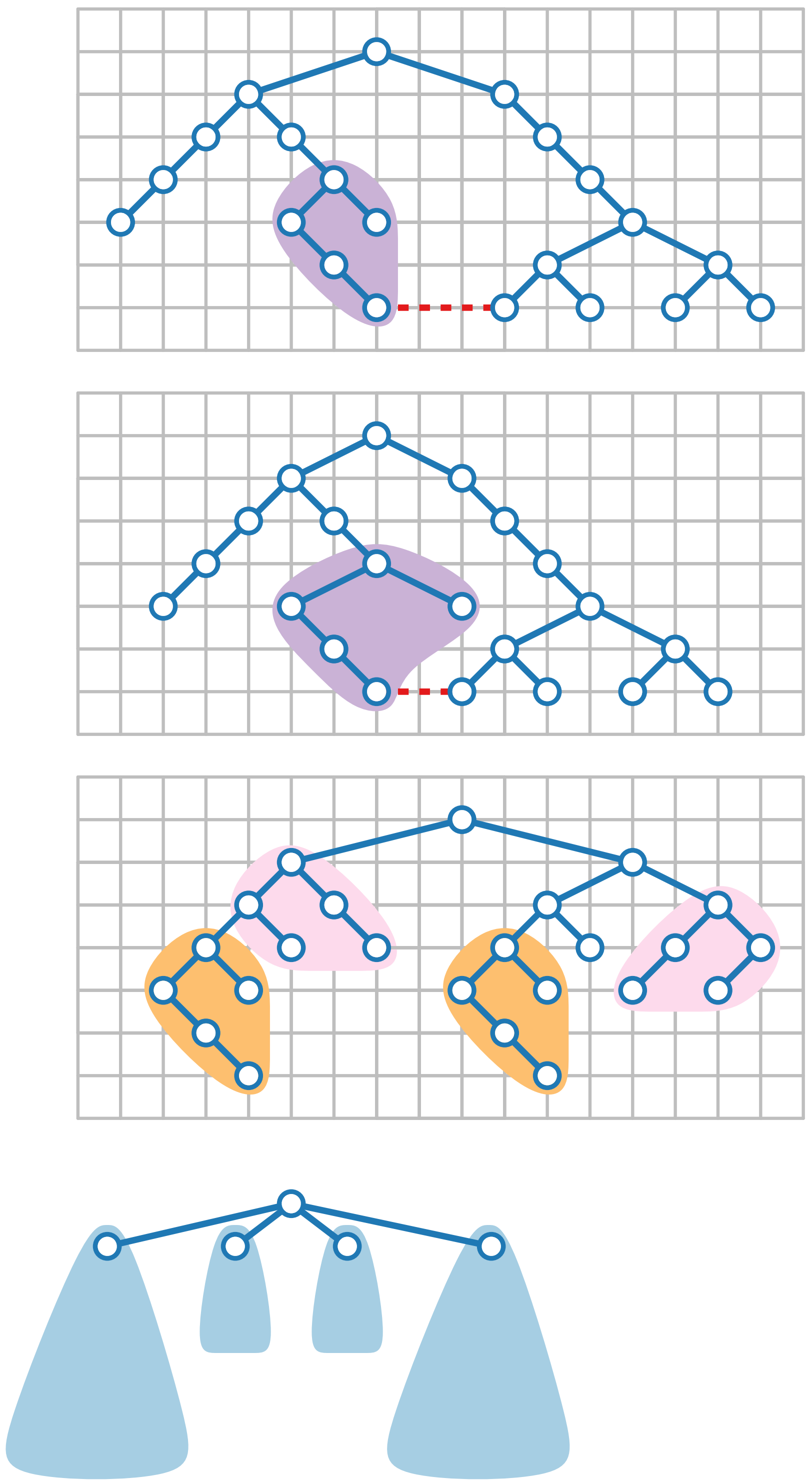

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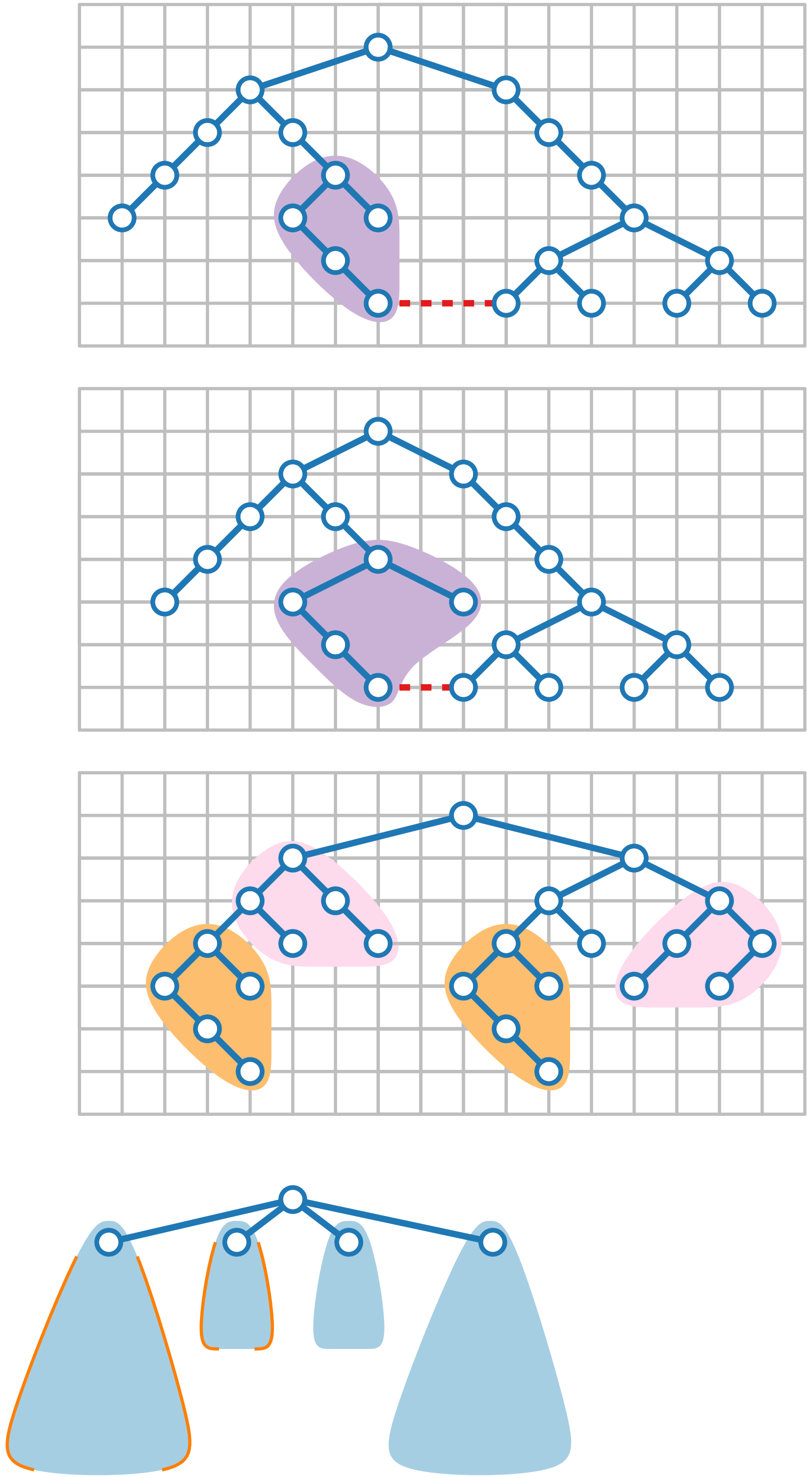

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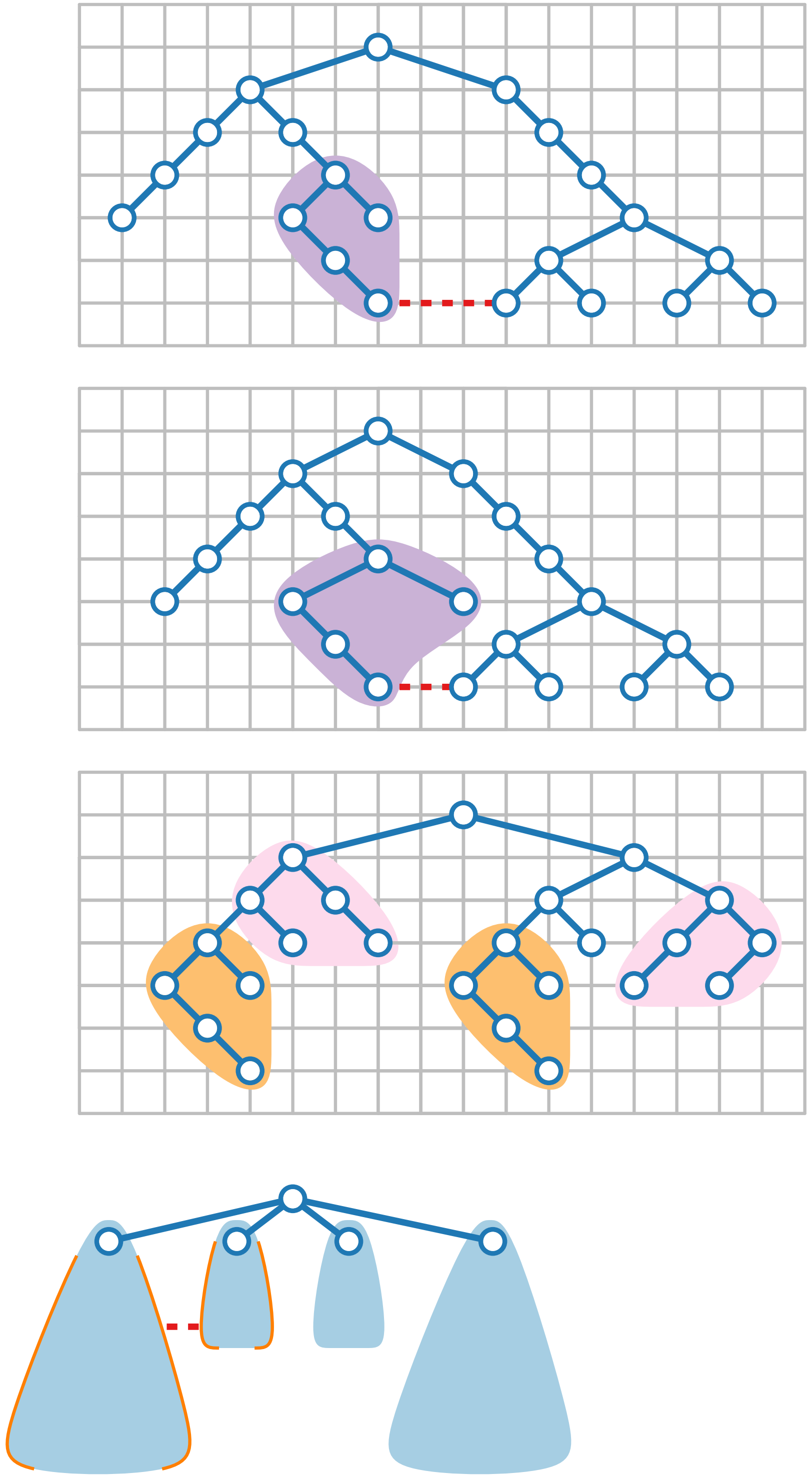

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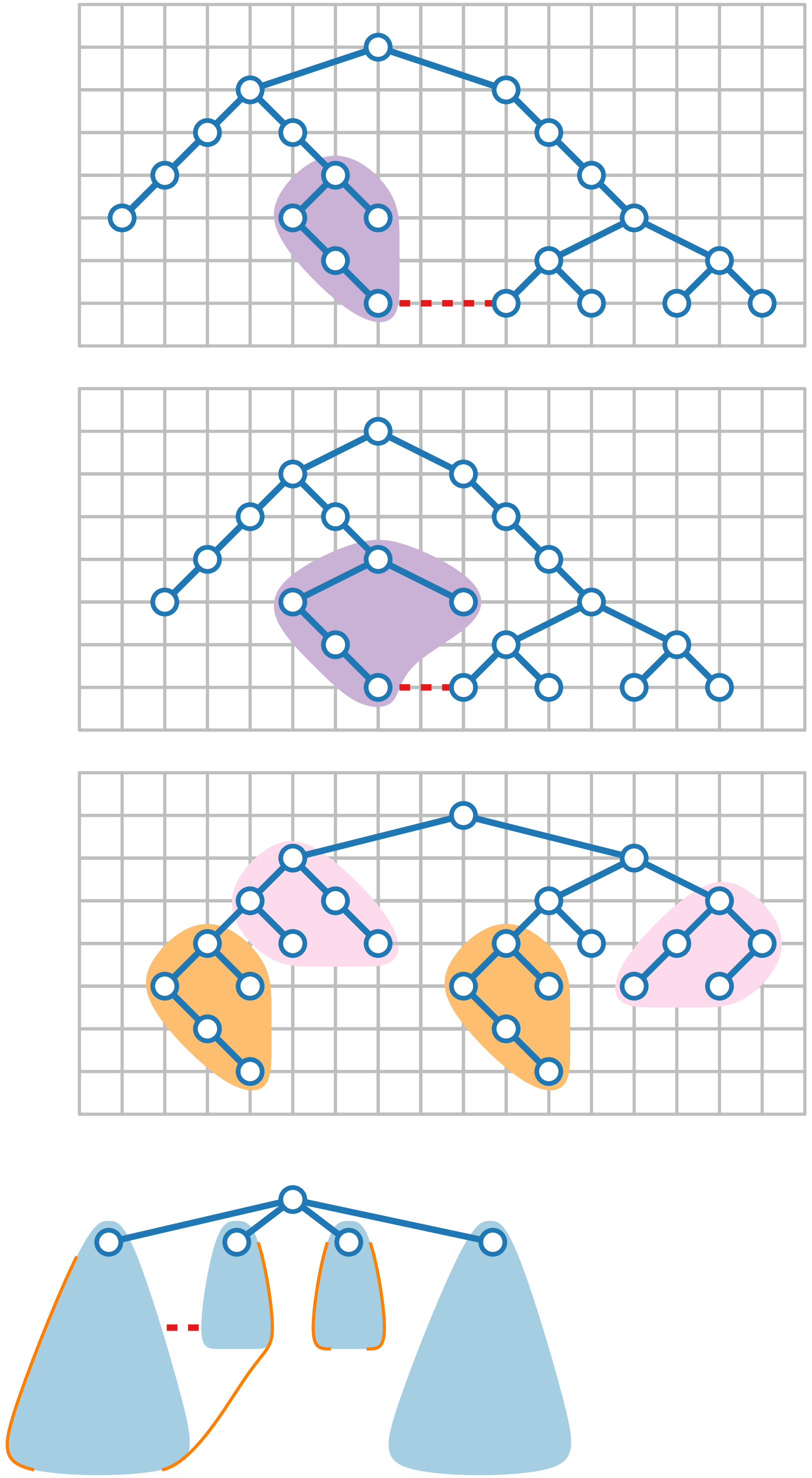

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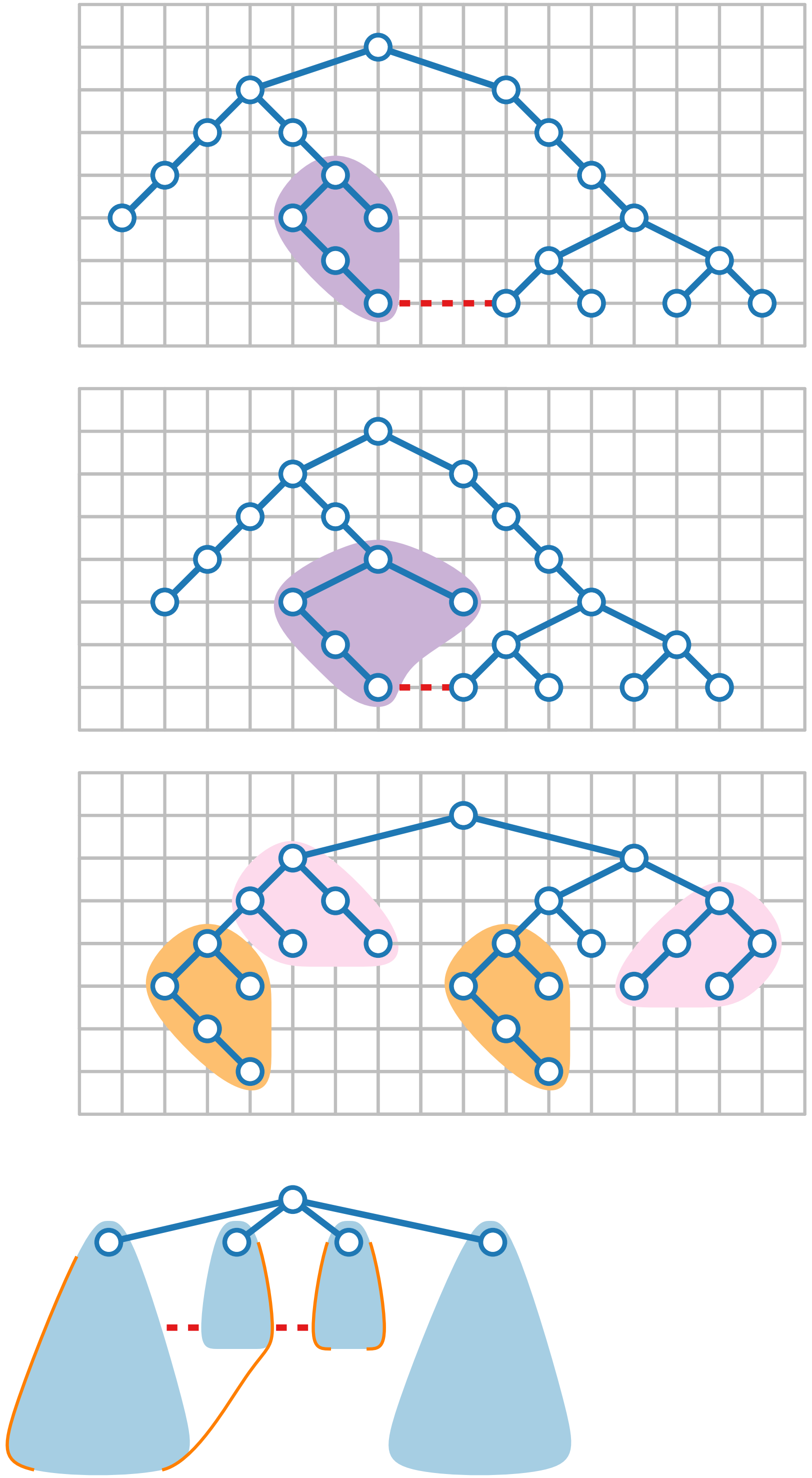

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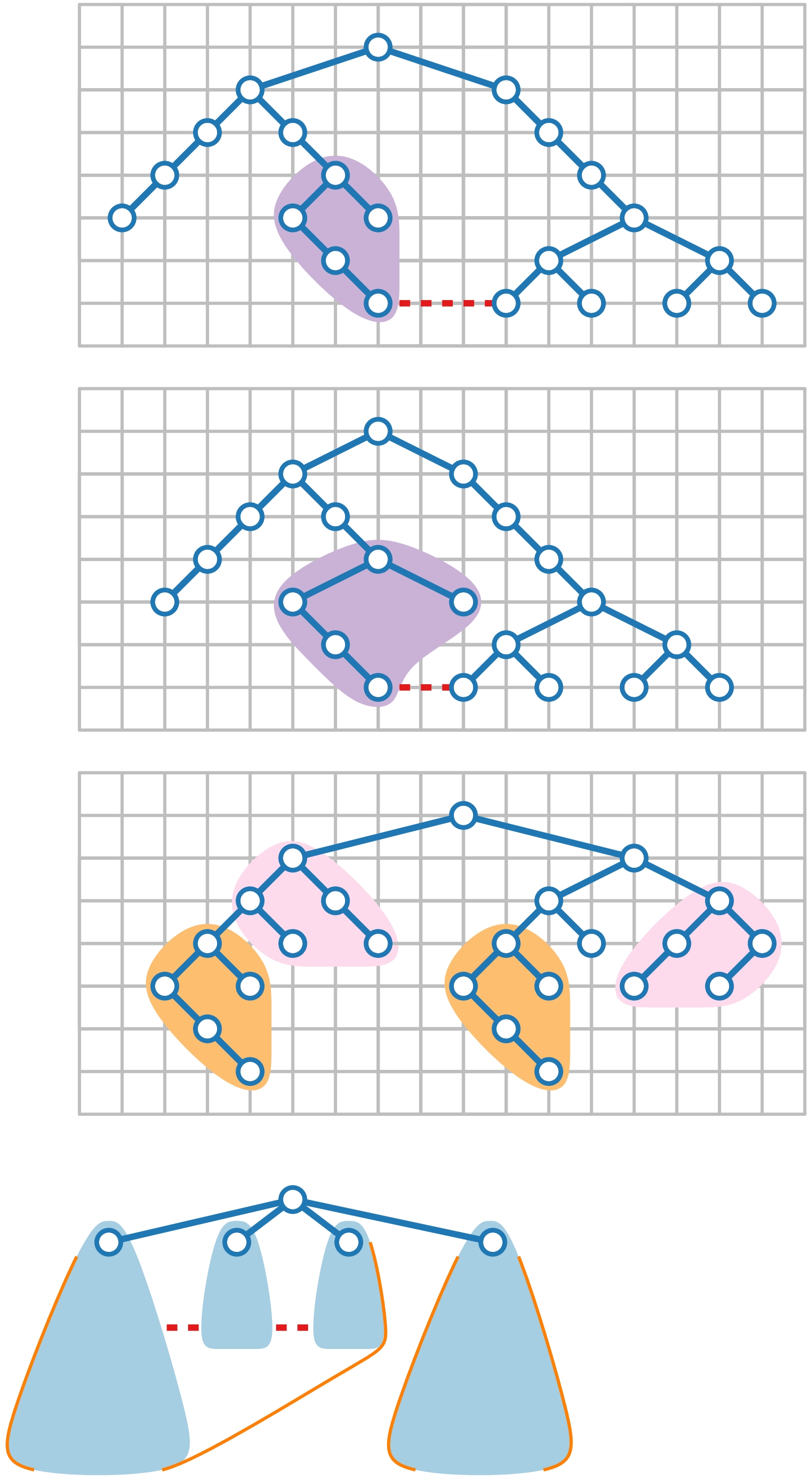

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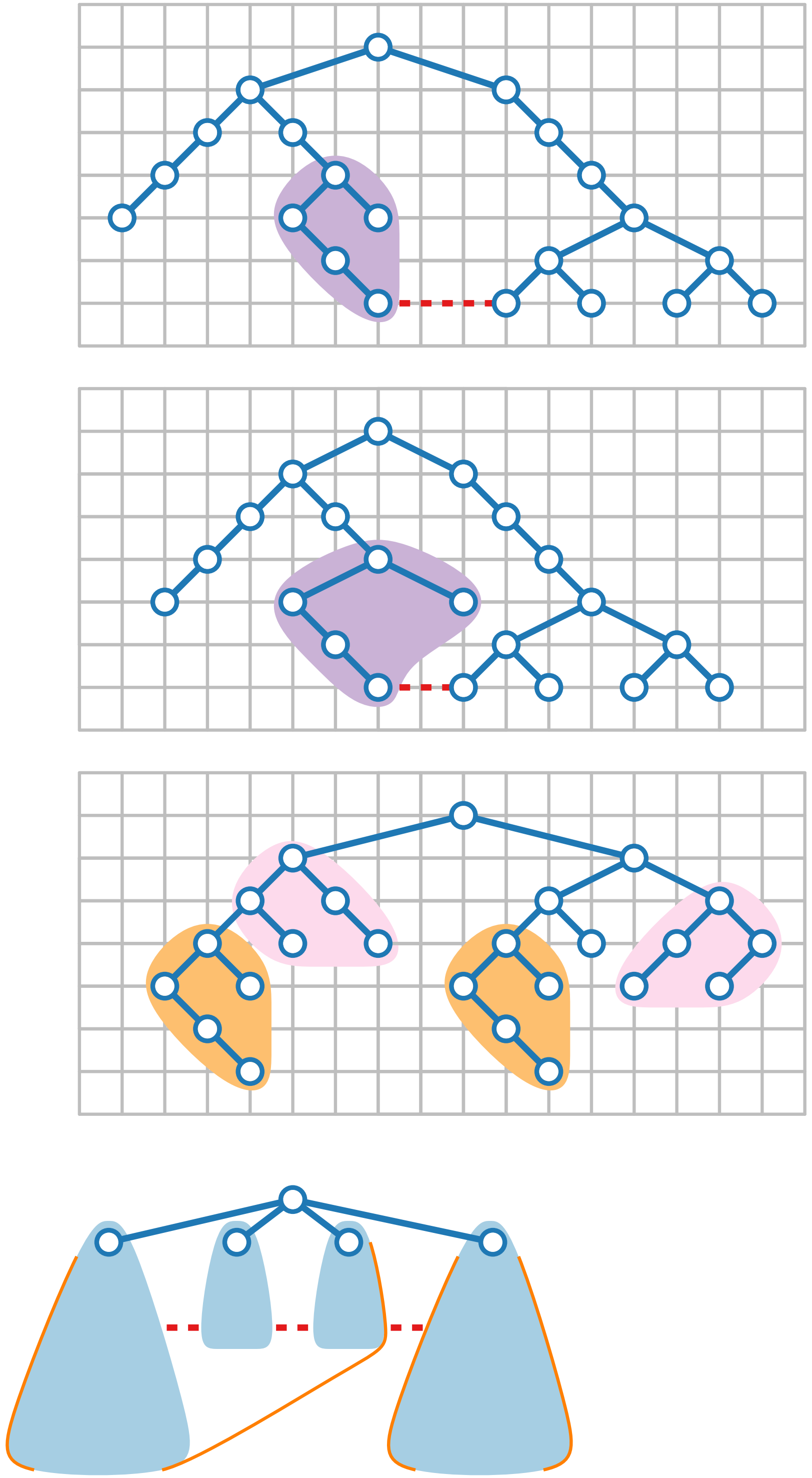

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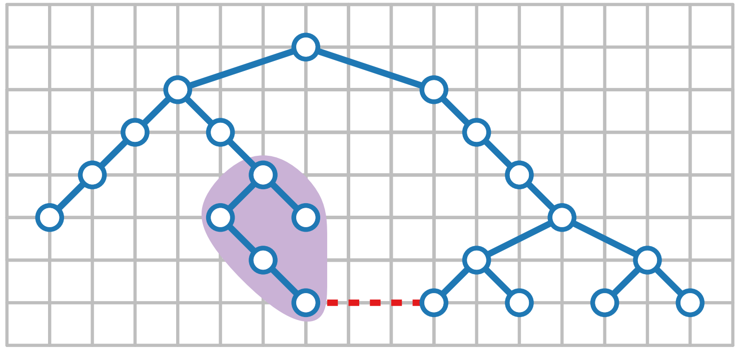
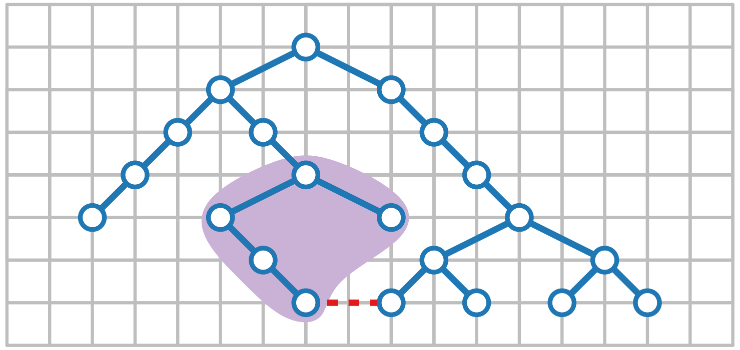
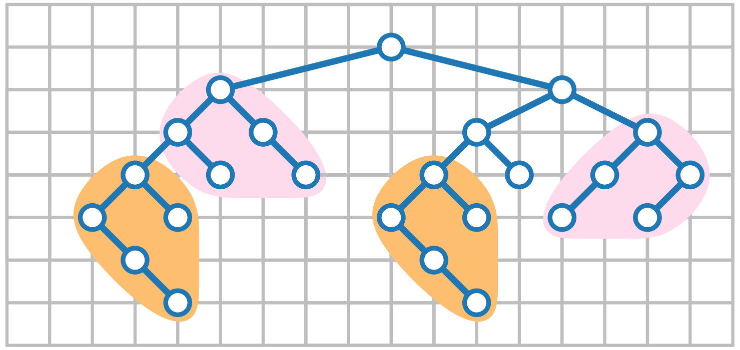
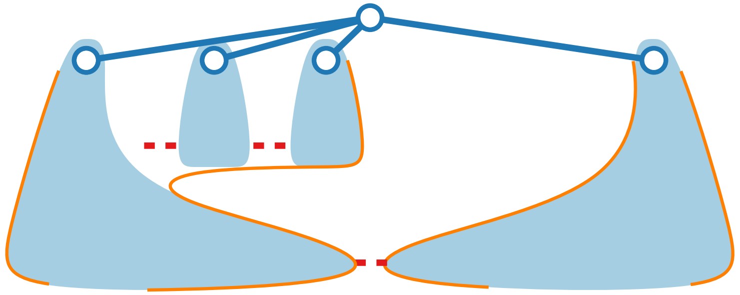

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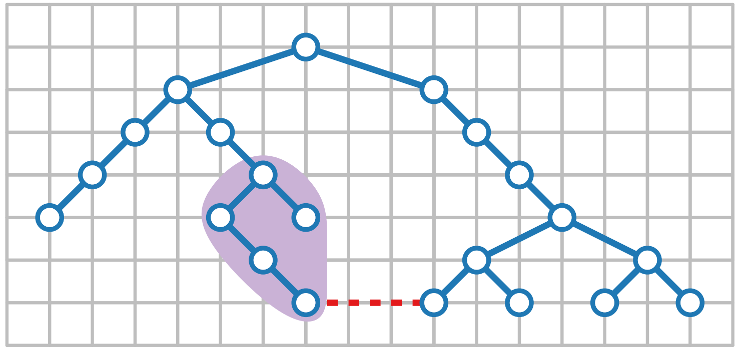
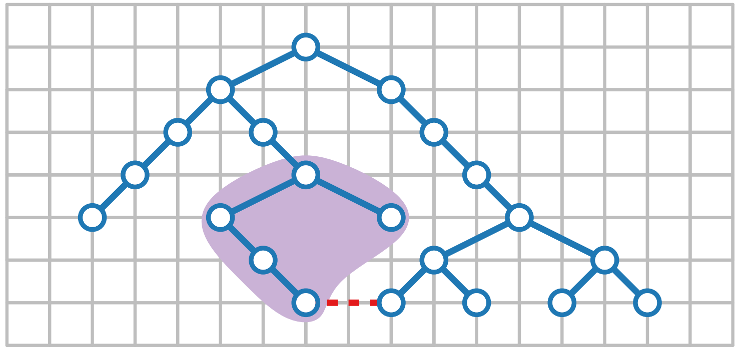
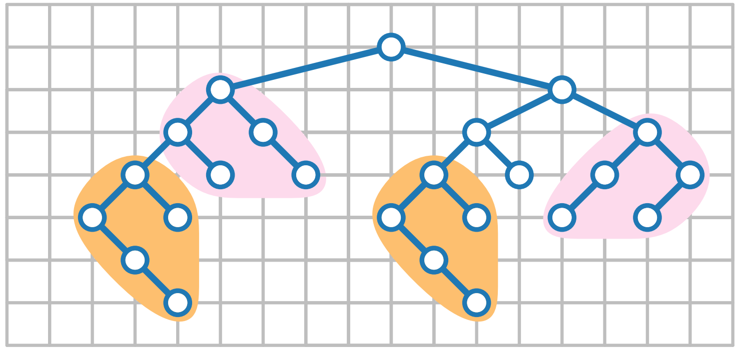
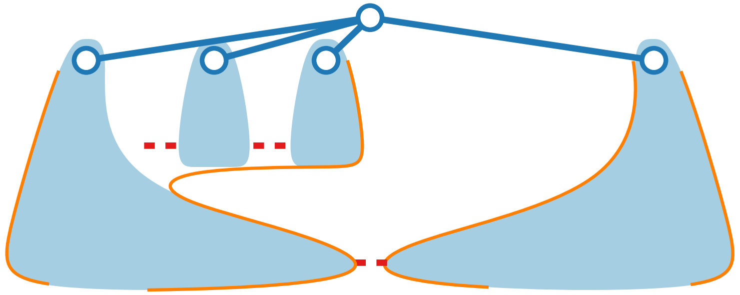
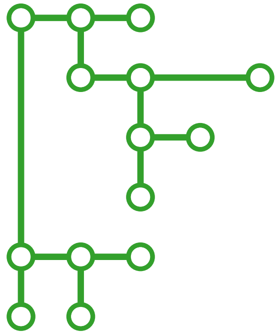
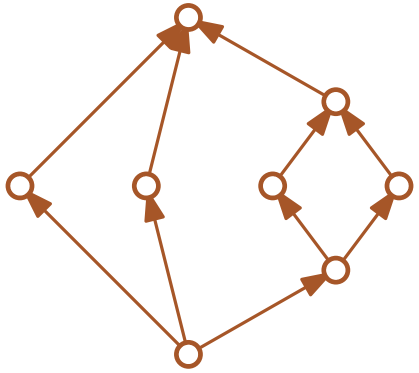





Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

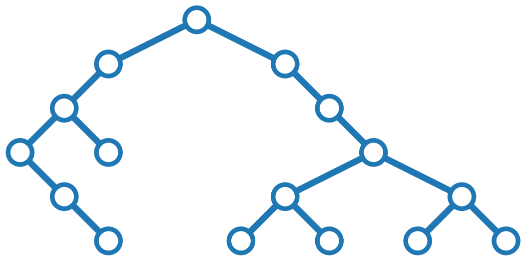
Visualization of Graphs

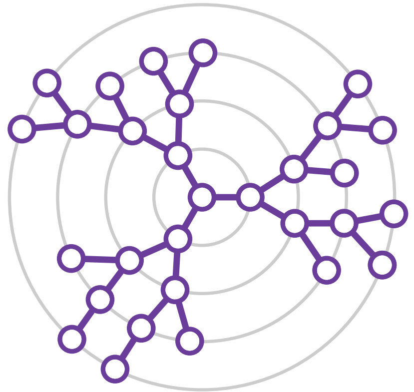
Lecture 1b:
Drawing Trees and Series-Parallel Graphs





Part II:
HV-Drawings





Jonathan Klawitter

HV-Drawings – Drawing Style

Applications

- Cons cell diagram in LISP

HV-Drawings – Drawing Style

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- Cons cell diagram in LISP
- *Cons*(constructs) are memory objects which hold two values or pointers to values

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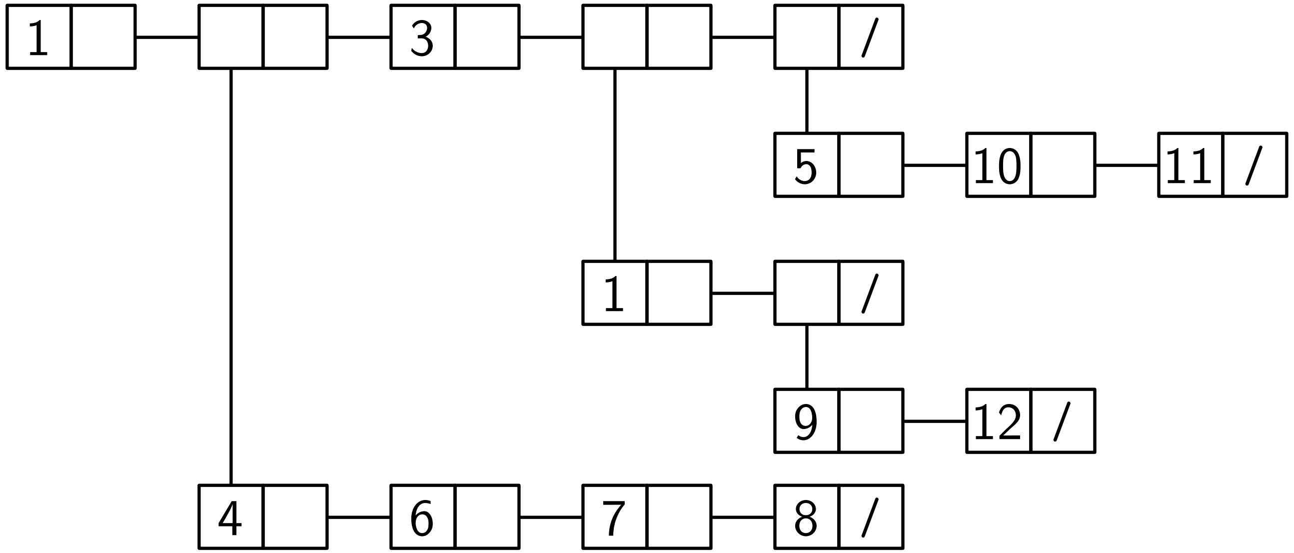

Source: after gajon.org/trees-linked-lists-common-lisp/

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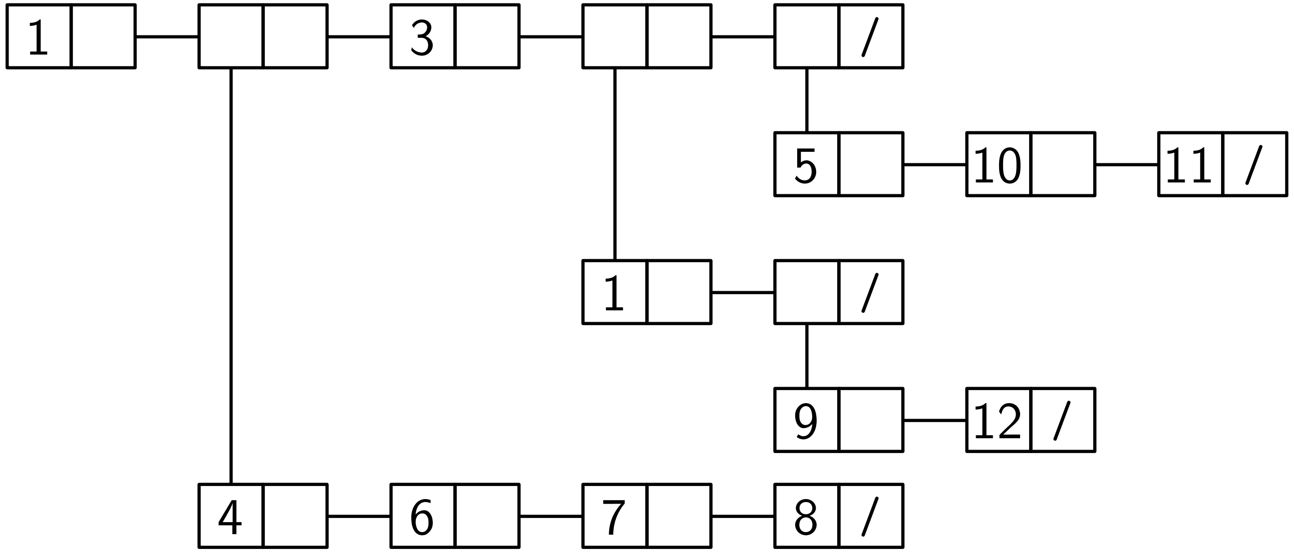

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Drawing conventions

Drawing aesthetics

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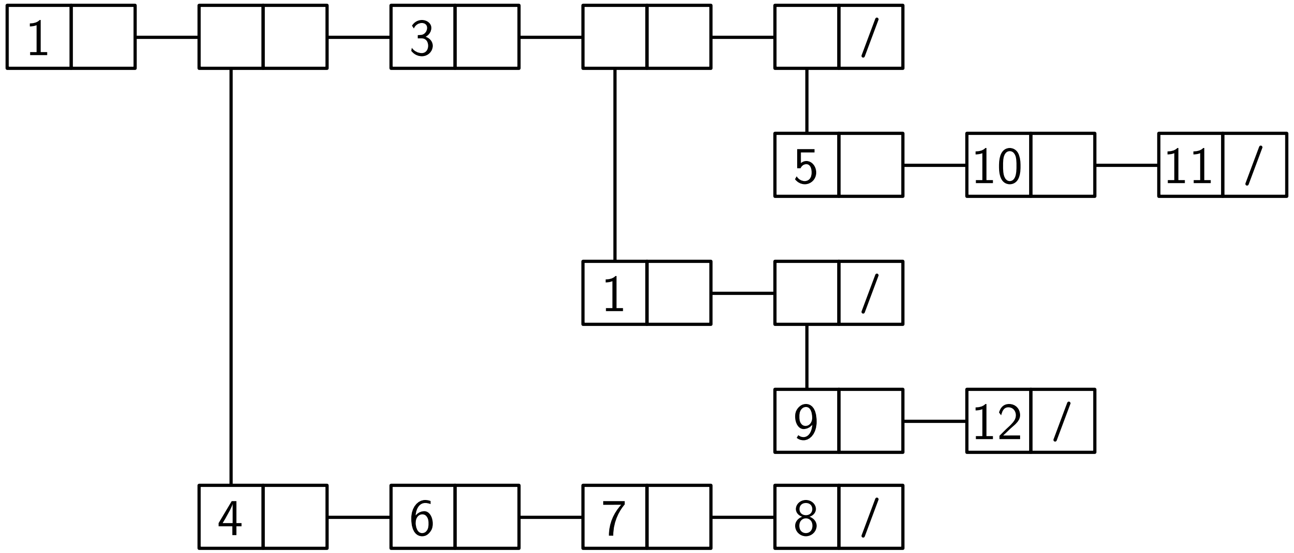

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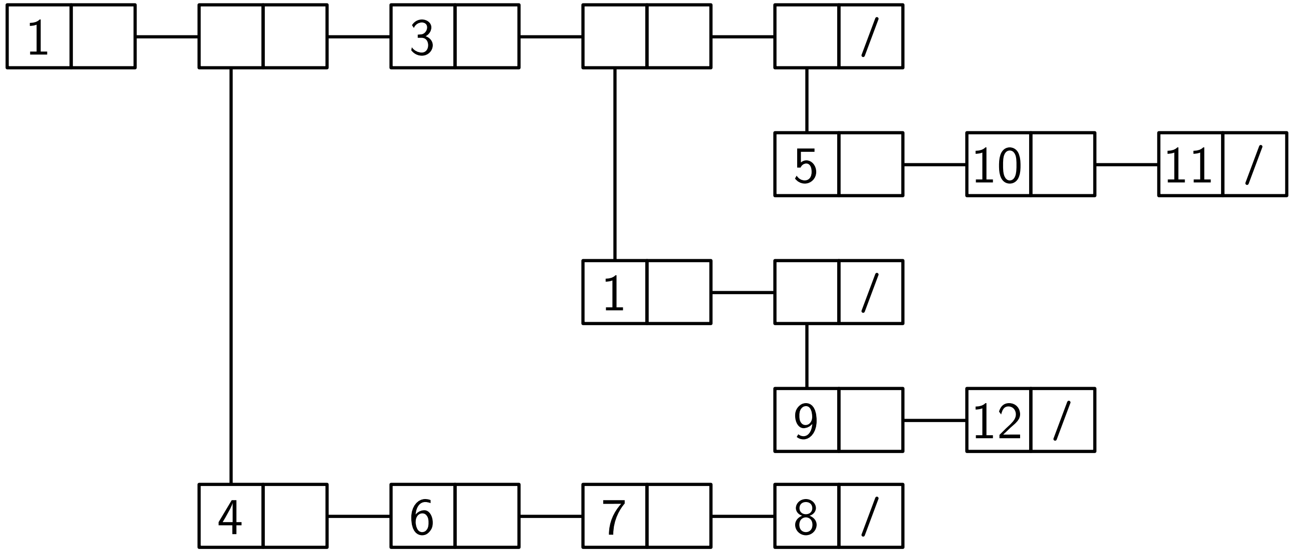

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- Children are vertically or horizontally aligned with their parent
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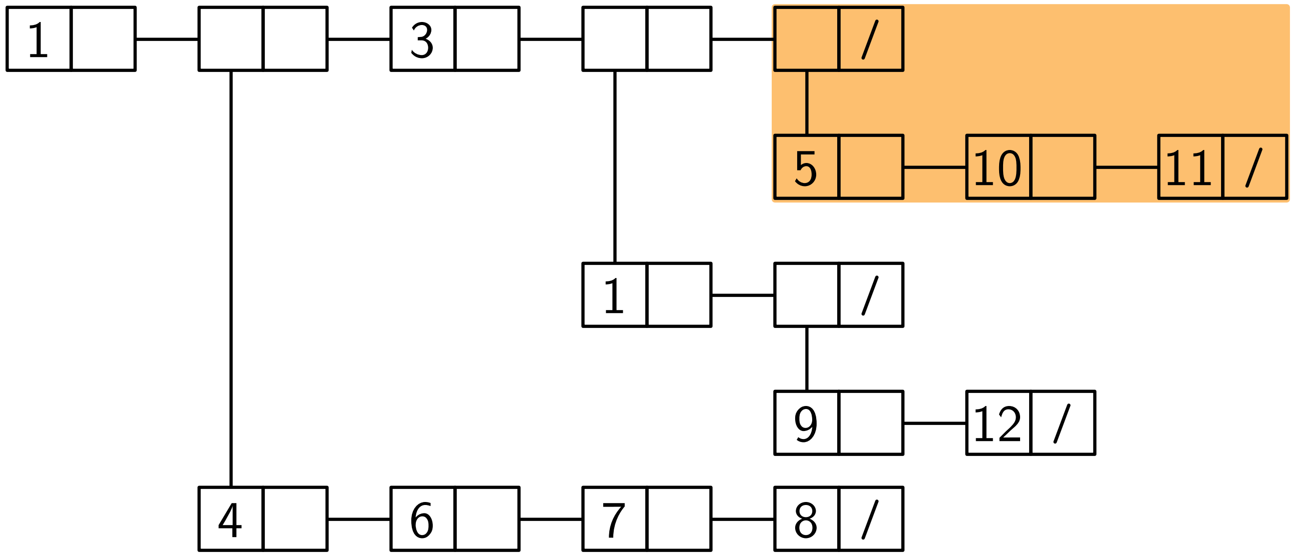

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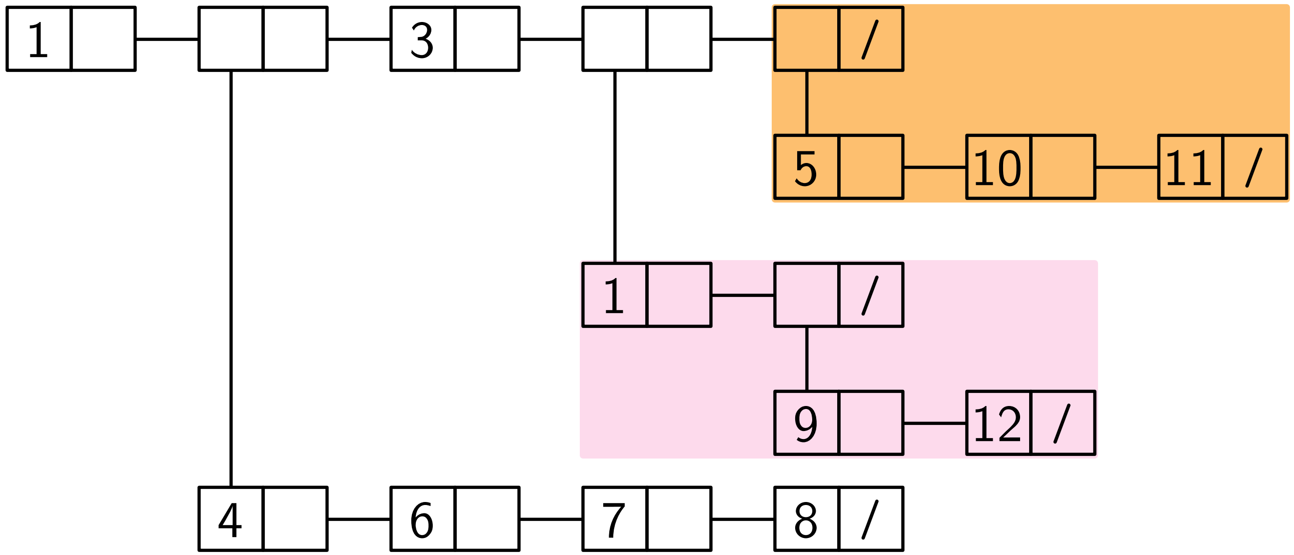

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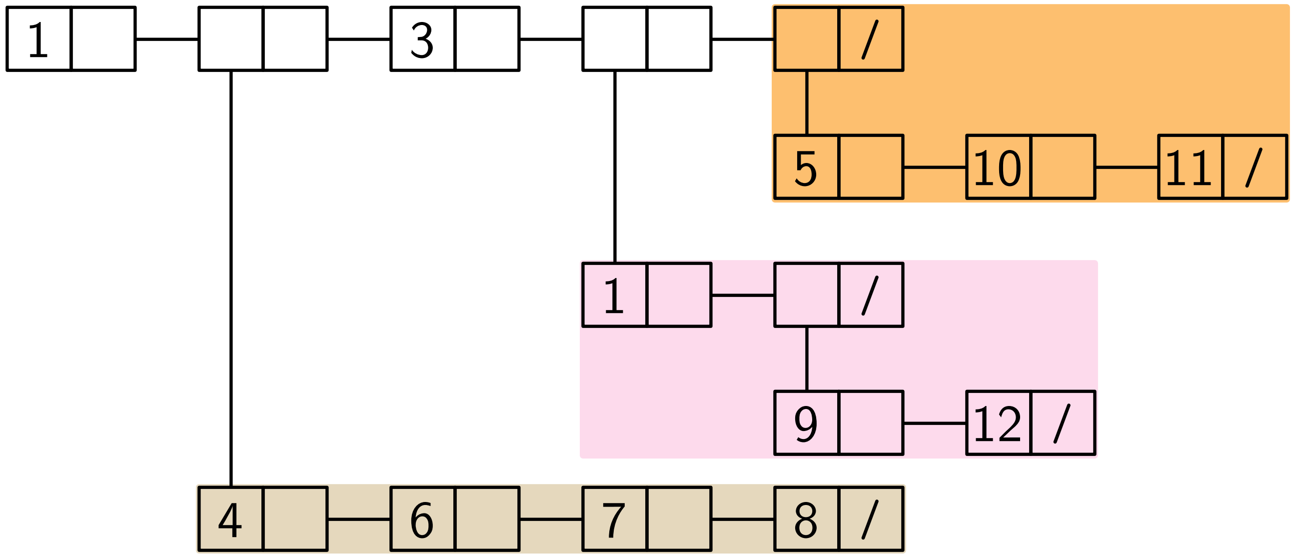

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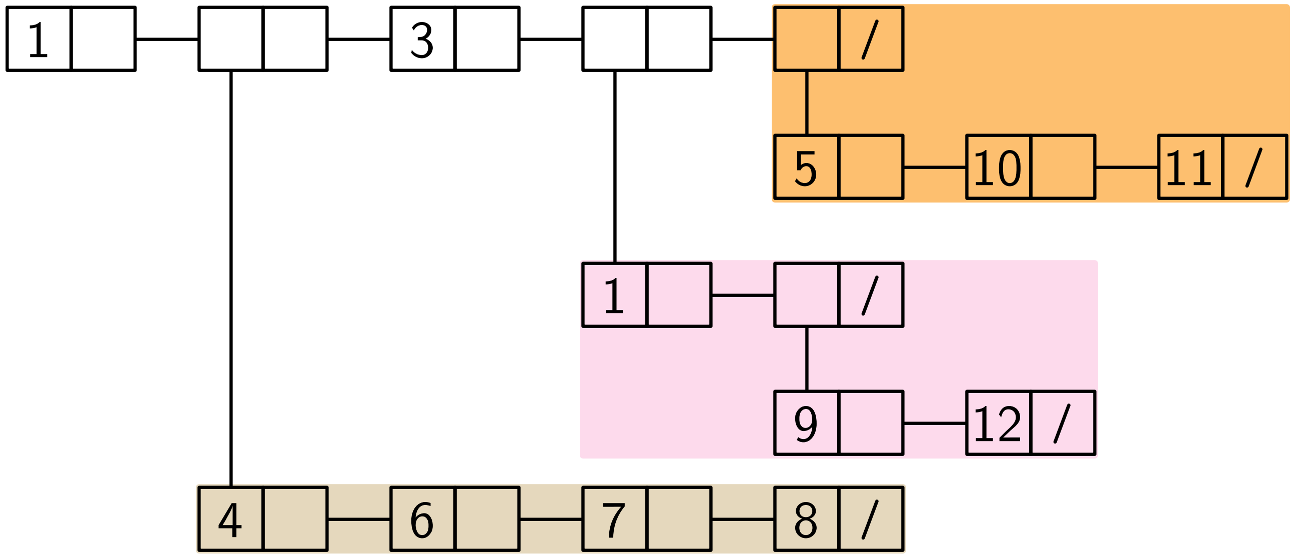

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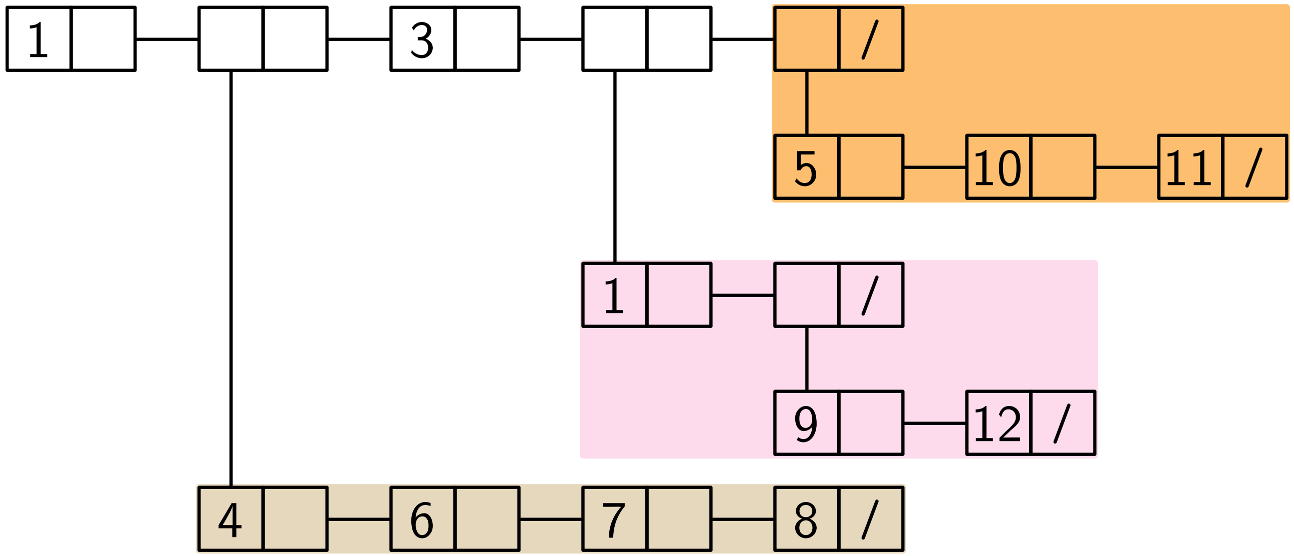

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Drawing aesthetics

- Height, width, area

HV-Drawings – Algorithm

Input: A binary tree T
Output: An HV-drawing of T

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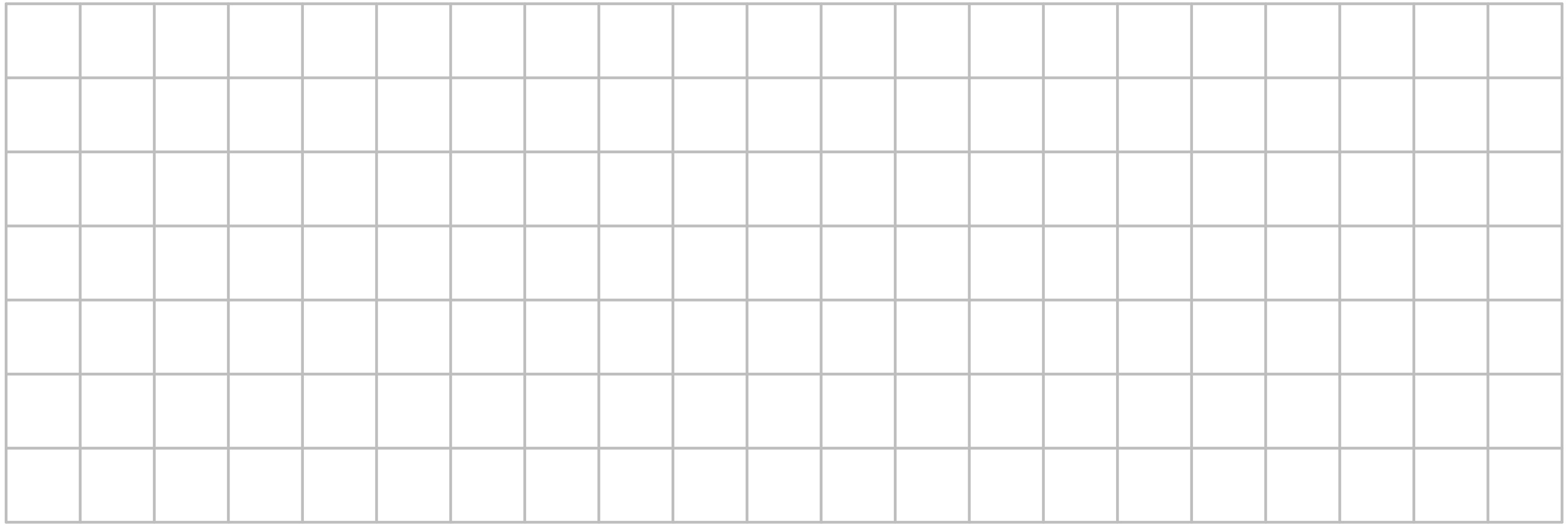
Conquer:



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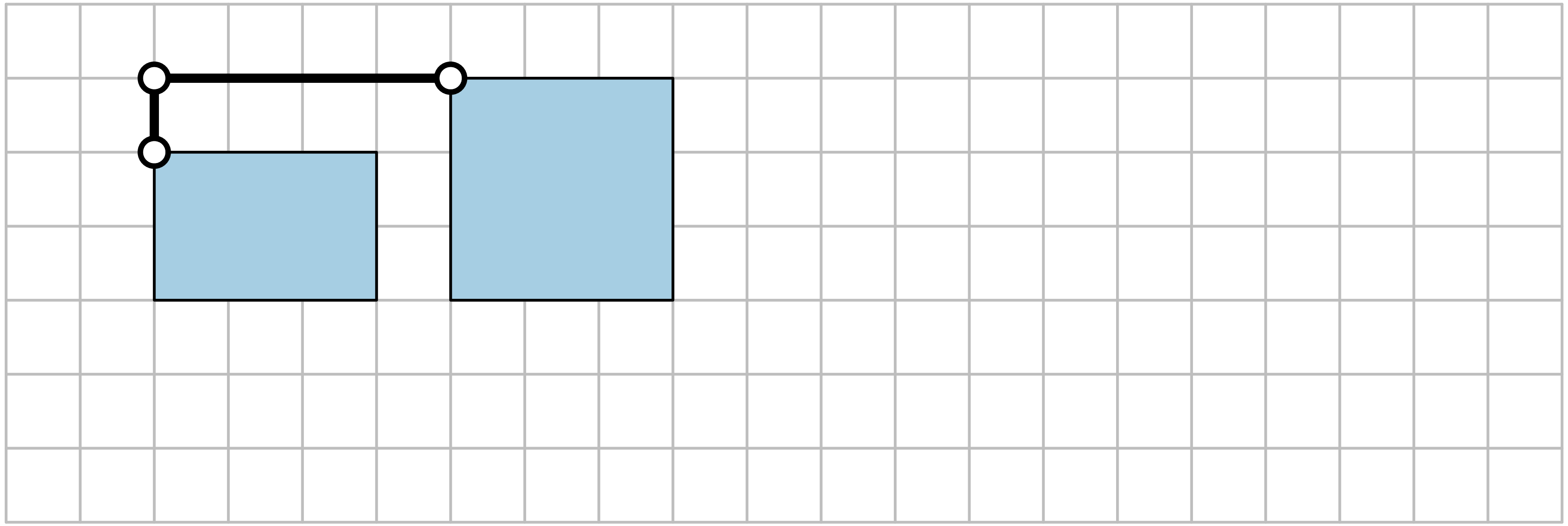
Conquer: horizontal combination



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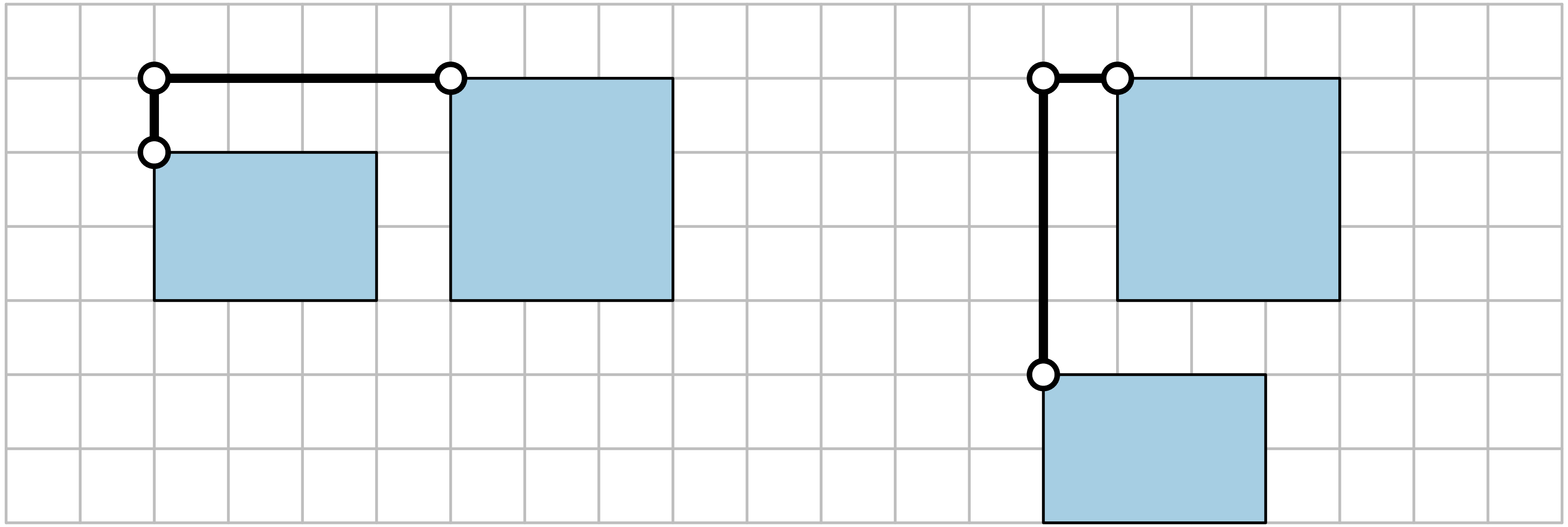

HV-Drawings – Right-Heavy HV-Layout

Right-heavy approach

- Always apply horizontal combination

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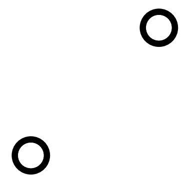

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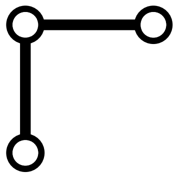

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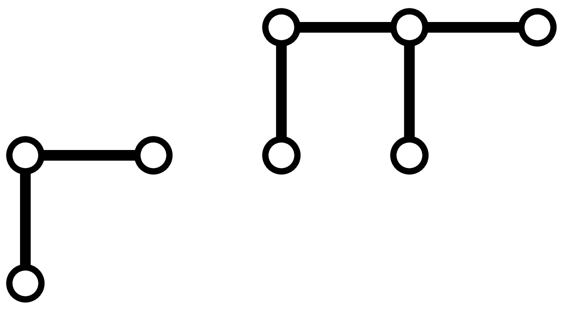

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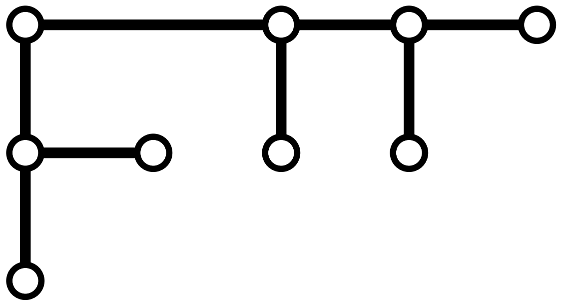

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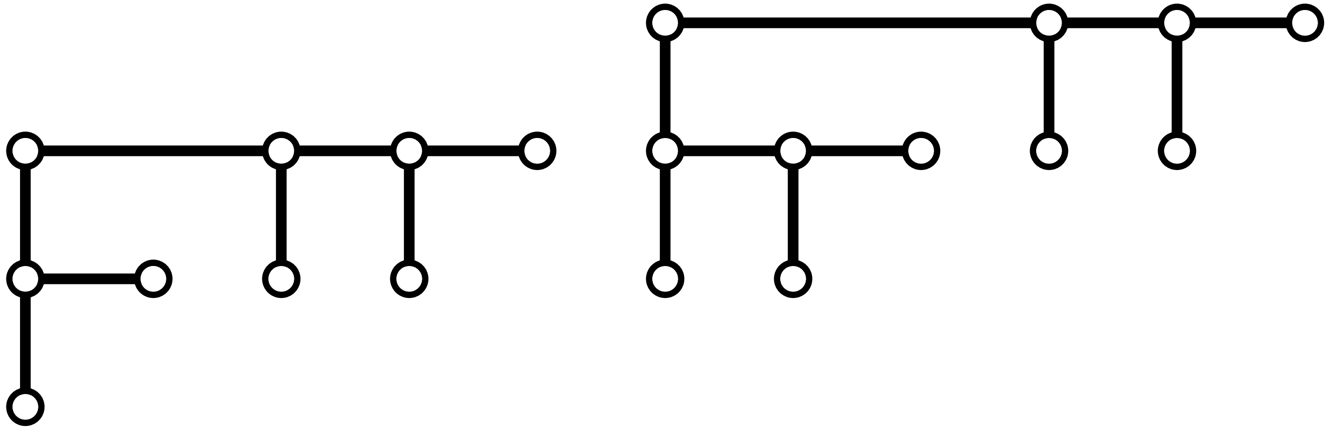

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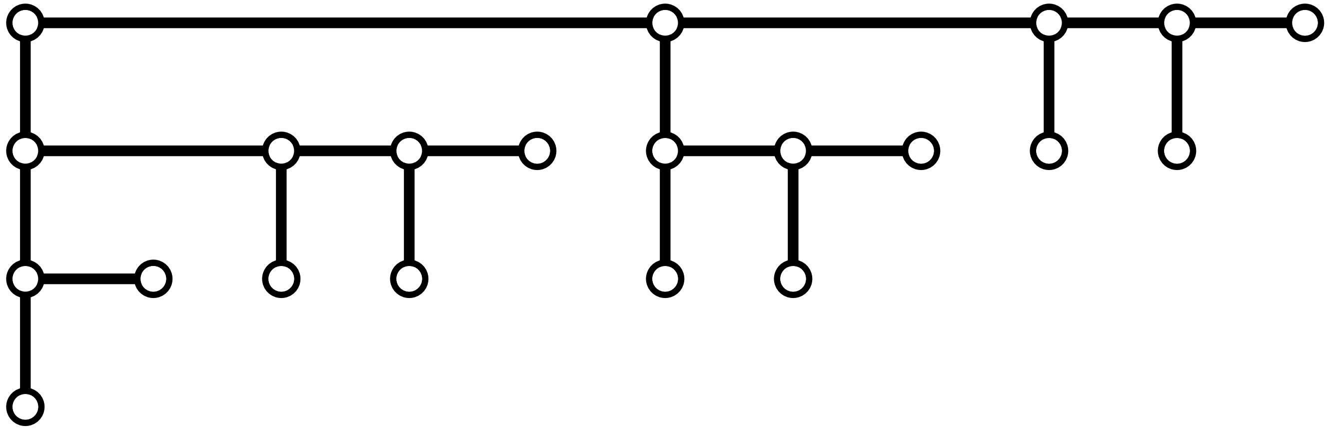

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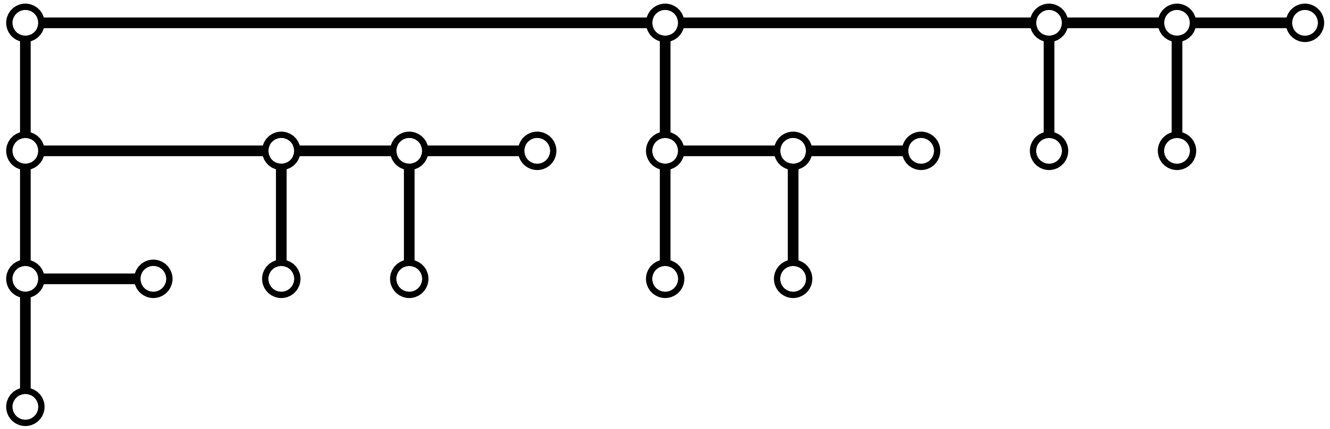

Lemma. Let T be a binary tree. The drawing constructed by the right-heavy approach has

- width at most and
- height at most

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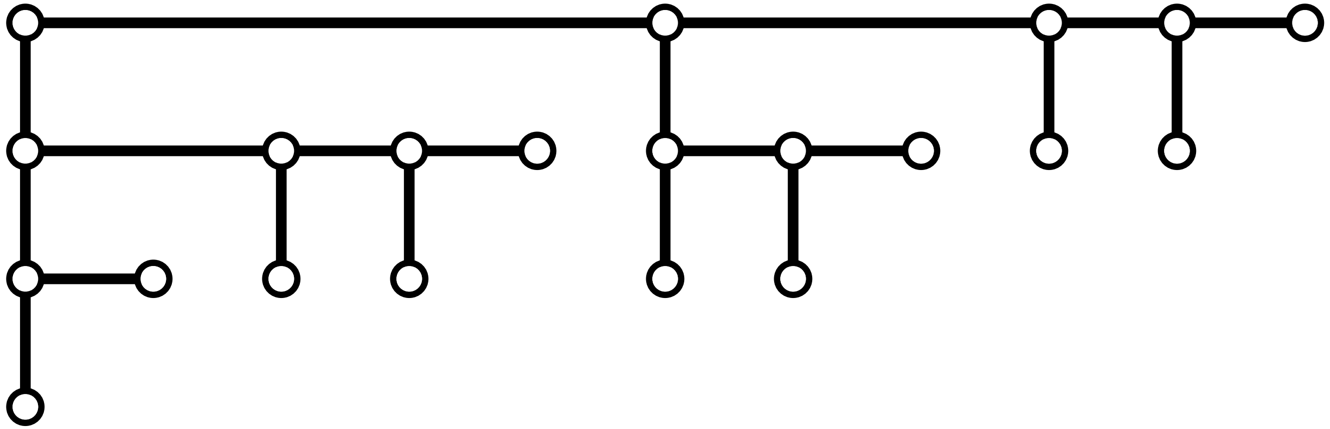

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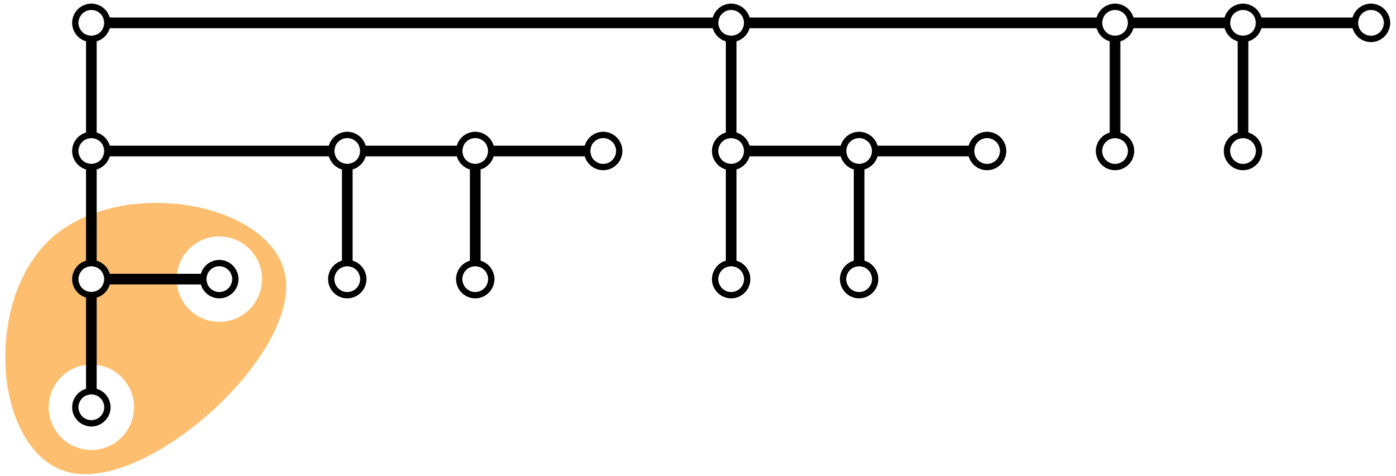

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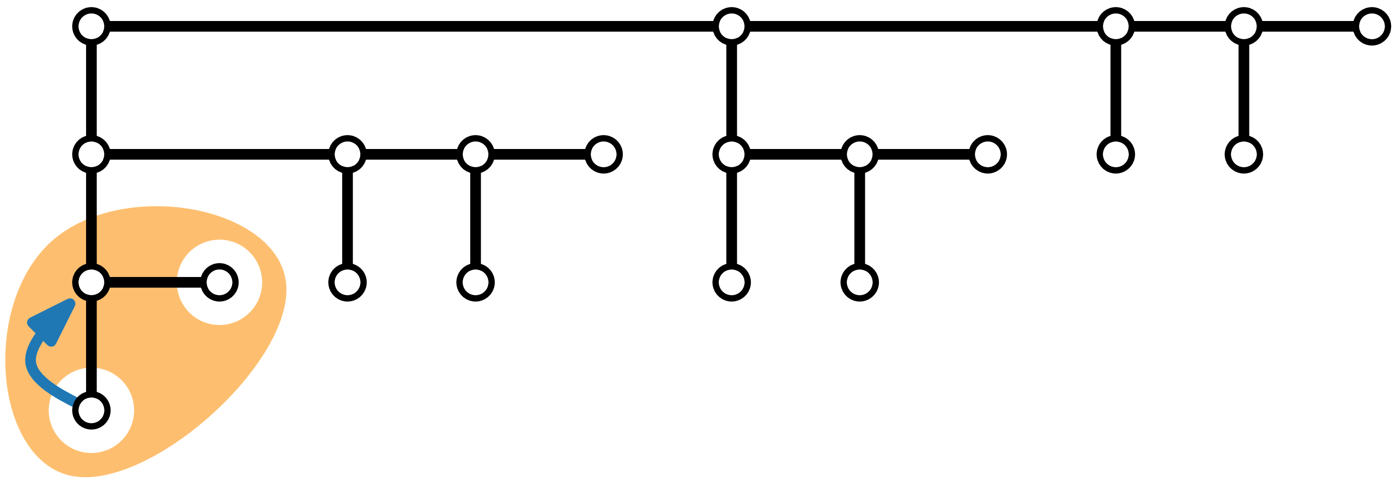

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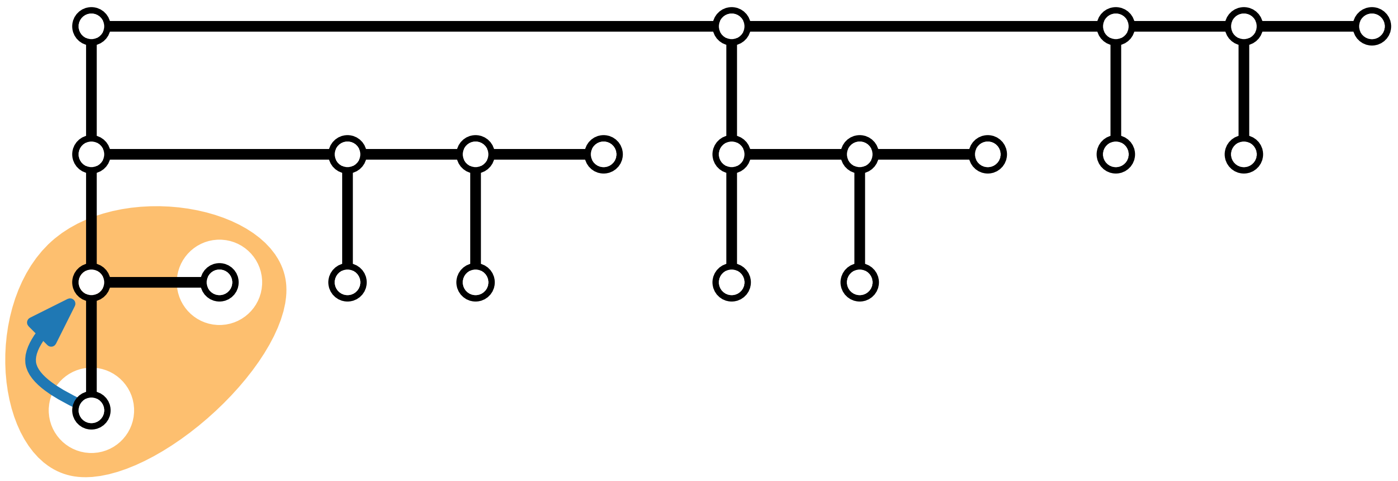
at least ·2



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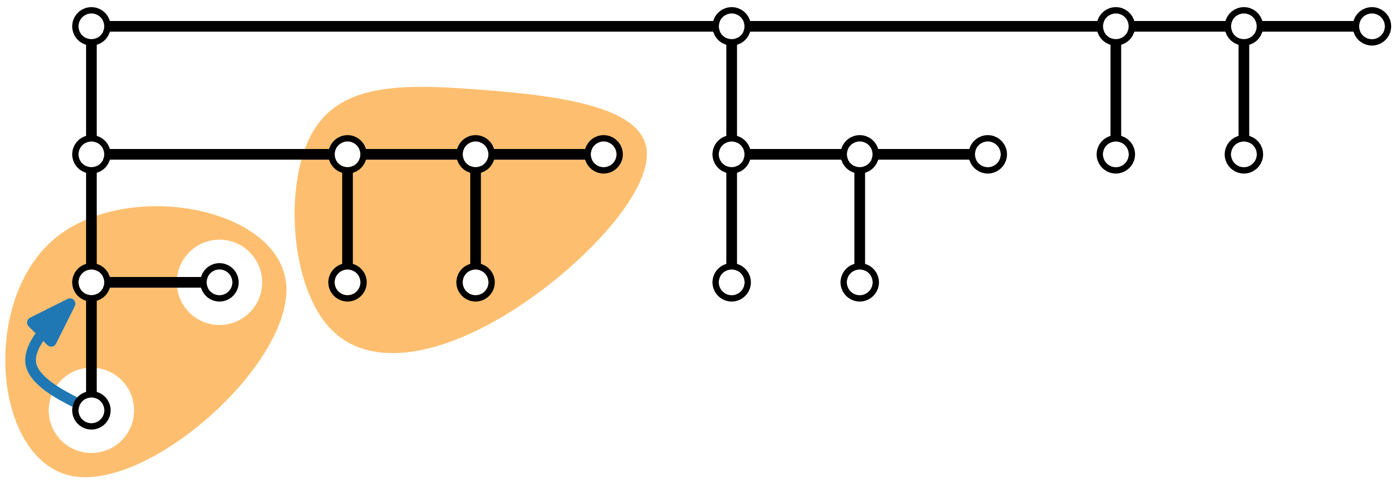

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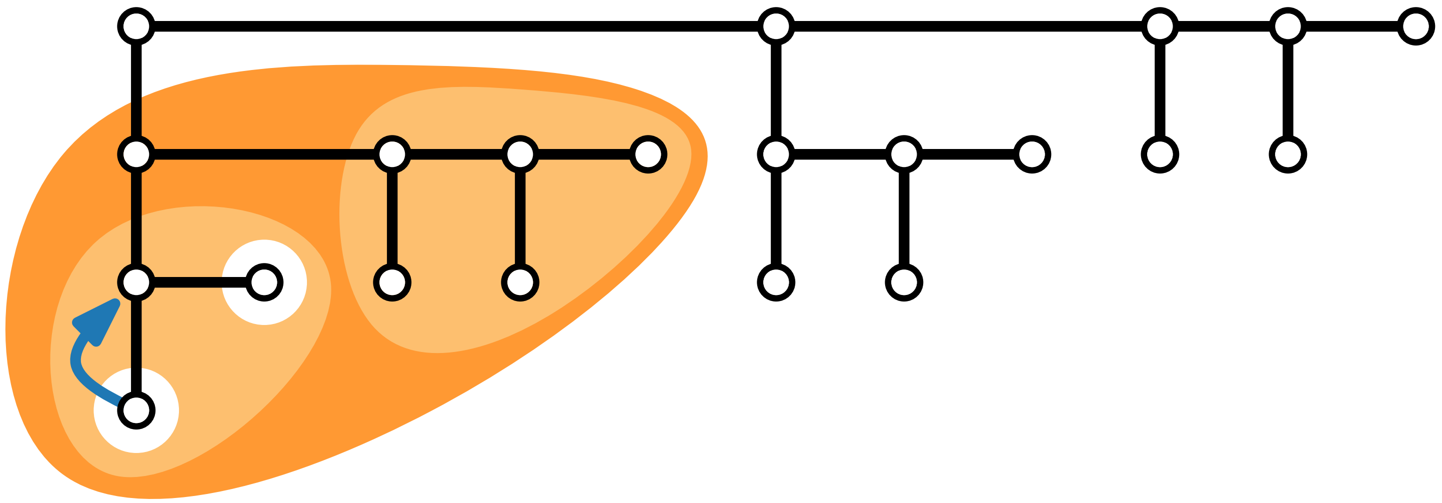

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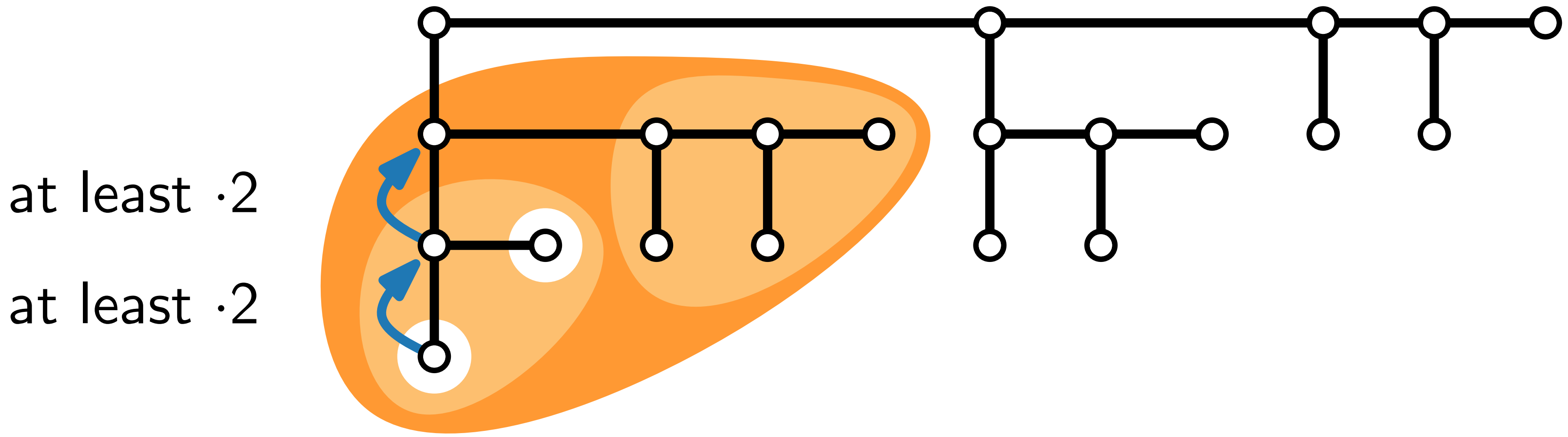

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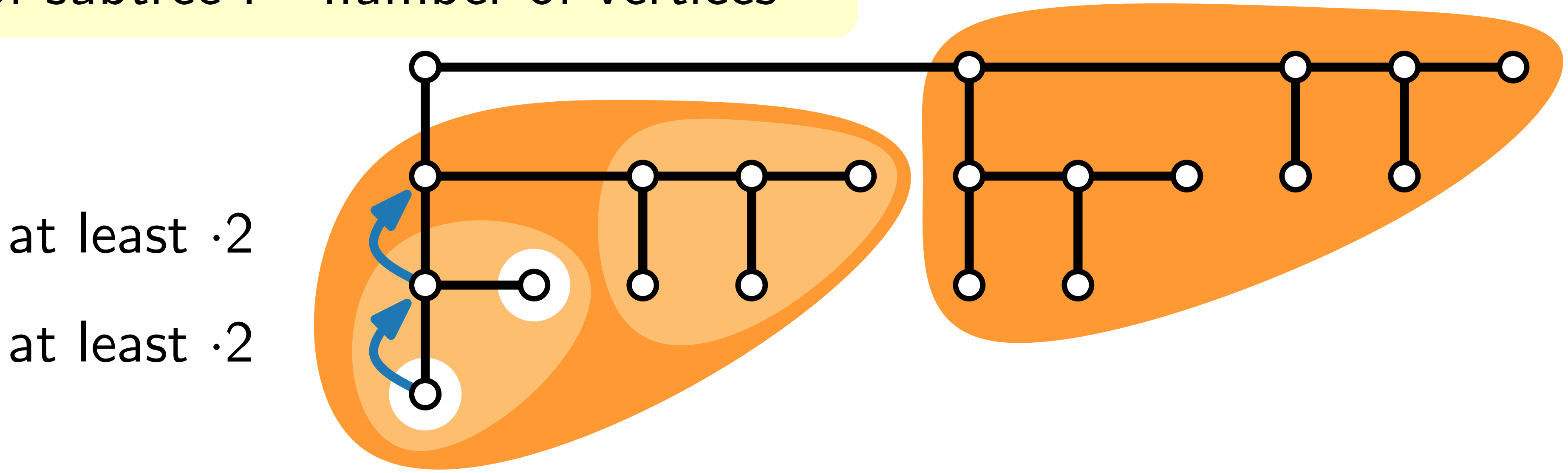

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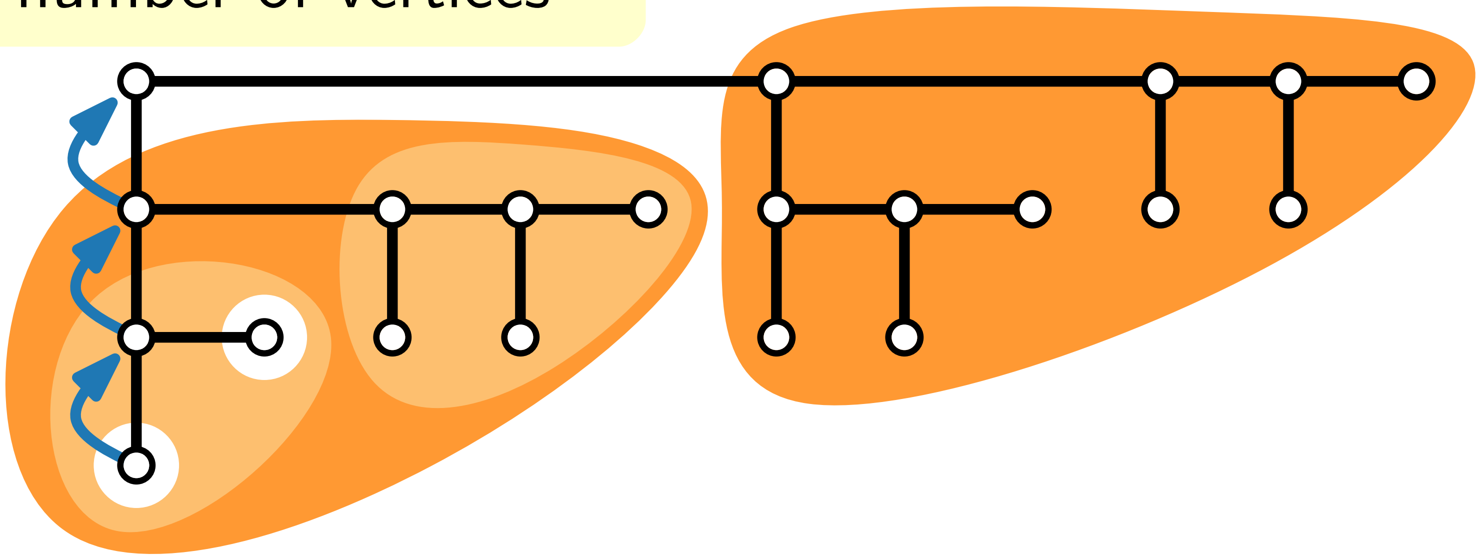

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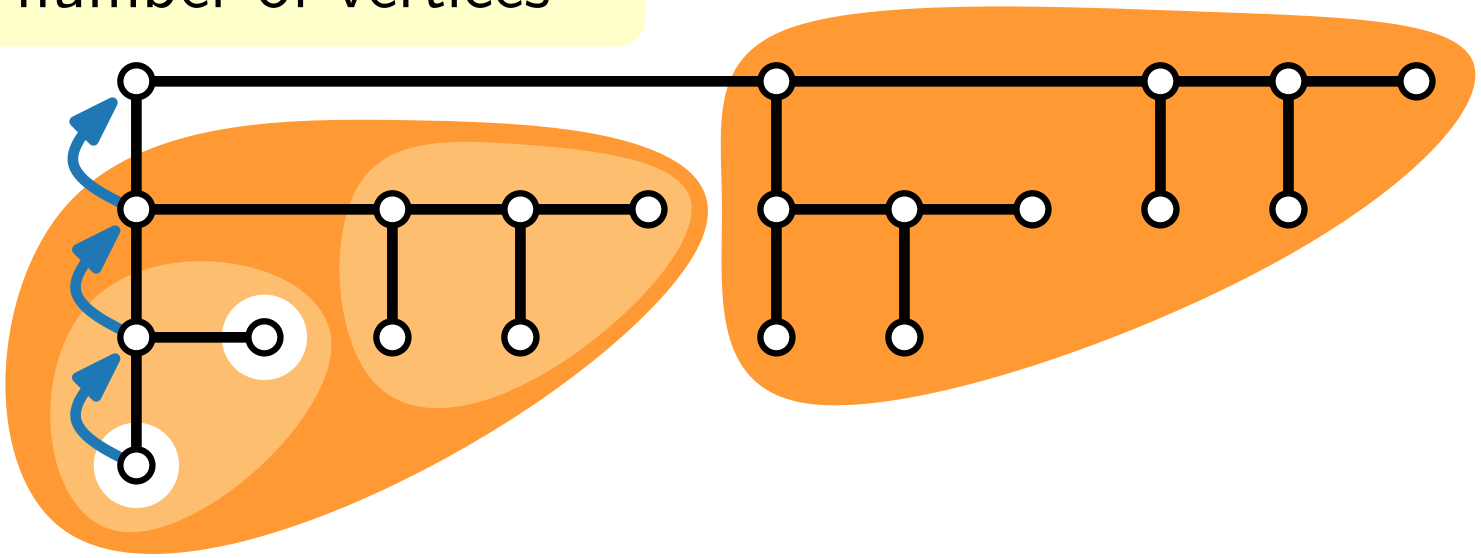

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How to implement this in linear time?

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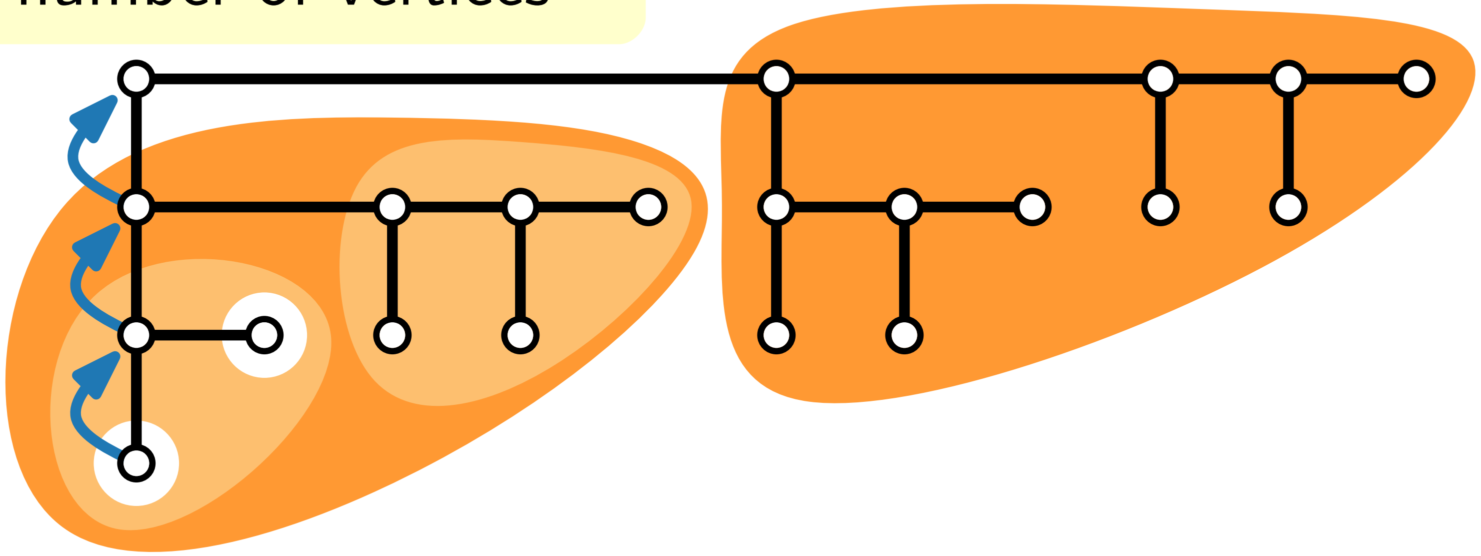

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Theorem.
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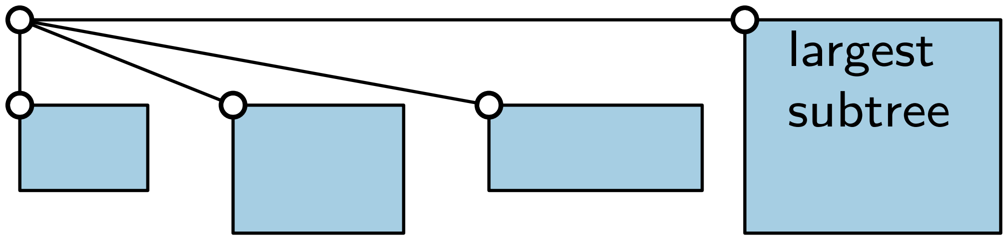

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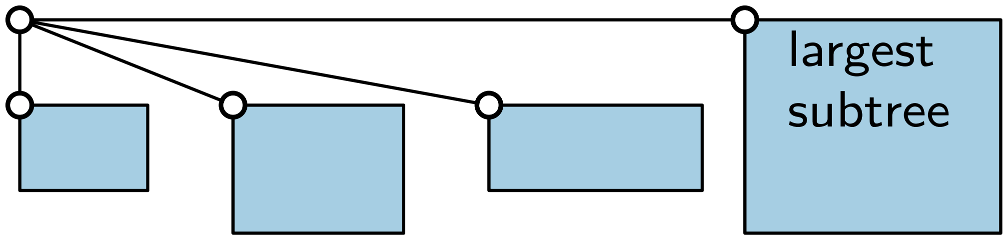

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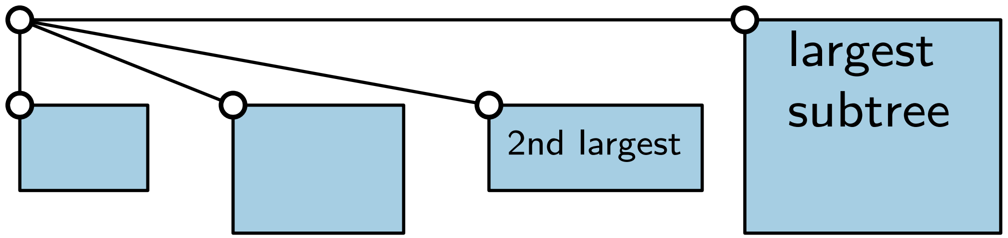

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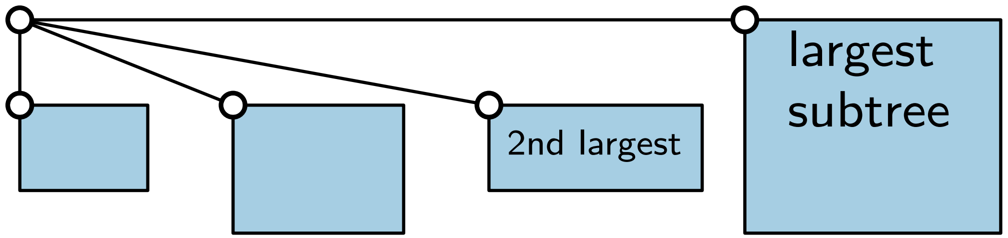


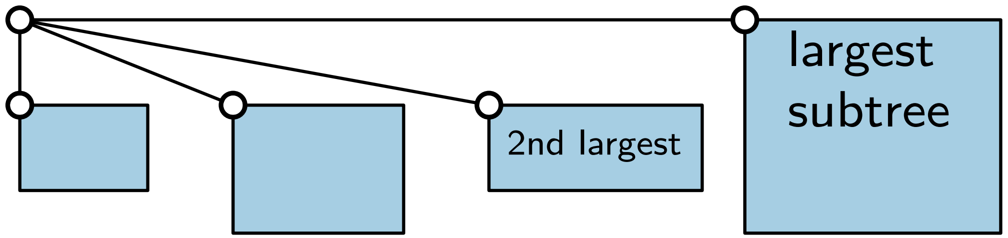
Optimal area?

HV-Drawings – Result

Theorem.
Let T be a ~~binary~~ rooted tree with n vertices. The right-heavy algorithm constructs in $O(n)$ time a drawing Γ of T s.t.:

- Γ is an HV-drawing (planar, ~~orthogonal,~~ strictly right-/downward)
- Width is at most $n-1$
- Height is at most $\log n$
- Area is in $\mathcal{O}(n \log n)$
- Simply and axially isomorphic subtrees have congruent drawings up to translation

General rooted tree

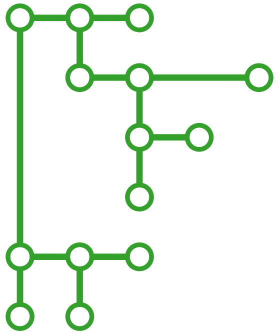


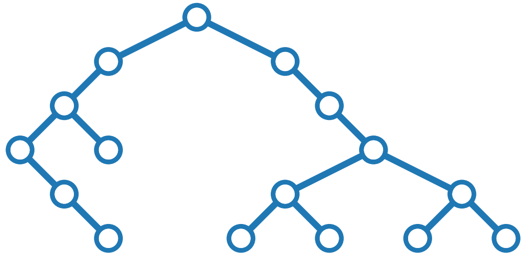
Optimal area?

Not with divide & conquer approach, but can be computed with Dynamic Programming.

Visualization of Graphs

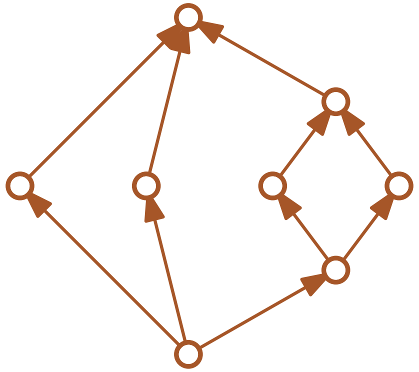
Lecture 1b:
Drawing Trees and Series-Parallel Graphs

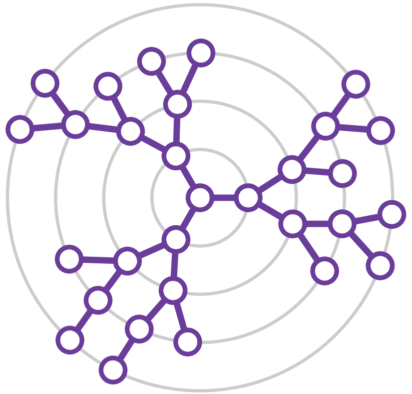




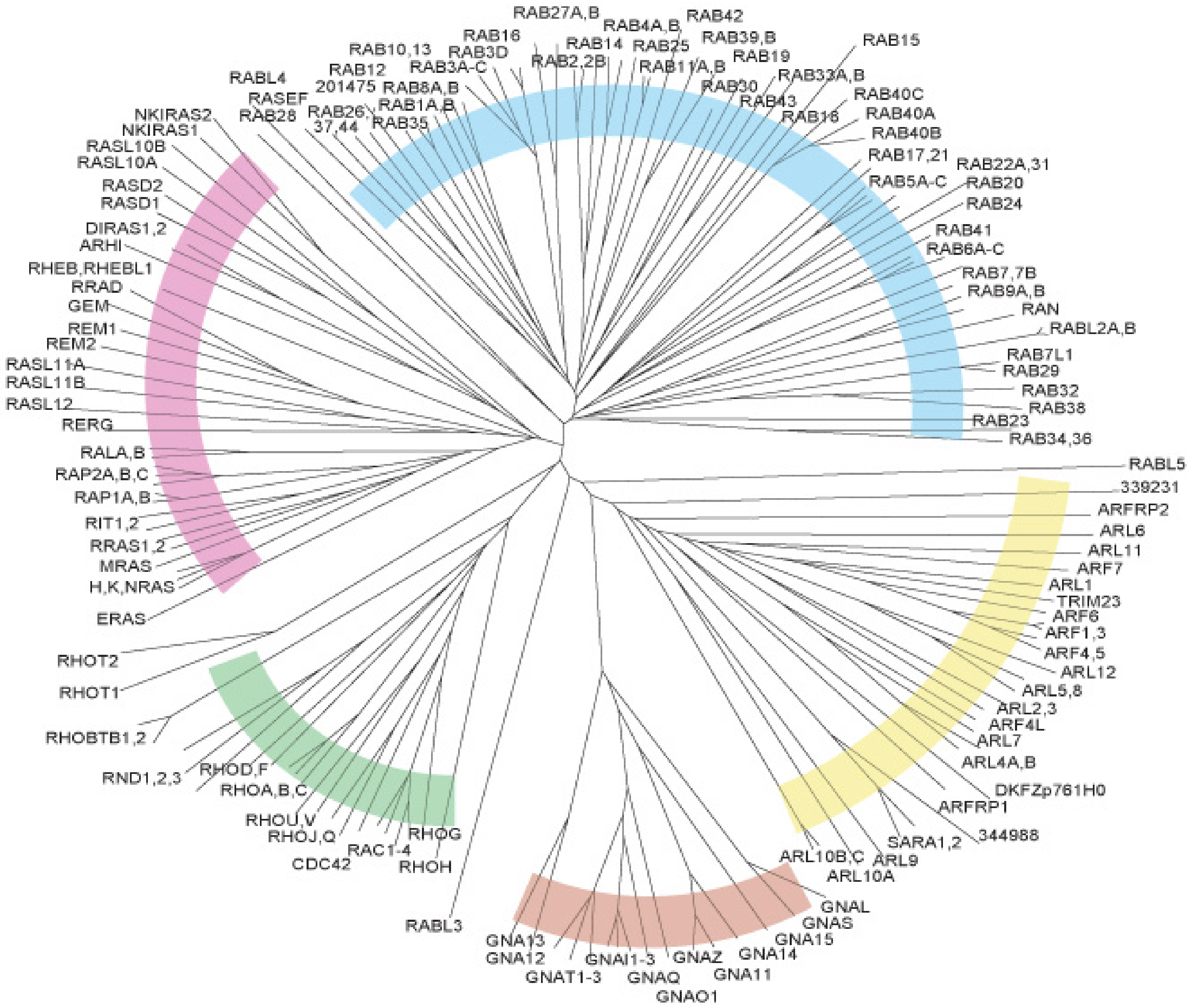
Part III:
Radial Layouts

Jonathan Klawitter



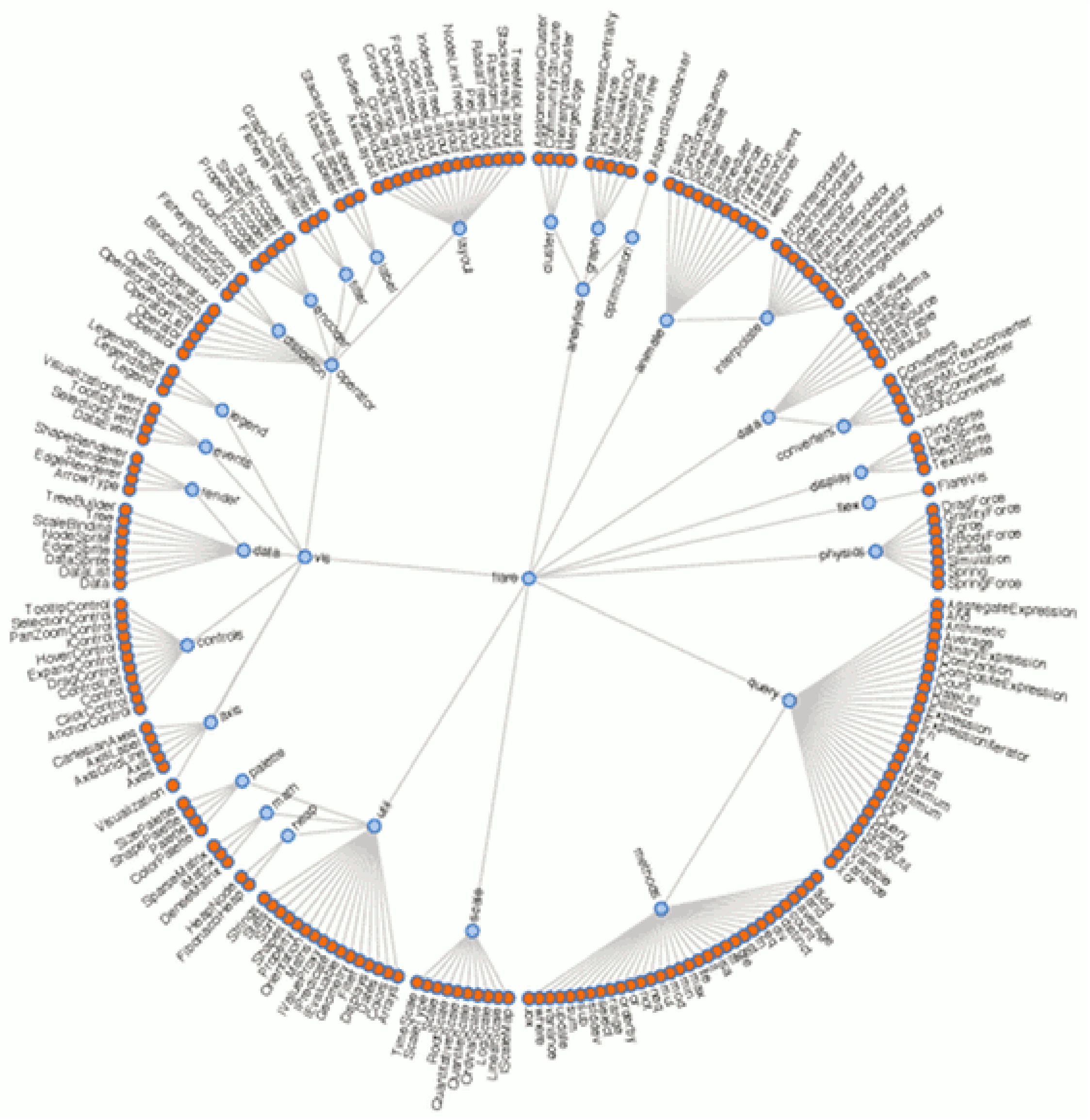


Radial Layouts – Applications

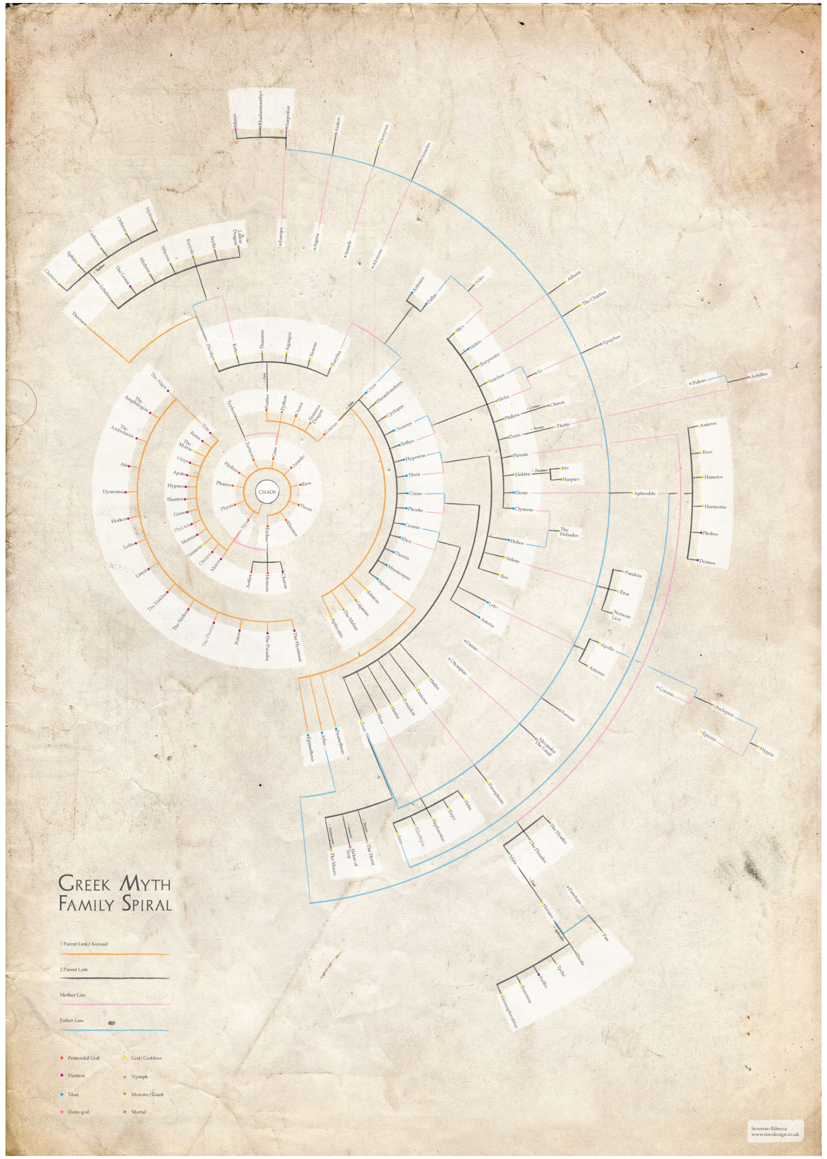


Phylogenetic tree
by Colicelli, ScienceSignaling, 2004

Radial Layouts – Applications

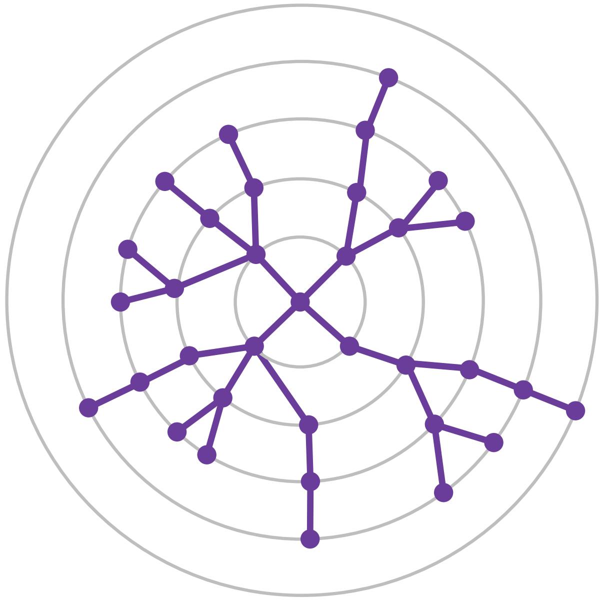


Flare Visualization Toolkit code structure
by Heer, Bostock and Ogievetsky, 2010



Greek Myth Family
by Ribecca, 2011

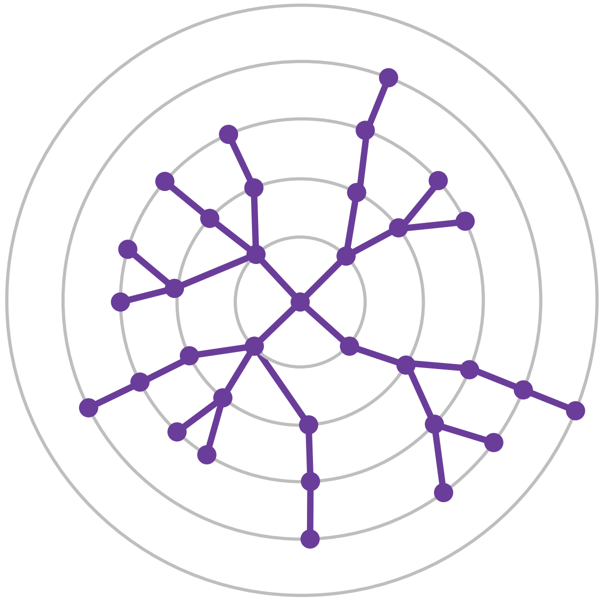
Radial Layouts – Drawing Style



Drawing conventions

Drawing aesthetics

Radial Layouts – Drawing Style

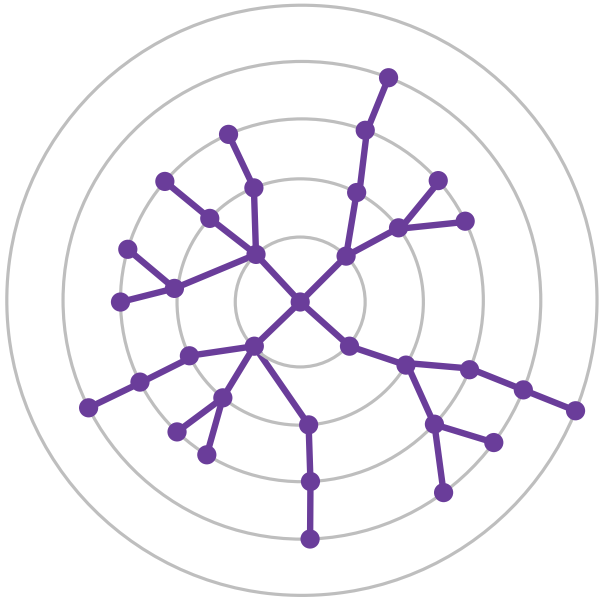


Drawing conventions

- Vertices lie on circular layers according to their depth

Drawing aesthetics

Radial Layouts – Drawing Style

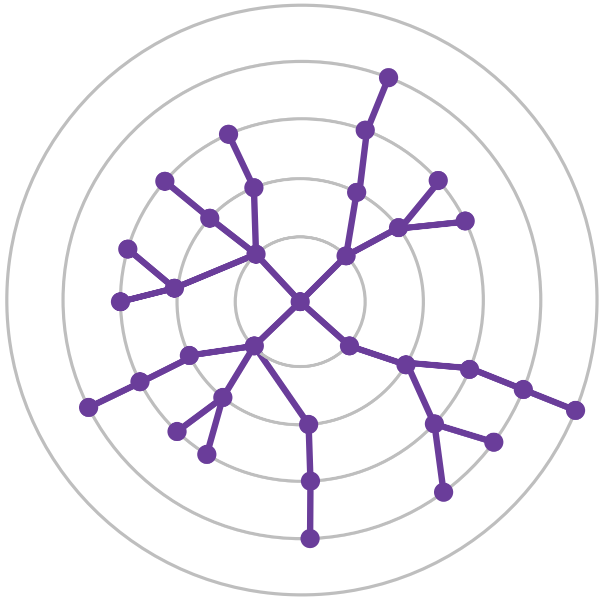


Drawing conventions

- Vertices lie on circular layers according to their depth
- Drawing is planar

Drawing aesthetics

Radial Layouts – Drawing Style



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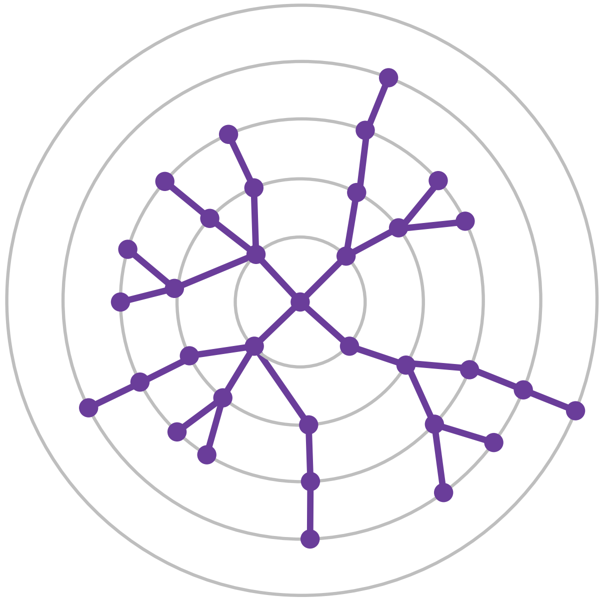
Drawing aesthetics

- Distribution of the vertices

Radial Layouts – Drawing Style



Drawing conventions

- Vertices lie on circular layers according to their depth
- Drawing is planar

Drawing aesthetics

- Distribution of the vertices

How can an algorithm optimize the distribution of the vertices?

Radial Layouts – Algorithm Attempt

Idea

- Reserve area corresponding to size $\ell(u)$ of $T(u)$:

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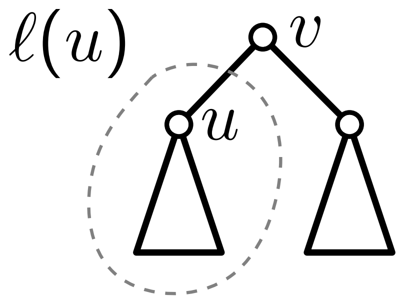

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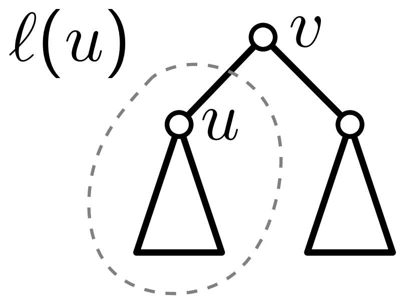

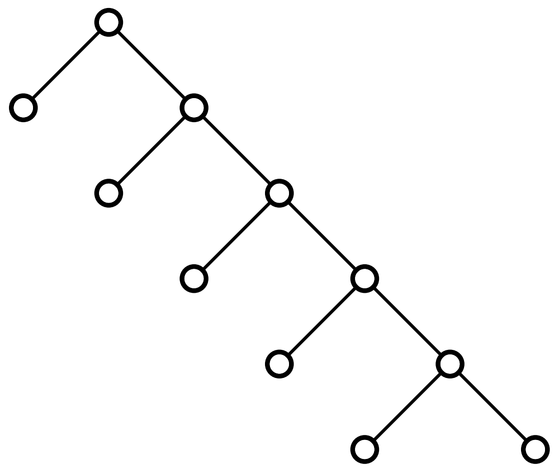

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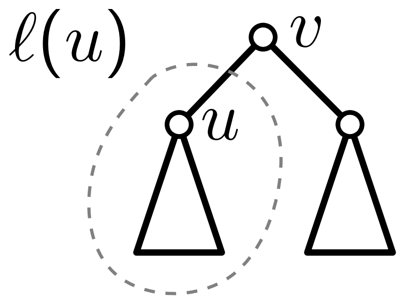

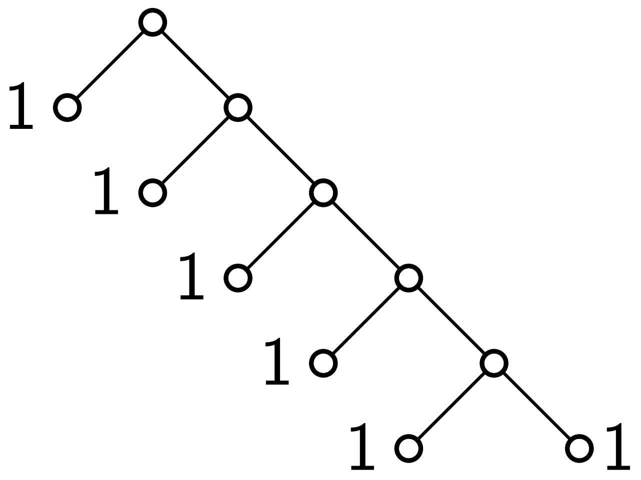

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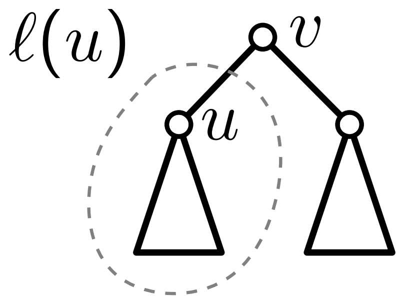

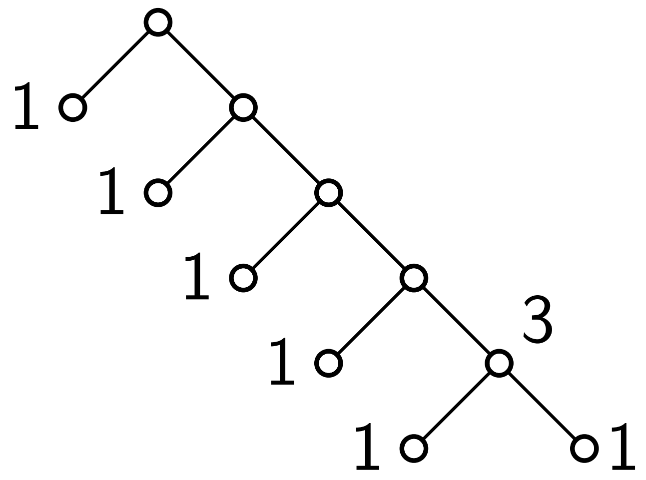

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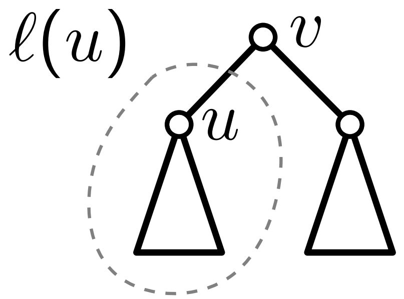

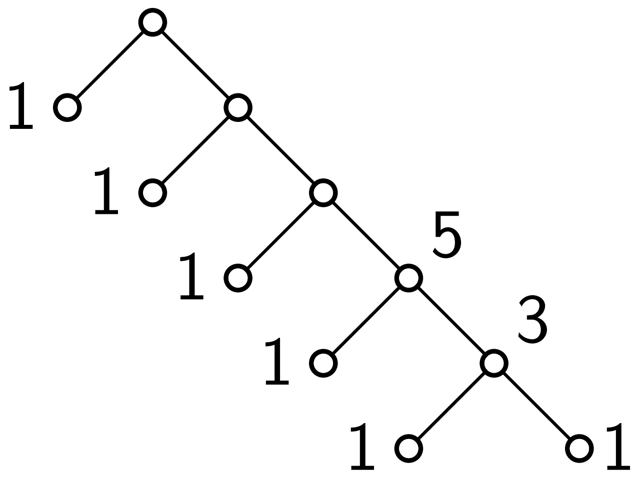

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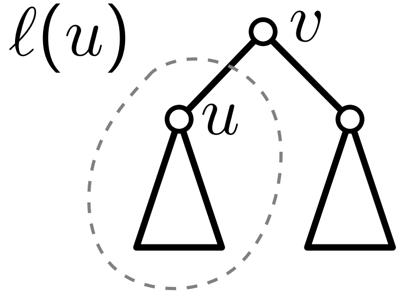

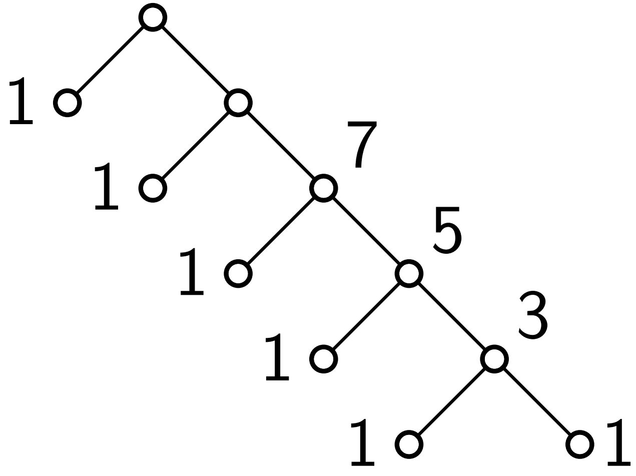

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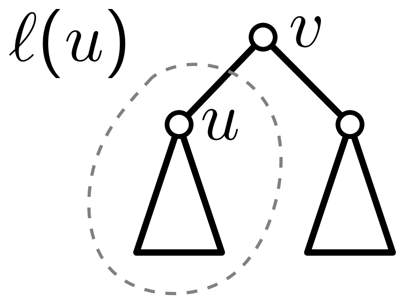

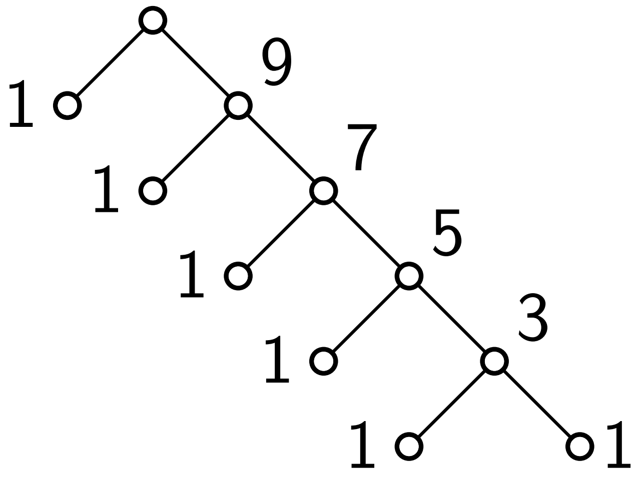

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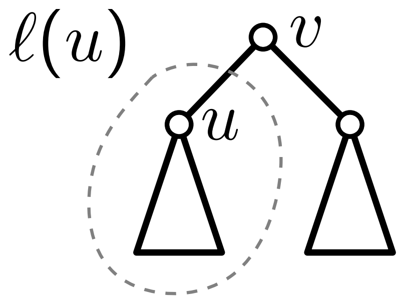

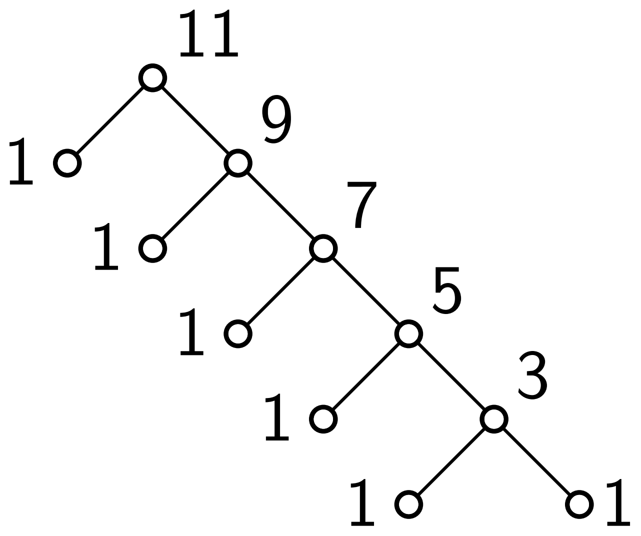

Radial Layouts – Algorithm Attempt

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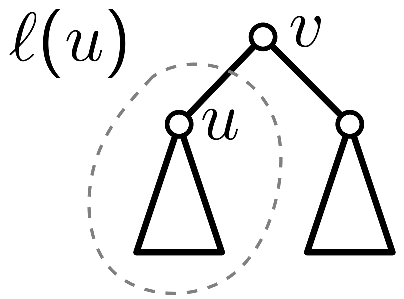
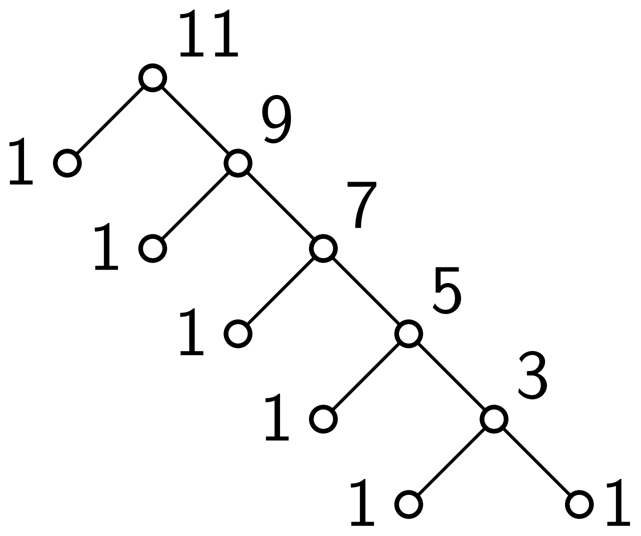
$$\tau_u = \frac{\ell(u)}{\ell(v) - 1}$$





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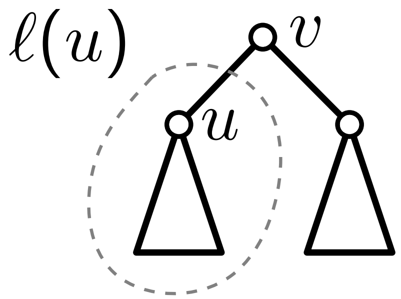
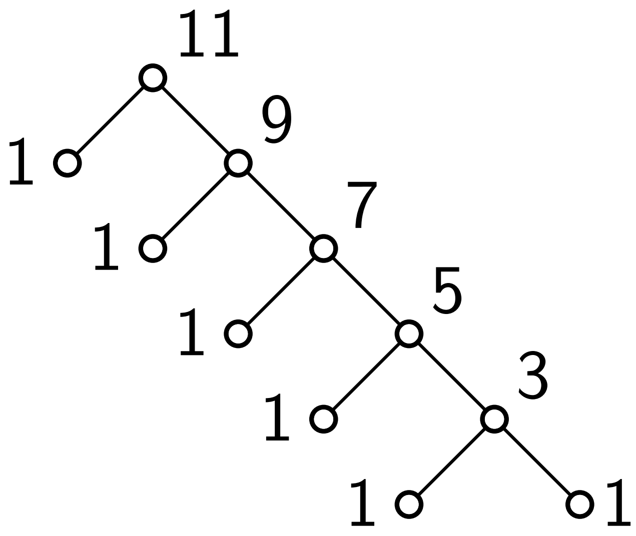


- Place u in middle of area



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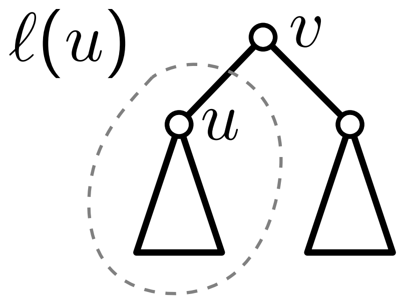

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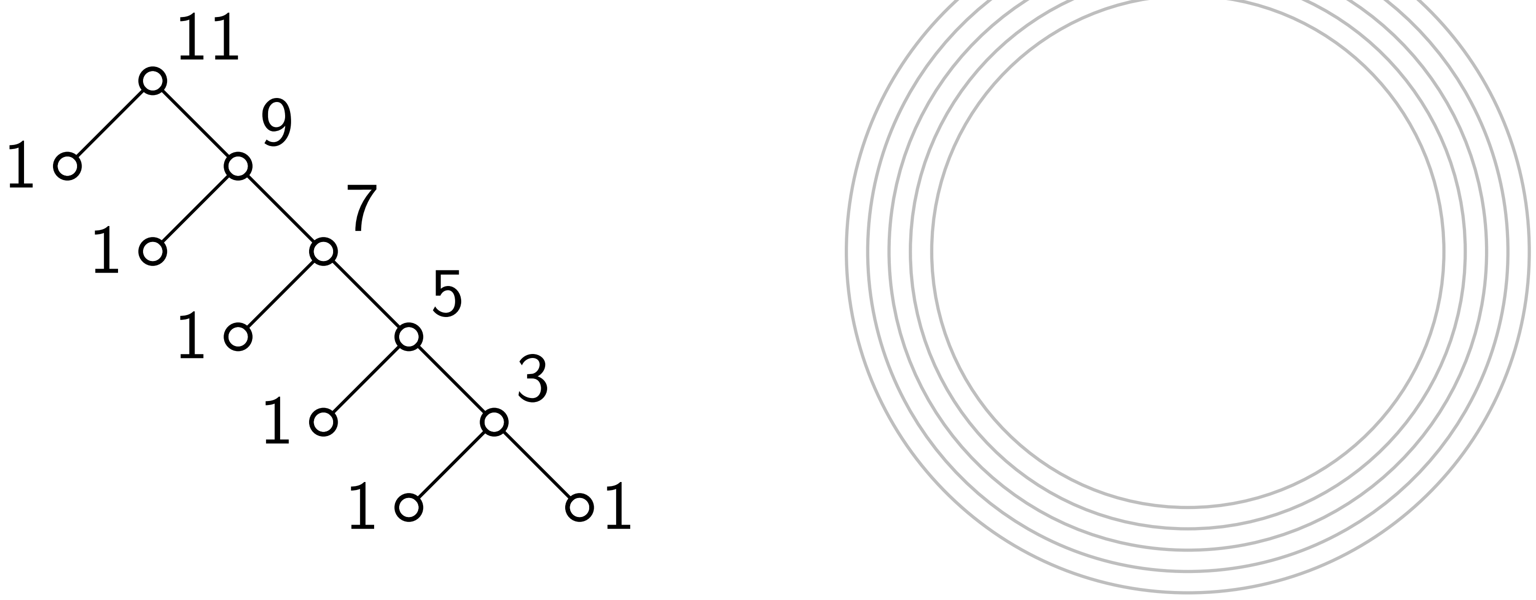

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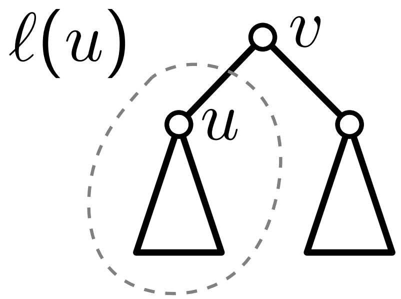

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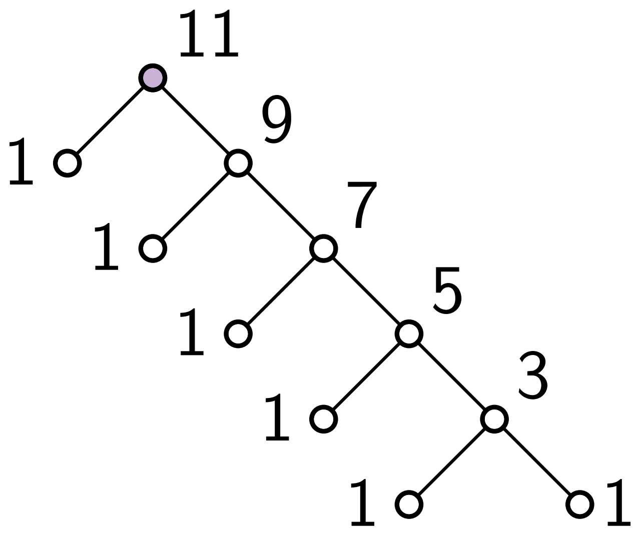

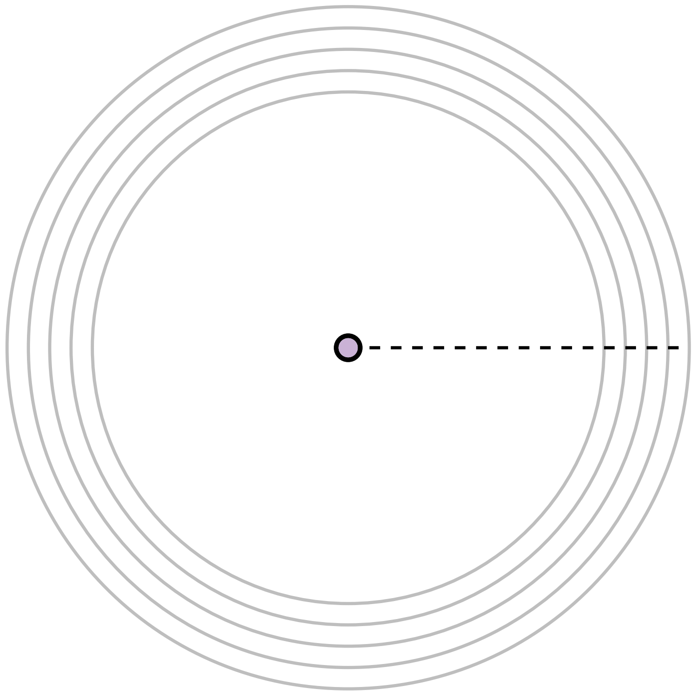

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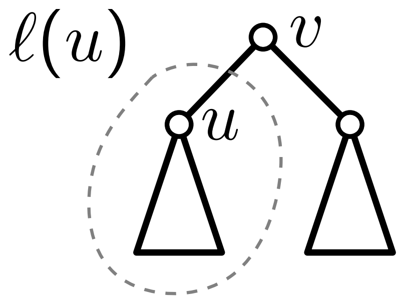

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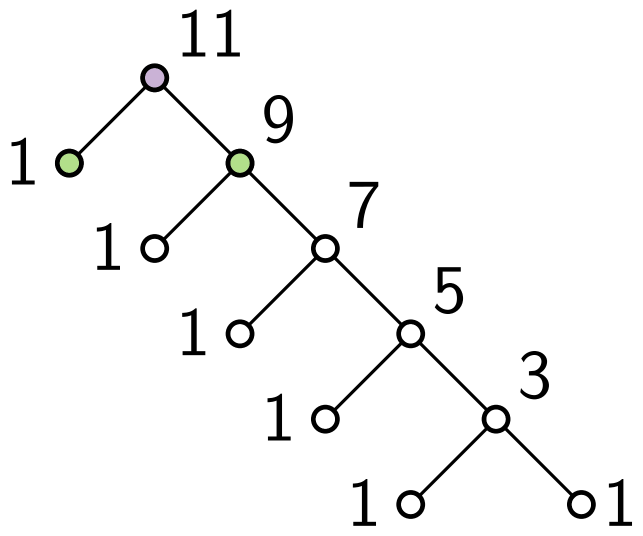

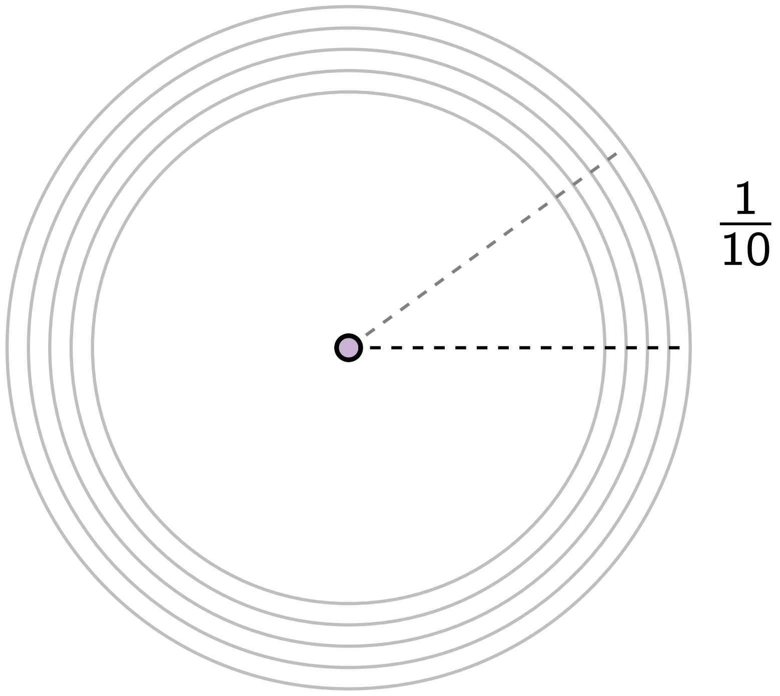

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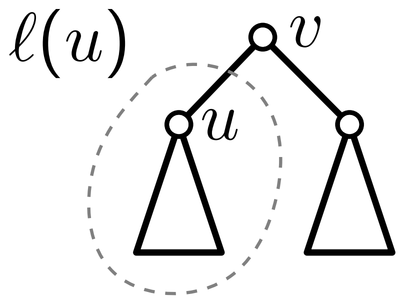

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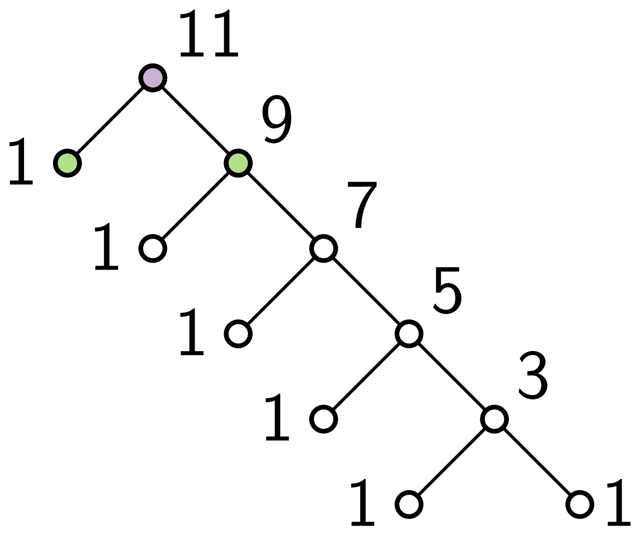

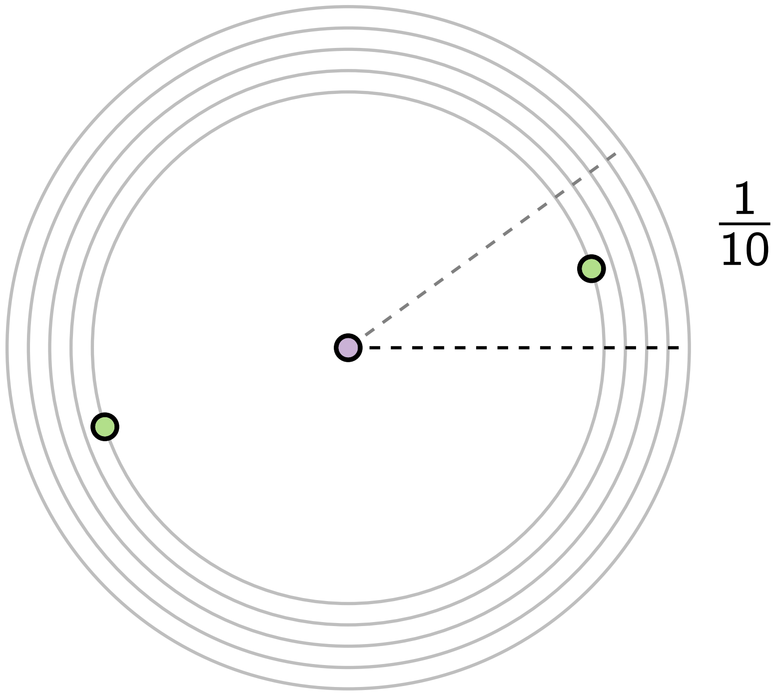

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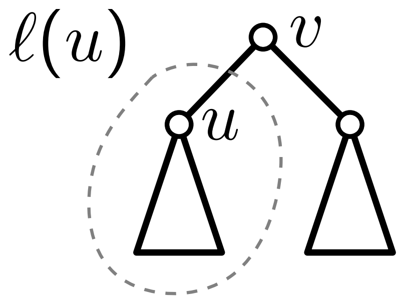

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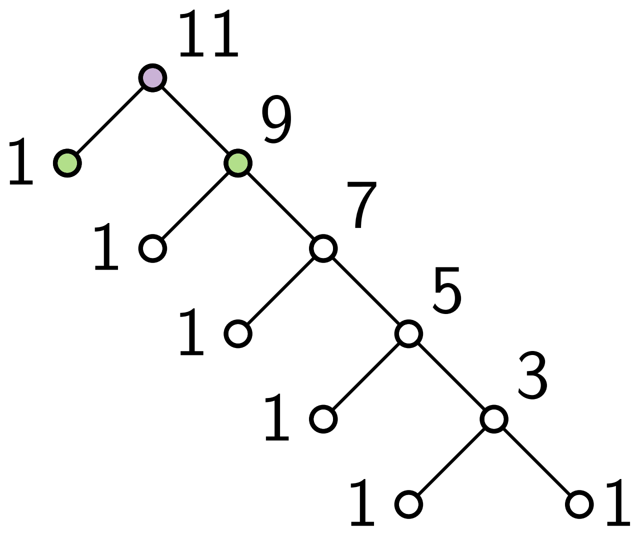

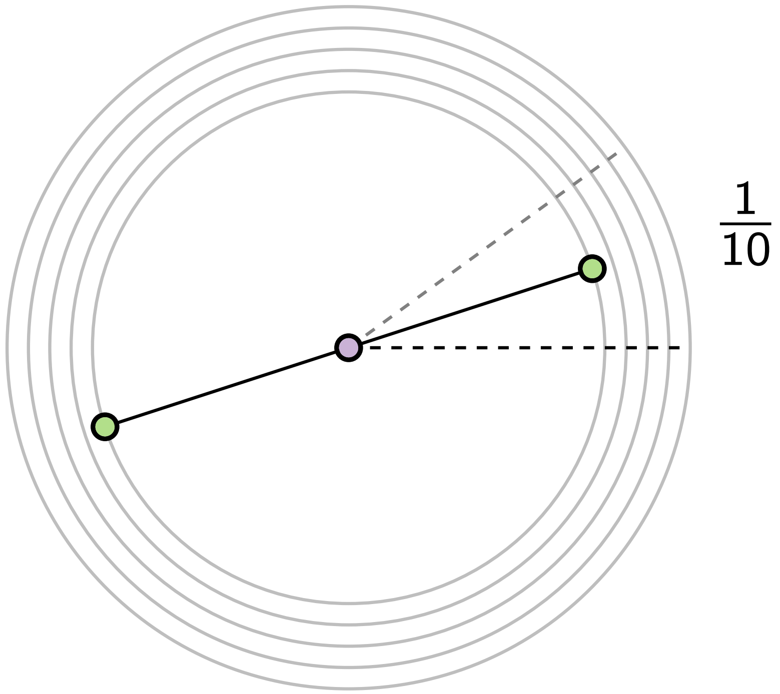

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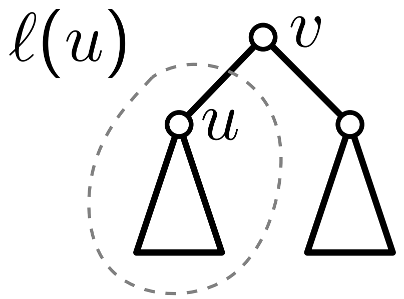

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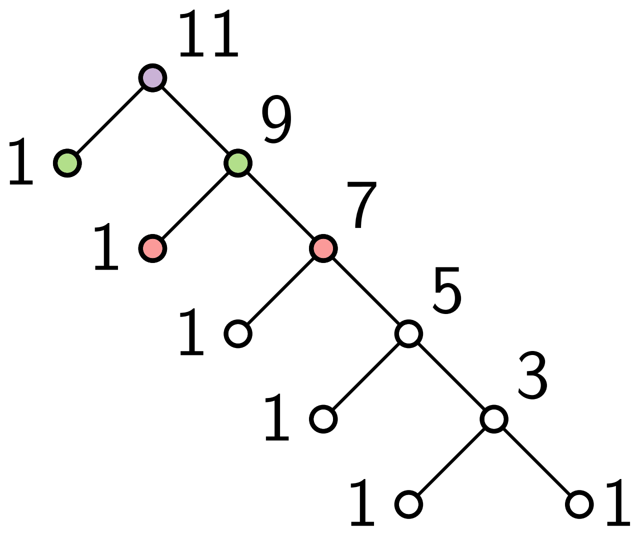

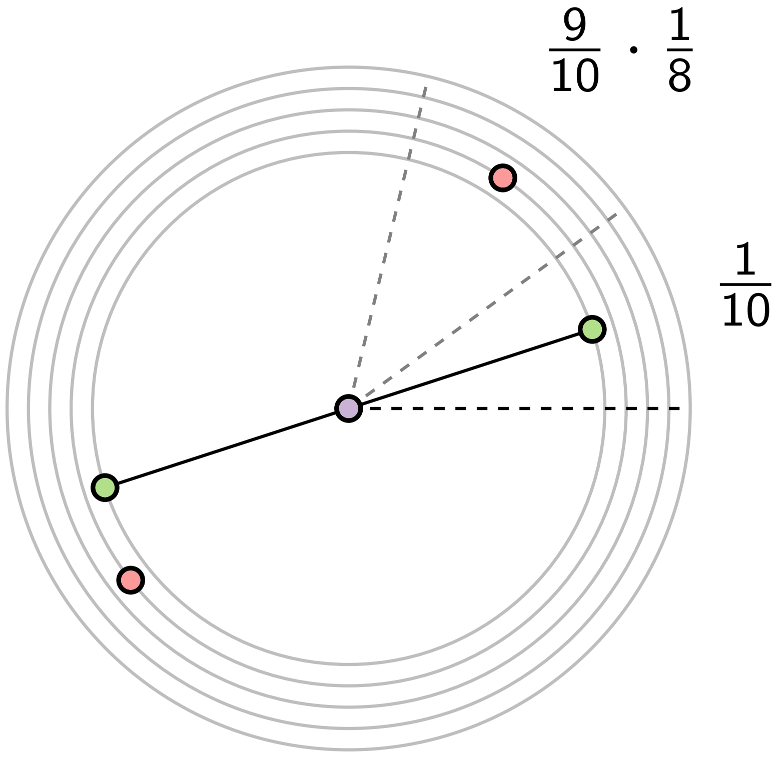

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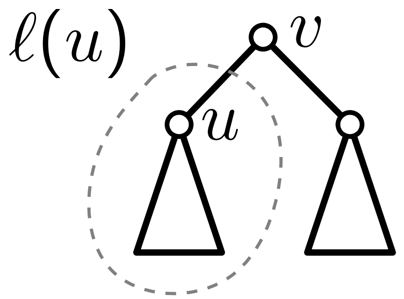

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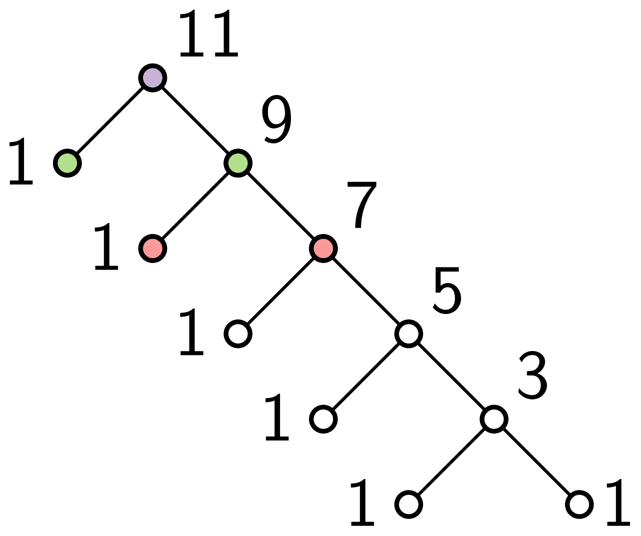

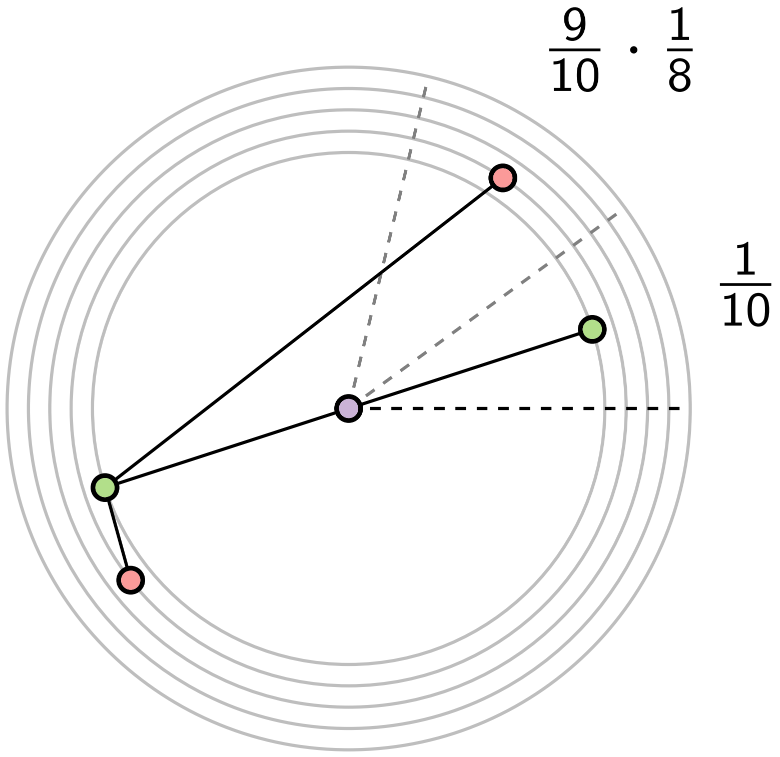

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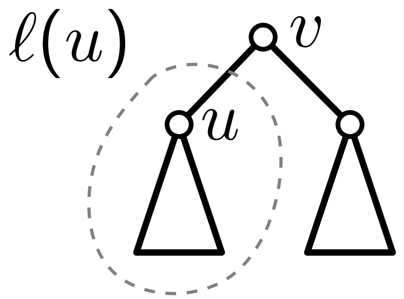

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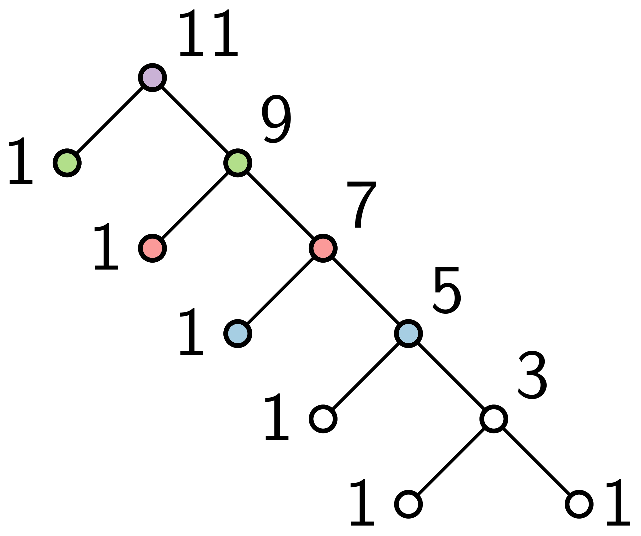

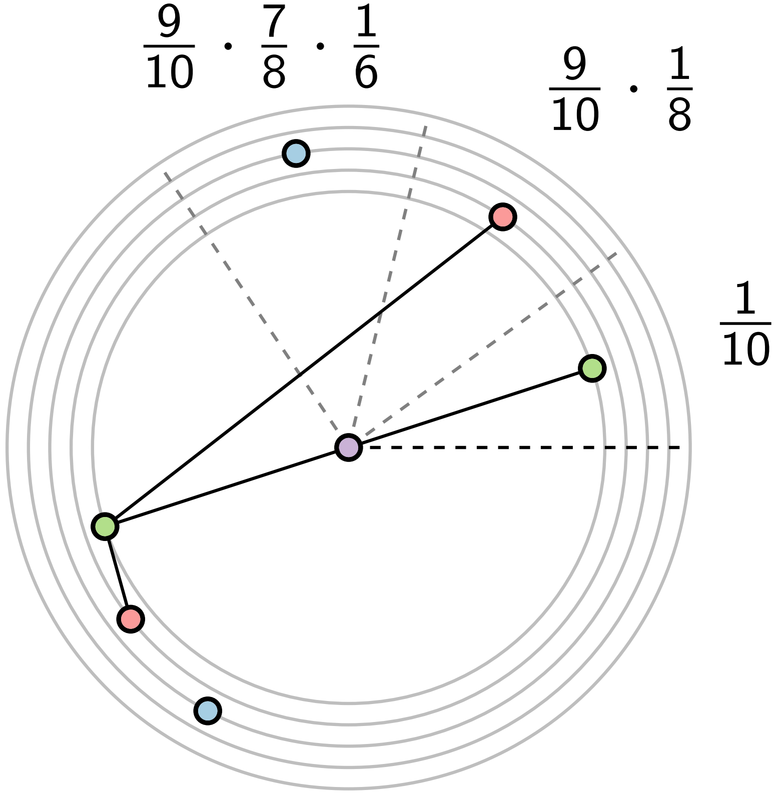

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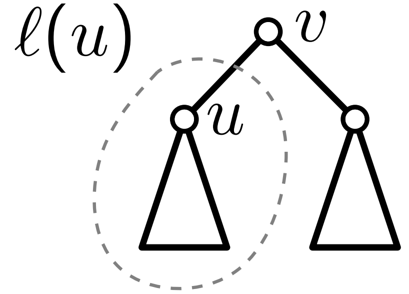

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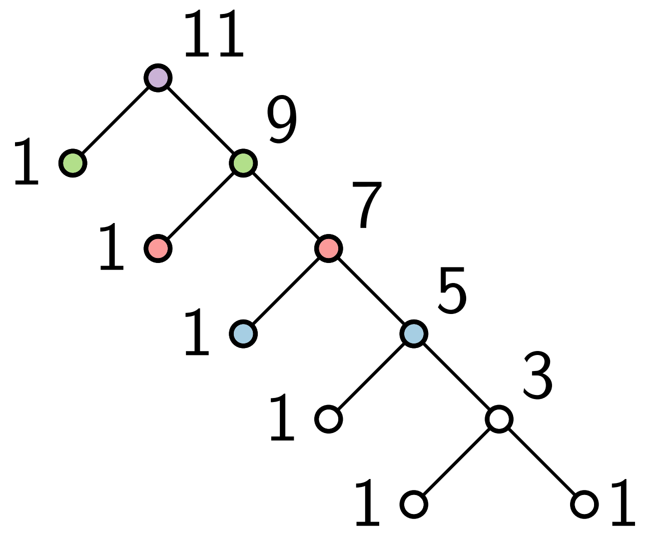
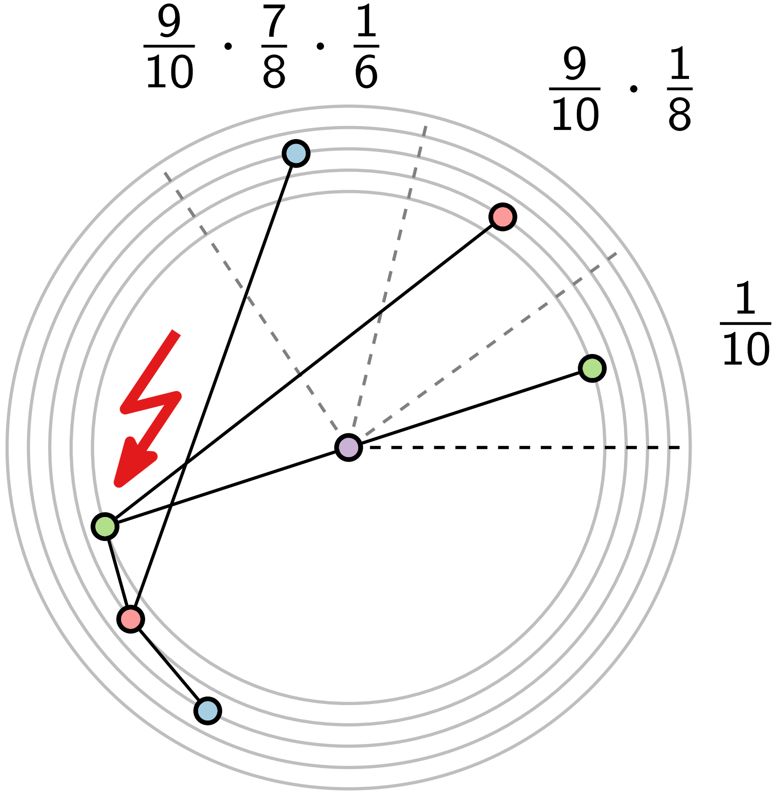
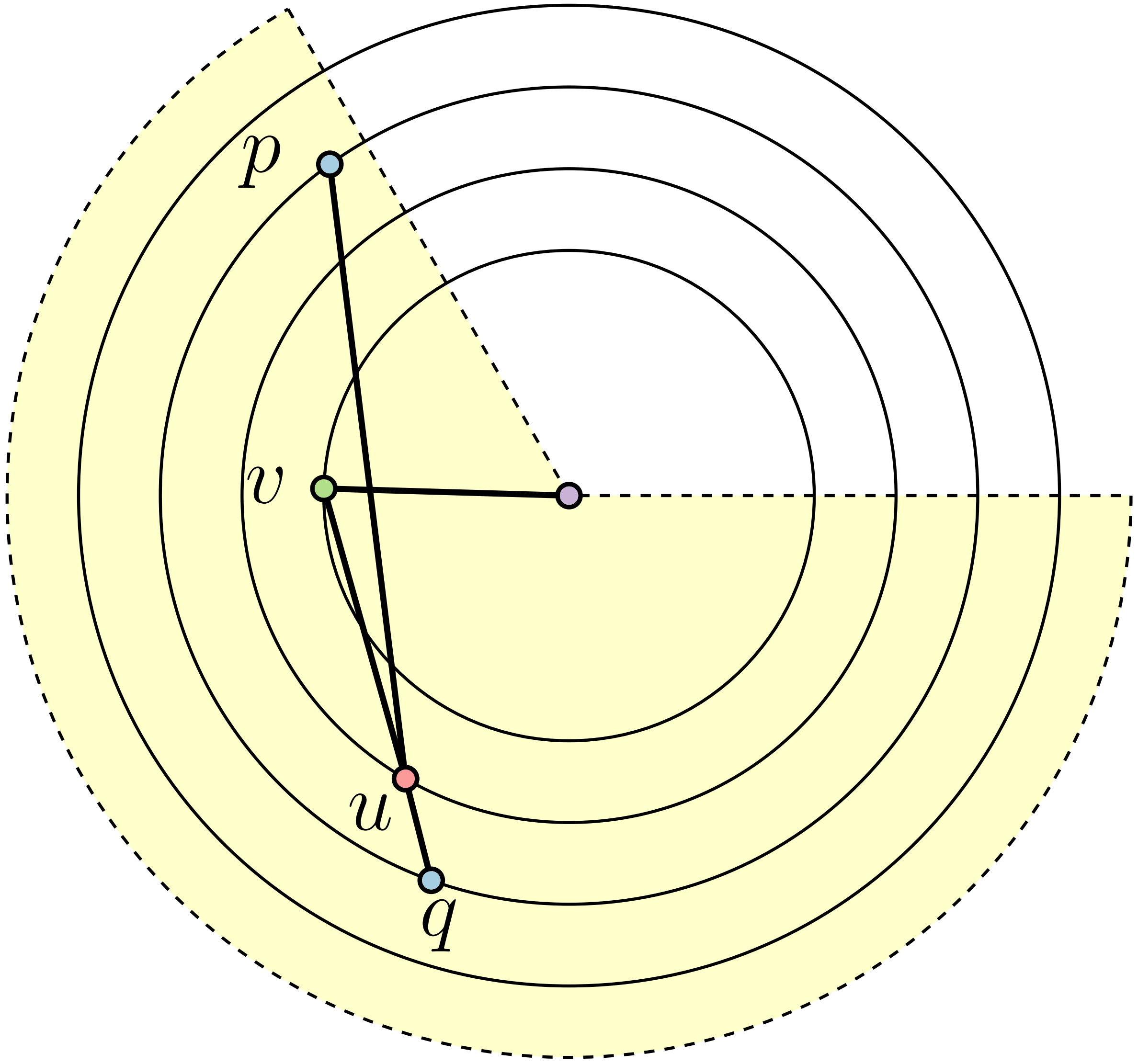




Radial Layouts – How To Avoid Crossings



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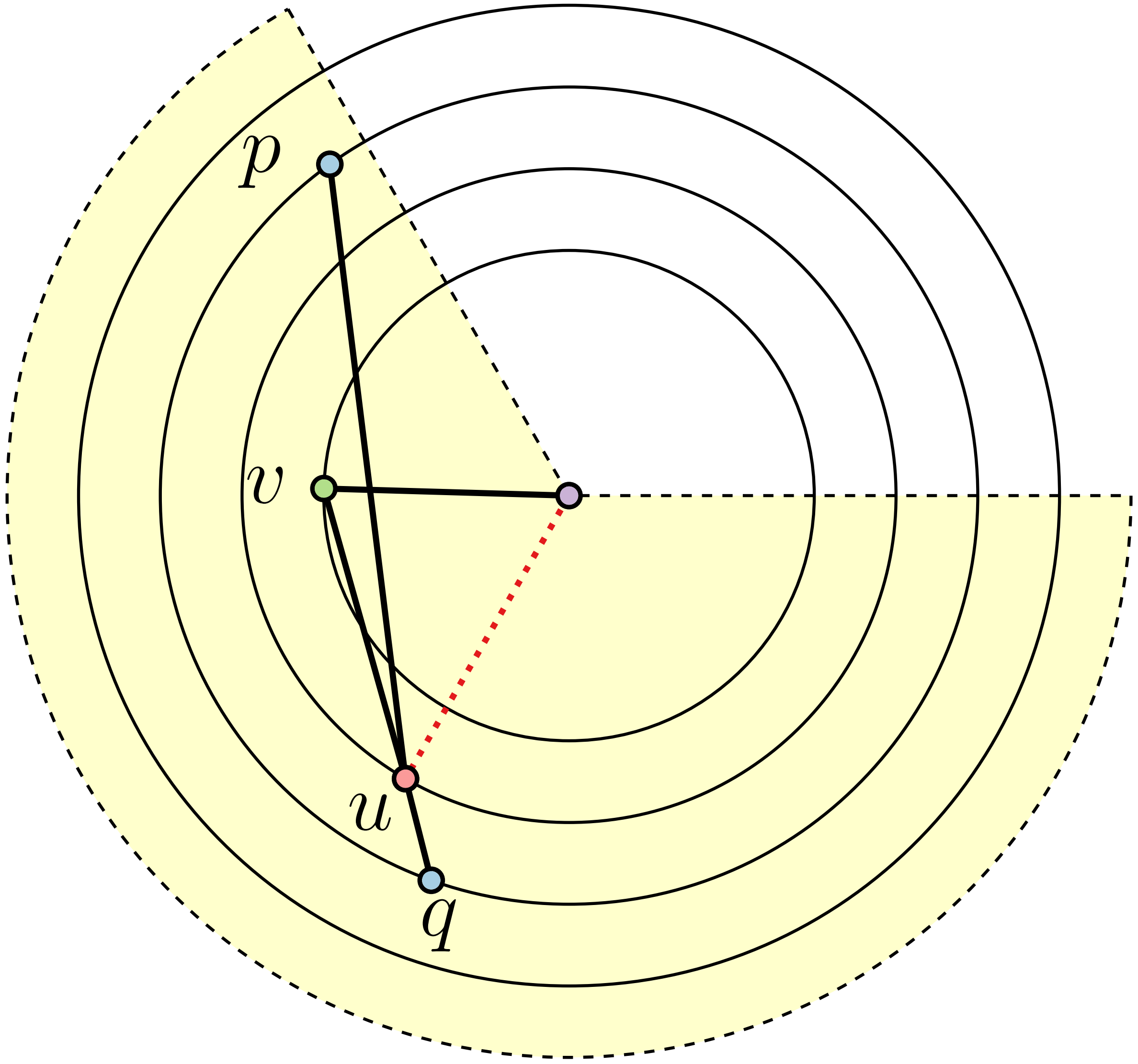

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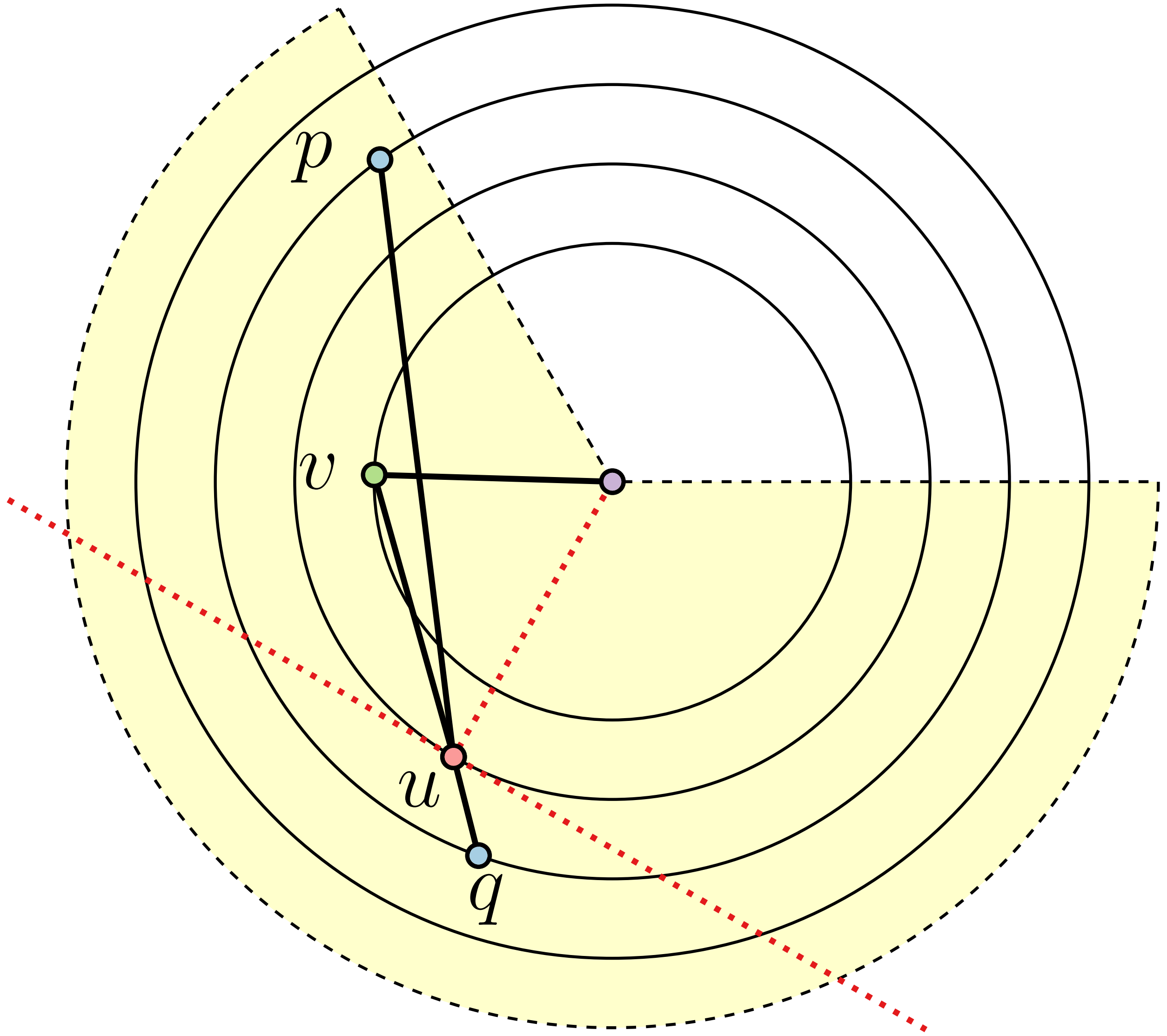

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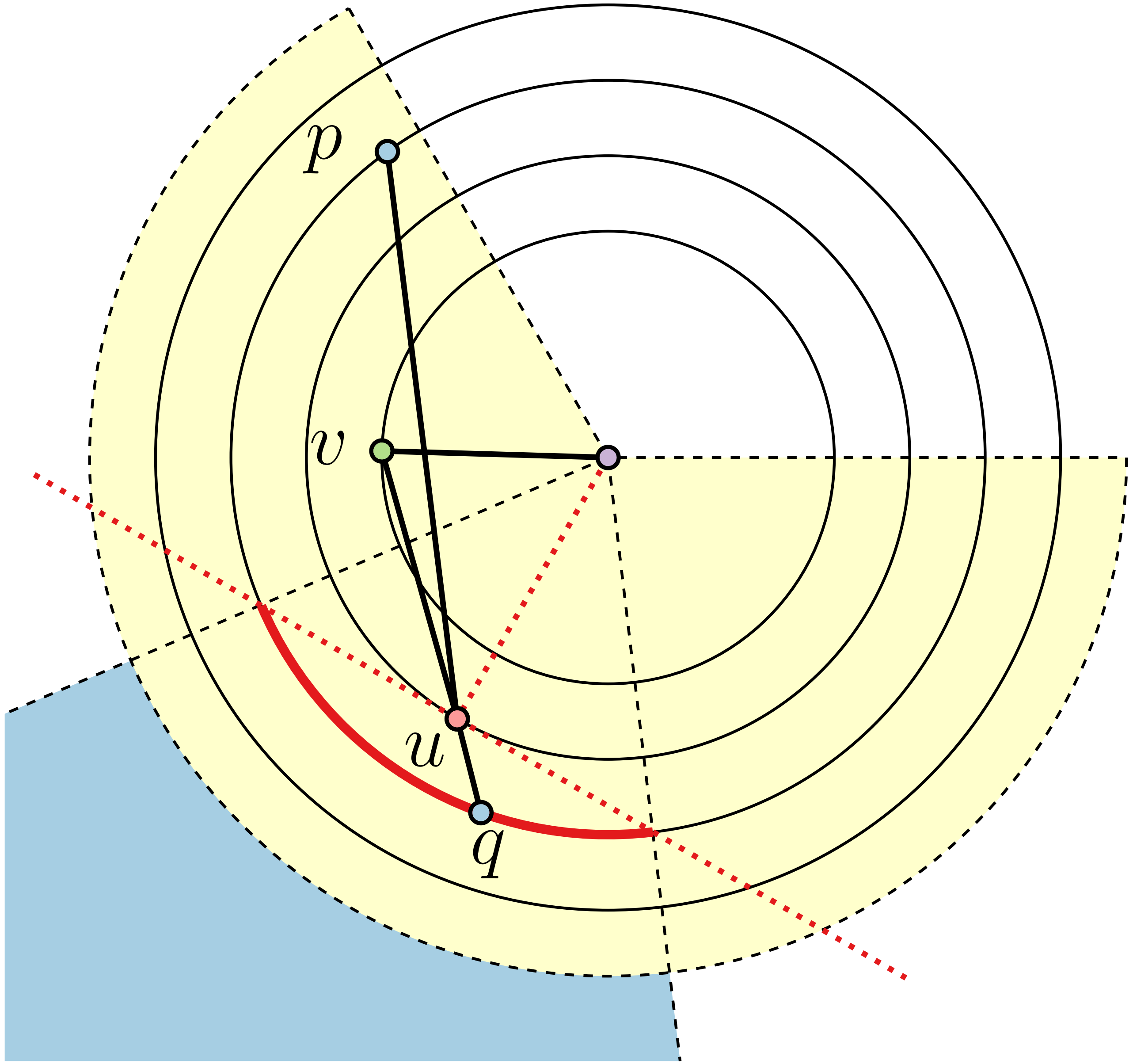
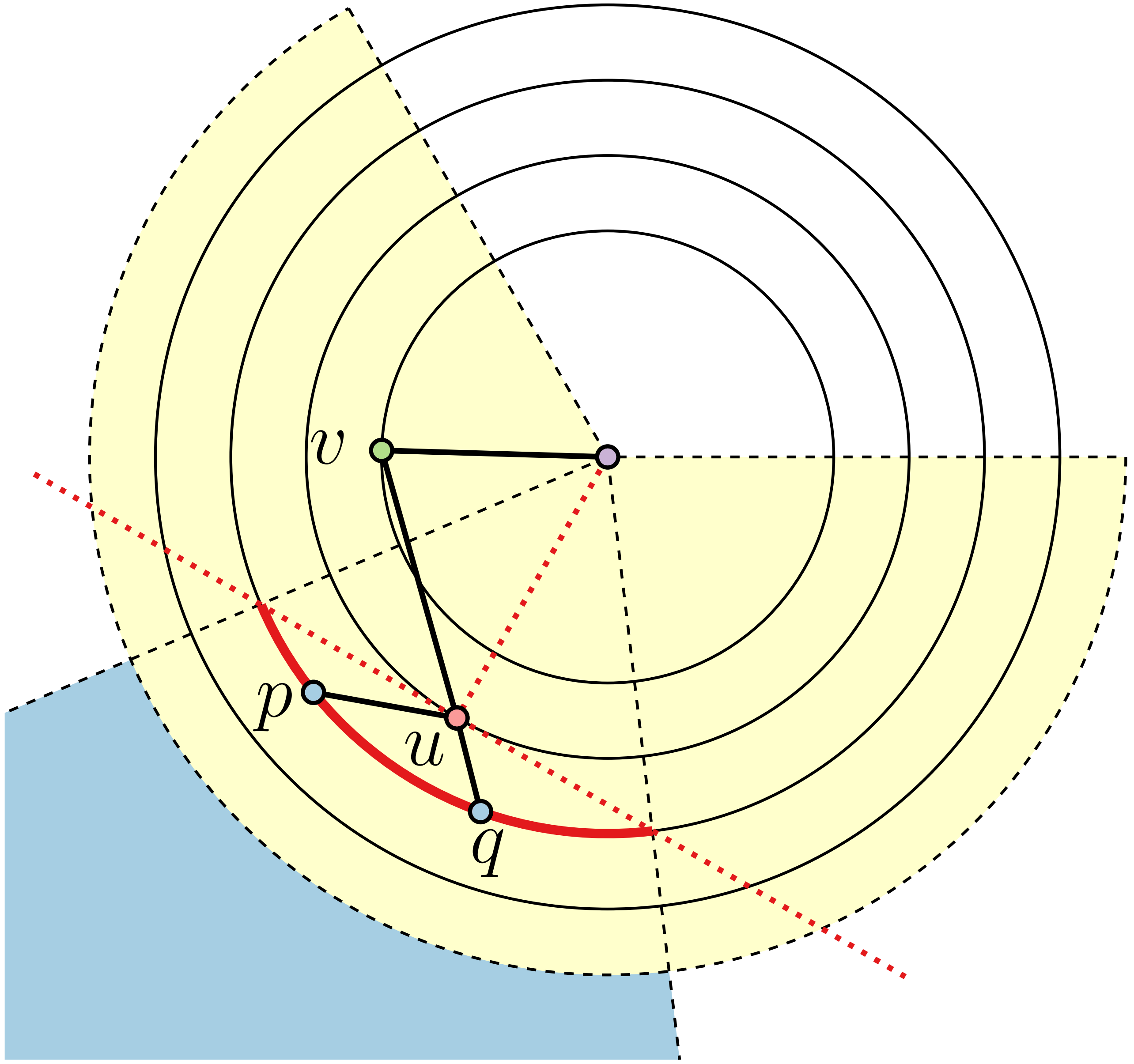
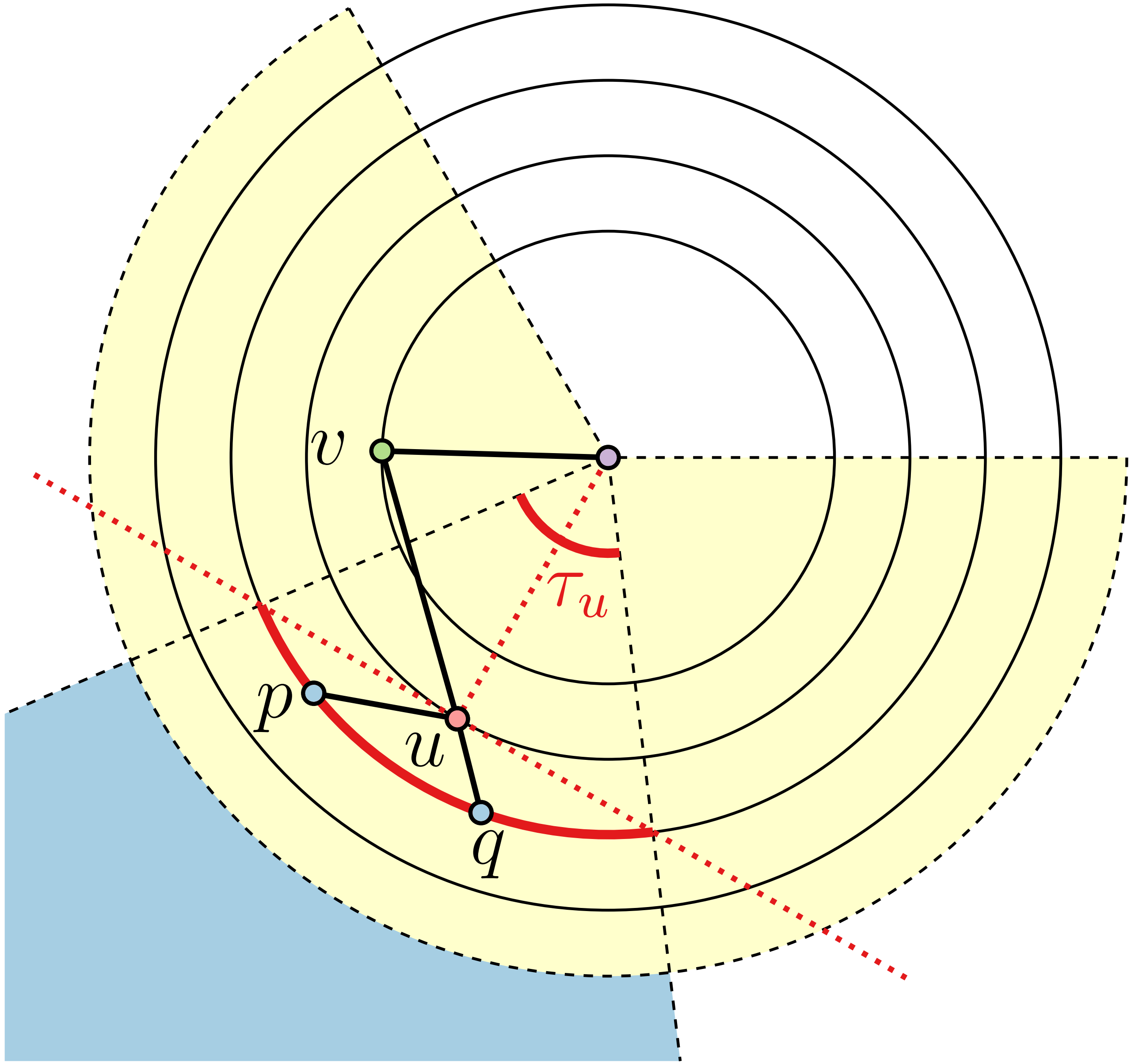


Radial Layouts – How To Avoid Crossings

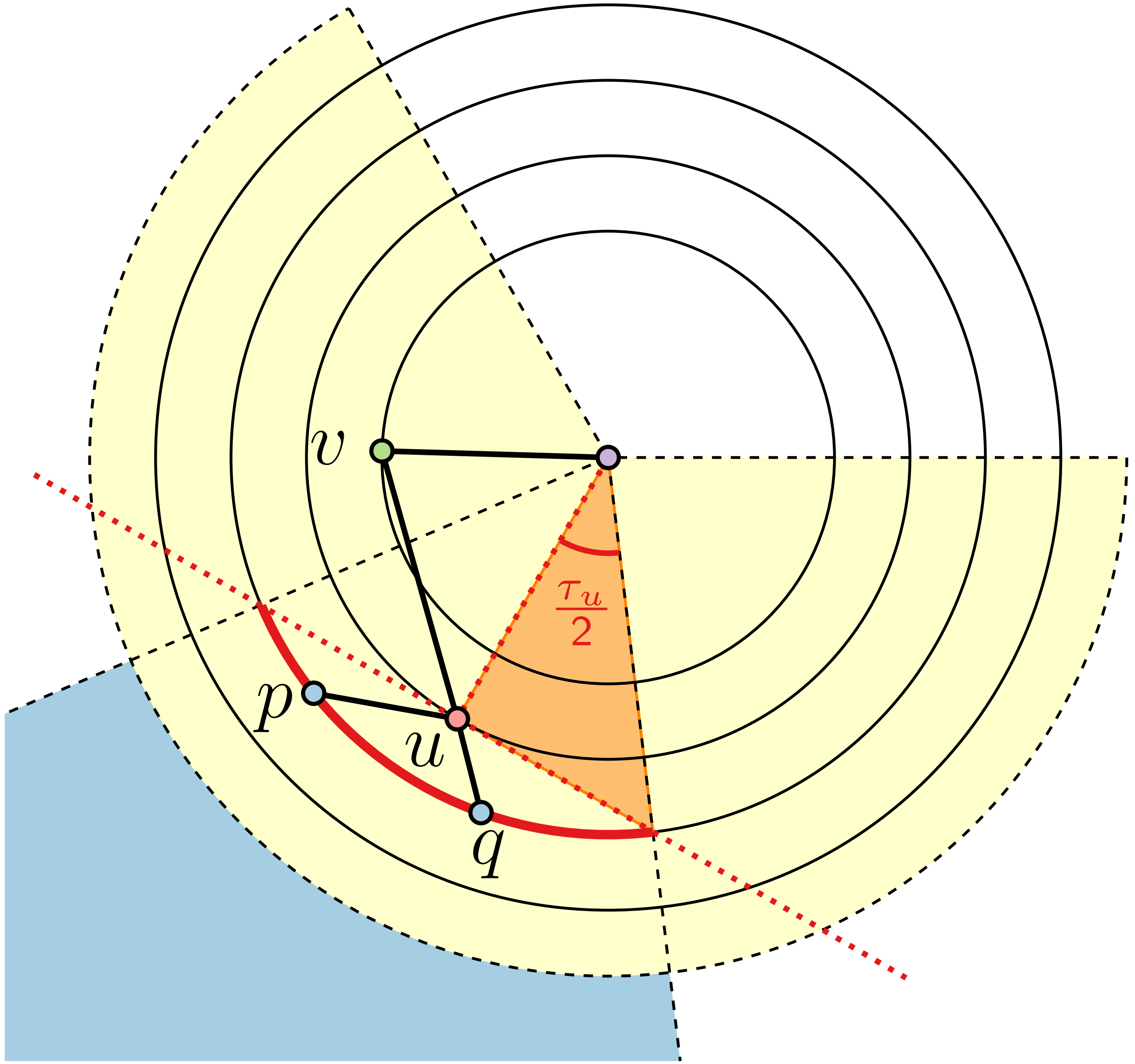


Radial Layouts – How To Avoid Crossings



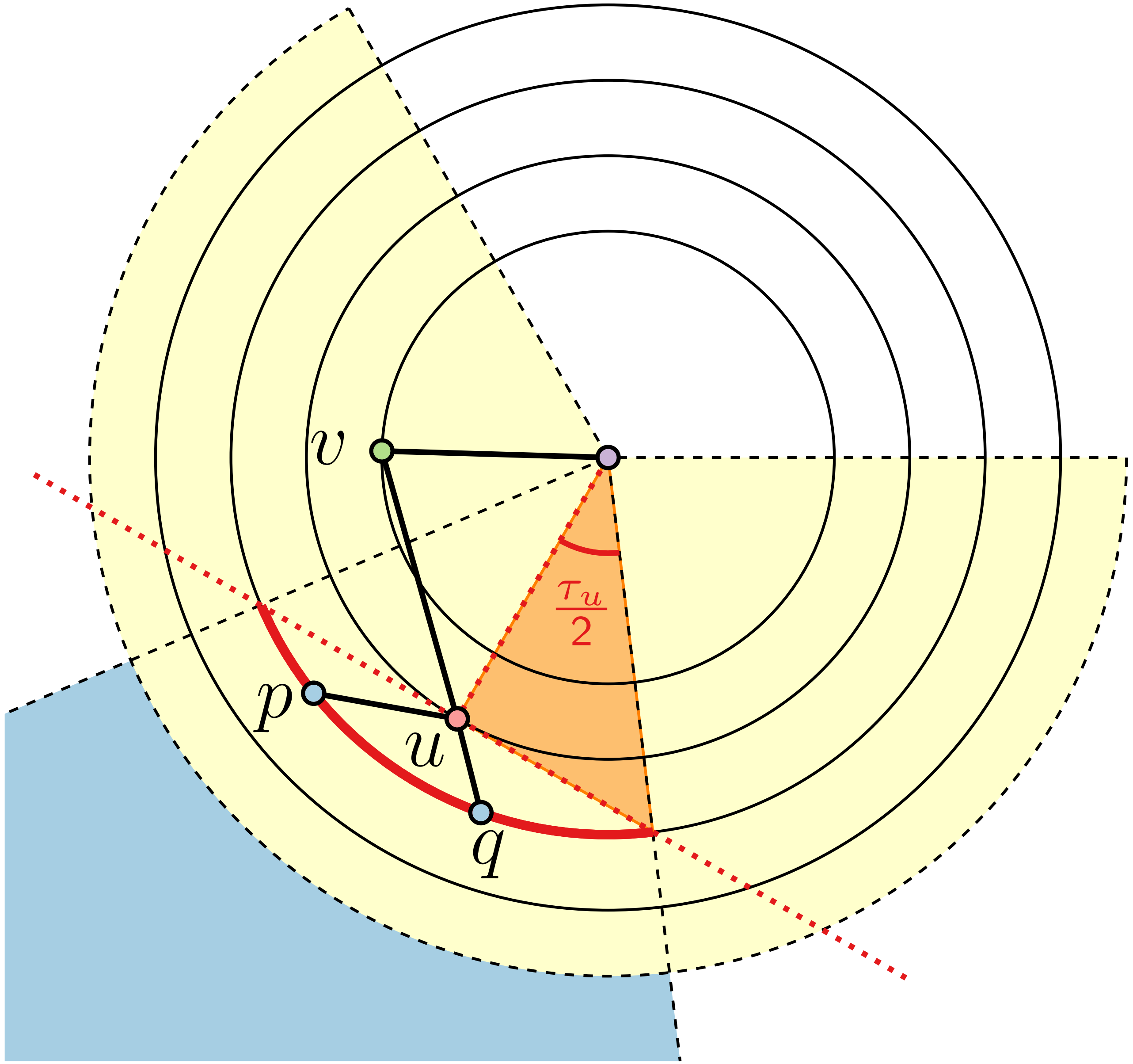
- τ_u – angle of the wedge corresponding to vertex u

Radial Layouts – How To Avoid Crossings



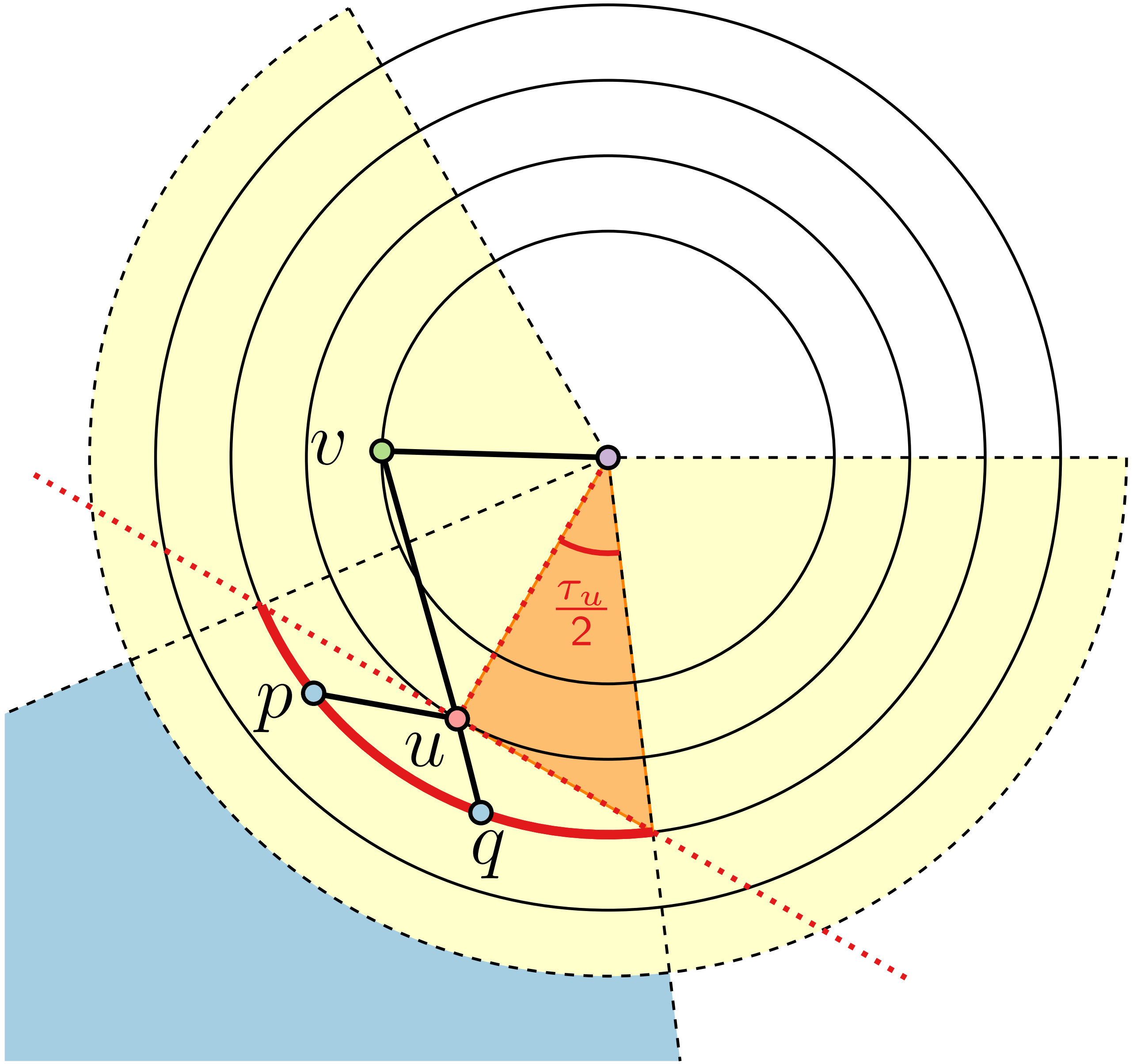
- τ_u – angle of the wedge corresponding to vertex u

Radial Layouts – How To Avoid Crossings



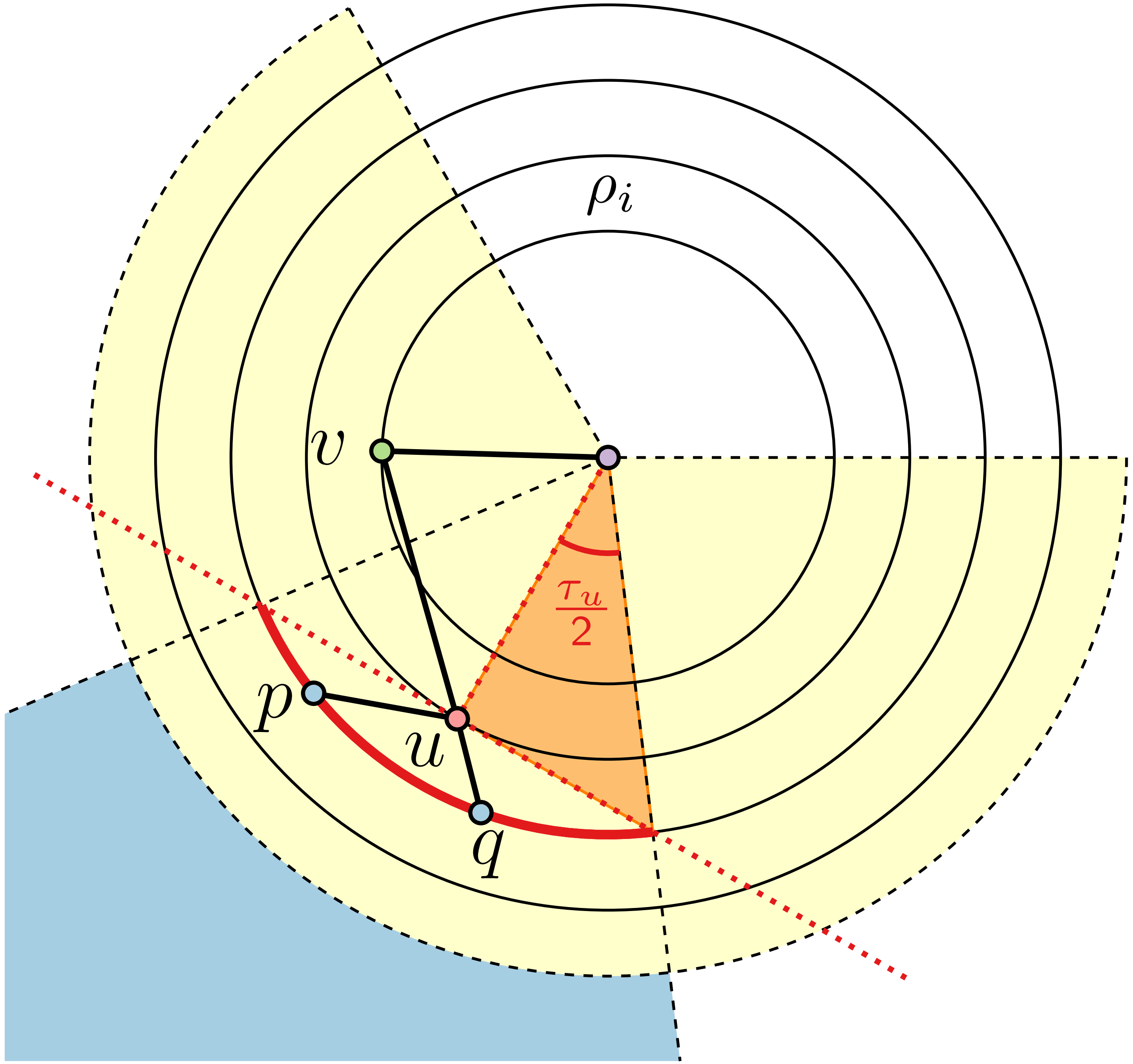
- τ_u – angle of the wedge corresponding to vertex u
- $\ell(u)$ – number of nodes in the subtree rooted at u

Radial Layouts – How To Avoid Crossings



- τ_u – angle of the wedge corresponding to vertex u
- $\ell(u)$ – number of nodes in the subtree rooted at u
- ρ_i – radius of layer i

Radial Layouts – How To Avoid Crossings



- τ_u – angle of the wedge corresponding to vertex u
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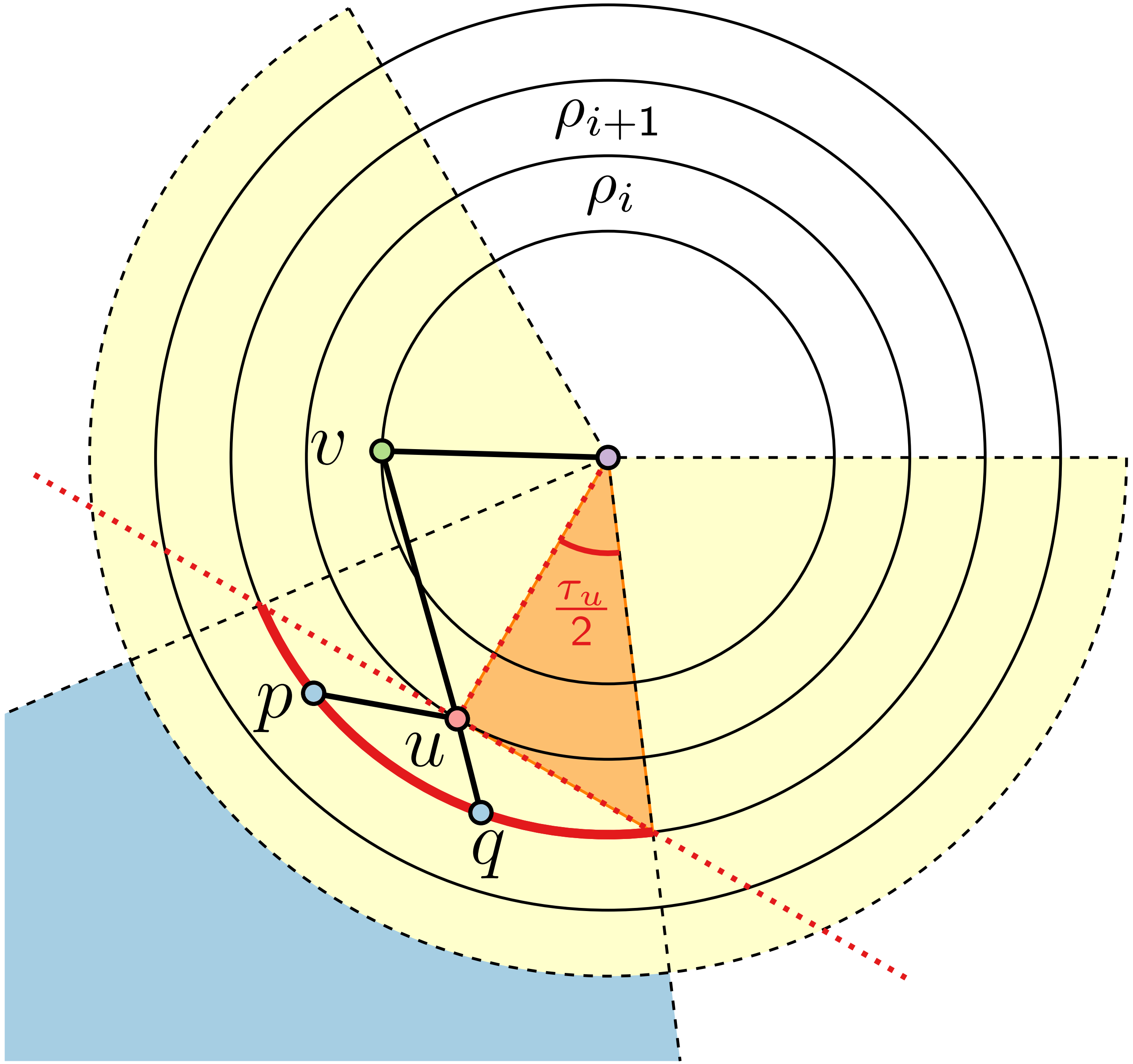
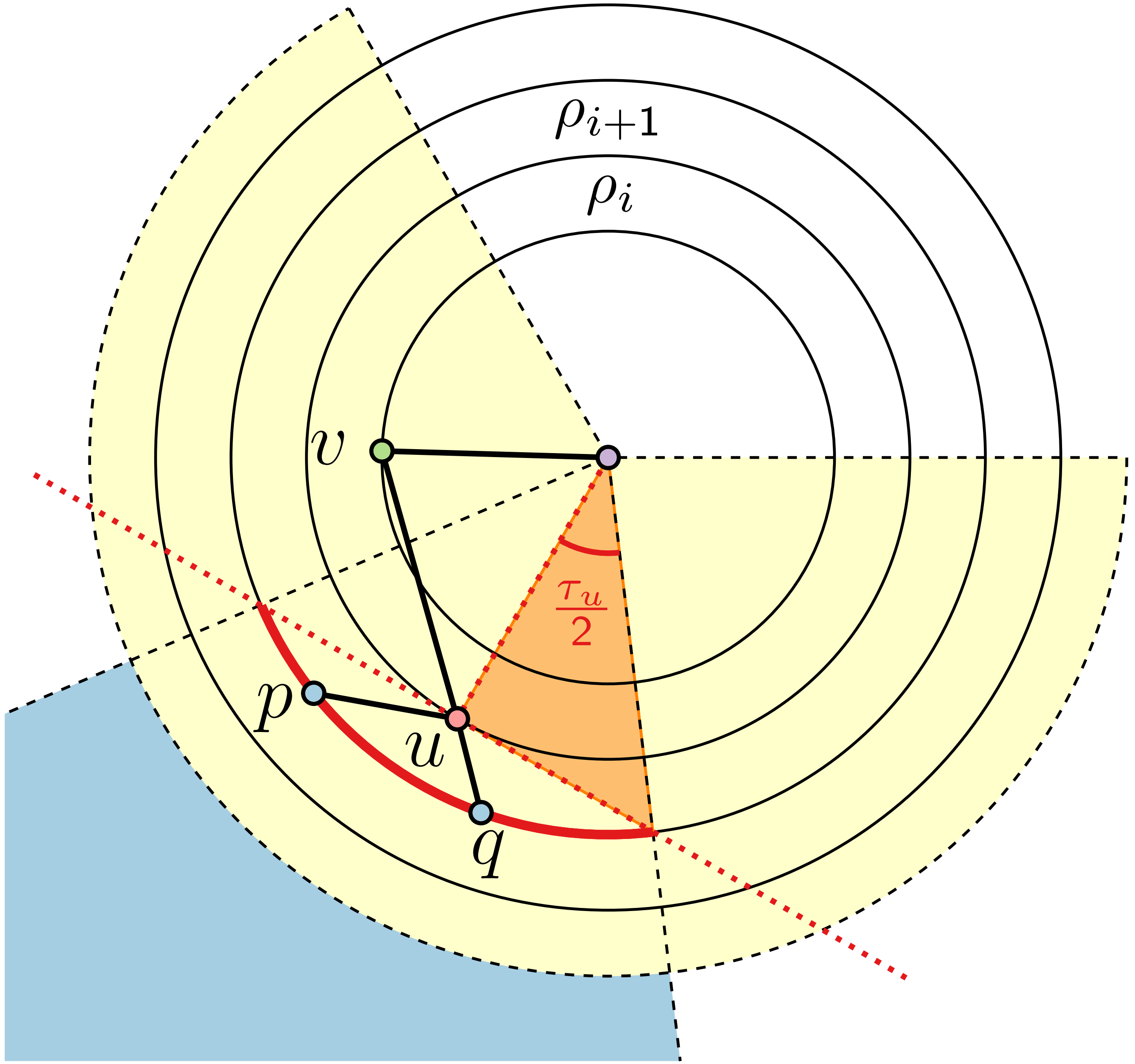
- ρ_i – radius of layer i

Radial Layouts – How To Avoid Crossings



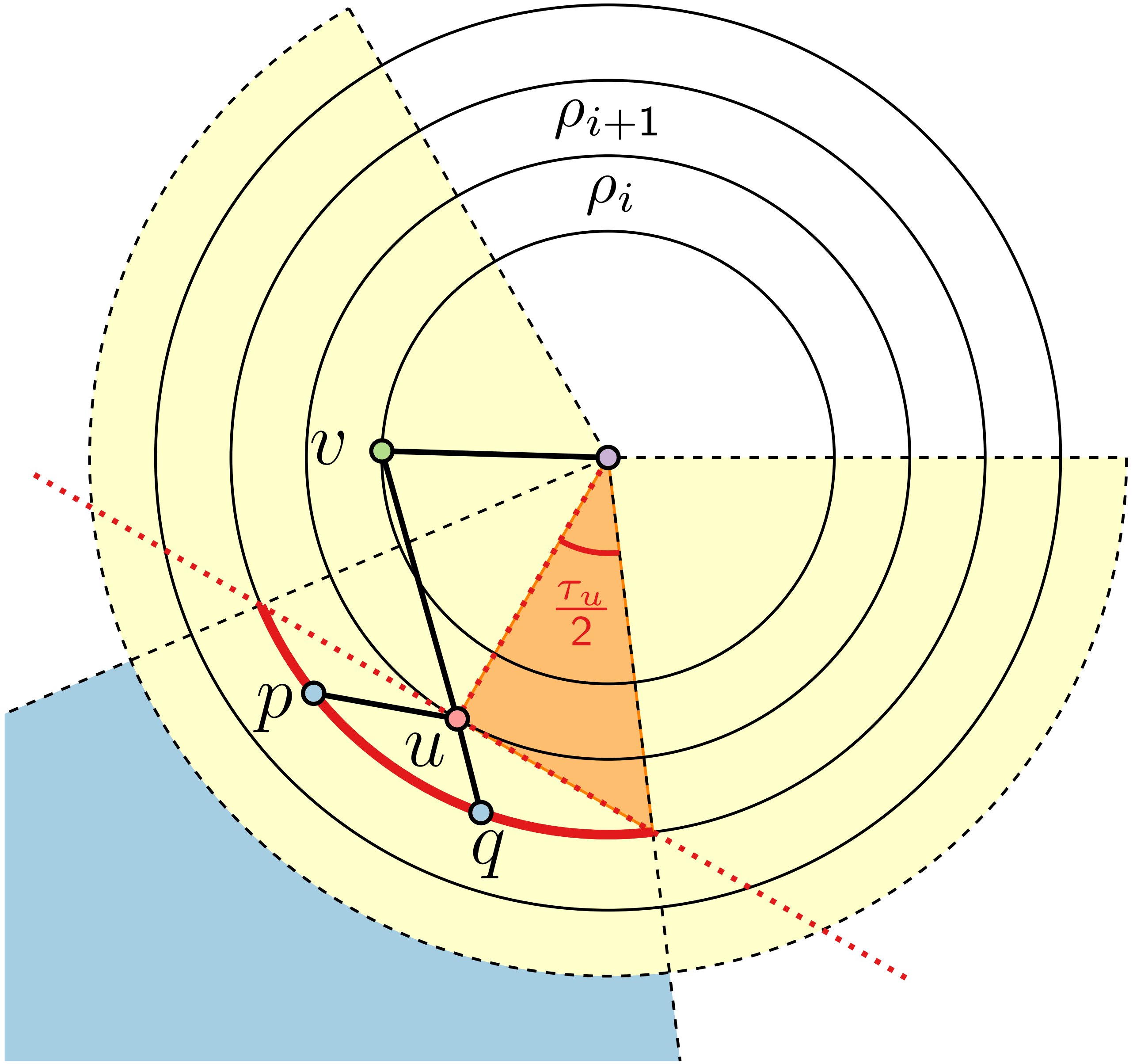
- τ_u – angle of the wedge corresponding to vertex u
- $\ell(u)$ – number of nodes in the subtree rooted at u
- ρ_i – radius of layer i

Radial Layouts – How To Avoid Crossings



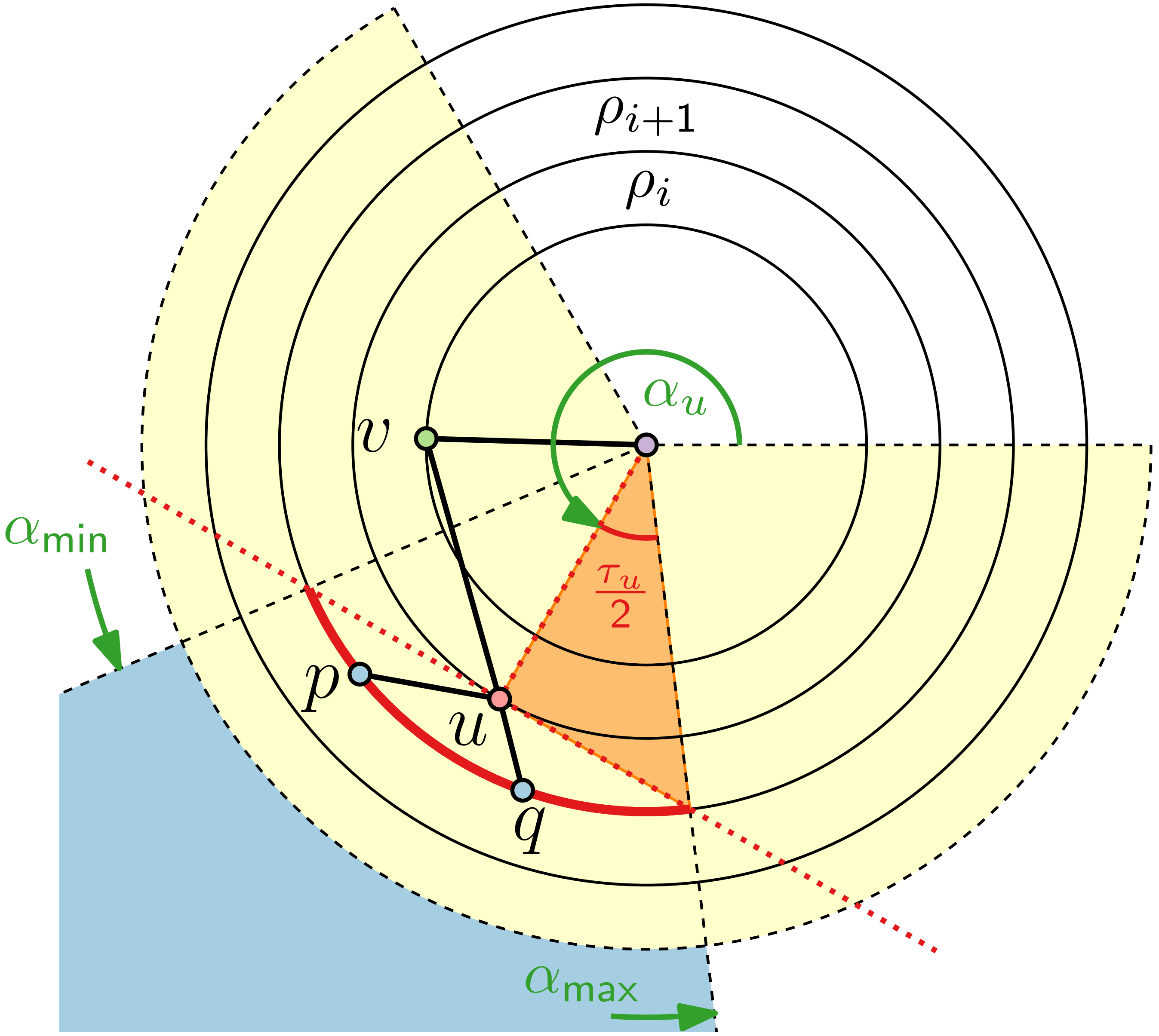
- τ_u – angle of the wedge corresponding to vertex u
- $\ell(u)$ – number of nodes in the subtree rooted at u
- ρ_i – radius of layer i
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$

Radial Layouts – How To Avoid Crossings



- τ_u – angle of the wedge corresponding to vertex u
- $\ell(u)$ – number of nodes in the subtree rooted at u
- ρ_i – radius of layer i
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$
- $\tau_u = \min\{\frac{\ell(u)}{\ell(v)-1}, 2\arccos \frac{\rho_i}{\rho_{i+1}}\}$

Radial Layouts – How To Avoid Crossings



- τ_u – angle of the wedge corresponding to vertex u
- $\ell(u)$ – number of nodes in the subtree rooted at u
- ρ_i – radius of layer i
- $\cos \frac{\tau_u}{2} = \frac{\rho_i}{\rho_{i+1}}$
- $\tau_u = \min\{\frac{\ell(u)}{\ell(v)-1}, 2\arccos \frac{\rho_i}{\rho_{i+1}}\}$
- Alternative:

$$\alpha_{\text{min}} = \alpha_u - \arccos \frac{\rho_i}{\rho_{i+1}}$$

$$\alpha_{\text{max}} = \alpha_u + \arccos \frac{\rho_i}{\rho_{i+1}}$$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$postorder(r)$

$preorder(r, 0, 0, 2\pi)$

return $(d_v, \alpha_v)_{v \in V(T)}$

// vertex pos./polar coord.

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

    $postorder(r)$
    $preorder(r, 0, 0, 2\pi)$
    return $(d_v, \alpha_v)_{v \in V(T)}$
    // vertex pos./polar coord.

postorder(vertex v)

    $\ell(v) \leftarrow 1$
    foreach child w of v **do**
        calculate the size of the subtree recursively

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$postorder(r)$

$preorder(r, 0, 0, 2\pi)$

return $(d_v, \alpha_v)_{v \in V(T)}$

// vertex pos./polar coord.

postorder(vertex v)

$\ell(v) \leftarrow 1$

foreach child w of v **do**

$postorder(w)$

$\ell(v) \leftarrow \ell(v) + \ell(w)$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

  postorder(r)

  preorder$(r, 0, 0, 2\pi)$

  return $(d_v, \alpha_v)_{v \in V(T)}$

  // vertex pos./polar coord.

postorder(vertex v)

  $\ell(v) \leftarrow 1$

  foreach child w of v **do**

    postorder(w)

    $\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\text{min}}, \alpha_{\text{max}}$)

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$postorder(r)$

$preorder(r, 0, 0, 2\pi)$

return $(d_v, \alpha_v)_{v \in V(T)}$

// vertex pos./polar coord.

postorder(vertex v)

$\ell(v) \leftarrow 1$

foreach child w of v **do**

$postorder(w)$

$\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\text{min}}, \alpha_{\text{max}}$)

$d_v \leftarrow \rho_t$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$\quad postorder(r)$

$\quad preorder(r, 0, 0, 2\pi)$

$\quad$ **return** $(d_v, \alpha_v)_{v \in V(T)}$

$\quad$ // vertex pos./polar coord.

postorder(vertex v)

$\quad \ell(v) \leftarrow 1$

$\quad$ **foreach** child w of v **do**

$\quad\quad postorder(w)$

$\quad\quad \ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

$\quad d_v \leftarrow \rho_t$

$\quad \alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

```
    postorder(r)
    preorder(r, 0, 0, 2π)
    return (d_v, α_v)_{v∈V(T)}
    // vertex pos./polar coord.
```

postorder(vertex v)

```
    ℓ(v) ← 1
    foreach child w of v do
        postorder(w)
        ℓ(v) ← ℓ(v) + ℓ(w)
```

preorder(vertex v, $t, \alpha_{\text{min}}, \alpha_{\text{max}}$)

```
    d_v ← ρ_t
    α_v ← (α_min + α_max)/2
```

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$postorder(r)$

$preorder(r, 0, 0, 2\pi)$

return $(d_v, \alpha_v)_{v \in V(T)}$

// vertex pos./polar coord.

postorder(vertex v)

$\ell(v) \leftarrow 1$

foreach child w of v **do**

$postorder(w)$

$\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\text{min}}, \alpha_{\text{max}}$)

$d_v \leftarrow \rho_t$

$\alpha_v \leftarrow (\alpha_{\text{min}} + \alpha_{\text{max}})/2$

//*output*

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin
- $postorder(r)$
- $preorder(r, 0, 0, 2\pi)$
- **return** $(d_v, \alpha_v)_{v \in V(T)}$
- // vertex pos./polar coord.

postorder(vertex v)
- $\ell(v) \leftarrow 1$
- **foreach** child w of v **do**
  - $postorder(w)$
  - $\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)
- $d_v \leftarrow \rho_t$
- $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$ *//output*
- **if** $t > 0$ **then**

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin
  $\mathit{postorder}(r)$
  $\mathit{preorder}(r, 0, 0, 2\pi)$
  return $(d_v, \alpha_v)_{v \in V(T)}$
  // vertex pos./polar coord.

postorder(vertex v)
  $\ell(v) \leftarrow 1$
  foreach child w of v **do**
    $\mathit{postorder}(w)$
    $\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)
  $d_v \leftarrow \rho_t$
  $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$
  //output
  if $t > 0$ **then**
    $\alpha_{\min} \leftarrow \max\{\alpha_{\min}, \alpha_v - \arccos \frac{\rho_t}{\rho_{t+1}}\}$
    $\alpha_{\max} \leftarrow \min\{\alpha_{\max}, \alpha_v + \arccos \frac{\rho_t}{\rho_{t+1}}\}$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$postorder(r)$

$preorder(r, 0, 0, 2\pi)$

return $(d_v, \alpha_v)_{v \in V(T)}$

// vertex pos./polar coord.

postorder(vertex v)

$\ell(v) \leftarrow 1$

foreach child w of v **do**

$postorder(w)$

$\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

$d_v \leftarrow \rho_t$

$\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$

//output

if $t > 0$ **then**

$\alpha_{\min} \leftarrow \max\{\alpha_{\min}, \alpha_v - \arccos \frac{\rho_t}{\rho_{t+1}}\}$

$\alpha_{\max} \leftarrow \min\{\alpha_{\max}, \alpha_v + \arccos \frac{\rho_t}{\rho_{t+1}}\}$

$left \leftarrow \alpha_{\min}$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$postorder(r)$

$preorder(r, 0, 0, 2\pi)$

return $(d_v, \alpha_v)_{v \in V(T)}$

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postorder(vertex v)

$\ell(v) \leftarrow 1$

foreach child w of v **do**

$postorder(w)$

$\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

$d_v \leftarrow \rho_t$

$\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$

//output

if $t > 0$ **then**

$\alpha_{\min} \leftarrow \max\{\alpha_{\min}, \alpha_v - \arccos \frac{\rho_t}{\rho_{t+1}}\}$

$\alpha_{\max} \leftarrow \min\{\alpha_{\max}, \alpha_v + \arccos \frac{\rho_t}{\rho_{t+1}}\}$

$left \leftarrow \alpha_{\min}$

foreach child w of v **do**

Radial Layouts – Pseudocode

```
RadialTreeLayout(tree T, root r ∈ T, radii ρ_1 < ··· < ρ_k)
begin
    postorder(r)
    preorder(r, 0, 0, 2π)
    return (d_v, α_v)_{v∈V(T)}
    // vertex pos./polar coord.
```

```
postorder(vertex v)
    ℓ(v) ← 1
    foreach child w of v do
        postorder(w)
        ℓ(v) ← ℓ(v) + ℓ(w)
```

```
preorder(vertex v, t, α_min, α_max)
    d_v ← ρ_t                          //output
    α_v ← (α_min + α_max)/2
    if t > 0 then
        α_min ← max{α_min, α_v − arccos ρ_t/ρ_{t+1}}
        α_max ← min{α_max, α_v + arccos ρ_t/ρ_{t+1}}
    left ← α_min
    foreach child w of v do
        right ← left + ℓ(w)/(ℓ(v)−1) · (α_max − α_min)
```

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

```
begin
    postorder(r)
    preorder(r, 0, 0, 2π)
    return (d_v, α_v)_{v∈V(T)}
    // vertex pos./polar coord.
```

postorder(vertex v)

```
    ℓ(v) ← 1
    foreach child w of v do
        postorder(w)
        ℓ(v) ← ℓ(v) + ℓ(w)
```

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

```
    d_v ← ρ_t                              //output
    α_v ← (α_min + α_max)/2
    if t > 0 then
        α_min ← max{α_min, α_v − arccos ρ_t/ρ_{t+1}}
        α_max ← min{α_max, α_v + arccos ρ_t/ρ_{t+1}}
    left ← α_min
    foreach child w of v do
        right ← left + ℓ(w)/(ℓ(v)−1) · (α_max − α_min)
        preorder(w, t + 1, left, right)
```

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$postorder(r)$

$preorder(r, 0, 0, 2\pi)$

return $(d_v, \alpha_v)_{v \in V(T)}$

// vertex pos./polar coord.

postorder(vertex v)

$\ell(v) \leftarrow 1$

foreach child w of v **do**

$postorder(w)$

$\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

$d_v \leftarrow \rho_t$

$\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$ //output

if $t > 0$ **then**

$\alpha_{\min} \leftarrow \max\{\alpha_{\min}, \alpha_v - \arccos \frac{\rho_t}{\rho_{t+1}}\}$

$\alpha_{\max} \leftarrow \min\{\alpha_{\max}, \alpha_v + \arccos \frac{\rho_t}{\rho_{t+1}}\}$

$left \leftarrow \alpha_{\min}$

foreach child w of v **do**

$right \leftarrow left + \frac{\ell(w)}{\ell(v)-1} \cdot (\alpha_{\max} - \alpha_{\min})$

$preorder(w, t+1, left, right)$

$left \leftarrow right$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$postorder(r)$

$preorder(r, 0, 0, 2\pi)$

return $(d_v, \alpha_v)_{v \in V(T)}$

// vertex pos./polar coord.

postorder(vertex v)

$\ell(v) \leftarrow 1$

foreach child w of v **do**

$postorder(w)$

$\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

$d_v \leftarrow \rho_t$

$\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$

//output

if $t > 0$ **then**

$\alpha_{\min} \leftarrow \max\{\alpha_{\min}, \alpha_v - \arccos \frac{\rho_t}{\rho_{t+1}}\}$

$\alpha_{\max} \leftarrow \min\{\alpha_{\max}, \alpha_v + \arccos \frac{\rho_t}{\rho_{t+1}}\}$

$left \leftarrow \alpha_{\min}$

foreach child w of v **do**

$right \leftarrow left + \frac{\ell(w)}{\ell(v)-1} \cdot (\alpha_{\max} - \alpha_{\min})$

$preorder(w, t+1, left, right)$

$left \leftarrow right$

Runtime?

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

   $postorder(r)$

   $preorder(r, 0, 0, 2\pi)$

   return $(d_v, \alpha_v)_{v \in V(T)}$

   // vertex pos./polar coord.

postorder(vertex v)

   $\ell(v) \leftarrow 1$

   foreach child w of v **do**

      $postorder(w)$

      $\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

   $d_v \leftarrow \rho_t$

   $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$ //output

   if $t > 0$ **then**

      $\alpha_{\min} \leftarrow \max\{\alpha_{\min}, \alpha_v - \arccos \frac{\rho_t}{\rho_{t+1}}\}$

      $\alpha_{\max} \leftarrow \min\{\alpha_{\max}, \alpha_v + \arccos \frac{\rho_t}{\rho_{t+1}}\}$

   $left \leftarrow \alpha_{\min}$

   foreach child w of v **do**

      $right \leftarrow left + \frac{\ell(w)}{\ell(v)-1} \cdot (\alpha_{\max} - \alpha_{\min})$

      $preorder(w, t+1, left, right)$

      $left \leftarrow right$

Runtime? $\mathcal{O}(n)$

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \dots < \rho_k$)

begin

 $postorder(r)$

 $preorder(r, 0, 0, 2\pi)$

 return $(d_v, \alpha_v)_{v \in V(T)}$

 // vertex pos./polar coord.

postorder(vertex v)

 $\ell(v) \leftarrow 1$

 foreach child w of v **do**

  $postorder(w)$

  $\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

 $d_v \leftarrow \rho_t$

 $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$ //output

 if $t > 0$ **then**

  $\alpha_{\min} \leftarrow \max\{\alpha_{\min}, \alpha_v - \arccos \frac{\rho_t}{\rho_{t+1}}\}$

  $\alpha_{\max} \leftarrow \min\{\alpha_{\max}, \alpha_v + \arccos \frac{\rho_t}{\rho_{t+1}}\}$

 $left \leftarrow \alpha_{\min}$

 foreach child w of v **do**

  $right \leftarrow left + \frac{\ell(w)}{\ell(v)-1} \cdot (\alpha_{\max} - \alpha_{\min})$

  $preorder(w, t+1, left, right)$

  $left \leftarrow right$

Runtime? $\mathcal{O}(n)$

Correctness?

Radial Layouts – Pseudocode

RadialTreeLayout(tree T, root $r \in T$, radii $\rho_1 < \cdots < \rho_k$)

begin

$\quad$ $postorder(r)$

$\quad$ $preorder(r, 0, 0, 2\pi)$

$\quad$ **return** $(d_v, \alpha_v)_{v \in V(T)}$

$\quad$ // vertex pos./polar coord.

postorder(vertex v)

$\quad$ $\ell(v) \leftarrow 1$

$\quad$ **foreach** child w of v **do**

$\quad\quad$ $postorder(w)$

$\quad\quad$ $\ell(v) \leftarrow \ell(v) + \ell(w)$

preorder(vertex v, $t, \alpha_{\min}, \alpha_{\max}$)

$\quad$ $d_v \leftarrow \rho_t$

$\quad$ $\alpha_v \leftarrow (\alpha_{\min} + \alpha_{\max})/2$ $\quad$ *//output*

$\quad$ **if** $t > 0$ **then**

$\quad\quad$ $\alpha_{\min} \leftarrow \max\{\alpha_{\min}, \alpha_v - \arccos \frac{\rho_t}{\rho_{t+1}}\}$

$\quad\quad$ $\alpha_{\max} \leftarrow \min\{\alpha_{\max}, \alpha_v + \arccos \frac{\rho_t}{\rho_{t+1}}\}$

$\quad$ $left \leftarrow \alpha_{\min}$

$\quad$ **foreach** child w of v **do**

$\quad\quad$ $right \leftarrow left + \frac{\ell(w)}{\ell(v)-1} \cdot (\alpha_{\max} - \alpha_{\min})$

$\quad\quad$ $preorder(w, t+1, left, right)$

$\quad\quad$ $left \leftarrow right$

Runtime? $\mathcal{O}(n)$

Correctness? ✓

Radial Layouts – Result

Theorem.
Let T be a tree with n vertices. The RadialTreeLayout algorithm constructs in $O(n)$ time a drawing Γ of T s.t.:

Radial Layouts – Result

Theorem.
Let T be a tree with n vertices. The RadialTreeLayout algorithm constructs in $O(n)$ time a drawing Γ of T s.t.:

- Γ is radial drawing

Radial Layouts – Result

Theorem.
Let T be a tree with n vertices. The RadialTreeLayout algorithm constructs in $O(n)$ time a drawing Γ of T s.t.:

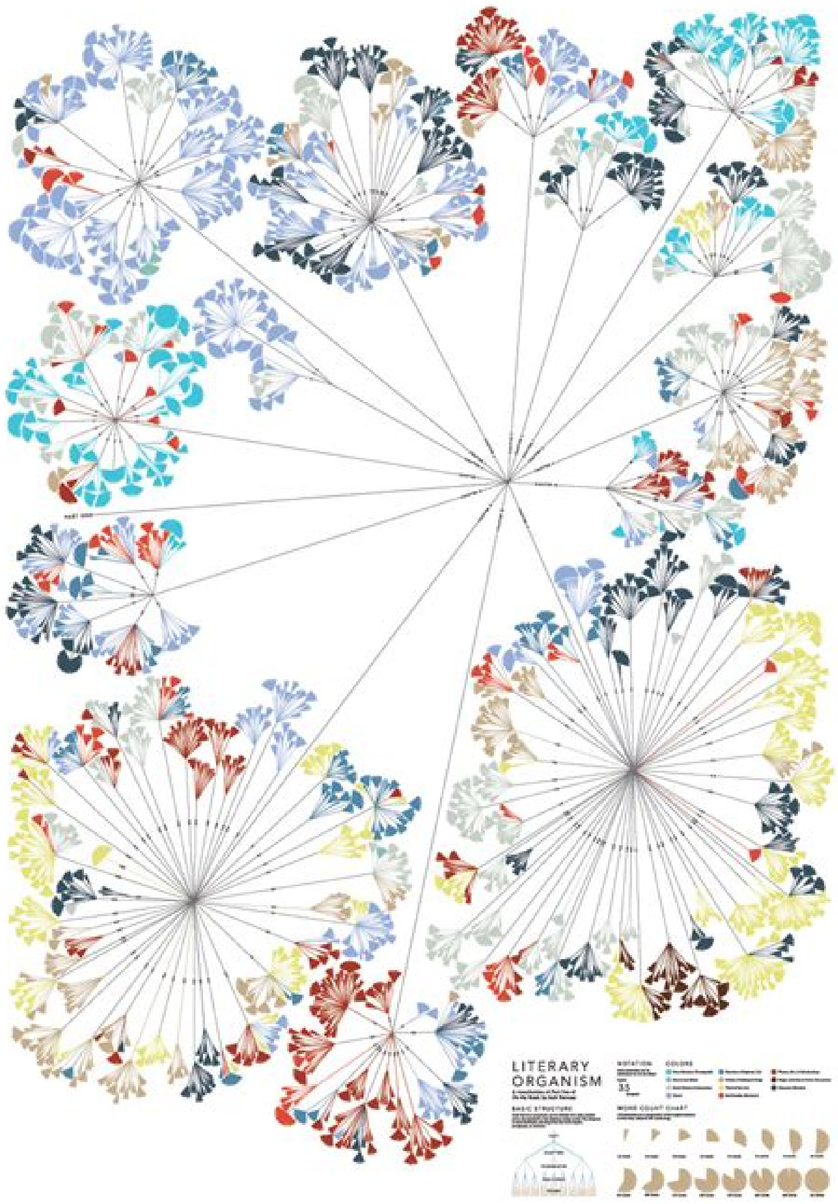
- Γ is radial drawing
- Vertices lie on circle according to their depth

Radial Layouts – Result

Theorem.
Let T be a tree with n vertices. The RadialTreeLayout algorithm constructs in $O(n)$ time a drawing Γ of T s.t.:

- Γ is radial drawing
- Vertices lie on circle according to their depth
- Area quadratic in max degree times height of T
  (see [GD Ch. 3.1.3] if interested)

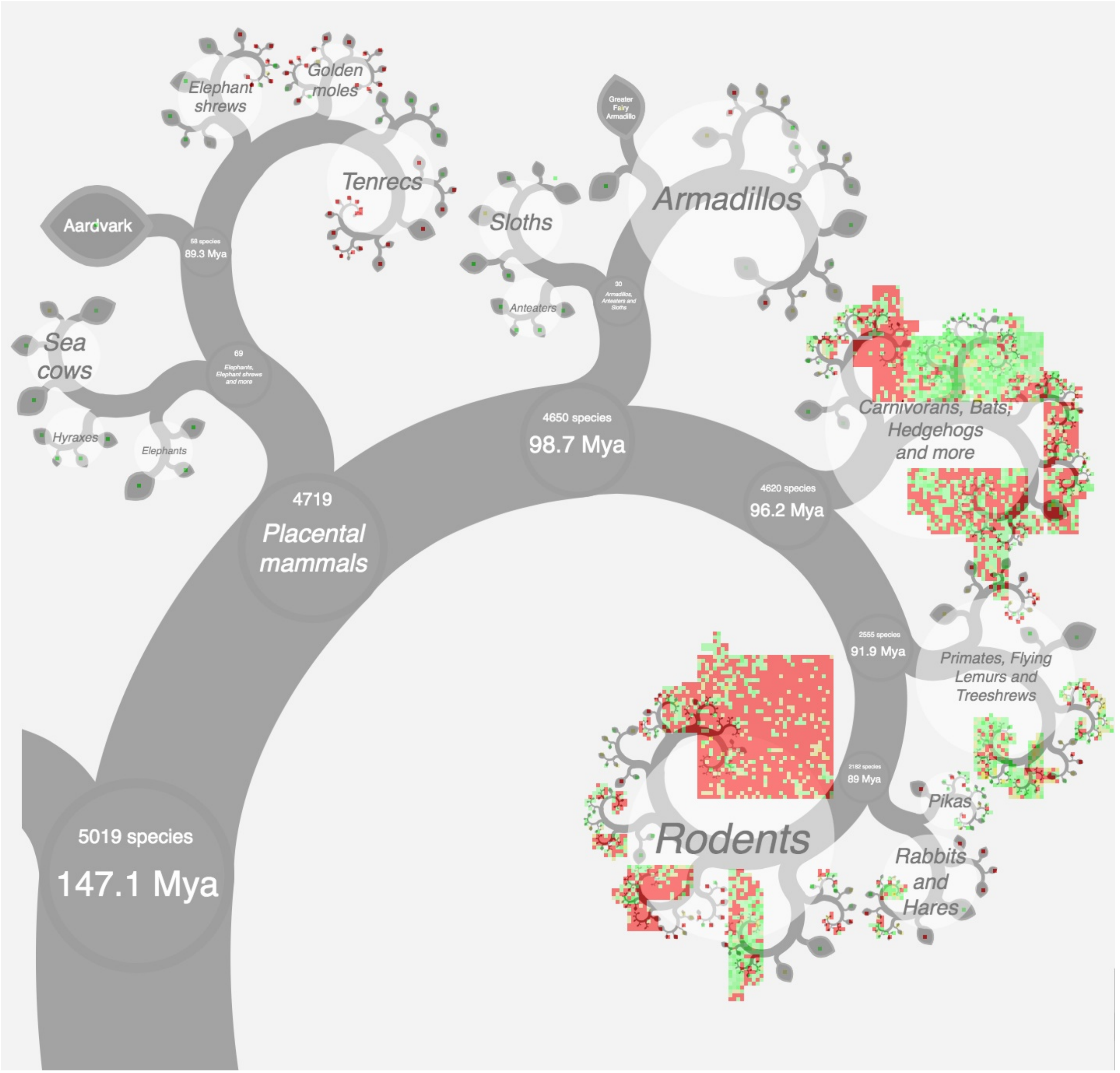
Other tree visualisation styles



Writing Without Words:
The project explores methods to visualises the differences in writing styles of different authors.

Similar to ballon layout

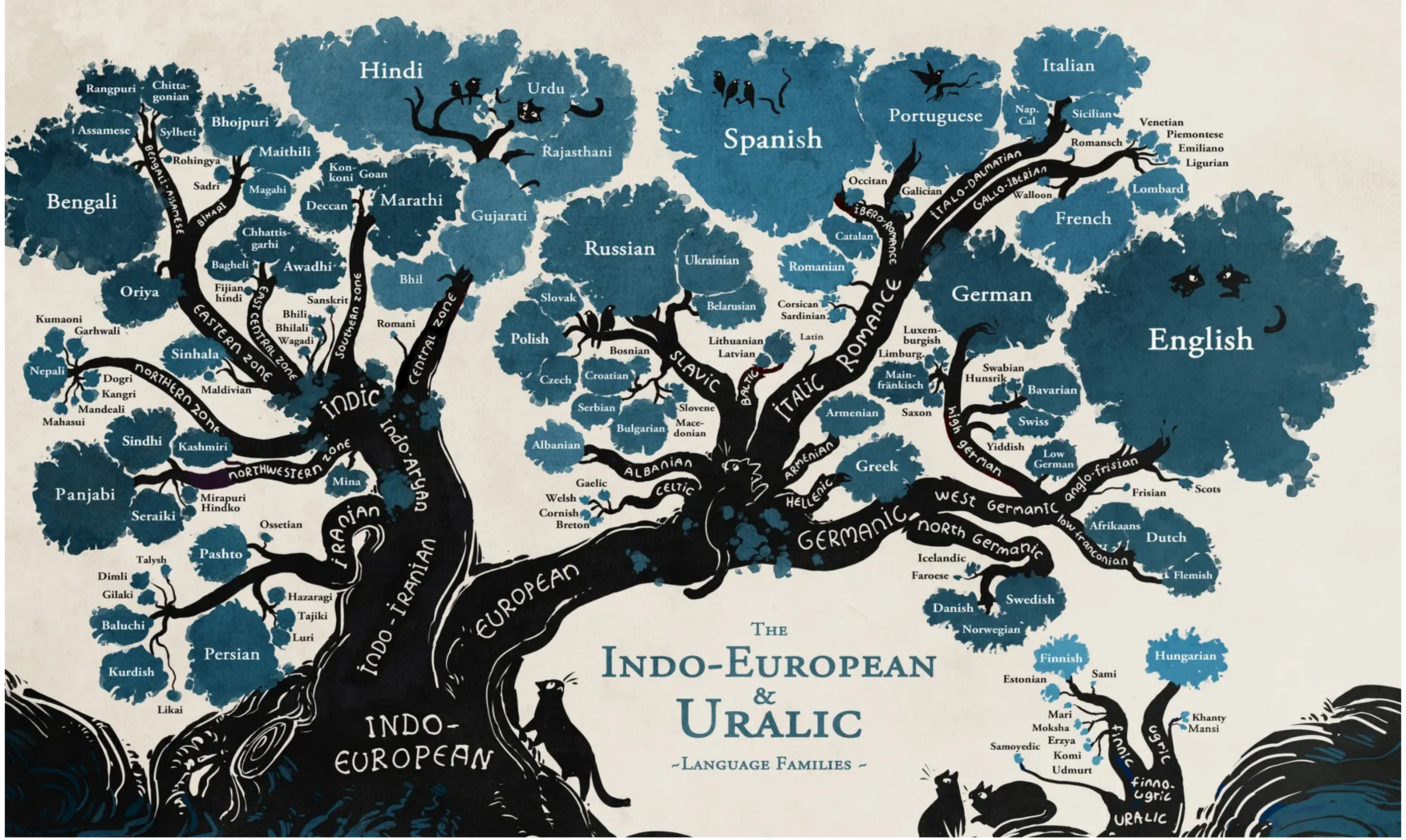
Other tree visualisation styles



A phylogenetically organised display of data for all placental mammal species.

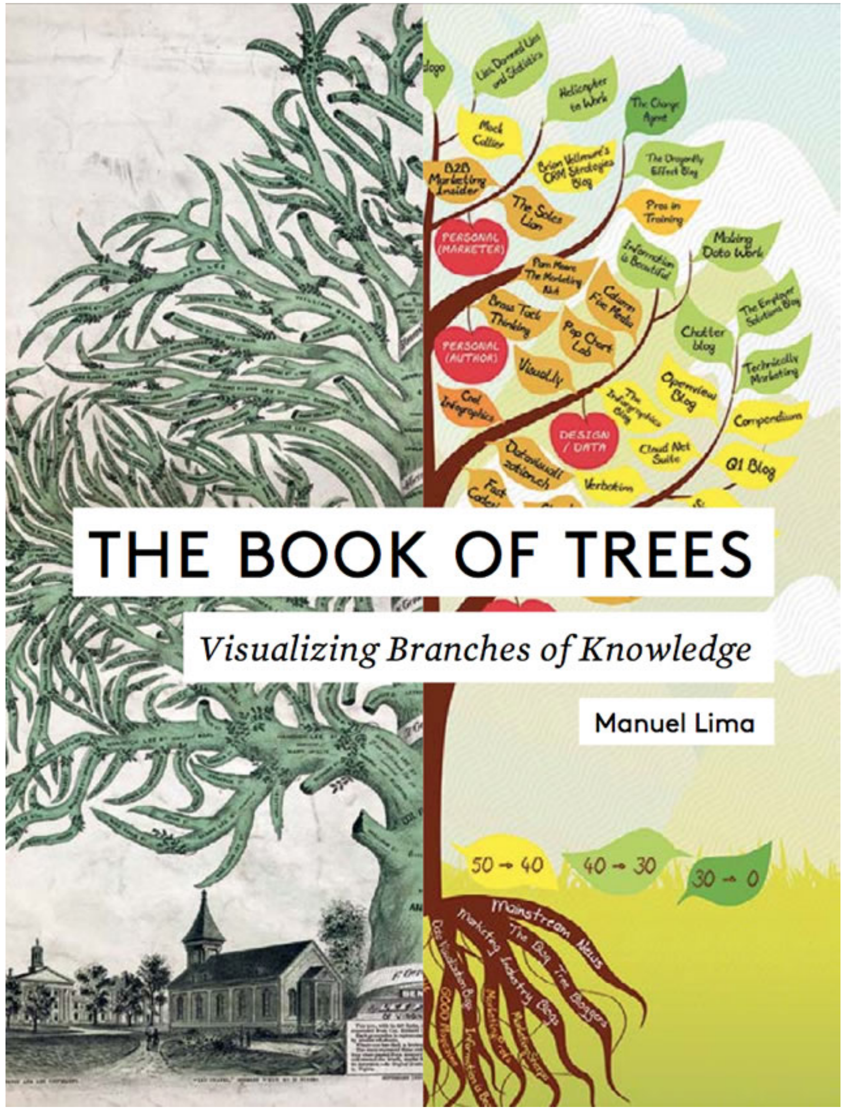
Fractal layout

Other tree visualisation styles

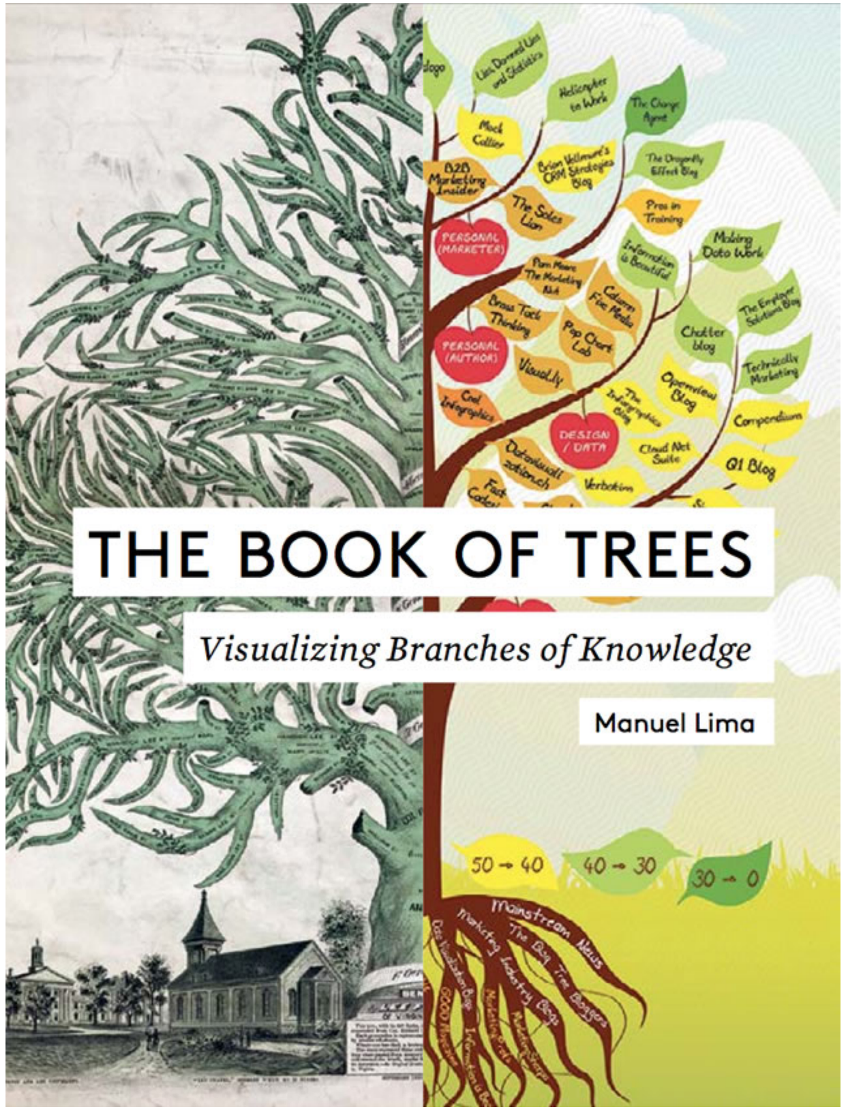


A language family tree – in pictures

Other tree visualisation styles



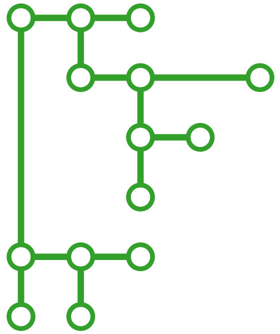
Other tree visualisation styles

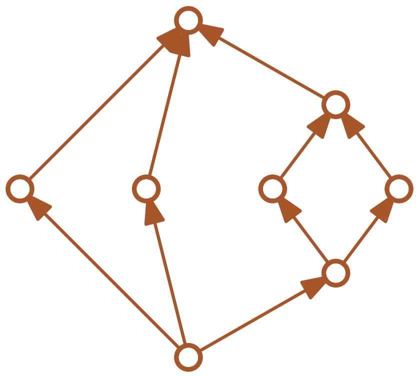


treevis.net

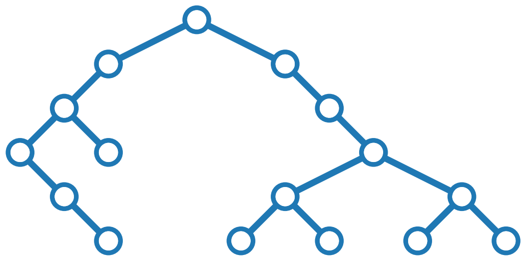
Visualization of Graphs

Lecture 1b:
Drawing Trees and Series-Parallel Graphs

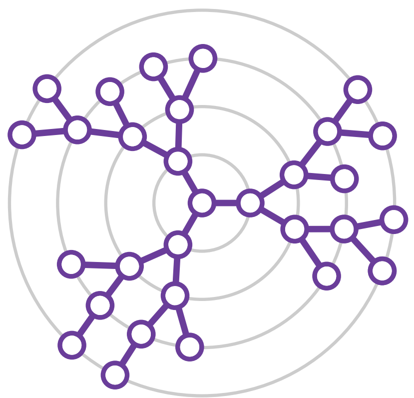




Part IV:
Series-Parallel Graphs



Jonathan Klawitter



Series-Parallel Graphs

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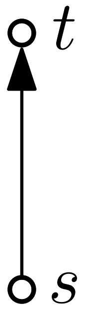

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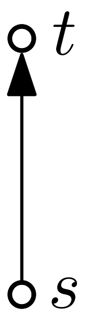

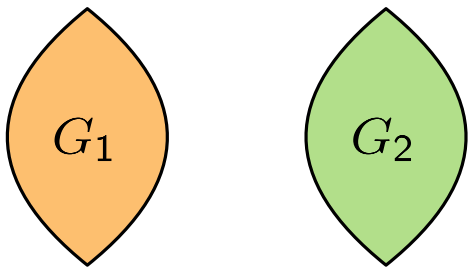

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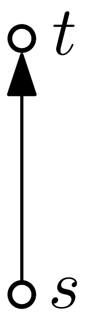

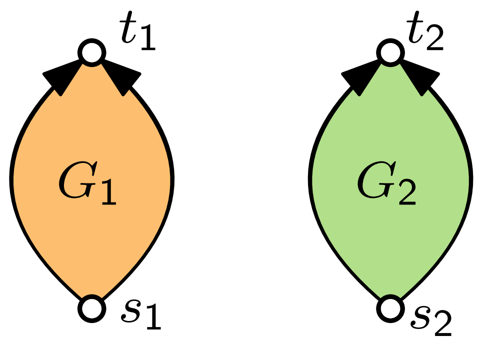

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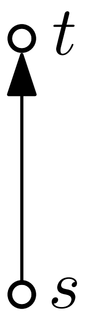

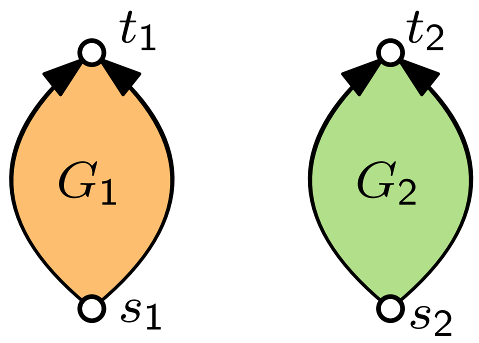

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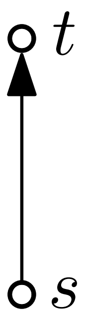
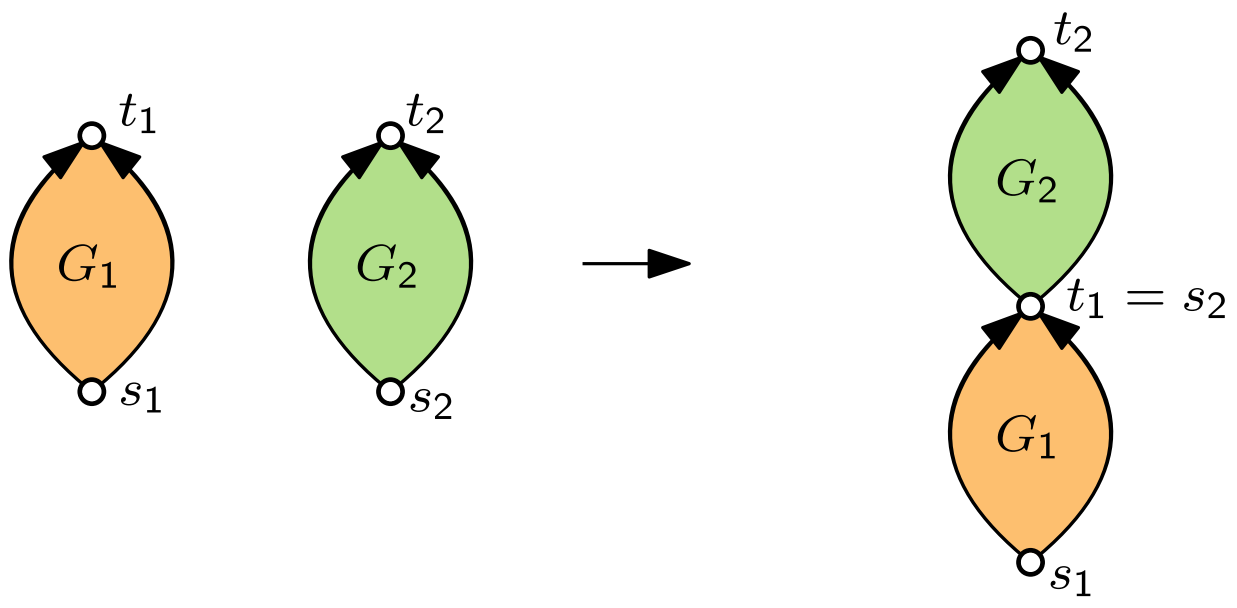


Series composition



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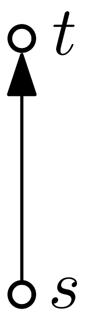
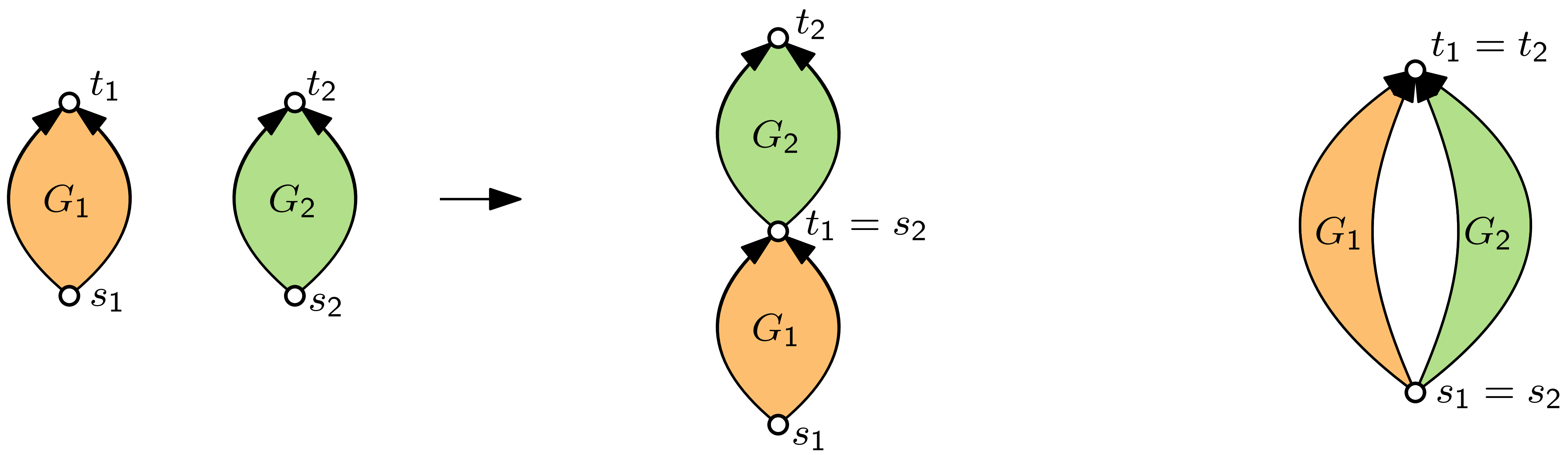


Series composition **Parallel composition**



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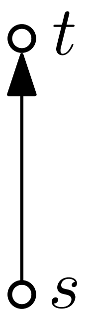
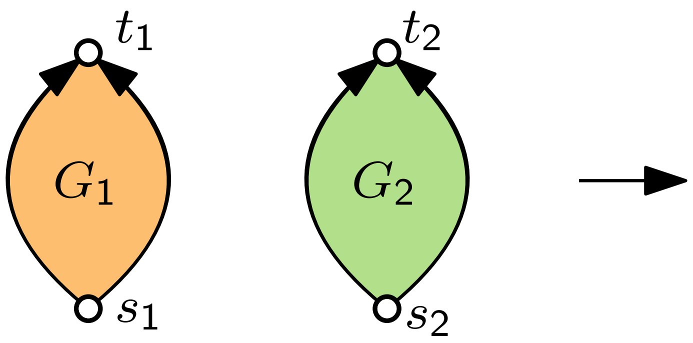
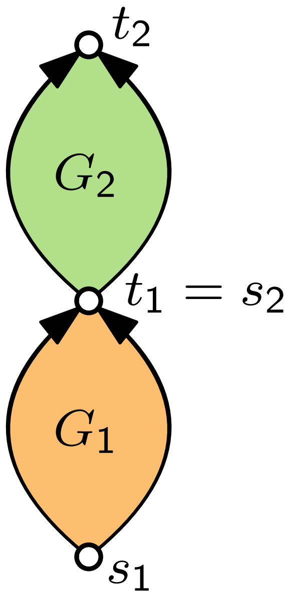


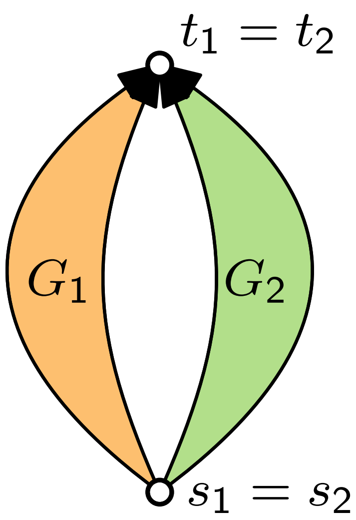
convince yourself that series-parallel graphs are planar



Series composition



Parallel composition



Series-Parallel Graphs – Decomposition Tree

A **decomposition tree** of G is a binary tree T with nodes of three types: **S**, **P** and **Q**-type

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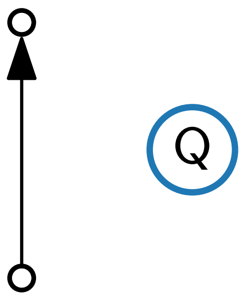

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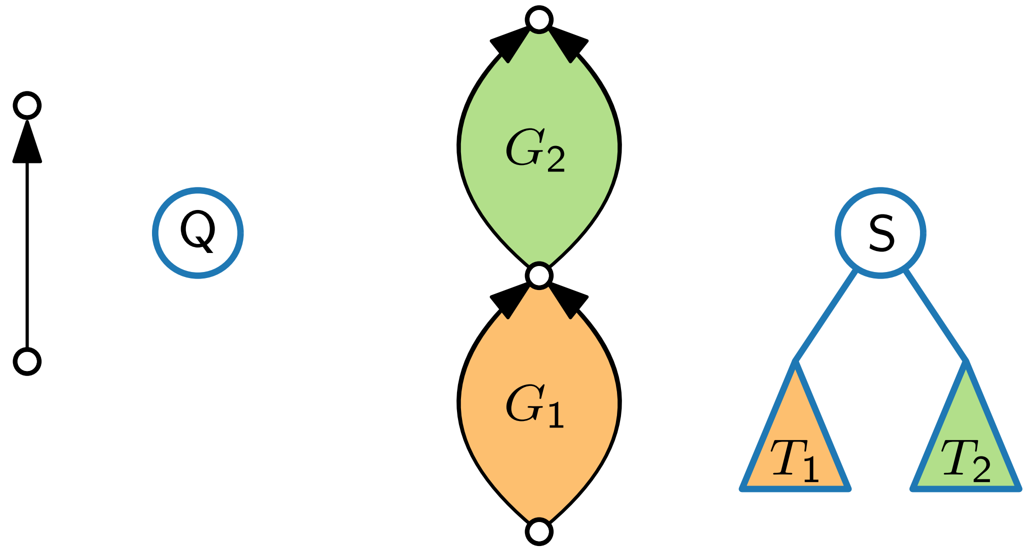

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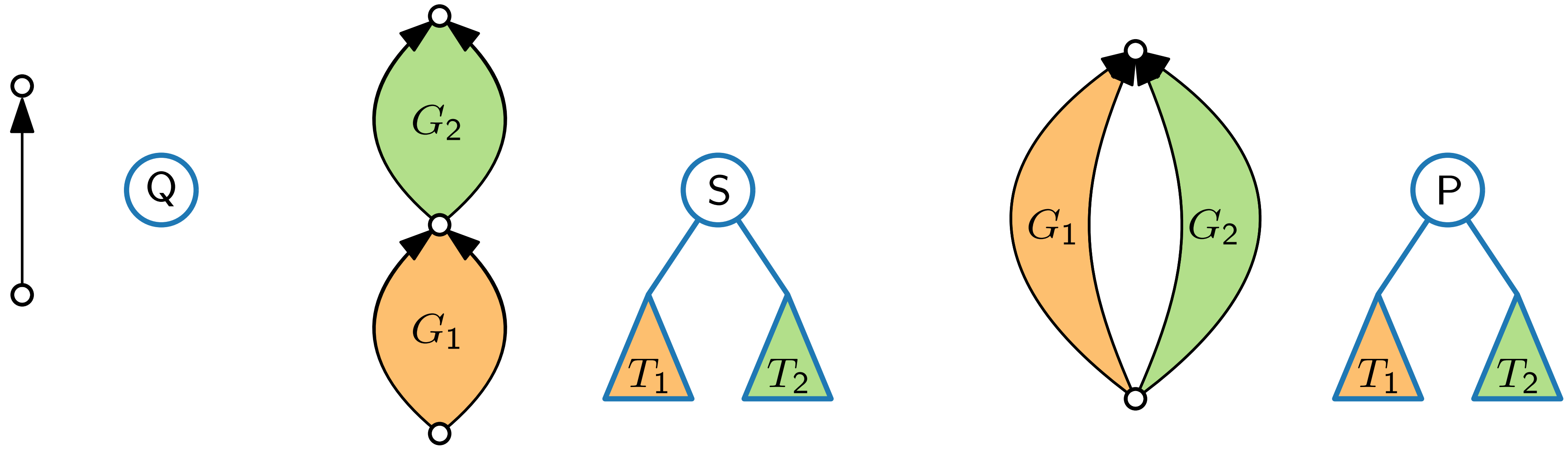
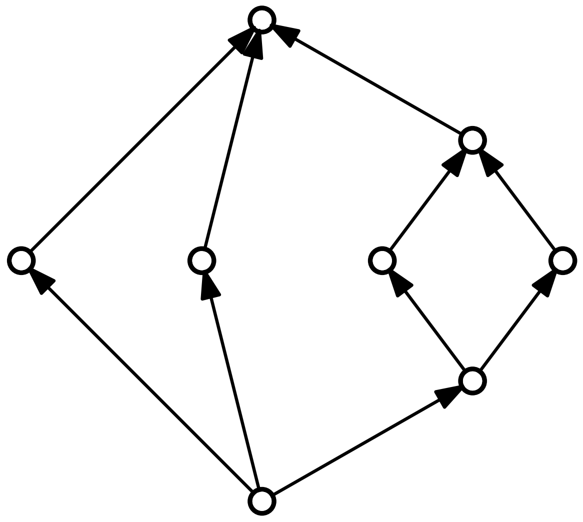
- A **P**-node represents a parallel composition; its children T_1 and T_2 represent G_1 and G_2



Series-Parallel Graphs – Decomposition Example



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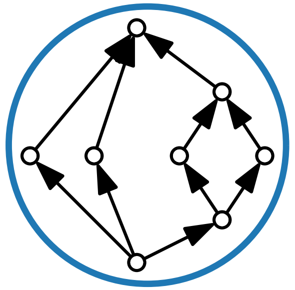

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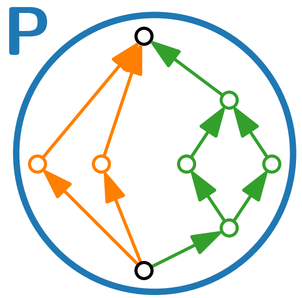

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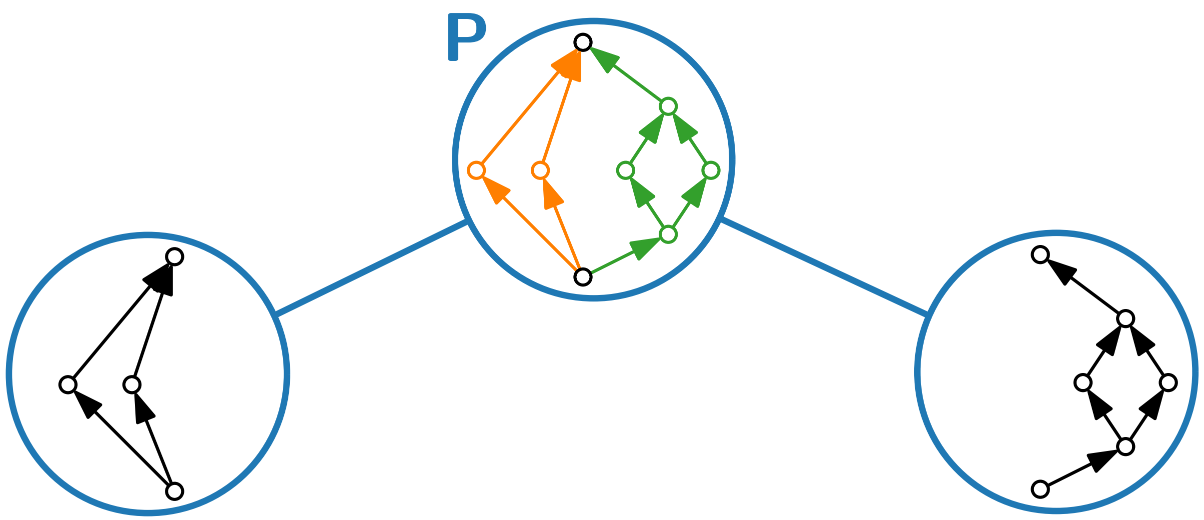

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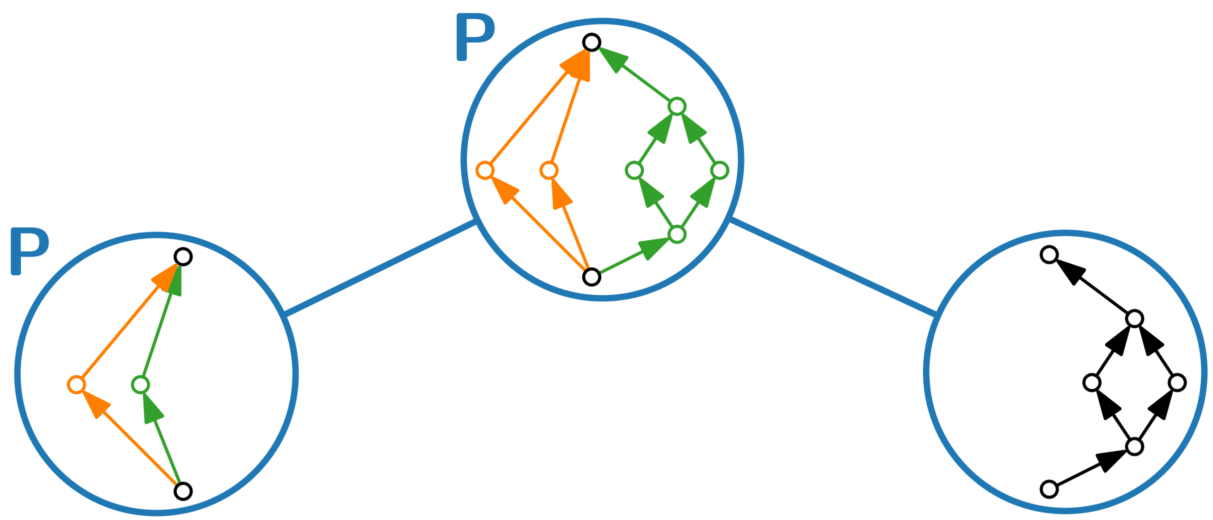

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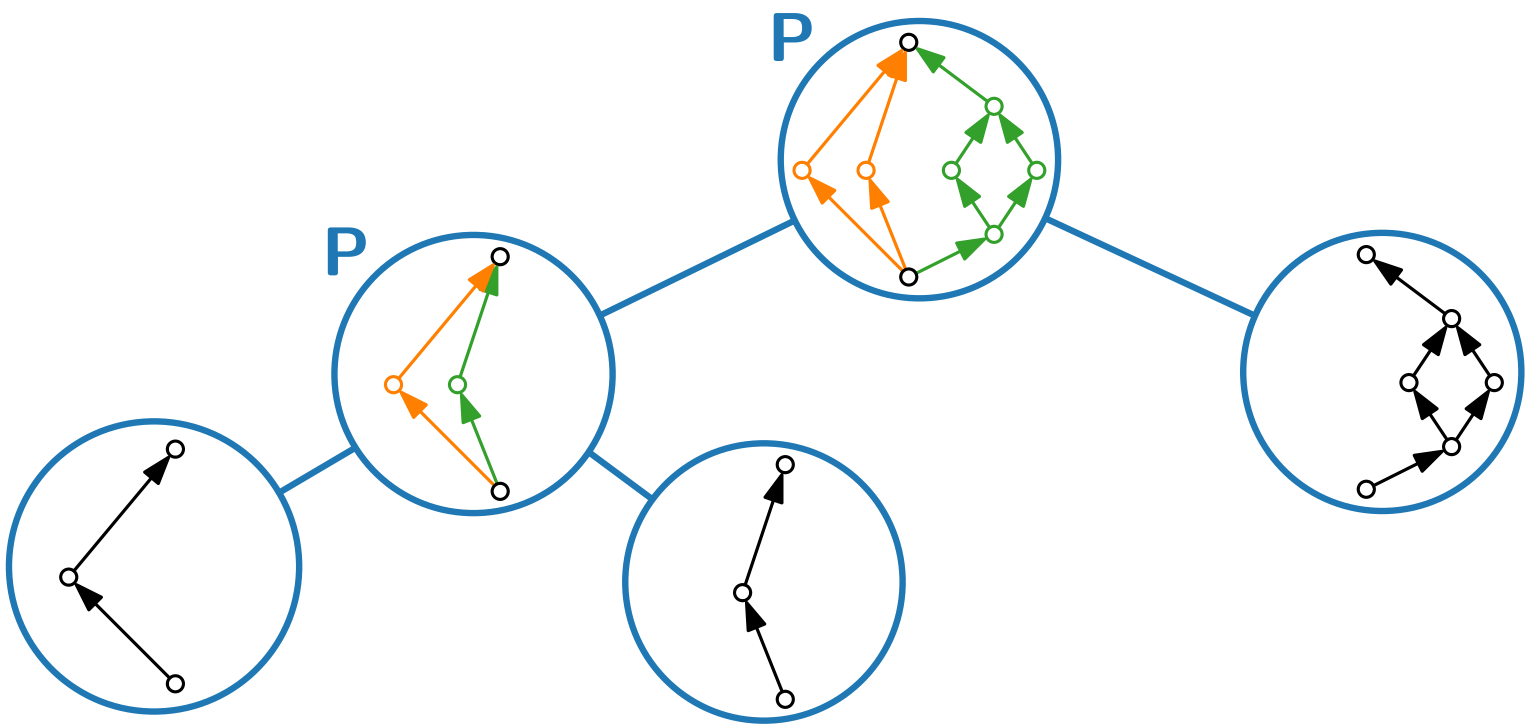

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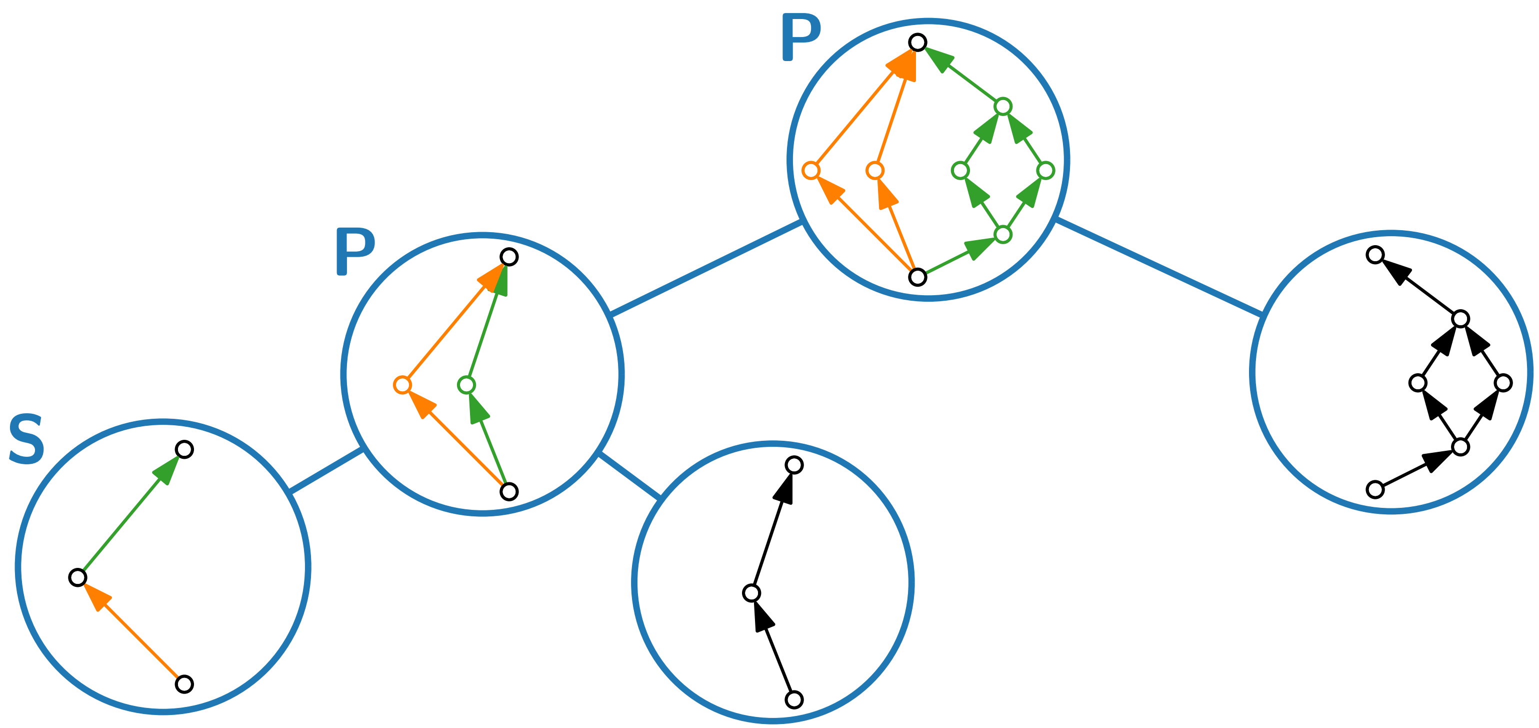

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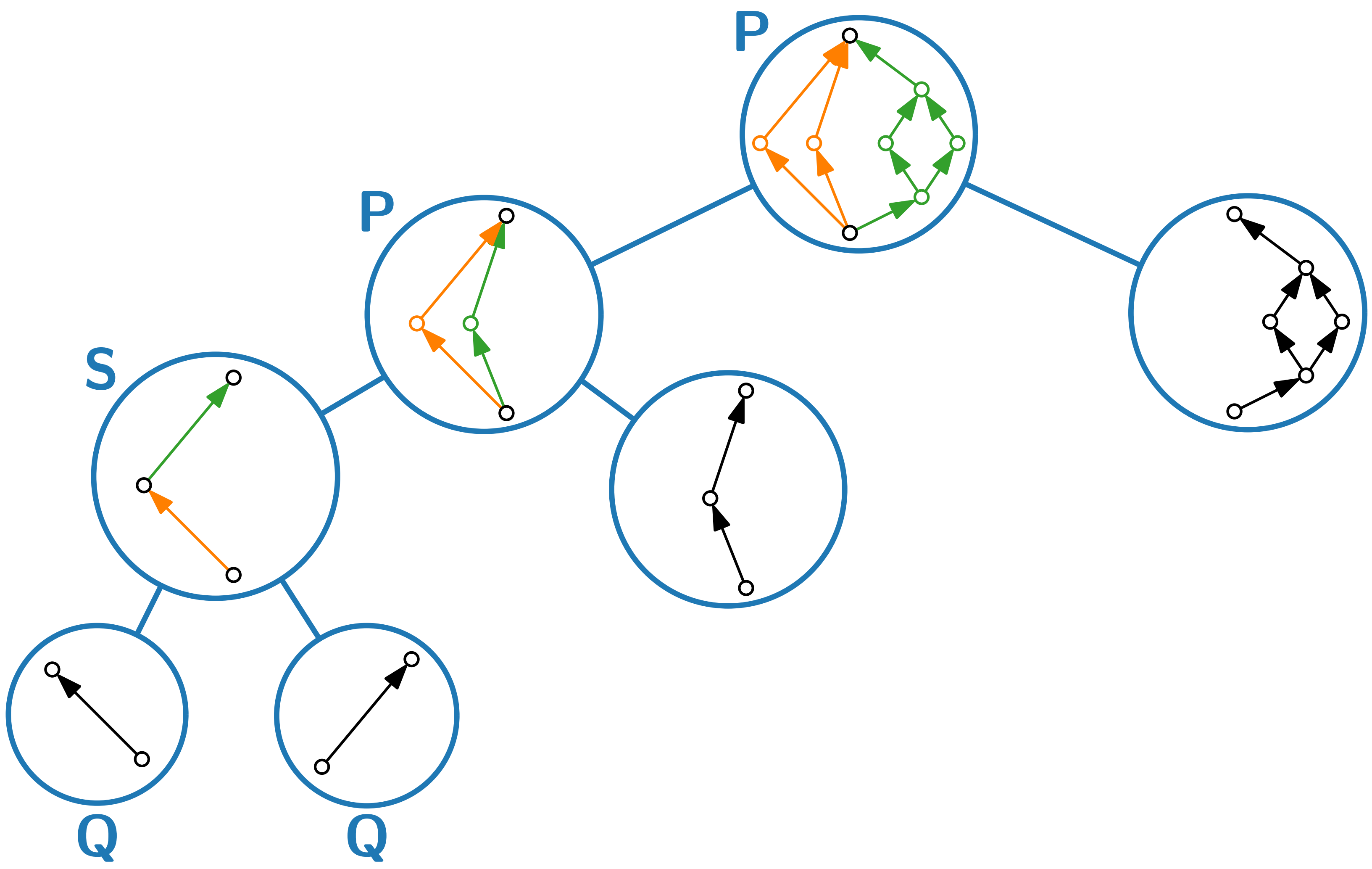

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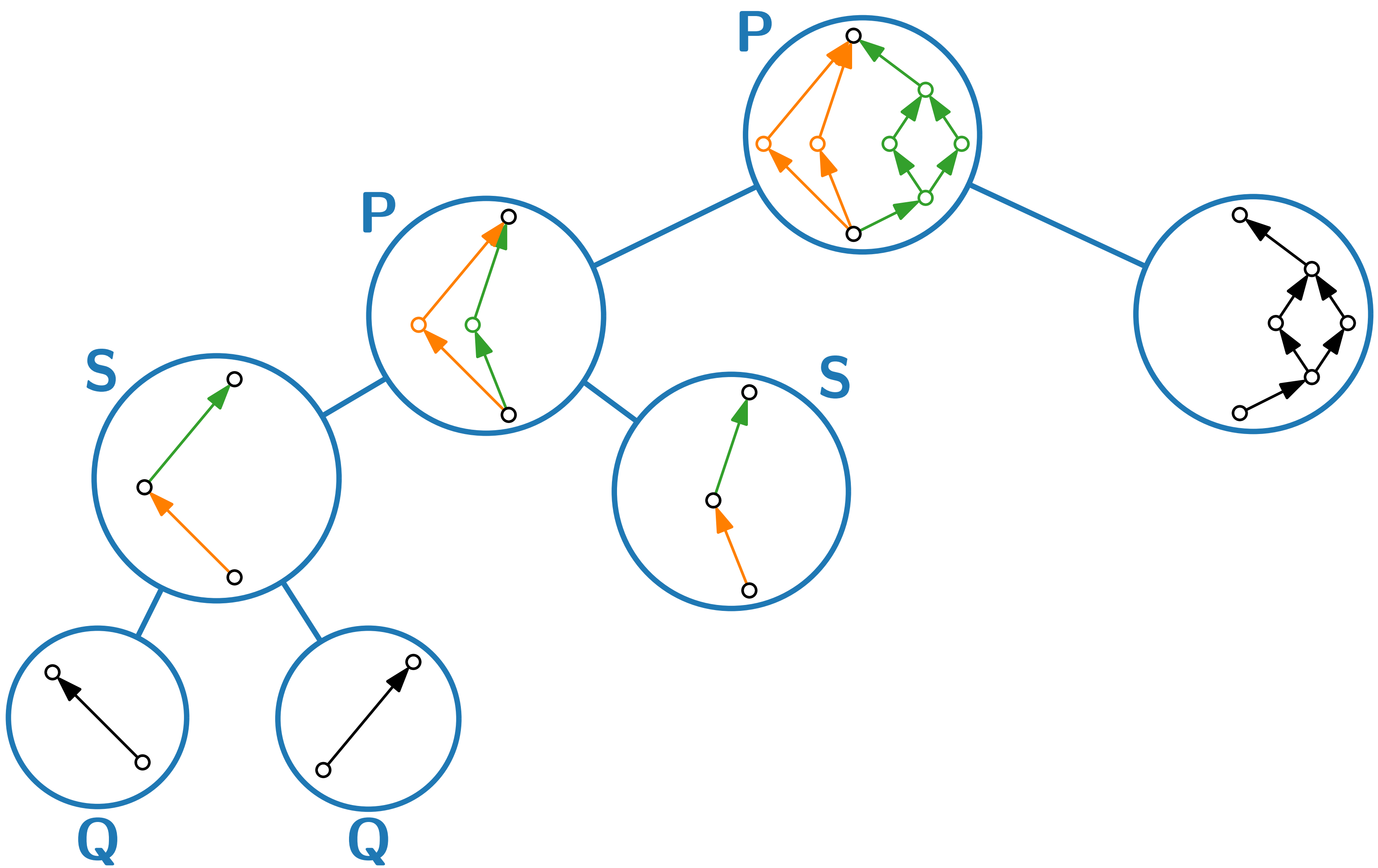

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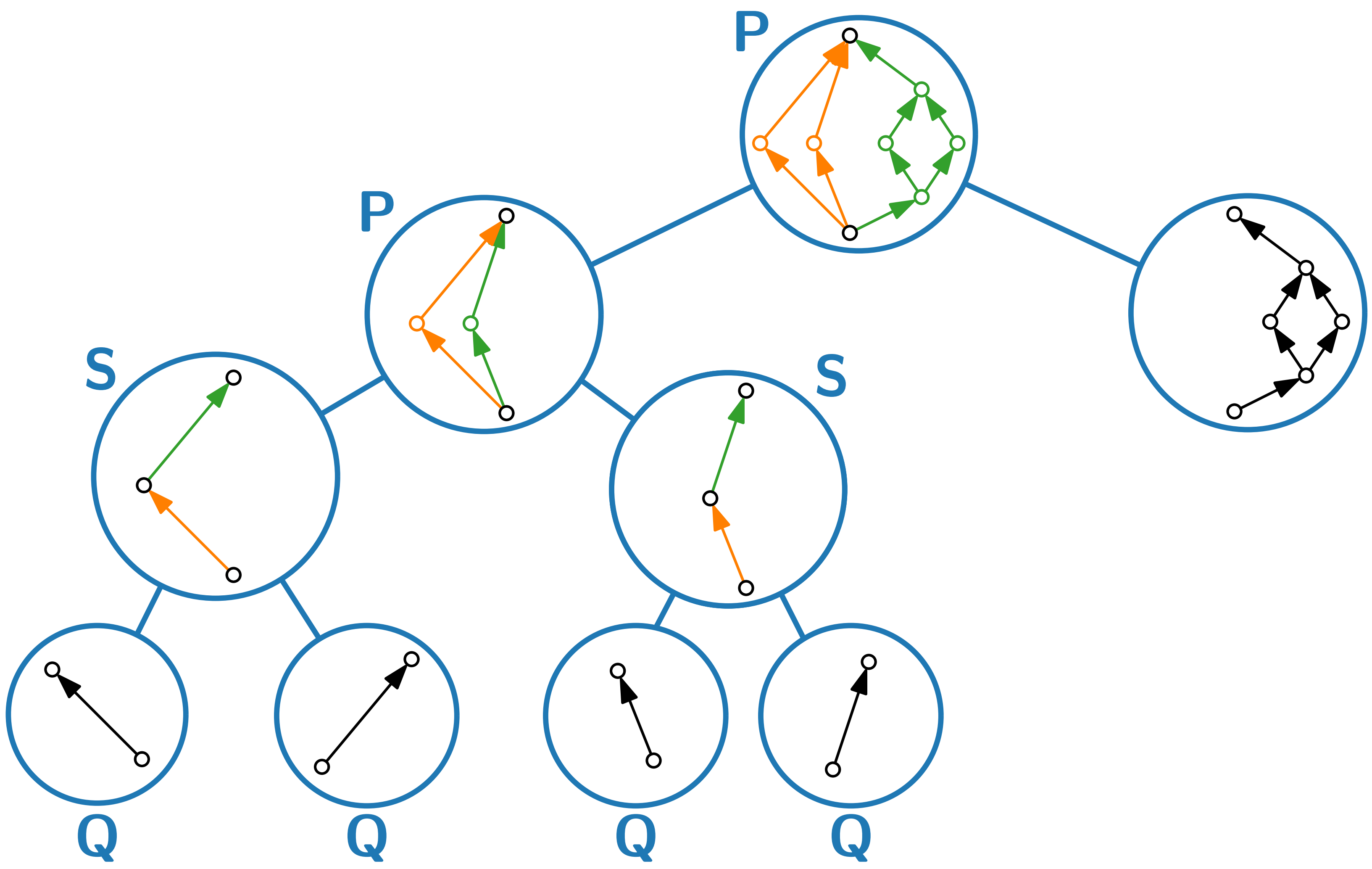

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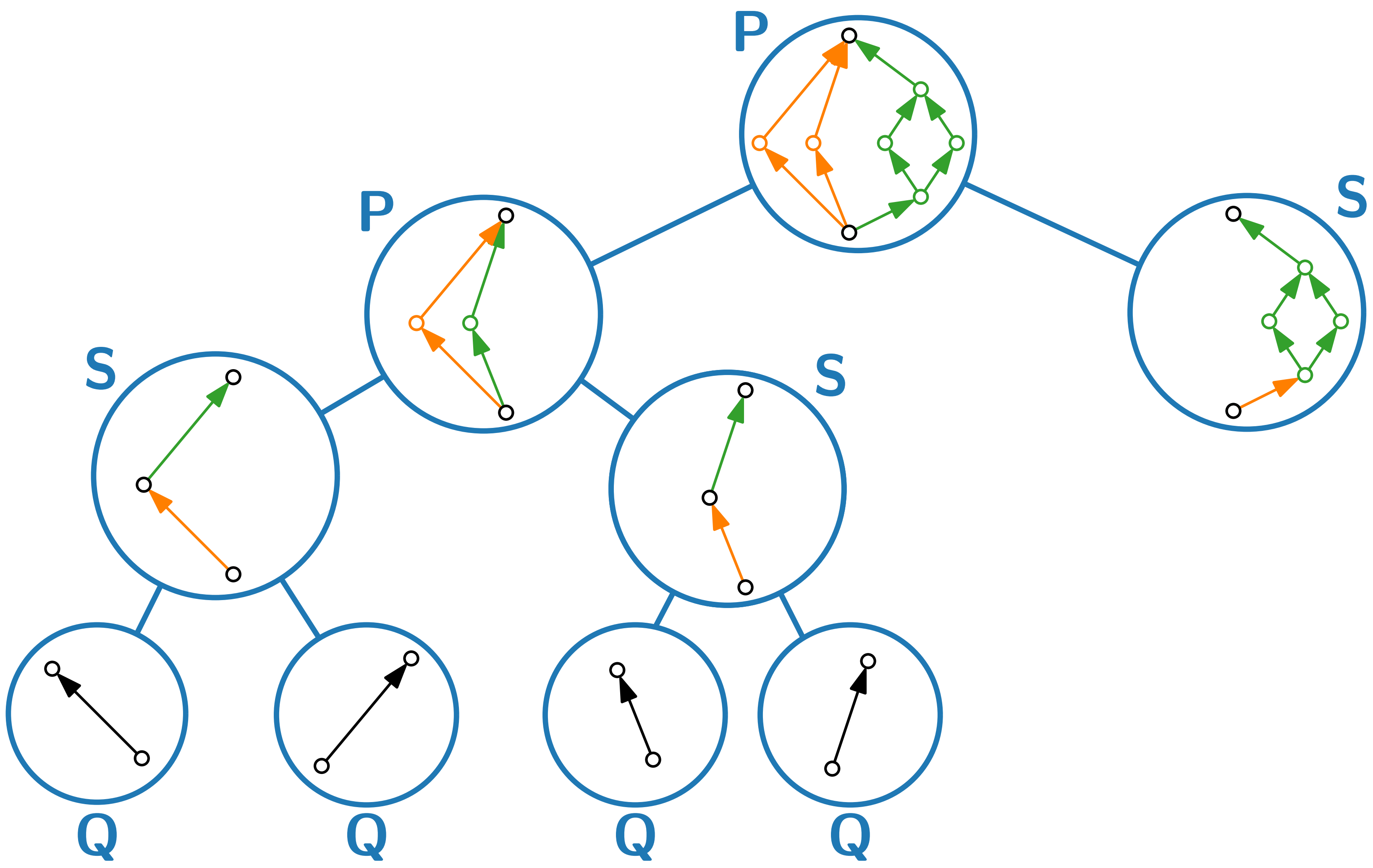

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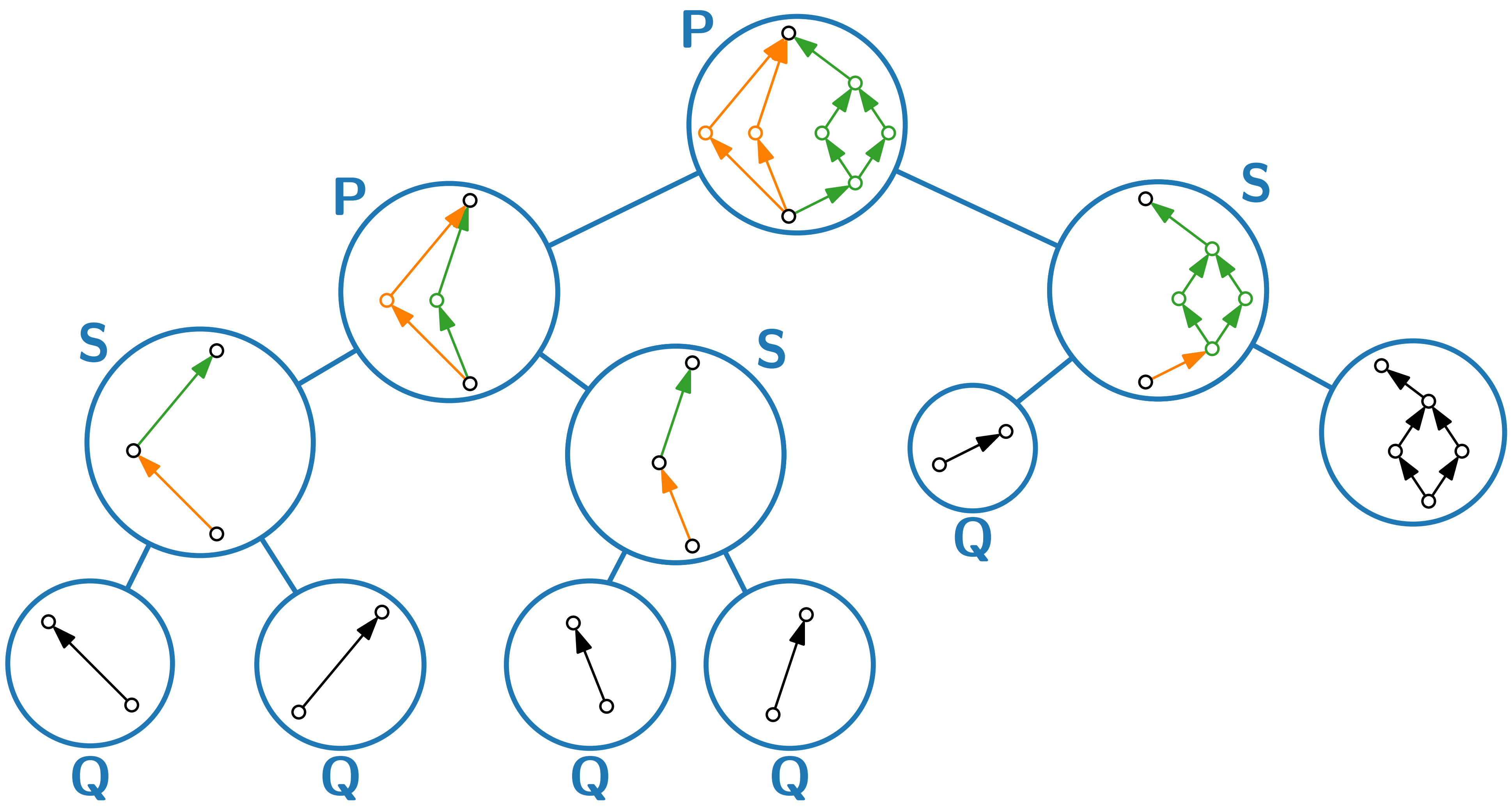

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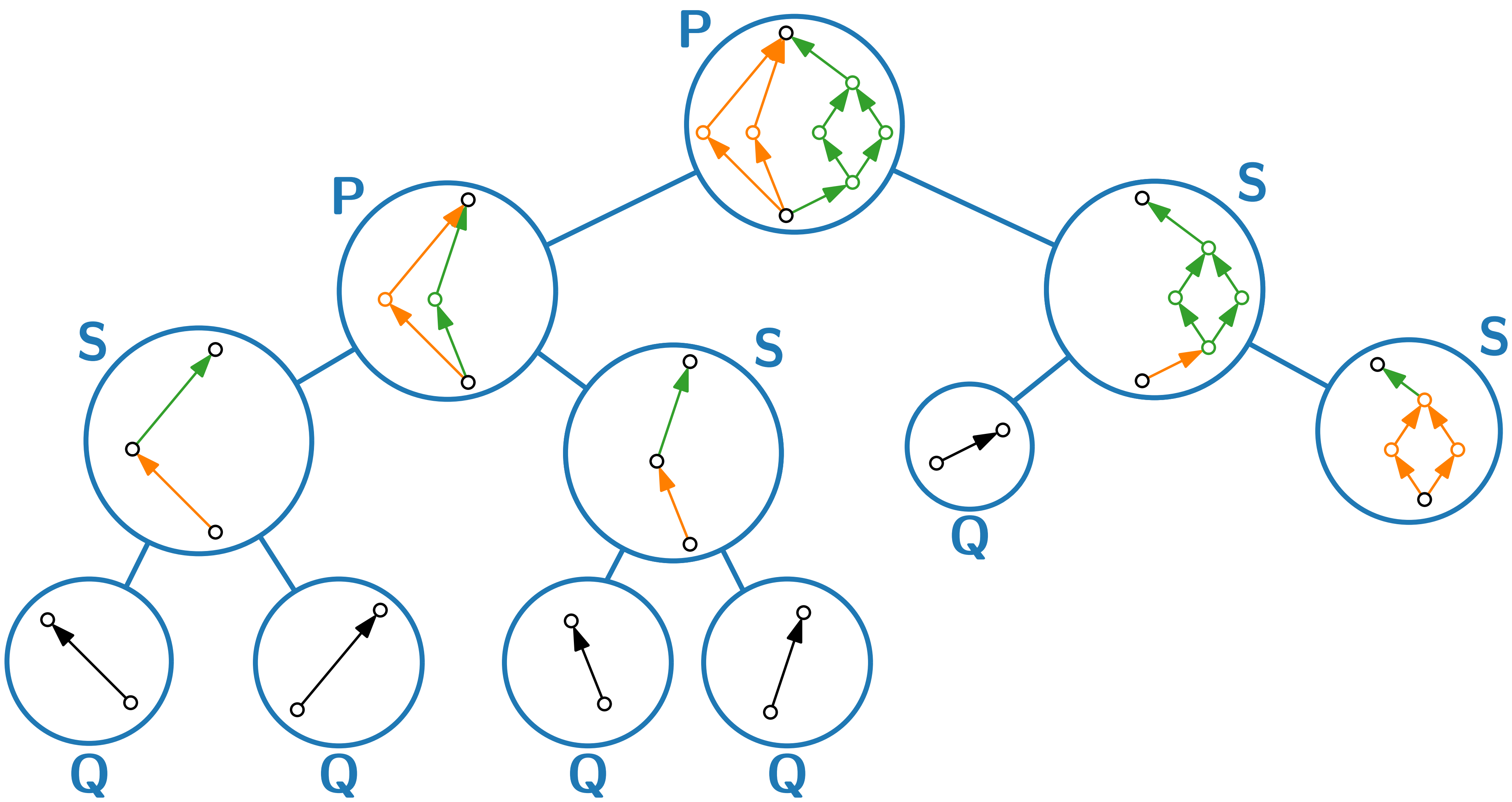

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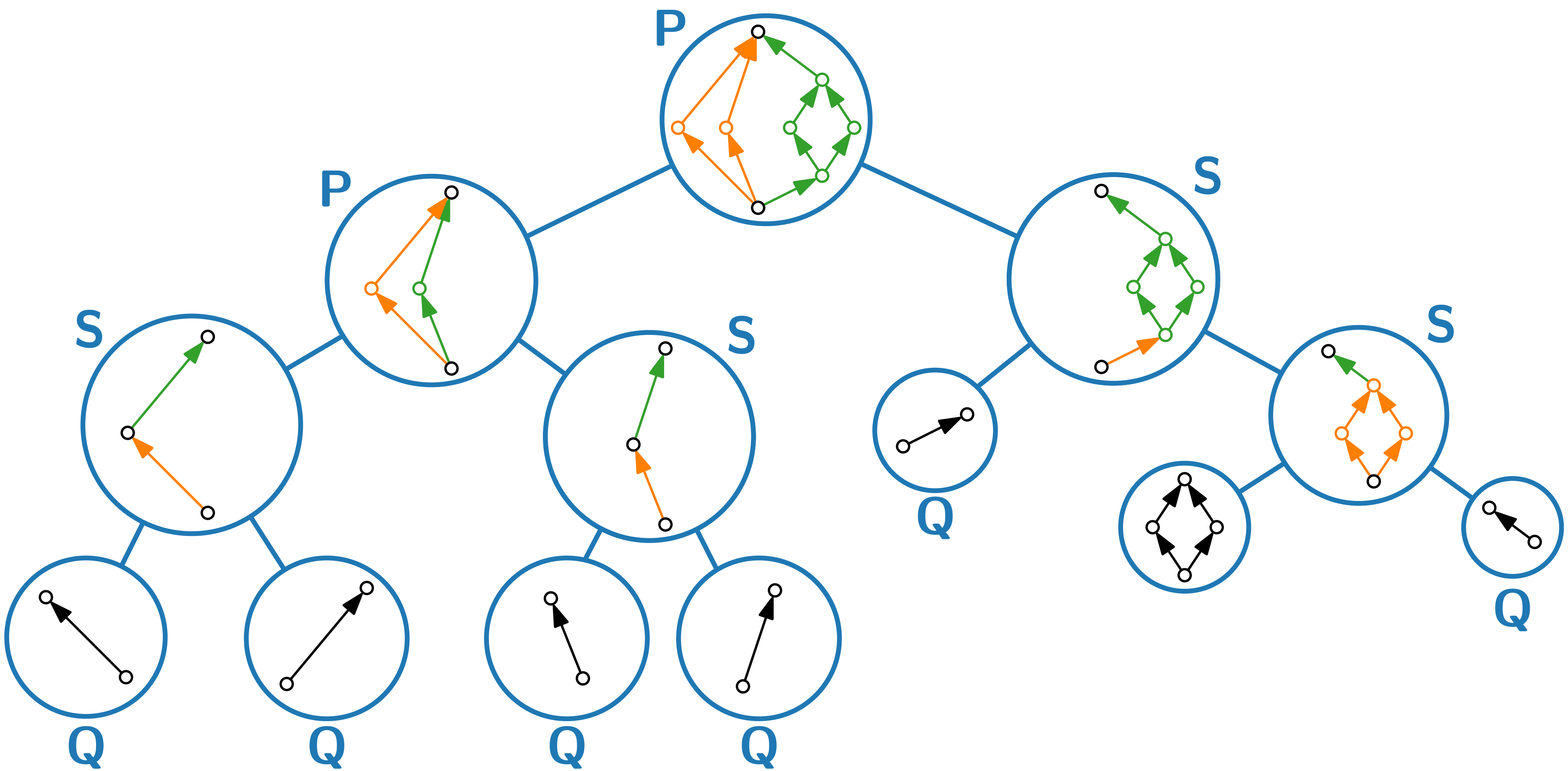

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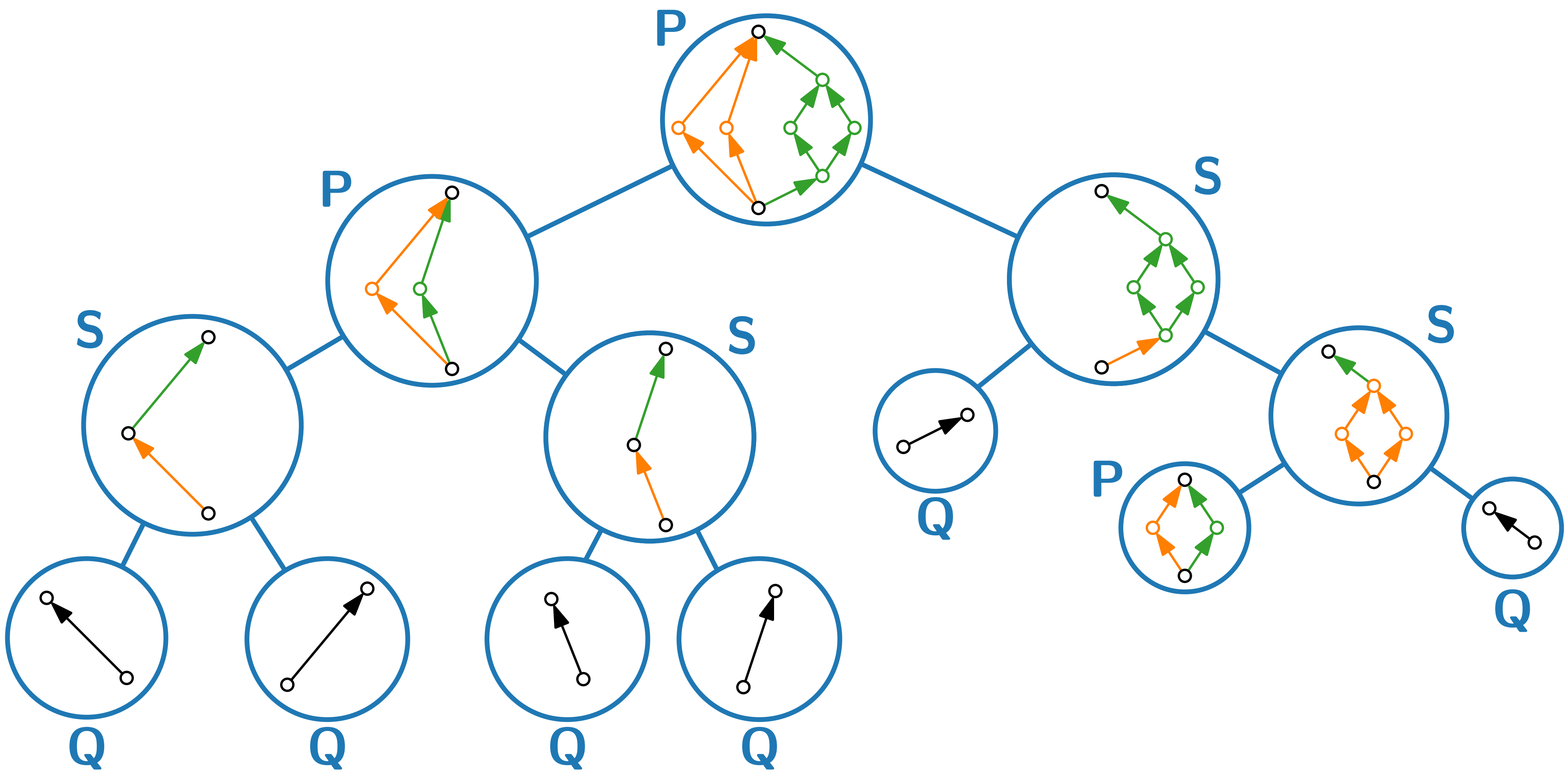

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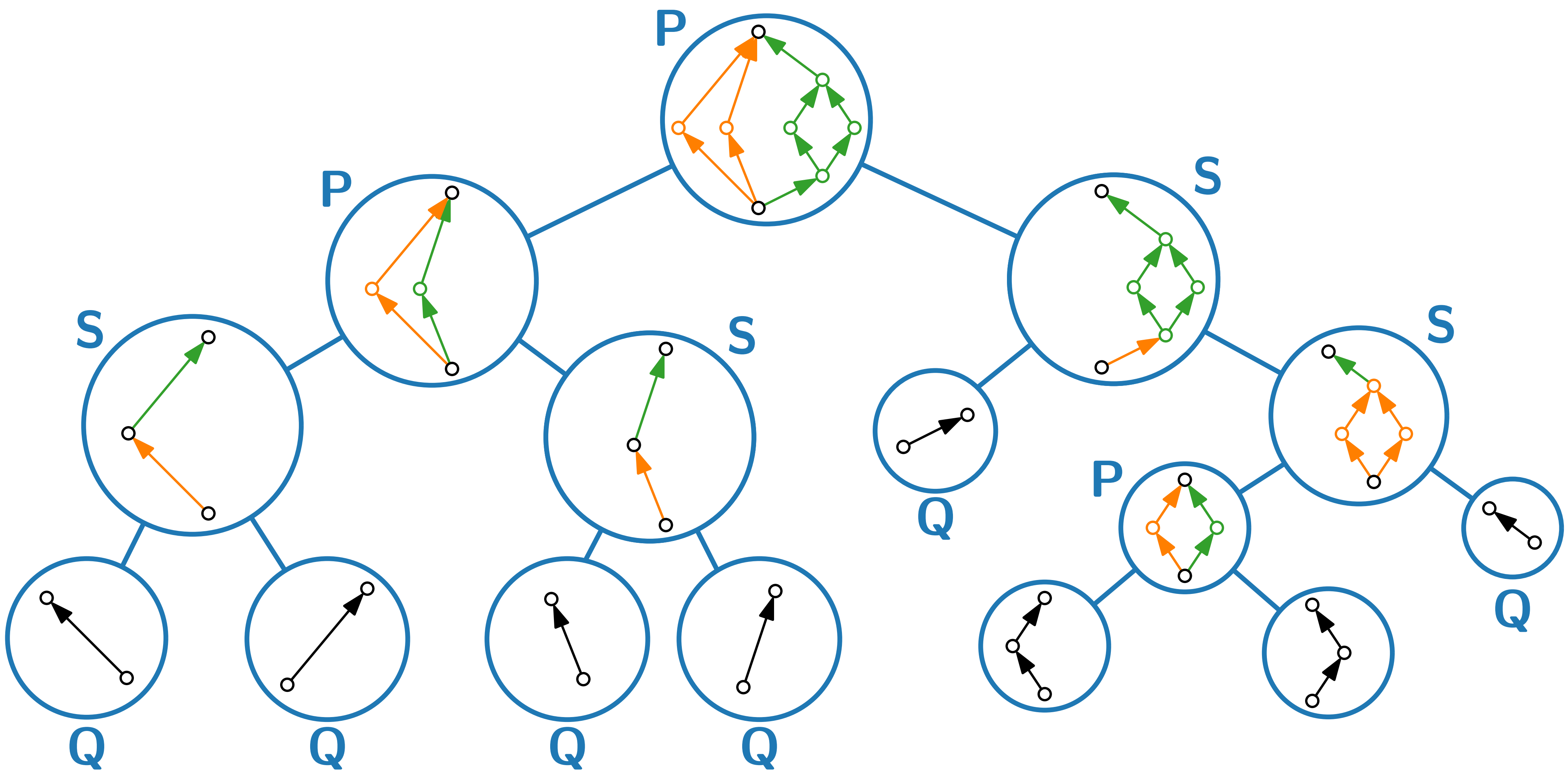

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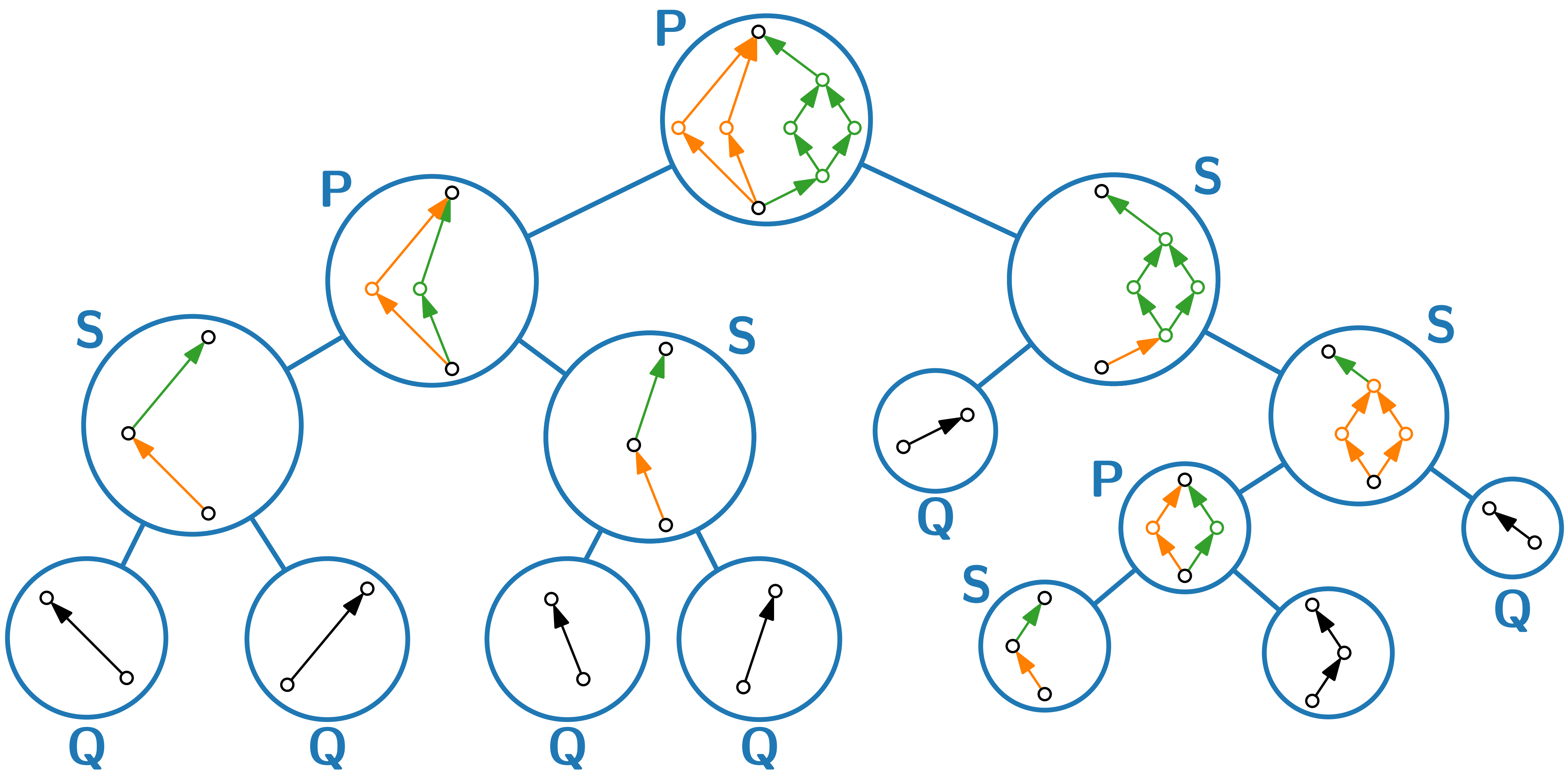

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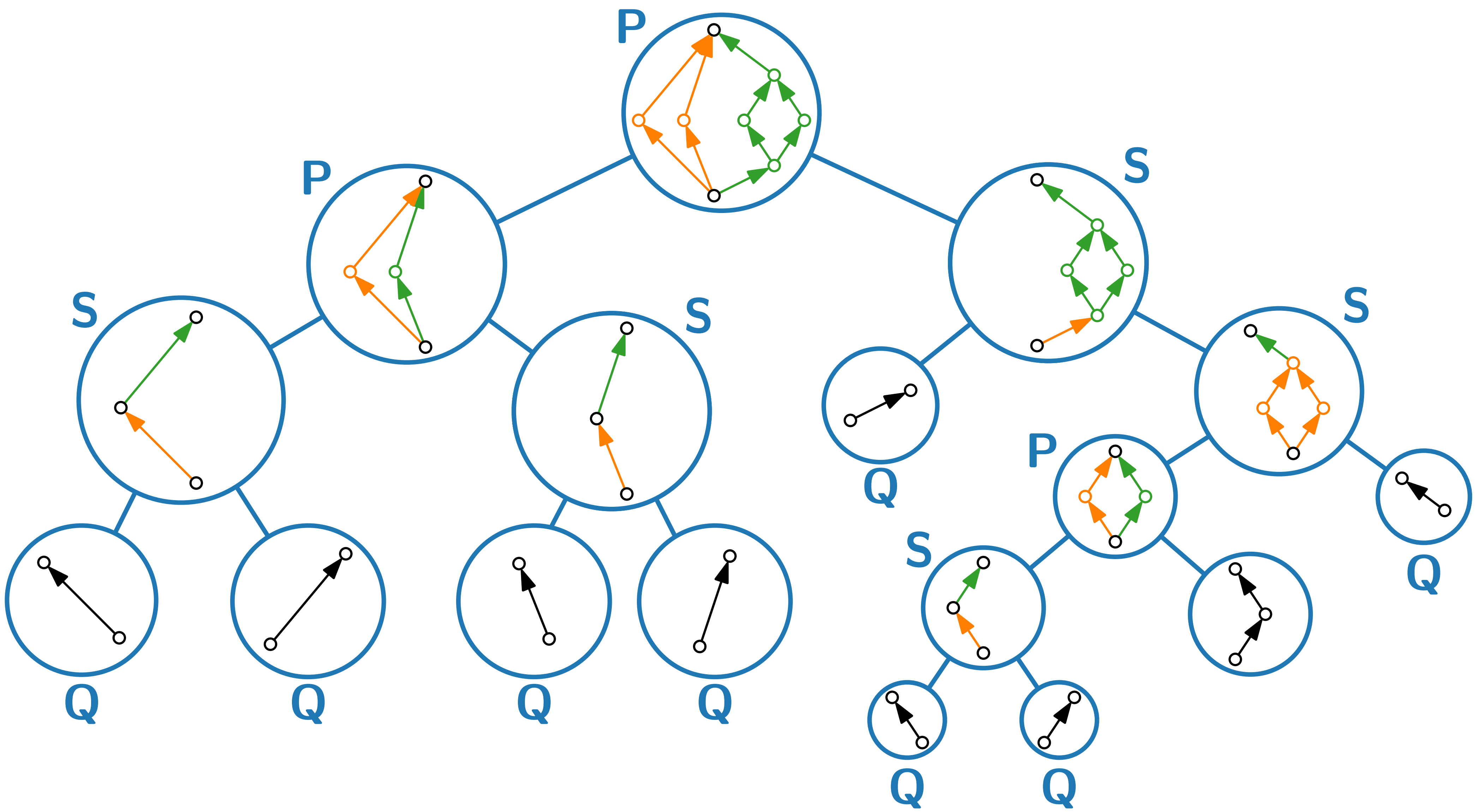

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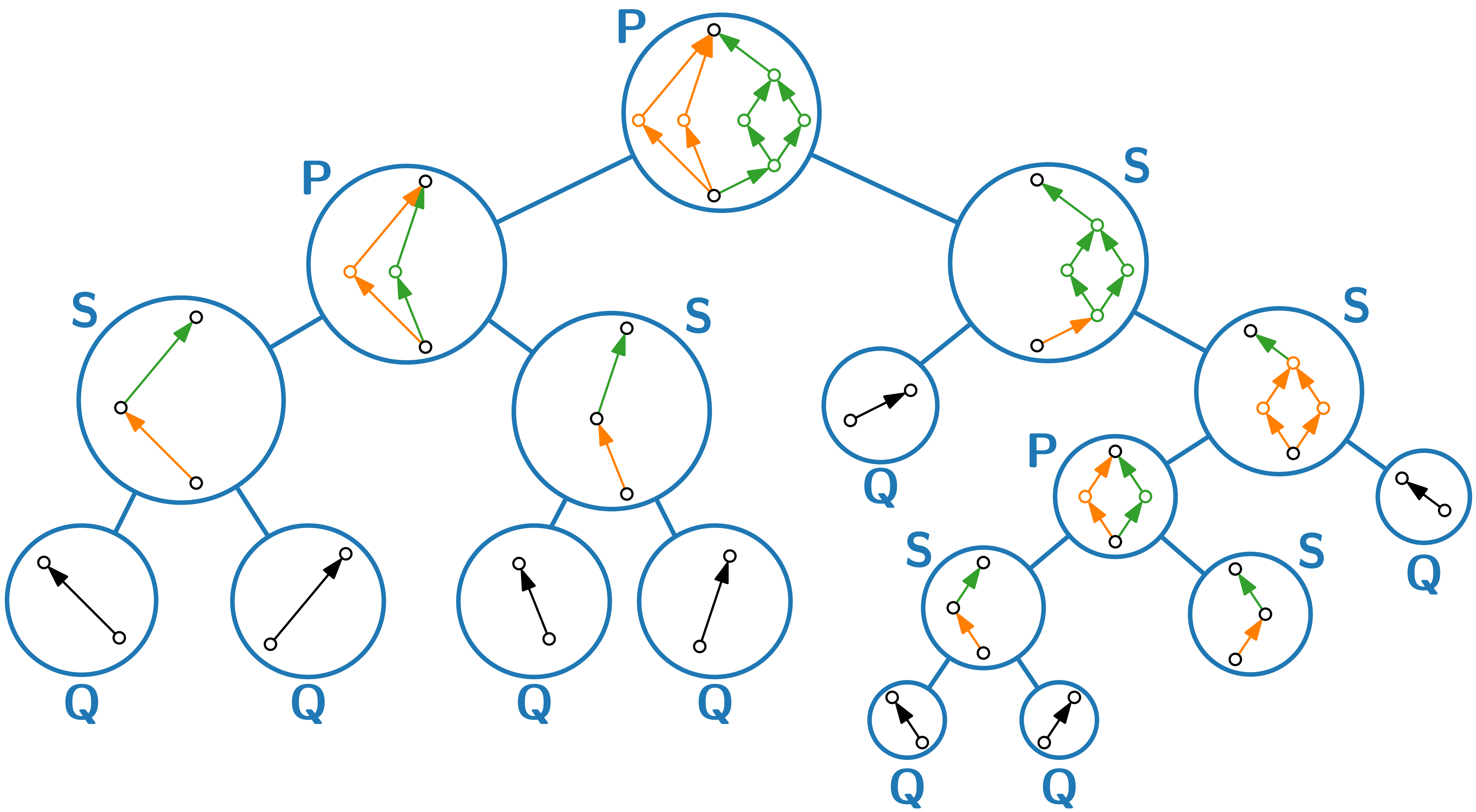

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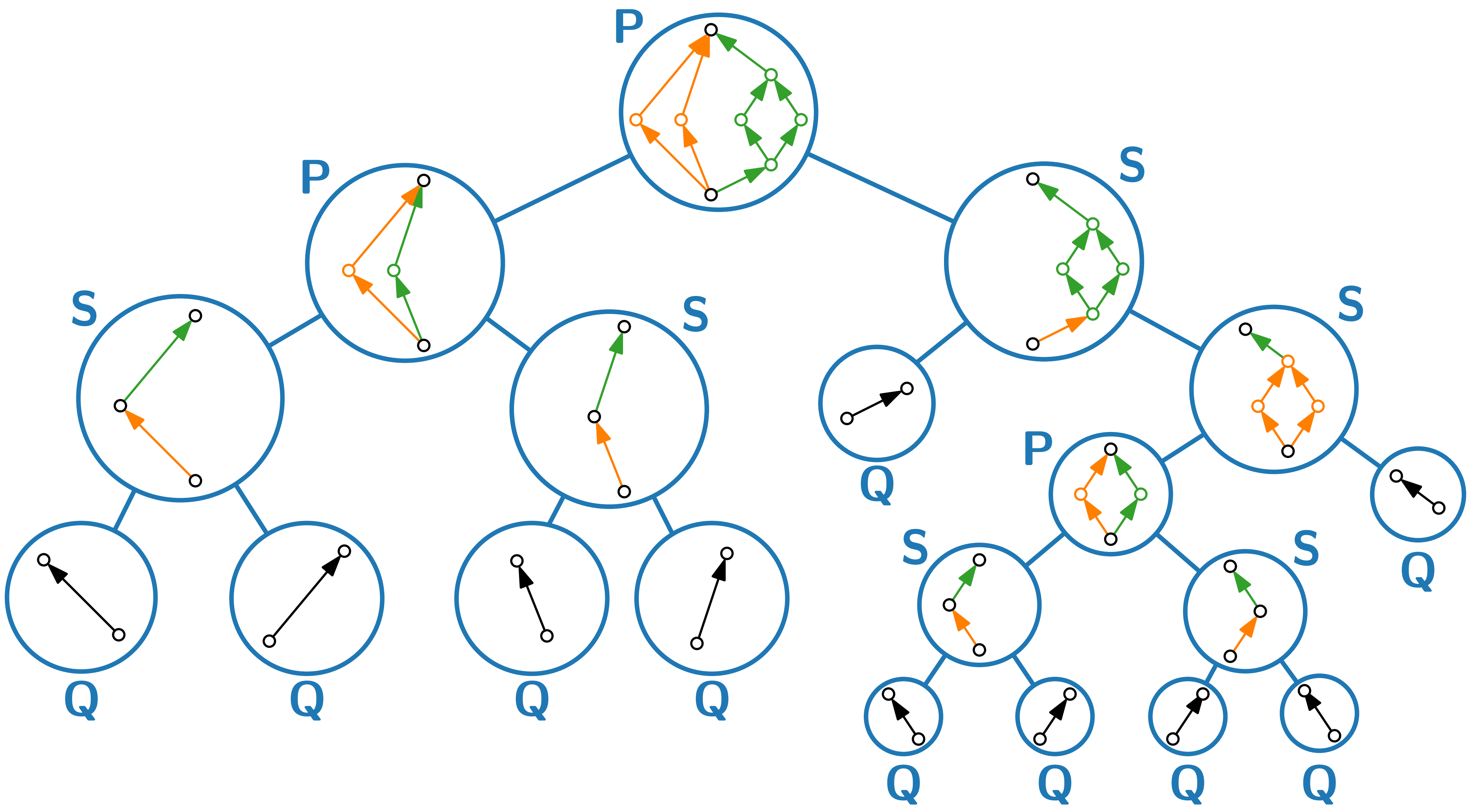
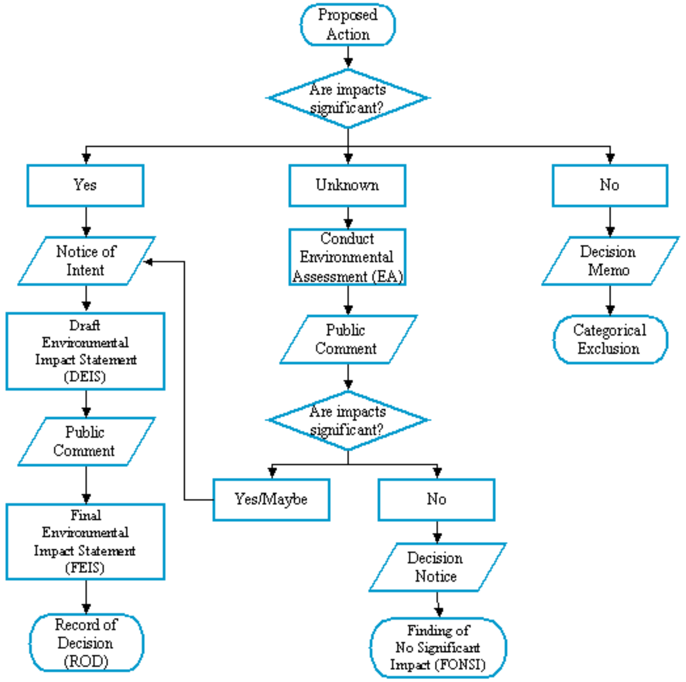
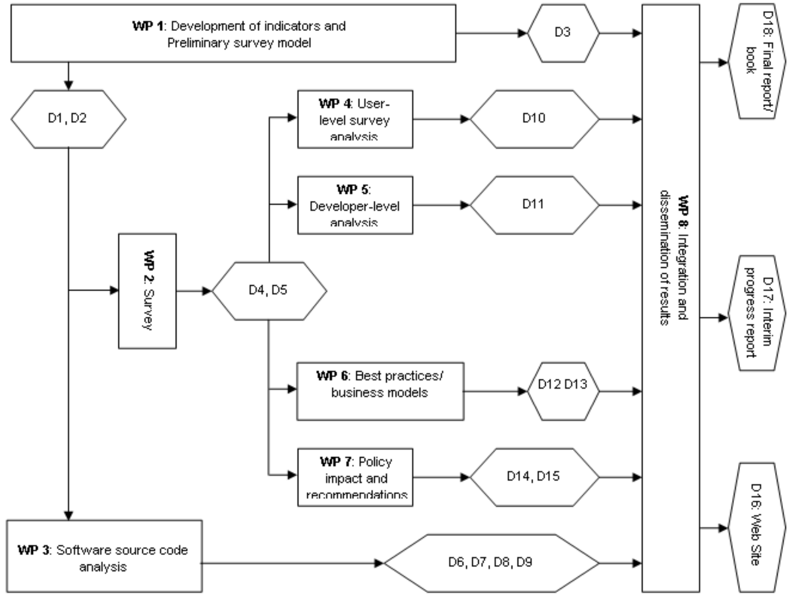


Series-Parallel Graphs – Applications



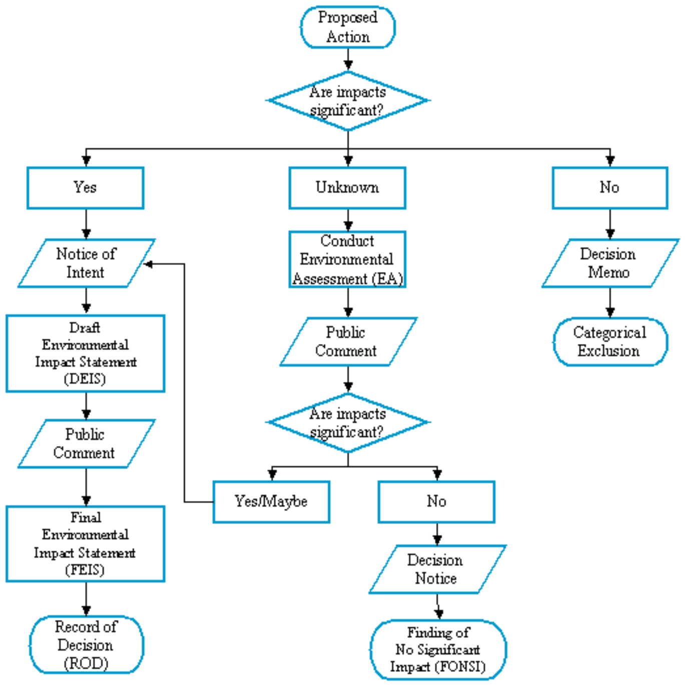
Flowcharts



PERT-Diagrams

(Program Evaluation and Review Technique)

Series-Parallel Graphs – Applications



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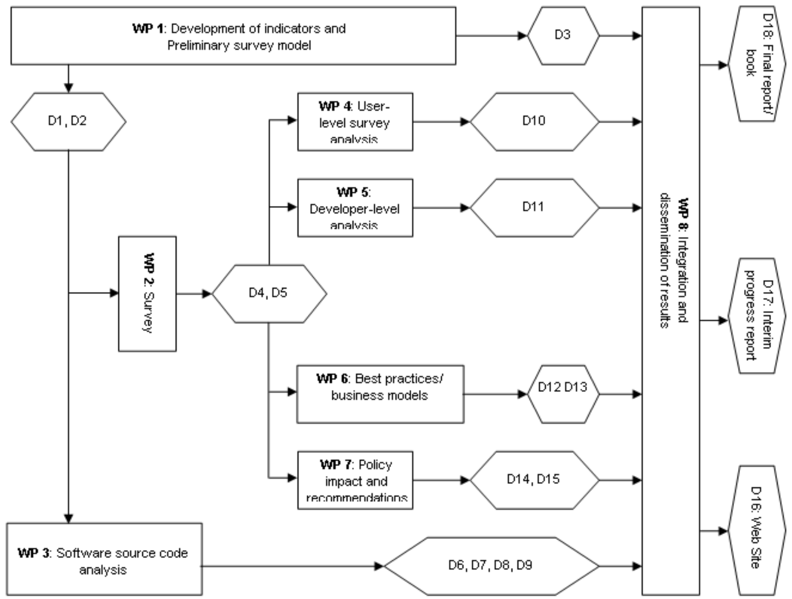


PERT-Diagrams

(Program Evaluation and Review Technique)

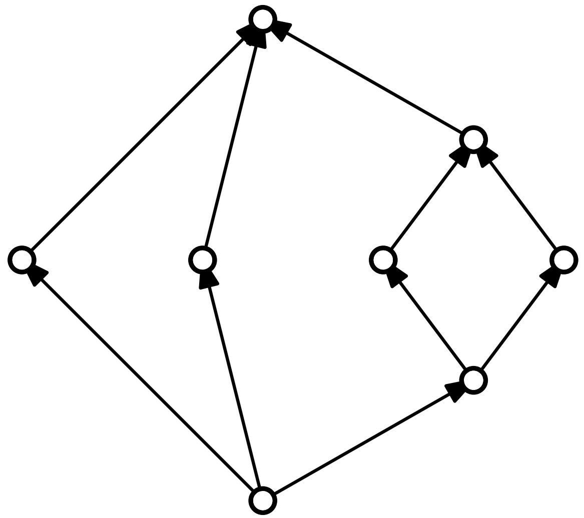
Computational complexity:

Linear time algorithms for $\mathcal{NP}$-hard problems

(e.g. Maximum Matching, MIS, Hamiltonian Completion)

Series-Parallel Graphs – Drawing Style

Drawing conventions

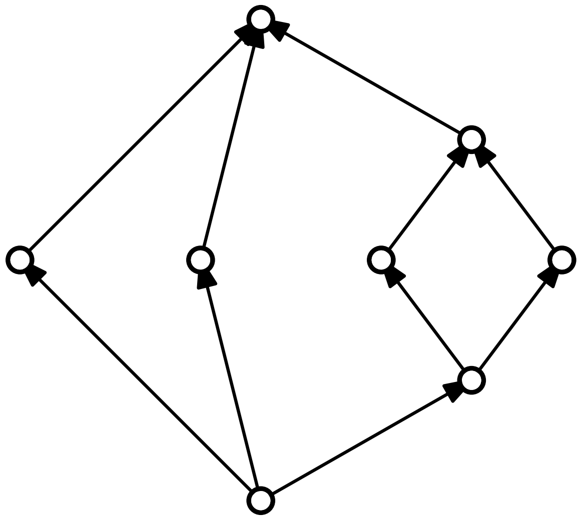


Drawing aesthetics

Series-Parallel Graphs – Drawing Style

Drawing conventions

- Planarity



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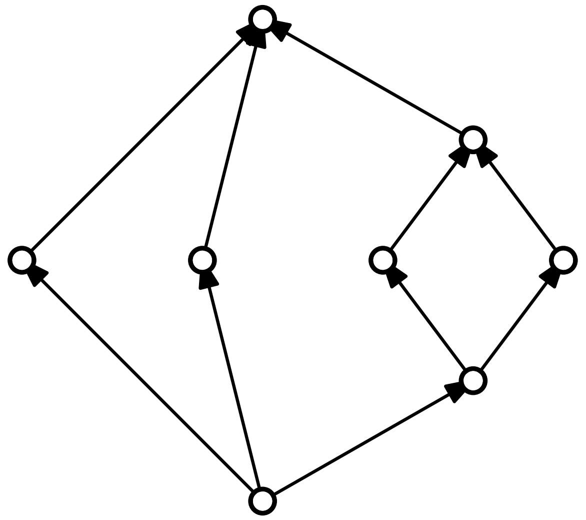
Series-Parallel Graphs – Drawing Style

Drawing conventions

- Planarity
- Straight-line edges



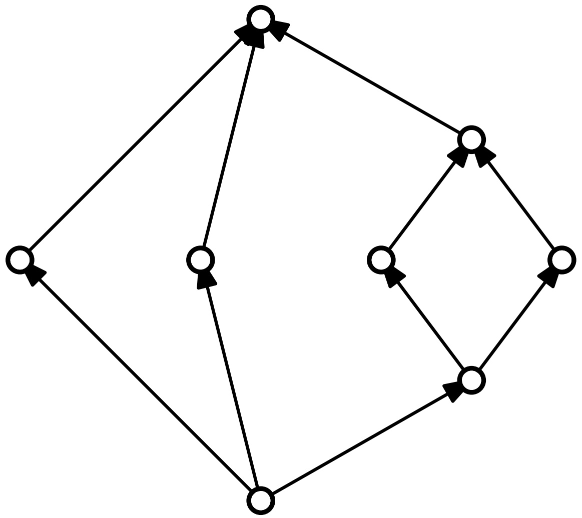
Drawing aesthetics

Series-Parallel Graphs – Drawing Style

Drawing conventions

- Planarity
- Straight-line edges
- Upward

Drawing aesthetics



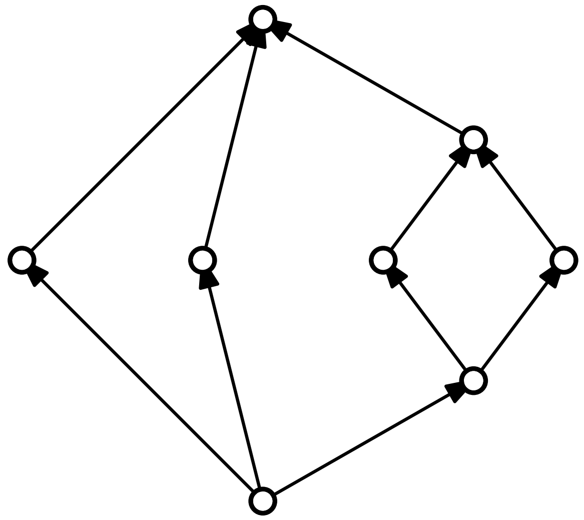
Series-Parallel Graphs – Drawing Style

Drawing conventions

- Planarity
- Straight-line edges
- Upward

Drawing aesthetics

- Area



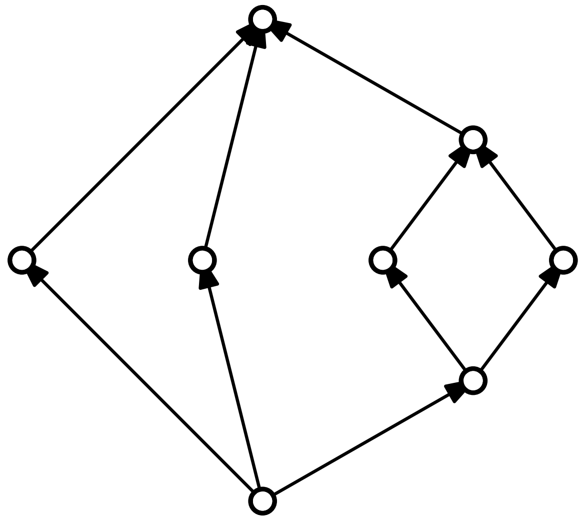
Series-Parallel Graphs – Drawing Style

Drawing conventions

- Planarity
- Straight-line edges
- Upward

Drawing aesthetics

- Area
- Symmetry



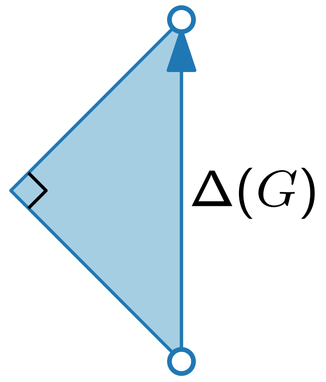
Series-Parallel Graphs – Straight-Line Drawings

Divide & conquer algorithm using the decomposition tree

Series-Parallel Graphs – Straight-Line Drawings

Divide & conquer algorithm using the decomposition tree

- Draw G inside a right-angled isosceles bounding triangle $\Delta(G)$



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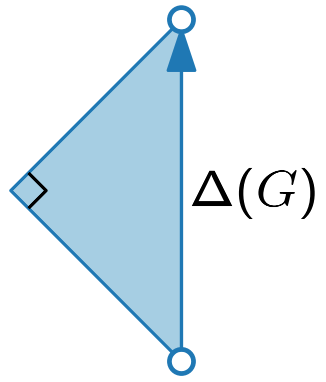
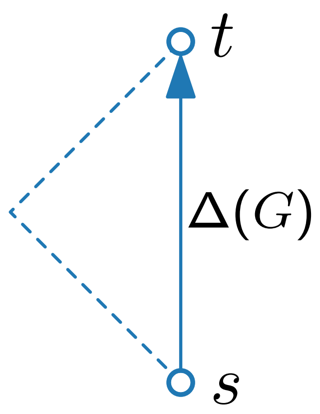


Base case: Q-nodes



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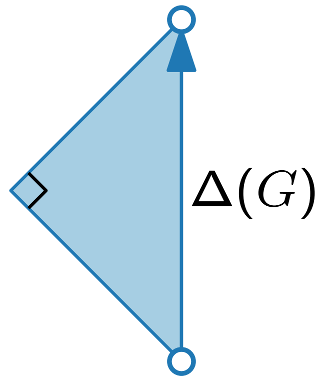
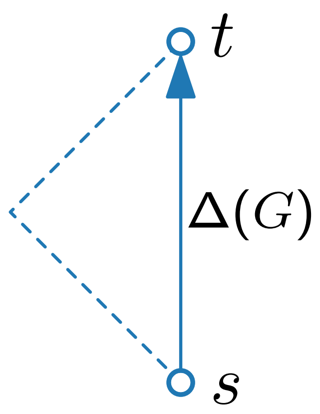


Base case: Q-nodes

Divide: Draw G_1 and G_2 first



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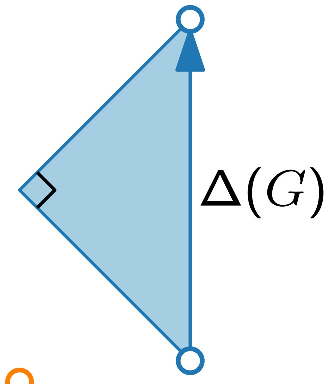

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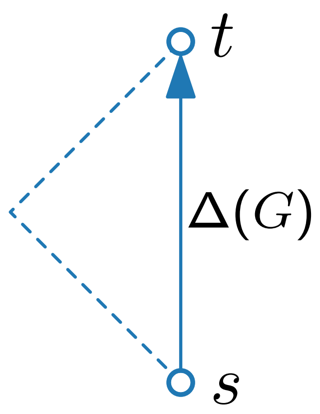

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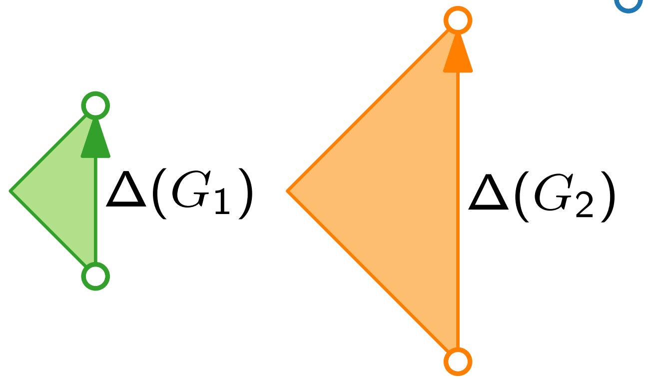

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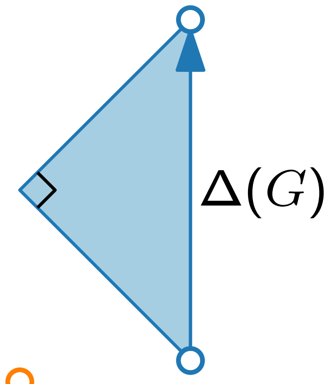

Base case: Q-nodes

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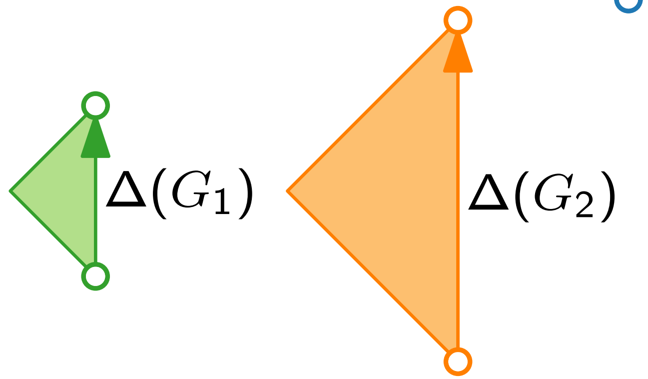
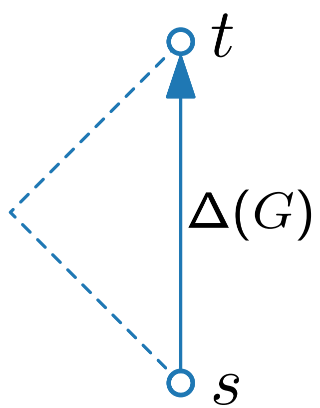


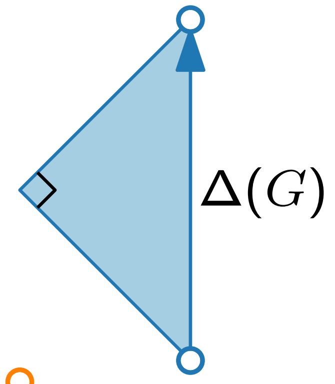
Conquer:



Series-Parallel Graphs – Straight-Line Drawings

Divide & conquer algorithm using the decomposition tree

- Draw G inside a right-angled isosceles bounding triangle $\Delta(G)$



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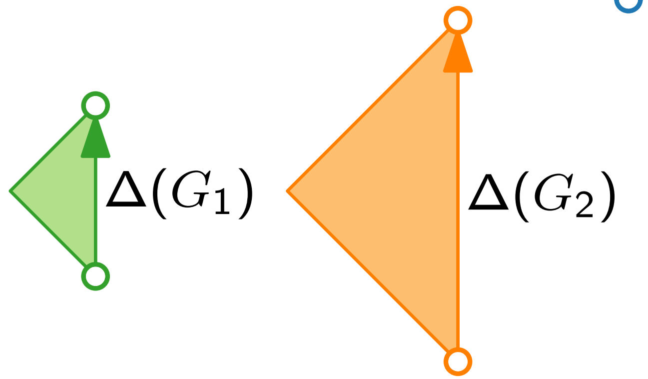
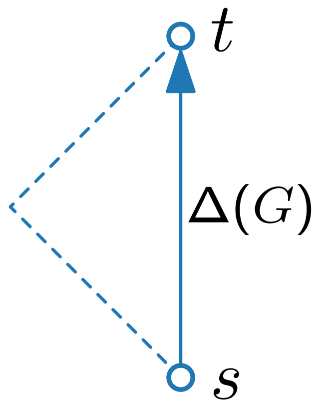


Conquer:

- S-nodes / series composition



Series-Parallel Graphs – Straight-Line Drawings

Divide & conquer algorithm using the decomposition tree

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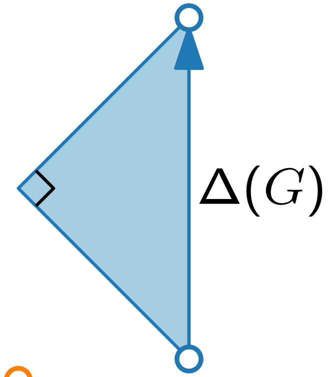

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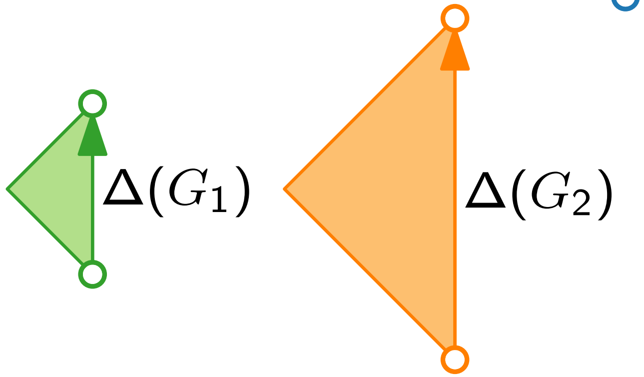

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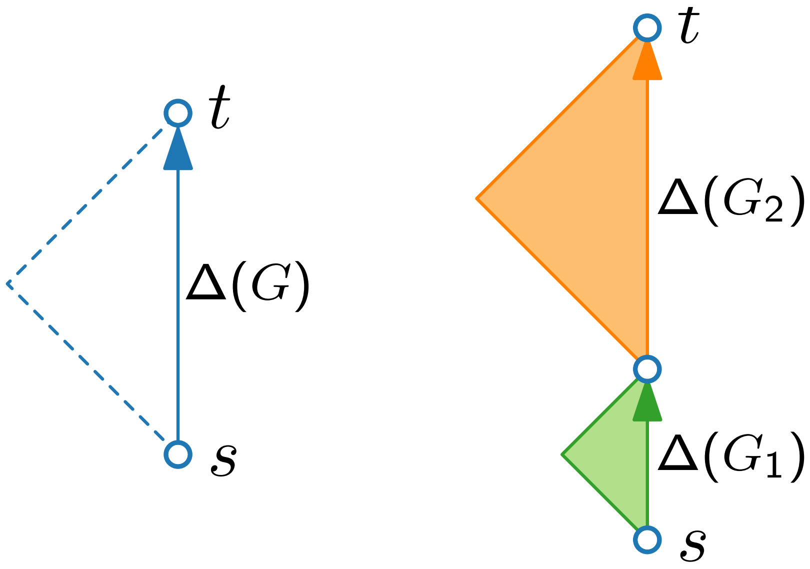
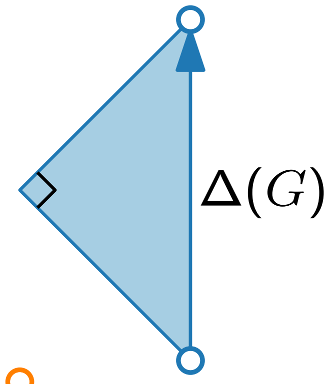


Series-Parallel Graphs – Straight-Line Drawings



Divide & conquer algorithm using the decomposition tree

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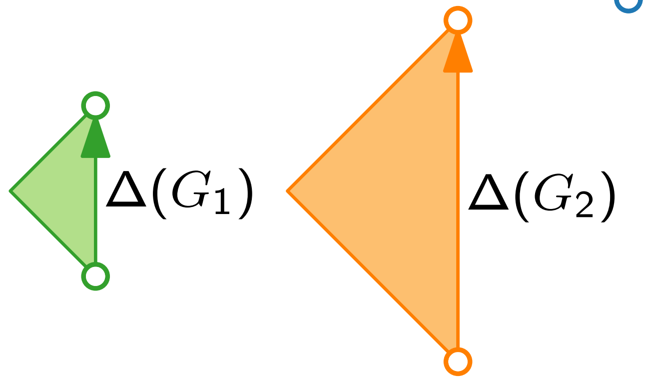

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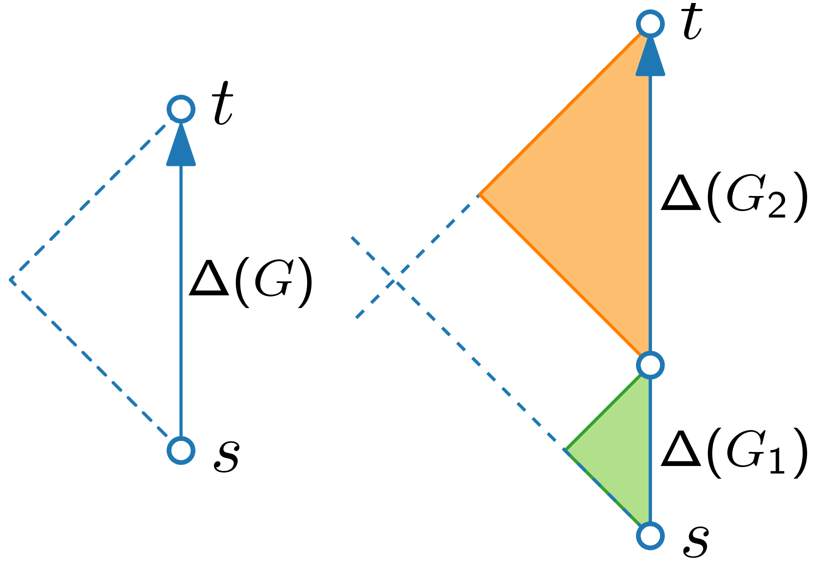

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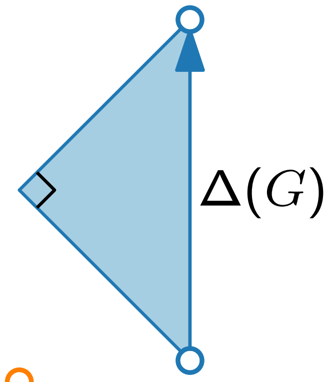

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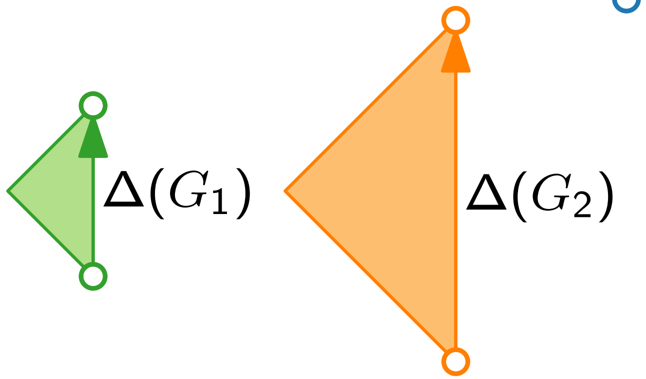
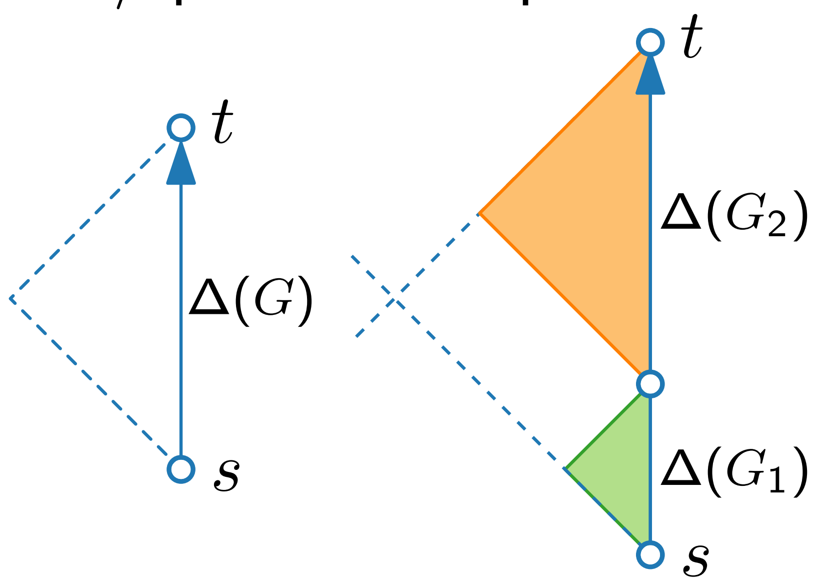


Conquer:

- S-nodes / series composition
- P-nodes / parallel composition



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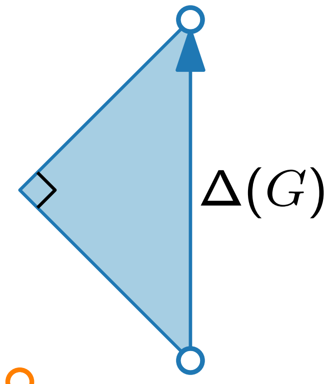

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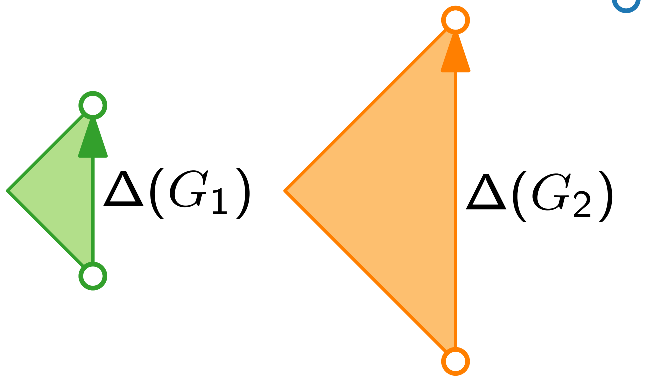

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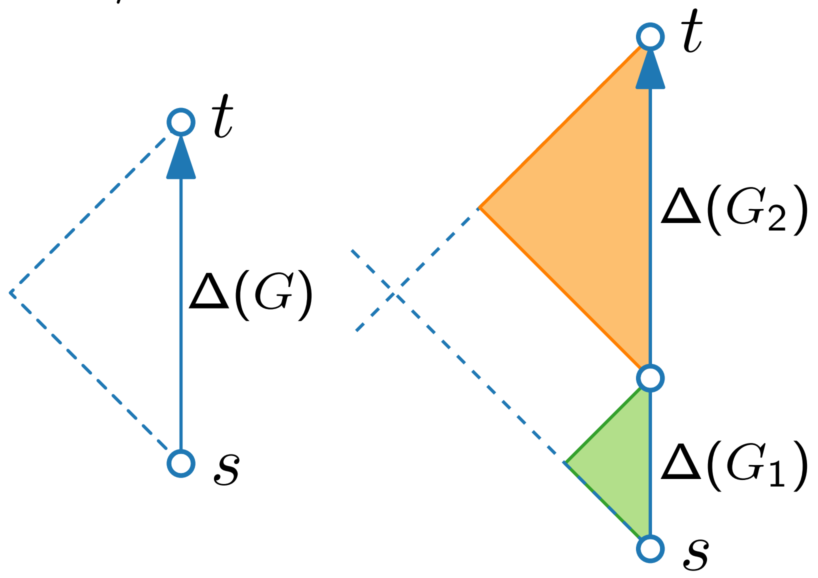

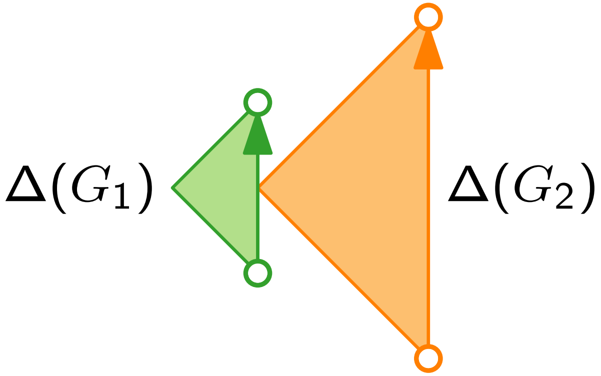

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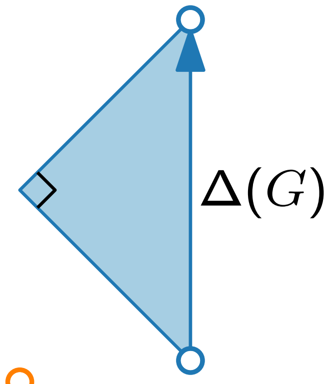

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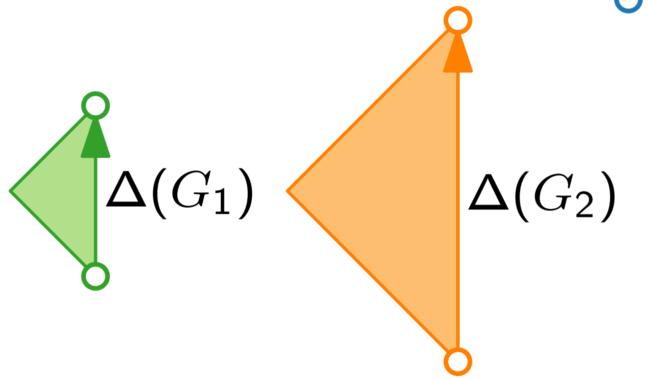

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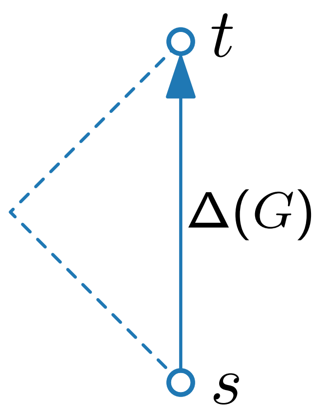

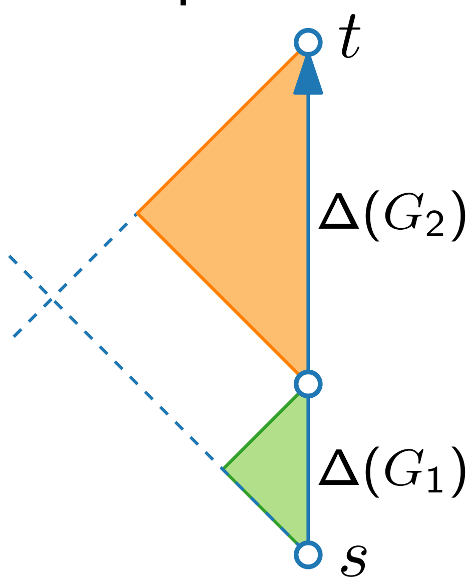

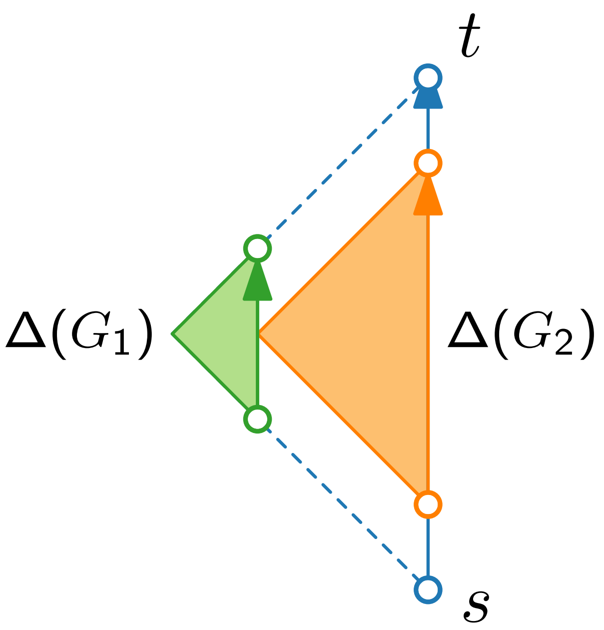

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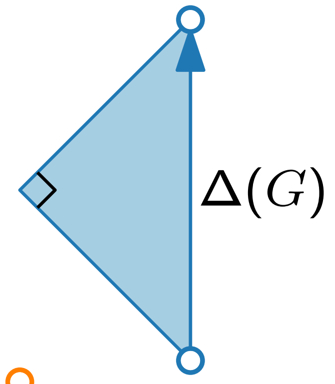

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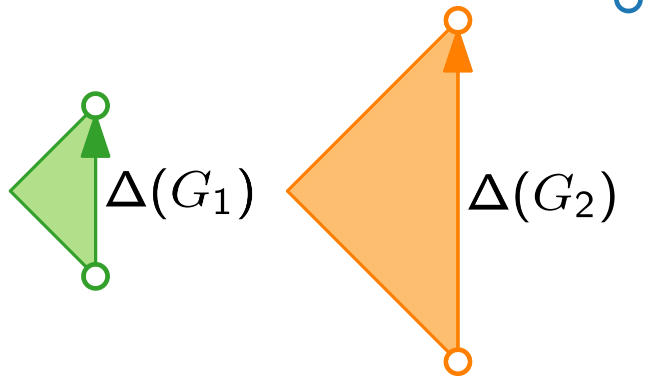

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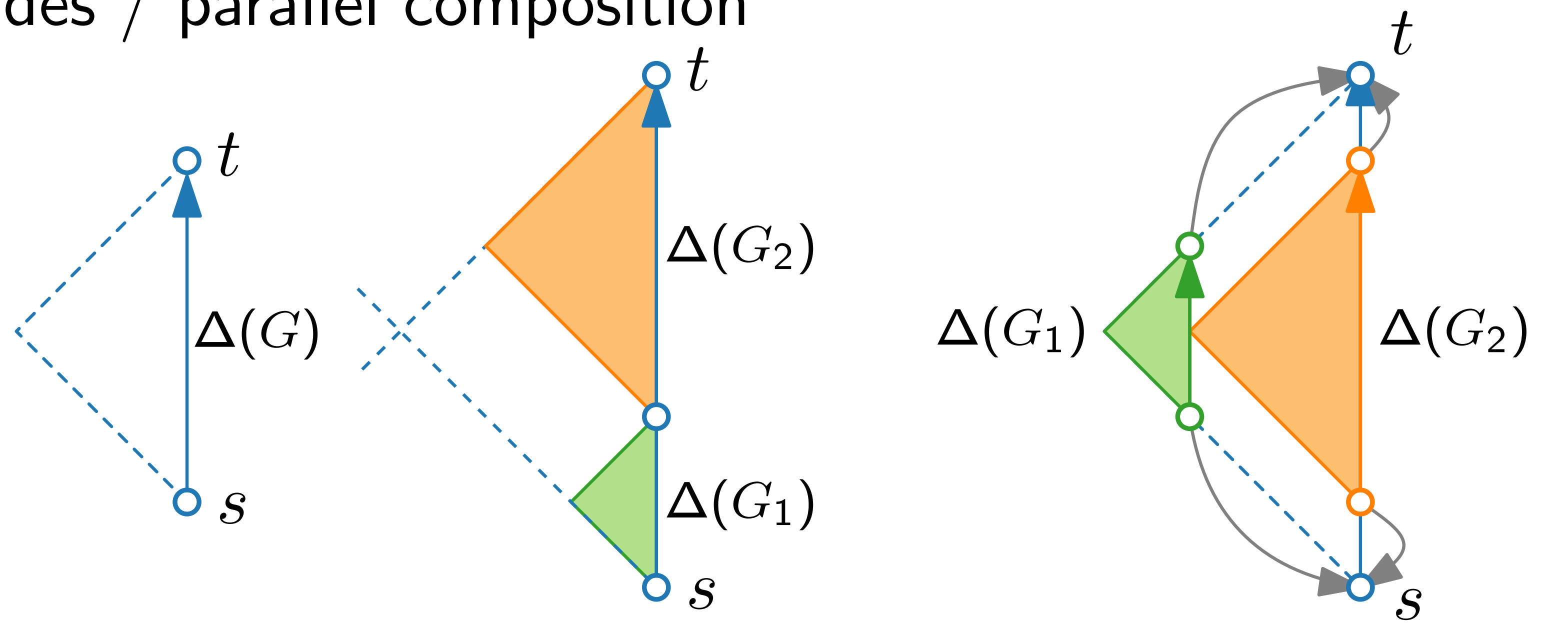

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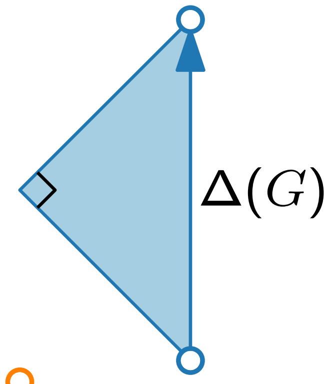

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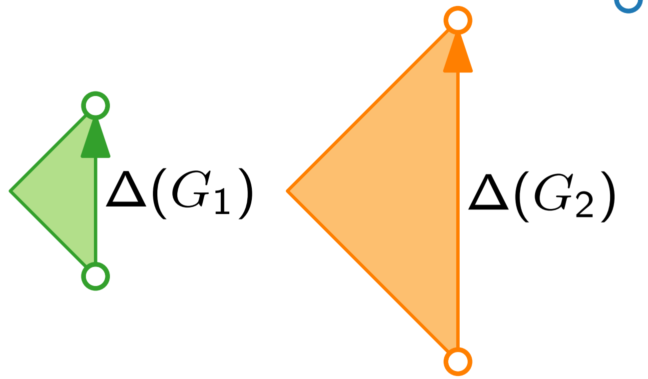

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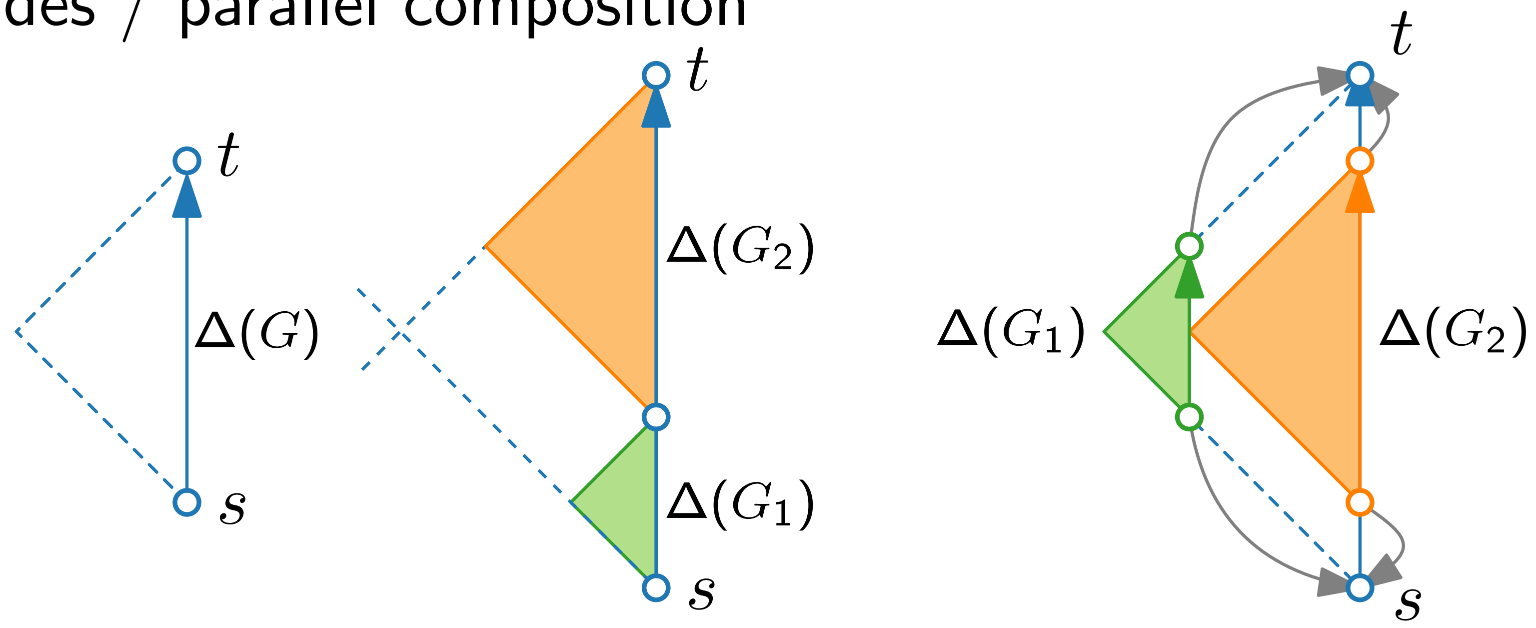
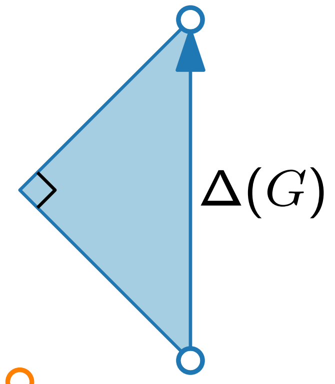


Do you see any problem?

Series-Parallel Graphs – Straight-Line Drawings



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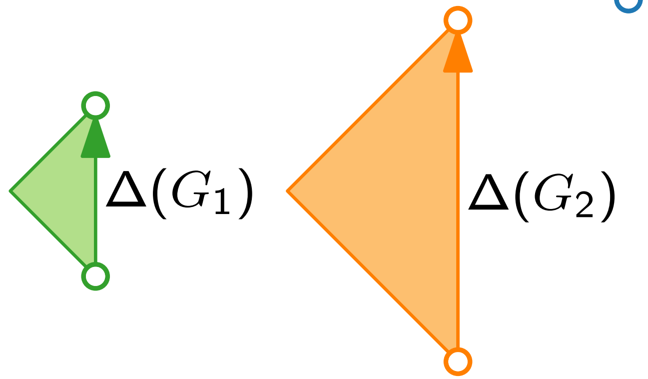

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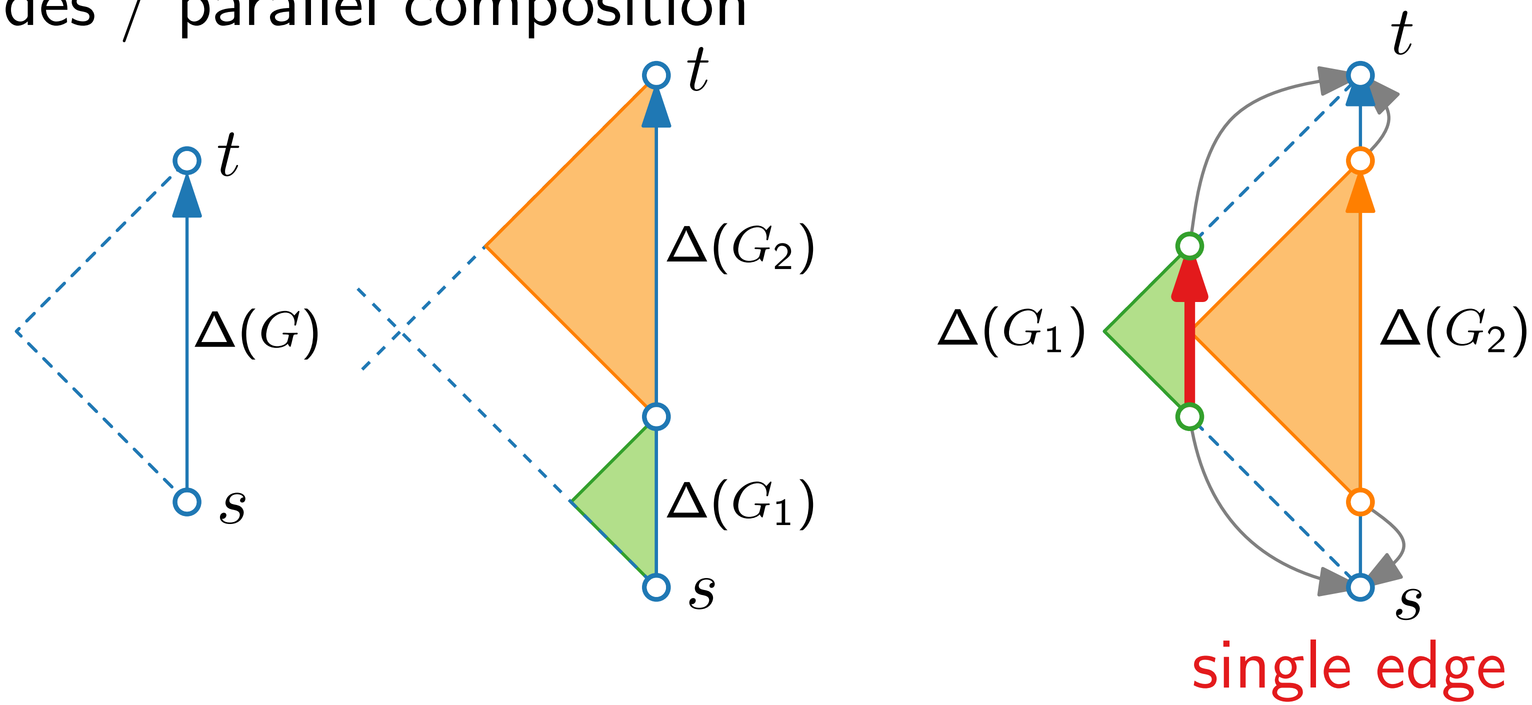

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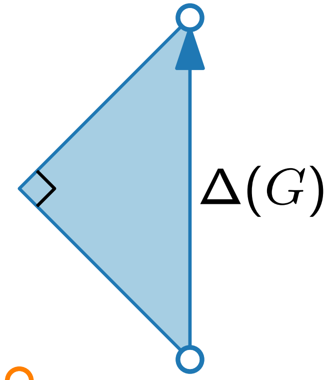

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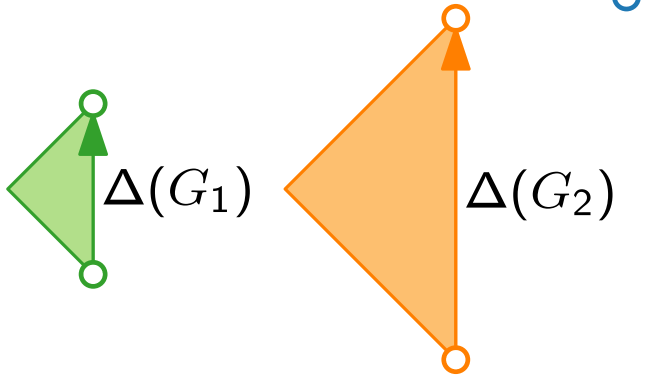

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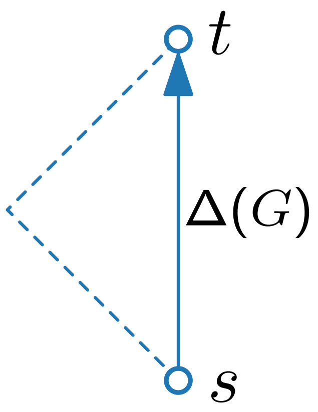

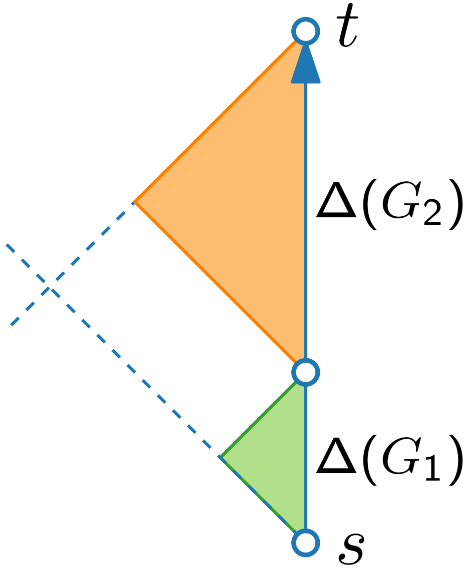

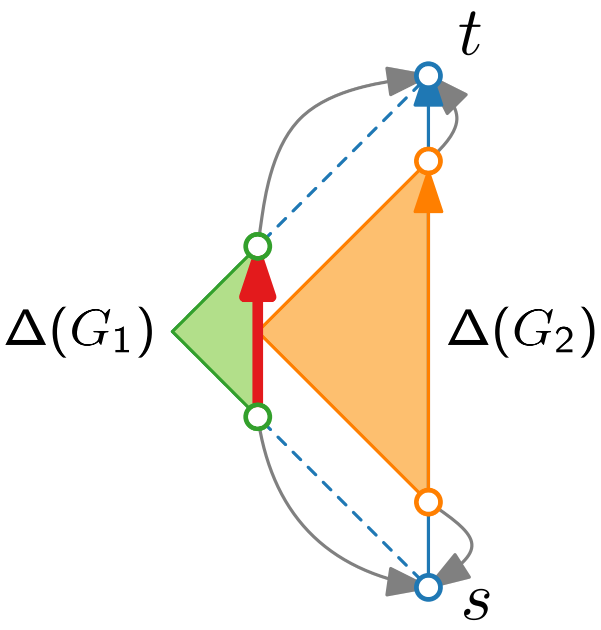

change embedding!

Series-Parallel Graphs – Straight-Line Drawings

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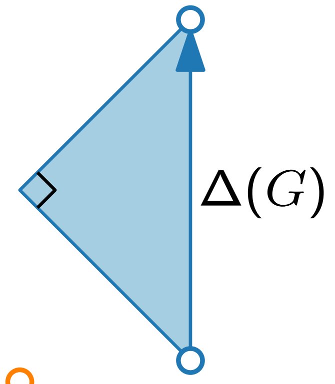

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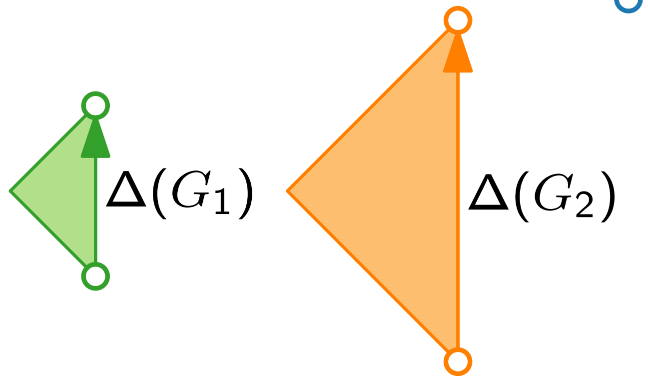

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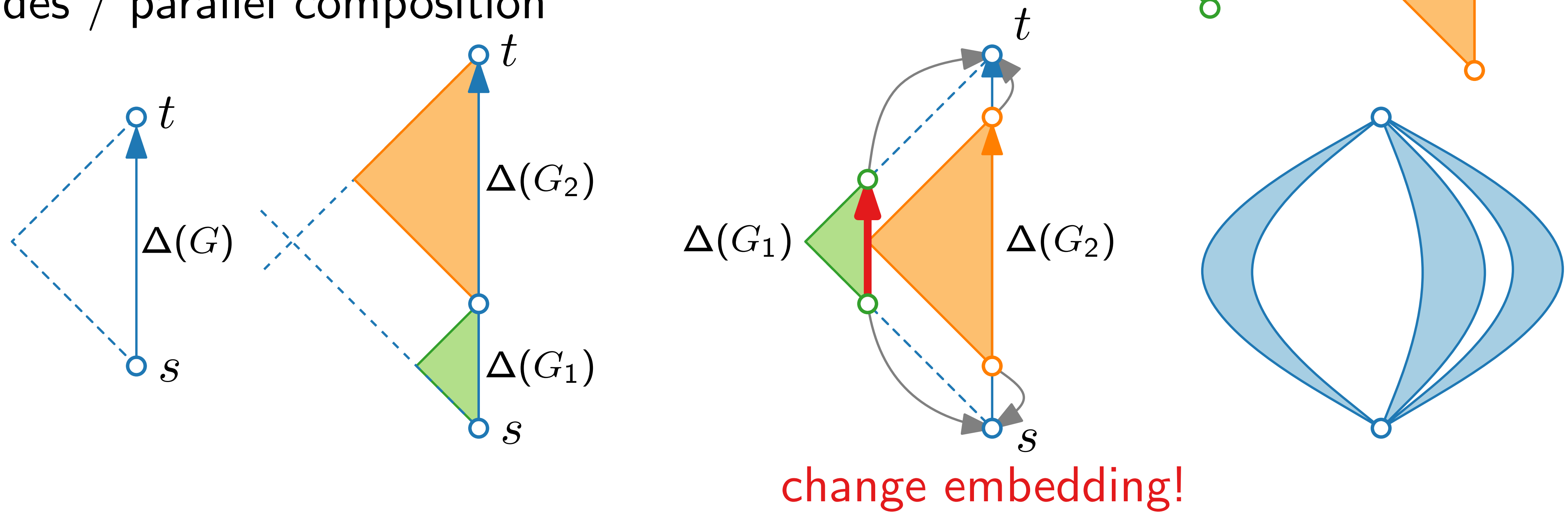

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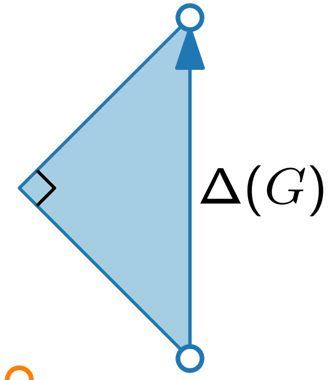

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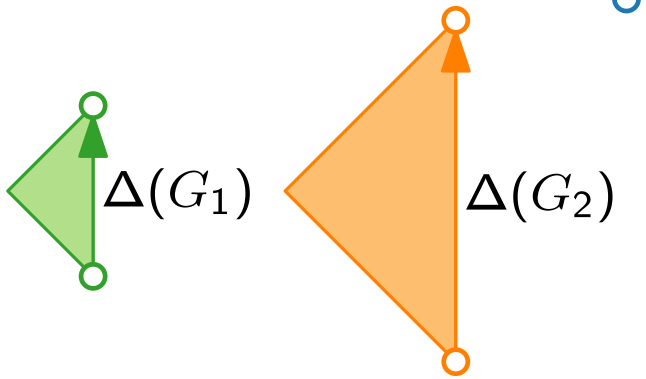

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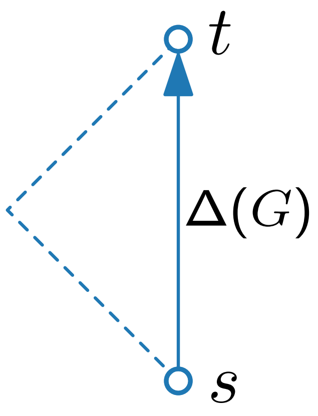

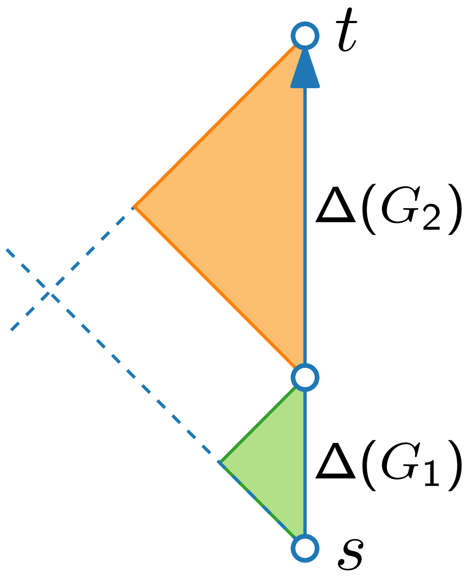

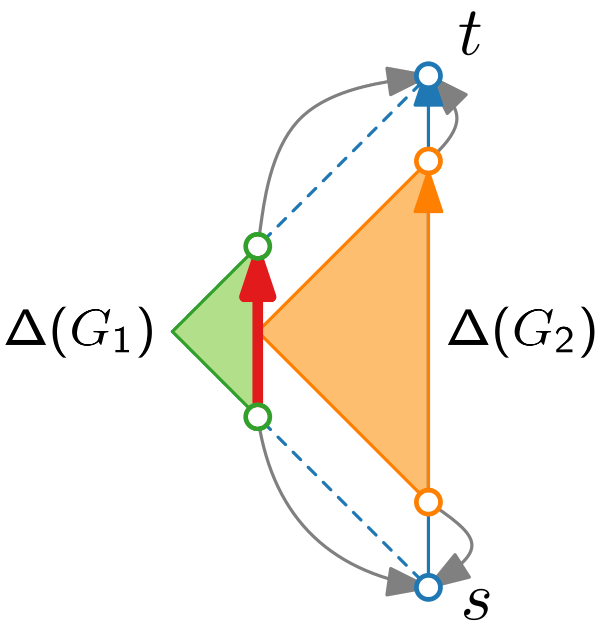

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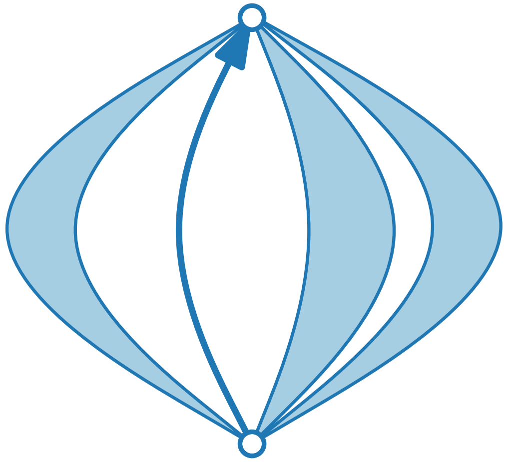

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Divide & conquer algorithm using the decomposition tree

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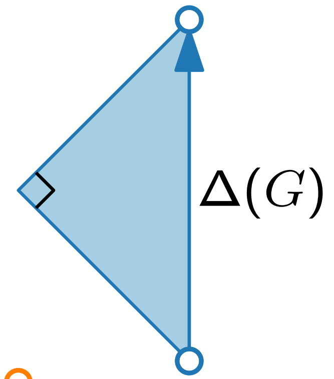

Base case: Q-nodes

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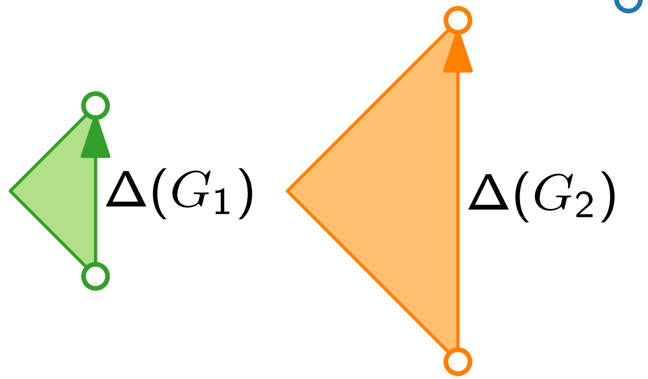

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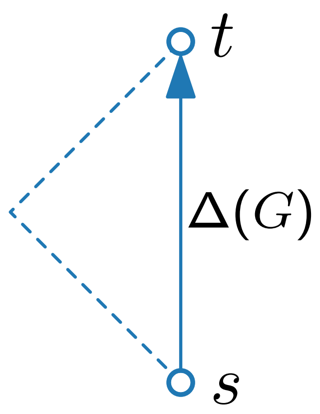

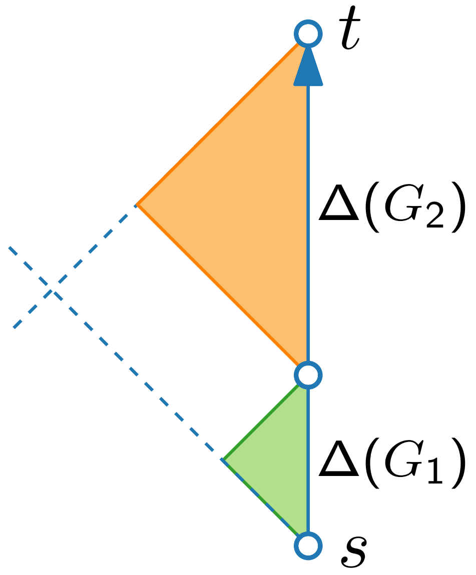

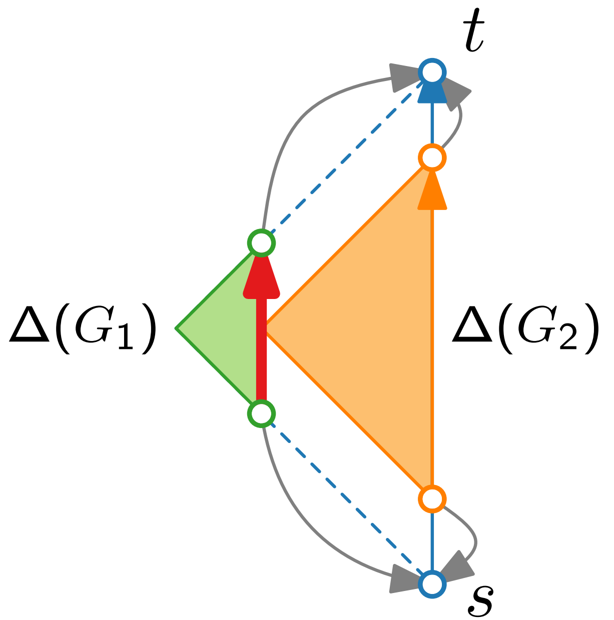

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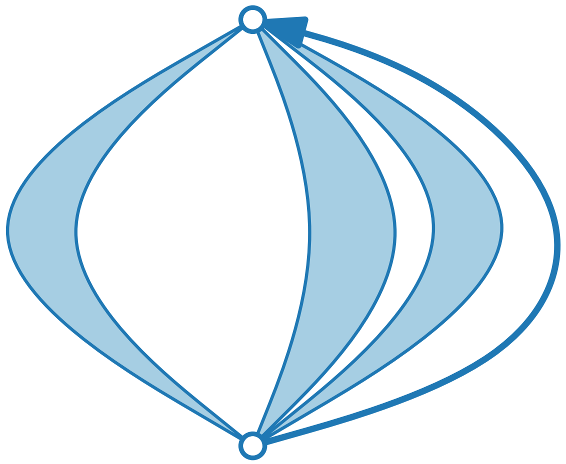

Series-Parallel Graphs – Straight-Line Drawings

- What makes parallel composition possible without creating crossings?

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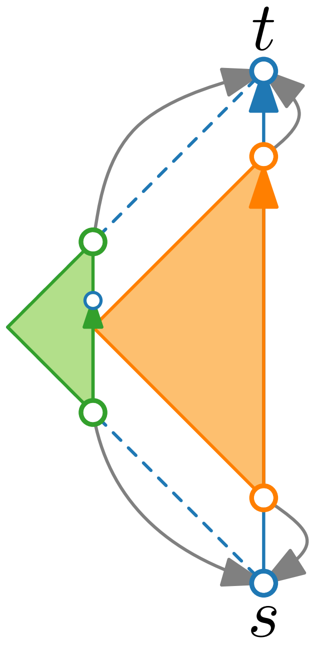

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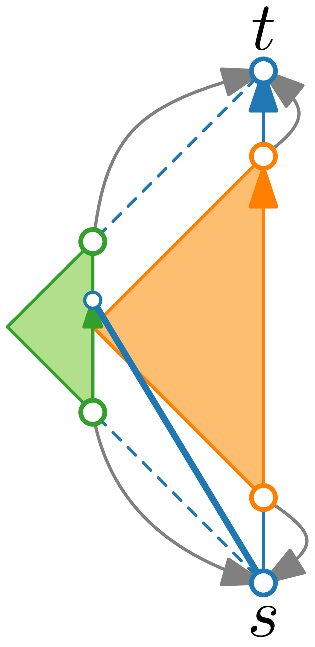

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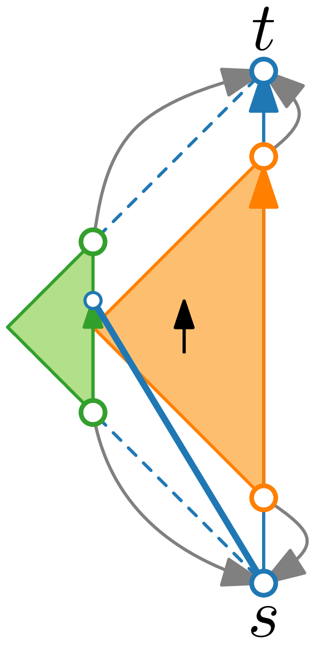

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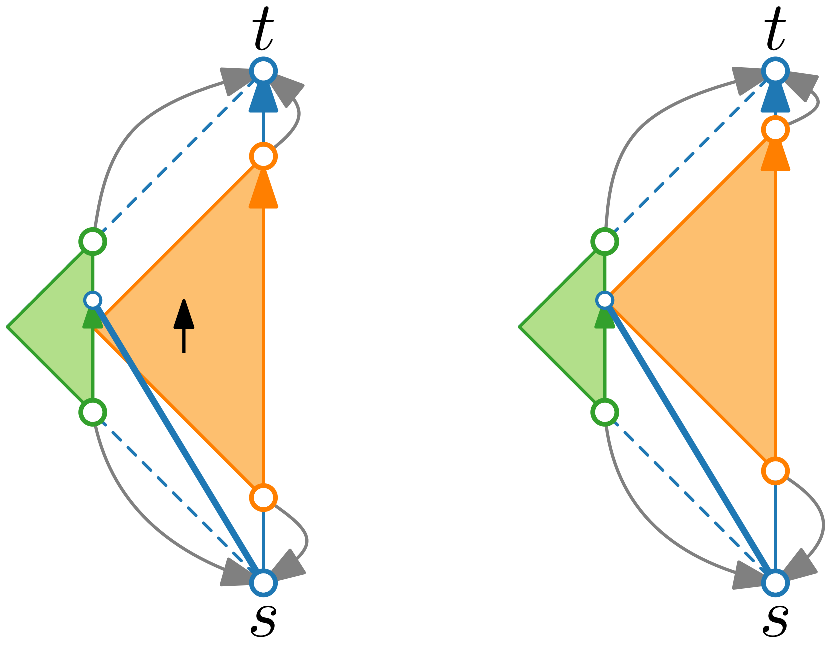

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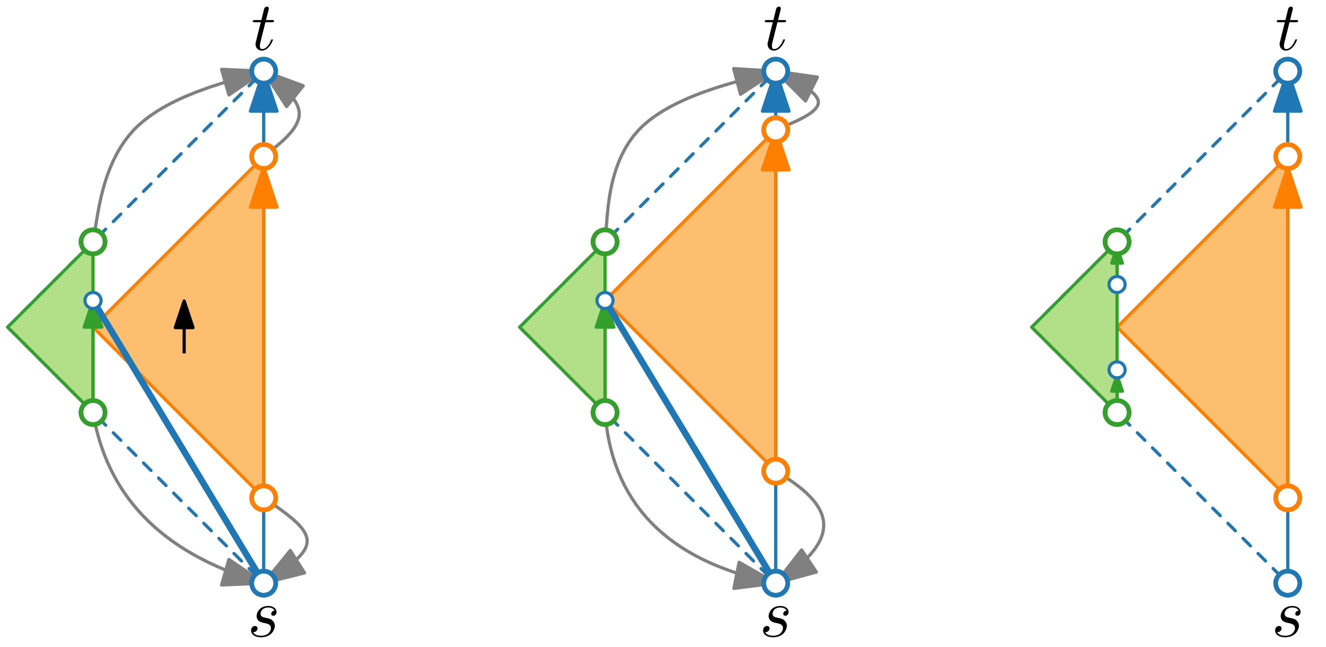

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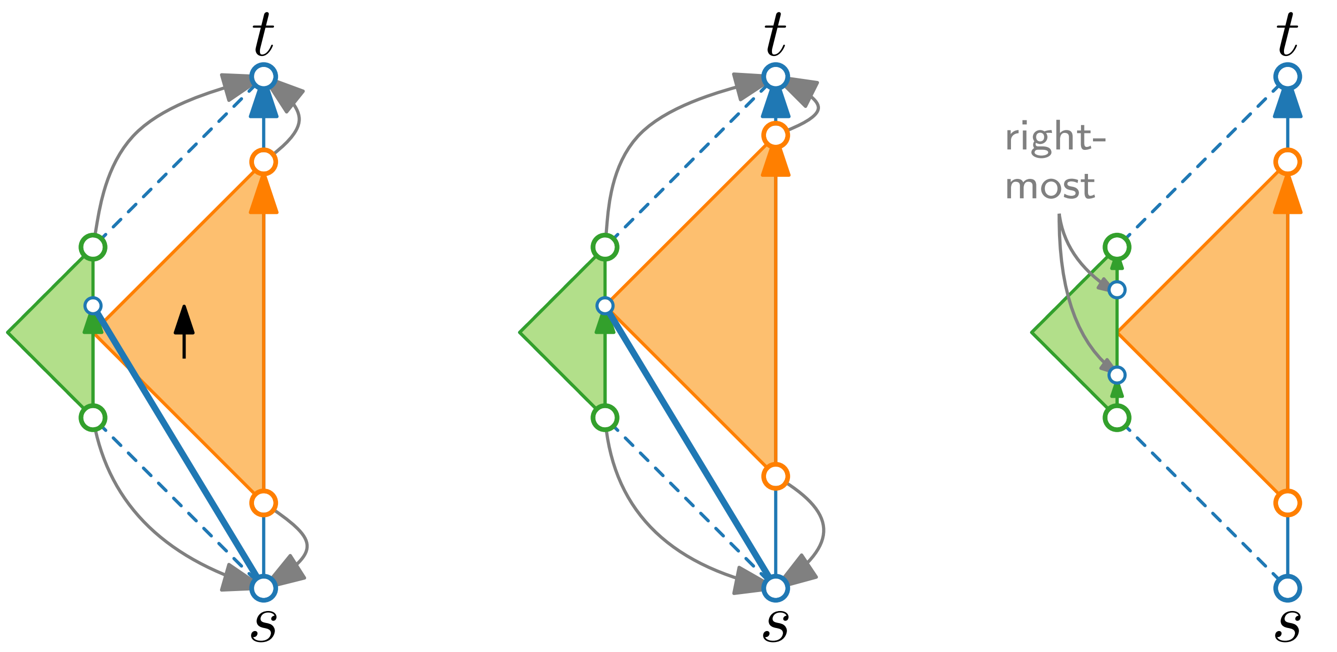

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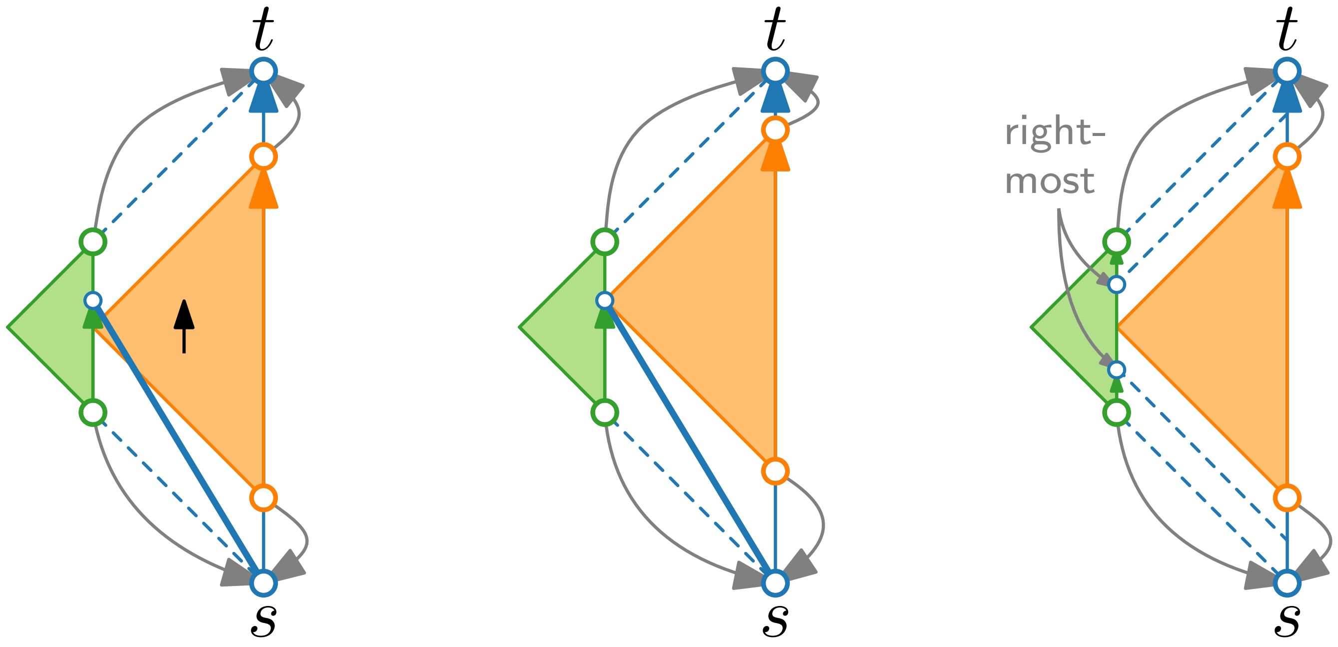

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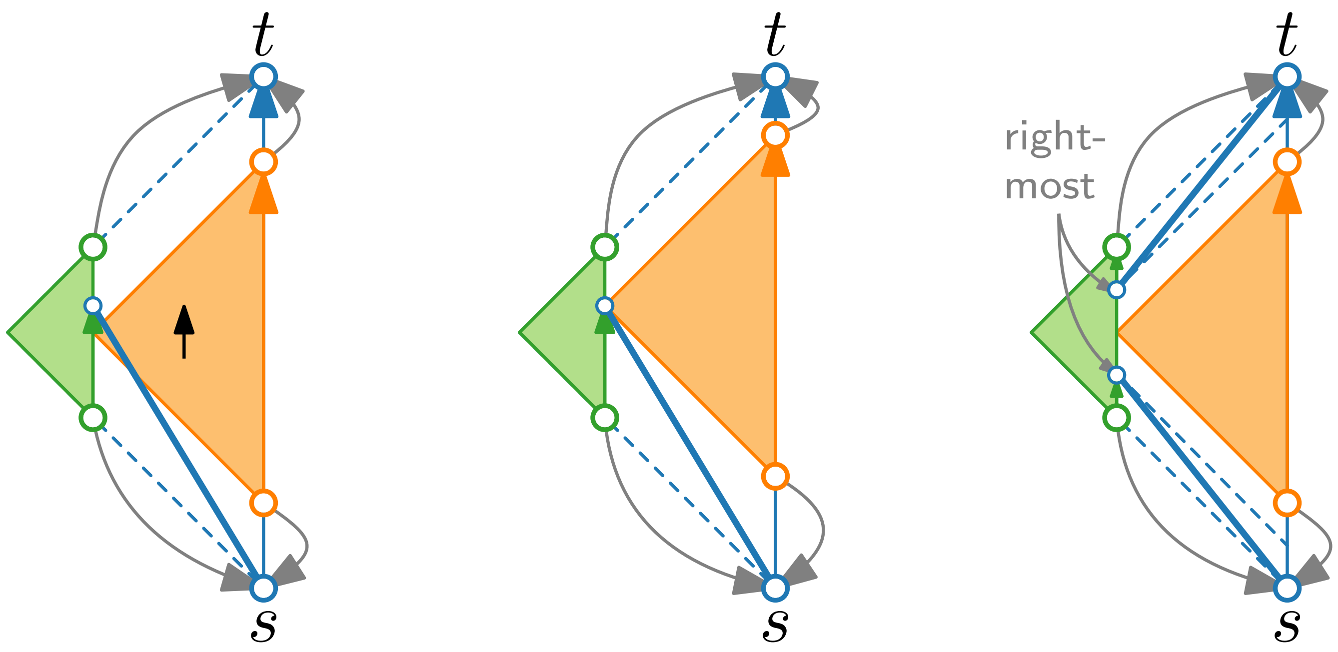

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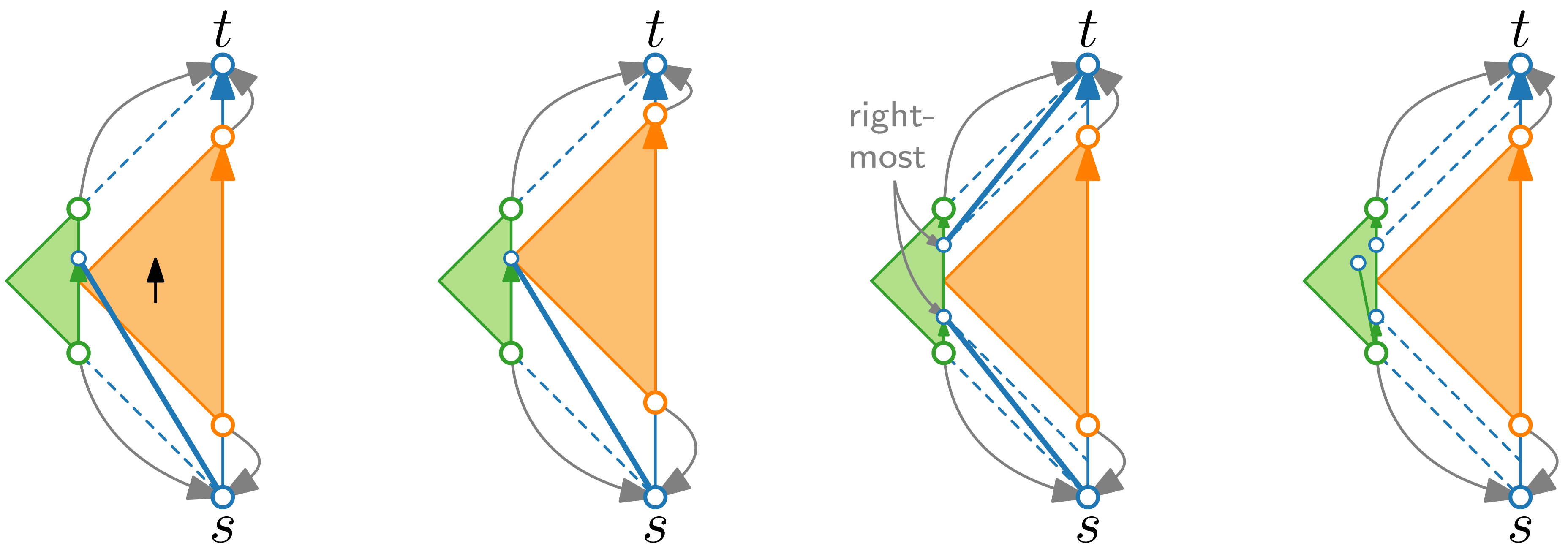

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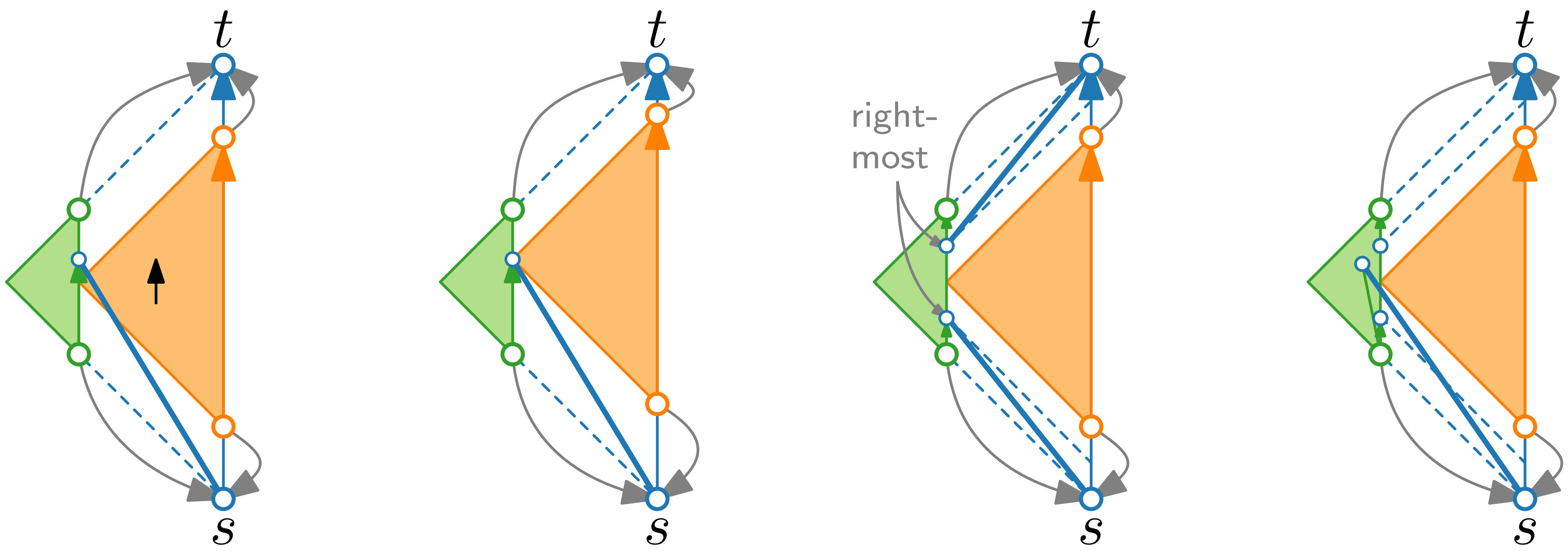

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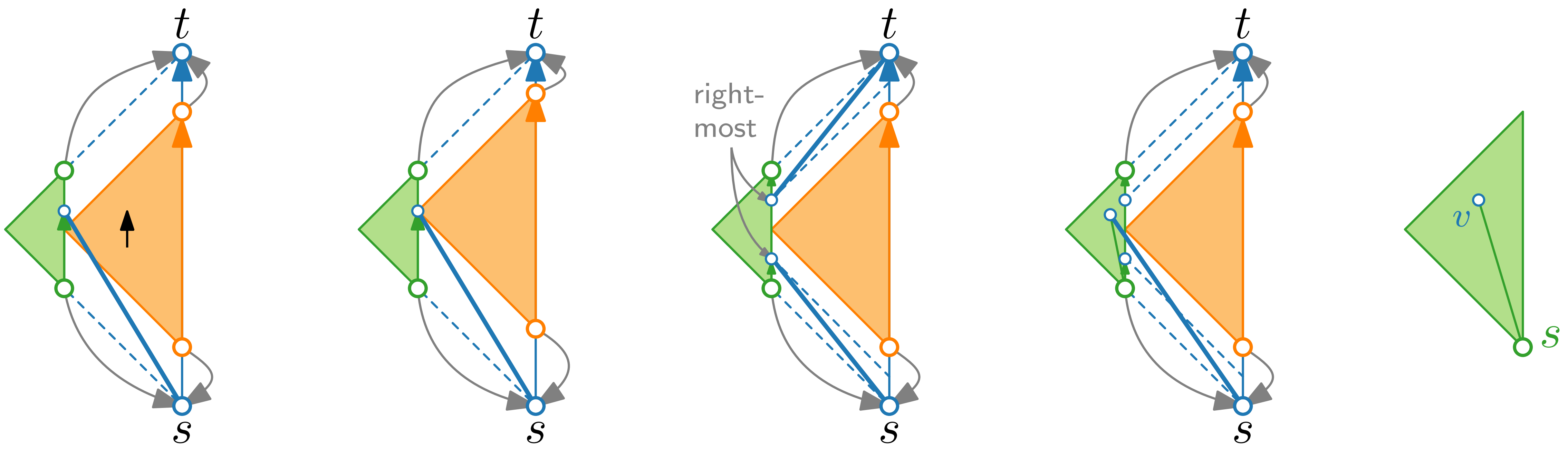

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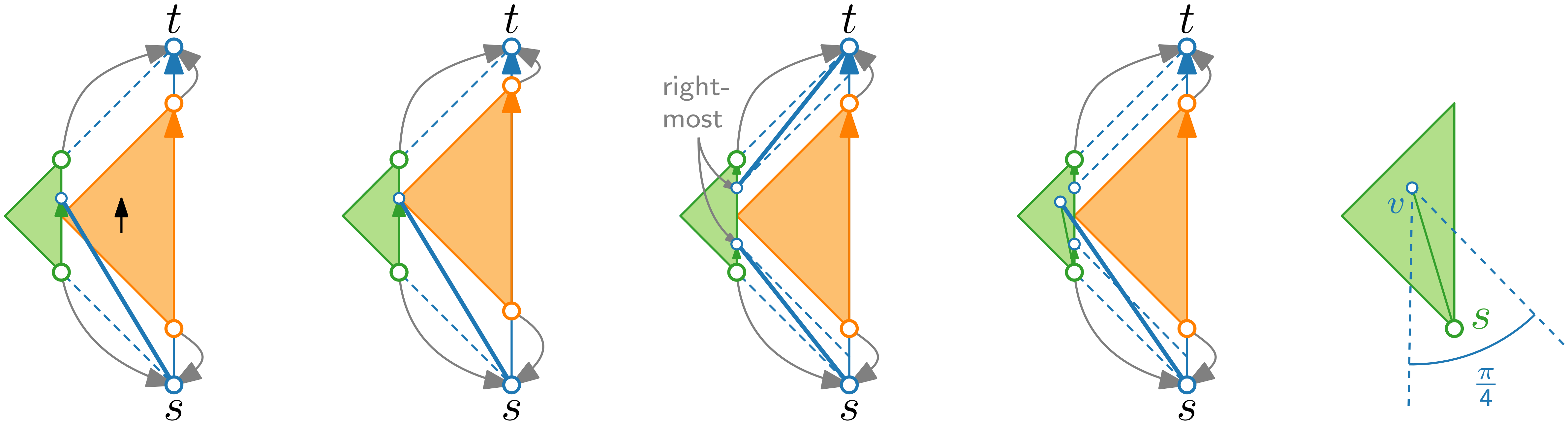

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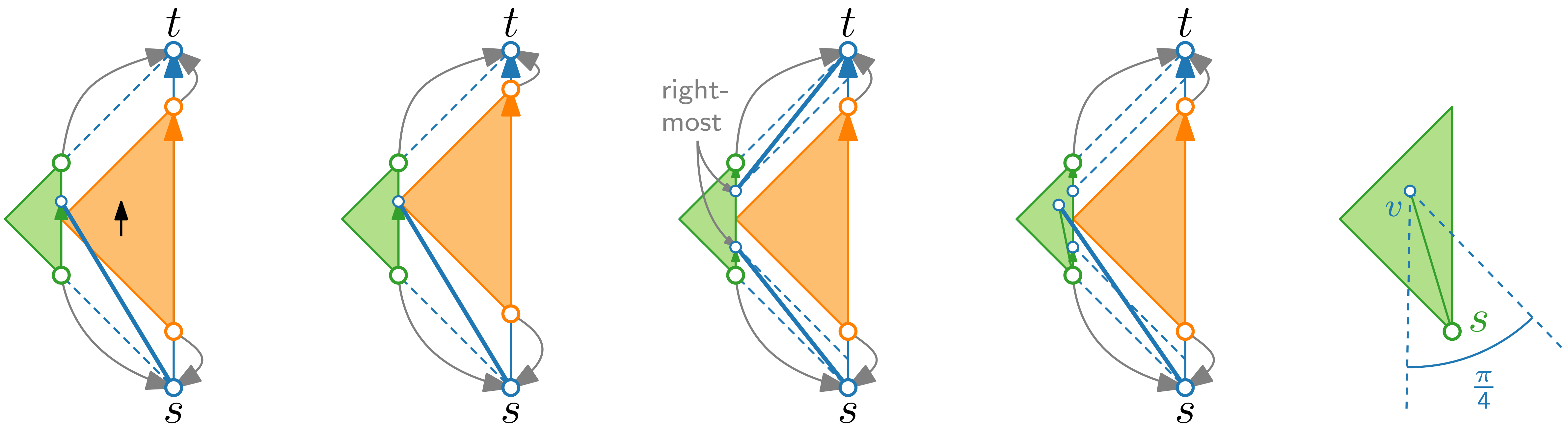

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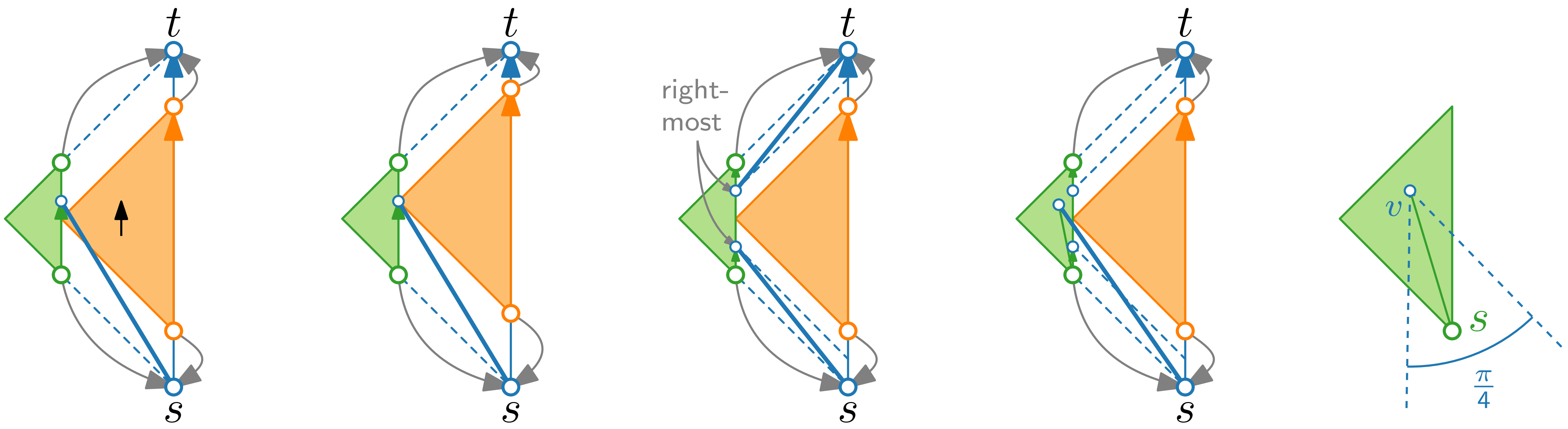

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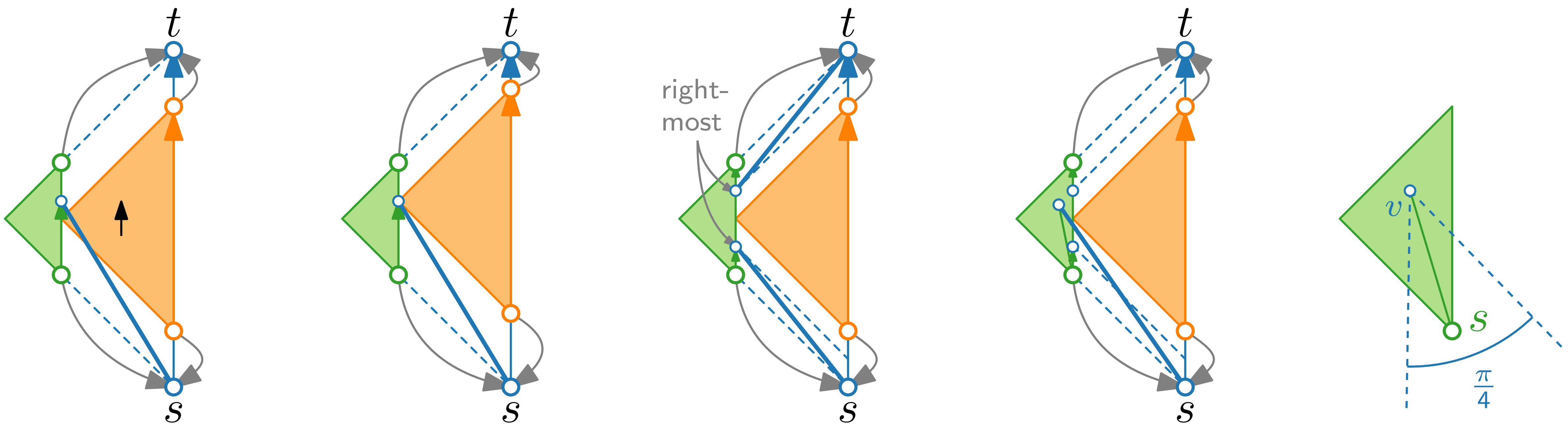

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Lemma.
The drawing produced by the algorithm is planar.

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Γ can be computed in $\mathcal{O}(n)$ time.

Series-Parallel Graphs – Fixed Embedding

Theorem. [Bertolazzi et al. 94]

There exists a $2n$-vertex series-parallel graph G_n such that any upward planar drawing of G_n that respects the embedding requires $\Omega(4^n)$ area.

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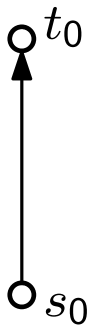

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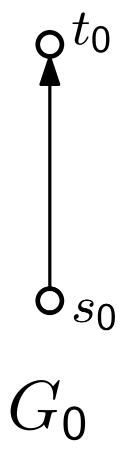

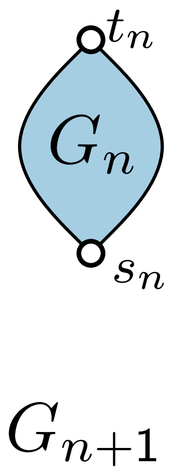

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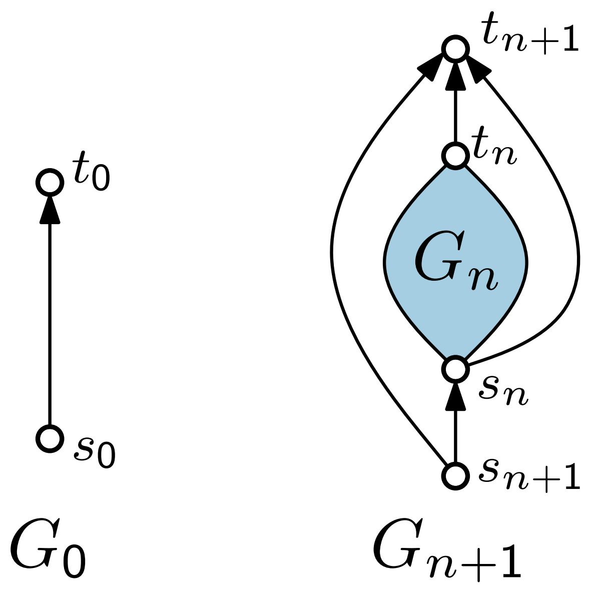

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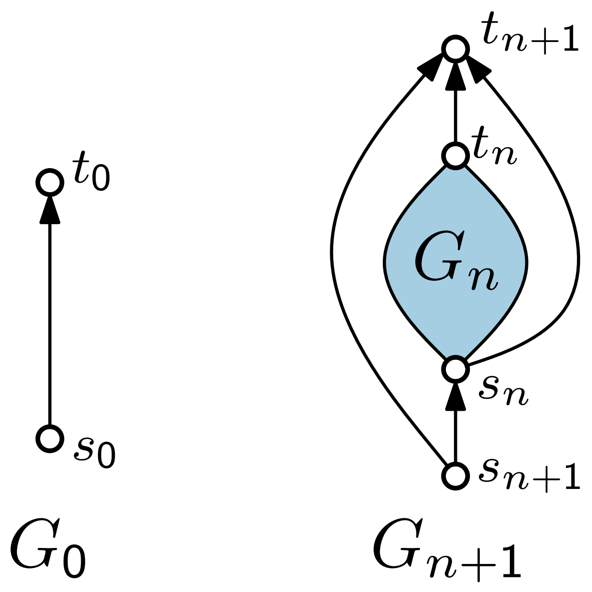

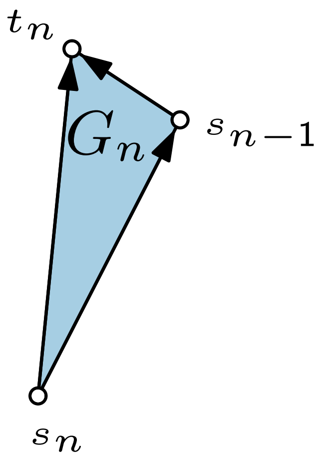

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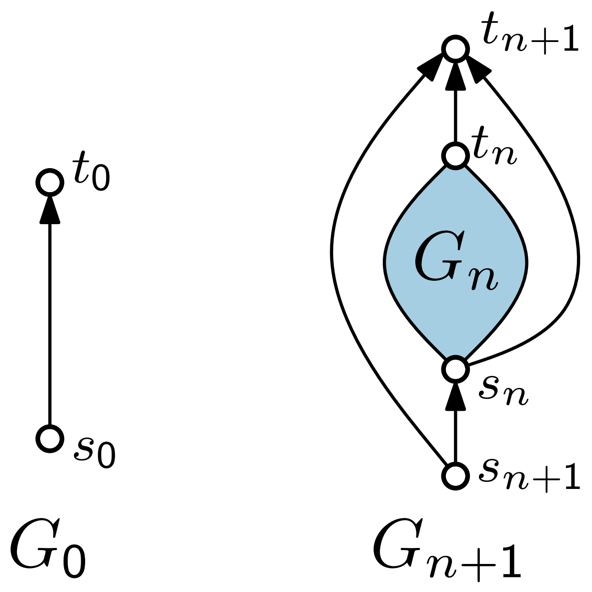

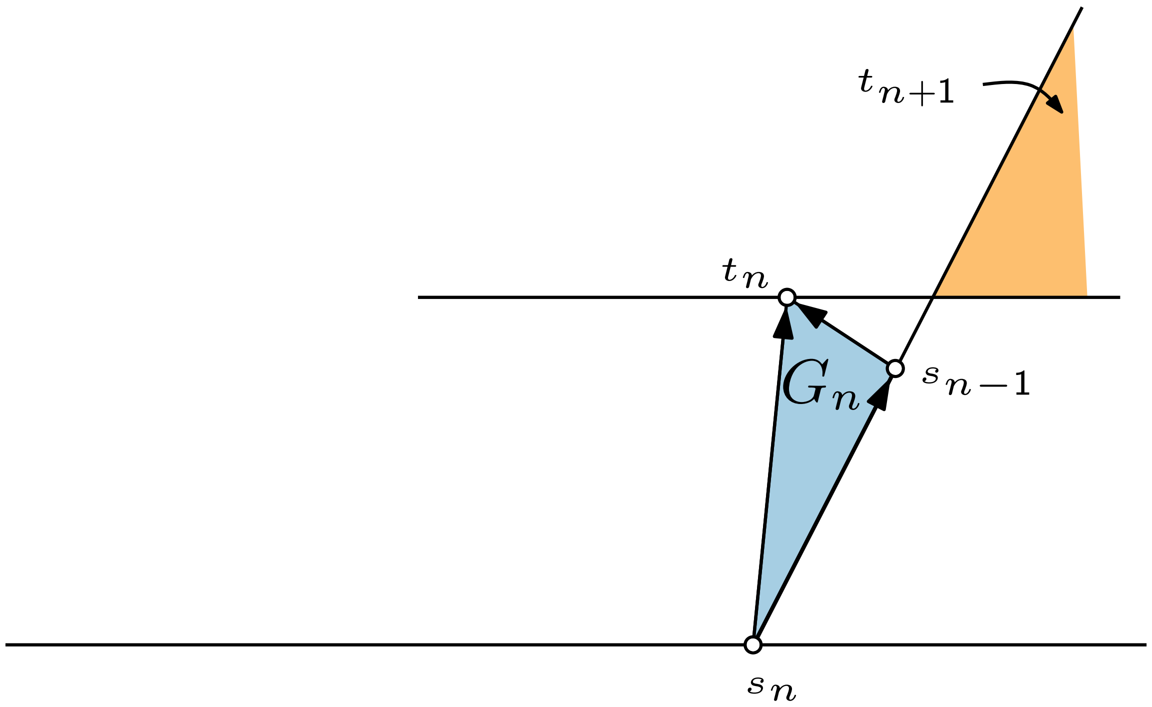

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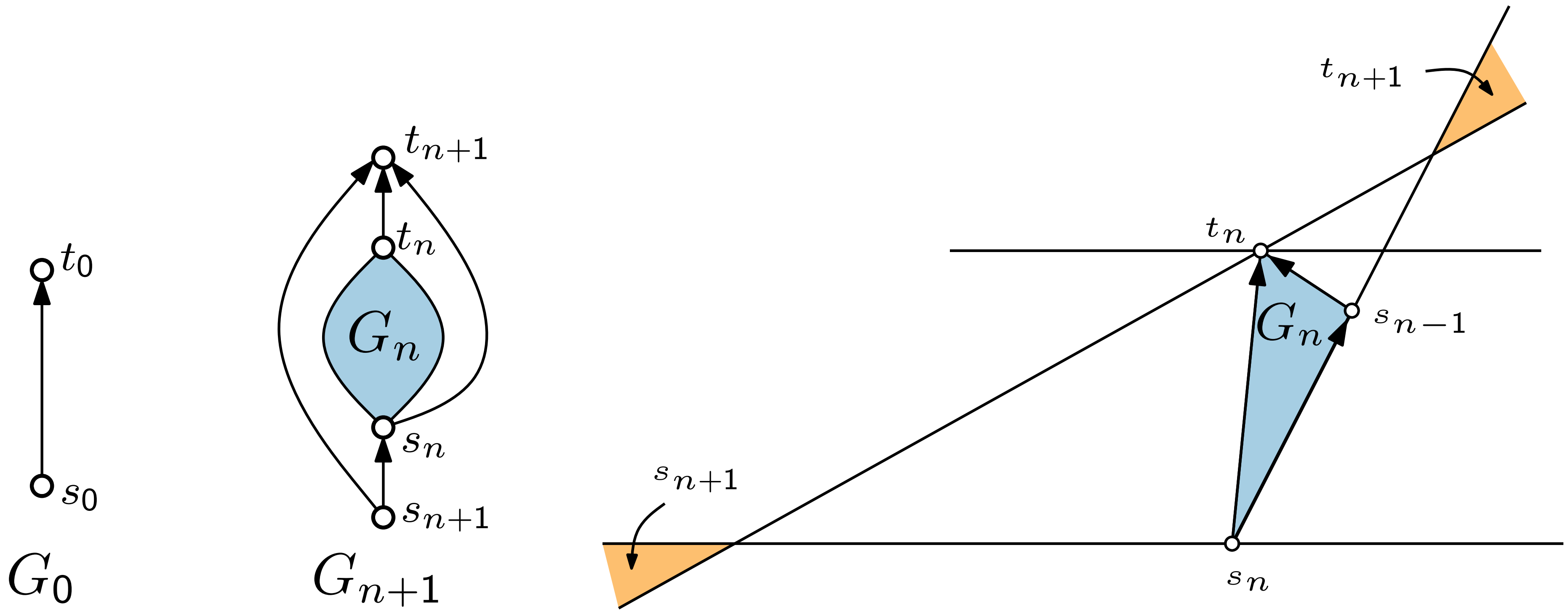

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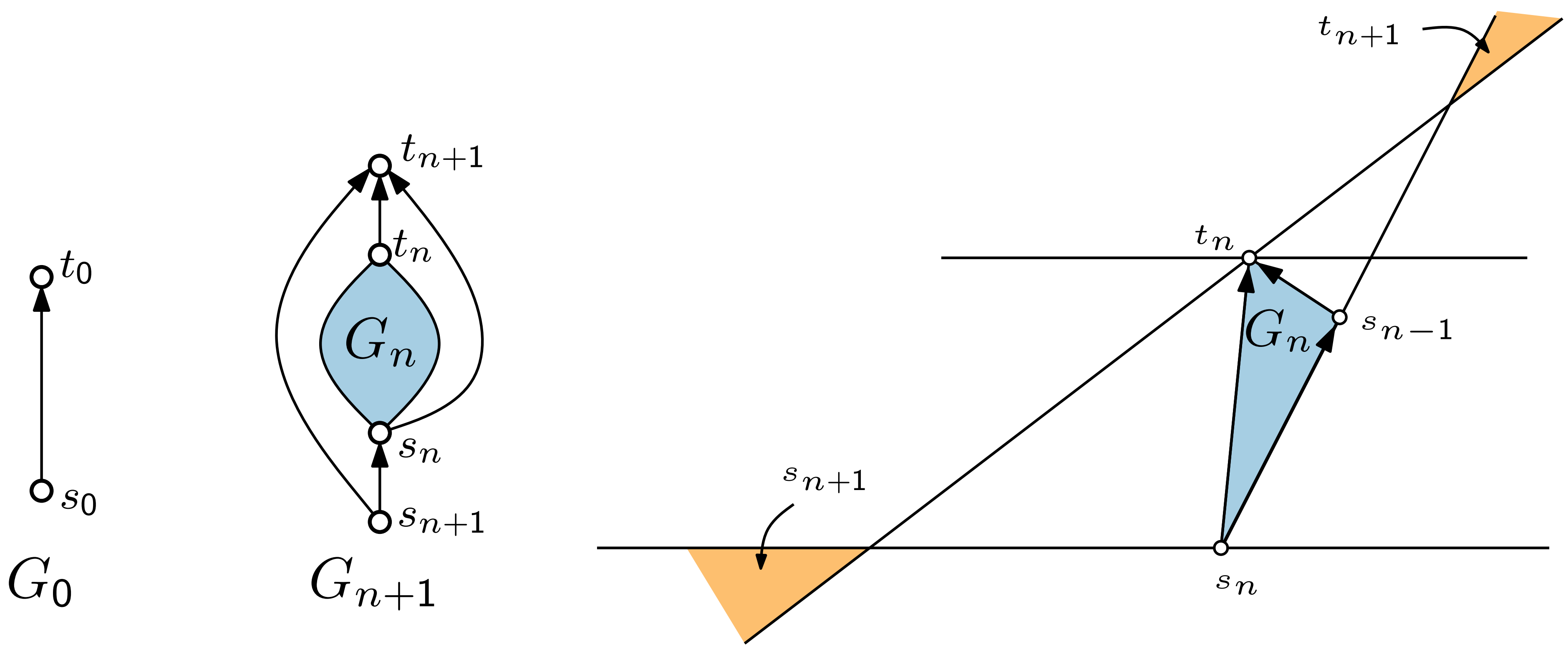

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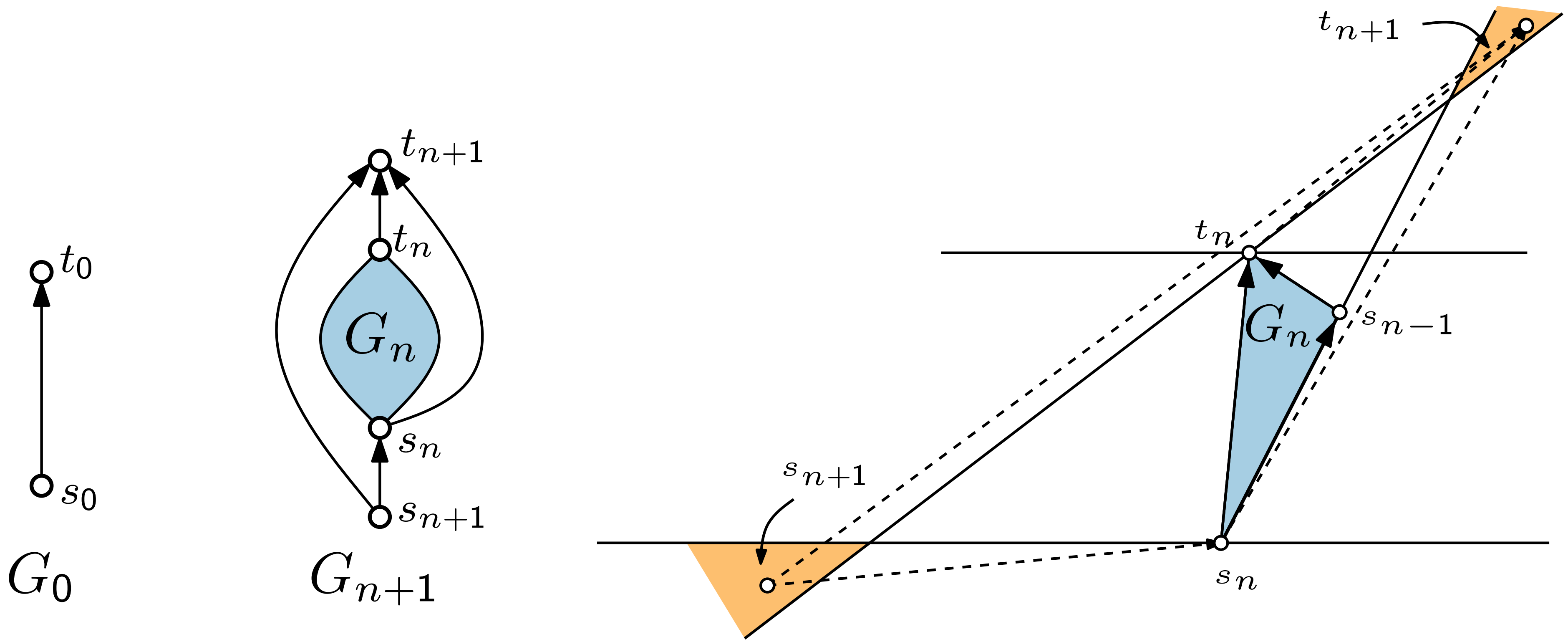

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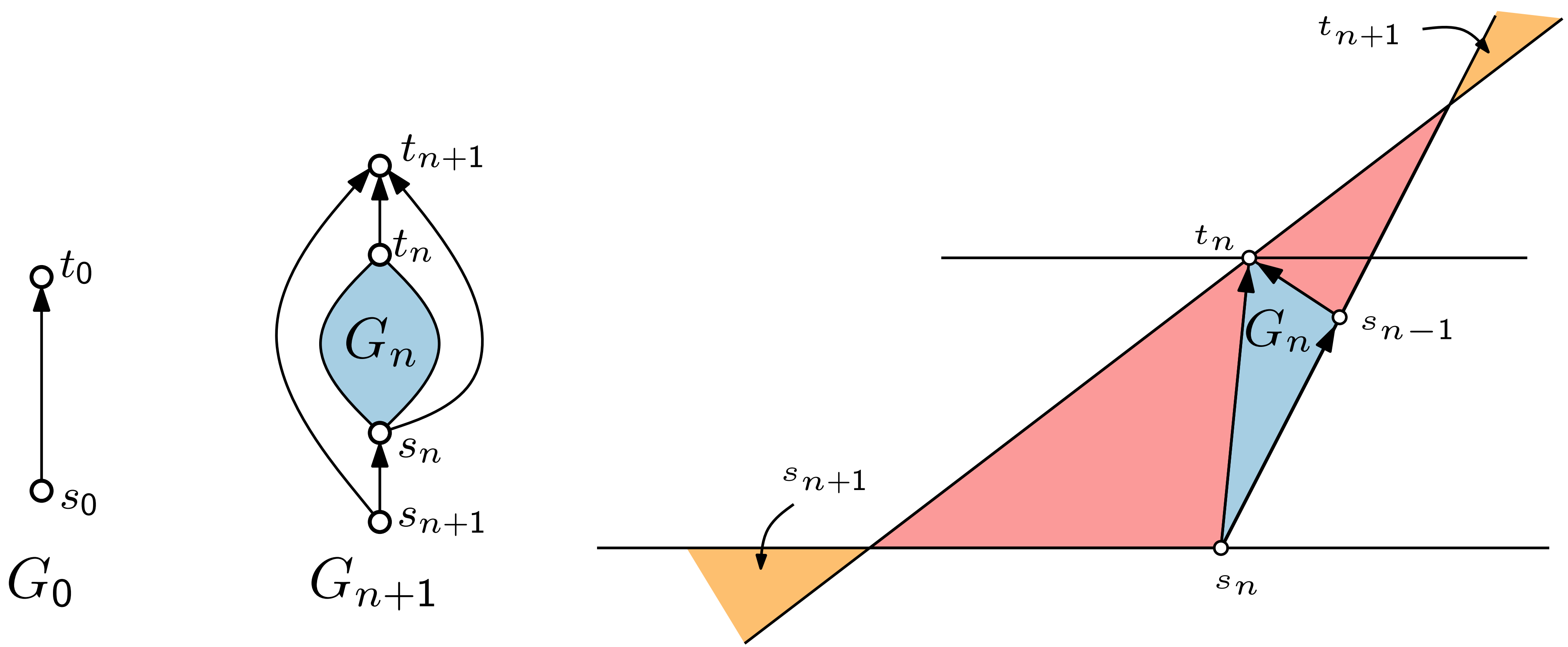

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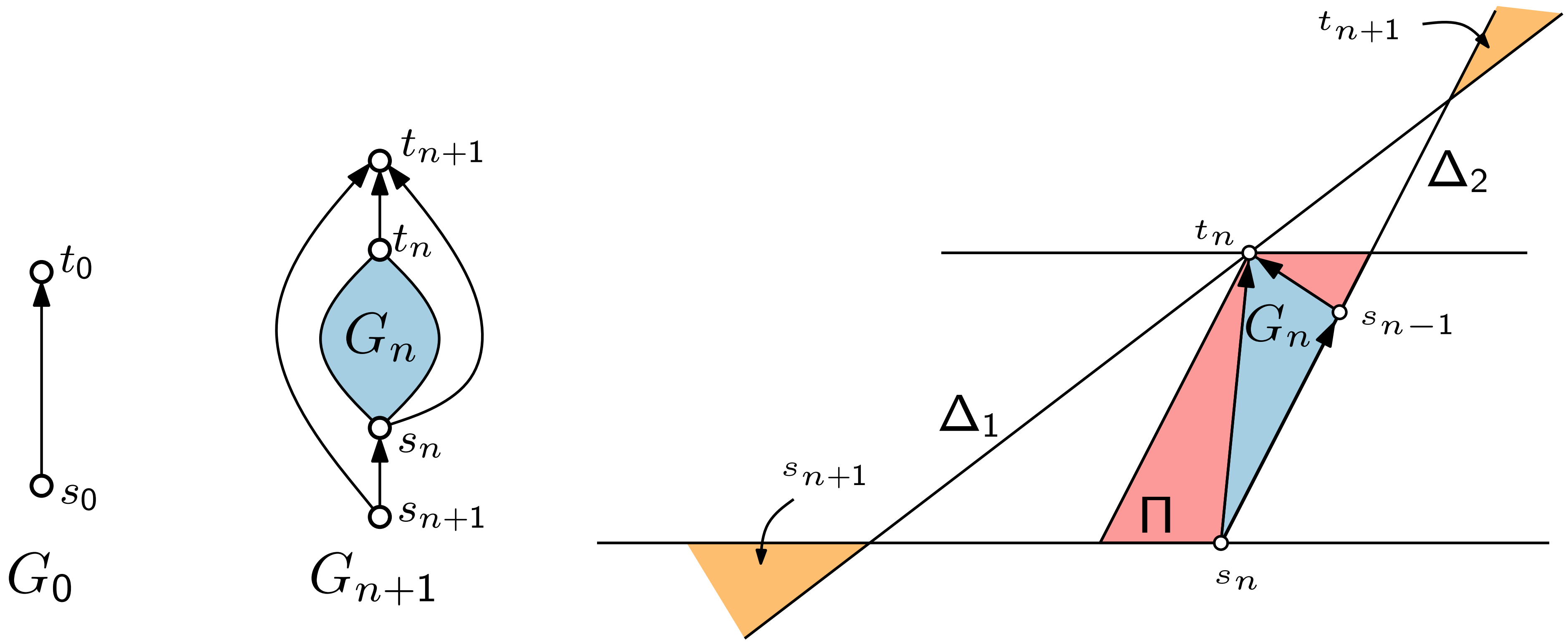

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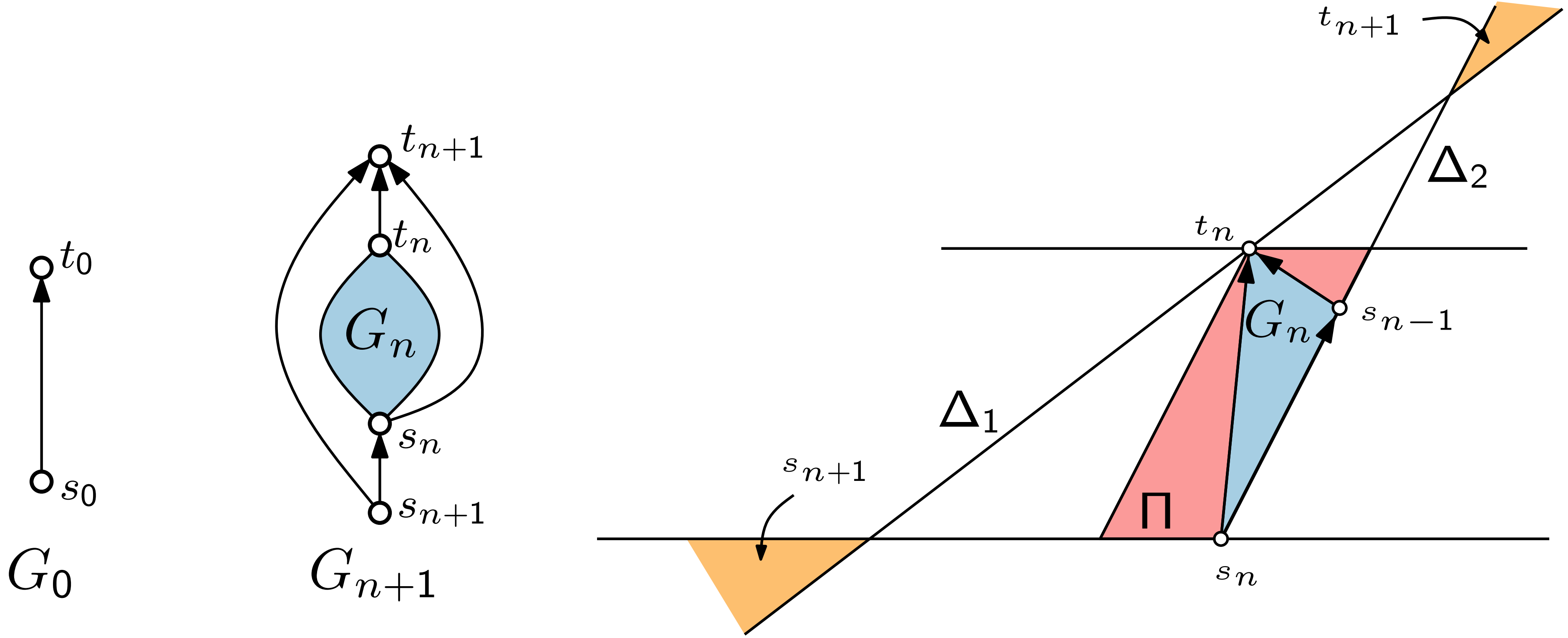
- $2 \cdot Area(G_n) < Area(\Pi)$



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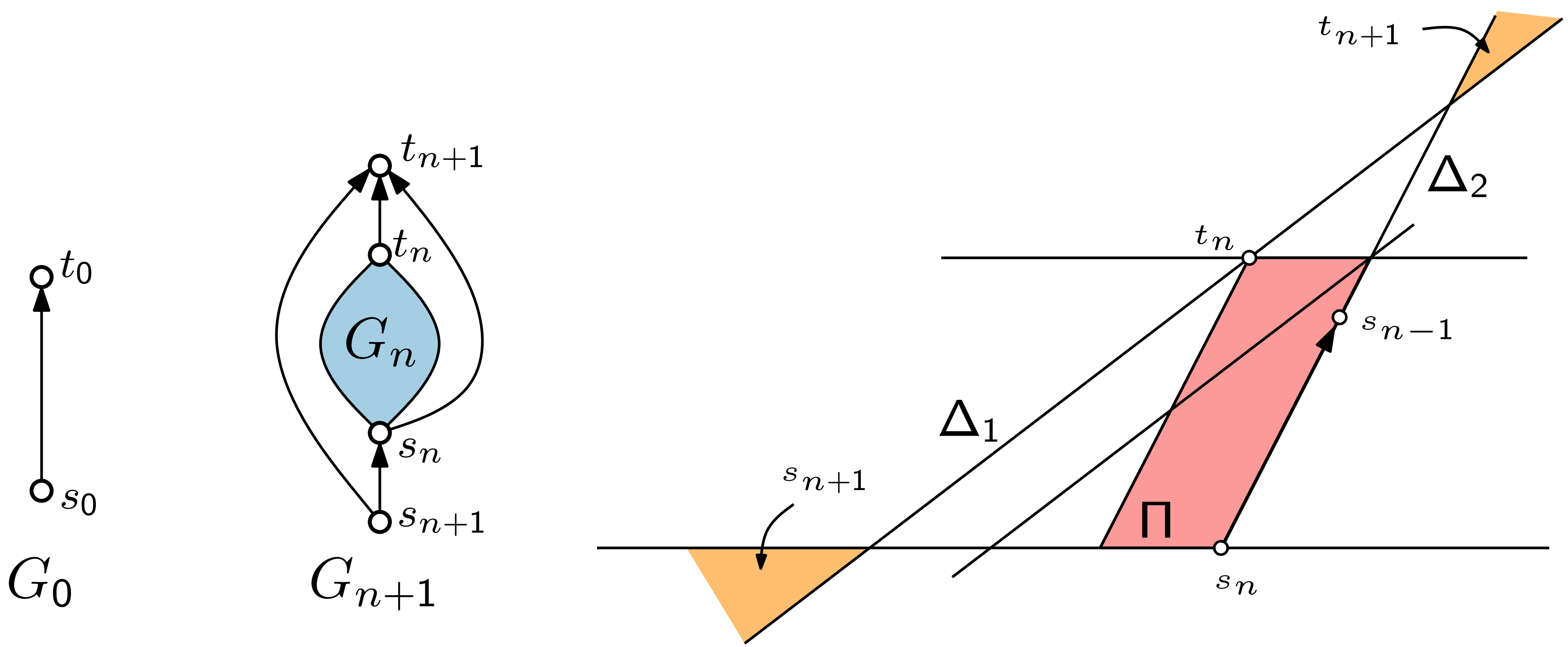

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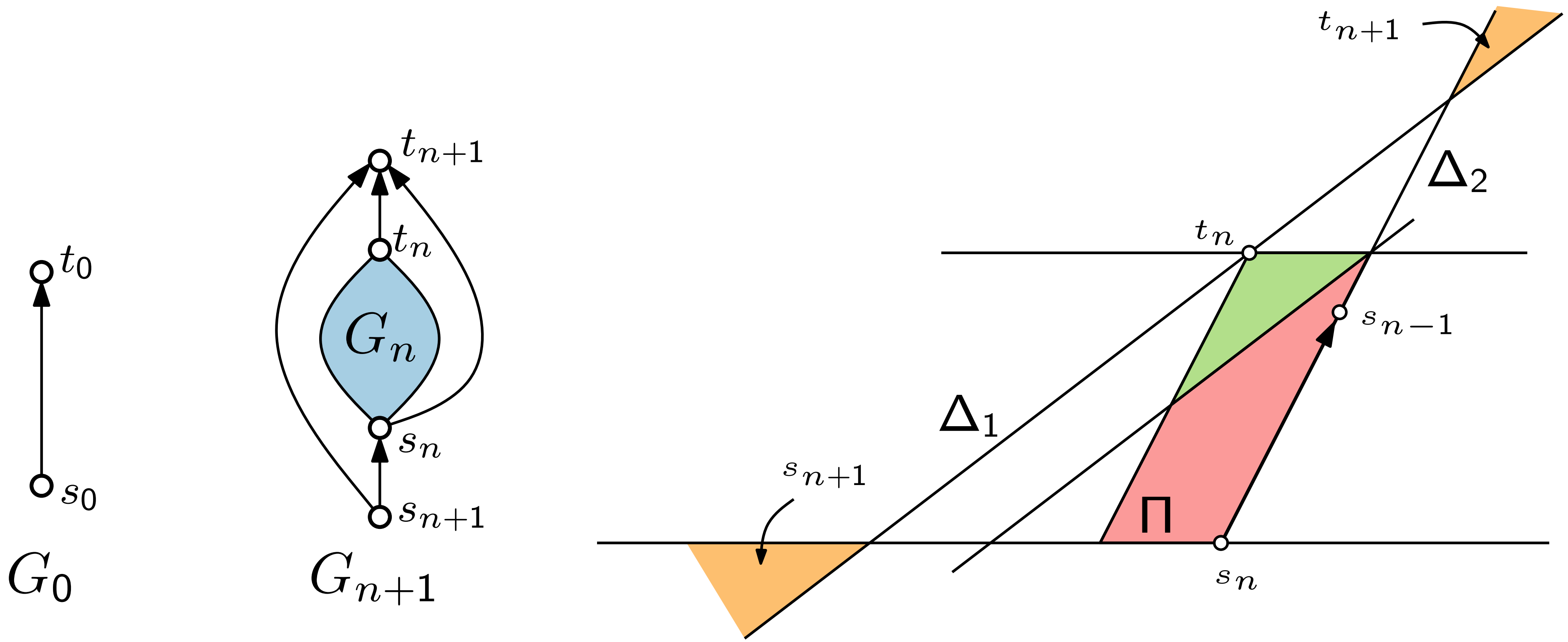

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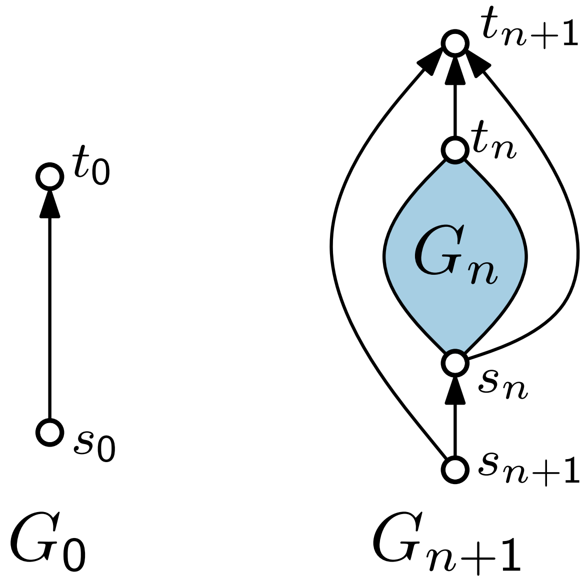

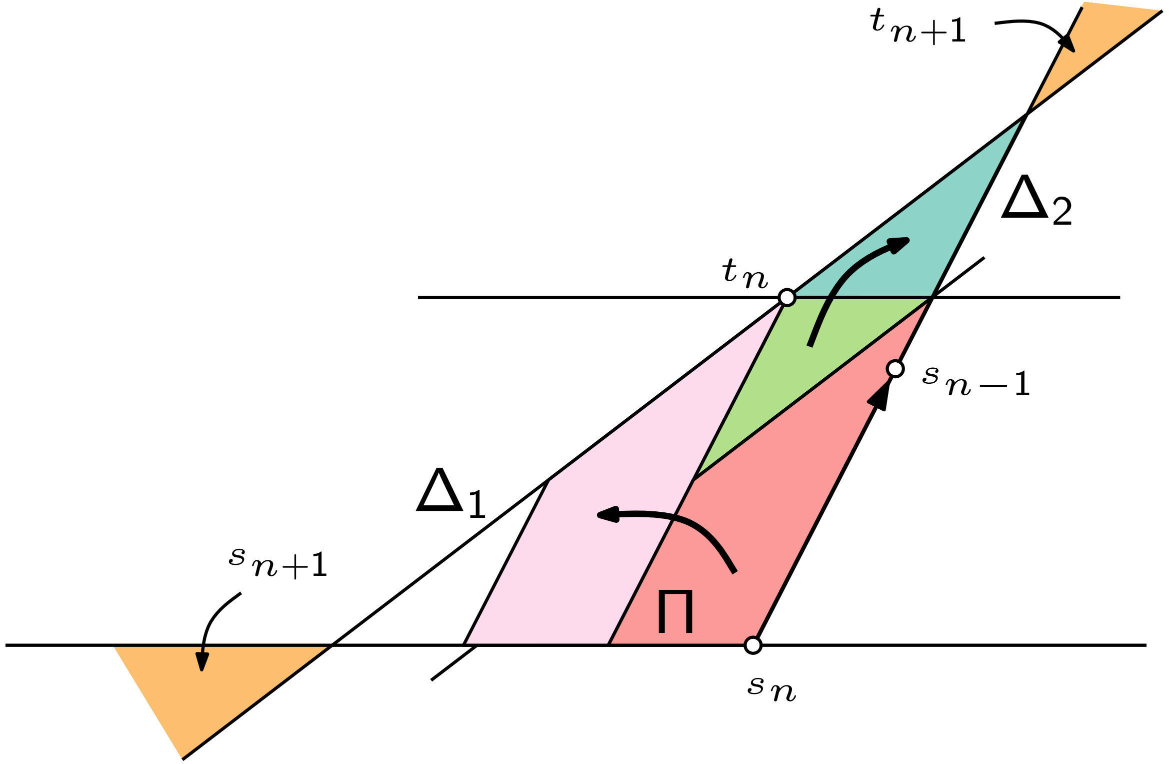

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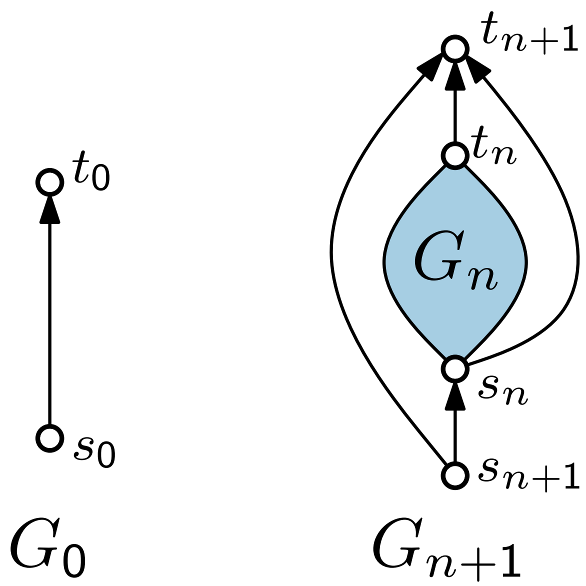

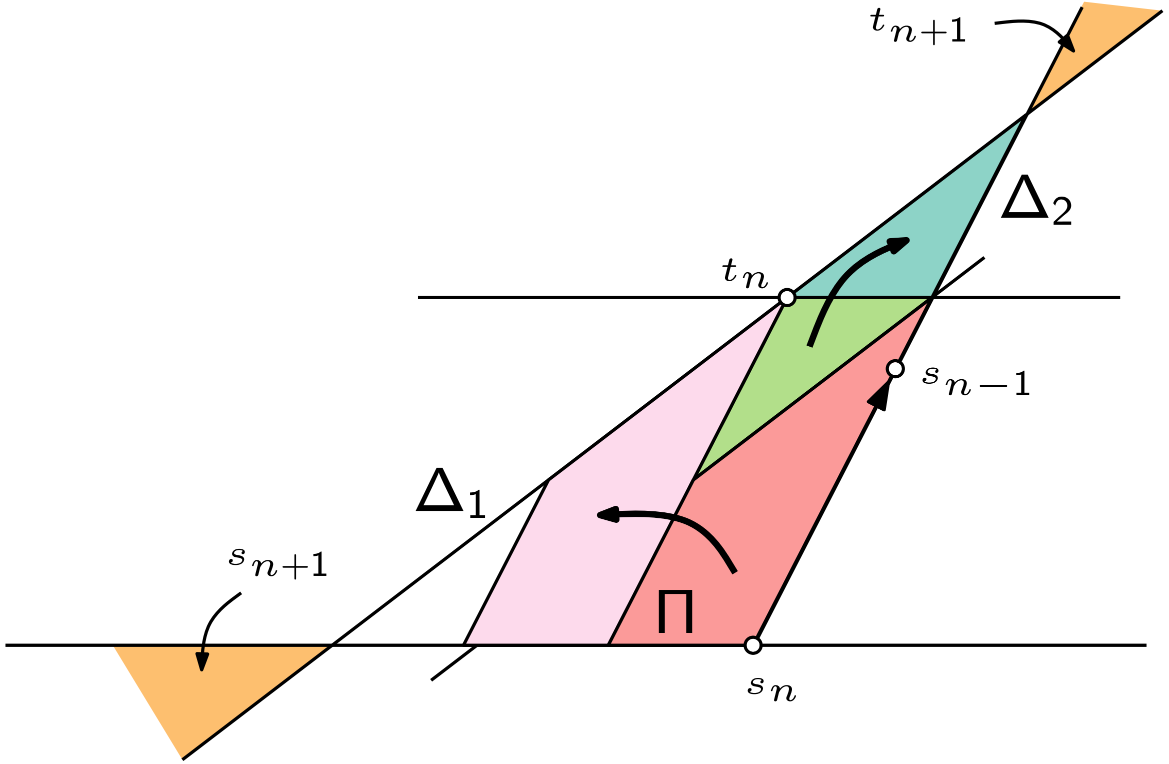

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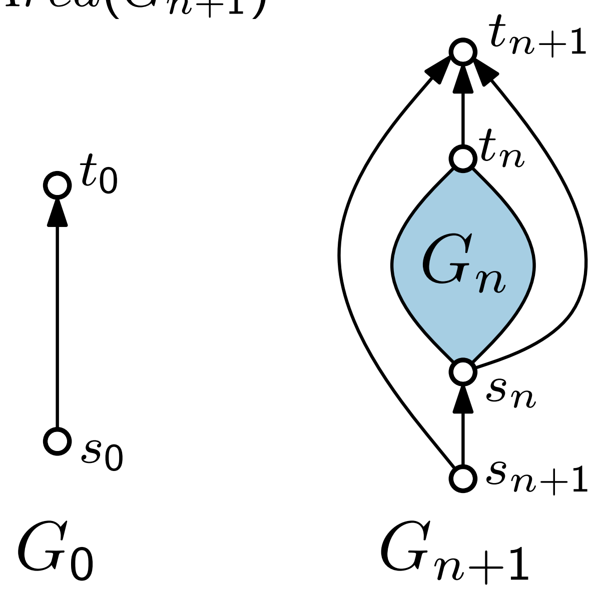

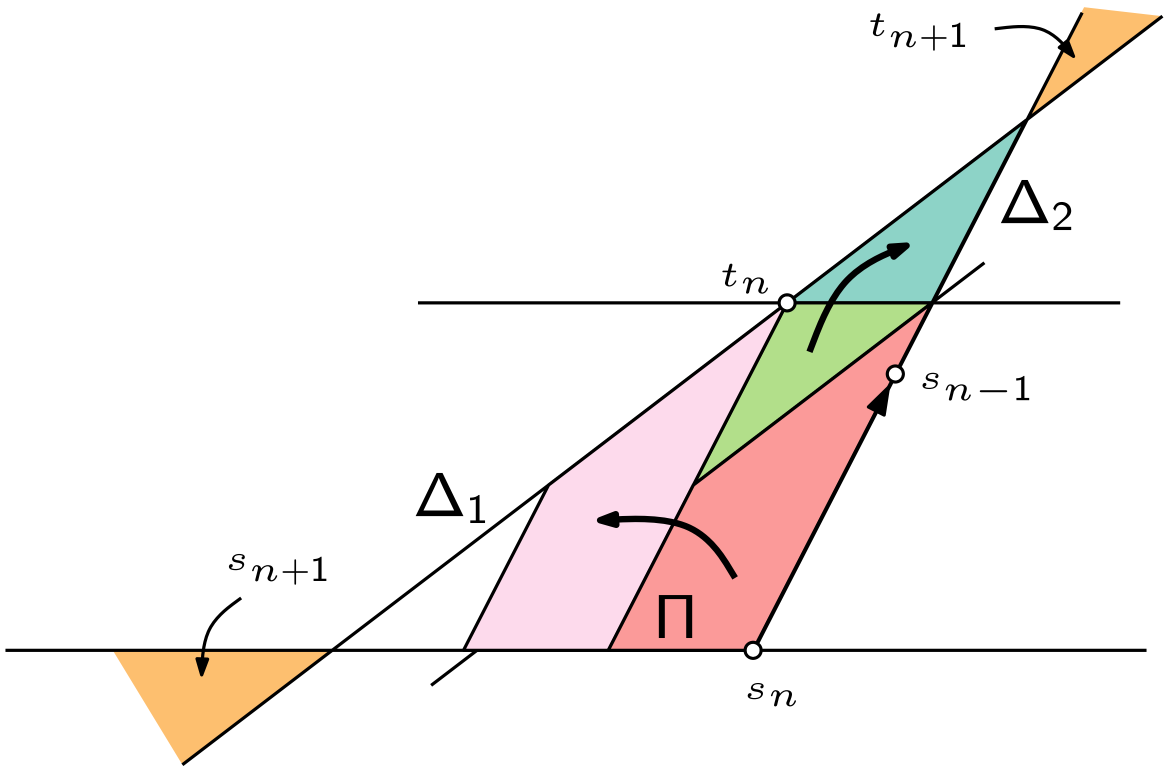

Literature

- [GD Chapter 3] for divide and conquer methods for rooted trees and series-parallel graphs
- [Reingold, Tilford '81] "Tidier Drawings of Trees"
  original paper for level-based layout algo
- [Reingold, Supowit '83] "The complexity of drawing trees nicely"
  NP-hardness proof for area minimisation & LP
- `treevis.net` – compendium of drawing methods for trees