Bonus Homework Assignment
Computational Geometry (Winter Semester 2020/21)

Exercise 1

Let S be a set of n points in the plane.

a) Suppose all points from S are collinear. Let r be a parameter with $1 \le r \le n$. Draw a fine simplicial partition for S of size r. What is the crossing number of your partition?

b) Now suppose that the points in S lie on a $\sqrt{n} \times \sqrt{n}$ grid. (Assume for simplicity that n is a square number.) Let r be a parameter with $1 \le r \le n$. Draw a fine simplicial partition for S of size r and crossing number $O(\sqrt{r})$.

Exercise 2

In this exercise we want to formally prove the size of a partition tree and analyze how the requirement of a fine simplicial partition influences the storage and query time.

a) Let $M(n)$ be the maximum number of nodes in a partition tree for a set of n points in the plane. $M(n)$ satisfies the following recurrence:

$$M(n) \le \begin{cases} 1 & \text{if } n = 1, \\ 1 + \sum_v M(|S(v)|) & \text{if } n > 1. \end{cases},$$

where we sum over all children v of the root of the tree. Prove that this recurrence solves to $M(n) \in O(n)$.

b) Suppose that we have a partition tree, except that the simplicial partitions used in the construction are not necessarily fine (but there must be at least one point in each triangle). What does this mean for the amount of storage used by the partition tree? And for its query time?

Exercise 3

Let T be a set of n triangles in the plane. An *inverse range counting* query asks to count the number of triangles from T containing a query point p.

   a) Let O and U be the set of upper and lower edges from T. Further, let P be the ray starting in the query point p and moving vertically upwards. Describe how to determine the desired number of triangles from the number of intersections between P and the triangle edges in O and U, respectively.

   b) Now design a data structure for inverse range counting queries by augmenting the multi-level partition trees from the lecture.

   c) Analyze the memory consumption and the query time of your data structure.

This assignment will not be graded. The solutions will be made available on February 5th.