Chair of Computer Science I
Algorithms and Complexity
University of Würzburg

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Homework Assignment #10
Computational Geometry (Winter Semester 2020/2021)

Exercise 1

Draw the Minkowski sum $P_1 \oplus P_2$ for the case where

a) both P_1 and P_2 are unit discs (radius 1); **[1 point]**

b) both P_1 and P_2 are unit squares (side length 1); **[1 point]**

c) P_1 is a unit disc and P_2 is a unit square; **[1 point]**

d) P_1 is a unit square and P_2 is a triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$. **[1 point]**

Exercise 2

Let P_1 and P_2 be two sets of points in the plane. Prove that the "shape" of the Minkowski sum $P_1 \oplus P_2$ is invariant under translations of P_1 and P_2. More precisely: Let P'_1 and P'_2 be two sets of points resulting from translating P_1 and P_2, respectively. Prove that the Minkowski sum $P'_1 \oplus P'_2$ results from translating $P_1 \oplus P_2$. **[3 points]**

Exercise 3

Let P_1 and P_2 be two convex polygons with vertex sets S_1 and S_2, respectively. Further, let $\mathrm{CH}(S)$ be the convex hull of a set of points S.

a) Prove that $\mathrm{CH}(S_1 \oplus S_2) \subseteq P_1 \oplus P_2$. **[3 points]**

b) Does even $\mathrm{CH}(S_1 \oplus S_2) = P_1 \oplus P_2$ hold? **[4 points]**

Exercise 4

In the lecture we gave an $O(n)$ bound on the complexity of the union of a set $\mathcal{P}$ of polygonal pseudodiscs with n vertices in total. We are interested in the precise bound.

a) Assume that the union boundary contains m original vertices of the polygons in $\mathcal{P}$. Show that the complexity of the union boundary is at most $2n-m$. Use this to prove an upper bound of $2n-3$ on the complexity of the union boundary. **[3 points]**

b) Prove a lower bound of $2n-6$ by constructing an example that has this complexity. **[3 points]**

This assignment is due on January 27th. The solutions will be made available on January 29th.