Lehrstuhl für Informatik I
Effiziente Algorithmen und
wissensbasierte Systeme
Universität Würzburg

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9th Exercise Sheet

Advanced Algorithms (WS20)

Exercise 1 – Rank structure

Suppose we want to build the rank structure for a bit string of length $n = 256$. Let A be the array that stores the cumulative rank of chunks and let B be the array that stores the cumulative rank of subchunks. Compute the lengths of A and B; show your calculations. **2 Points**

Explicitly write the first 10 rows of the lookup table for the bitstringts of length $\frac{1}{2}\log n$. One row may be build up like this:

$$\texttt{[bitstring]}\,\texttt{[rank(0)]}\,\texttt{[rank(1)]}\,\texttt{[rank(2)]}\ldots$$

You may write the query answers $\texttt{rank}(j)$ in decimal notation. **2 Points**

Give pseudocode for the following methods:

- $\texttt{predecessor}(i)$, which returns the index of the predecessor of the element indexed by i;
- $\texttt{successor}(i)$, which returns the index of the successor of the element index by i.

3 Points

Exercise 2 – The child operation on succinct binary trees

Consider the succinct representation of binary trees from the lecture. Let i be the index (position of its representative 1 in the bitstring) of a vertex v that has two children. Prove that the indices of the children of v are then given by $2\texttt{rank}(i)$ and $2\texttt{rank}(i) + 1$. **6 Points**

Exercise 3 – Operations on LOUDS

Consider the LOUDS representation of a tree from the lecture. Let i be the index of a vertex v (i.e. the position of its "1" in the LOUDS bitstring). Give pseudocode for the following methods, which should run in constant time:

- `outDegree`(i), which returns the outdegree of v;
- `childNum`(i, j), which returns the index of the j-th child of v if it exists and otherwise -1;
- `isRoot`(i), which returns whether vertex v is the (real) root; and
- `isLeaf`(i), which returns whether vertex v is a leaf.

Your pseudocode should not only contain the calculation, but also make sure the input is valid; for example, does i even have j children? **7 Points**

This assignment is due on January 18 at 10 am. Please submit your solutions via WueCampus. The exercises will be discussed in the tutorial session on January 18.