Lehrstuhl für Informatik I
Effiziente Algorithmen und
wissensbasierte Systeme
Universität Würzburg

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8$^{\text{th}}$ Exercise Sheet

Advanced Algorithms (WS20)

Exercise 1 – Ski Rental

Consider the following randomised strategy for the ski rental problem. We throw a fair coin. If it shows HEAD we buy skis after $M/2$ days, otherwise we buy skis after M days.

   a) Show that this strategy is in expectation not better than the 2-competitive deterministic ones. **4 Points**

Now we change our strategy and buy skis after $M/2$ days only if we throw HEAD twice in a row and otherwise we buy skis after M days.

   b) What is the expected competitive ratio of this strategy? **4 Points**

Exercise 2 – Online Bin Packing

In the offline bin packing problem, you are given a set of items each with a size in $[0, 1]$. The goal is to pack these items into bins of size 1, such that no bin is overpacked and the total number of used bins is minimized. In the online version, the items arrive in a stream-like fashion. Each item has to be packed immediately and later repacking is not allowed. We consider the *FirstFit* strategy. Here, the item is packed in the first bin where it still fits. If it fits in no bin, a new bin is opened.

Prove that FirstFit is 2-competitive (up to some additive constant). Show with an example that it can not be better than 3/2-competitive. **5 Points**

Exercise 3 – Online tree colouring game

In the game of online tree colouring, a *forest warden* and a *painter* play against each other. The game starts with an empty graph G_0. In turn i, the forest warden adds a vertex v_i to G_{i-1} and any edges from v_i to vertices in G_{i_1}. The resulting graph G_i has to be a forest. The painter than has to assign a colour to v_i such that G_i is properly coloured, that is, no two adjacent vertices have the same colour.

a) Show that there is a strategy with which the forest warden can enforce an arbitrarily large number of colours. **4 Points**

b) What is the best the forest warden can hope for, if the degree of vertex v_i in G_i may be at most k? **1 Point**

This assignment is due on January 11 at 10 am. Please submit your solutions via WueCampus. The exercises will be discussed in the tutorial session on January 11.