

## Homework Assignment #8

### Computational Geometry (Winter Semester 2020/2021)

#### Exercise 1

In this exercise, we look at the number of different triangulations that a set of  $n$  points in the plane admits.

- Prove that no set of  $n$  points can be triangulated in more than  $2^{\binom{n}{2}}$  ways. **[2 points]**
- Can you give an (asymptotically) sharper upper bound on the number of possible triangulations? **[2 extrapoints]**
- Prove that there are sets of  $n$  points that can be triangulated in at least  $2^{n-2\sqrt{n}+1}$  different ways. **[4 points]**
- Can you give an example that the points can be triangulated in more different ways (asymptotically)? **[2 extrapoints]**

#### Exercise 2

Prove that any two triangulations of a planar point set can be transformed into each other by edge flips.

*Hint:* Start by showing that any two triangulations of a convex polygon can be transformed into each other by edge flips. **[5 extrapoints]**

#### Exercise 3

Prove that the smallest angle of any triangulation of a convex polygon whose vertices lie on a circle is the same. This implies that *any* completion of the Delaunay *graph* of a set of points to a Delaunay *triangulation* maximizes the minimum angle.

*Hint:* You can use the result of Exercise 2. **[4 points]**

#### Exercise 4

Let  $P$  be a set of points in the plane and let  $G_P = (P, E_P)$  be the graph with  $\{p, q\} \in E_P$  if and only if  $p$  and  $q$  are the only points in  $P$  that are contained inside the disk  $D_{pq}$  with diameter  $\overline{pq}$ , i.e.,  $D_{pq} \cap P = \{p, q\}$ .

- a) Prove that the Delaunay graph of  $P$  contains the graph  $G_P$ . **[3 points]**
- b) Prove that  $\{p, q\}$  is an edge of  $G_P$  if and only if the Delaunay edge between  $p$  and  $q$  intersects its dual Voronoi edge. **[3 points]**
- c) Show how to compute  $G_P$  for a set  $P$  of  $n$  points in  $O(n \log n)$  time. **[4 points]**

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This assignment is due on January 13th. The solutions will be made available on January 15th.