7$^{\text{th}}$ Exercise Sheet

Advanced Algorithms (WS20)

Exercise 1 – Fair coins and fair dice

We have a coin that shows tails with probably p and heads with probably $1 - p$. Suppose the coin is biased, that is, $p \neq 0.5$

- Describe a strategy to get a fair result from the biased coin.   **1 Point**

- Can we use an analogous strategy to make a biased k-sided die fair? Why or why not? What practical problem would we run into?   **2 Points**

Exercise 2 – Randomised 3-colouring

Given a graph $G = (V, E)$ and the three colours red, green, and blue. Colouring the vertices, our goal is to maximise the number of bichromatic edges, that is, edges whose incident vertices have a different colour. Design a Monte Carlo algorithm and analyse the expected number of bichromatic edges.   **3 Points**

Exercise 3 – FindMinSlowly

Consider the following (inefficient) algorithm to compute the minimum of a set S of pairwise distinct integers: If $|S| = 1$, return its unique element. Otherwise, pick a random element x and recursively compute the minimum x' of $S \setminus \{x\}$. If $x' < x$, return x'. Otherwise, check if x is smaller than all elements in $S \setminus \{x\}$ and if so, return x.

Is the algorithm correct and guaranteed to terminate? What is the worst-case running time of the algorithm? What is the expected running time of the algorithm?   **6 Points**

Exercise 4 – Packing argument

Let δ be a positive real number. Show that a $\delta \times \delta$ square can contain at most $\mathcal{O}(1)$ points such that the distance between any pair of these points is at least δ.   **3 Points**

Exercise 5 – Random permutation

Suppose you are given an array A of length n and you want to compute a random permutation of its elements. How can you do this with a fair coin (that can generate a random bit in $\mathcal{O}(1)$ time)? Describe an algorithm, why it works, and the (expected) running time.   **5 Points**

This assignment is due on December 21 at 10 am. Please submit your solutions via WueCampus. The exercises will be discussed in the tutorial session on December 21.