Lehrstuhl für Informatik I
Algorithmen und Komplexität
Universität Würzburg

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5th Exercise Sheet

Advanced Algorithms (WS20)

Exercise 1 – Pathwidth

In this exercise, we study the pathwidth of some basic graph classes.

a) Show that the pathwidth of a simple cycle is at most 2. **1 Point**

b) A *caterpillar* is a tree in which there exists a path that contains every vertex of degree two or more. Show that caterpillars have pathwidth at most 1 (and exactly 1, if they contain at least one edge). **2 Points**

c) Prove the existence of trees that have pathwidth at least 2. **3 Points**

d) Show that the pathwidth of a $k \times N$ grid graph is at most k. **4 Points**

Exercise 2 – Nice path decompositions

Let $G = (V, E)$ be a graph. Suppose you are given a path decomposition $P = (X_1, X_2, \ldots, X_r)$ of G with width p. Show that it can be turned into a *nice* path decomposition of G with width p in $\mathcal{O}(p^2 \cdot \max\{r, |V|\})$ time. **5 Points**

Exercise 3 – MaxCut in graphs of bounded pathwidth

In the fourth lecture, we studied approximation algorithms for the MaxCut problem.

Show that the unweighted version of this problem (where all edge weights are 1) is FPT with respect to the pathwidth. You may assume that you are given a nice path decomposition of width k. **5 Points**

This assignment is due on December 7 at 10 am. Please submit your solutions via WueCampus. The exercises on this assignment will be discussed in the tutorial session on December 7.