

3rd Exercise Sheet

Advanced Algorithms (WS20)

Exercise 1 – Lucky vertex order for Greedy Vertex Coloring

Show that there exists an ordering v_1, \dots, v_n of the vertices of G such that the algorithm `GREEDYVERTEXCOLORING` from the lecture computes an optimal solution. (Finding such an order remains of course hard.) **5 Points**

Exercise 2 – Min Vertex Coloring in planar graphs

We consider the subproblem of Min Vertex Coloring where the input is restricted to planar graphs. By the famous four color theorem every planar graph can be colored with just four colors. Eventhough the proof involves a case distinction with 1476 cases (or 633 in an improved version), the proof is constructive and gives a polynomial-time algorithm that colors every planar graph with four colors. However, the decision problem of whether a planar graph is 3-colorable is NP-complete.

Using the 4-color algorithm as a black box, describe an approximation algorithm for planar graphs with additive approximation guarantee 1 that also computes a coloring. **4 Points**

Exercise 3 – Nearest Addition Algorithm for Metric TSP

Prove that the `NEARESTADDITIONALGORITHM` from the lecture is a 2-approximation algorithm for Metric TSP.

Hint: Take a look at Prim's algorithm.

5 Points

Exercise 4 – Non-approximability of Multiprocessor Scheduling

Show that there can be no approximation algorithm \mathcal{A} for `MULTIPROCESSORSCHEDULING`, with $|\mathcal{A}(I) - \text{OPT}(I)| \leq k$ for a constant k (unless $P = NP$).

Hint 1: How could such an algorithm, if it would exist, be used to solve this NP-hard problem?

Hint 2: Scale the problem instance.

6 Points

This assignment is due on November 23 at 9 am in the morning. Please submit your solutions via VueCampus. The exercises on this assignment will be discussed in the tutorial session on November 23.