

1st Exercise Sheet

Advanced Algorithms (WS20)

Exercise 1 – Flow problem with vertex capacities

Let $G = (V, E)$ be a flow network where the capacity function c is not only defined on the edges but also on the vertices. More precisely, for a flow f in G there is a capacity constraint $\sum_{u \in V} f(u, v) = \sum_{u \in V} f(v, u) \leq c(v)$ for each $v \in V$.

Show that a maximum flow problem on a flow network with vertex and edge capacities can be reduced to a maximum flow problem in a normal flow network G' , i.e. without vertex capacities. How many more vertices and edges does G' require precisely? **3 Points**

Exercise 2 – Implementing the generic push-relabel algorithm

In the lecture we have only outlined how the methods `PUSH` and `RELABEL` work and not discussed how we actually find the next applicable operation at all. Show how to implement `PUSH` in $\mathcal{O}(1)$ time, `RELABEL` in $\mathcal{O}(|V|)$, and how to select an applicable operation in $\mathcal{O}(1)$ time. In particular, what data structure would you use to maintain overflowing vertices?

6 Points

Using these implementations, what is the resulting runtime of the push-relabel algorithm?

1 Point

Exercise 3 – Maximum bipartite matching

A *matching* in a graph $G = (V, E)$ is a subset of edges $M \subset E$ such that for all $v \in V$, at most one edge of M is incident to v . A *maximum matching* is a matching with maximum cardinality, that is, a matching M such that for any matching M' , we have $|M| \geq |M'|$. A *bipartite* graph $G = (L, R, E)$ has two vertex sets L and R and each edge in E connects a vertex in L to a vertex in R .

- Explain how we can transform the problem of finding a maximum matching in a bipartite graph $G = (L, R, E)$ into a maximum flow problem on a flow network N .
- Prove that the cardinality of a maximum matching M in G equals the value of a maximum flow f in your constructed flow network N .

5 Points

Exercise 4 – From maximum flow to minimum cut

Give an efficient push-relabel algorithm to find a maximum matching in a bipartite graph $G = (L, R, E)$ and analyze it. Aim for a running time in $\mathcal{O}(|V|^2 + |V||E|)$. **5 Points**

This assignment is due at the beginning of the next lecture, that is, on November 9 at 10:15. Please submit your solutions via WueCampus. The exercises on this assignment will be discussed in the first tutorial session.