Julius-Maximilians-UNIVERSITÄT WÜRZBURG

Lehrstuhl für INFORMATIK I Algorithmen & Komplexität

Institut für Informatik

Exakte Algorithmen

Lecture 9.2 Reductions and the $W[t]$ Hierarchy

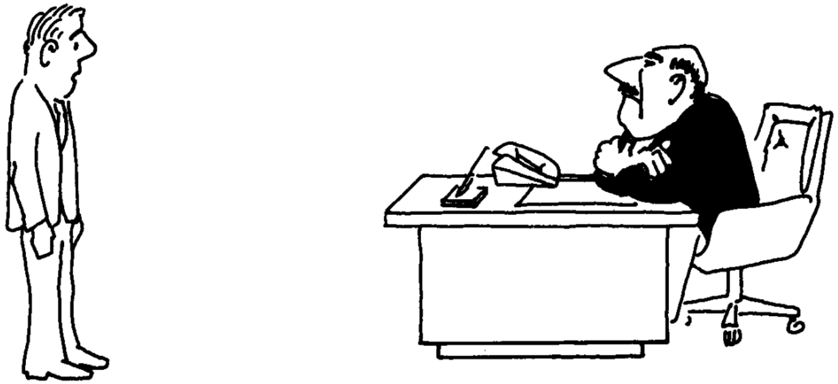
Based on: [Parameterized Algorithms: §13]

(slides by Thomas Bläsius)

Alexander Wolff

Lehrstuhl für Informatik I

How to obtain lower bounds?



“I can’t find an efficient algorithm, I guess I’m just too dumb.”

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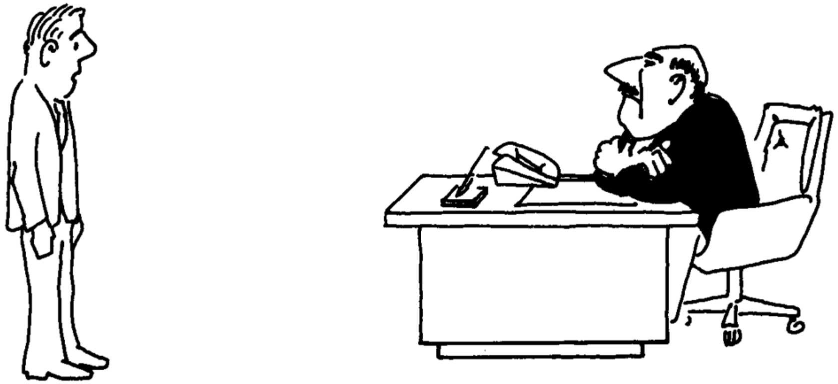

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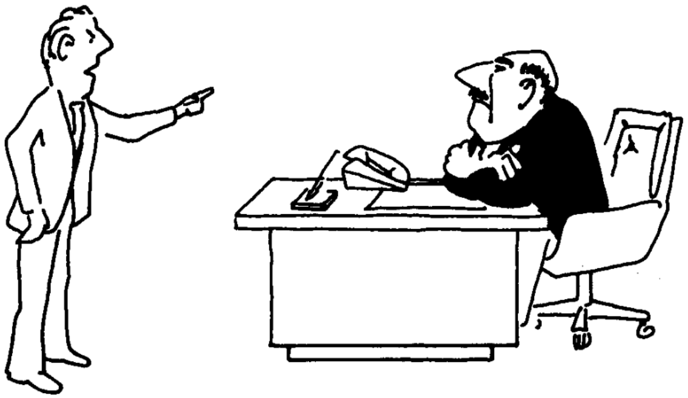

"I can't find an efficient algorithm, because no such algorithm is possible!"

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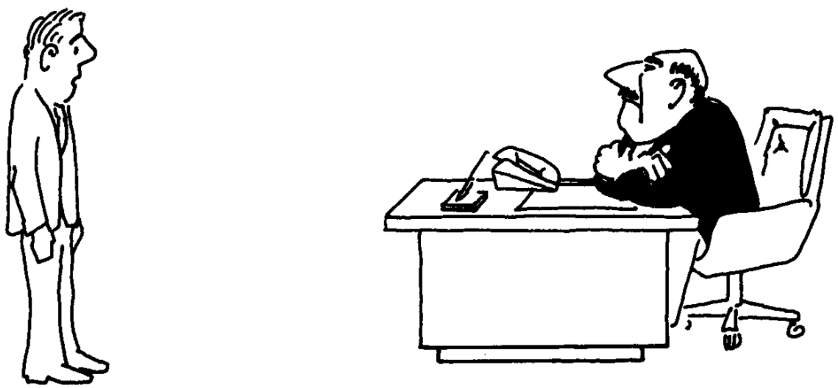

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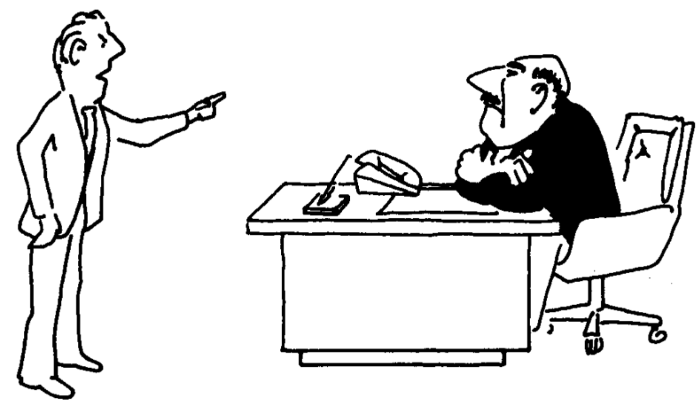

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Problem

• difficult to show absence of “nice” algorithms

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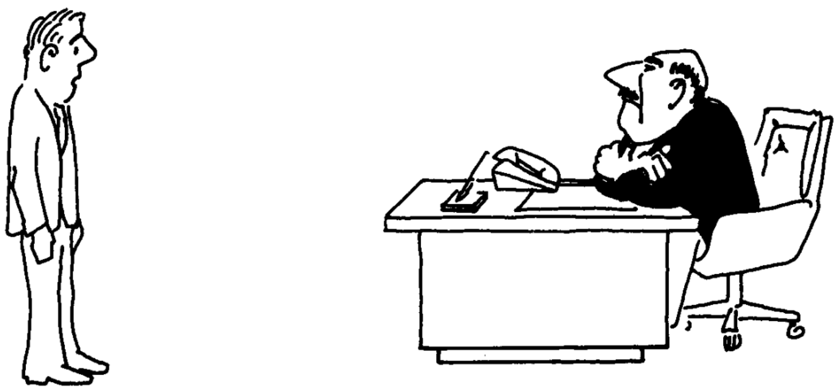

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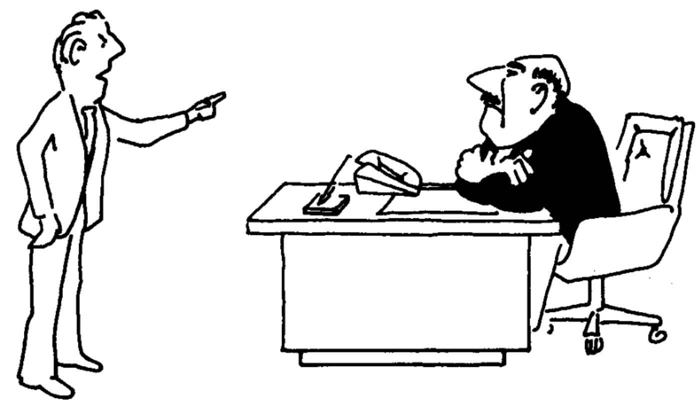

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- difficult to show absence of "nice" algorithms

Solution

- argue that having an efficient algorithm⇒ some well studied diffcult problem can be solved efficiently

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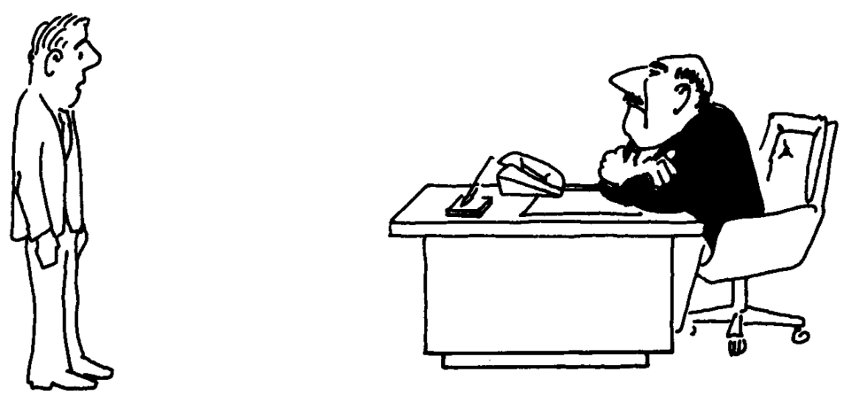

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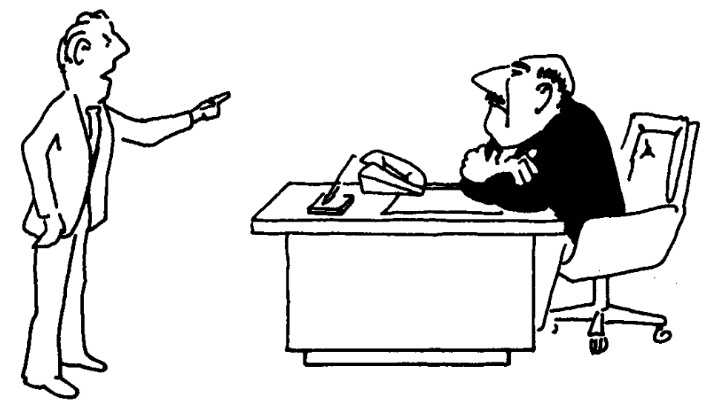


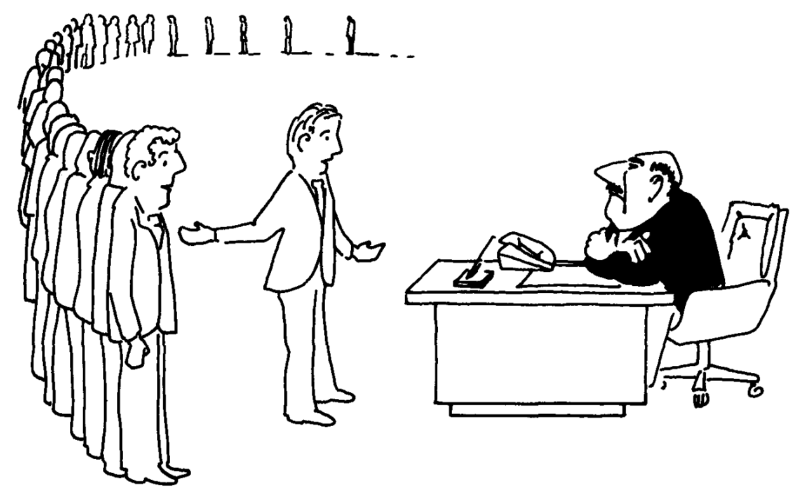
"I can't find an efficient algorithm, because no such algorithm is possible!"

Problem

- difficult to show absence of "nice" algorithms

Solution

- argue that having an efficient algorithm$\Rightarrow$ some well studied diffcult problem can be solved efficiently
- Tool: reduction between problems



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Polynomial Reduction

Reduction from Problem $\mathcal{L}$ to Problem $\mathcal{L}'$

- map each instance I of $\mathcal{L}$ to an instance I' of $\mathcal{L}'$ so that
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- polynomial time algorithm for $\mathcal{L}'$ $\Rightarrow$ polynomial time algorithm for $\mathcal{L}$

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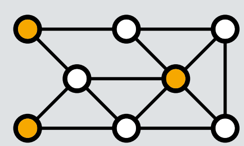
Easy Example

- reduce INDEPENDENT SET to VERTEX COVER

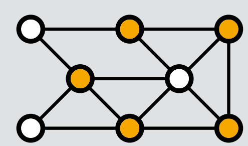
INDEPENDENT SET



$V' \subseteq V$, so that

$\forall e \in E\colon |e \cap V'| \leq 1$

VERTEX COVER



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- reduce Independent Set to Vertex Cover
- trivial reduction: G has IS of size k $\Leftrightarrow$ G has VC of size $n - k$

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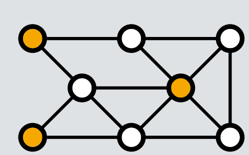

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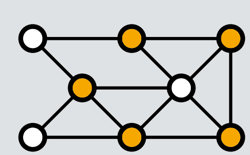

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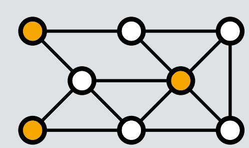
- also: VC $\in$ P $\Rightarrow$ IS $\in$ P

INDEPENDENT SET



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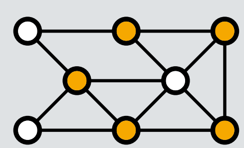

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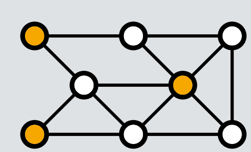
- what about VC $\in$ FPT $\Rightarrow$ IS $\in$ FPT?

Independent Set



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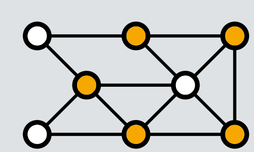

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  $\rightarrow$ NO, the parameter depends on n

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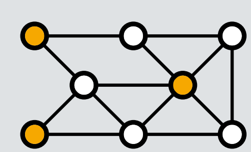

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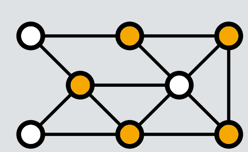

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  $\rightarrow$ NO, the parameter depends on n
- for "efficient" $\sim$ "FPT" we need a different type of reduction

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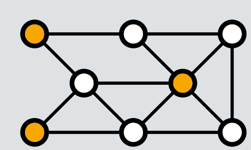

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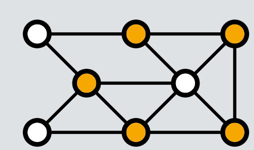

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Reduction from Problem $\mathcal{L}$ to Problem $\mathcal{L}'$

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(f and g must also be computable functions)

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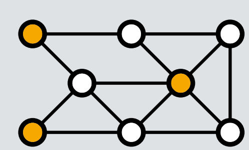
Easy Example

- reduce Independent Set to Clique

Independent Set



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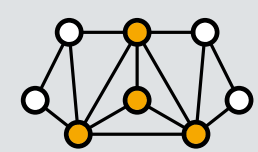
Clique



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$G[V']$ complete

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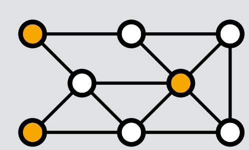
- G has a size k IS $\Leftrightarrow$ edge-complement of G has a size k Clique.

Independent Set



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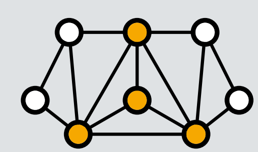

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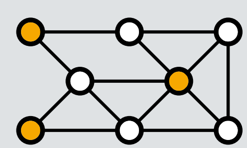

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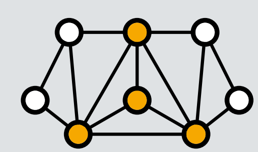

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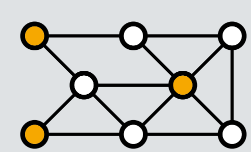
- reduce INDEPENDENT SET to CLIQUE
- G has a size k IS $\Leftrightarrow$ edge-complement of G has a size k Clique.
- also: CLIQUE $\in$ FPT $\Rightarrow$ IS $\in$ FPT
- reverse reduction also applies:
  CLIQUE $\in$ FPT $\Leftrightarrow$ IS $\in$ FPT

INDEPENDENT SET



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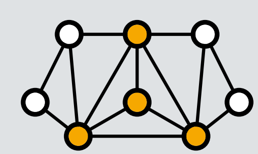

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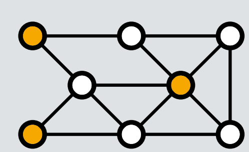
- reverse reduction also applies:
  CLIQUE $\in$ FPT $\Leftrightarrow$ IS $\in$ FPT
- expectation: CLIQUE, IS $\notin$ FPT

INDEPENDENT SET



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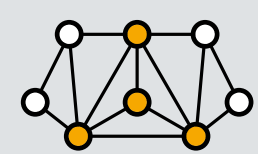
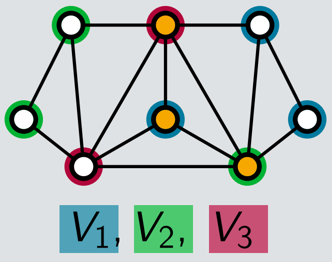
CLIQUE



$V' \subseteq V$, so that
$G[V']$ complete

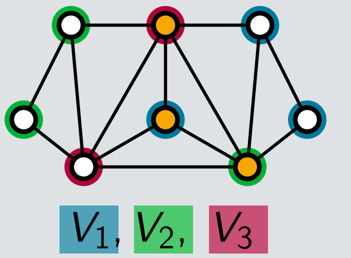
Colored Cliques

Problem: Multicolored Clique
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.

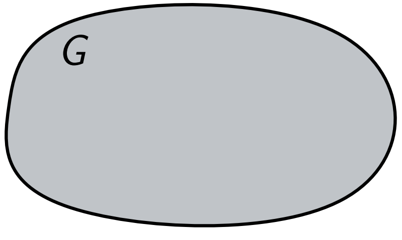


Colored Cliques

Problem: **MULTICOLORED CLIQUE**
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



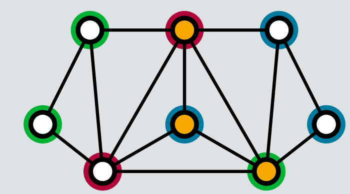
Reduction: from CLIQUE to MULTICOLORED CLIQUE



Instance $(G, 3)$ of CLIQUE

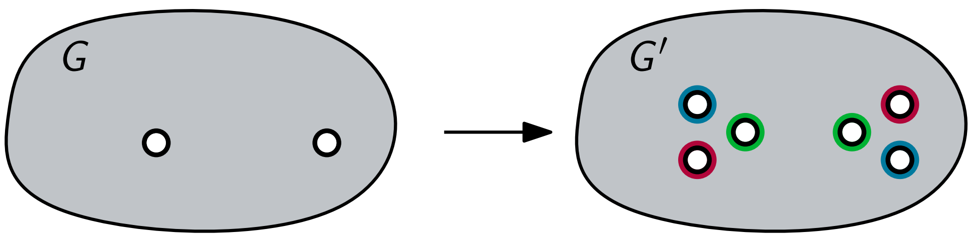
Colored Cliques

Problem: MULTICOLORED CLIQUE
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



V_1, V_2, V_3

Reduction: from CLIQUE to MULTICOLORED CLIQUE



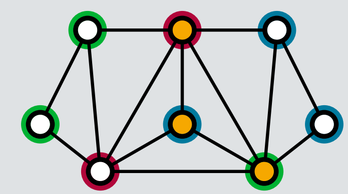
Instance $(G, 3)$ of CLIQUE

Instance of MC CLIQUE: $(G', 3, (V_1, V_2, V_3))$

- copy each $v \in V$ to $v^1, \ldots, v^k$, and set $v^i \in V_i$

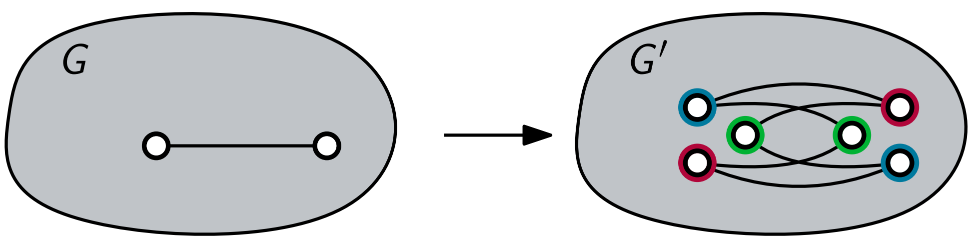
Colored Cliques

Problem: **Multicolored Clique**
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



V_1, V_2, V_3

Reduction: from Clique to Multicolored Clique



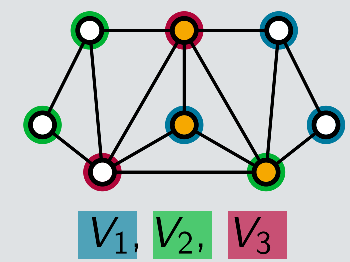
Instance $(G, 3)$ of Clique

Instance of MC Clique: $(G', 3, (V_1, V_2, V_3))$

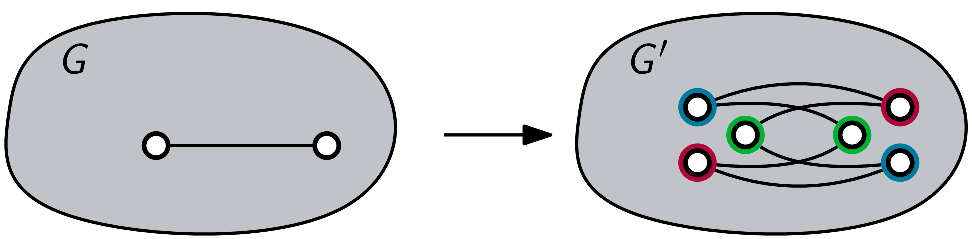
- copy each $v \in V$ to $v^1, \ldots, v^k$, and set $v^i \in V_i$
- for each $uv \in E$ connect u^i with v^j for each $i \neq j$
- $k = k'$

Colored Cliques

Problem: MULTICOLORED CLIQUE
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



Reduction: from CLIQUE to MULTICOLORED CLIQUE



Instance $(G, 3)$ of CLIQUE

Instance of MC CLIQUE: $(G', 3, (V_1, V_2, V_3))$

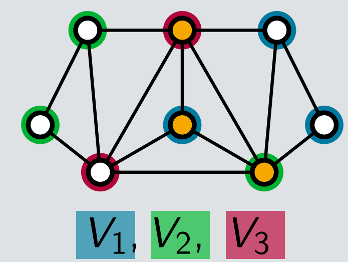
- copy each $v \in V$ to $v^1, \ldots, v^k$, and set $v^i \in V_i$
- for each $uv \in E$ connect u^i with v^j for each $i \neq j$
- $k = k'$

G has a size k clique $\Rightarrow$ G' has a size k colored clique

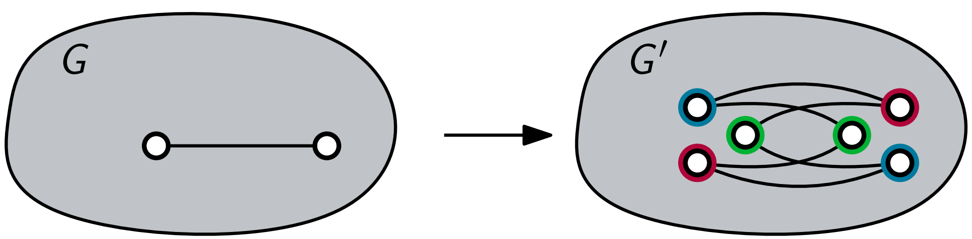
- Let $v_1, \ldots, v_k$ be a clique in G
- then $v_1^1, \ldots, v_k^k$ is a clique in G'

Colored Cliques

Problem: **Multicolored Clique**
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



Reduction: from Clique to Multicolored Clique



Instance $(G, 3)$ of Clique

Instance of MC Clique: $(G', 3, (V_1, V_2, V_3))$

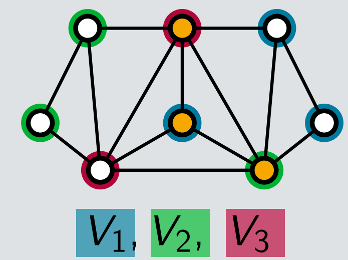
- copy each $v \in V$ to $v^1, \ldots, v^k$, and set $v^i \in V_i$
- for each $uv \in E$ connect u^i with v^j for each $i \neq j$
- $k = k'$

G has a size k clique $\Rightarrow$ G' has a size k colored clique

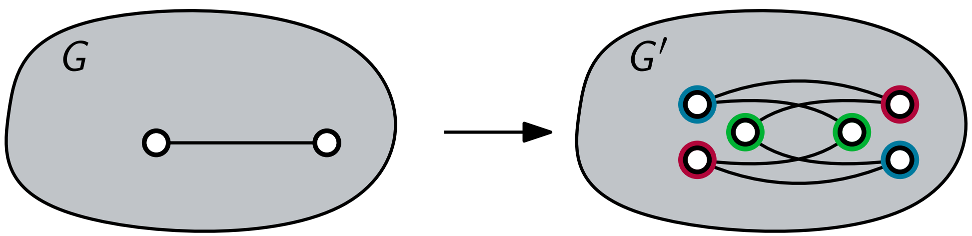
- Let $v_1, \ldots, v_k$ be a clique in G
- then $v_1^1, \ldots, v_k^k$ is a clique in G'
- all these vertices have distinct colors

Colored Cliques

Problem: Multicolored Clique
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \dots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



Reduction: from Clique to Multicolored Clique



Instance $(G, 3)$ of Clique

Instance of MC Clique: $(G', 3, (V_1, V_2, V_3))$

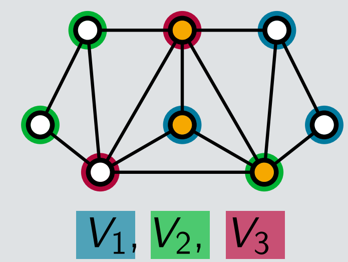
- copy each $v \in V$ to $v^1, \dots, v^k$, and set $v^i \in V_i$
- for each $uv \in E$ connect u^i with v^j for each $i \neq j$
- $k = k'$

G' has a size k colored clique $\Rightarrow$ G has a size k clique

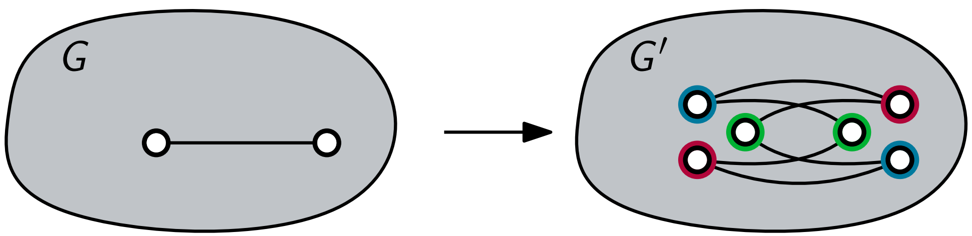
- Let $v^1_{\pi(1)}, \dots, v^k_{\pi(k)}$ be a colored clique in G with $\pi\colon \{1, \dots, k\} \to \{1, \dots n\}$

Colored Cliques

Problem: **Multicolored Clique**
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



Reduction: from Clique to Multicolored Clique



Instance $(G, 3)$ of Clique

Instance of MC Clique: $(G', 3, (V_1, V_2, V_3))$

- copy each $v \in V$ to $v^1, \ldots, v^k$, and set $v^i \in V_i$
- for each $uv \in E$ connect u^i with v^j for each $i \neq j$
- $k = k'$

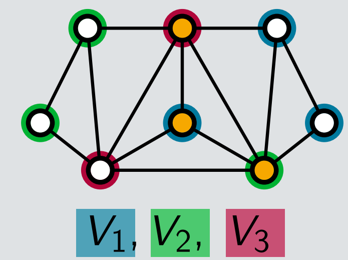
G' has a size k colored clique $\Rightarrow$ G has a size k clique

- Let $v^1_{\pi(1)}, \ldots, v^k_{\pi(k)}$ be a colored clique in G with $\pi\colon \{1, \ldots, k\} \to \{1, \ldots n\}$
- π is injective

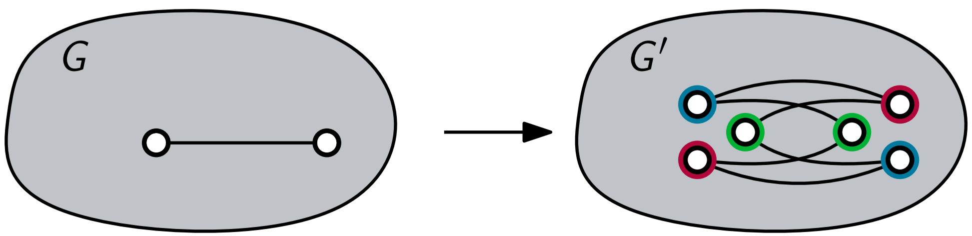
(the colored clique does not contain two copies of the same vertex)

Colored Cliques

Problem: Multicolored Clique
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



Reduction: from Clique to Multicolored Clique



Instance $(G, 3)$ of Clique

Instance of MC Clique: $(G', 3, (V_1, V_2, V_3))$

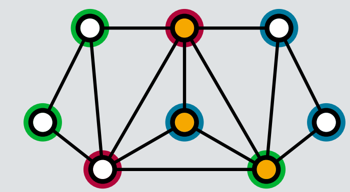
- copy each $v \in V$ to $v^1, \ldots, v^k$, and set $v^i \in V_i$
- for each $uv \in E$ connect u^i with v^j for each $i \neq j$
- $k = k'$

G' has a size k colored clique $\Rightarrow$ G has a size k clique

- Let $v^1_{\pi(1)}, \ldots, v^k_{\pi(k)}$ be a colored clique in G with $\pi\colon \{1, \ldots, k\} \to \{1, \ldots n\}$
- π is injective (the colored clique does not contain two copies of the same vertex)
- thus, $v_{\pi(1)}, \ldots, v_{\pi(k)}$ are k distinct nodes that form a clique in G

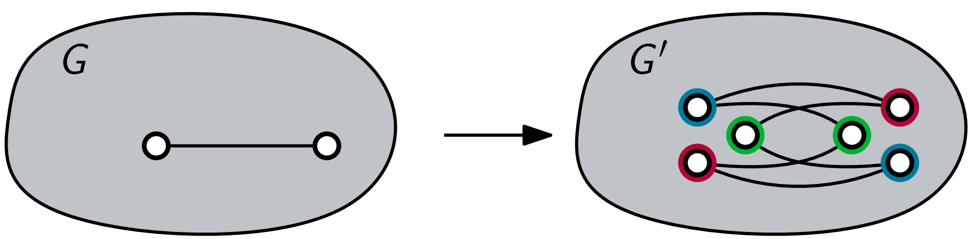
Colored Cliques

Problem: MULTICOLORED CLIQUE
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



V_1, V_2, V_3

Reduction: from CLIQUE to MULTICOLORED CLIQUE



Instance $(G, 3)$ of CLIQUE

Instance of MC CLIQUE: $(G', 3, (V_1, V_2, V_3))$

- copy each $v \in V$ to $v^1, \ldots, v^k$, and set $v^i \in V_i$
- for each $uv \in E$ connect u^i with v^j for each $i \neq j$
- $k = k'$

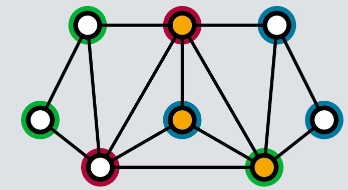
***G'* has a size *k* colored clique $\Rightarrow$ *G* has a size *k* clique**

- Let $v^1_{\pi(1)}, \ldots, v^k_{\pi(k)}$ be a colored clique in G with $\pi\colon \{1, \ldots, k\} \to \{1, \ldots n\}$
- π is injective
  (the colored clique does not contain two copies of the same vertex)
- thus, $v_{\pi(1)}, \ldots, v_{\pi(k)}$ are k distinct nodes that form a clique in G

$\square$

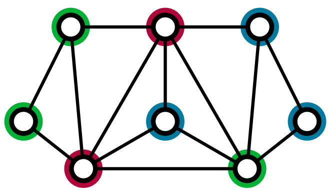
Colored Cliques

Problem: Multicolored Clique
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



V_1, V_2, V_3

Reduction: from Multicolored Clique to Clique

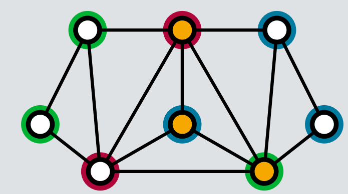


$\longrightarrow$

Instance of MC Clique:
$(G, 3, (V_1, V_2, V_3))$

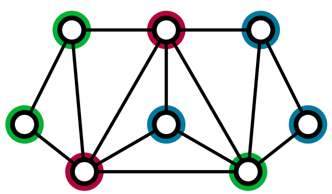
Colored Cliques

Problem: **MULTICOLORED CLIQUE**
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.

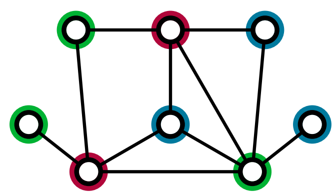


V_1, V_2, V_3

Reduction: from MULTICOLORED CLIQUE to CLIQUE



Instance of MC CLIQUE:
$(G, 3, (V_1, V_2, V_3))$

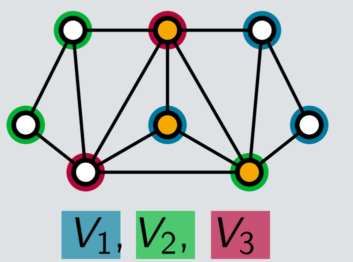


Instance $(G', 3)$ of CLIQUE

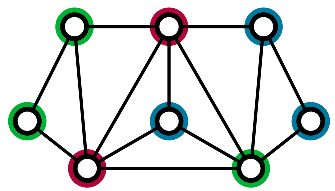
- delete edges within each color class
- $k' = k$

Colored Cliques

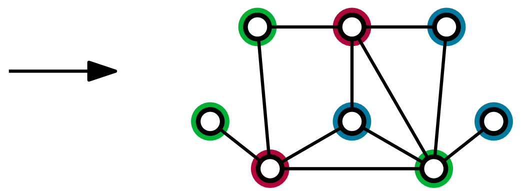
Problem: MULTICOLORED CLIQUE
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \dots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



Reduction: from MULTICOLORED CLIQUE to CLIQUE



Instance of MC CLIQUE: $(G, 3, (V_1, V_2, V_3))$



Instance $(G', 3)$ of CLIQUE

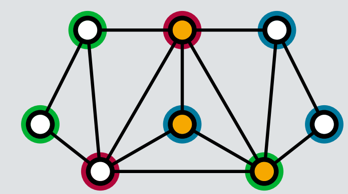
- delete edges within each color class
- $k' = k$

G has a colored size k clique $\Rightarrow$ G' has a size k clique

- the colored clique does not use any edges inside a color class
- thus, G' contains a size k clique

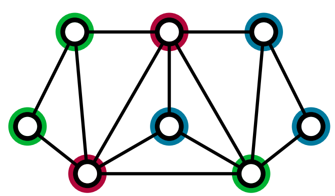
Colored Cliques

Problem: MULTICOLORED CLIQUE
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.

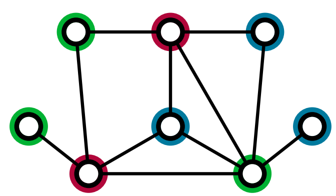


V_1, V_2, V_3

Reduction: from MULTICOLORED CLIQUE to CLIQUE



Instance of MC CLIQUE: $(G, 3, (V_1, V_2, V_3))$



Instance $(G', 3)$ of CLIQUE

- delete edges within each color class
- $k' = k$

G has a colored size k clique $\Rightarrow$ G' has a size k clique

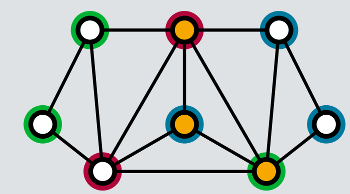
- the colored clique does not use any edges inside a color class
- thus, G' contains a size k clique

G' has a size k clique $\Rightarrow$ G has a colored size k clique

- in G', no monochromatic vertices are adjacent
- thus, each clique must be a colored clique

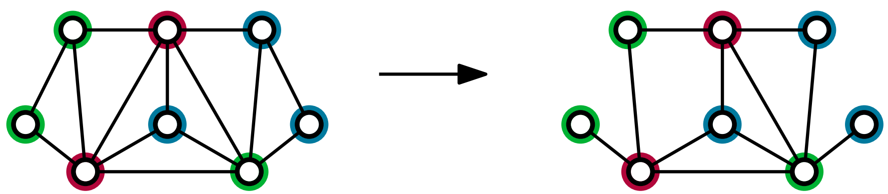
Colored Cliques

Problem: **MULTICOLORED CLIQUE**
Given: Graph $G = (V, E)$, parameter k,
and partition $(V_1, \ldots, V_k)$ of V.
Find: Clique $V' \subseteq V$ of size k, so that $|V' \cap V_i| = 1$ for each i.



V_1, V_2, V_3

Reduction: from MULTICOLORED CLIQUE to CLIQUE



Instance of MC CLIQUE: $(G, 3, (V_1, V_2, V_3))$

Instance $(G', 3)$ of CLIQUE

- delete edges within each color class
- $k' = k$

G has a colored size k clique $\Rightarrow$ G' has a size k clique

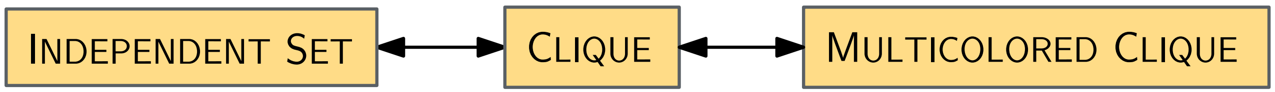
- the colored clique does not use any edges inside a color class
- thus, G' contains a size k clique

G' has a size k clique $\Rightarrow$ G has a colored size k clique

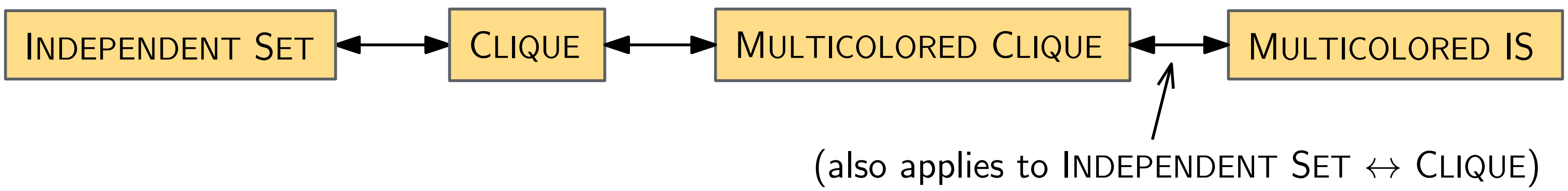
- in G', no monochromatic vertices are adjacent
- thus, each clique must be a colored clique

$\square$

FPT-Reductions so far



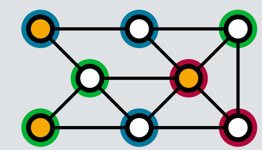
FPT-Reductions so far



Dominating Set

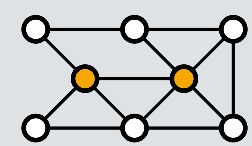
Reduce Multicolored Independent Set to Dominating Set

MC Independent Set



$V' \subseteq V$ colored, so that
$\forall e \in E \colon |e \cap V'| \leq 1$

Dominating Set



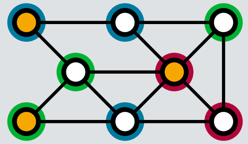
$V' \subseteq V$, so that
$\forall v \in V \colon |N[v] \cap V'| \geq 1$

DOMINATING SET

Reduce MULTICOLORED INDEPENDENT SET to DOMINATING SET

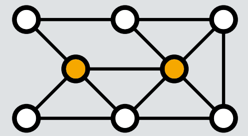
- enforce the following properties on the DOMINATING SET instance
  - select exactly one element from each color class
  - avoid simultaneous selection of designated node pairs

MC INDEPENDENT SET



$V' \subseteq V$ colored, so that
$\forall e \in E\colon |e \cap V'| \leq 1$

DOMINATING SET



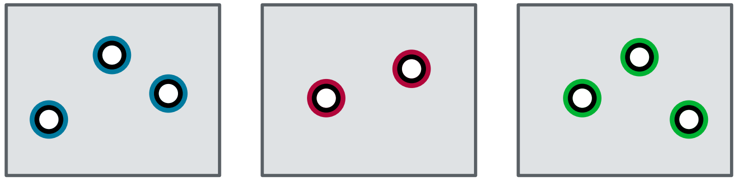
$V' \subseteq V$, so that
$\forall v \in V\colon |N[v] \cap V'| \geq 1$

Dominating Set

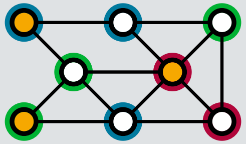
Reduce Multicolored Independent Set to Dominating Set

- enforce the following properties on the Dominating Set instance
  - select exactly one element from each color class
  - avoid simultaneous selection of designated node pairs

One element in each color class



MC Independent Set



$V' \subseteq V$ colored, so that
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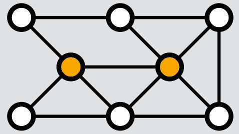
Dominating Set



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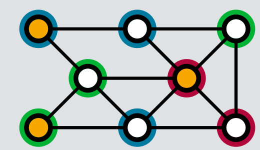

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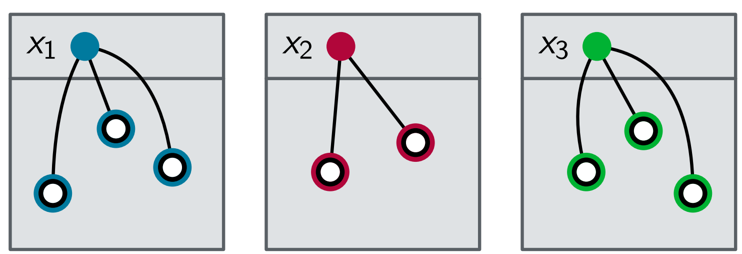

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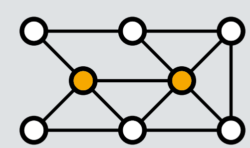

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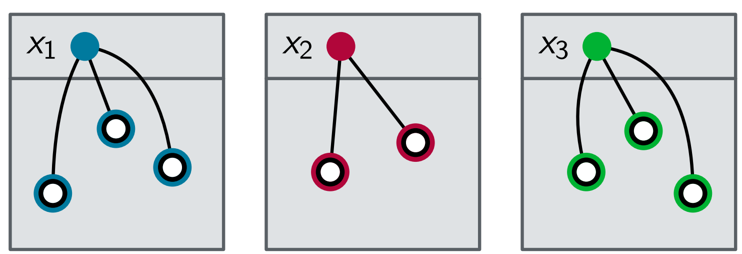

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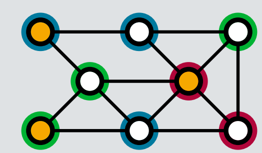

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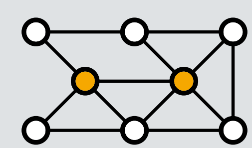

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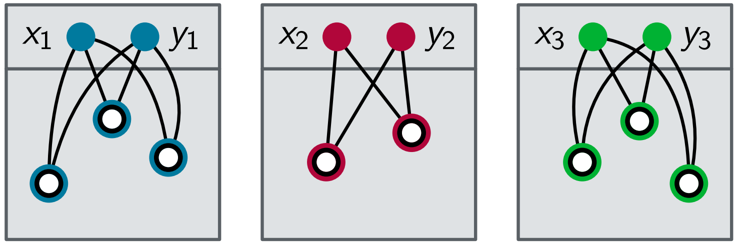

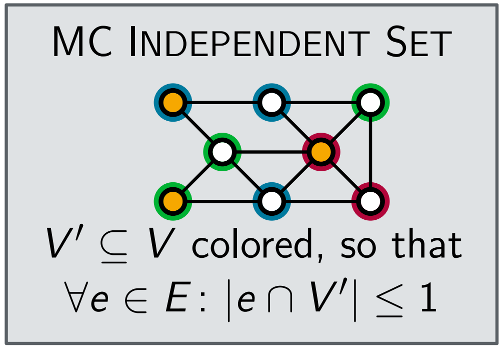

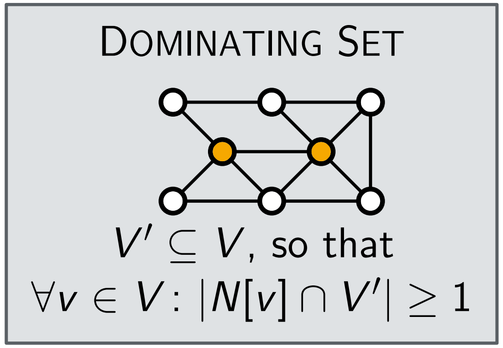

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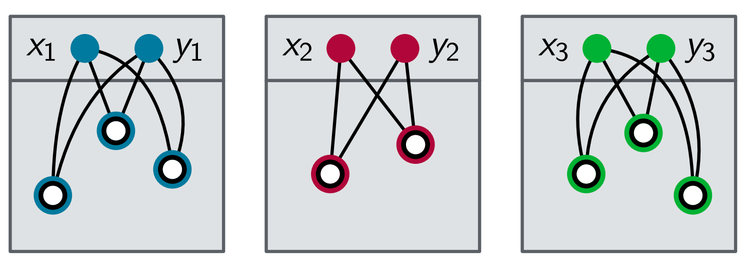

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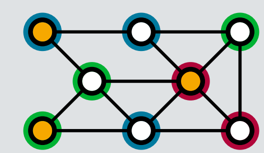

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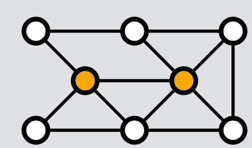

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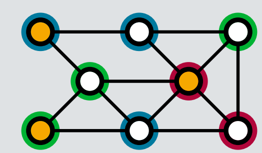

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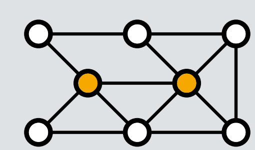

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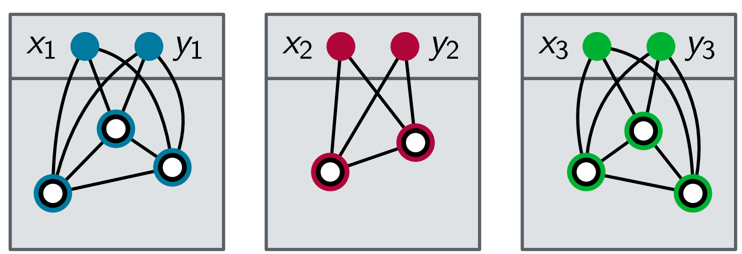

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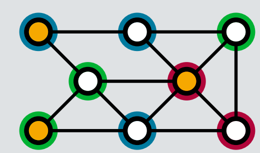

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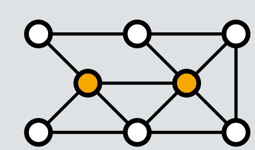

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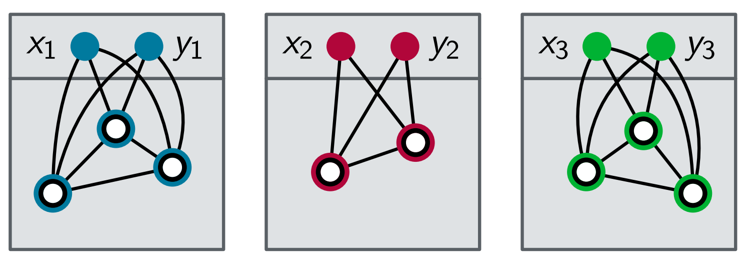

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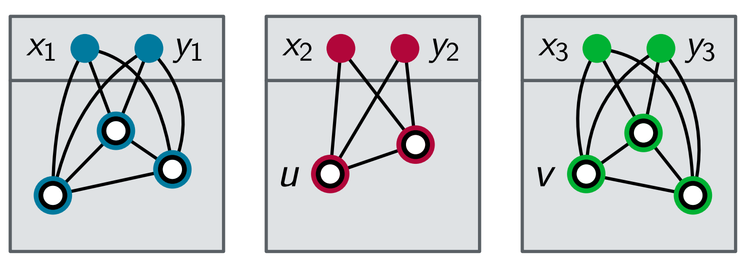

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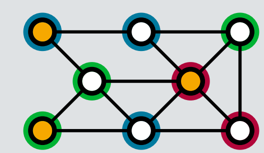

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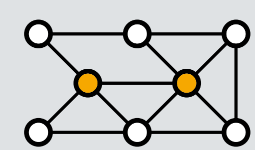

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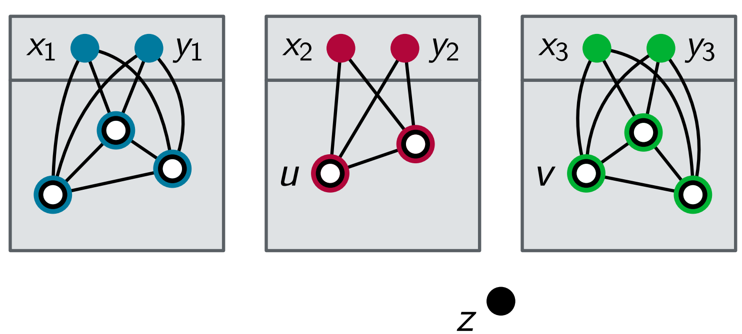

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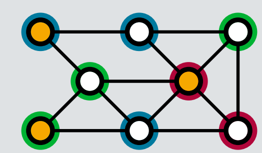

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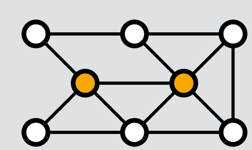

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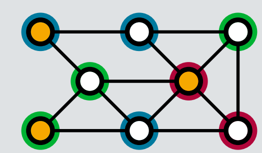

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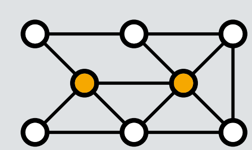

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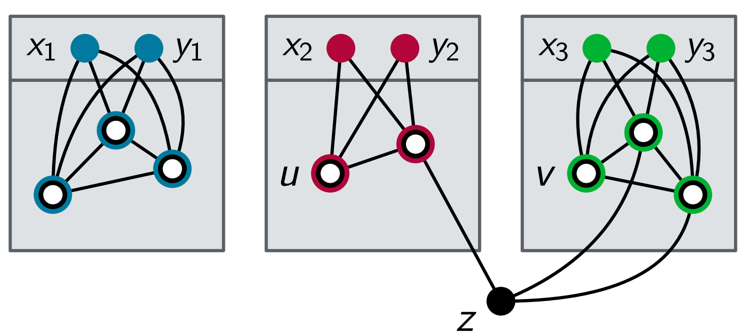

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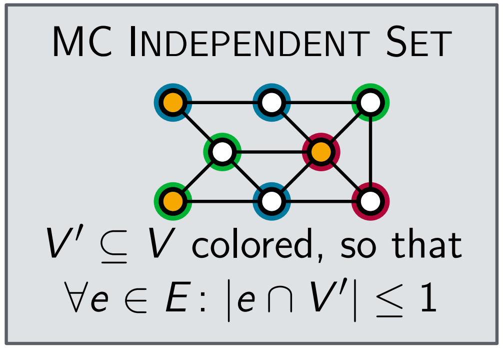
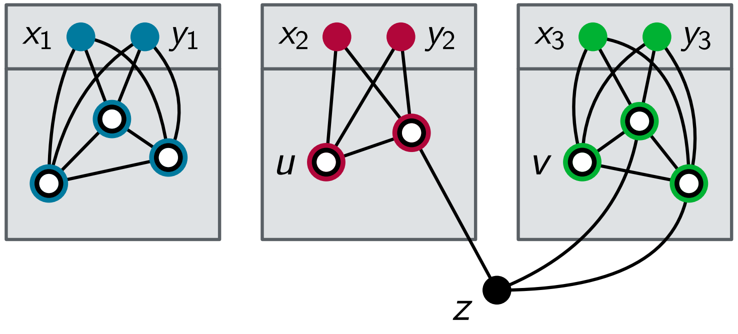
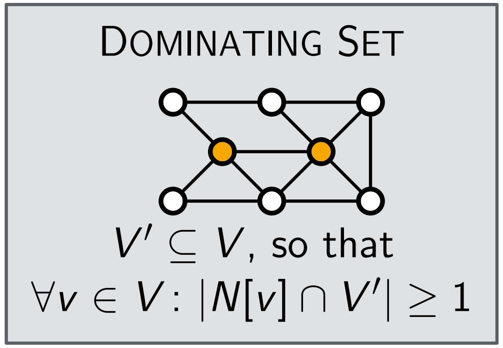
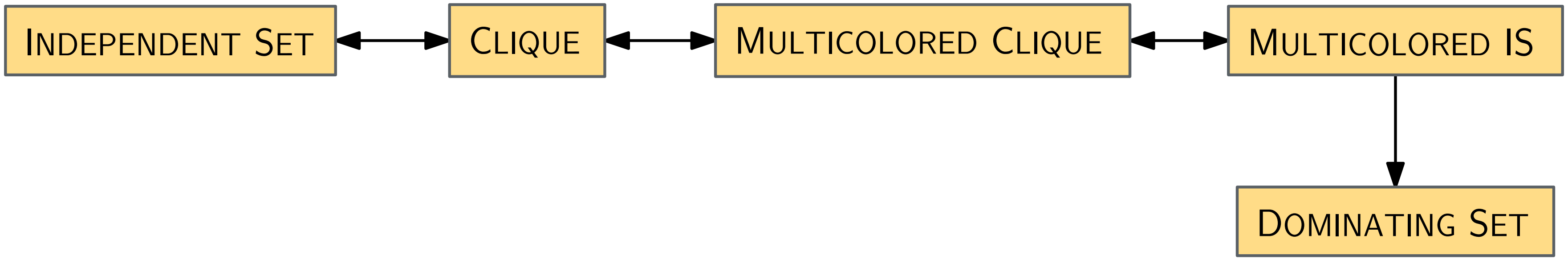






FPT-Reductions so far



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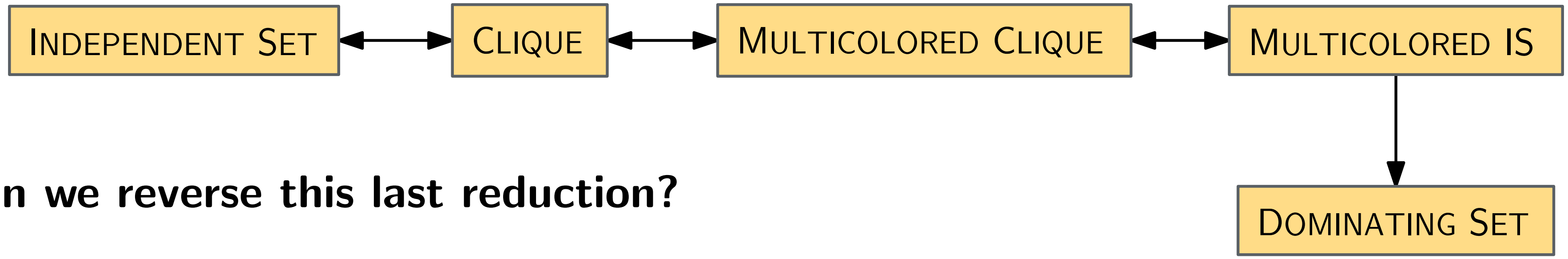

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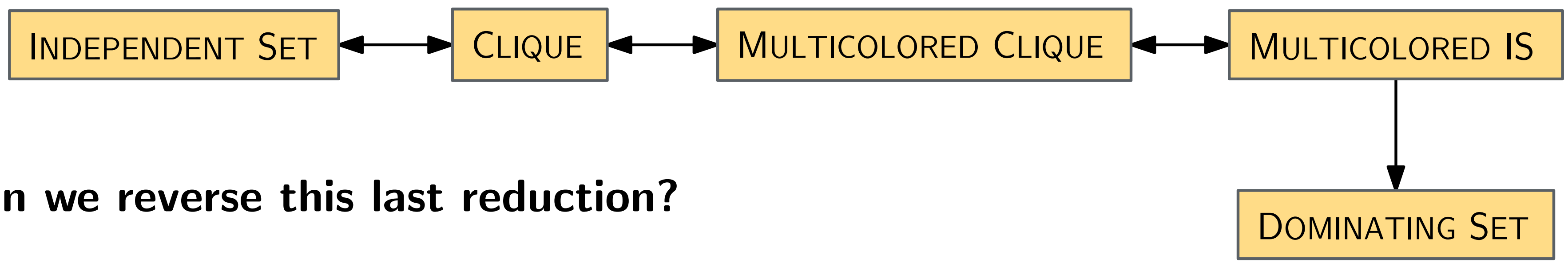

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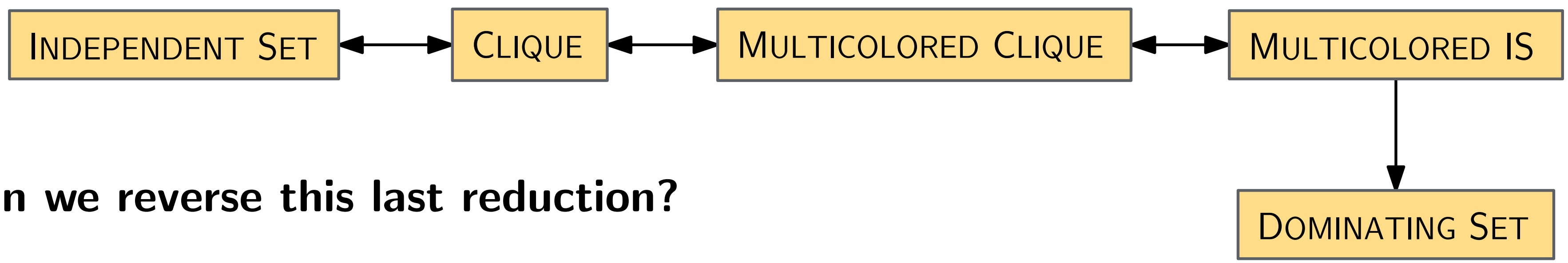

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$$\begin{tikzcd} \textsc{Independent Set} \arrow[r, leftrightarrow] & \textsc{Clique} \arrow[r, leftrightarrow] & \textsc{Multicolored Clique} \arrow[r, leftrightarrow] & \textsc{Multicolored IS} \arrow[d] \\ & & & \textsc{Dominating Set} \end{tikzcd}$$

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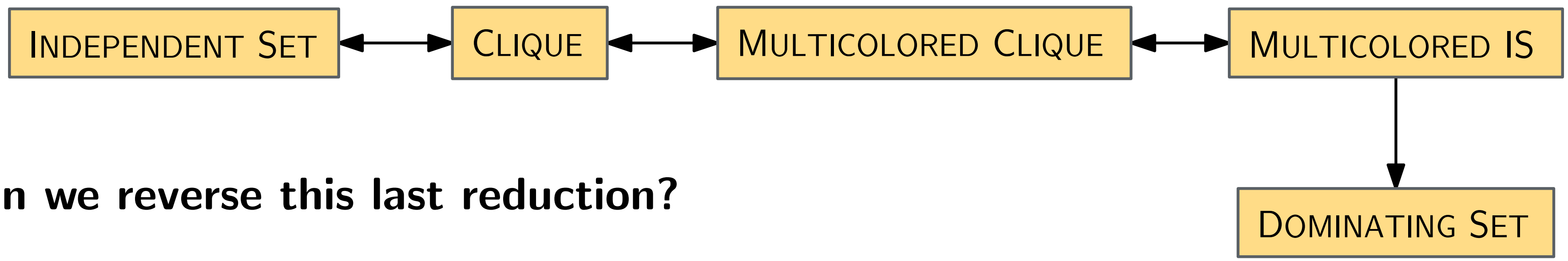

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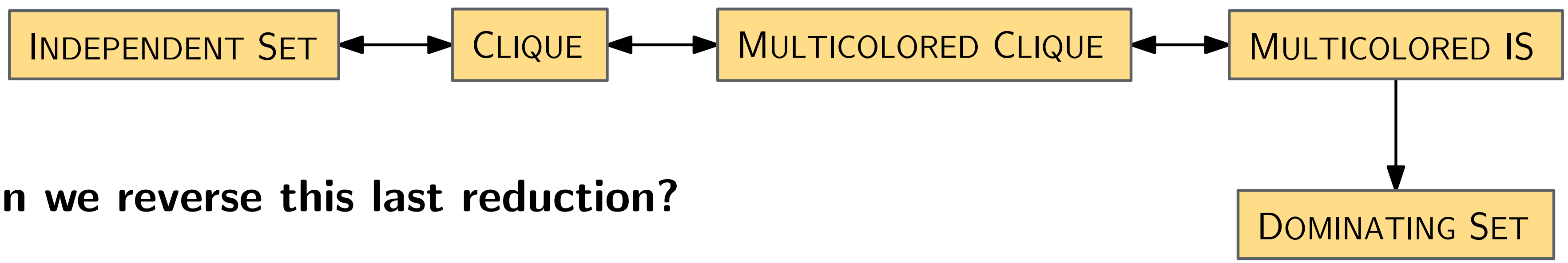

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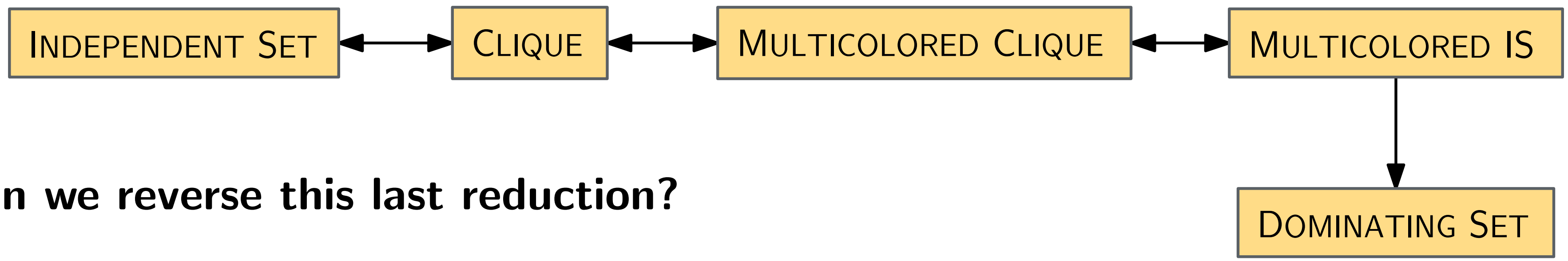

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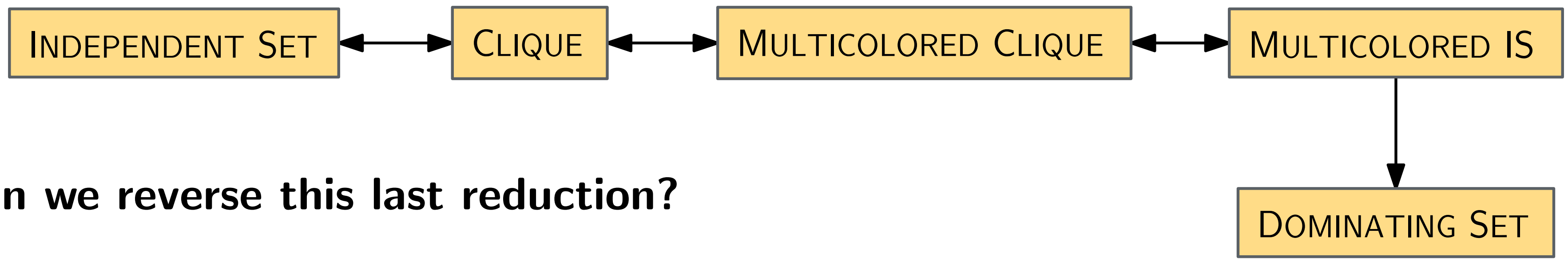

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And now? What about FPT?

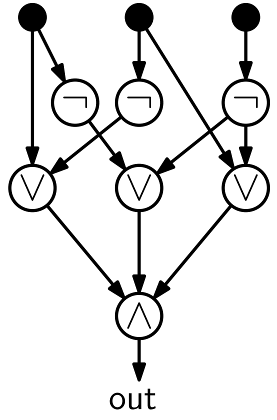
- define natural hierarchy of complexity classes
- establish prototypical problem for each level

Weighted Circuit Satisfiability

Boolean Circuits

- directed acyclic graph (DAG) with the following node types:

example



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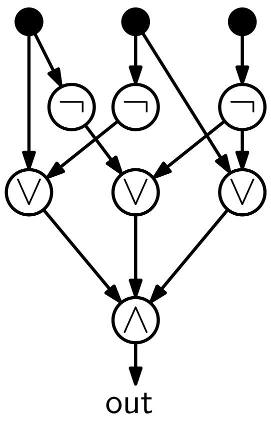

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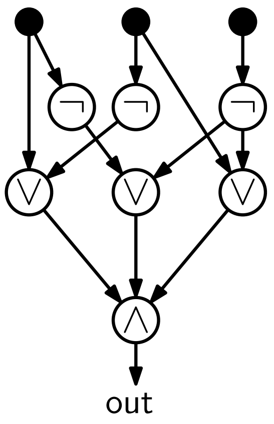

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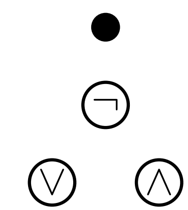

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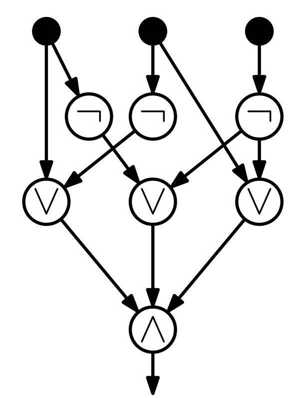

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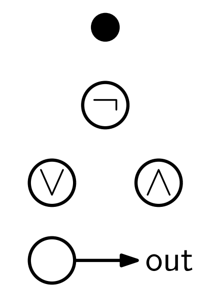
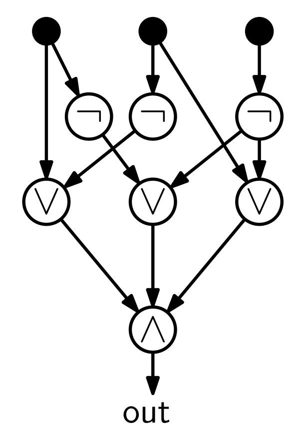
  - one sink (out-degree 0) is the **output node**



example



Weighted Circuit Satisfiability

Boolean Circuits

- directed acyclic graph (DAG) with the following node types:
  - sources (in-degree 0) are **input nodes**
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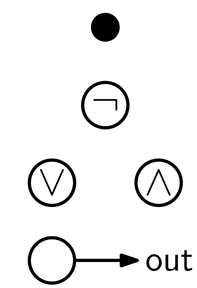
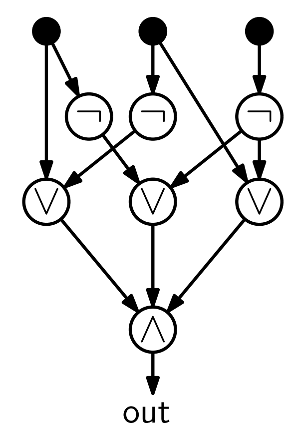


- an assignment of 0 or 1 to each input node propogates through the other nodes, providing an output value. (in the natural way)
- an assignment **statisfying** when the output value is 1

example



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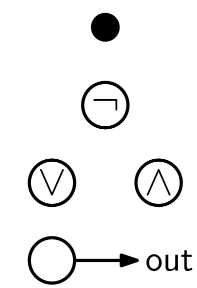

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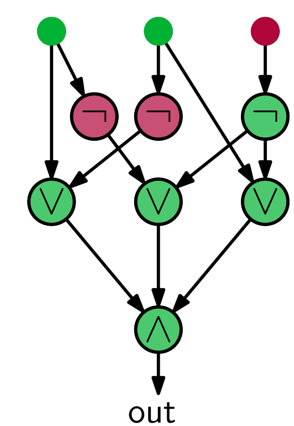

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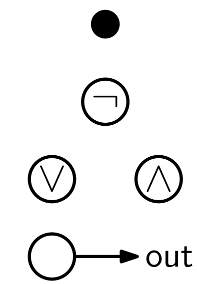

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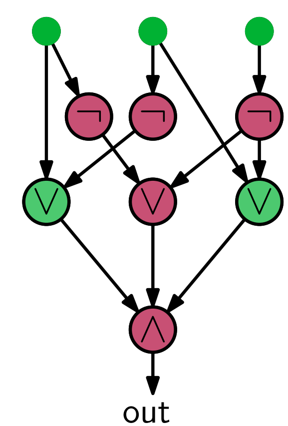

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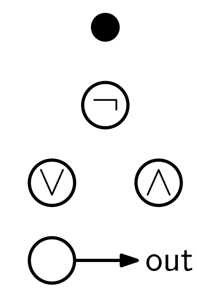

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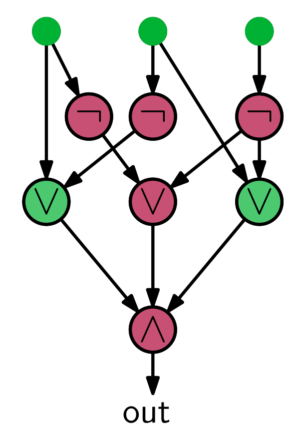

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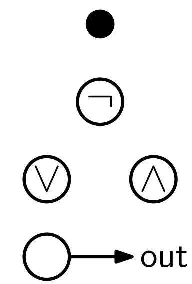

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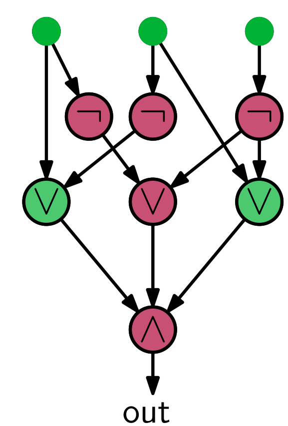
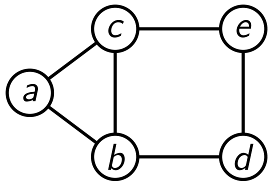


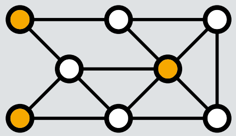
Problem: Weighted Circuit Satisfiability (WCS)
Given a boolean circuit and a parameter k
Find: A weight k satisfying assignment

Reductions visualized

Independent Set → WCS



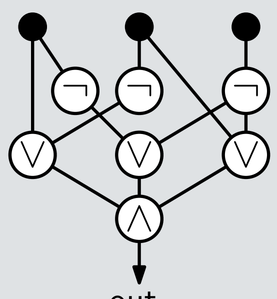
Independent Set



$V' \subseteq V$, so that

$\forall e \in E\colon |e \cap V'| \leq 1$

WCS



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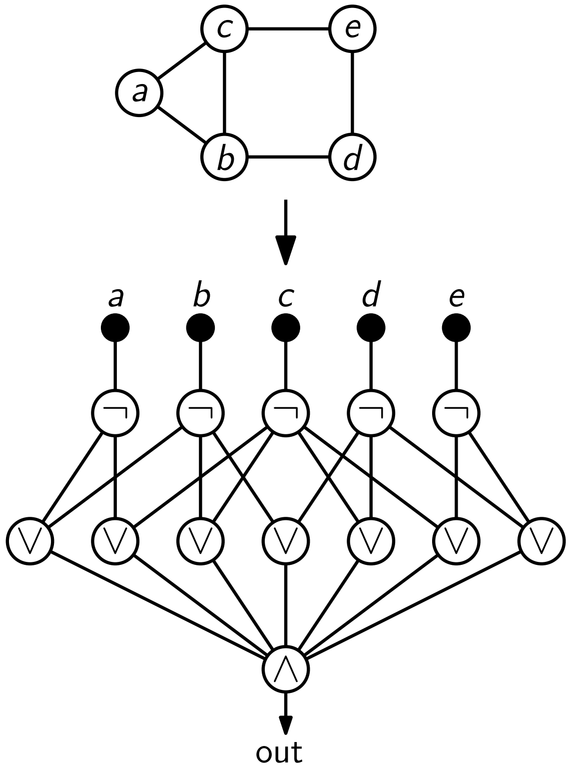

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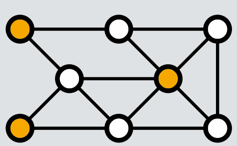

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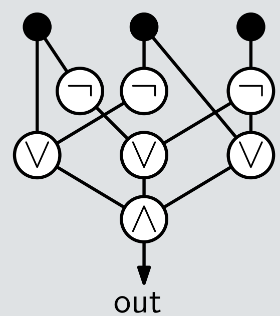
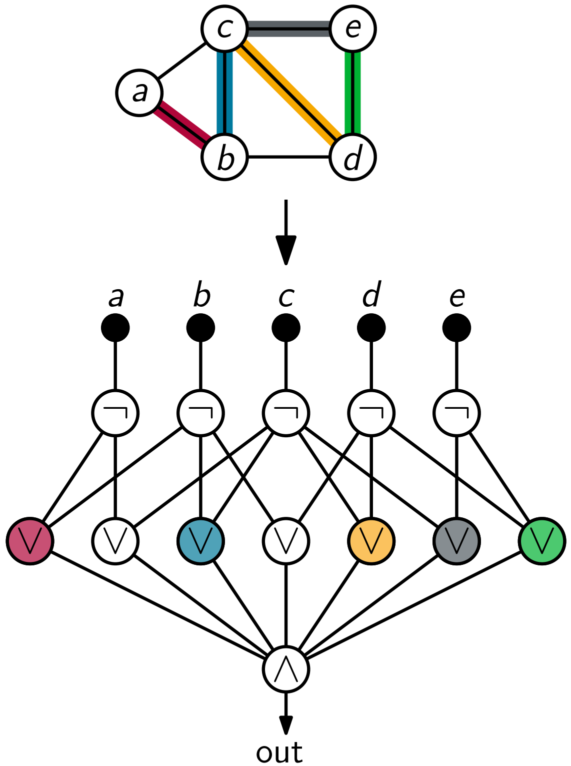


Reductions visualized

INDEPENDENT SET → WCS



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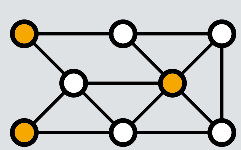

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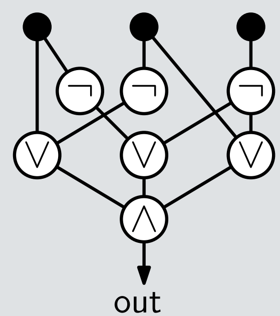

Reductions visualized

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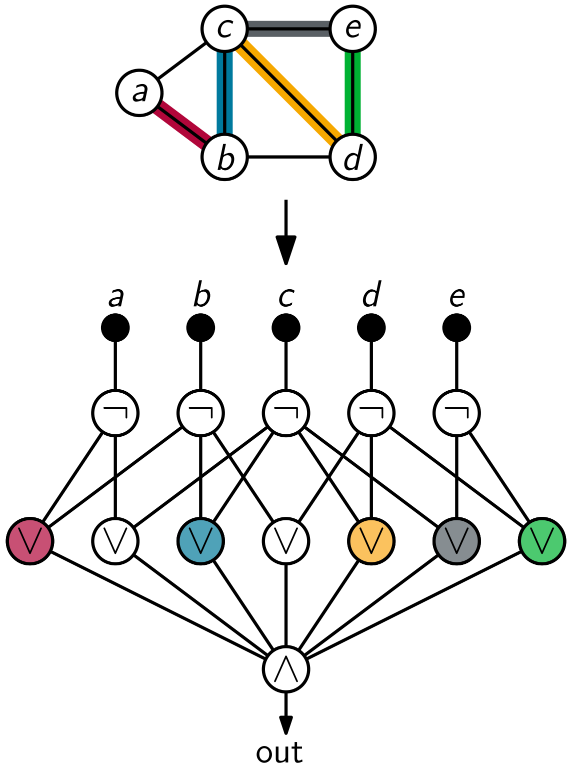
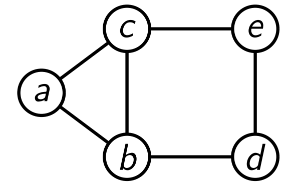


Dominating Set → WCS



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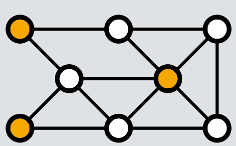

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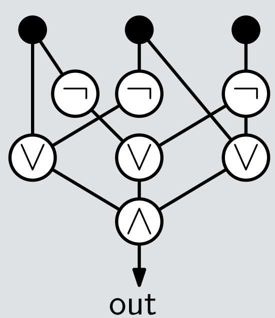

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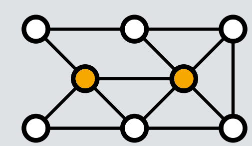

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Reductions visualized

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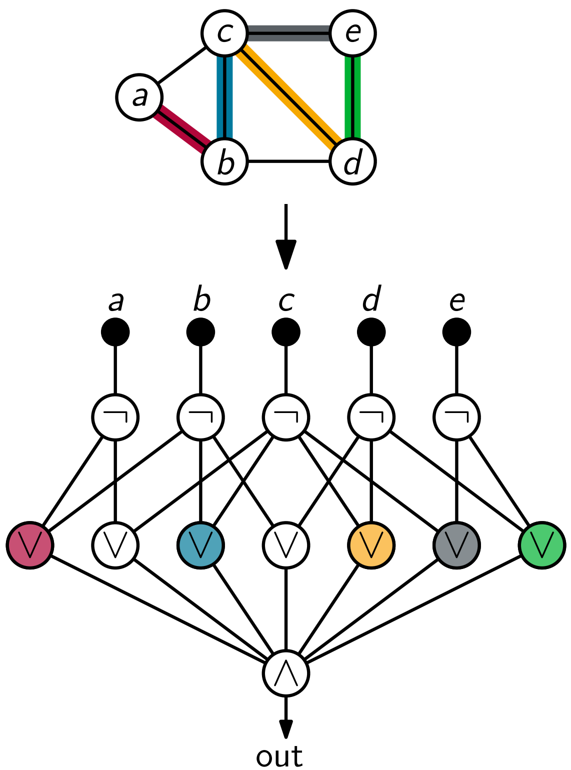

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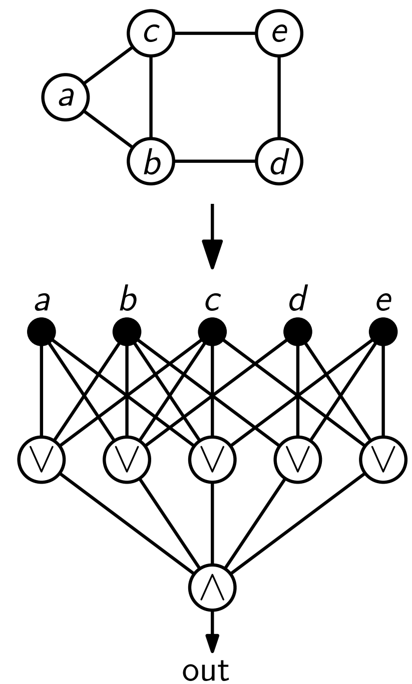

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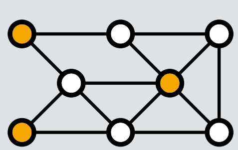

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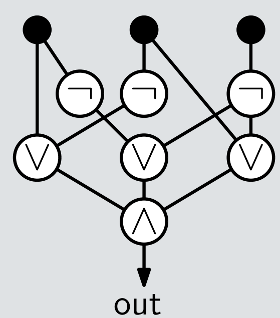

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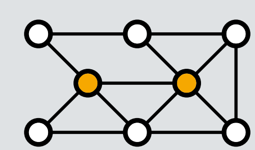

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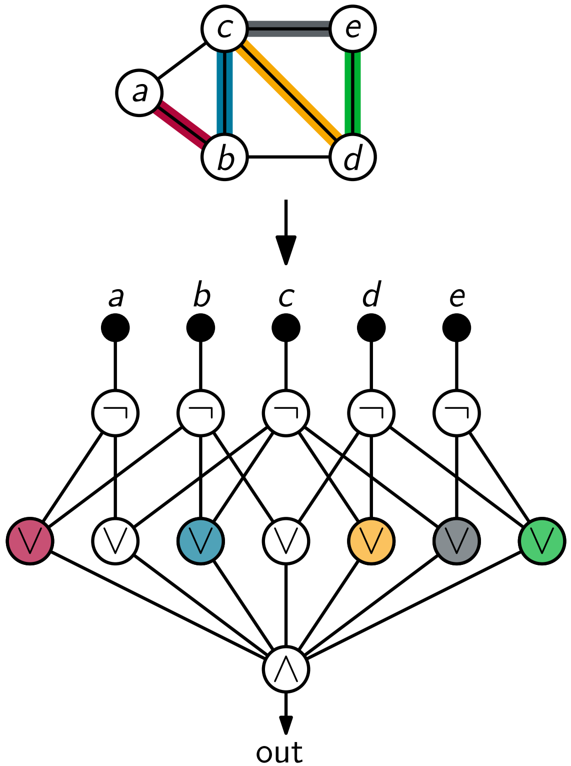

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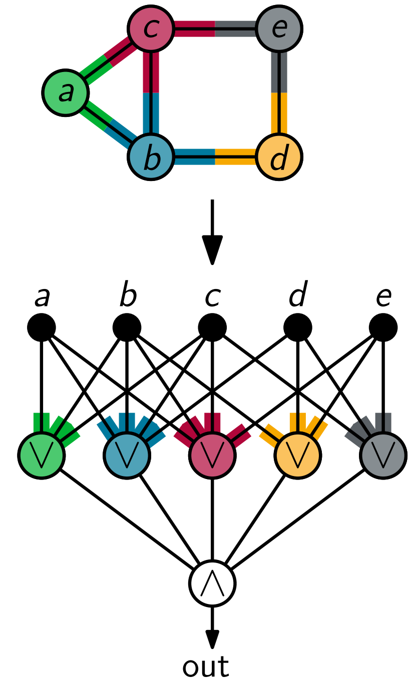
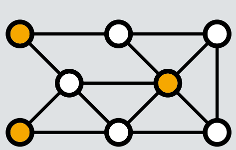


INDEPENDENT SET



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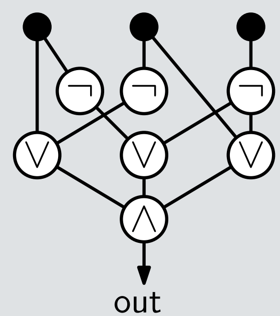

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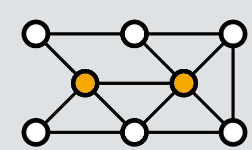

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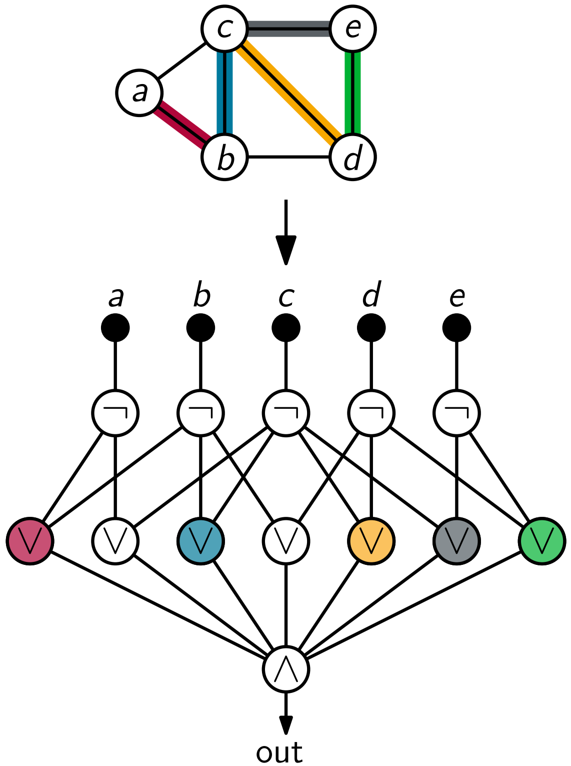

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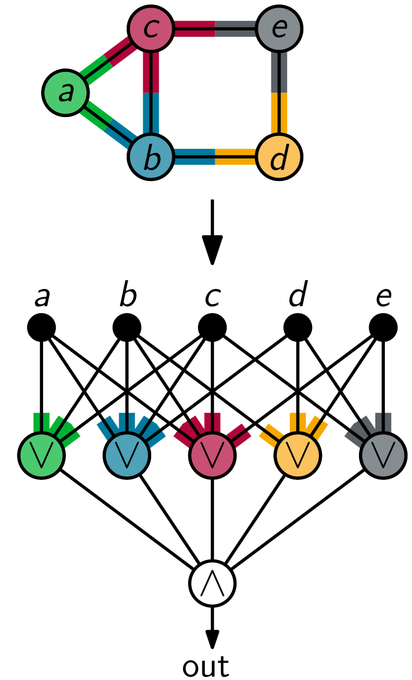
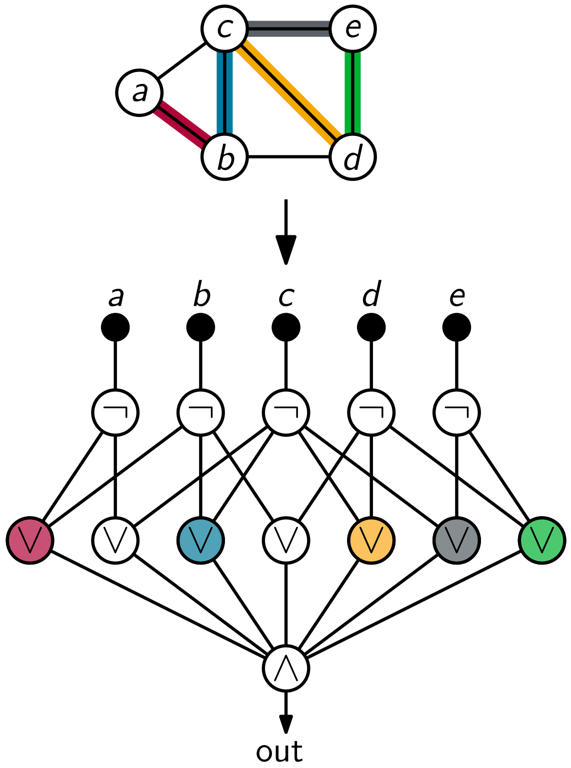


Observations

- the circuits have constant depth

Reductions visualized

Independent Set → WCS



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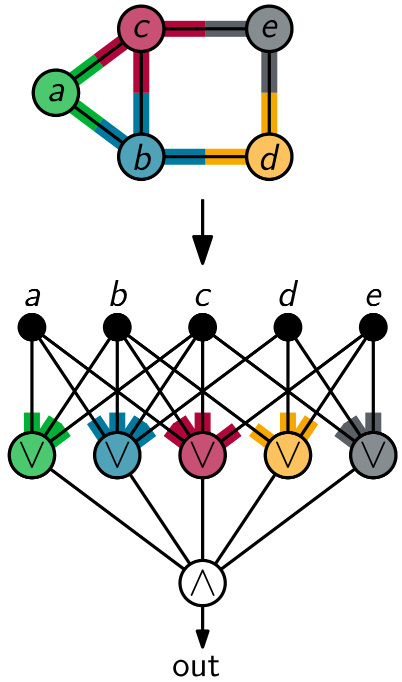

Observations

- the circuits have constant depth
- the circuit for DS contains more nodes with in-degree > 2

Reductions visualized

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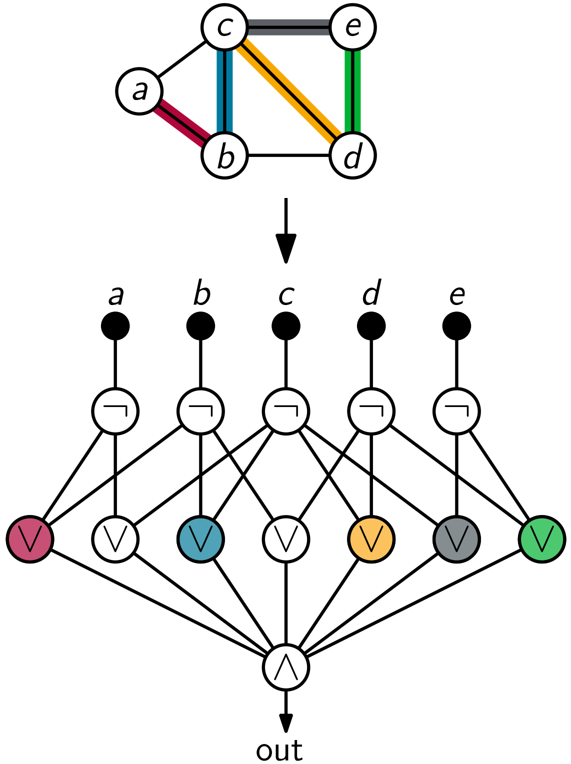
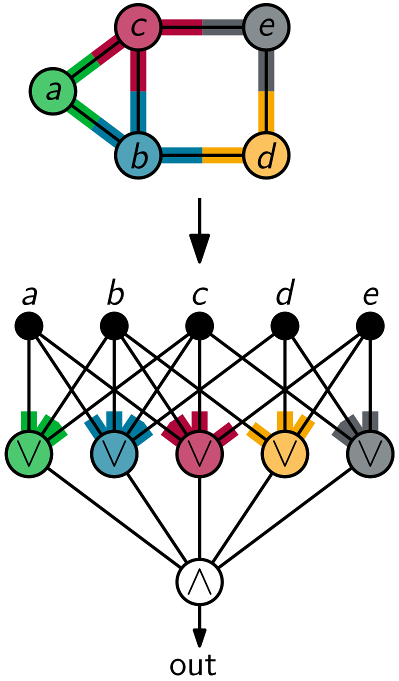


Dominating Set → WCS

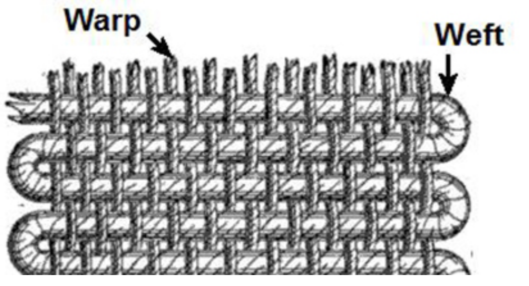


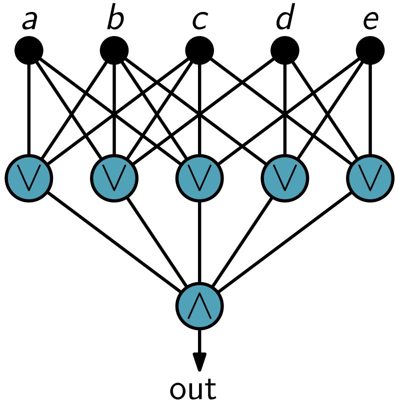
Observations

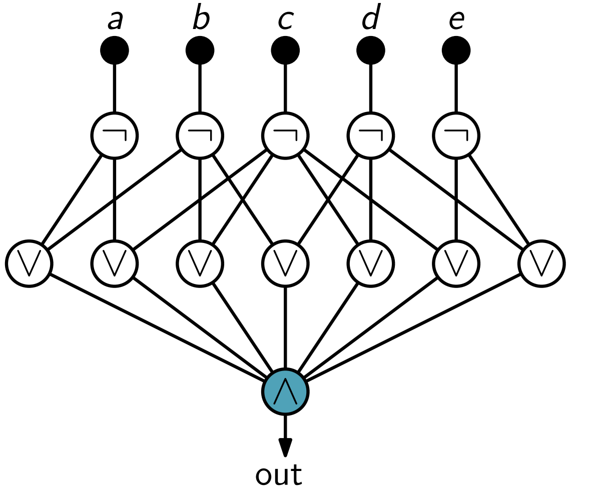
- the circuits have constant depth
- the circuit for DS contains more nodes with in-degree > 2
- is DS harder (in terms of FPT) than IS?

Weft

Definition
The **Weft** of a boolean circuit is the maximum number of nodes with in-degree > 2 on a directed path.







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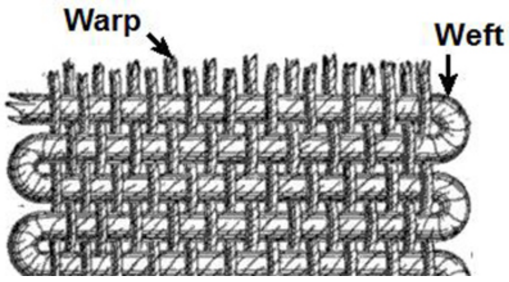

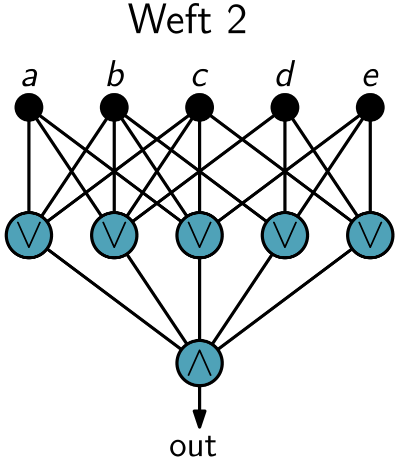

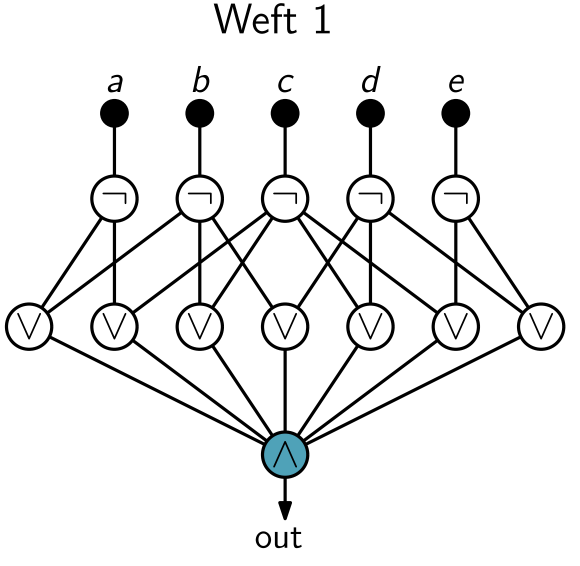

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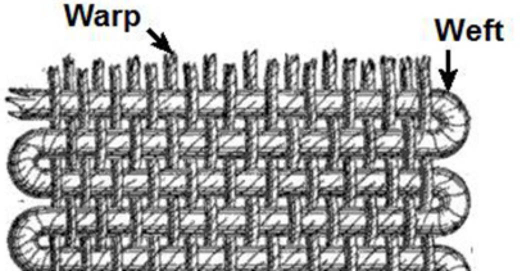
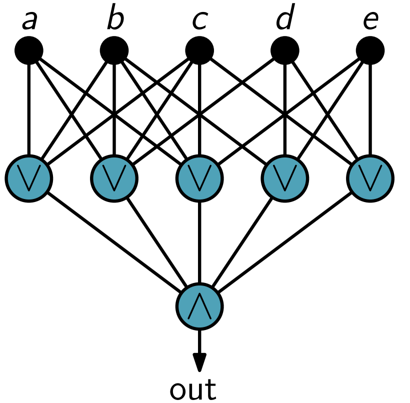
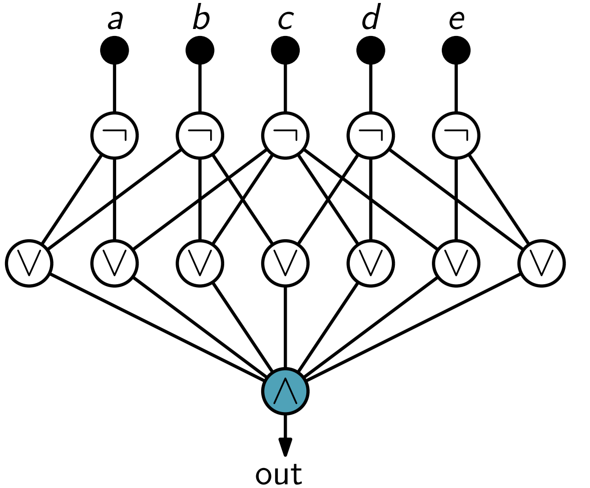
Problem
WCS[*t*] is WCS limited to circuits with constant depth and weft at most t.



Weft 2



Weft 1



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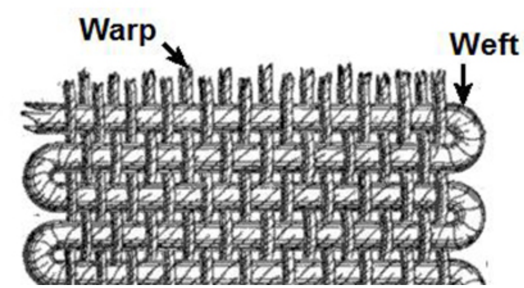
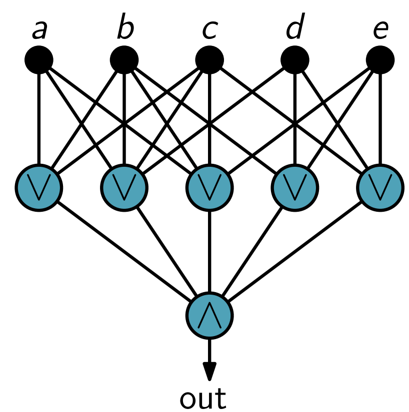


Definition
The class ***W*[*t*]** contains all problems with a paramterized reduction to WCS[t].

We have seen

- Independent Set $\in W[1] \subseteq W[2]$
- Dominating Set $\in W[2]$

Weft 2



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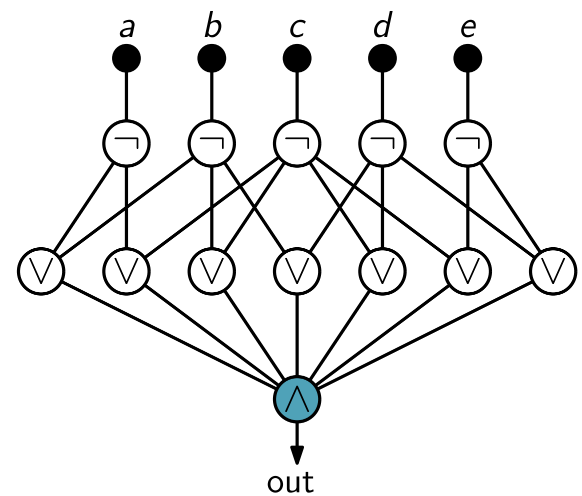
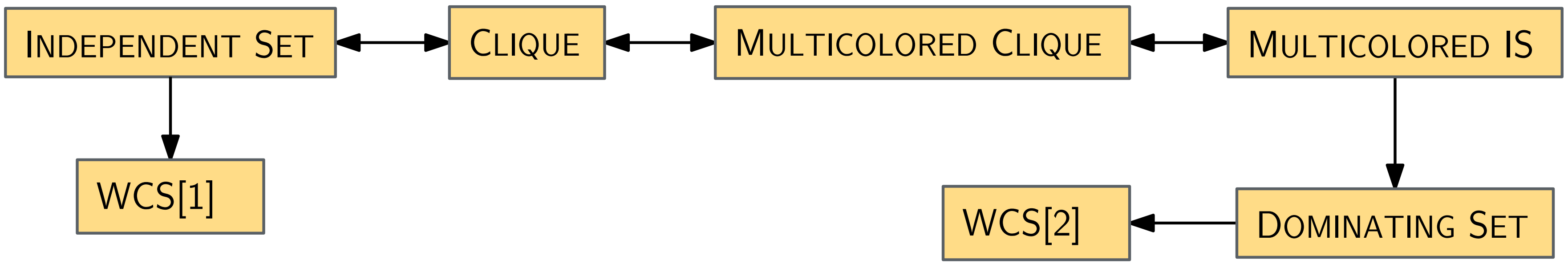


FPT-Reductions so far



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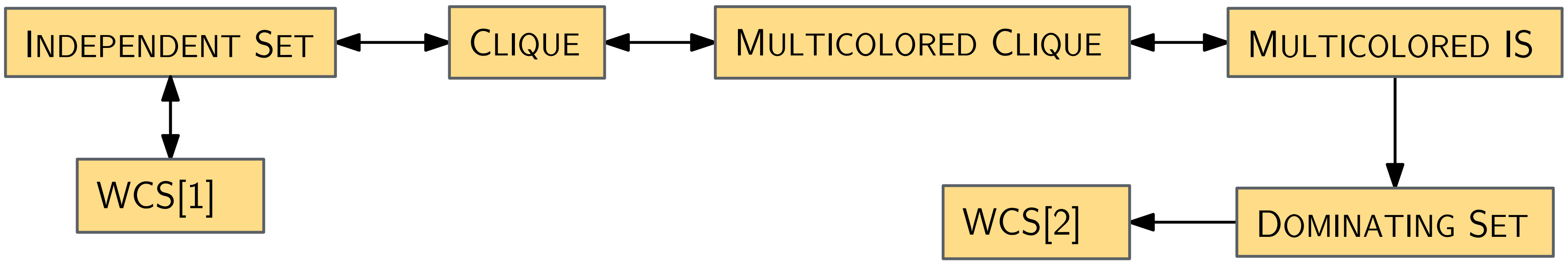

Further reductions

- one can also reduce WCS[1] to INDEPENDENT SET

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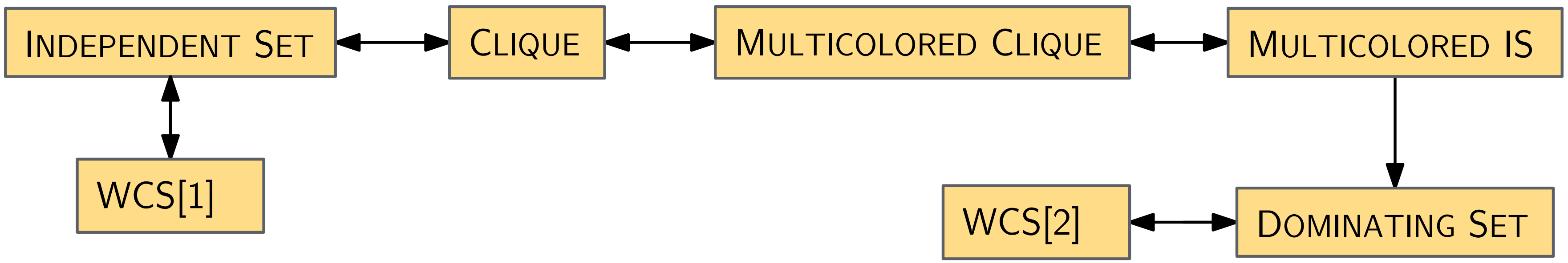
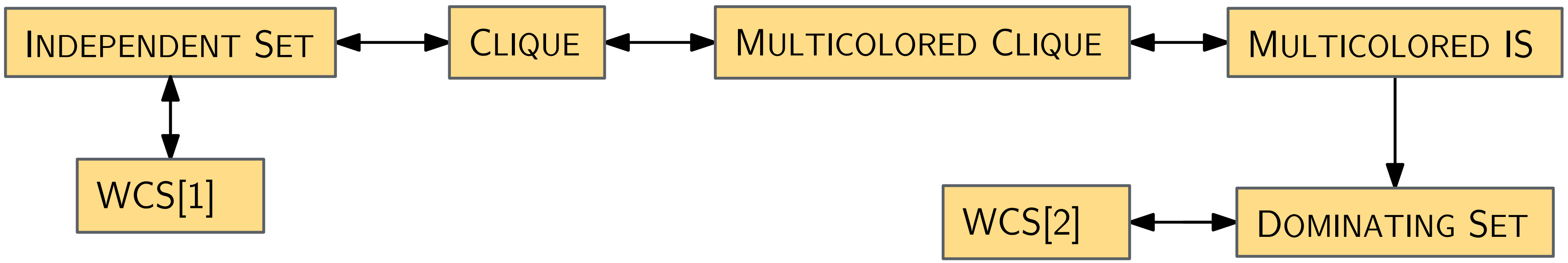


Further reductions

- one can also reduce WCS[1] to Independent Set
- and WCS[2] to Dominating Set

FPT-Reductions so far

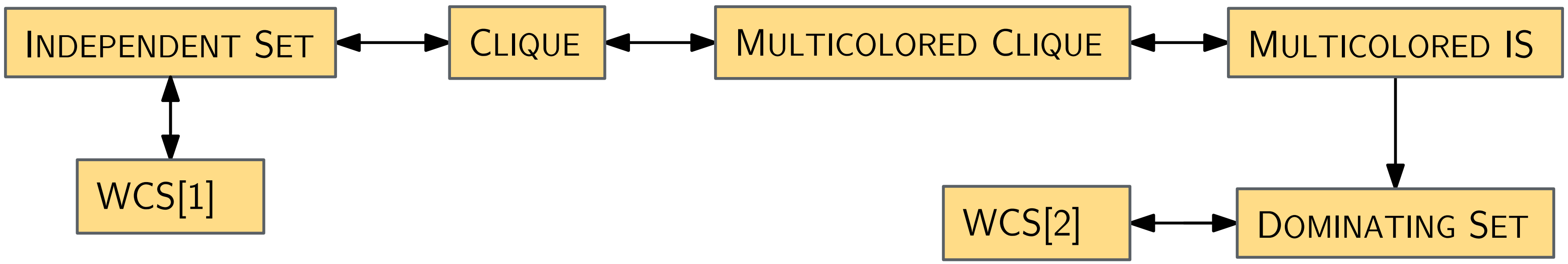


Further reductions

- one can also reduce WCS[1] to INDEPENDENT SET
- and WCS[2] to DOMINATING SET
- so, all problems in $W[1]$ reduce to IS

$\rightsquigarrow$ IS is $W[1]$-complete

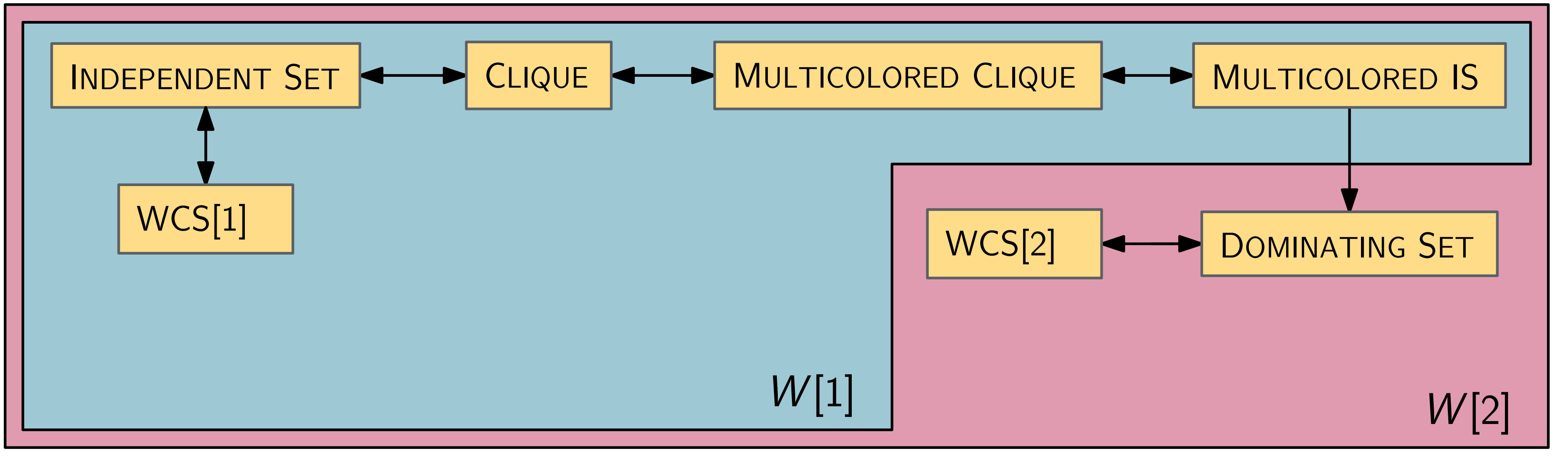
FPT-Reductions so far



Further reductions

- one can also reduce WCS[1] to Independent Set
- and WCS[2] to Dominating Set
- so, all problems in $W[1]$ reduce to IS
  $\rightsquigarrow$ IS is $W[1]$-complete
- similarly DS is $W[2]$-complete

FPT-Reductions so far



Further reductions

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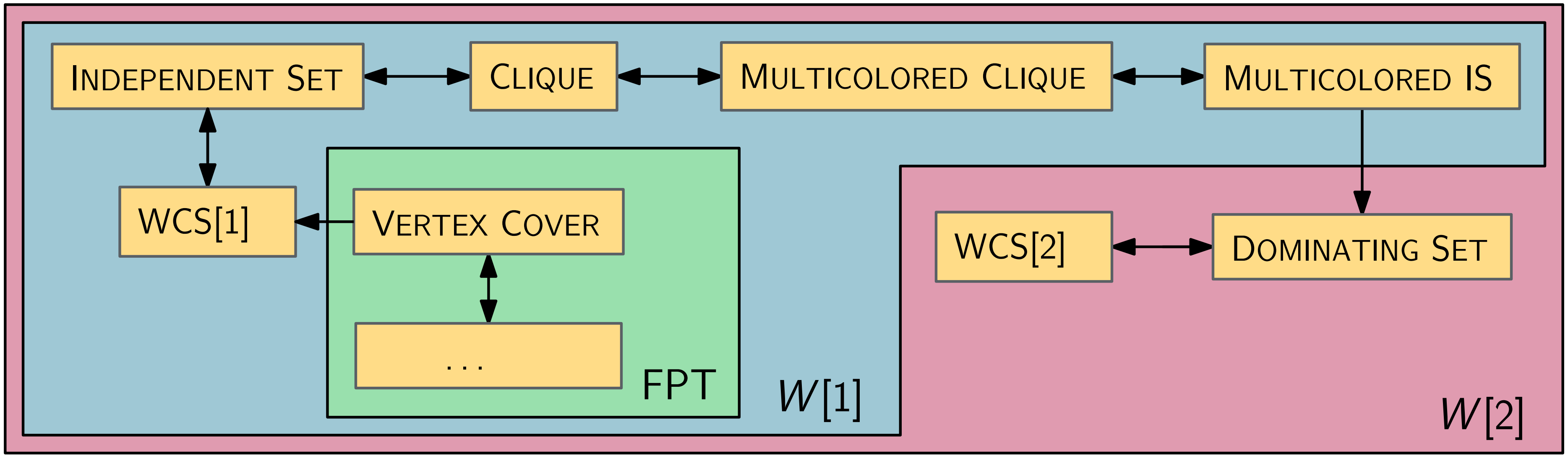

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Summary

The W-Hierarchy

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