Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Computational Geometry

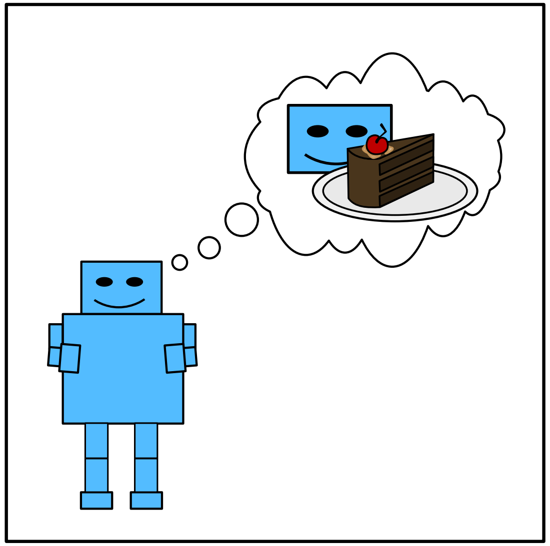
Motion Planning

Lecture #10

Thomas van Dijk

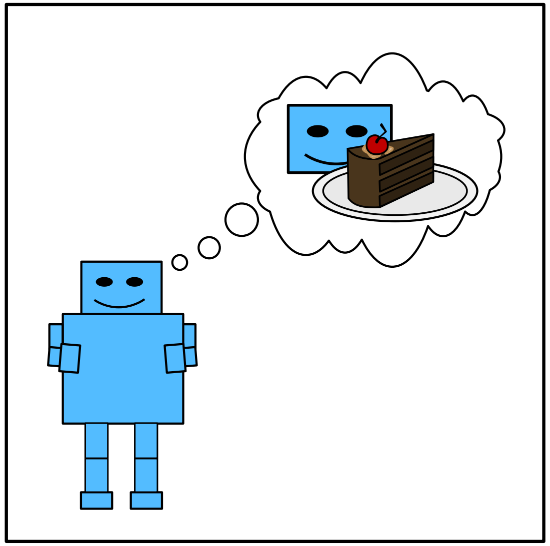
Winter Semester 2018/19

Planning



current situation,
desired situation

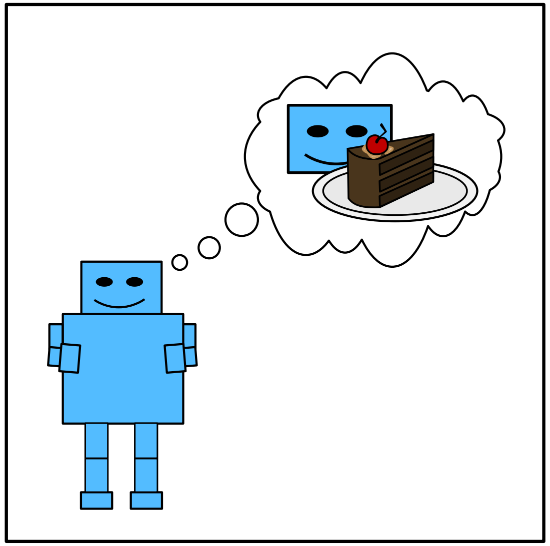
Planning



current situation,
desired situation

$\Rightarrow$

Planning



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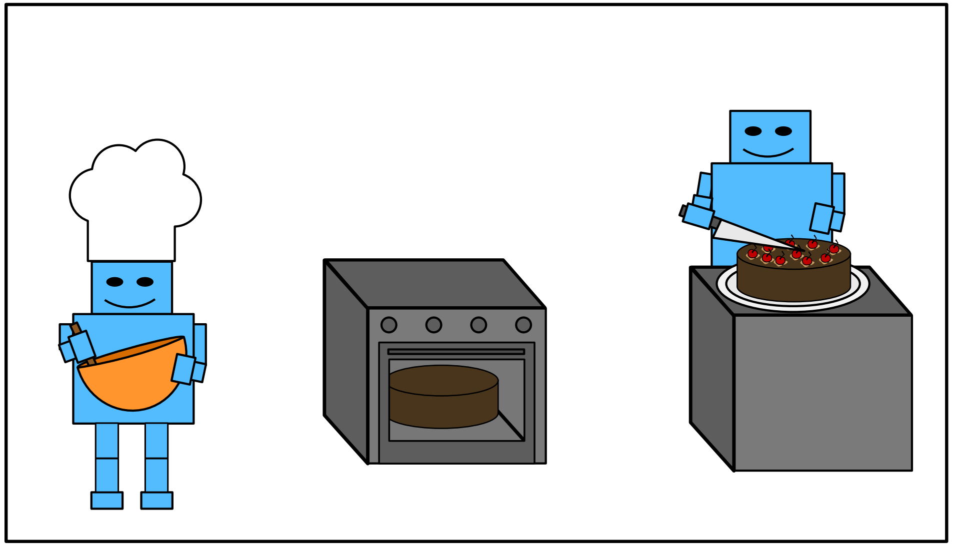
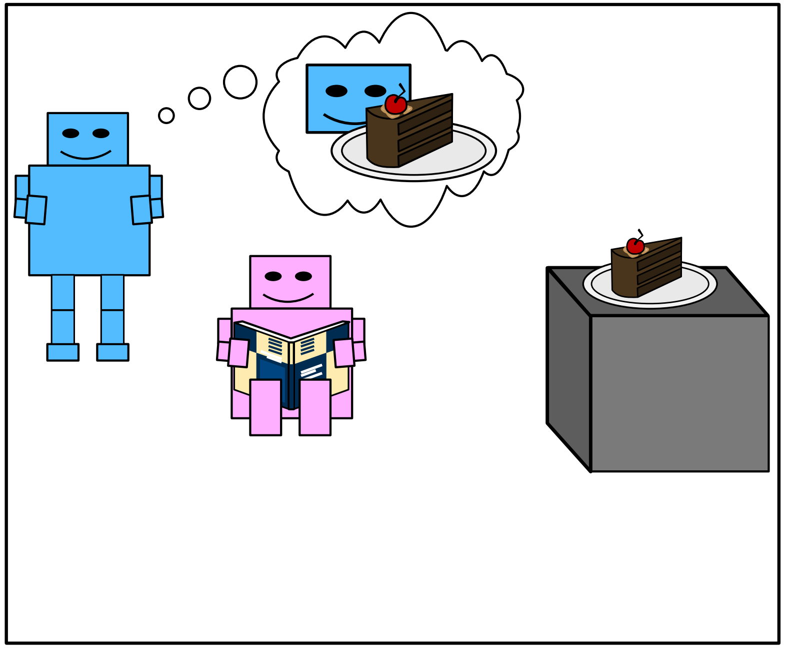


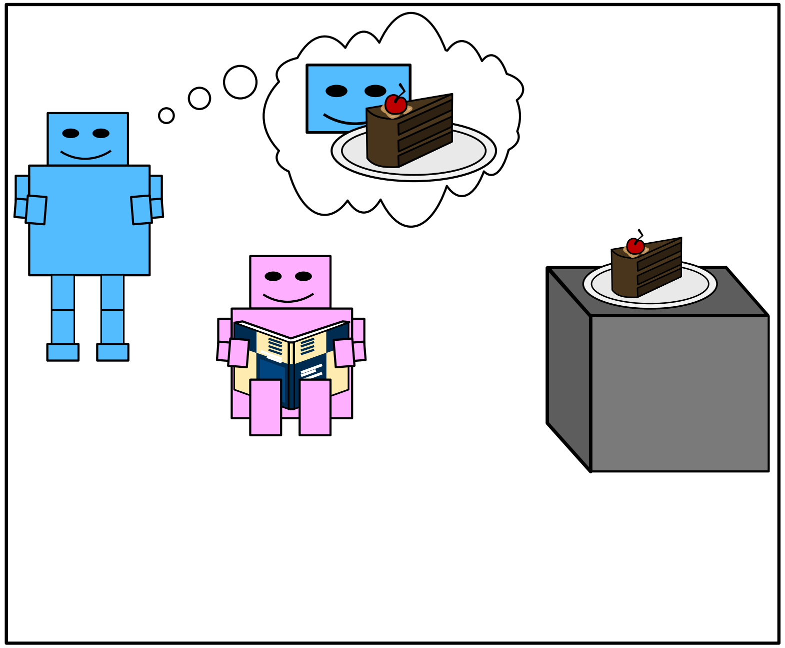
sequence of steps to reach
the one from the other

Path Planning



current location,
desired location

Path Planning



⇒

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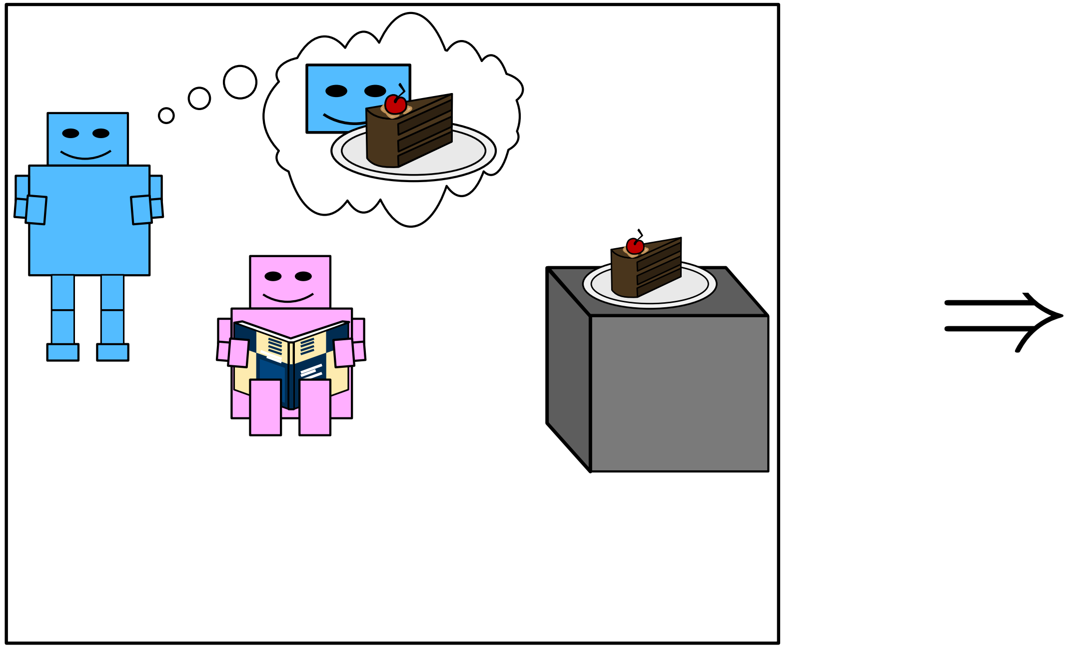
Path Planning



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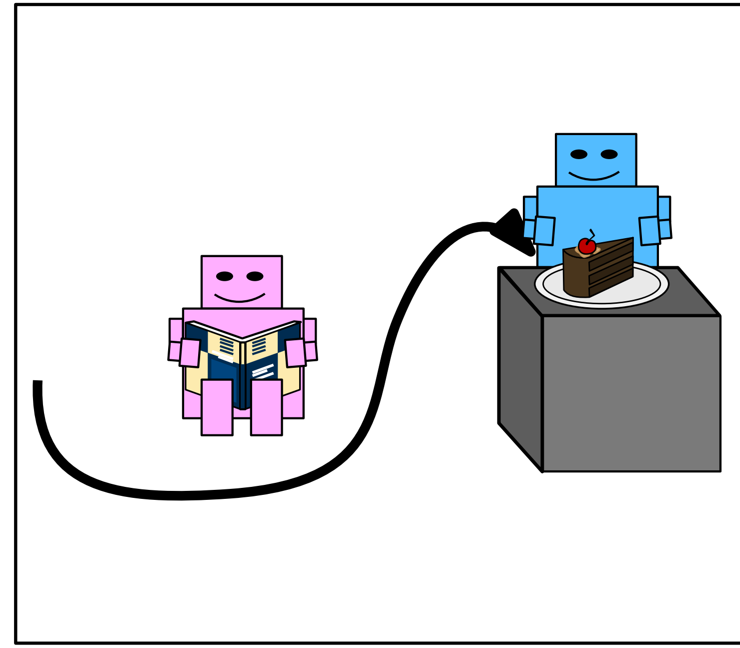

path to reach the
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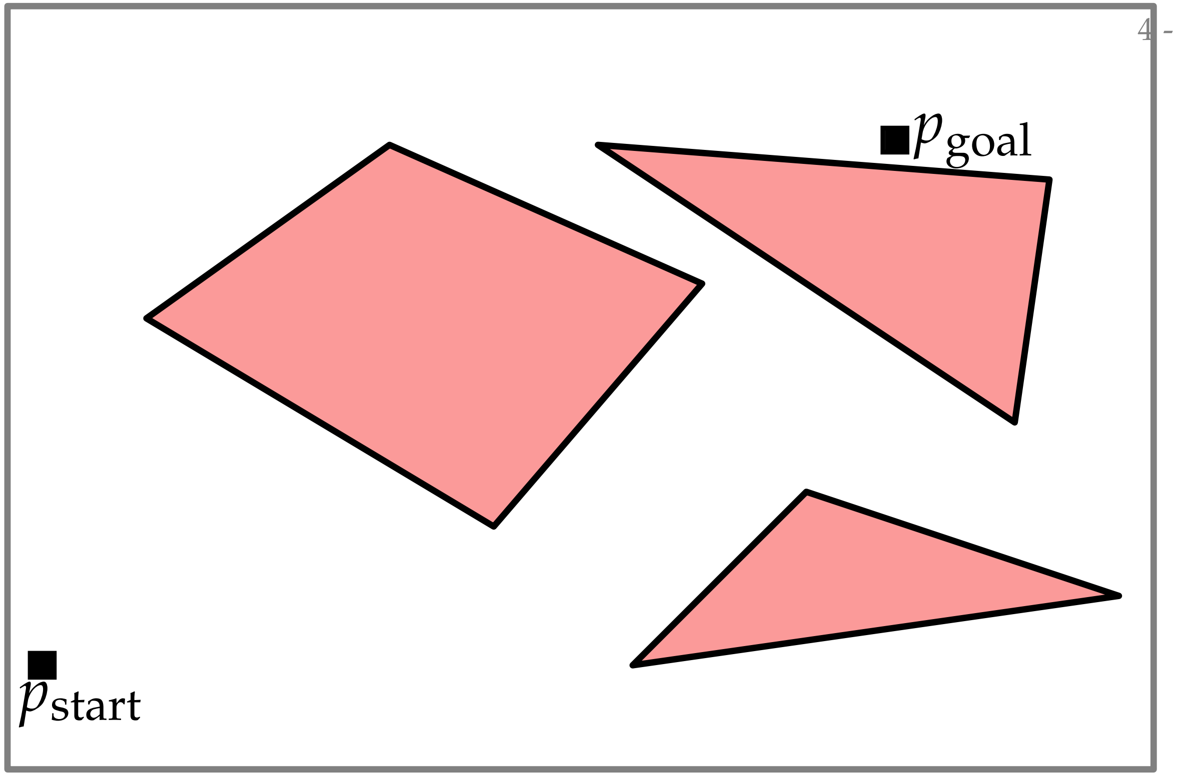
Point-Shaped Robots



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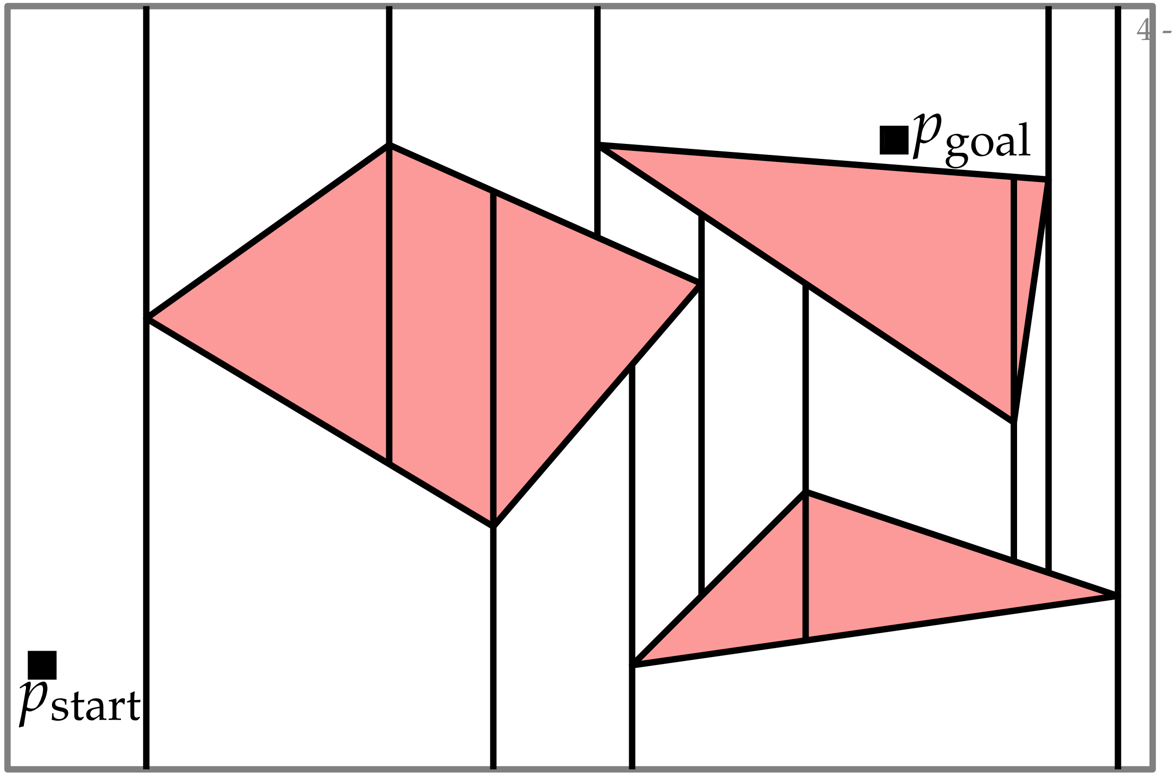
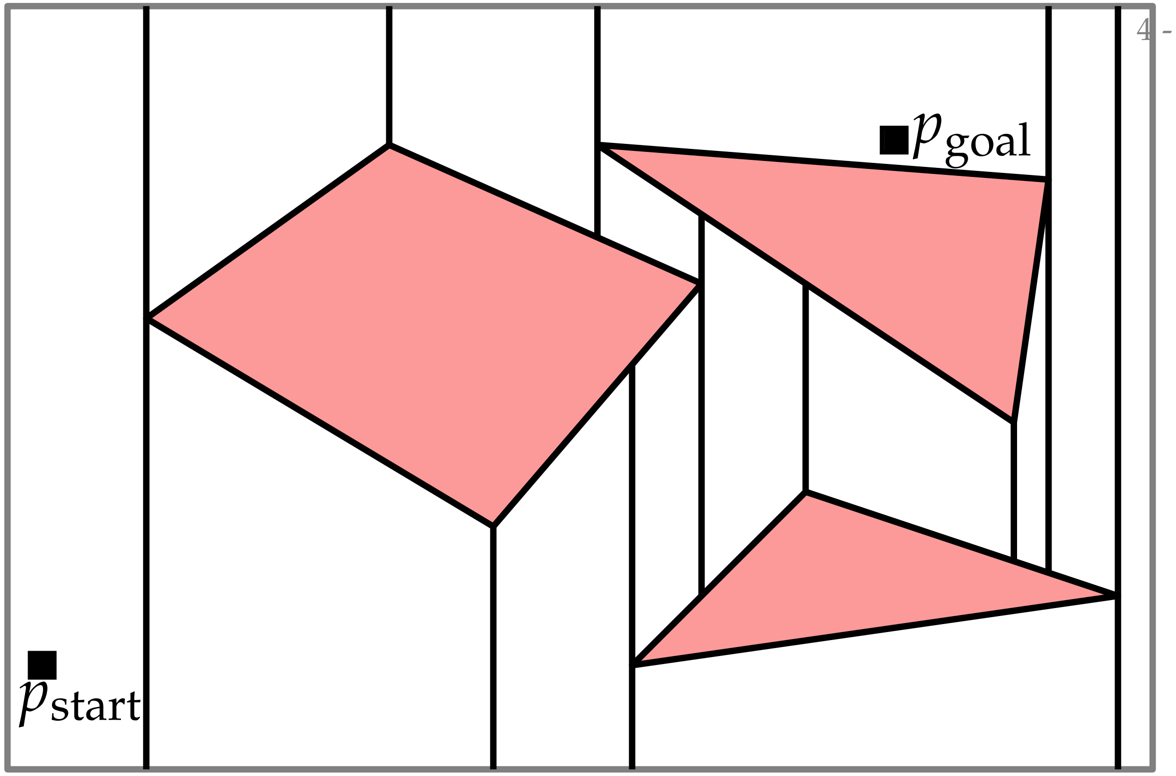


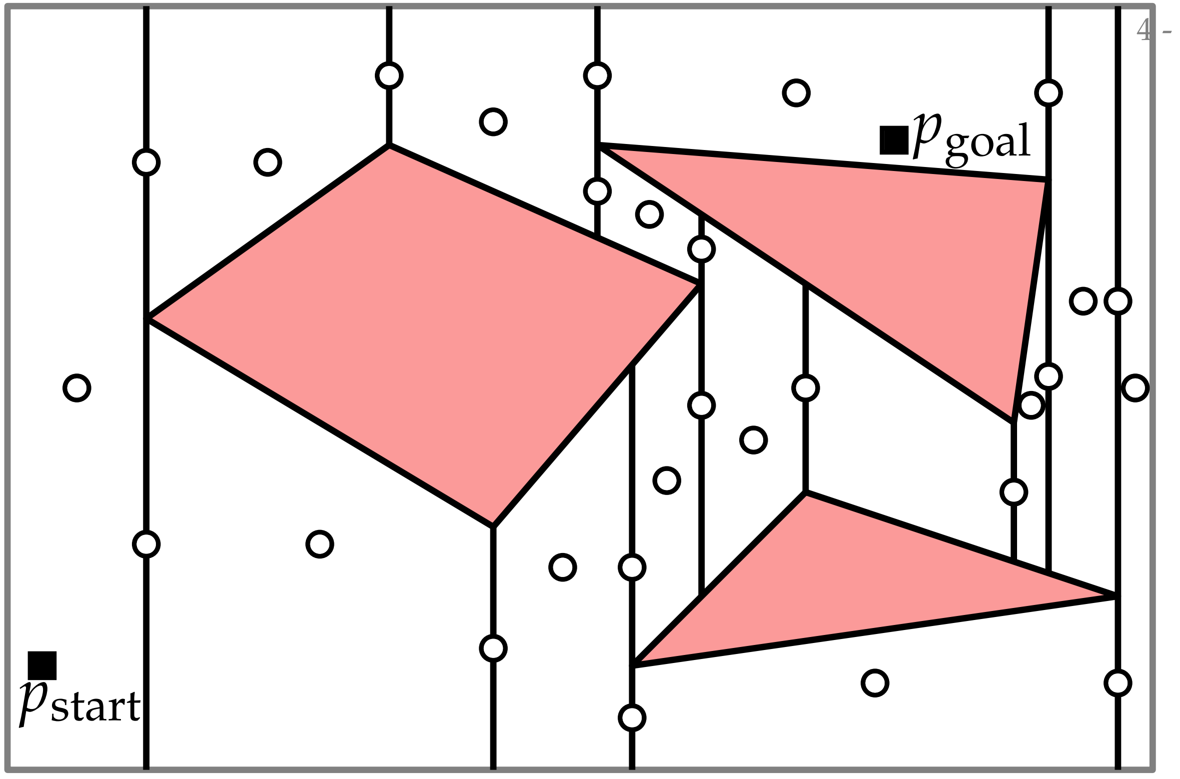
- Create trapezoidal map of obstacle edges.

Point-Shaped Robots



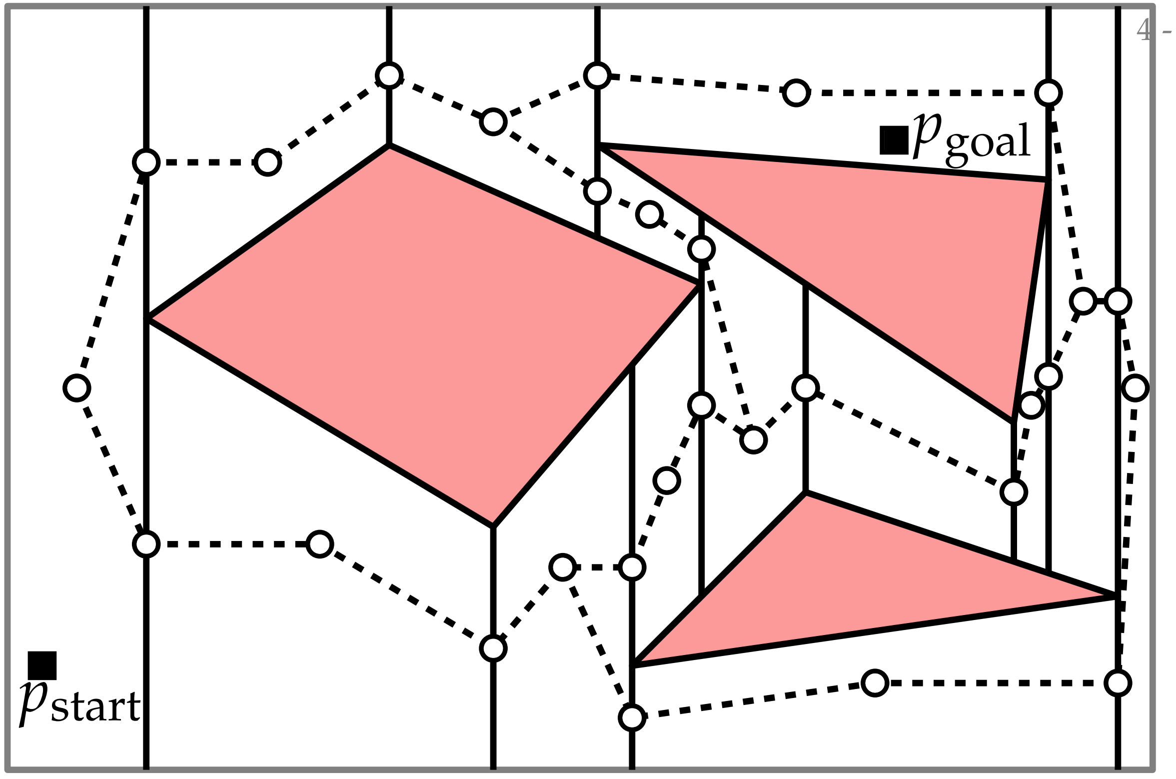
- Create trapezoidal map of obstacle edges.
- Remove vertical extensions inside obstacles.

Point-Shaped Robots



- Create trapezoidal map of obstacle edges.
- Remove vertical extensions inside obstacles.
- Vertices at centers of trapez. and vertical ext.

Point-Shaped Robots



- Create trapezoidal map of obstacle edges.
- Remove vertical extensions inside obstacles.
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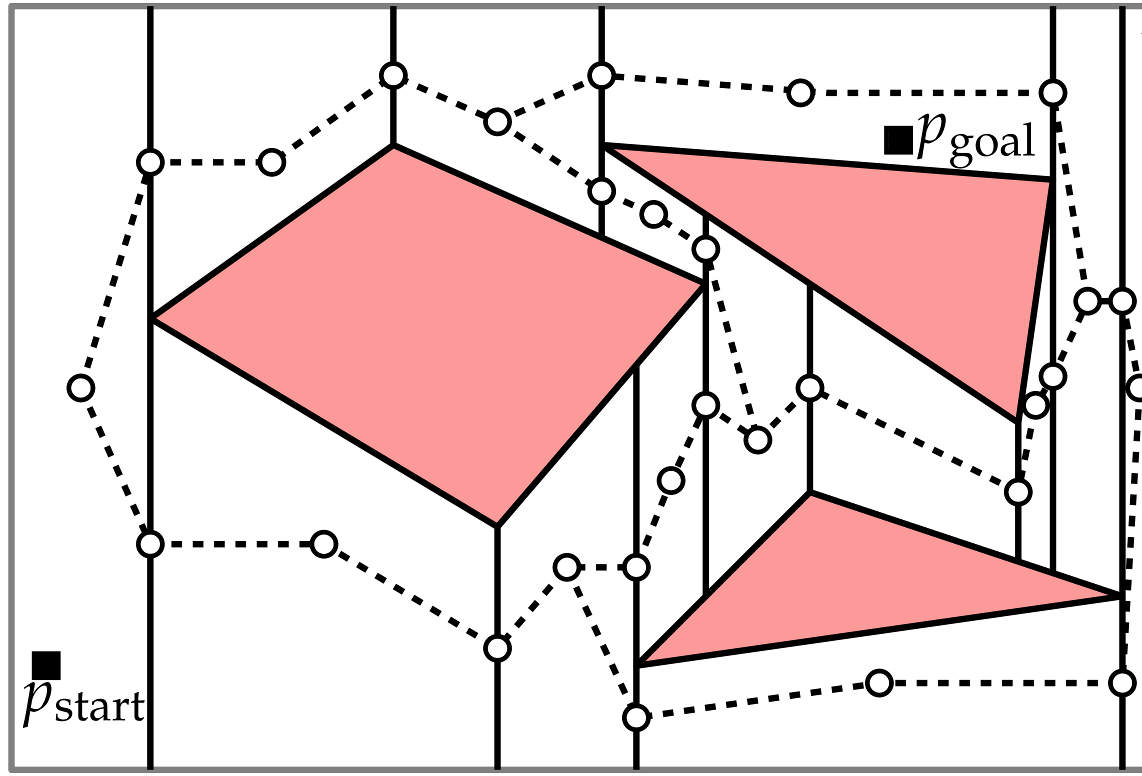
- Connect “neighboring” vertices by line segm.

Point-Shaped Robots



- Create trapezoidal map of obstacle edges.
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- Locate p_{start}, p_{goal} in map

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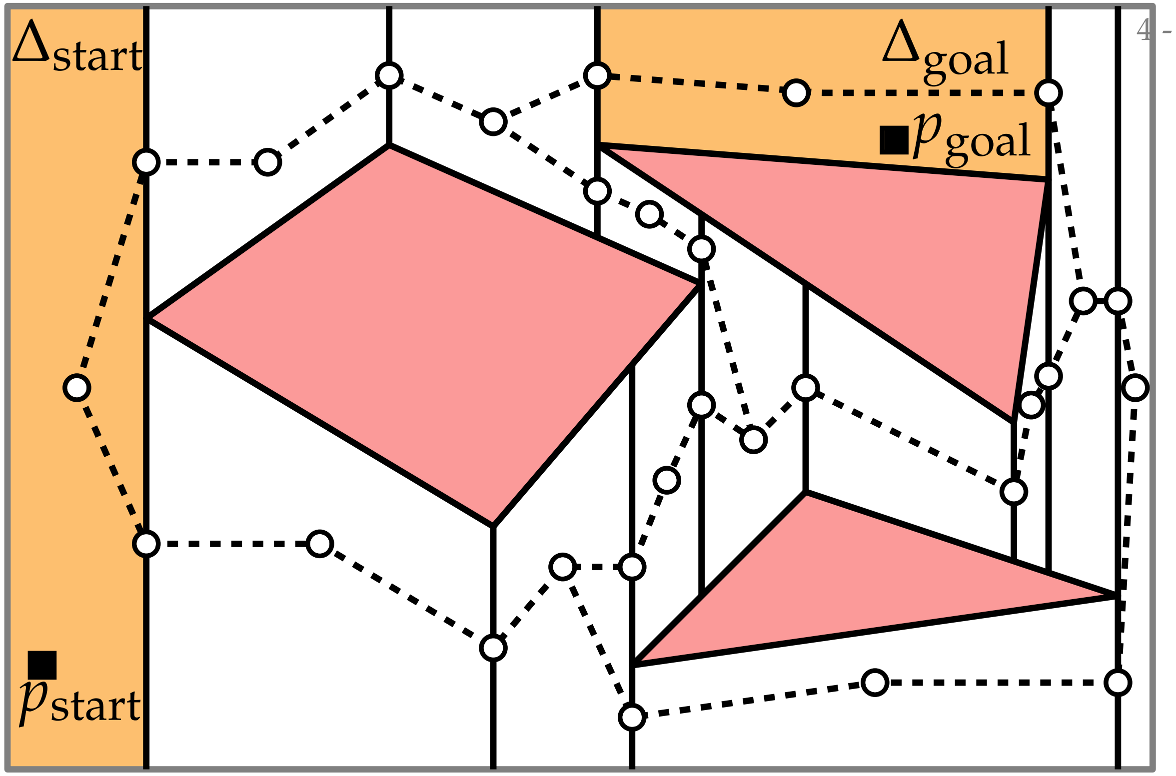

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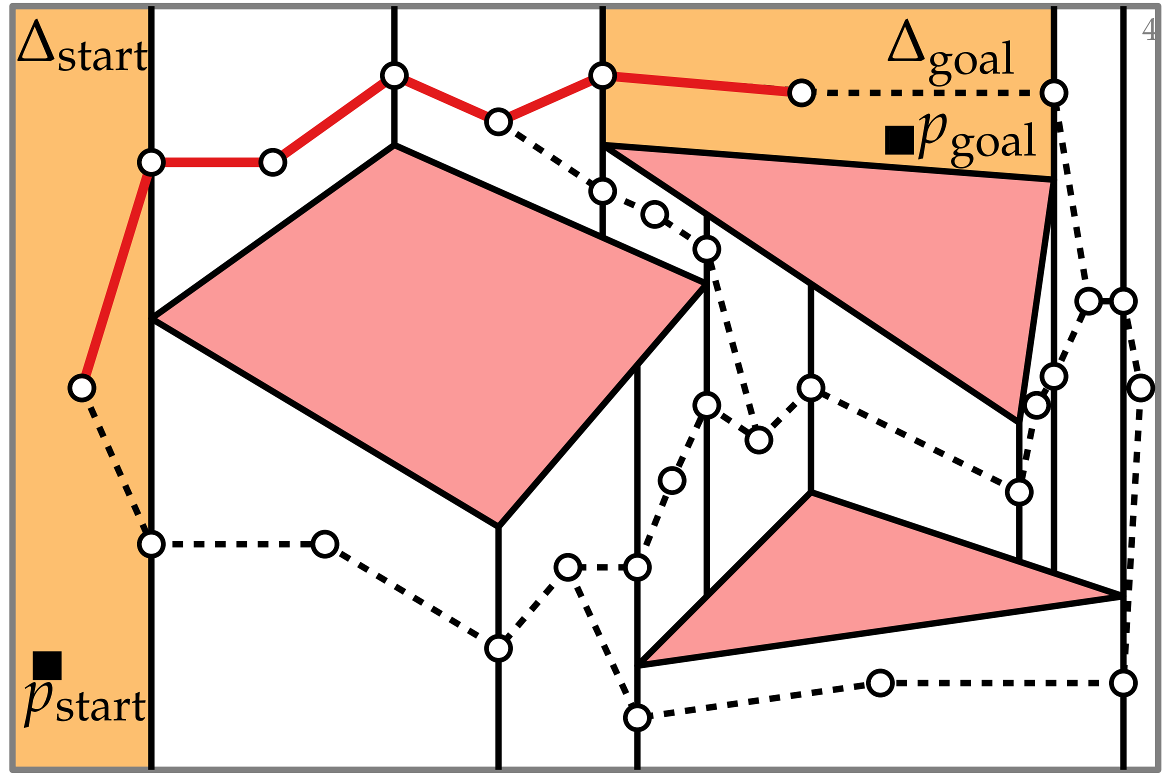

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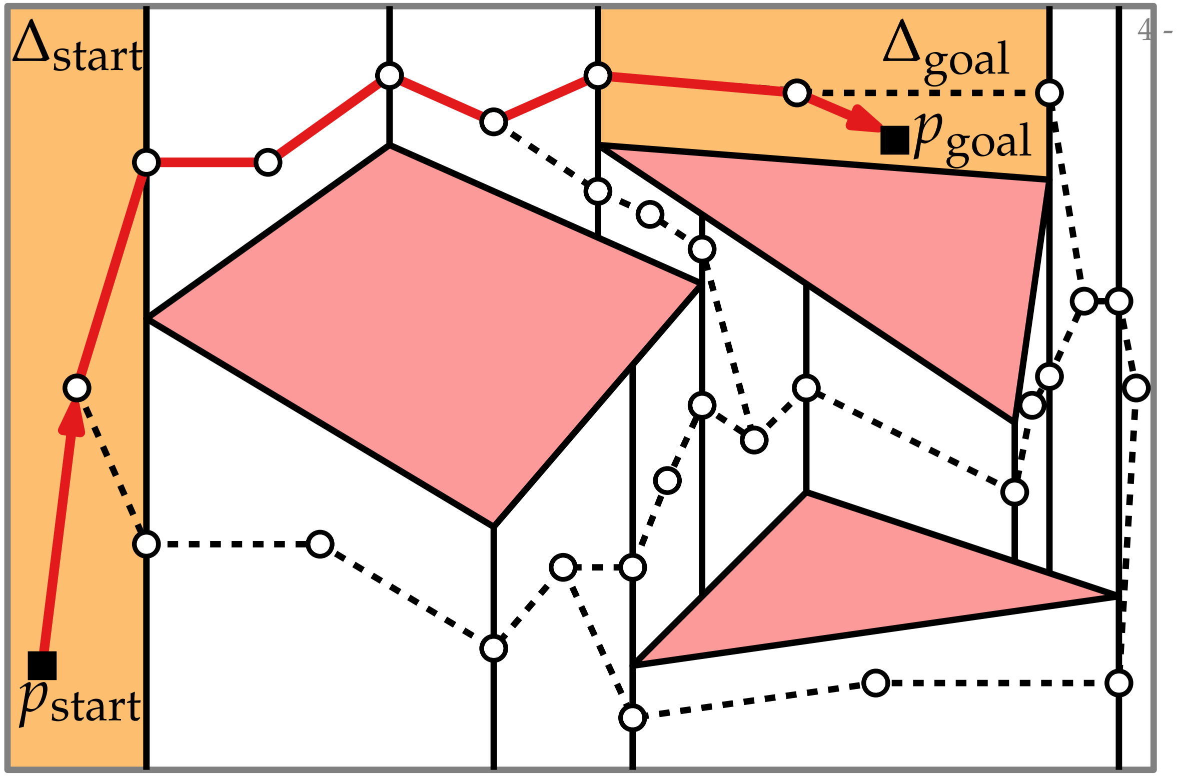
Point-Shaped Robots



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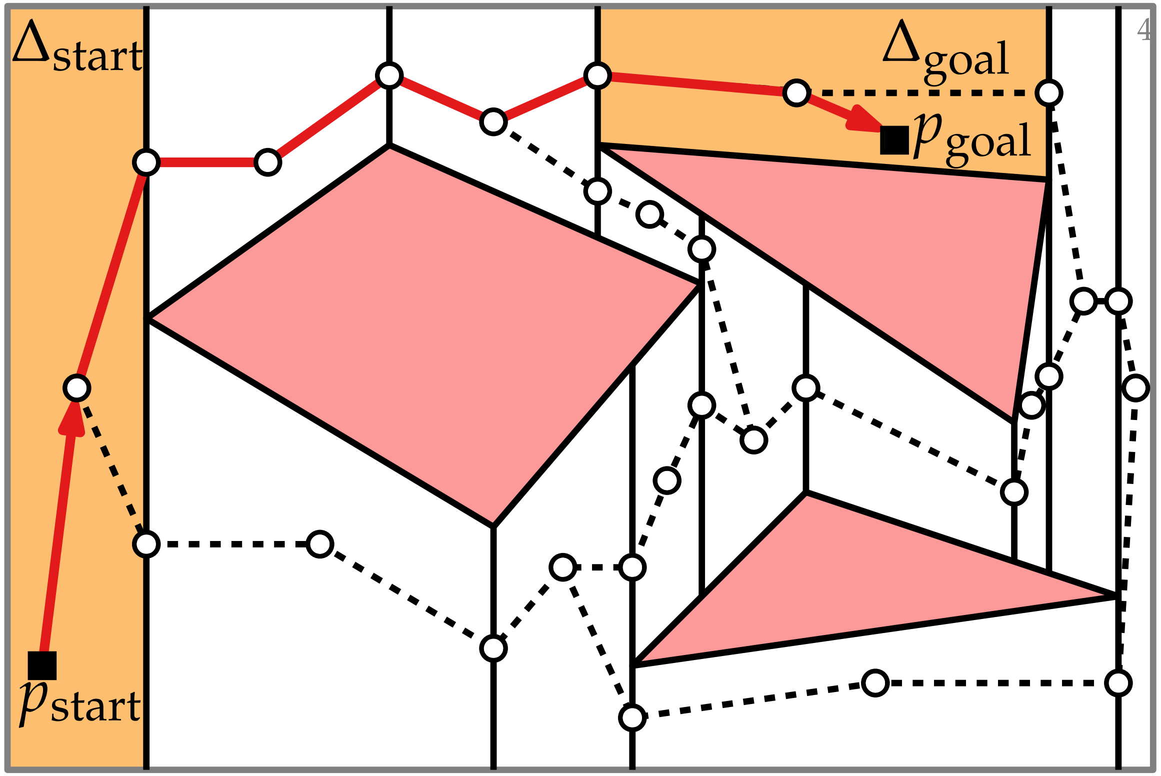

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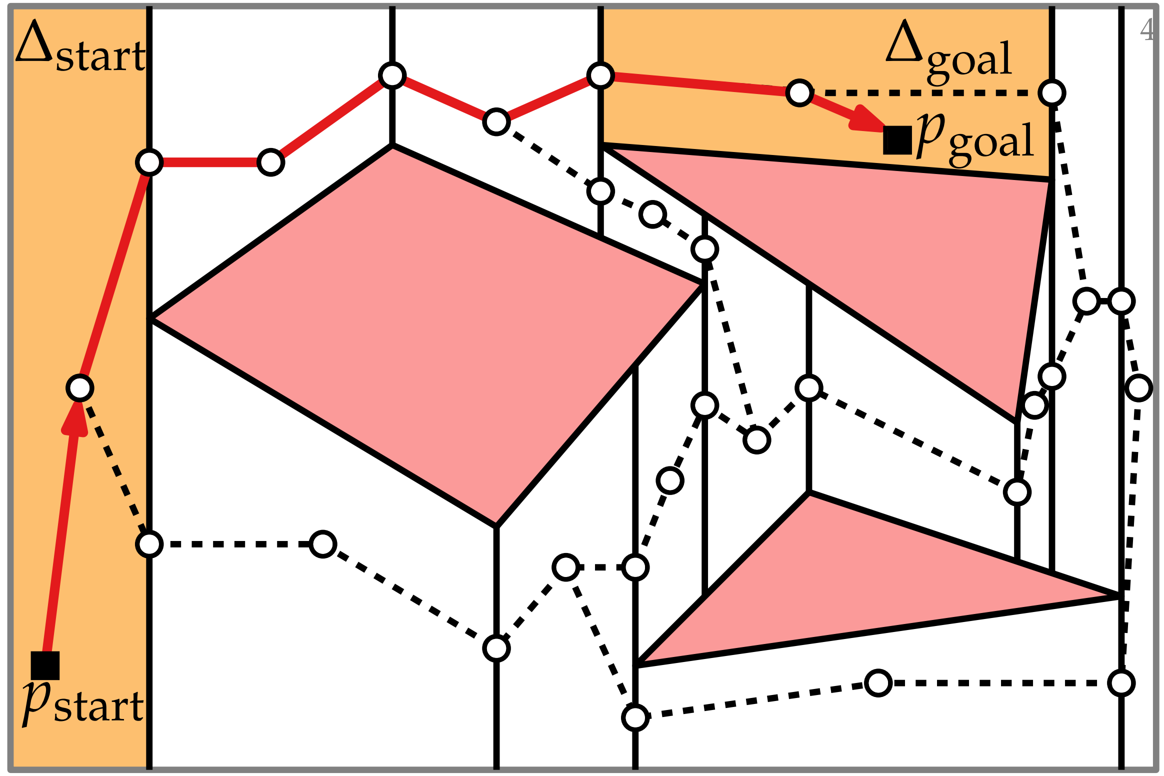

- Create trapezoidal map of obstacle edges. $O(n \log n)$
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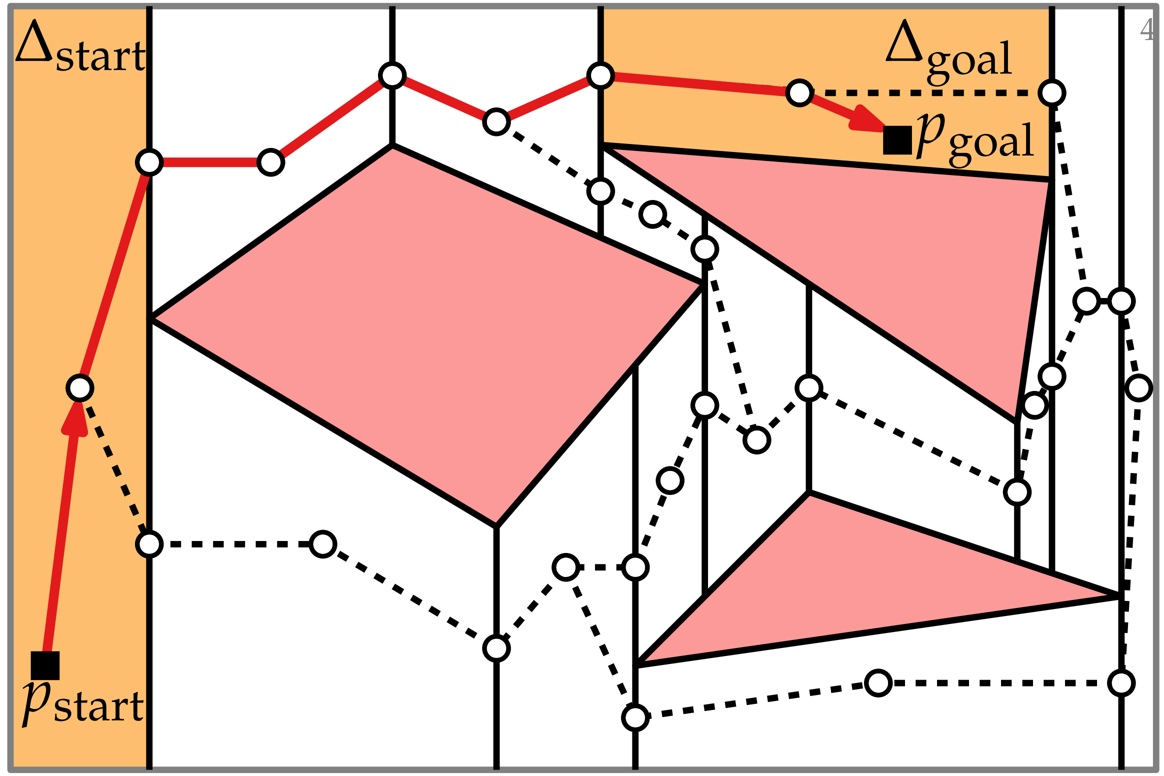

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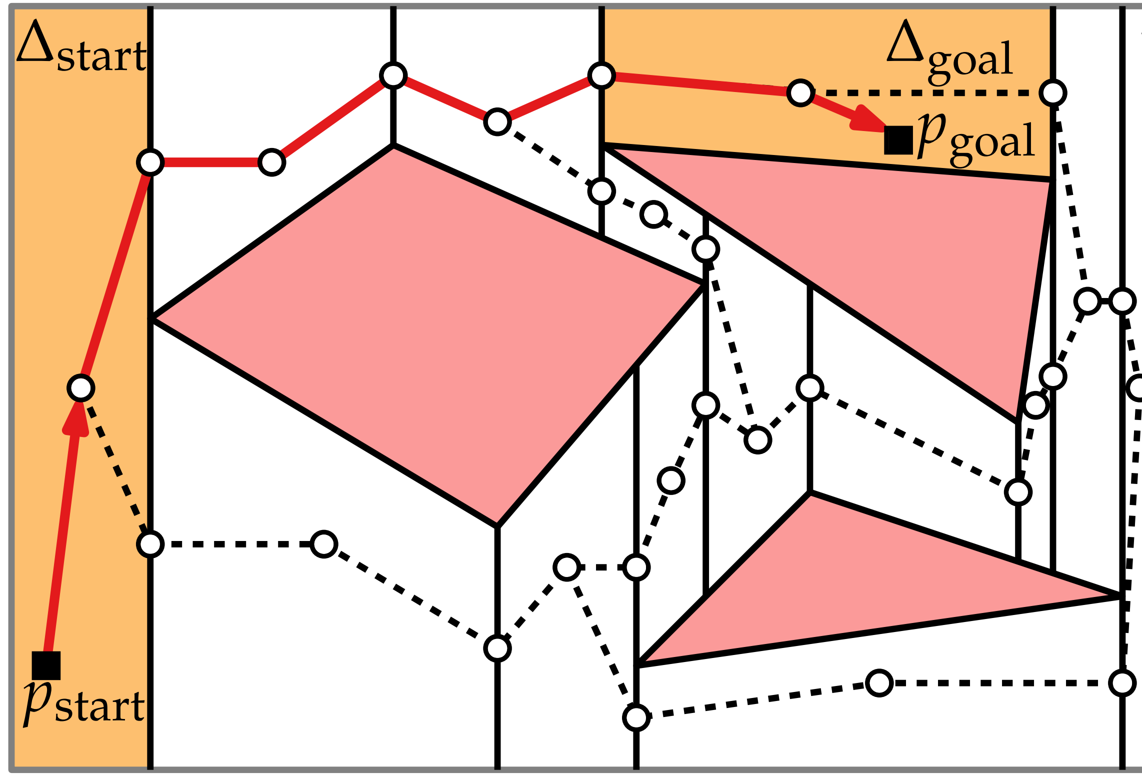

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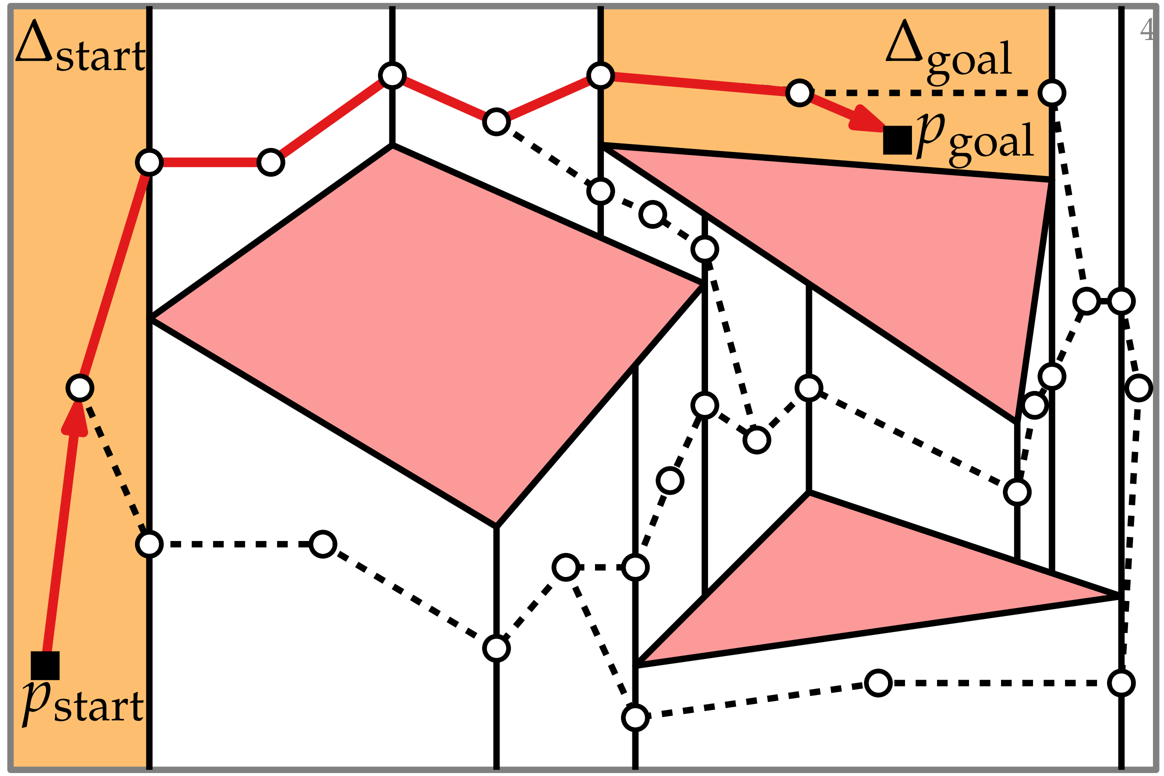

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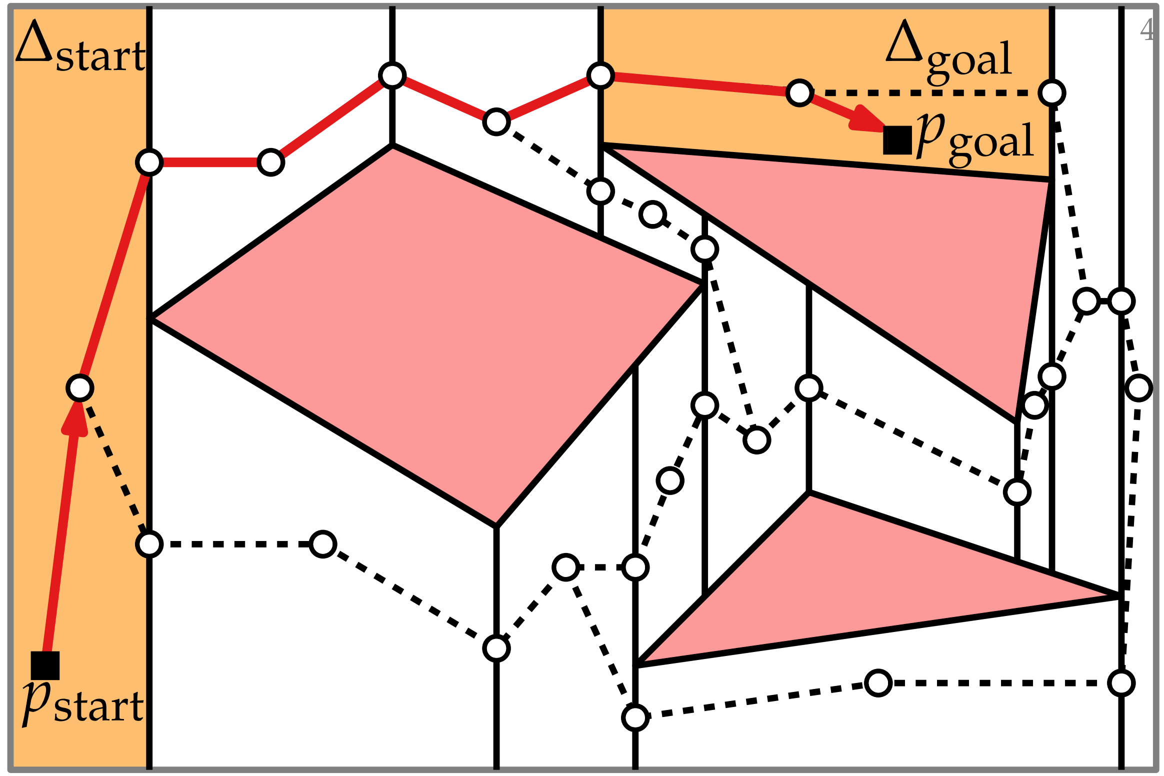

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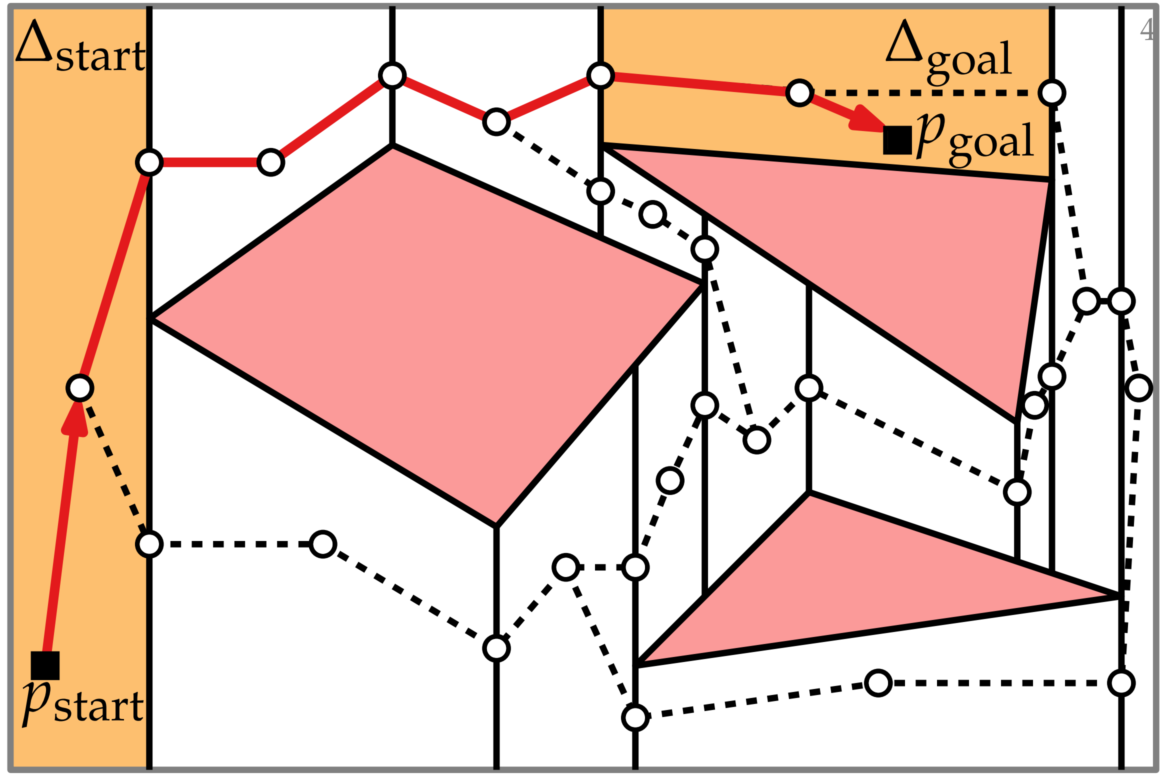

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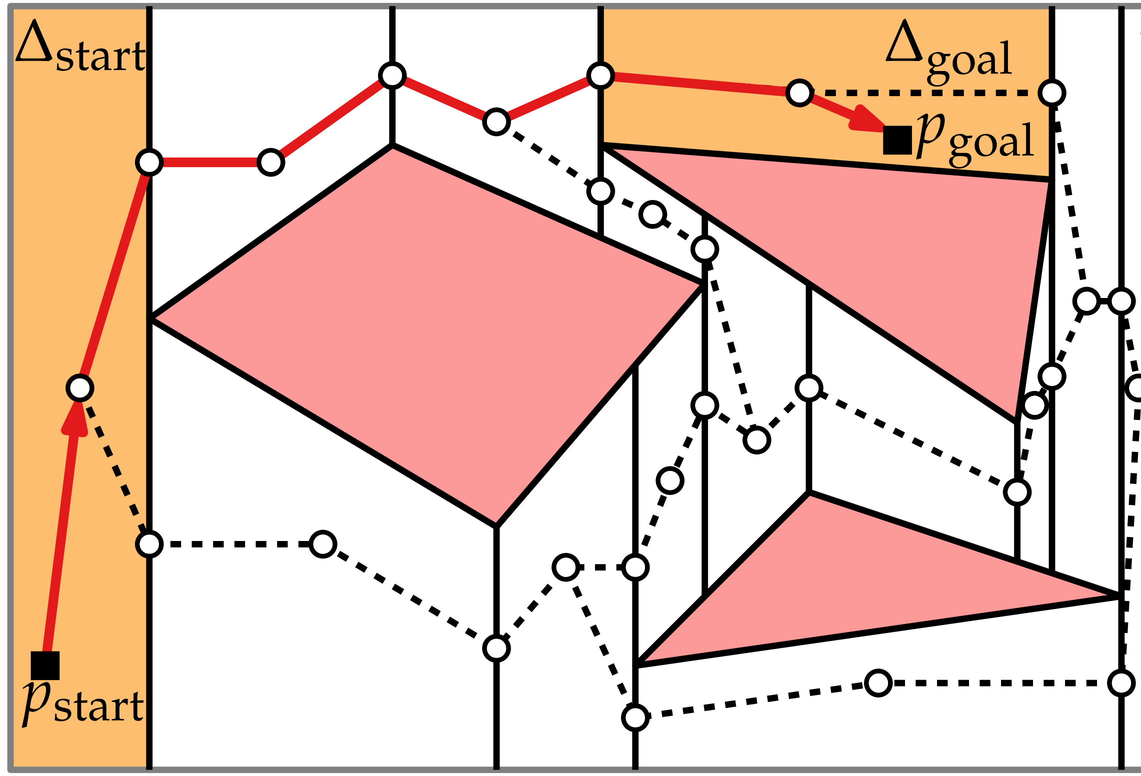

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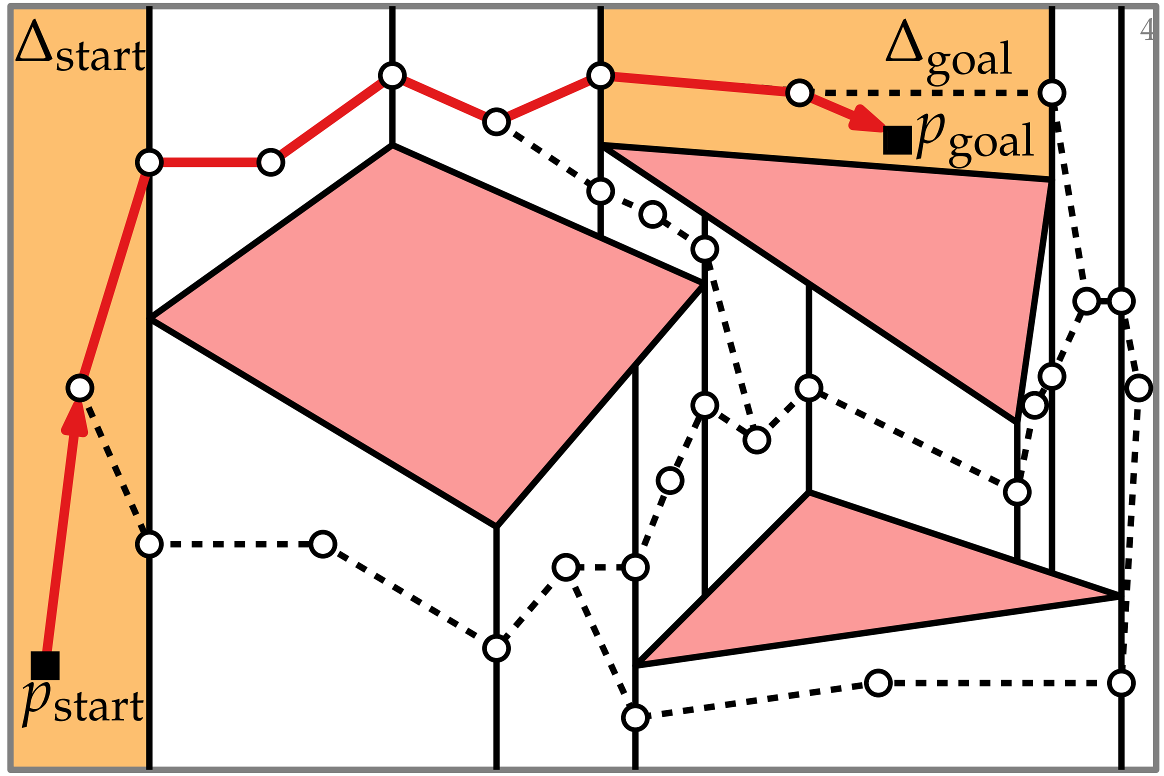
Point-Shaped Robots



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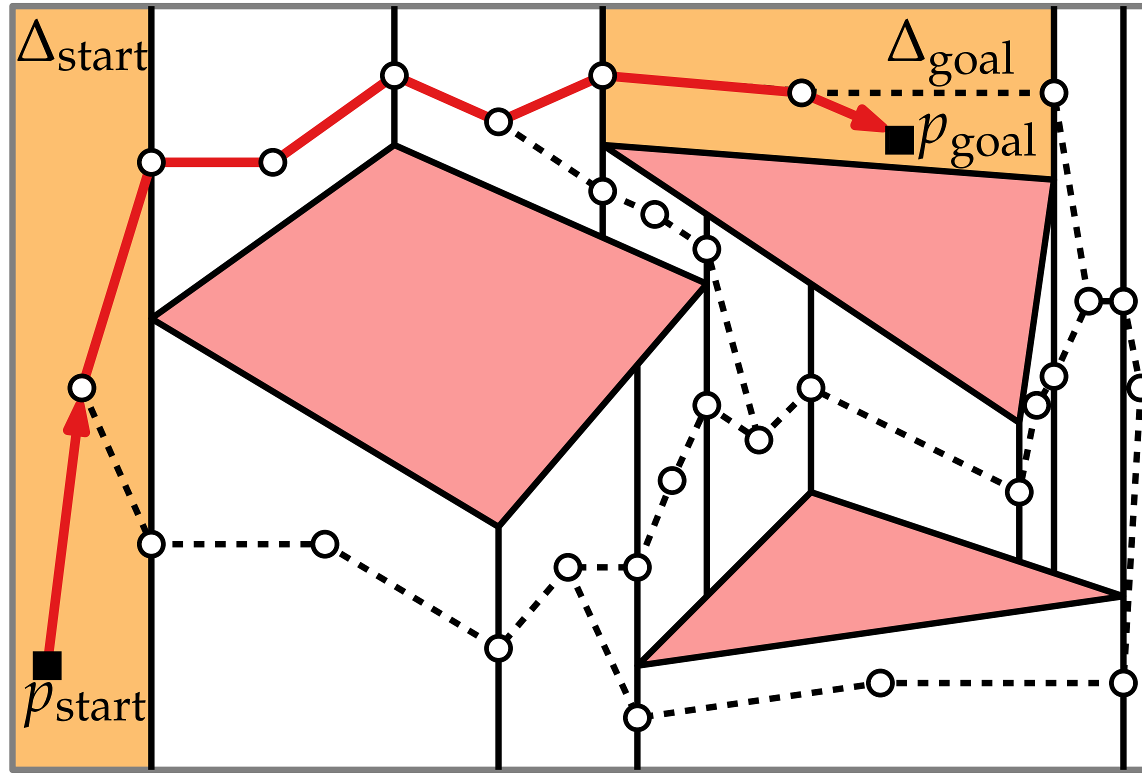

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querying	• Locate $p_{\text{start}}, p_{\text{goal}}$ in map $\rightarrow \Delta_{\text{start}}, \Delta_{\text{goal}}$.	$O(\log n)$
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A First Result

Theorem: We can preprocess a set of polygonal obstacles with a total of n edges in $O(n \log n)$ expected time such that, given a start and a goal position, we can find a collision-free path for a point robot in $O(n)$ time if it exists.

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What about, say, *polygonal* robots?

Degrees of Freedom

Every robot has some number d of *degrees of freedom*, meaning that its *configuration* with respect to the world can be specified by d parameters.

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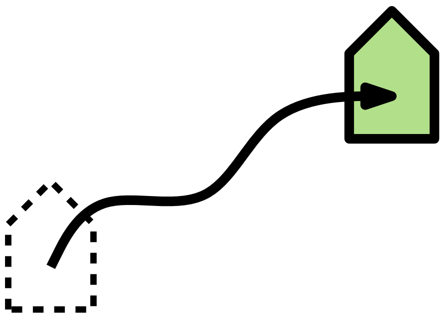

2D translating robot

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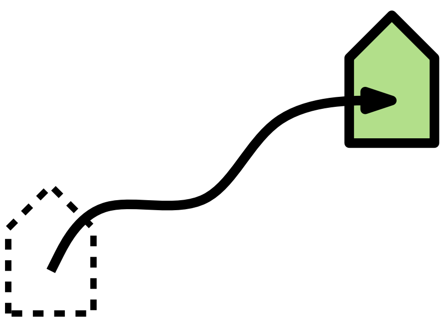

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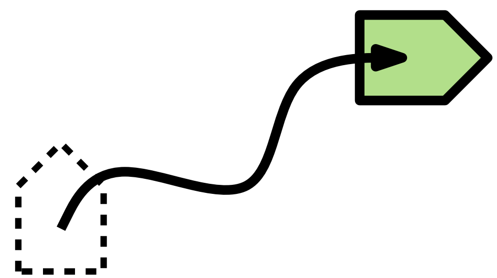

2D translating, rotating robot

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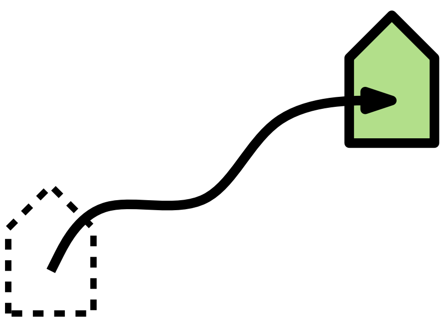

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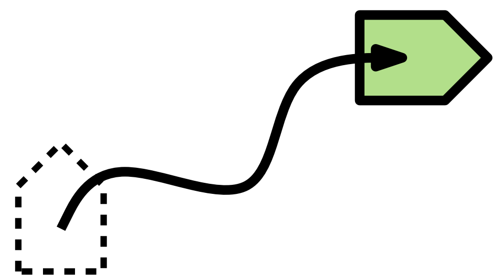

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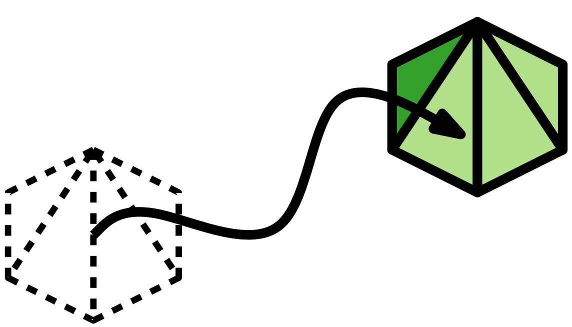

3D translating robot

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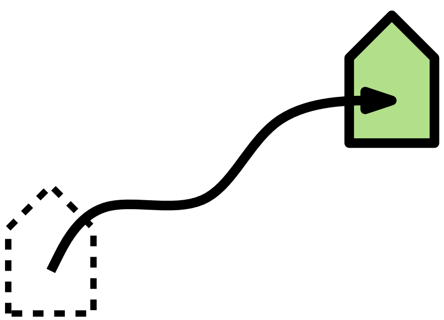

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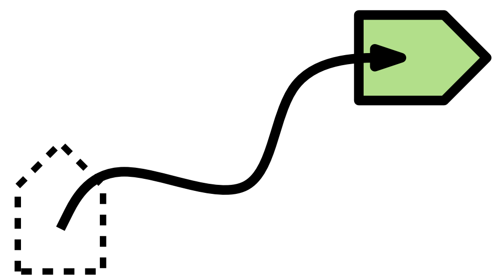
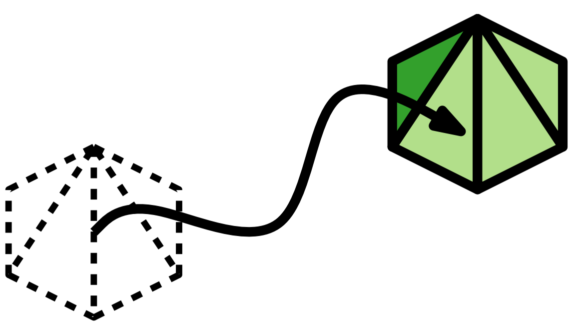


2D translating, rotating robot



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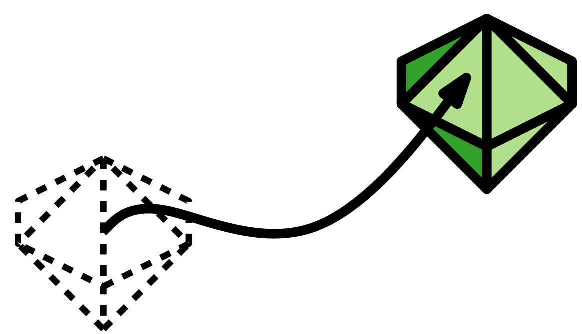
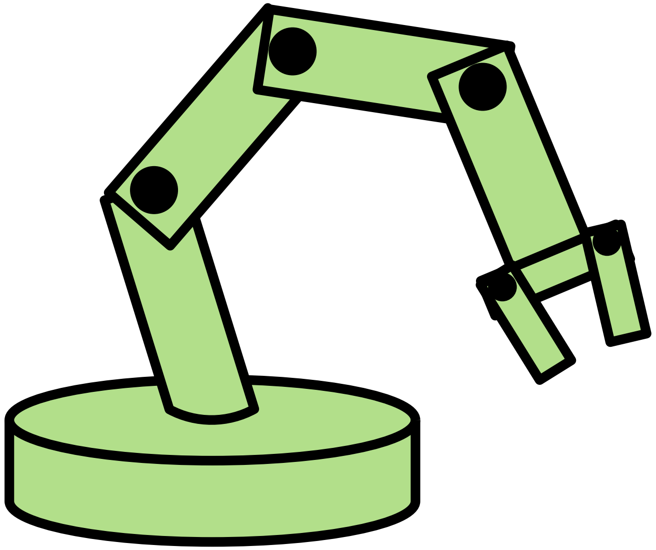


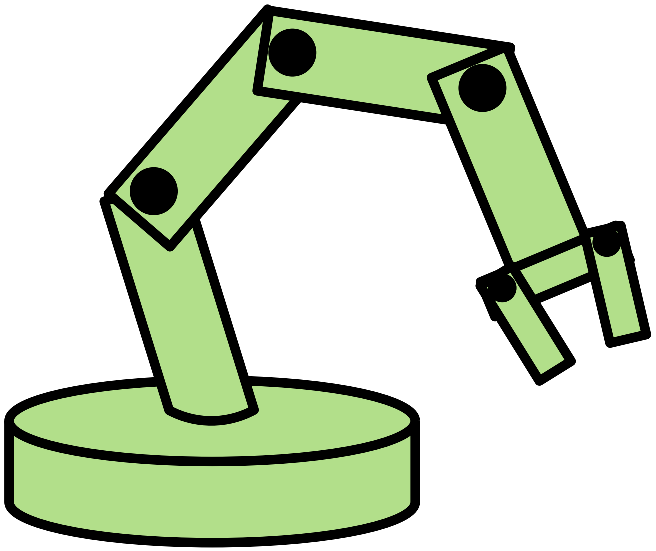
3D translating, rotating robot

Configuration Space



robotic arm

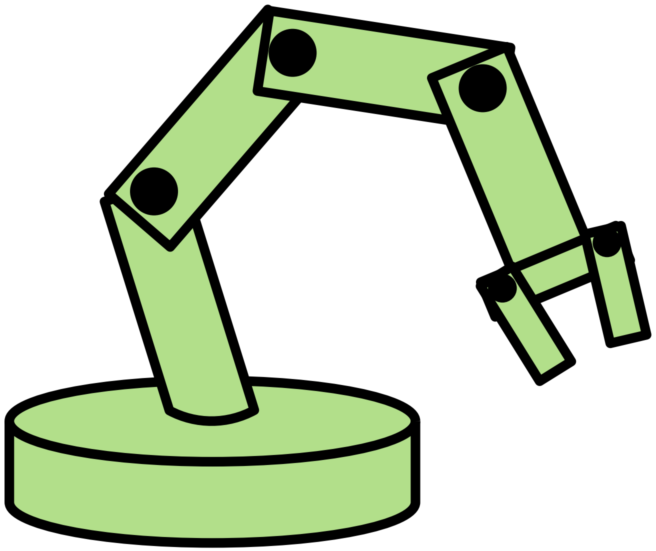
Configuration Space



robotic arm

The *configuration space* is the *d*-dimensional space of all possible (i.e., obstacle avoiding) parameter value combinations.

Configuration Space



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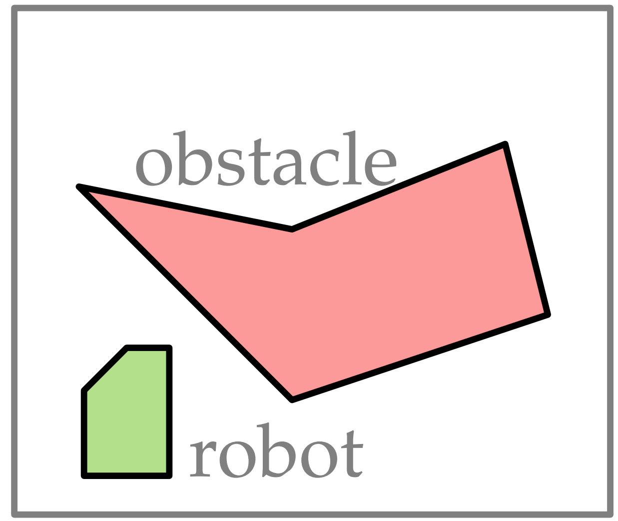

work space

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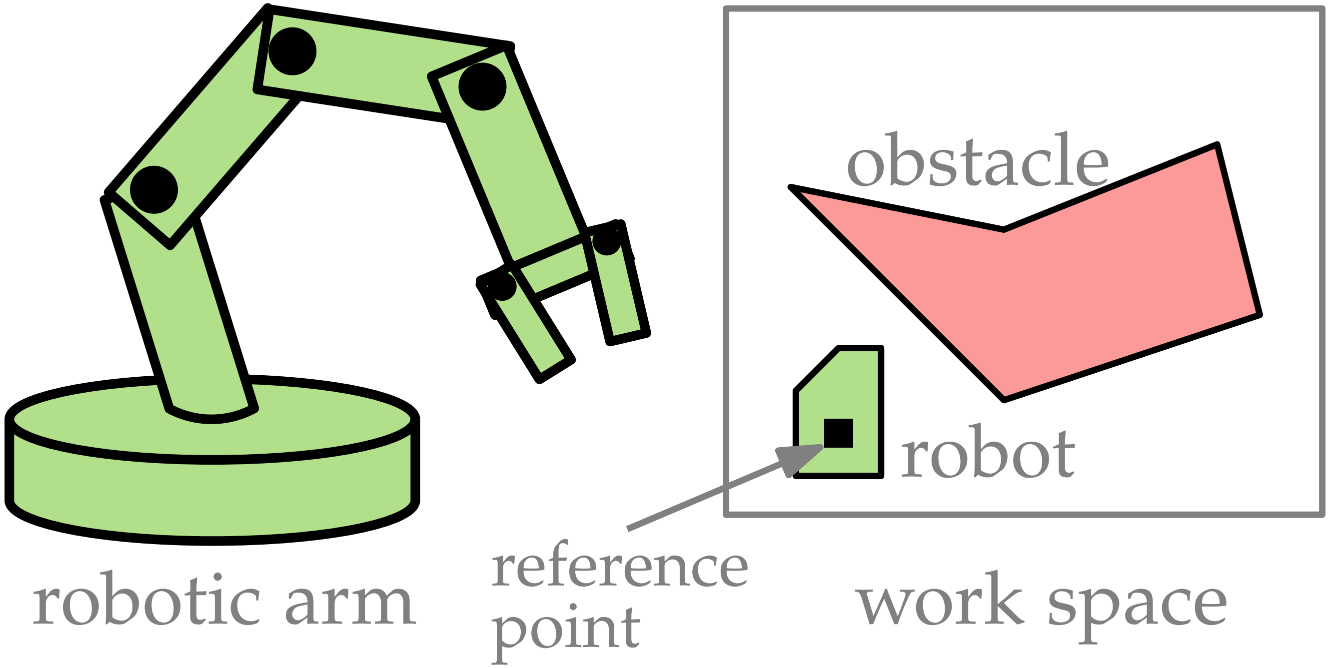
Configuration Space



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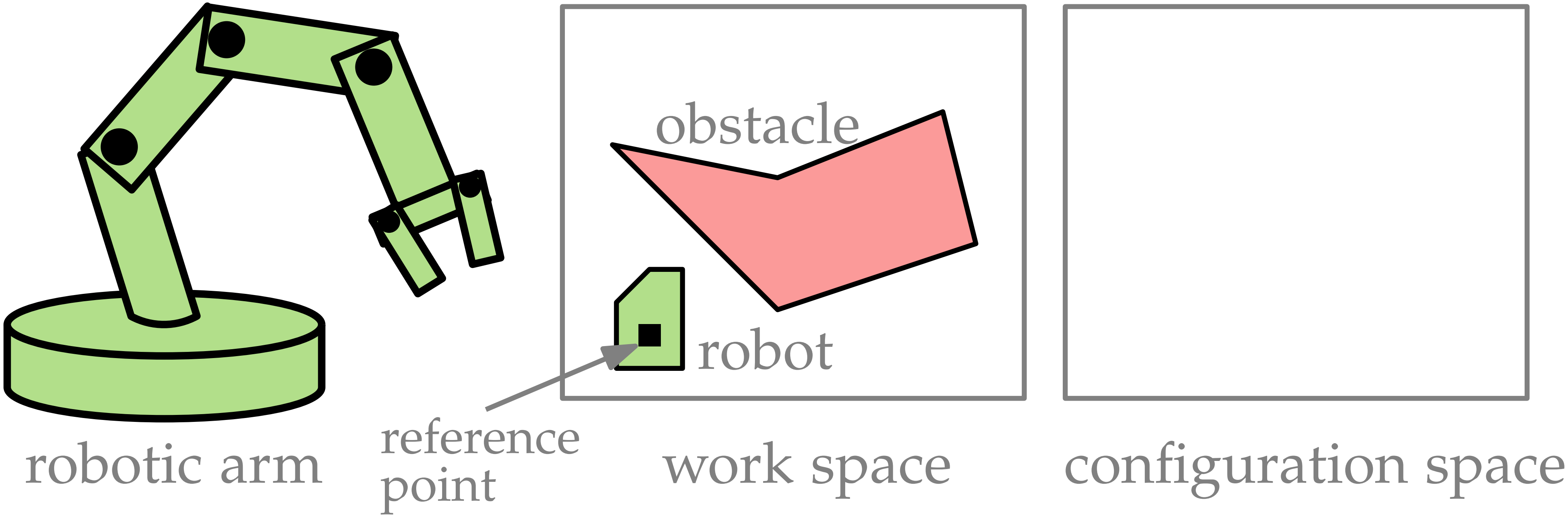

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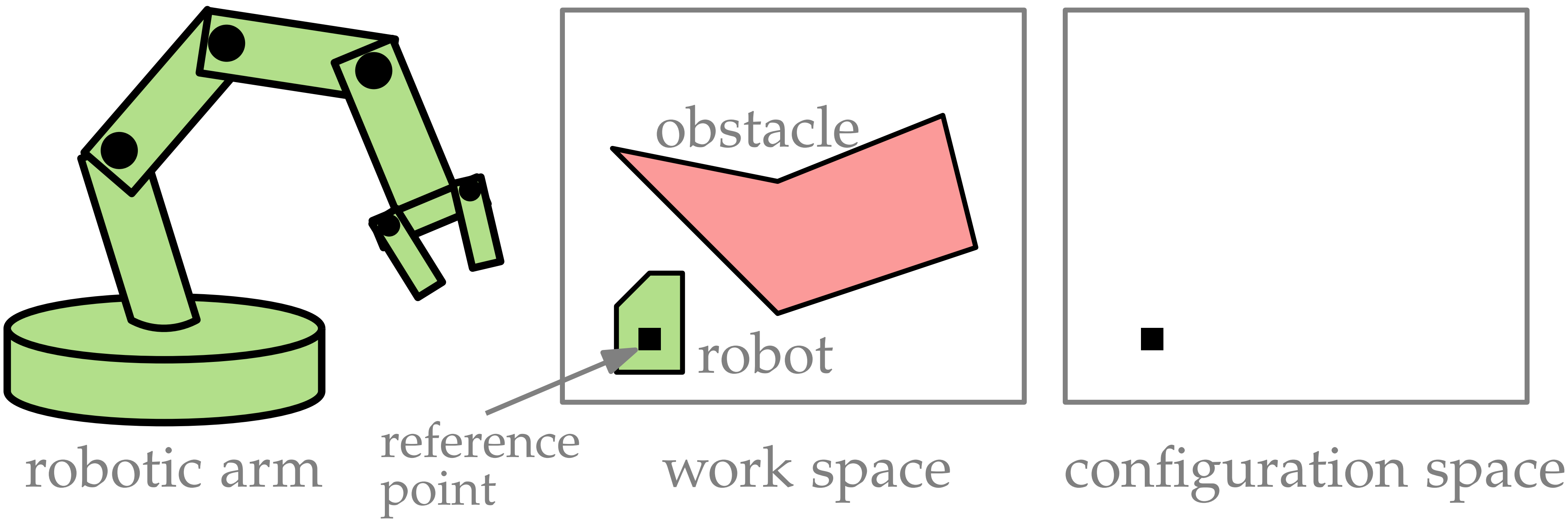

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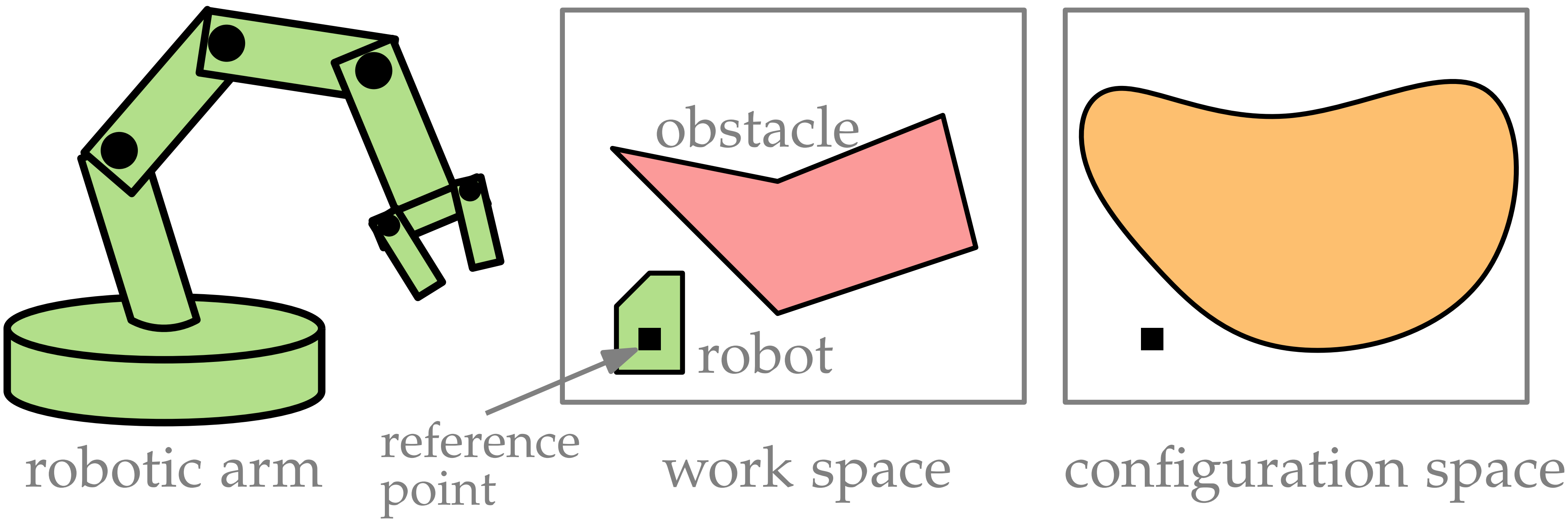

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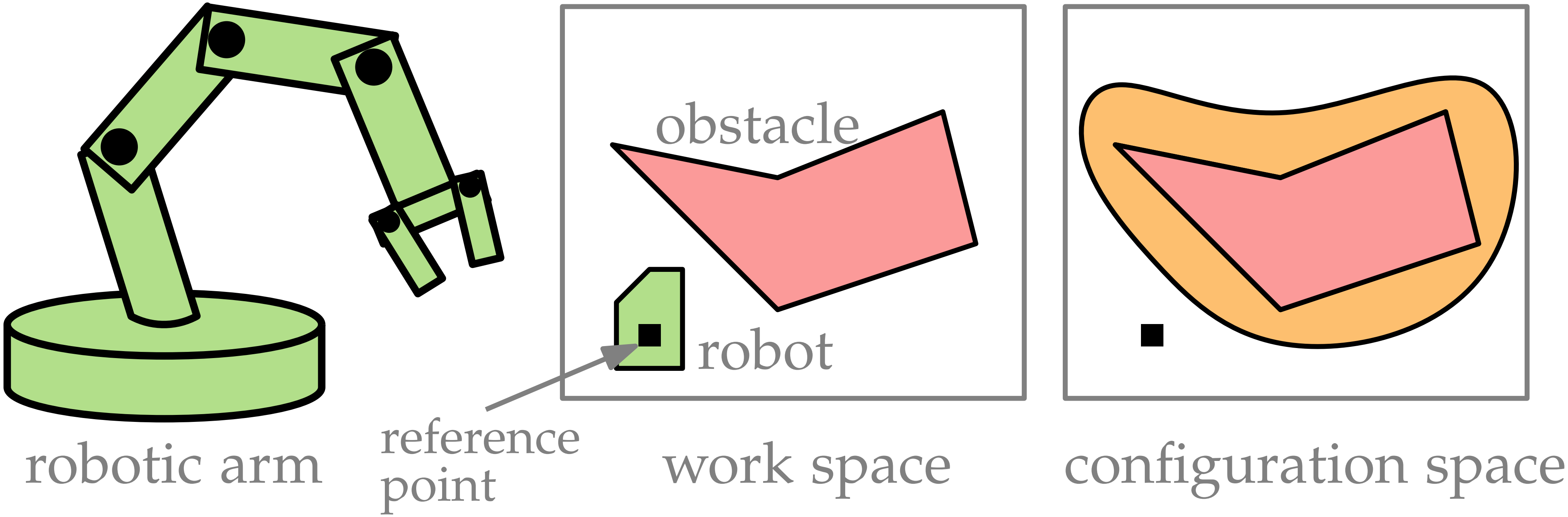

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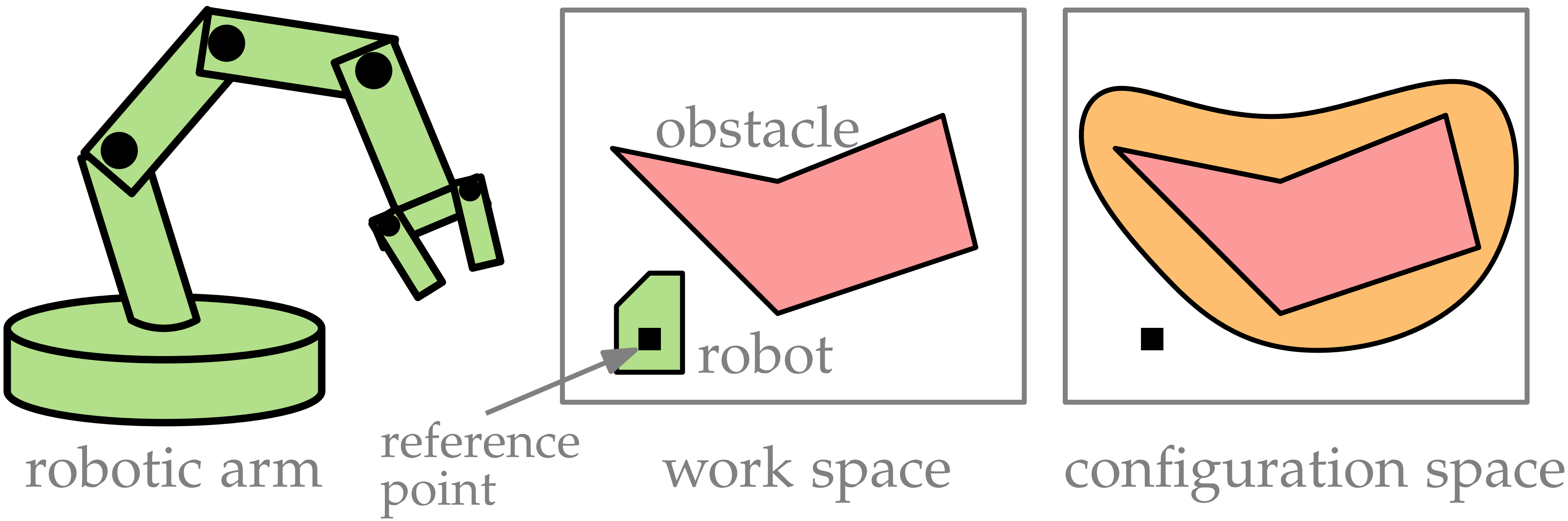

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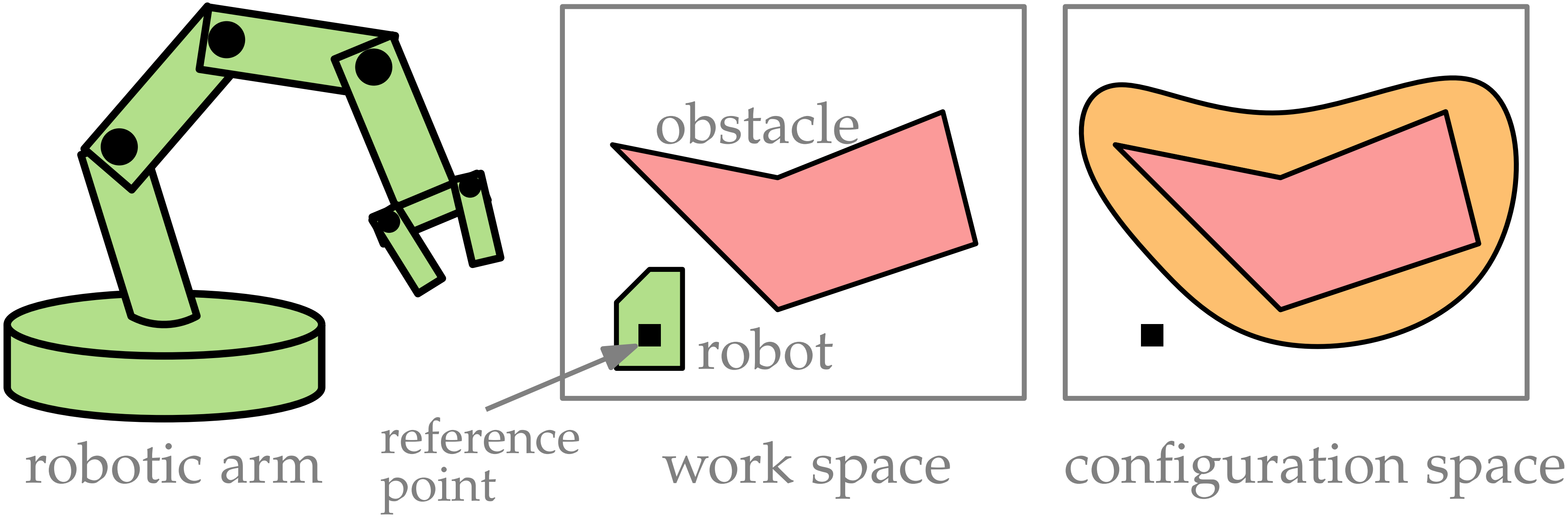
Path for a *point* through configuration space

Configuration Space

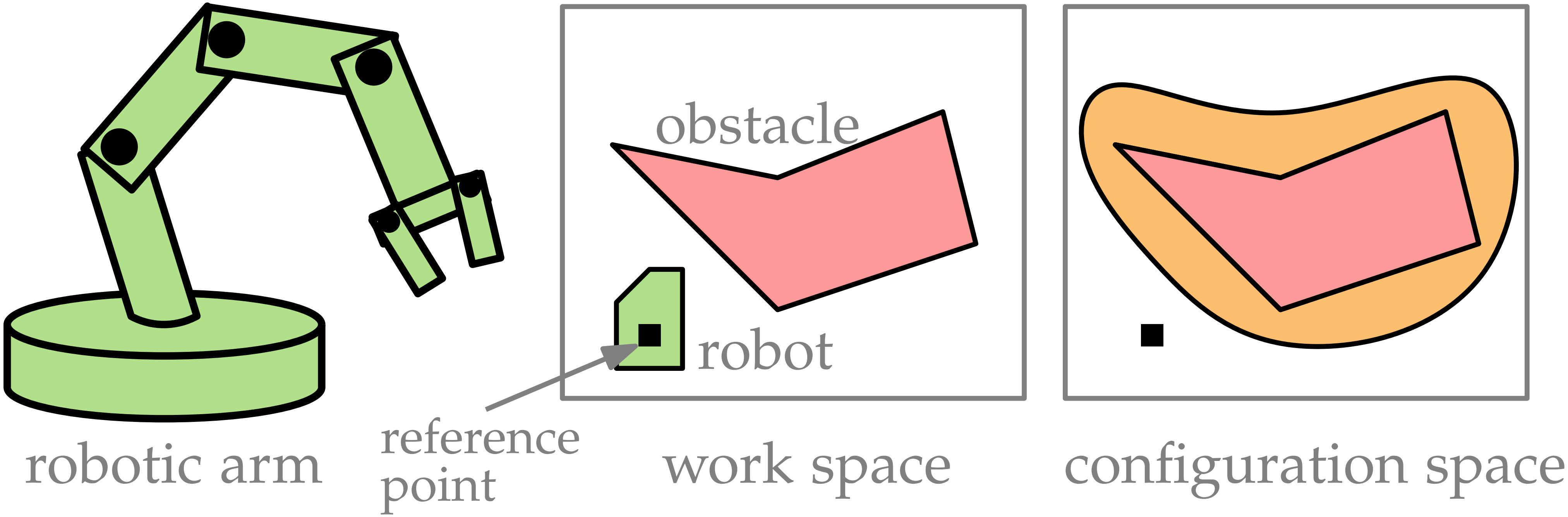


The *configuration space* is the *d*-dimensional space of all possible (i.e., obstacle avoiding) parameter value combinations.

Path for a *point* through configuration space

$$\Downarrow$$

Configuration Space



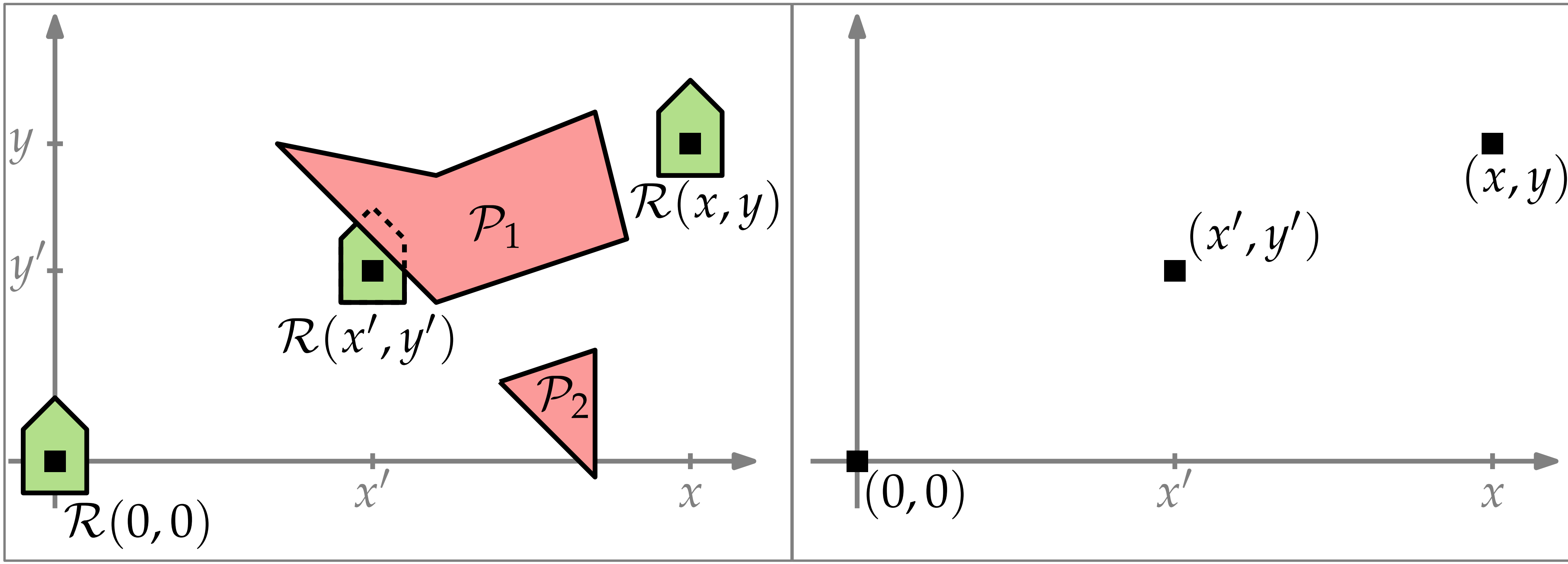
The *configuration space* is the *d*-dimensional space of all possible (i.e., obstacle avoiding) parameter value combinations.

Path for a *point* through configuration space

$$\Downarrow$$

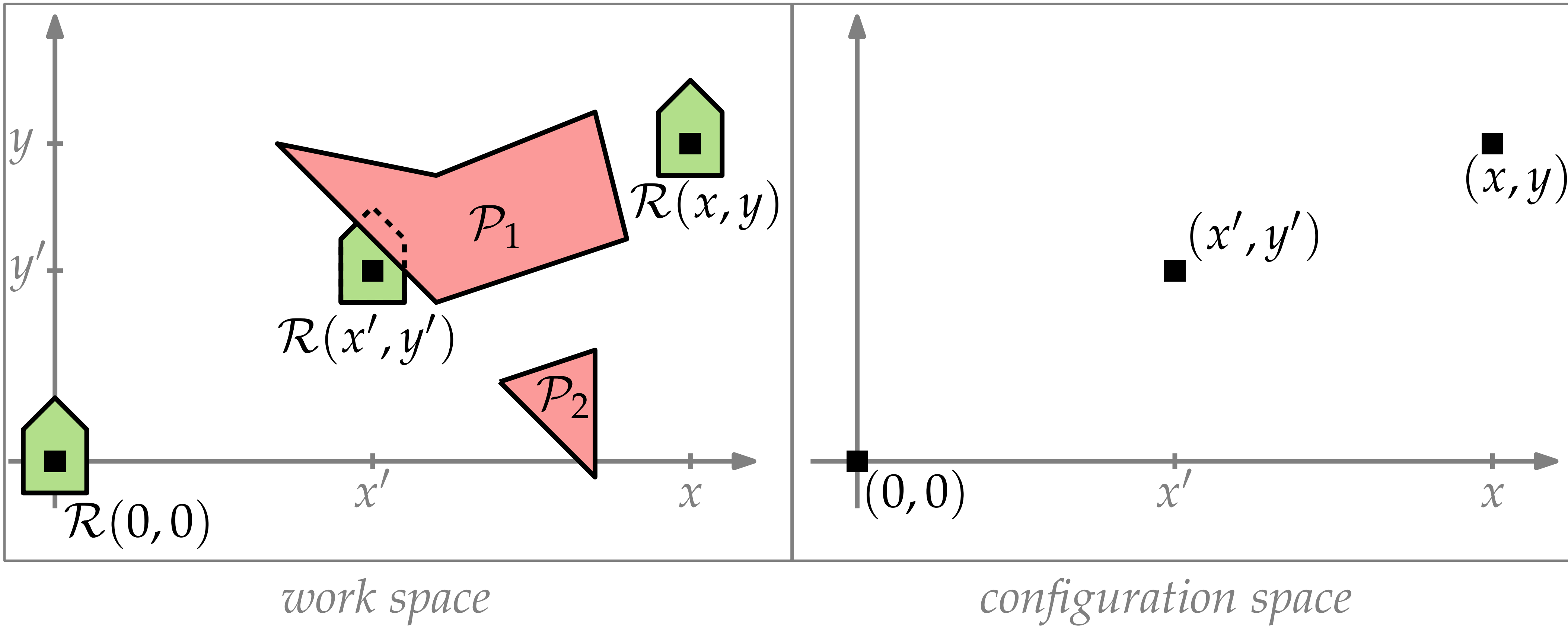
path for the *robot* in the original space.

Example: Translating 2D Polygonal Robots



work space *configuration space*

Example: Translating 2D Polygonal Robots



- Compute $\mathcal{CP}_i = \{(x,y) : \mathcal{R}(x,y) \cap \mathcal{P}_i \neq \emptyset\}$ for each $\mathcal{P}_i$.

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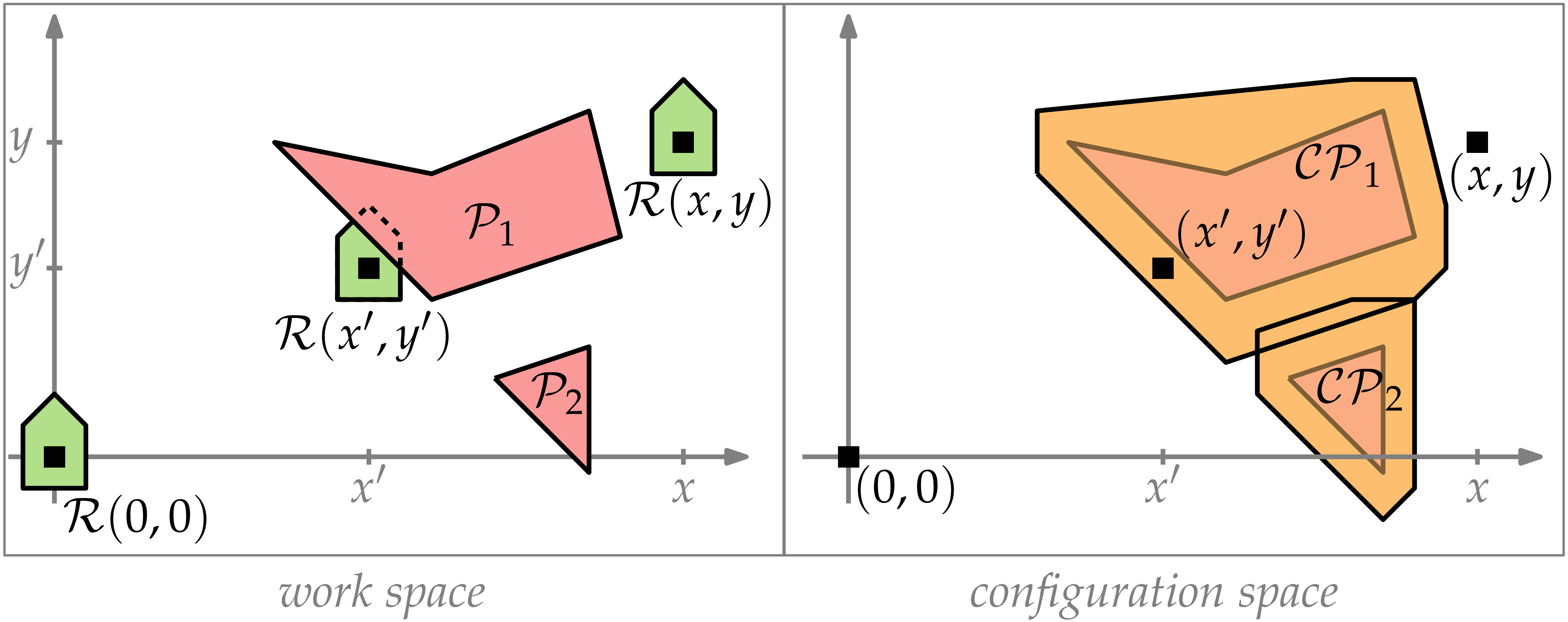

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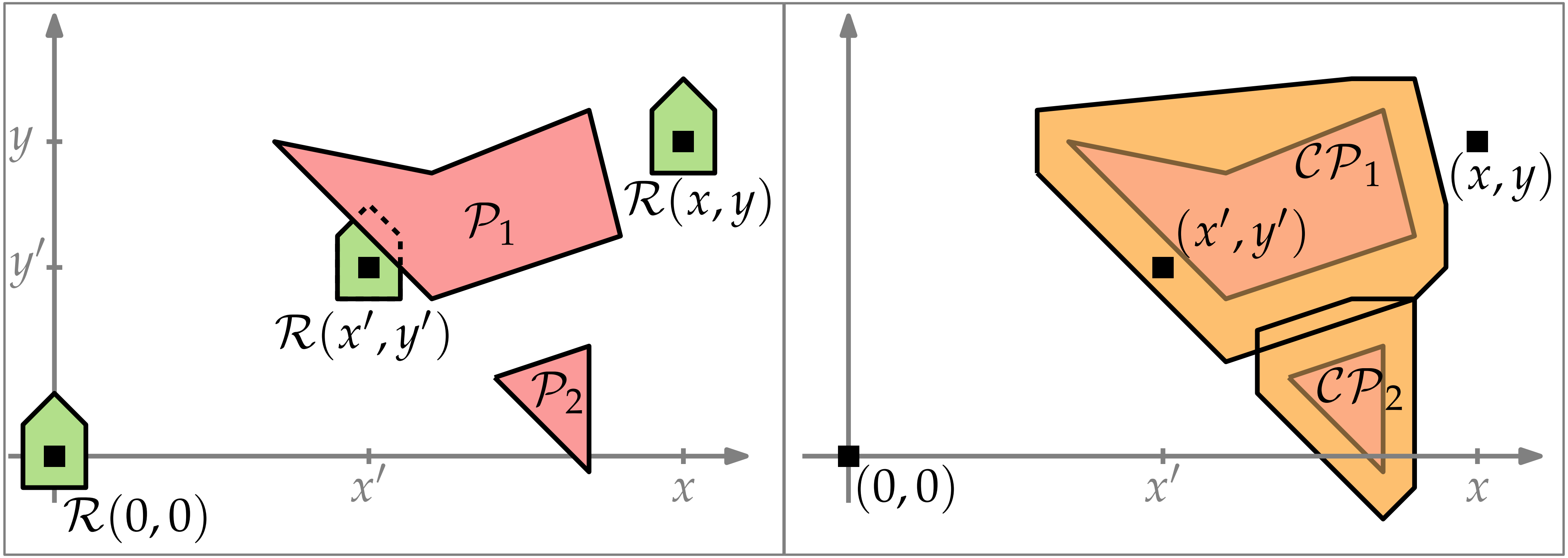

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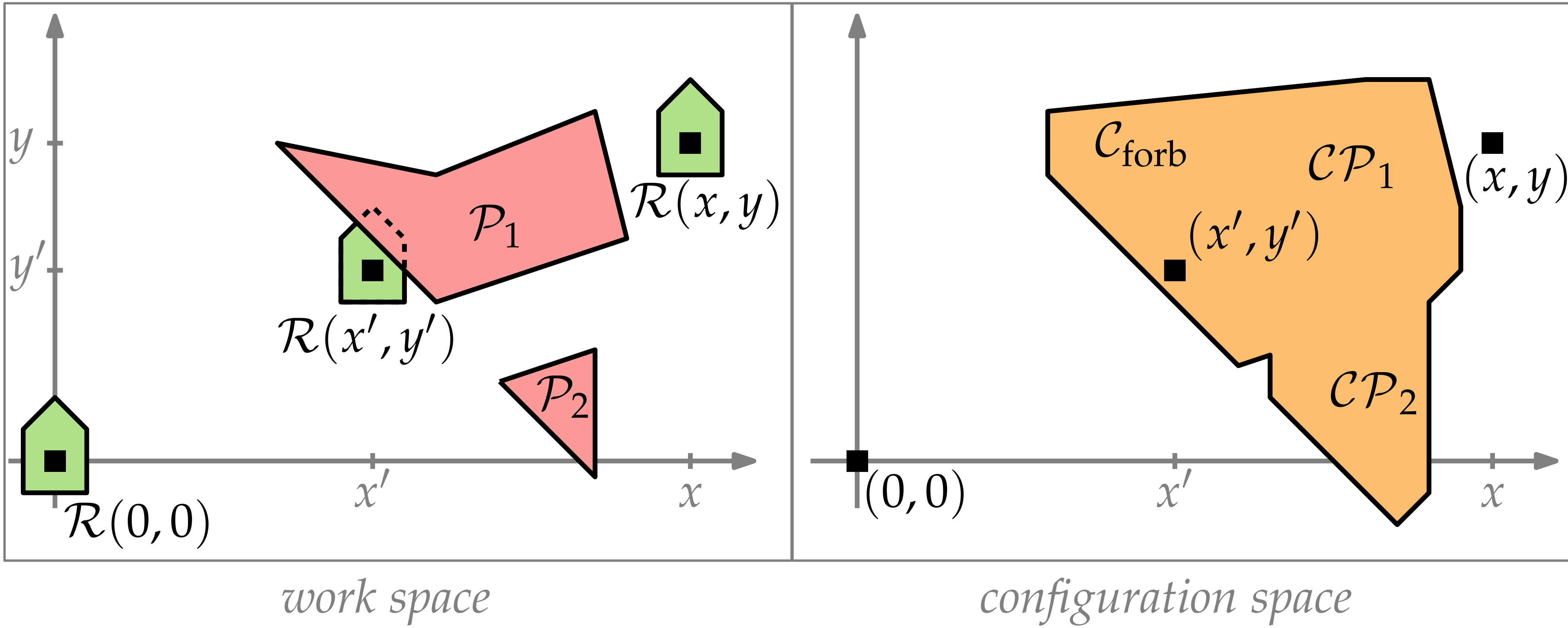

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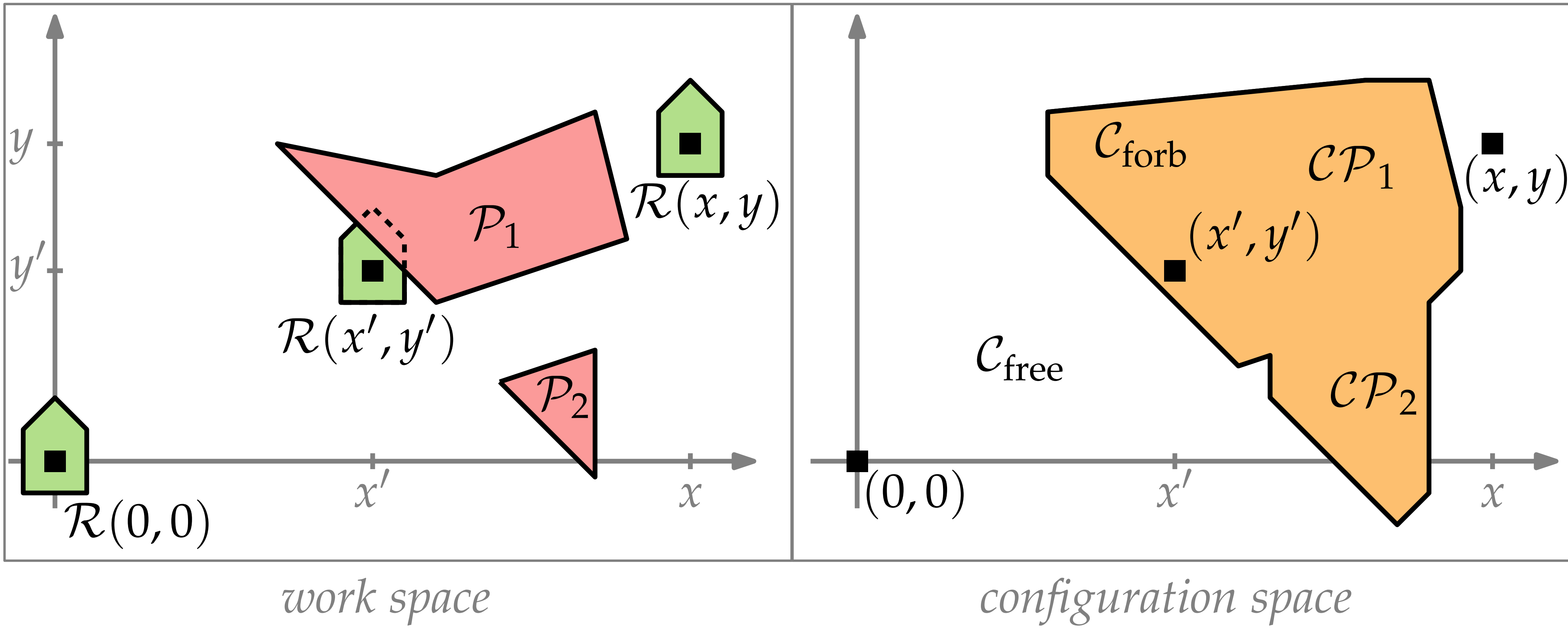

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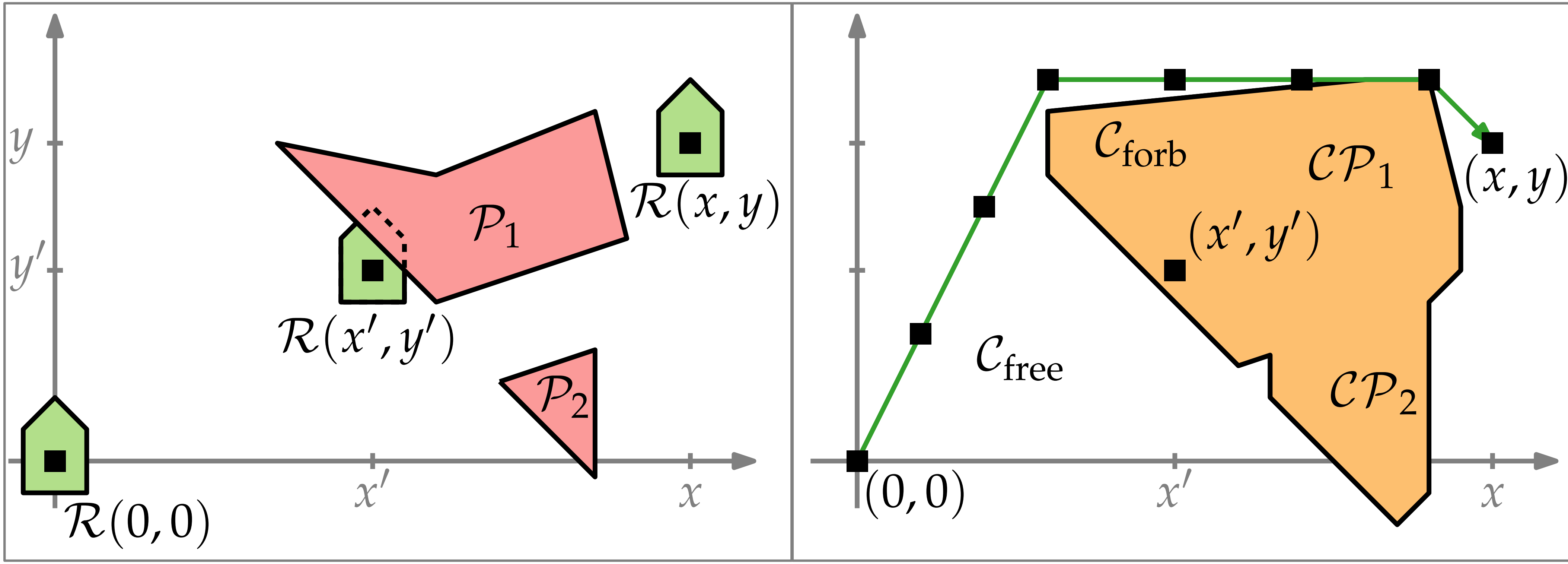

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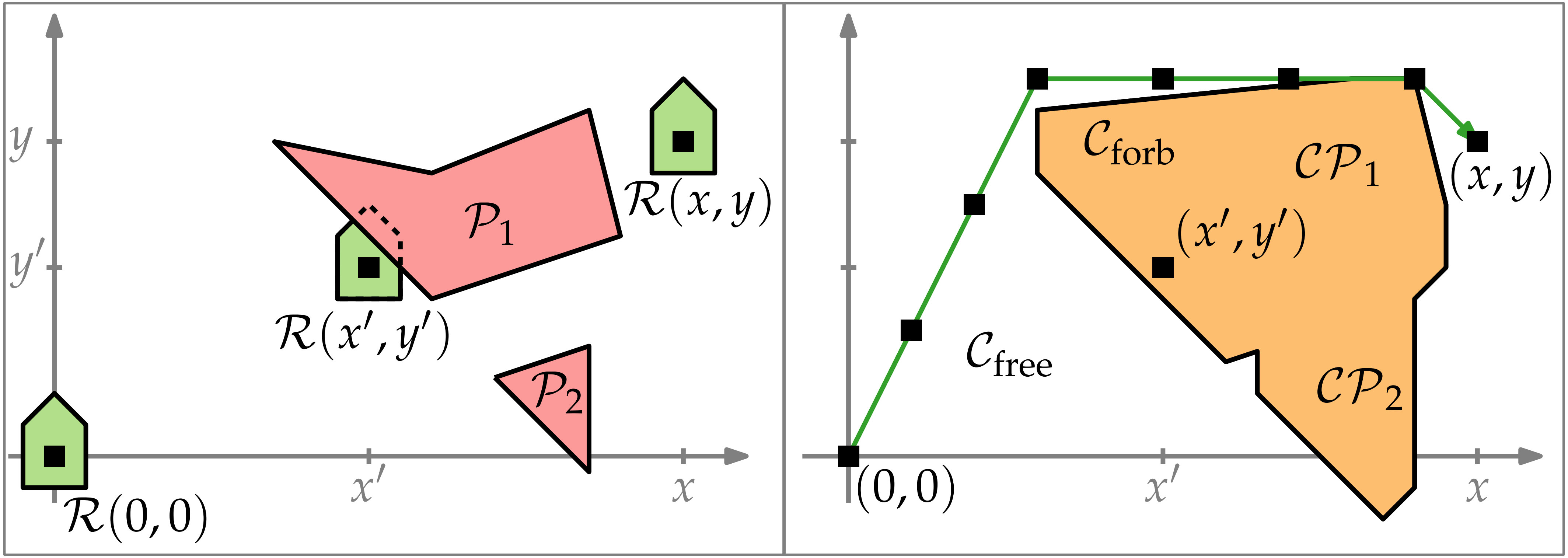

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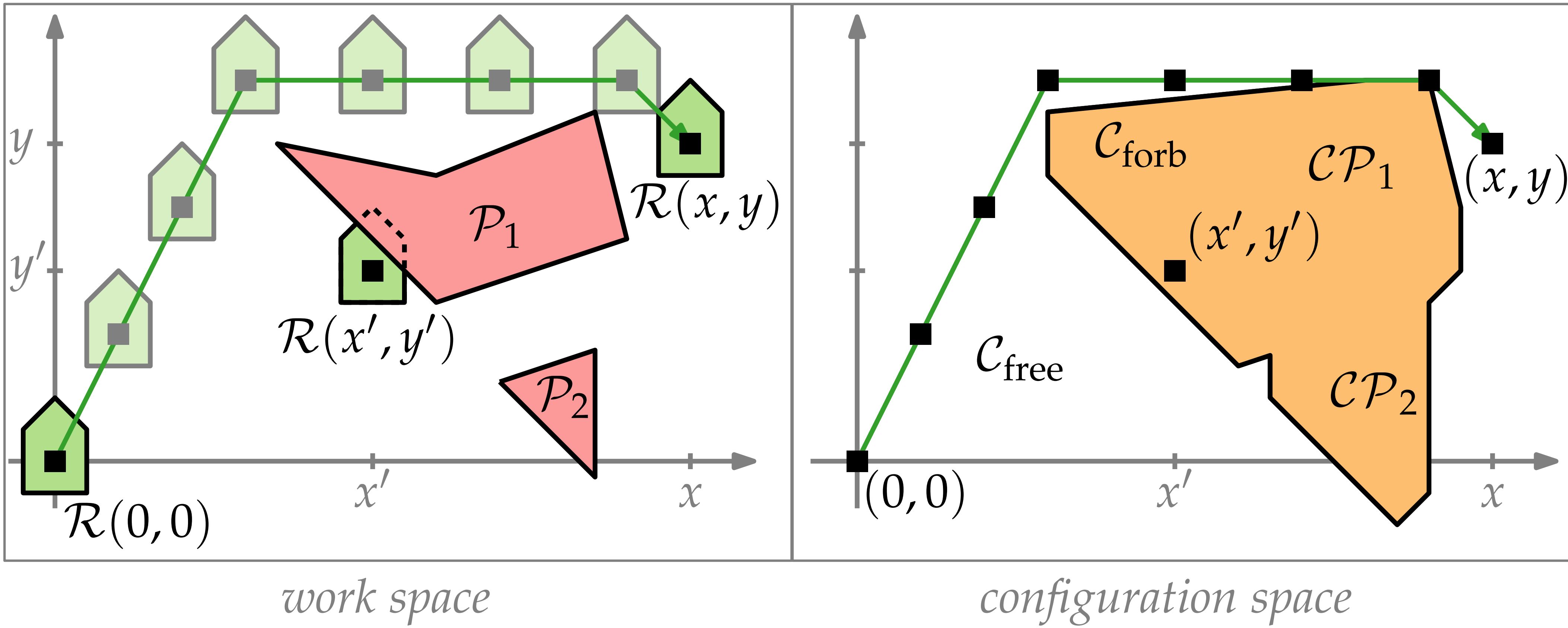

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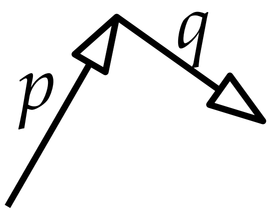
Some Linear Algebra

Vector sums



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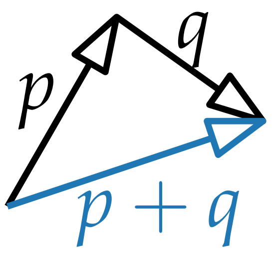

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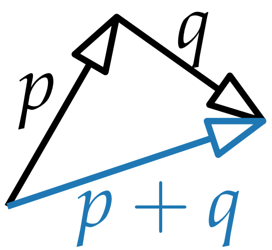

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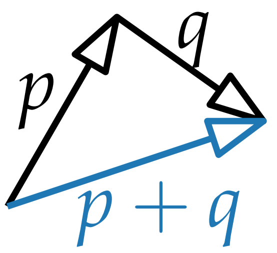

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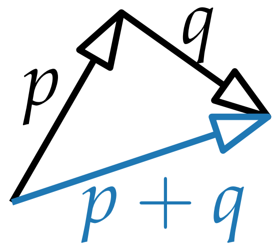

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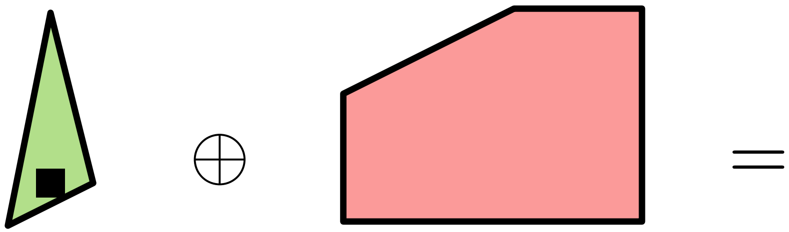

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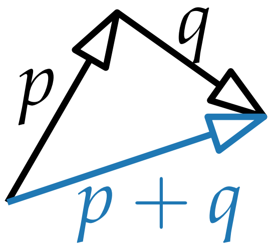

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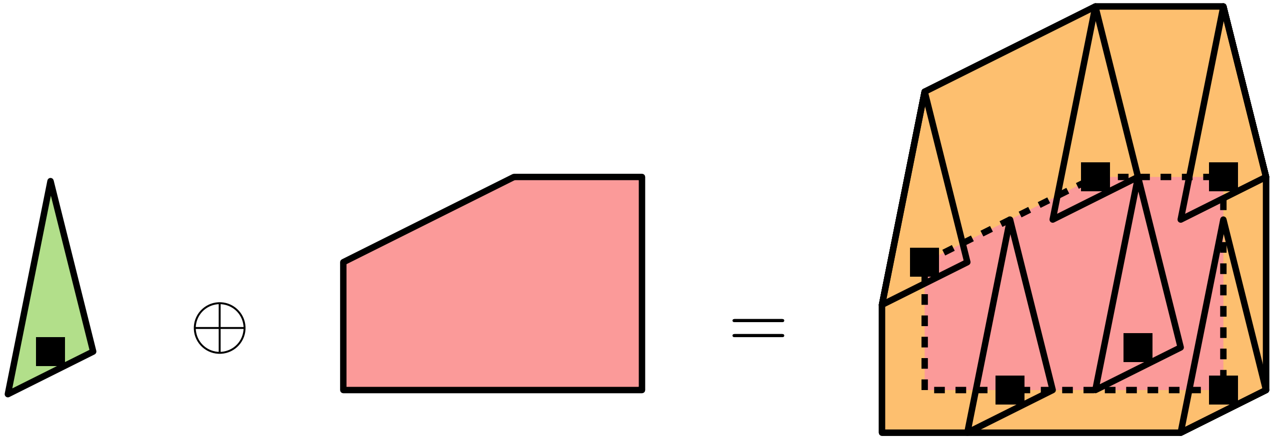

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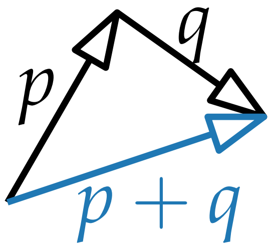

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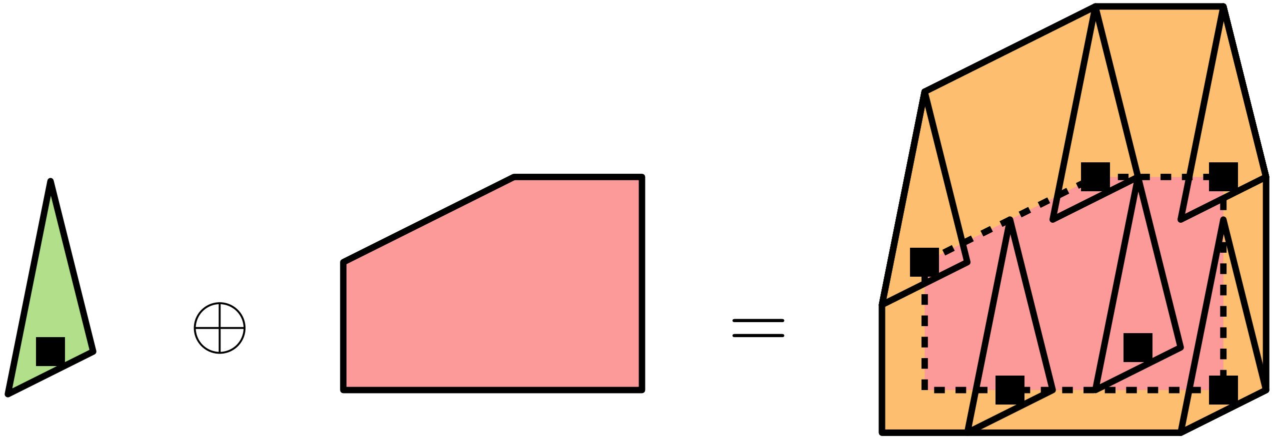

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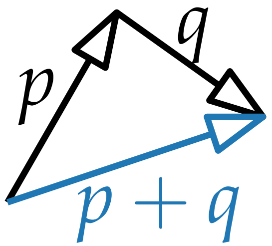

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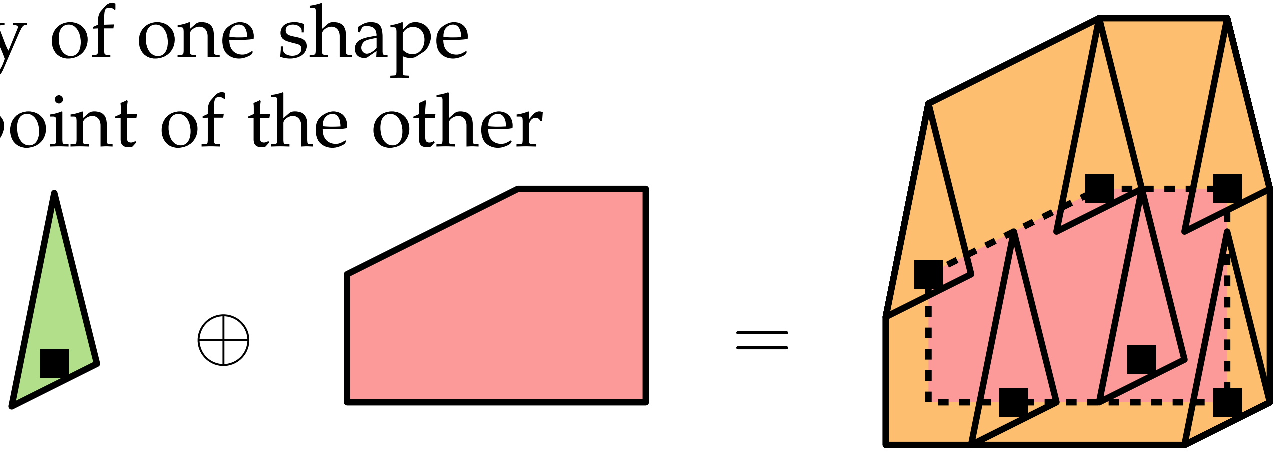

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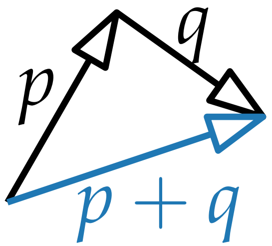

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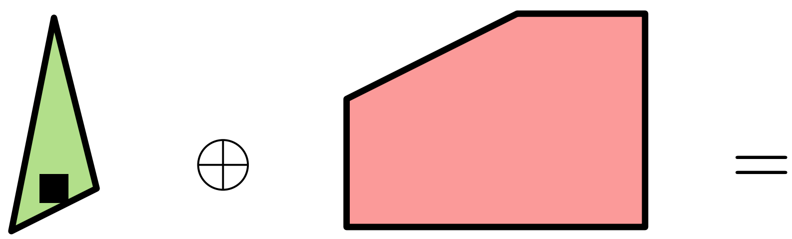
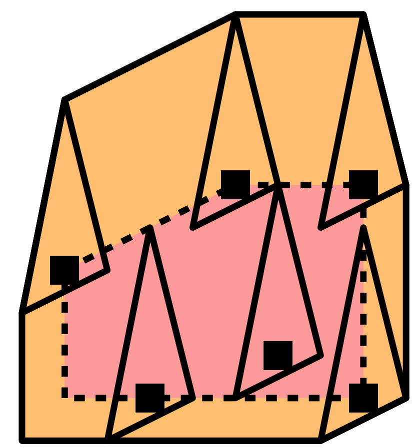
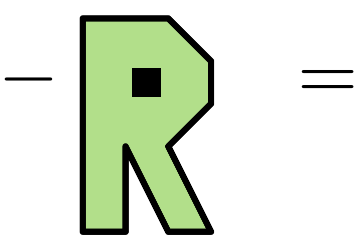




Inversion



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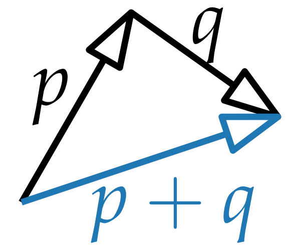

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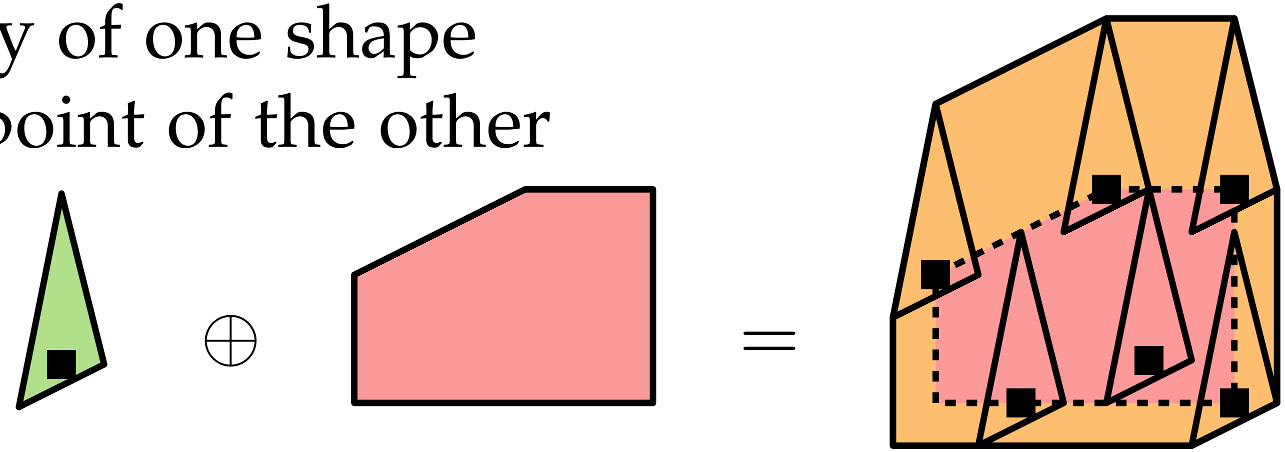

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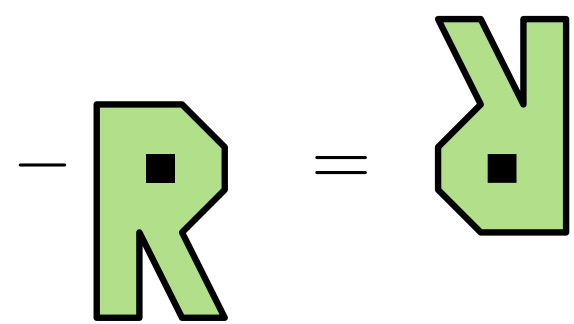

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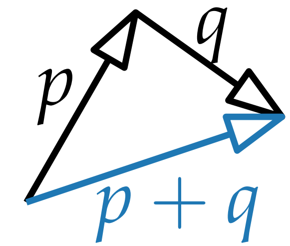

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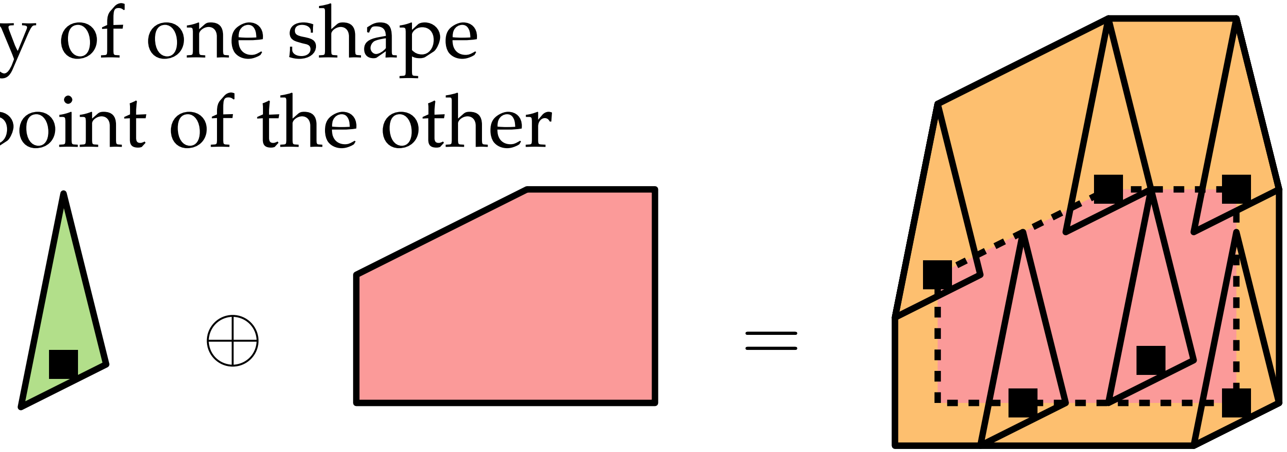
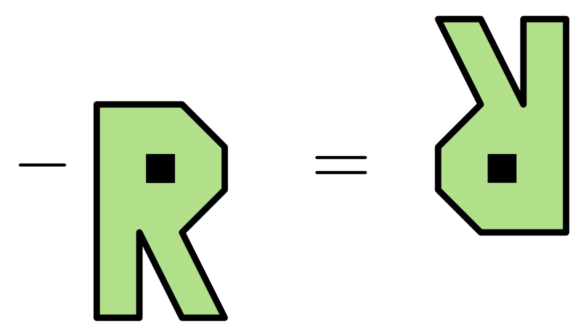


Inversion

Algebra: $-S = \{-p \mid p \in S\}$



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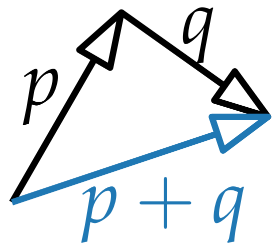

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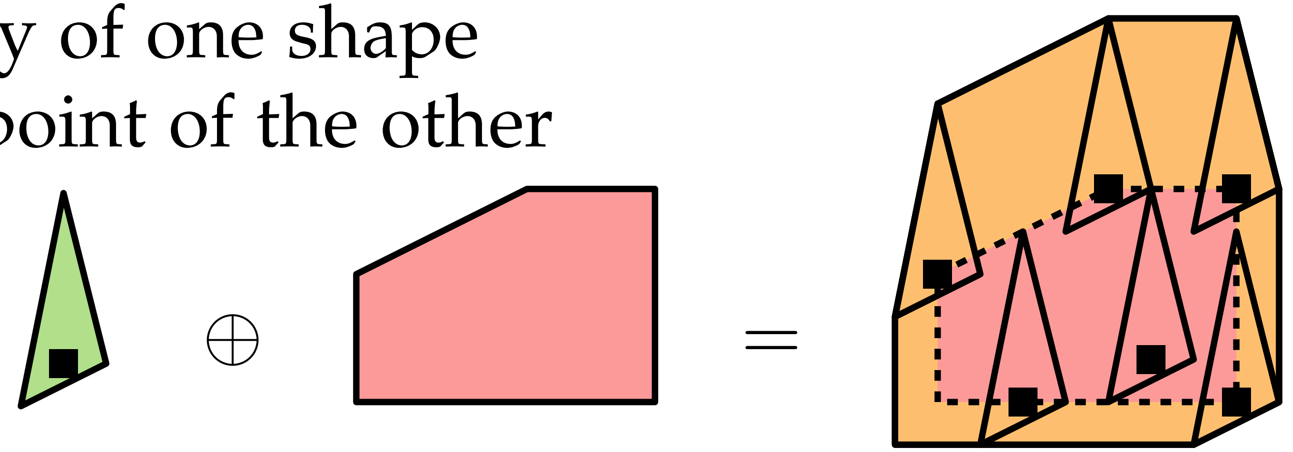

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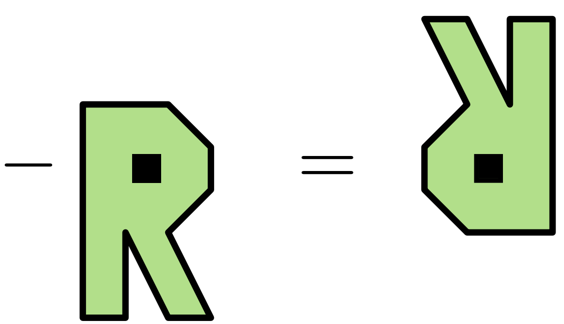

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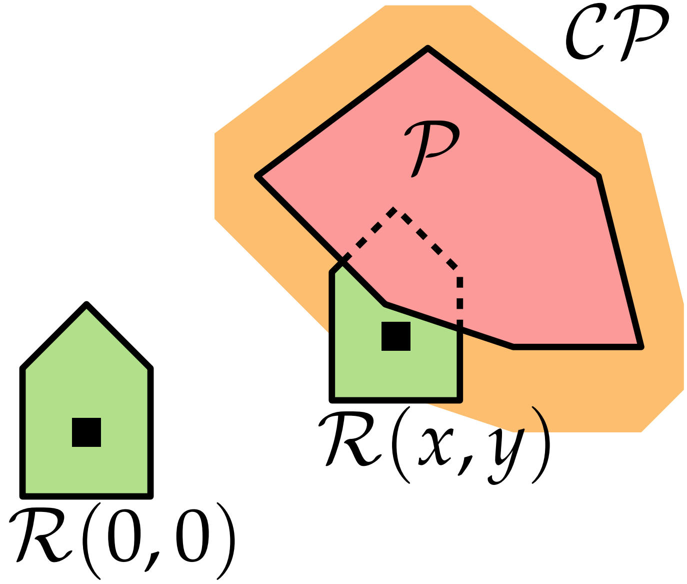
Recall that $\mathcal{CP} = \{(x, y) \colon \mathcal{R}(x, y) \cap \mathcal{P} \neq \emptyset\}$ for an obstacle $\mathcal{P}$.



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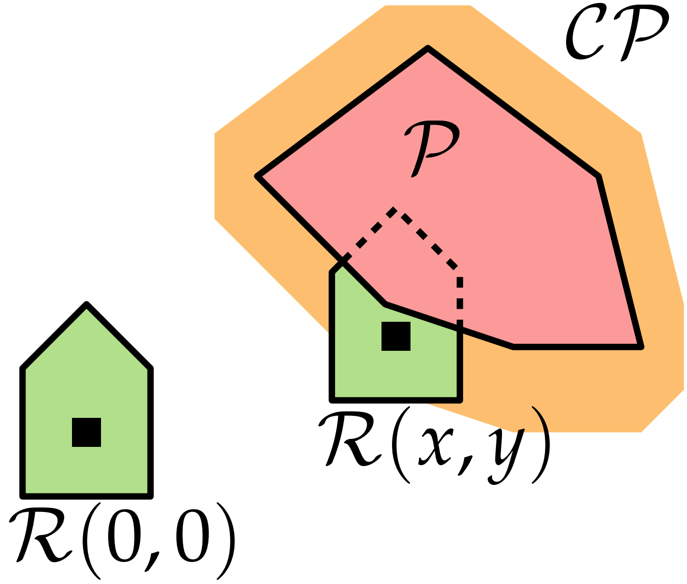

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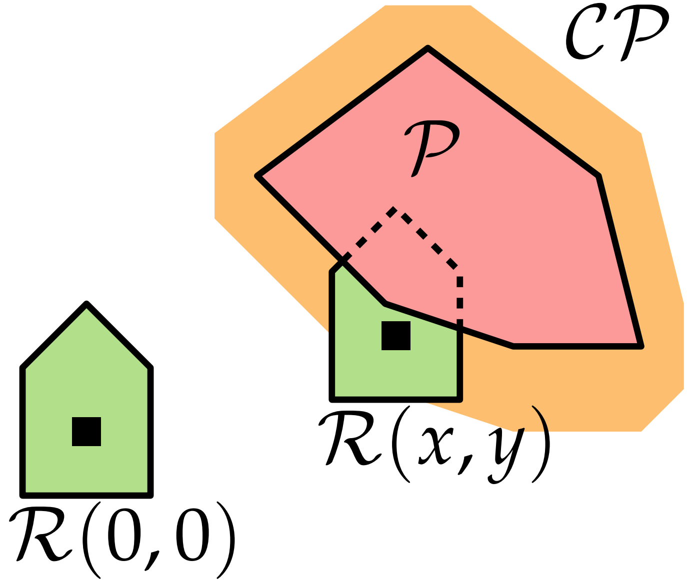
Theorem. $\mathcal{CP} = \mathcal{P} \oplus (-\mathcal{R}(0,0))$



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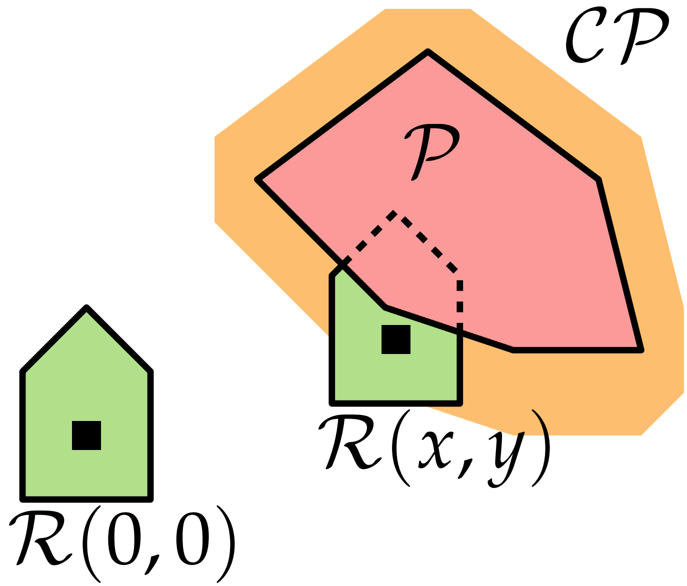

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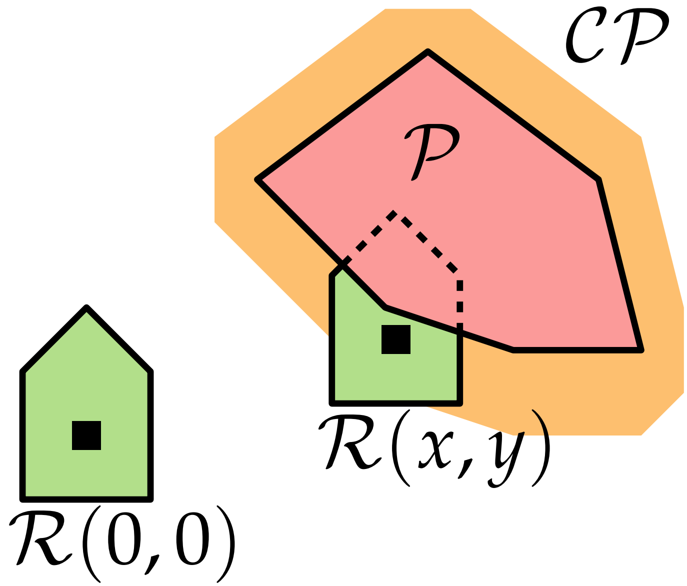

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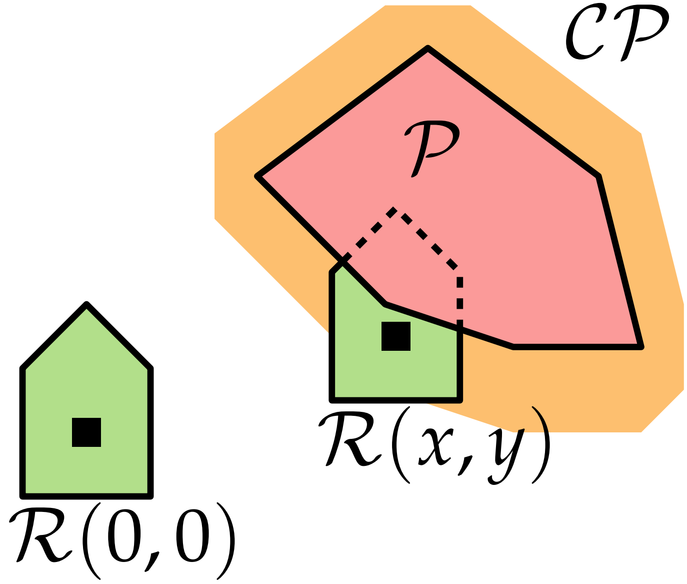

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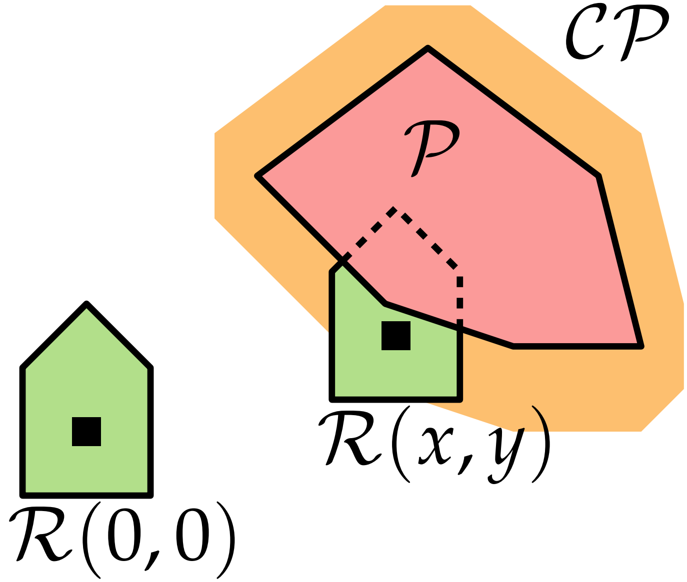

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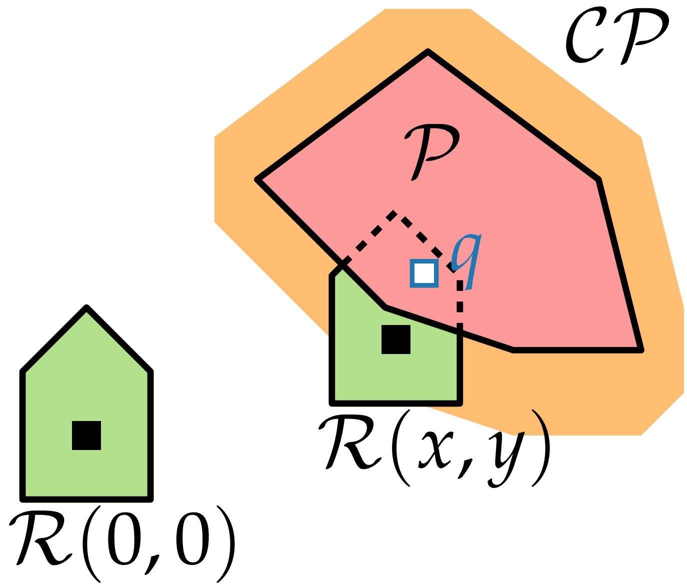

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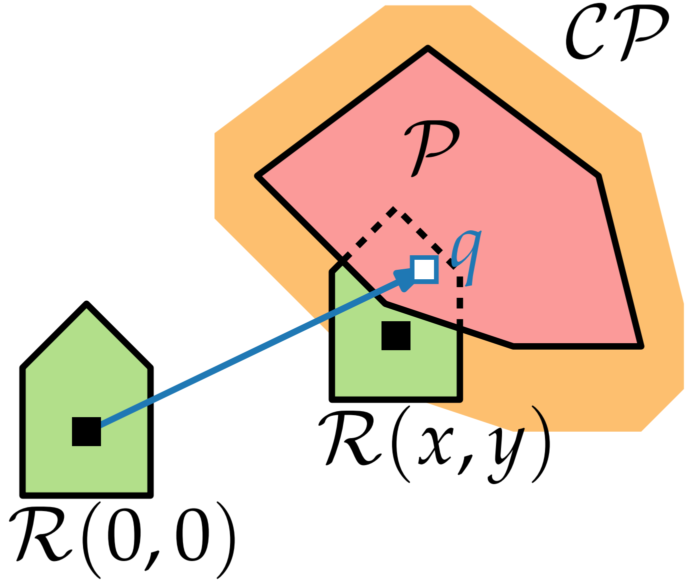

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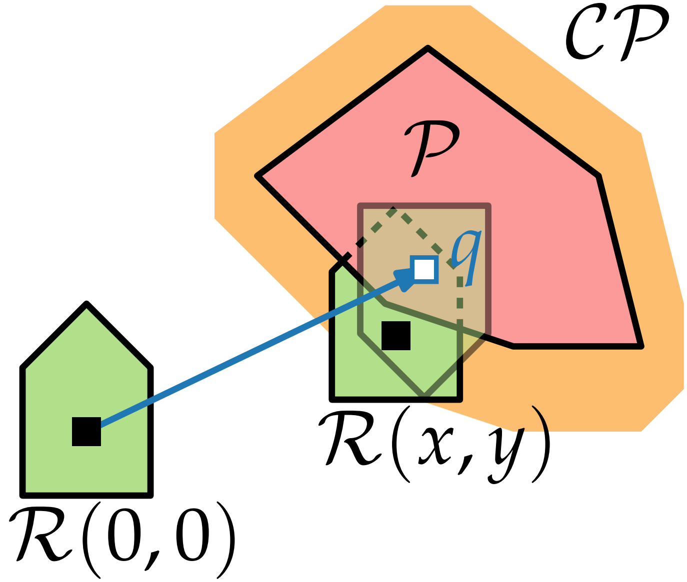

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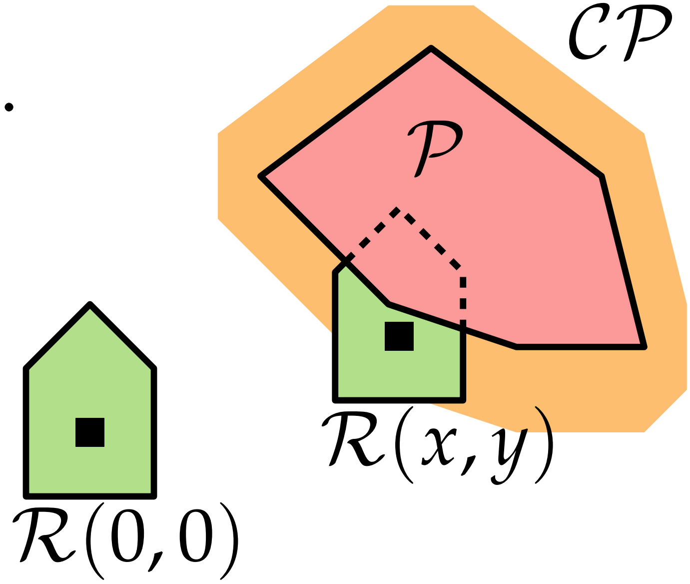

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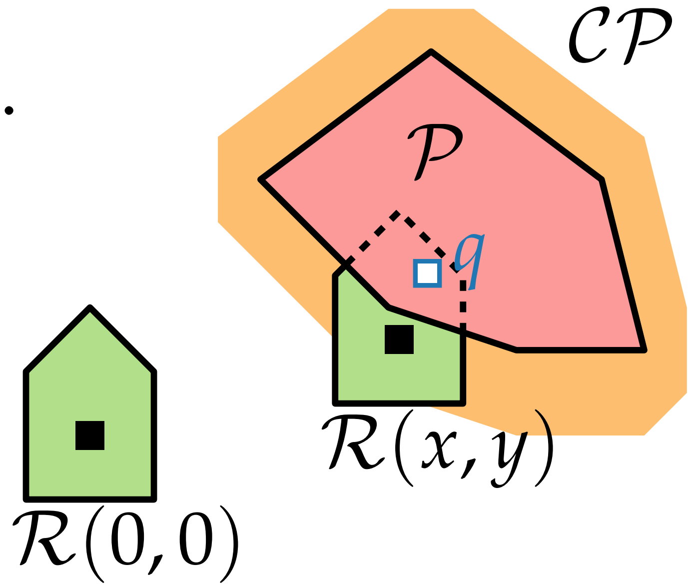

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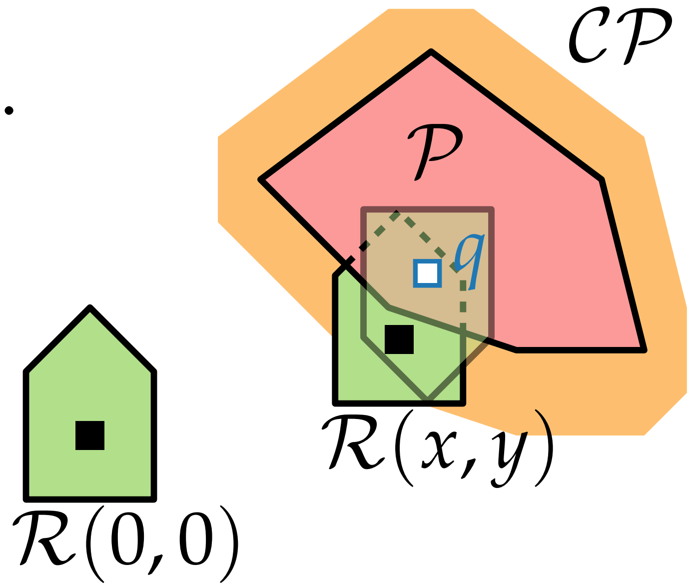

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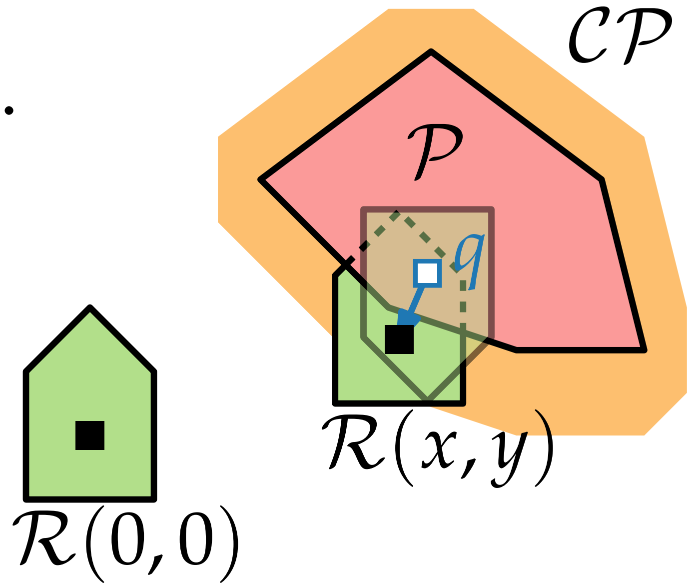

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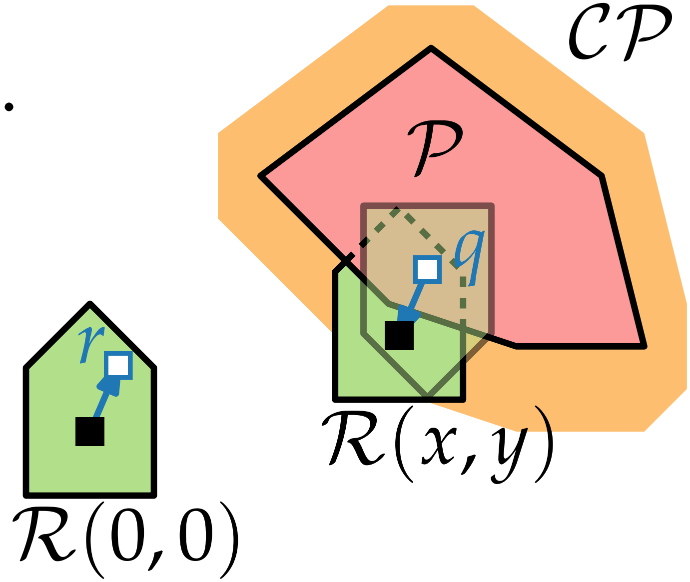

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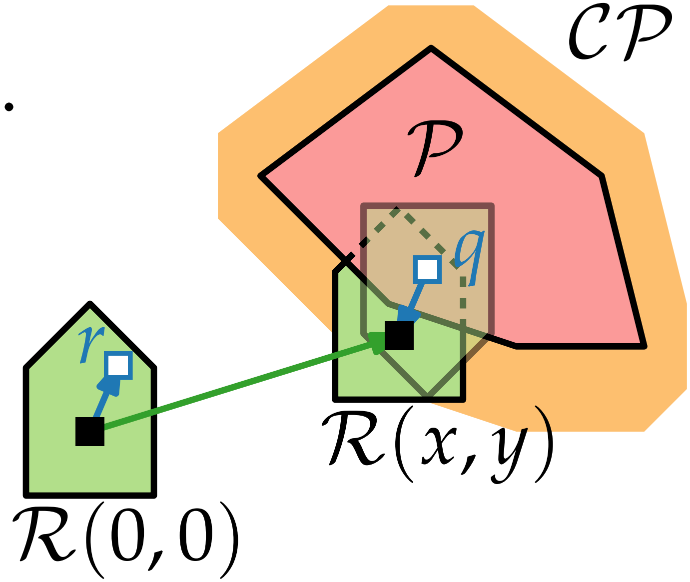

Minkowski Sums: Complexity

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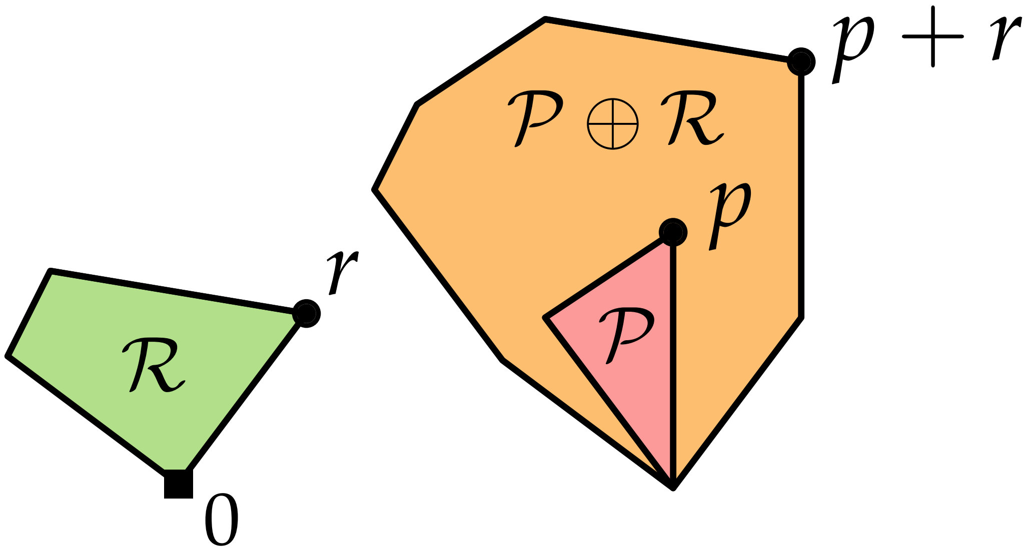

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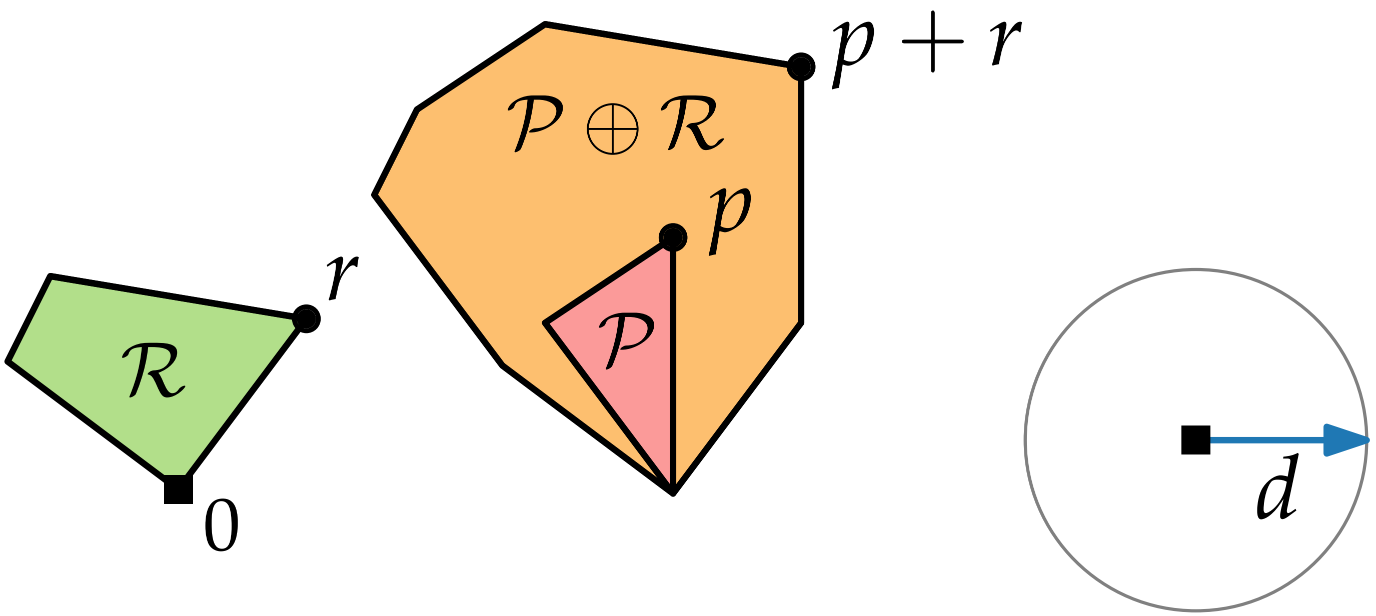

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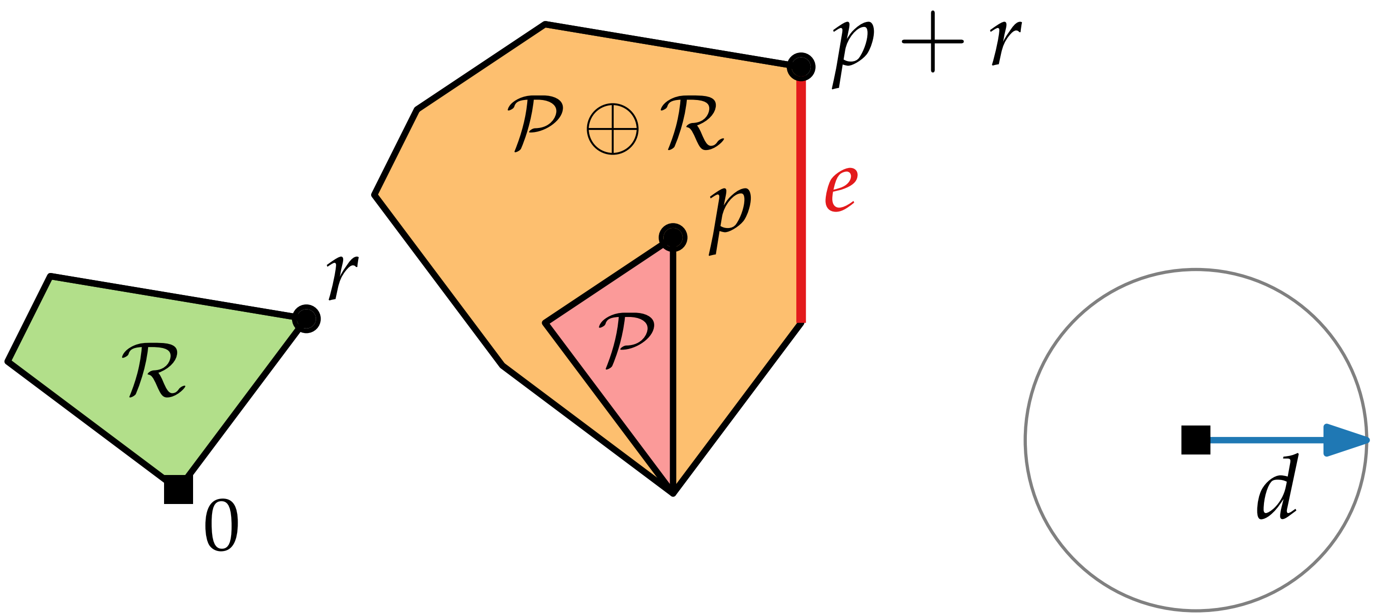

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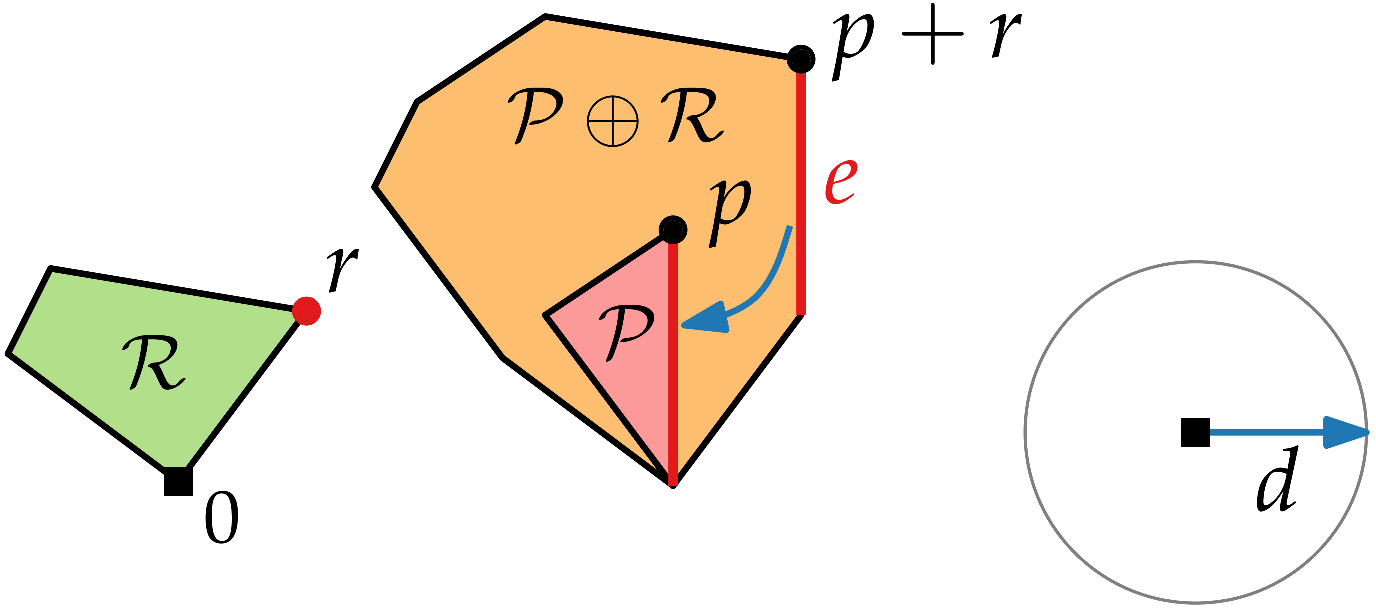

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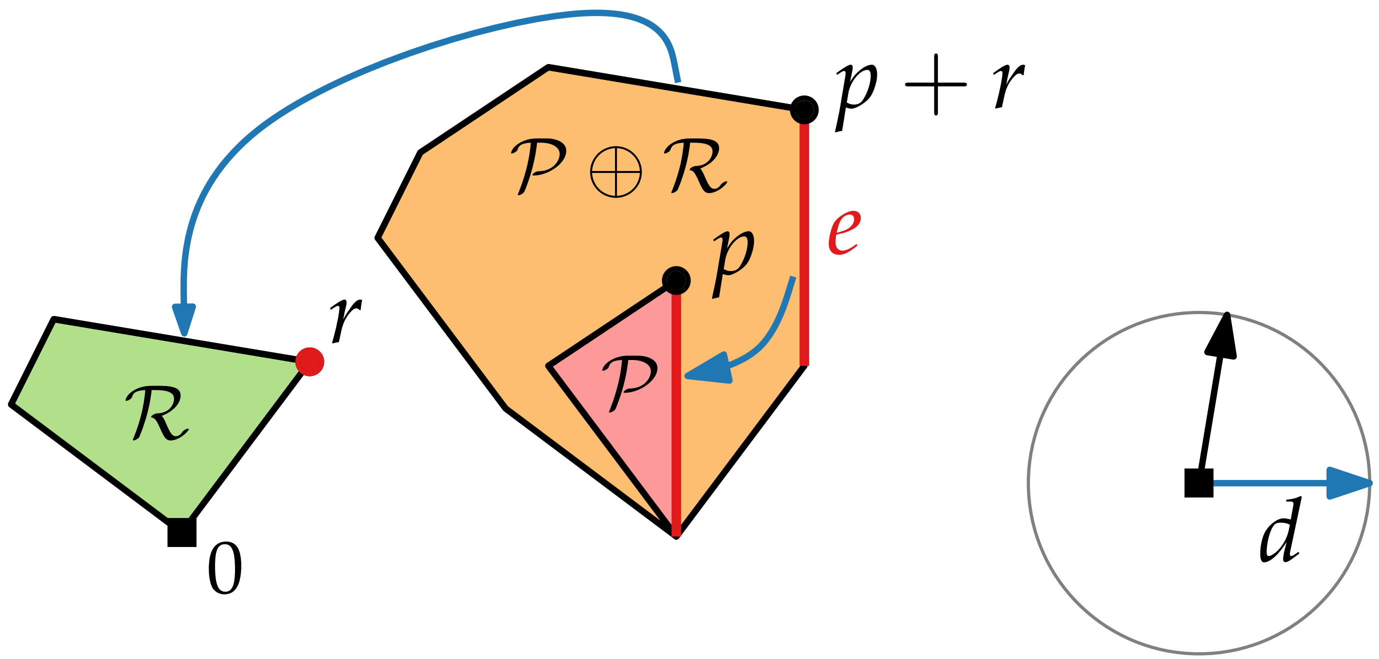

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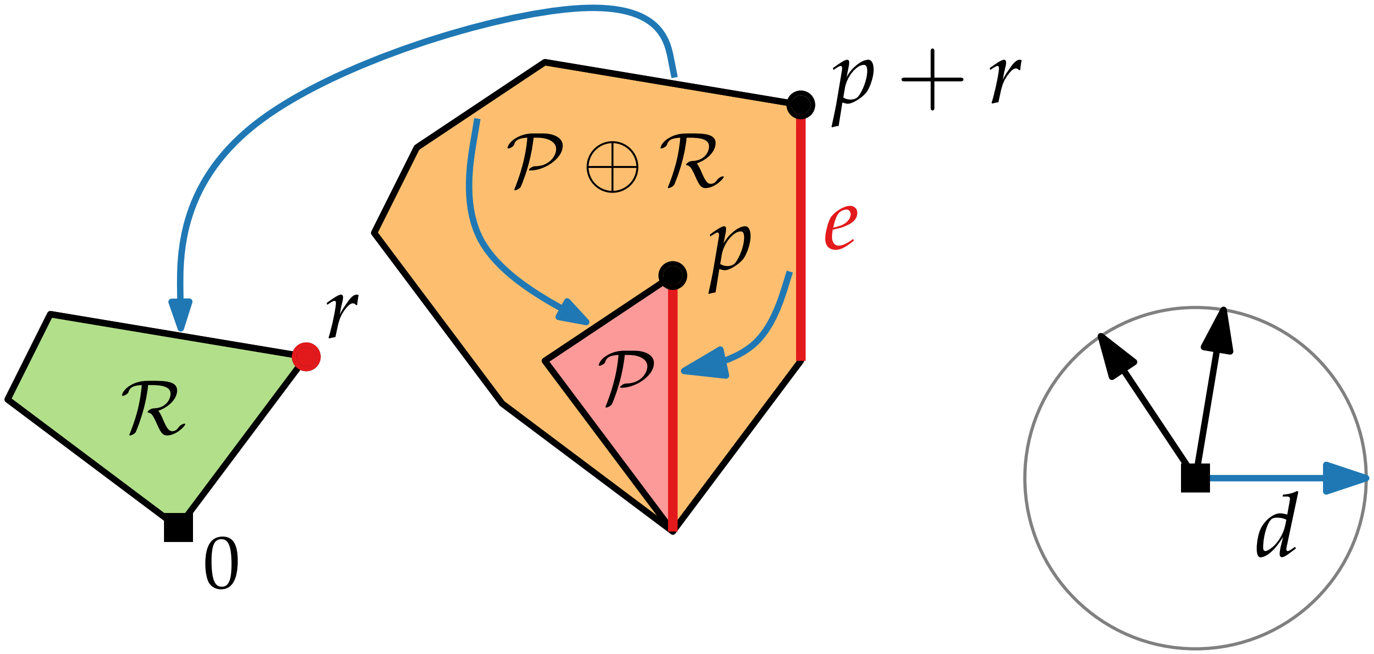

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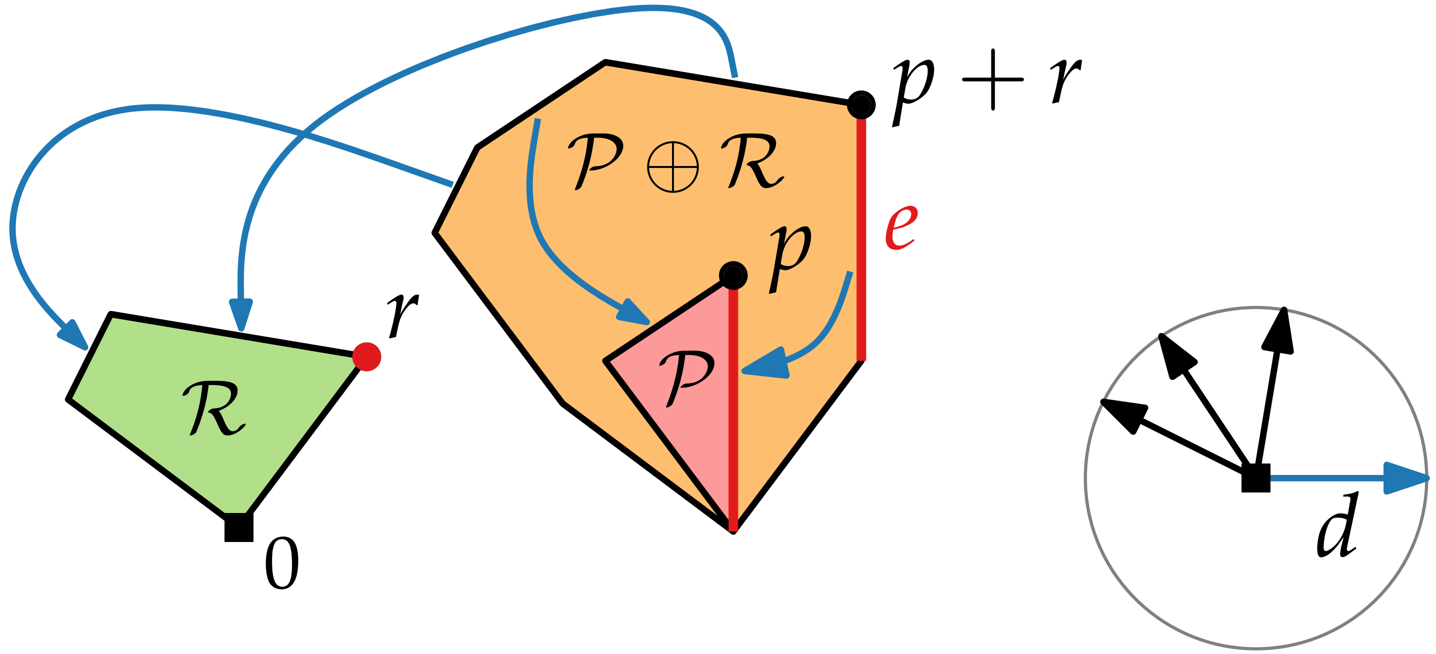

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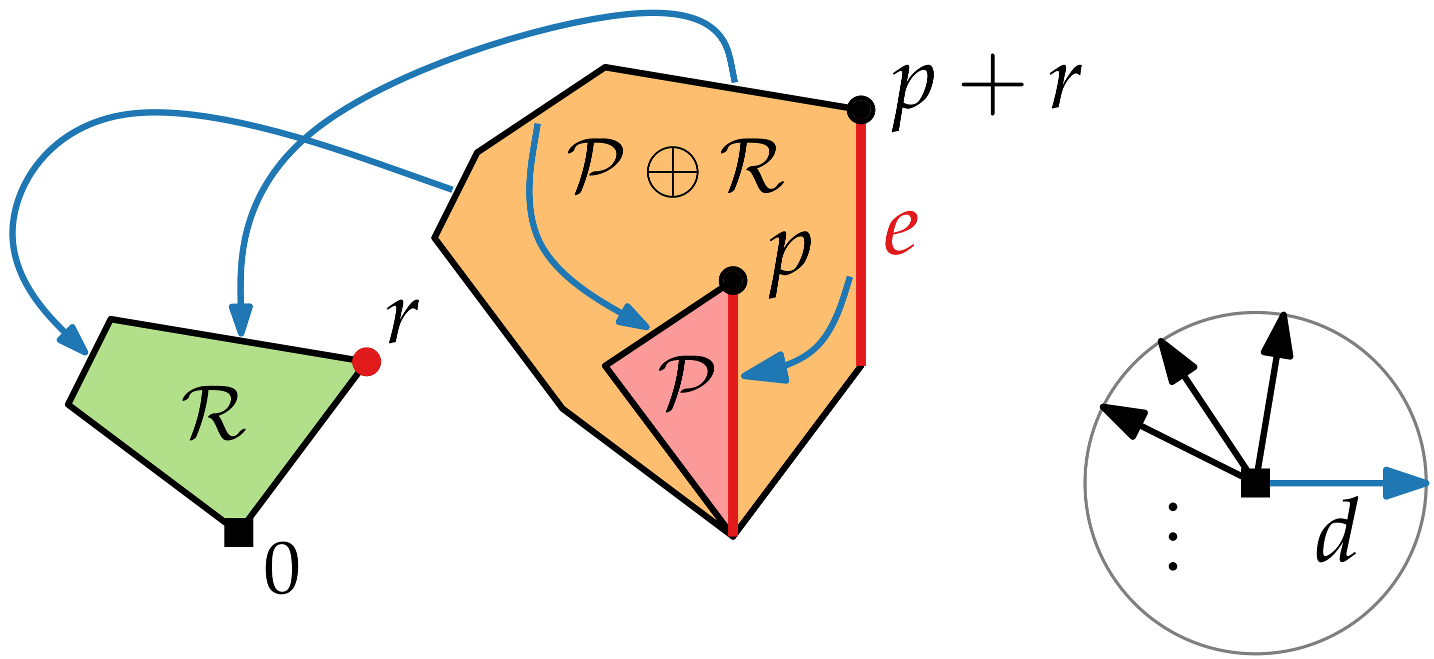

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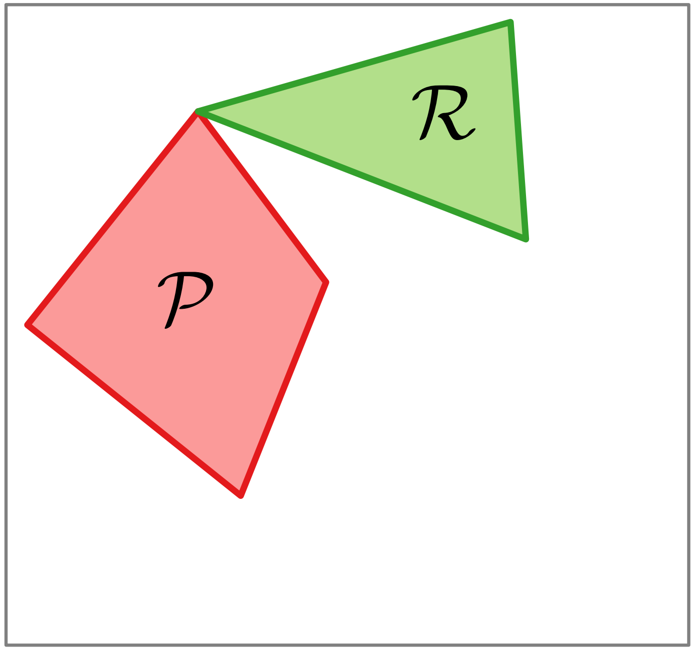

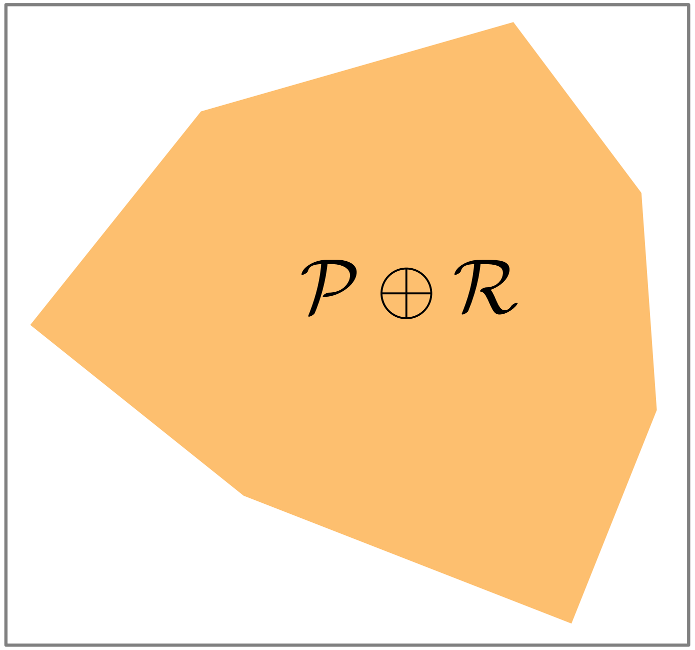

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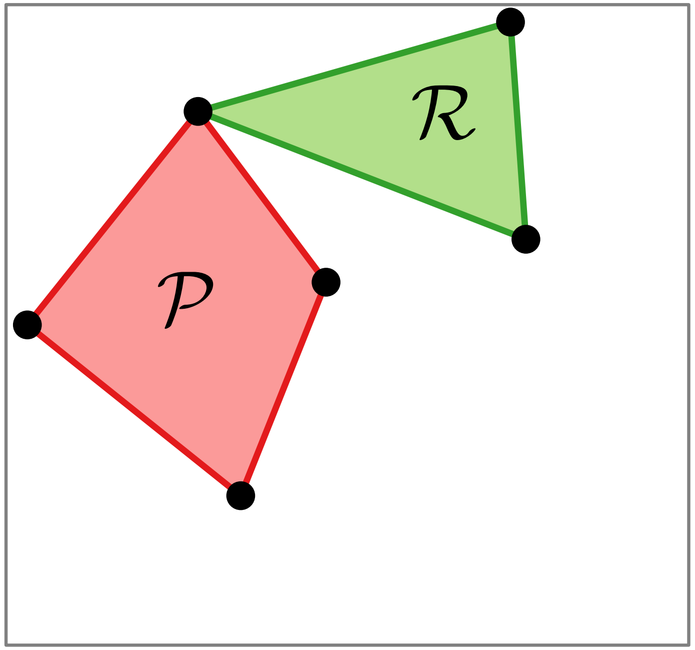
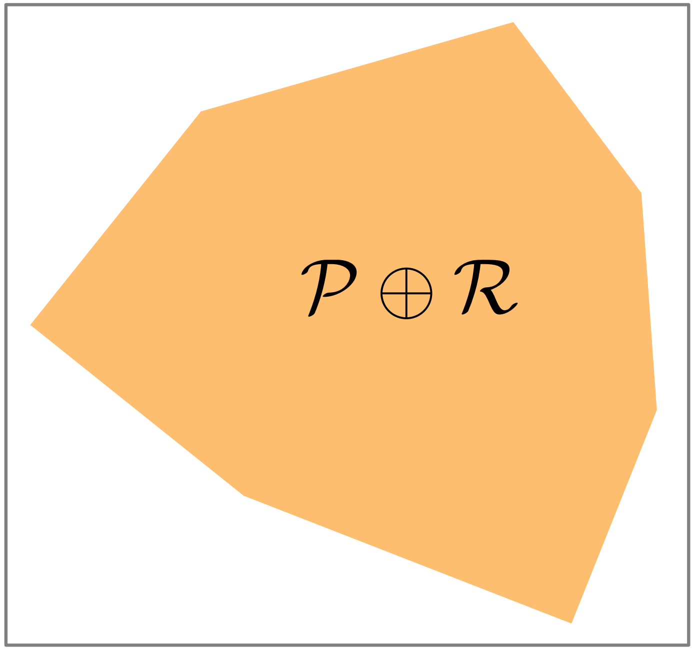
Task: How would you compute $\mathcal{P} \oplus \mathcal{R}$ given $\mathcal{P}$ and $\mathcal{R}$?





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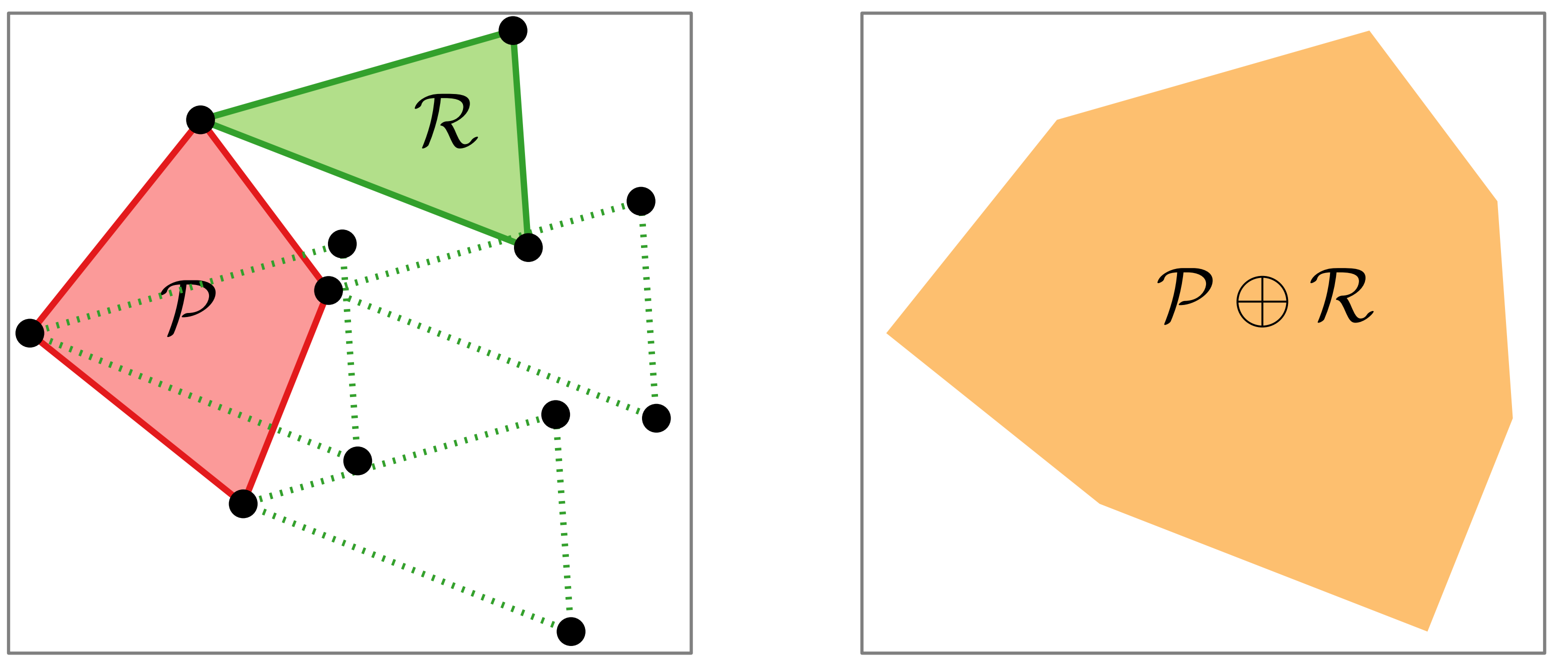

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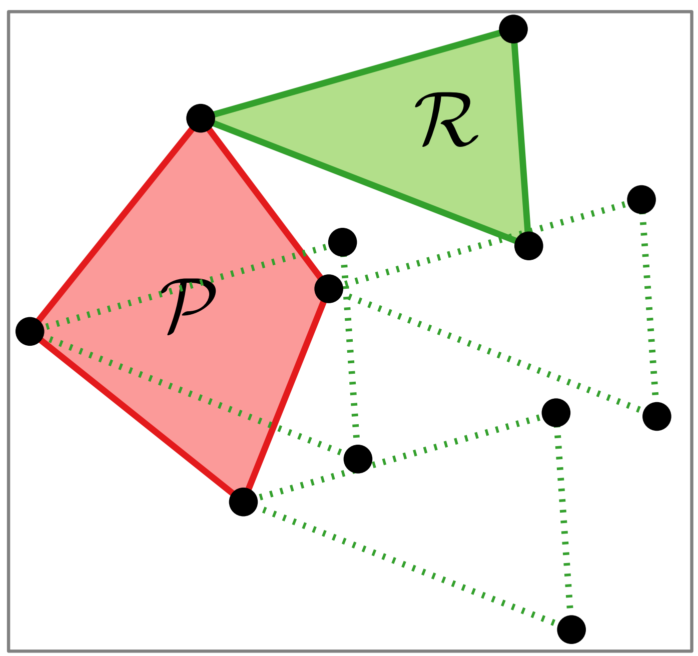

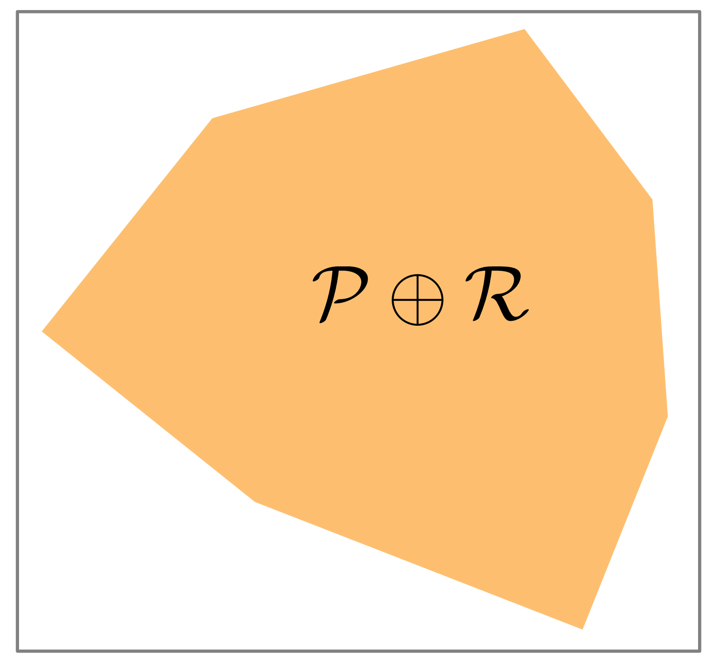

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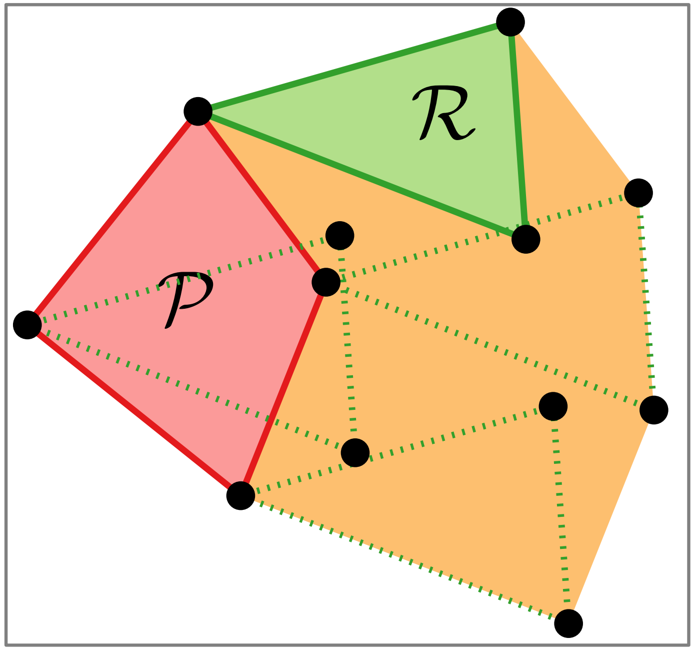

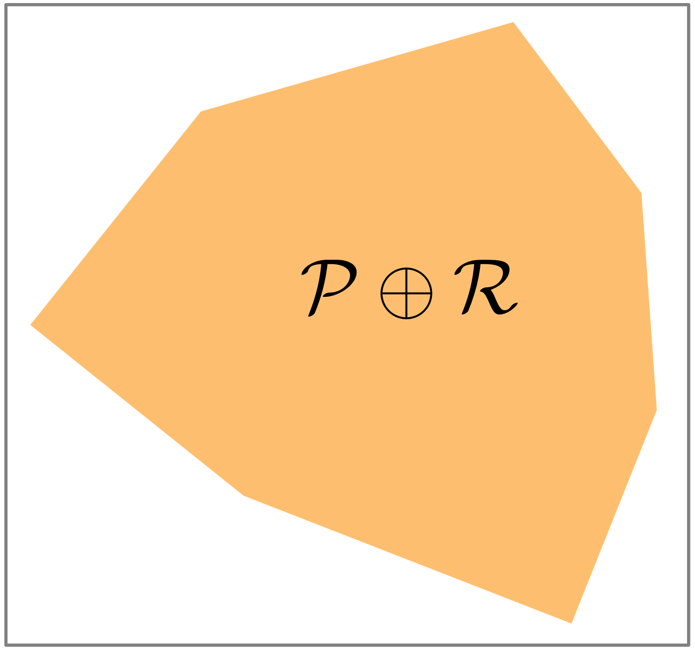

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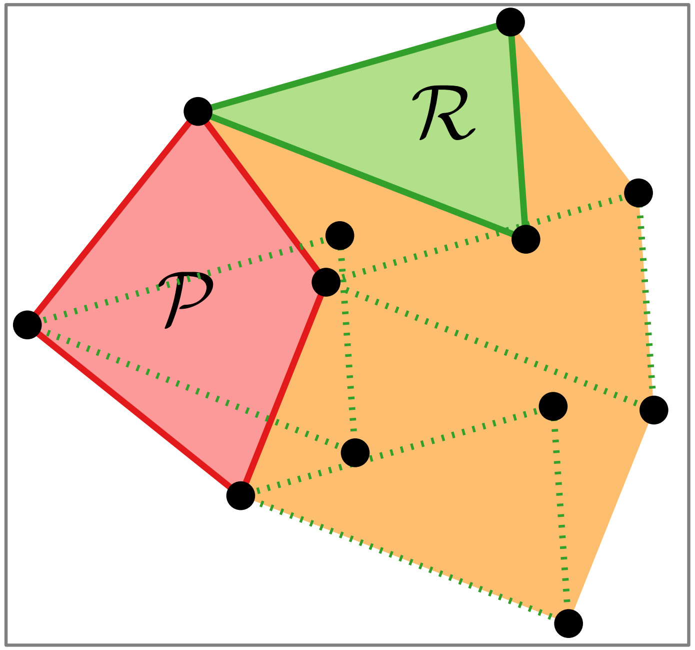

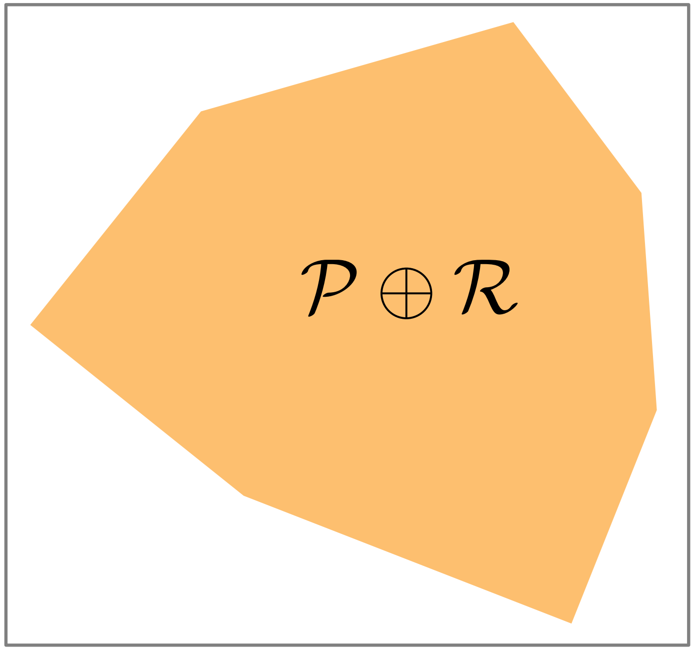

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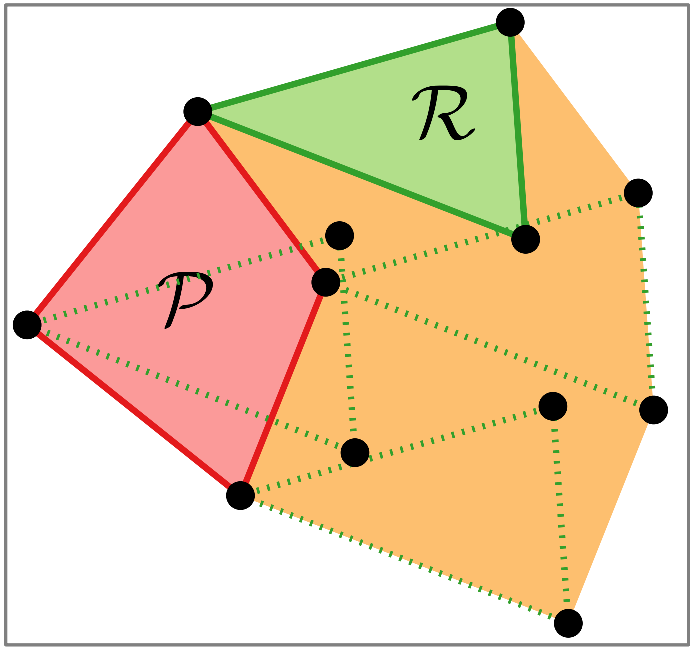

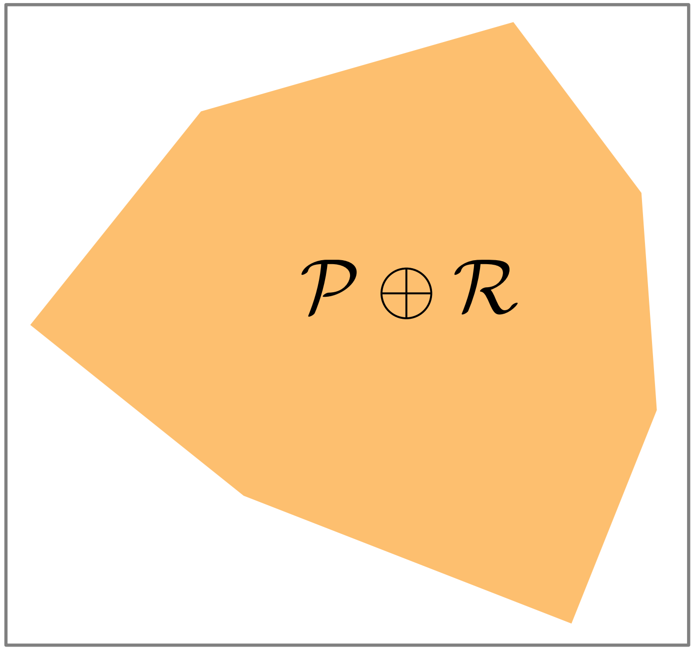

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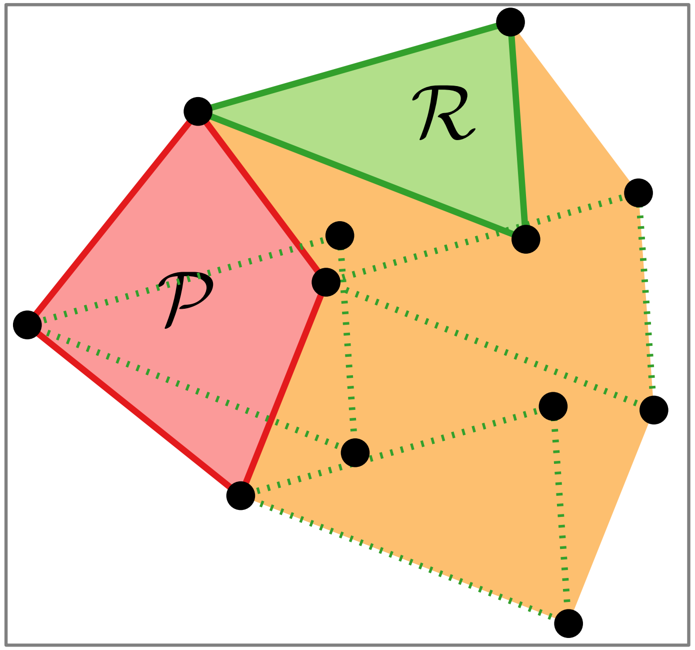

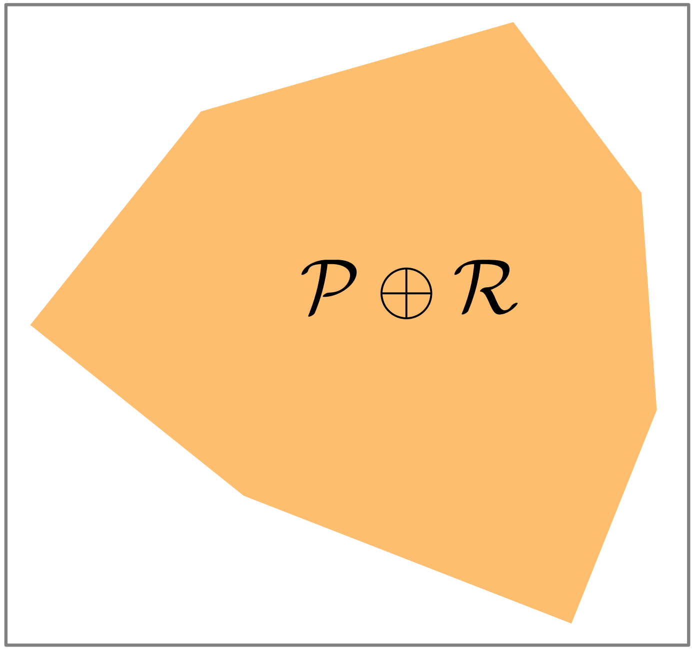

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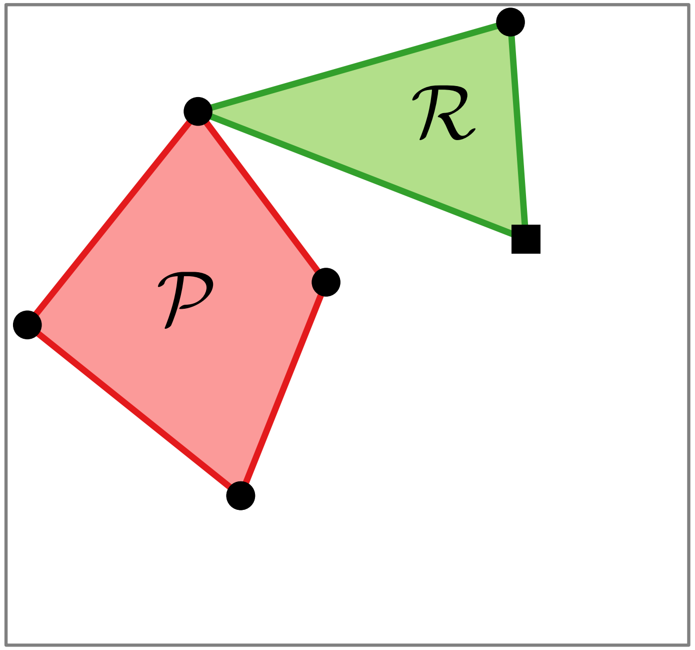

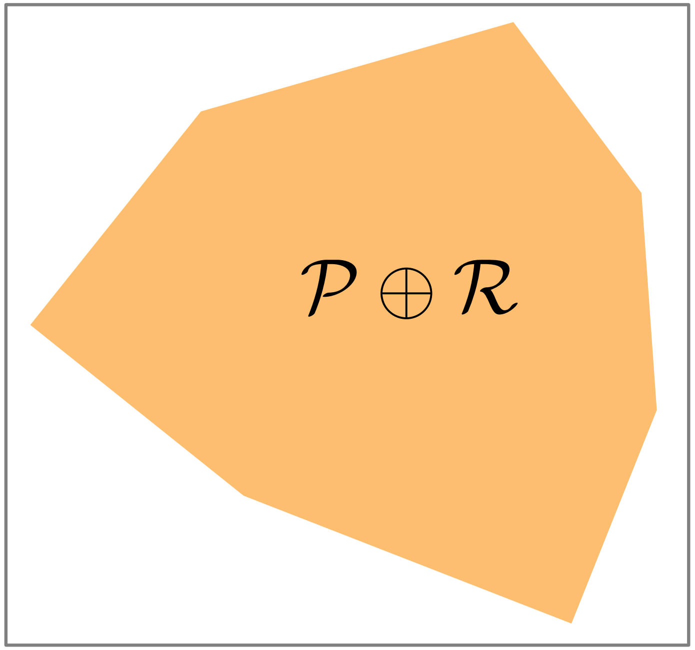

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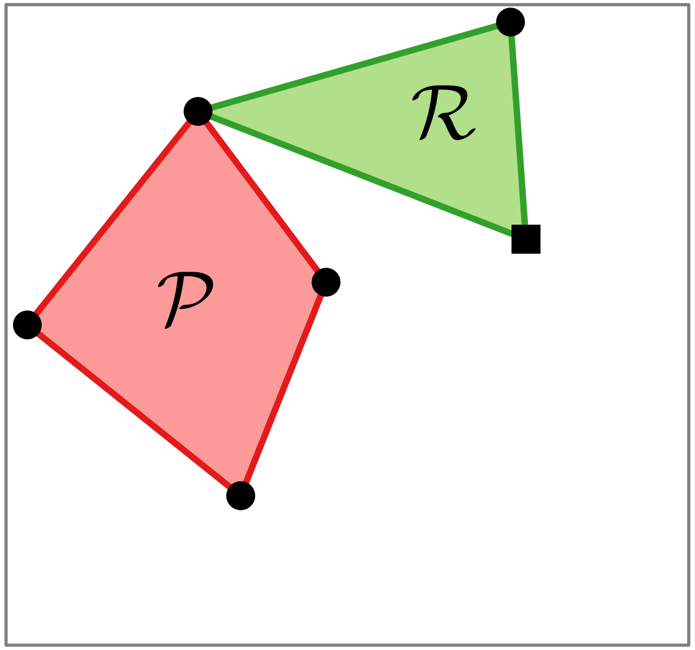

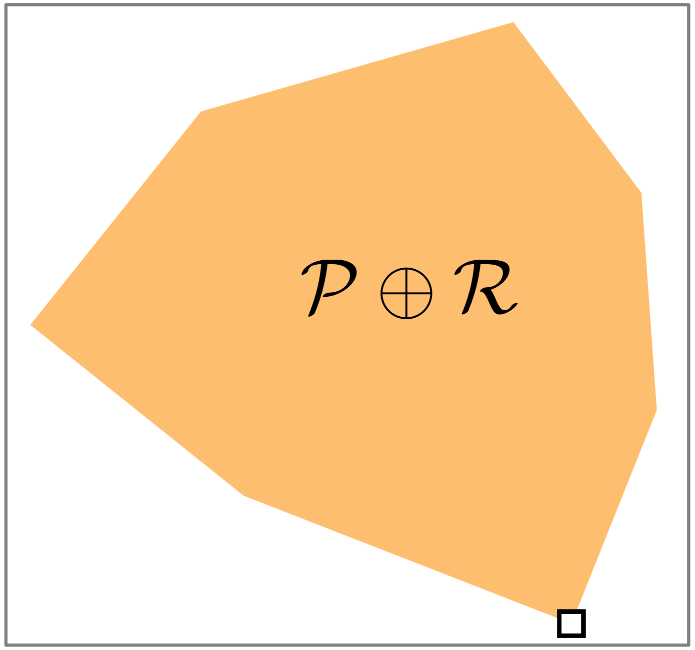

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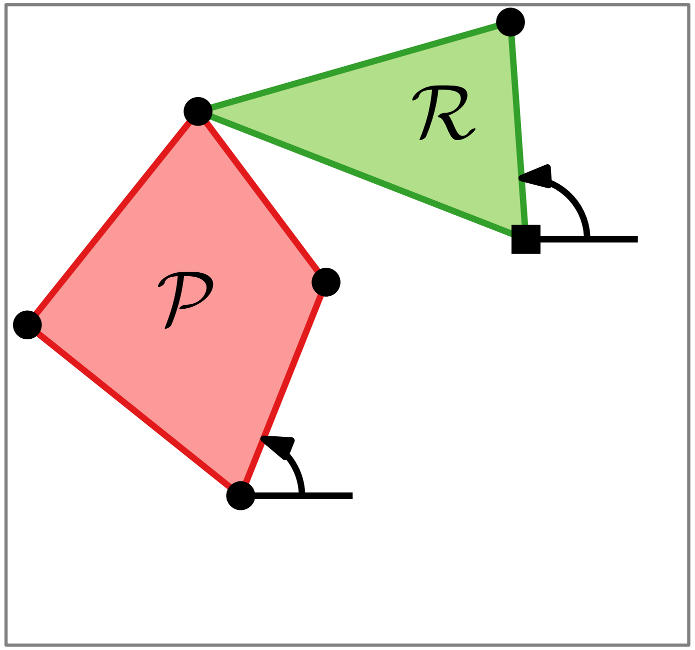

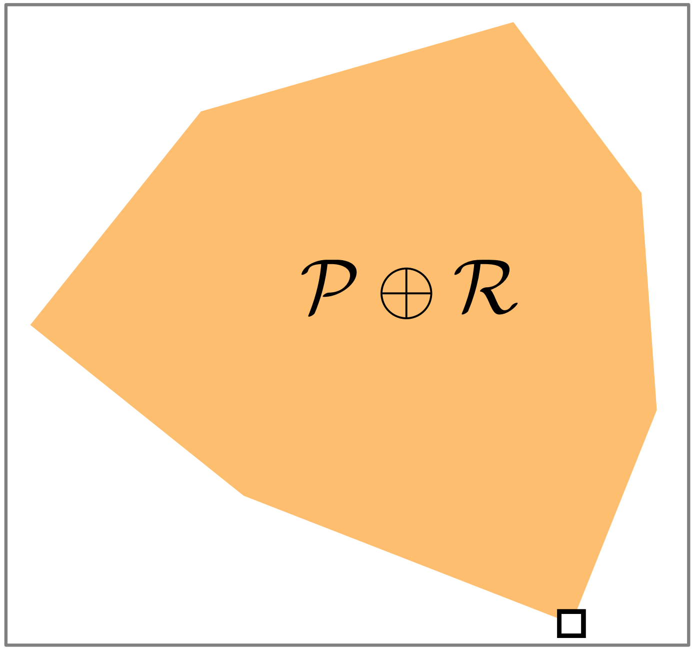

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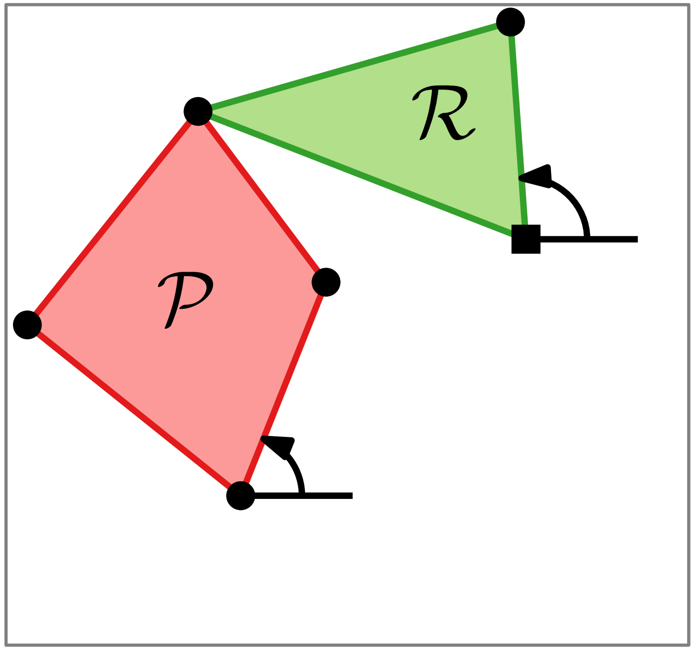

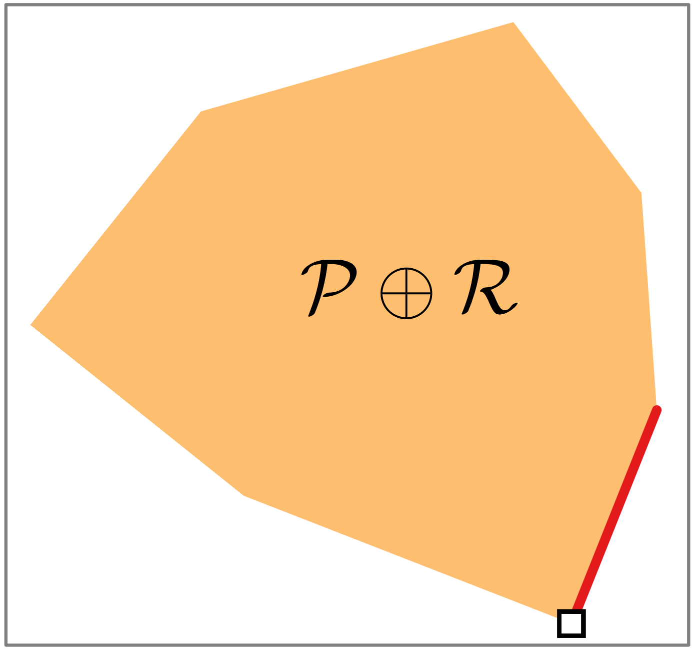

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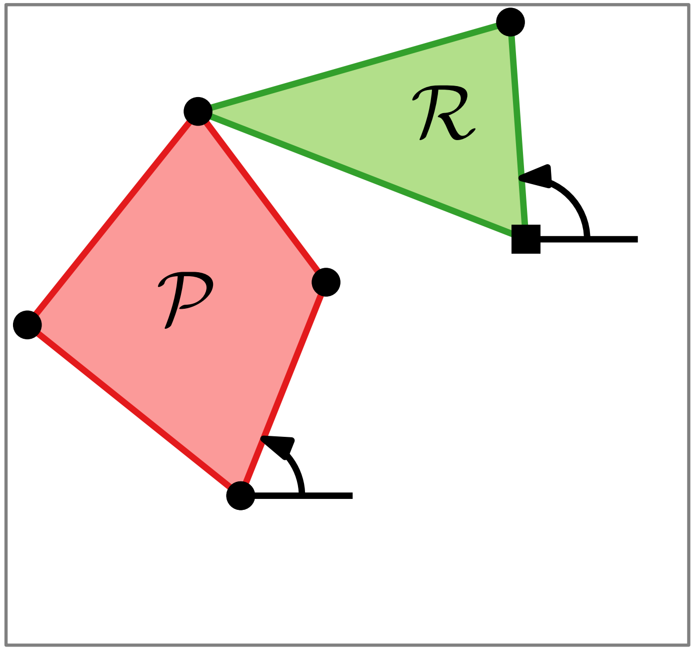

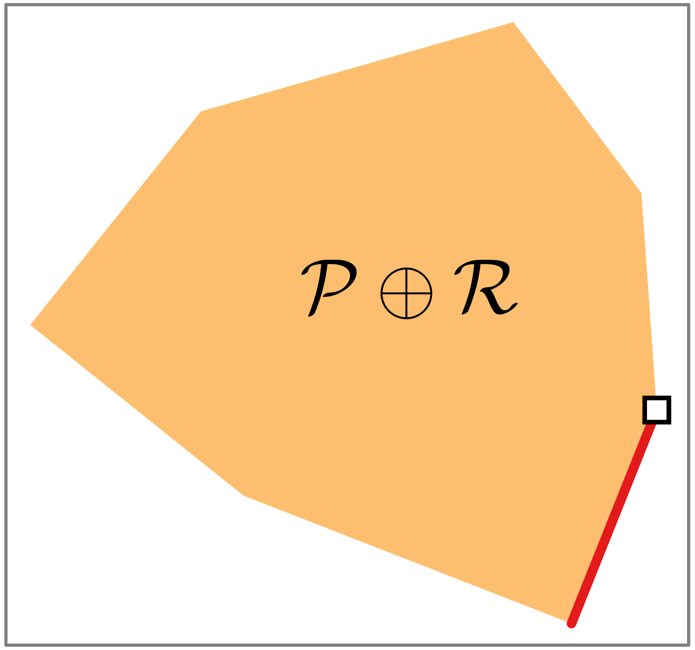

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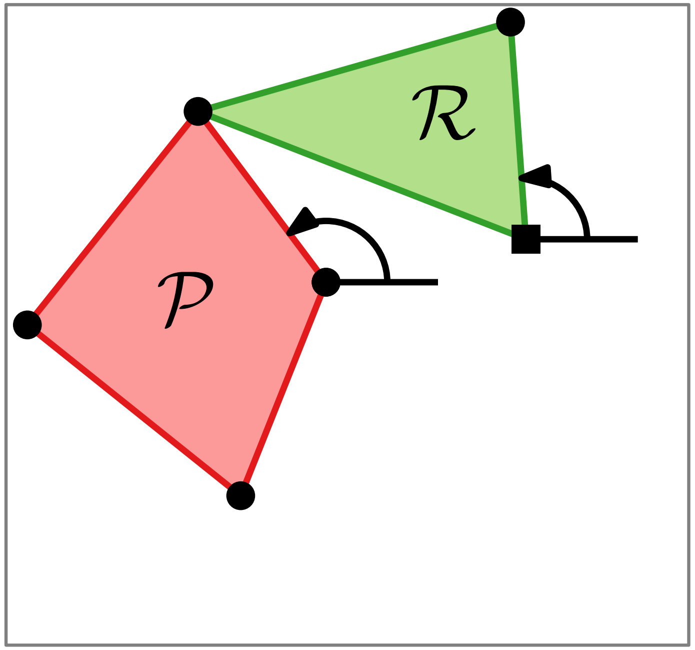
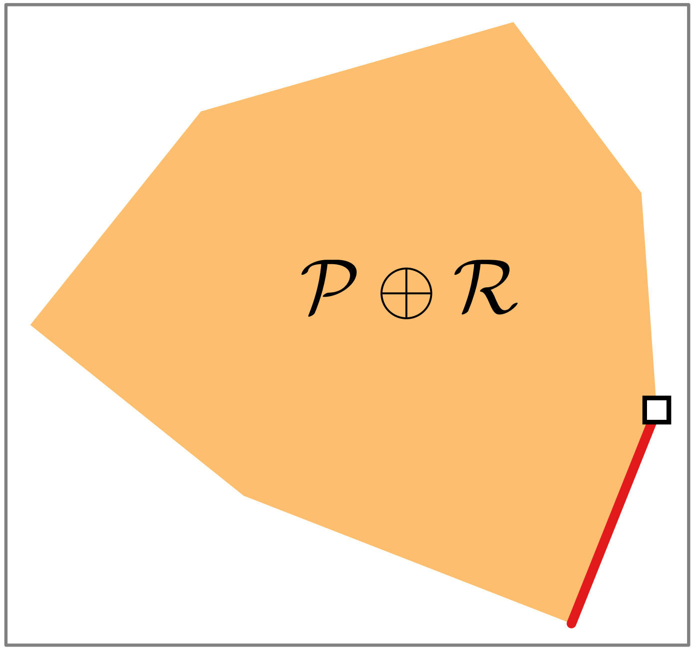
Idea: $\mathcal{P} \oplus \mathcal{R} = \mathrm{CH}\big(\underbrace{\{p + r \mid p \in \mathcal{P}, r \in \mathcal{R}\}}\big)$ *(Proof?)*

Problem: complexity $\in \Theta(|\mathcal{P}| \cdot |\mathcal{R}|)$:-(

Theorem. The Minkowski sum of two convex polygons $\mathcal{P}$ and $\mathcal{R}$ can be computed in $O(|\mathcal{P}| + |\mathcal{R}|)$ time. :-)





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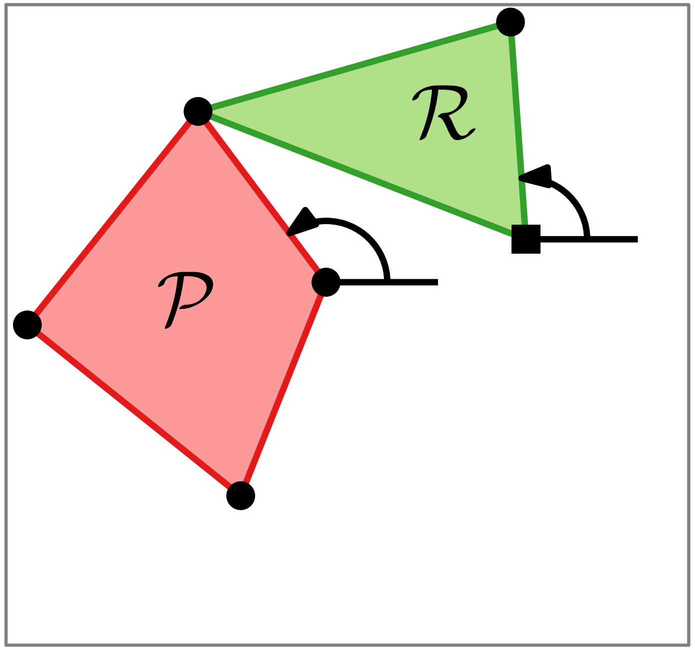

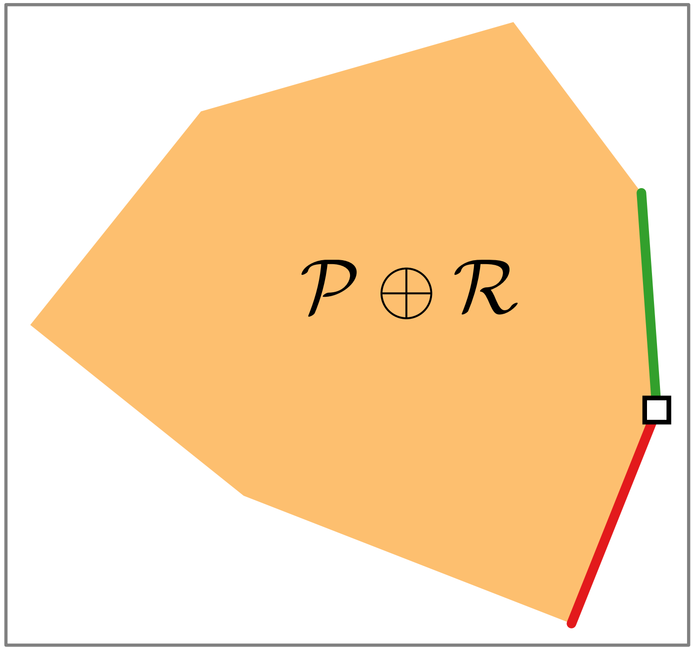

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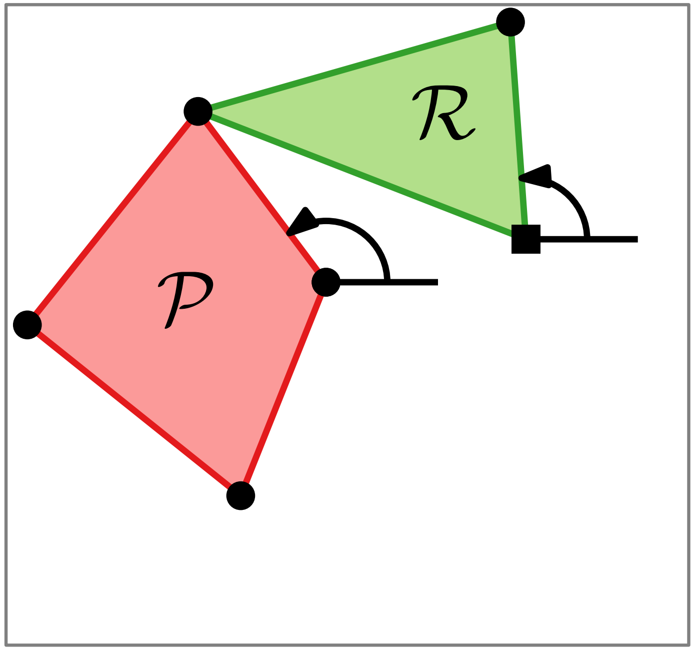

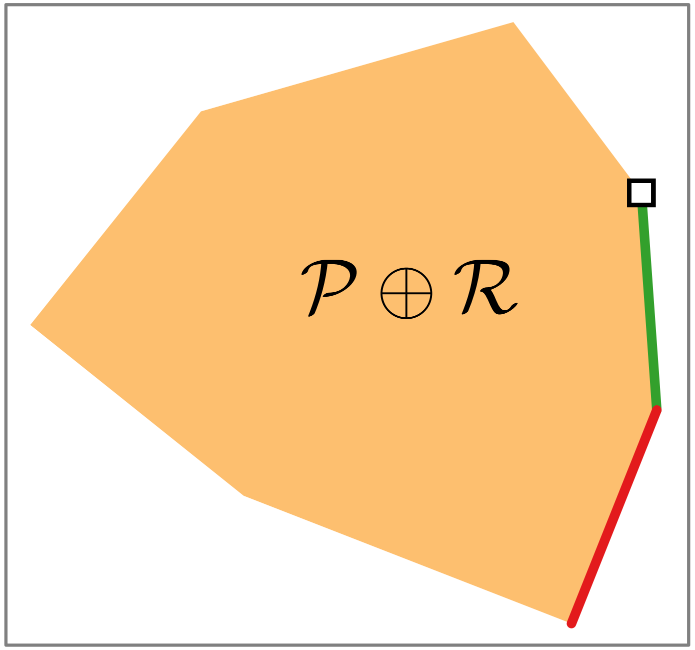

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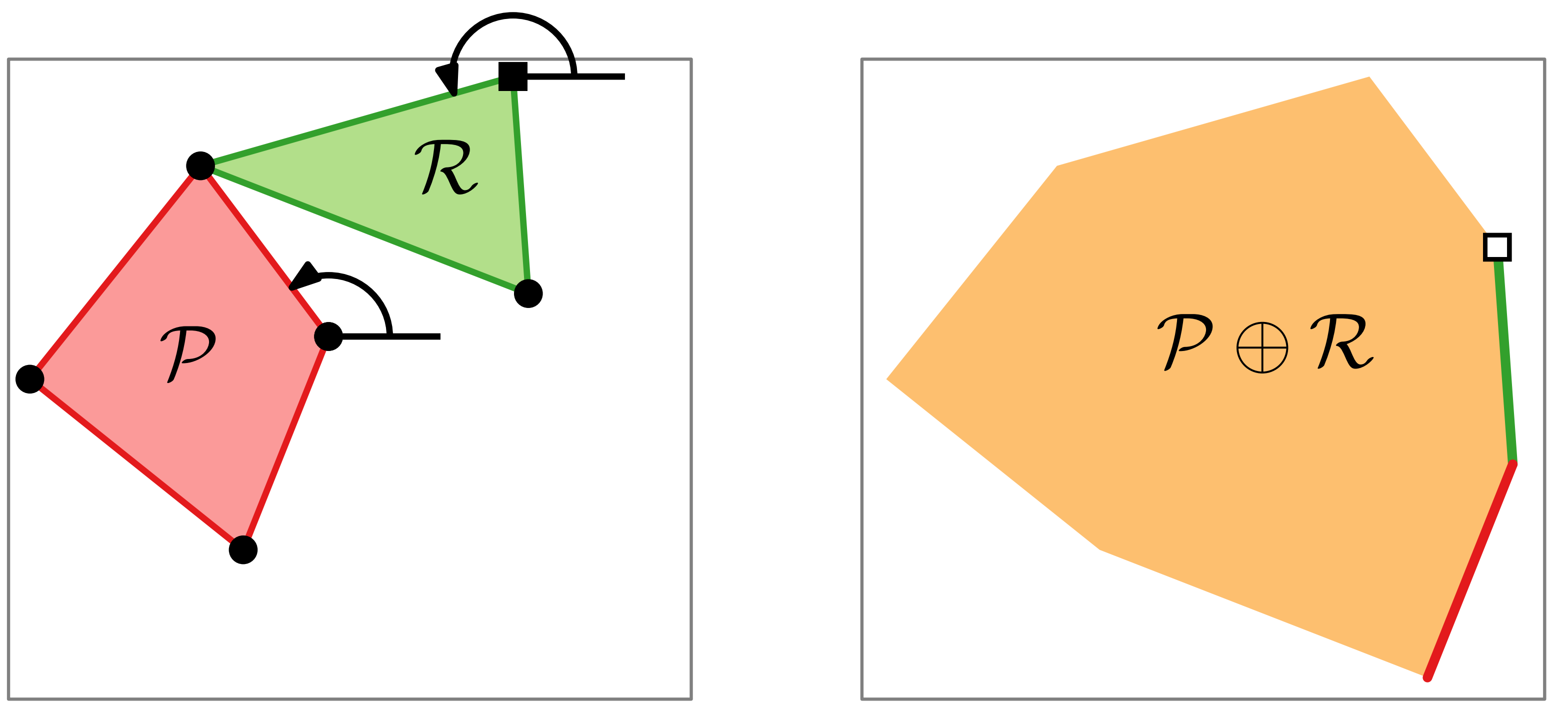

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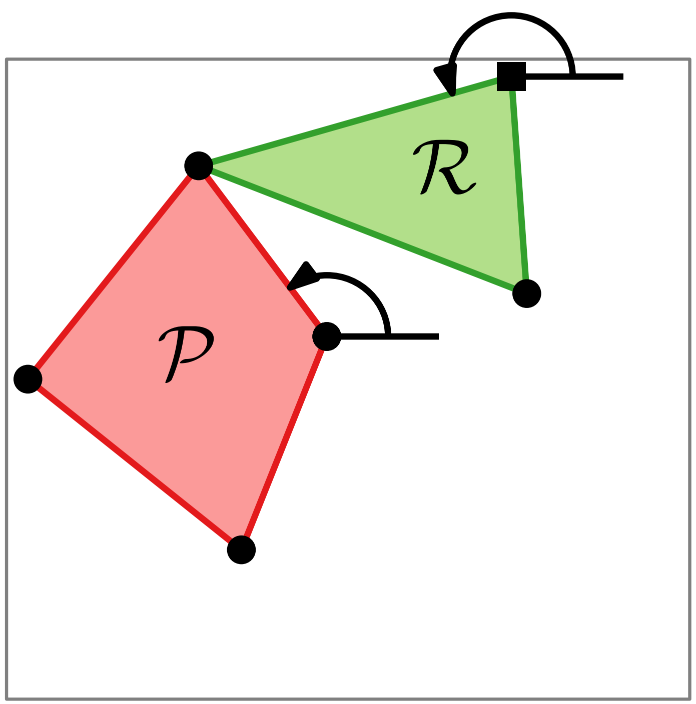

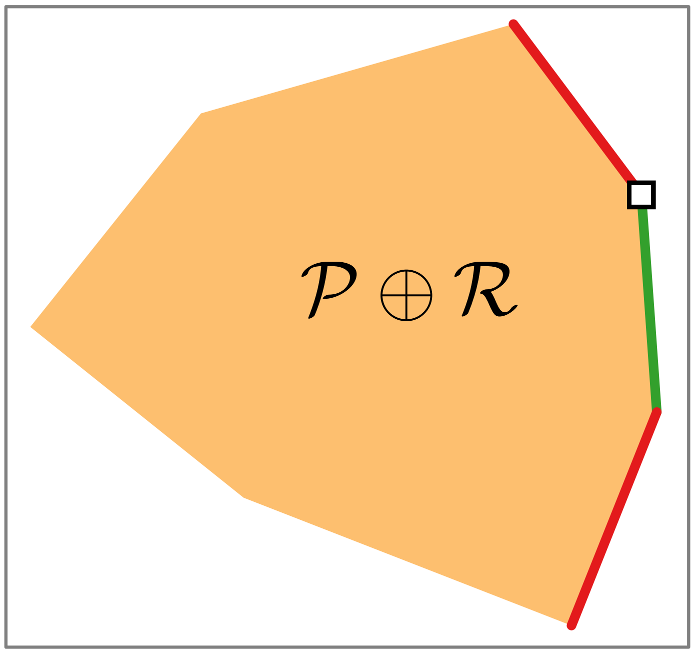

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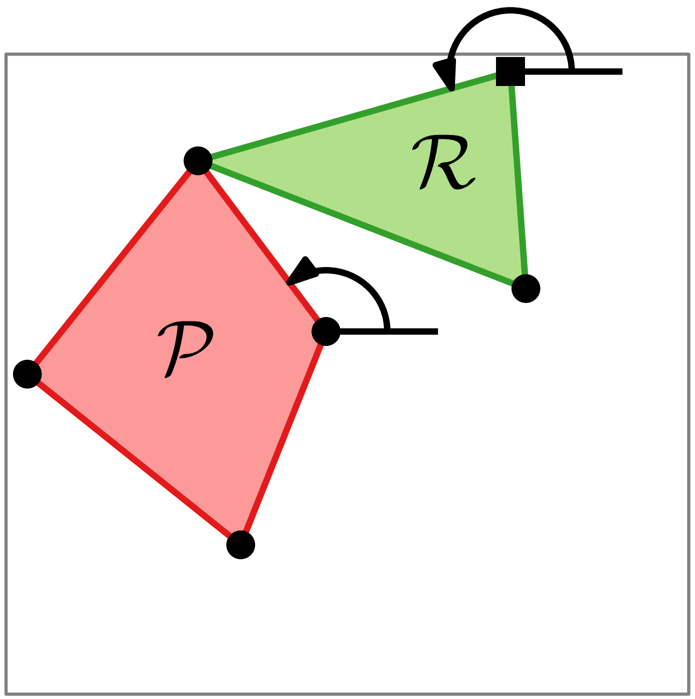

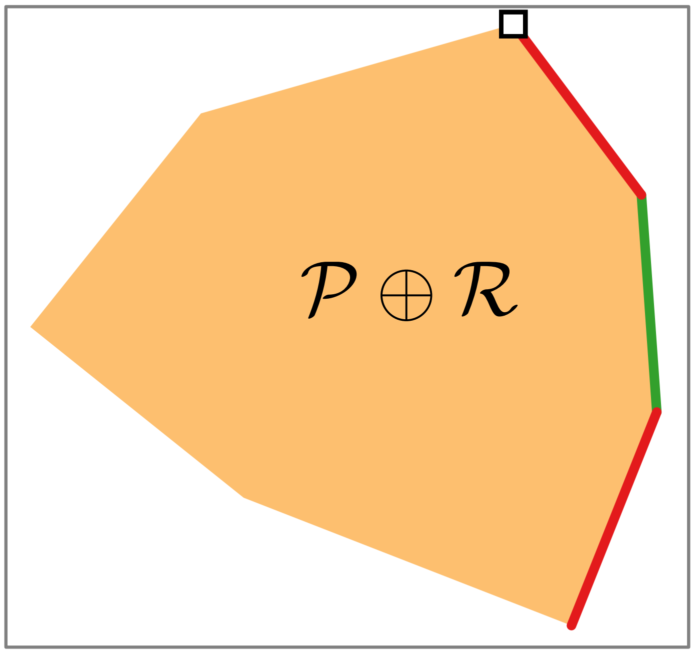

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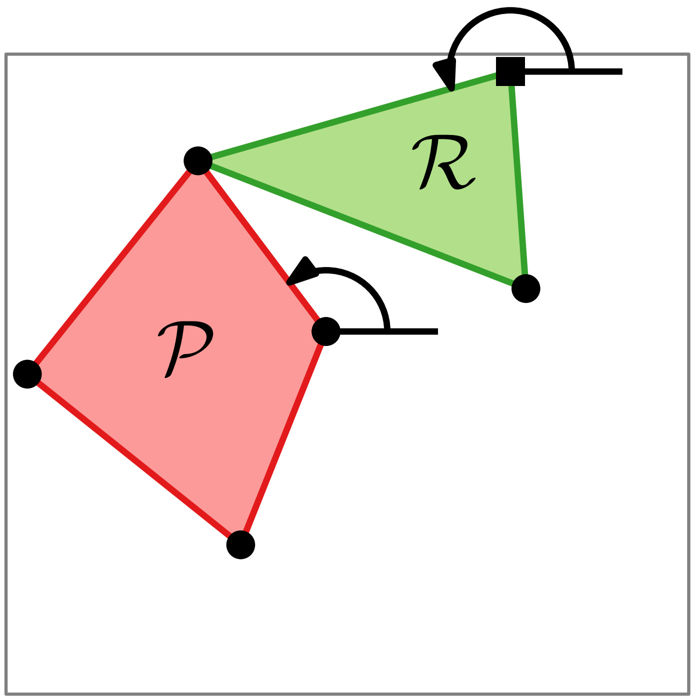
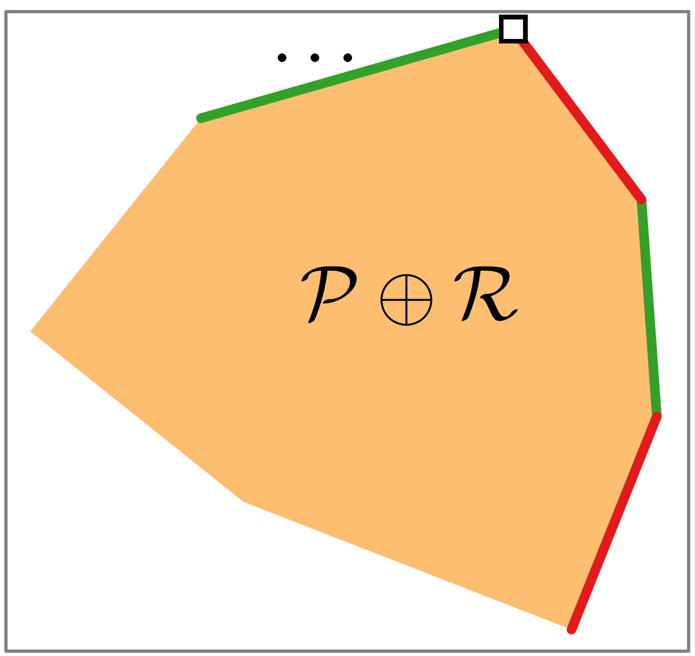
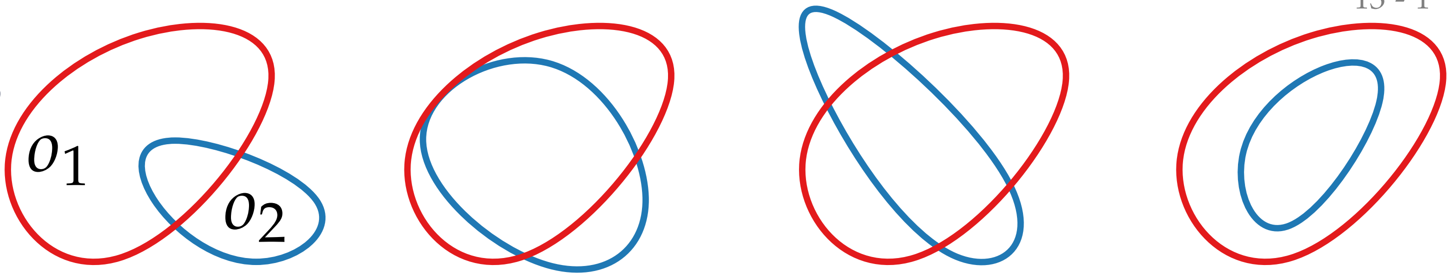




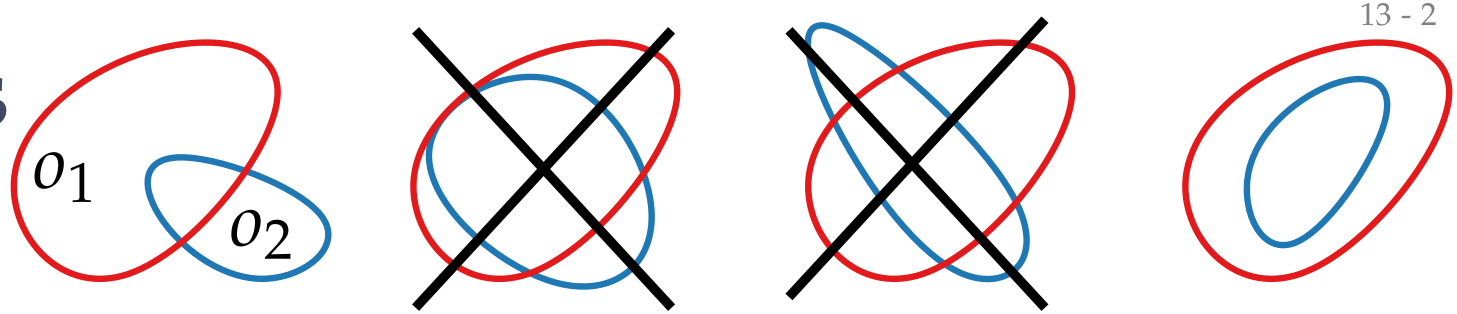
Pseudodisks



Definition: A pair of planar objects o_1 and o_2 is a pair of pseudodisks if:

- $\partial o_1 \cap \mathrm{int}(o_2)$ is connected, and
- $\partial o_2 \cap \mathrm{int}(o_1)$ is connected.

Pseudodisks



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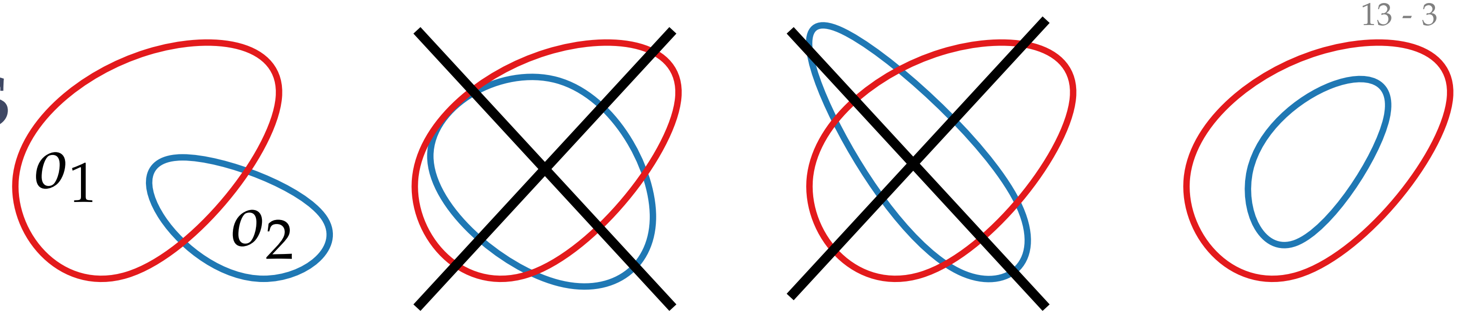

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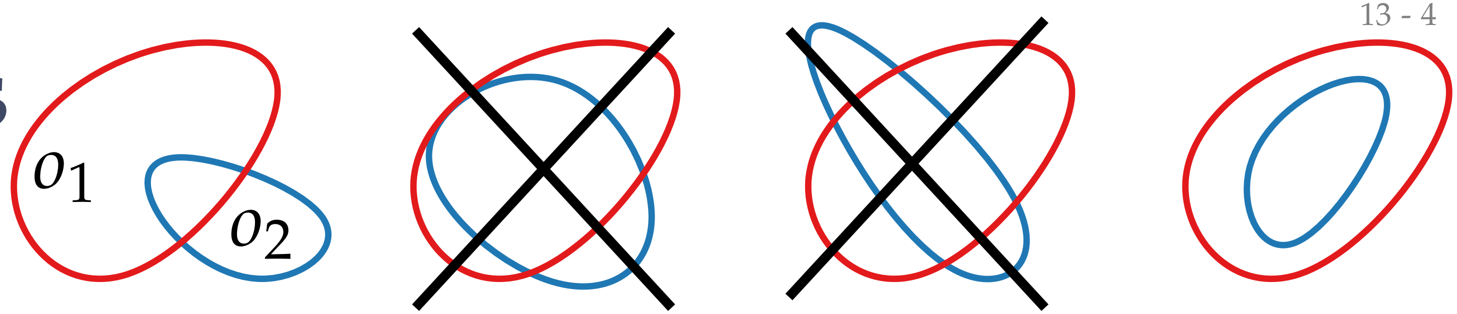

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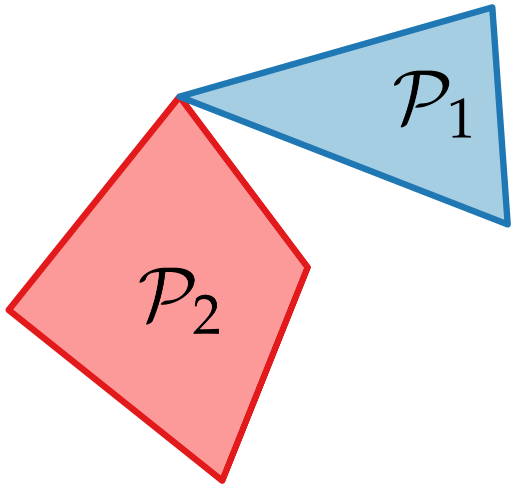
Observation: A pair of polygonal pseudodisks defines at most two boundary crossings.

Extreme Directions

Observation: Let $\mathcal{P}_1$, $\mathcal{P}_2$ be interior-disjoint convex polygons



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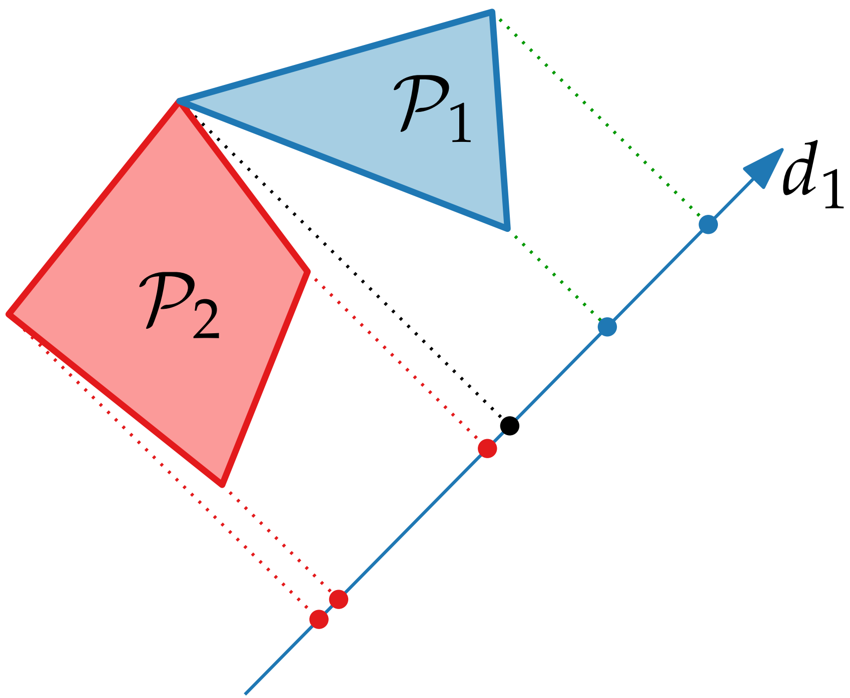
Observation: Let $\mathcal{P}_1$, $\mathcal{P}_2$ be interior-disjoint convex polygons. Let d_1 and d_2 be directions in which $\mathcal{P}_1$ is more extreme than $\mathcal{P}_2$.



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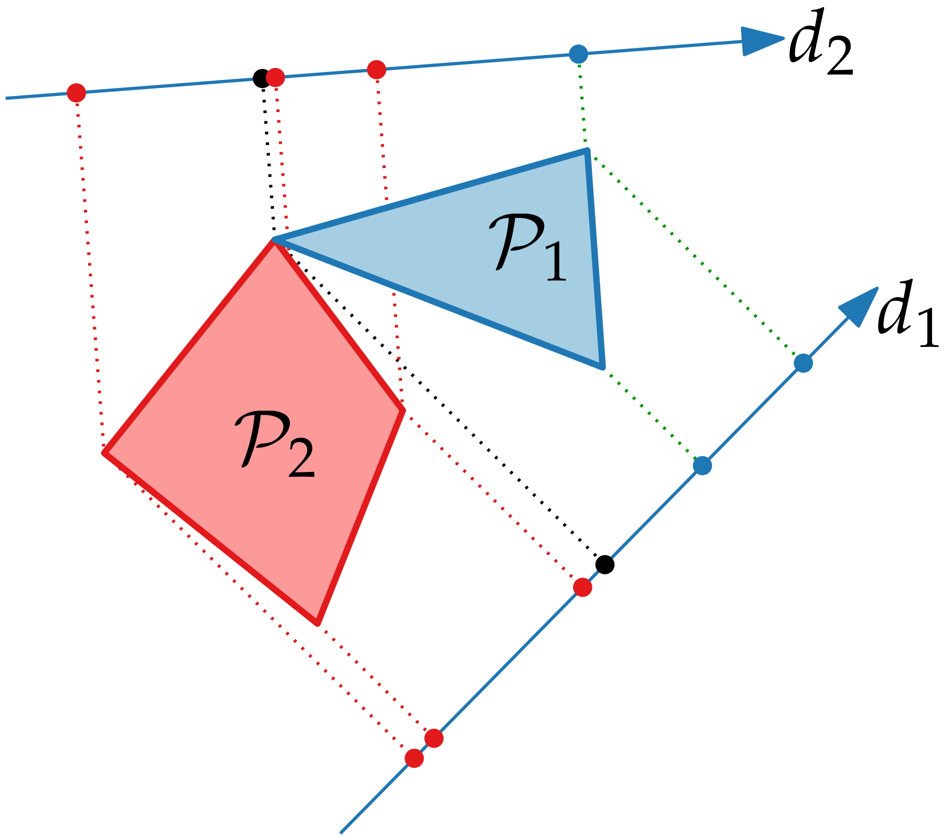

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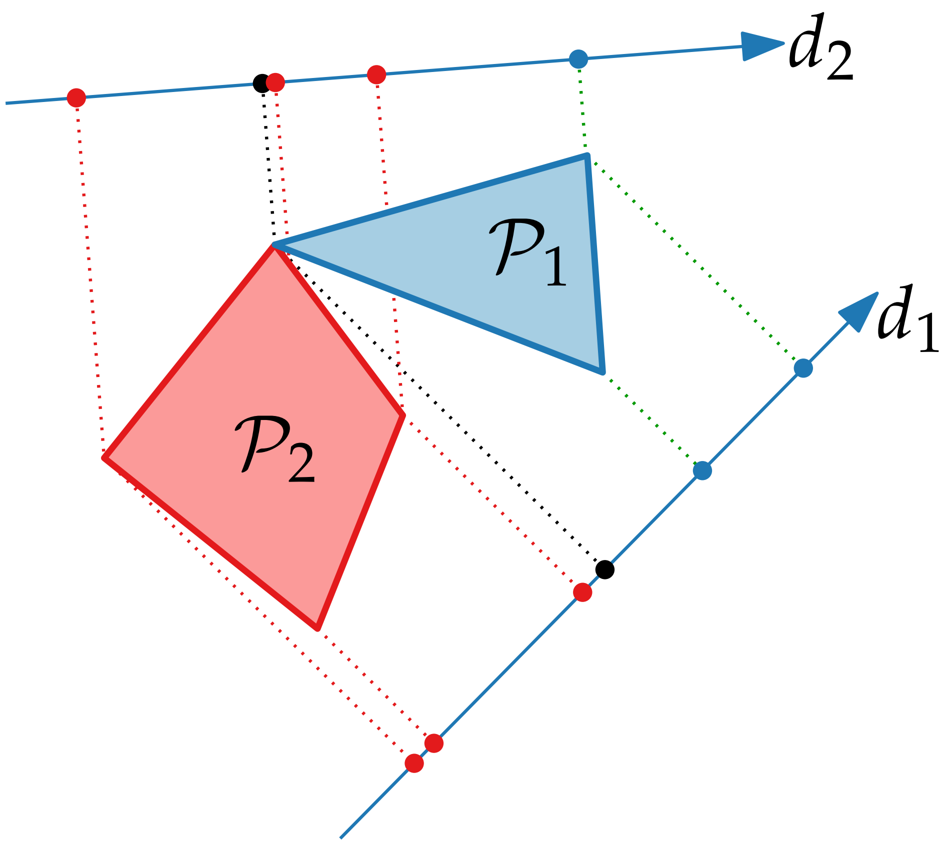

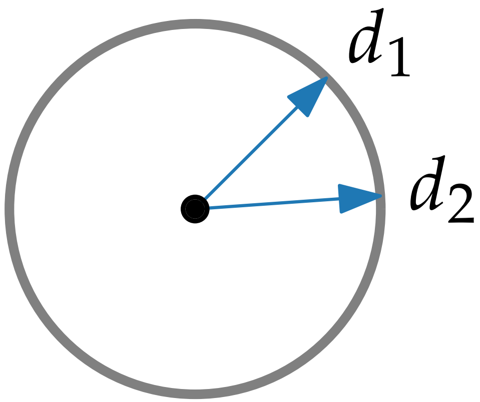

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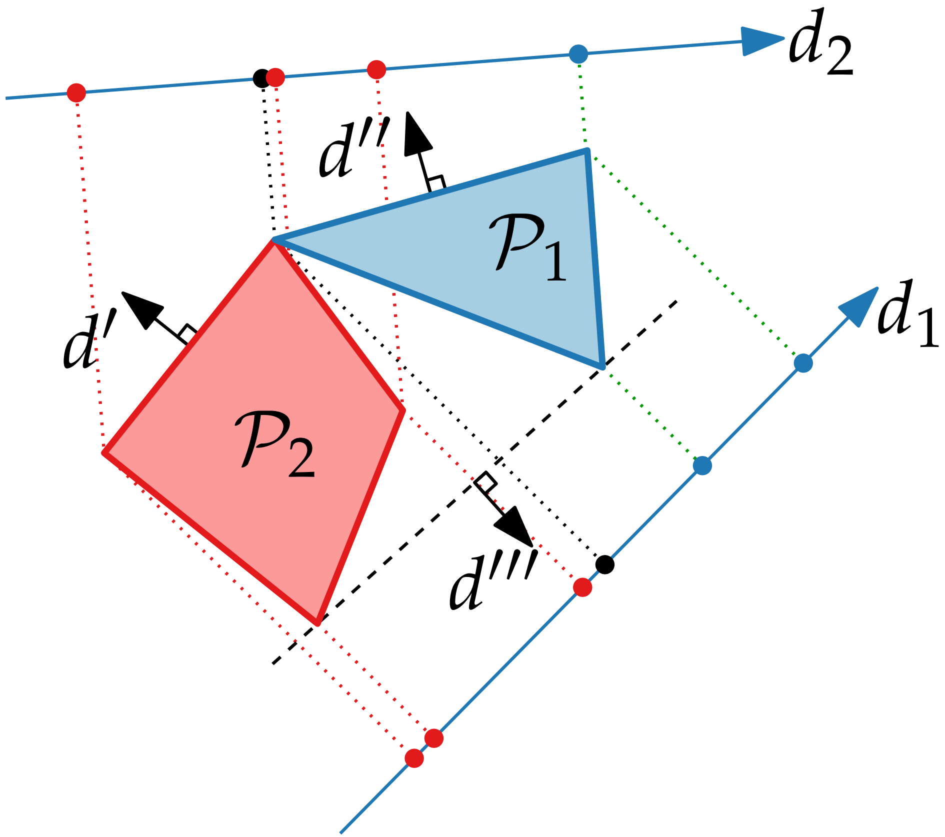

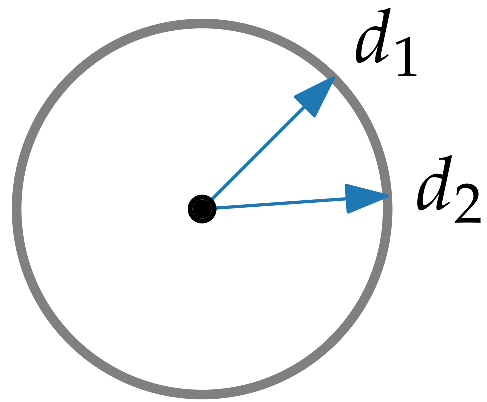

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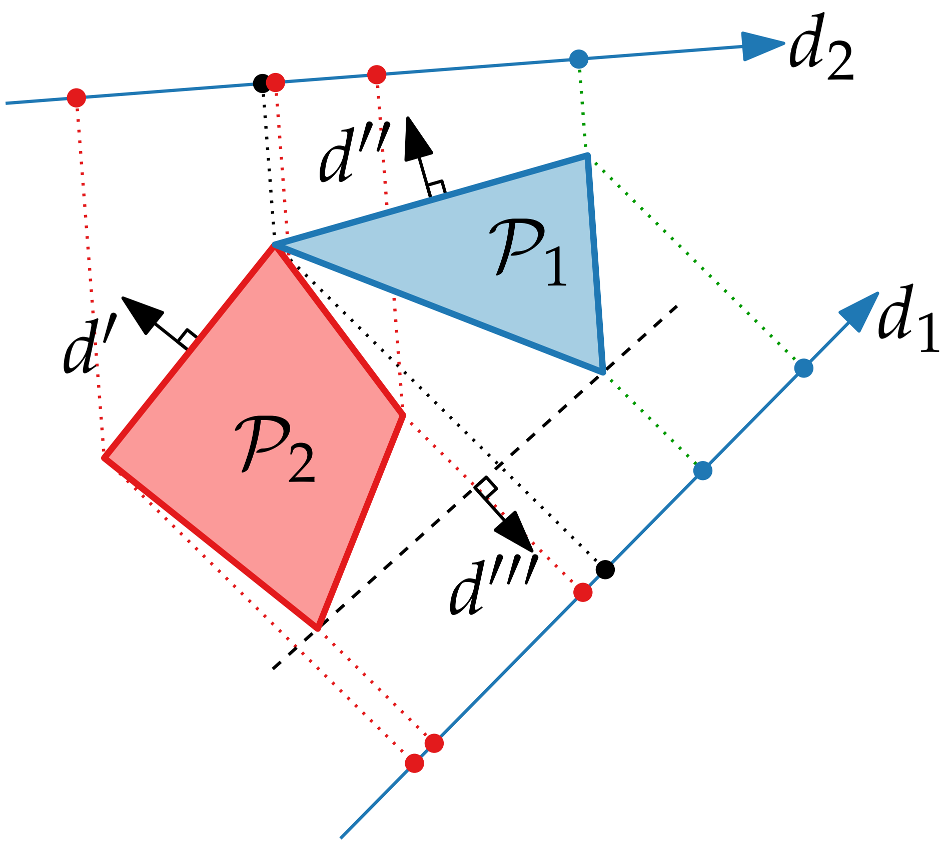

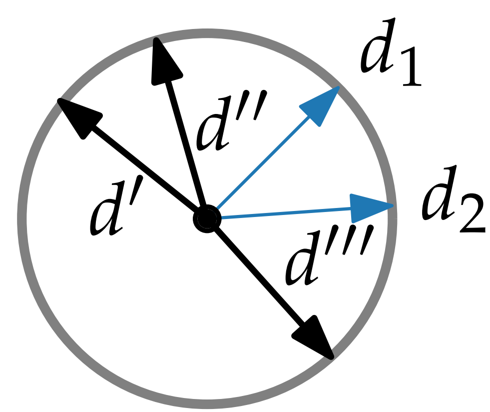

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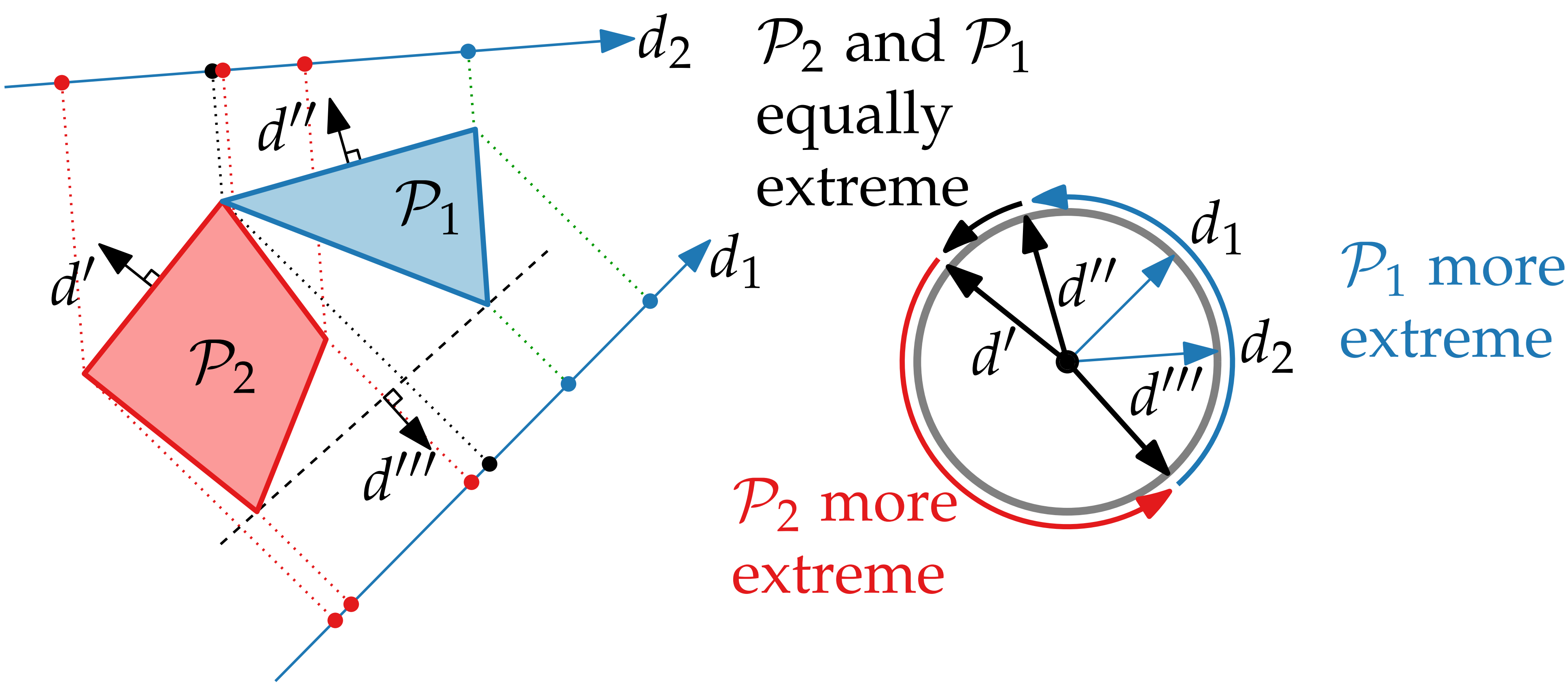

Polygonal Pseudodisks

Theorem: If $\mathcal{P}_1$ and $\mathcal{P}_2$ are convex polygons with disjoint interiors, and $\mathcal{R}$ is another convex polygon, then $\mathcal{P}_1 \oplus \mathcal{R}$ and $\mathcal{P}_2 \oplus \mathcal{R}$ is a pair of pseudodisks.

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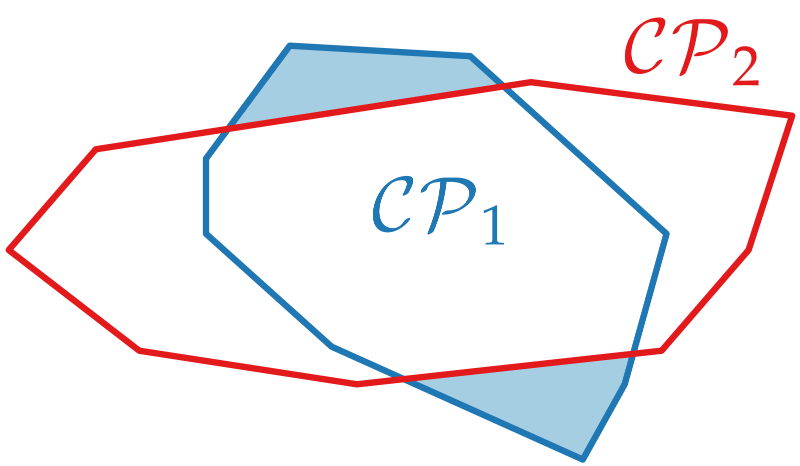

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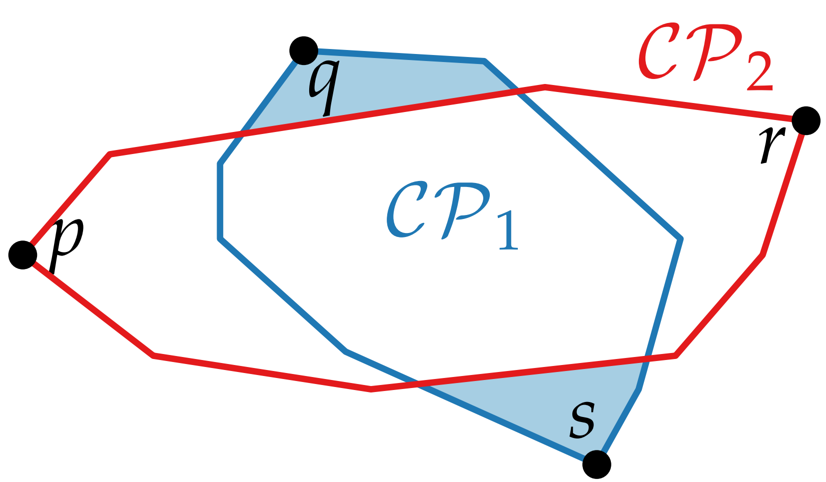

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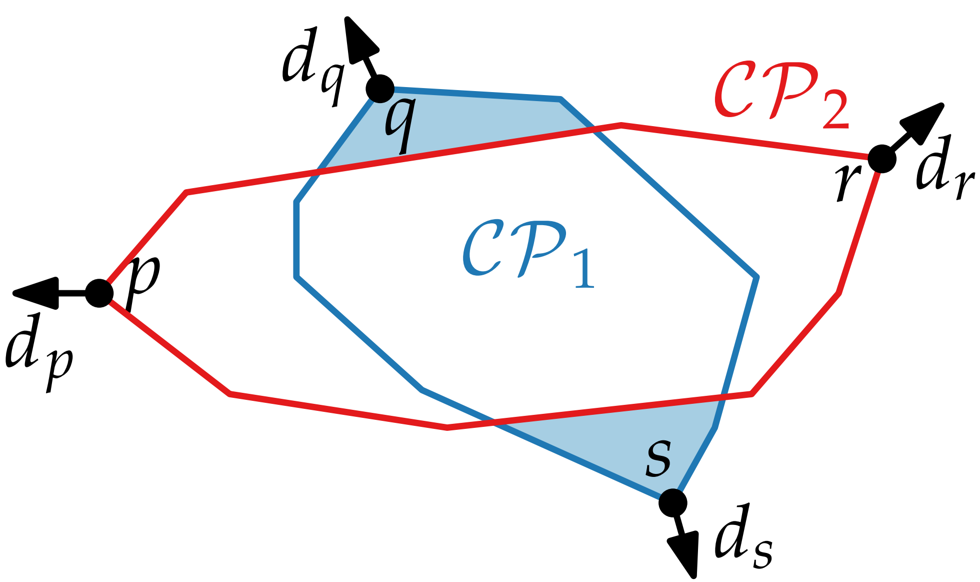

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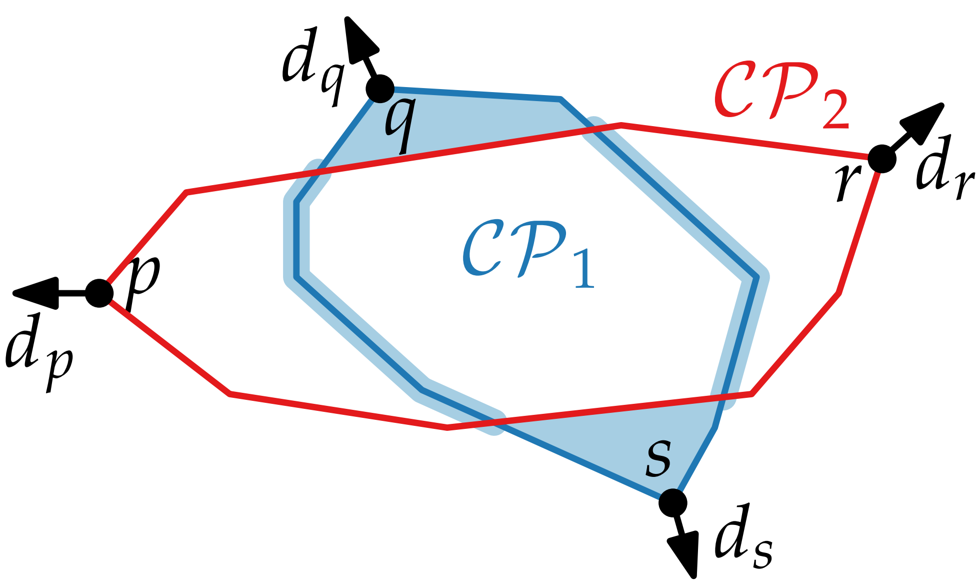

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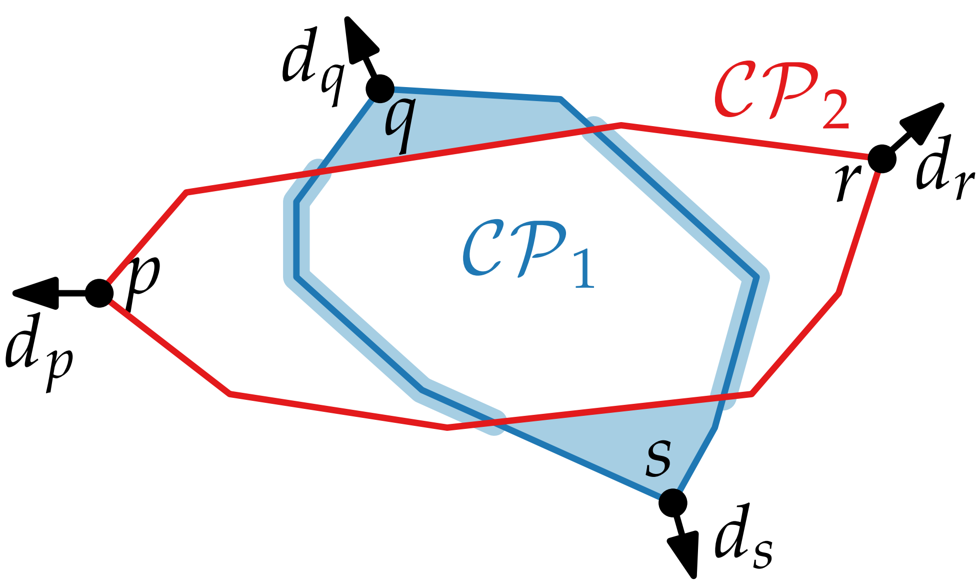

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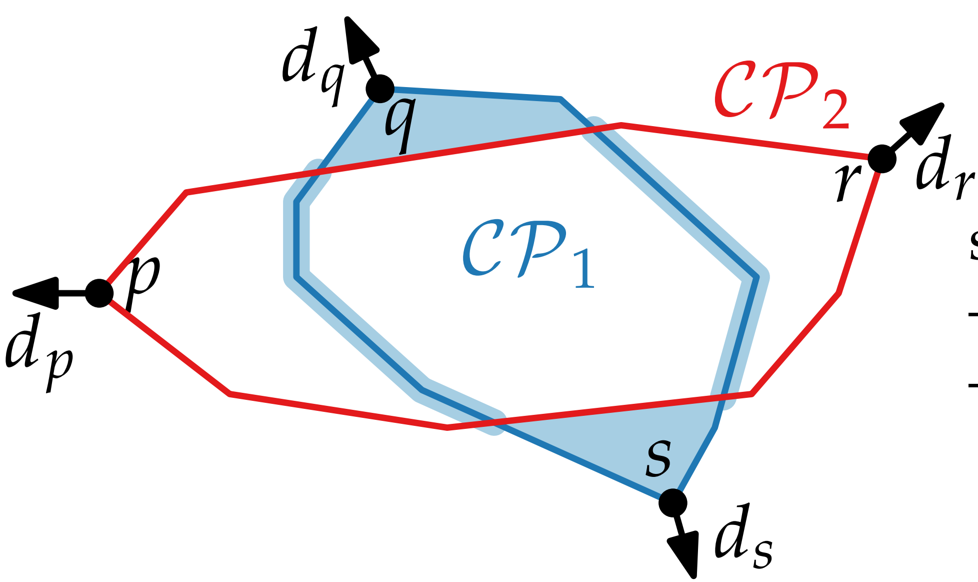

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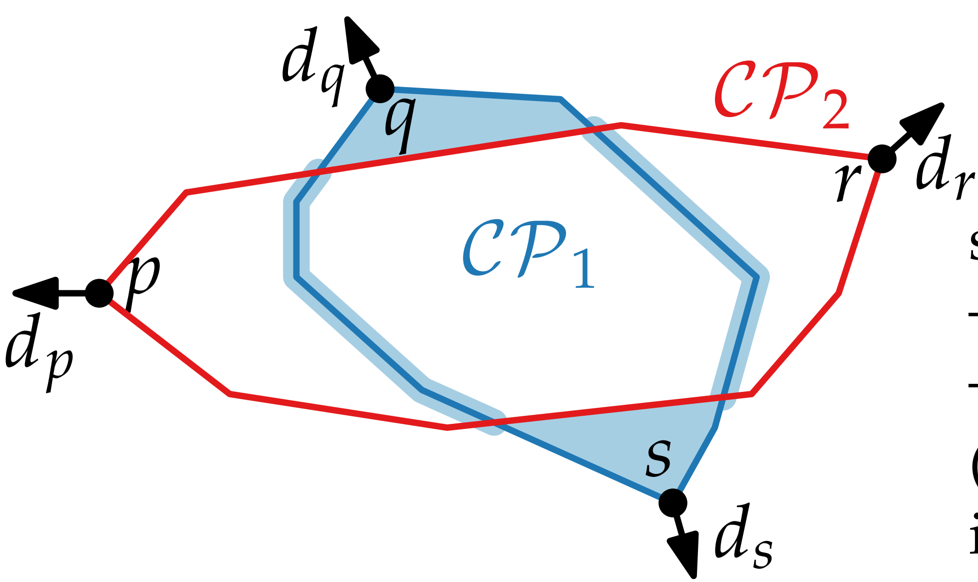

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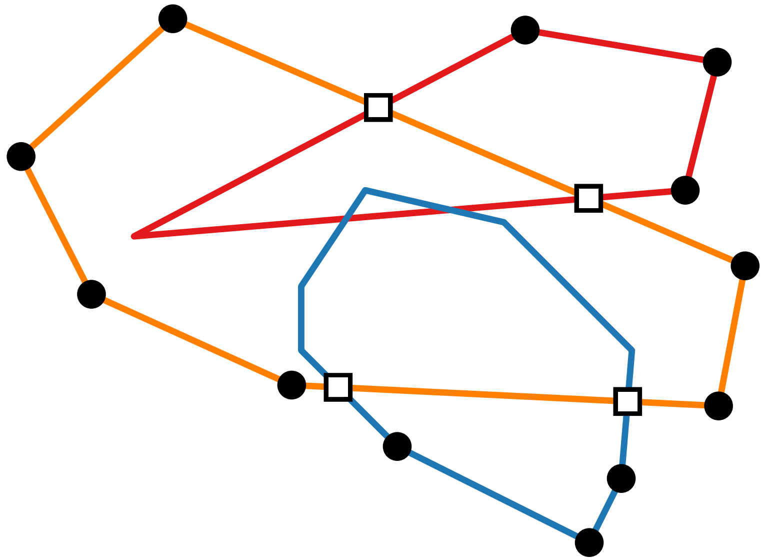
(and $\mathcal{P}_1$ and $\mathcal{P}_2$ are convex and interior-disjoint).

Union Complexity

Theorem: A collection S of convex polygonal pseudodiscs with n vtc in total has a union with $\leq 2n$ vtc.



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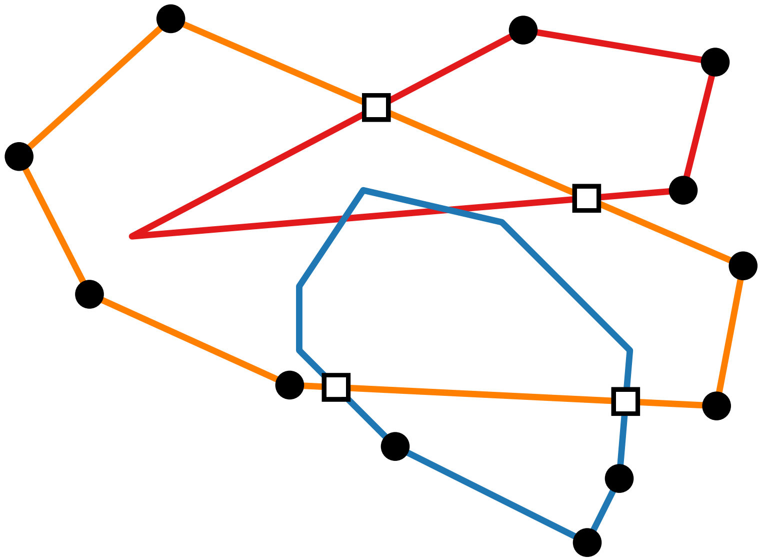

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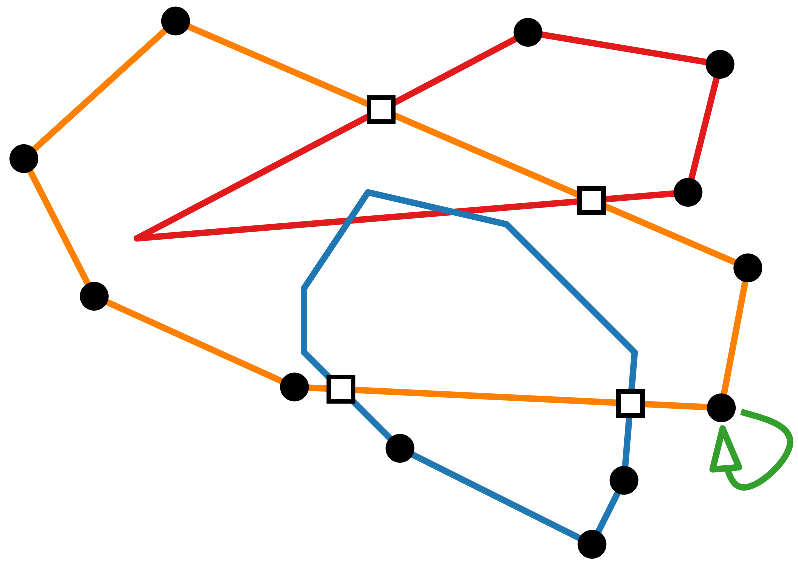

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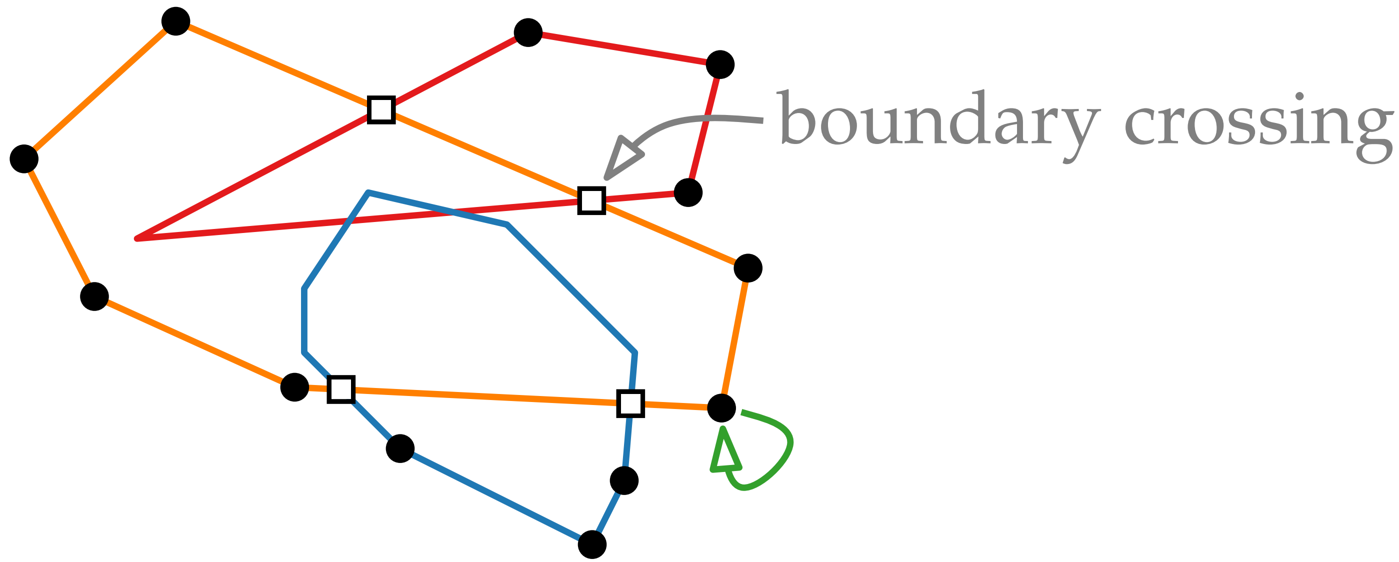

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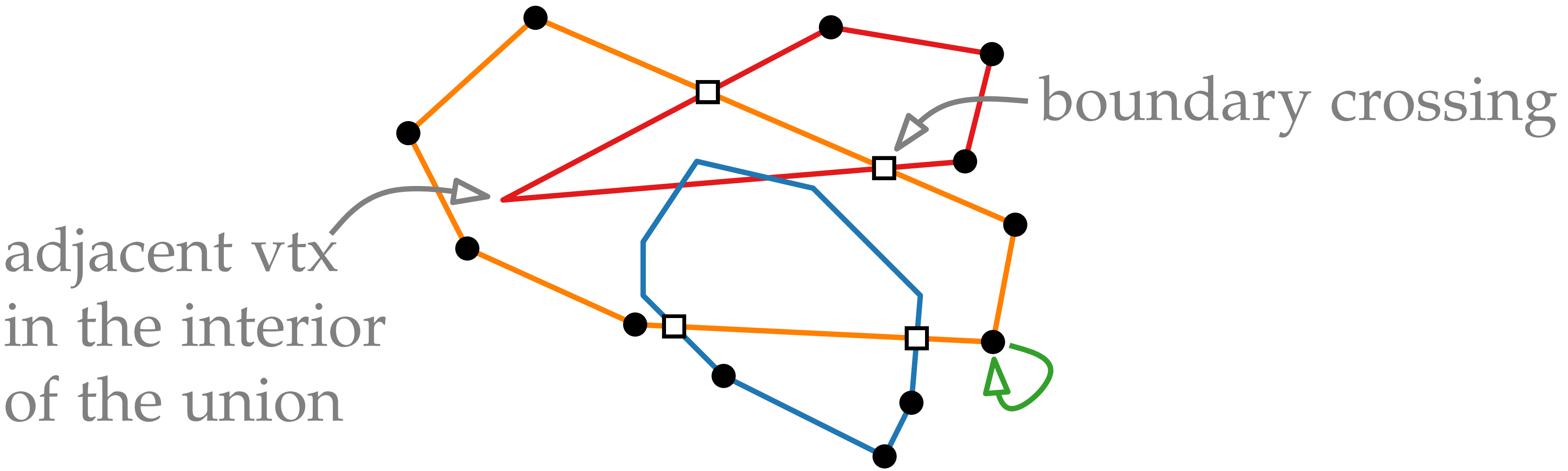

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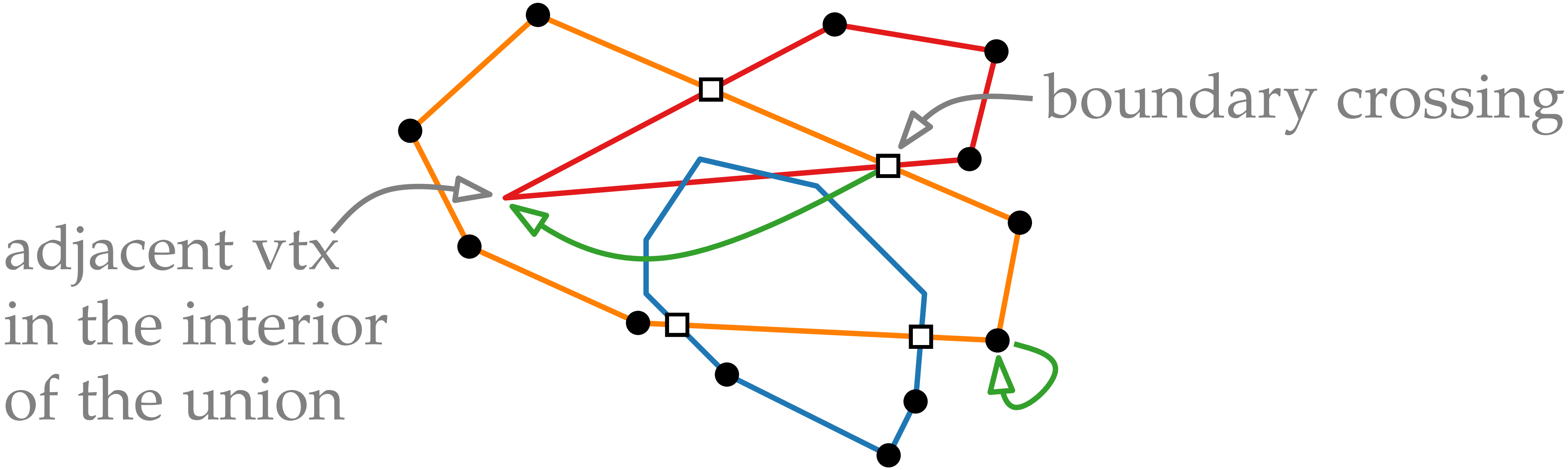

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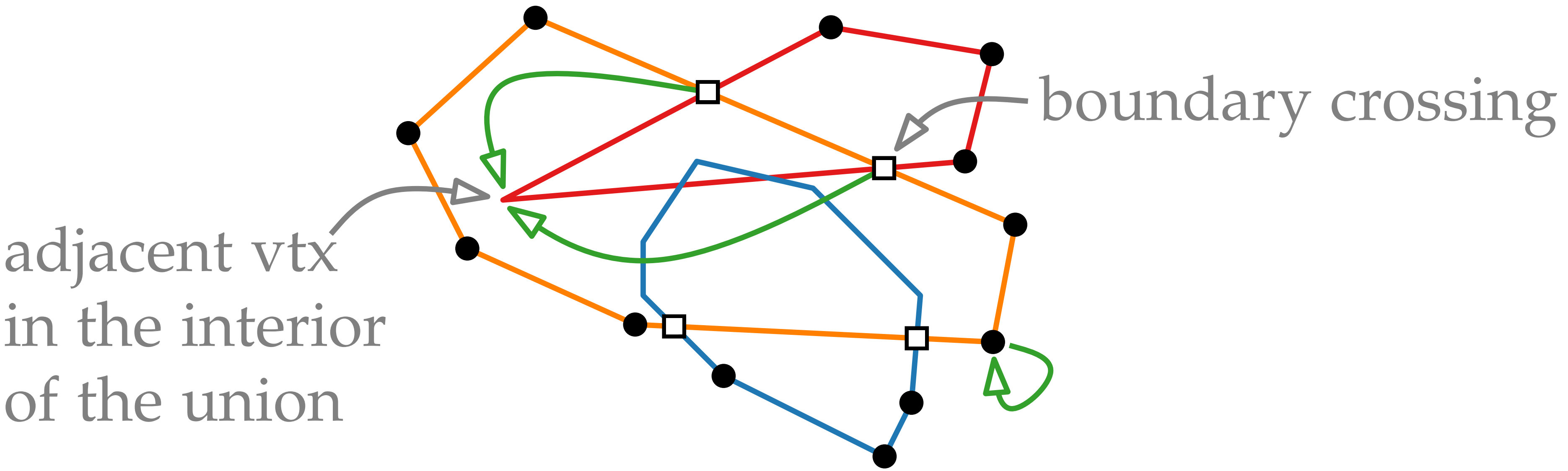

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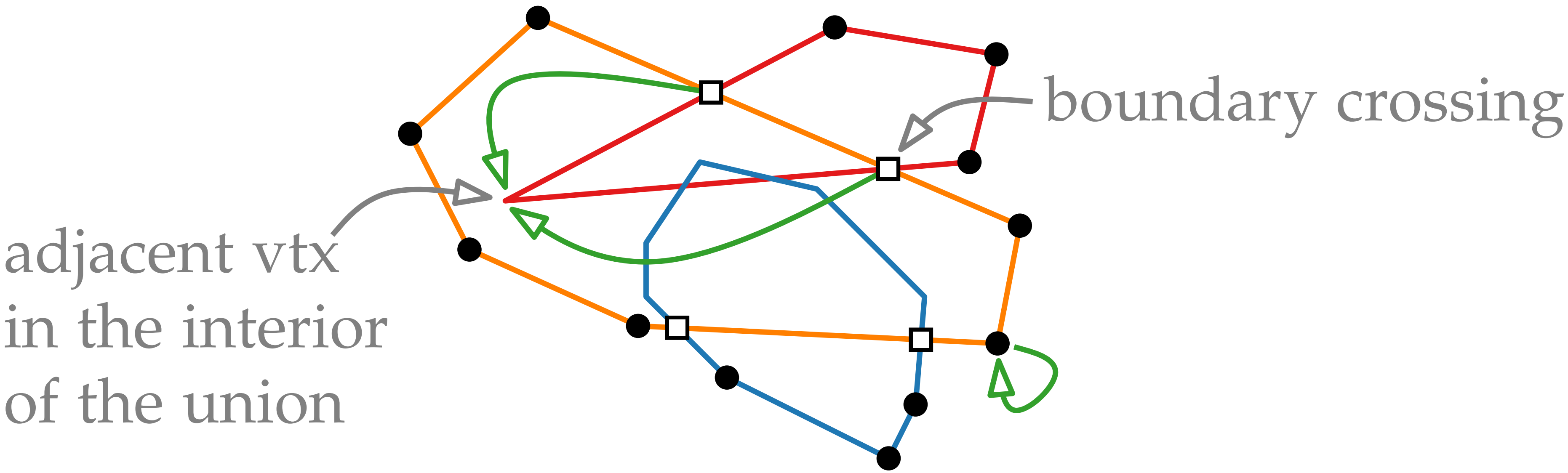

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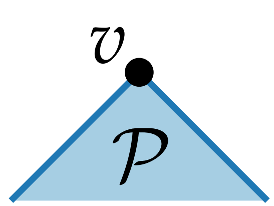

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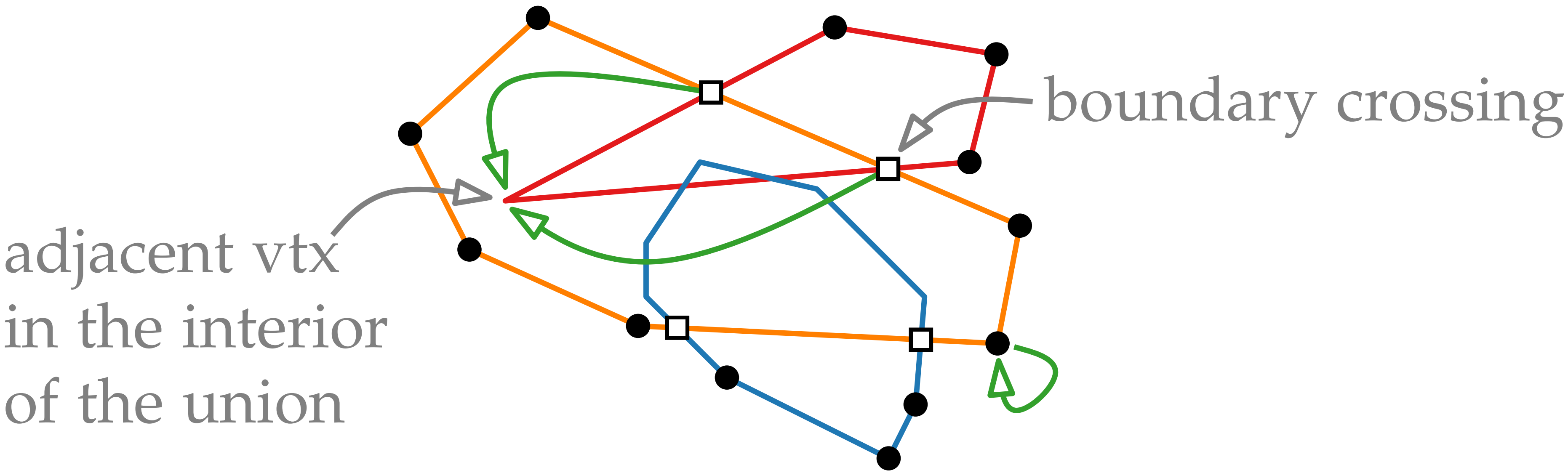

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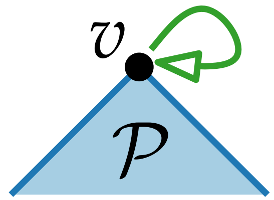

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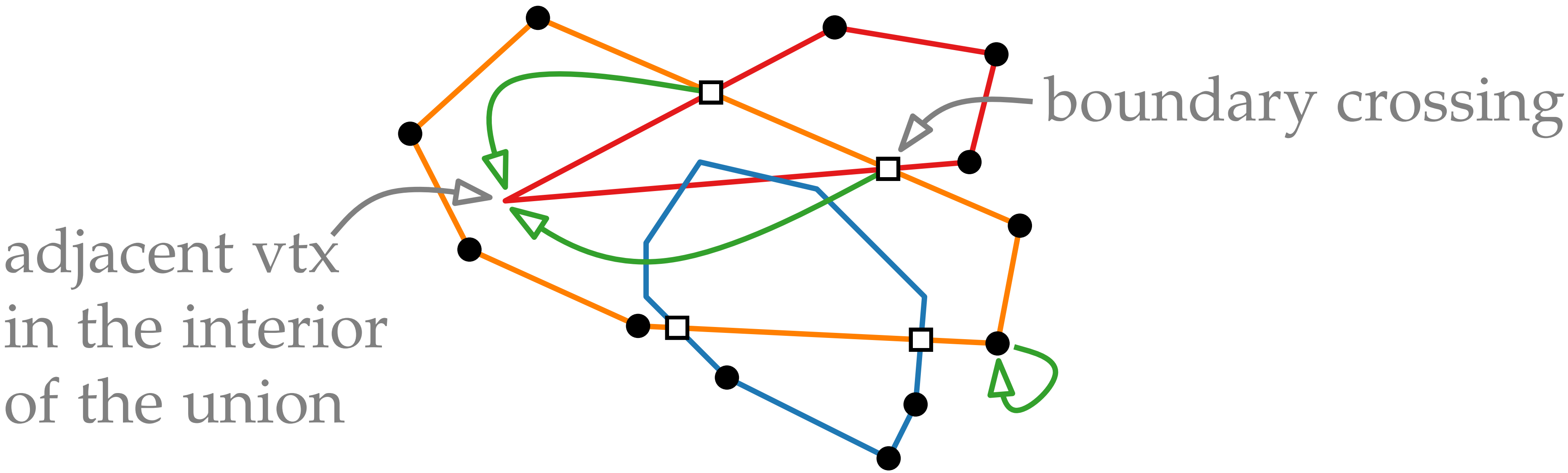

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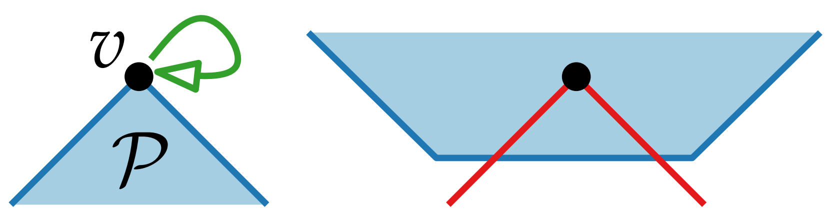

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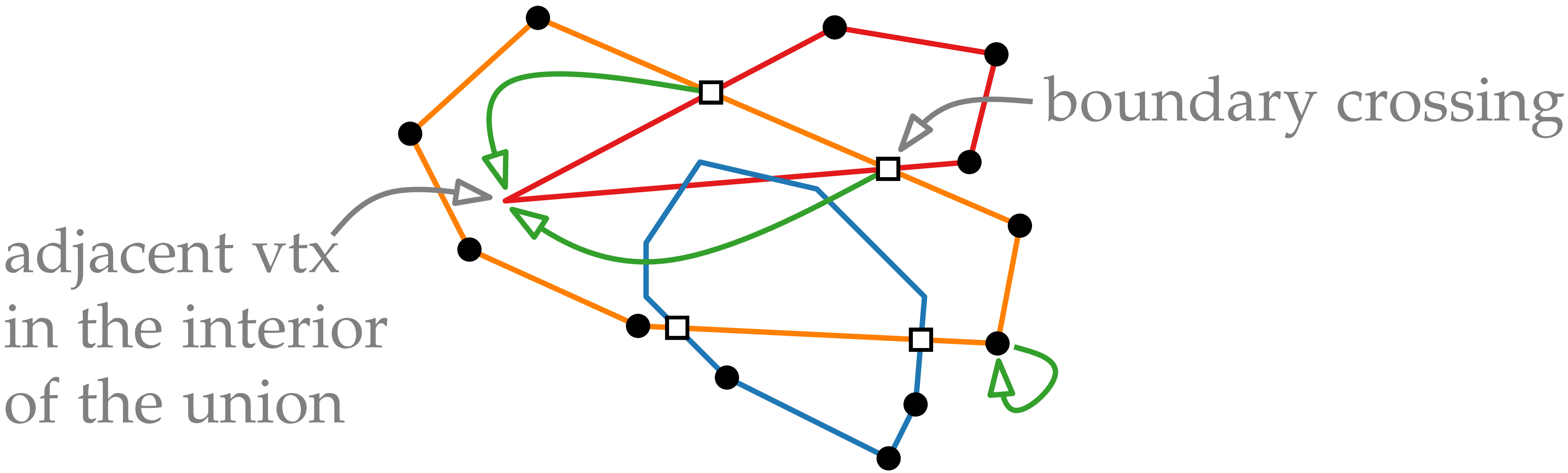

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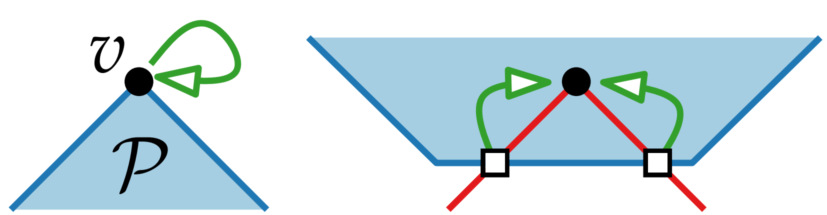

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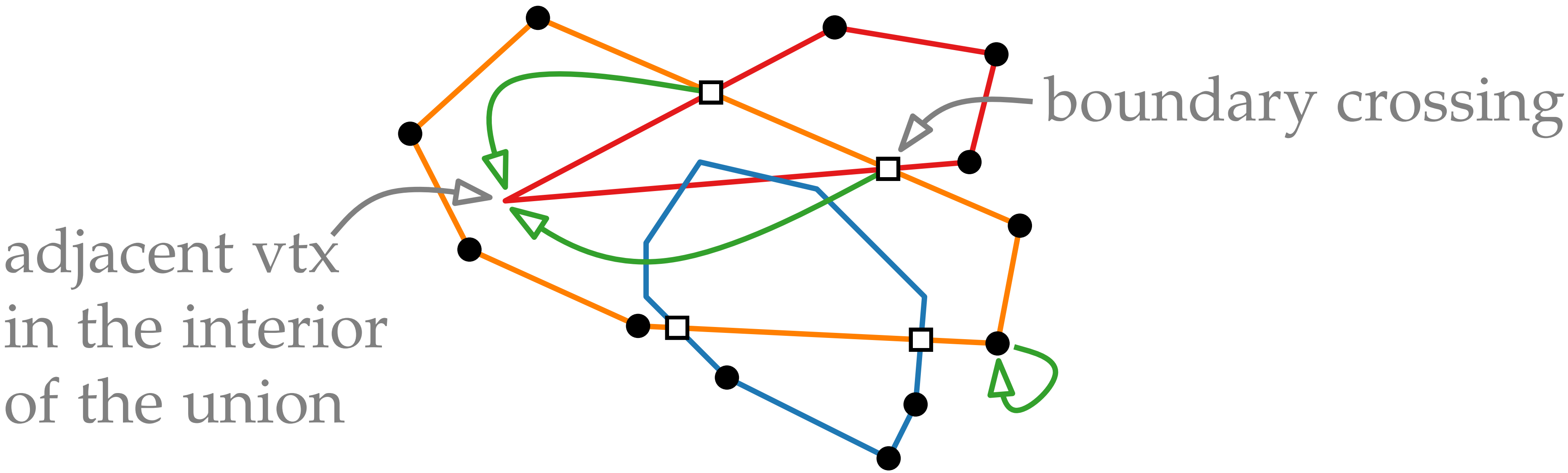

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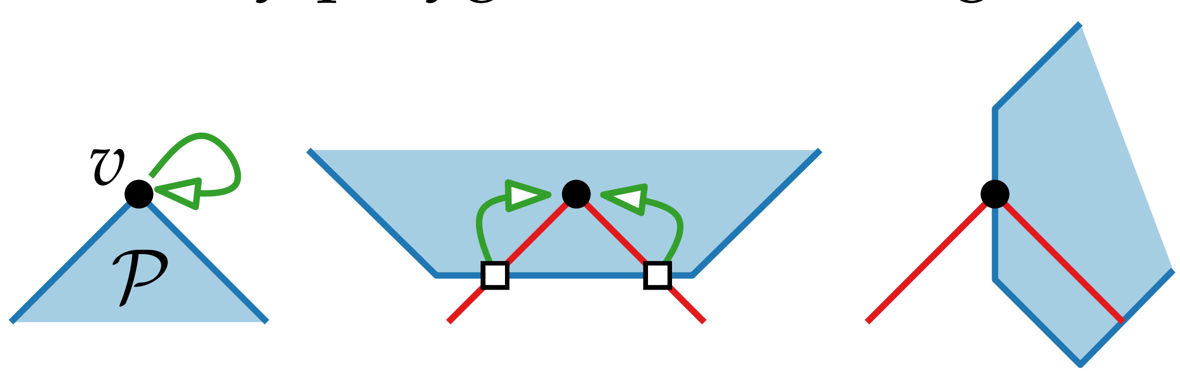

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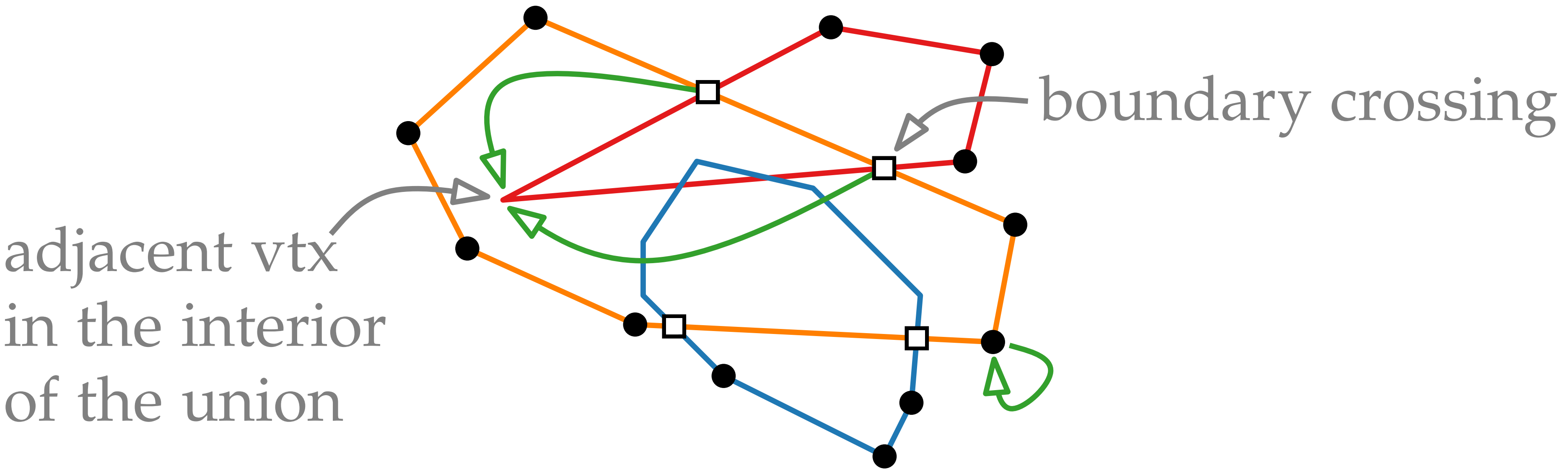

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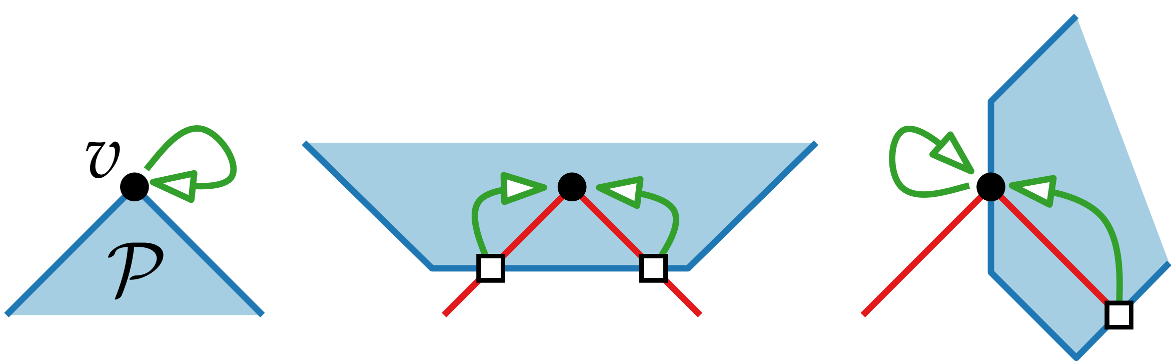

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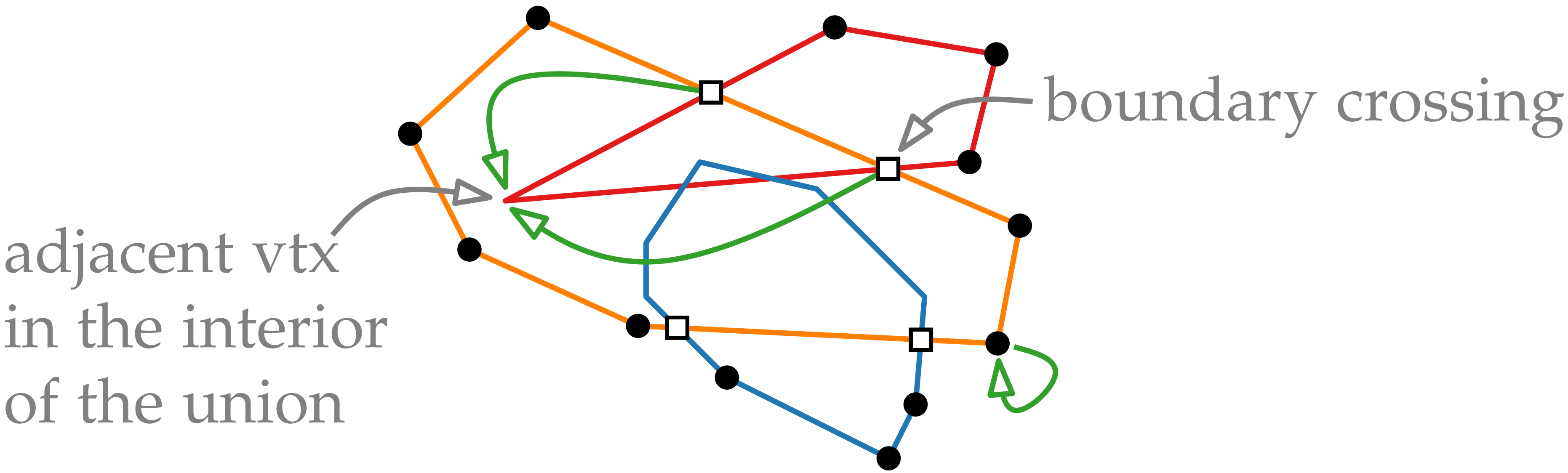

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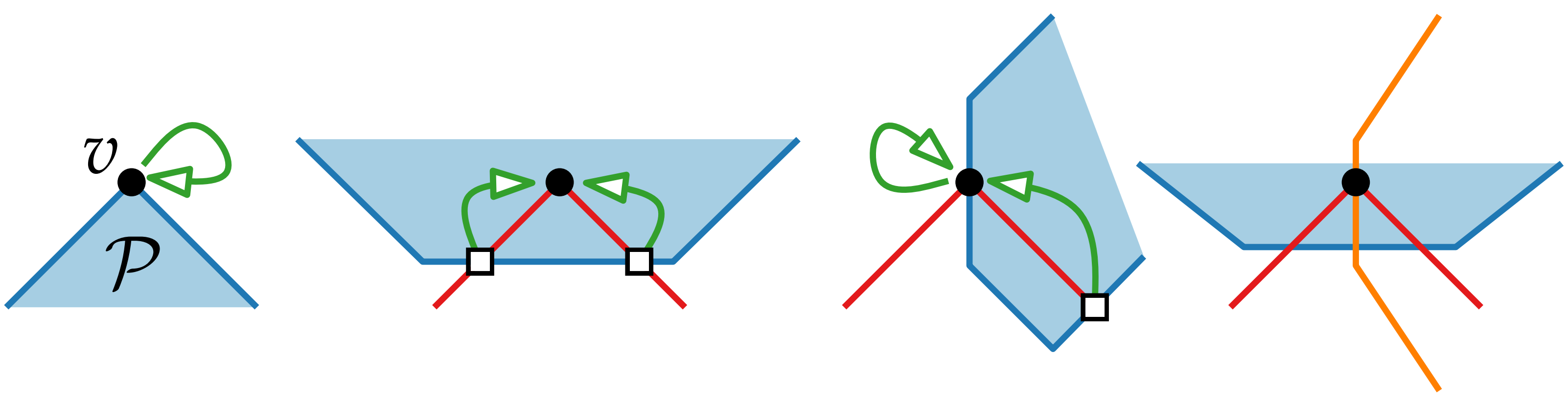

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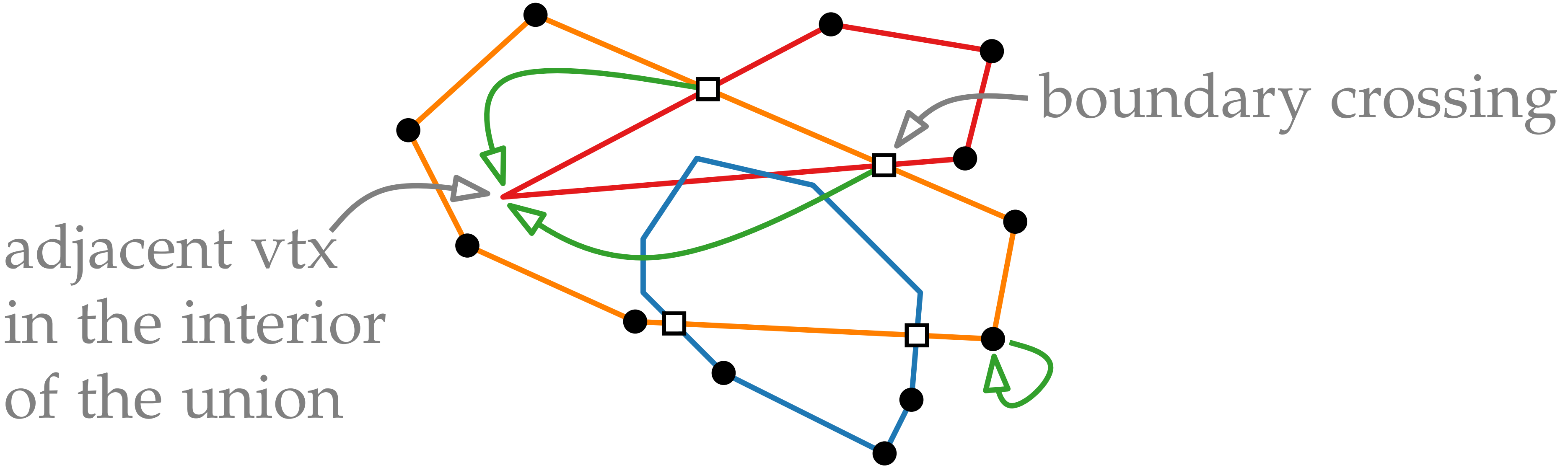

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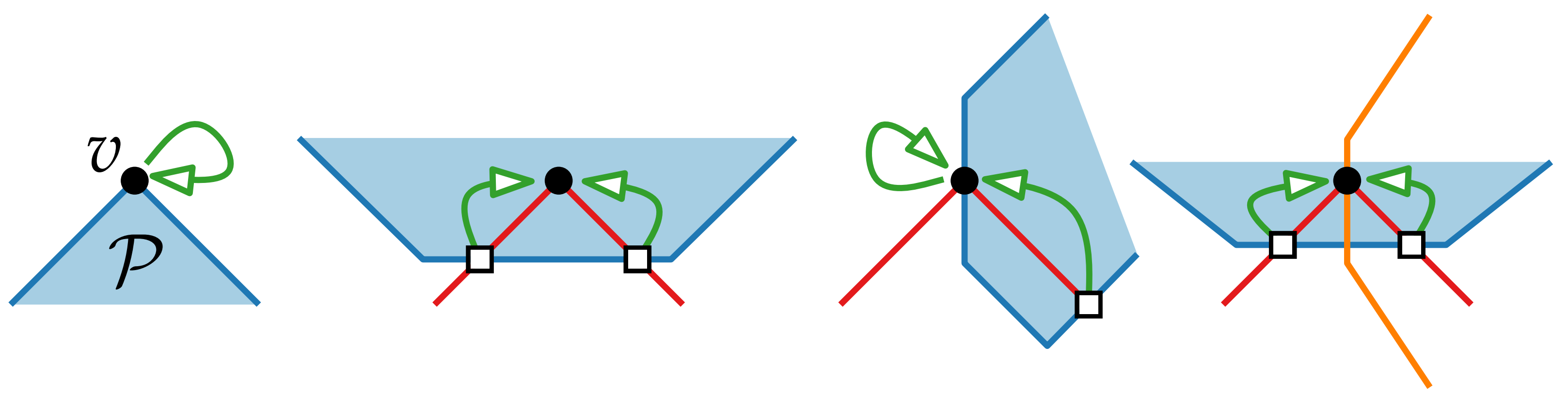

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