Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

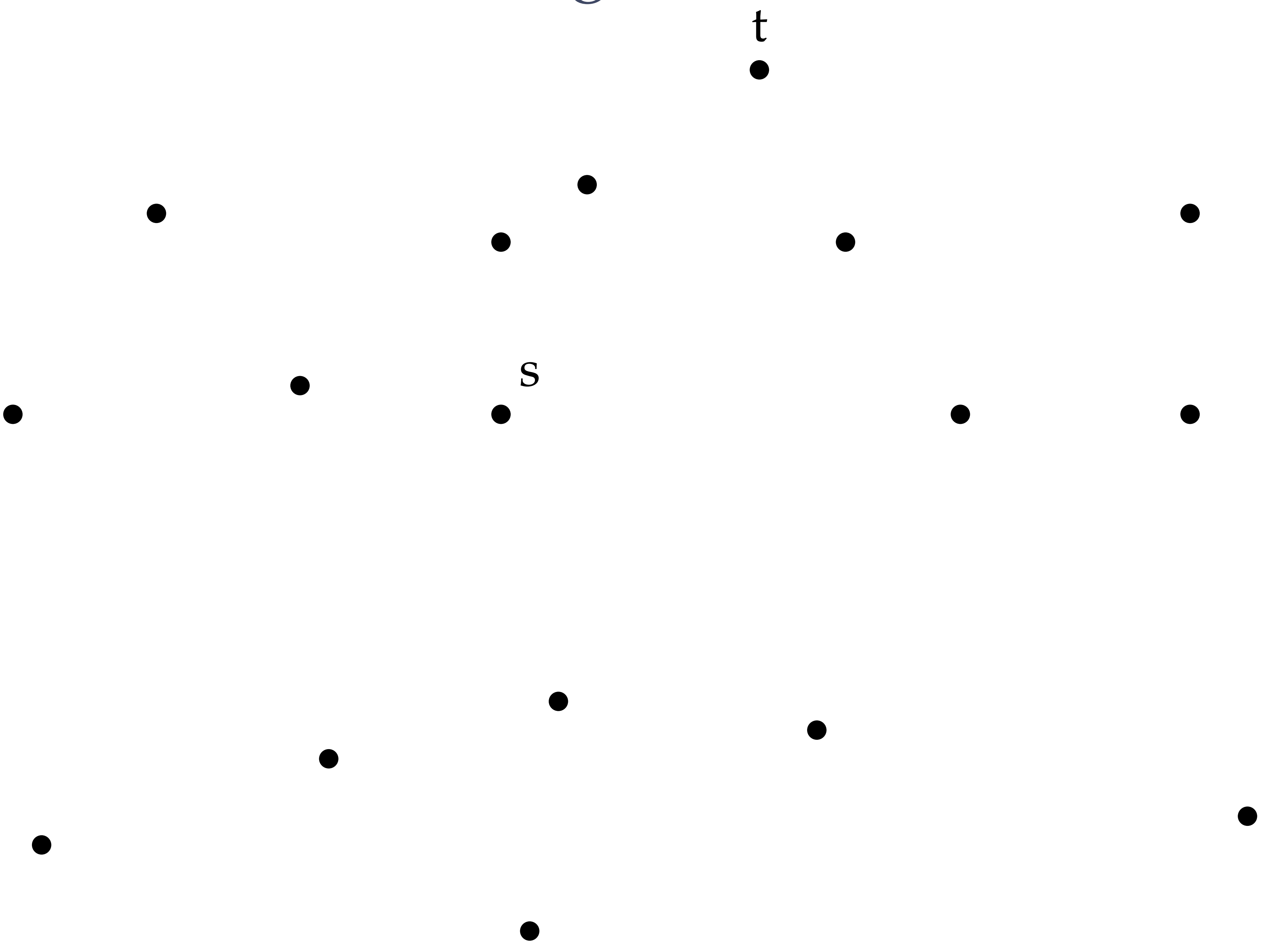
Computational Geometry

Convex Partition
or
Oblivious Routing

Thomas van Dijk

Winter Semester 2019/20

Oblivious routing



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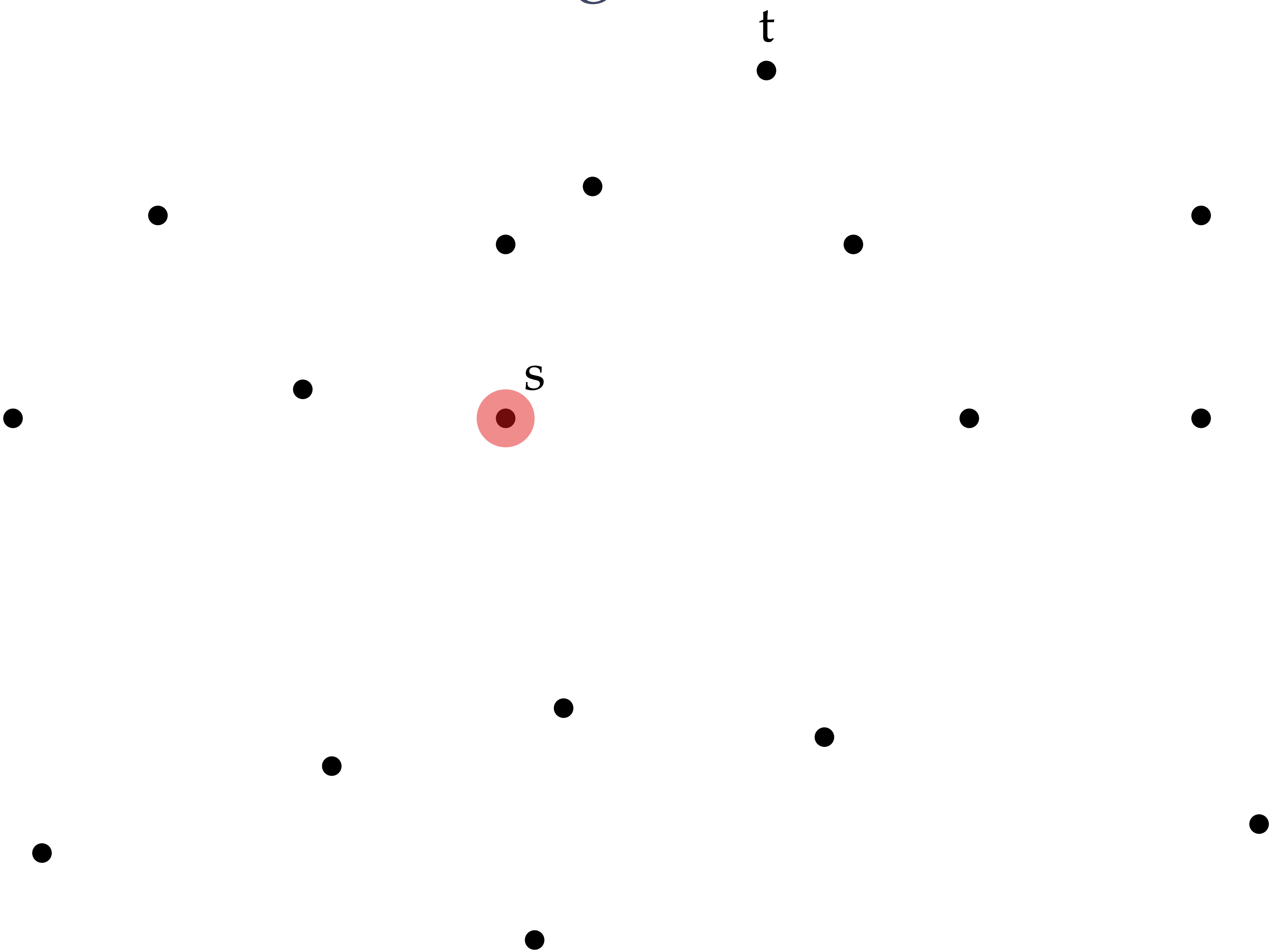

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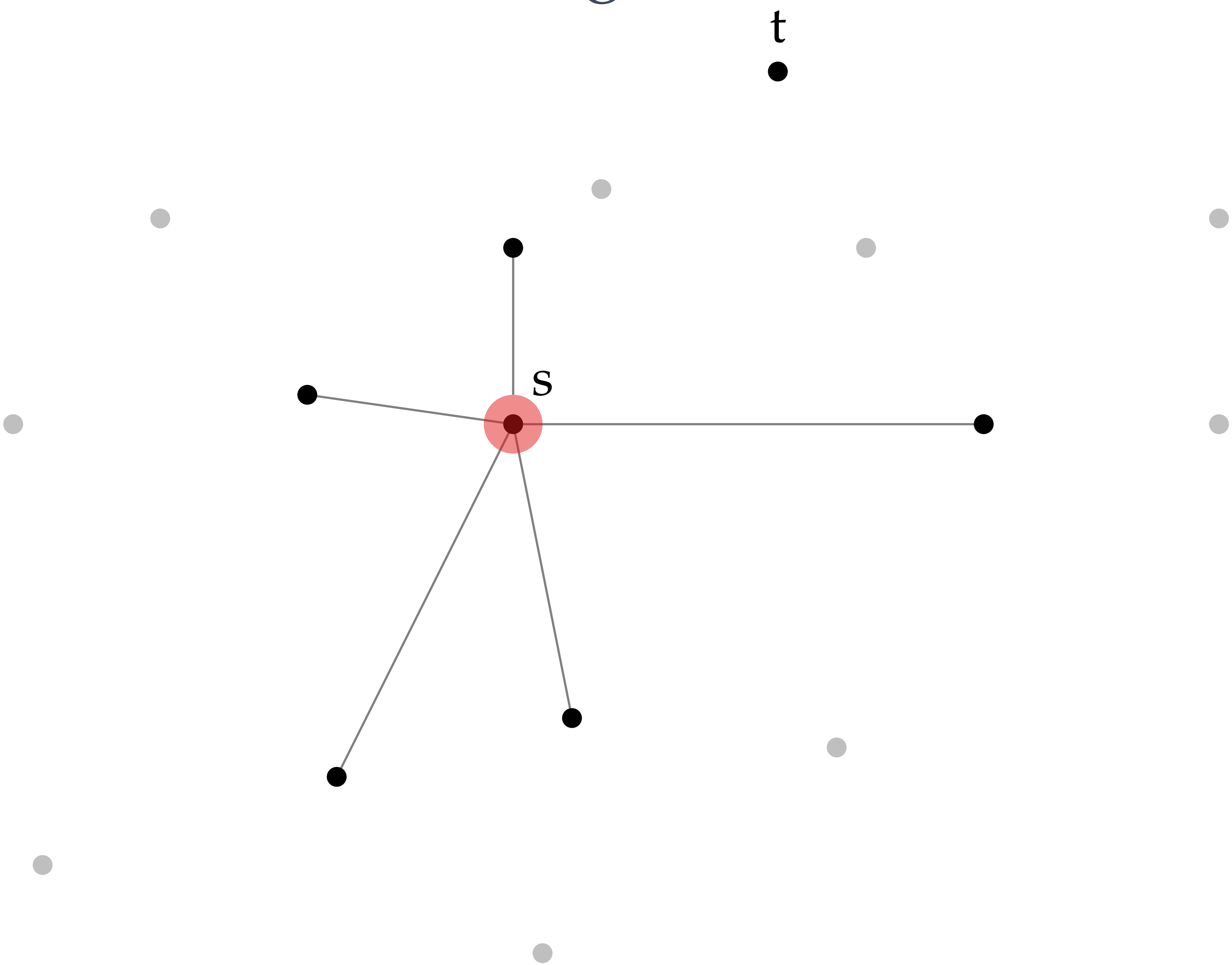

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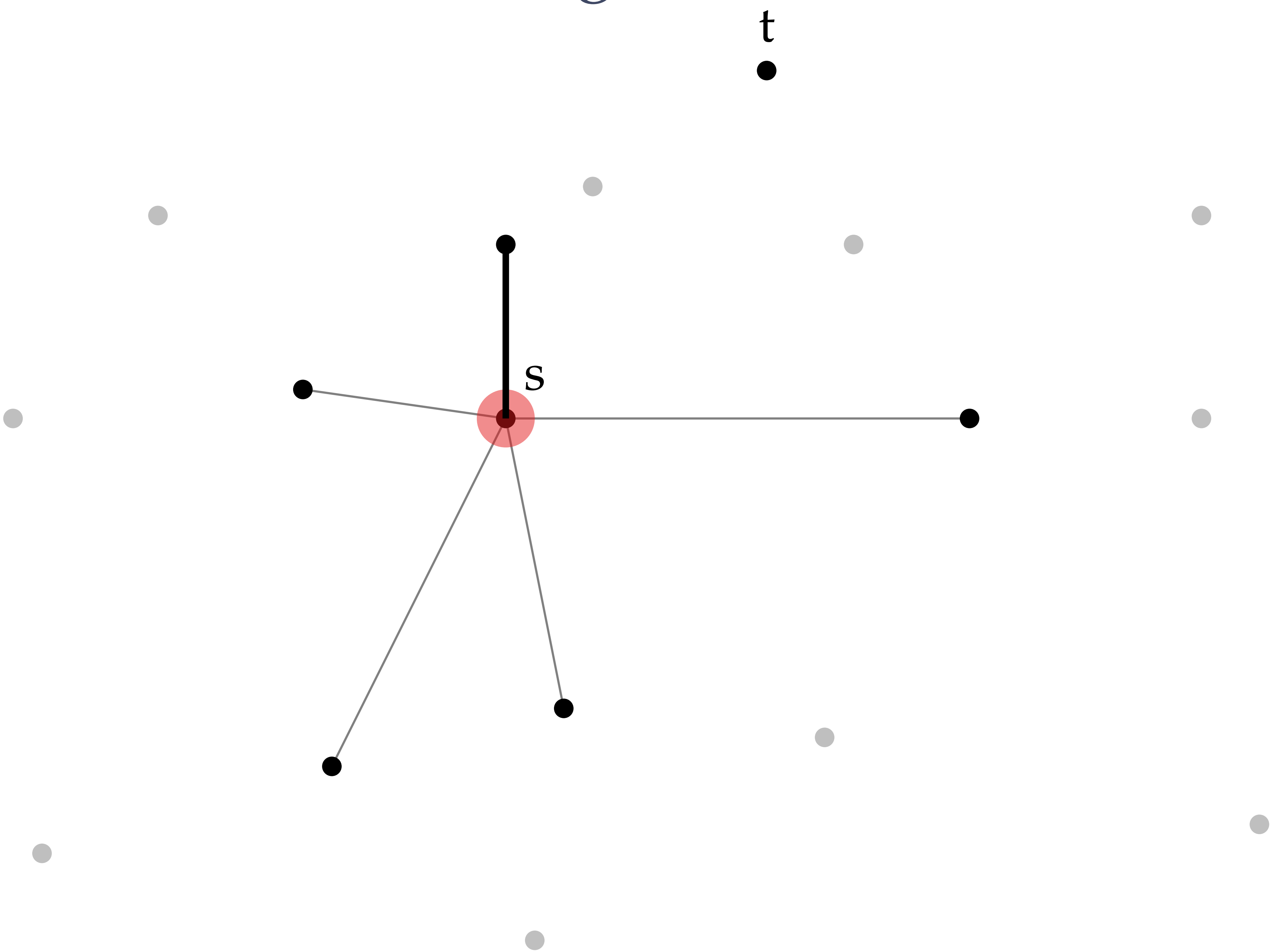

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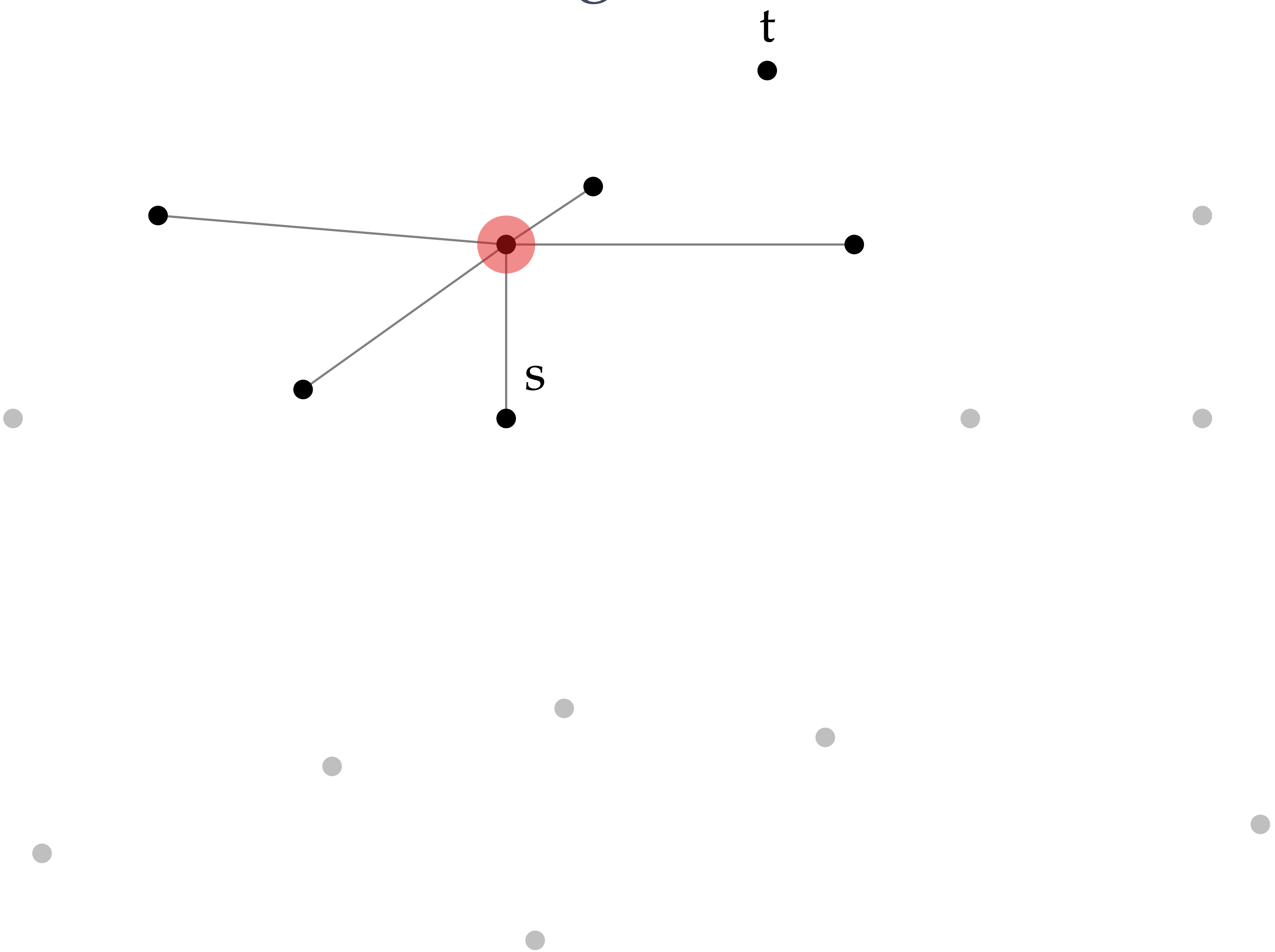

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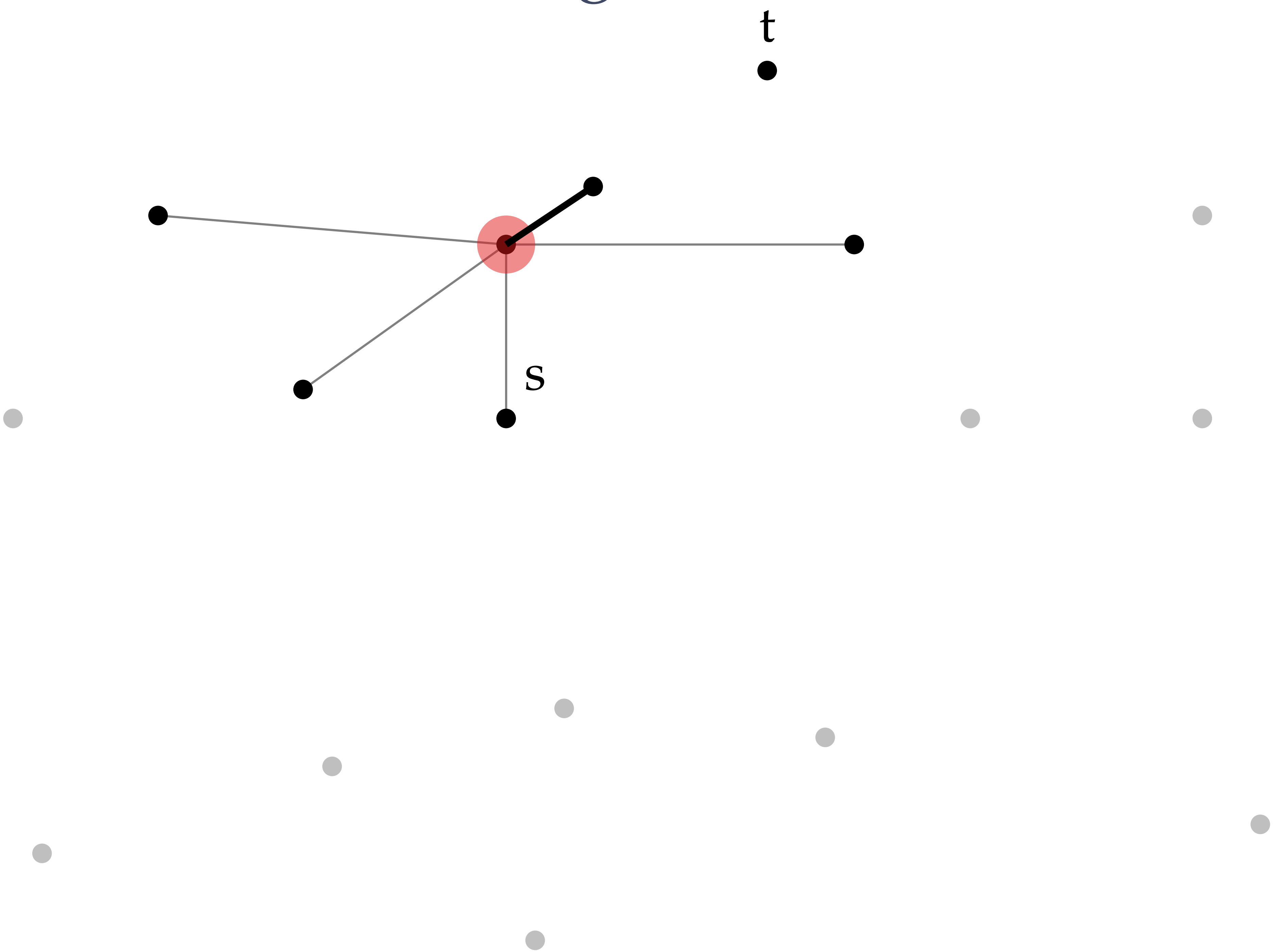

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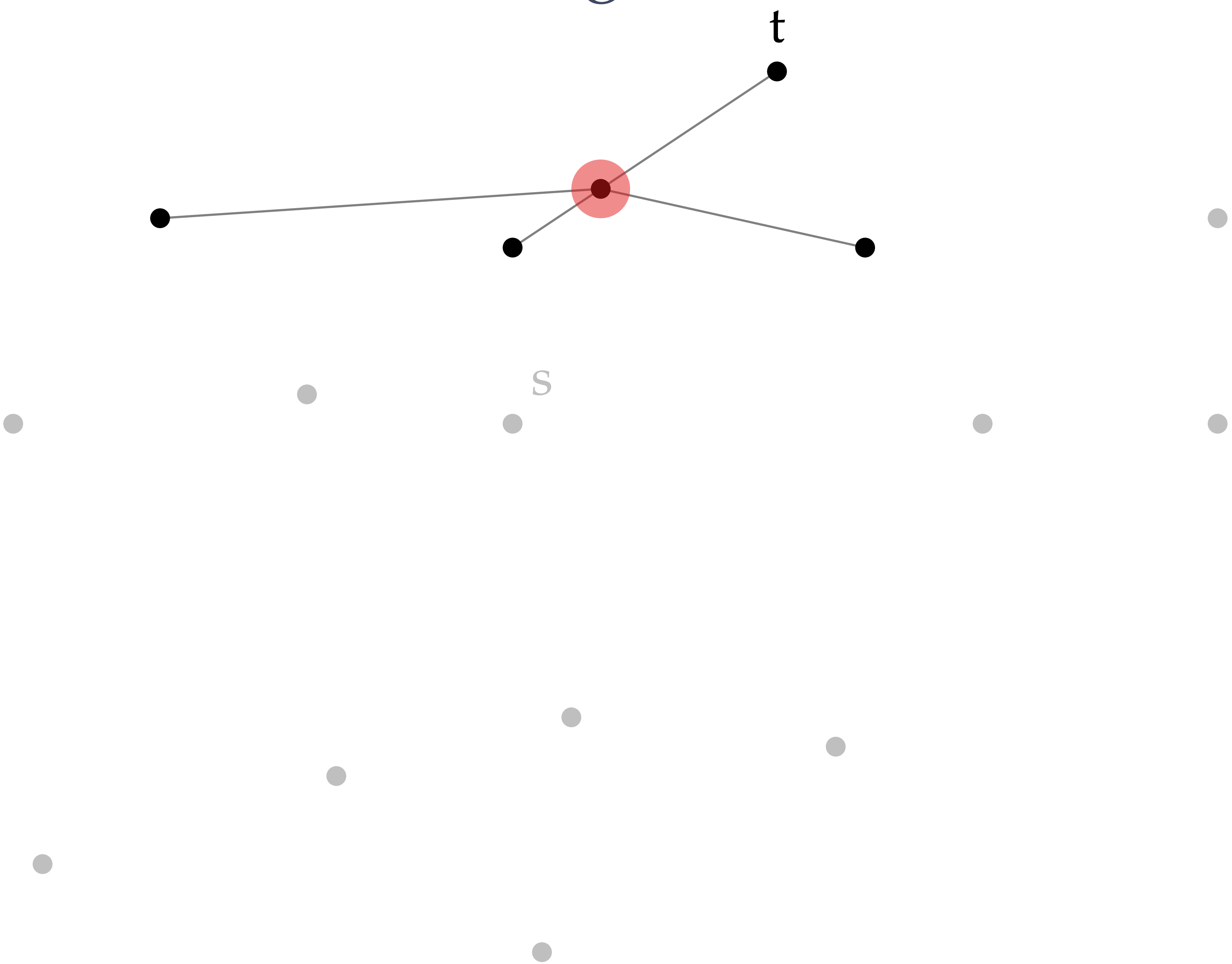

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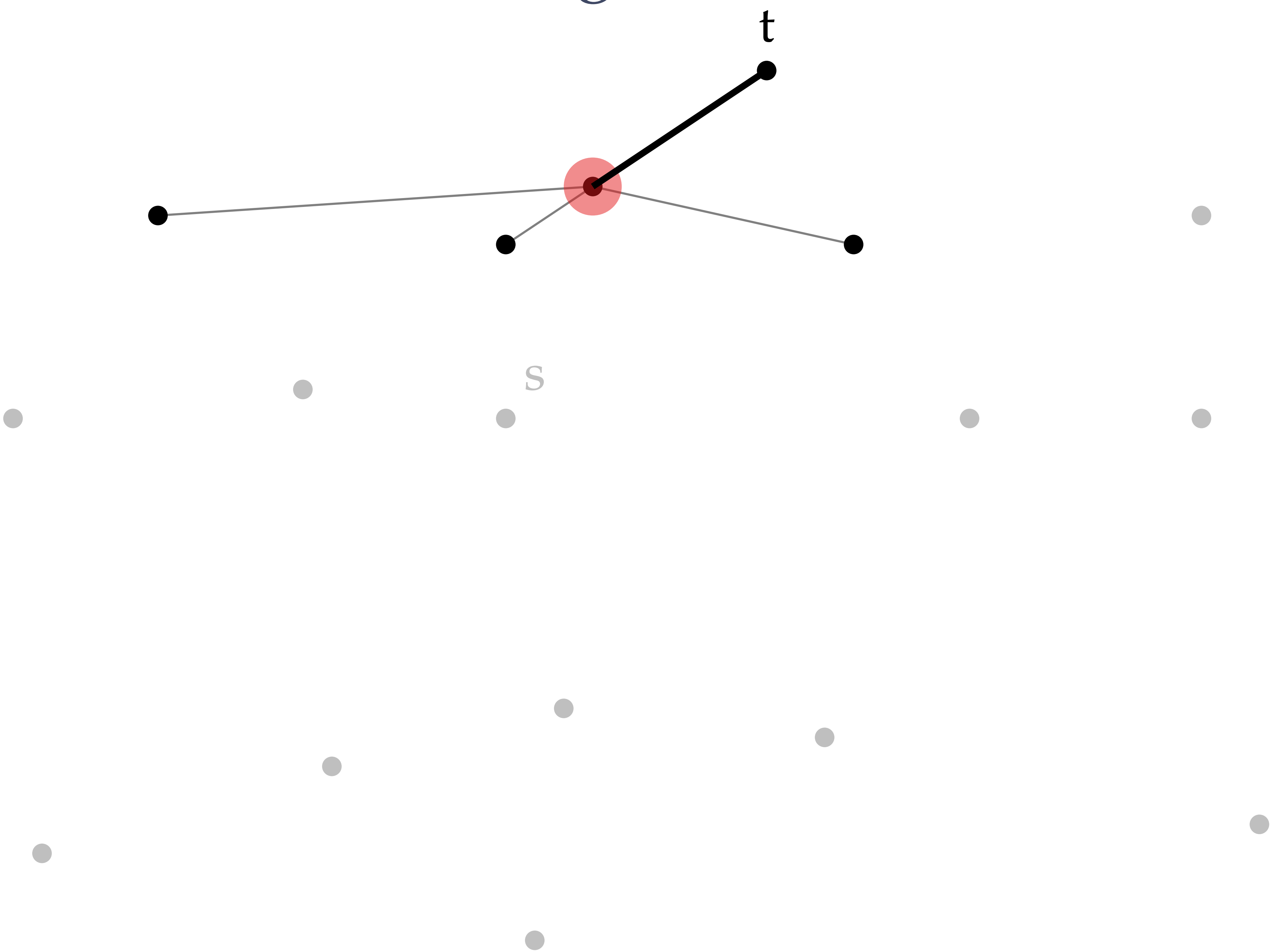

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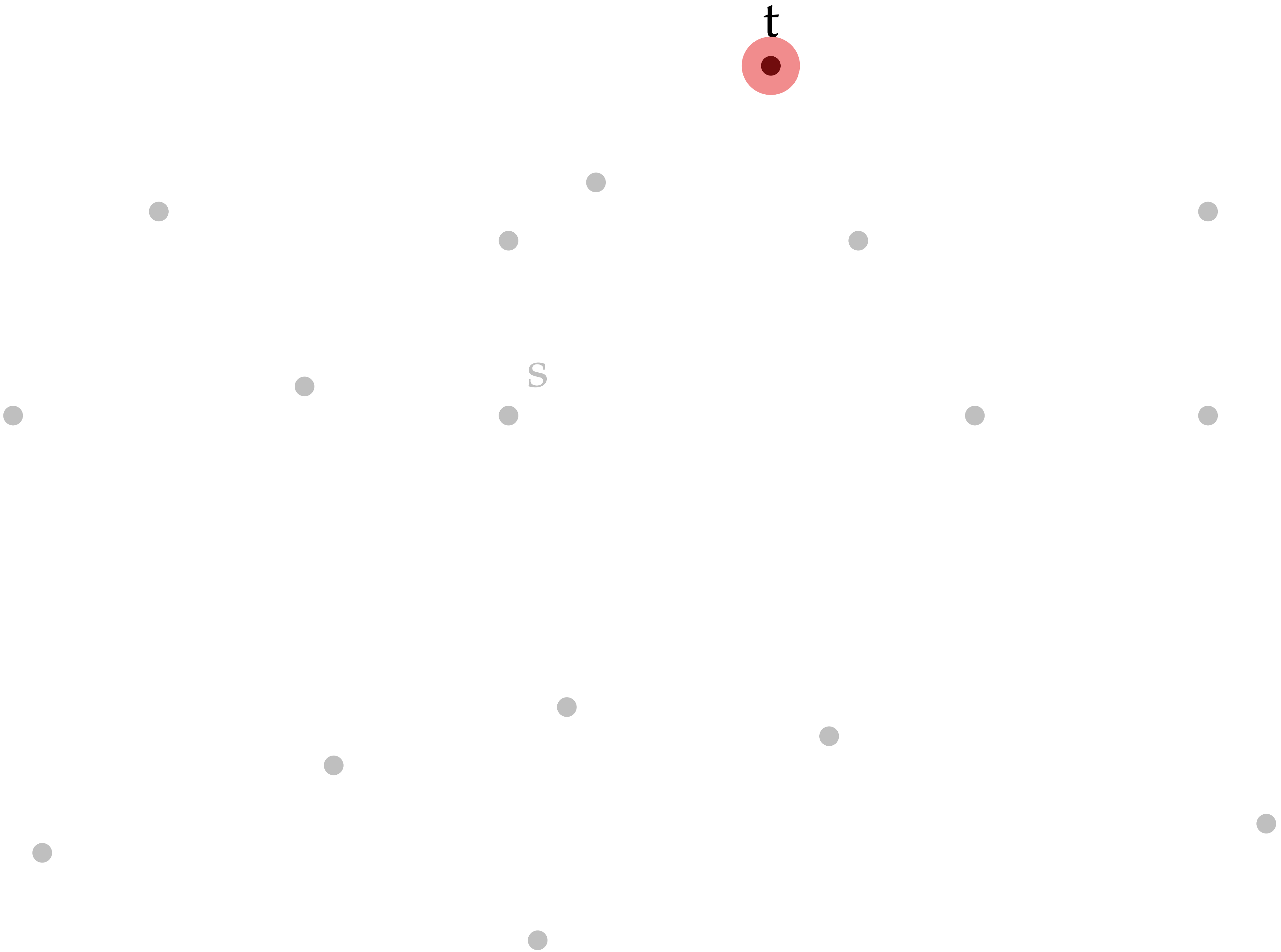

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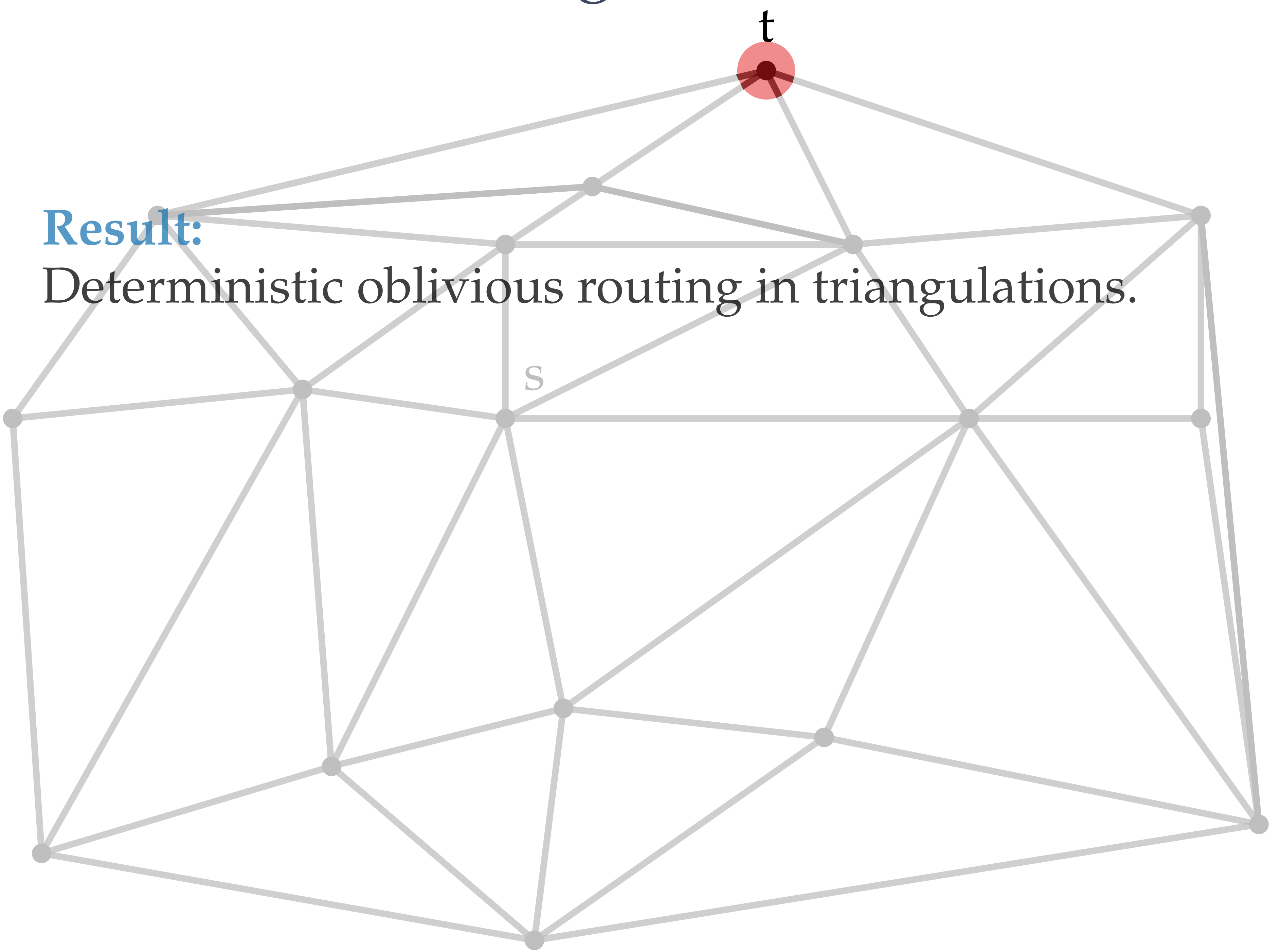
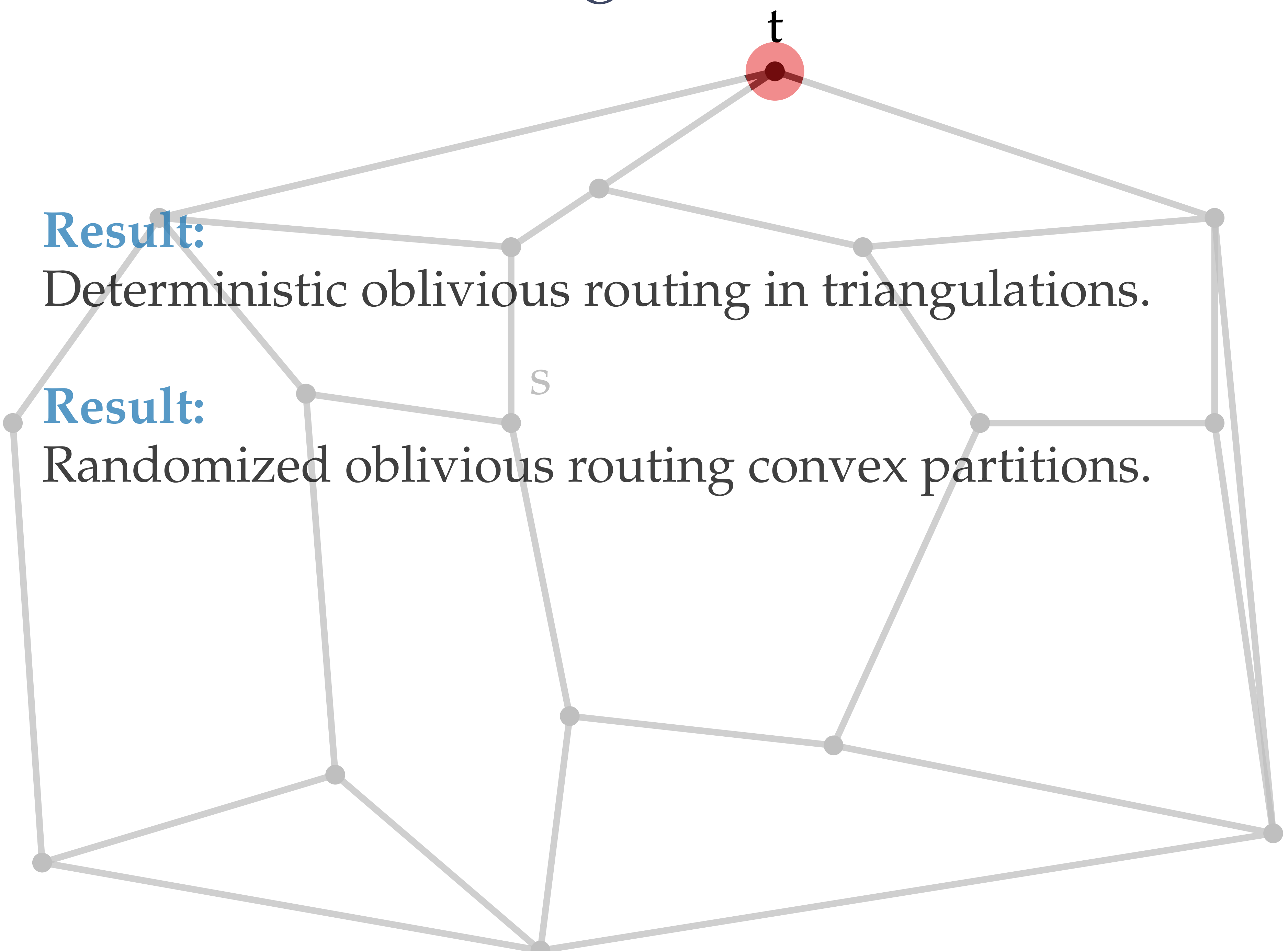


Result:

Deterministic oblivious routing in triangulations.

Oblivious routing

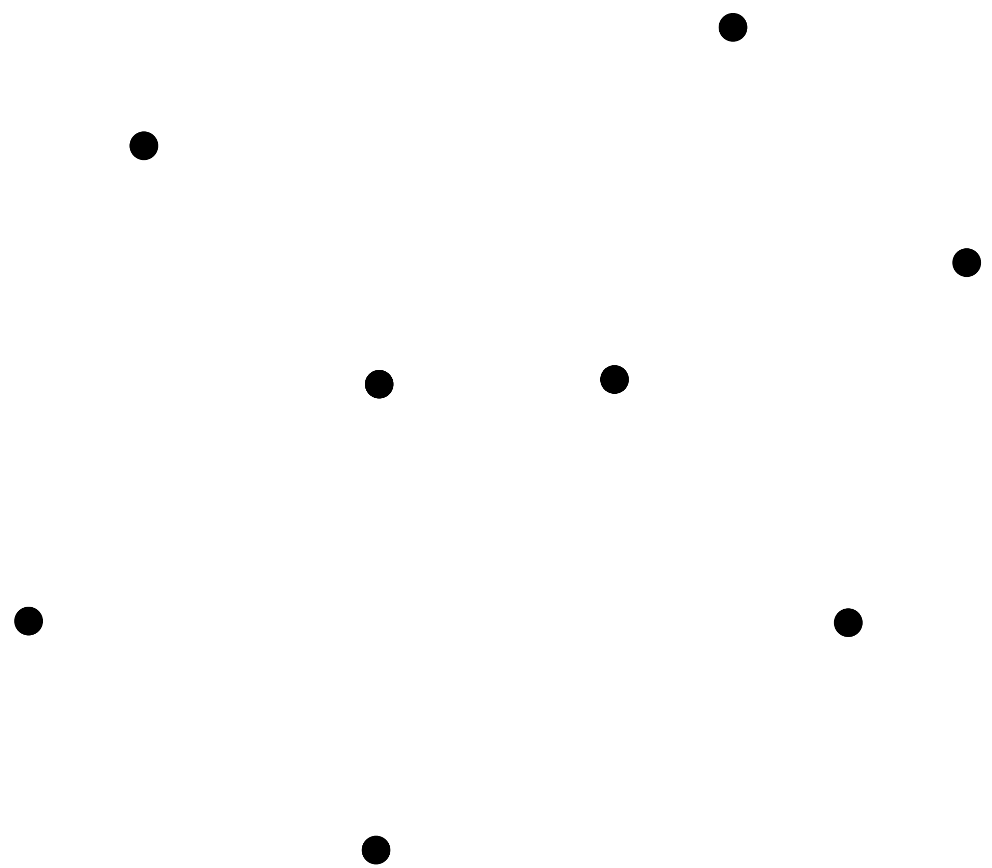


Result:
Deterministic oblivious routing in triangulations.

Result:
Randomized oblivious routing convex partitions.

Minimum Convex Partition of Points Sets

(Often: assume no colinear points.)



Point set P

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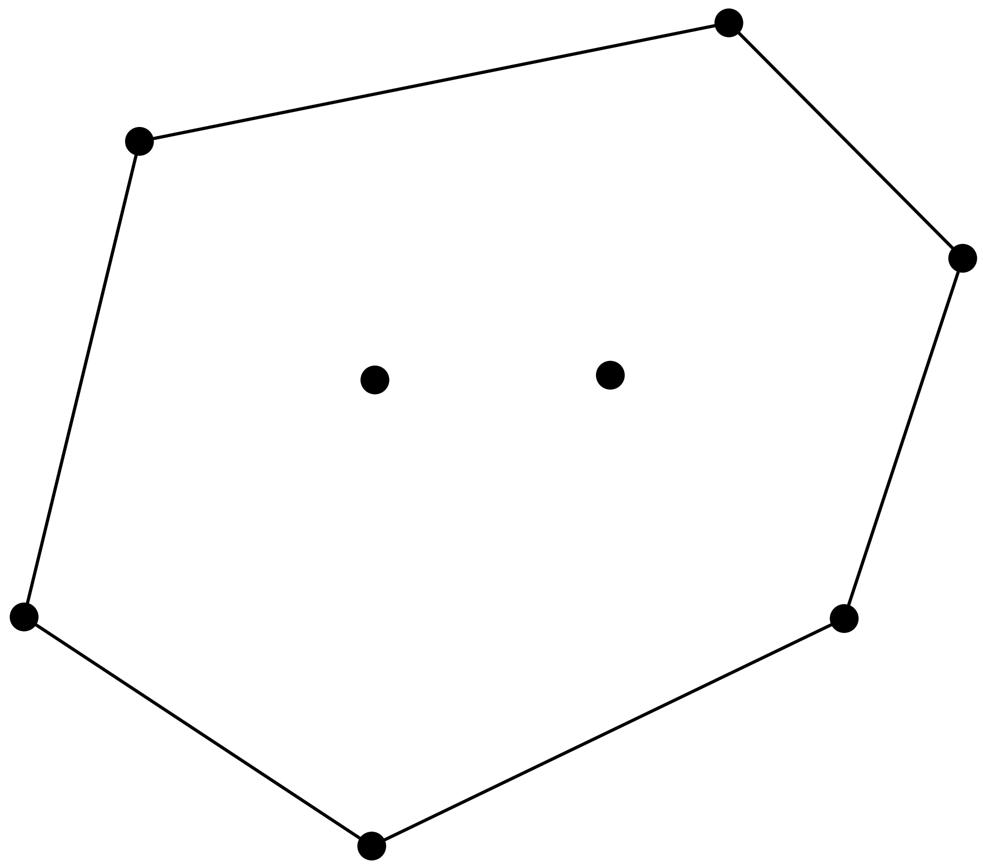

Convex hull $CH(P)$

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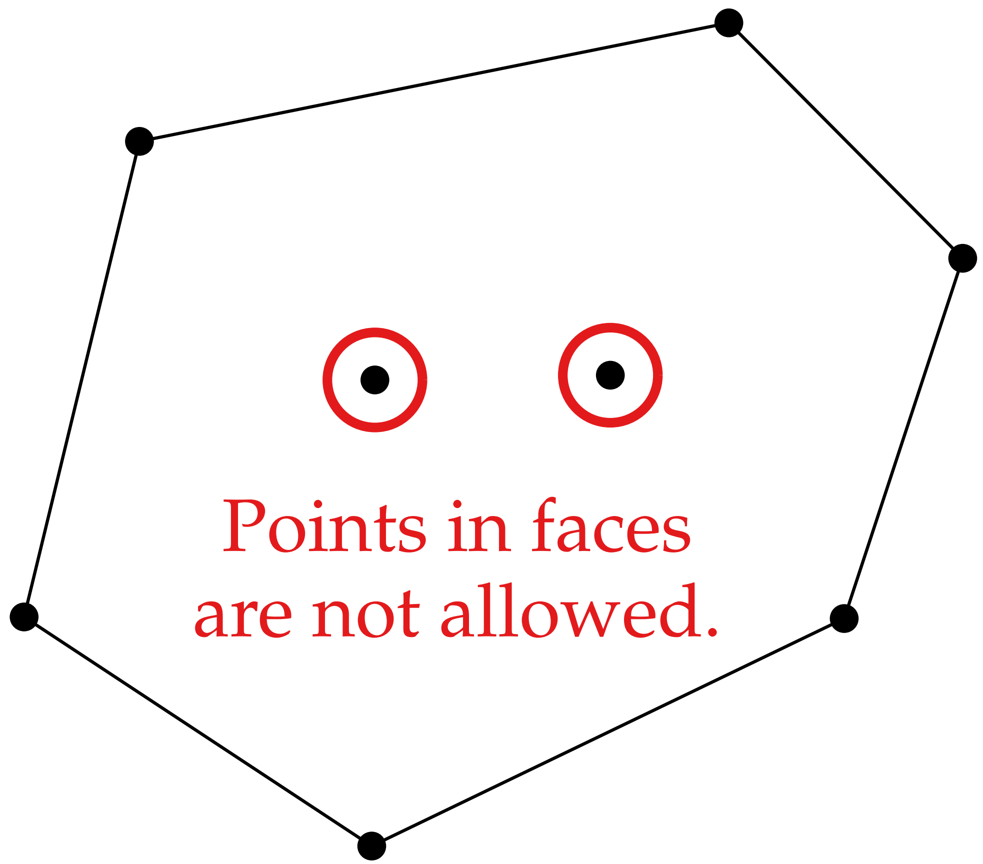

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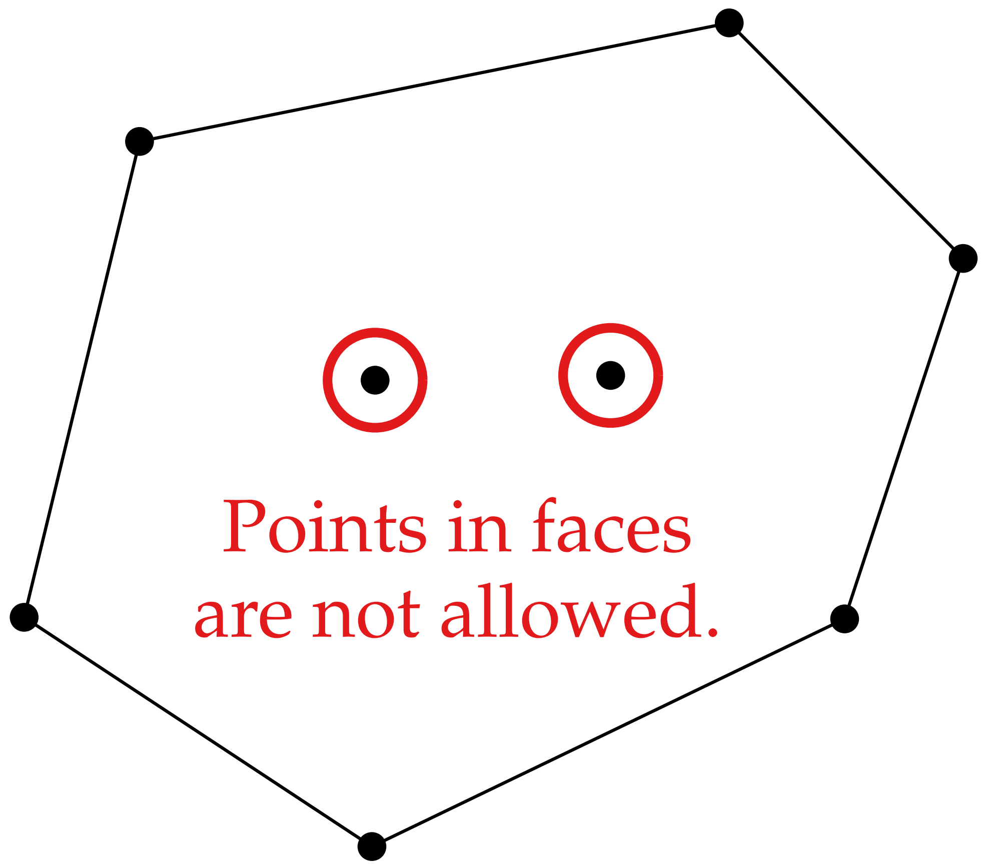

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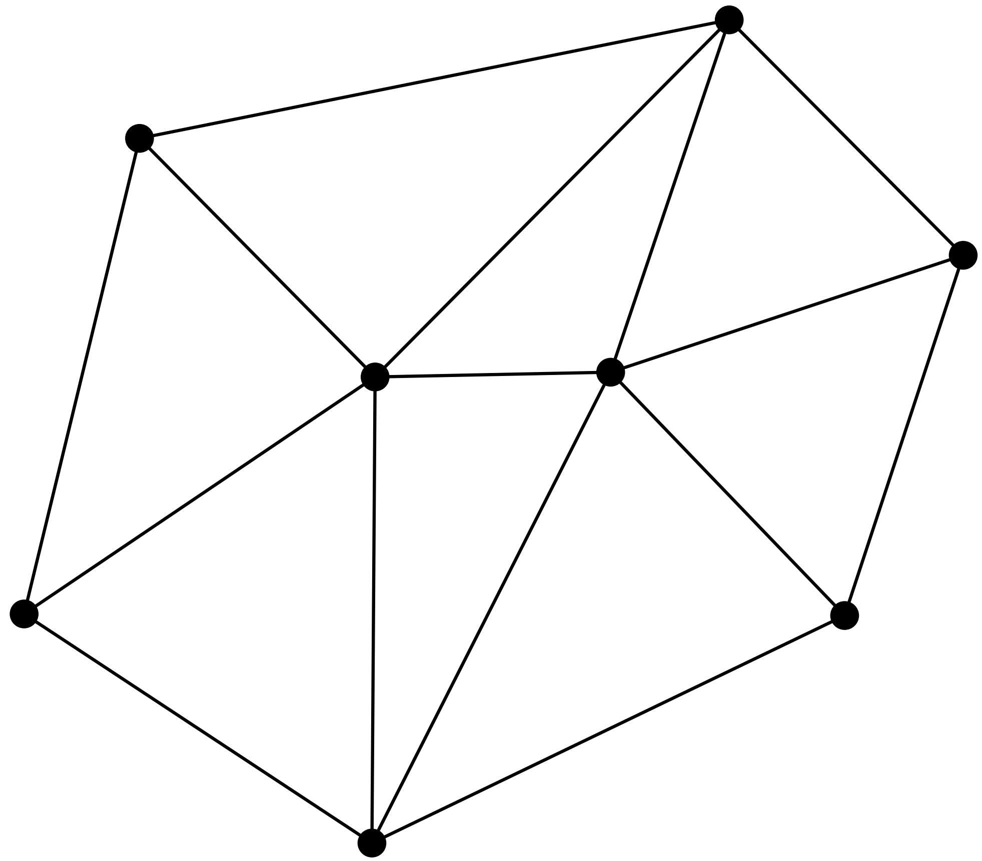

A triangulation of P:

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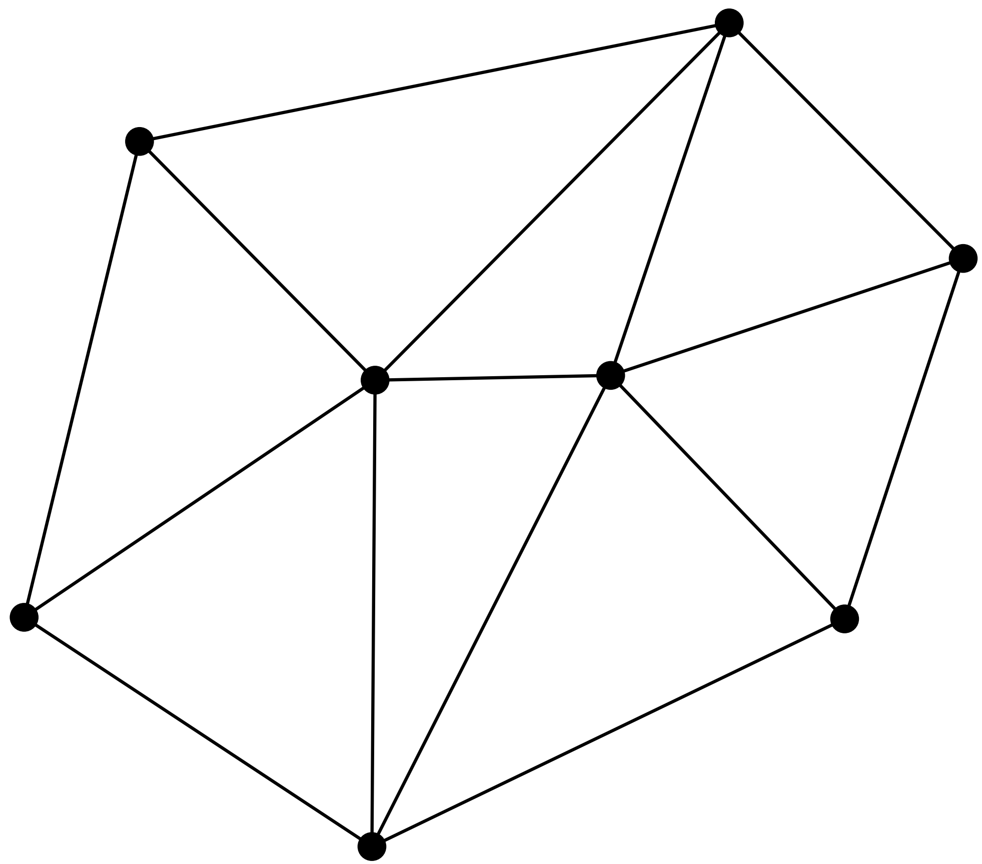

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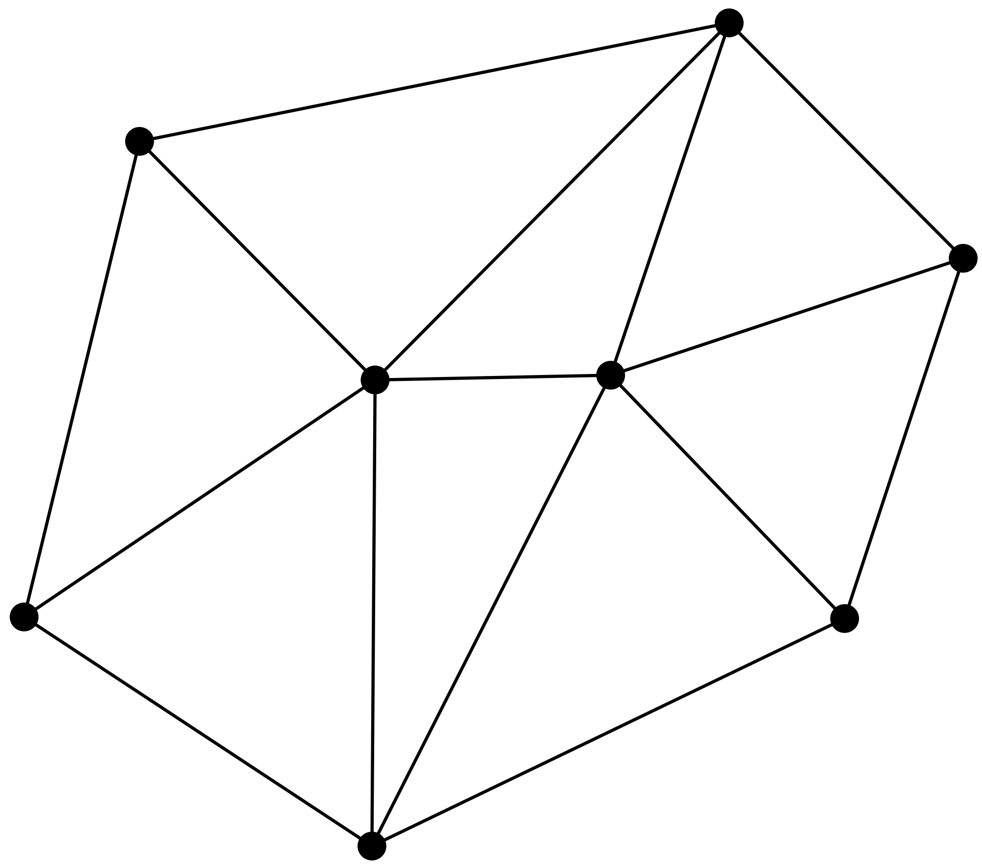

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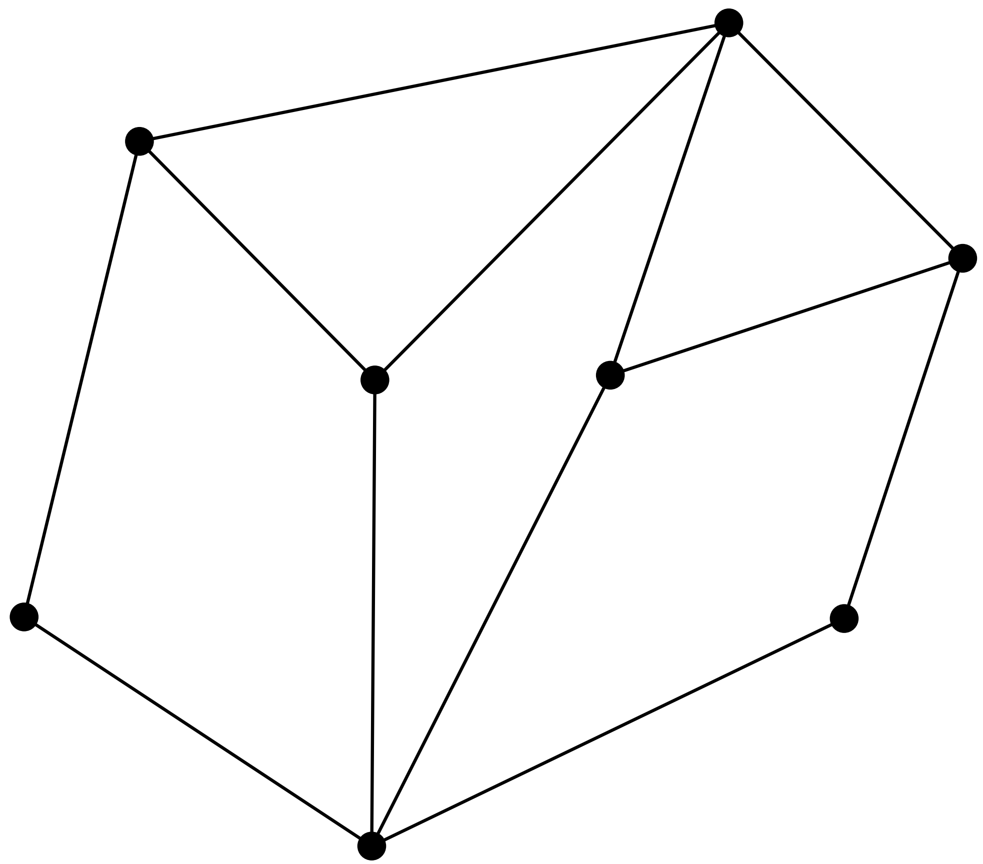

A minimal convex partition: 5 faces

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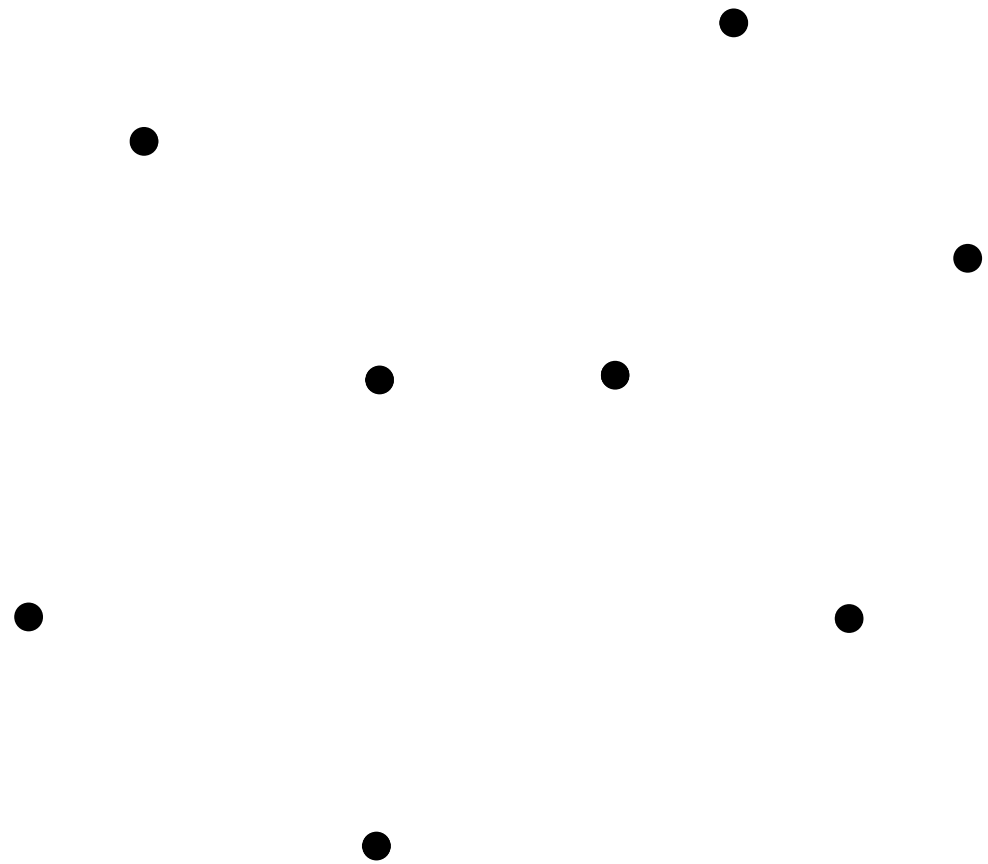

Minimum number of faces?

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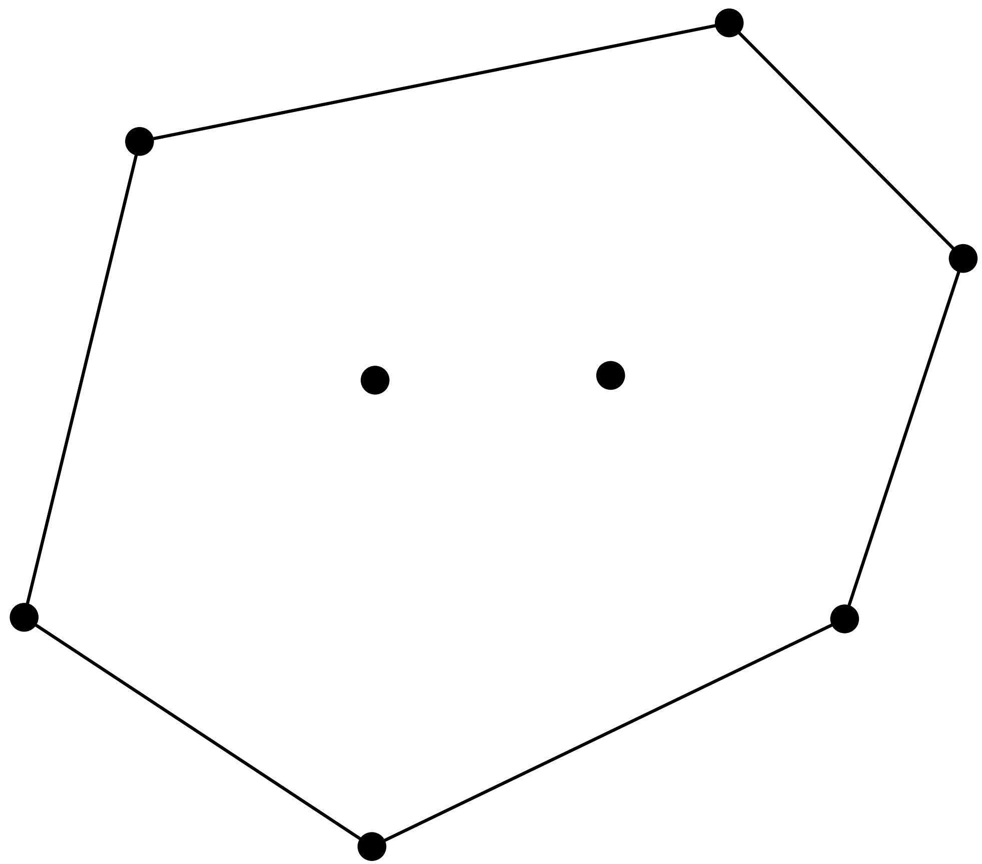

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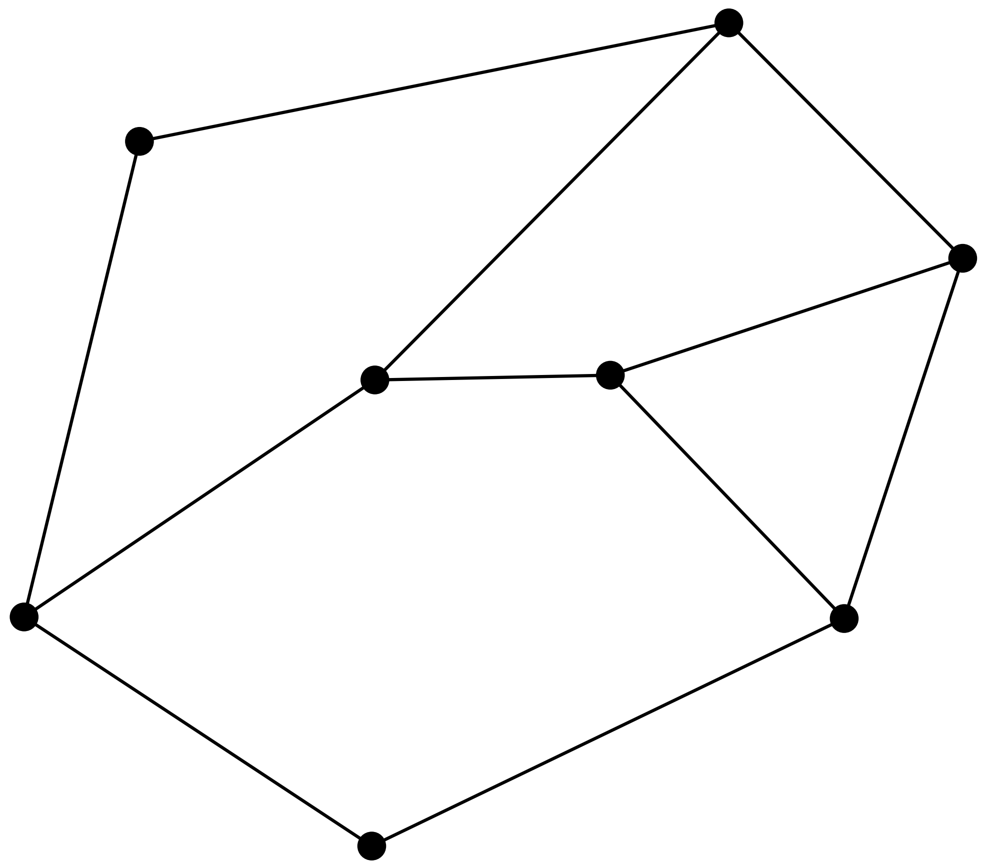

Minimum number of faces? 4 faces

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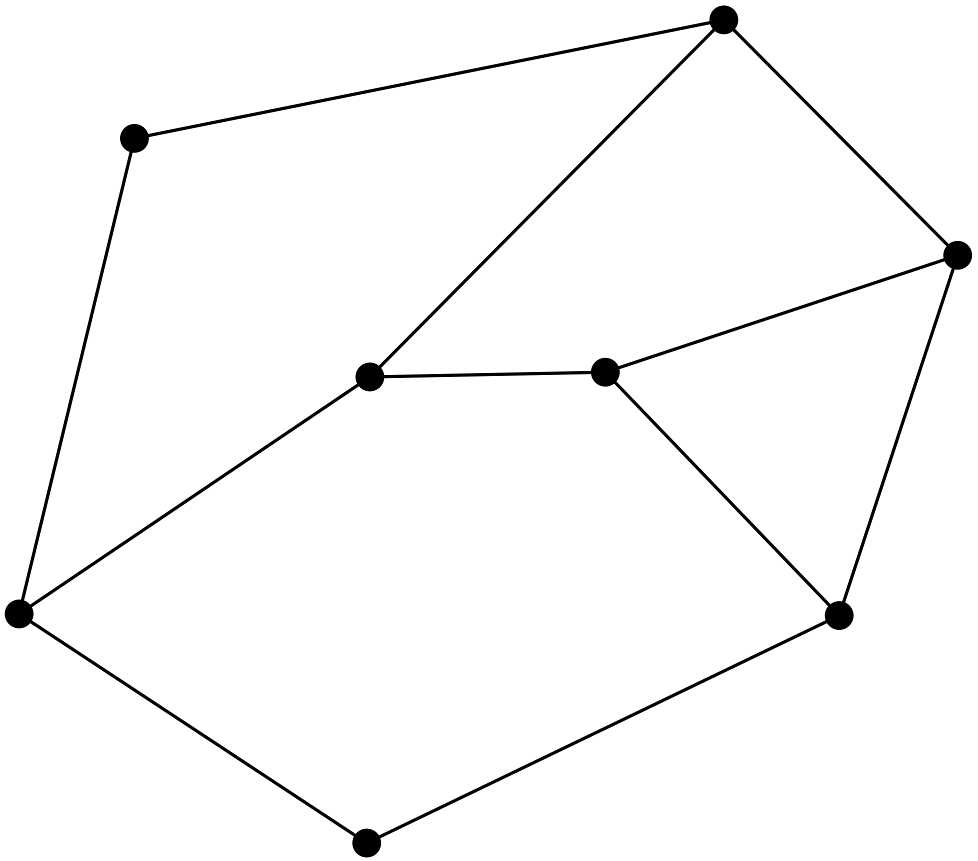

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This lecture

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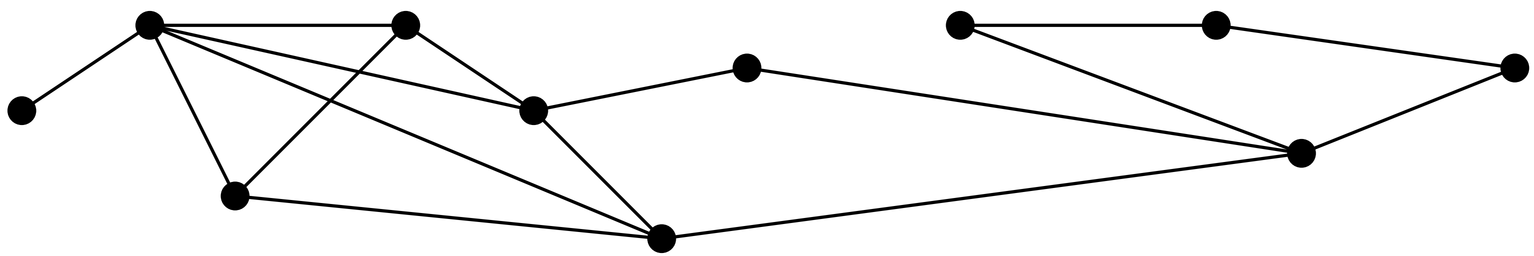

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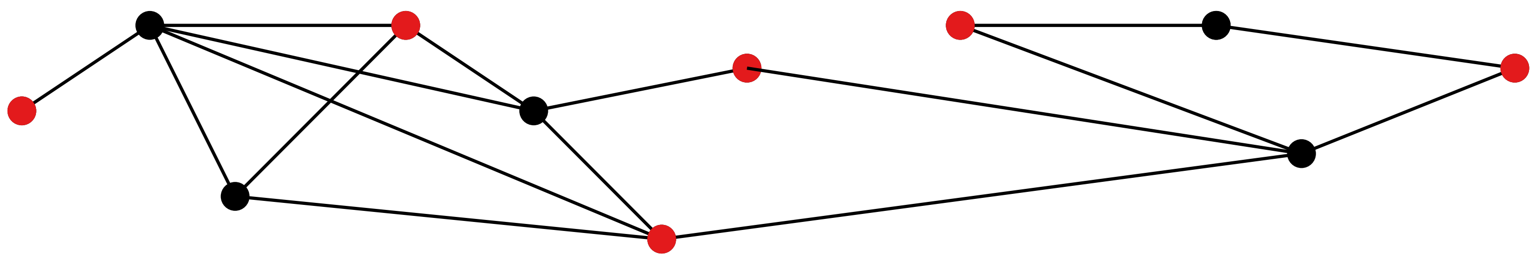

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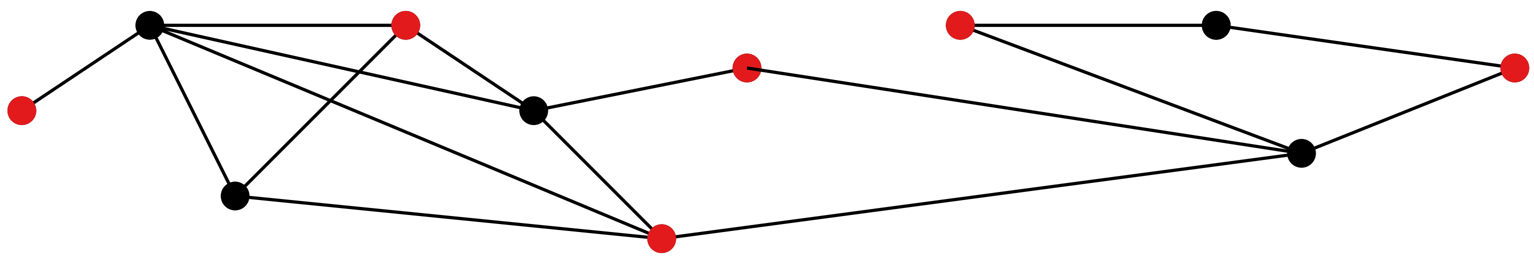

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$x_v \in \mathbb{Z}, \quad 0 \leq x_v \leq 1$ MAX INDEPENDENT SET

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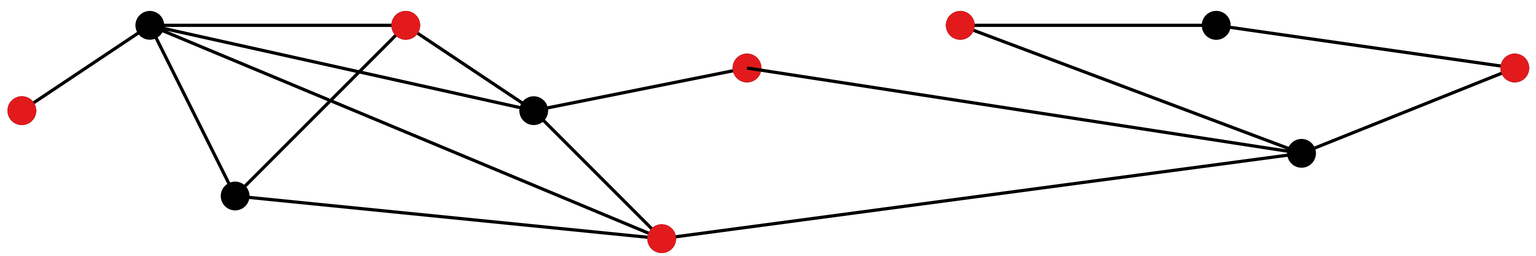

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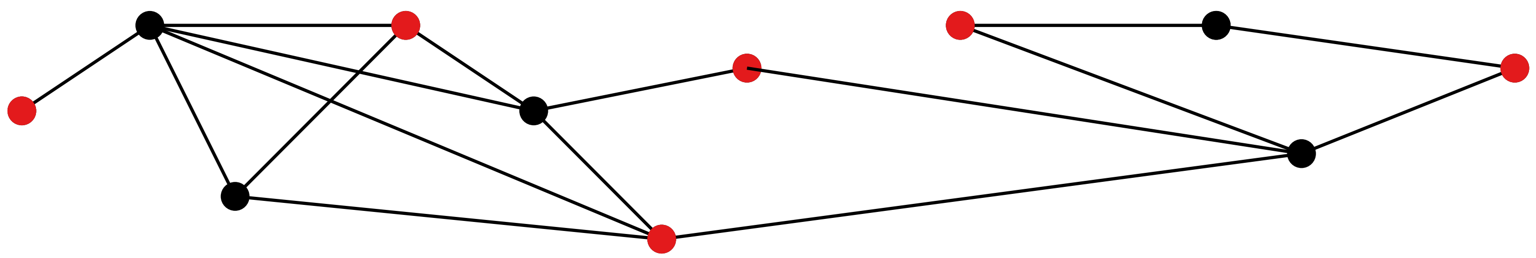

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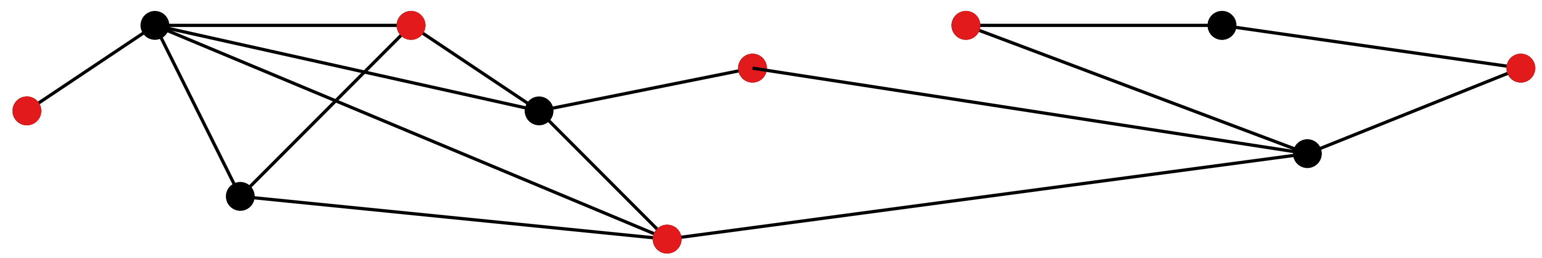

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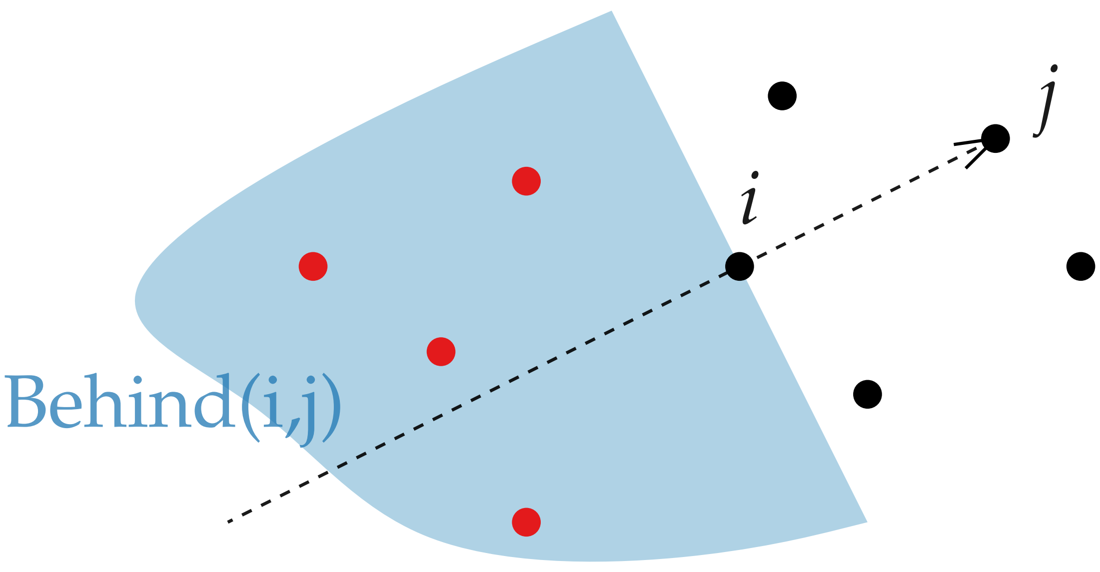

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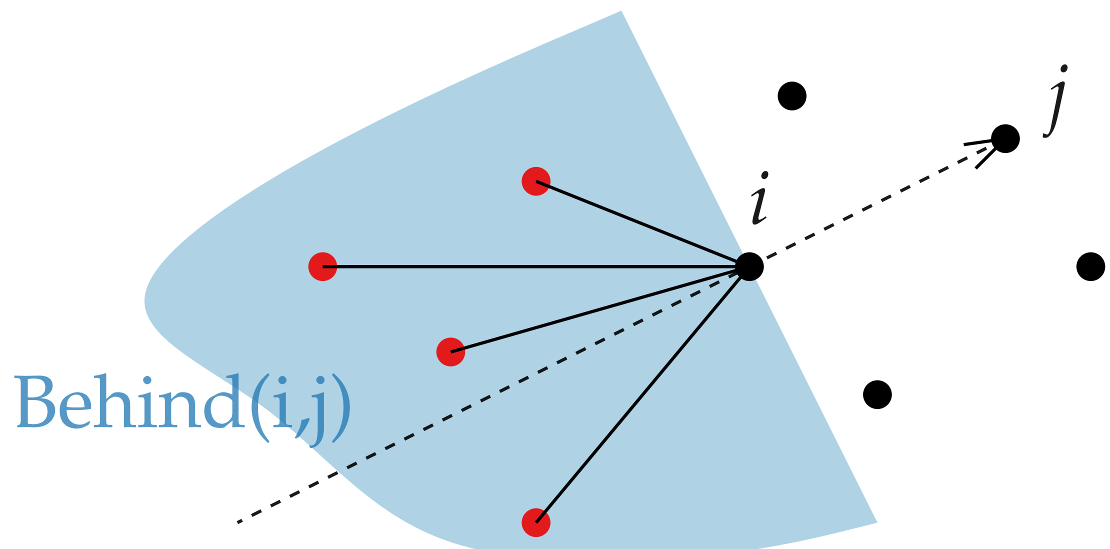

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3-Approximation

Theorem A 3-approximation of a minimum convex partition of P can be computed in $O(n \log n)$ time. (This assumes no colinear points.)

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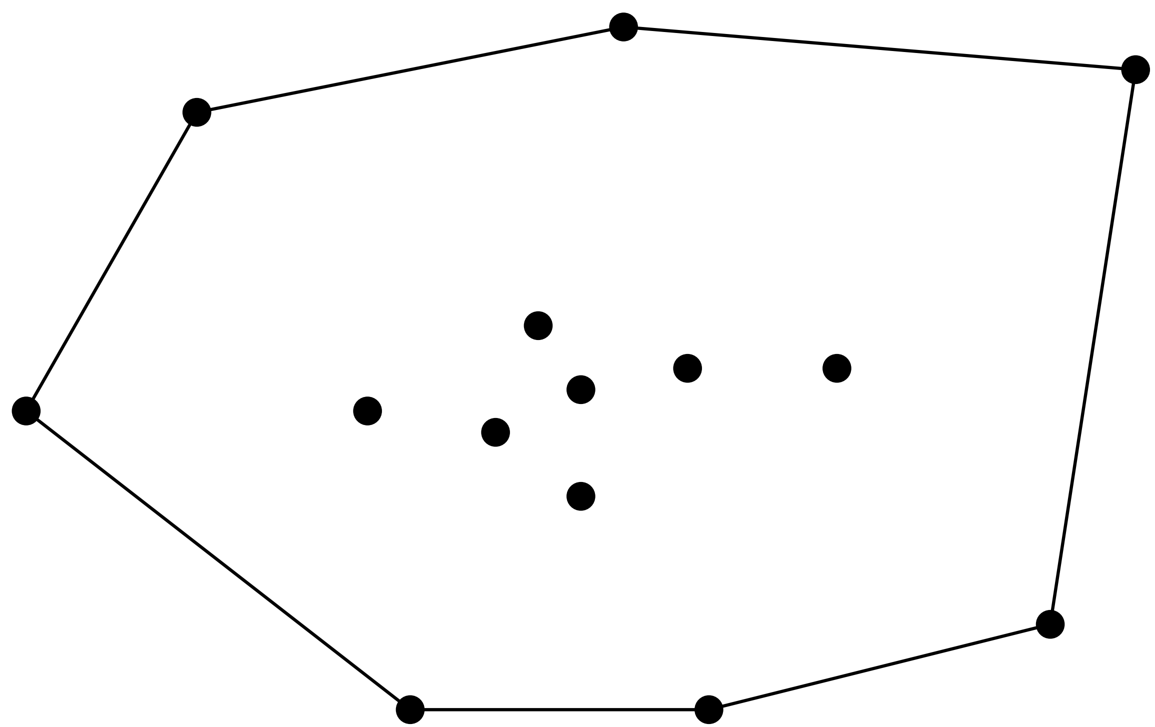
A lowerbound

Let $k \geq 3$ be number of interior points; $n - k$ outer points.



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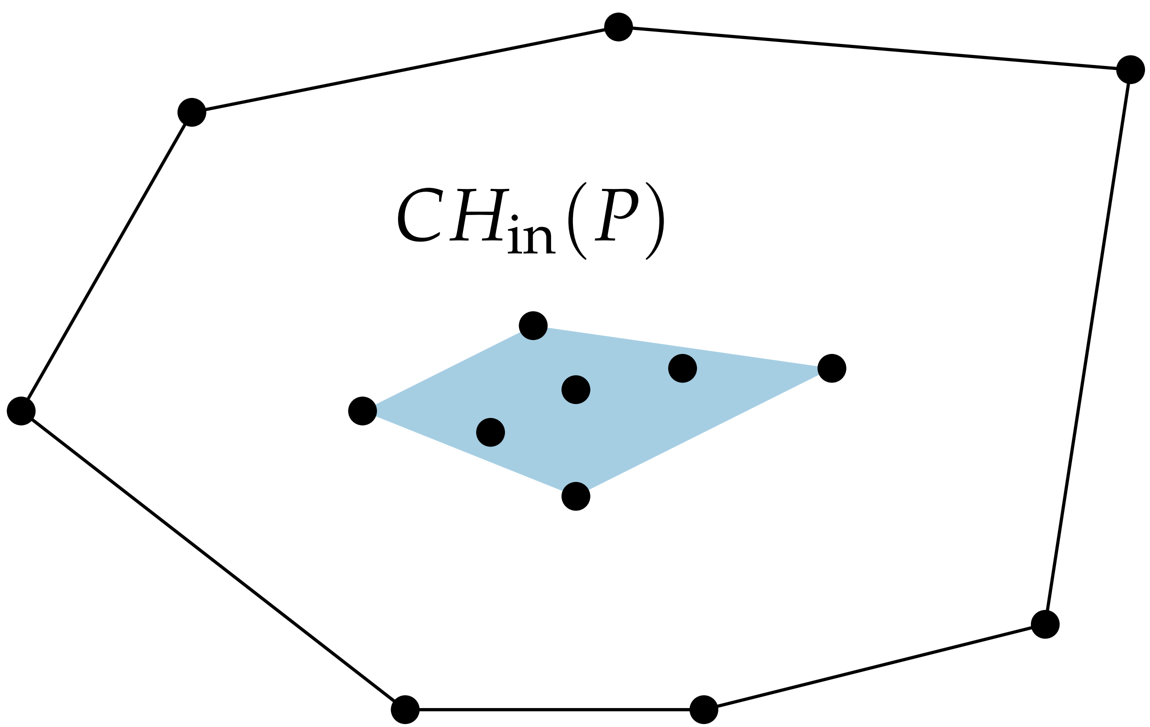

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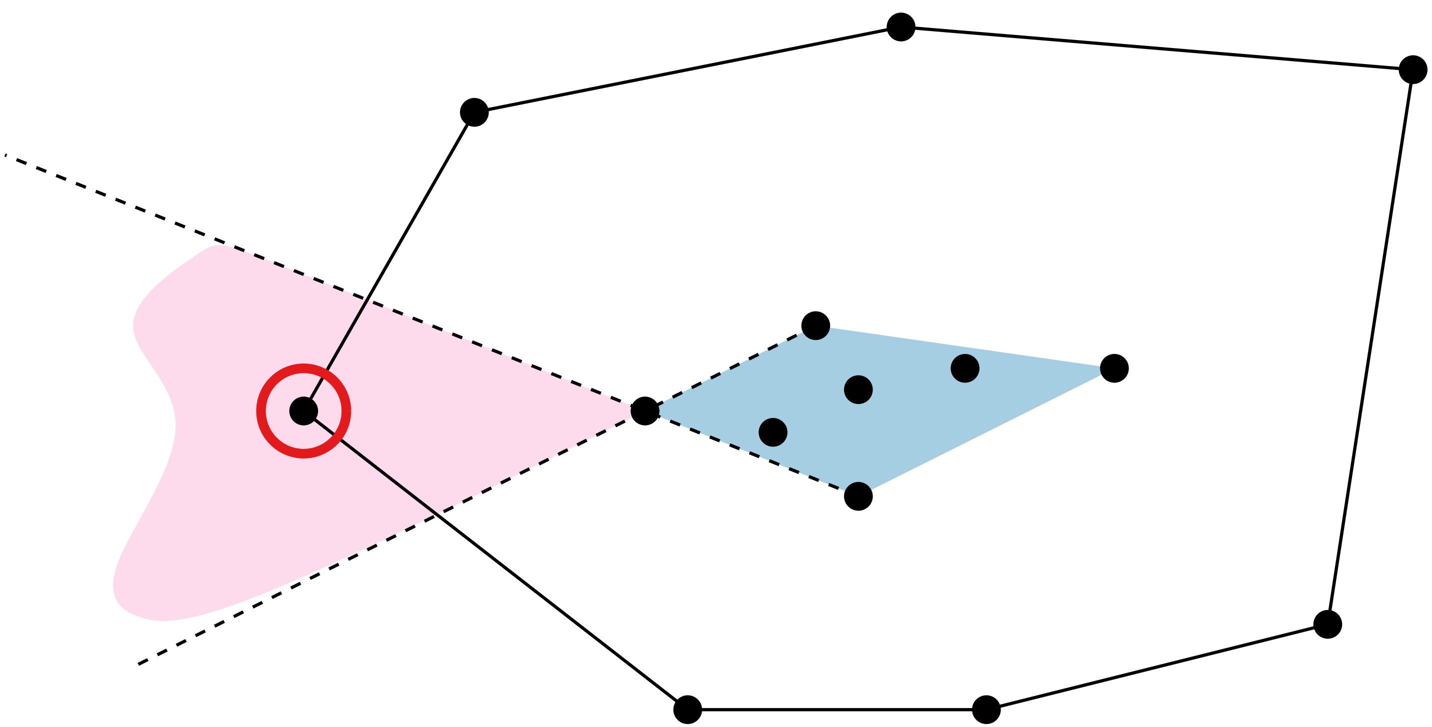

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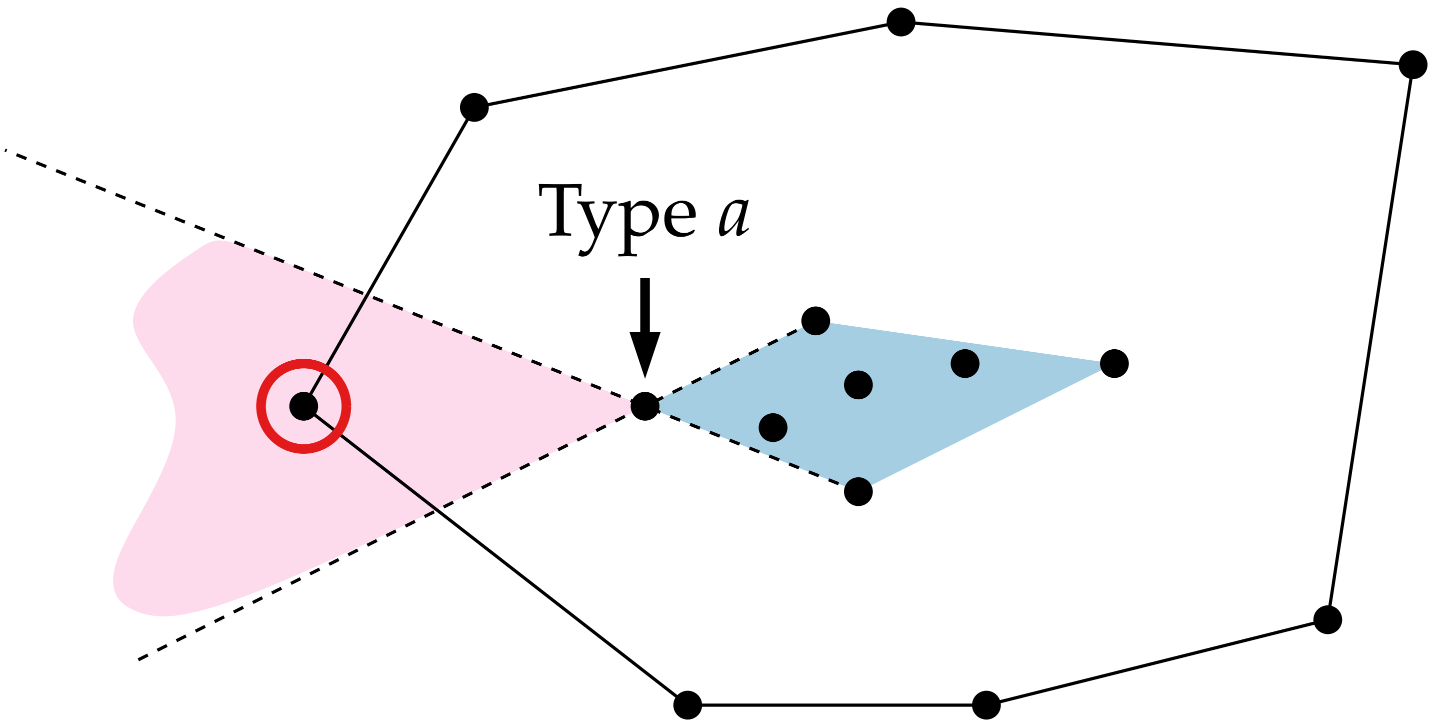

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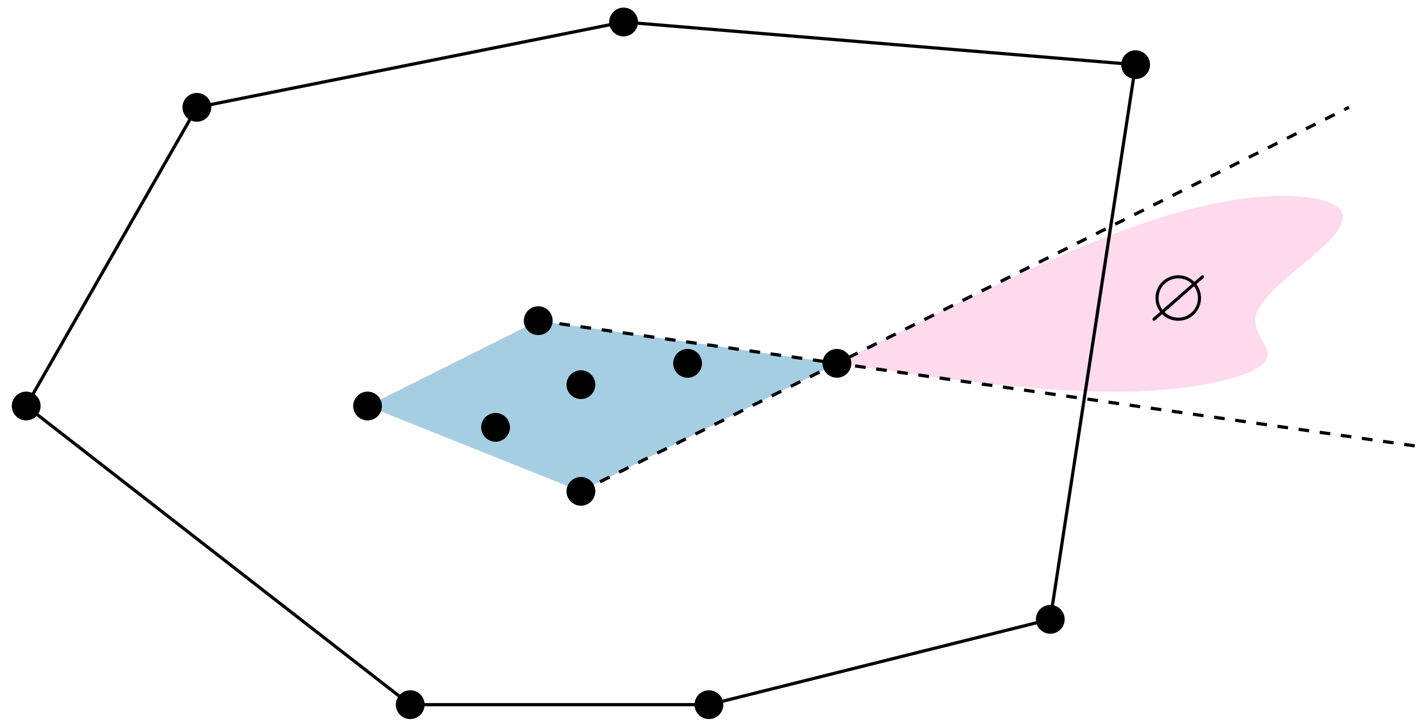

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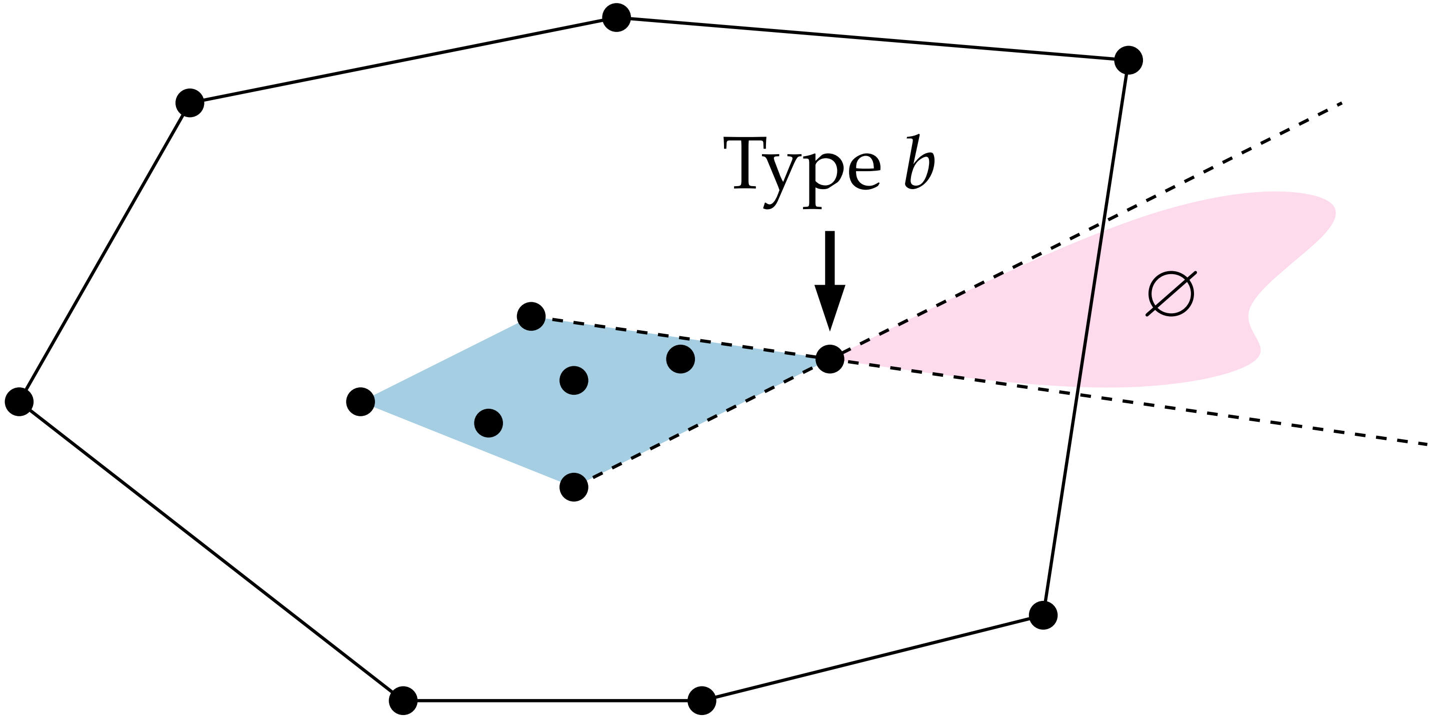

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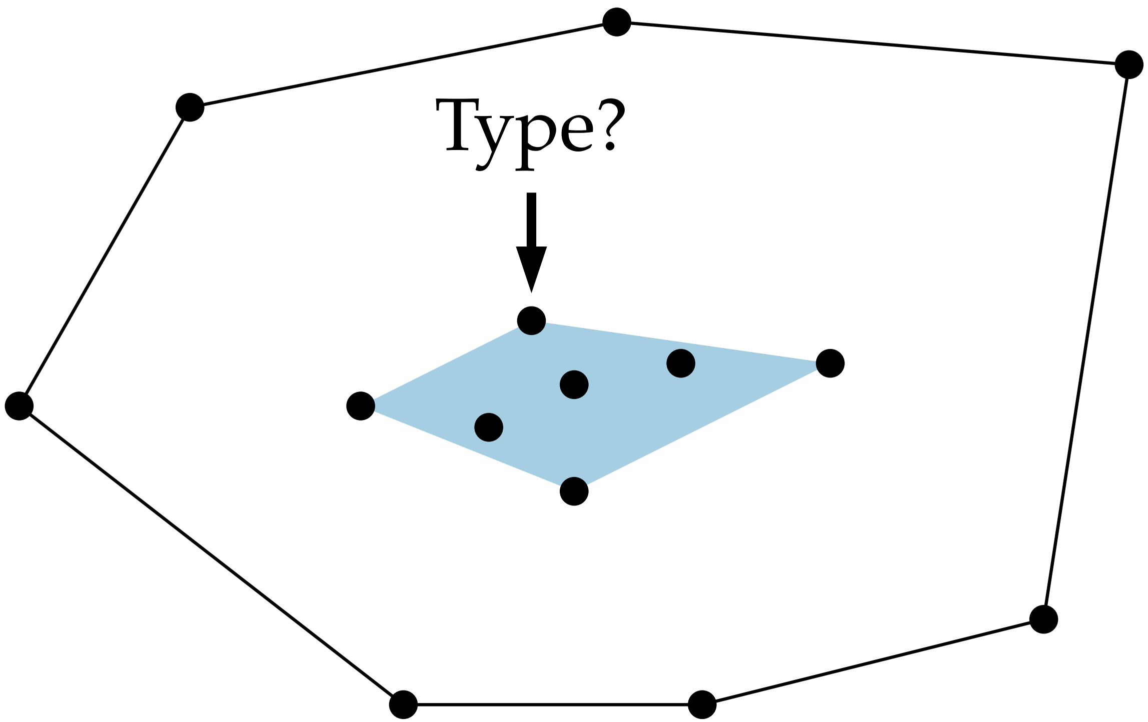

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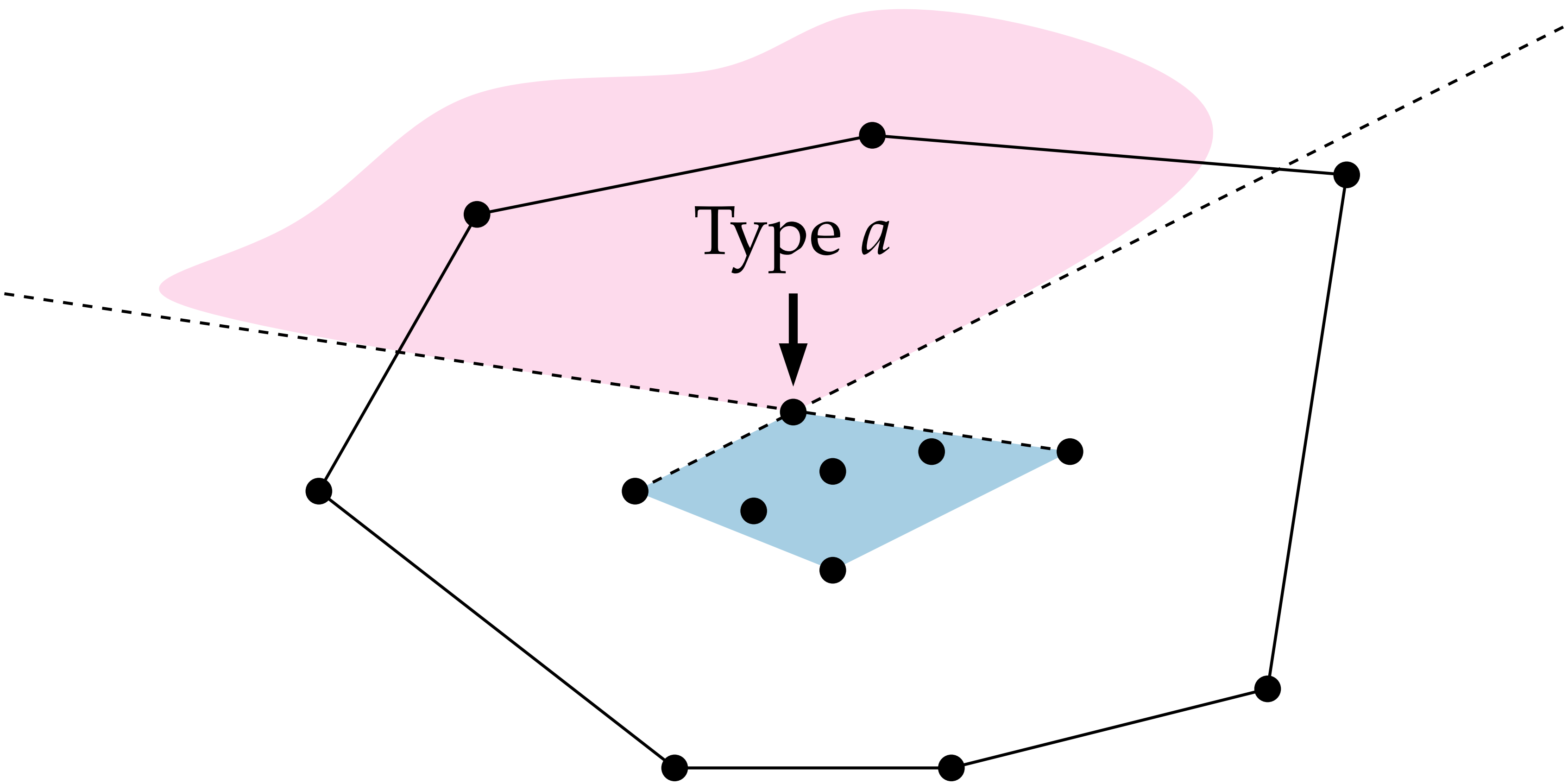

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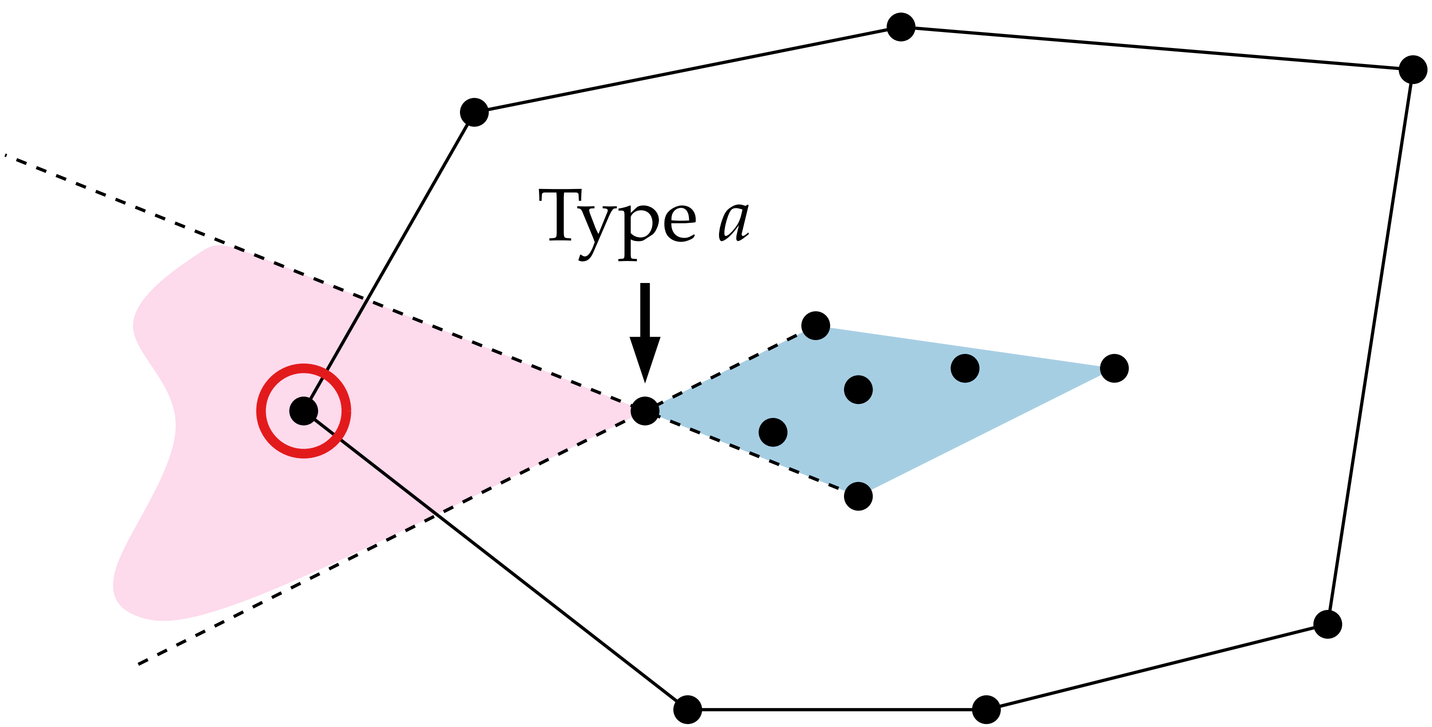

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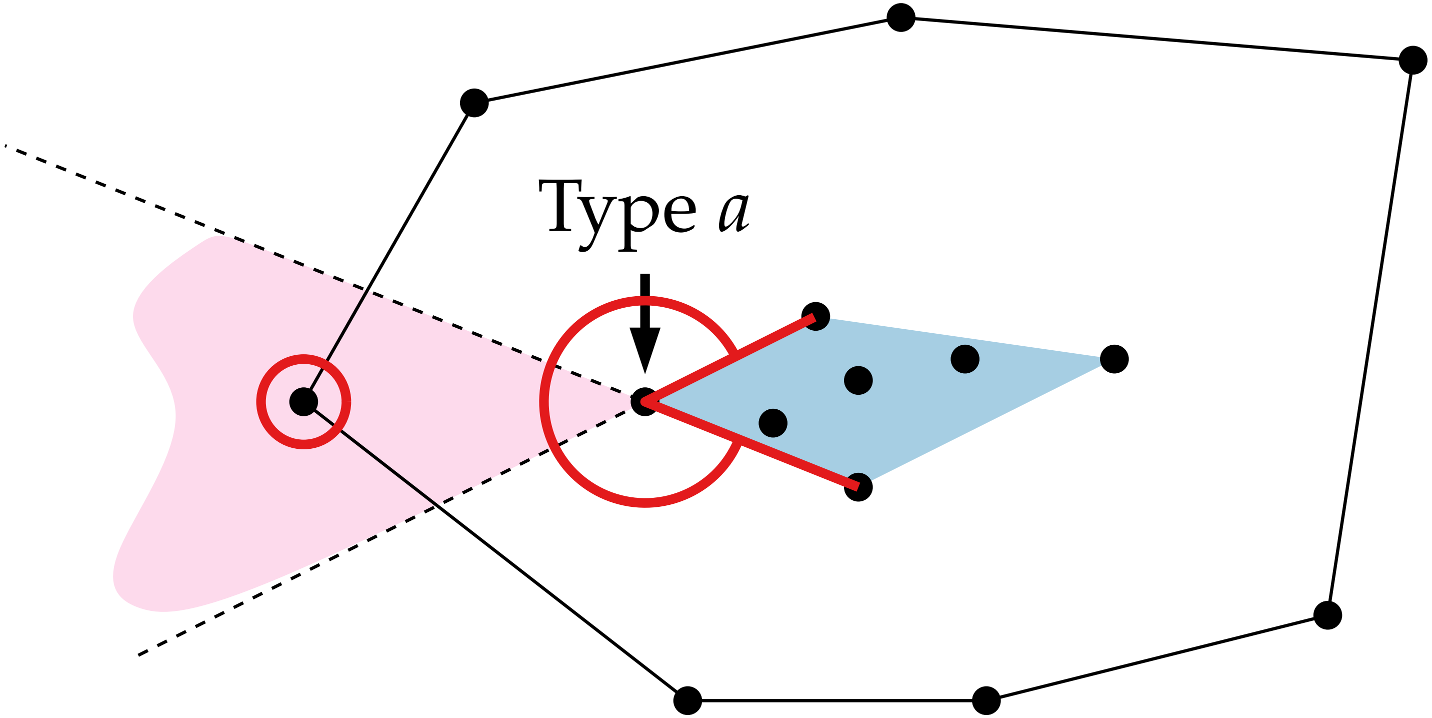

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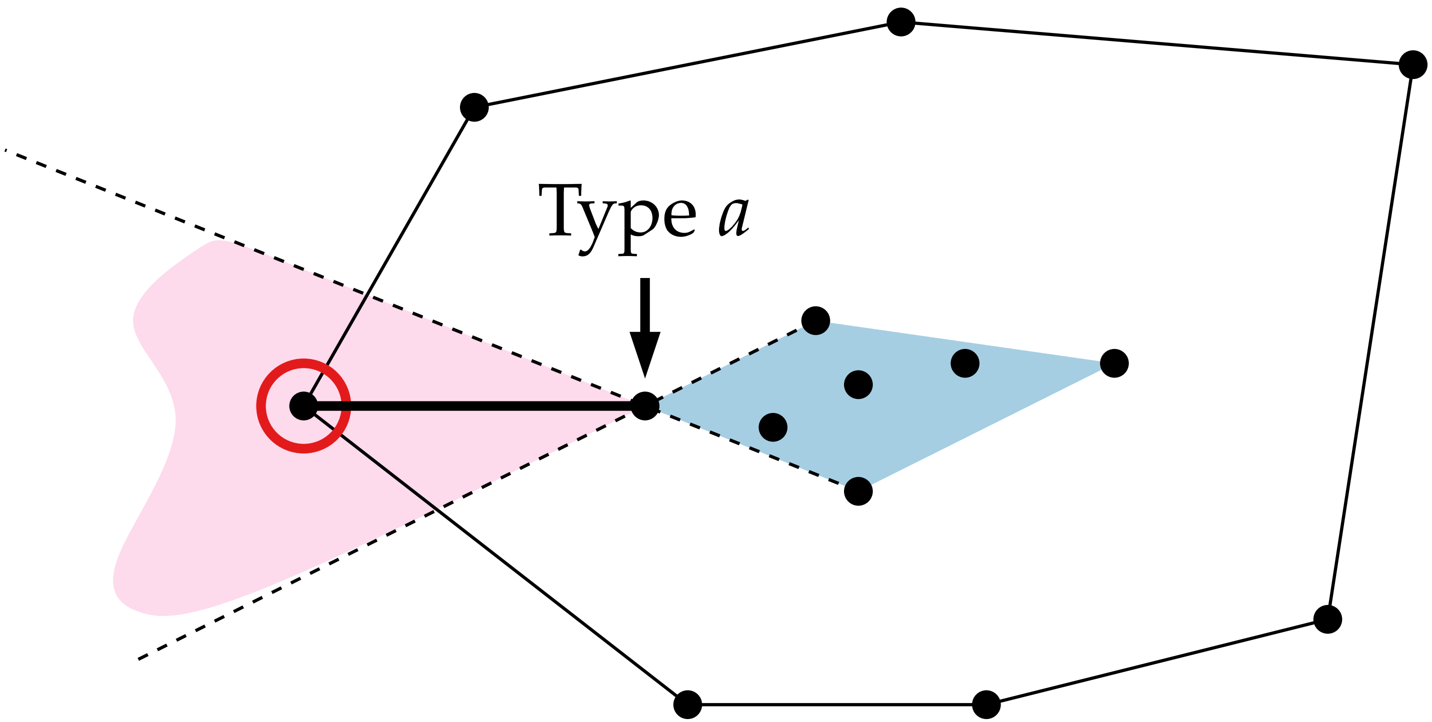

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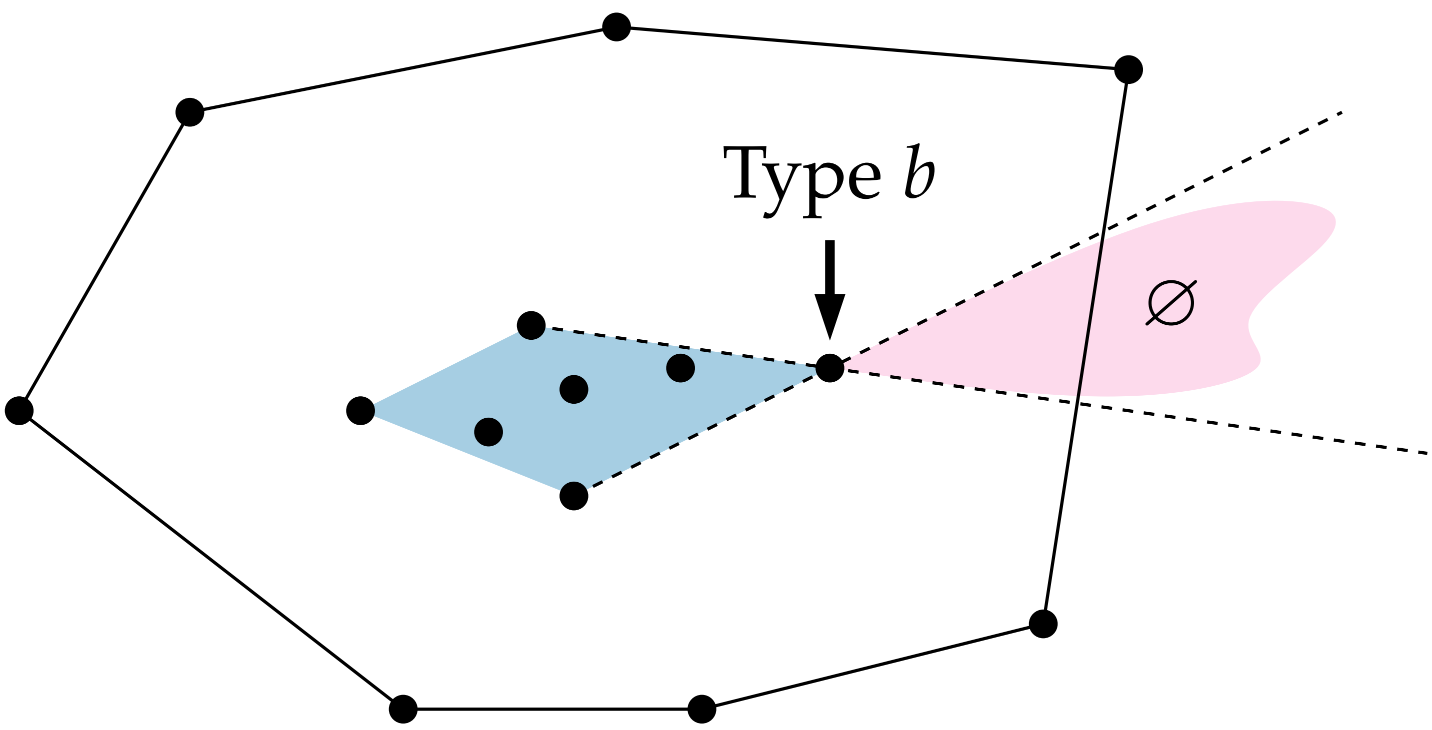

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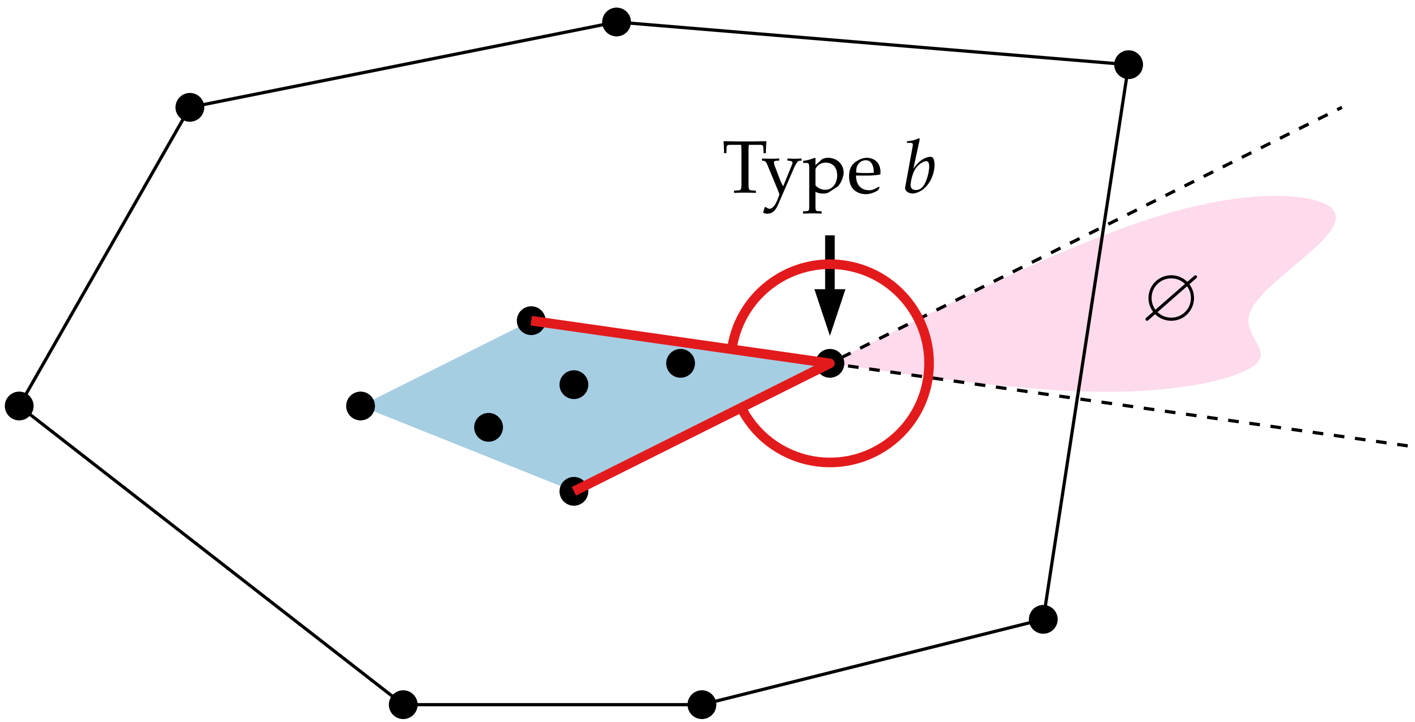

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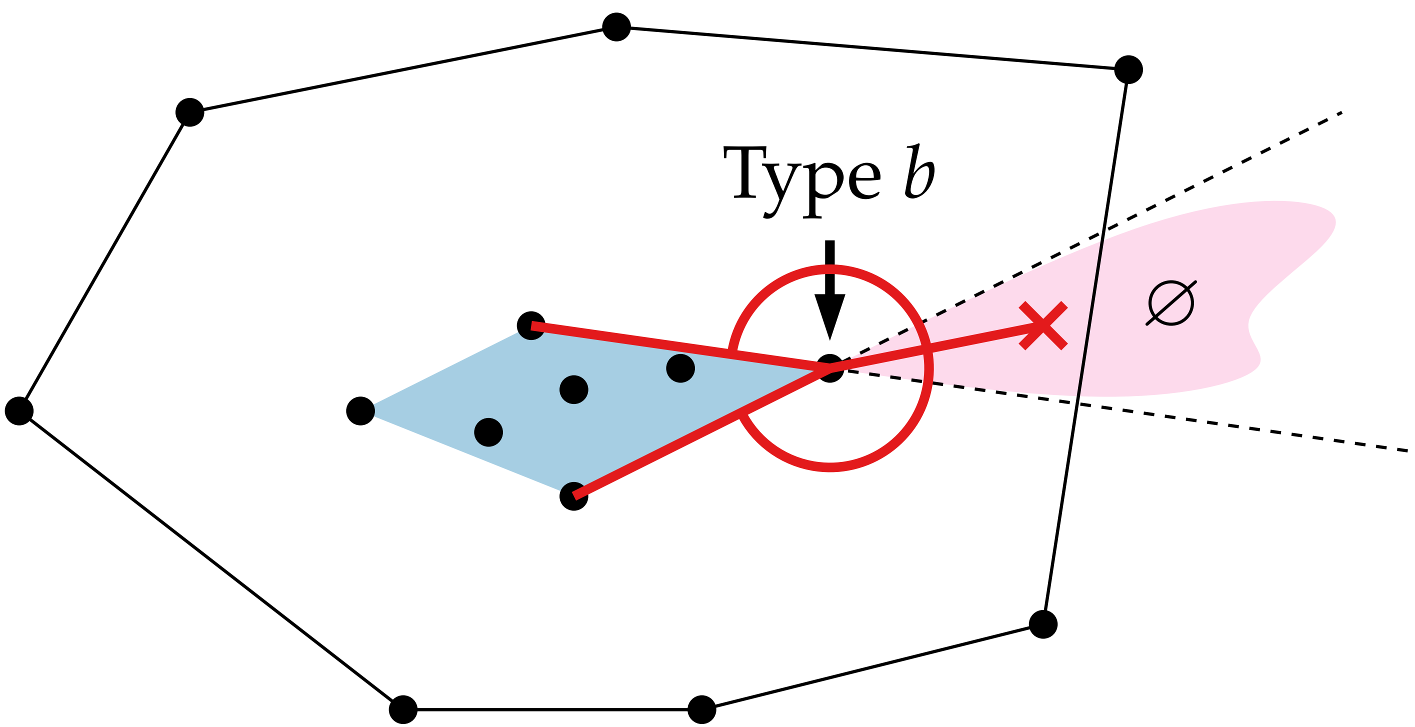

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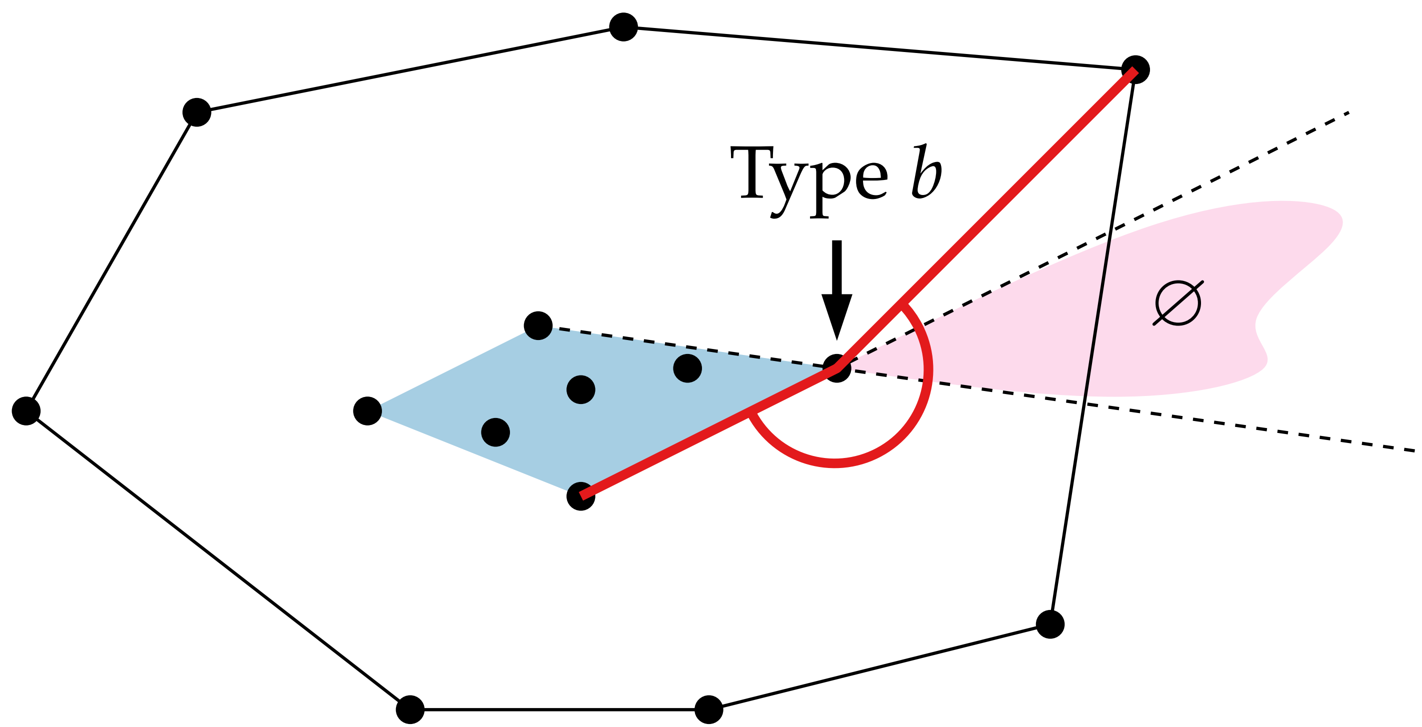

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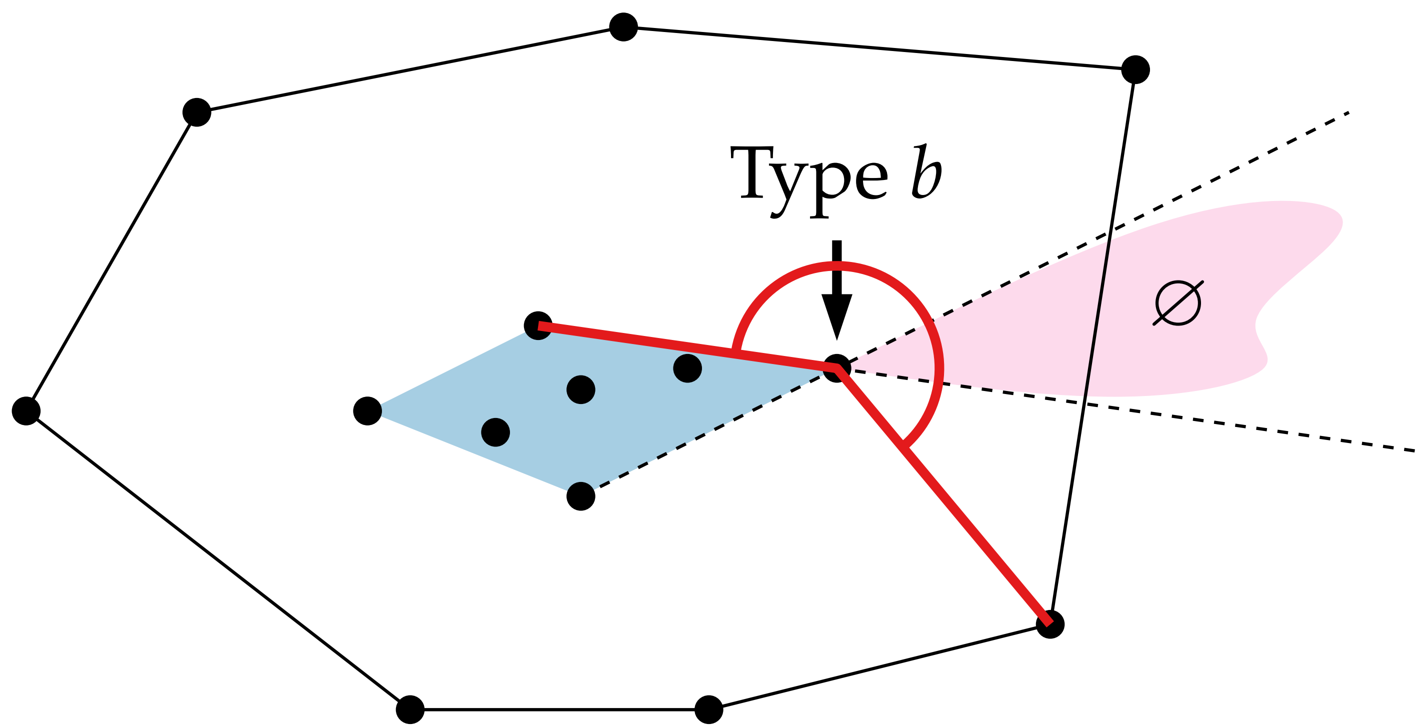

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Every outer point has degree two on the convex hull.

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Degree sum: $2(n-k) \; + 3k$

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Every interior point has degree at least 3.

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$$\# \text{ edges} \geq n + \frac{k}{2} + \frac{a}{2} + b \xrightarrow{\textit{Euler}} \# \text{ faces} \geq \frac{k}{2} + \frac{a}{2} + b + 1$$

$\square$

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Lemma There is a convex partition E of P such that $|R(E)| \leq \frac{3}{2}k + \frac{3}{2}$.

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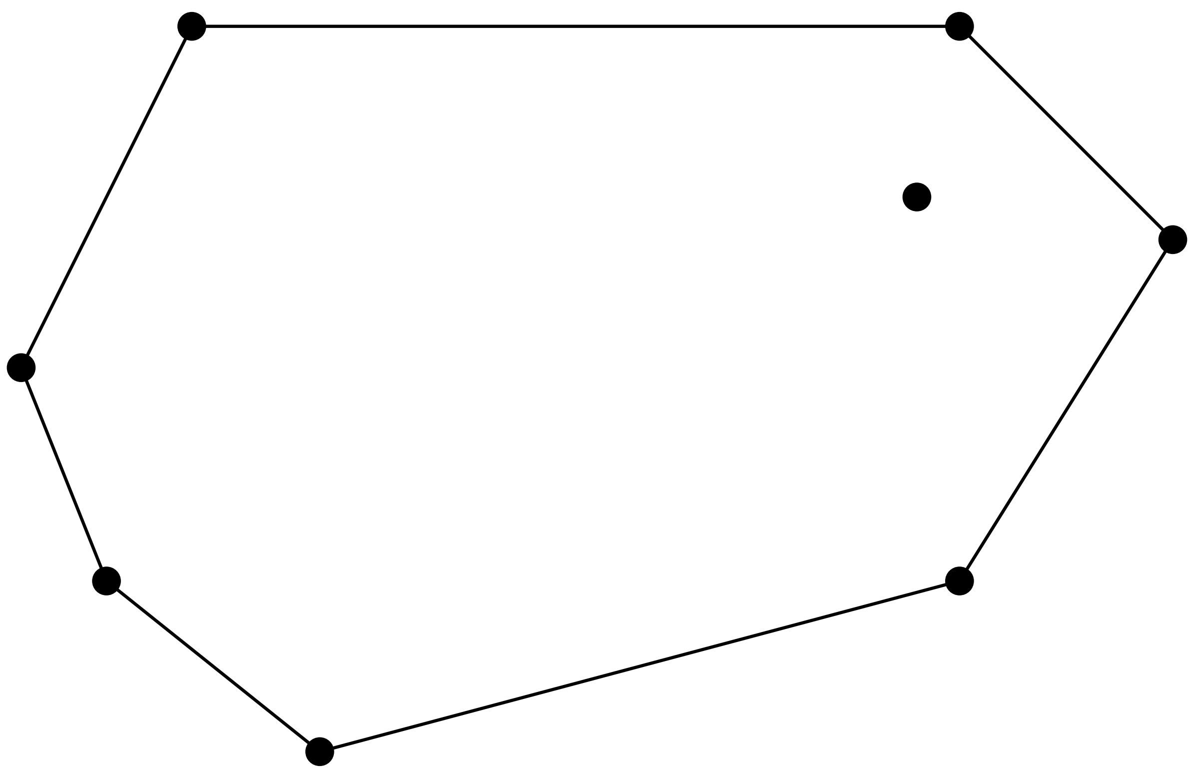

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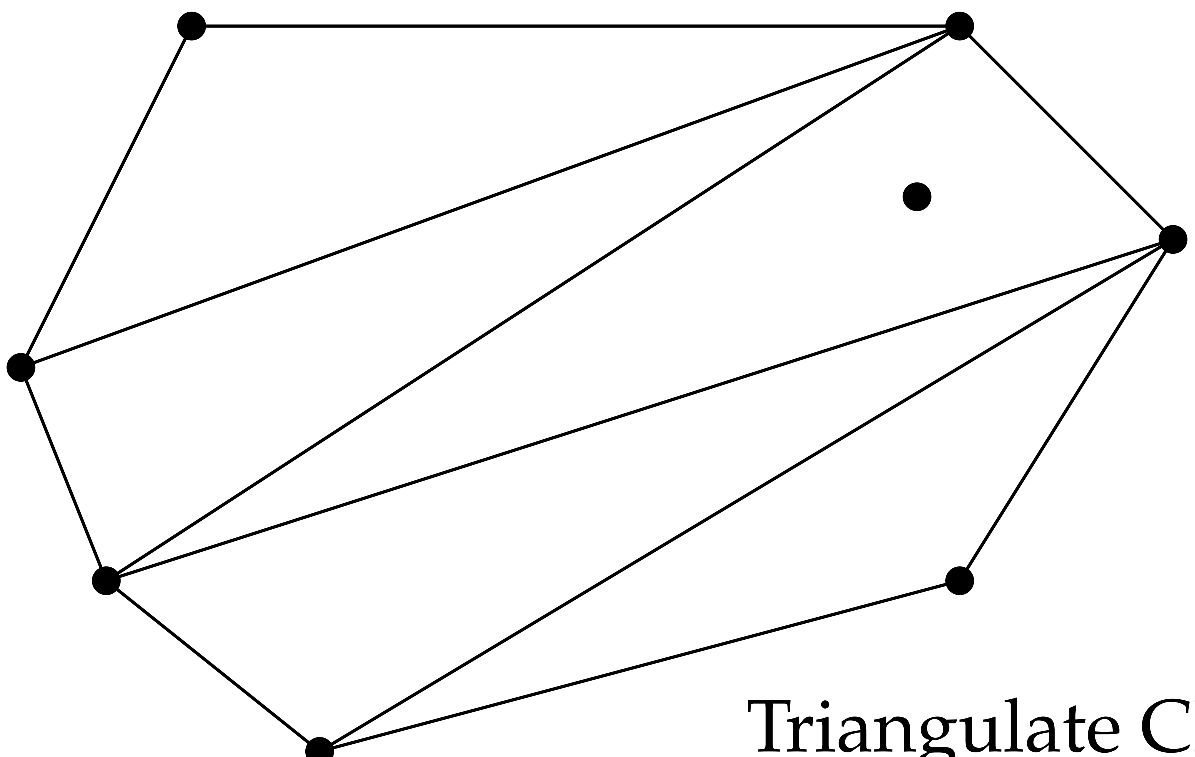

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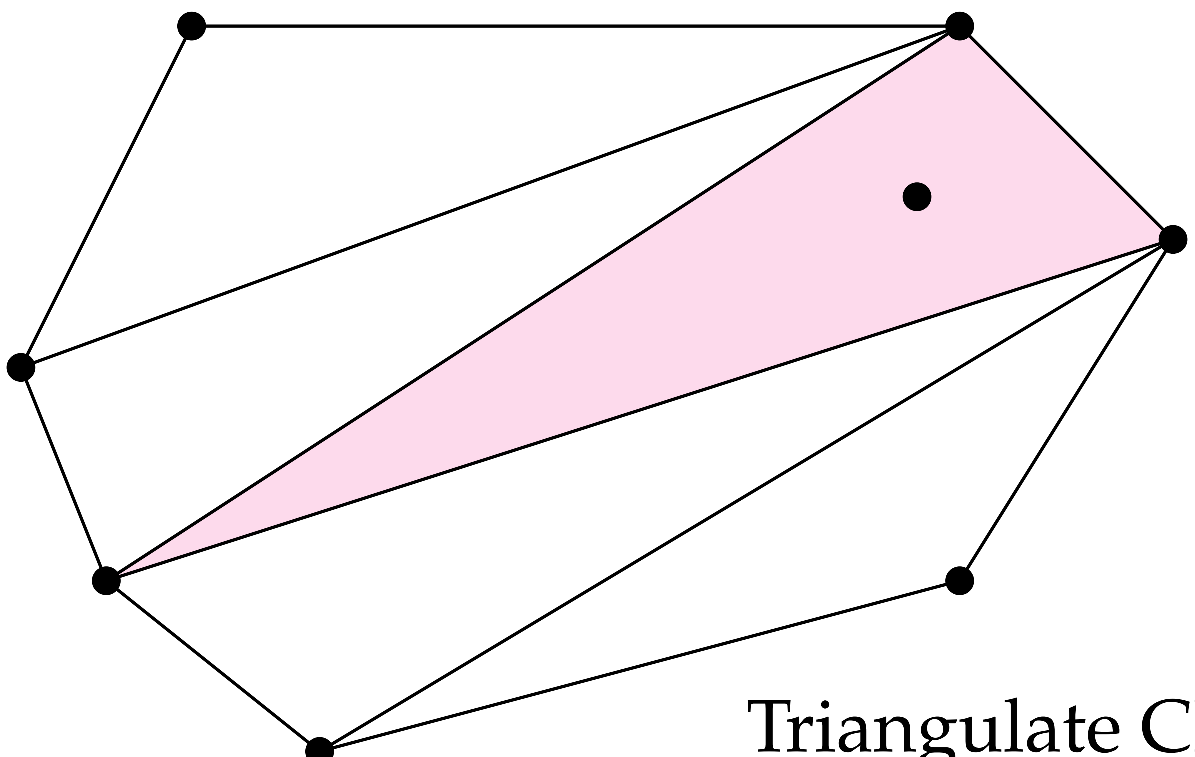

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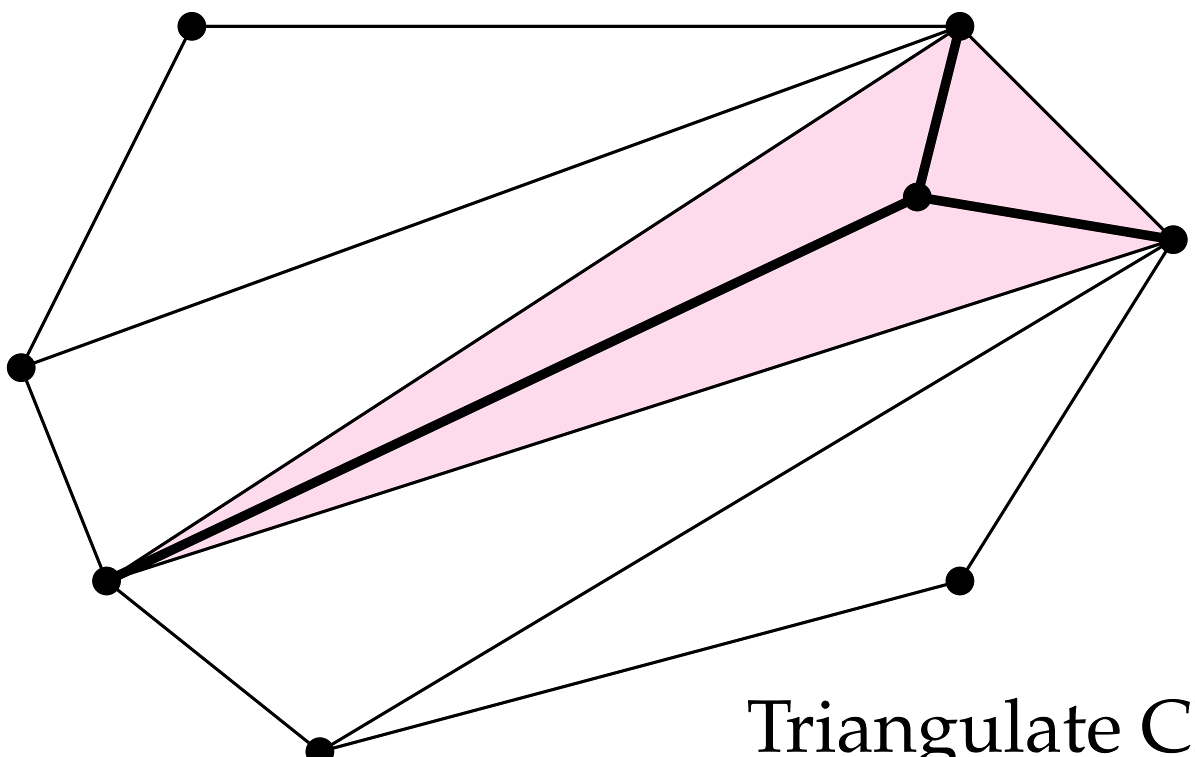

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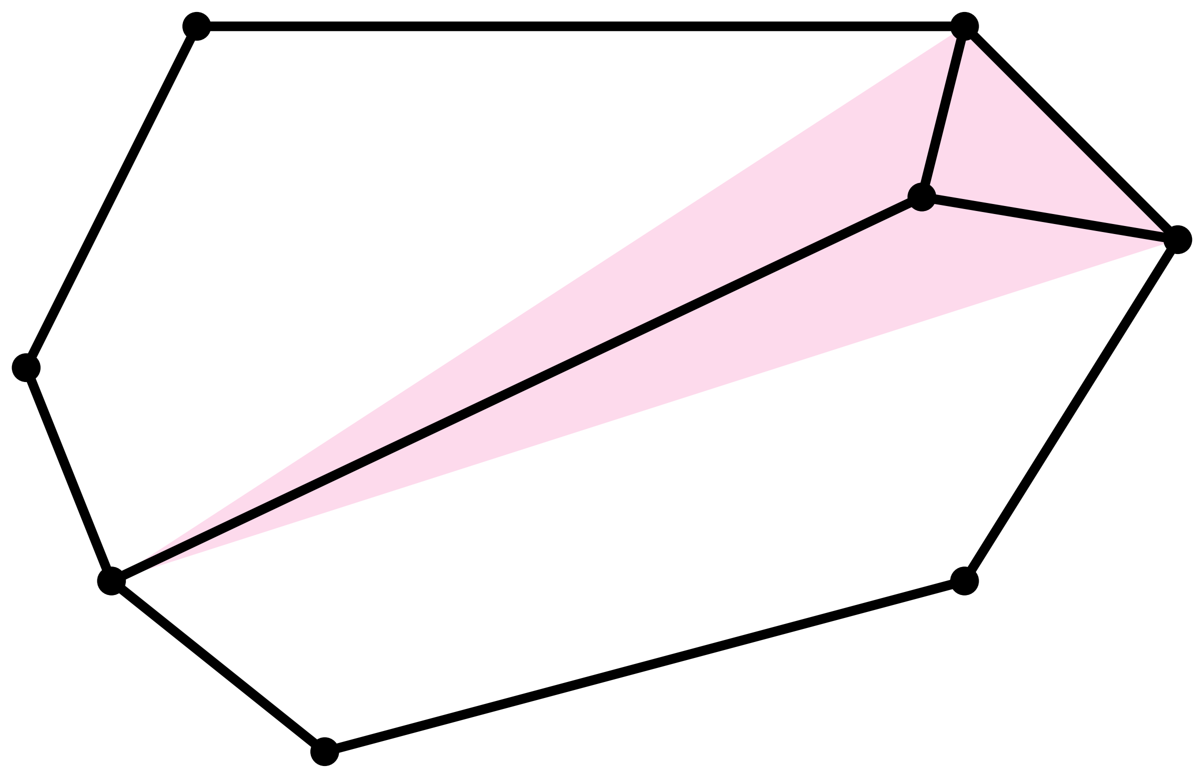

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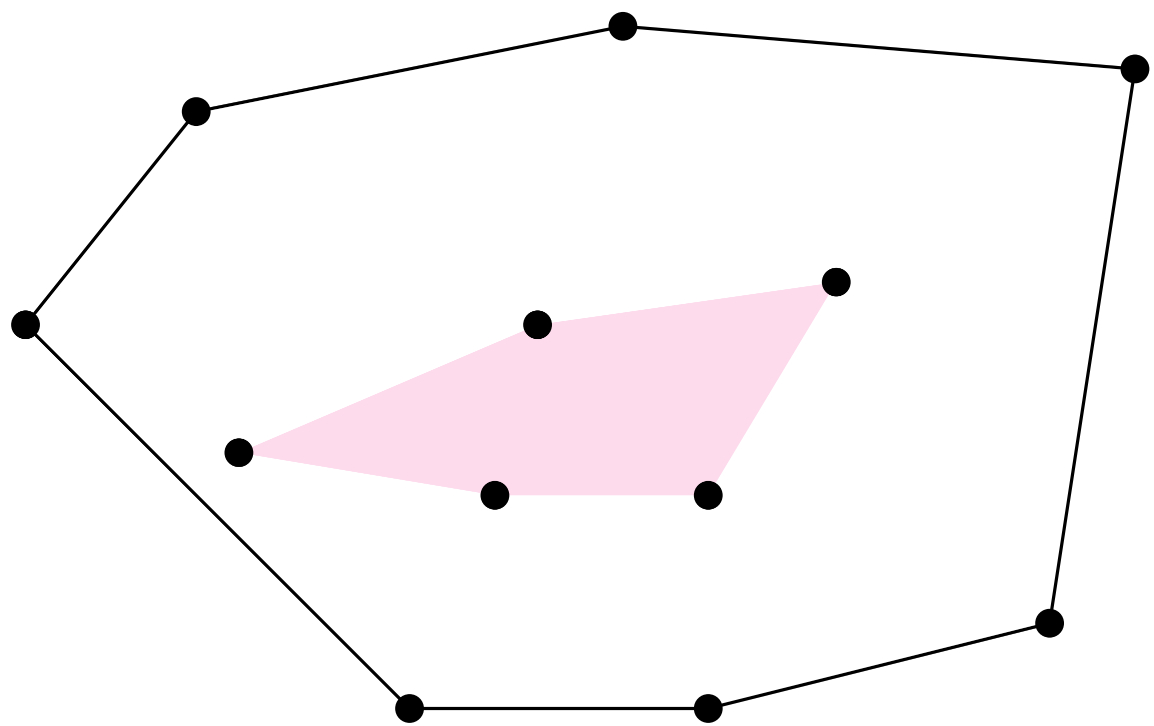

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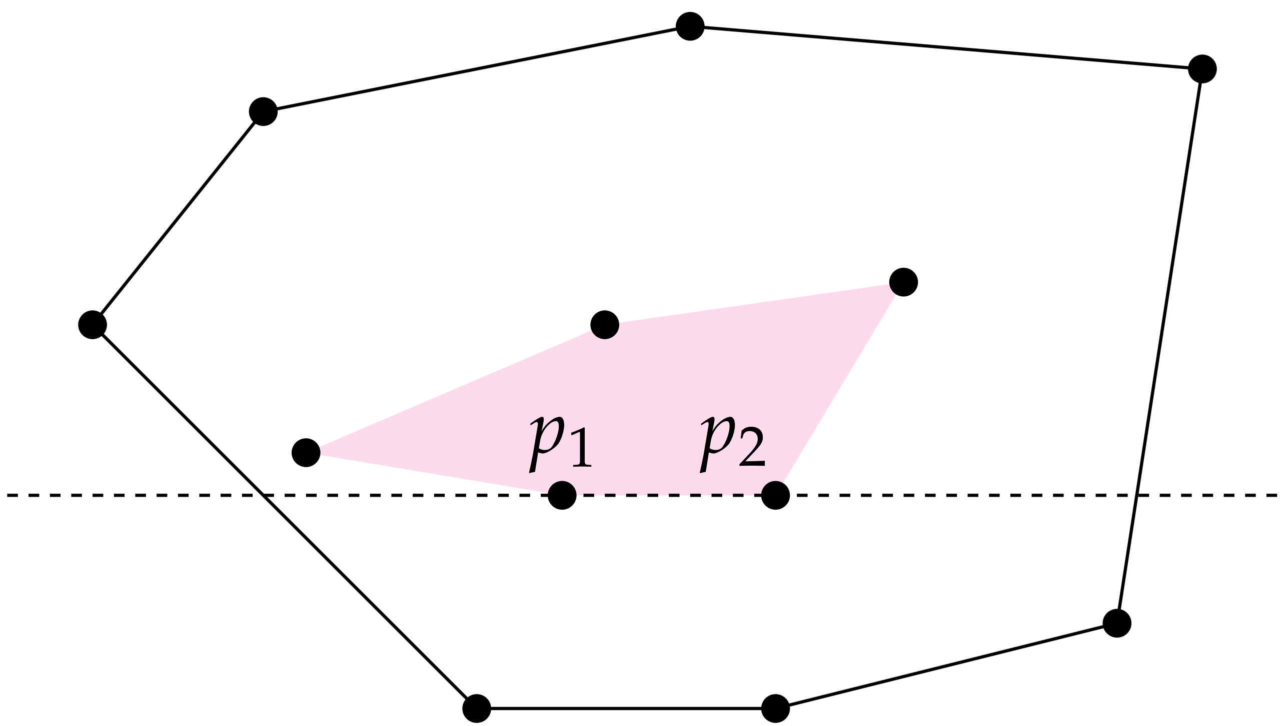

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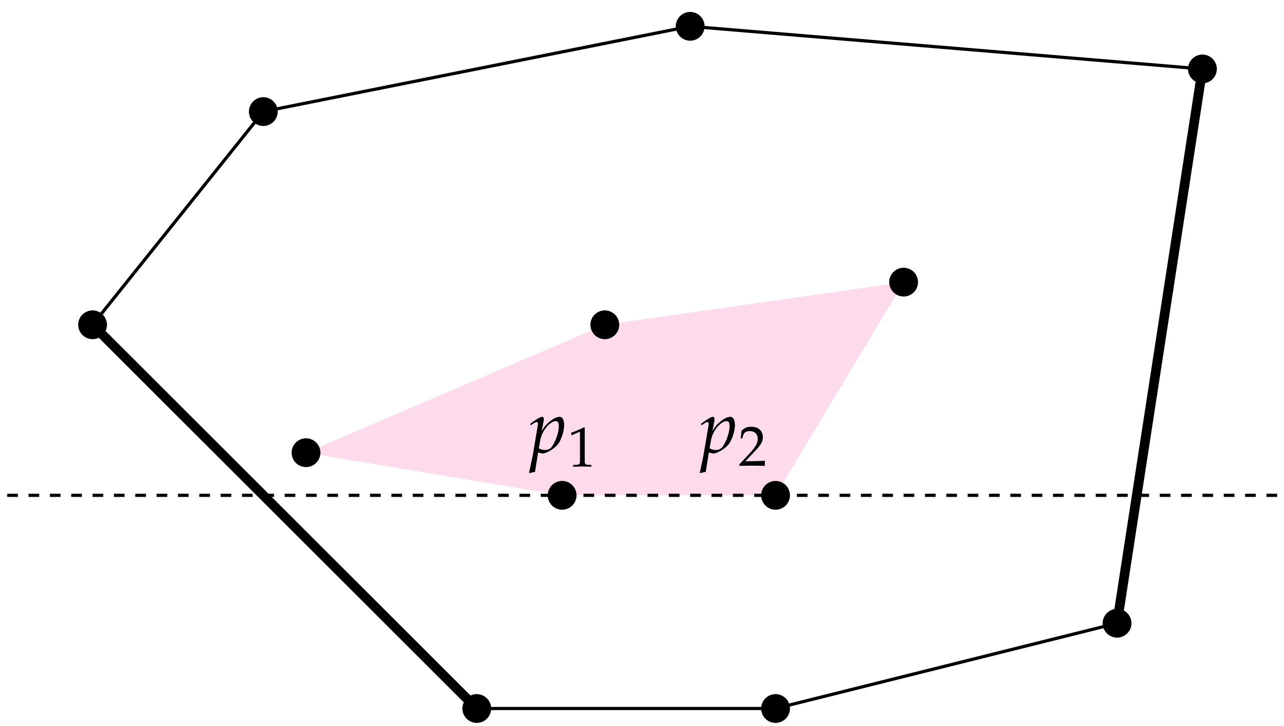

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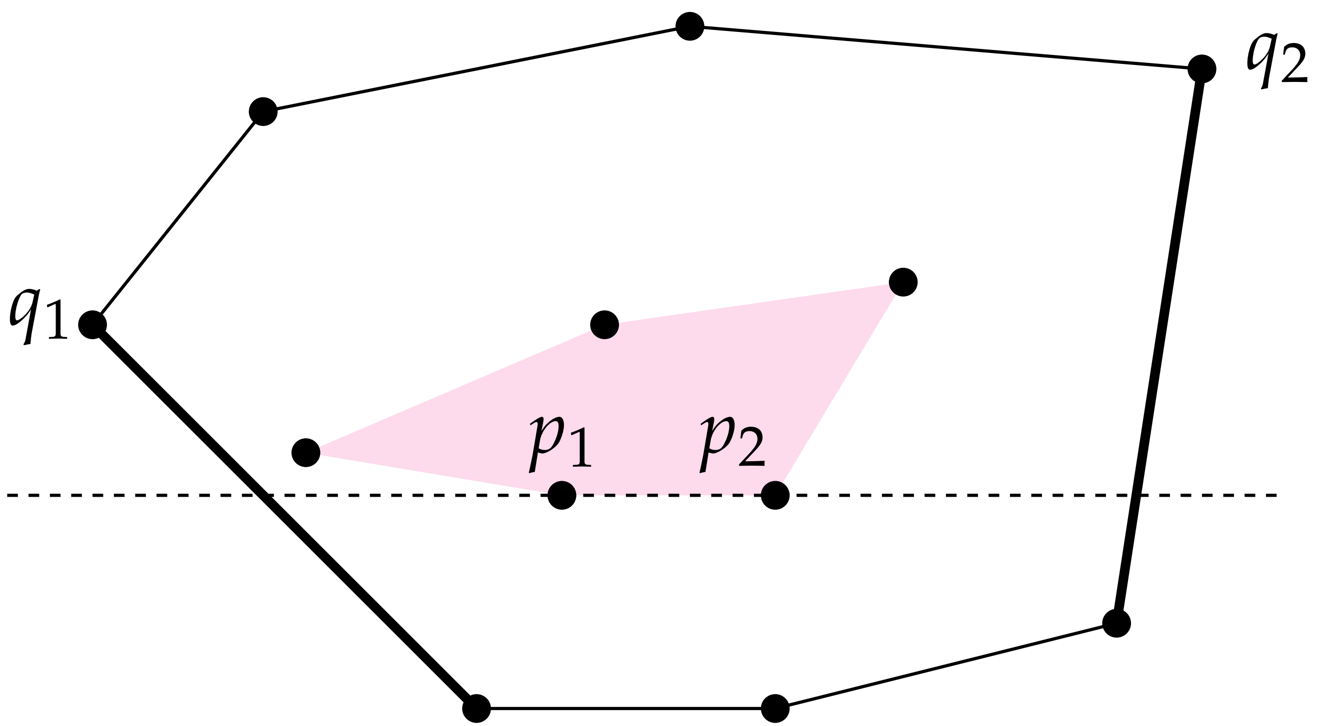

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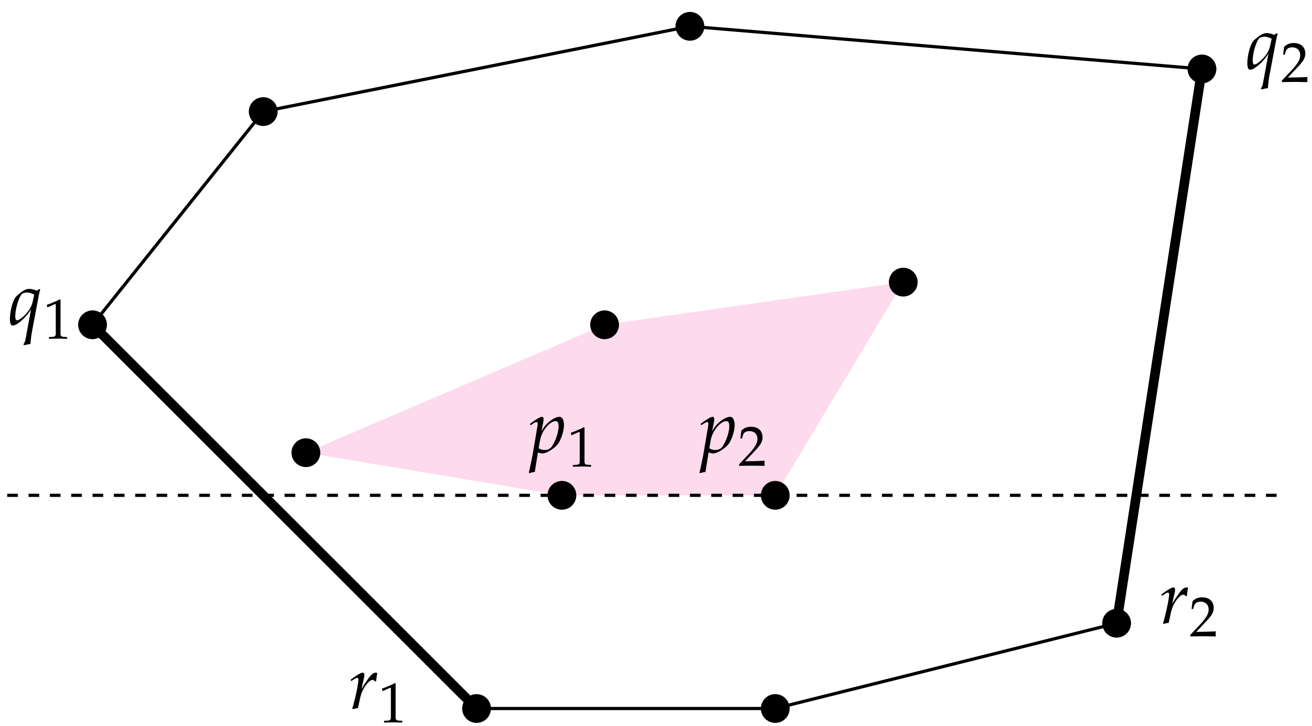

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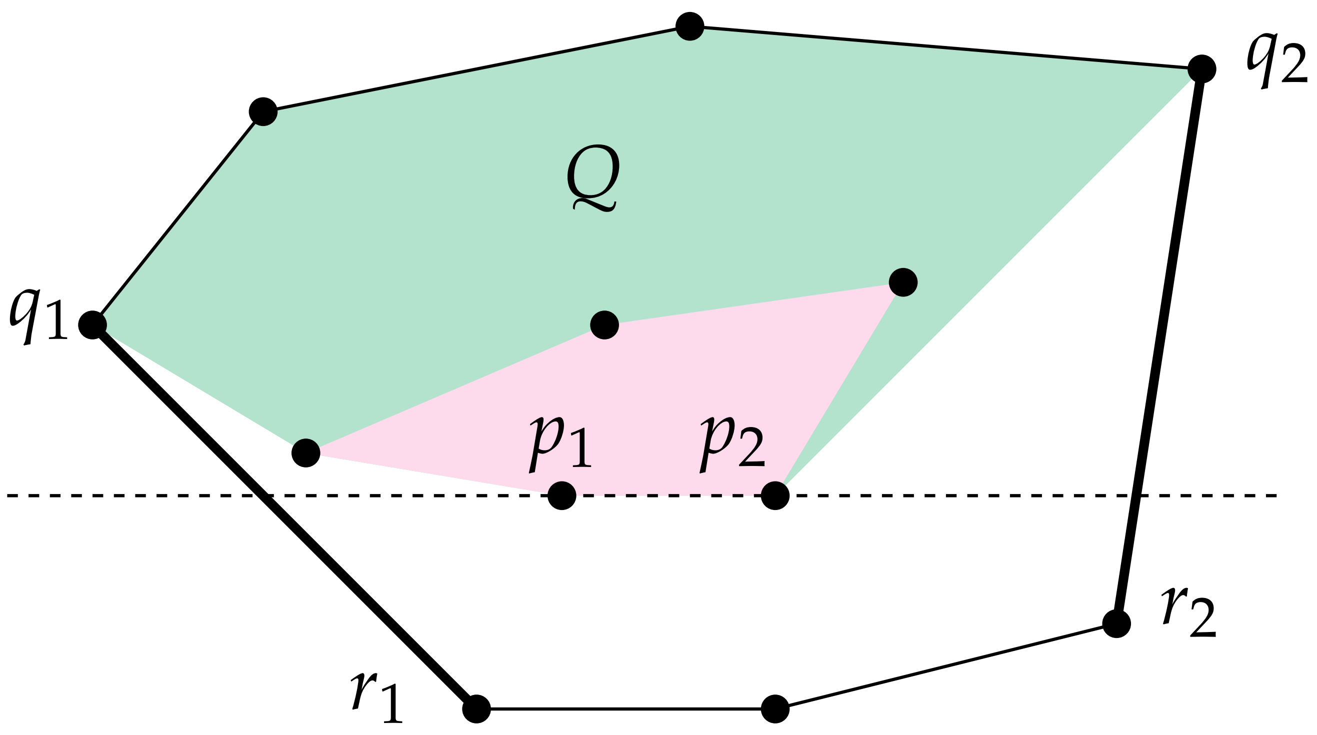

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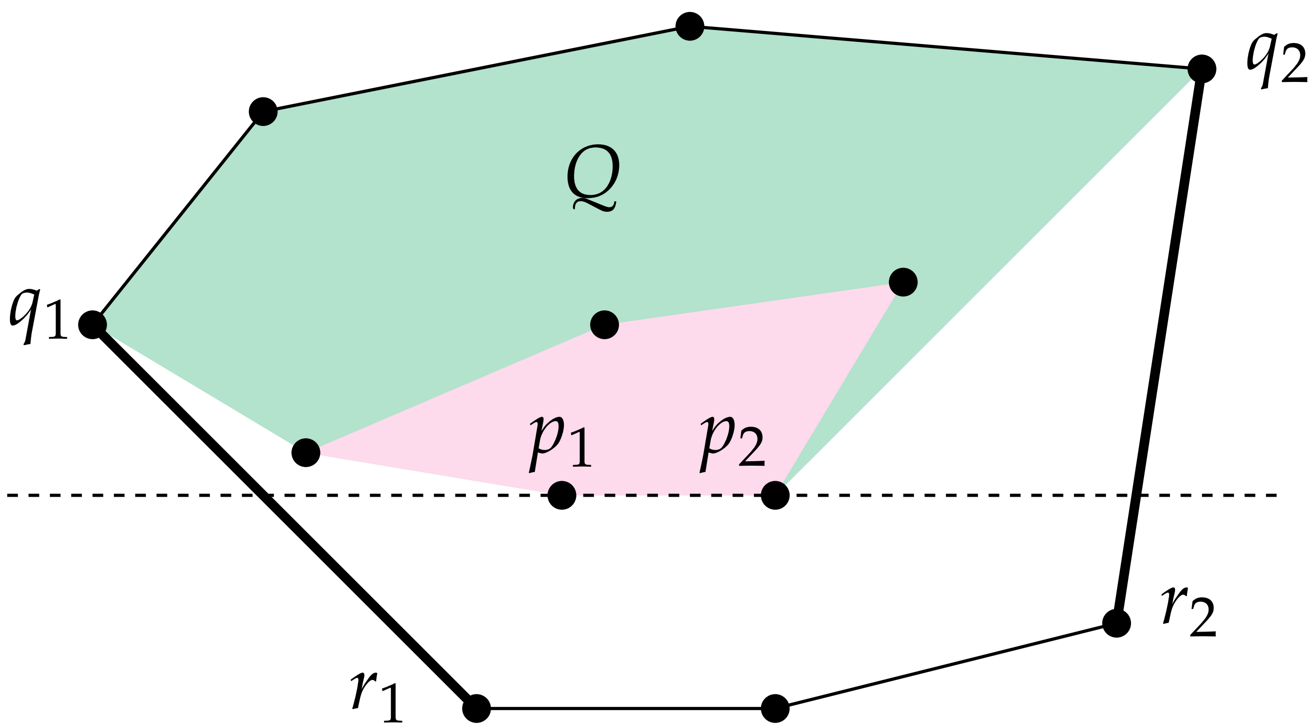

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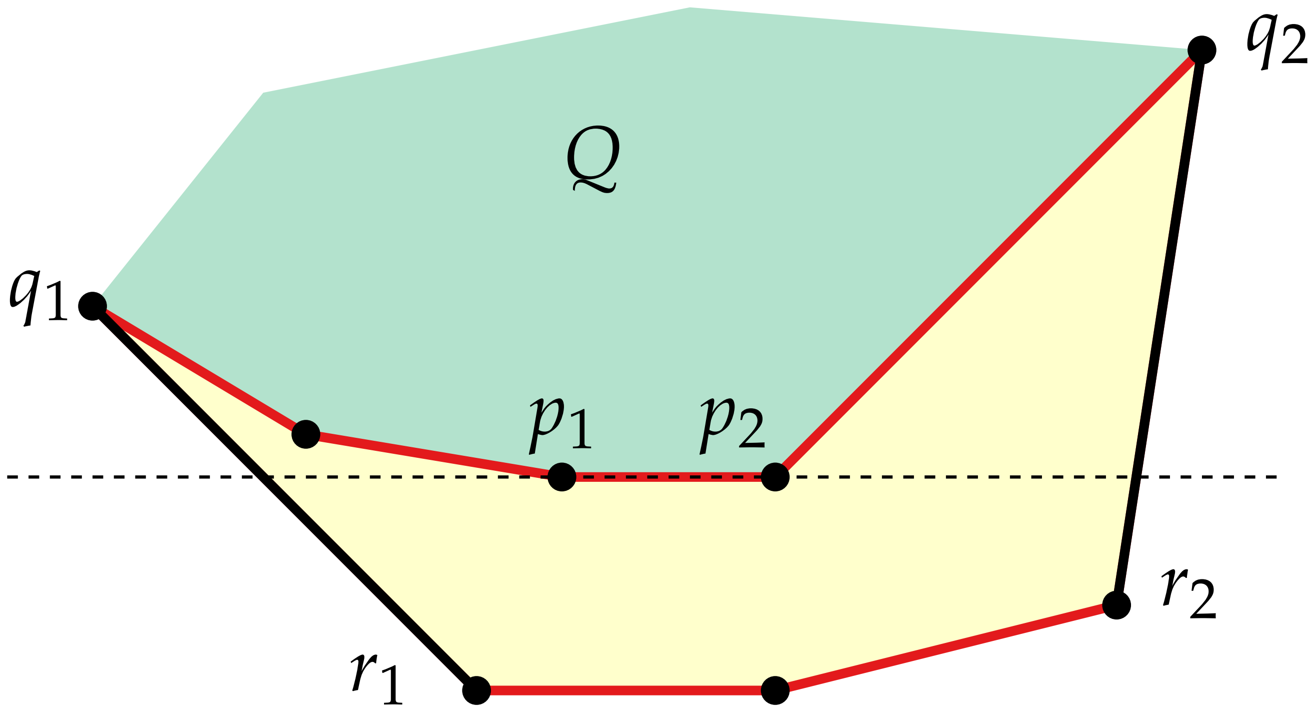

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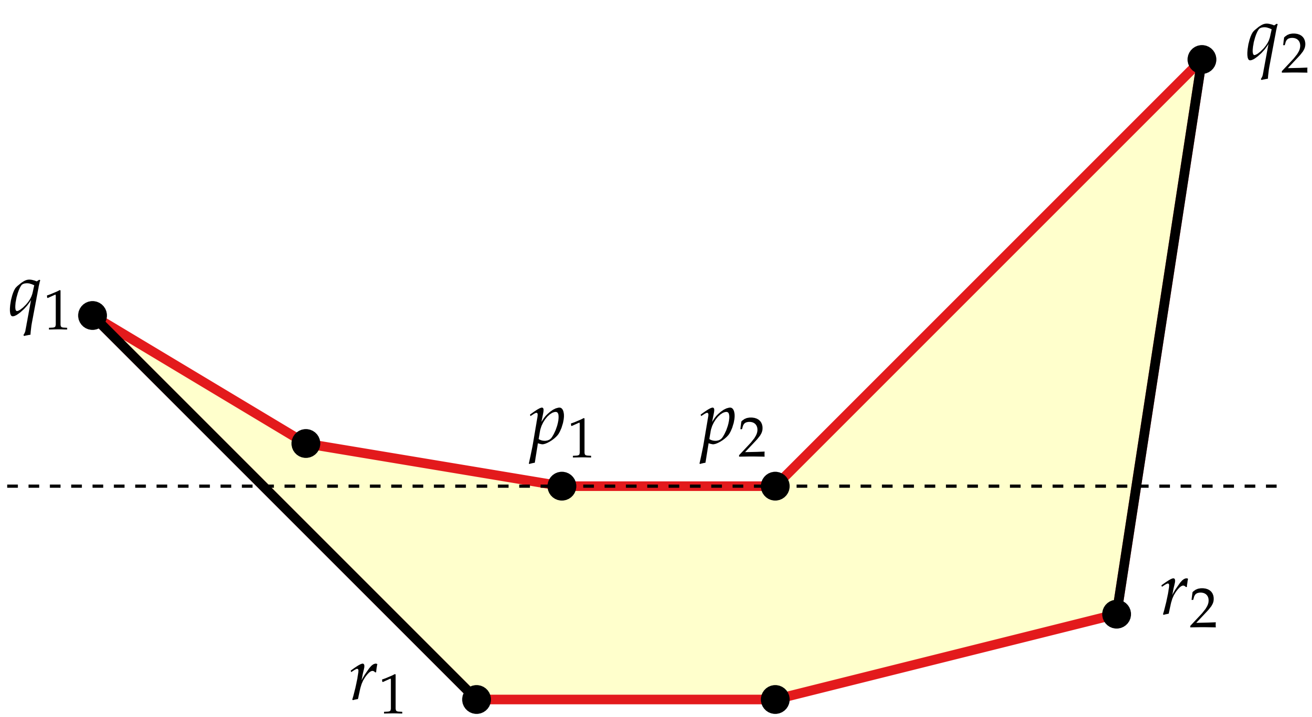

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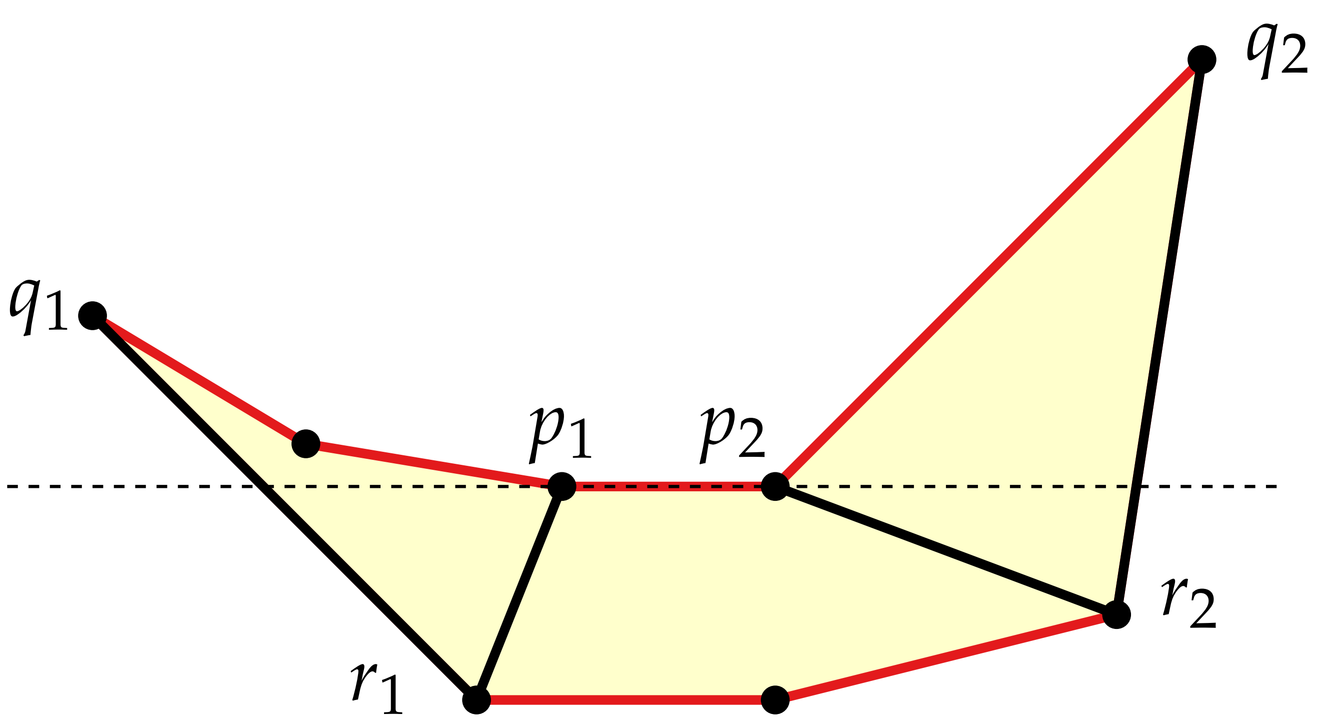

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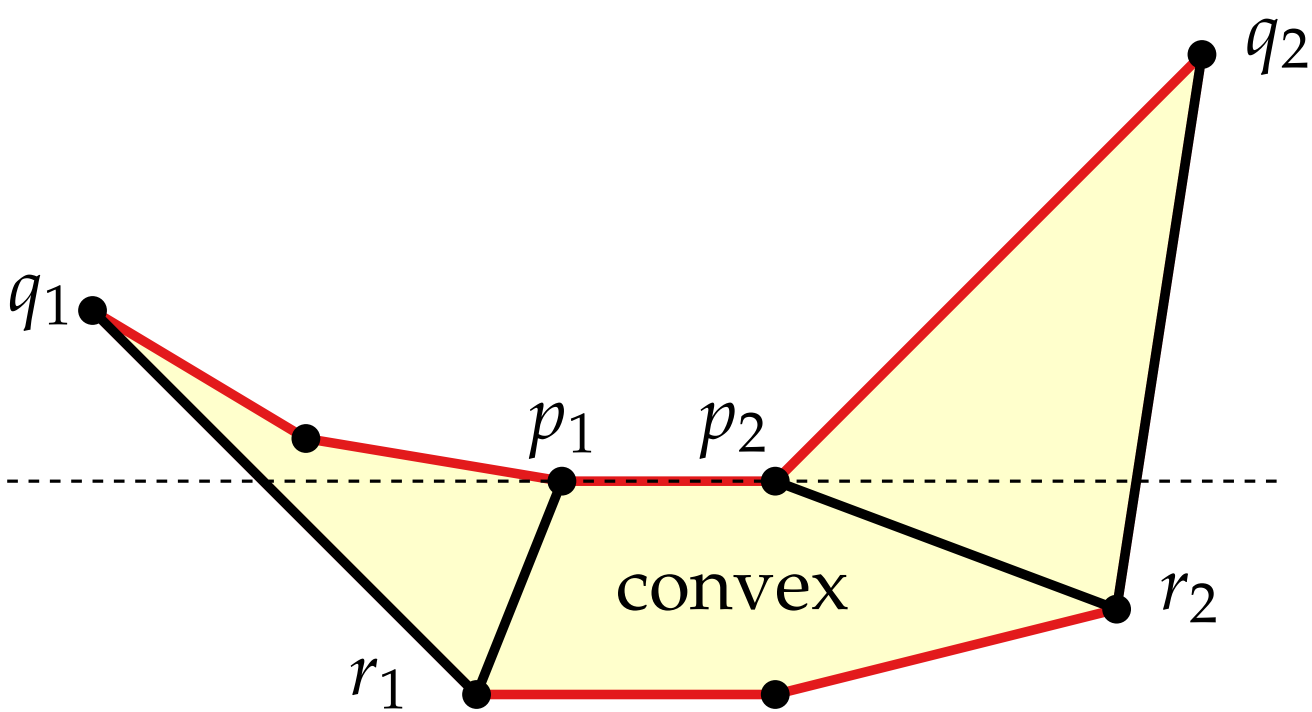

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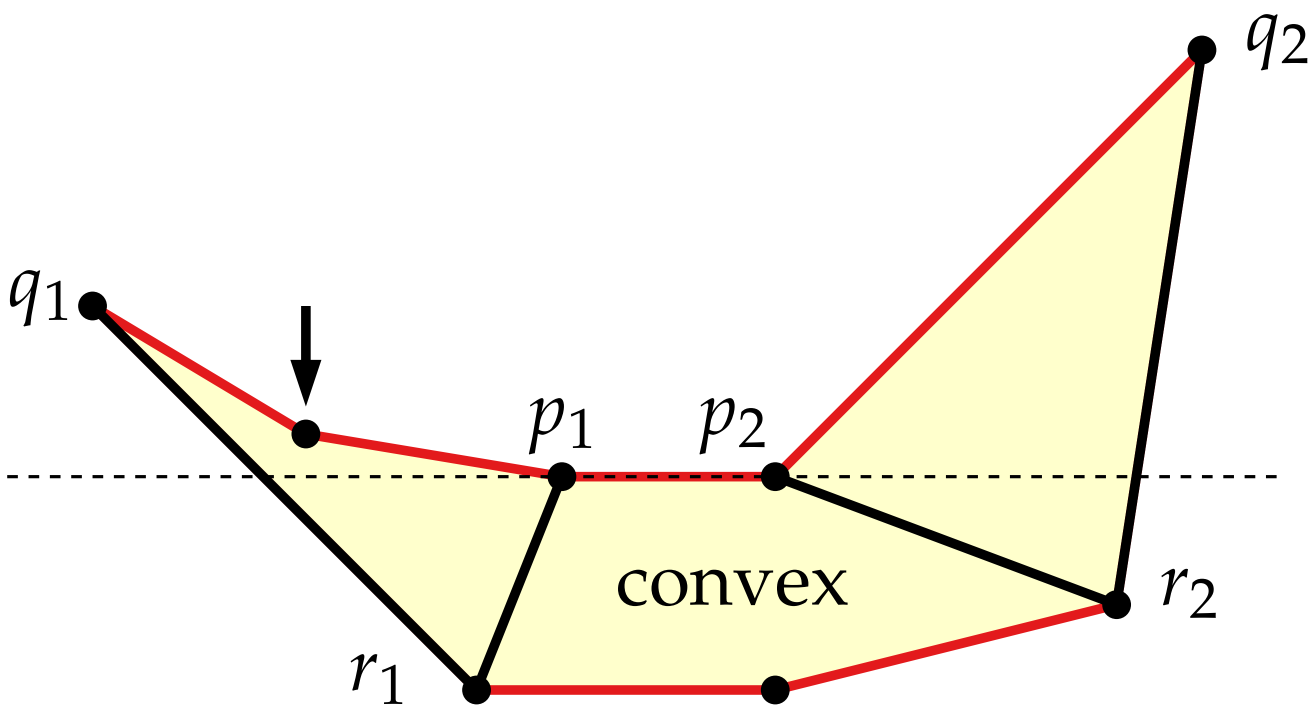

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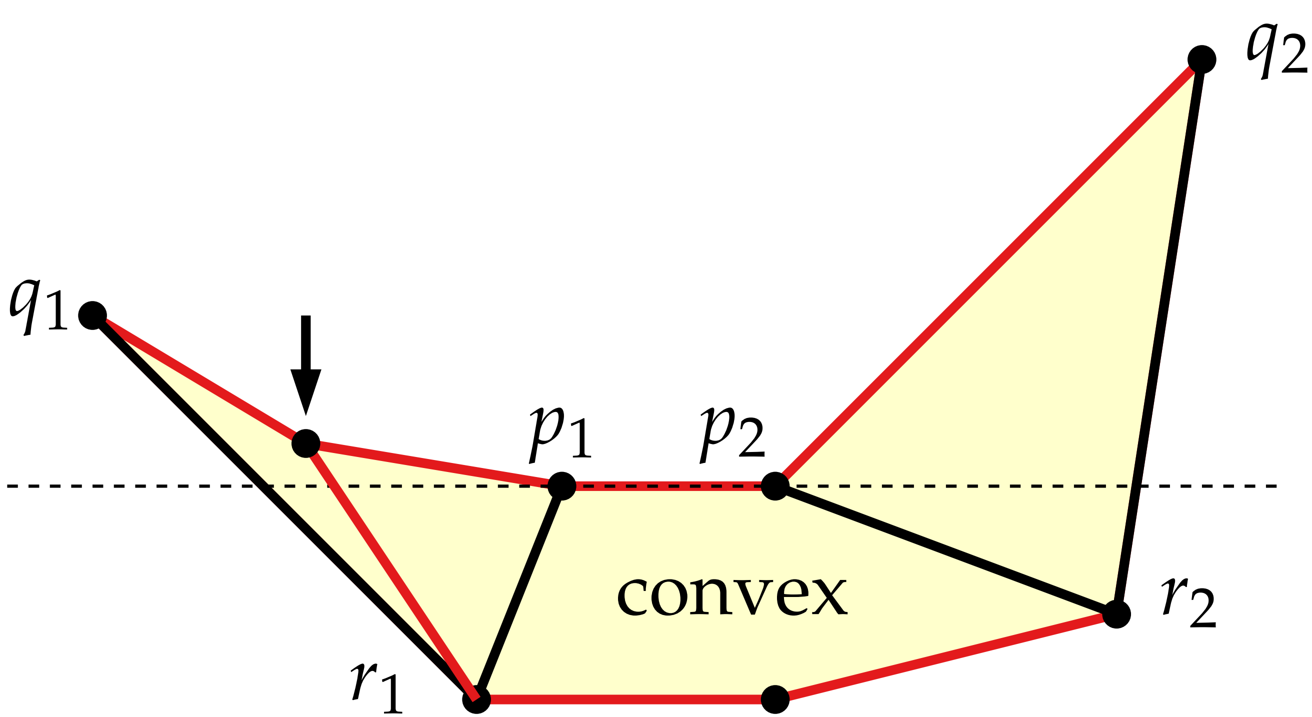

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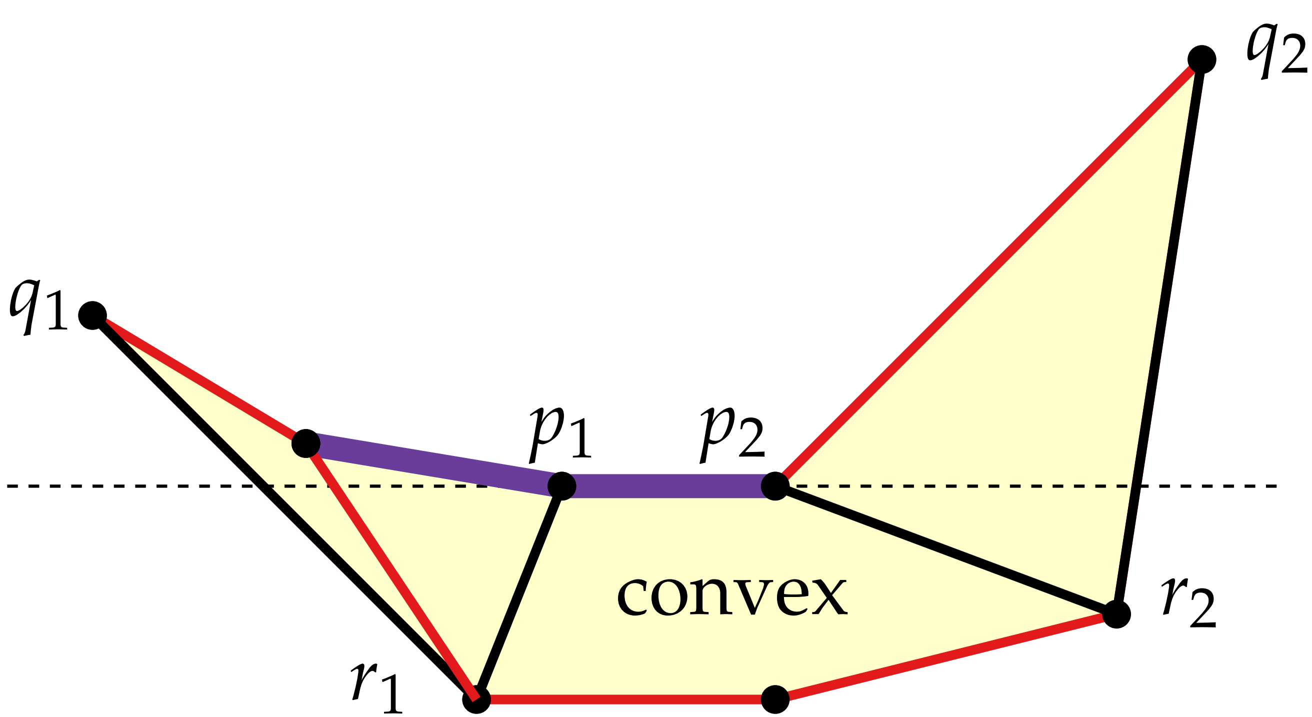

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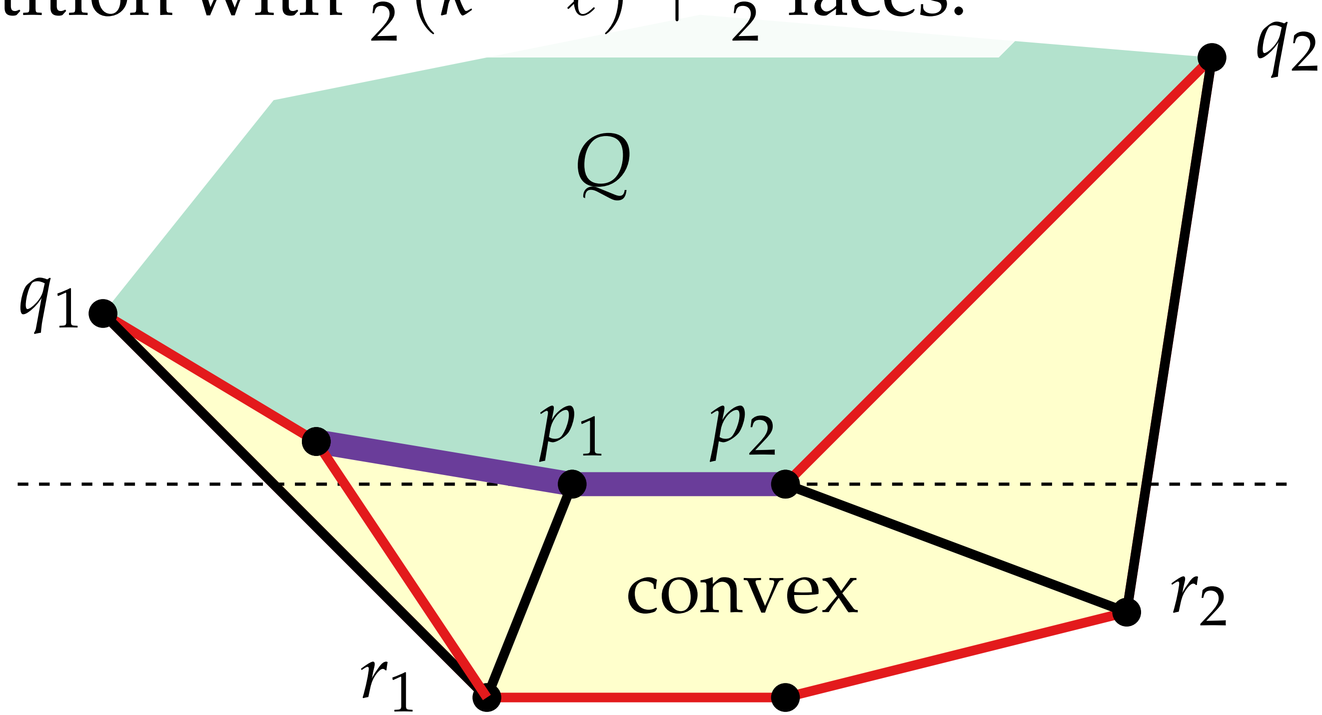

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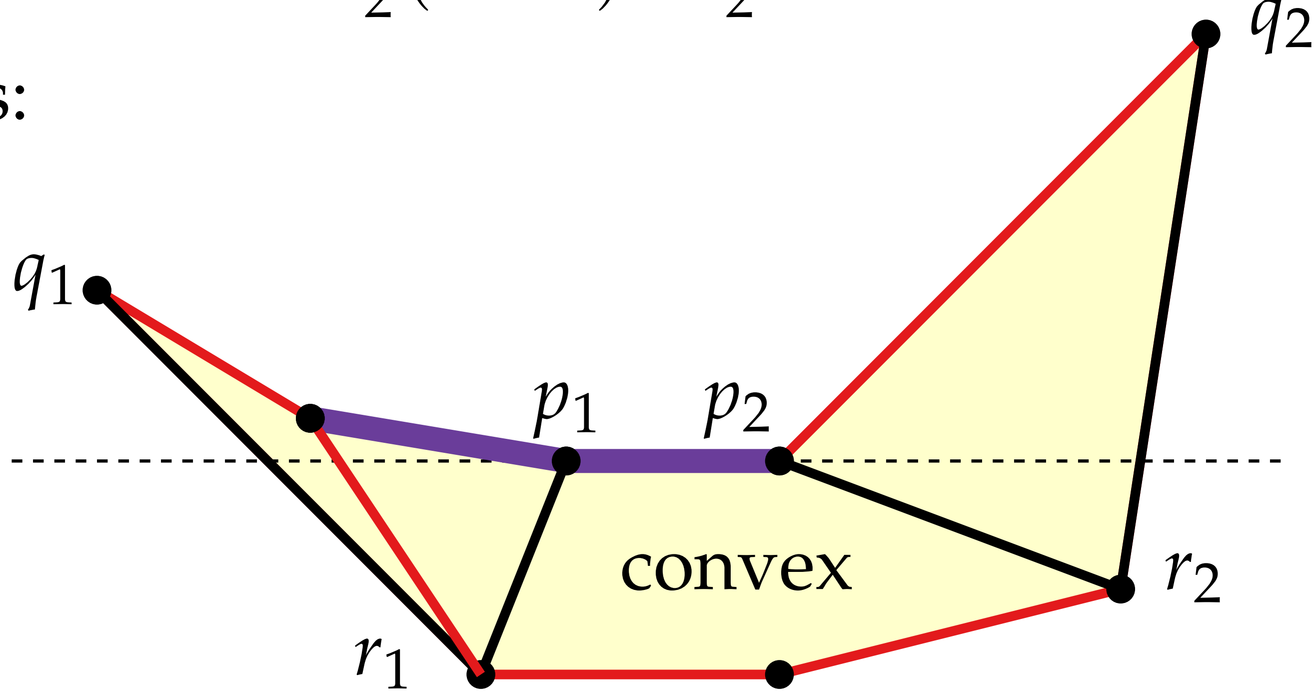

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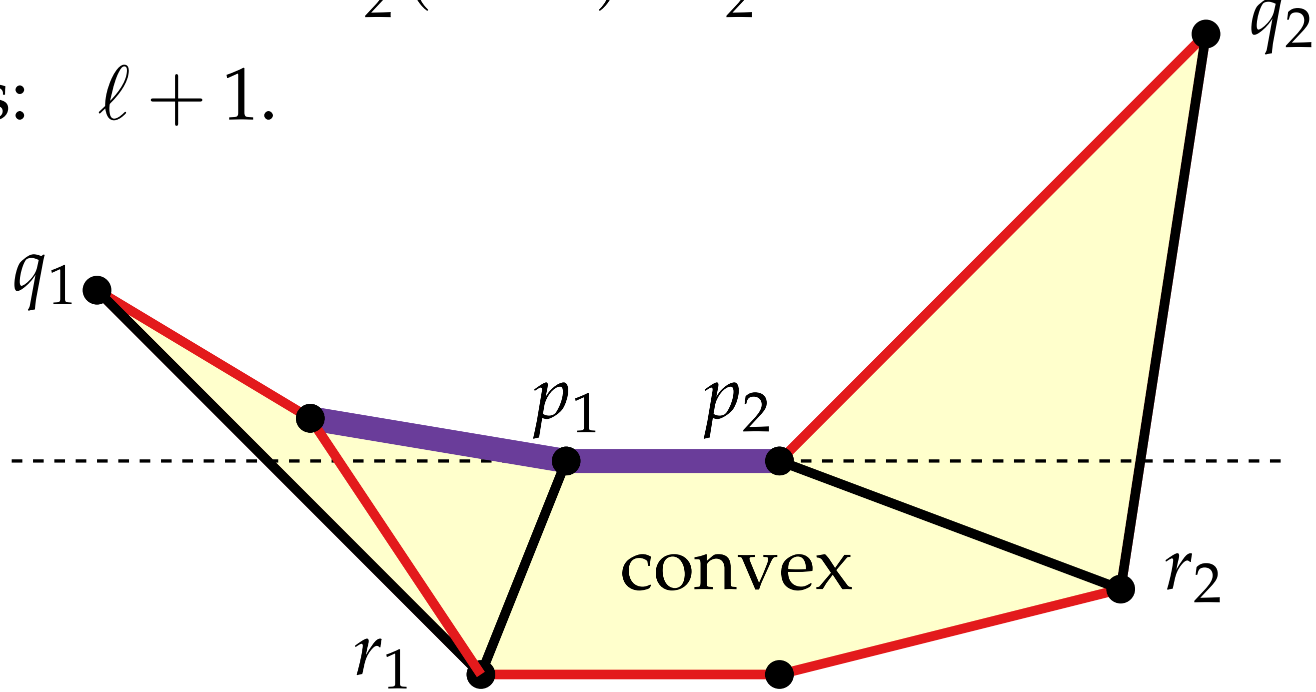

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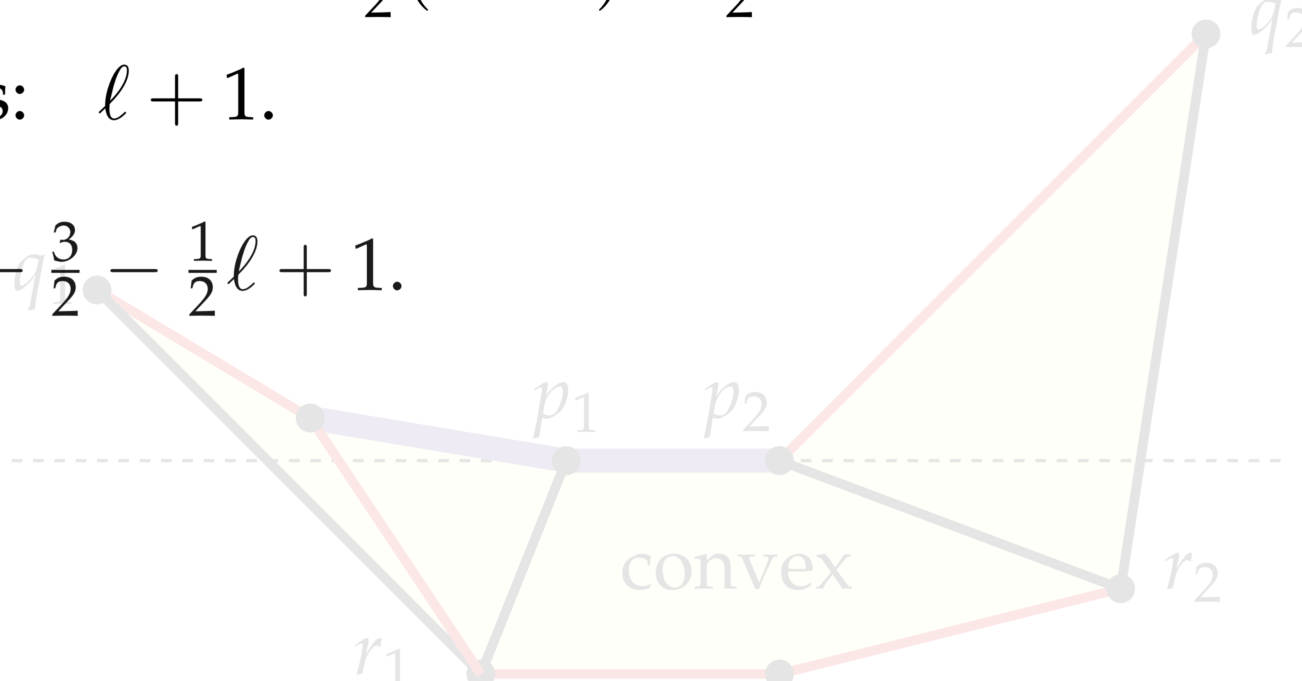

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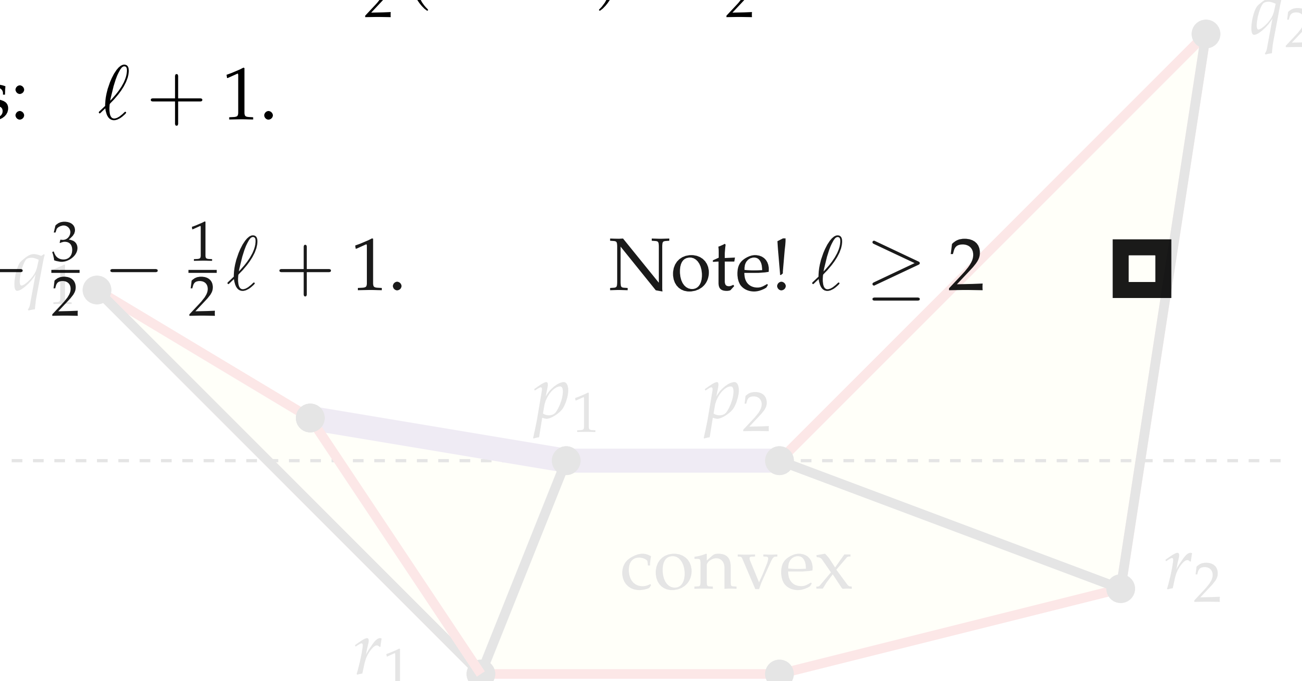
Total: $\frac{3}{2}k + \frac{3}{2} - \frac{1}{2}\ell + 1.$ Note! $\ell \geq 2$ $\square$



Putting it together

Theorem A 3-approximation of a minimum convex partition of P can be computed in $O(n \log n)$ time. (This assumes no colinear points.)

Proof:

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Proof: At the excercise sheet!