Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Computational Geometry

Triangulating Polygons
or
Guarding Art Galleries

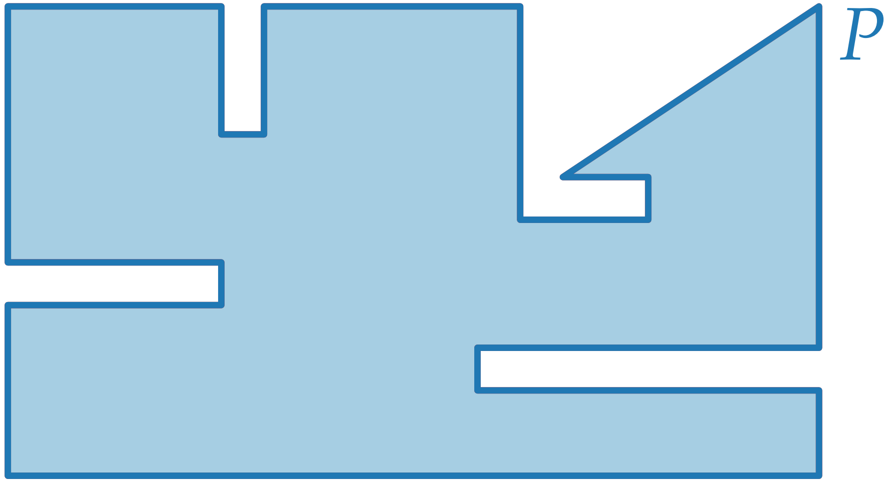
Lecture #2

Thomas van Dijk

Winter Semester 2019/20

Guarding an Art Gallery

Given a *simple* polygon P (i.e., no holes, no self-intersection)...



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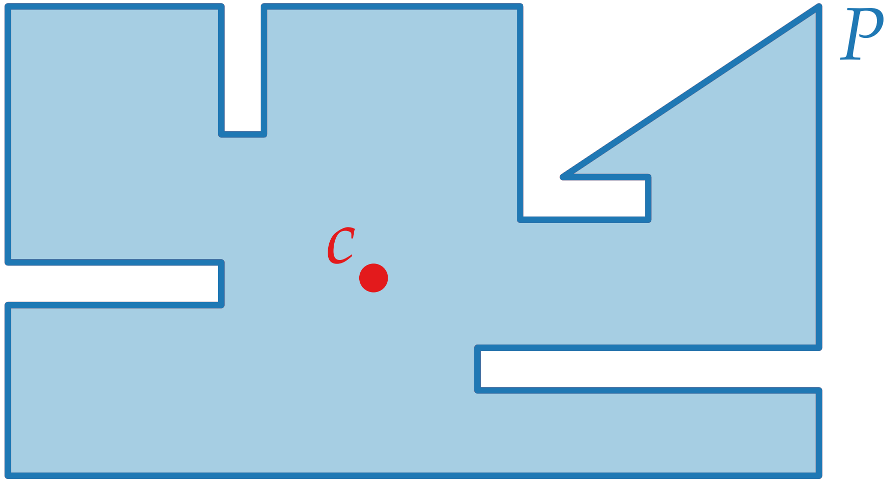

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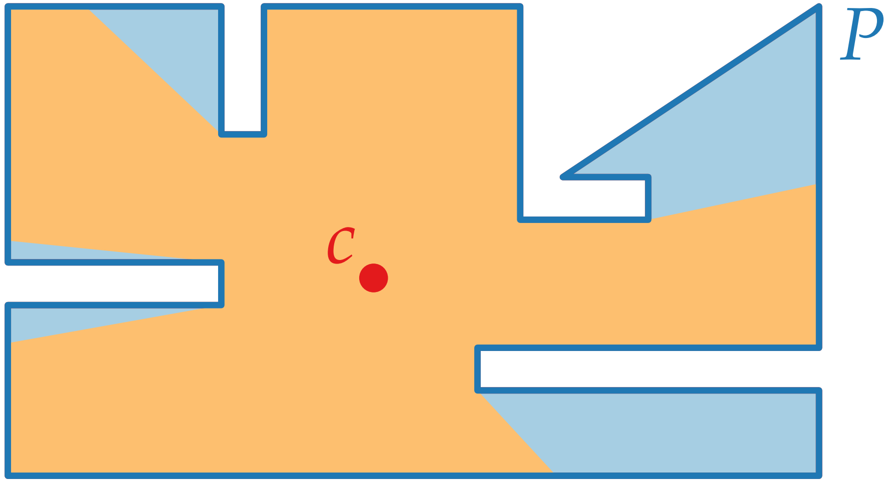

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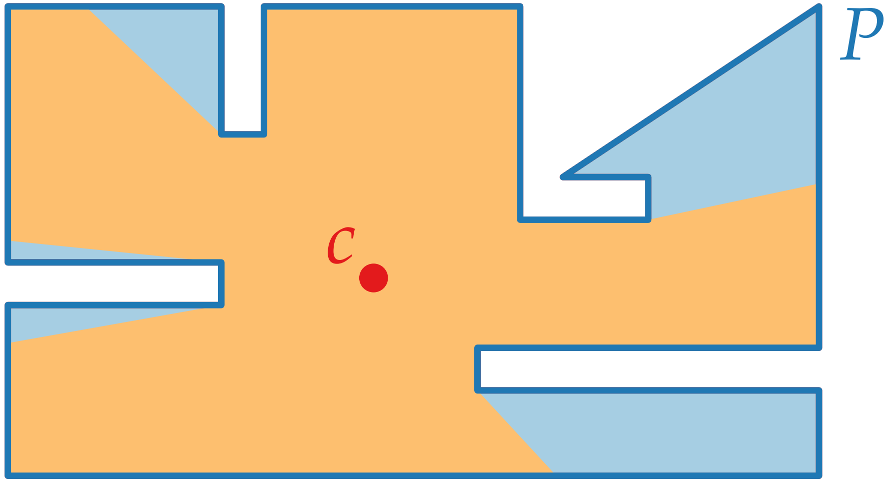

Observation. Camera c "sees" a star-shaped region

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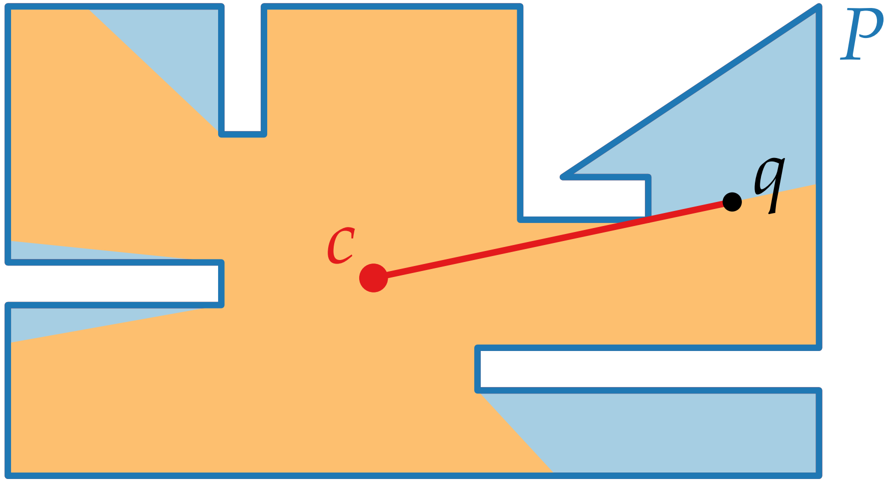

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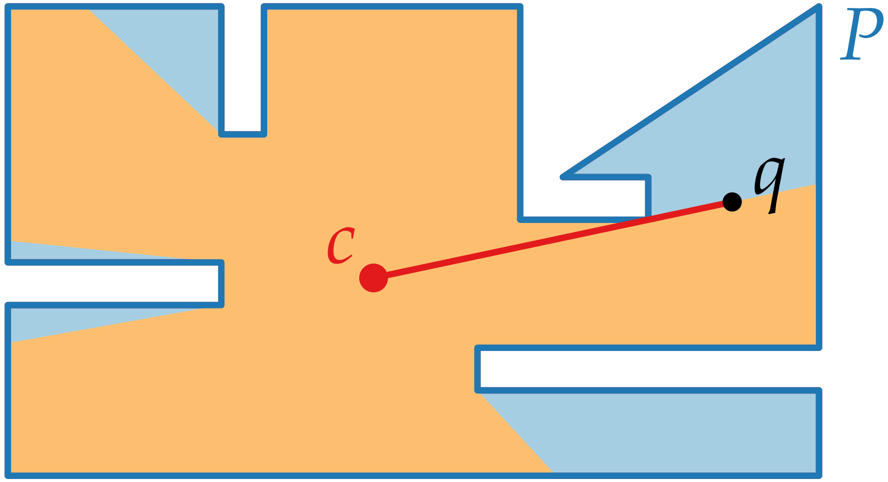

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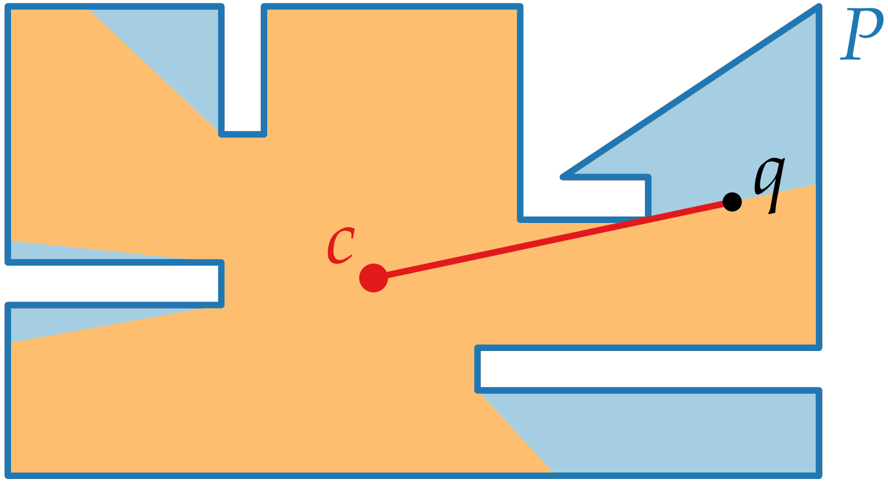

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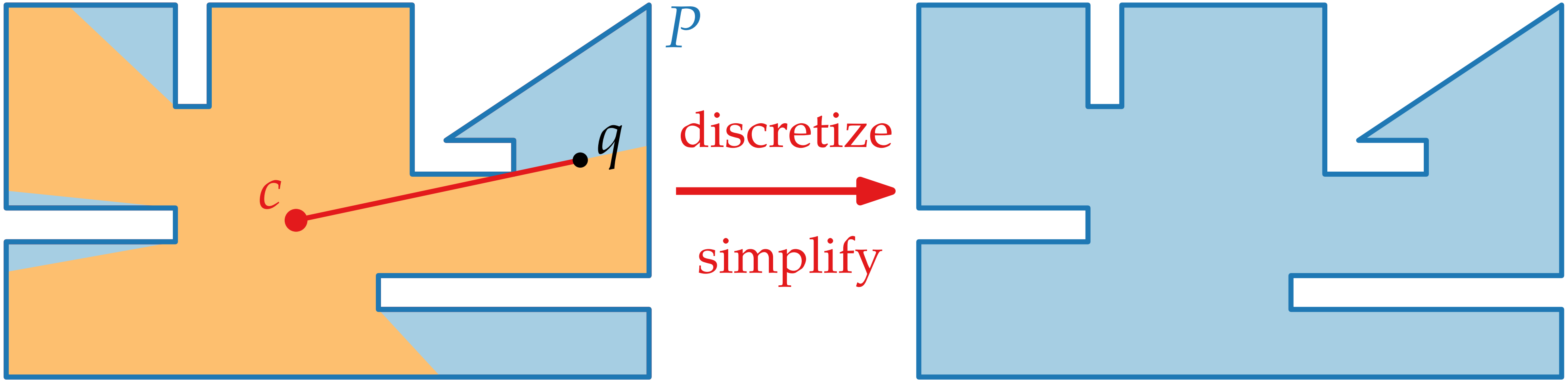

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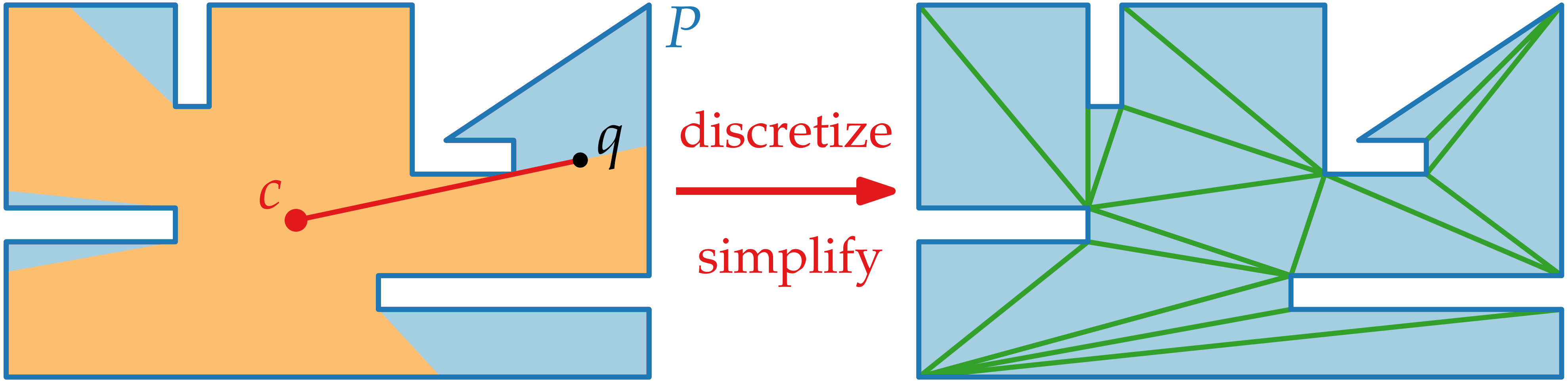

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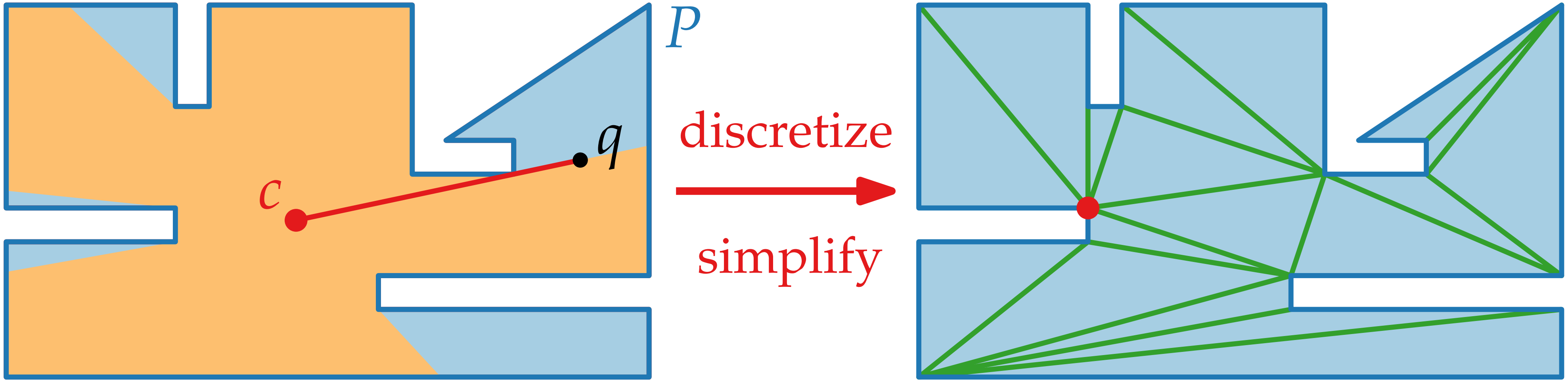

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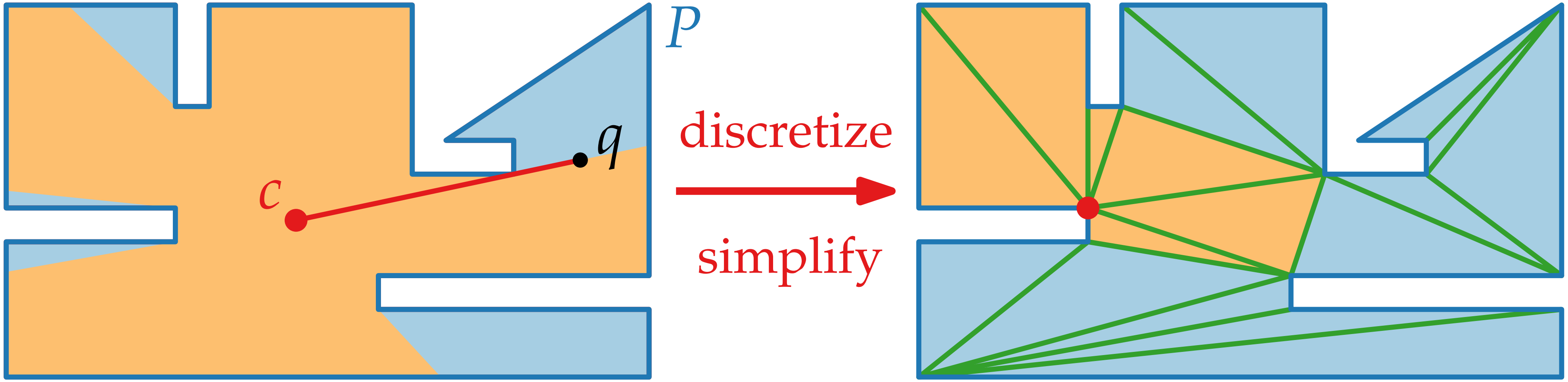

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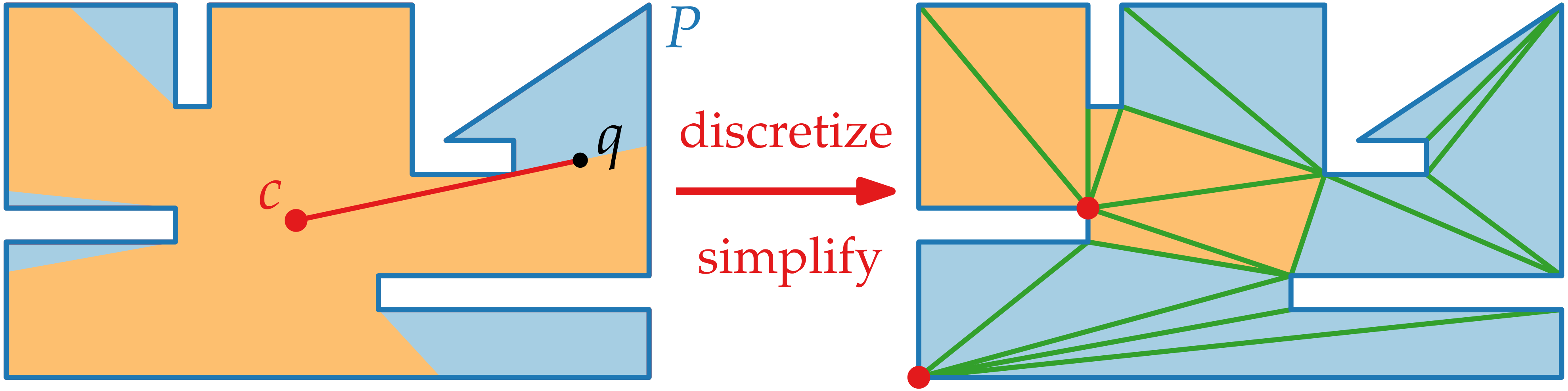

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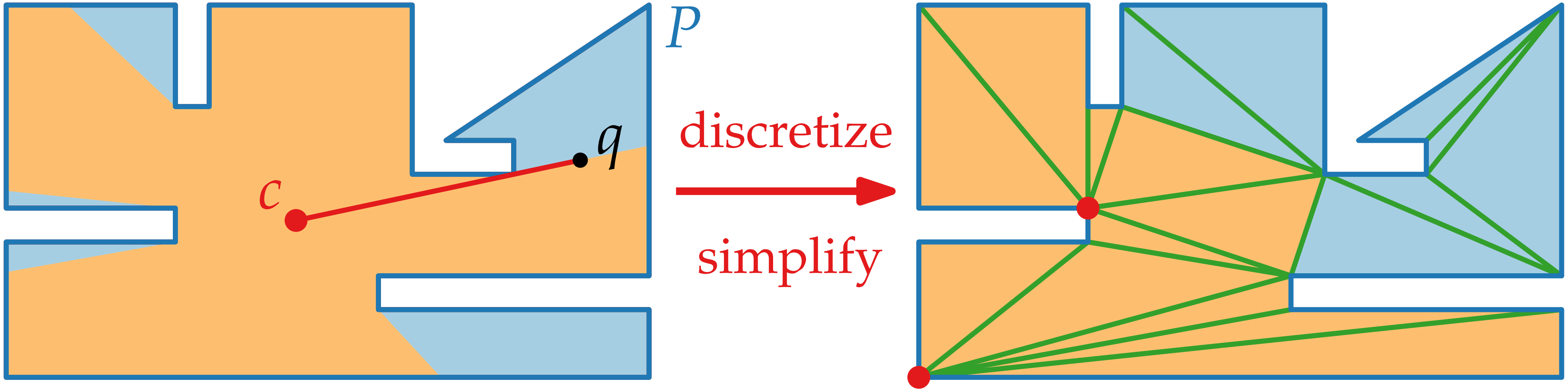

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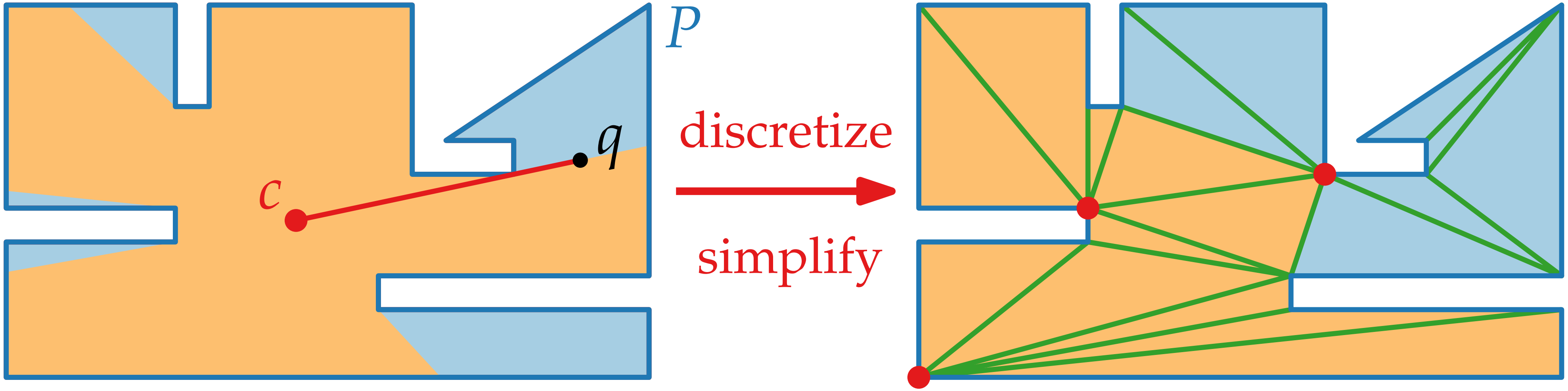

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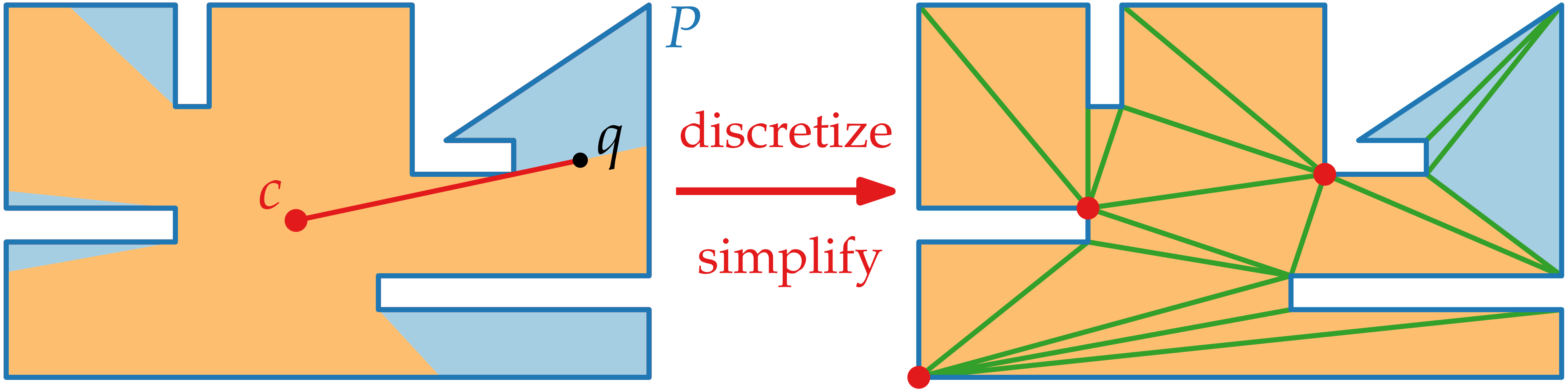

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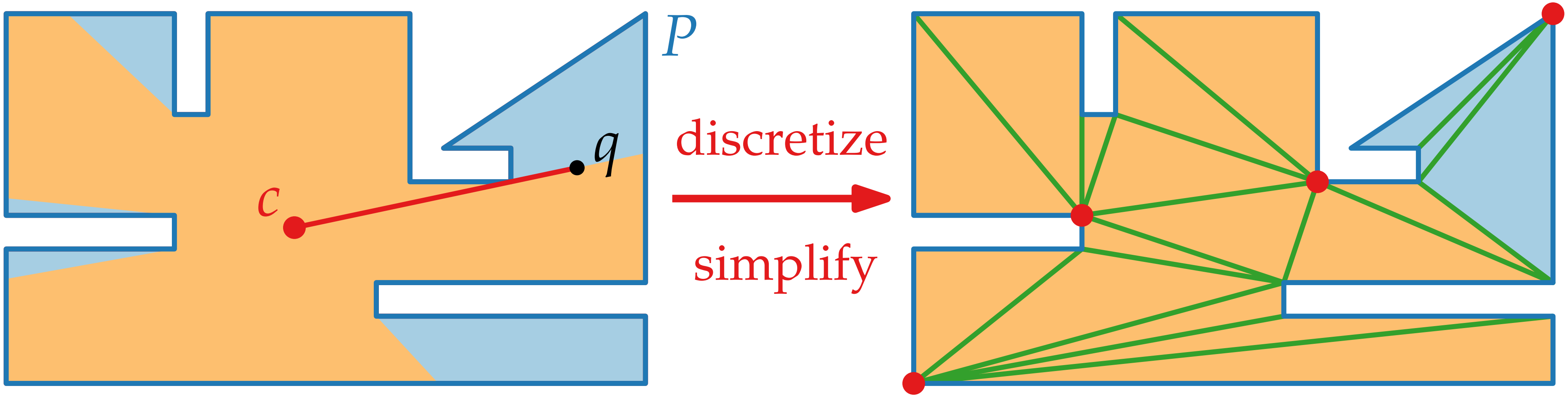

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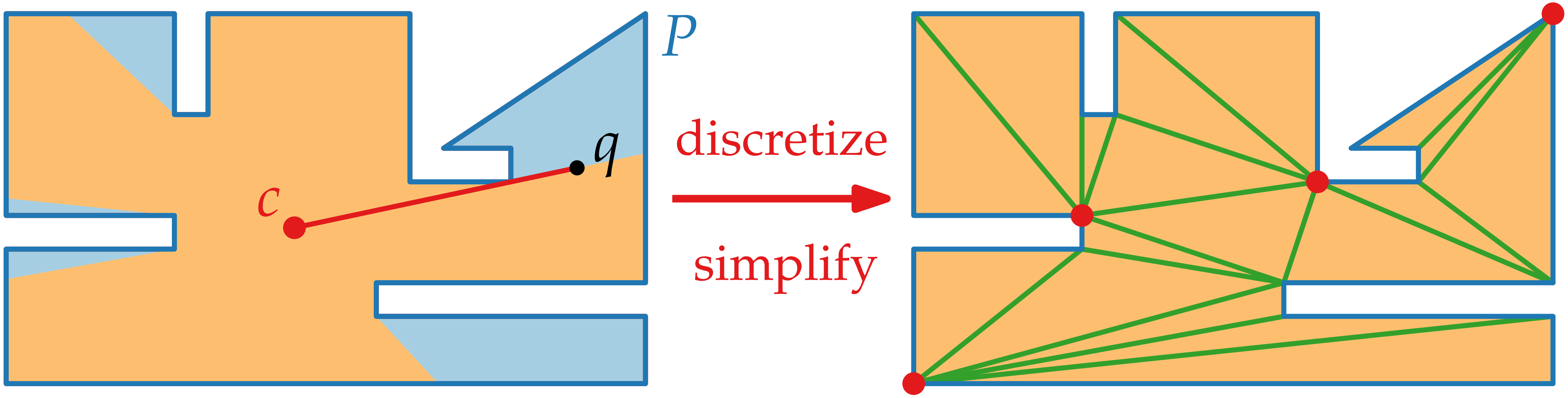

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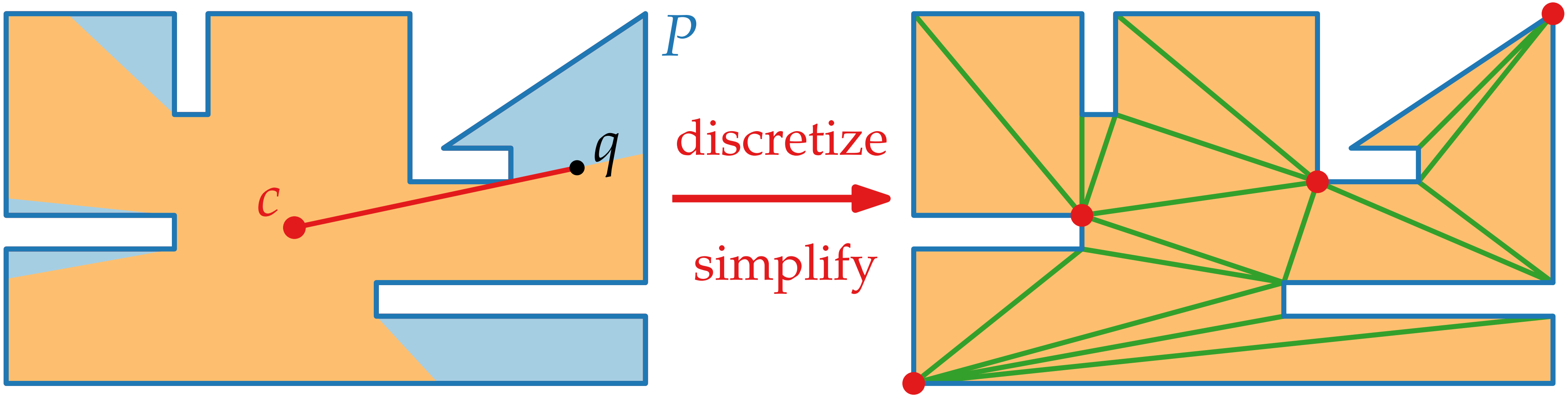

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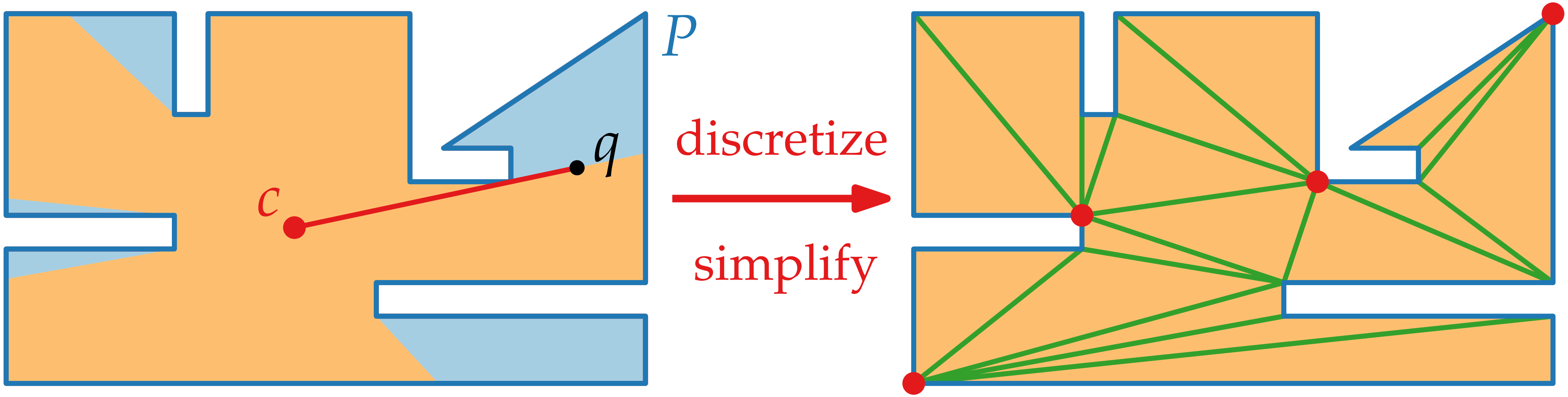

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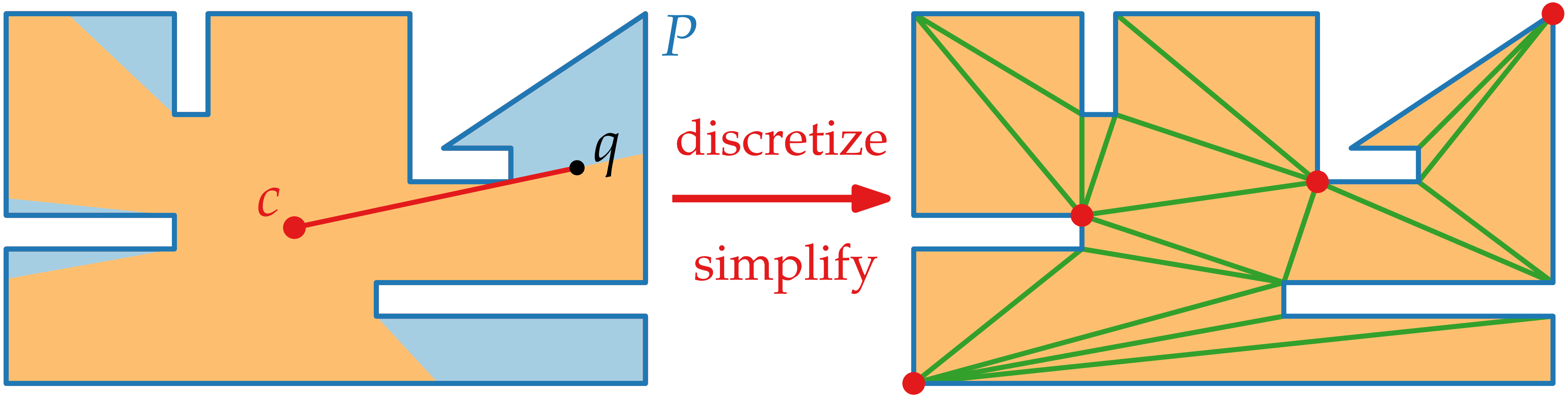

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How can we prove these?

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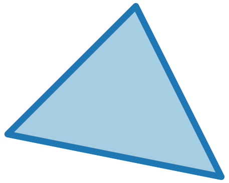
$n = 3$:  1 triangle ✓

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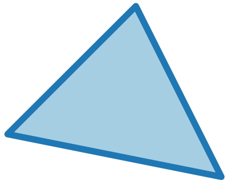

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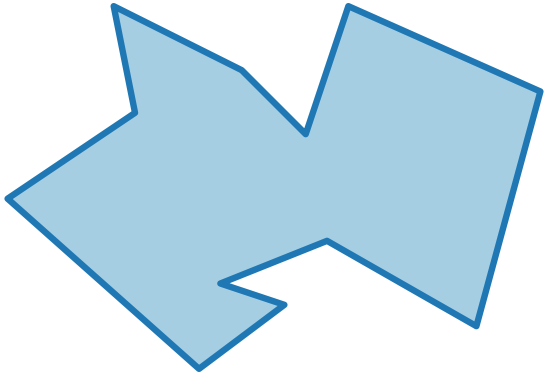

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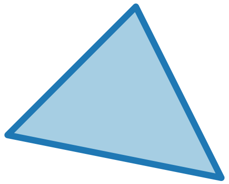

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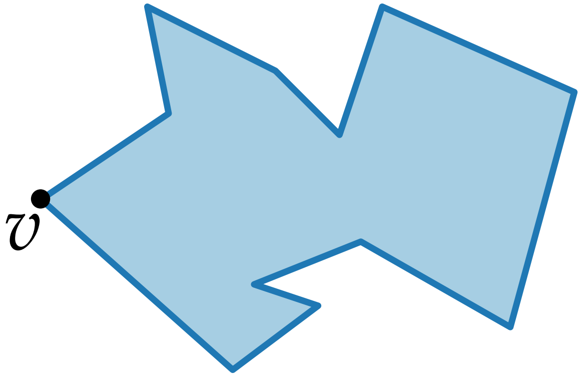

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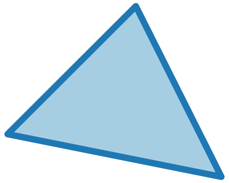
$n = 3$:  1 triangle ✓

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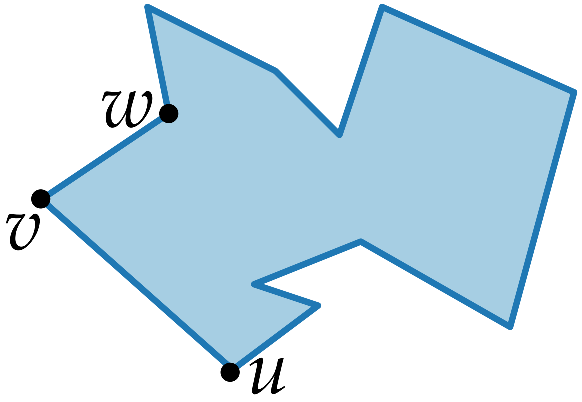

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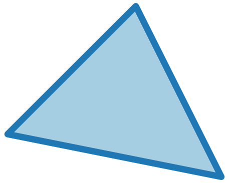
$n = 3$:  1 triangle ✓

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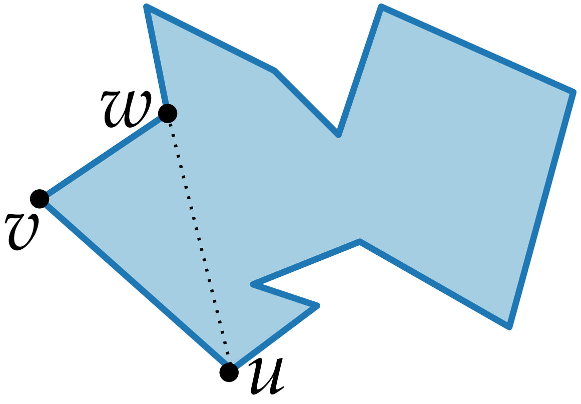

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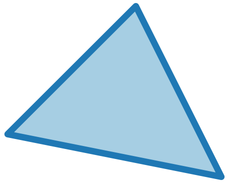
$n = 3$:  1 triangle ✓

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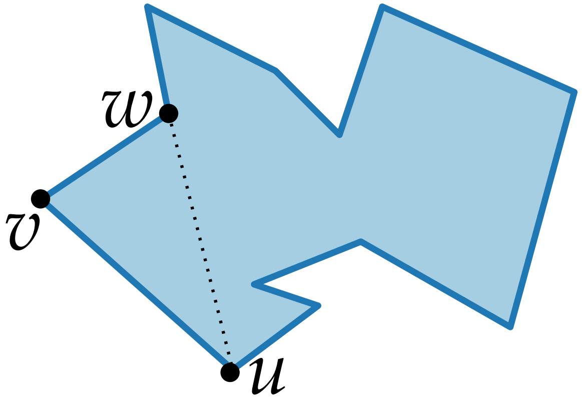

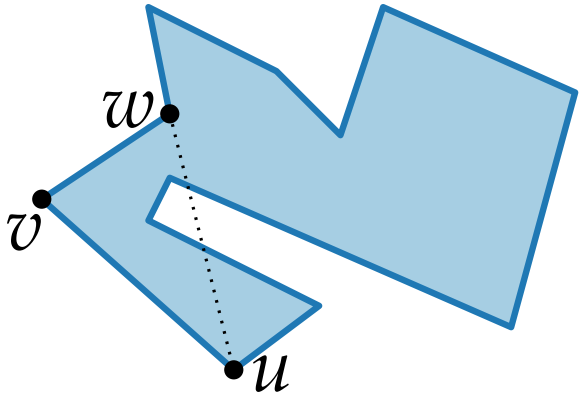

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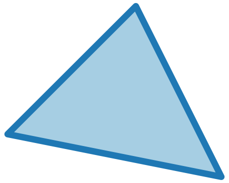

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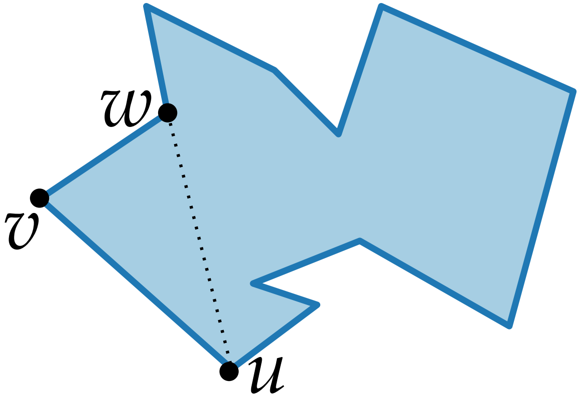
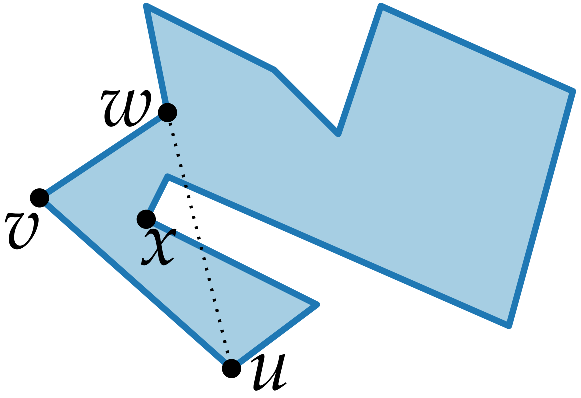


x furthest from uw



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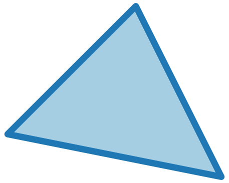

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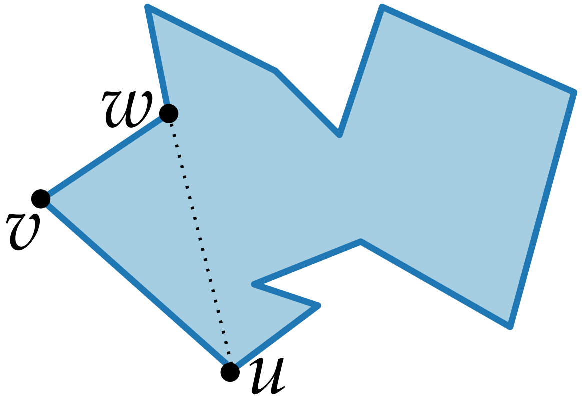

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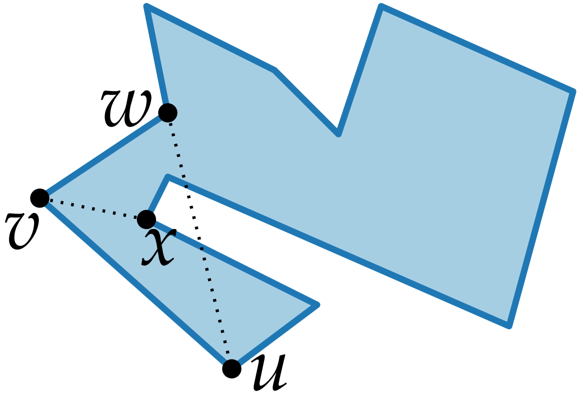

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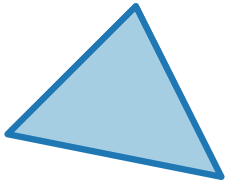

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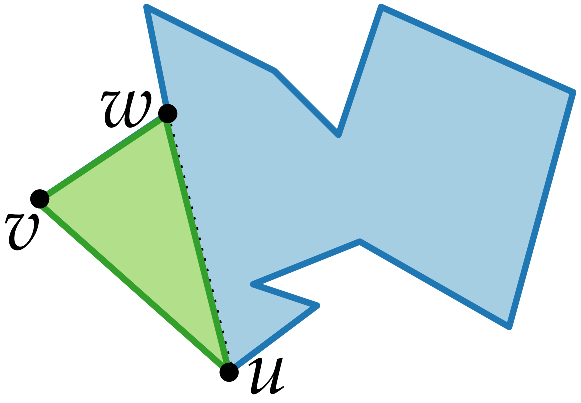

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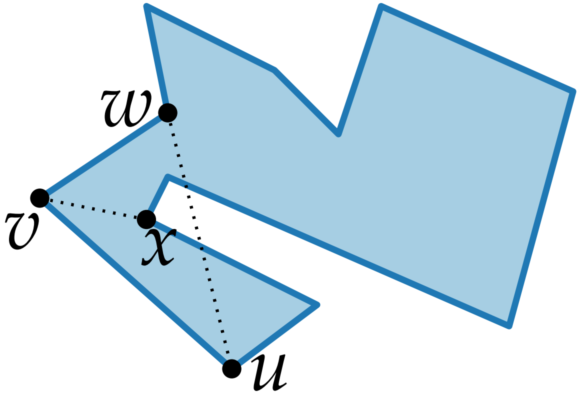

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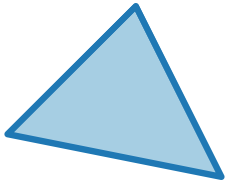
$n = 3$:  1 triangle ✓

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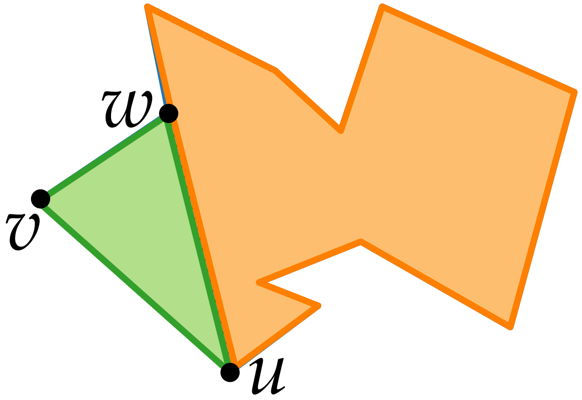

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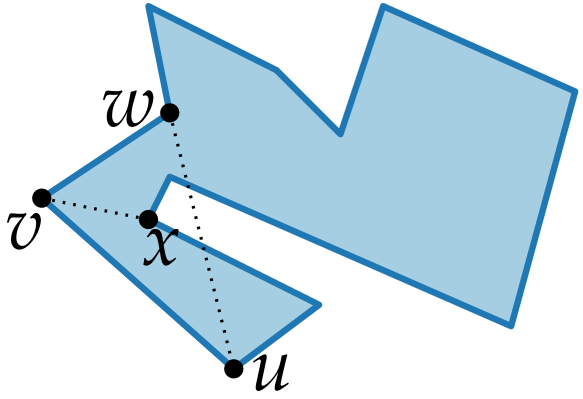

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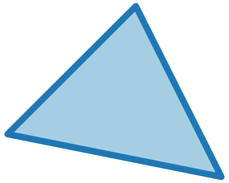
$n = 3$:  1 triangle ✓

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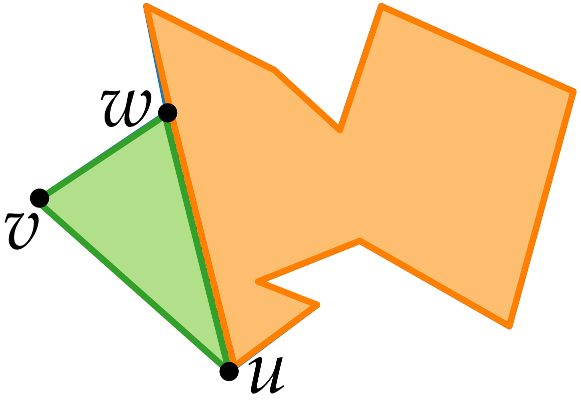

3 vtcs ⇒ 1 triangle

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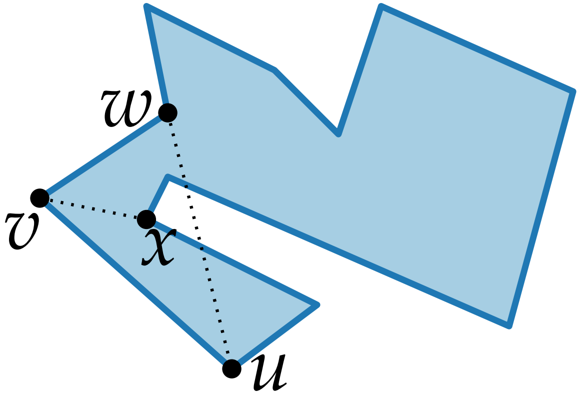

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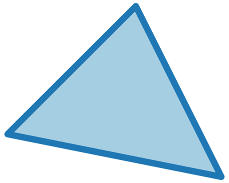

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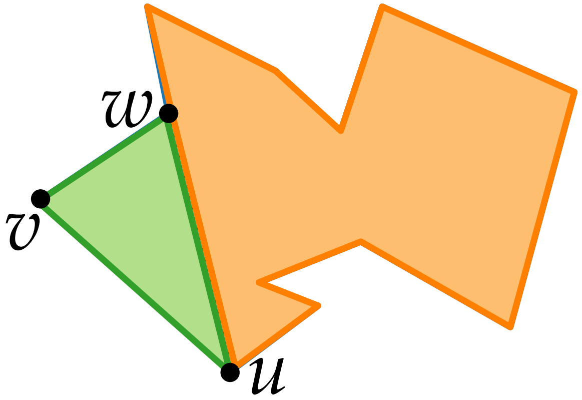

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$n-1$ vtcs ⇒ $n-3$ triangles

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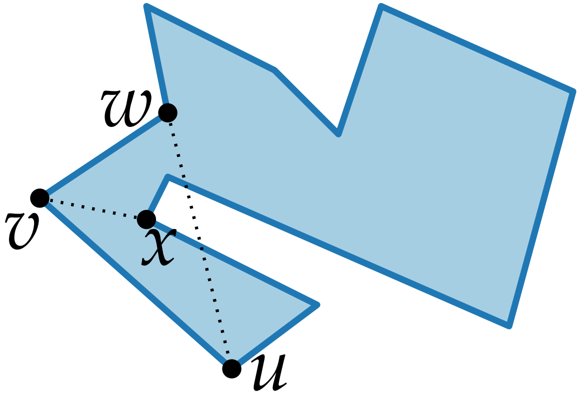

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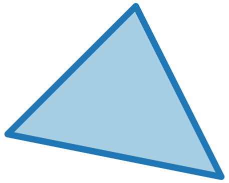
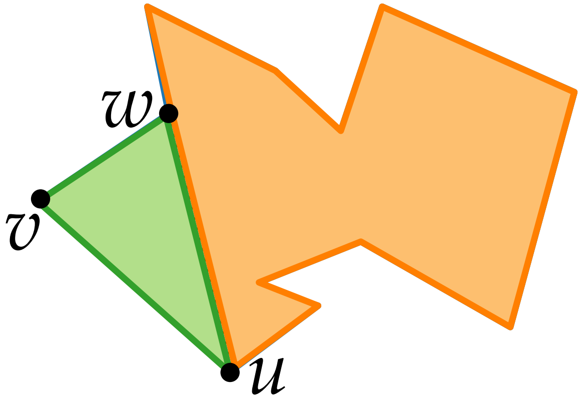
$n = 3$:  1 triangle ✓

$3, \ldots, n-1 \to n$:



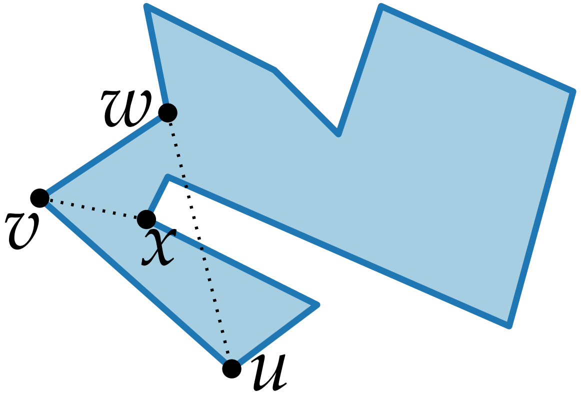
3 vtcs ⇒ 1 triangle

$n-1$ vtcs ⇒ $n-3$ triangles

⇒ $n-2$ triangles

✓

x furthest from uw



Existence of Triangulation

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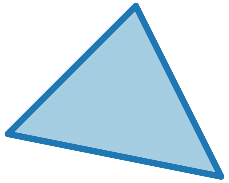
$n = 3$:  1 triangle ✓

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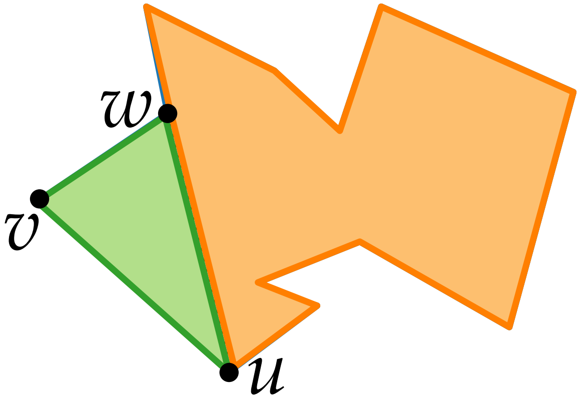

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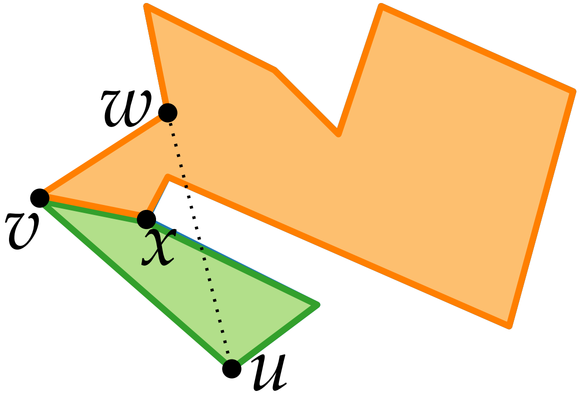
x furthest from uw



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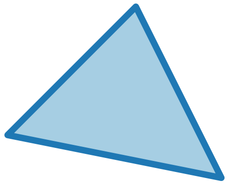
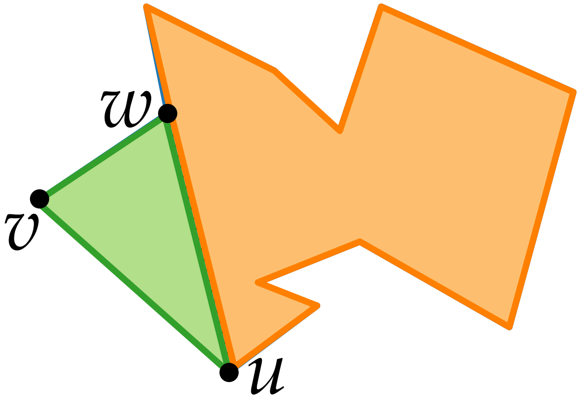
$n = 3$:  1 triangle ✓

$3, \ldots, n - 1 \to n$:



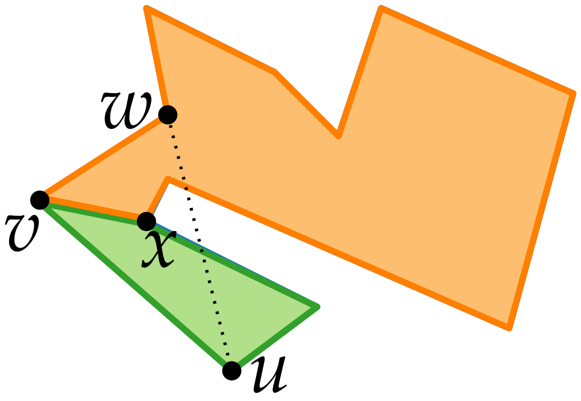
3 vtcs $\Rightarrow$ 1 triangle

$n{-}1$ vtcs $\Rightarrow$ $n{-}3$ triangles

$\Rightarrow$ $n{-}2$ triangles

✓

x furthest from uw



m vtcs $\Rightarrow$ $m{-}2$ triangles

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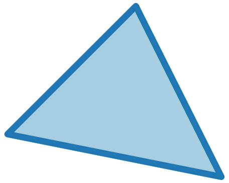

1 triangle ✓

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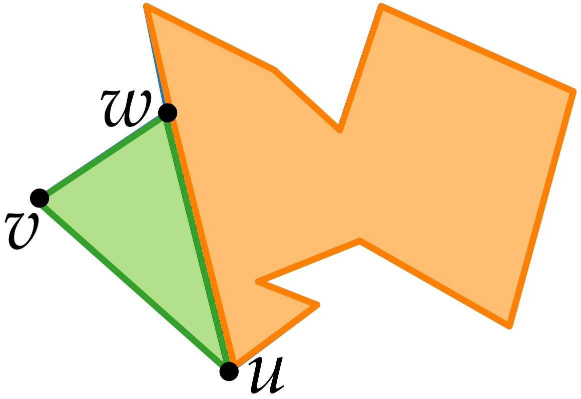

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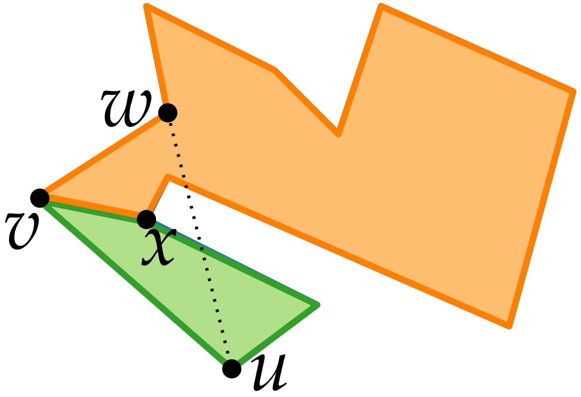
⇒ $n-2$ triangles
✓

x furthest from uw



m vtcs ⇒ $m-2$ triangles
$n-m+2$ vtcs ⇒ $n-m$ triangles

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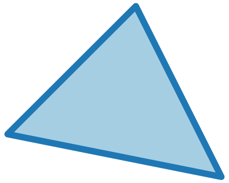
$n = 3$:  1 triangle ✓

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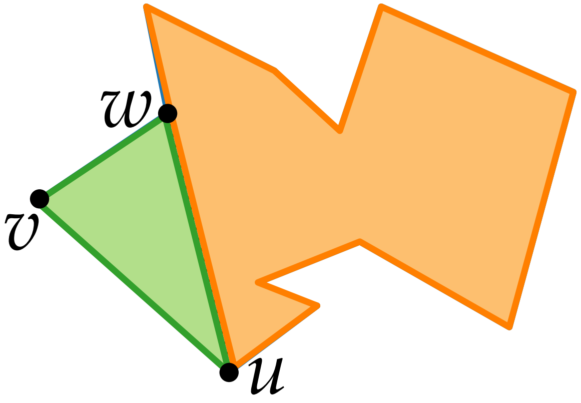

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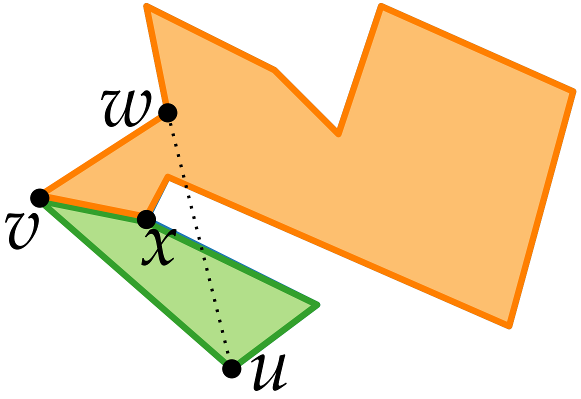

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$n-m+2$ vtcs ⇒ $n-m$ triangles

⇒ $n-2$ triangles

✓

The Art Gallery Theorem [Chvátal ’75]

Theorem. For surveilling a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient.

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Exercise. Find, for arbitrarily large n, a polygon with n vertices, where $\approx n/3$ cameras are necessary.

[2 minutes]

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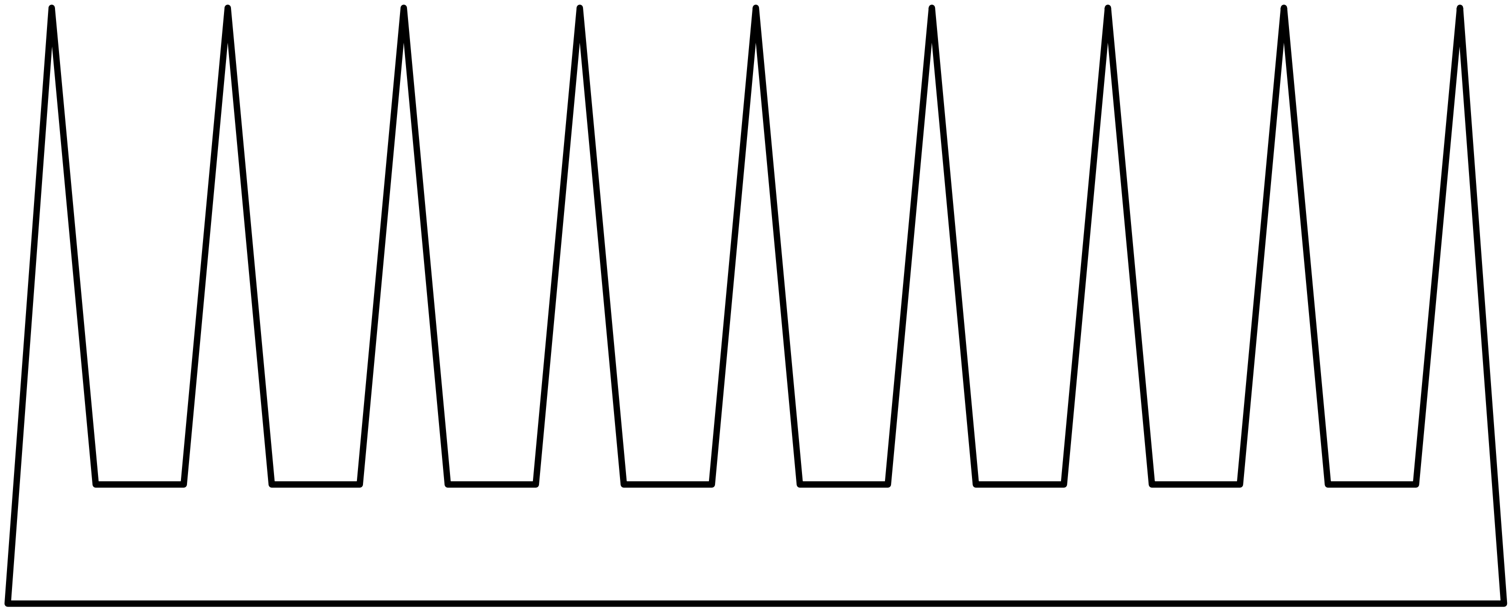

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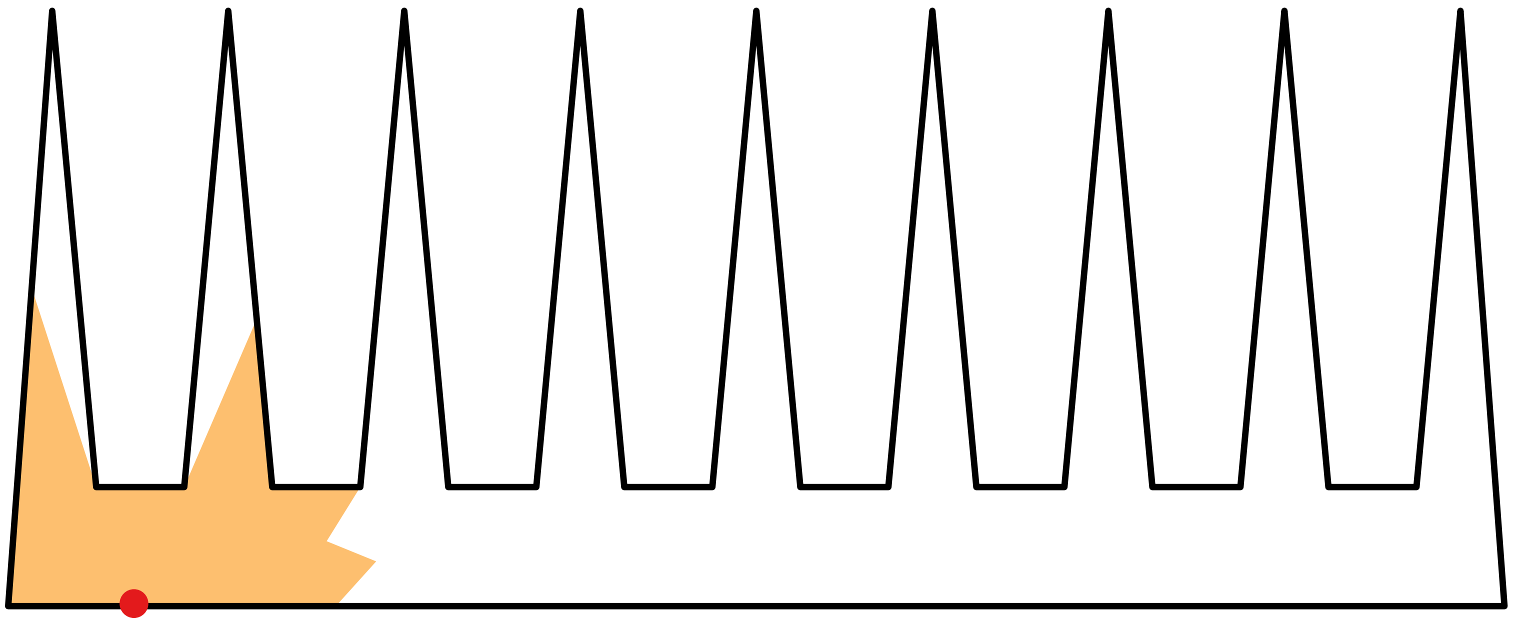

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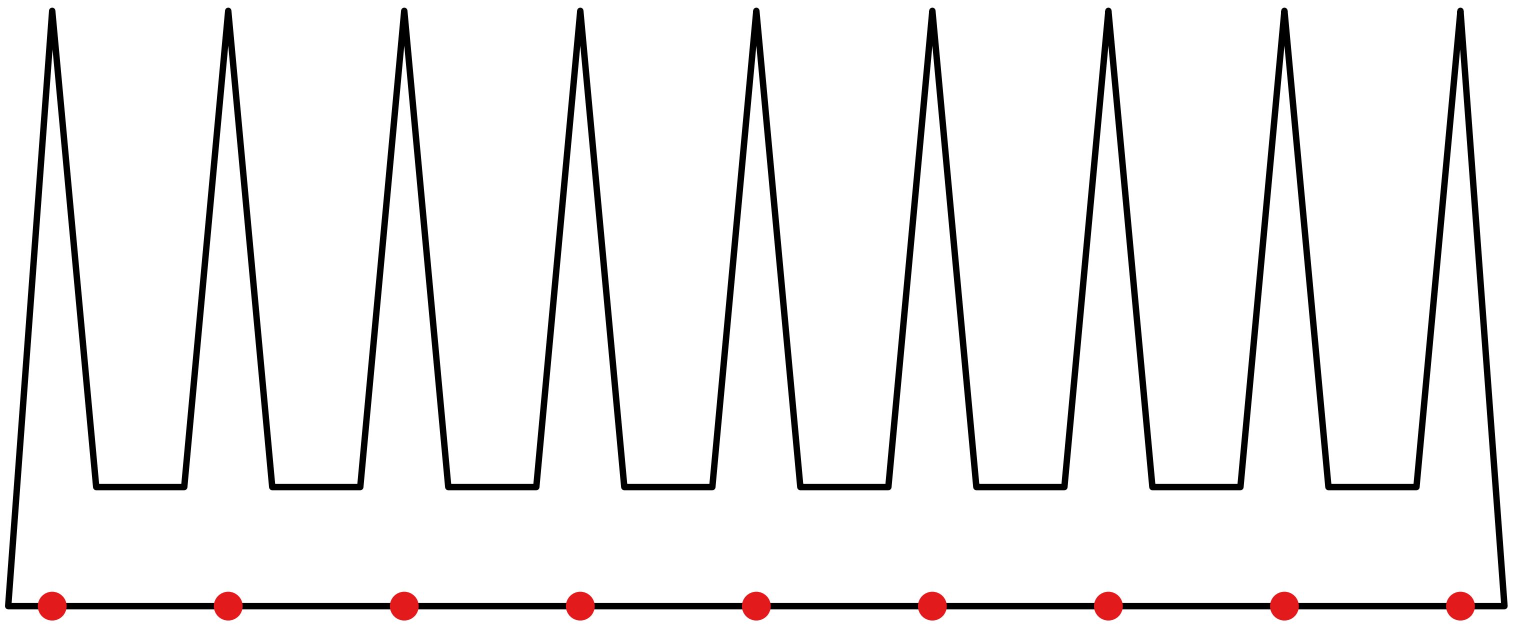

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rectilinear

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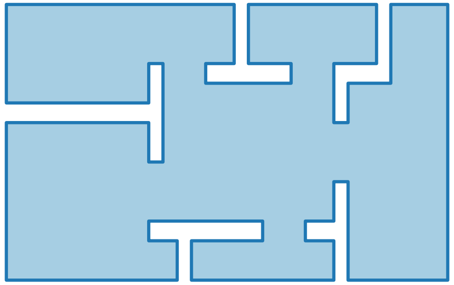
$n/4$



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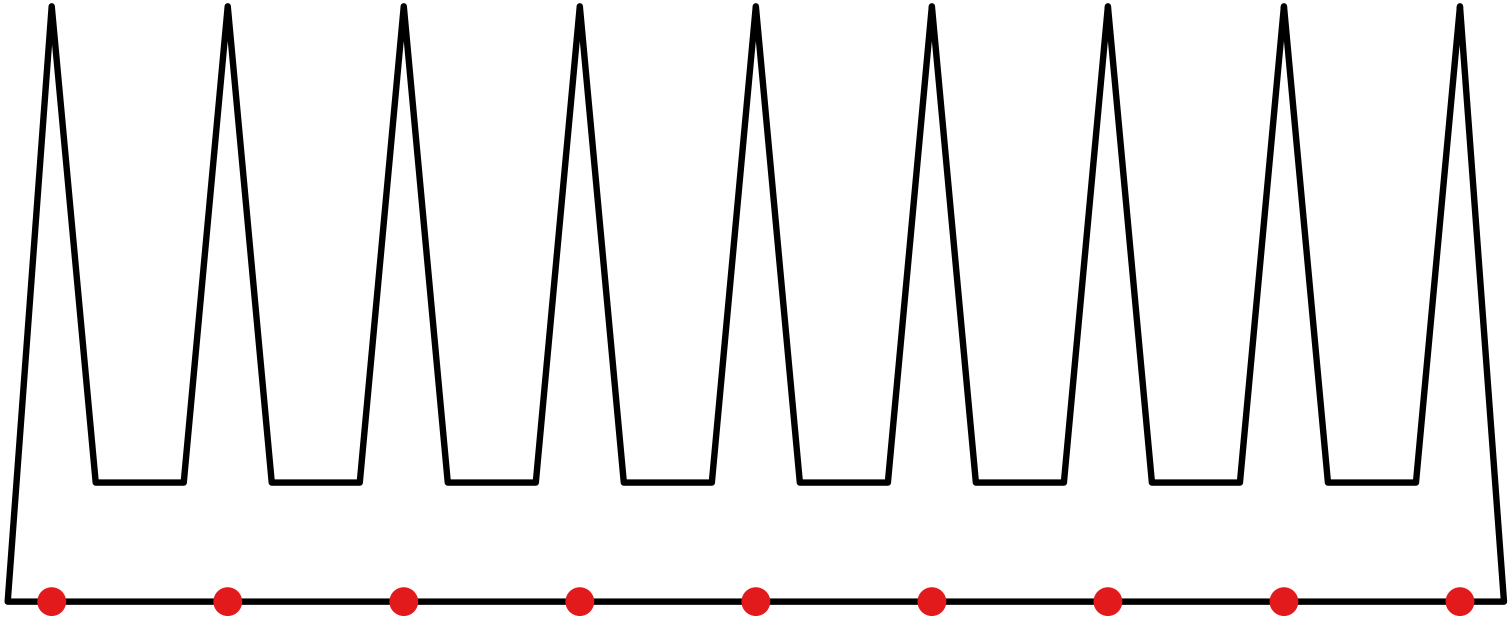

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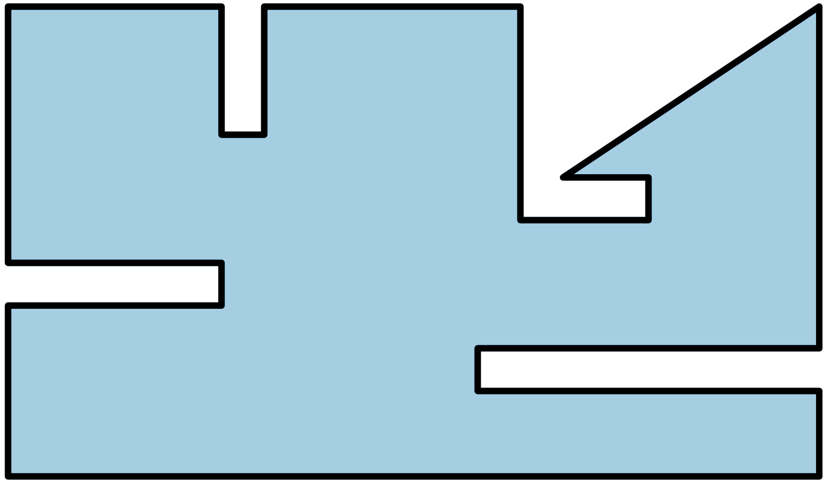

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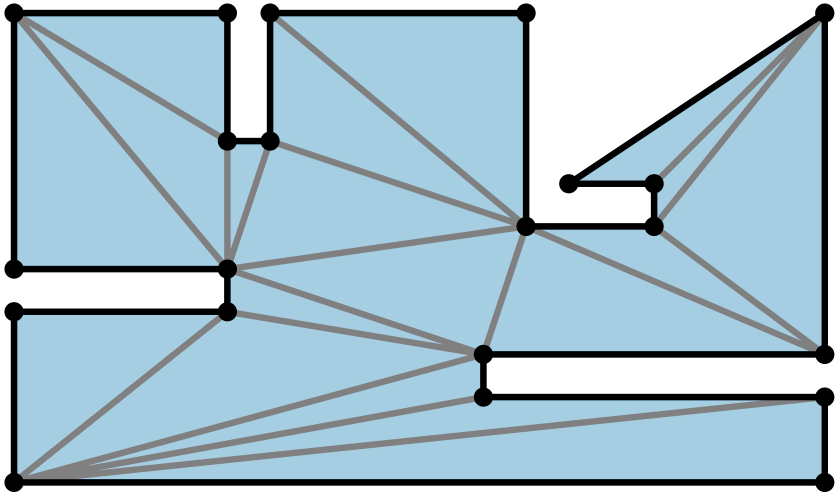

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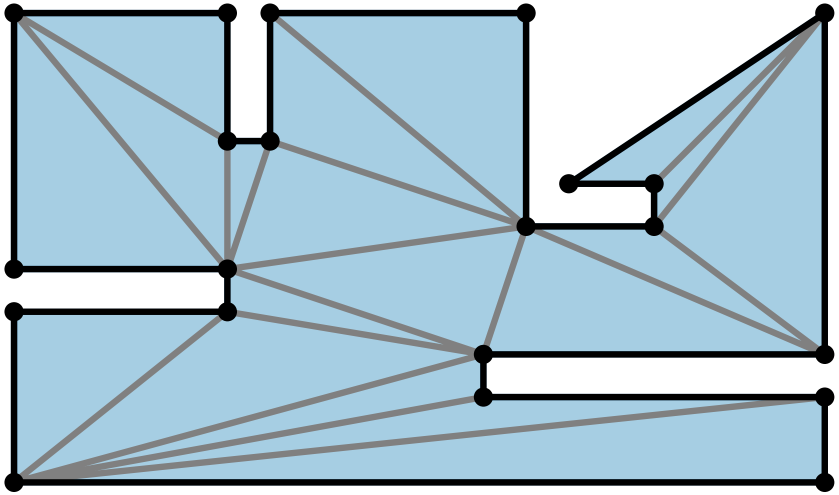

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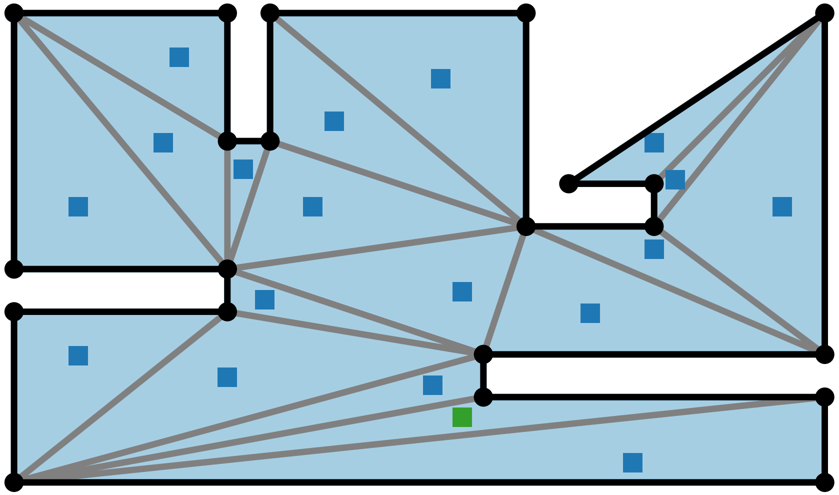

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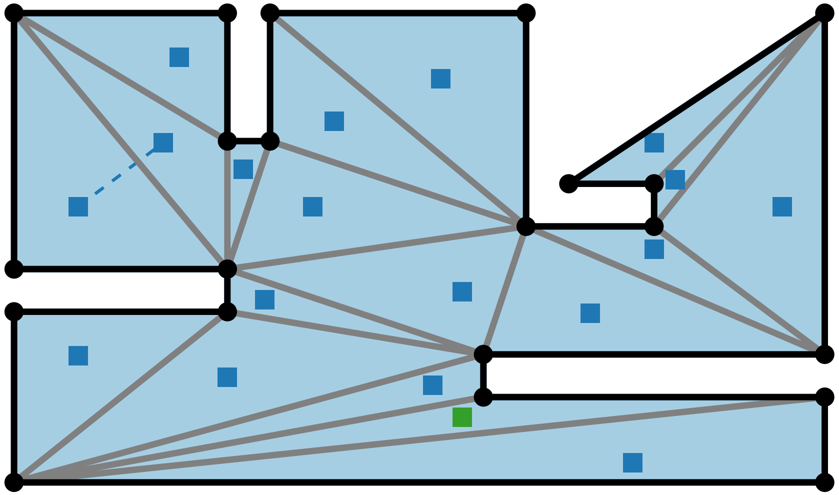

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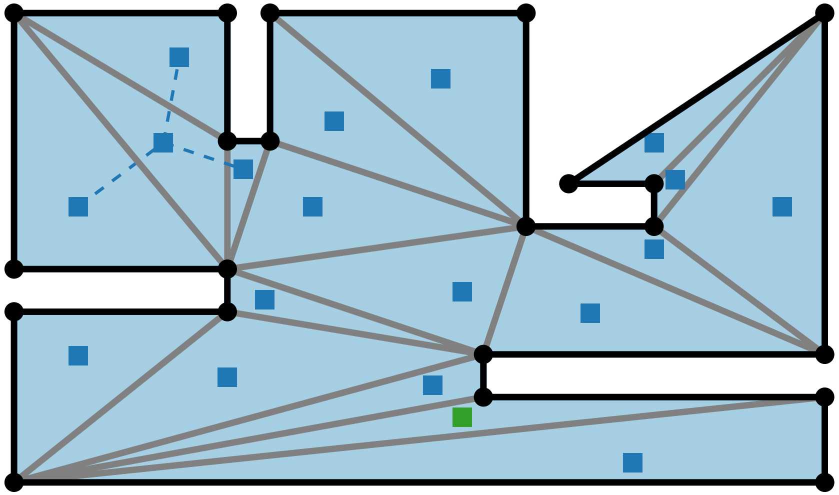

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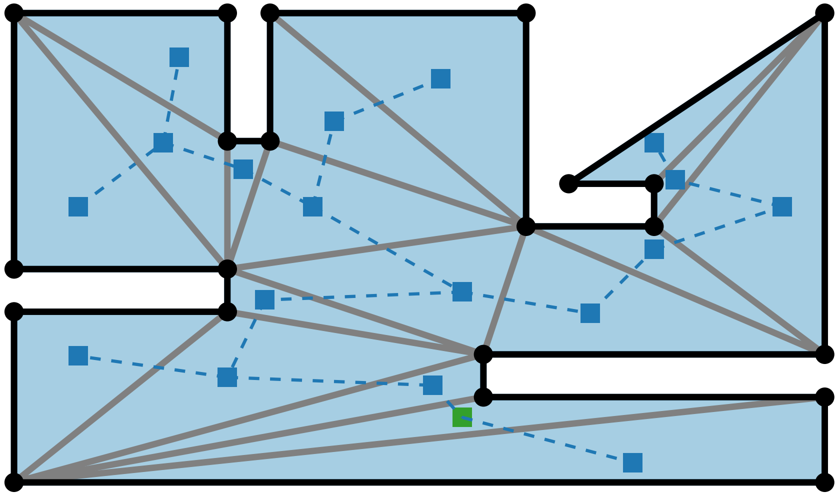
dual tree



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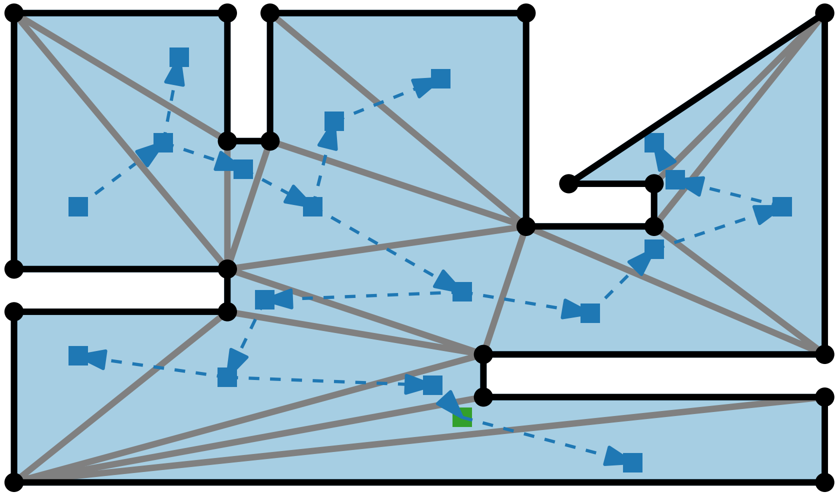

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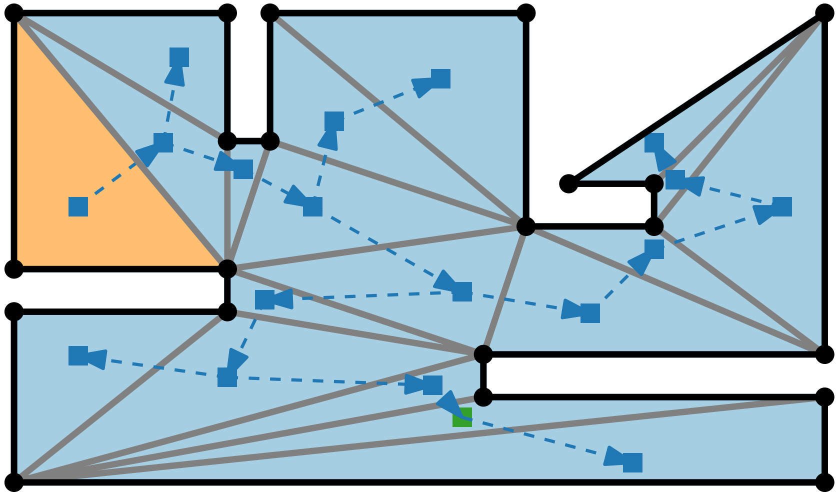

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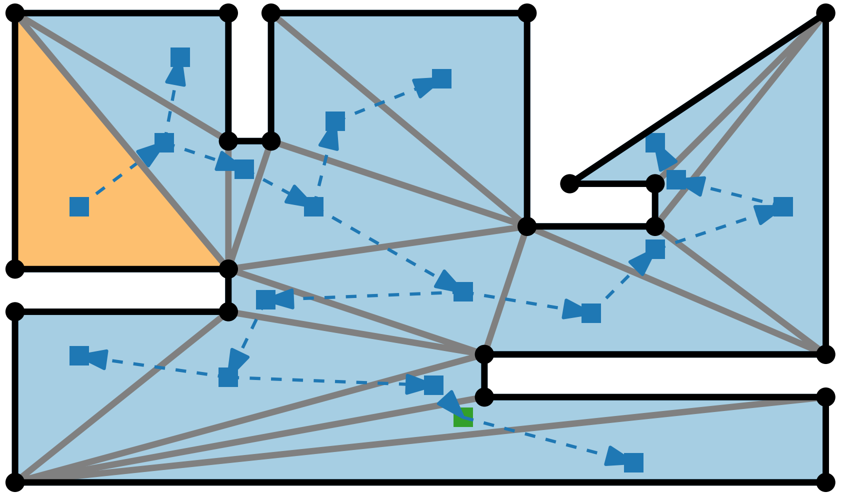

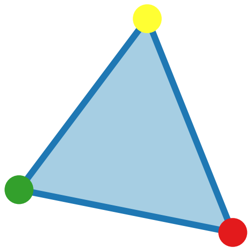

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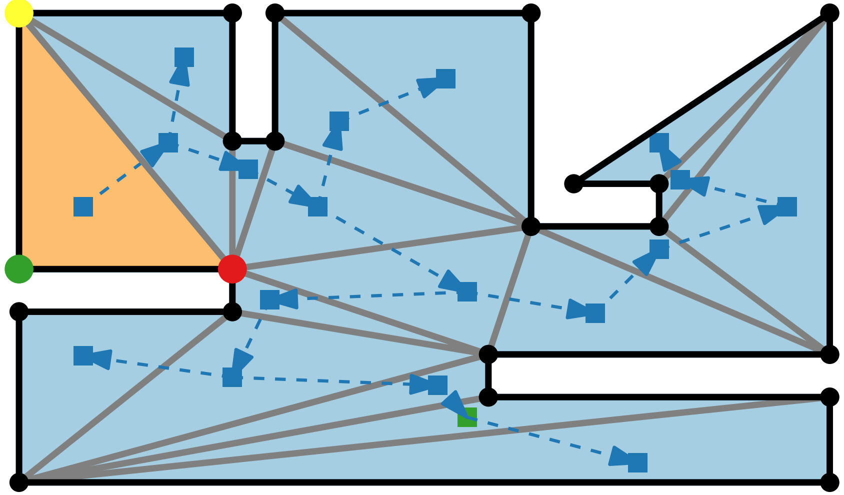

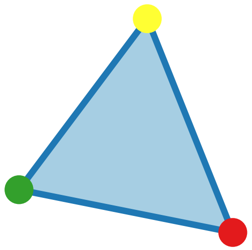

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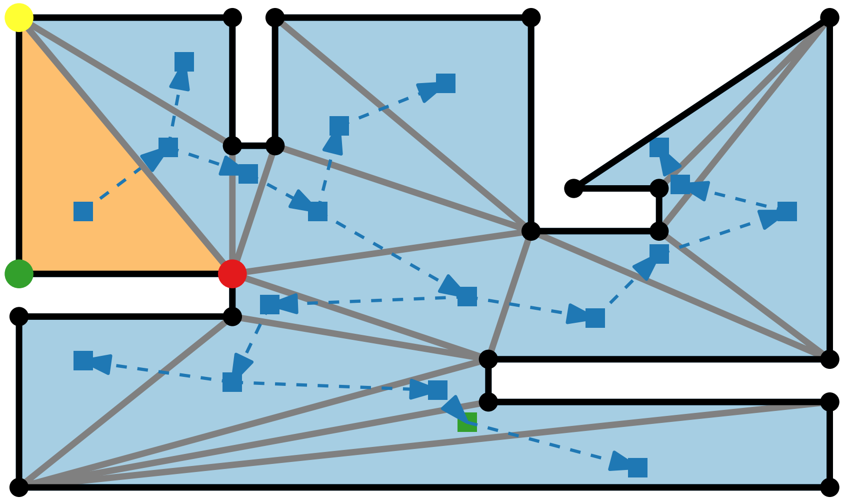

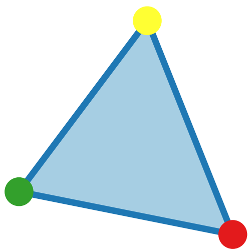

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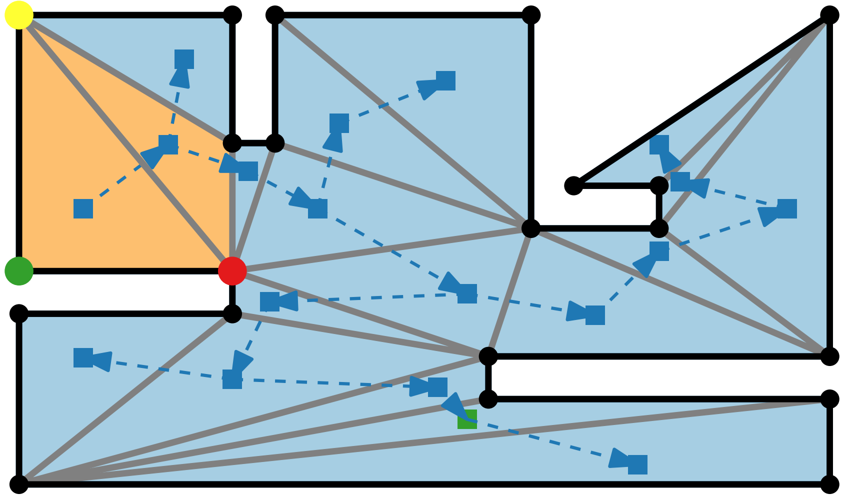

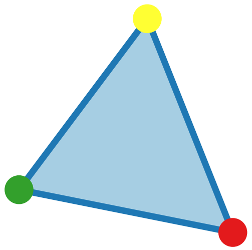

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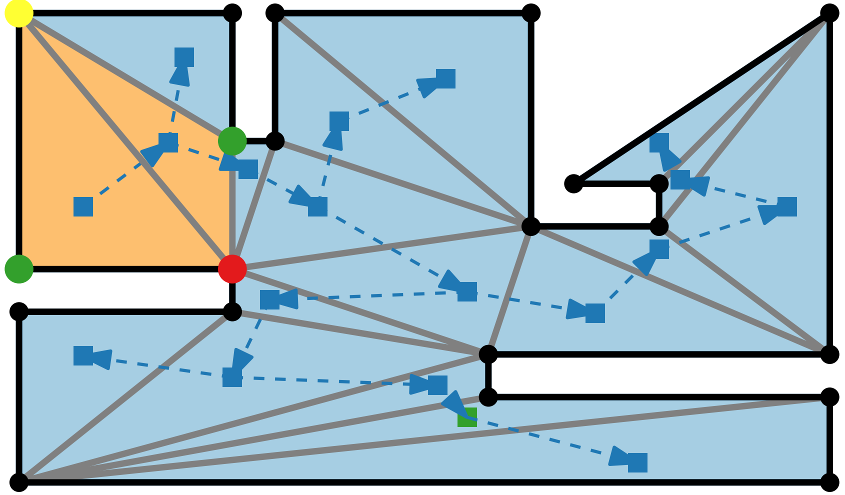

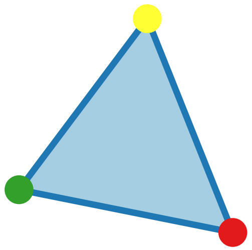

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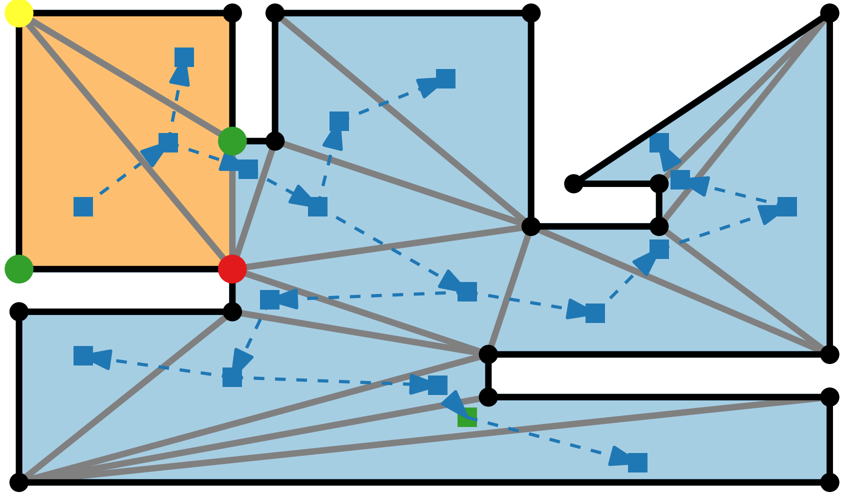

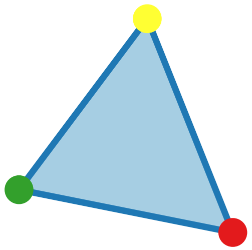

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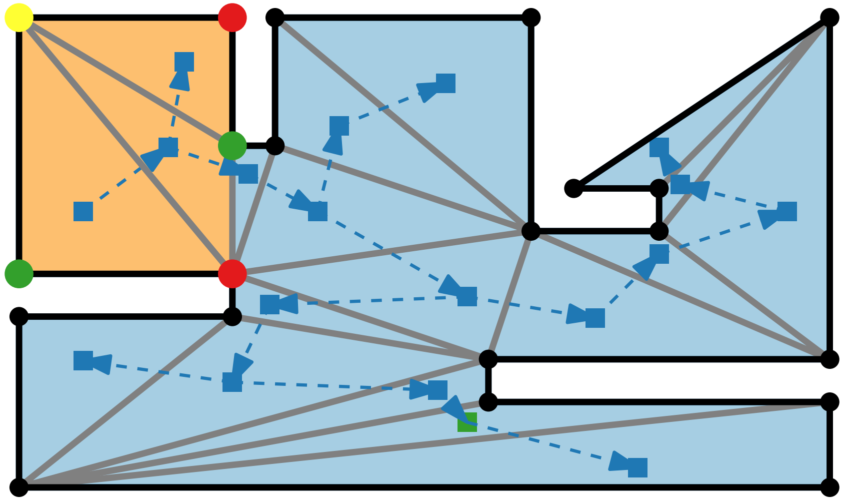

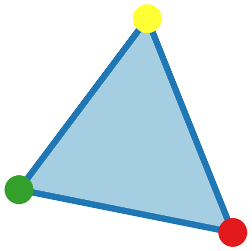

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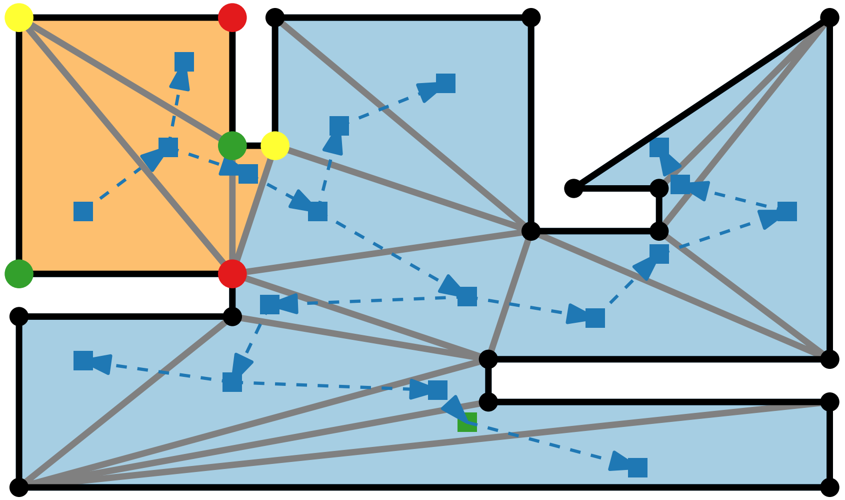

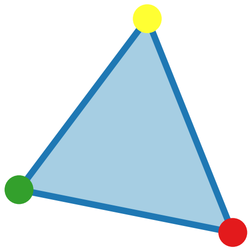

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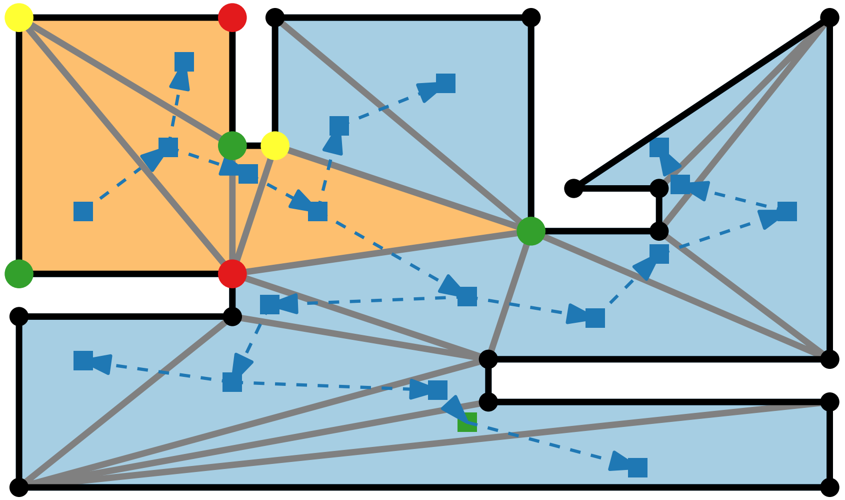

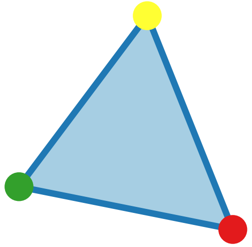

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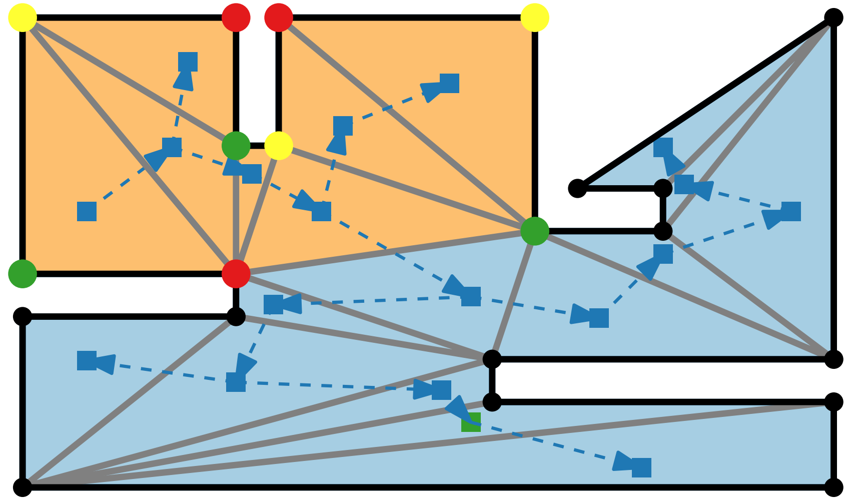

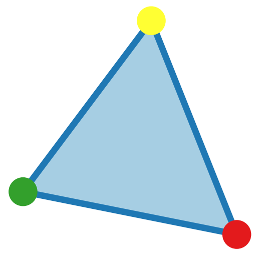

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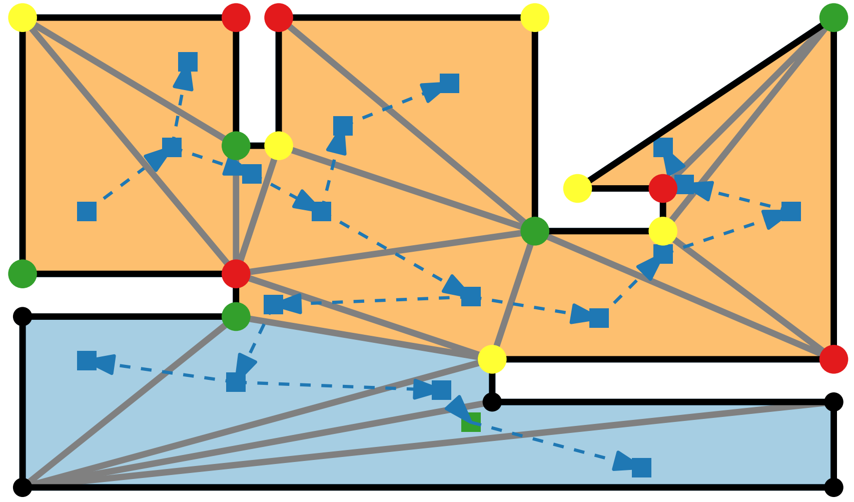

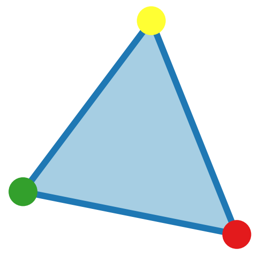

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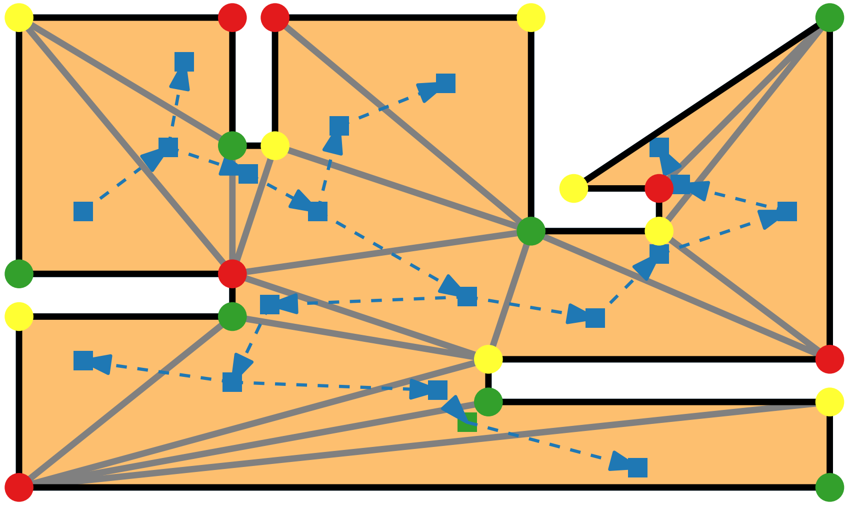

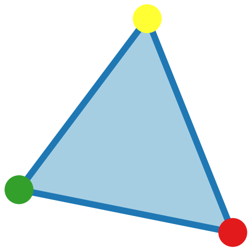

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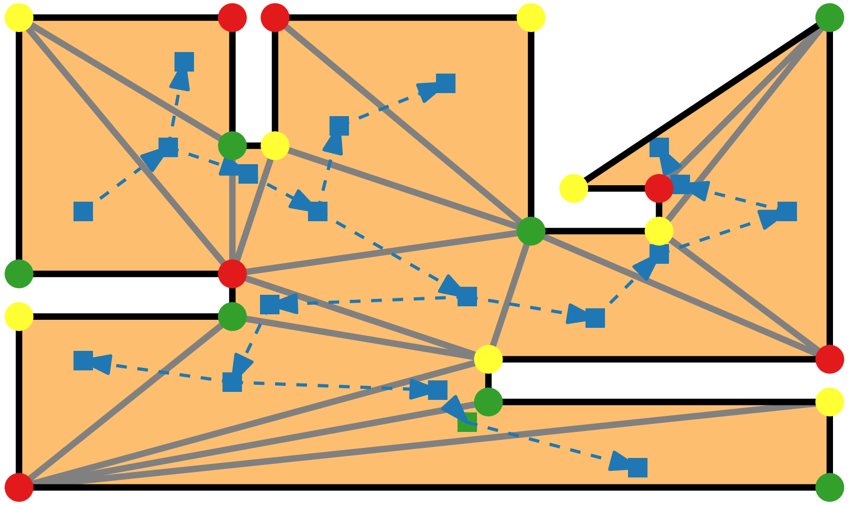
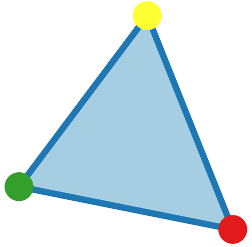




Pick “smallest” color

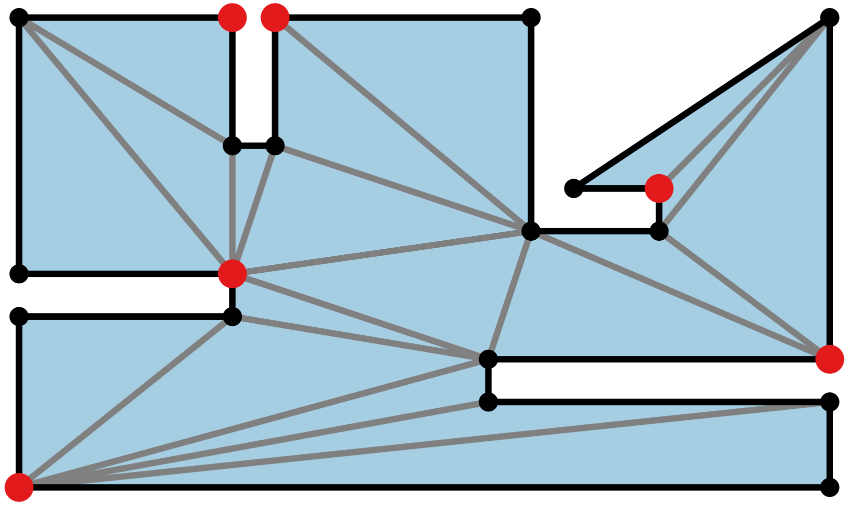
The Art Gallery Theorem

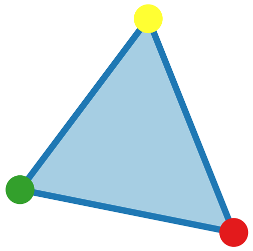
[Chvátal ’75]

Theorem. For surveilling a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient.

3-color the vtcs

Traverse the dual tree





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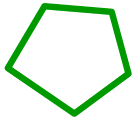

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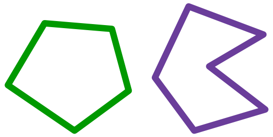

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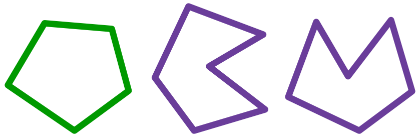

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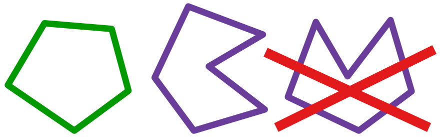

Part. a Polygon into Monotone Pieces

Idea: Classify vertices of given simple polygon P

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– *turn* vertices:

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vertical component of walking direction changes

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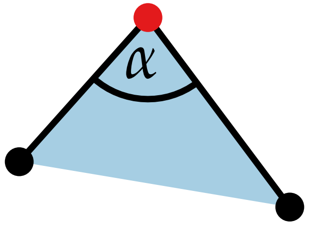

if $\alpha < 180°$

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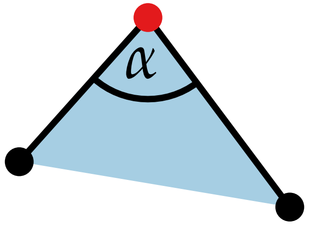
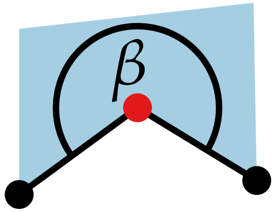


if $\alpha < 180^\circ$

- *split* vertex



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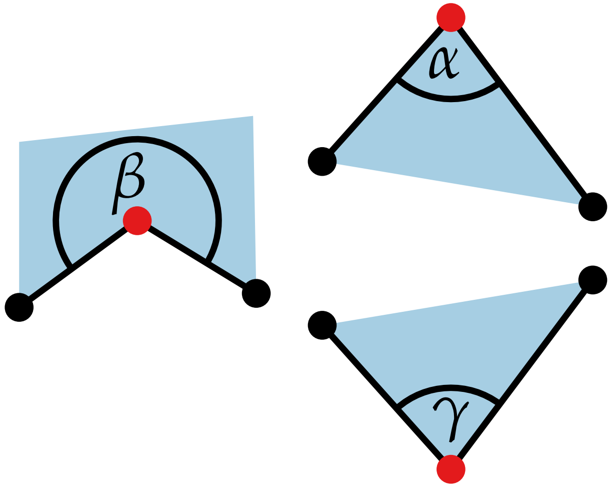

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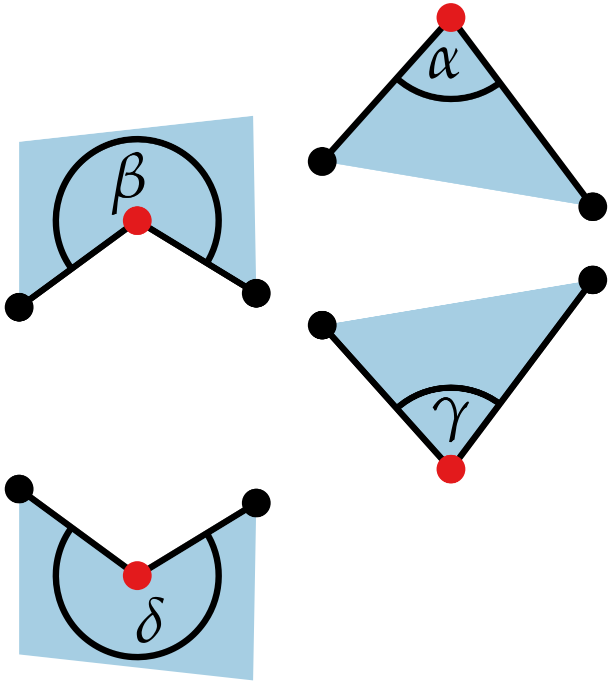

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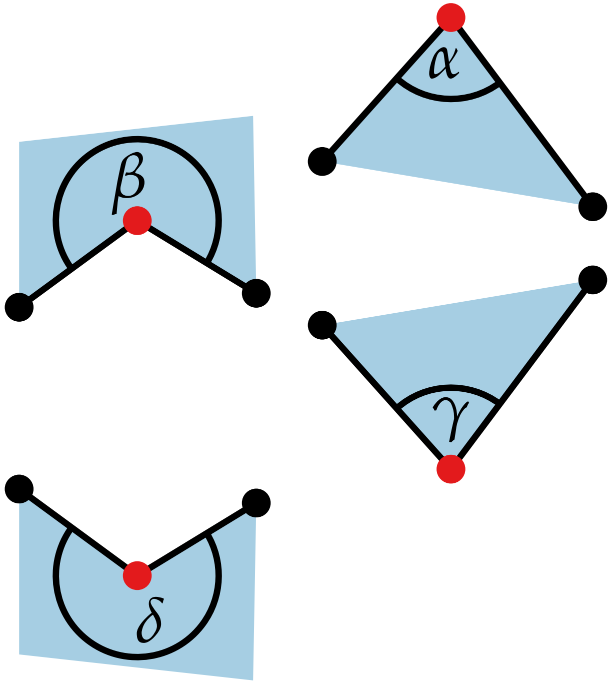

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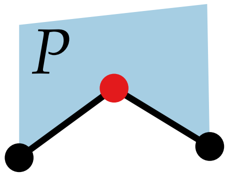
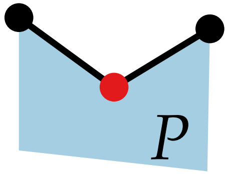
Lemma: Let P be a simple polygon. Then P is y-monotone $\Leftrightarrow$ P has neither split vertices nor merge vertices.

Towards an Algorithm





Idea: Add diagonals to “destroy” split and merge vtcs.

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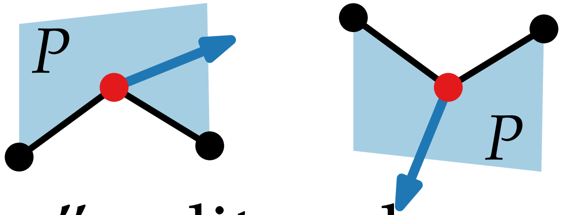

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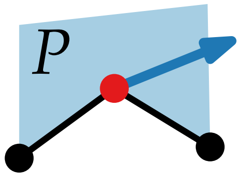

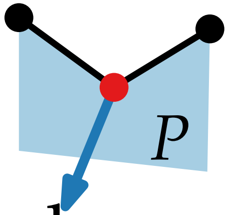

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Problem: Diagonals must not cross:

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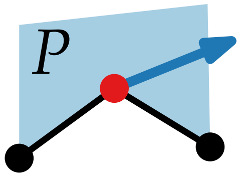

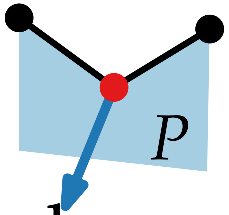

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Problem: Diagonals must not cross:
– each other
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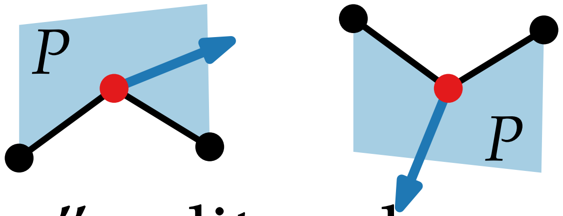
Towards an Algorithm



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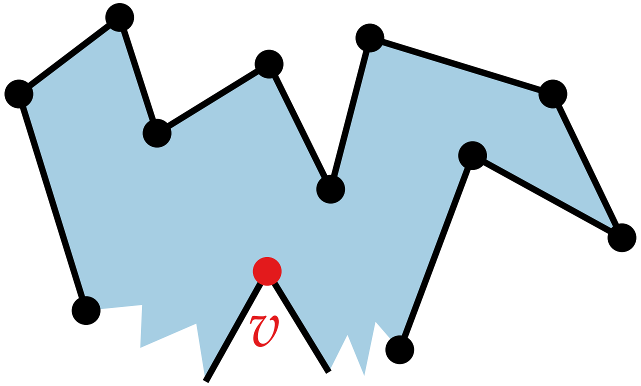
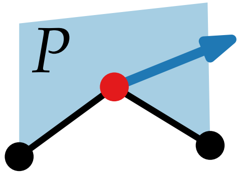
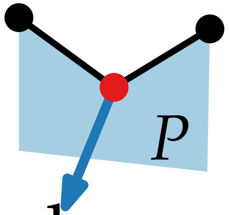
1) Treating split vertices



Towards an Algorithm





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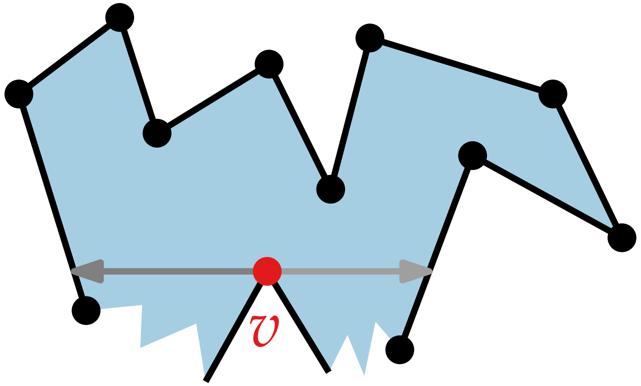

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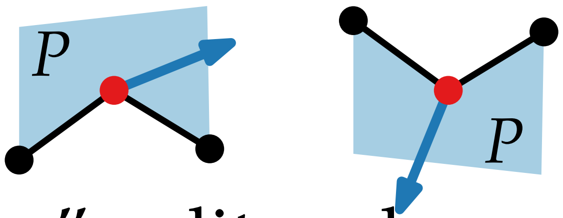

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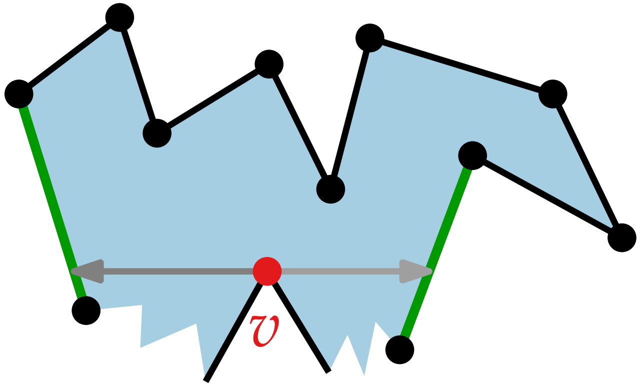

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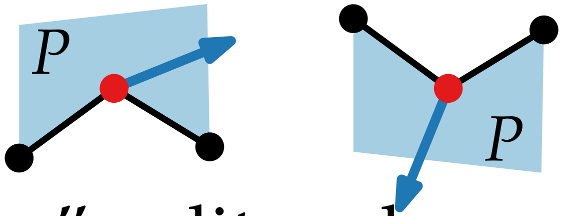

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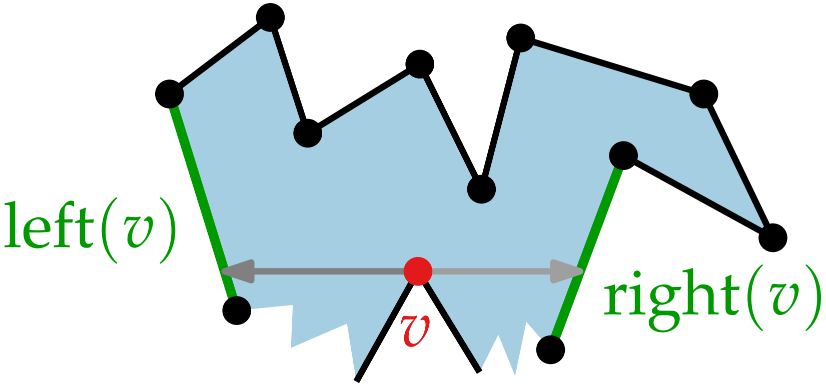

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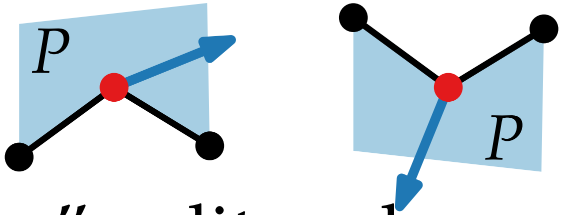

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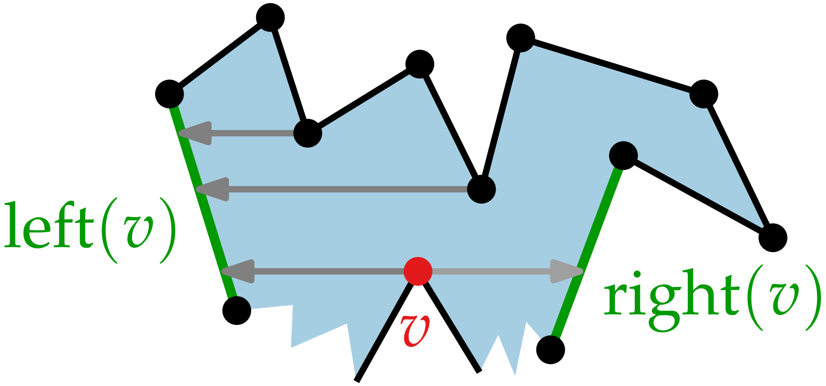

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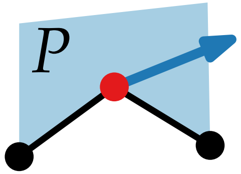

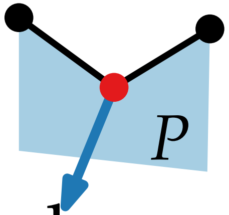

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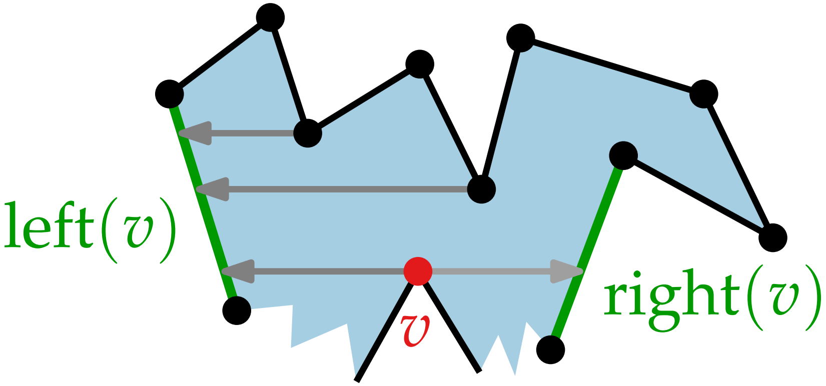

Connect v to vertex $w^\star$ having minimum y-coordinate among all vertices w above v and with $\text{left}(w) = \text{left}(v)$.

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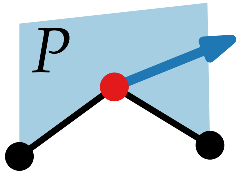

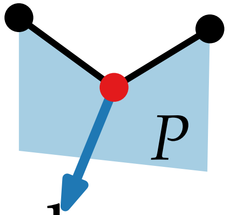

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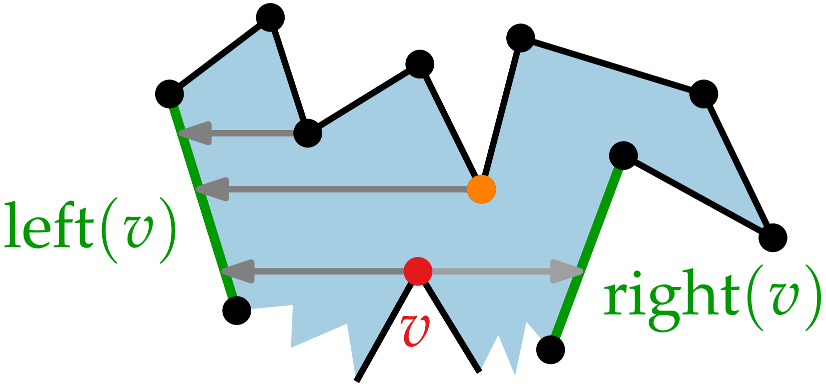

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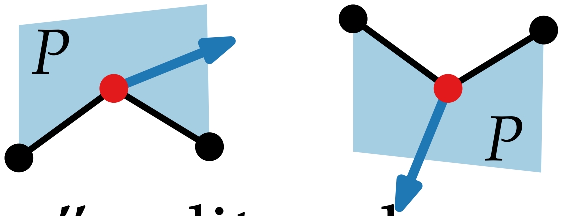

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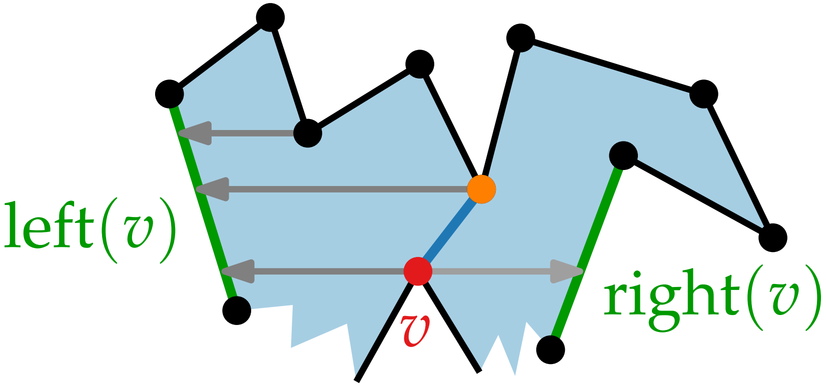

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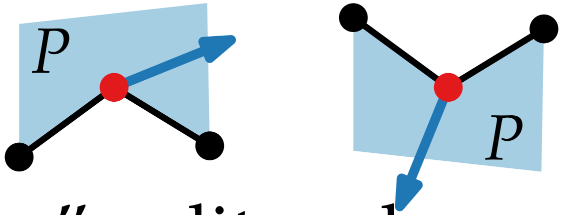

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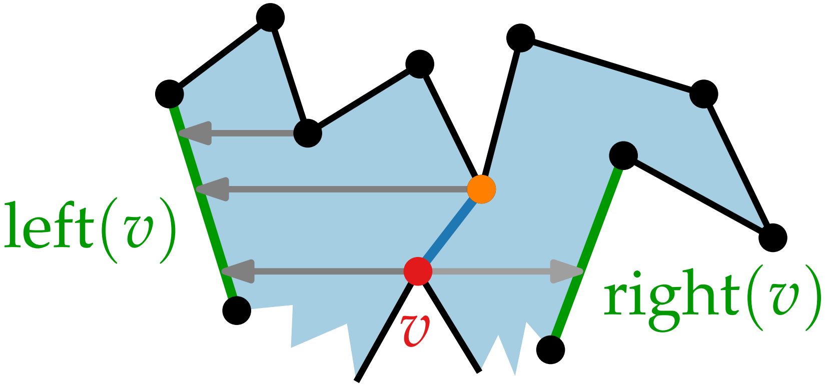

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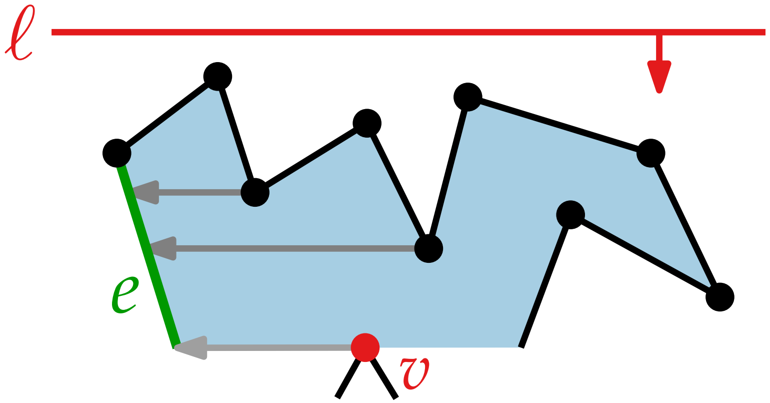
Think of a sweep-line algorithm:



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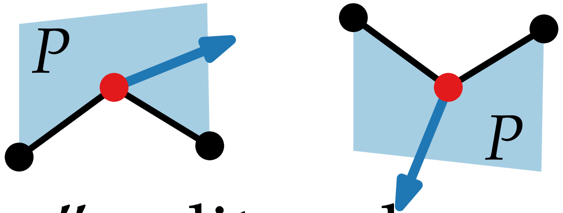

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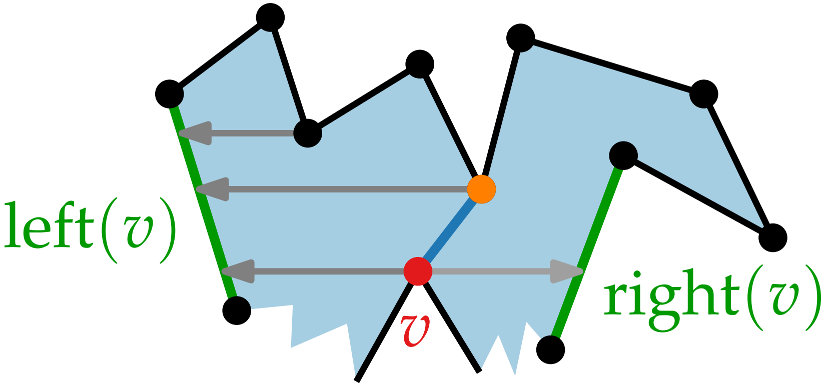

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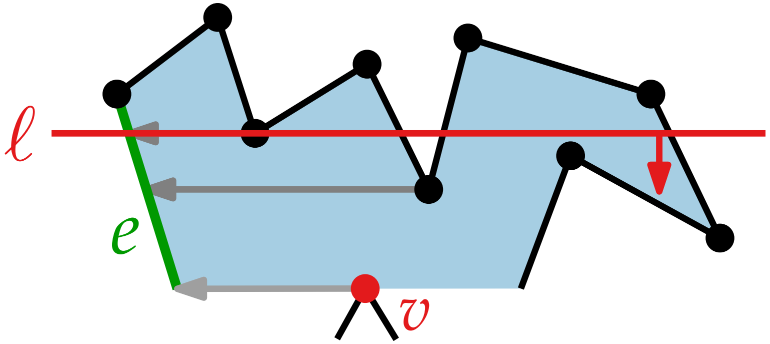
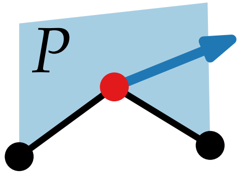
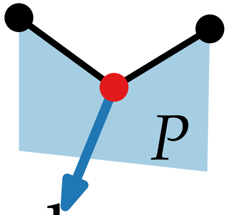
Think of a sweep-line algorithm:



Towards an Algorithm

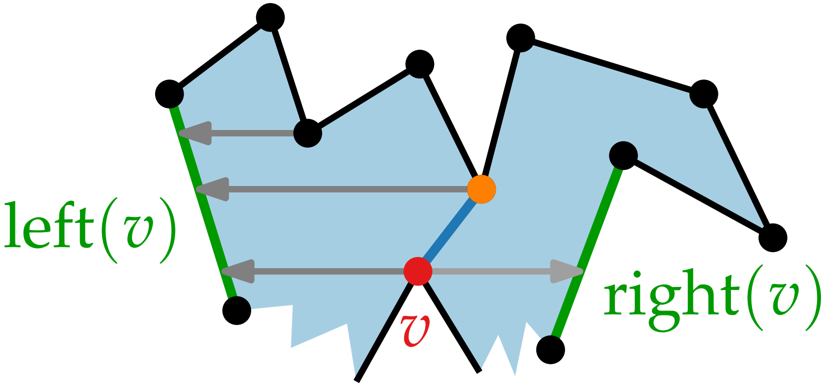




Idea: Add diagonals to “destroy” split and merge vtcs.

Problem: Diagonals must not cross:
– each other
– edges of P

1) Treating split vertices



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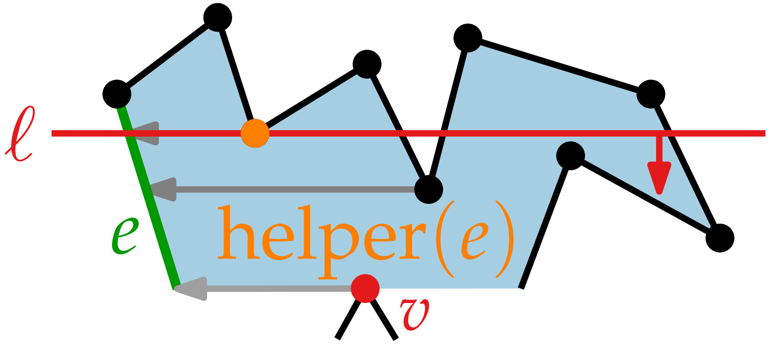
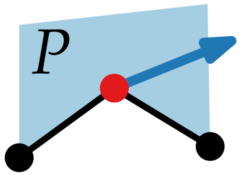
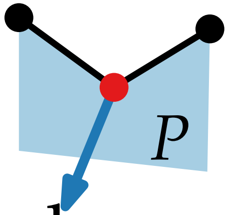


Towards an Algorithm





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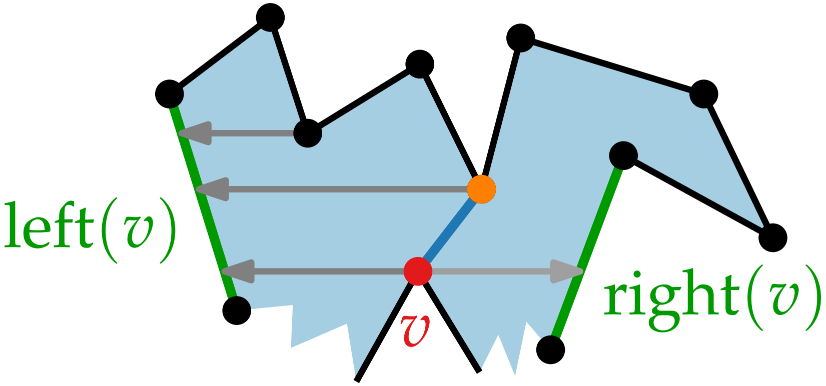

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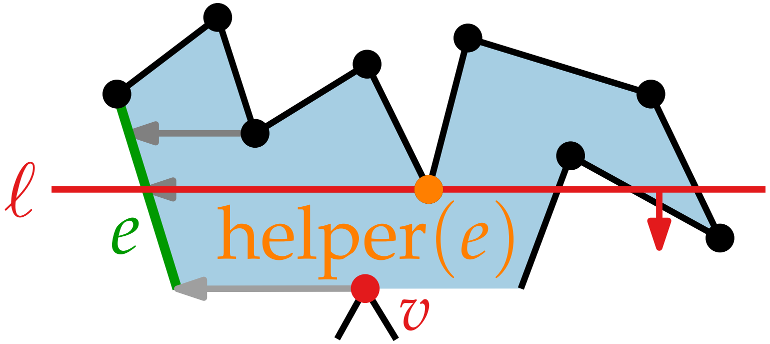
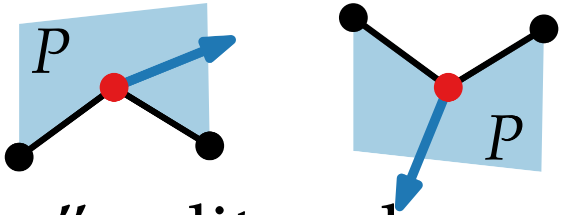


Towards an Algorithm



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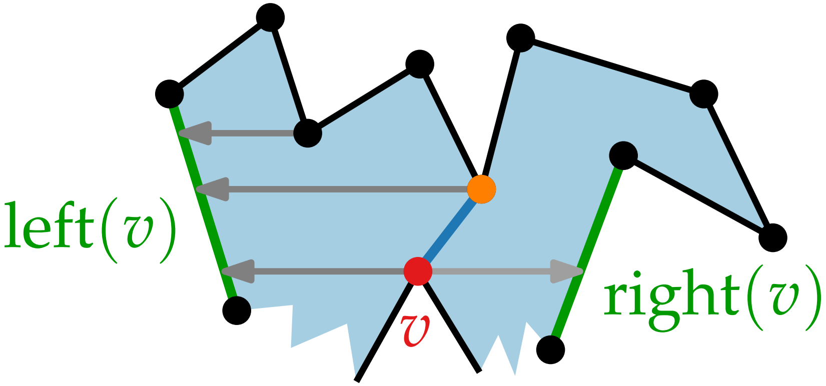
1) Treating split vertices



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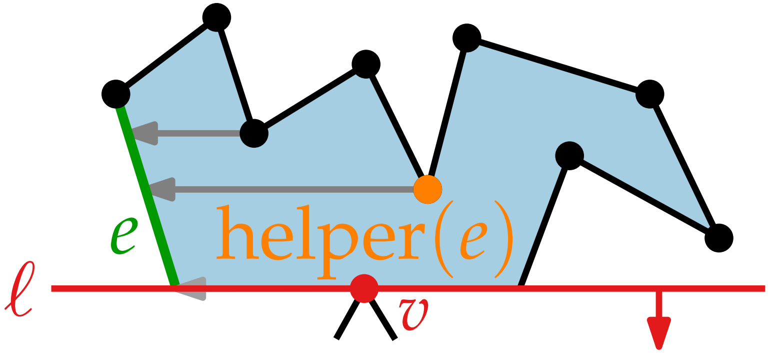
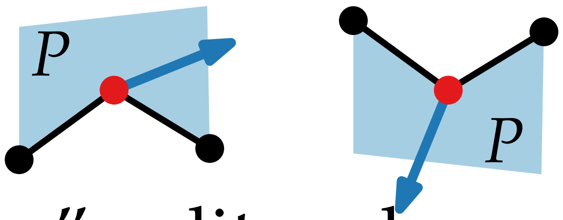


Towards an Algorithm



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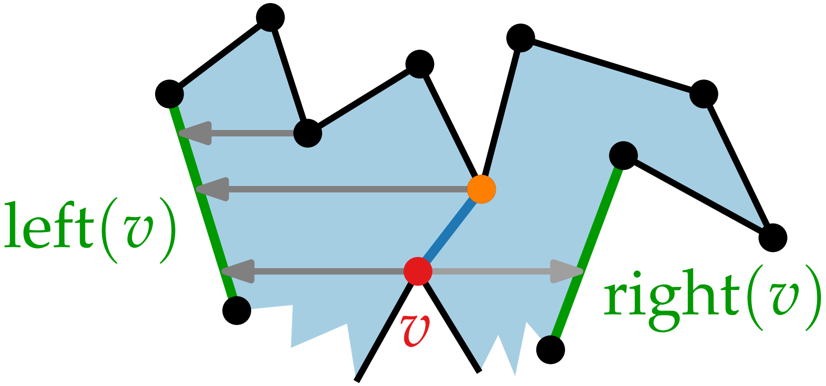
1) Treating split vertices



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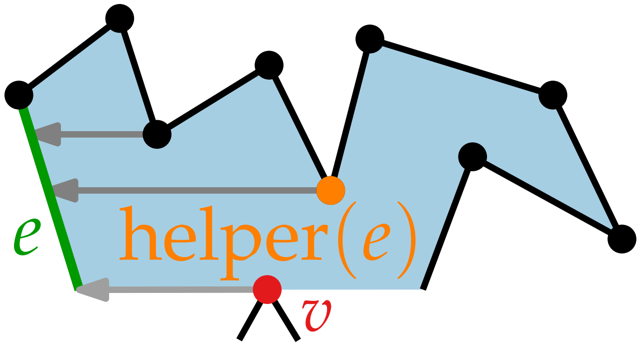
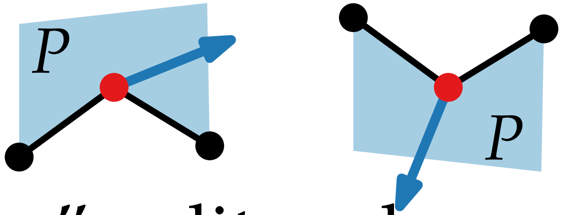
Think of a sweep-line algorithm:

Connect v to helper(left(v)).



Towards an Algorithm



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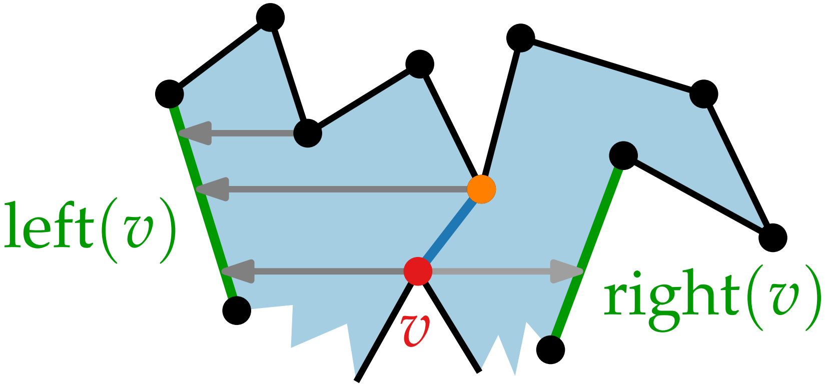

Connect v to vertex $w^\star$ having minimum y-coordinate among all vertices w above v and with left$(w) = $ left(v).

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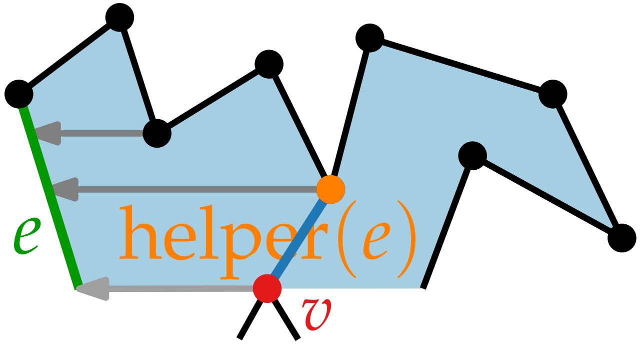
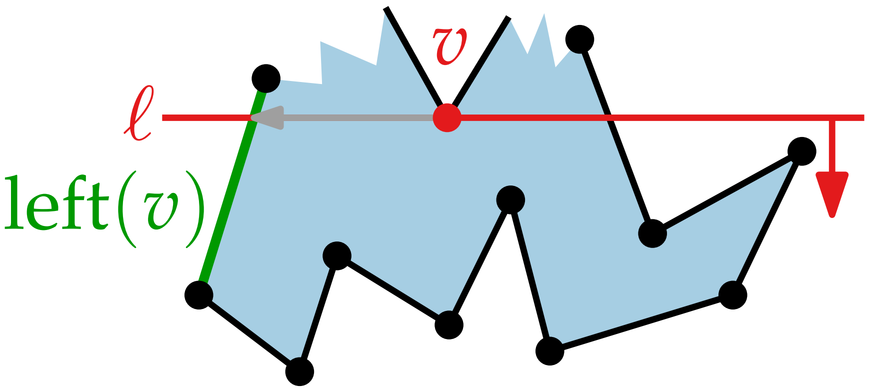


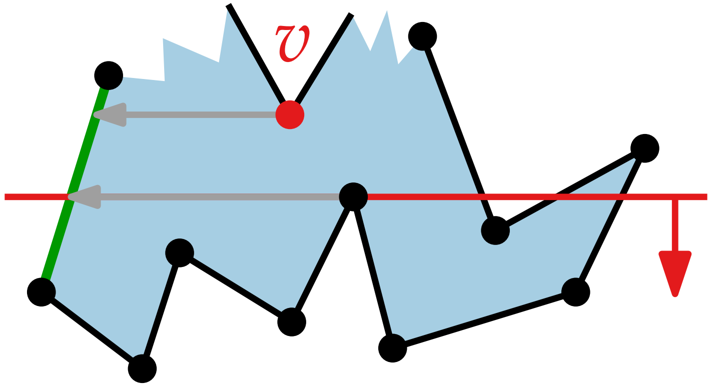
An Algorithm

2) Treating merge vertices



An Algorithm

2) Treating merge vertices



An Algorithm

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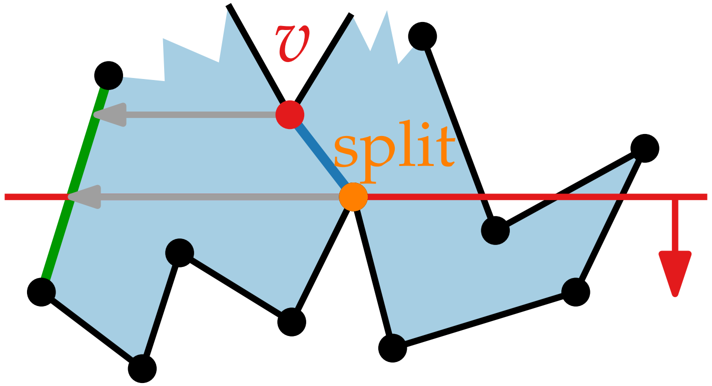
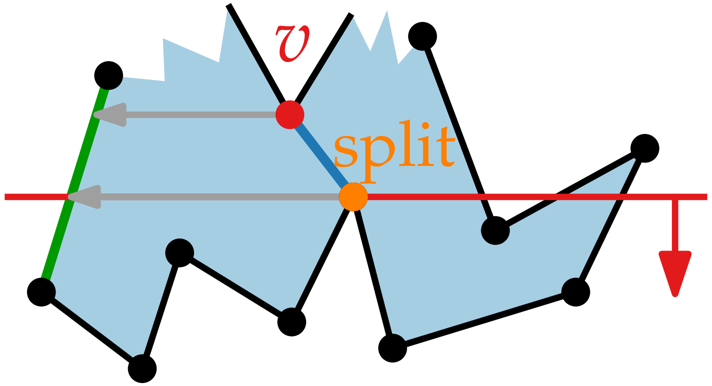
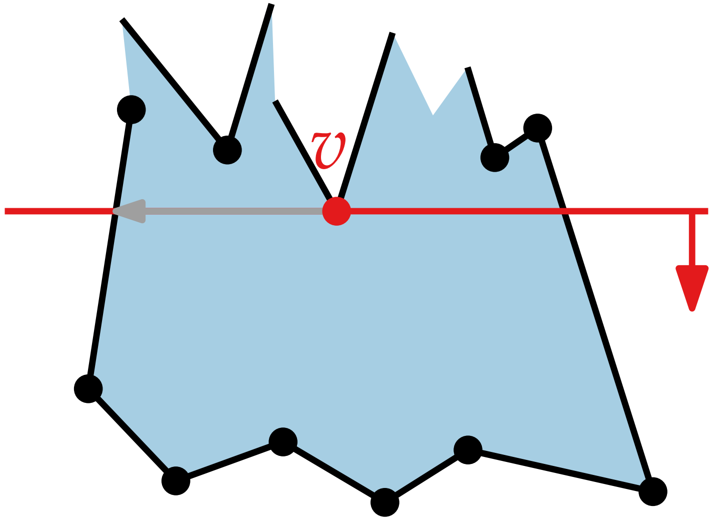


An Algorithm

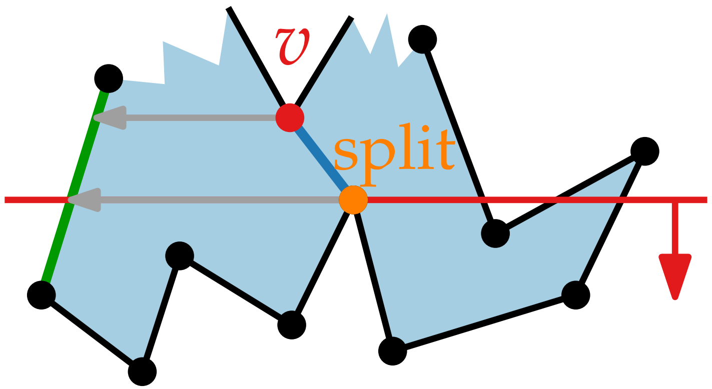
2) Treating merge vertices

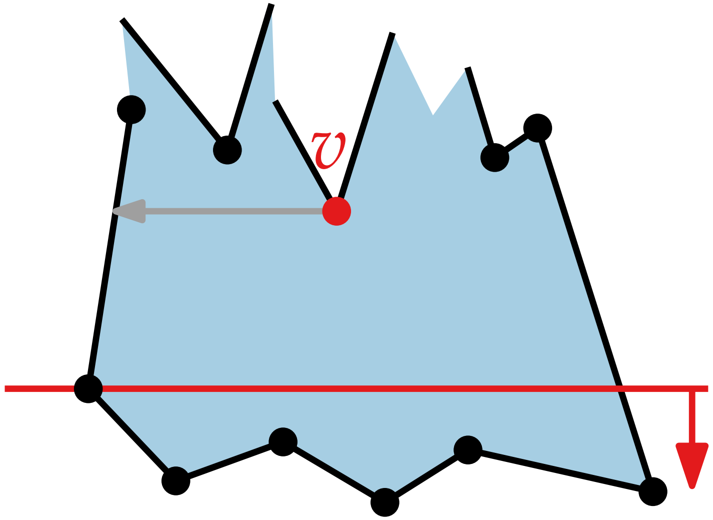




An Algorithm

2) Treating merge vertices





An Algorithm

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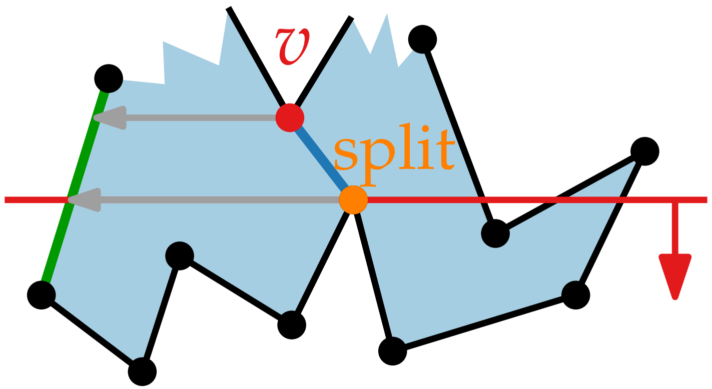
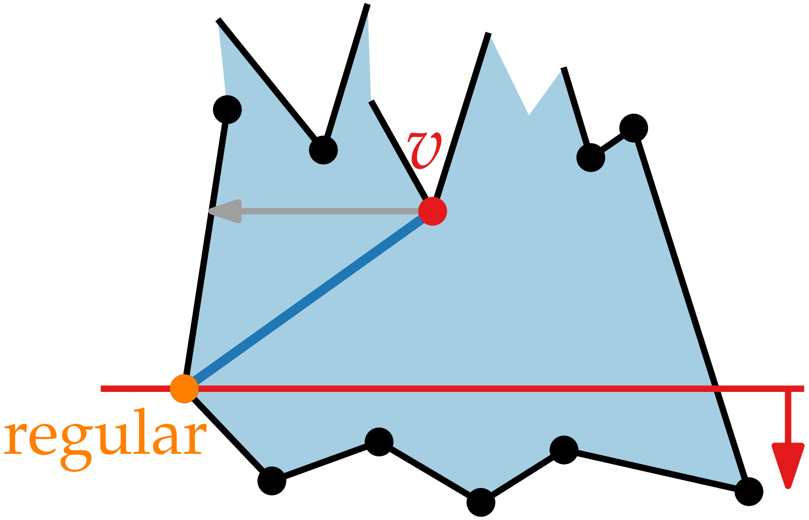
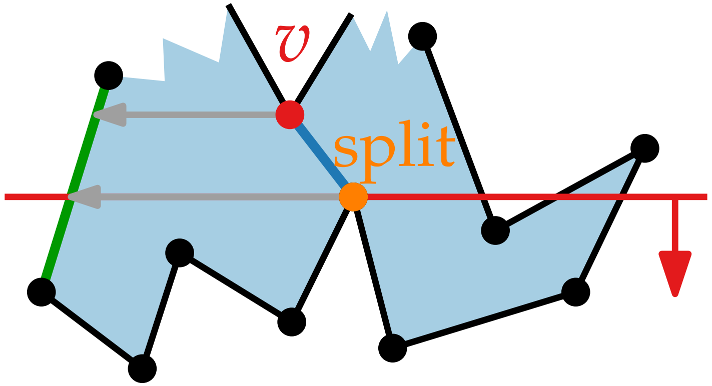
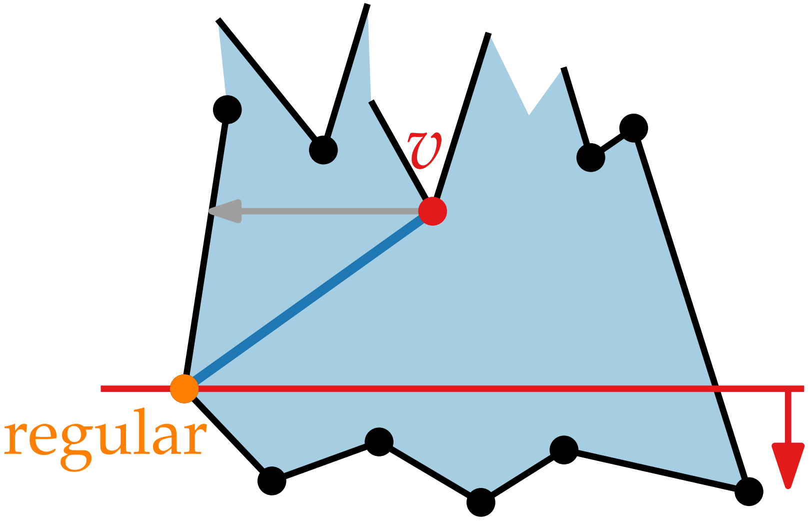




An Algorithm

2) Treating merge vertices

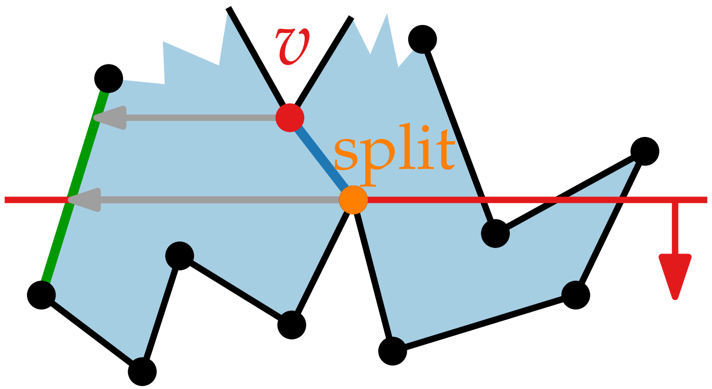


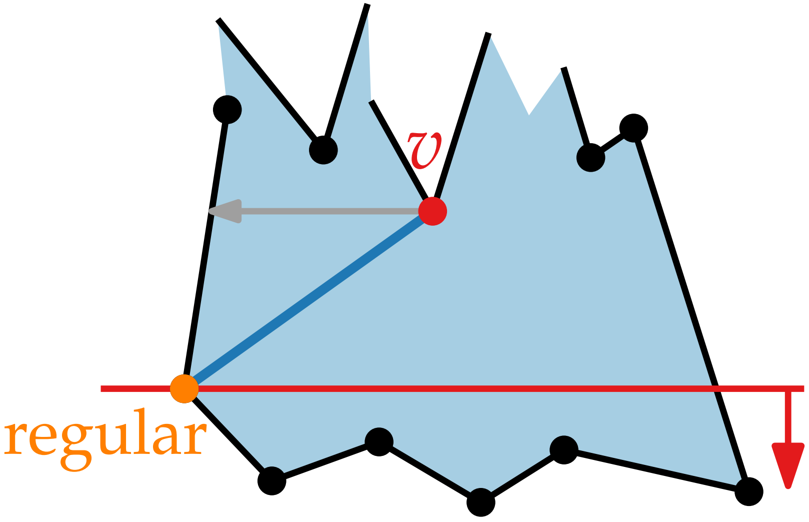


```
**makeMonotone**(polygon $P$)
$\mathcal{D} \leftarrow \text{DCEL}(V(P), E(P))$
$\mathcal{Q} \leftarrow$ priority queue on $V(P)$
$\mathcal{T} \leftarrow$ empty bin. search tree
```

An Algorithm

2) Treating merge vertices





```
**makeMonotone(**polygon $P$**)**
$\mathcal{D} \leftarrow \mathrm{DCEL}(V(P), E(P))$
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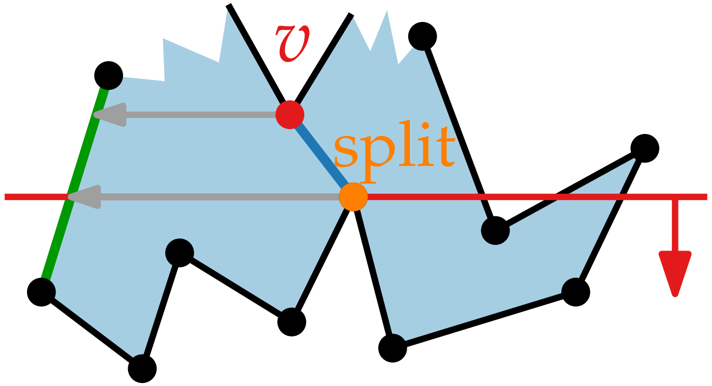
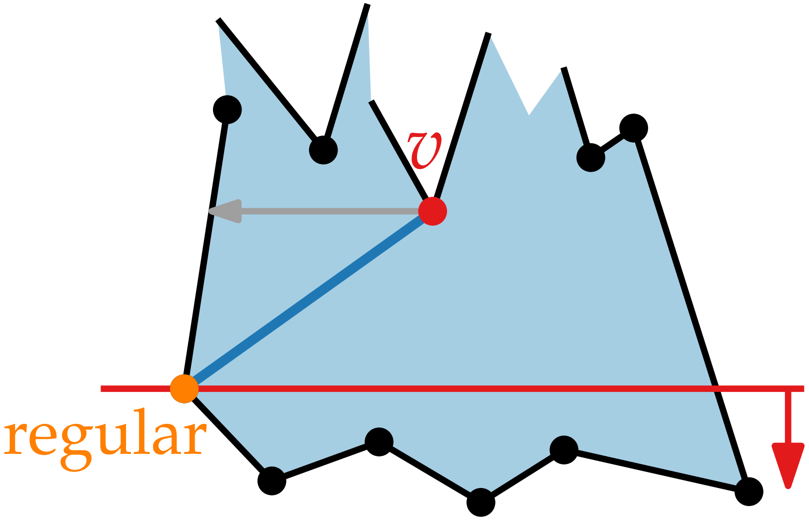
doubly-connected edge list:
data structure for planar subdivisions

An Algorithm

2) Treating merge vertices





```
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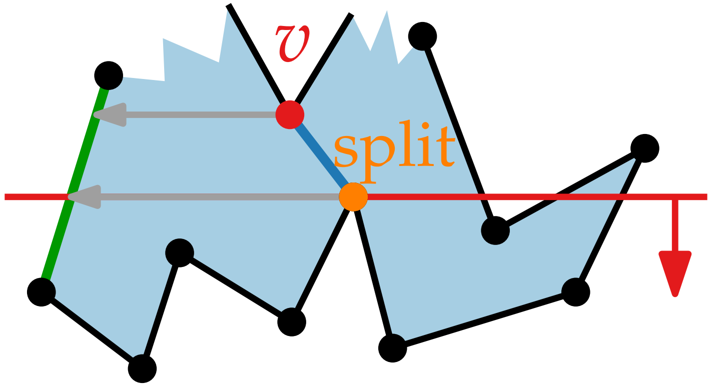
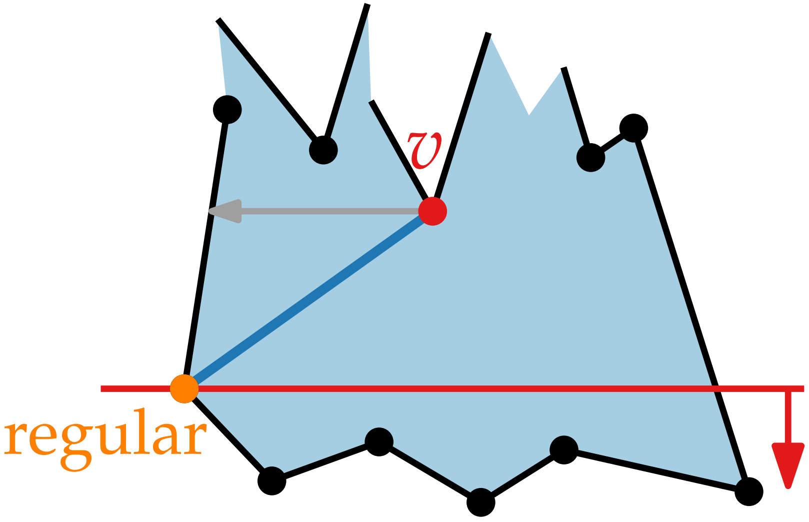
doubly-connected edge list:
data structure for planar subdivisions

$(x,y) \prec (x',y') \;:\Leftrightarrow$
$y > y' \;\vee\; (y = y' \wedge x < x')$

An Algorithm

2) Treating merge vertices





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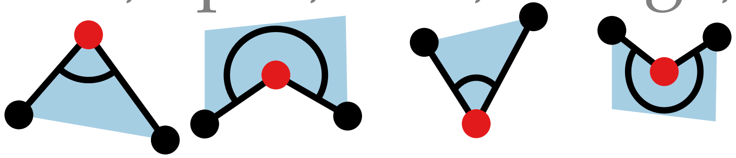
$\mathcal{Q} \leftarrow$ priority queue on $V(P)$
$\mathcal{T} \leftarrow$ empty bin. search tree
while $\mathcal{Q} \neq \emptyset$ do
    $v \leftarrow \mathcal{Q}$.extractMax()
    type $\leftarrow$ type of vertex $v \in$ start, split, end, merge, regular
    handleVertex$_{\text{type}}(v)$
return DCEL $\mathcal{D}$
```

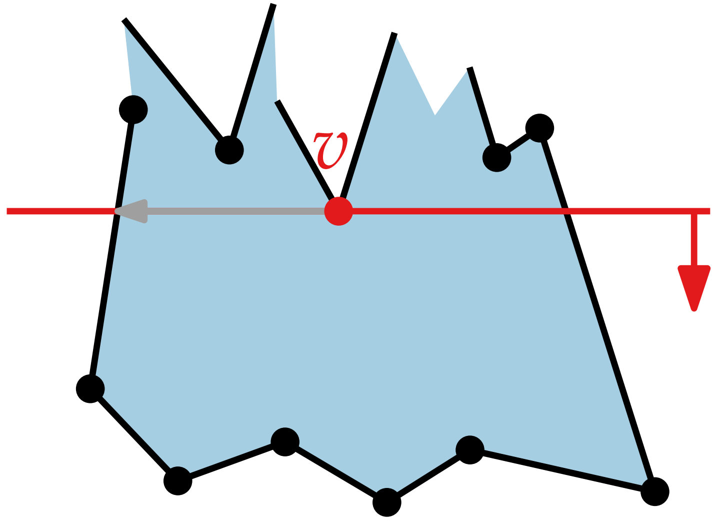
doubly-connected edge list:
data structure for planar subdivisions

$$(x,y) \prec (x',y') \;:\Leftrightarrow\; y > y' \;\vee\; (y = y' \wedge x < x')$$



An Algorithm

2) Treating merge vertices



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handleVertex$_{\text{merge}}$(vertex $v$)
$e \leftarrow$ edge following $v$ ccw
if helper$(e)$ merge vtx then
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$\mathcal{T}$.delete$(e)$
$e' \leftarrow \mathcal{T}$.edgeLeftOf$(v)$
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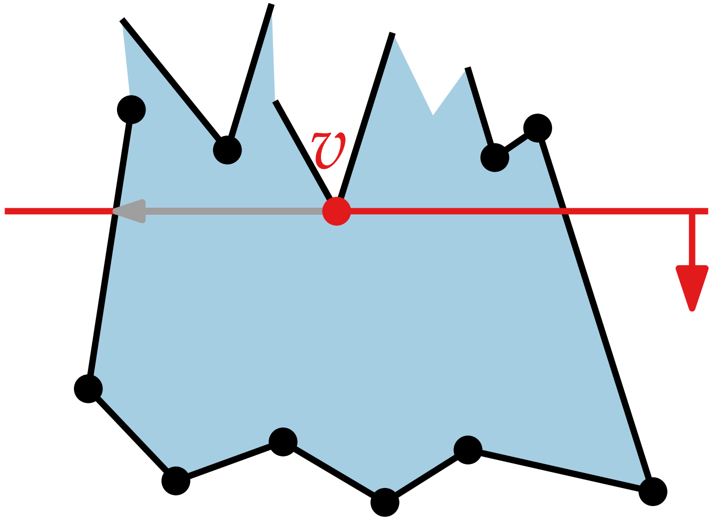
An Algorithm

2) Treating merge vertices



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$\mathcal{T}$.delete($e$)
$e' \leftarrow \mathcal{T}$.edgeLeftOf($v$)
if helper($e'$) merge vtx then
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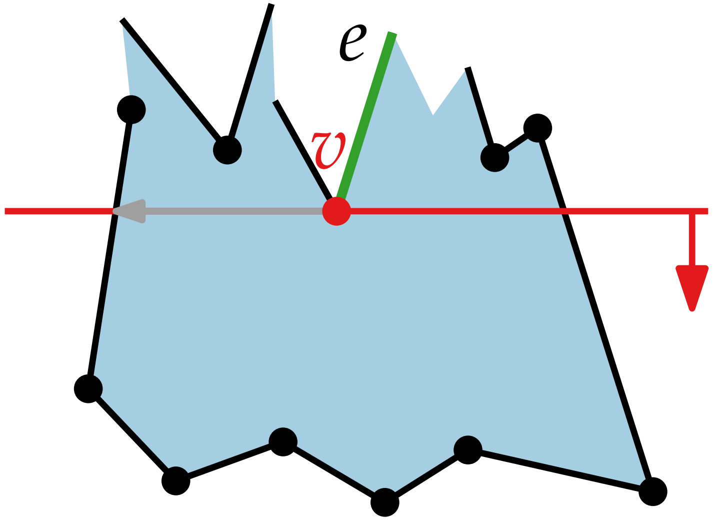
An Algorithm

2) Treating merge vertices



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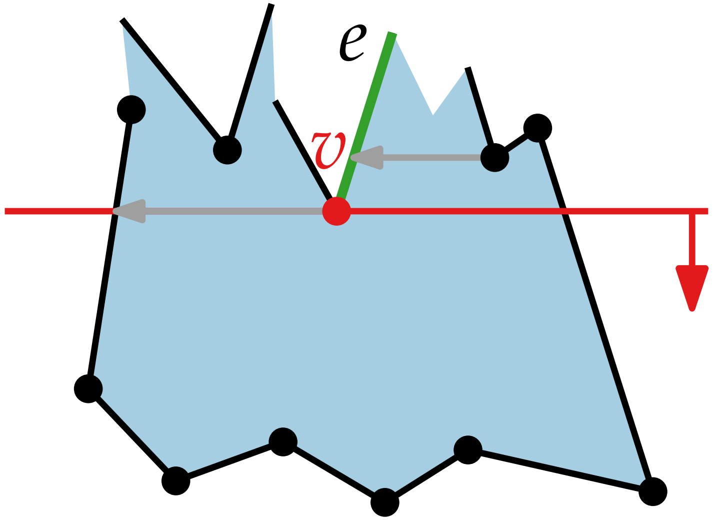
An Algorithm

2) Treating merge vertices



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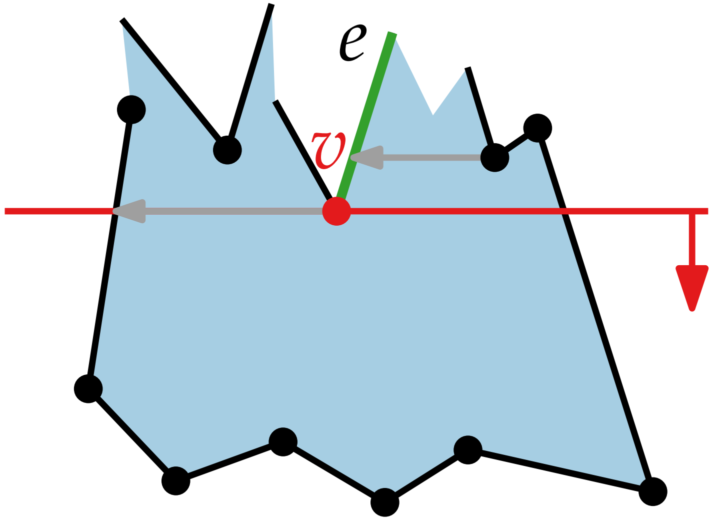
An Algorithm

2) Treating merge vertices



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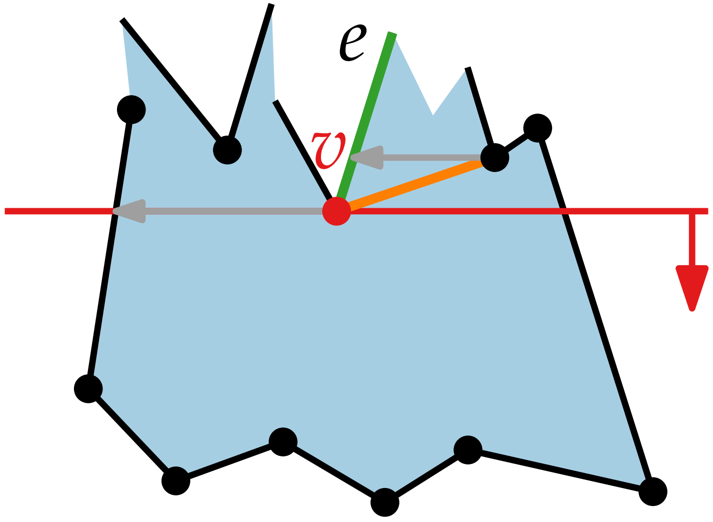
An Algorithm

2) Treating merge vertices



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return DCEL $\mathcal{D}$
```

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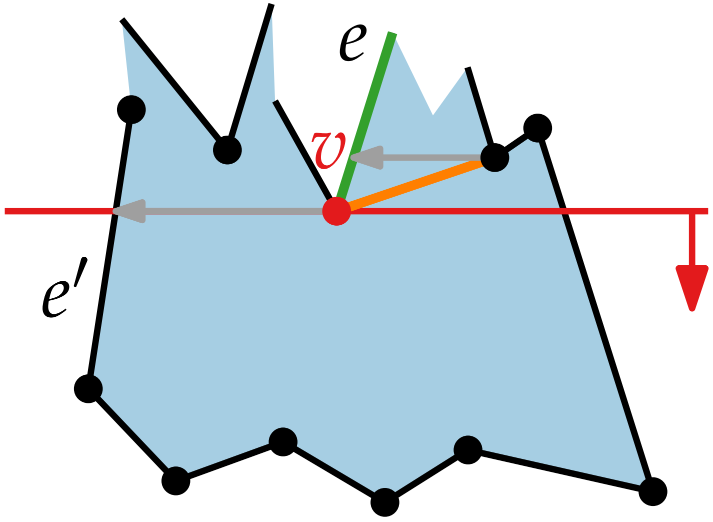
An Algorithm

2) Treating merge vertices



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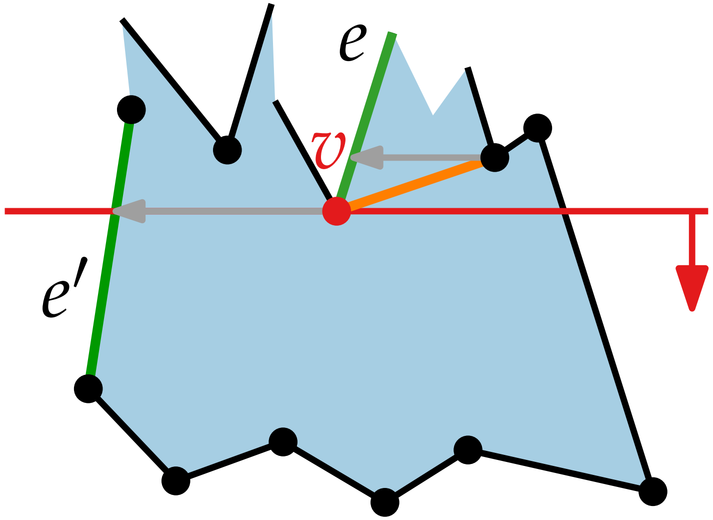
```

An Algorithm

2) Treating merge vertices



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$\mathcal{Q} \leftarrow$ priority queue on $V(P)$
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**while** $\mathcal{Q} \neq \emptyset$ **do**
    $v \leftarrow \mathcal{Q}$.extractMax()
    type $\leftarrow$ type of vertex $v$
    handleVertex$_{\text{type}}(v)$
**return** DCEL $\mathcal{D}$
```

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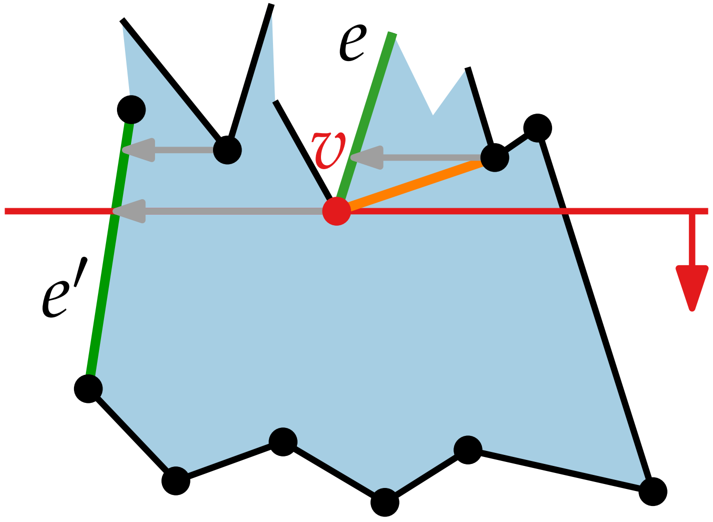
    $\mathcal{D}$.insert(diag($v$, helper$(e')$))
helper$(e') \leftarrow v$
```

An Algorithm

2) Treating merge vertices

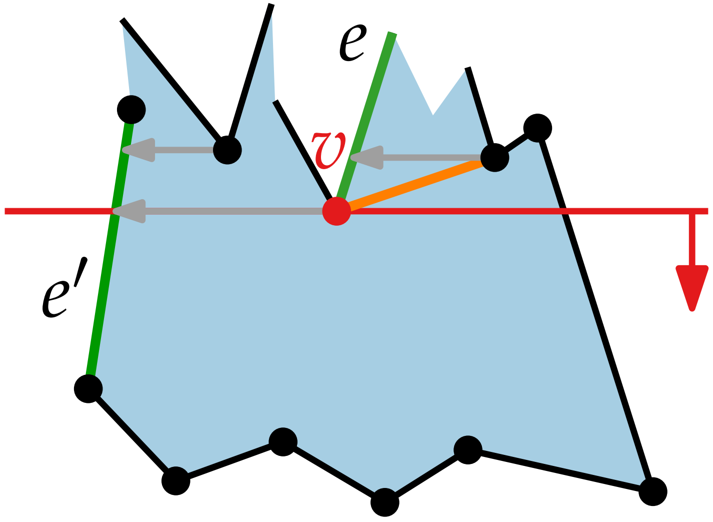


```
makeMonotone(polygon $P$)
$\mathcal{D} \leftarrow$ DCEL($V(P), E(P)$)
$\mathcal{Q} \leftarrow$ priority queue on $V(P)$
$\mathcal{T} \leftarrow$ empty bin. search tree
while $\mathcal{Q} \neq \emptyset$ do
    $v \leftarrow \mathcal{Q}$.extractMax()
    type $\leftarrow$ type of vertex $v$
    handleVertex$_{\text{type}}(v)$
return DCEL $\mathcal{D}$
```

```
handleVertex$_{\text{merge}}$(vertex $v$)
$e \leftarrow$ edge following $v$ ccw
if helper($e$) merge vtx then
    $\mathcal{D}$.insert(diag($v$, helper($e$)))
$\mathcal{T}$.delete($e$)
$e' \leftarrow \mathcal{T}$.edgeLeftOf($v$)
if helper($e'$) merge vtx then
    $\mathcal{D}$.insert(diag($v$, helper($e'$)))
helper($e'$) $\leftarrow v$
```

An Algorithm

2) Treating merge vertices



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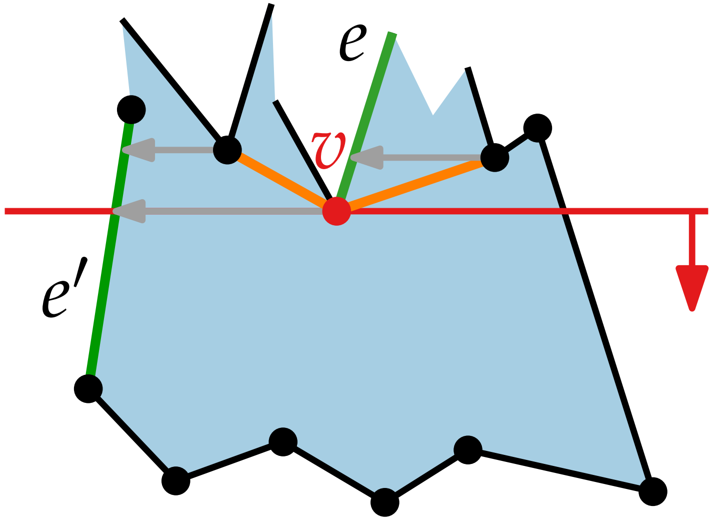
$e' \leftarrow \mathcal{T}$.edgeLeftOf$(v)$
if helper$(e')$ merge vtx then
    $\mathcal{D}$.insert(diag$(v$, helper$(e'))$)
helper$(e') \leftarrow v$
```

An Algorithm

2) Treating merge vertices



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```

Analysis

Lemma. makeMonotone() adds a set of non-intersecting diagonals to P such that P is partitioned into y-monotone subpolygons.

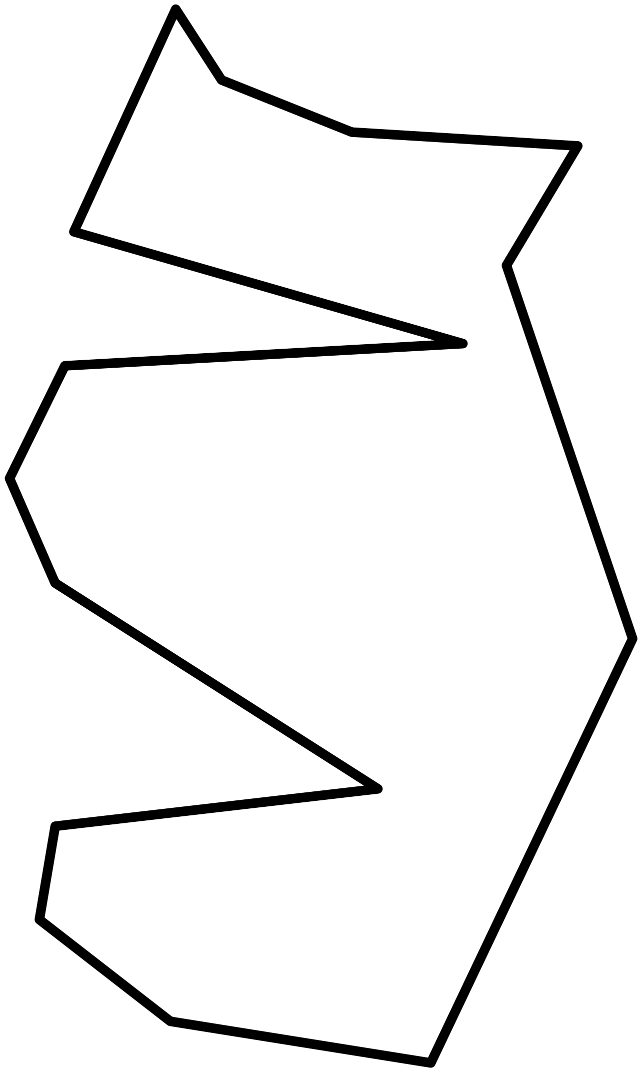
Analysis

Lemma. makeMonotone() adds a set of non-intersecting diagonals to P such that P is partitioned into y-monotone subpolygons.

Lemma. A simple polygon with n vertices can be subdivided into y-monotone polygons in $O(n \log n)$ time.

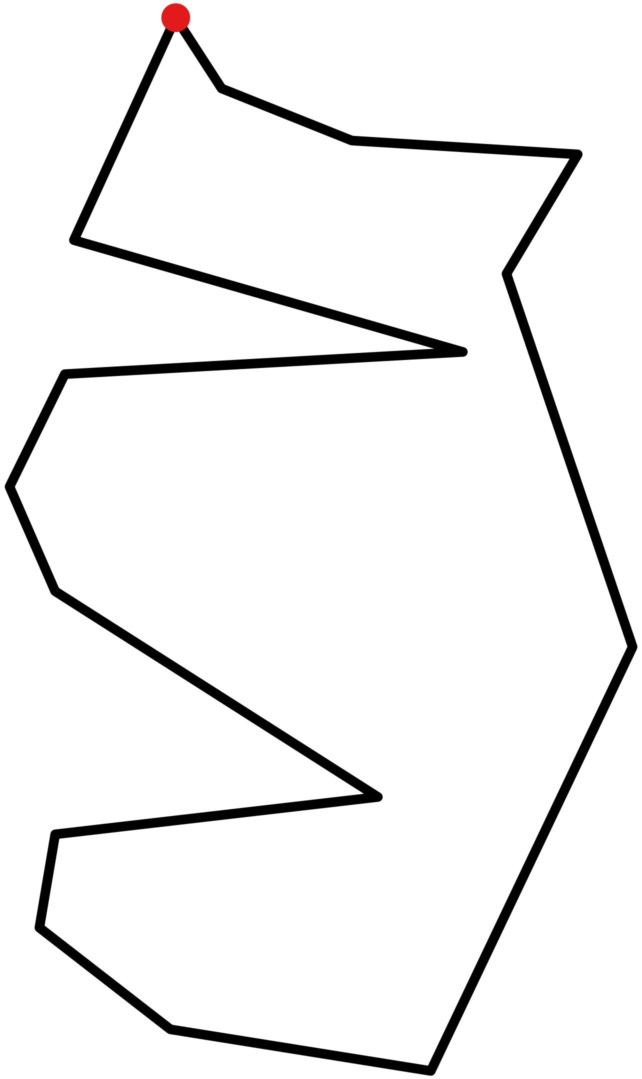
Triangulating a *y*-Monotone Polygon *P*

Approach: greedy, going from top to bottom



Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



Triangulating a y-Monotone Polygon P

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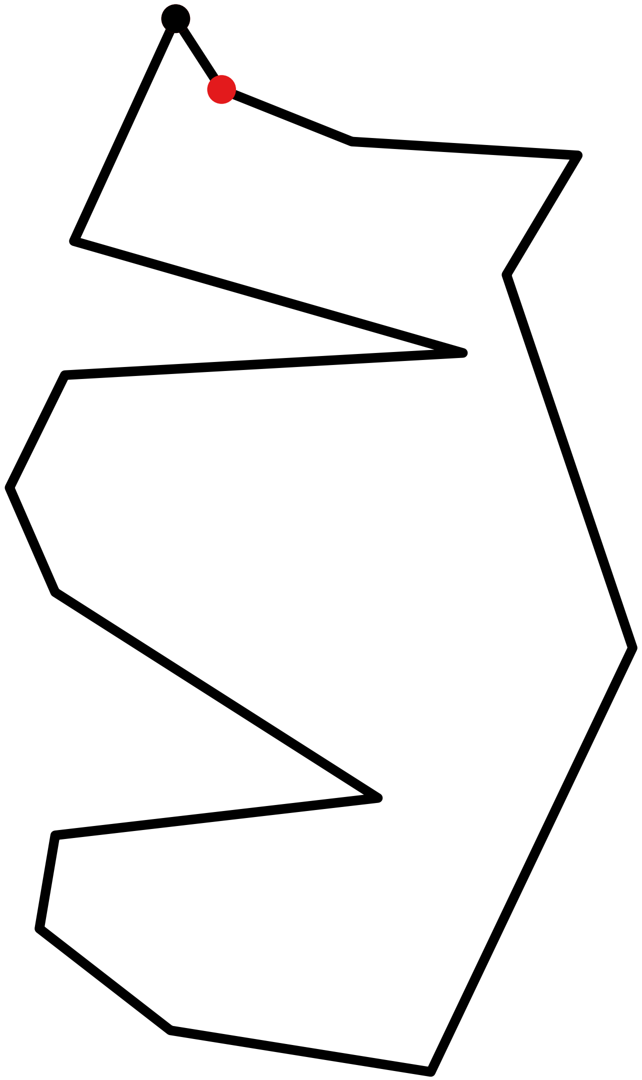

Triangulating a y-Monotone Polygon P

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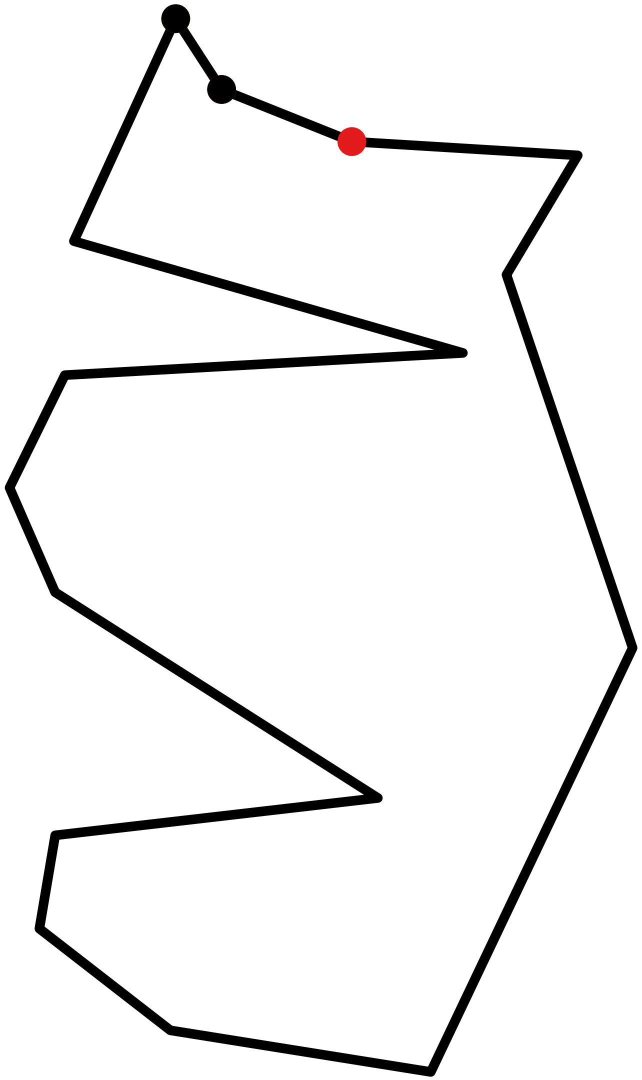
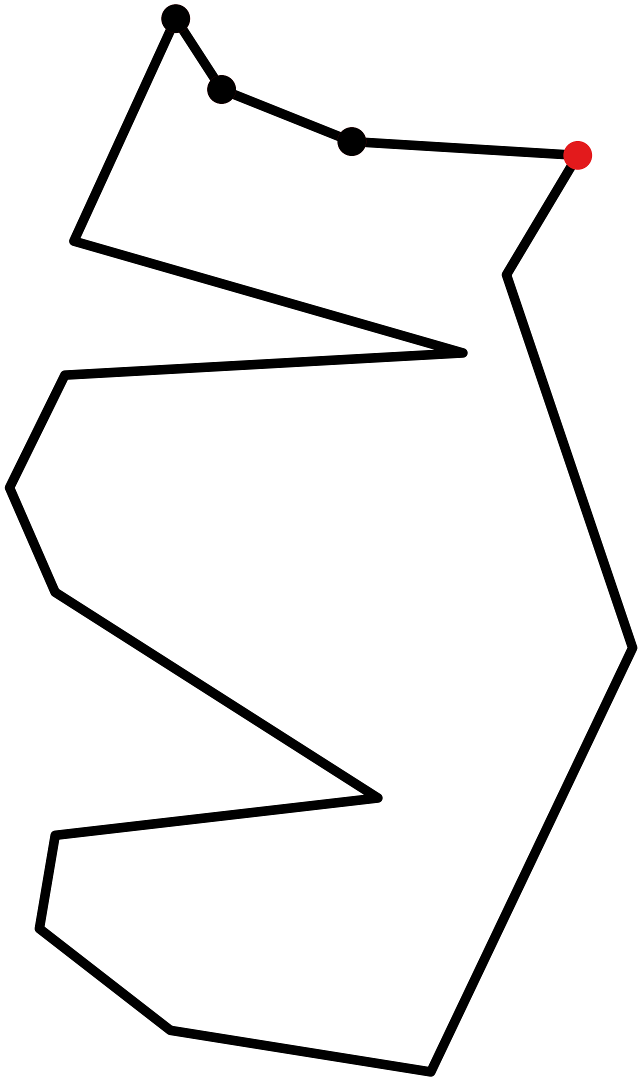


Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



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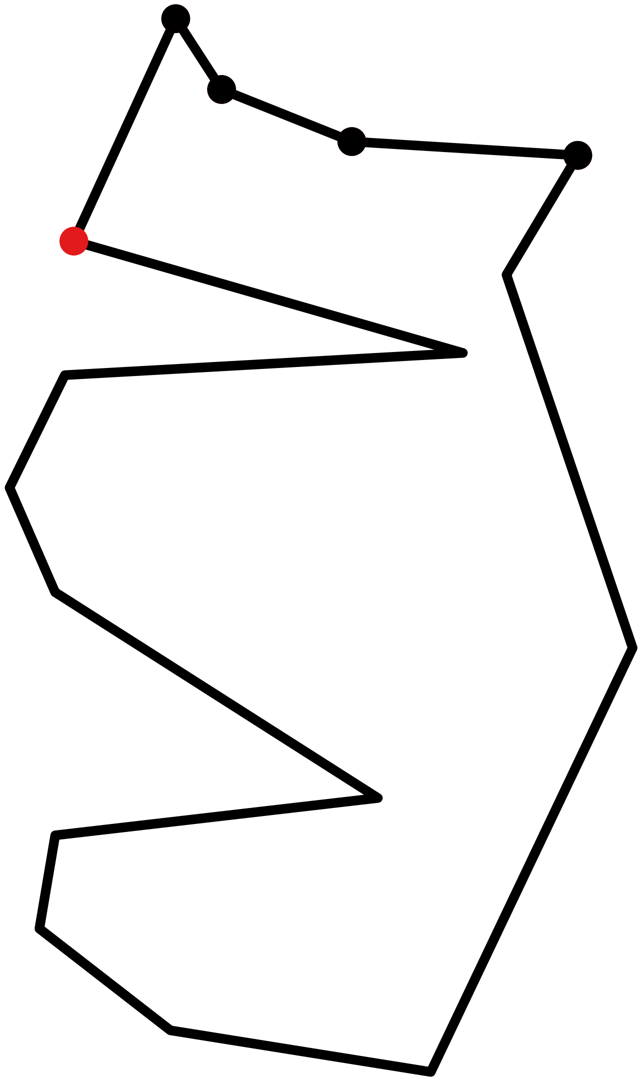

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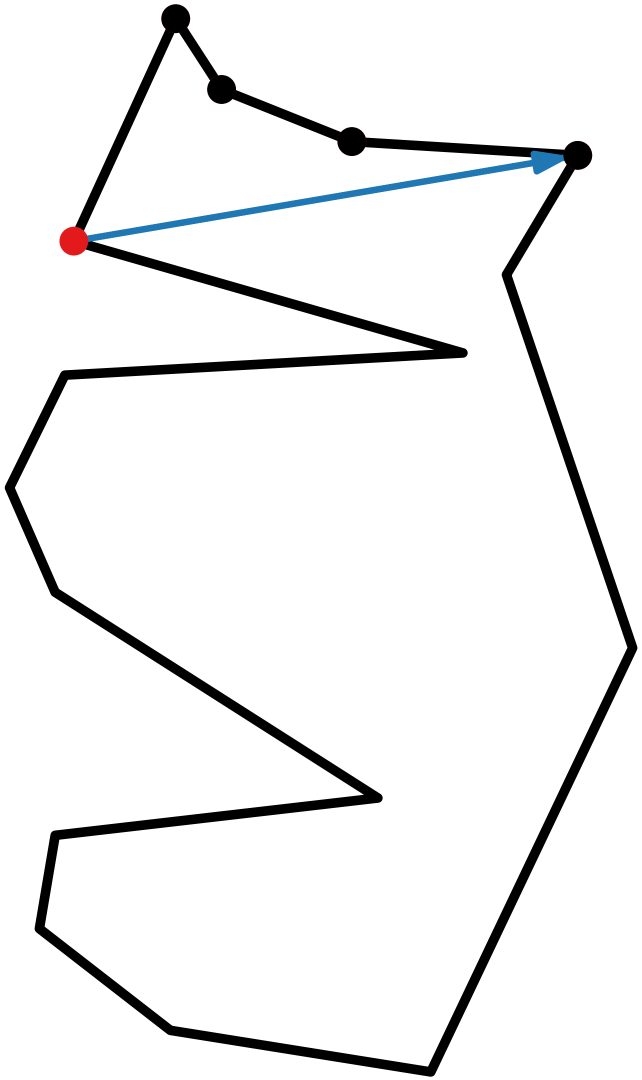
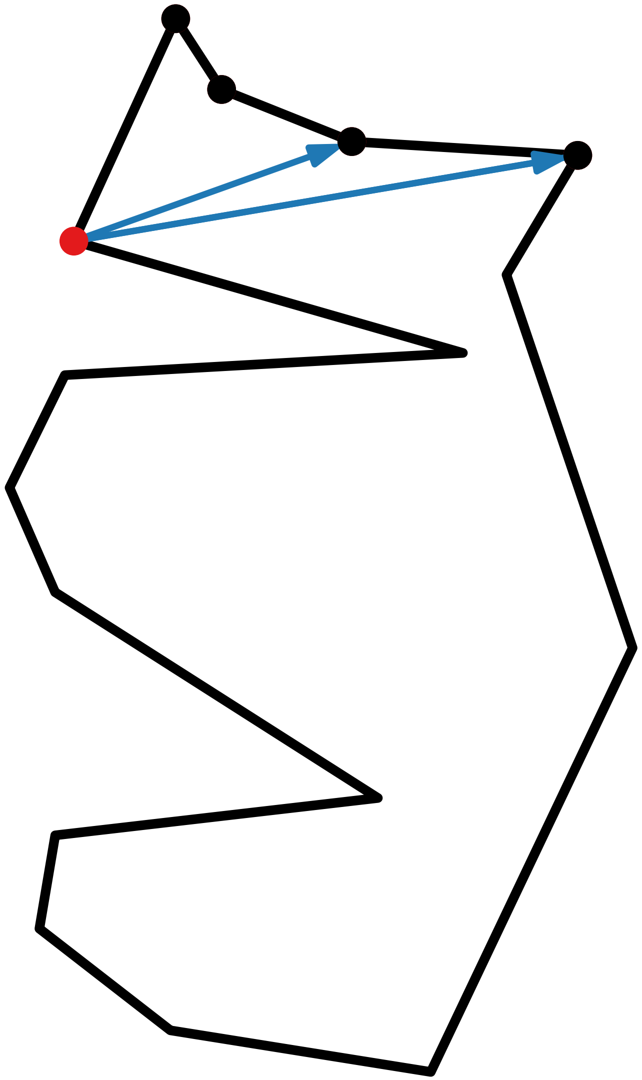


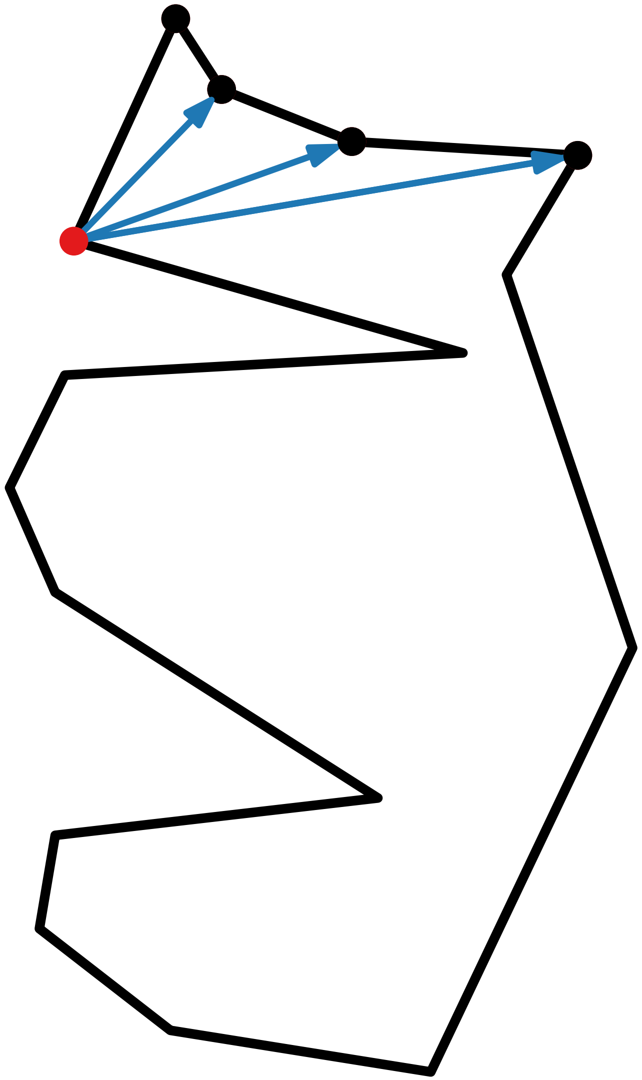
Triangulating a *y*-Monotone Polygon *P*

Approach: greedy, going from top to bottom



Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



Triangulating a y-Monotone Polygon P

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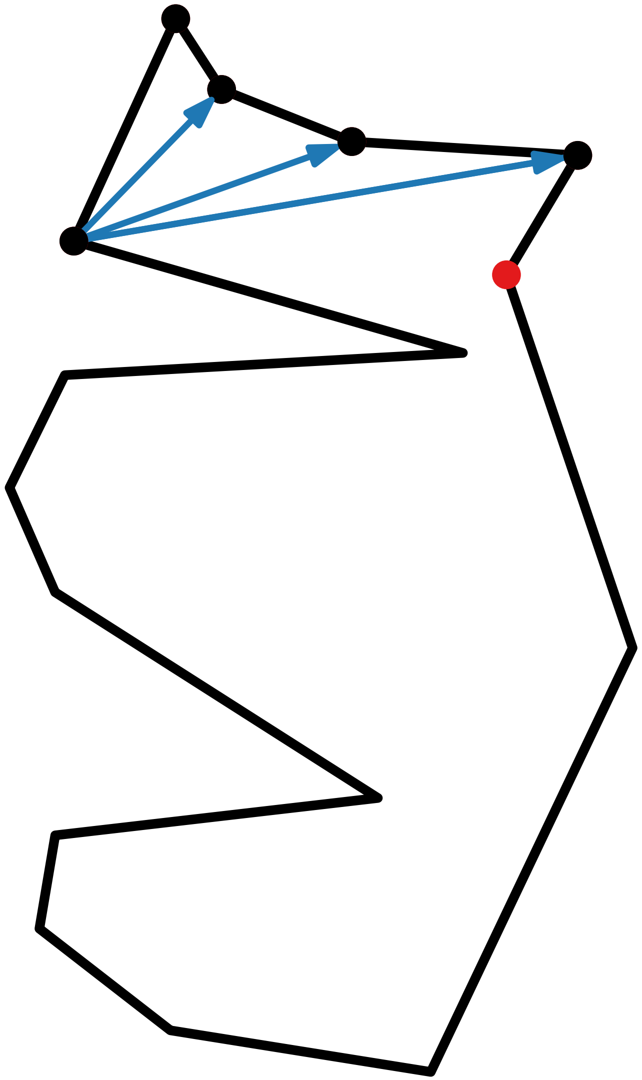

Triangulating a y-Monotone Polygon P

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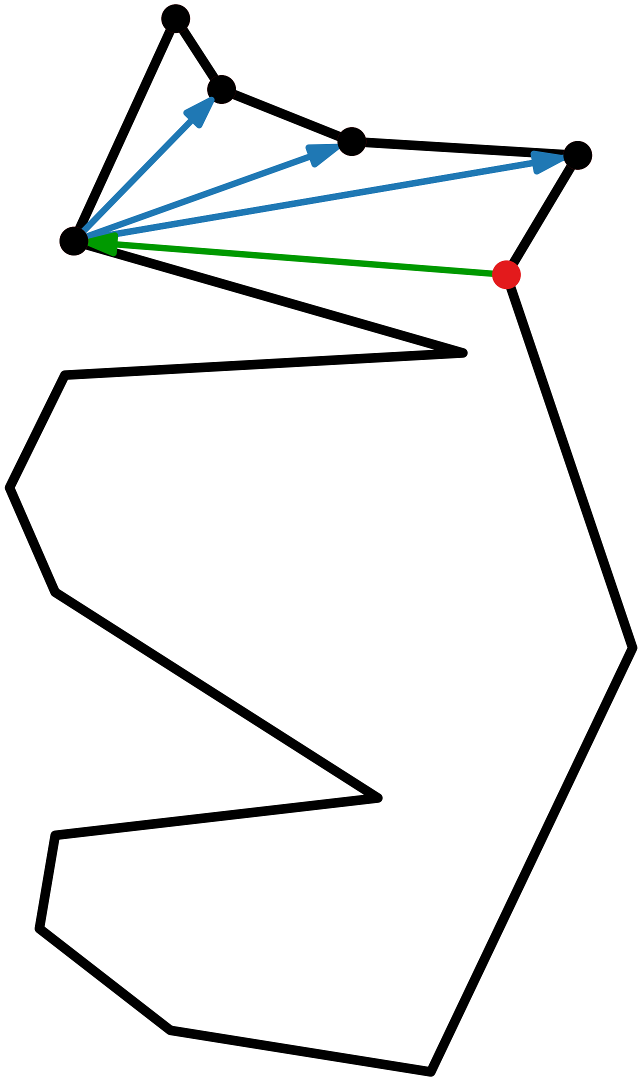
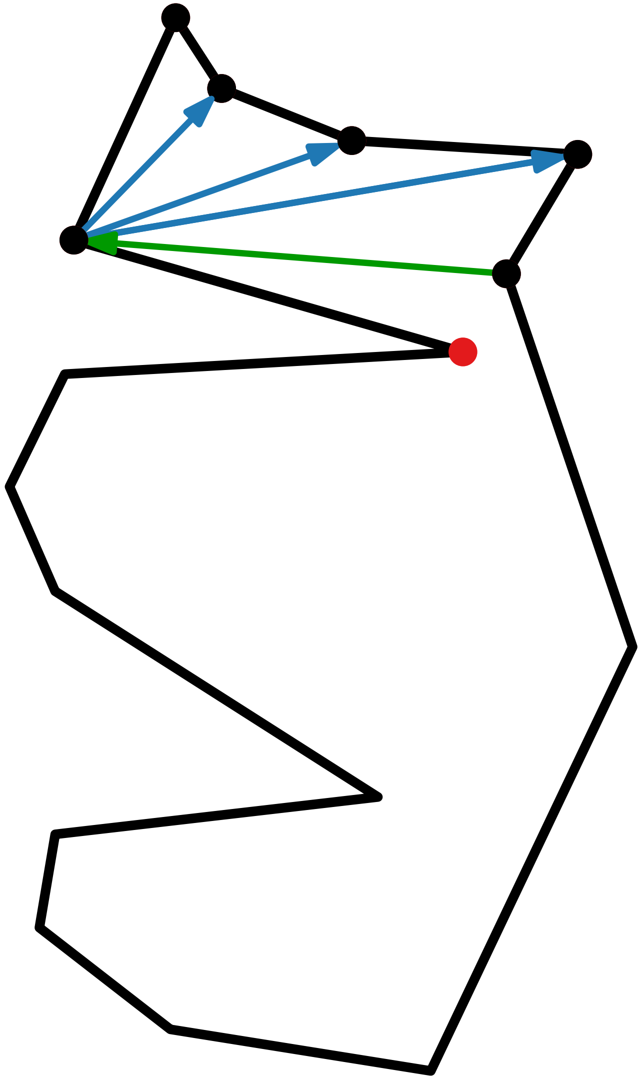


Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



Triangulating a *y*-Monotone Polygon *P*

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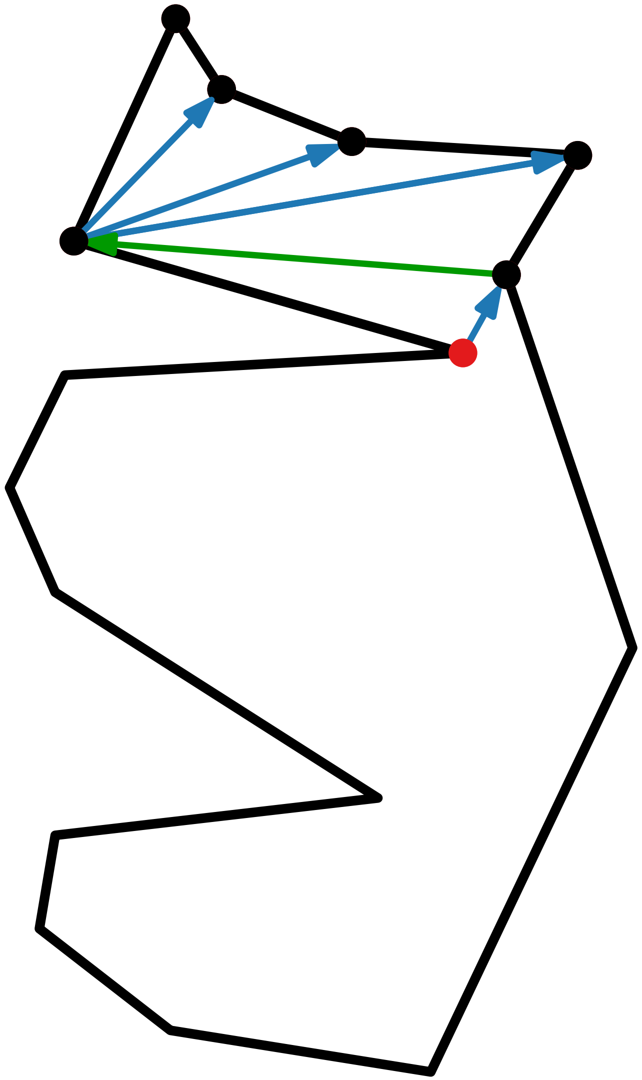
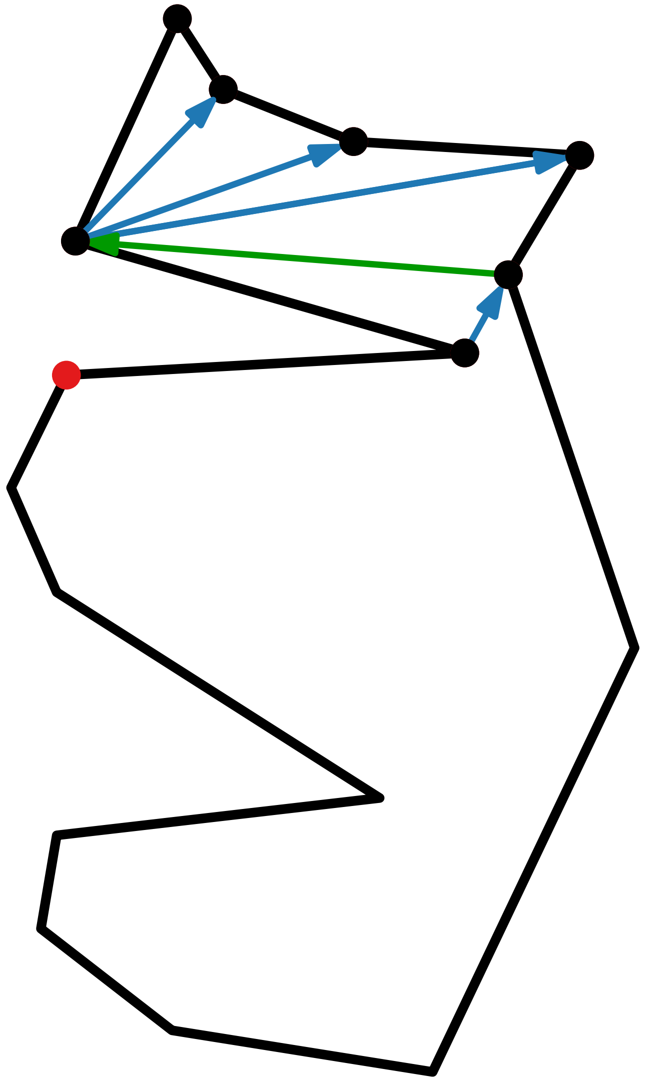


Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



Triangulating a y-Monotone Polygon P

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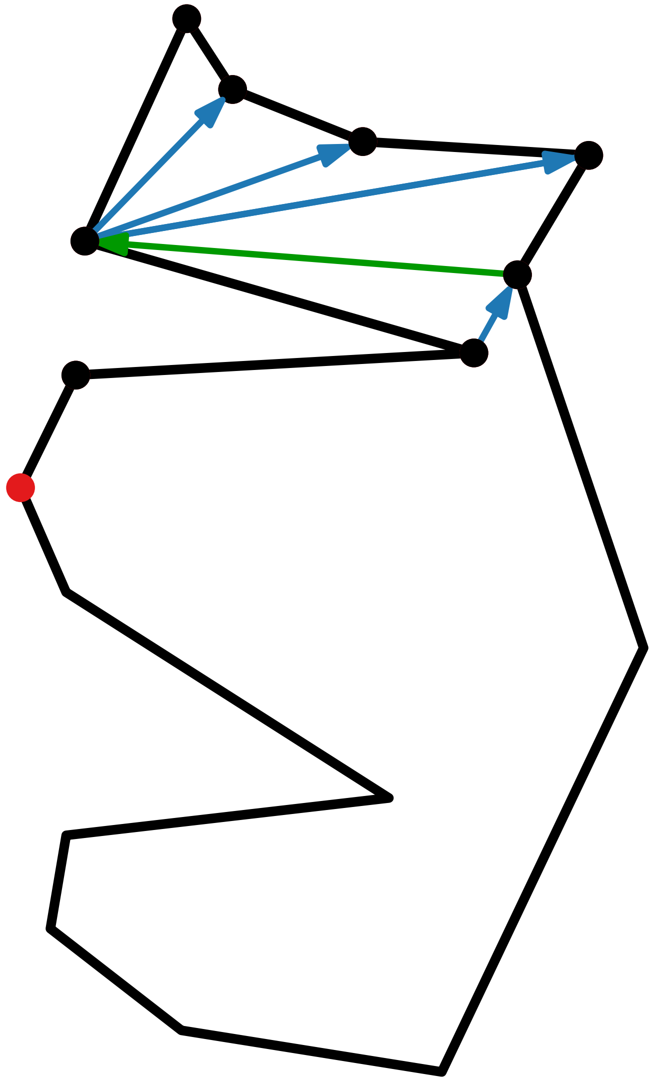

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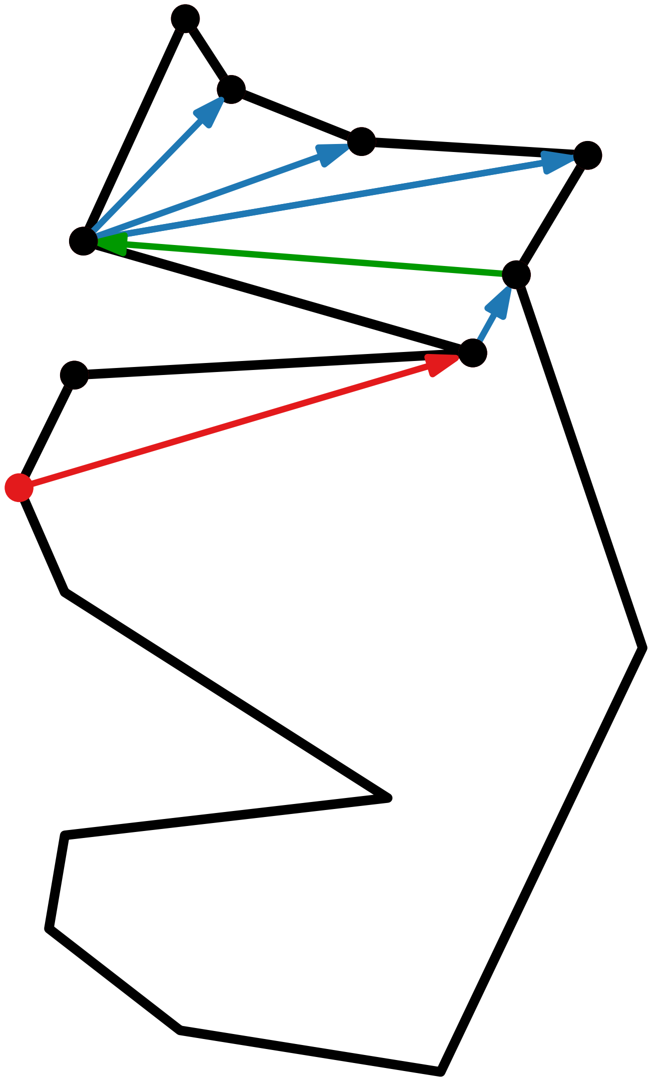

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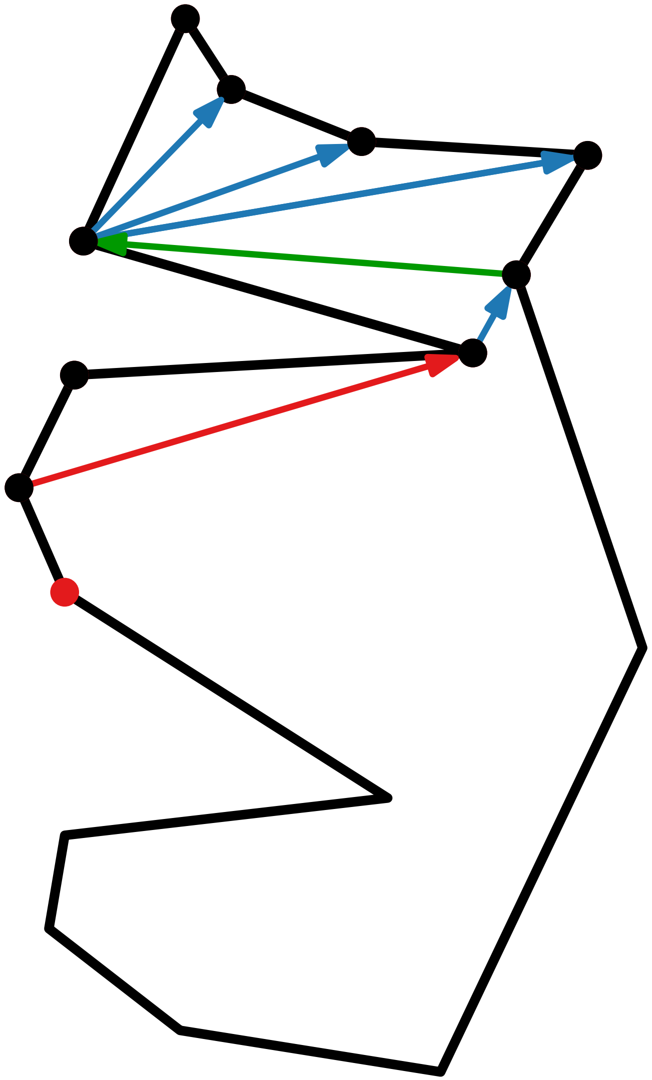
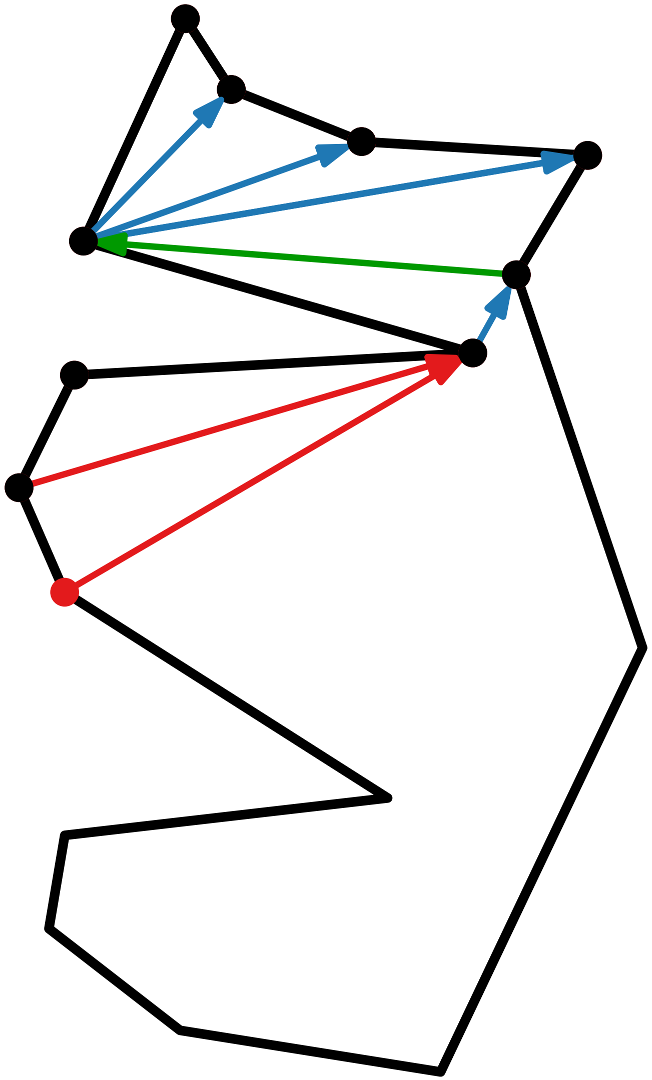


Triangulating a *y*-Monotone Polygon *P*

Approach: greedy, going from top to bottom



Triangulating a y-Monotone Polygon P

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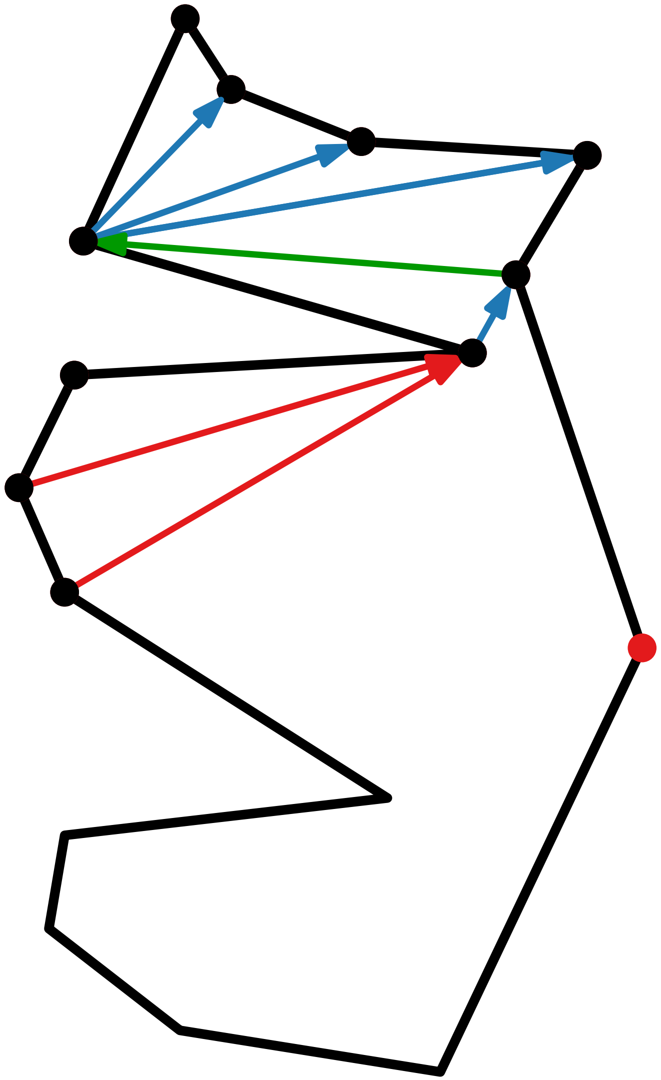

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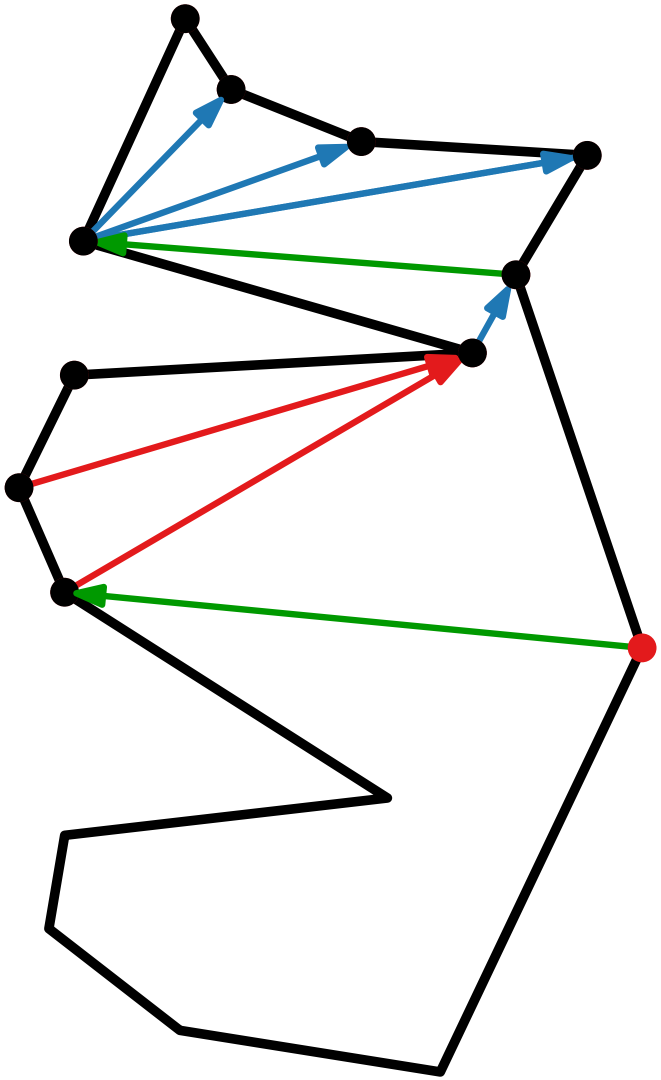

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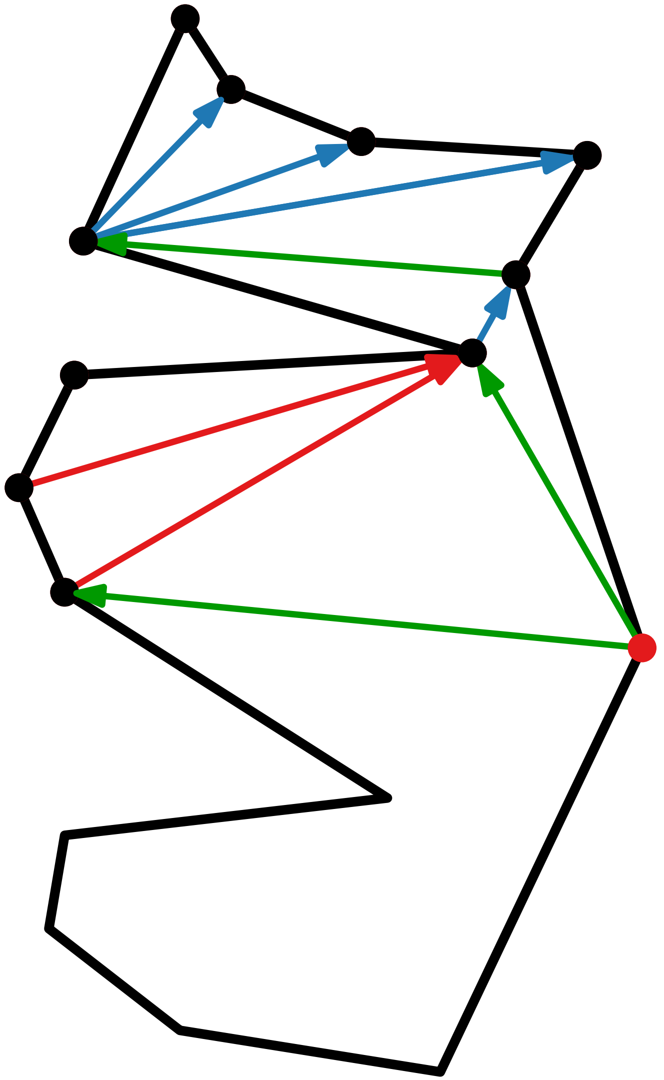

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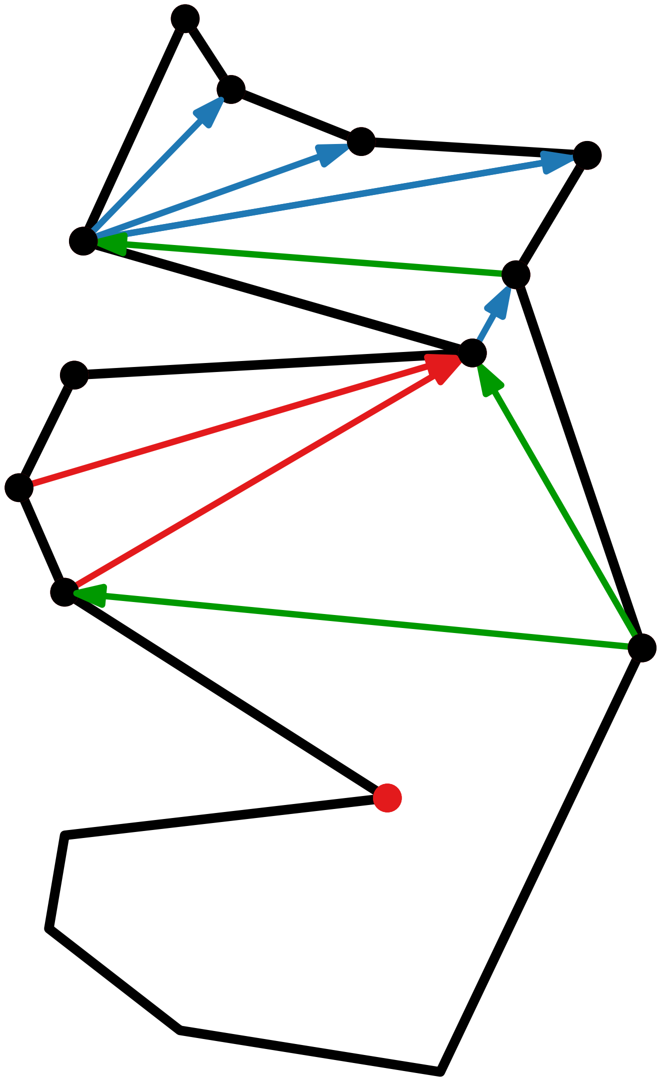

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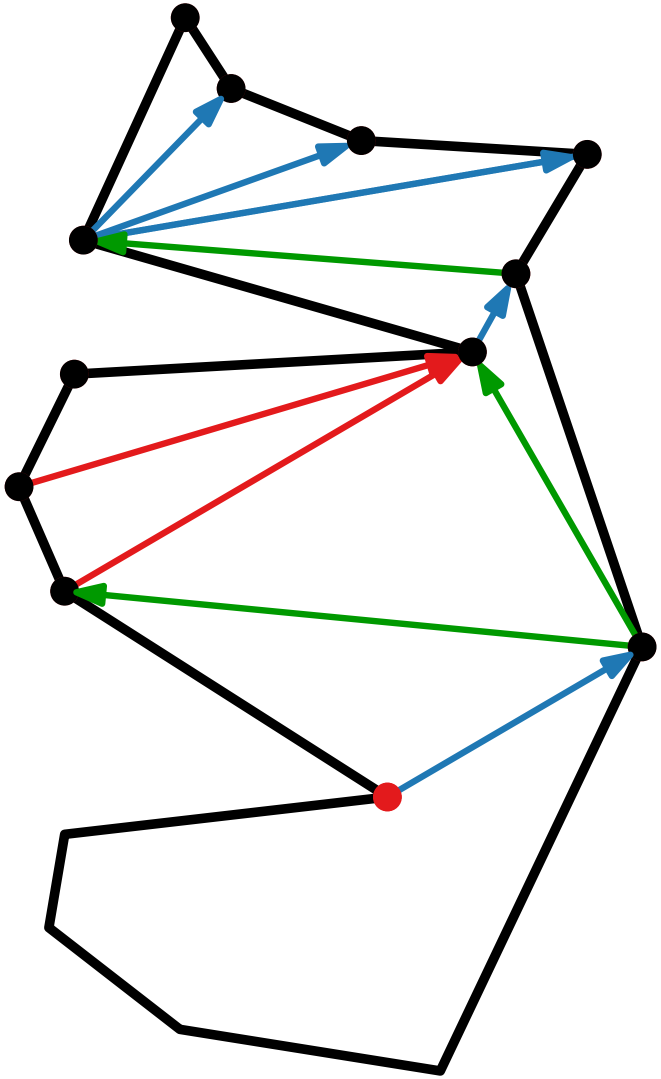

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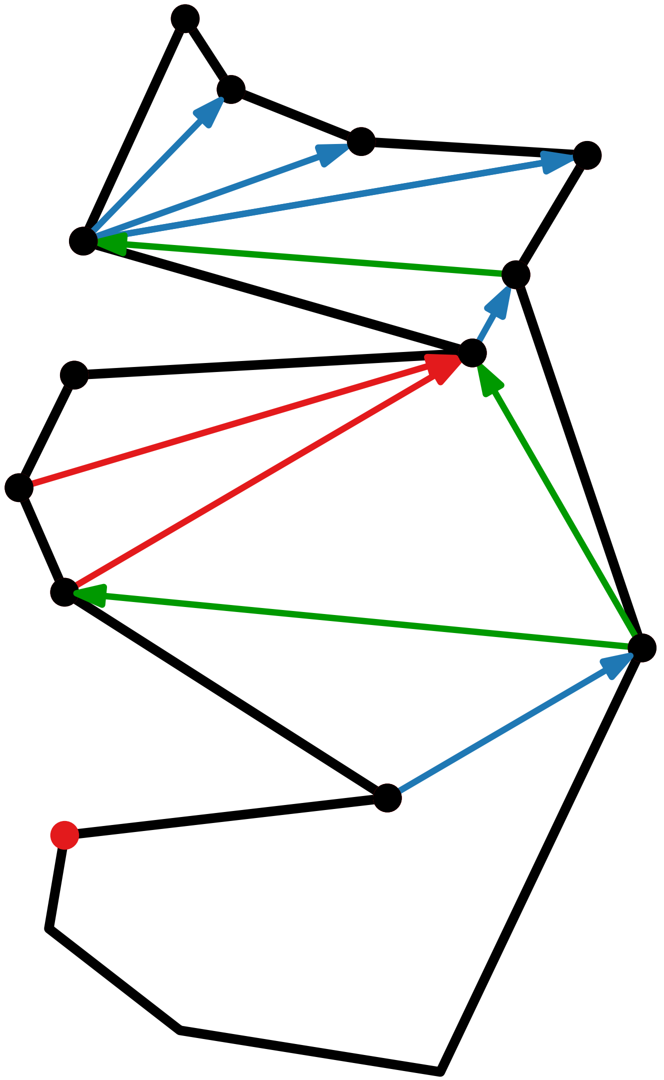
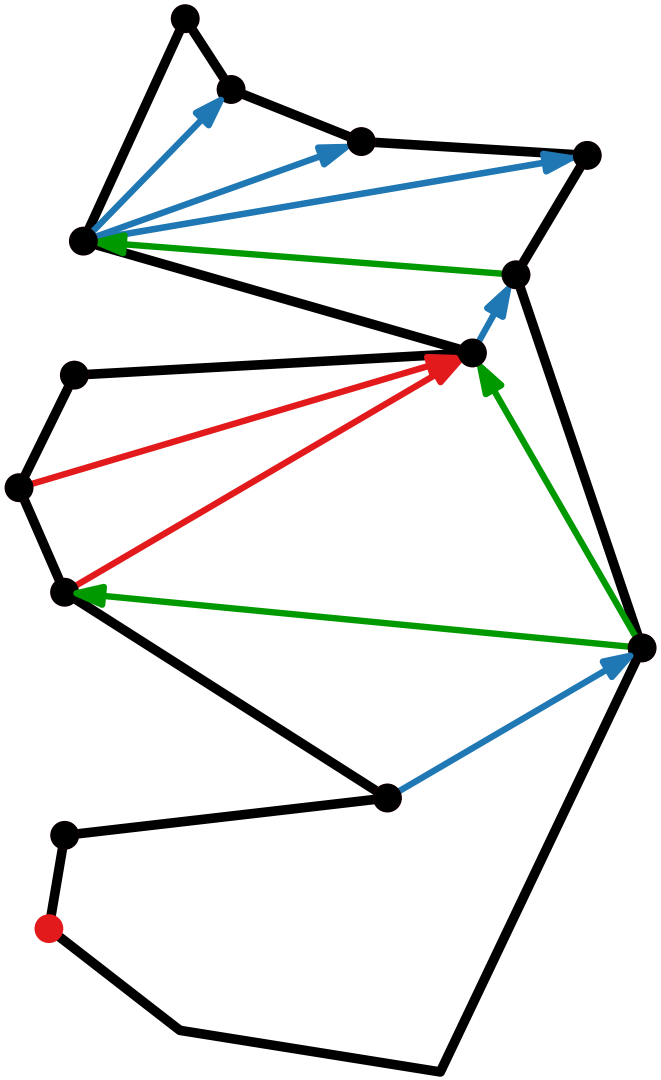


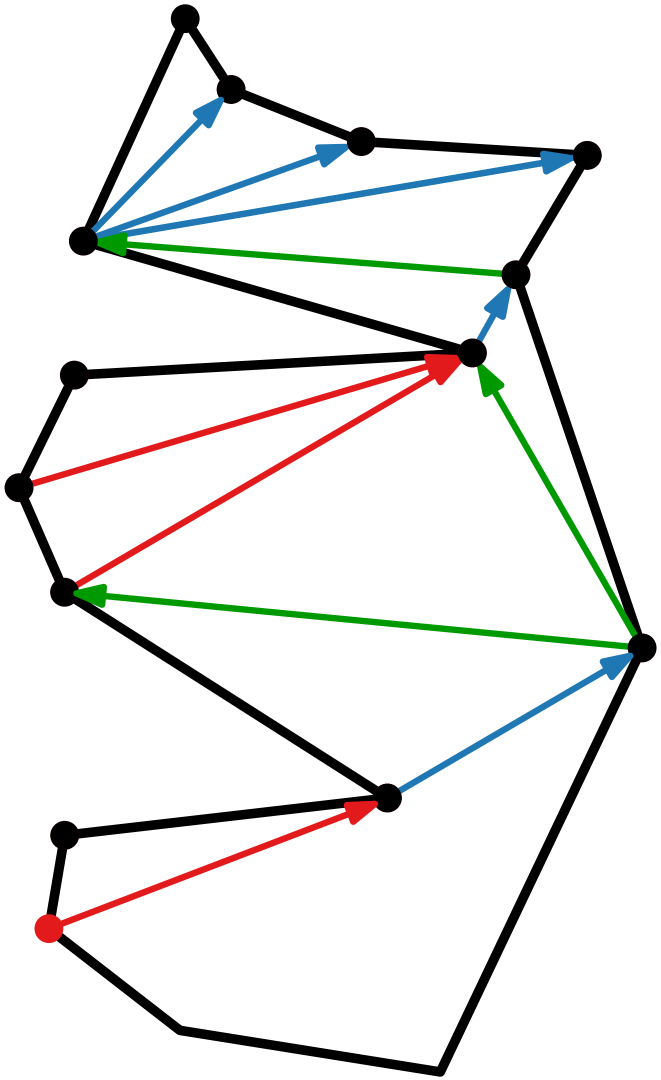
Triangulating a *y*-Monotone Polygon *P*

Approach: greedy, going from top to bottom



Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



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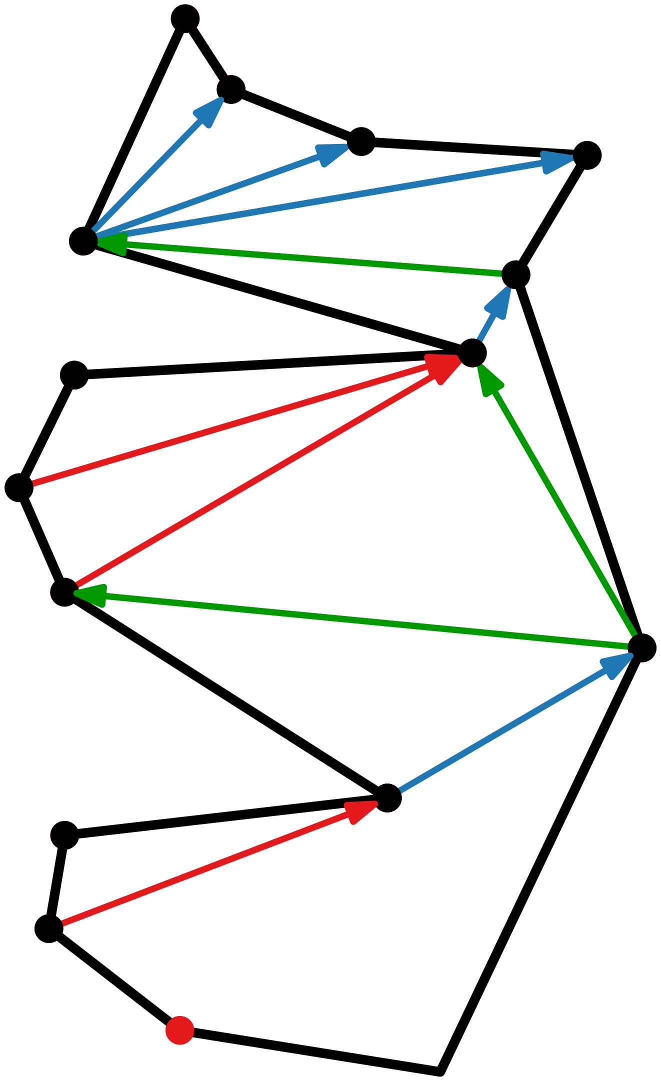

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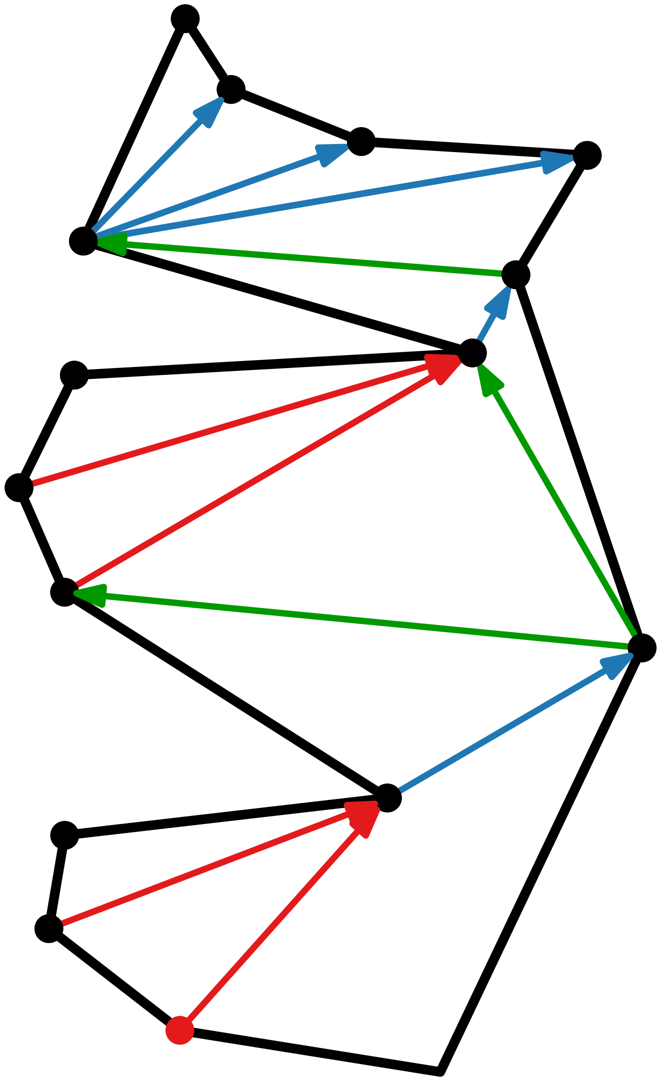

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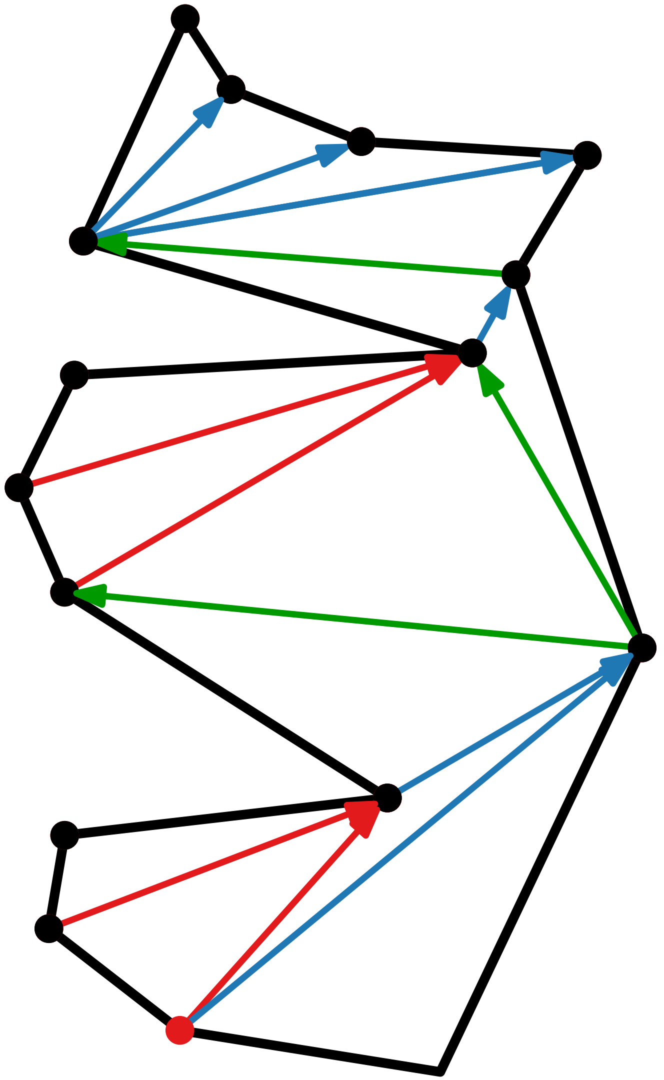

Triangulating a y-Monotone Polygon P

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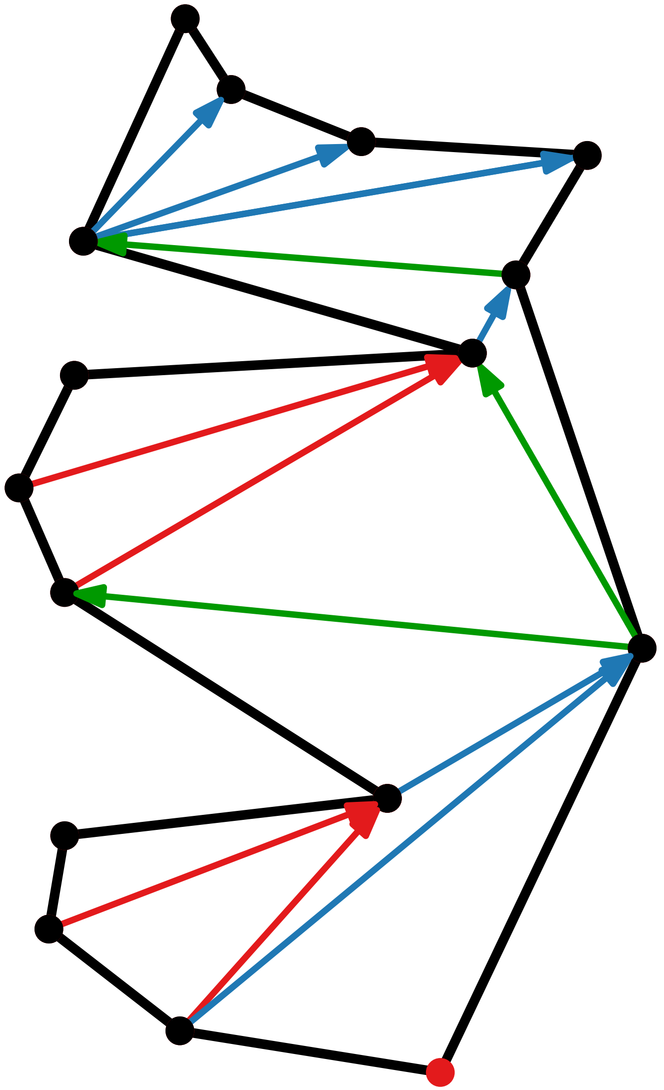
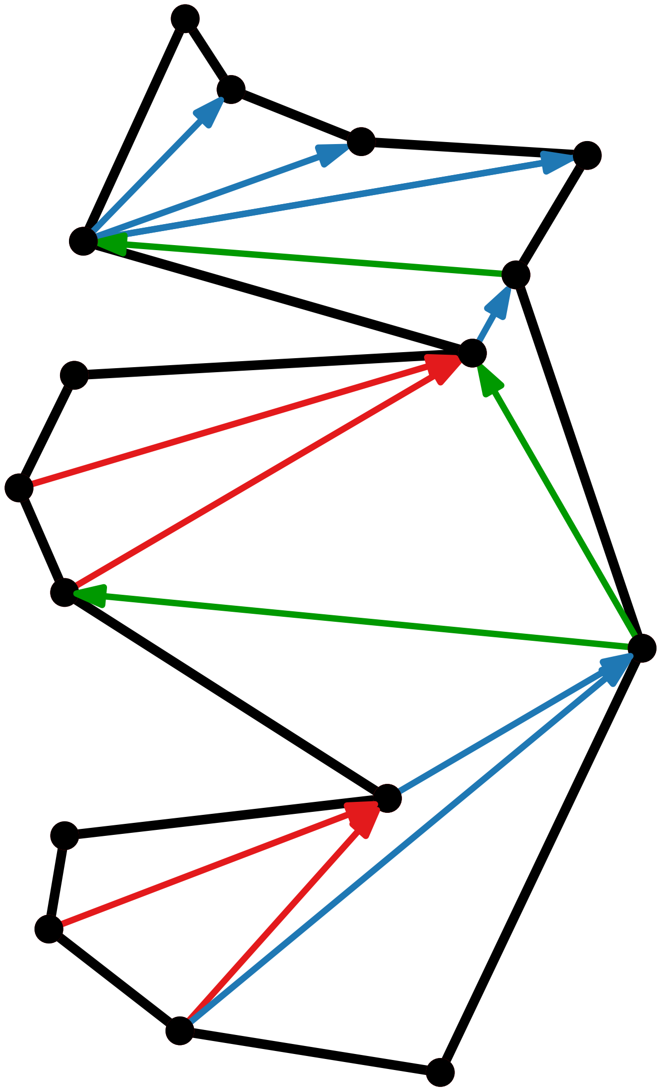


Triangulating a y-Monotone Polygon P

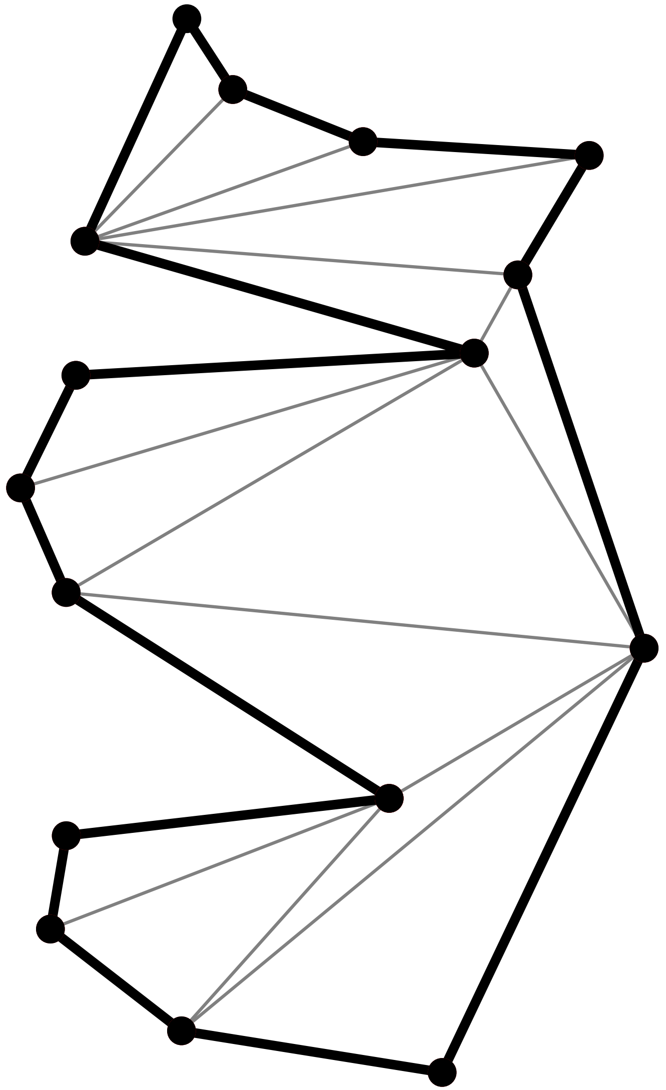
Approach: greedy, going from top to bottom



Invariant?

Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



Invariant?

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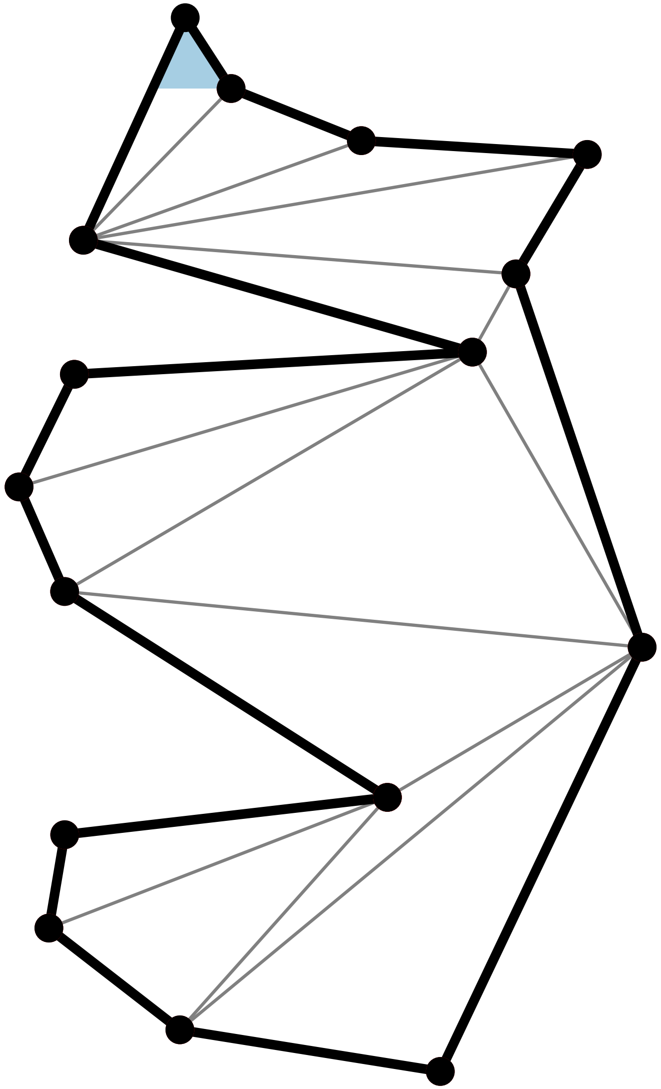
Approach: greedy, going from top to bottom



Invariant?

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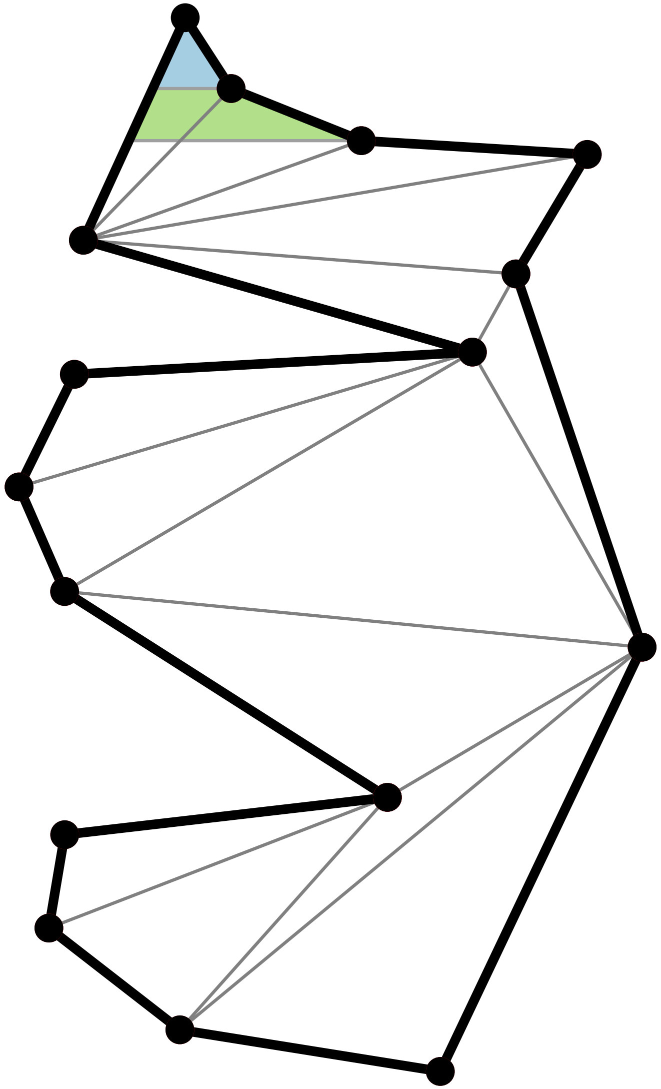

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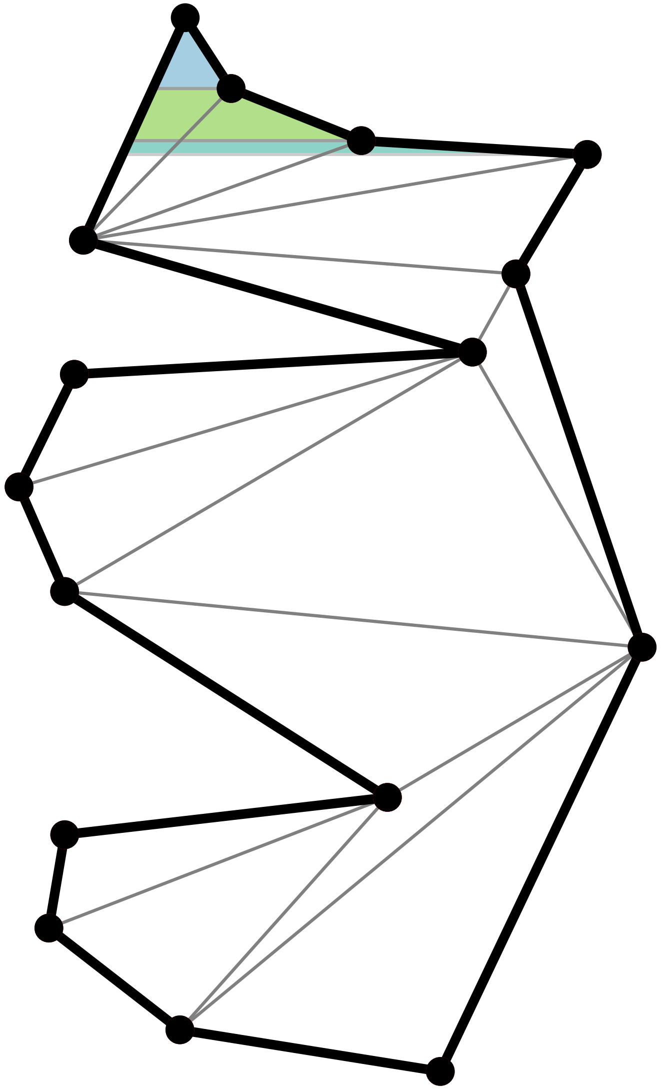
Approach: greedy, going from top to bottom



Invariant?

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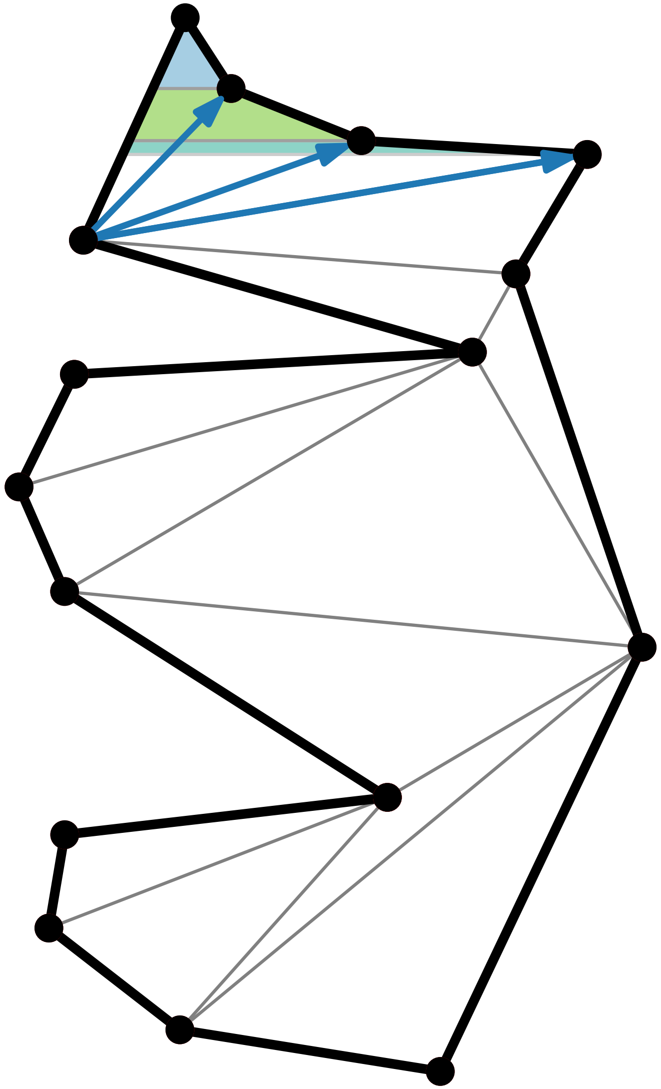
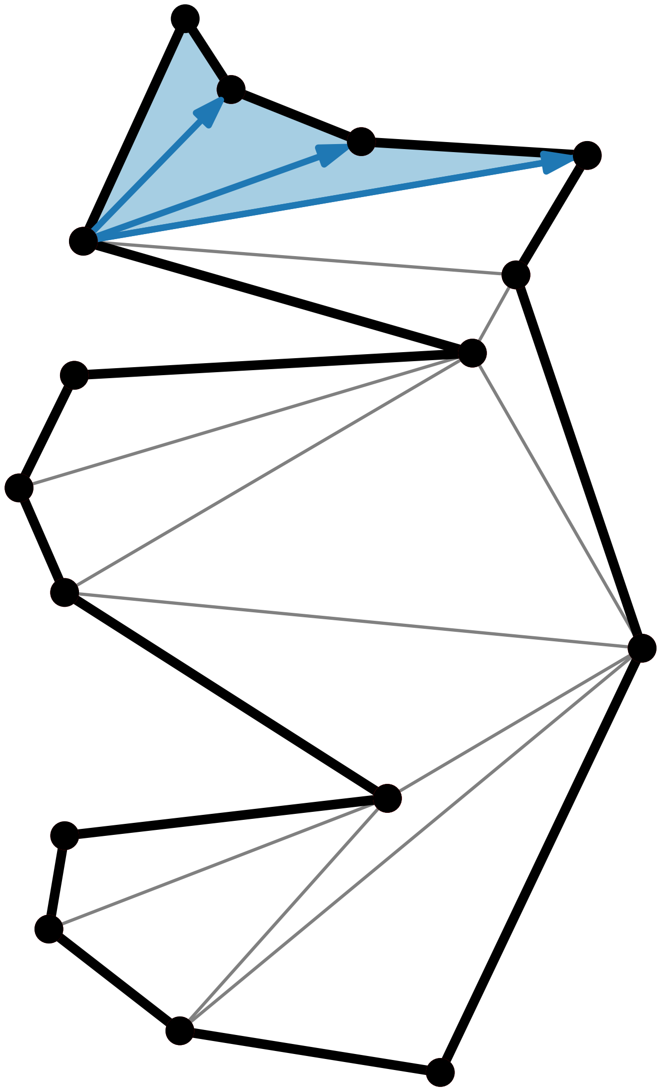
Approach: greedy, going from top to bottom



Invariant?

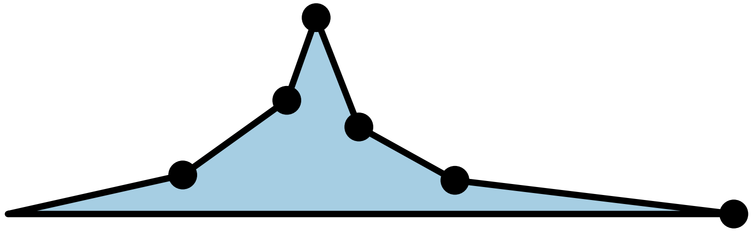
Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



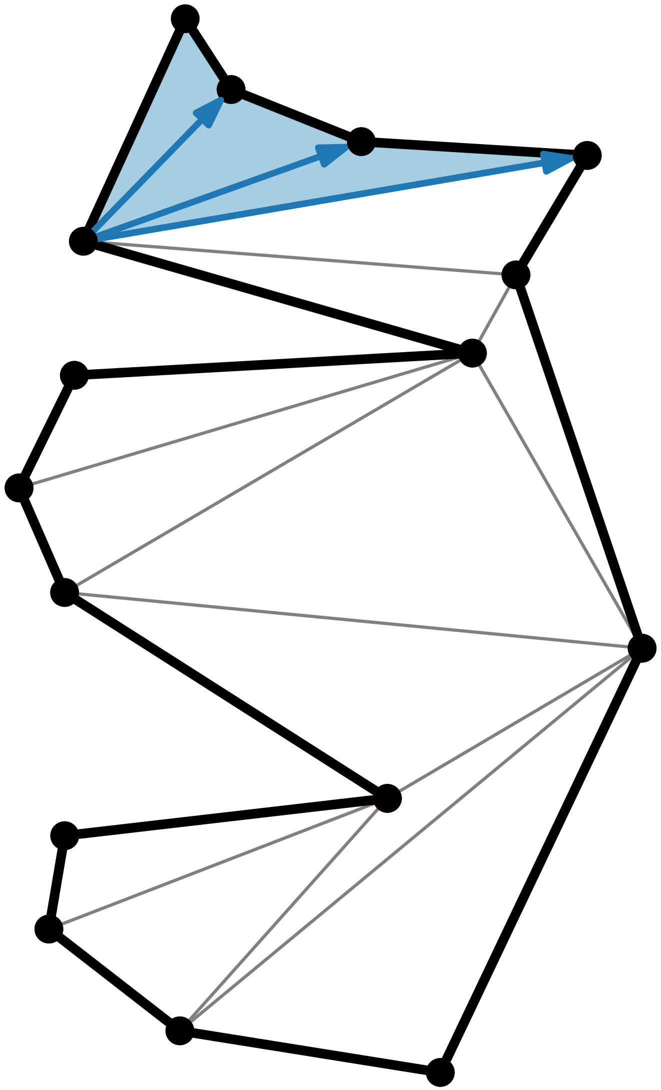
Invariant?

The part of P that we have seen but not yet triangulated is a *funnel*.



Triangulating a *y*-Monotone Polygon *P*

Approach: greedy, going from top to bottom



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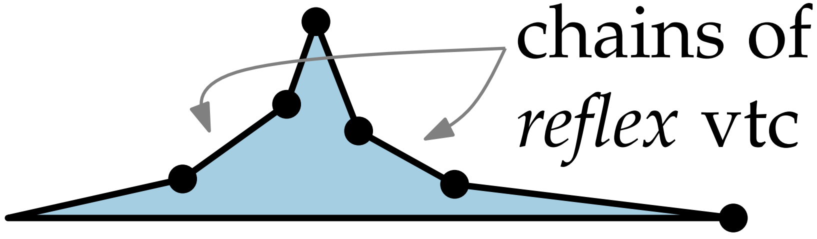
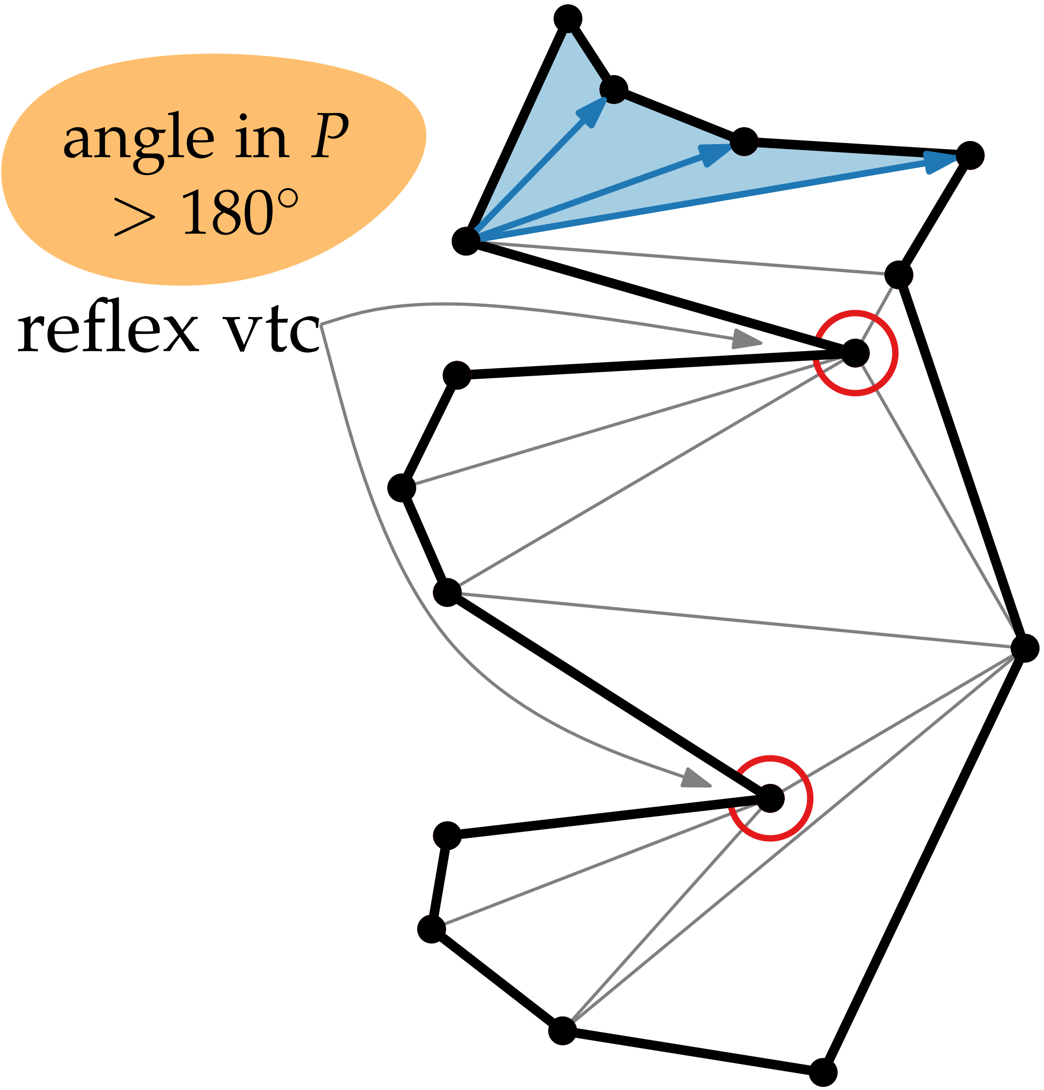


chains of *reflex* vtc

Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom

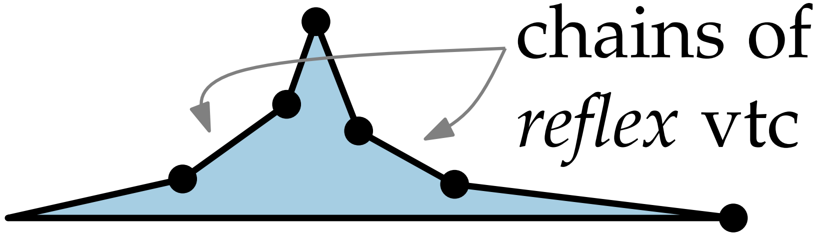


angle in P $> 180°$

reflex vtc

Invariant?

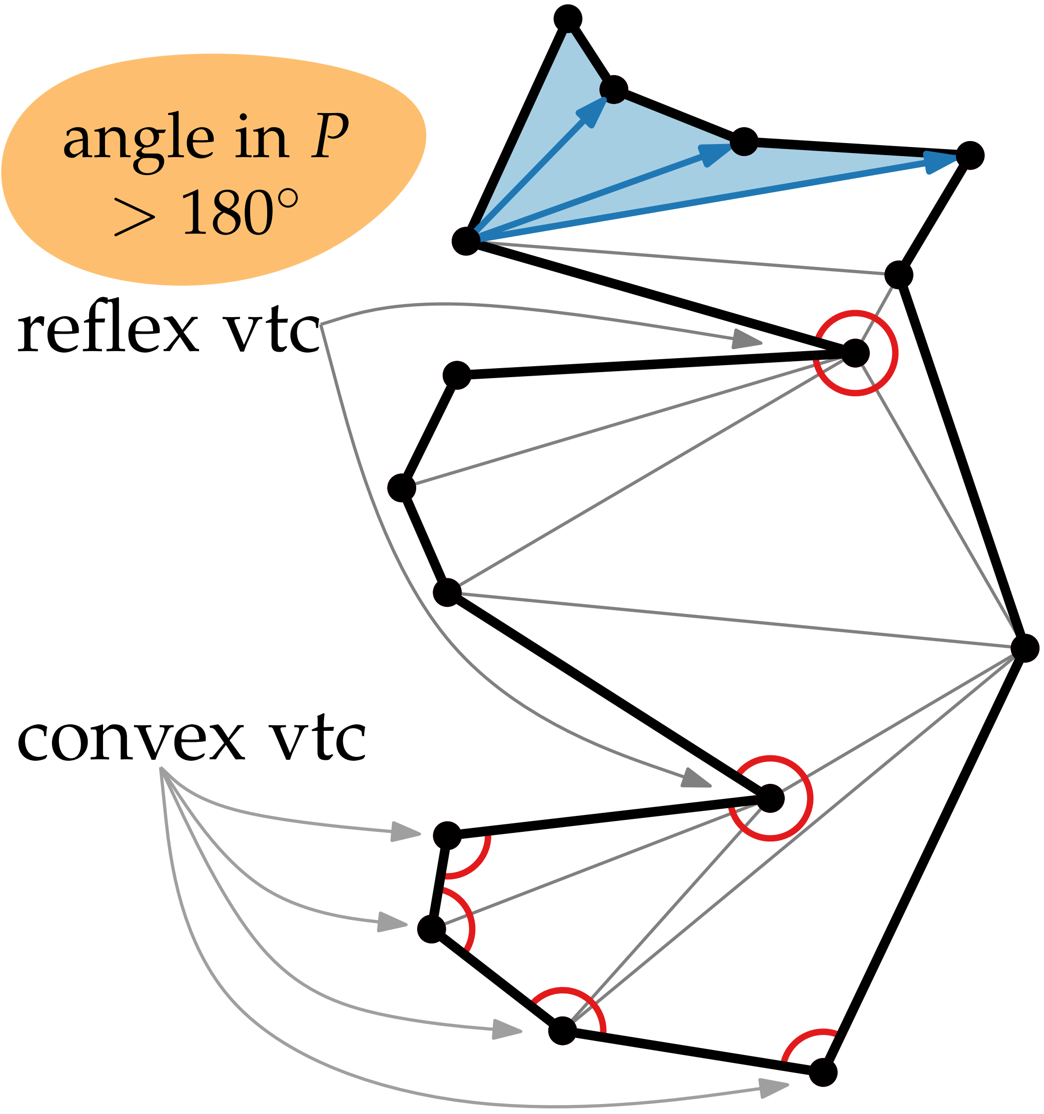
The part of P that we have seen but not yet triangulated is a *funnel*.



chains of *reflex* vtc

Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



Invariant?

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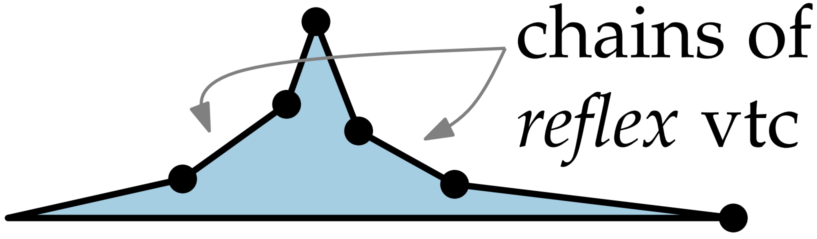
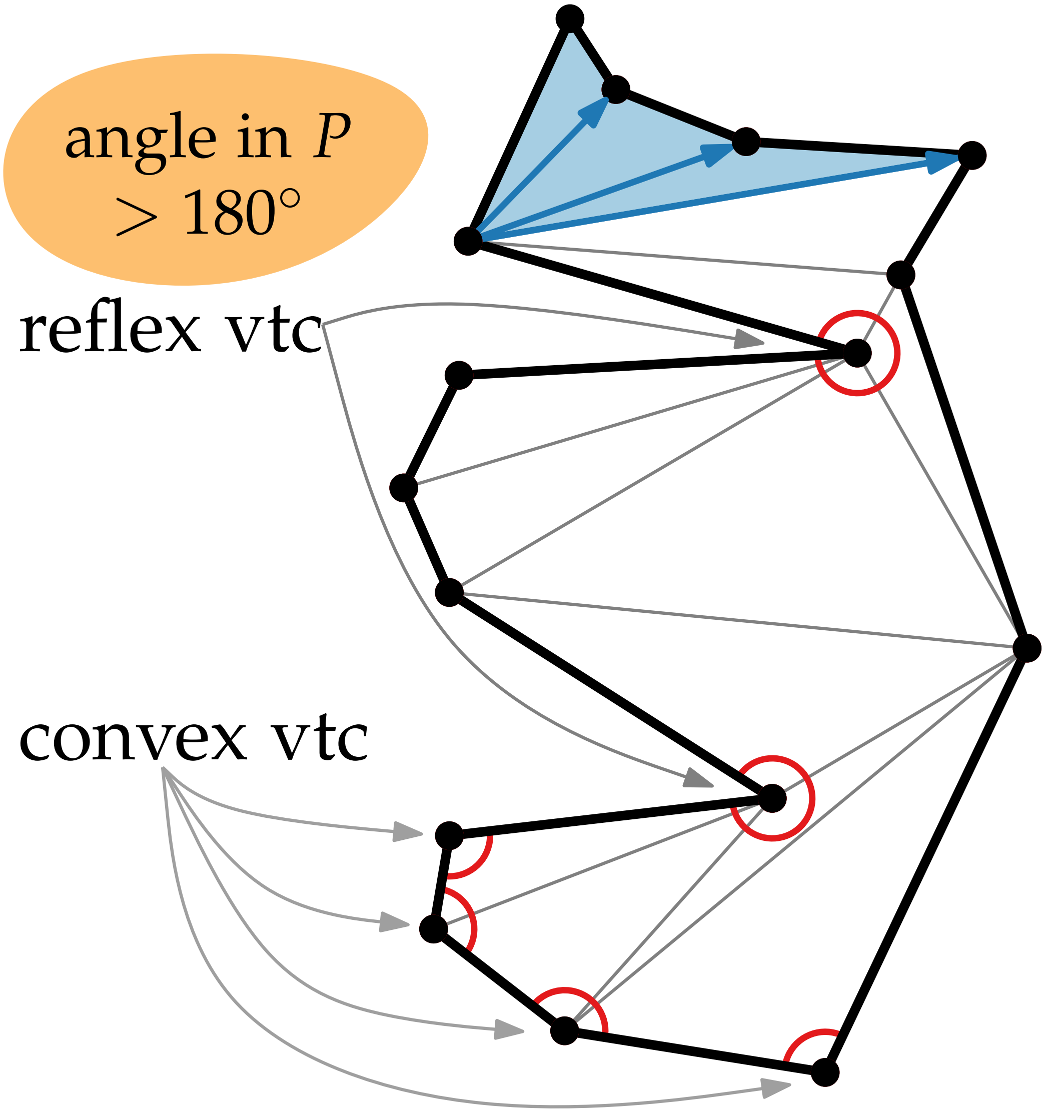


Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



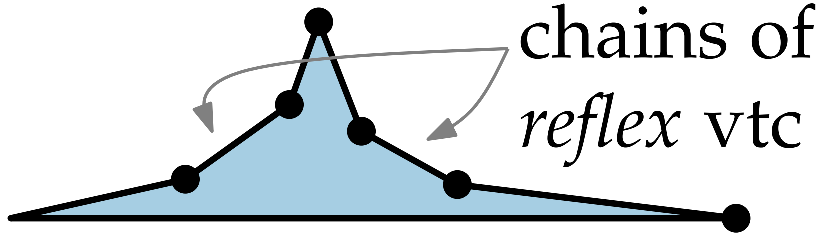
angle in P $> 180°$

reflex vtc

convex vtc

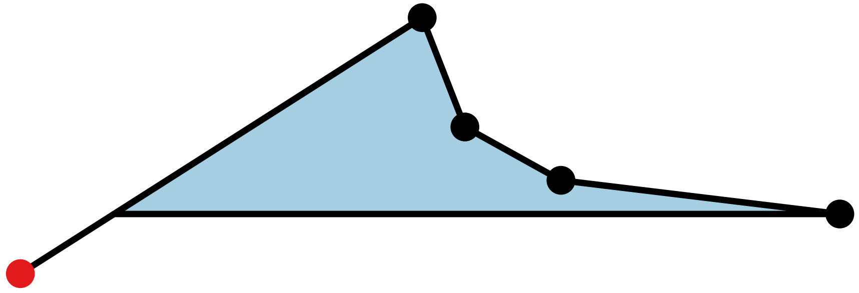
Invariant?

The part of P that we have seen but not yet triangulated is a *funnel*.



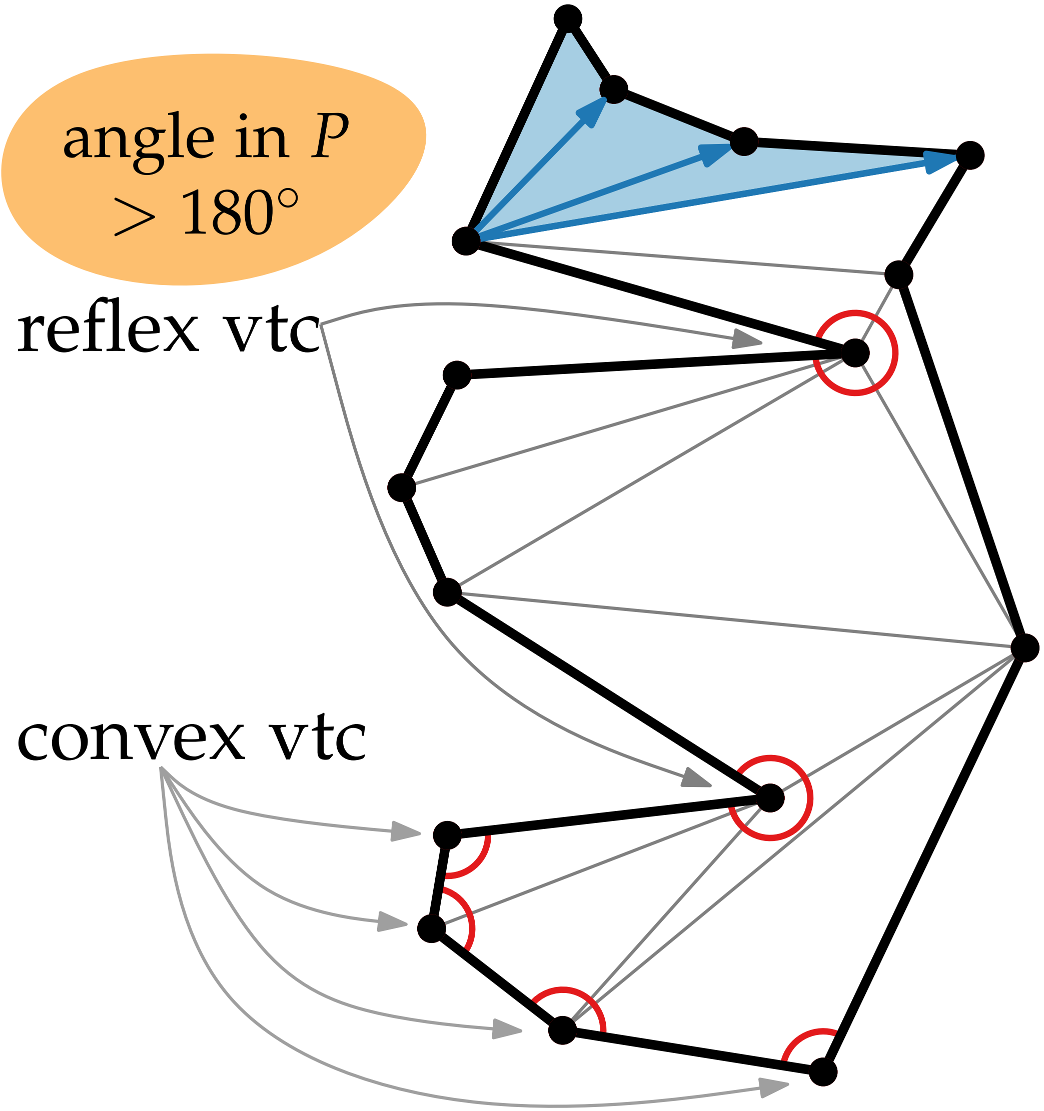
chains of *reflex* vtc

Our funnels are special:



Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



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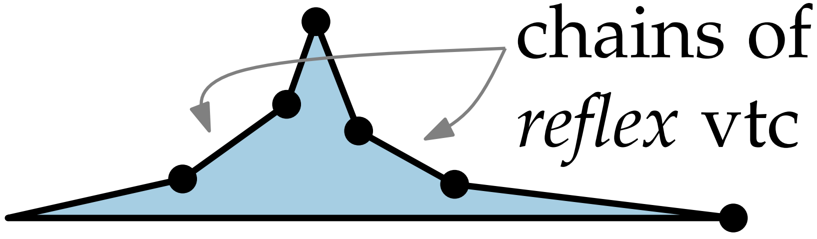

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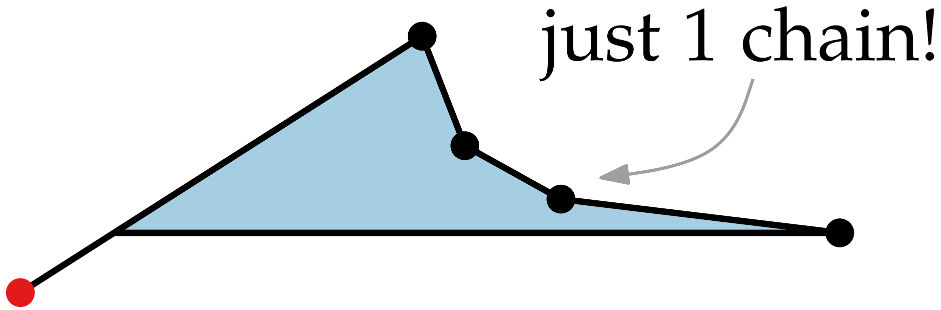
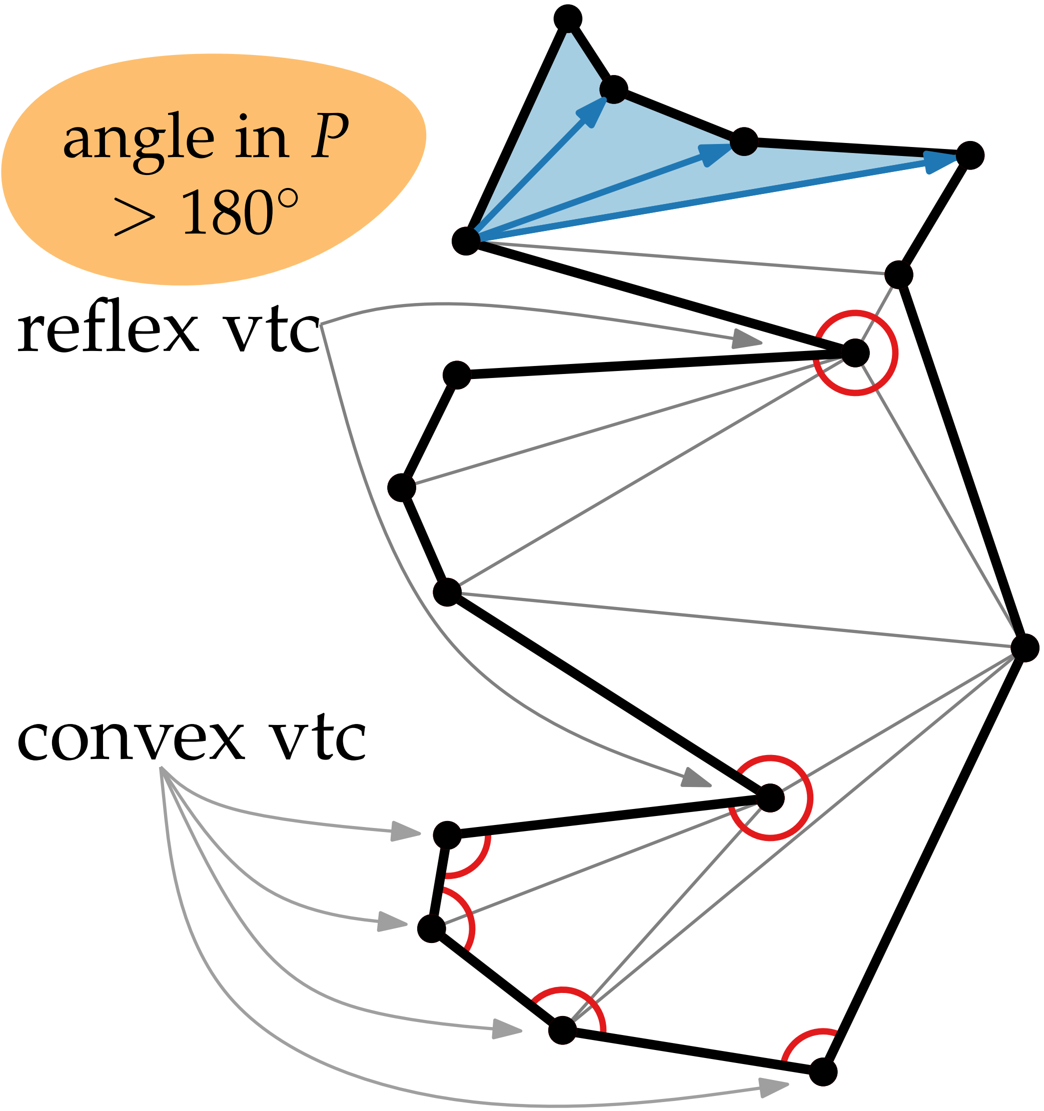


Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



Invariant?

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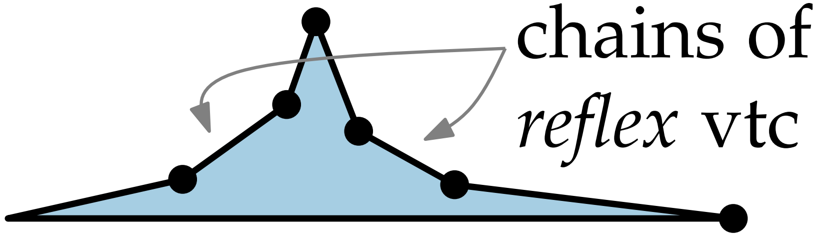

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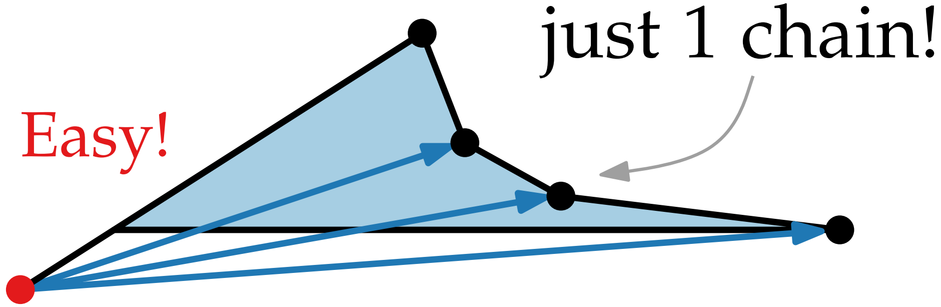

Algorithm

```
**TriangulateMonotonePolygon**(Polygon $P$ as circular vertex list)
    merge left and right chain → seq. $u_1, \ldots, u_n$ with $y_1 \geq \ldots \geq y_n$
    Stack $S$; $S$.push($u_1$); $S$.push($u_2$)
    **for** $j \leftarrow 3$ **to** $n - 1$ **do**
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Algorithm

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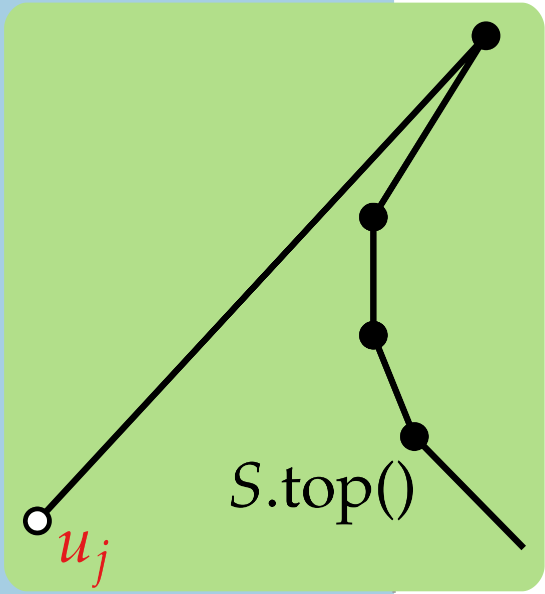

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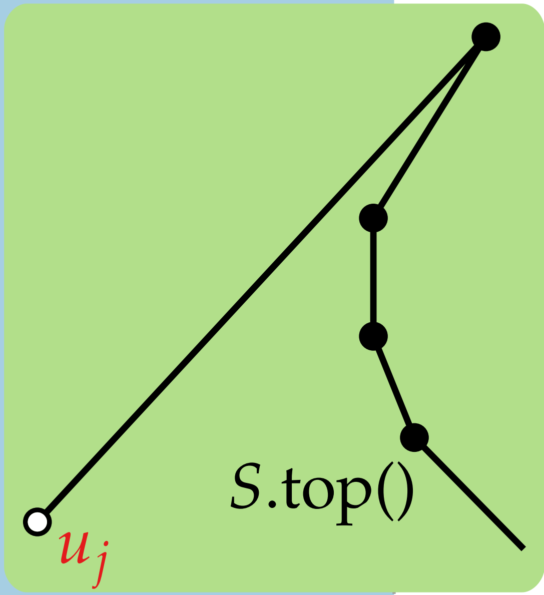

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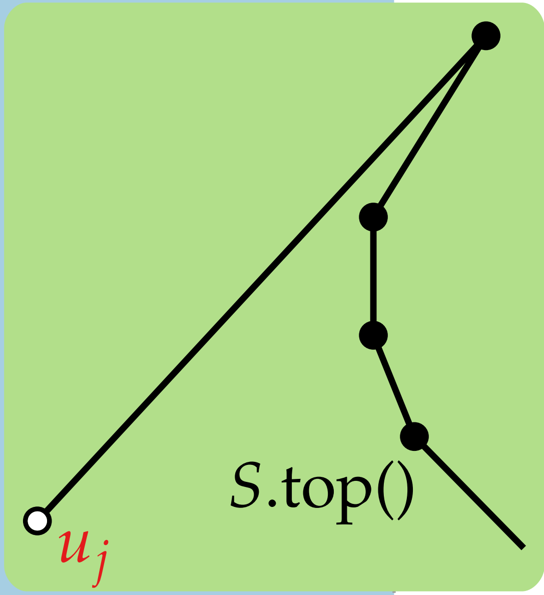

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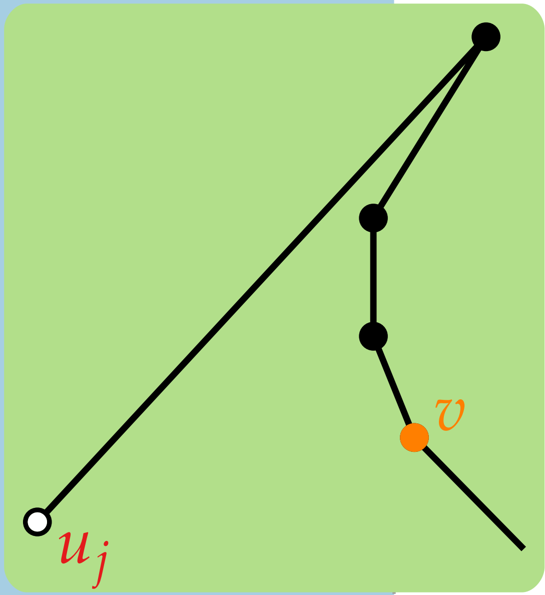

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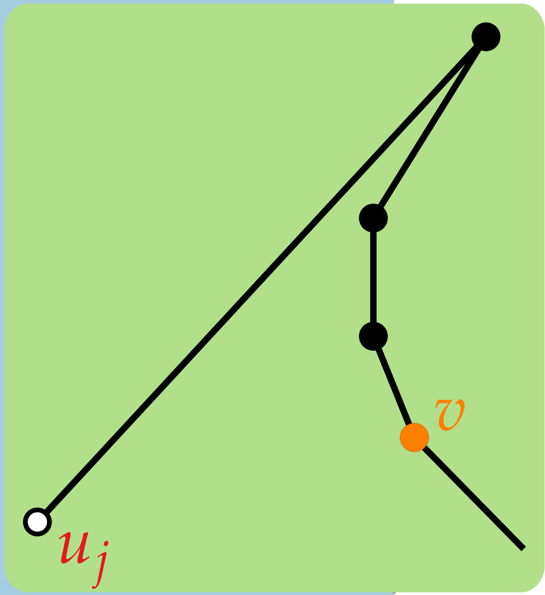

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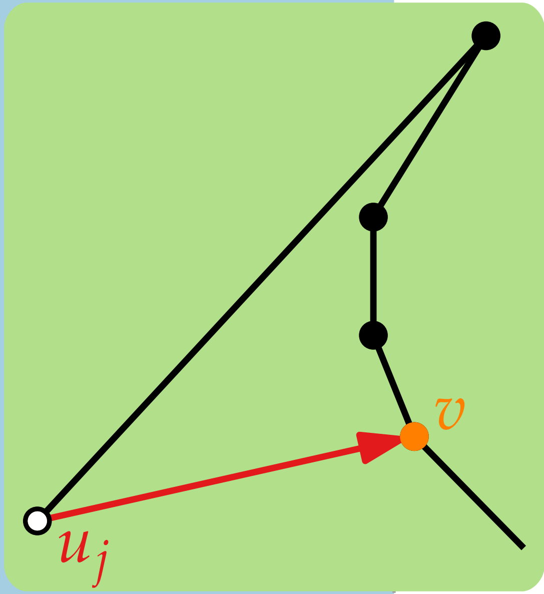

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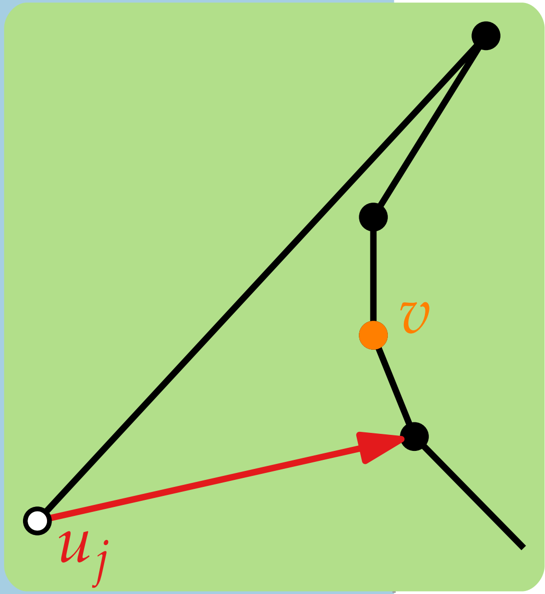

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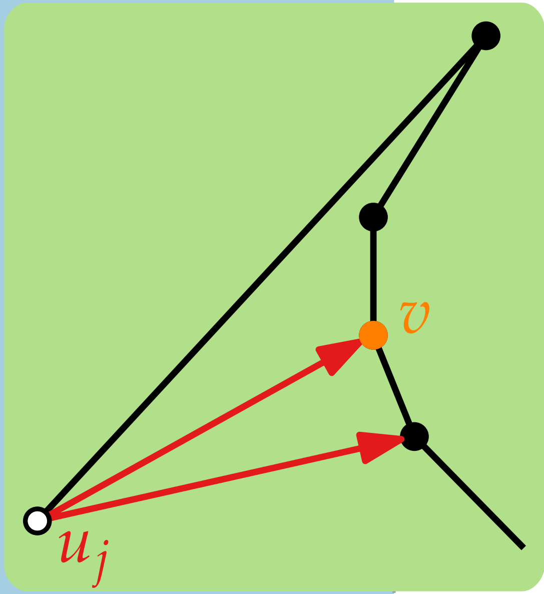

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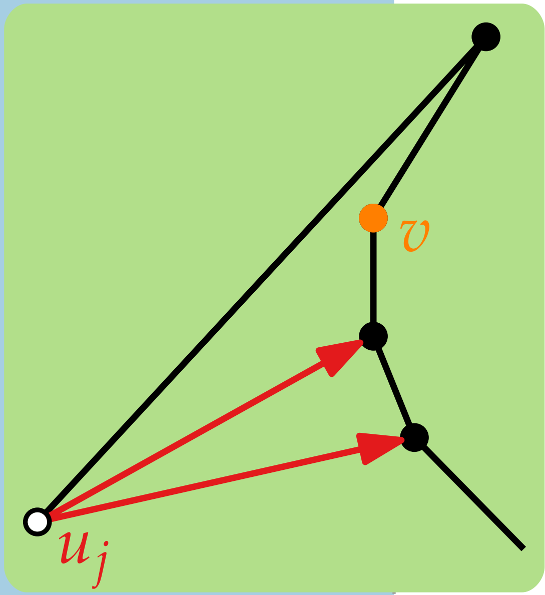

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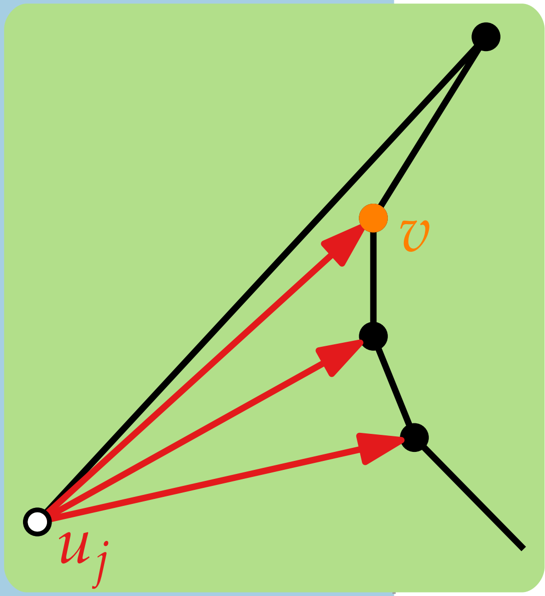

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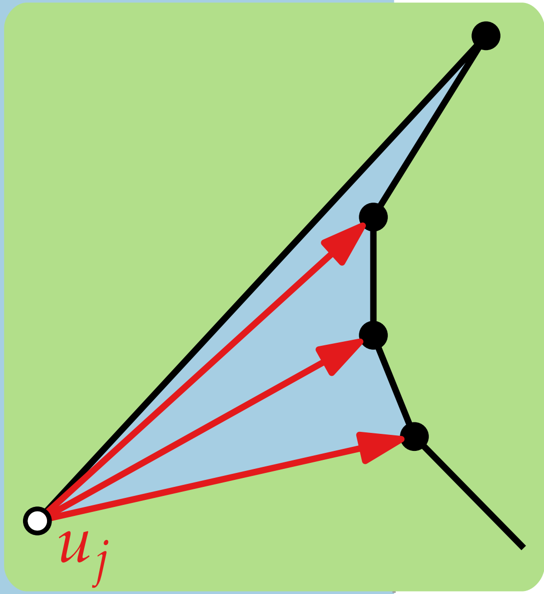

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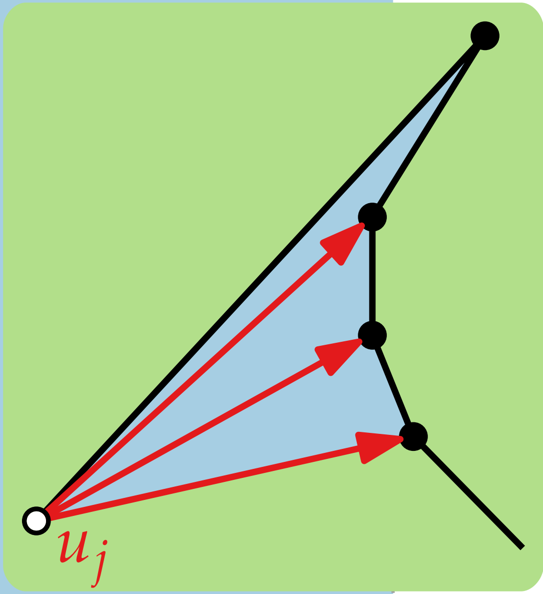

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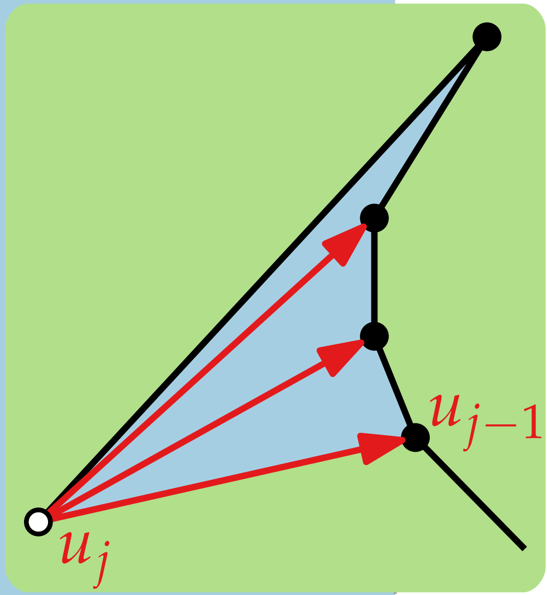

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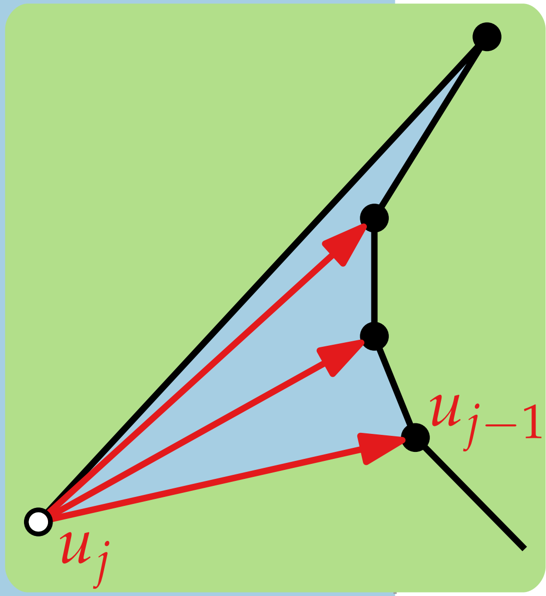

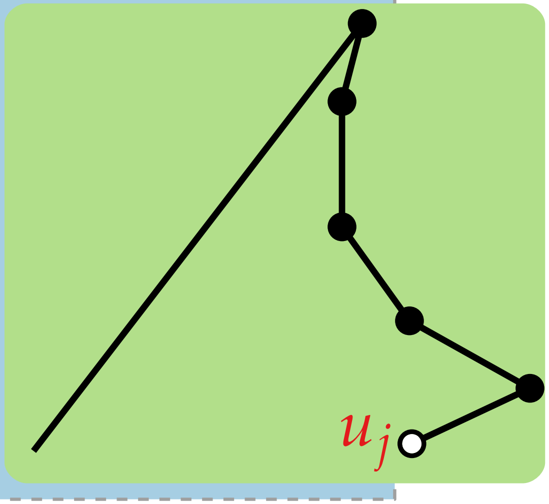

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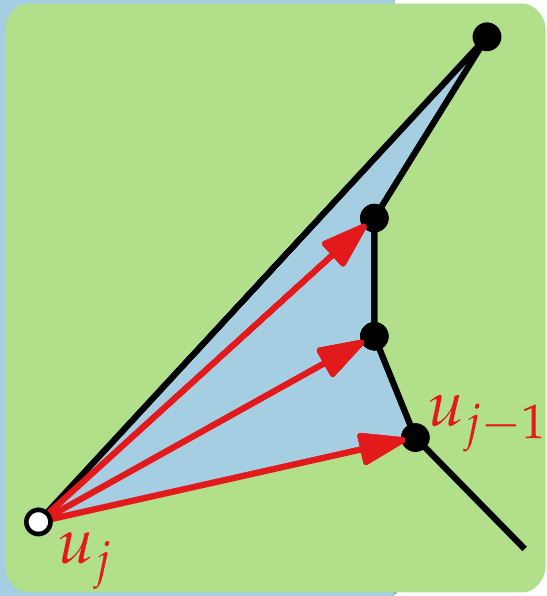

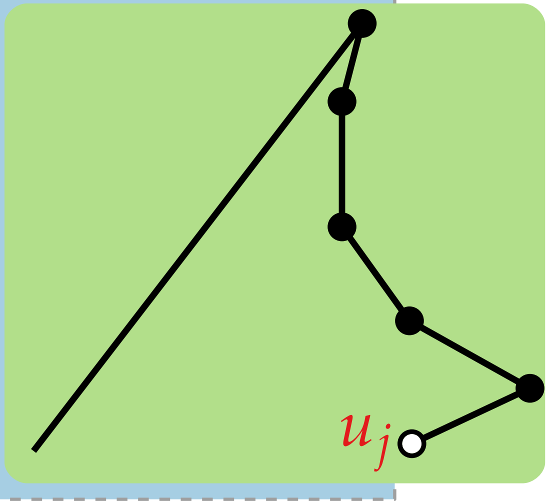

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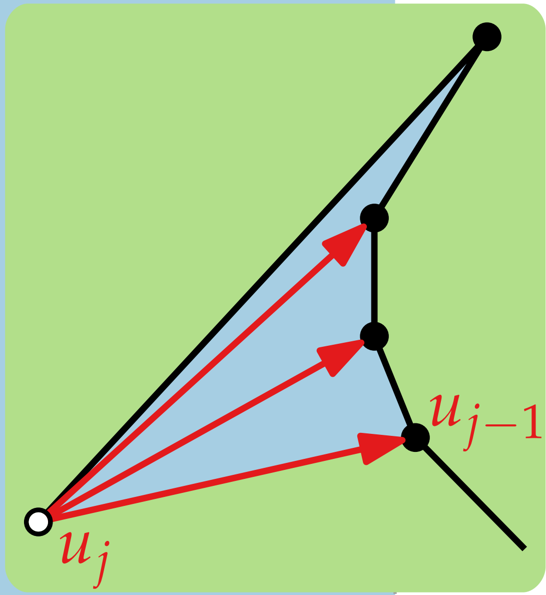

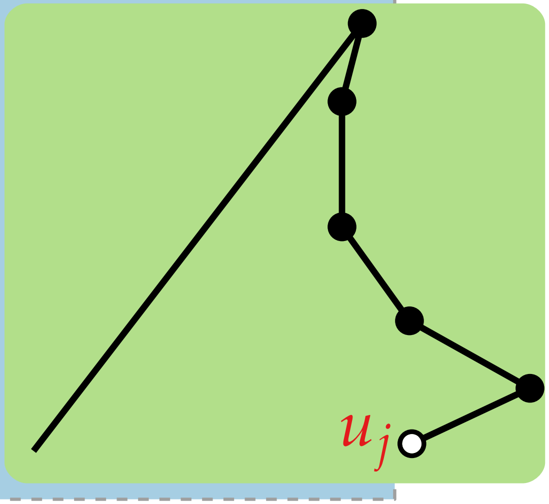

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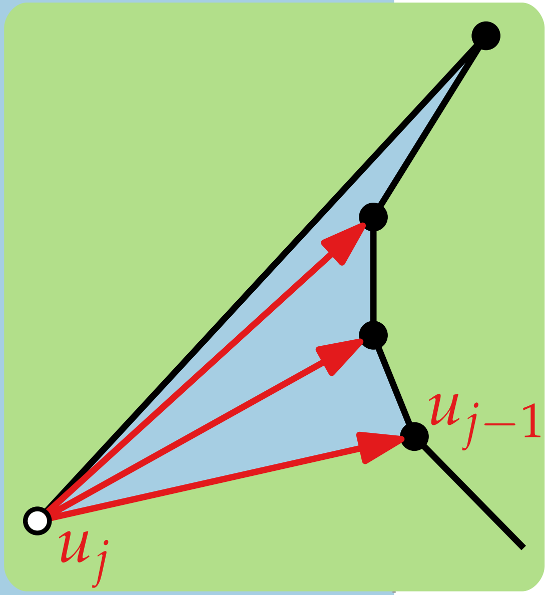

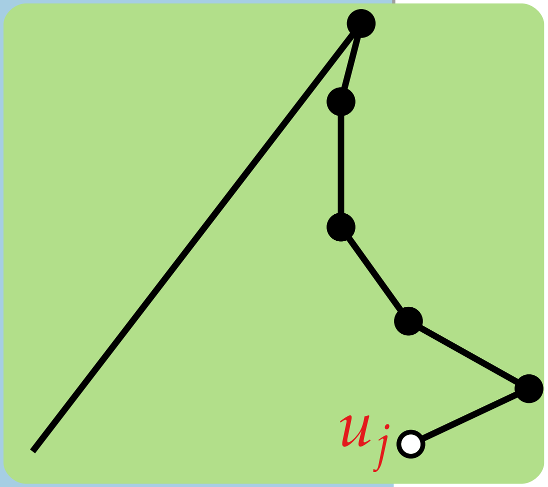

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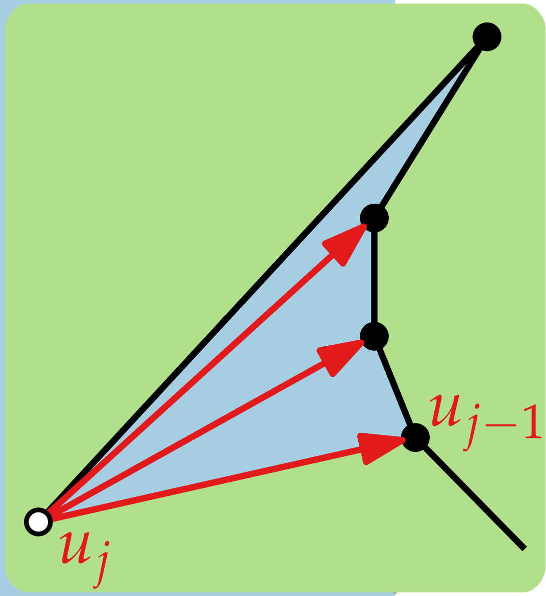

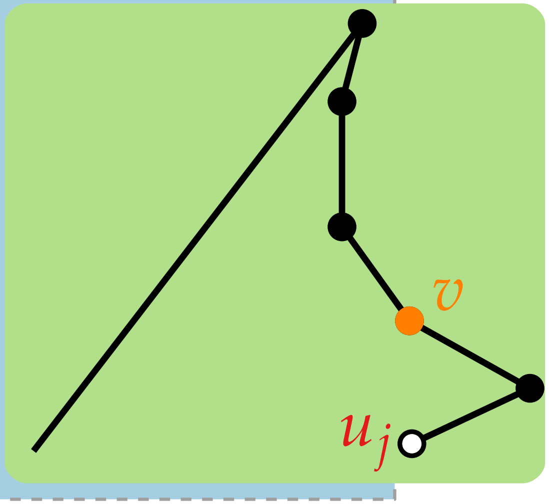

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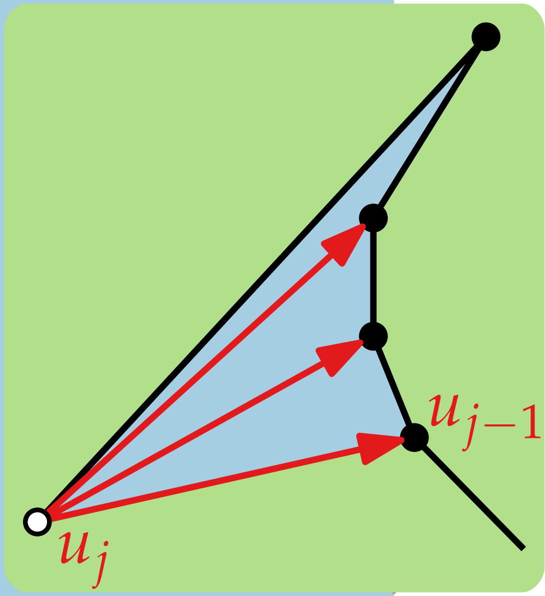

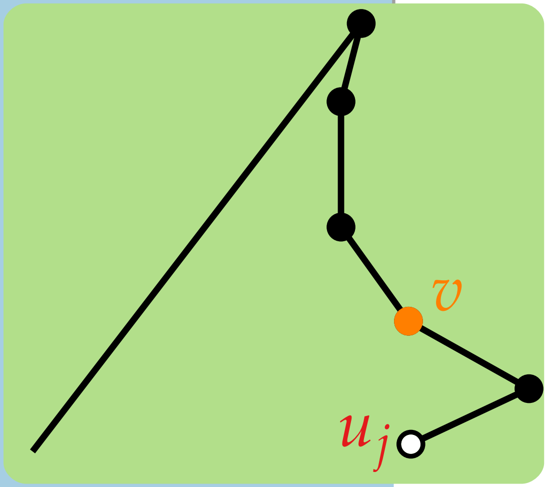

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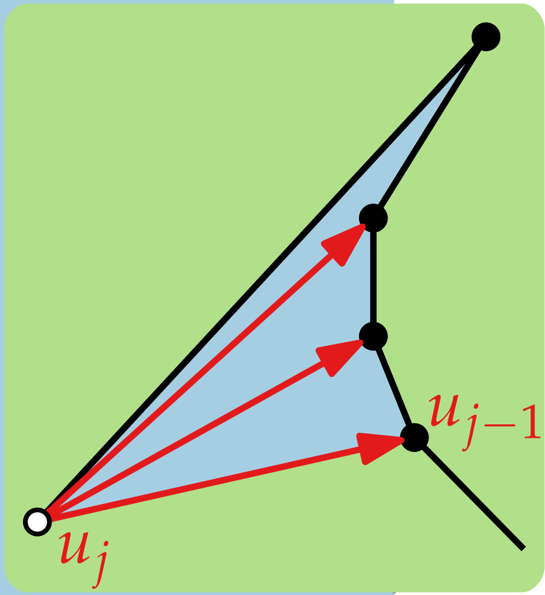

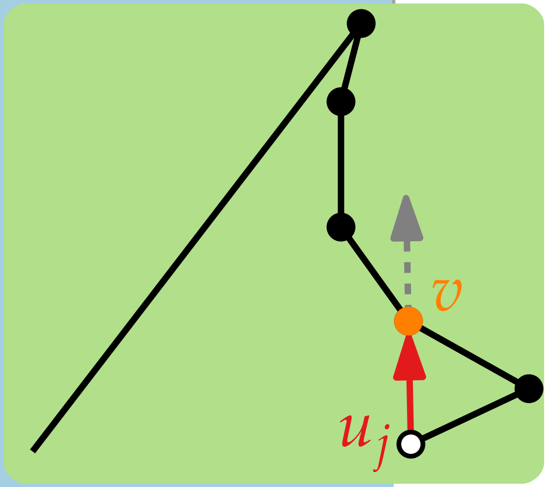

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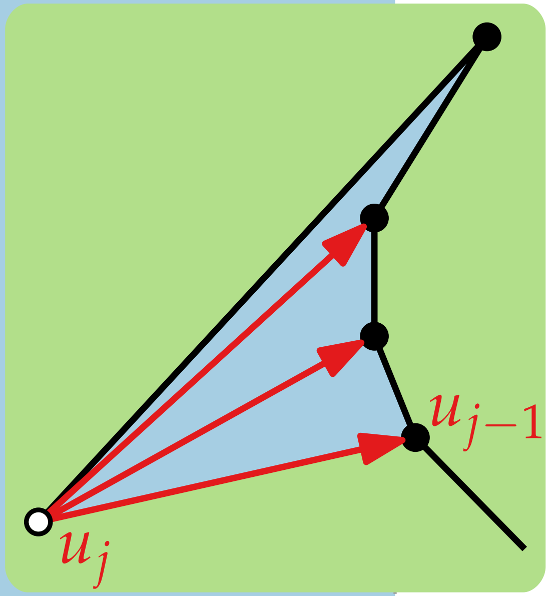

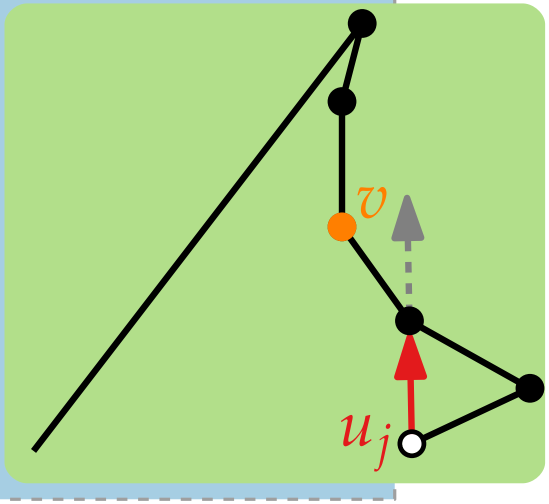

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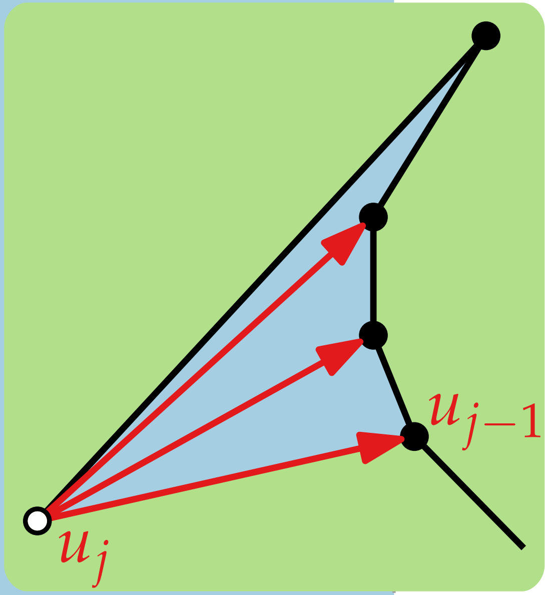

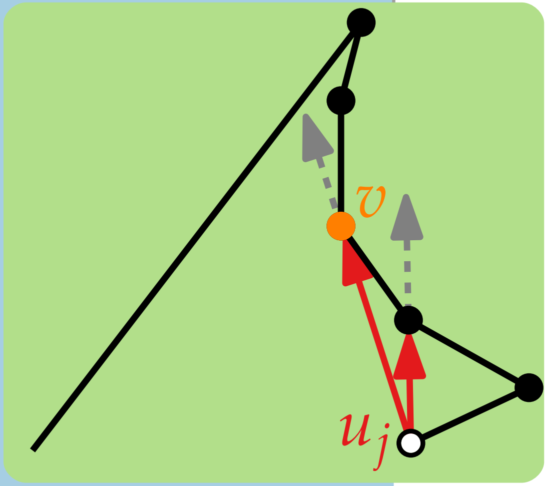

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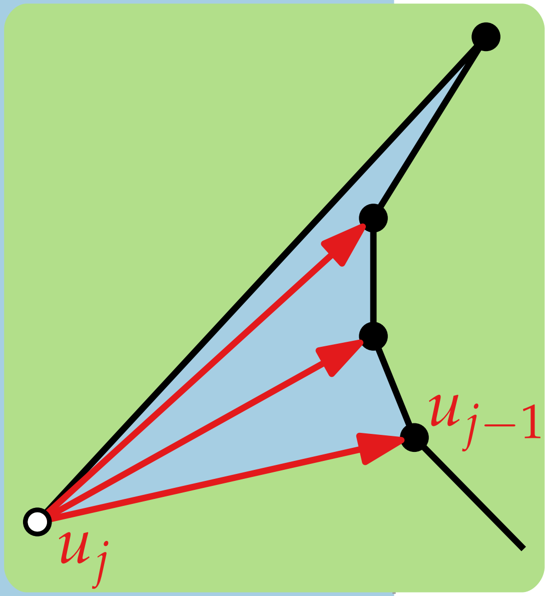

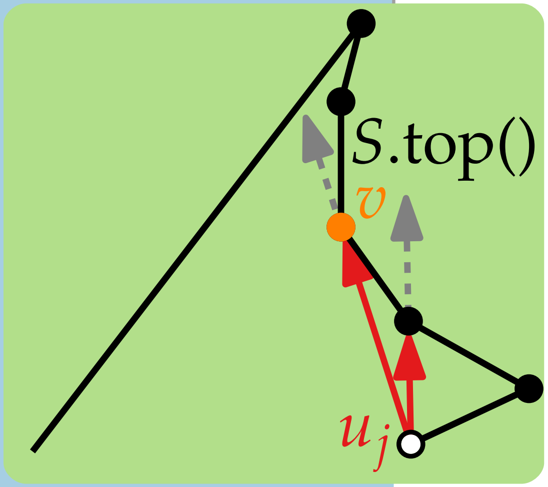

Algorithm

```
TriangulateMonotonePolygon(Polygon $P$ as circular vertex list)
  merge left and right chain → seq. $u_1, \ldots, u_n$ with $y_1 \geq \ldots \geq y_n$
  Stack $S$; $S$.push($u_1$); $S$.push($u_2$)
  for $j \leftarrow 3$ to $n-1$ do
      if $u_j$ and $S$.top() lie on different chains then
          while not $S$.empty() do
              $v \leftarrow S$.pop()
              if not $S$.empty() then draw diag. $(u_j, v)$
          $S$.push($u_{j-1}$); $S$.push($u_j$)
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          $v \leftarrow S$.pop()
          while not $S$.empty() and $u_j$ sees $S$.top() do
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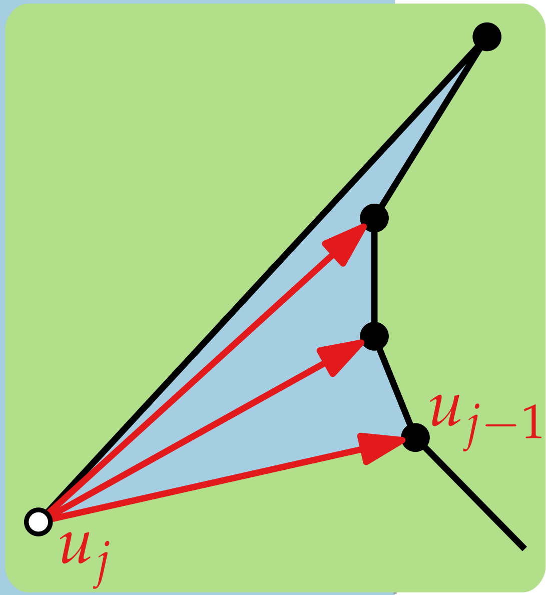

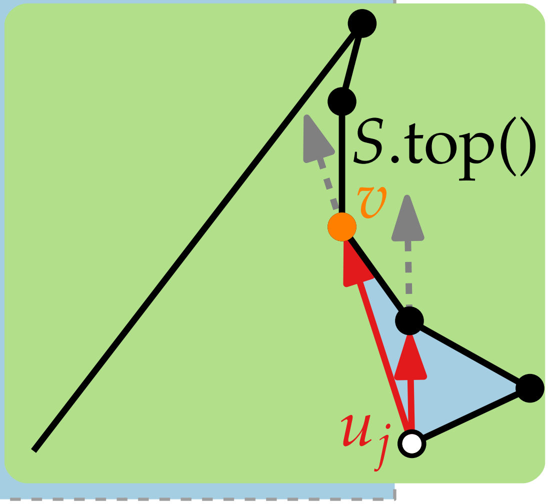

Algorithm

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              $v \leftarrow S$.pop()
              draw diagonal $(u_j, v)$
          $S$.push($v$); $S$.push($u_j$)
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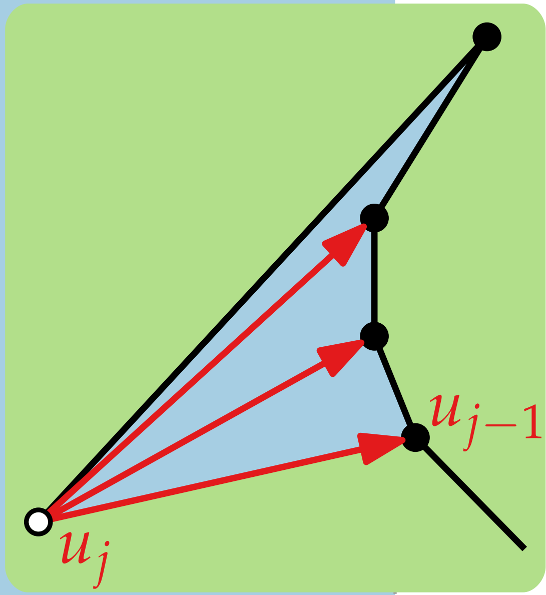

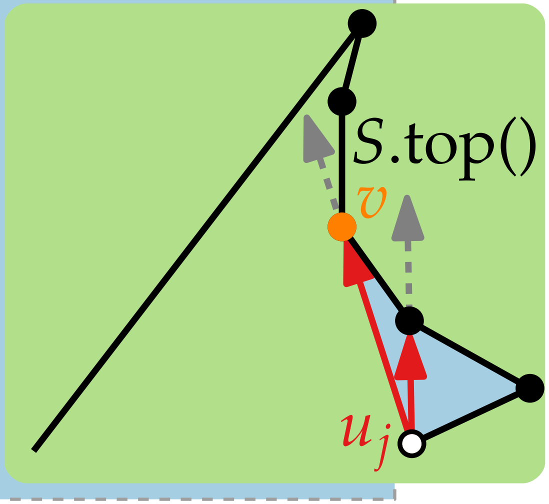

Algorithm

```
**TriangulateMonotonePolygon**(Polygon $P$ as circular vertex list)
  merge left and right chain → seq. $u_1, \ldots, u_n$ with $y_1 \geq \ldots \geq y_n$
  Stack $S$; $S$.push($u_1$); $S$.push($u_2$)
  for $j \leftarrow 3$ to $n-1$ do
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              if not $S$.empty() then draw diag. $(u_j, v)$
          $S$.push($u_{j-1}$); $S$.push($u_j$)
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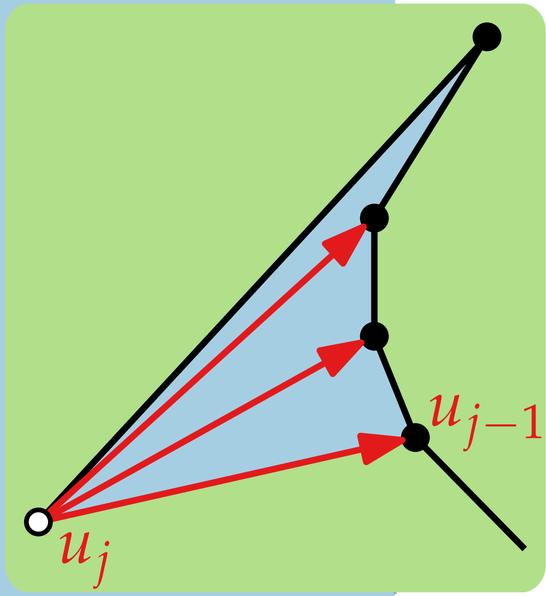
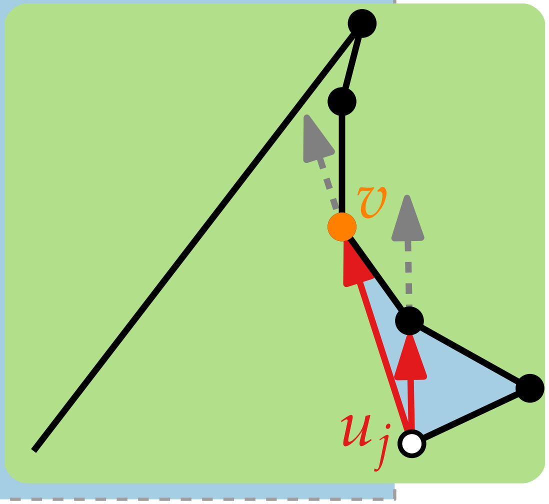
          while not $S$.empty() and $u_j$ sees $S$.top() do
              $v \leftarrow S$.pop()
              draw diagonal $(u_j, v)$
          $S$.push($v$); $S$.push($u_j$)
```





Algorithm

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TriangulateMonotonePolygon(Polygon $P$ as circular vertex list)
  merge left and right chain → seq. $u_1, \dots, u_n$ with $y_1 \geq \dots \geq y_n$
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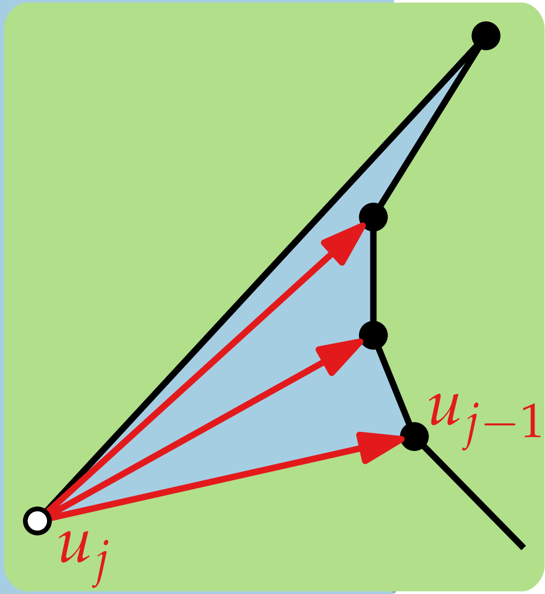
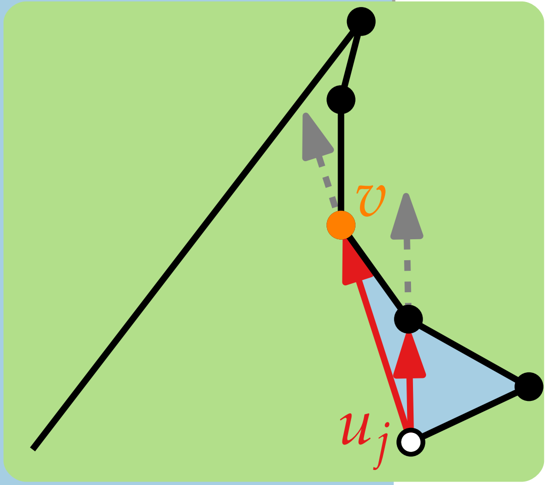
              $v \leftarrow S$.pop()
              draw diagonal $(u_j, v)$
          $S$.push($v$); $S$.push($u_j$)
  draw diagonals from $u_n$ to all vtc on $S$ except first
```





Algorithm

Running time?

```
TriangulateMonotonePolygon(Polygon $P$ as circular vertex list)
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  draw diagonals from $u_n$ to all vtc on $S$ except first
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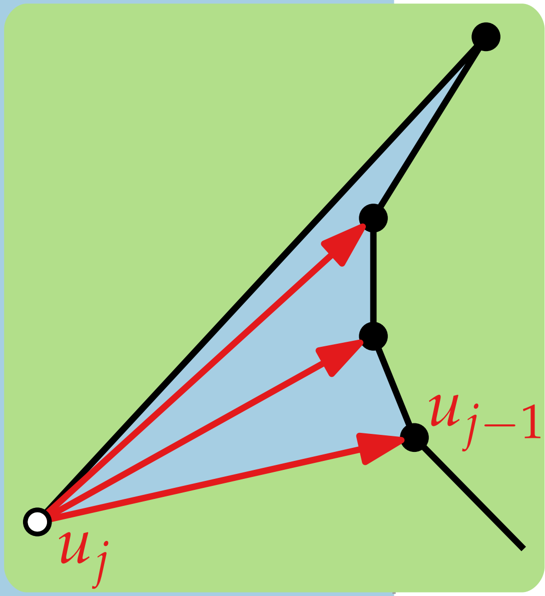

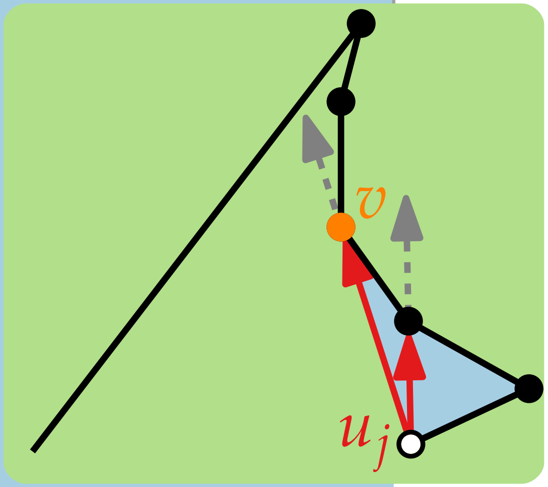

Algorithm

Running time? $\Theta(n)$

```
TriangulateMonotonePolygon(Polygon $P$ as circular vertex list)
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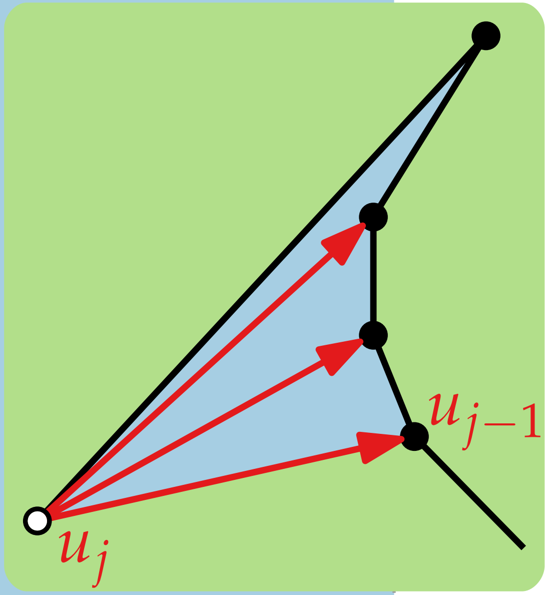
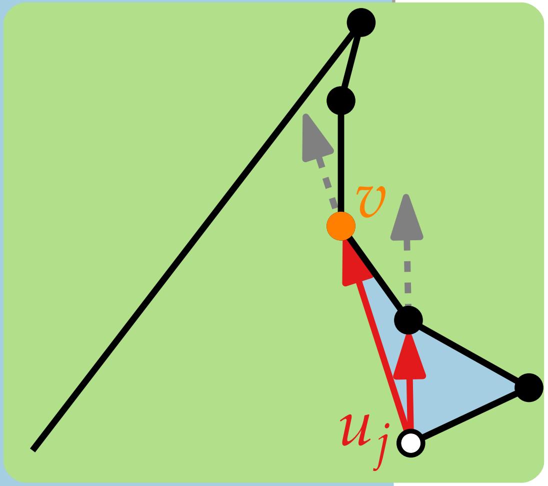
```





Summary

n-vtx polygon $\xrightarrow[O(n \log n)]{}$ “nice” pieces, n' vtc $\xrightarrow[O(n')]{}$ n'' triangles

Summary

n-vtx polygon $\xrightarrow[O(n \log n)]{}$ “nice” pieces, n' vtc $\xrightarrow[O(n')]{}$ n'' triangles

Lemma. A simple polygon with n vertices can be subdivided into y-monotone polygons in $O(n \log n)$ time.

old

Summary

n-vtx polygon $\xrightarrow[O(n \log n)]{}$ “nice” pieces, n' vtc $\xrightarrow[O(n')]{}$ n'' triangles

Lemma. A simple polygon with n vertices can be subdivided into y-monotone polygons in $O(n \log n)$ time.

old

Lemma. A y-monotone polygon with n vertices can be triangulated in $O(n)$ time.

new

Summary

n-vtx polygon $\xrightarrow[O(n \log n)]{}$ "nice" pieces, n' vtc $\xrightarrow[O(n')]{}$ n'' triangles

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A simple polygon with n vertices can be subdivided into y-monotone polygons in $O(n \log n)$ time.

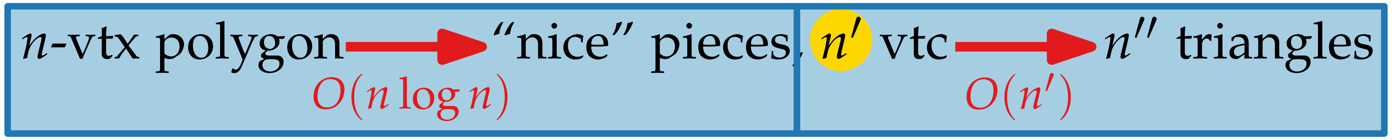
Lemma. new

A y-monotone polygon with n vertices can be triangulated in $O(n)$ time.

Lemma. homework

Subdividing a simple polygon with n vertices by drawing d (pairwise non-crossing) diagonals yields $d+1$ simple polygons of total complexity $O(n)$.

Summary



n-vtx polygon $\xrightarrow[O(n \log n)]{}$ “nice” pieces, n' vtc $\xrightarrow[O(n')]{}$ n'' triangles

Lemma. old

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Lemma. new

A y-monotone polygon with n vertices can be triangulated in $O(n)$ time.

Lemma. home-work

Subdividing a simple polygon with n vertices by drawing d (pairwise non-crossing) diagonals yields $d + 1$ simple polygons of total complexity $O(n)$.

Theorem. A simple polygon with n vertices can be triangulated in $O(n \log n)$ time.

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home-work

Theorem. A simple polygon with n vertices can be triangulated in $O(n \log n)$ time.

Is this it?

Summary

n-vtx polygon $\xrightarrow[O(n\log n)]{}$ "nice" pieces, n' vtc $\xrightarrow[O(n')]{}$ n'' triangles

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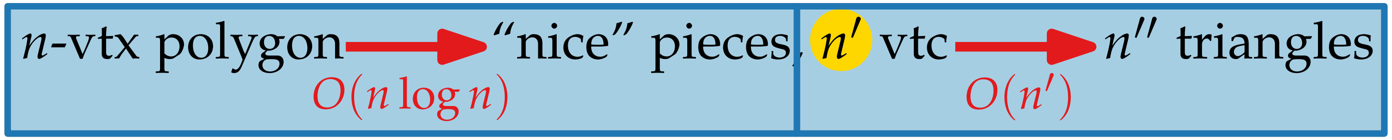
Is this it? Tarjan & van Wyk [1988]:

Summary



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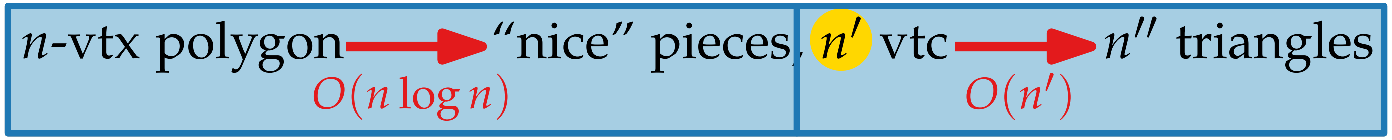
Is this it? Tarjan & van Wyk [1988]: $O(n \log \log n)$

Summary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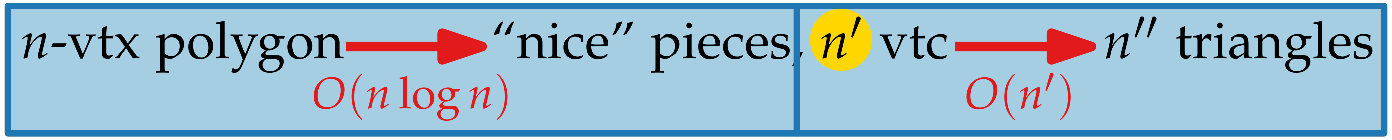
Tarjan & van Wyk [1988]: $O(n \log \log n)$

Clarkson, Tarjan, van Wyk [1989]:

Summary



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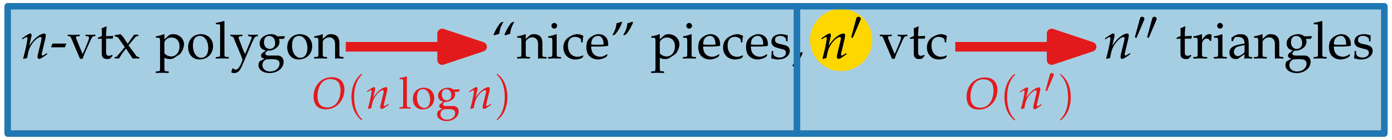
Is this it?

Tarjan & van Wyk [1988]:	$O(n \log \log n)$
Clarkson, Tarjan, van Wyk [1989]:	$O(n \log^* n)$

Summary



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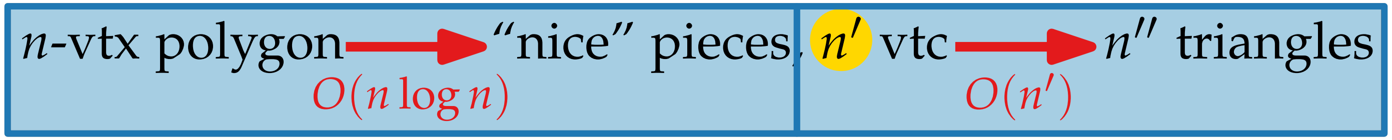

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Clarkson, Tarjan, van Wyk [1989]:	$O(n \log^* n)$	Seidel [1991]: *randomized*
Chazelle [1991]:	$O(n)$	