Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Computational Geometry

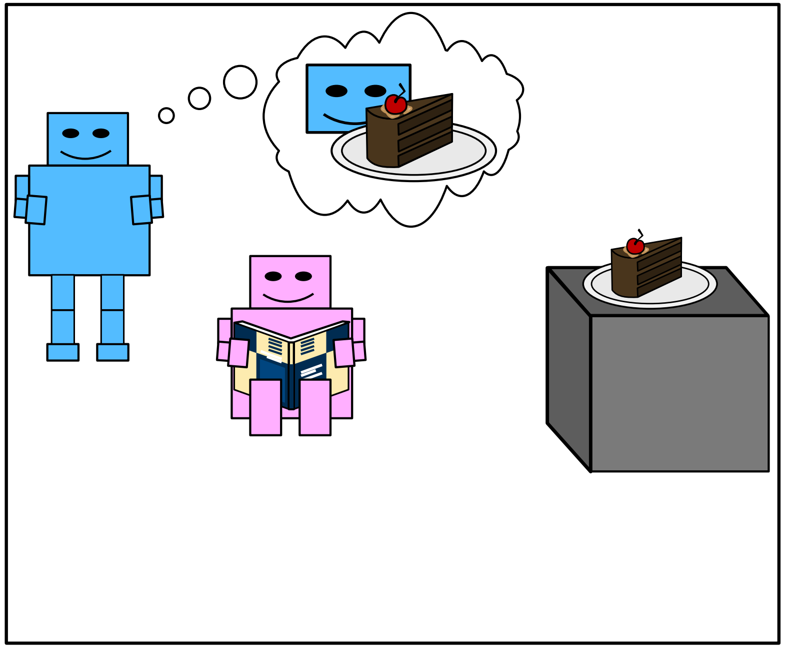
Visibility Graphs
or
Finding Shortest Paths

Lecture #12

Dr. Philipp Kindermann

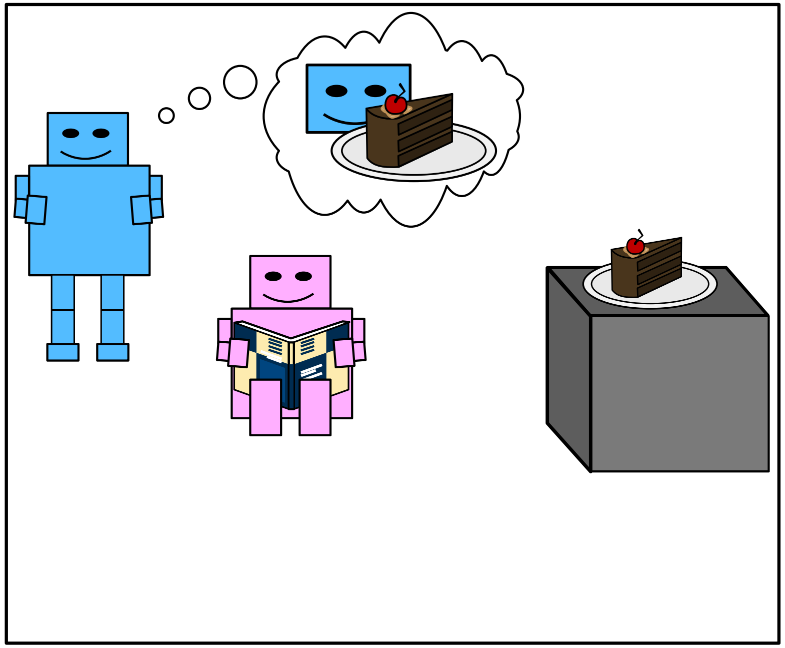
Winter Semester 2018/19

Path Planning



current location,
desired location

Path Planning



$\Rightarrow$

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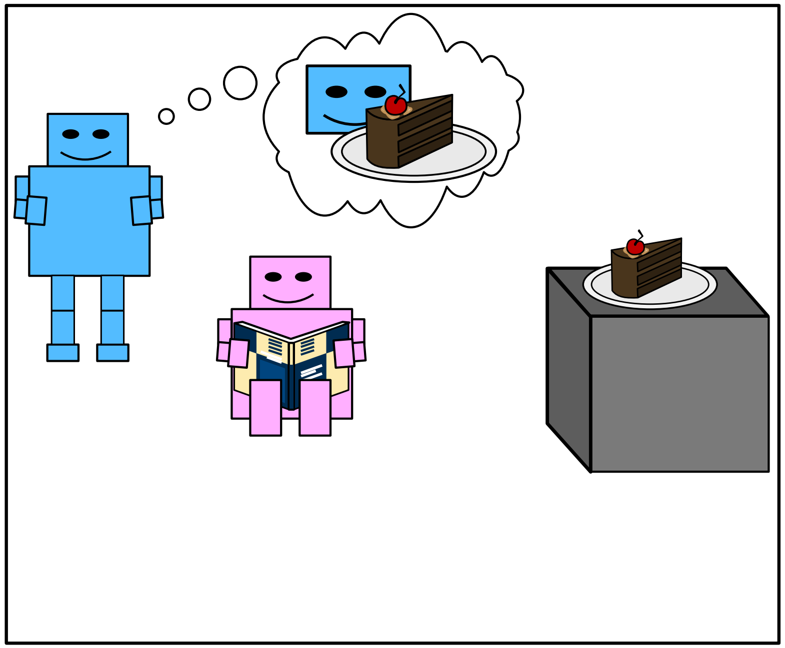

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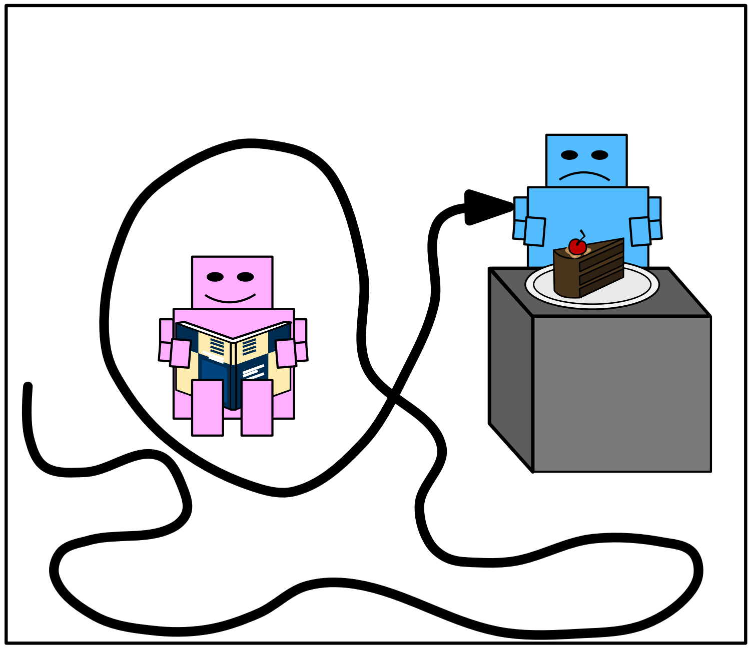
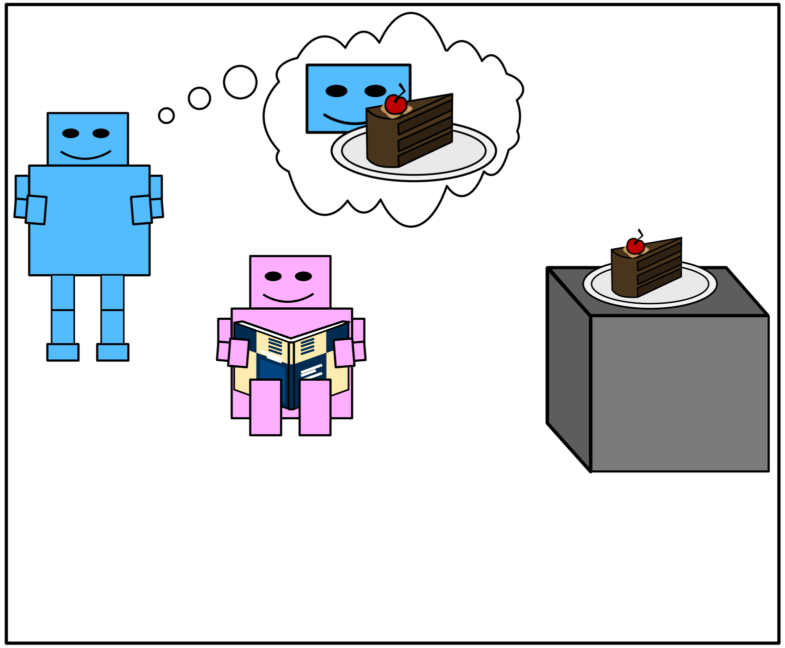
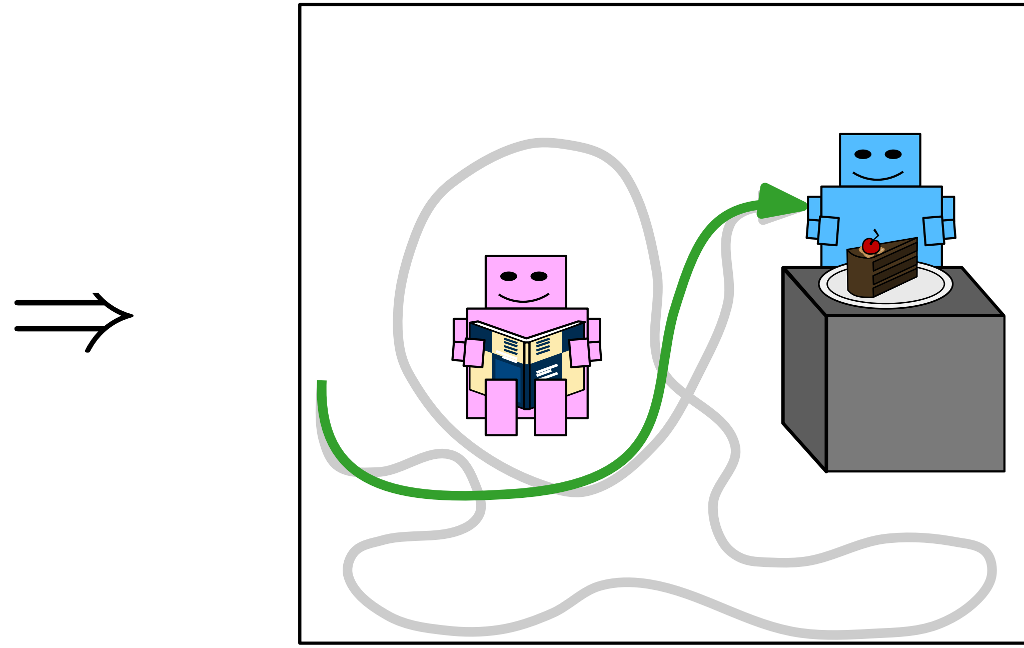


path to reach the
one from the other

Path Planning





current location,
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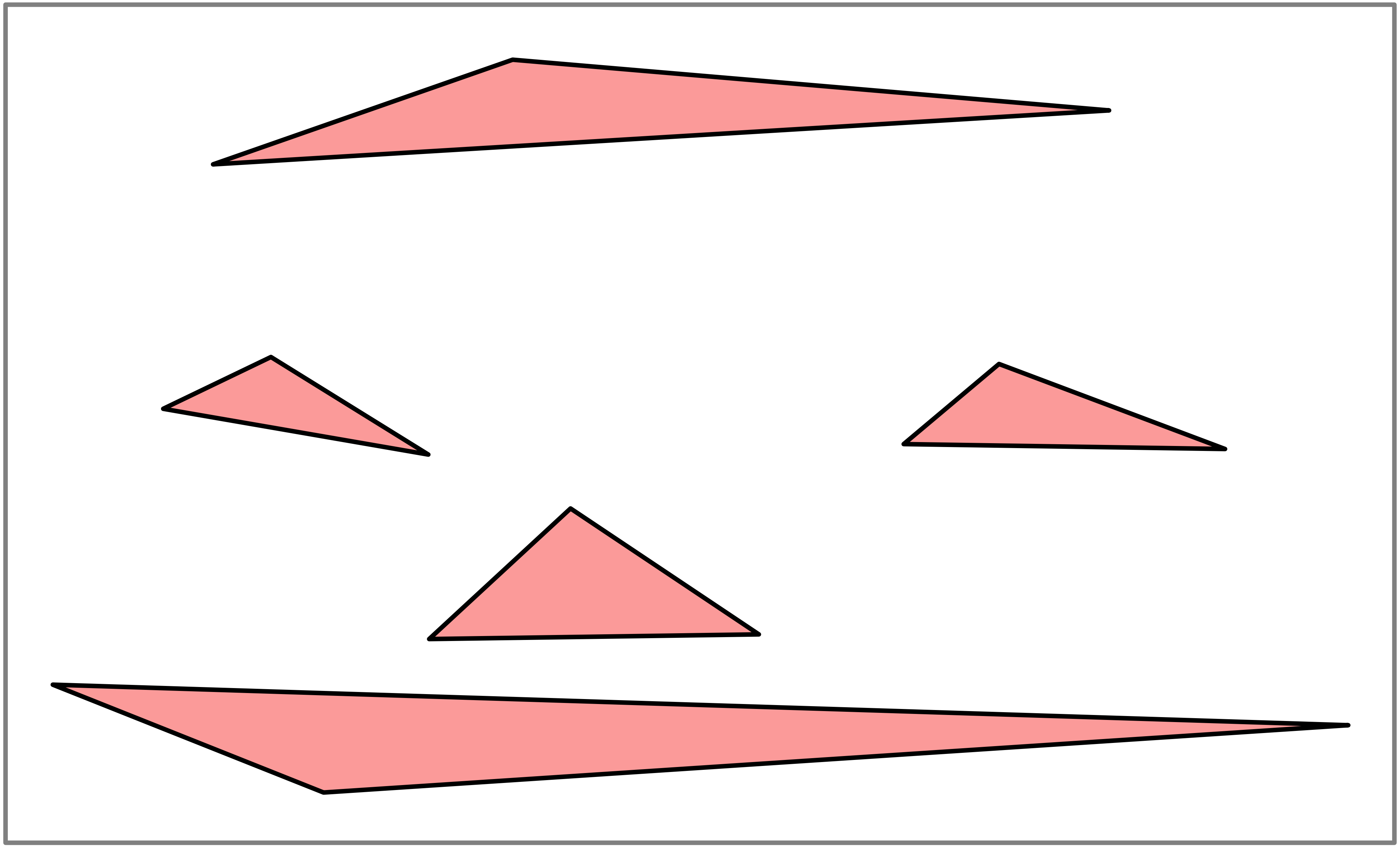
shortest path to reach the one from the other

Let's recap...



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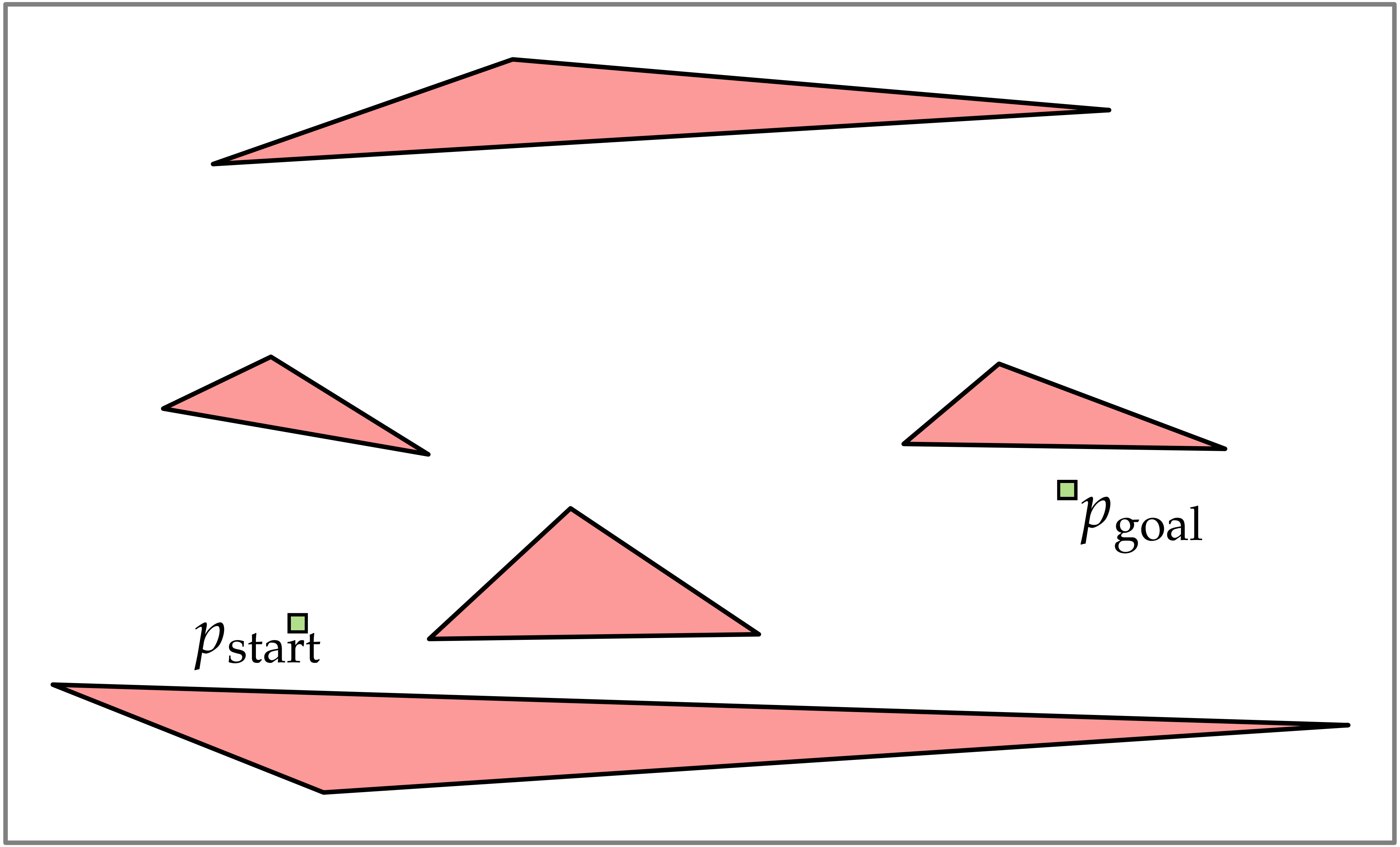

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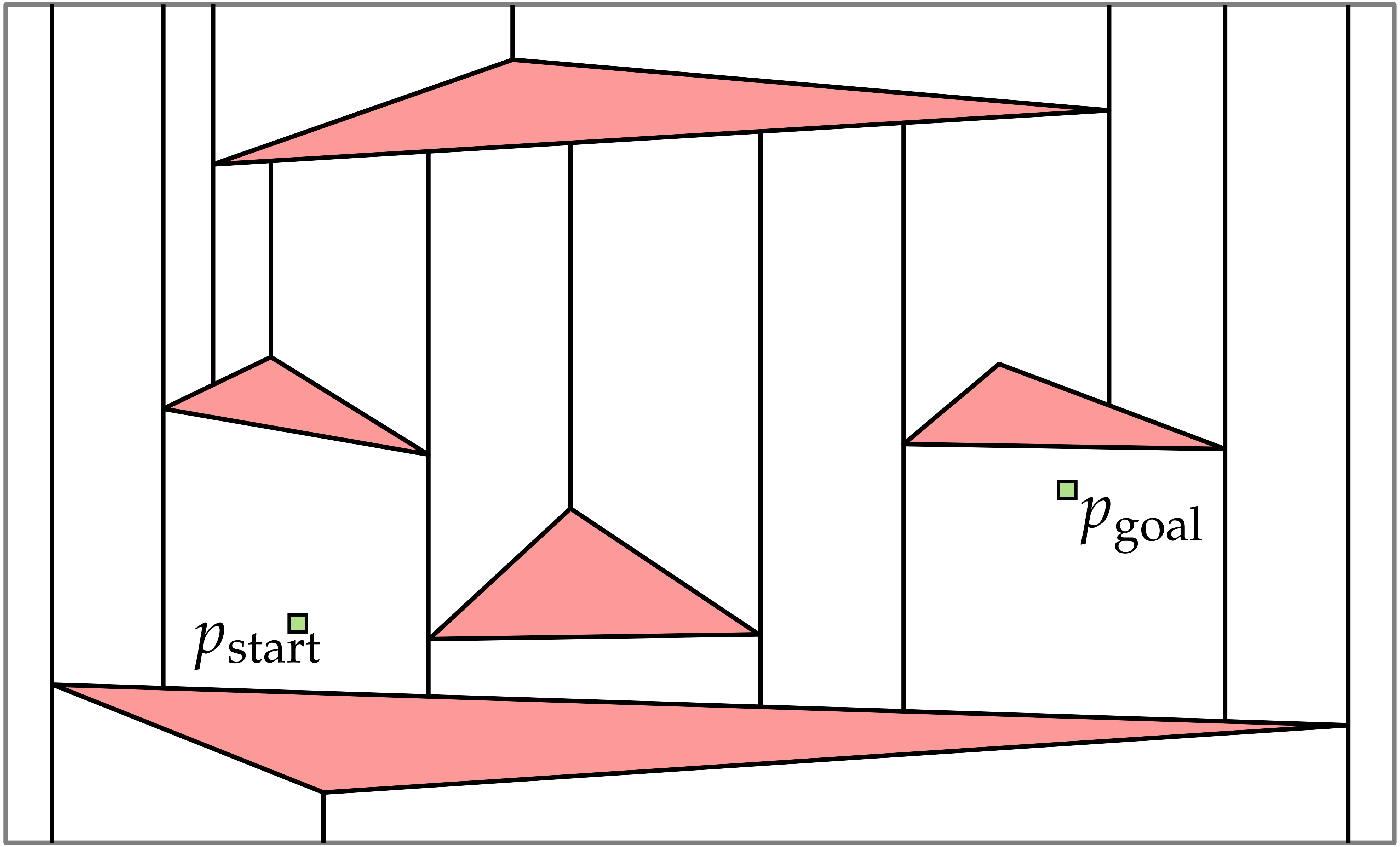

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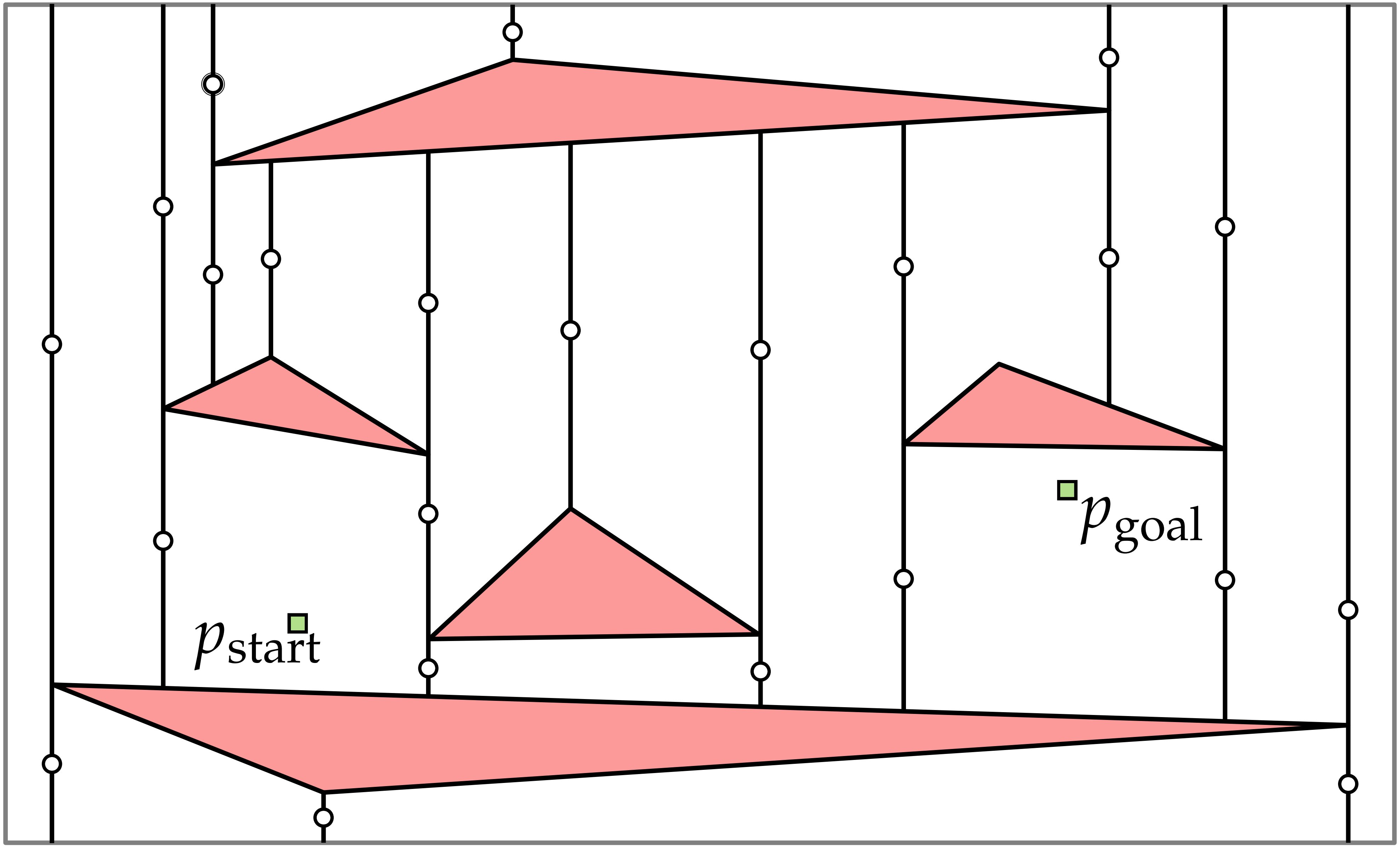

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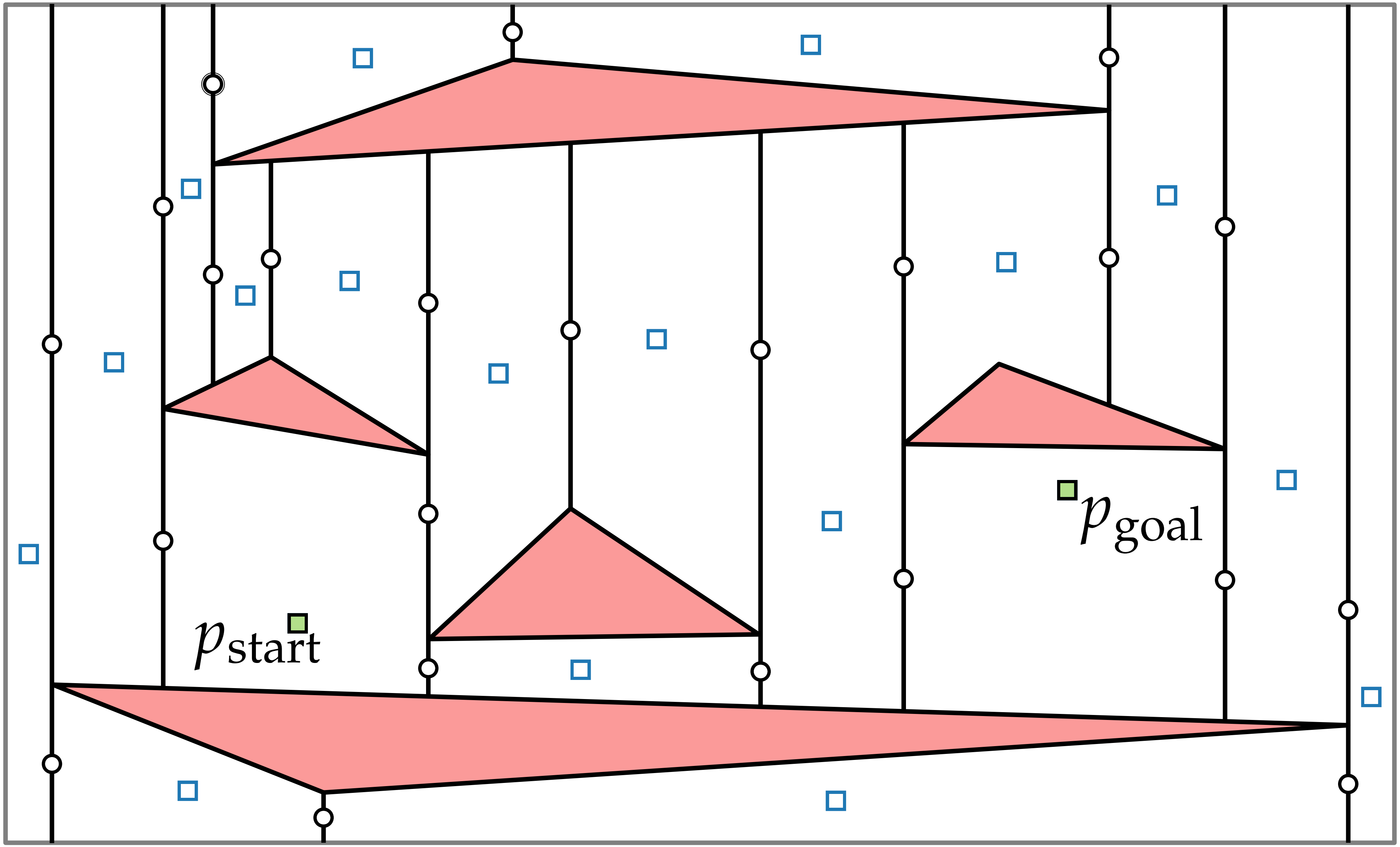

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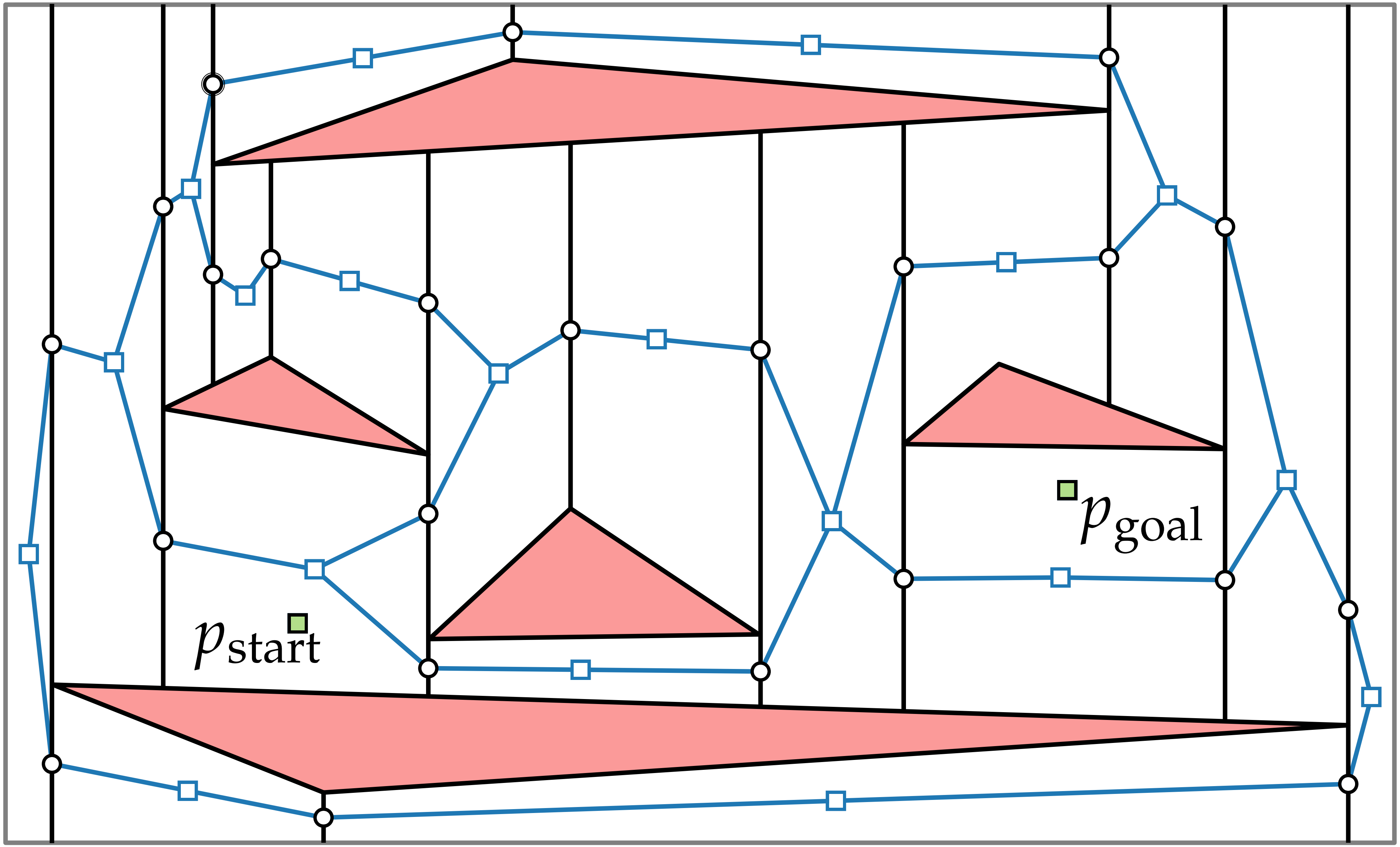

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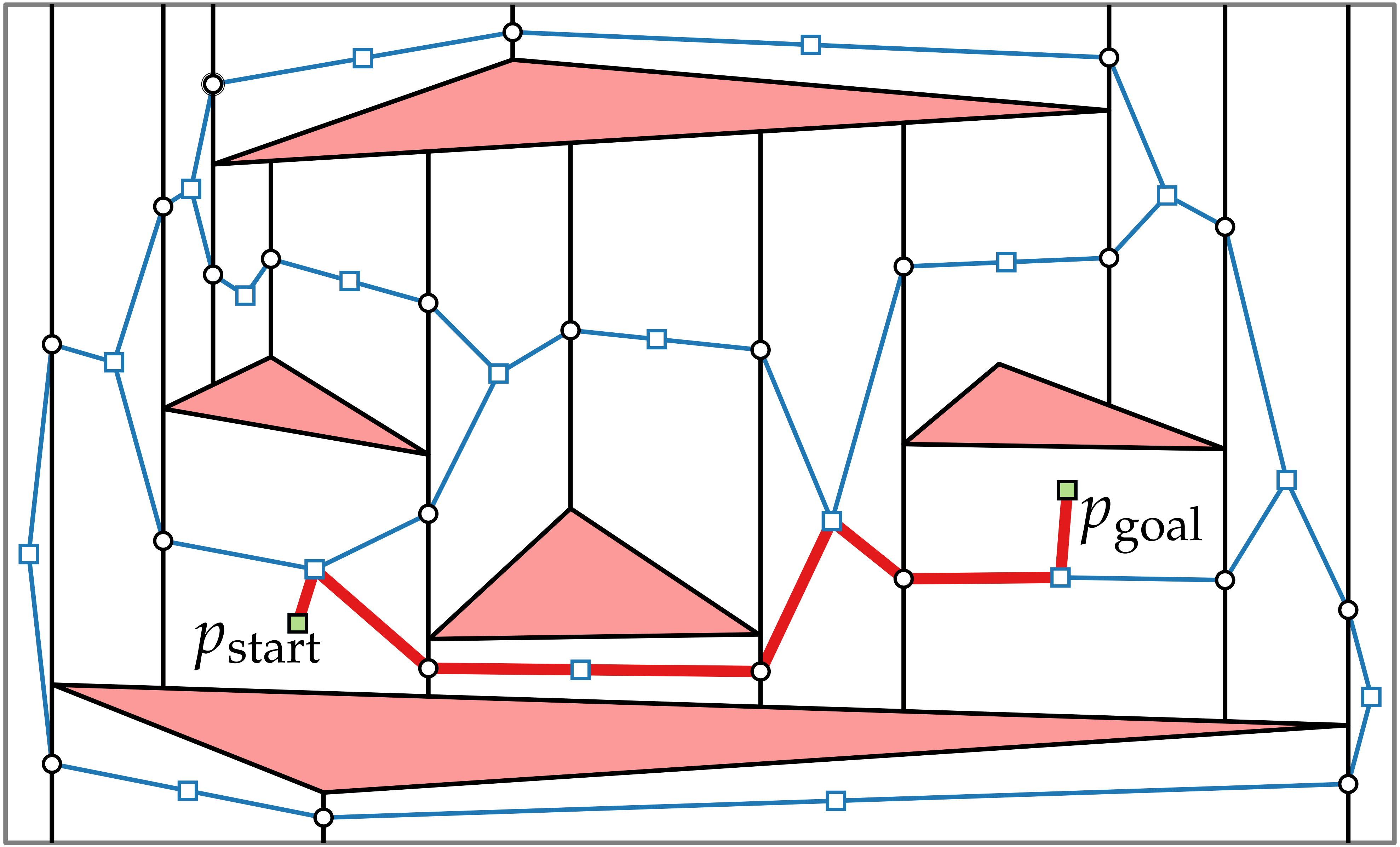

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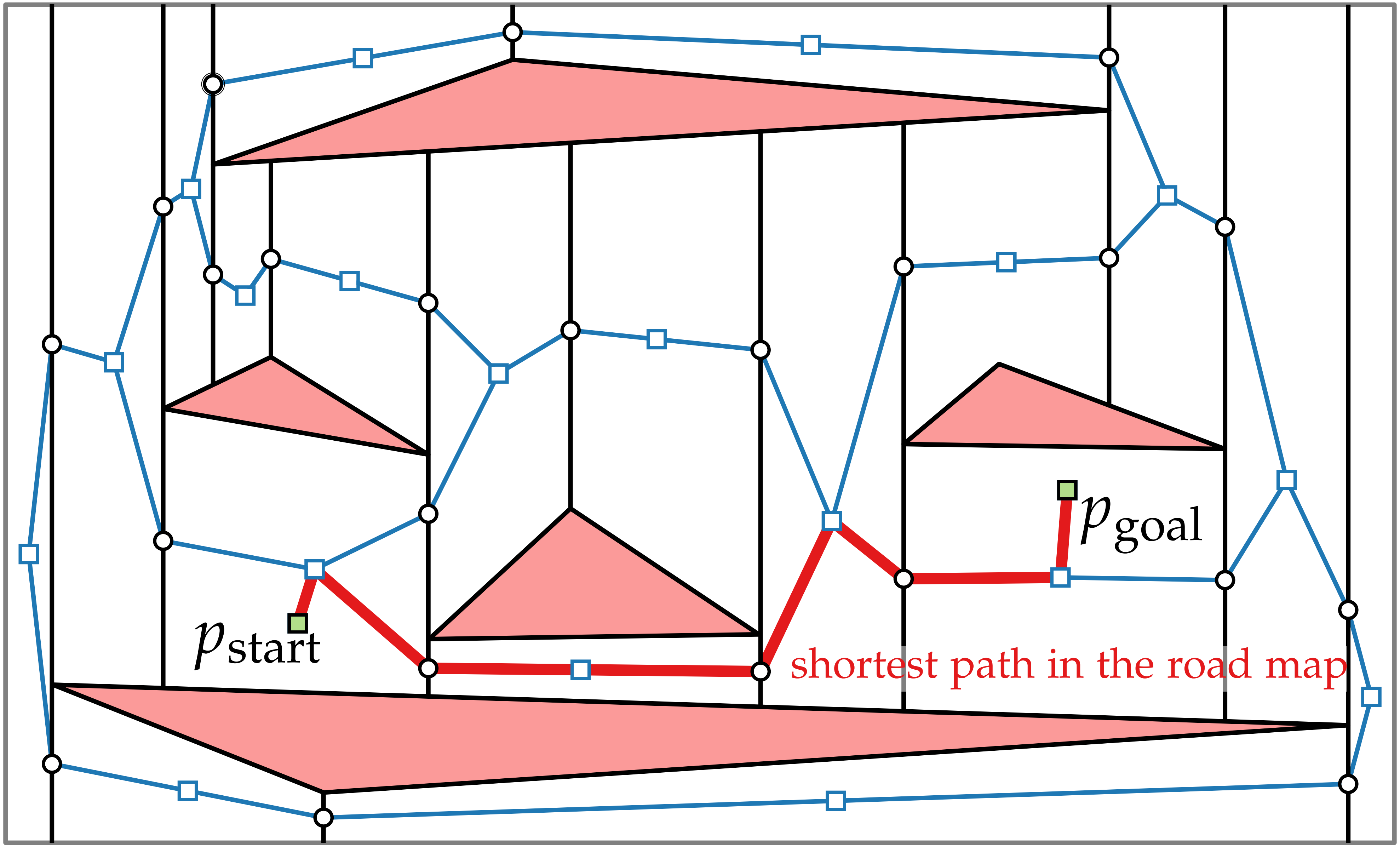

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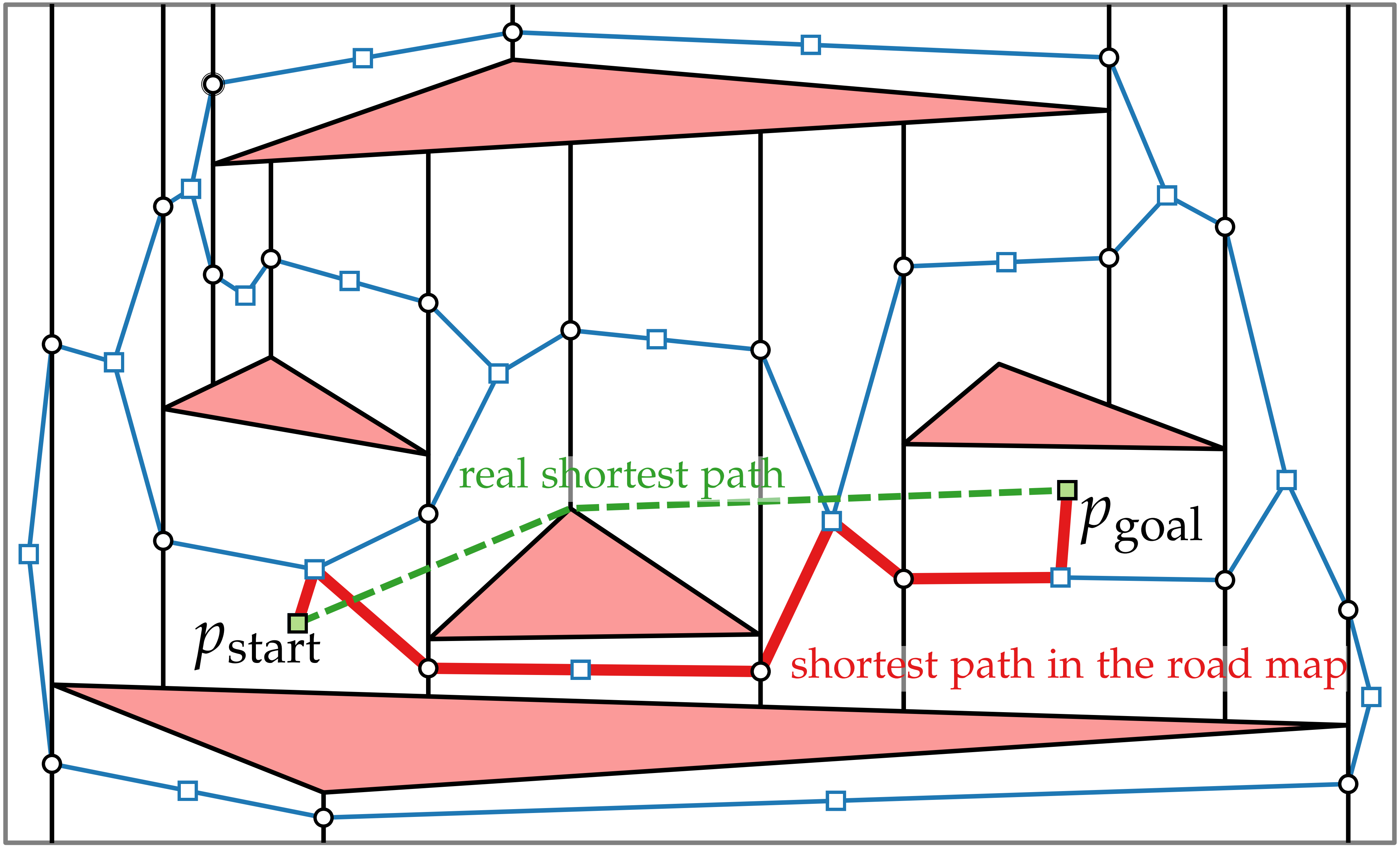
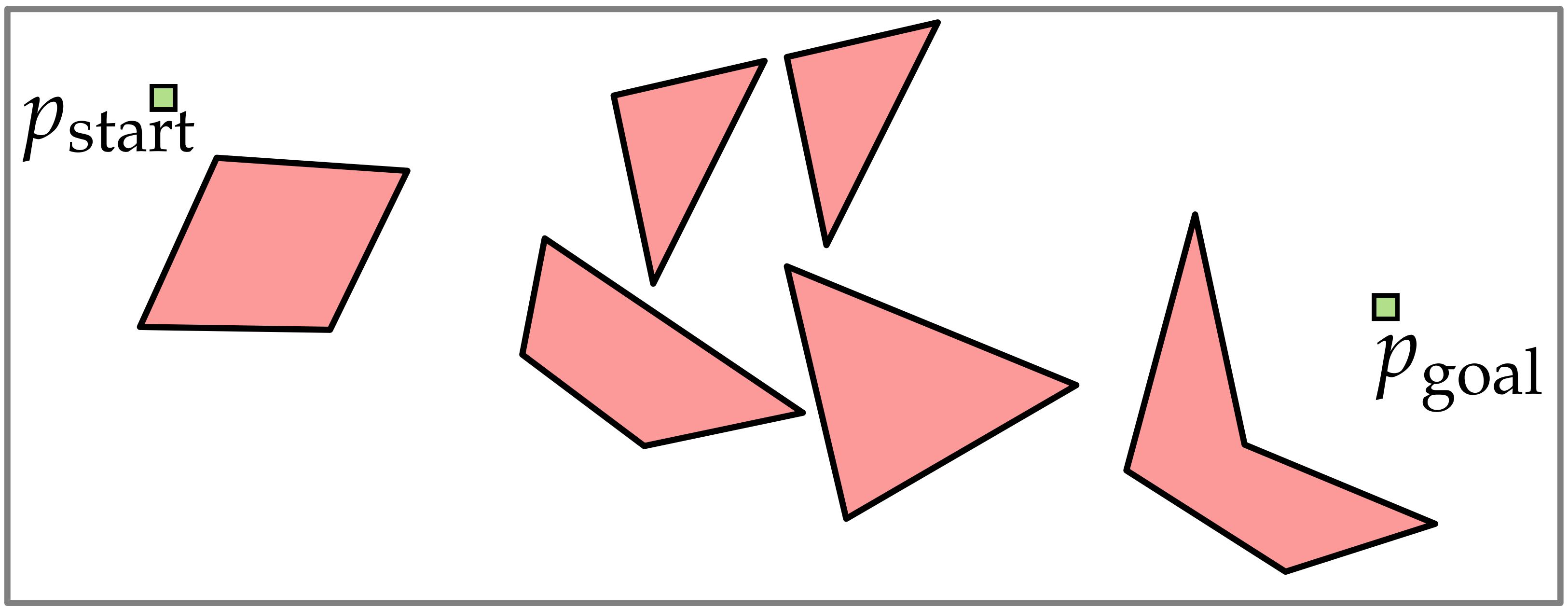

real shortest path
p_{goal}
p_{start}
shortest path in the road map

Characterization

Lemma. Given a set S of disjoint polygonal obstacles in $\mathbb{R}^2$ and points p_{start} and p_{goal} in the free space,



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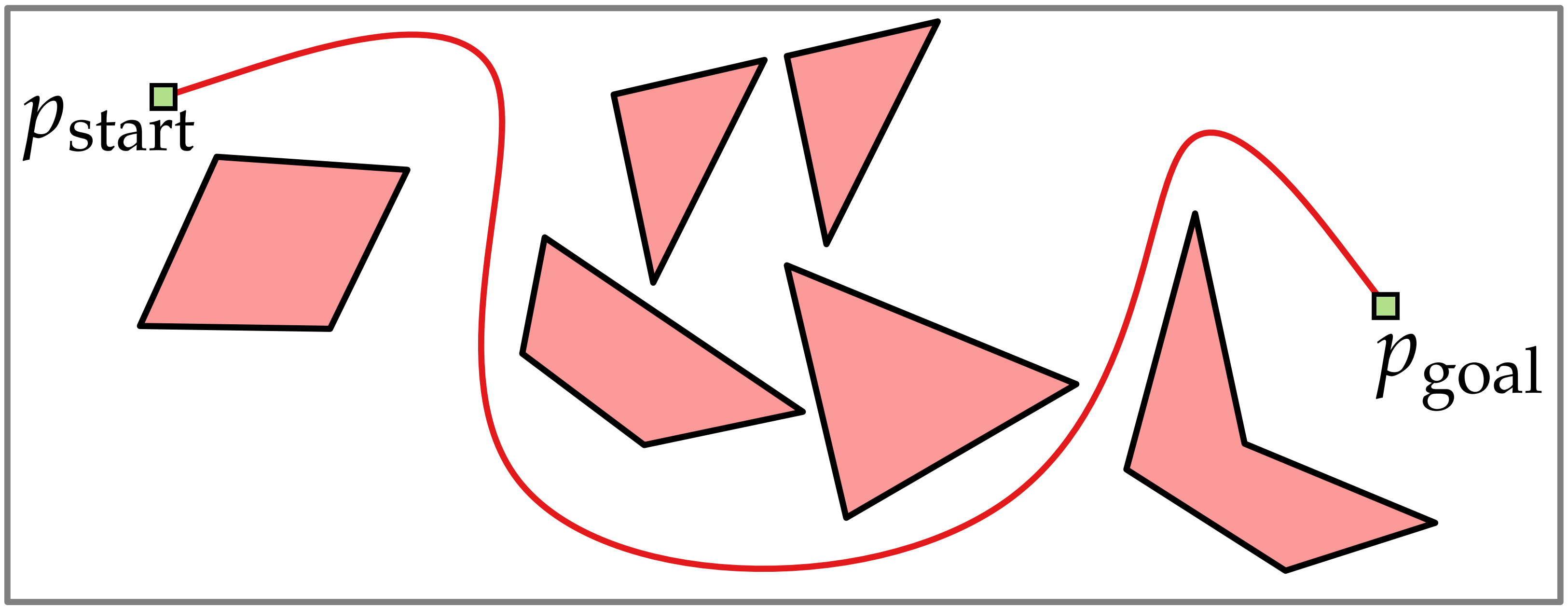

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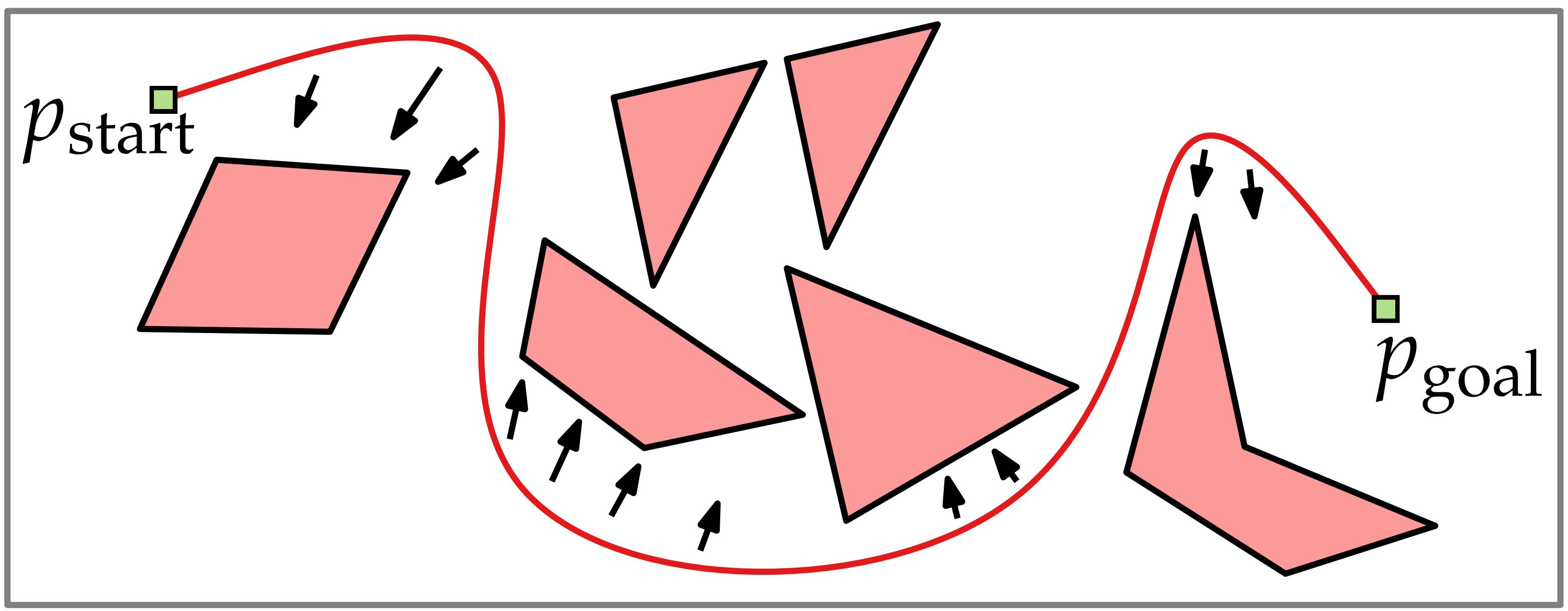

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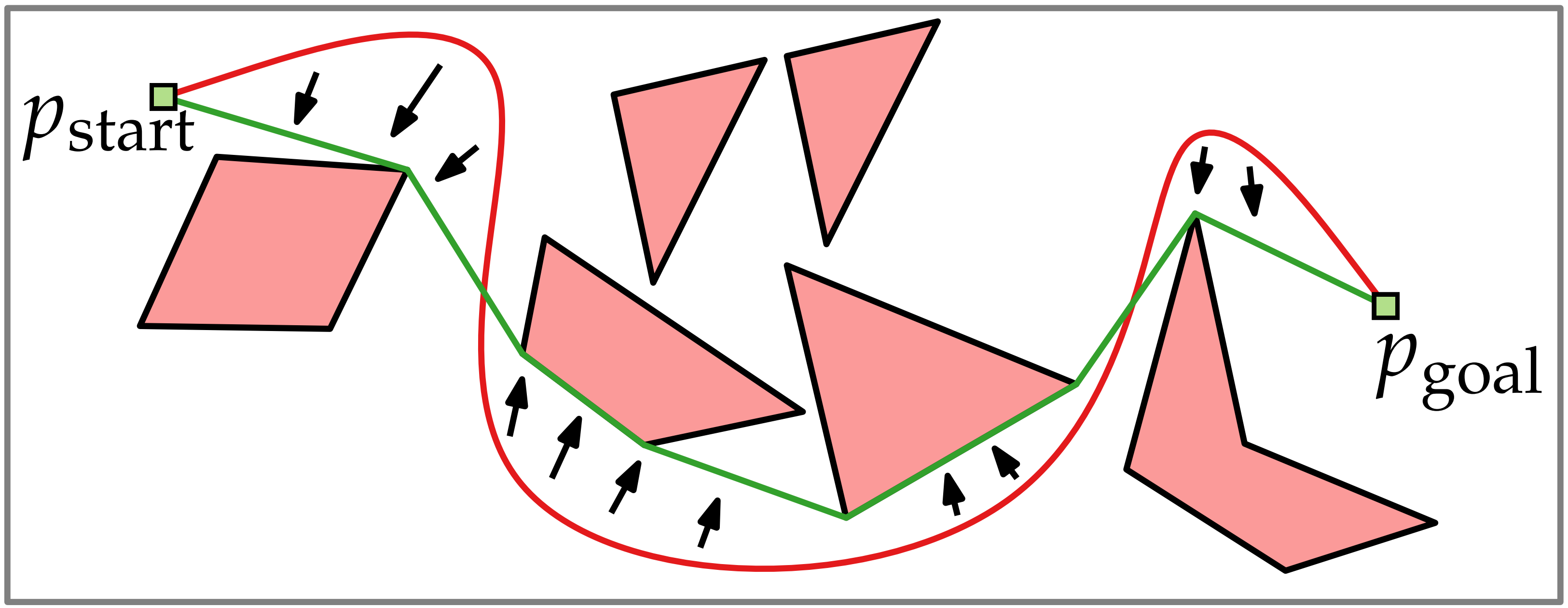

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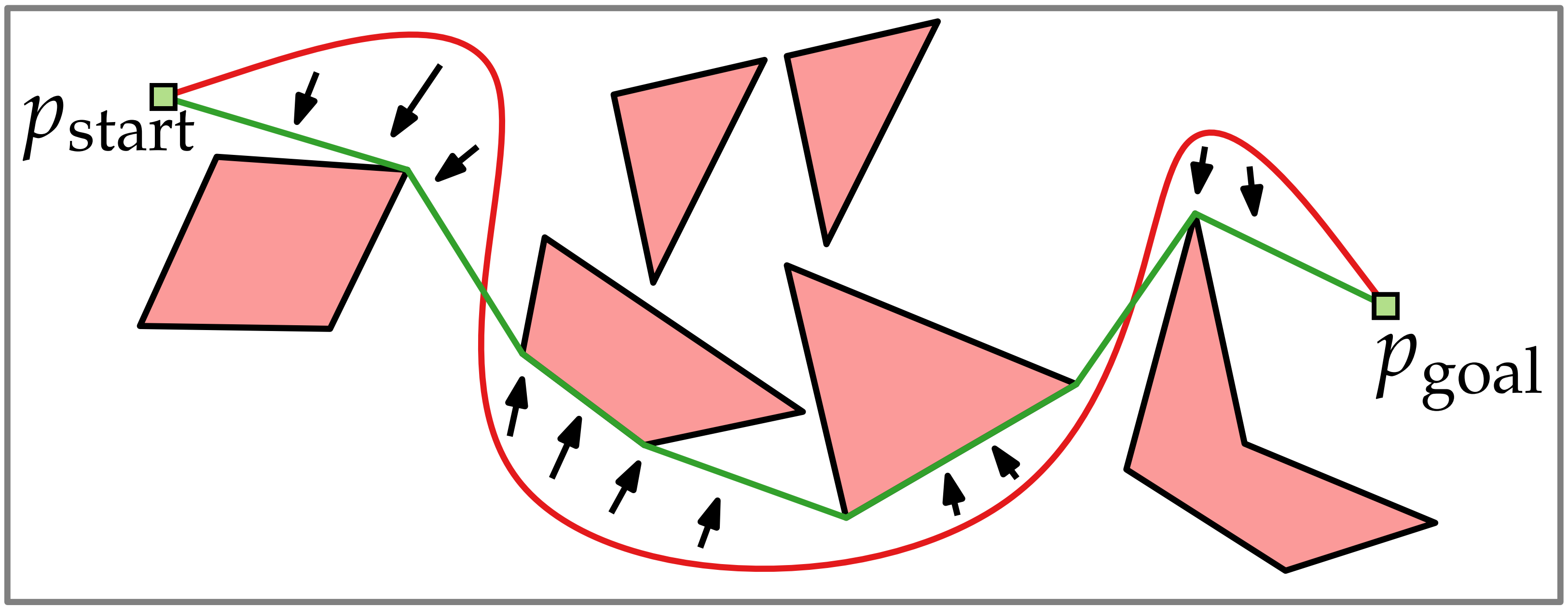

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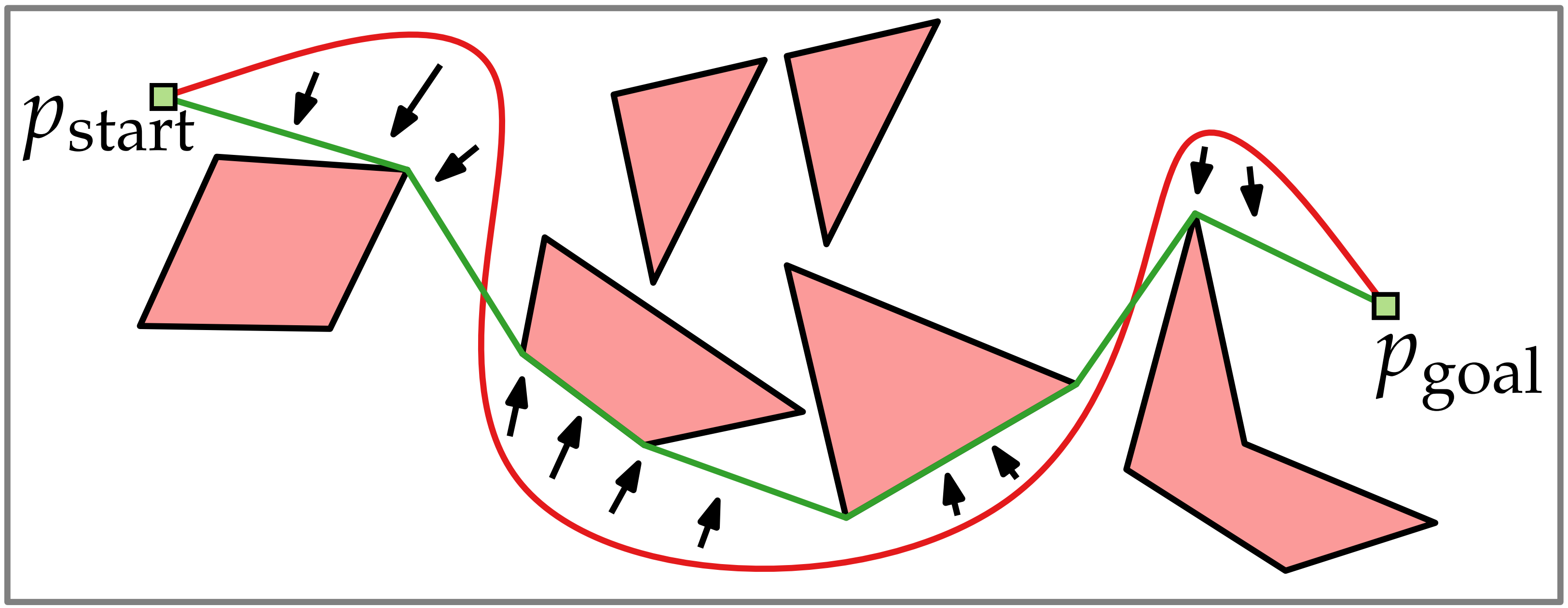

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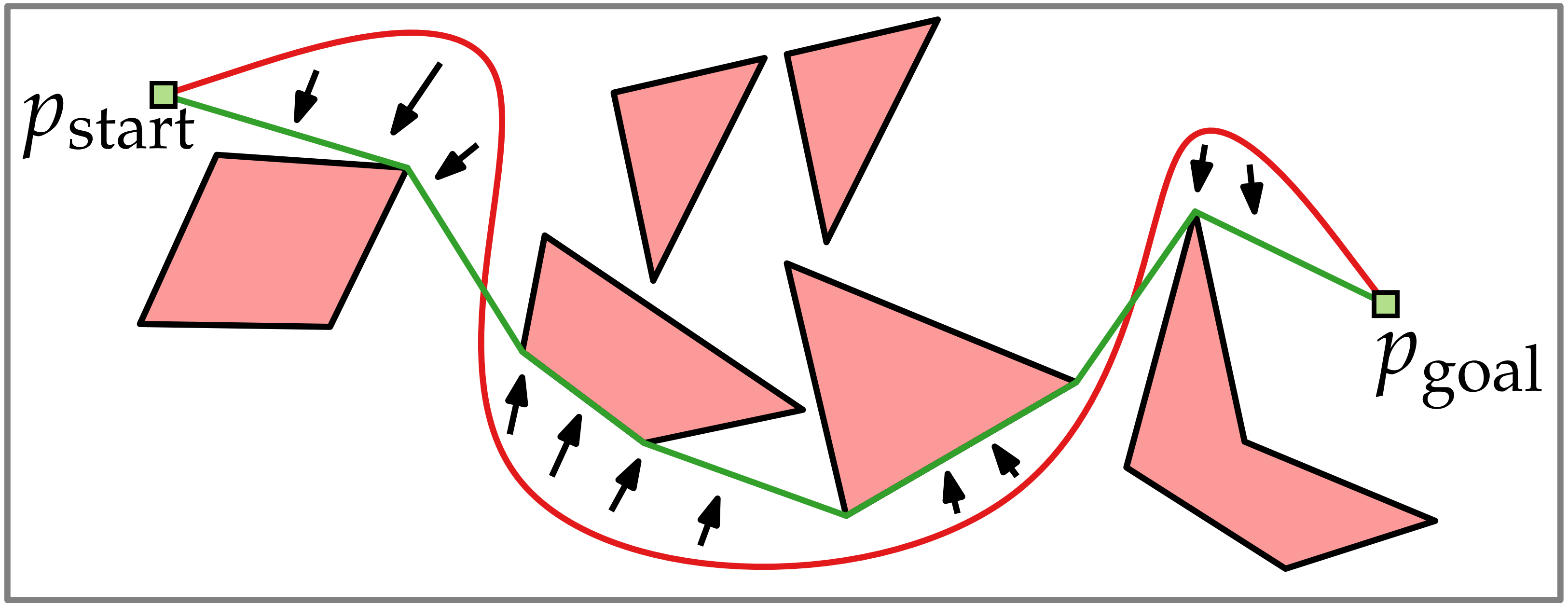

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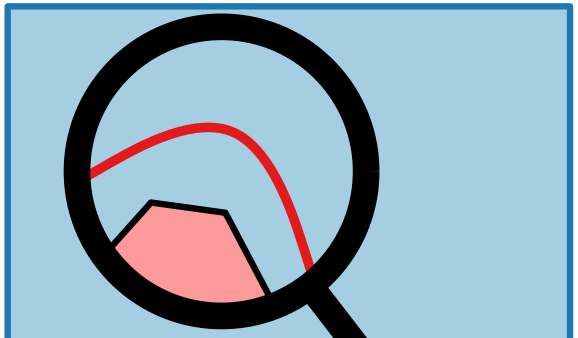

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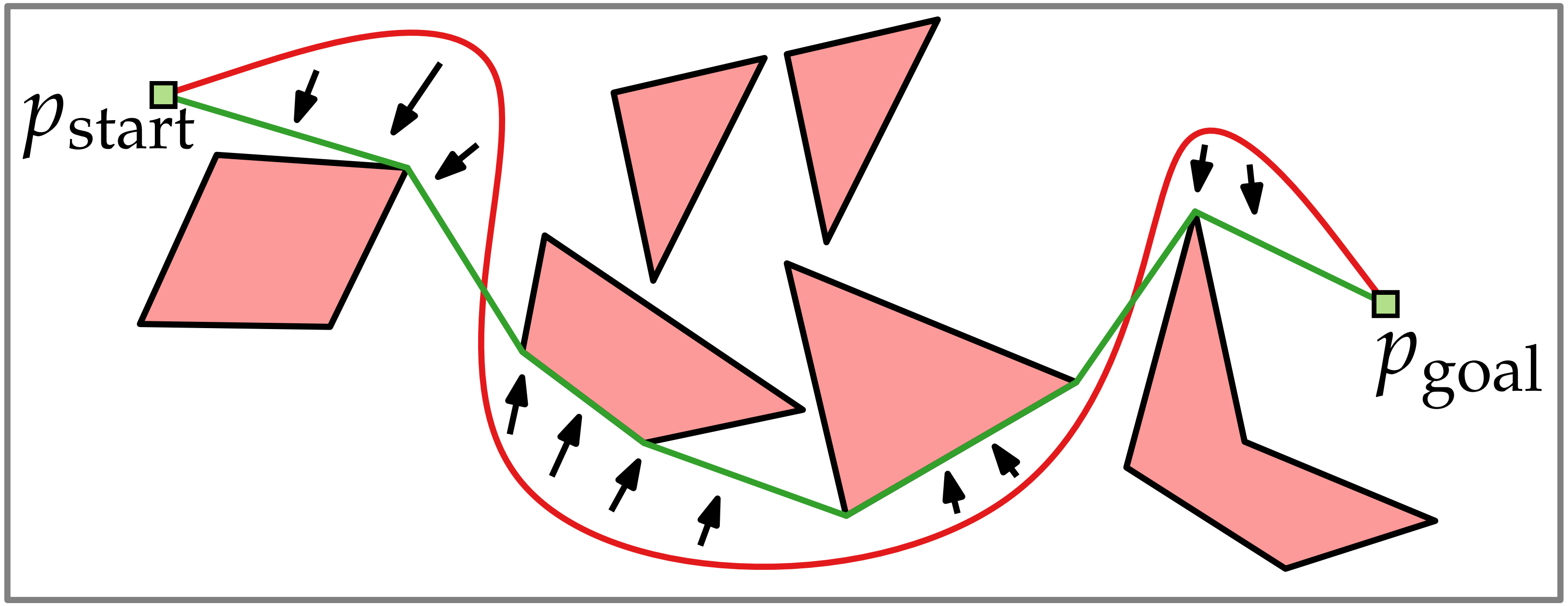

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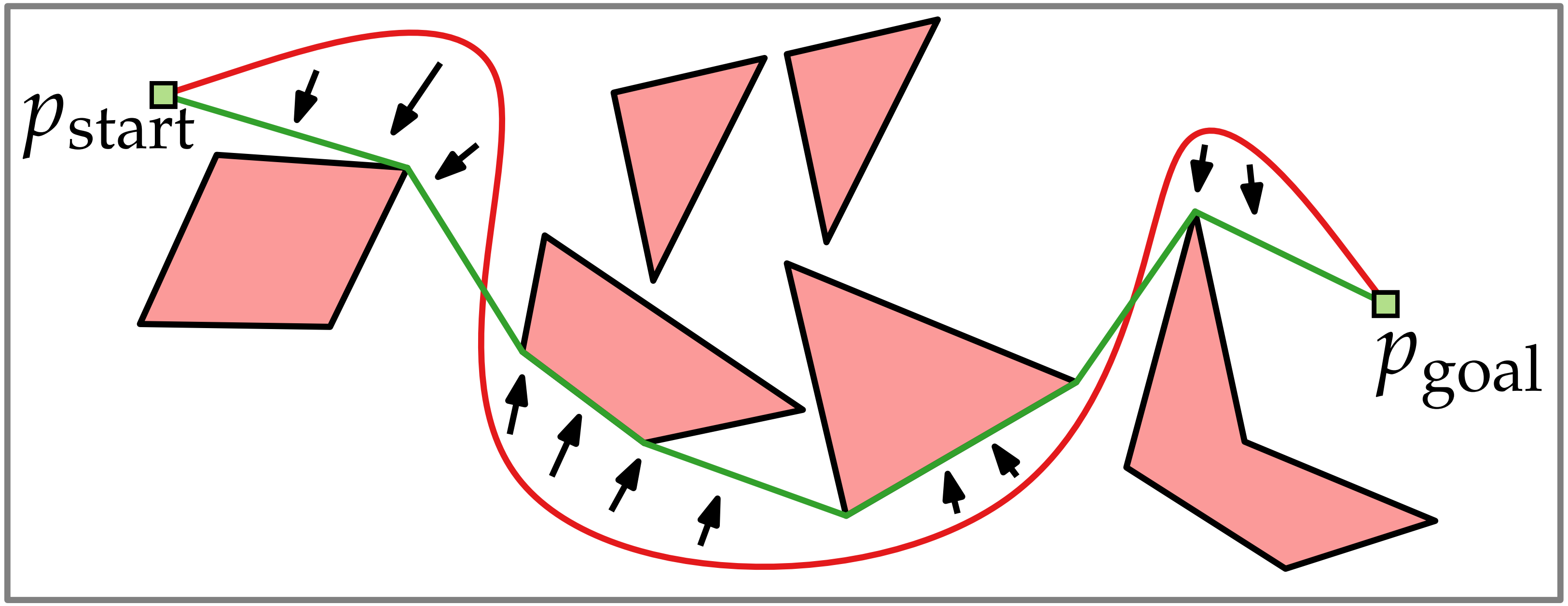

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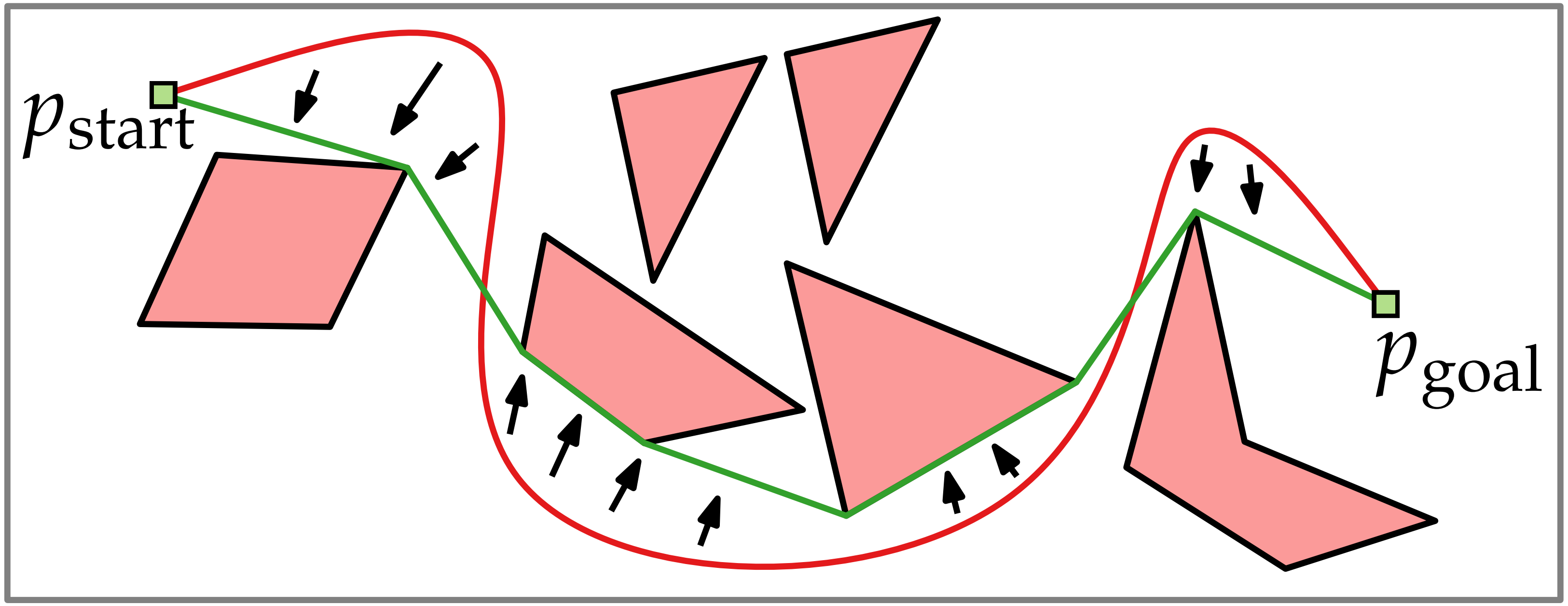

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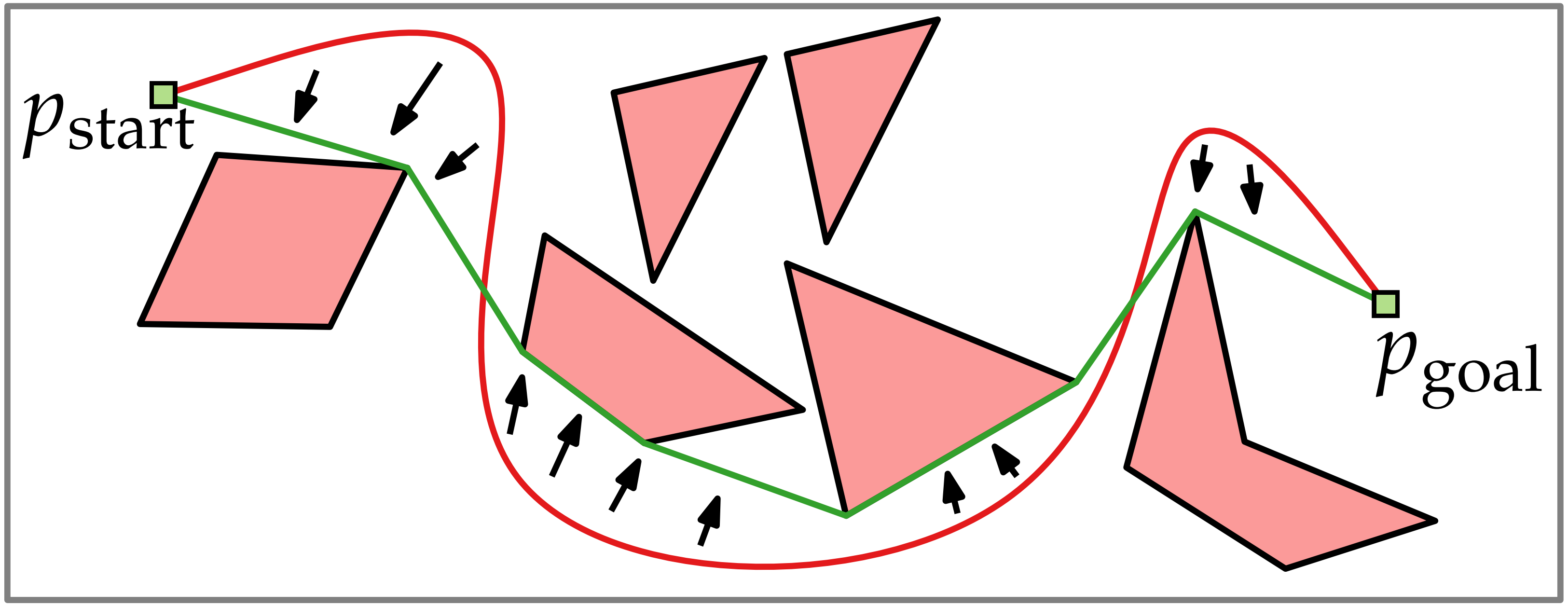

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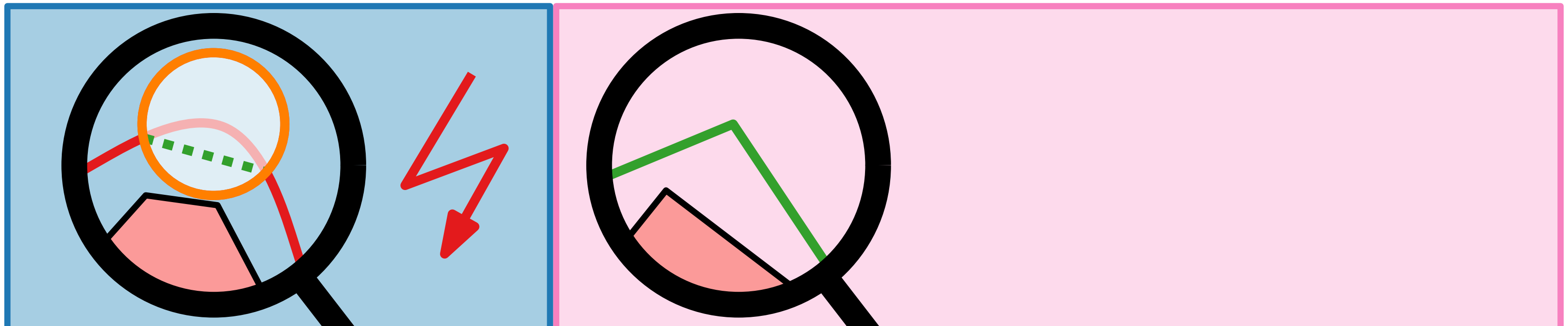

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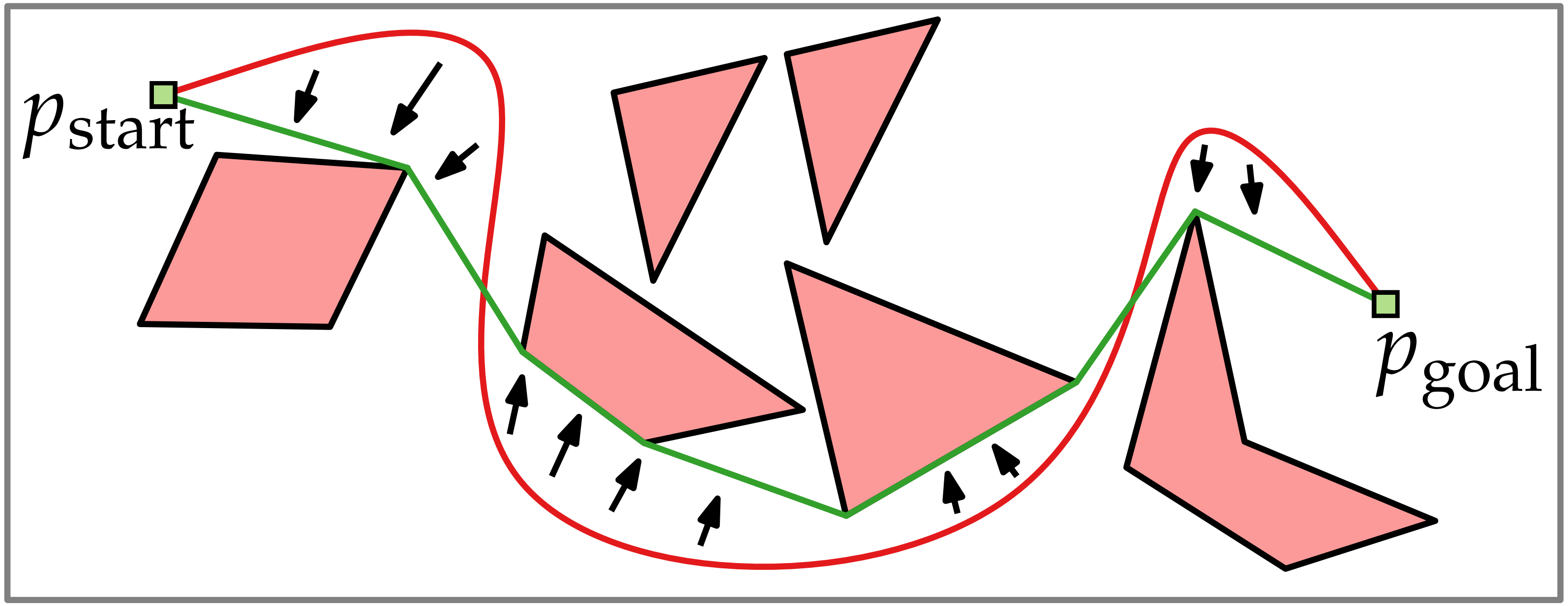

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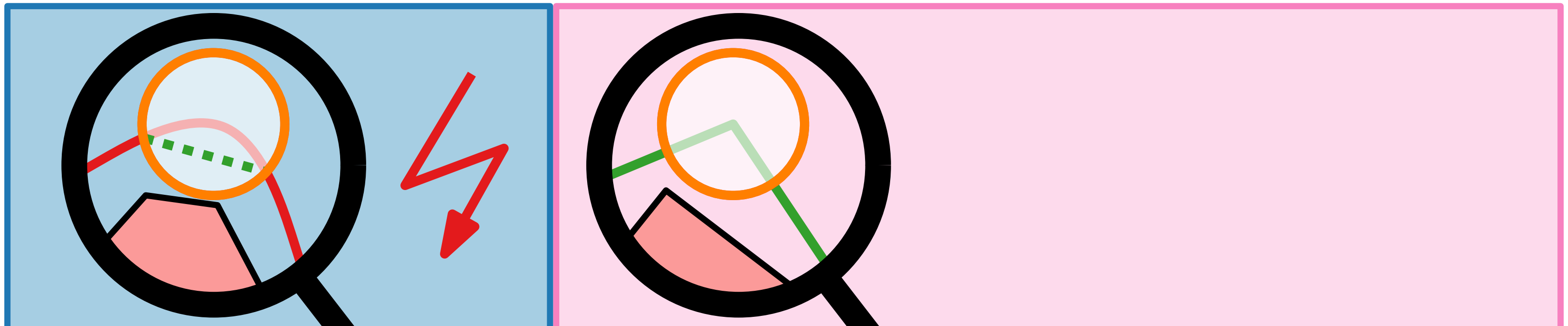

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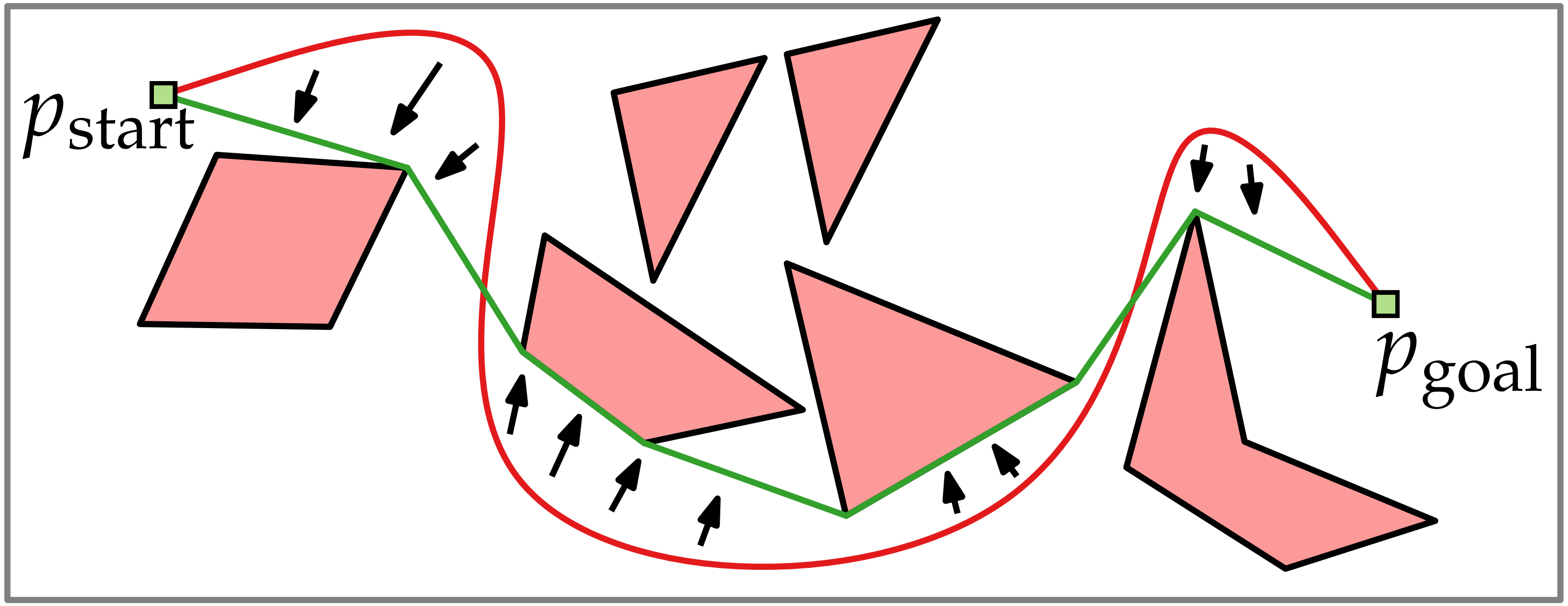

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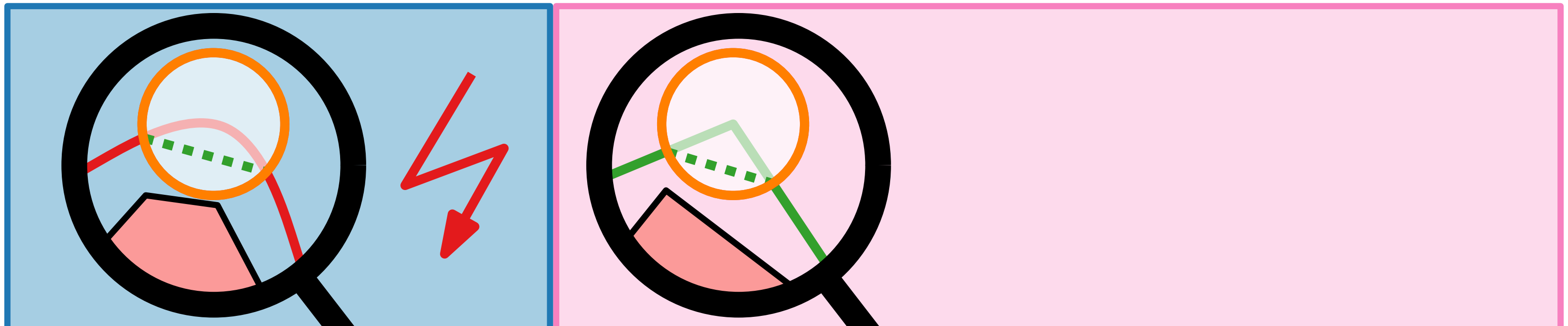

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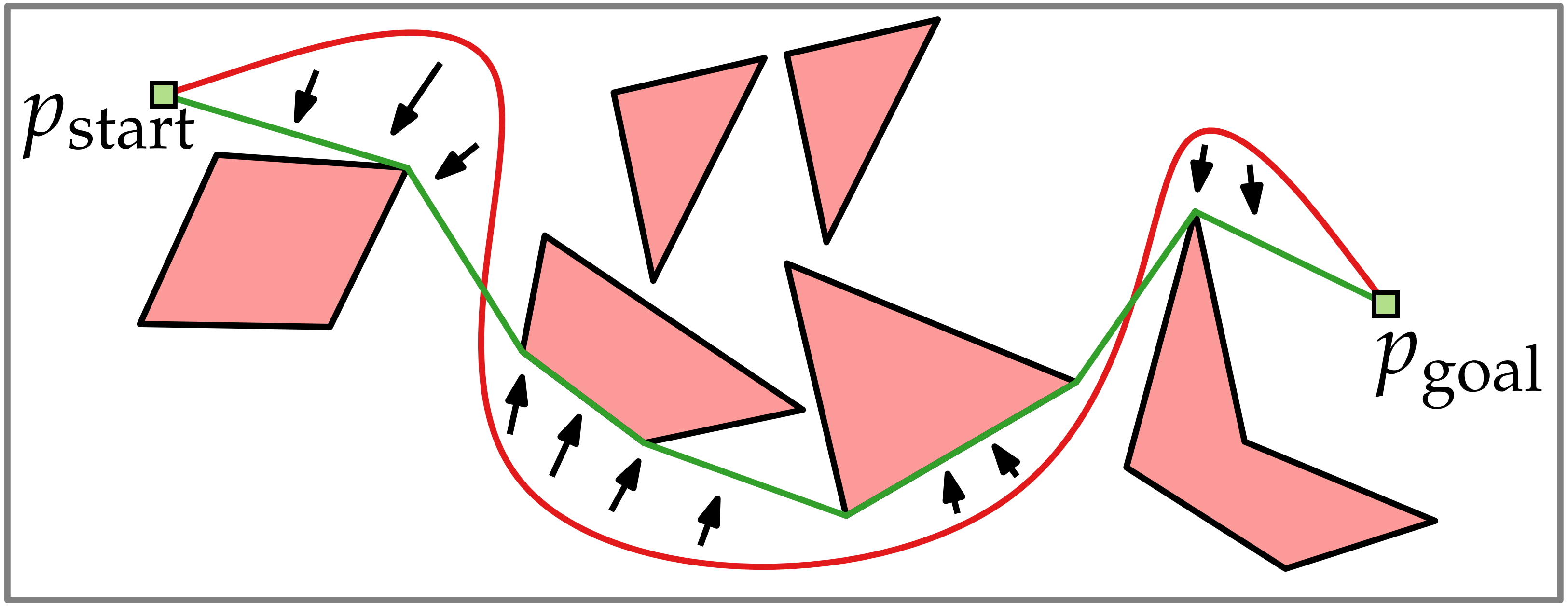

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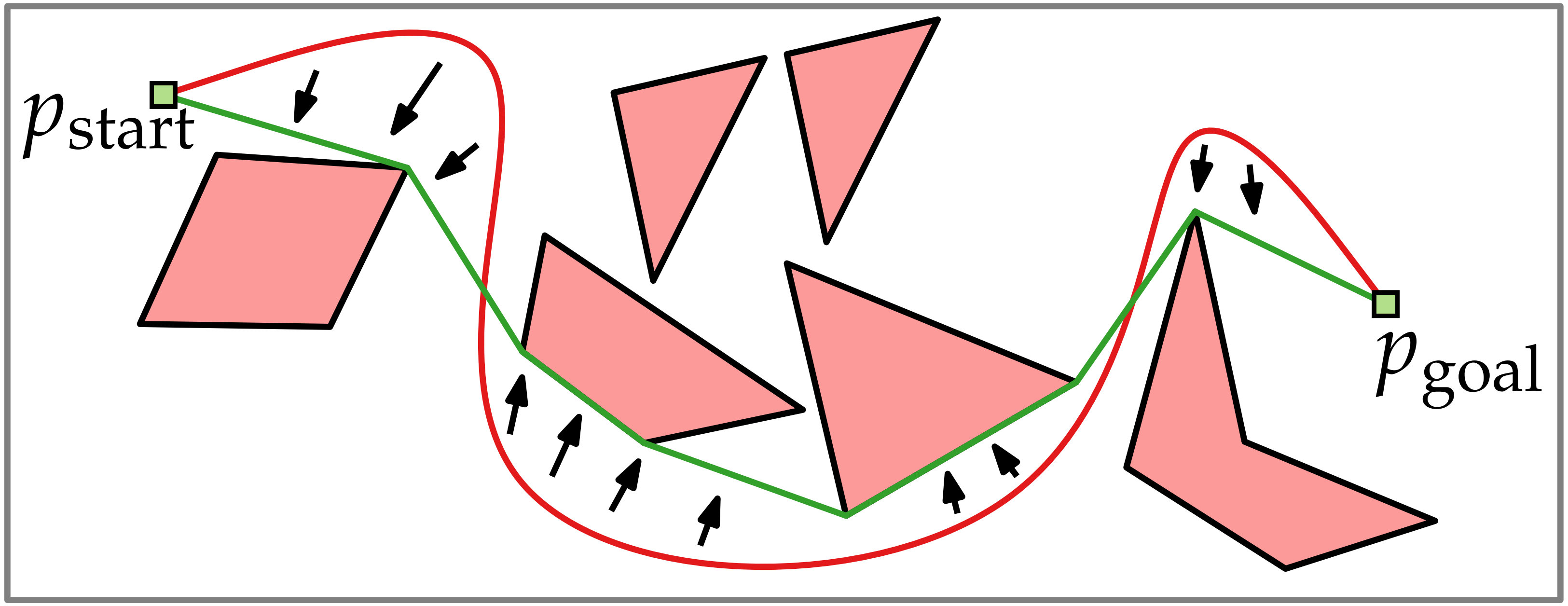

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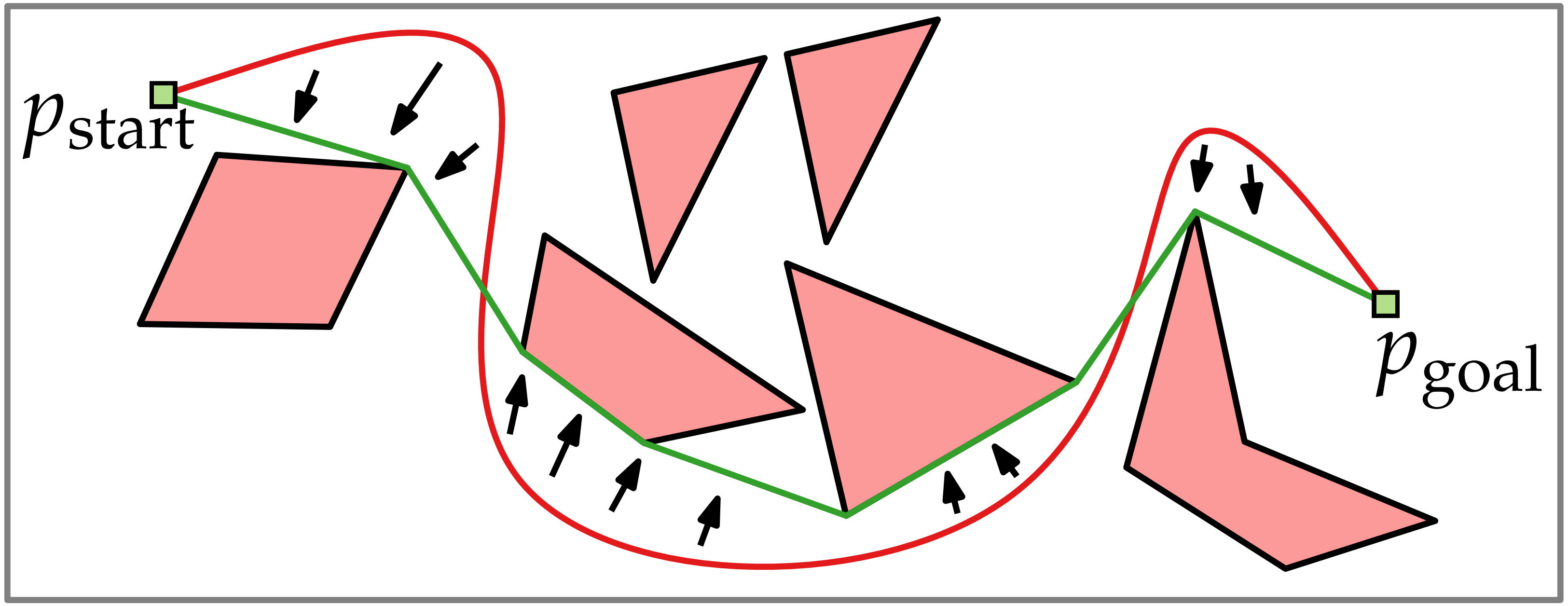

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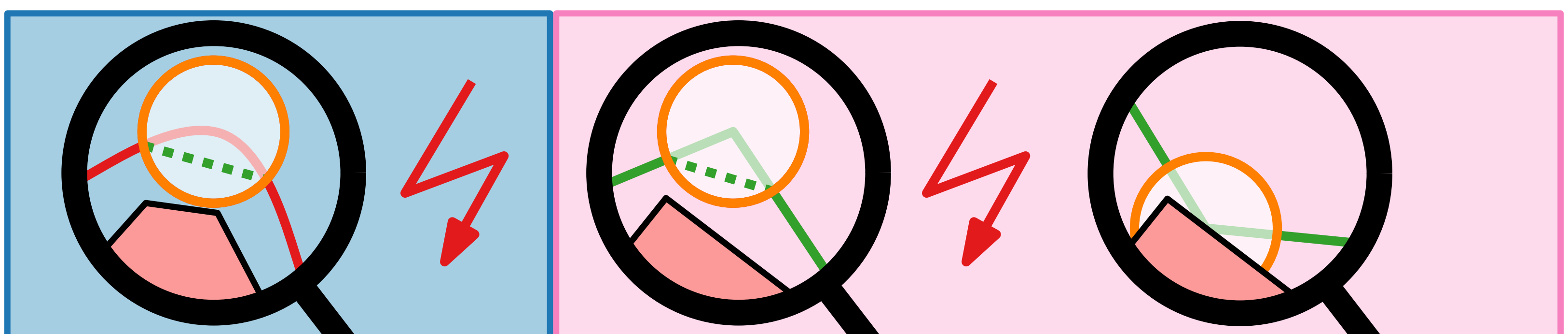

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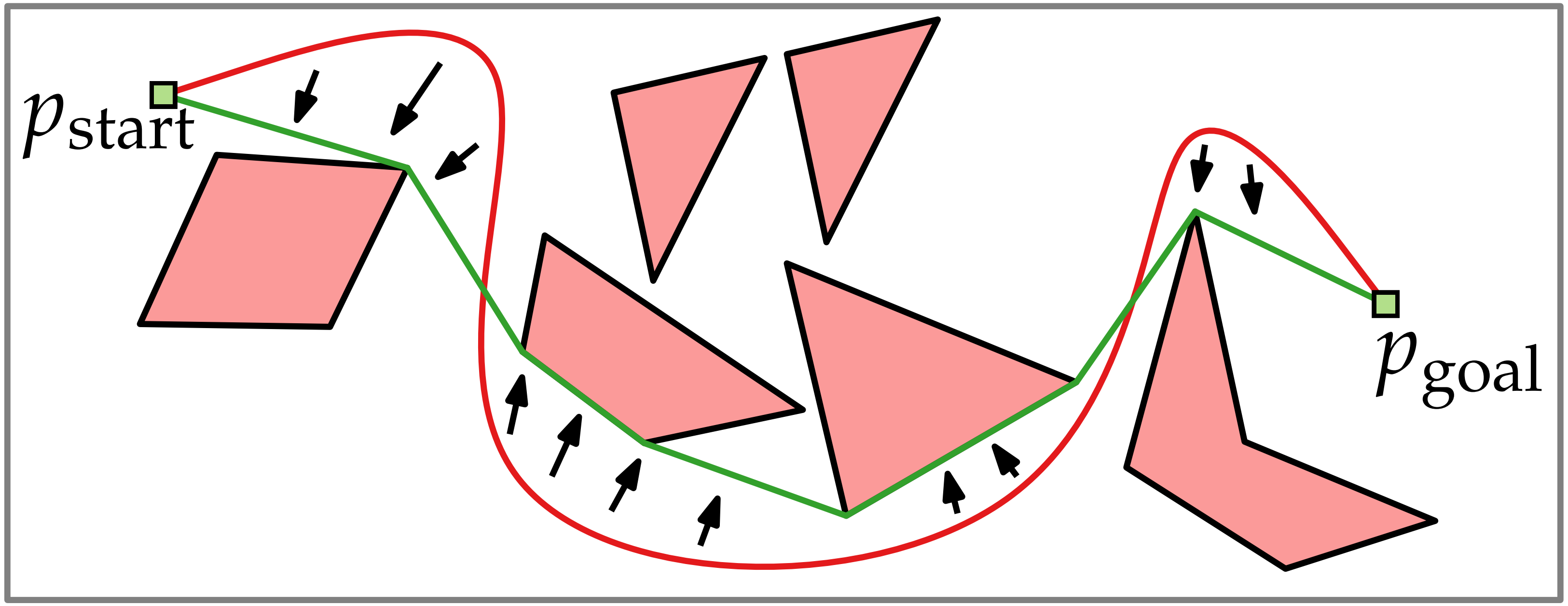

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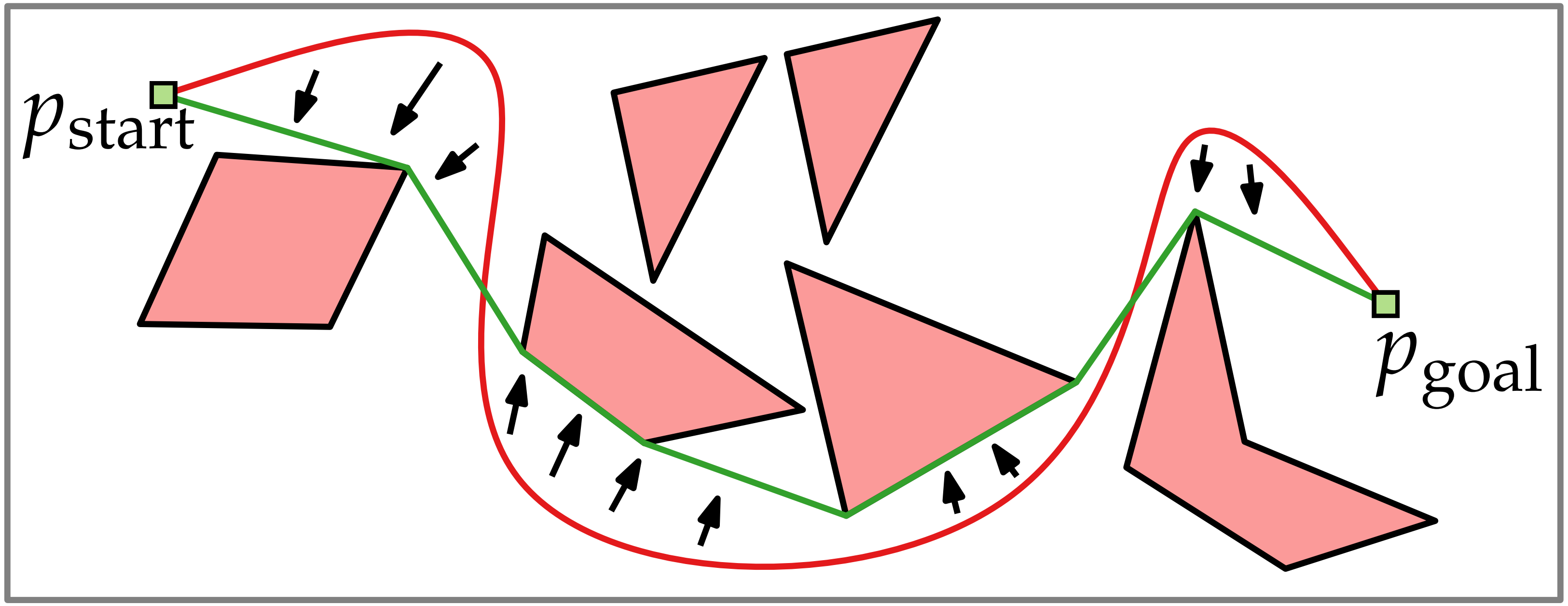

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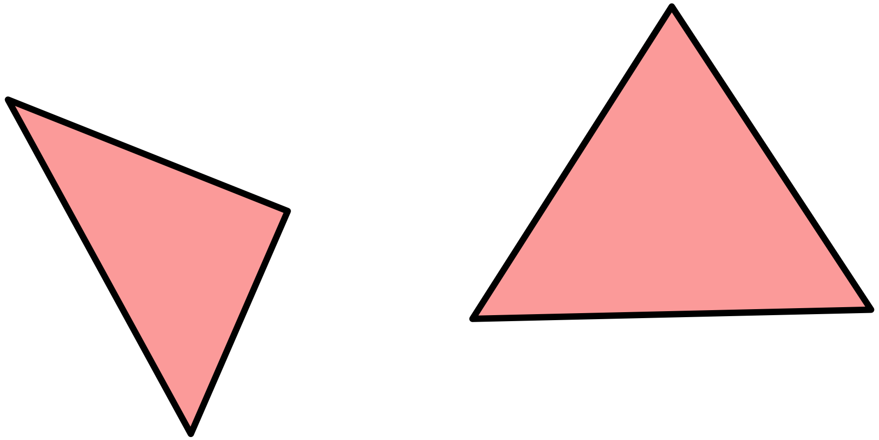
Visibility Graph

Given a set S of disjoint (open) polygons...



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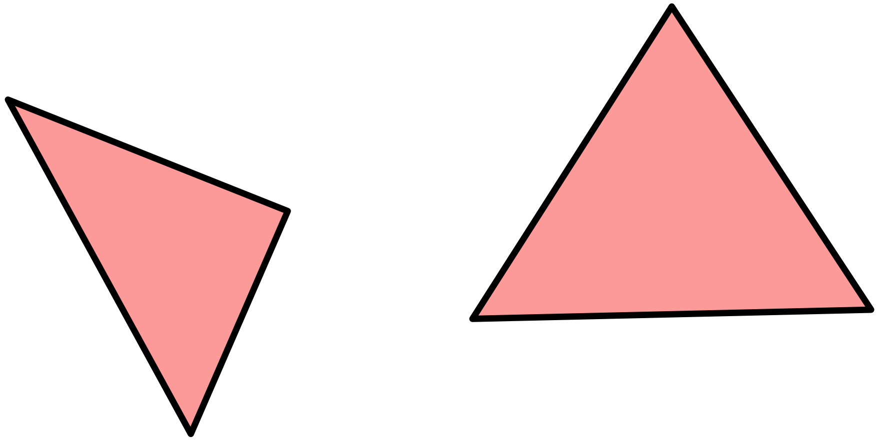

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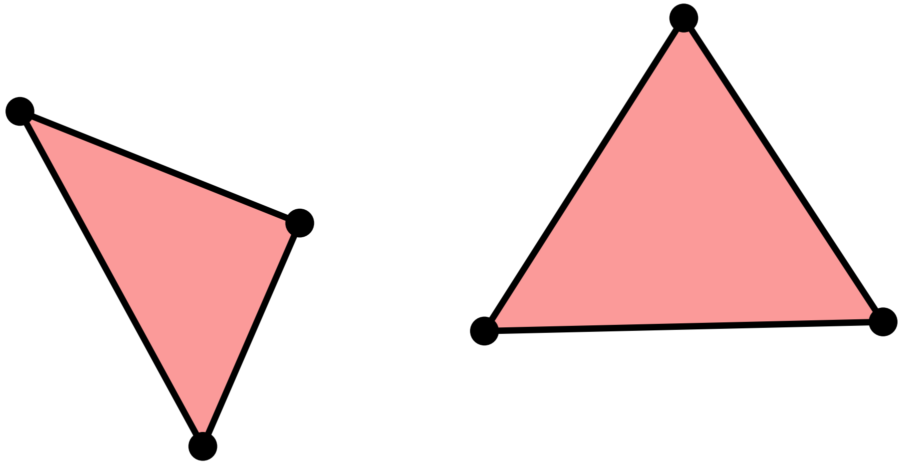

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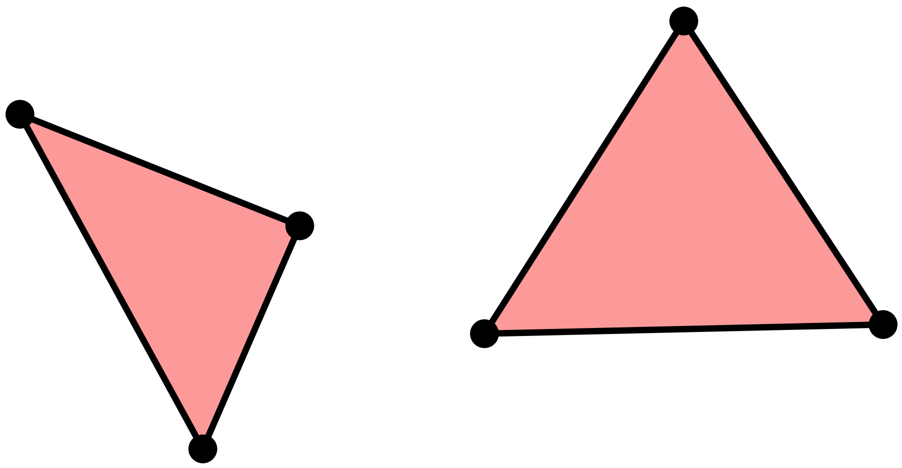

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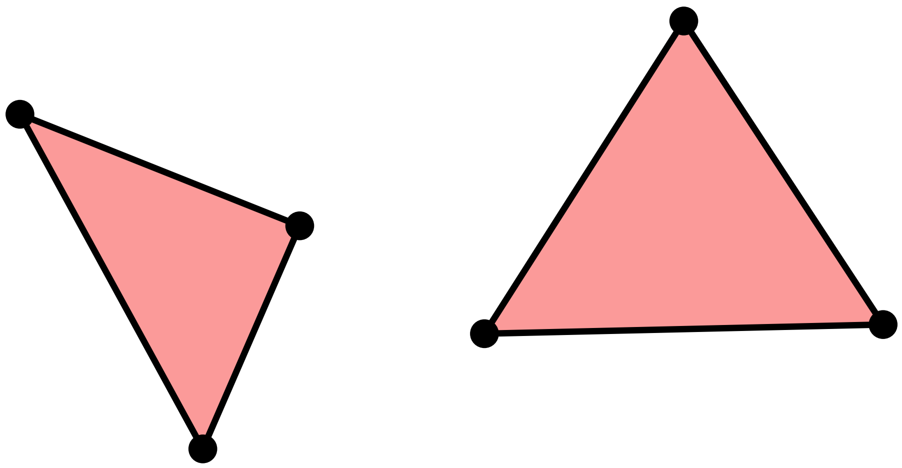

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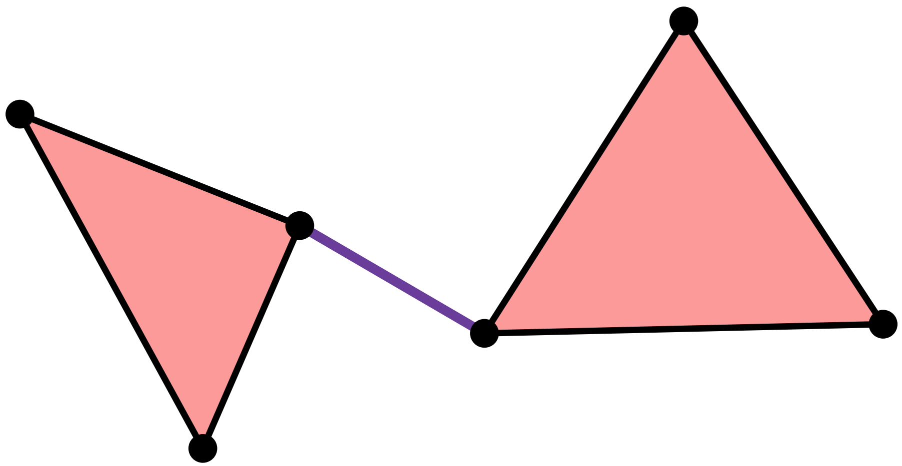

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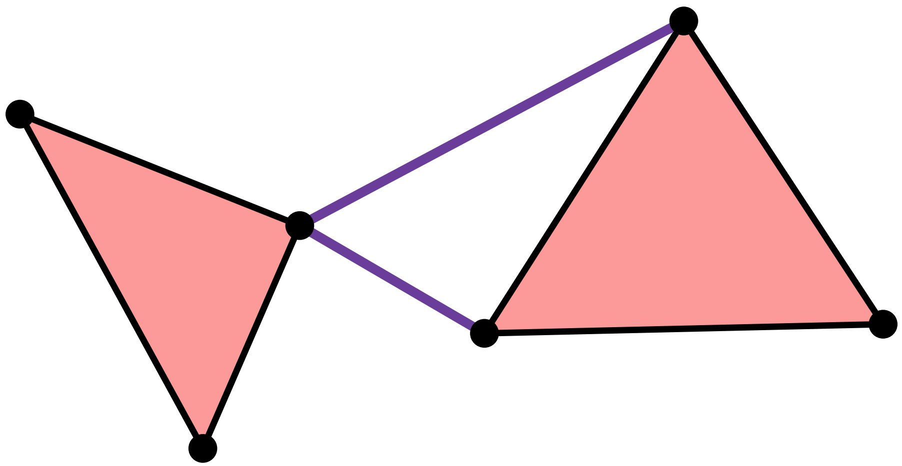

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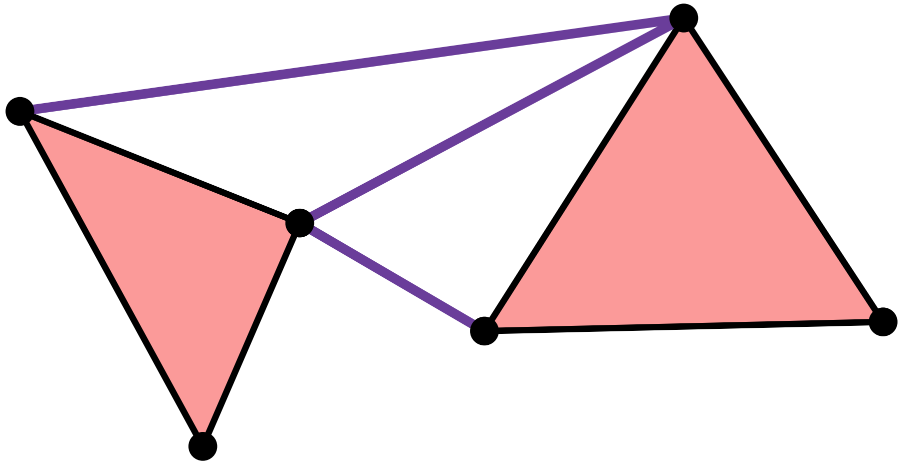

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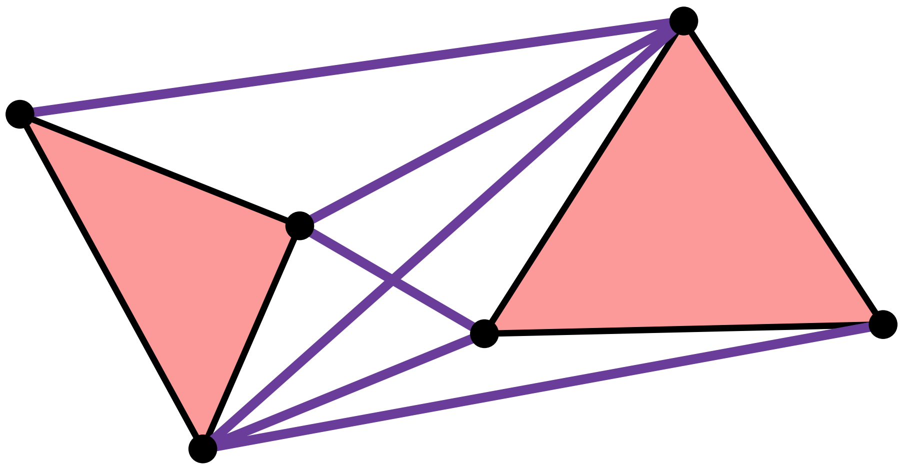

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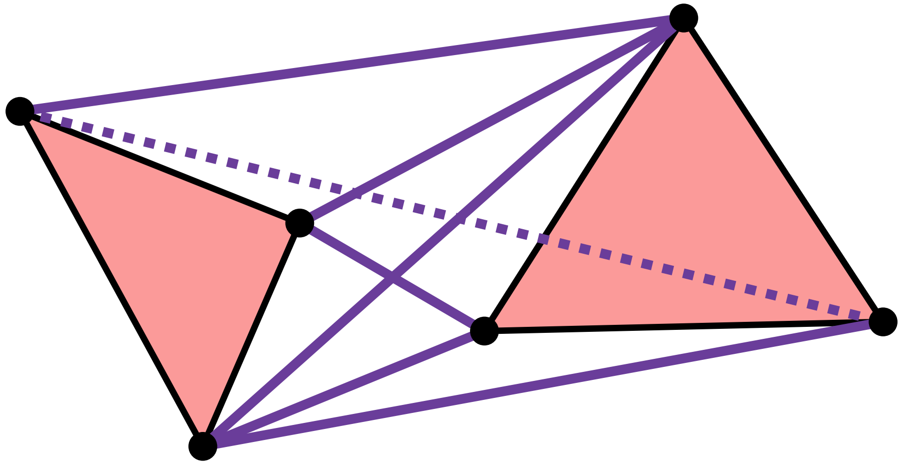

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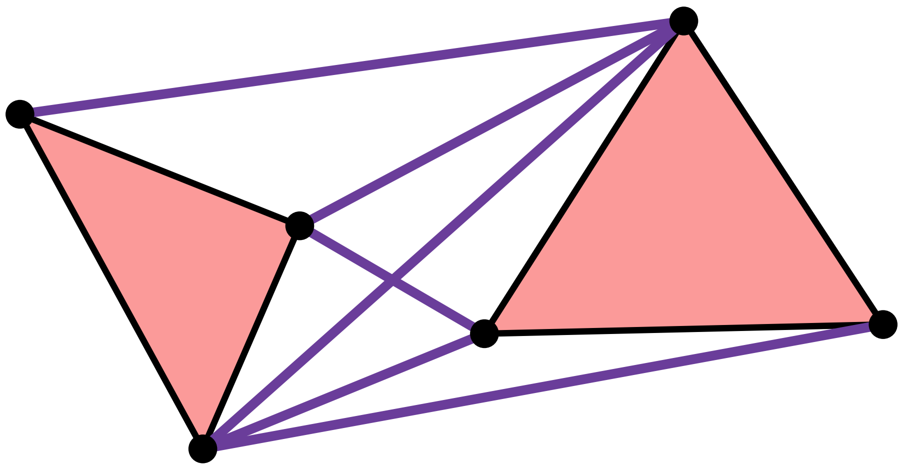

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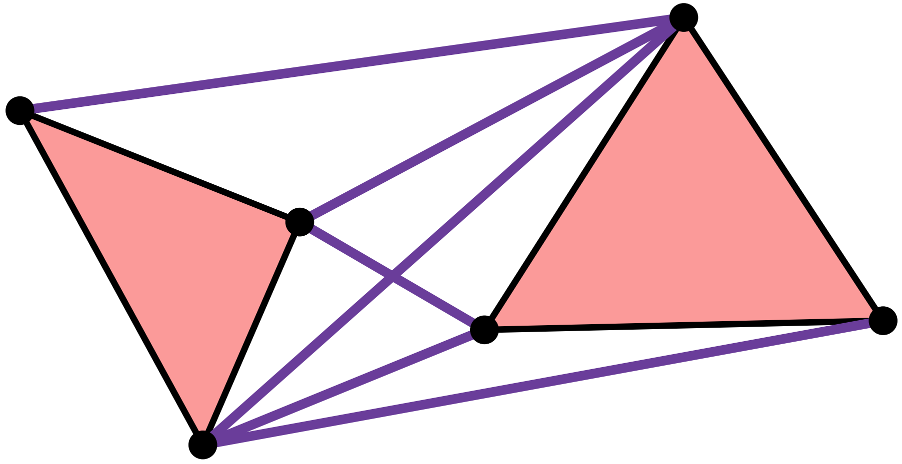

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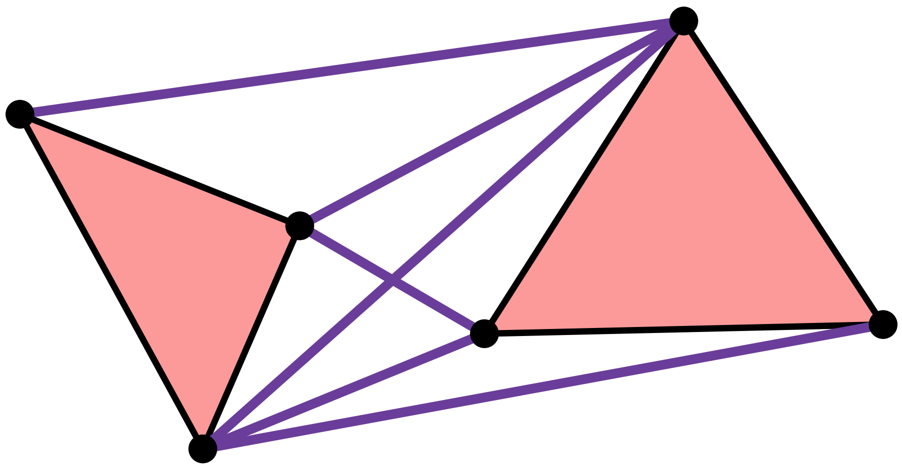

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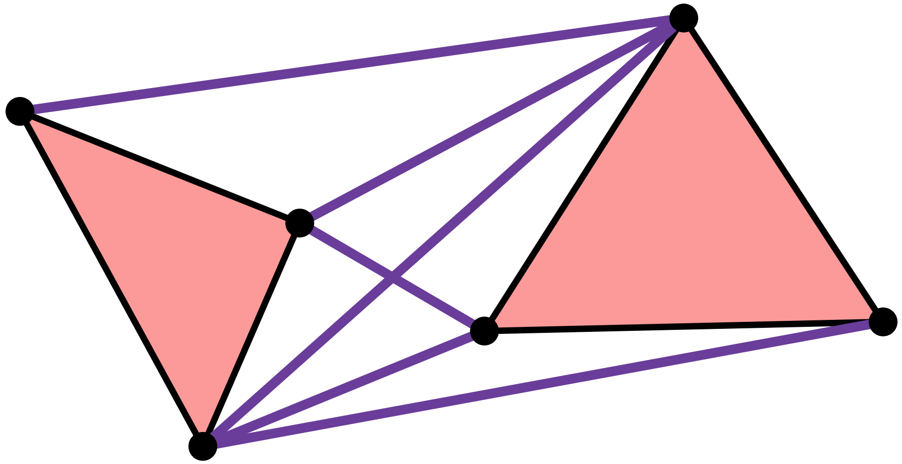

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Visibility Graph

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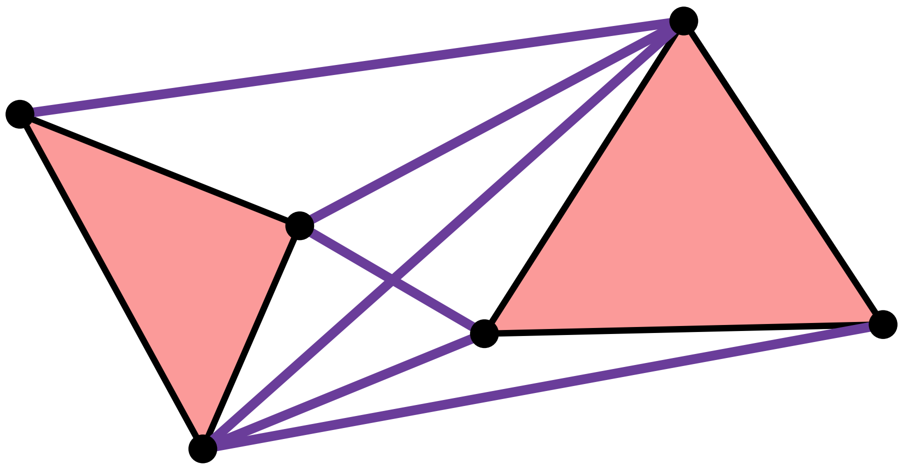

...let $V(S)$ be the vertex set of S.

Let $G_{\text{vis}}(S) = (V(S), E_{\text{vis}}(S))$ be the *visibility graph* of S,
where $E_{\text{vis}}(S) = \{uv \mid u, v \in V(S), u \text{ sees } v\}$ and $w(uv) = |uv|$.

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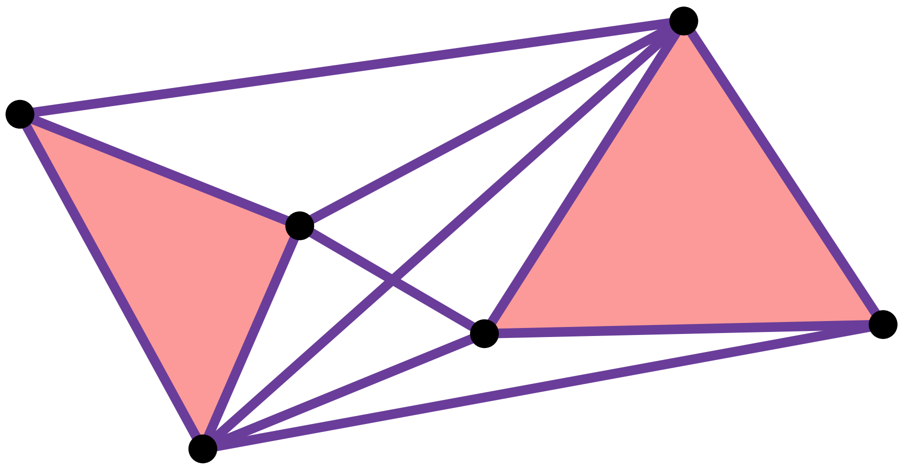

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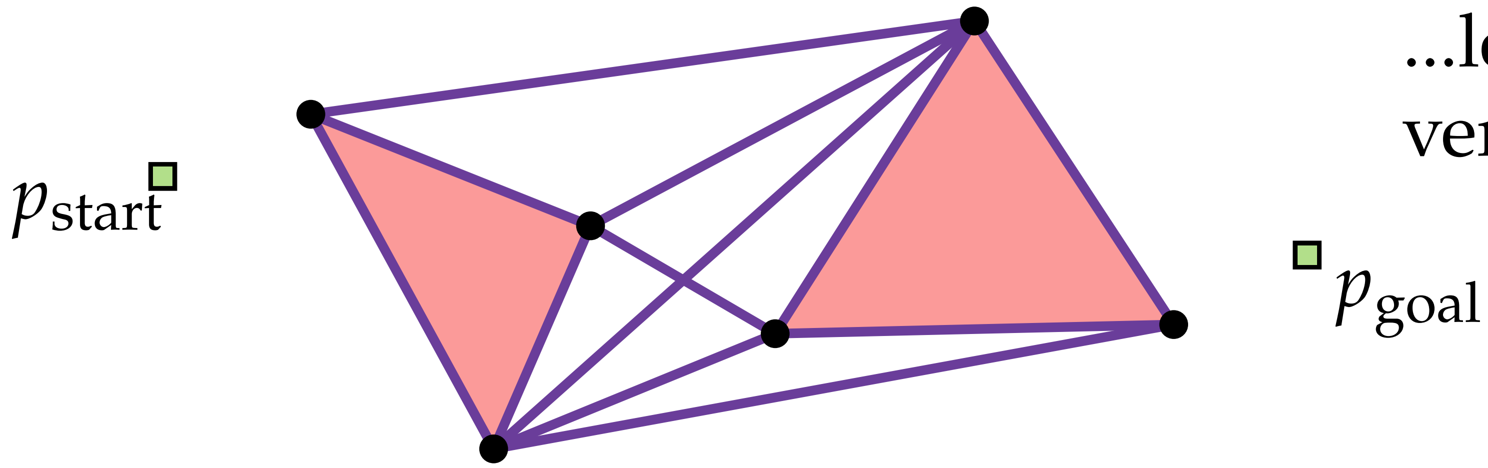

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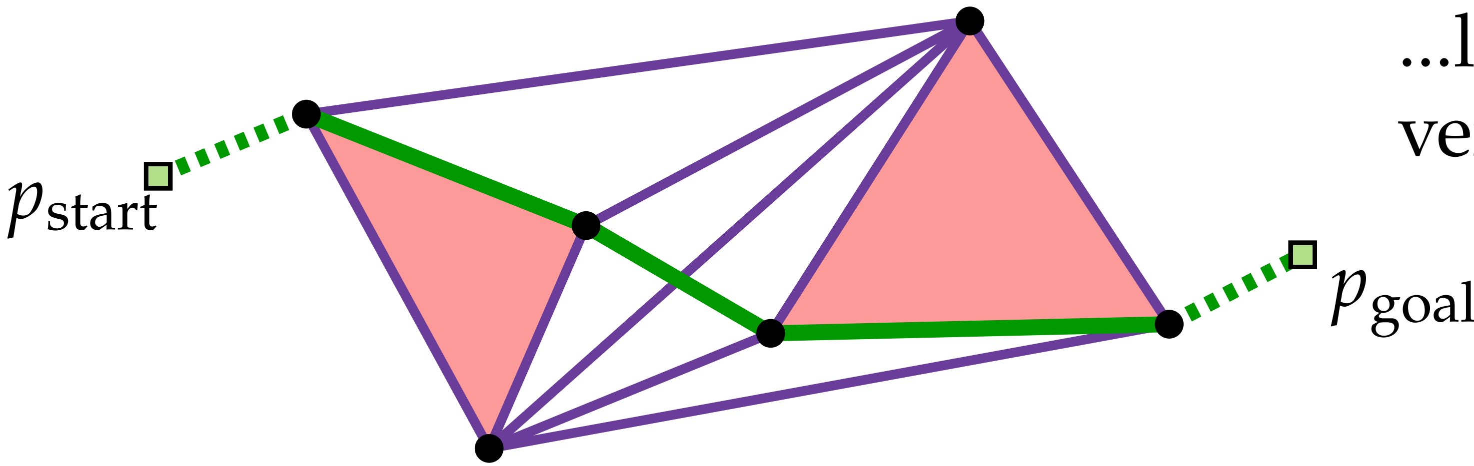

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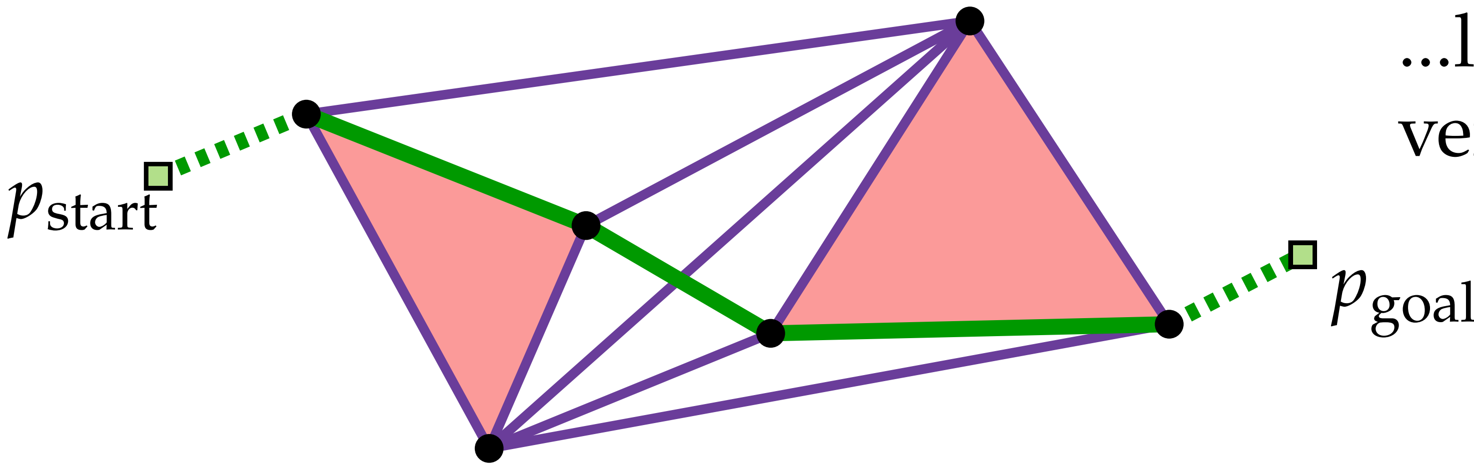

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$$S^\star = S \cup \{p_{\text{start}}, p_{\text{goal}}\}.$$

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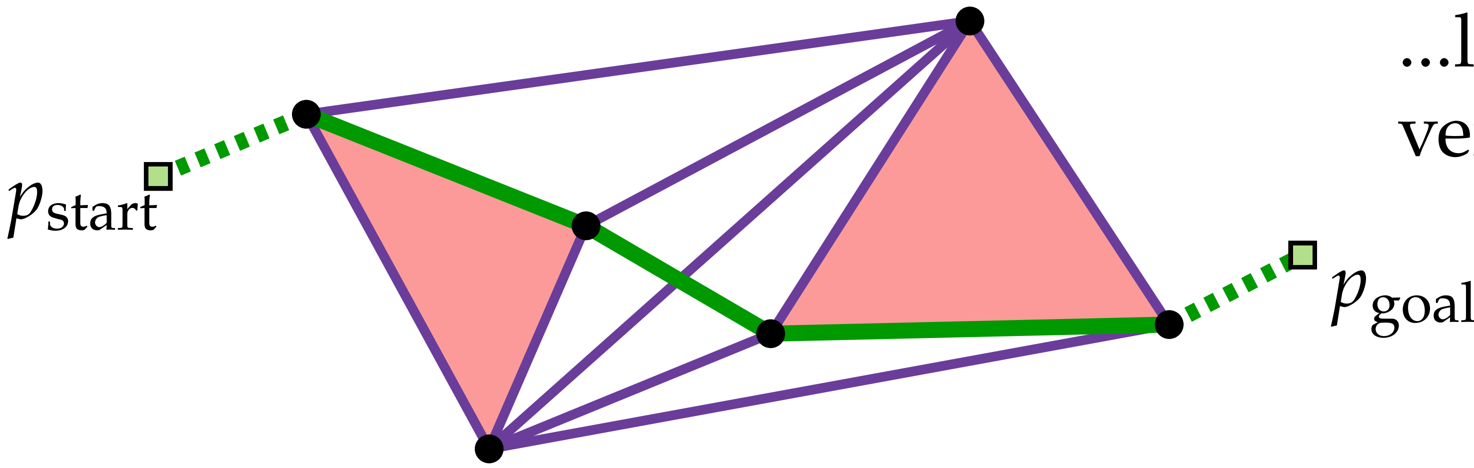

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$G_{\text{vis}}(S^\star)$

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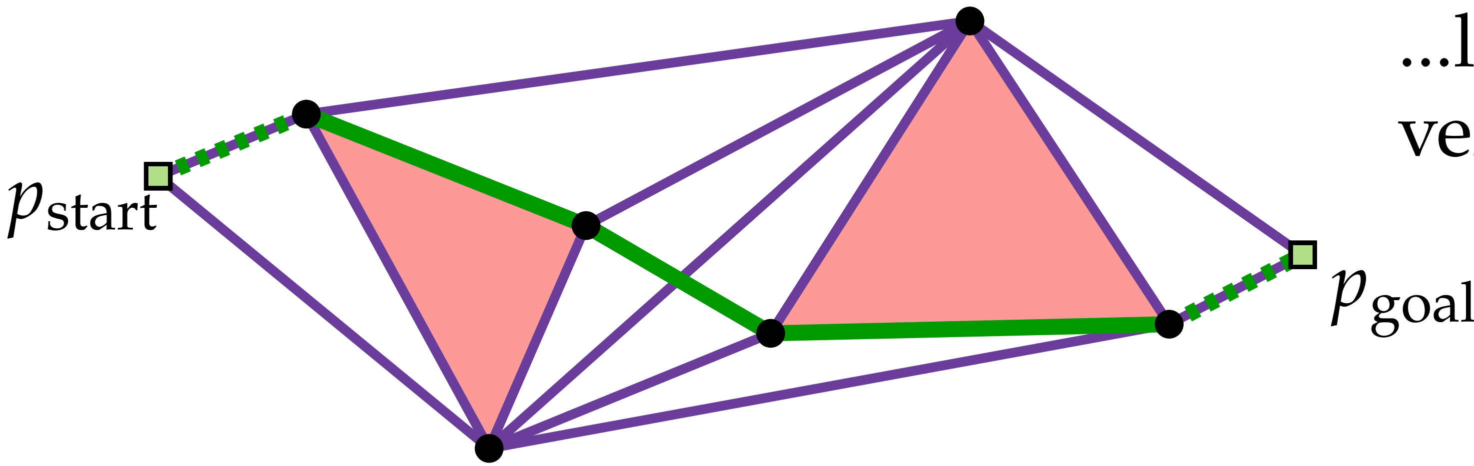

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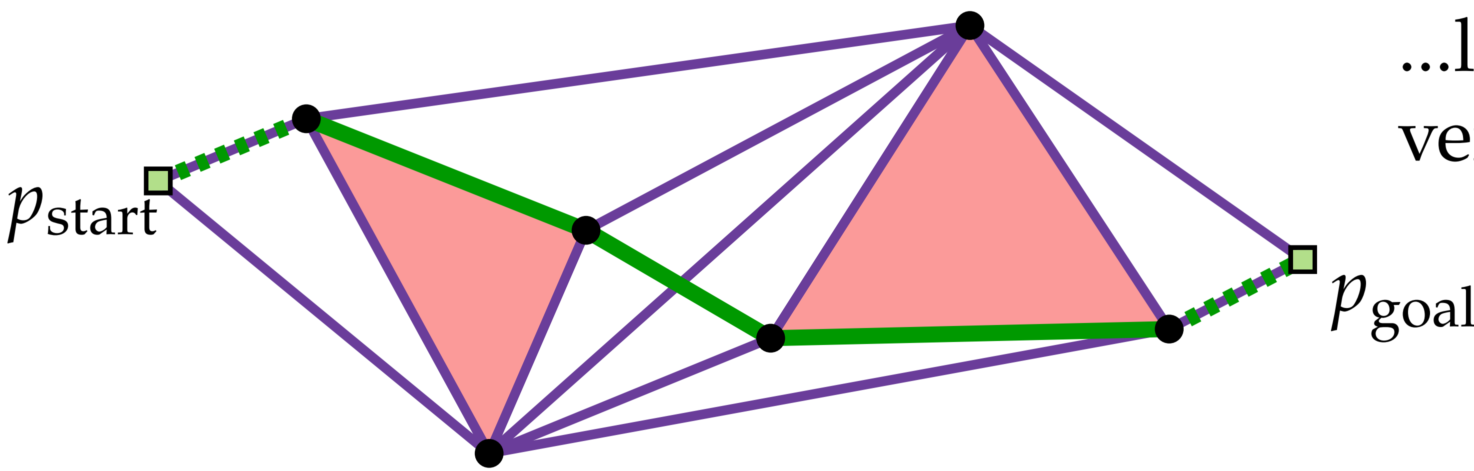

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Corollary. A shortest path between p_{start} and p_{goal} corresponds to a path in $G_{\text{vis}}(S^\star)$, where $S^\star = S \cup \{p_{\text{start}}, p_{\text{goal}}\}$.

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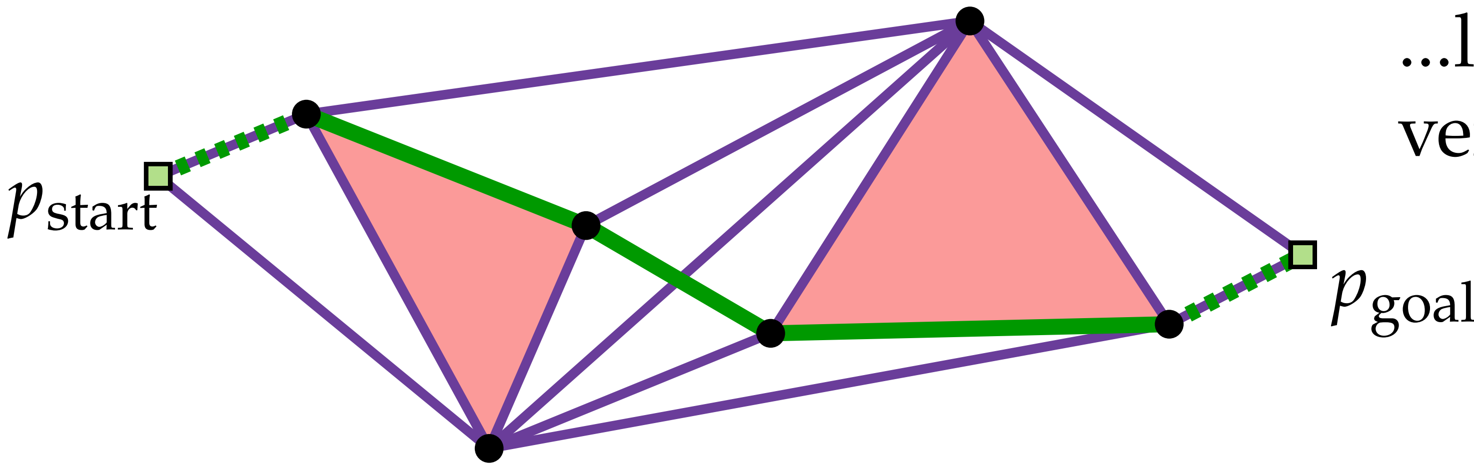

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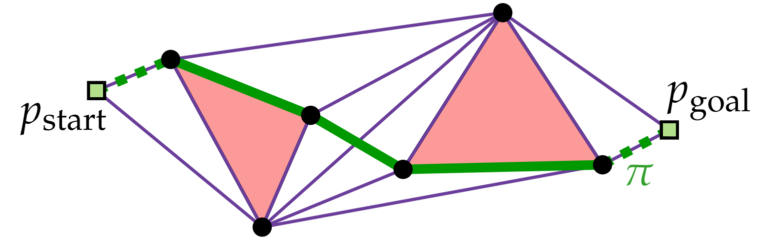
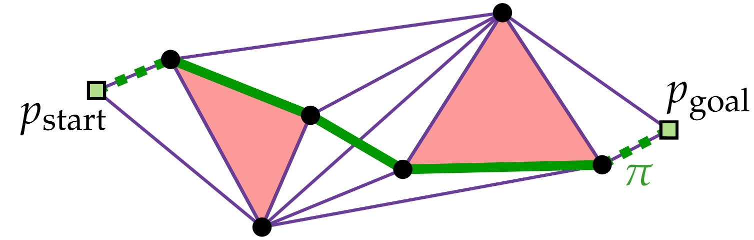
Corollary. A shortest path between p_{start} and p_{goal} corresponds to a *shortest* path in $G_{\text{vis}}(S^\star)$, where $S^\star = S \cup \{p_{\text{start}}, p_{\text{goal}}\}$.

Algorithm



SHORTESTPATH($S, p_{\text{start}}, p_{\text{goal}}$)

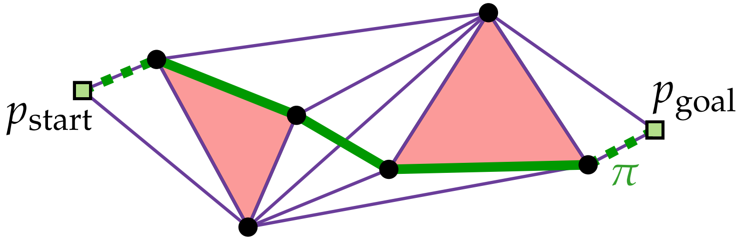
Algorithm



$\textsc{ShortestPath}(S, p_{\text{start}}, p_{\text{goal}})$

$G_{\text{vis}} \leftarrow \textsc{VisibilityGraph}(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$

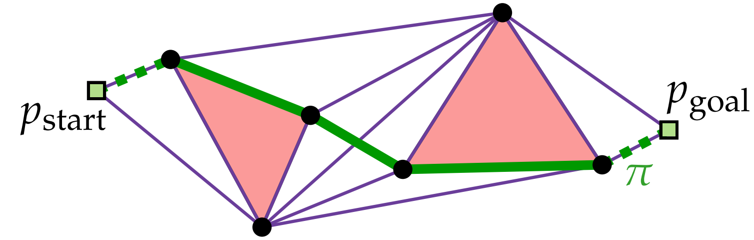
Algorithm



ShortestPath$(S, p_{\text{start}}, p_{\text{goal}})$

$G_{\text{vis}} \leftarrow$ **VisibilityGraph**$(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$
foreach $uv \in E_{\text{vis}}$ **do**

Algorithm



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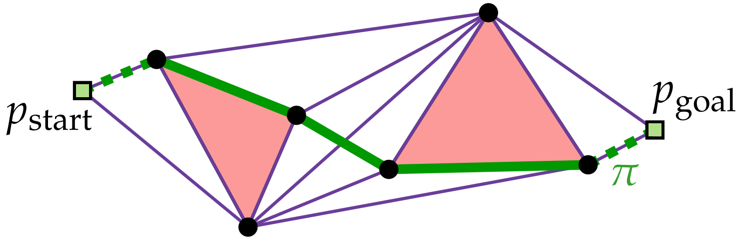
foreach $uv \in E_{\text{vis}}$ **do**

$w(uv) = d_{\text{Eucl.}}(u, v)$

Algorithm



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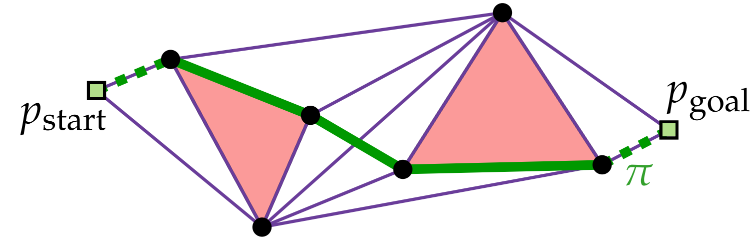
Algorithm



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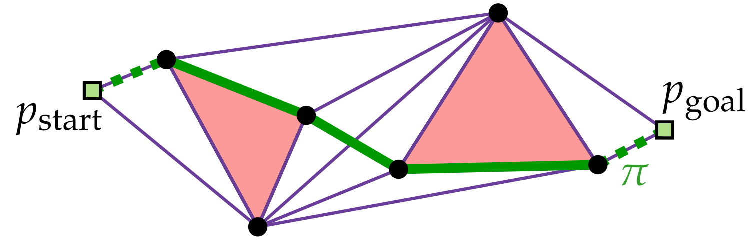
foreach $uv \in E_{\text{vis}}$ **do**

$w(uv) = d_{\text{Eucl.}}(u, v)$

$\pi \leftarrow \textsc{Dijkstra}(G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}})$

Algorithm



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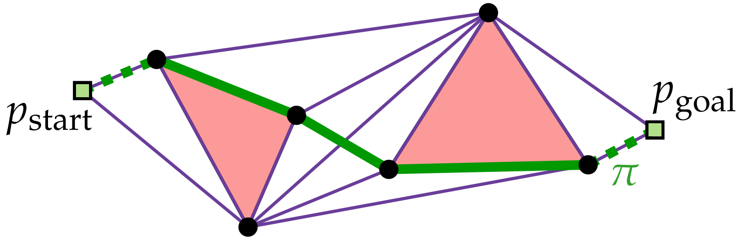
return π

Algorithm



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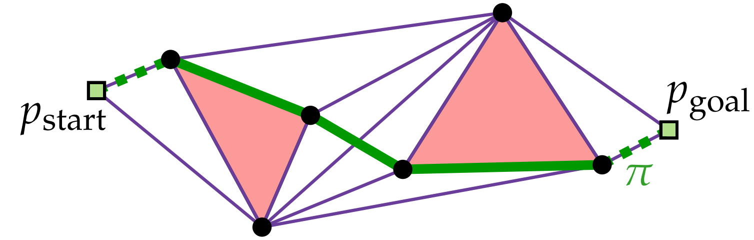
Running time?

Algorithm

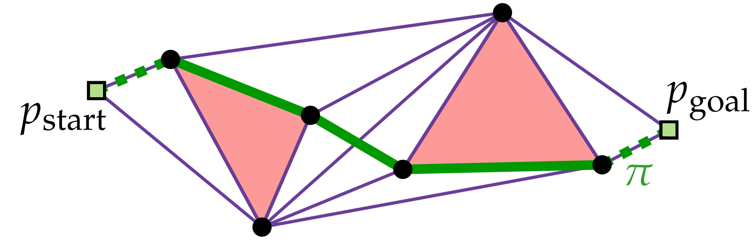


ShortestPath$(S, p_{\text{start}}, p_{\text{goal}})$ $\quad n = |V(S)|$

$G_{\text{vis}} \leftarrow$ VisibilityGraph$(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$
foreach $uv \in E_{\text{vis}}$ **do**
  $w(uv) = d_{\text{Eucl.}}(u, v)$
$\pi \leftarrow$ Dijkstra$(G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}})$
return π

Running time?

Algorithm



$\textsc{ShortestPath}(S, p_{start}, p_{goal})$ $\quad n = |V(S)|,\ m = |E_{vis}(S)|$

$G_{vis} \leftarrow \textsc{VisibilityGraph}(S \cup \{p_{start}, p_{goal}\})$

foreach $uv \in E_{vis}$ **do**

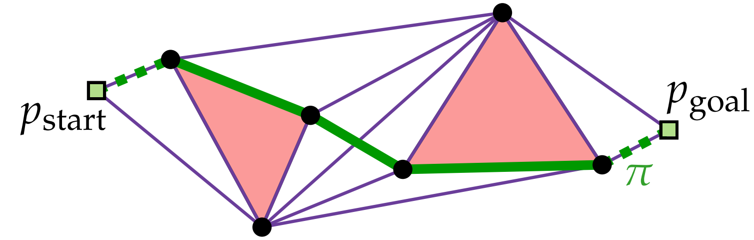
$\quad w(uv) = d_{Eucl.}(u, v)$

$\pi \leftarrow \textsc{Dijkstra}(G_{vis}, w, p_{start}, p_{goal})$

return π

Running time?

Algorithm



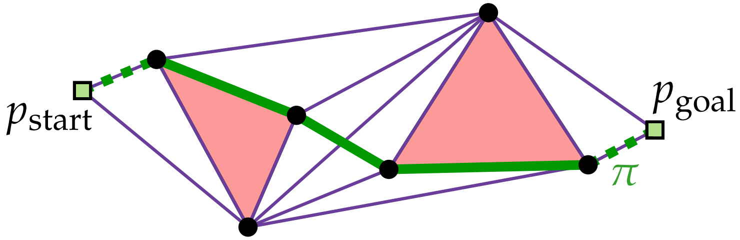
$n = |V(S)|$, $m = |E_{\text{vis}}(S)|$

ShortestPath$(S, p_{\text{start}}, p_{\text{goal}})$

- $G_{\text{vis}} \leftarrow$ VisibilityGraph$(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$
- **foreach** $uv \in E_{\text{vis}}$ **do**
  - $w(uv) = d_{\text{Eucl.}}(u, v)$ — $O(m)$
- $\pi \leftarrow$ Dijkstra$(G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}})$
- **return** π

Running time?

Algorithm



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ShortestPath($S, p_{\text{start}}, p_{\text{goal}}$)

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foreach $uv \in E_{\text{vis}}$ **do**

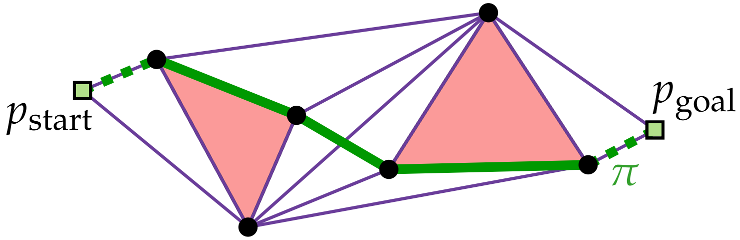
  $w(uv) = d_{\text{Eucl.}}(u, v)$ $O(m)$

$\pi \leftarrow$ Dijkstra($G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}}$) $O(m + n \log n)$

return π

Running time?

Algorithm



SHORTESTPATH$(S, p_{\text{start}}, p_{\text{goal}})$ $n = |V(S)|$, $m = |E_{\text{vis}}(S)|$

$G_{\text{vis}} \leftarrow$ VISIBILITYGRAPH$(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$?

foreach $uv \in E_{\text{vis}}$ **do** $O(m)$

$w(uv) = d_{\text{Eucl.}}(u, v)$

$\pi \leftarrow$ DIJKSTRA$(G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}})$ $O(m + n \log n)$

return π

Running time?

Computing the Visibility Graph

VisibilityGraph(S)

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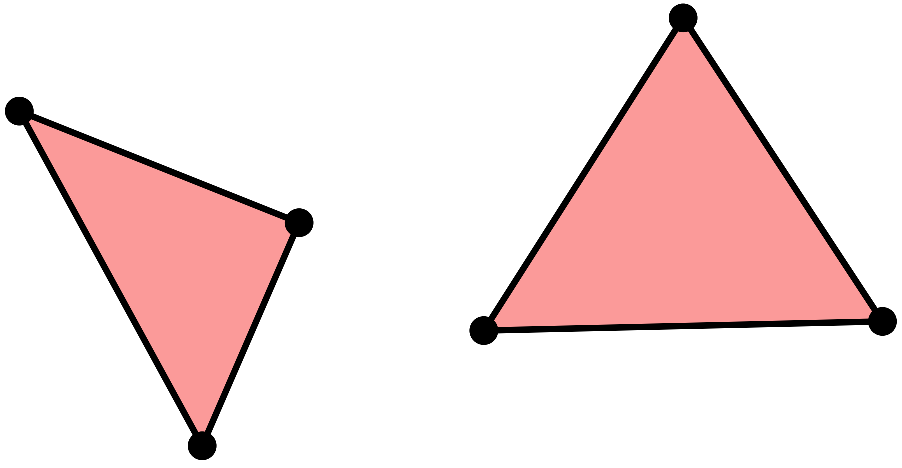
Input: a set S of disjoint polygons



Computing the Visibility Graph

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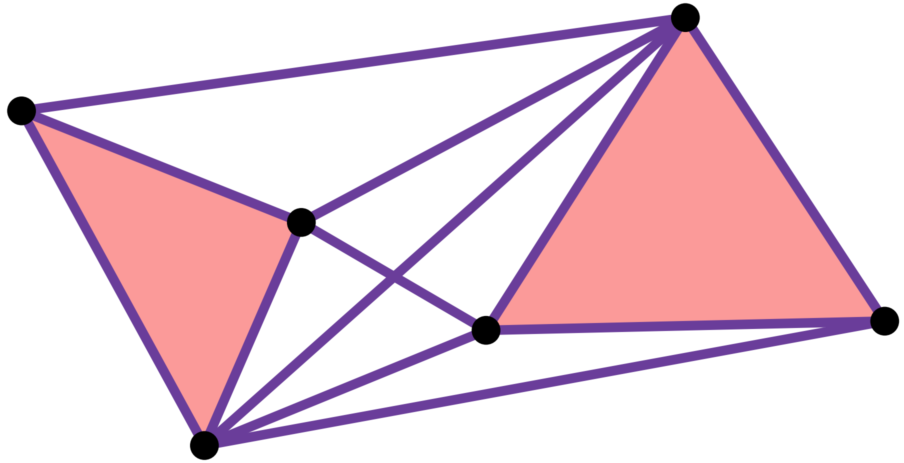
Output: $G_{\text{vis}}(S)$



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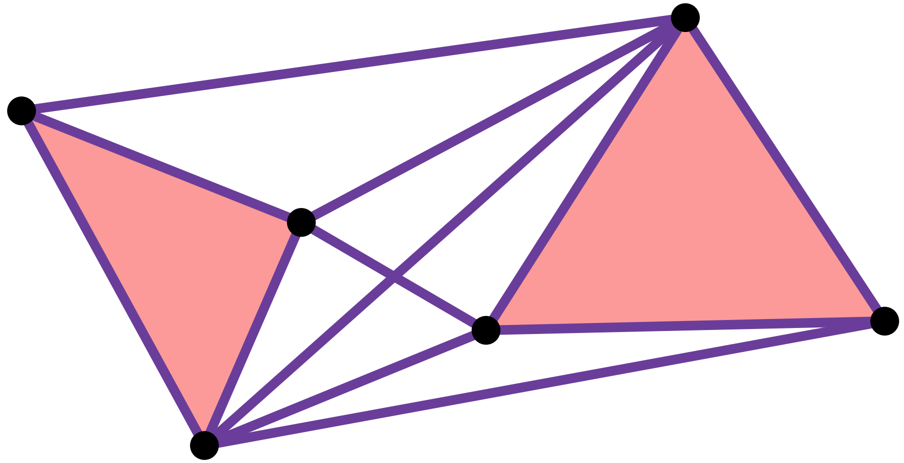
$E \leftarrow \emptyset$



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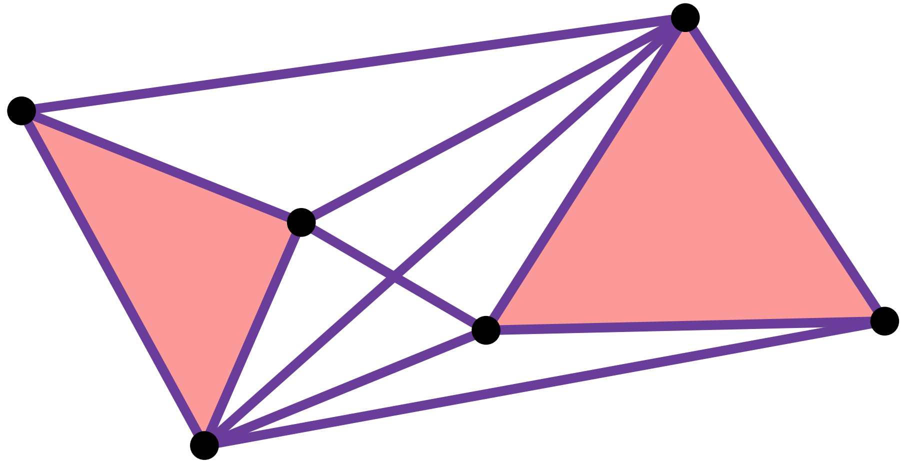

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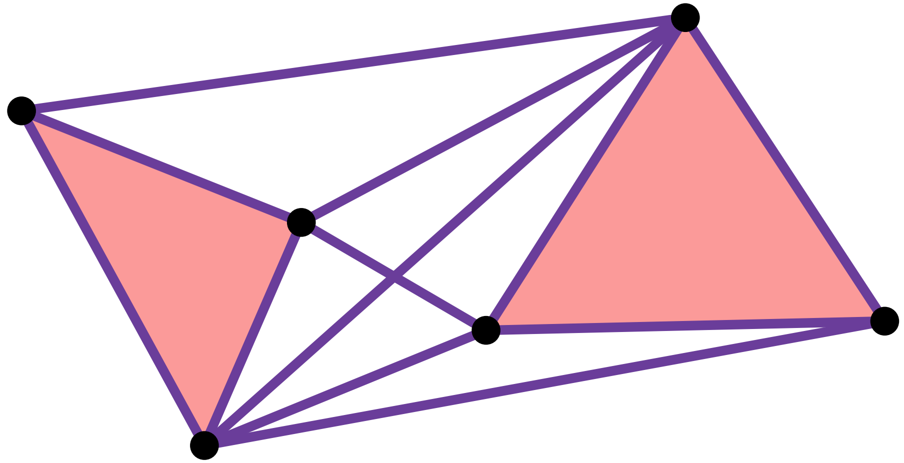
foreach $v \in V(S)$ **do**
    $W = $ VisibleVertices(v, S)



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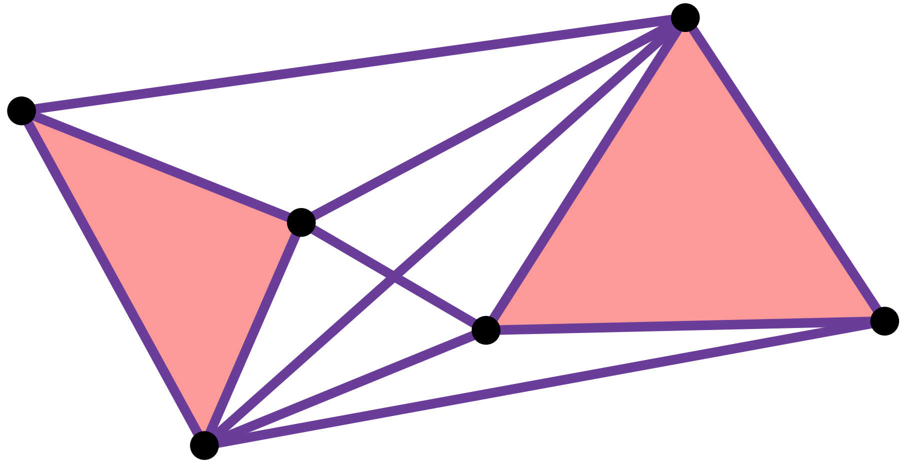

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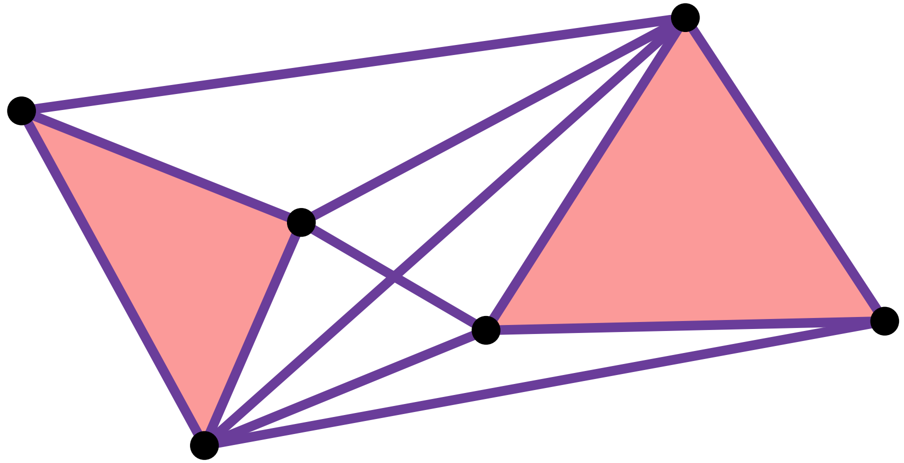
return $(V(S), E)$



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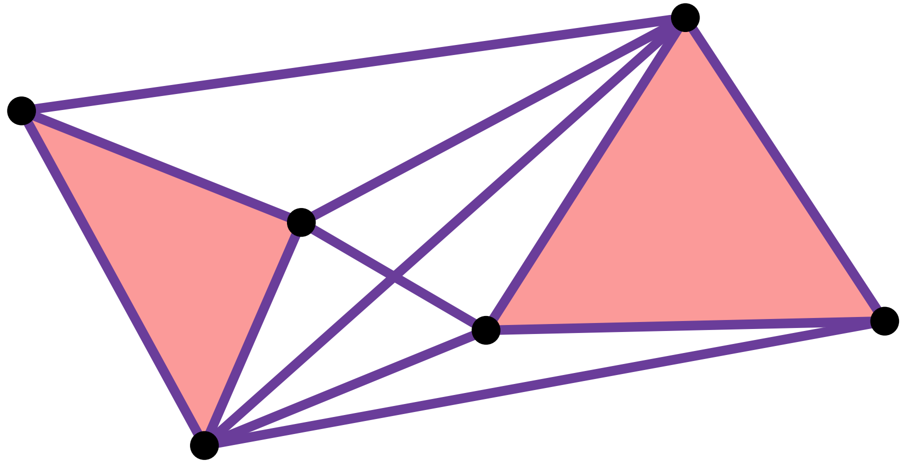
$O(n) \cdot$



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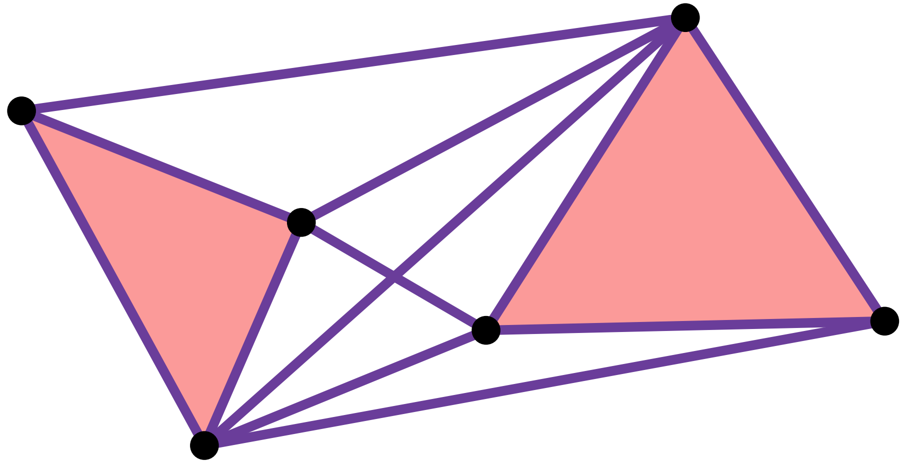

Computing Visible Vertices

VISIBLEVERTICES(p, S)

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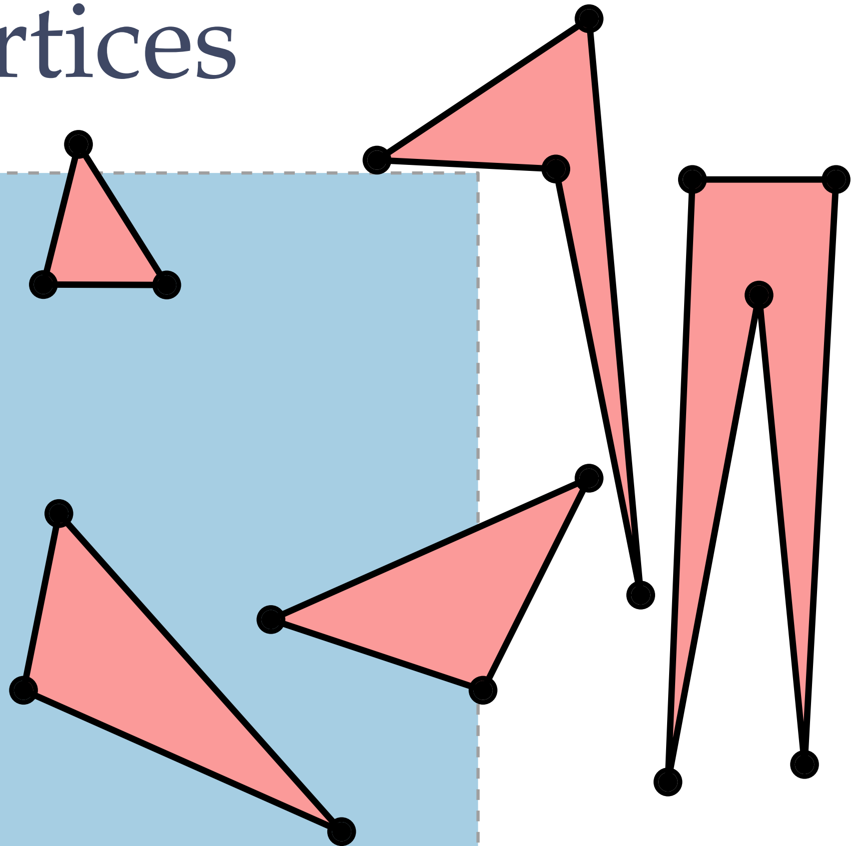

Computing Visible Vertices

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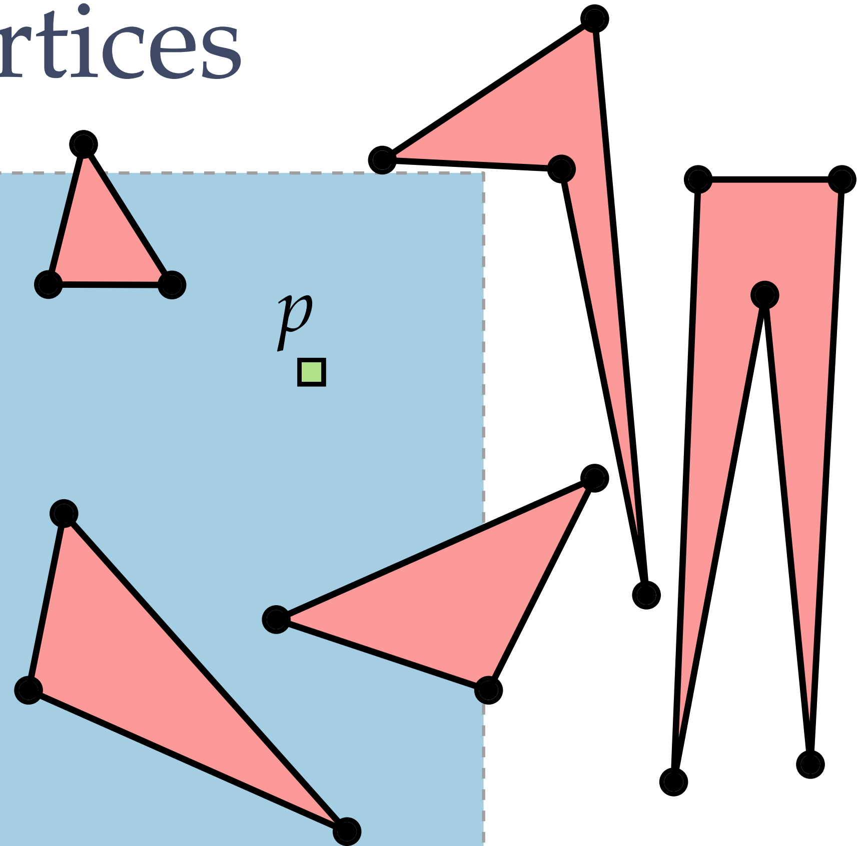

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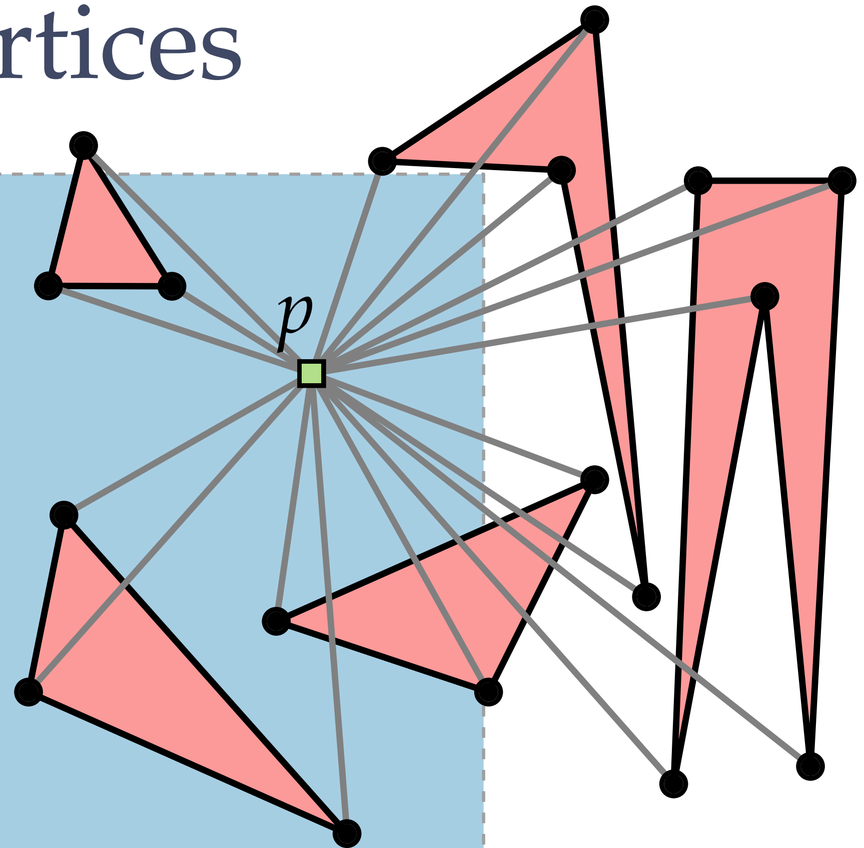

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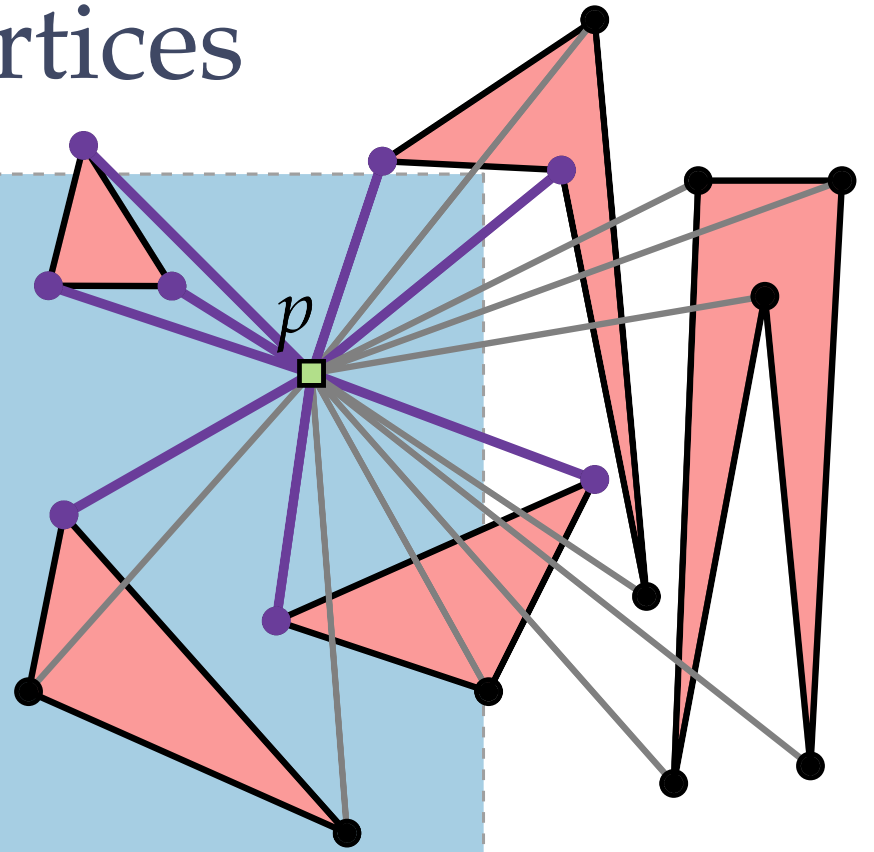
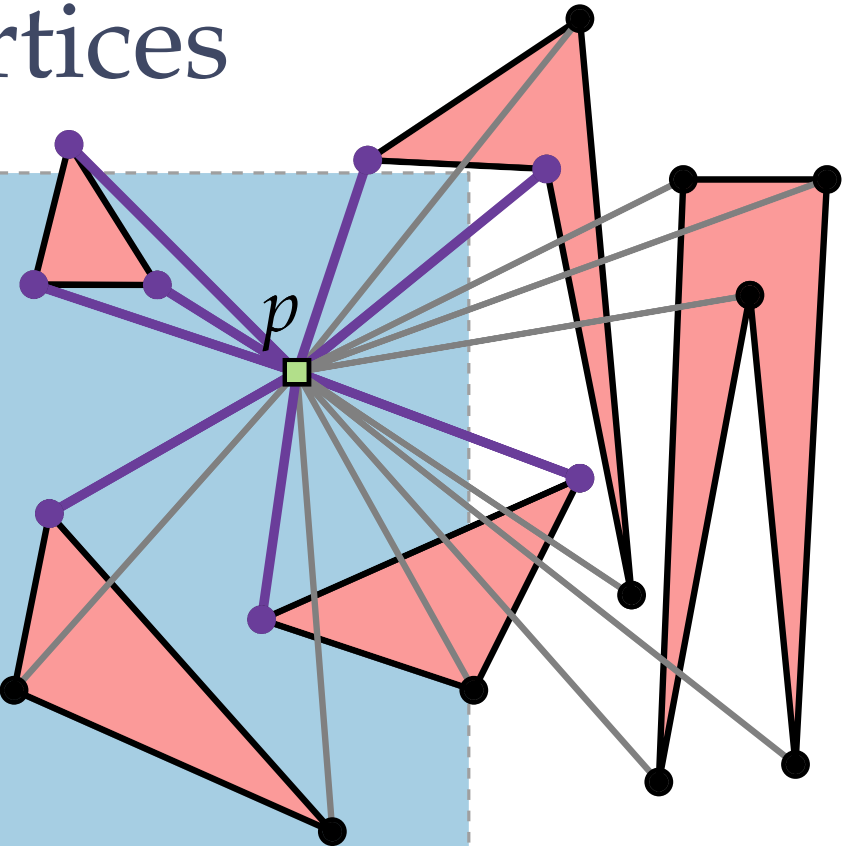


Computing Visible Vertices

VisibleVertices(p, S)

Task: Separate the "**good**" from the "**evil**"



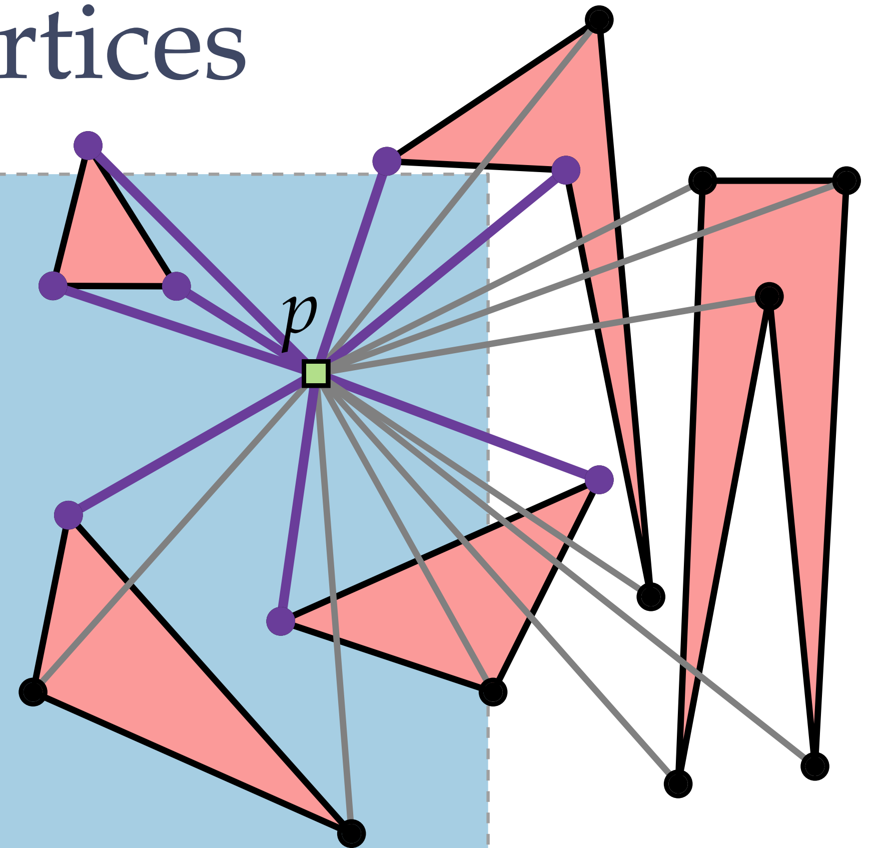
Computing Visible Vertices

VisibleVertices(p, S)

Task: Separate the "**good**" from the "**evil**":

Given p and S, find in $O(n \log n)$ time all vertices in $V(S)$ visible from p!

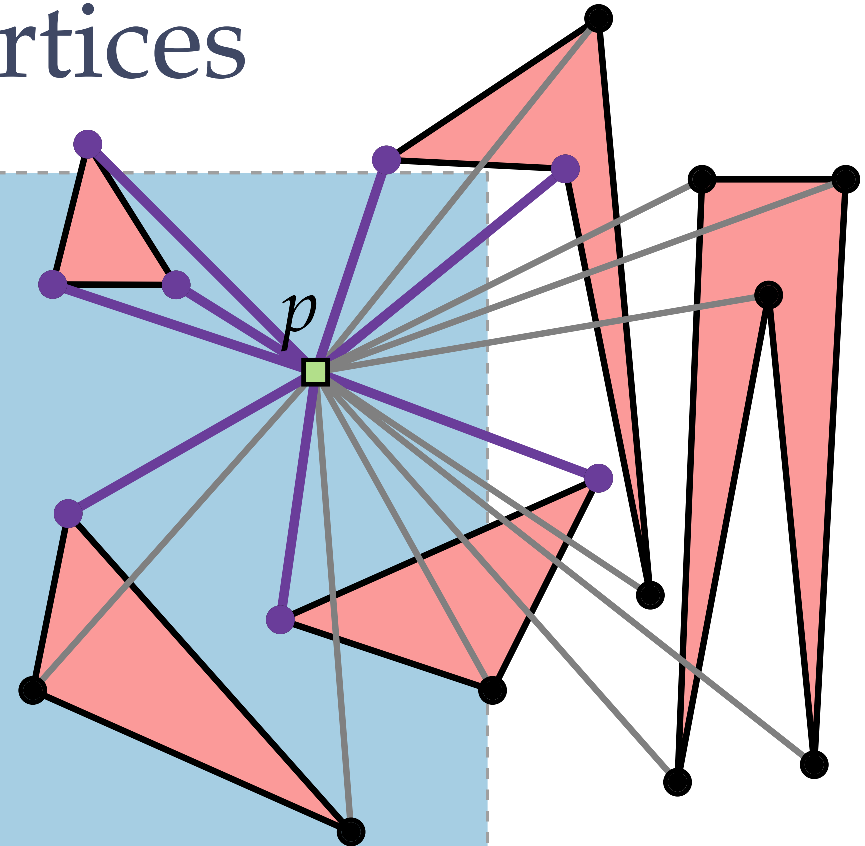
[3 min]



Computing Visible Vertices

VisibleVertices(p, S)

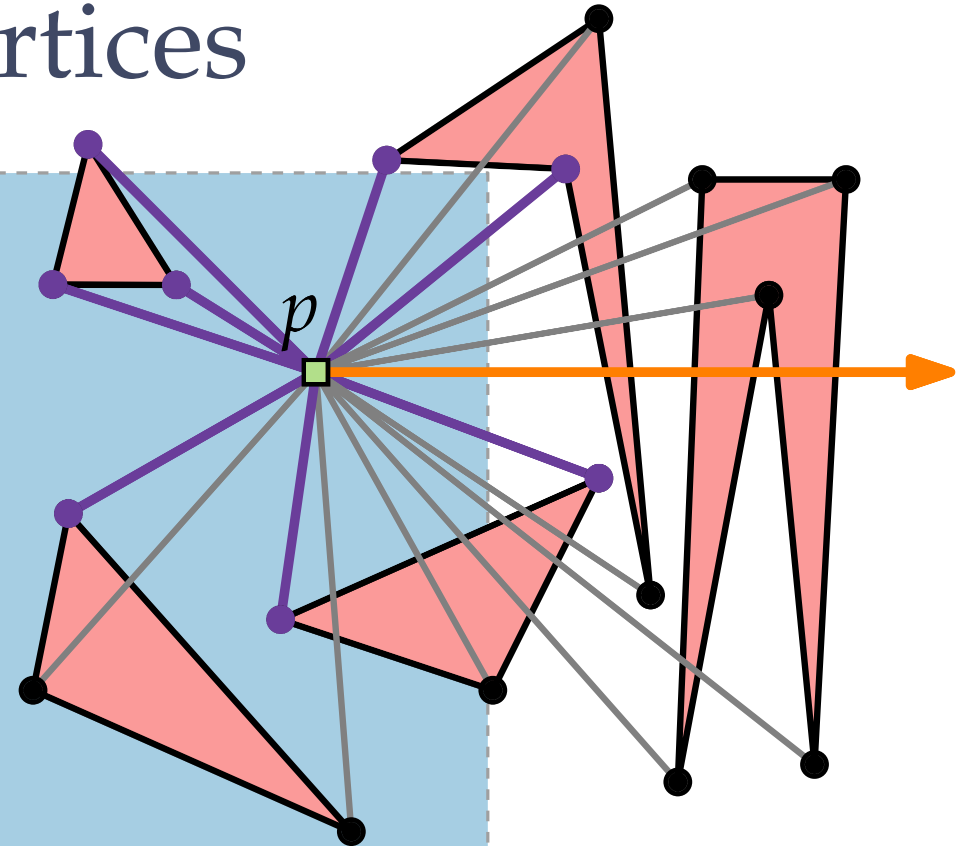
$r \leftarrow p + \mathbb{R}_{\geq 0}\binom{1}{0}$



Computing Visible Vertices

VISIBLEVERTICES(p, S)

$r \leftarrow p + \mathbb{R}_{\geq 0}\binom{1}{0}$



Computing Visible Vertices

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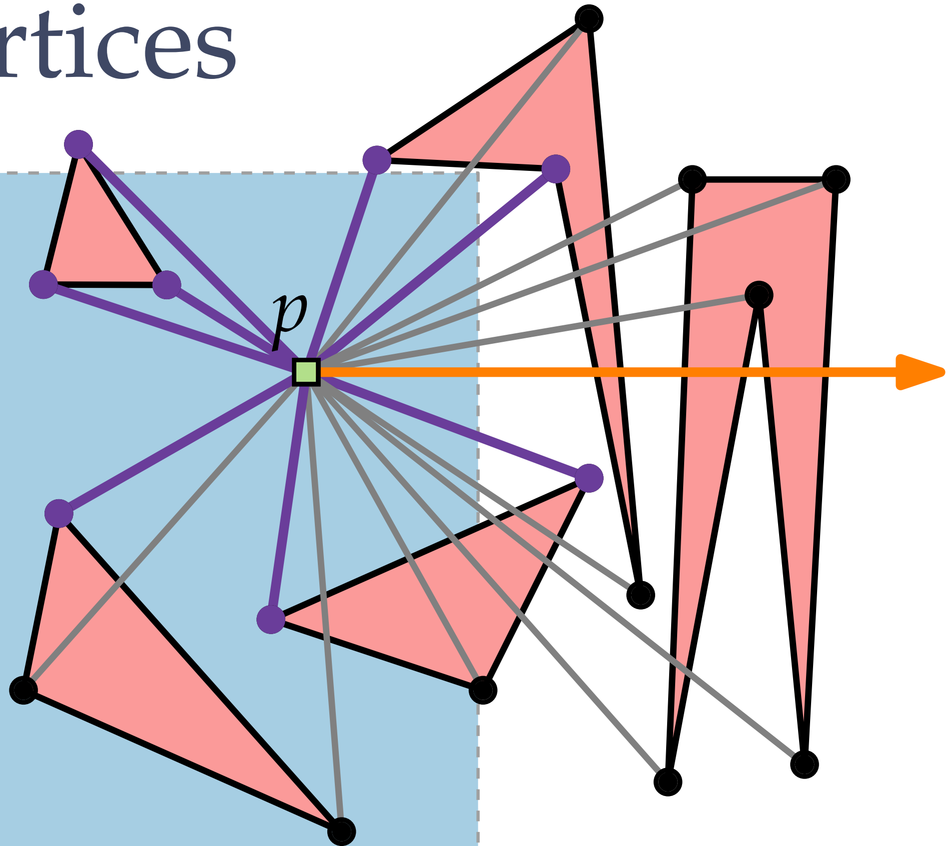
$r \leftarrow p + \mathbb{R}_{\geq 0}\binom{1}{0}$

$I \leftarrow \{e \in E(S) \mid e \cap r \neq \emptyset\}$



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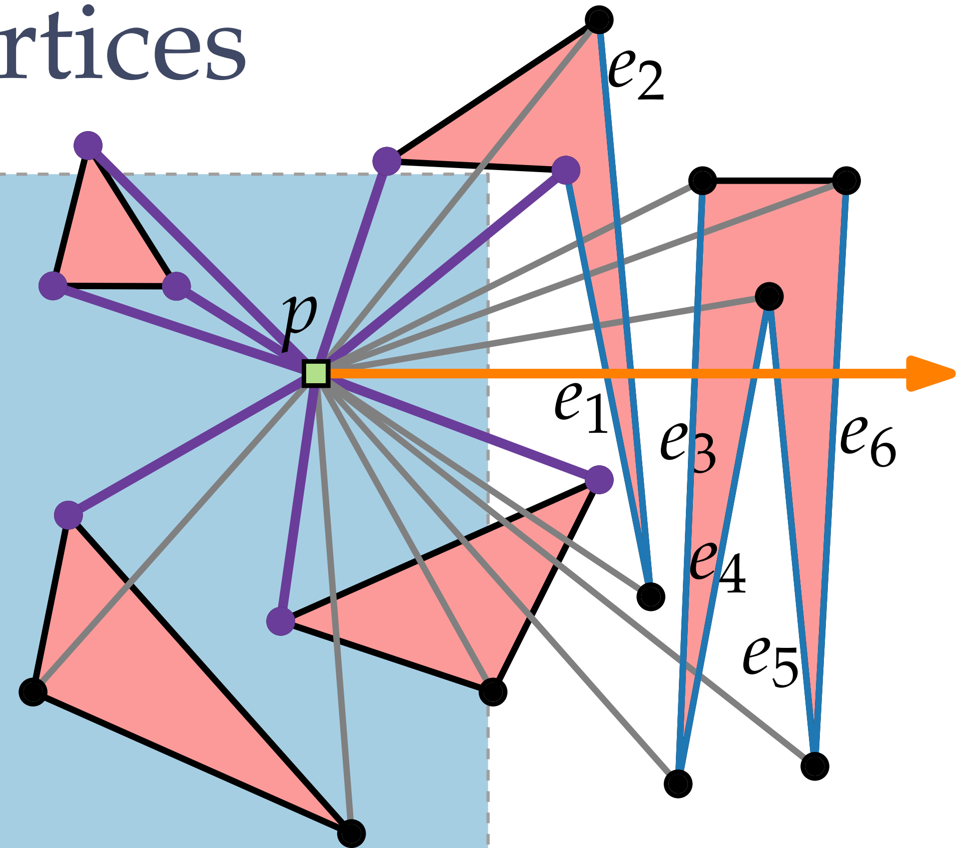

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$r \leftarrow p + \mathbb{R}_{\geq 0}\binom{1}{0}$

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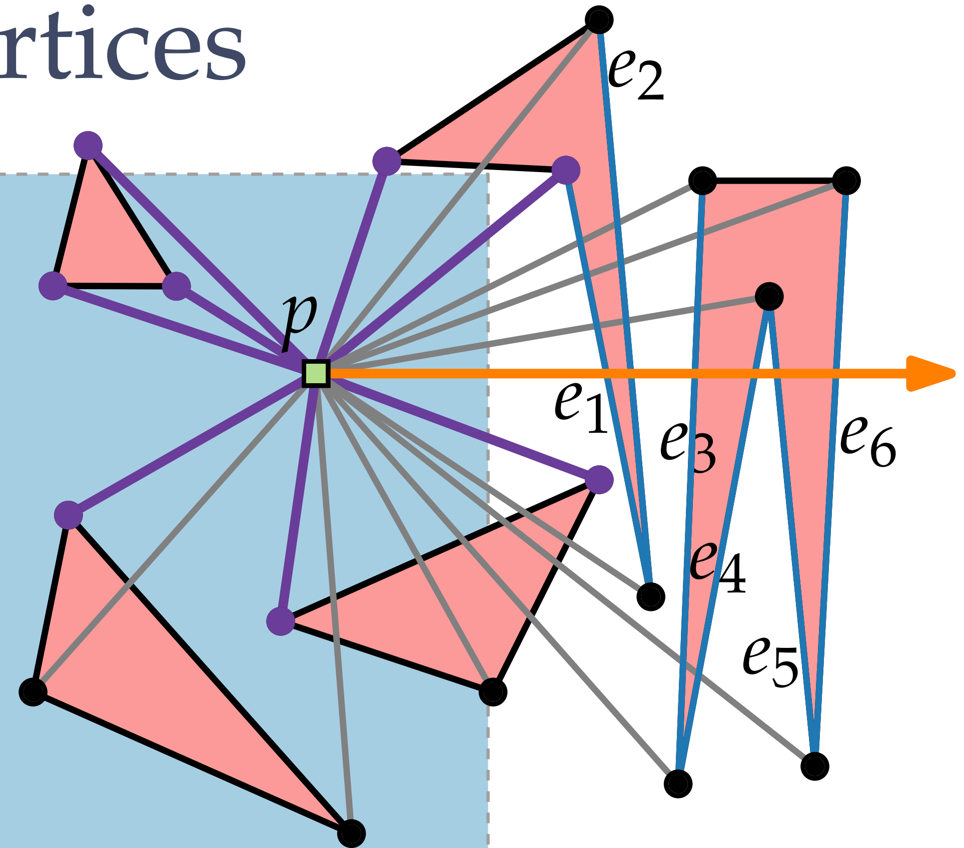
$\mathcal{T} \leftarrow \text{balancedBinaryTree}(I)$



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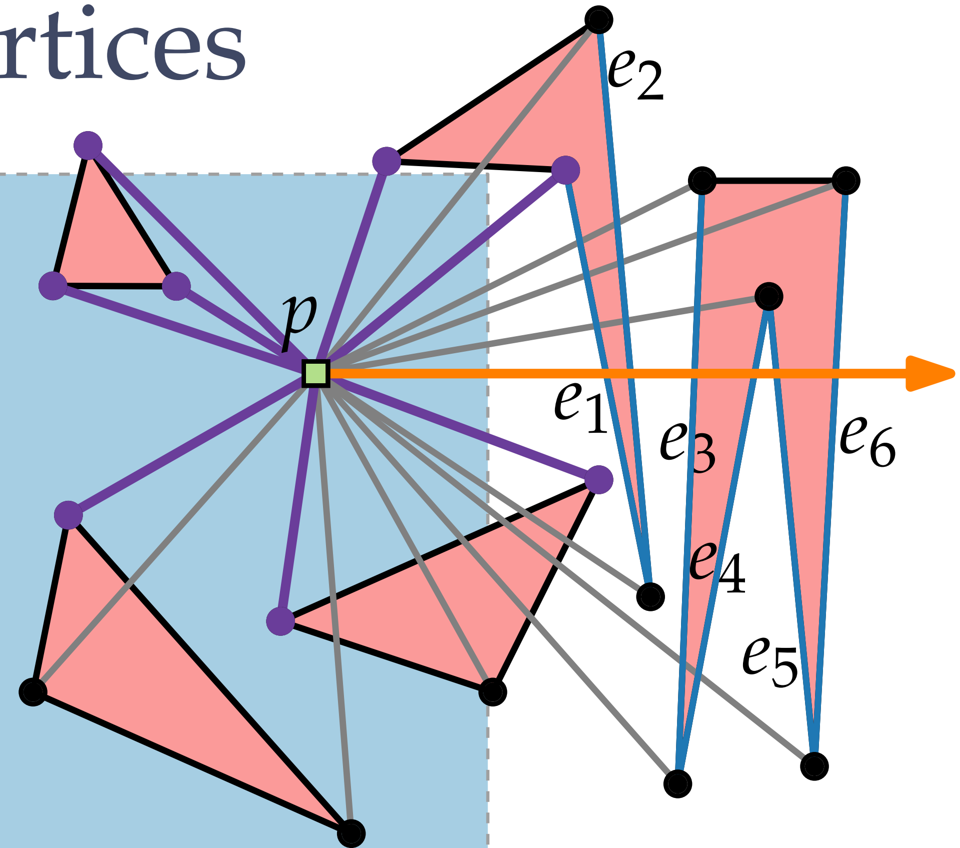

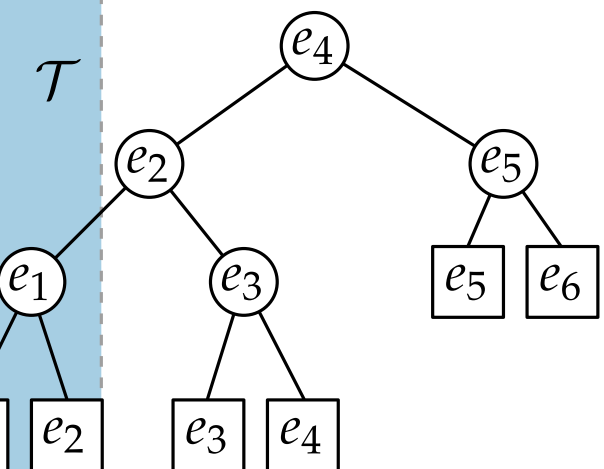

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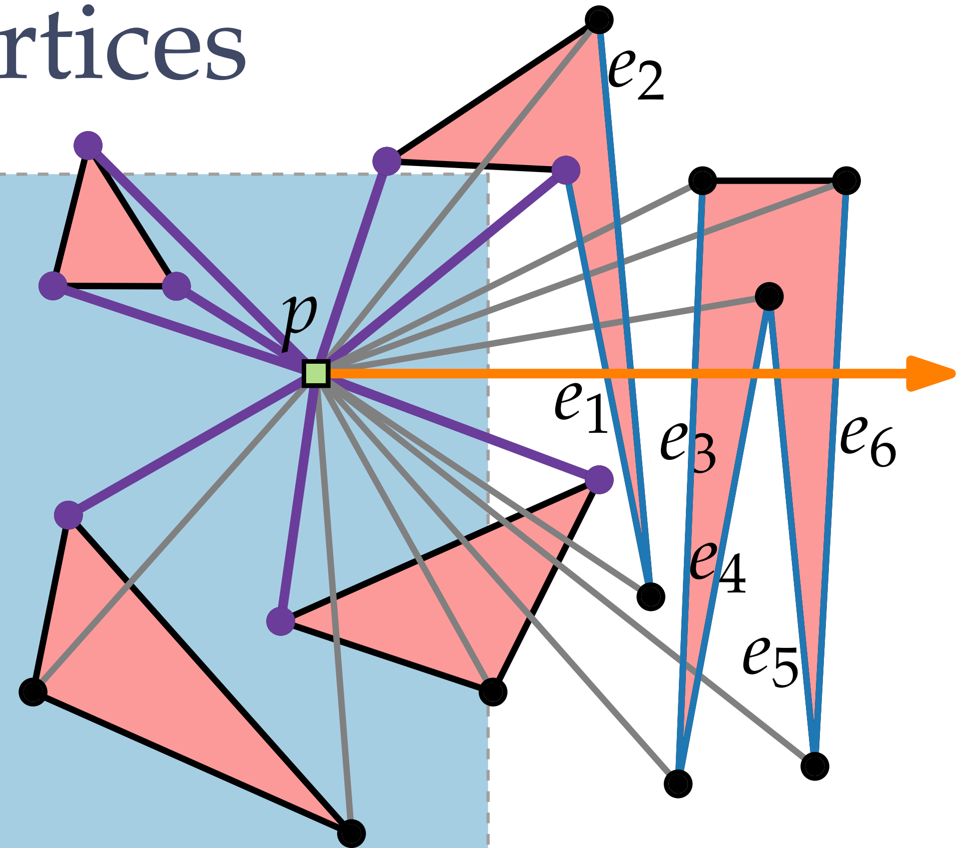
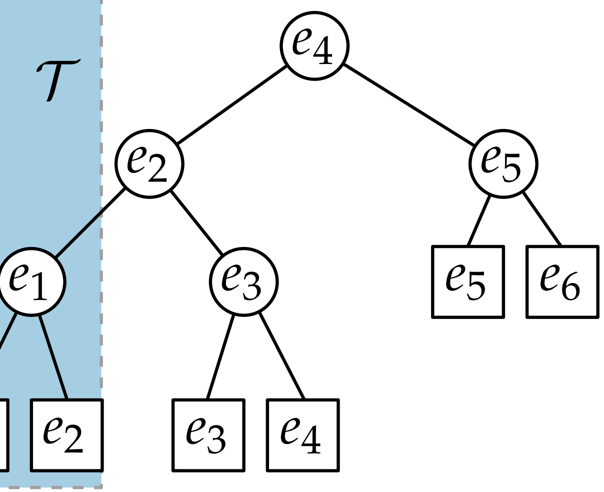
sort $V(S)$





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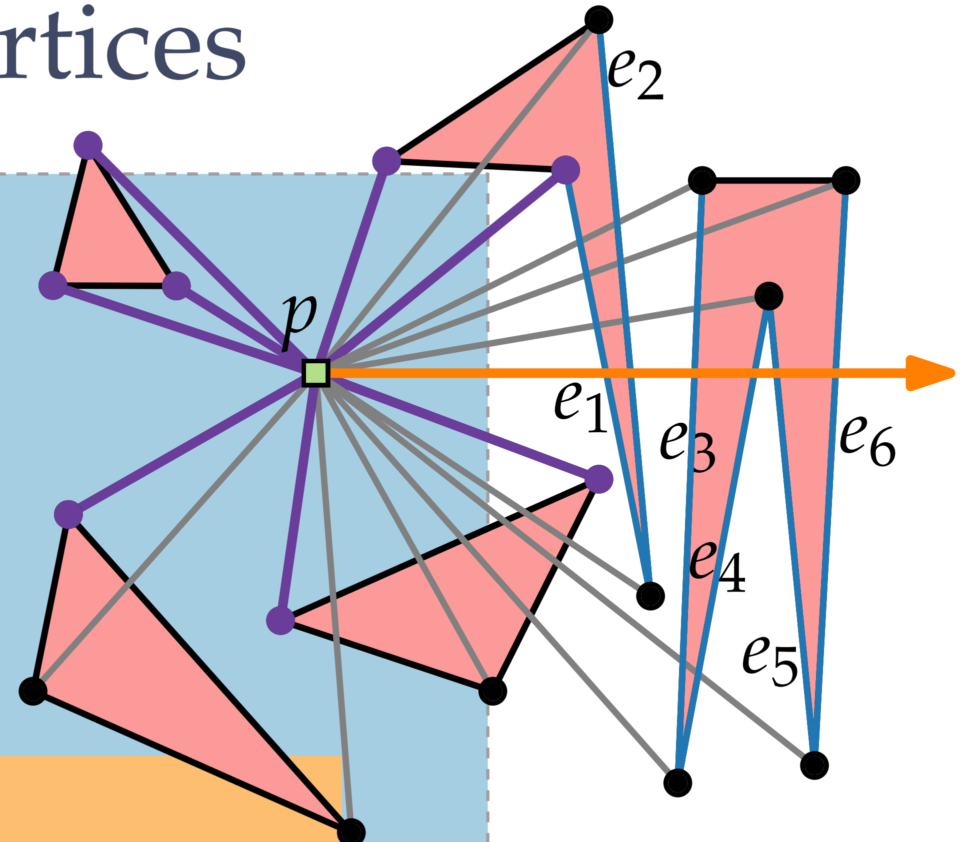
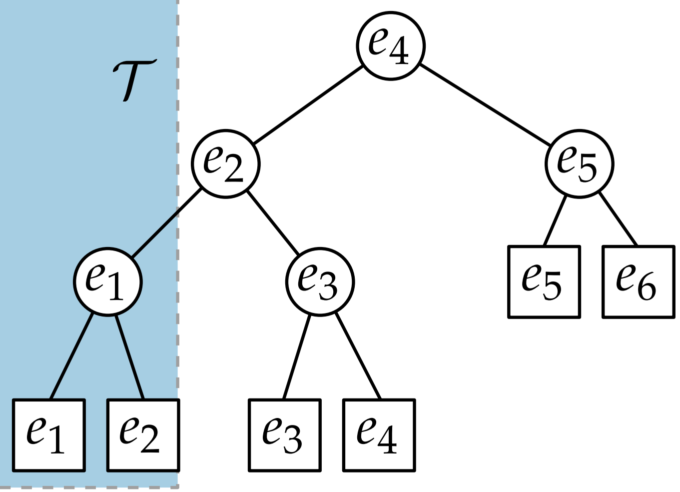
sort $V(S)$ $\quad v \prec v' :\Leftrightarrow$

$\angle v < \angle v'$ or
$(\angle v = \angle v'$ and $|pv| < |pv'|)$





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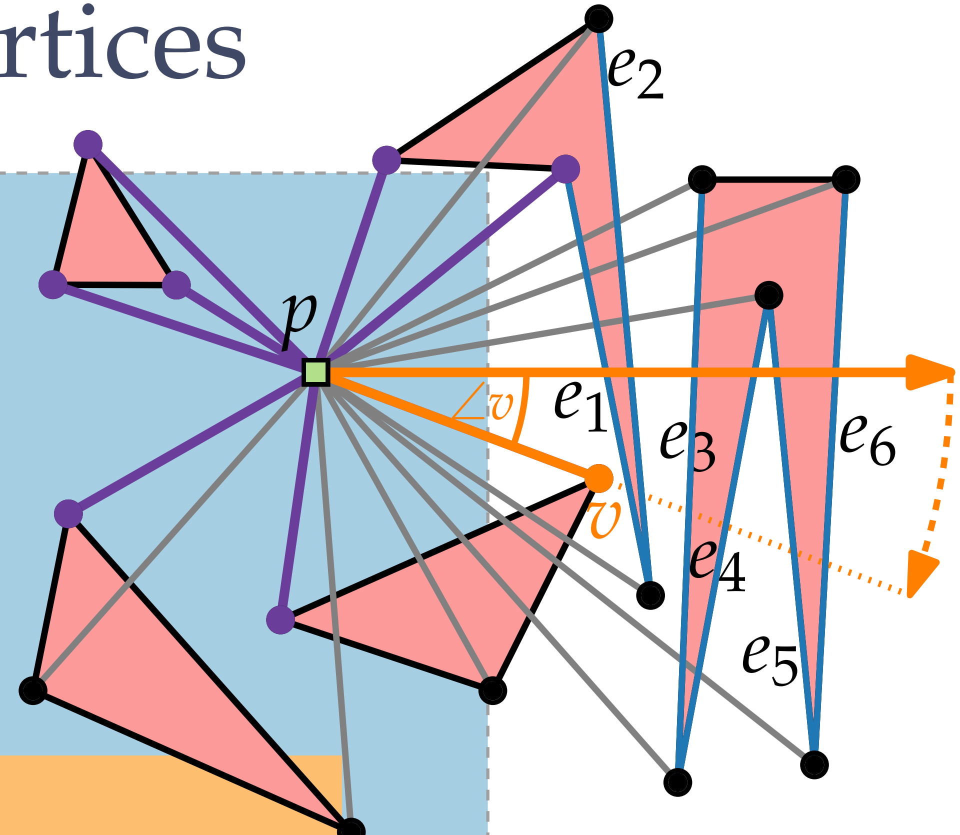
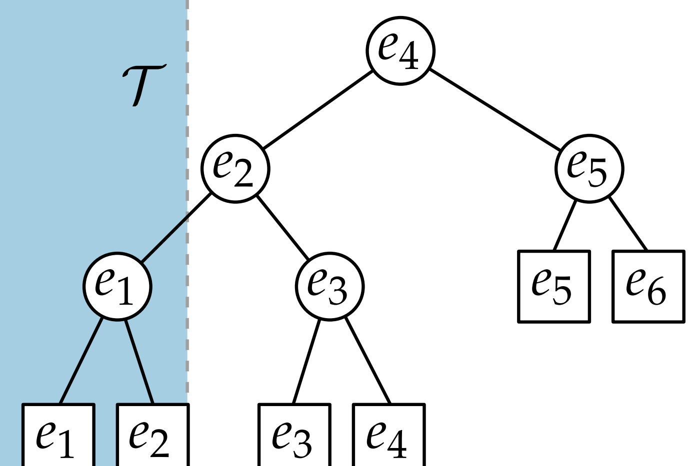
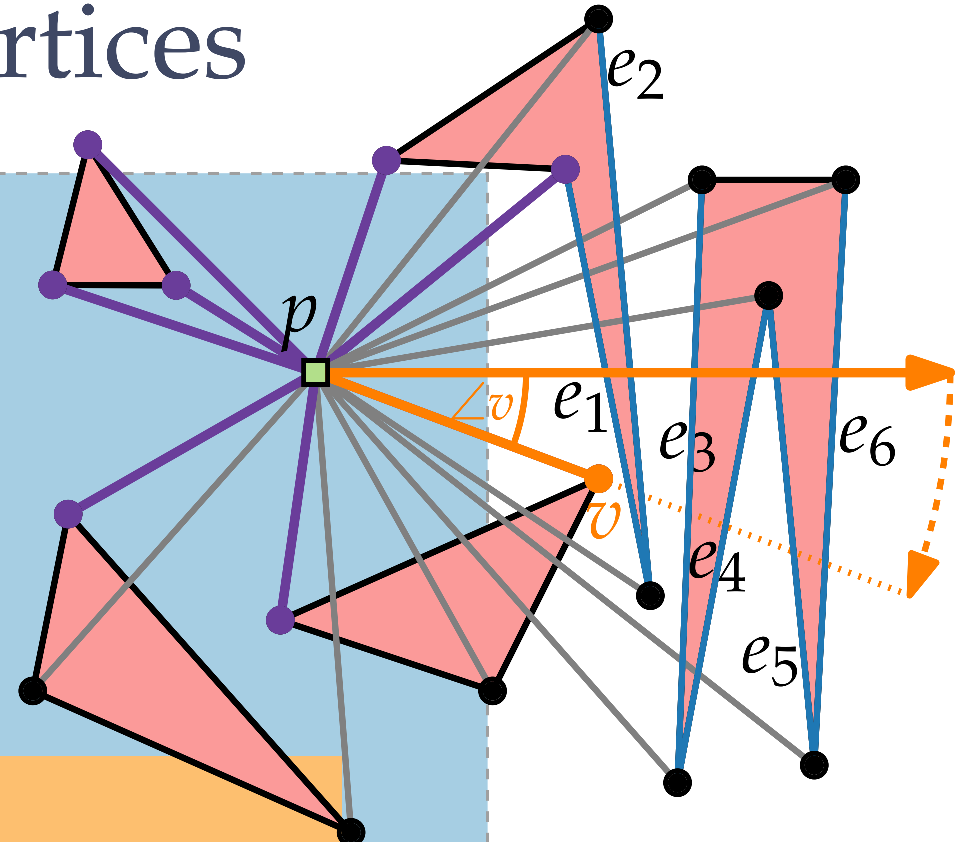




Computing Visible Vertices



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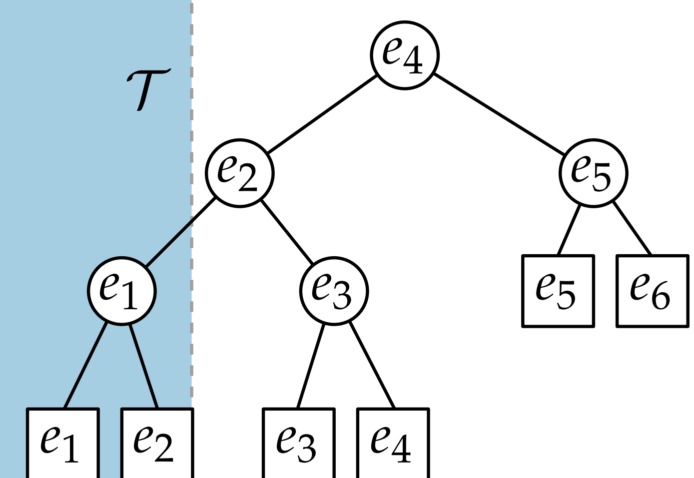
sort $V(S)$ $\quad v \prec v' :\Leftrightarrow$

$\angle v < \angle v'$ or
($\angle v = \angle v'$ and $|pv| < |pv'|$)

rotational plane sweep



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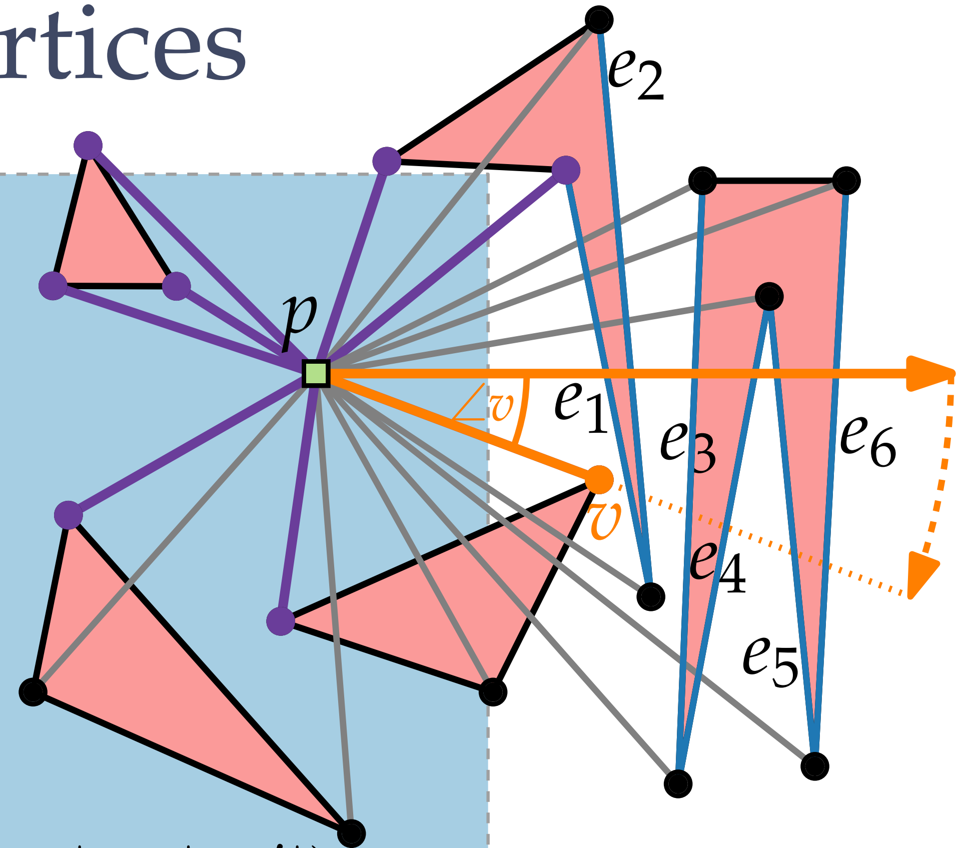

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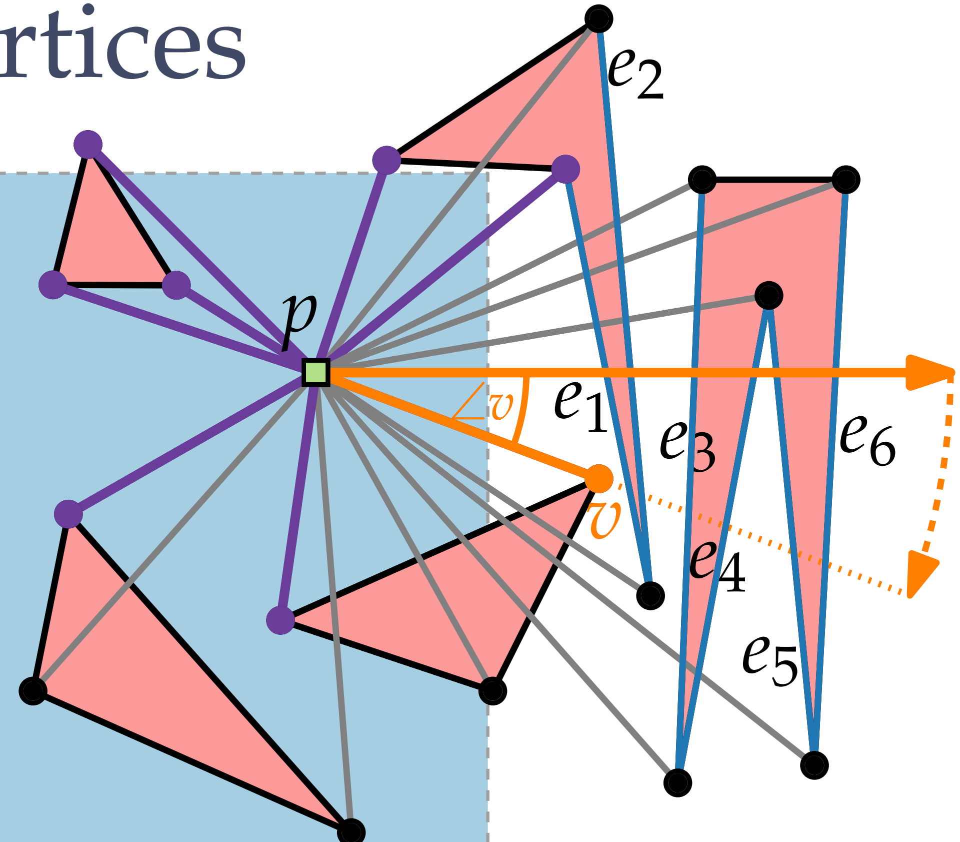

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$\angle v < \angle v'$ or
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$W \leftarrow \emptyset$

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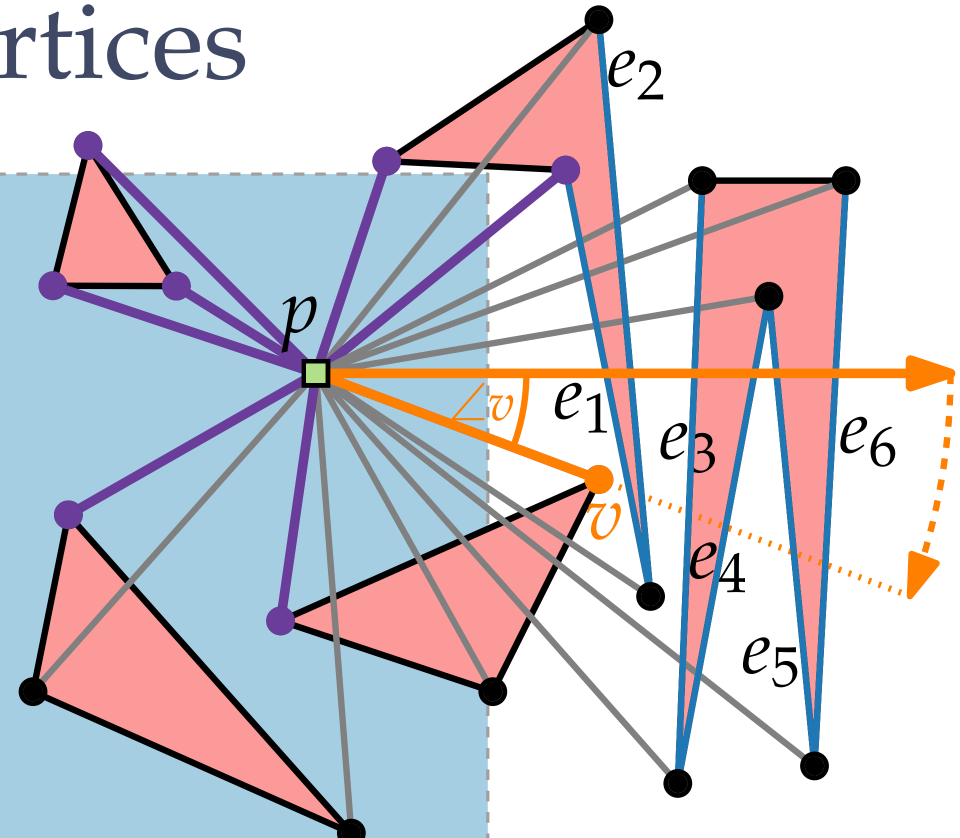
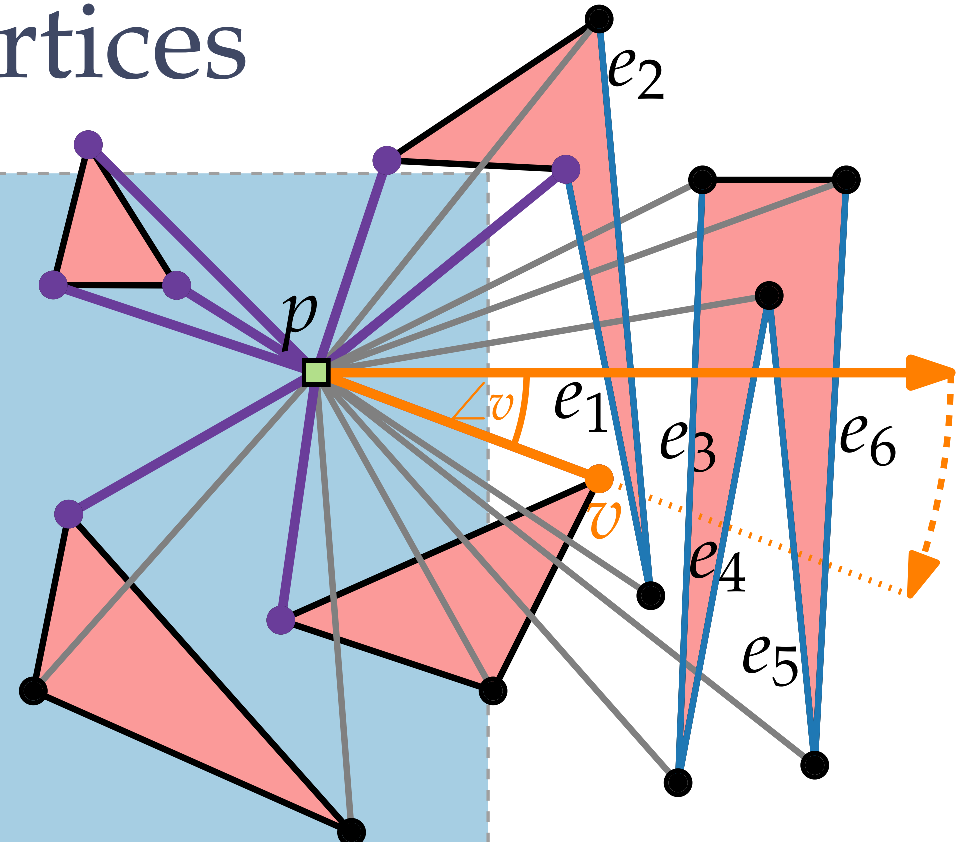
foreach $v \in V(S)$ **do**



Computing Visible Vertices



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foreach $v \in V(S)$ **do**

 if $\textsc{Visible}(v)$ **then**

Computing Visible Vertices

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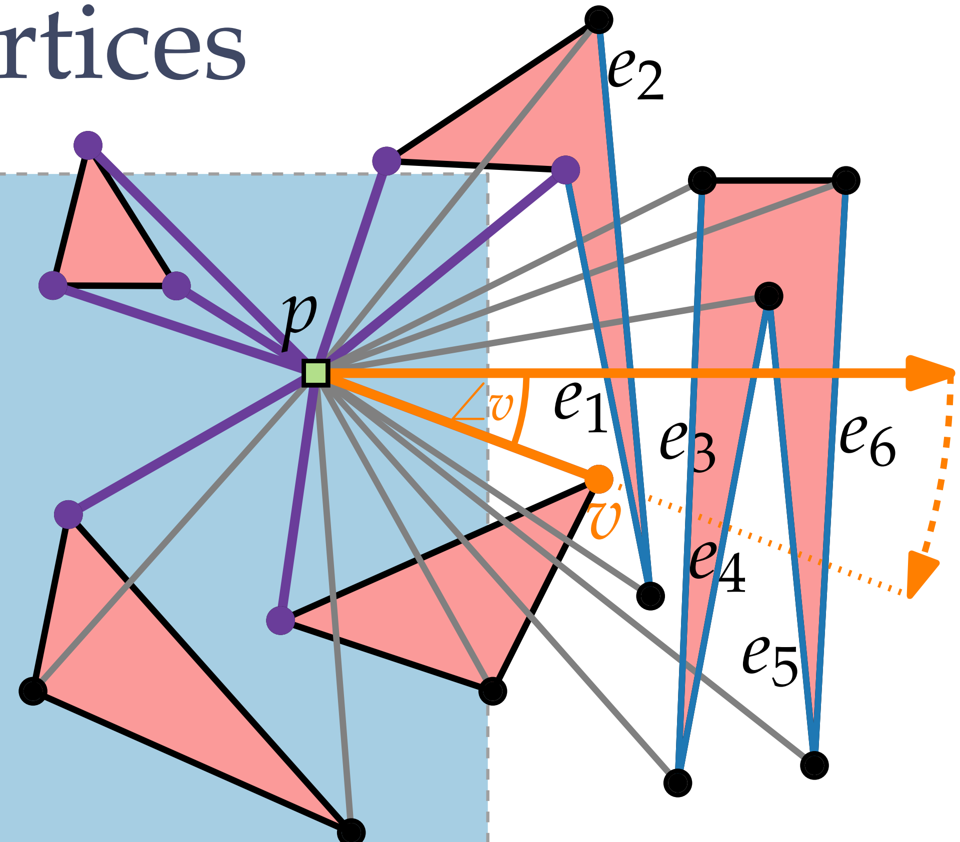
$W \leftarrow \emptyset$

foreach $v \in V(S)$ **do**
  if VISIBLE(v) **then**
    $W \leftarrow W \cup \{v\}$



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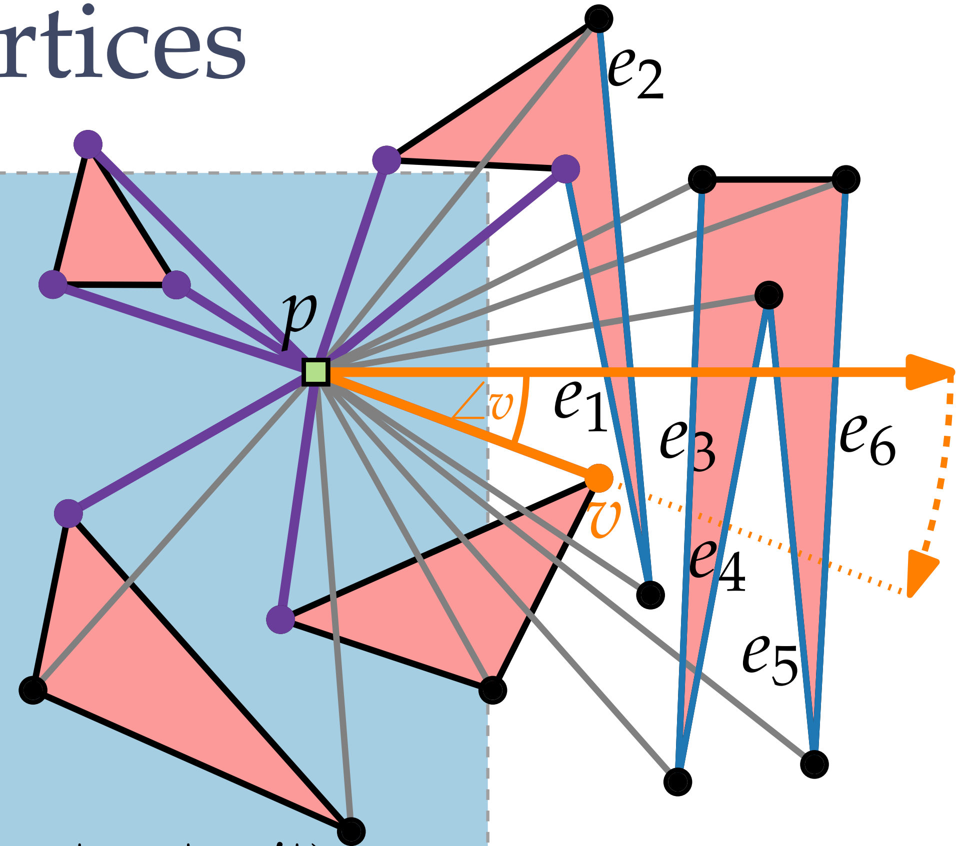

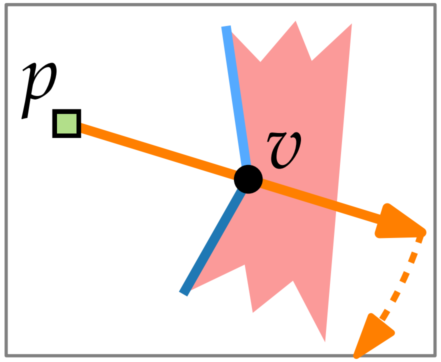

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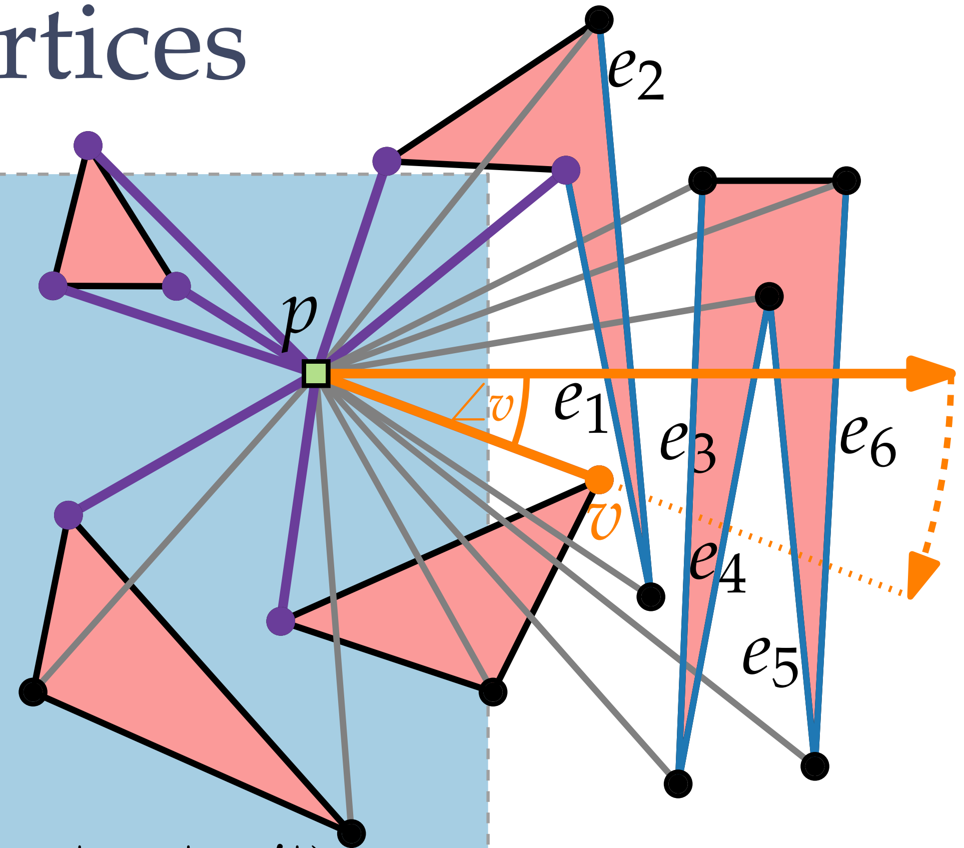
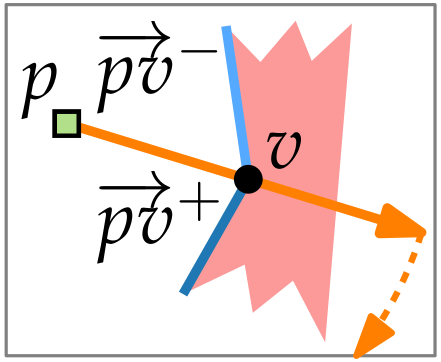
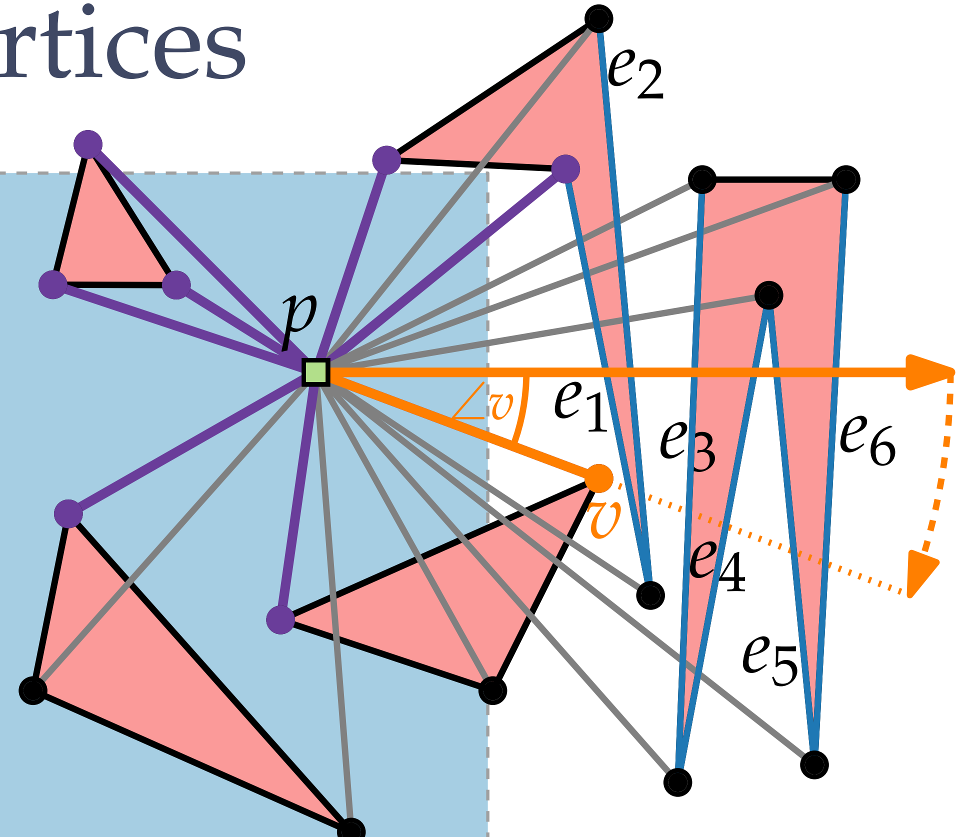




Computing Visible Vertices



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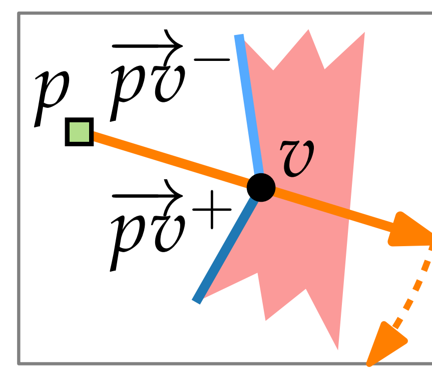
$W \leftarrow \emptyset$

foreach $v \in V(S)$ **do**

- **if** Visible(v) **then**
  - $W \leftarrow W \cup \{v\}$
- insert into $\mathcal{T}$ edges incident to v in $\overrightarrow{pv}^{+}$



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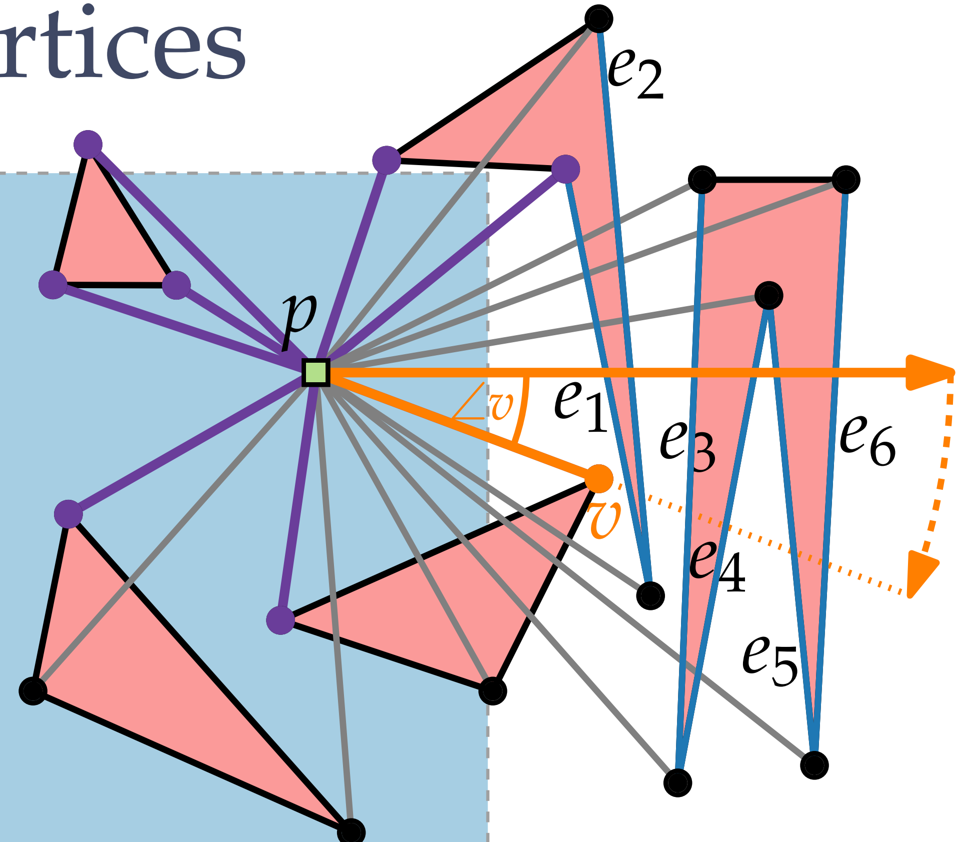

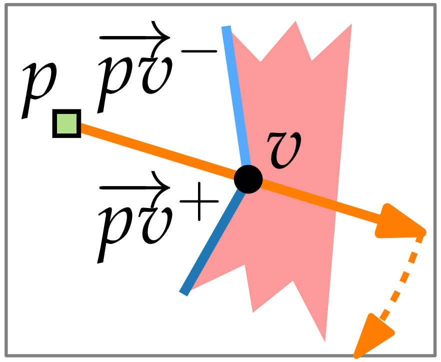

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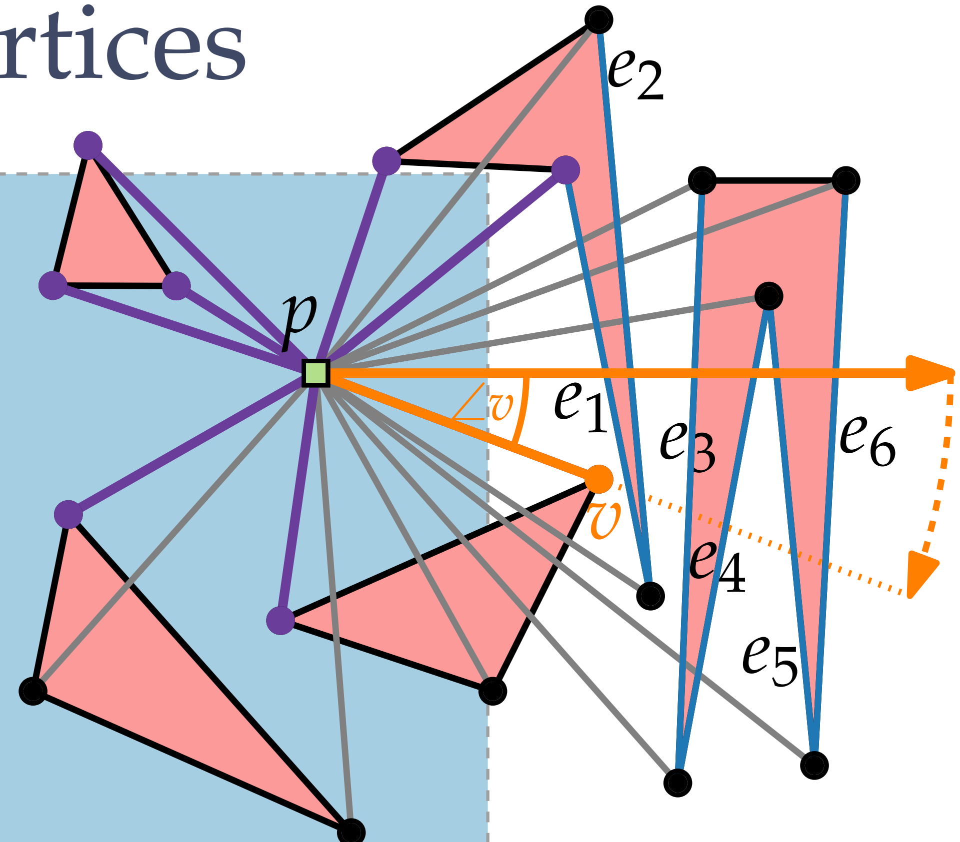
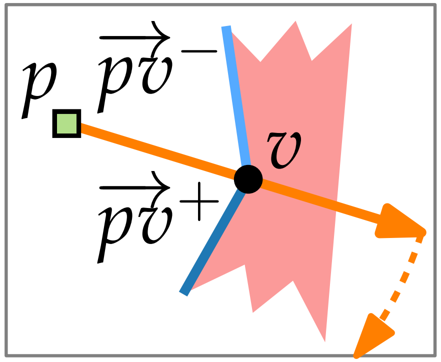
  delete from $\mathcal{T}$ edges incident to v in $\overrightarrow{pv}^{-}$

return W





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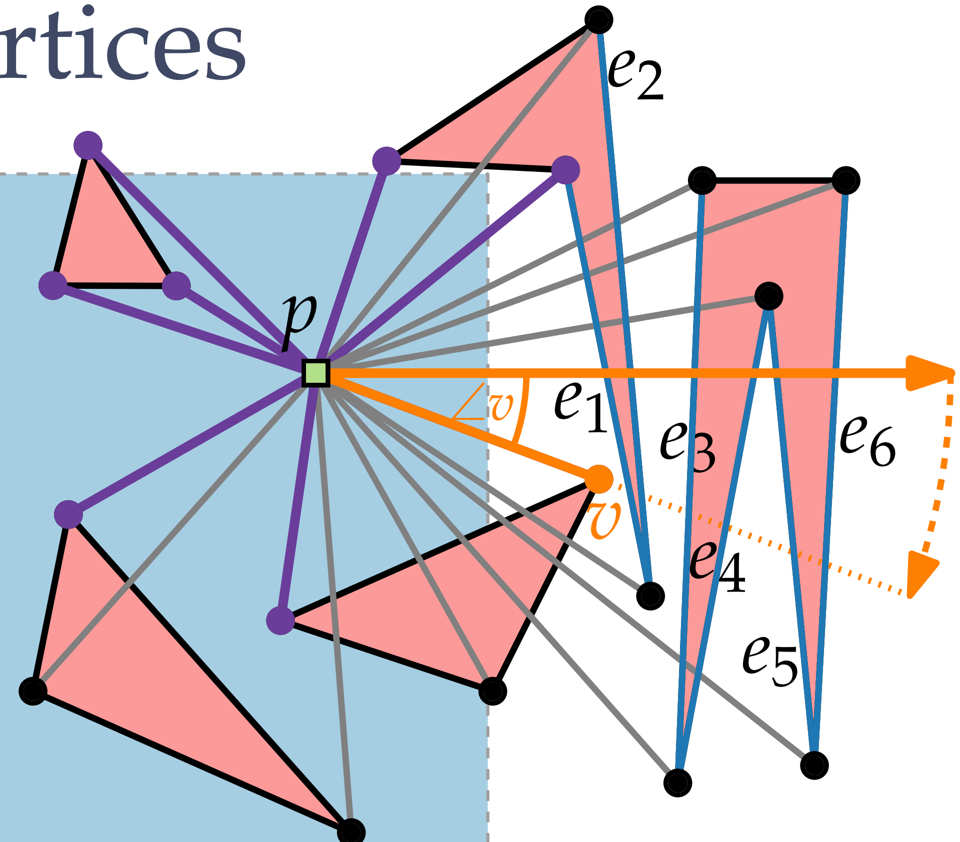

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$W \leftarrow \emptyset$

foreach $v \in V(S)$ **do**

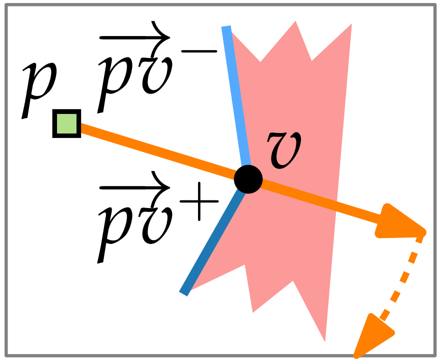
  if $\textsc{Visible}(v)$ **then**

    $W \leftarrow W \cup \{v\}$

  insert into $\mathcal{T}$ edges incident to v in $\overrightarrow{pv}^+$

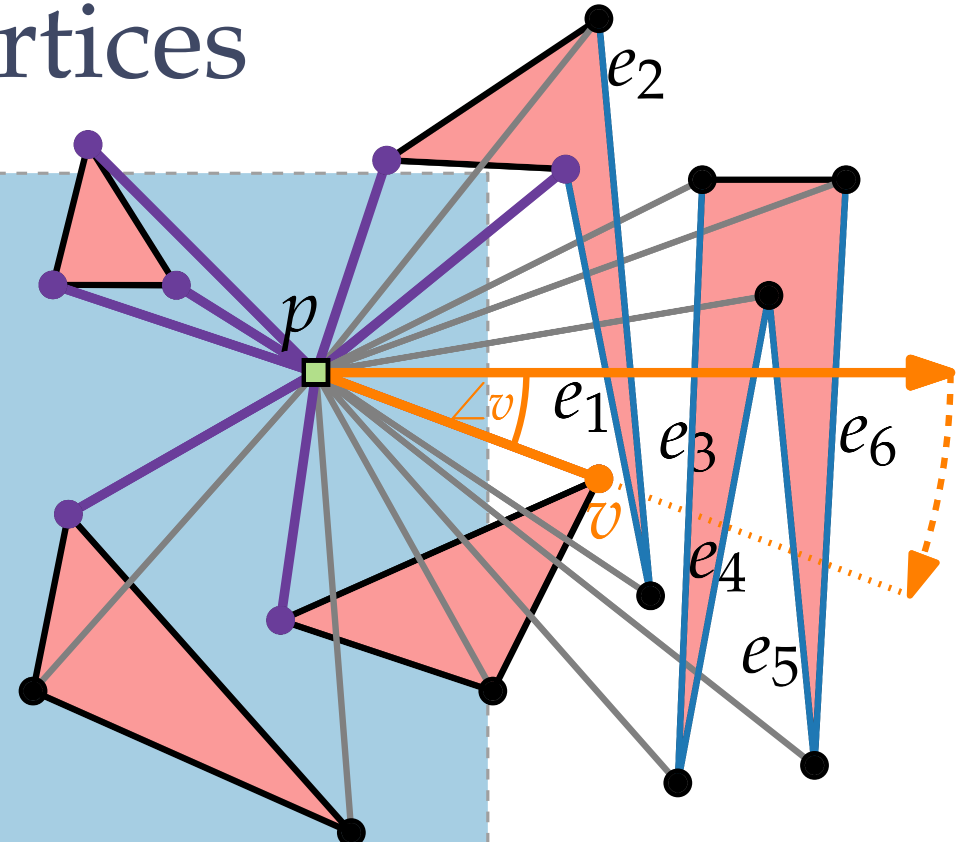
  delete from $\mathcal{T}$ edges incident to v in $\overrightarrow{pv}^-$

return W



$O(n \log n)$

Computing Visible Vertices



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$W \leftarrow \emptyset$

foreach $v \in V(S)$ **do**

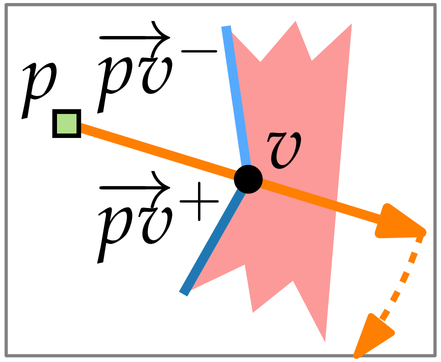
  if $\textsc{Visible}(v)$ **then** ?

    $W \leftarrow W \cup \{v\}$

  insert into $\mathcal{T}$ edges incident to v in $\overrightarrow{pv}^{+}$

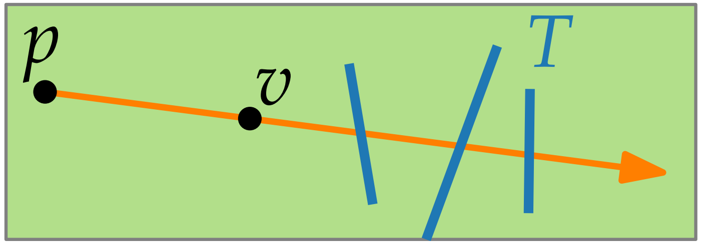
  delete from $\mathcal{T}$ edges incident to v in $\overrightarrow{pv}^{-}$

return W

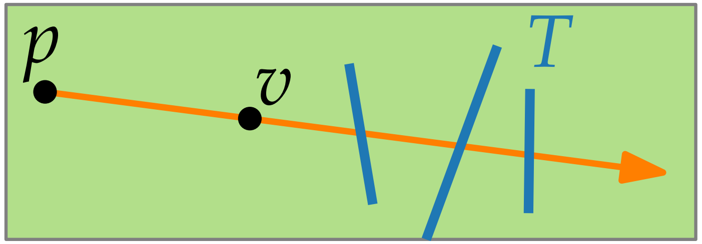


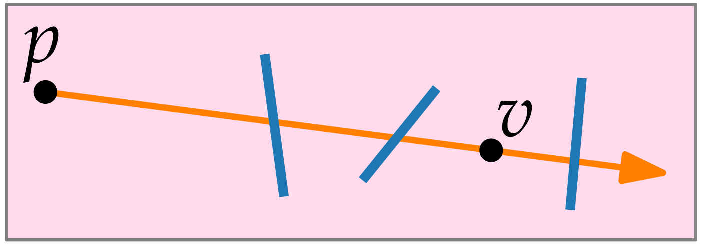
$O(n \log n)$

Cases

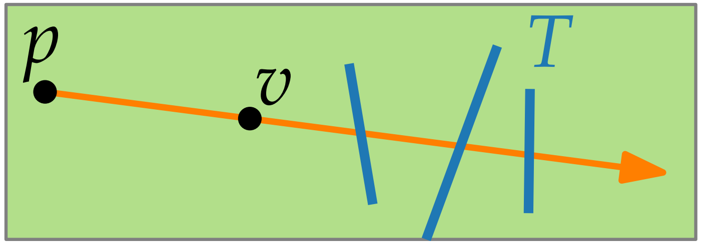


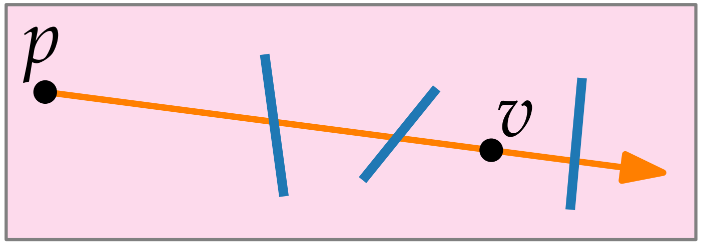
Cases

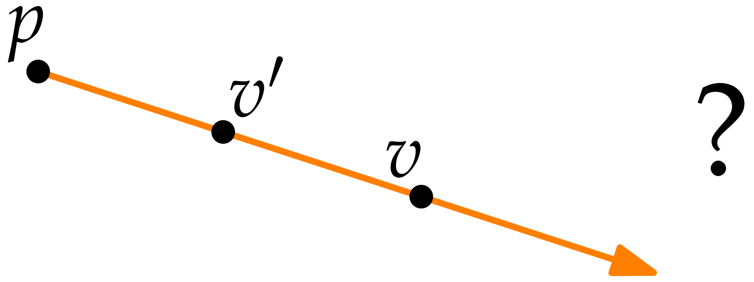




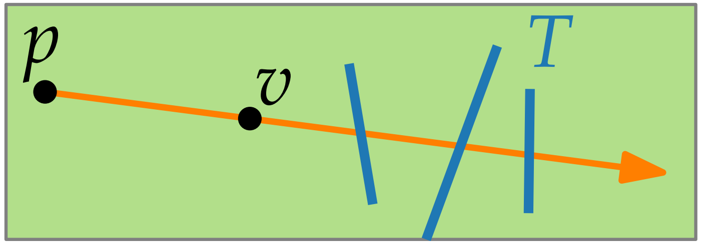
Cases

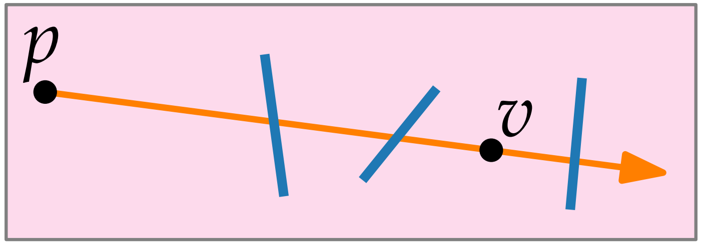






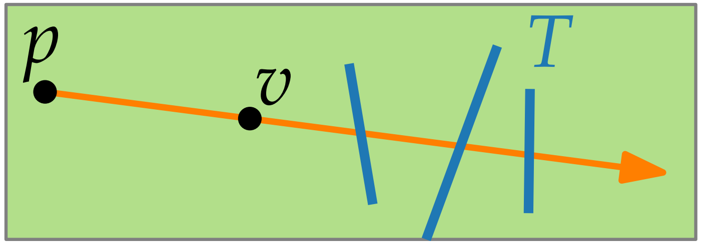
Cases

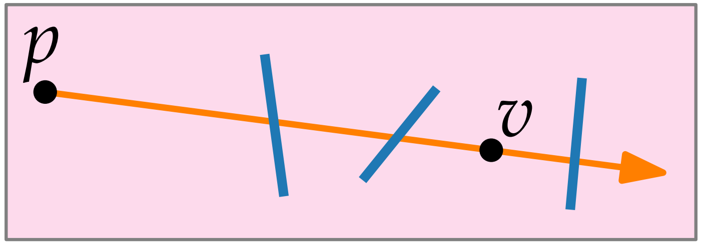




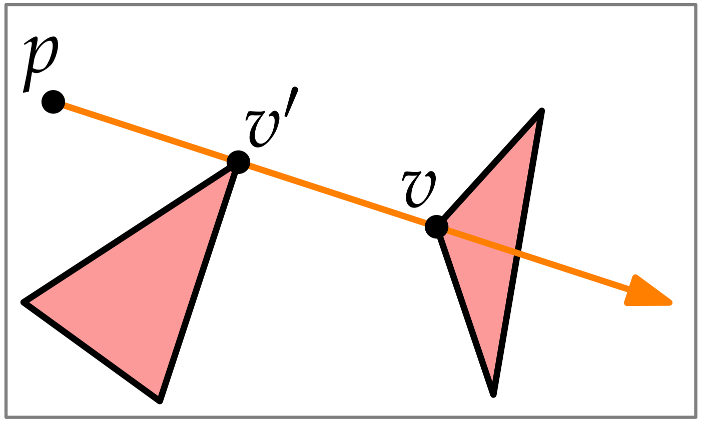
Let v' be the immediate predecessor of v according to $\prec$.

Cases

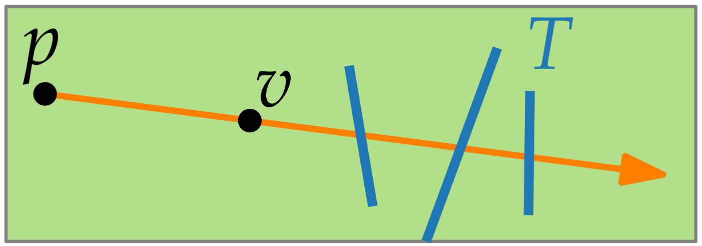


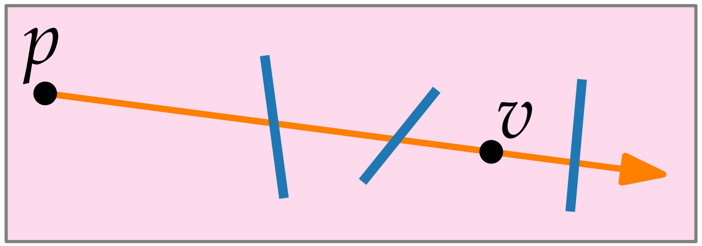


Let v' be the immediate predecessor of v according to $\prec$.



Cases





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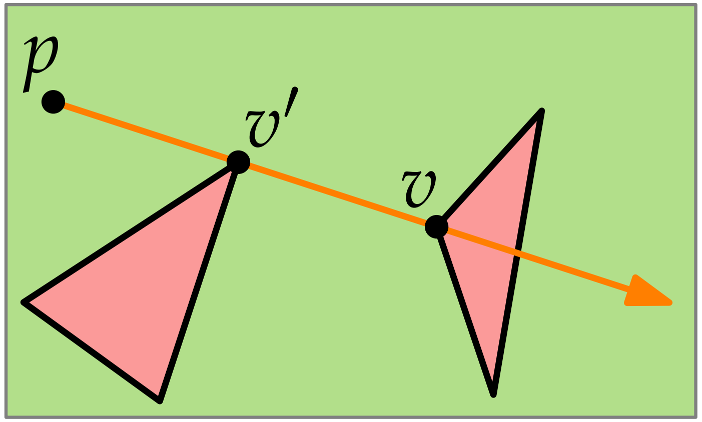
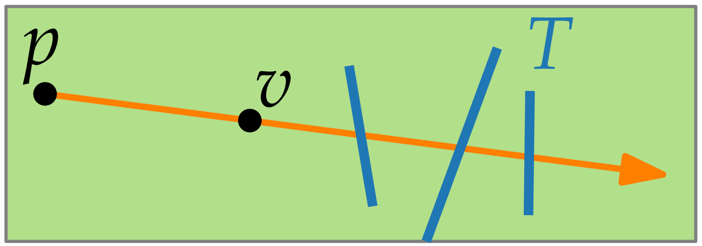
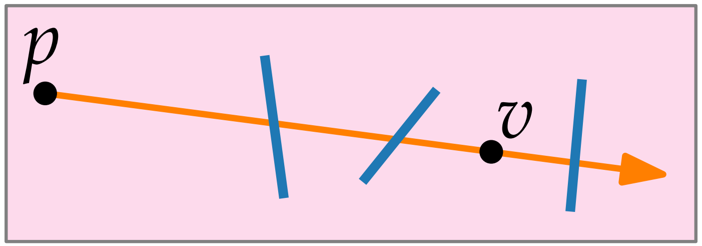
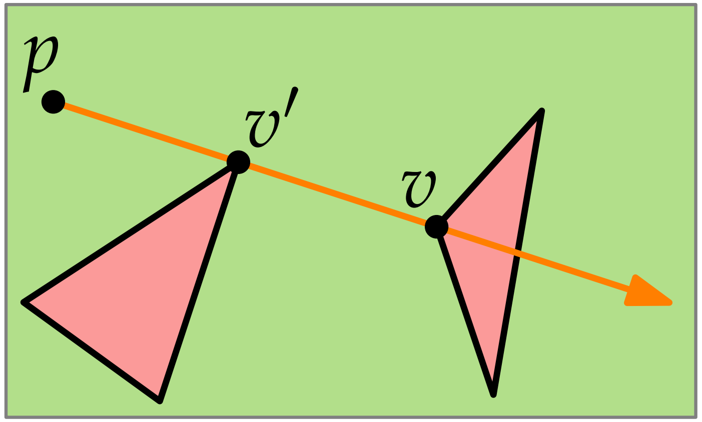


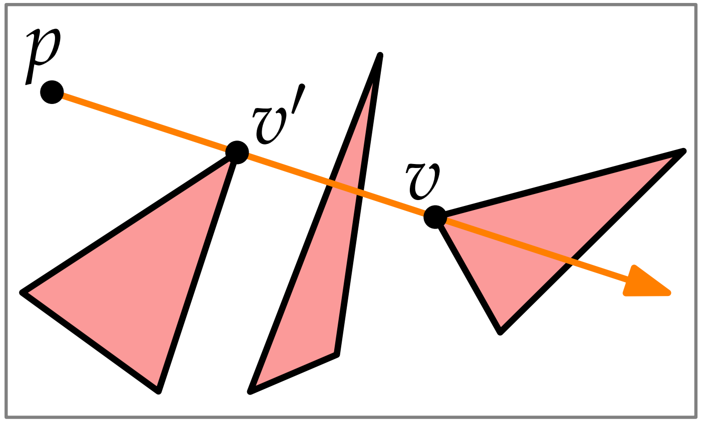
Cases



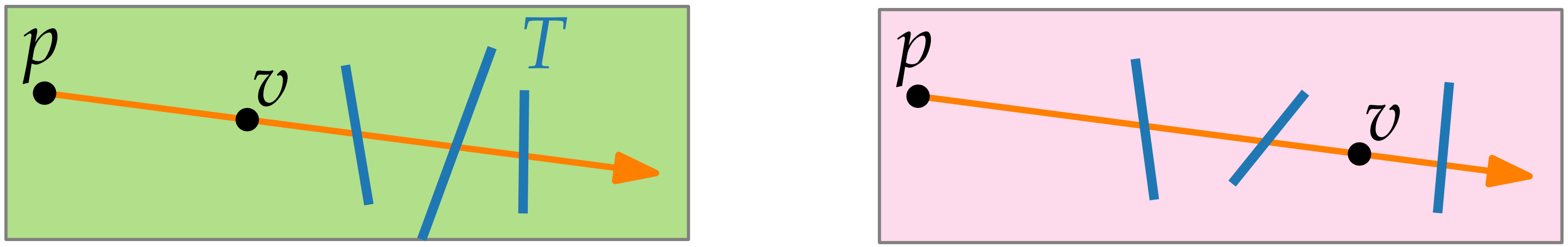


Let v' be the immediate predecessor of v according to $\prec$.

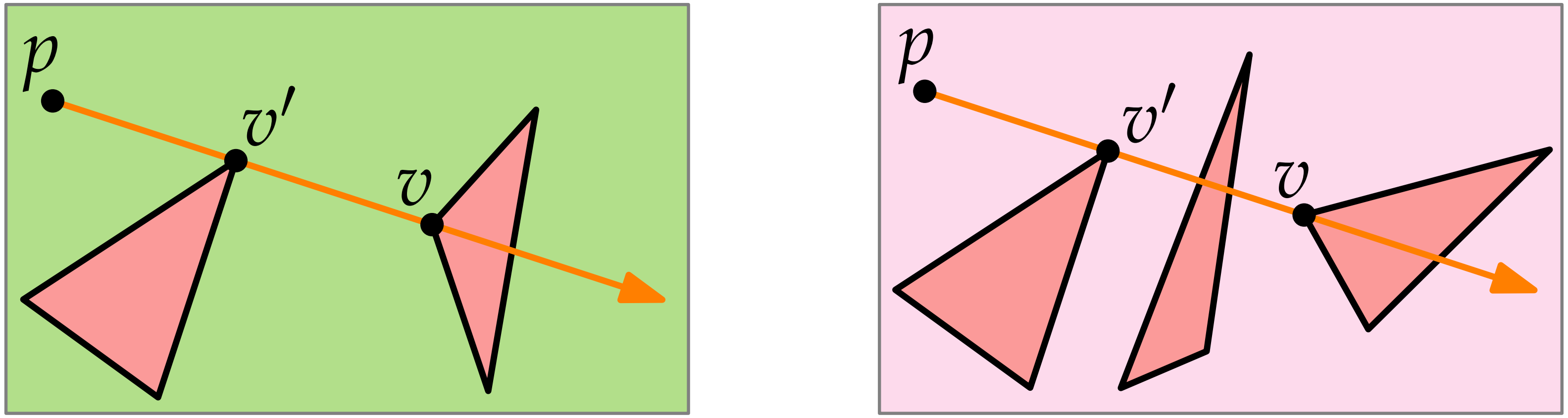




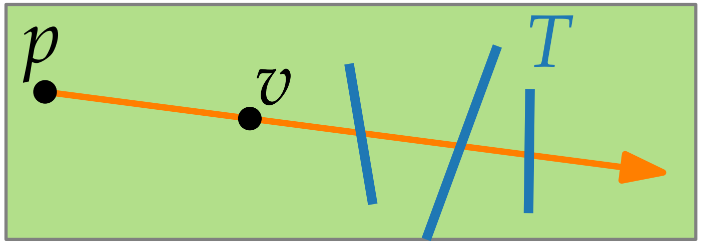
Cases

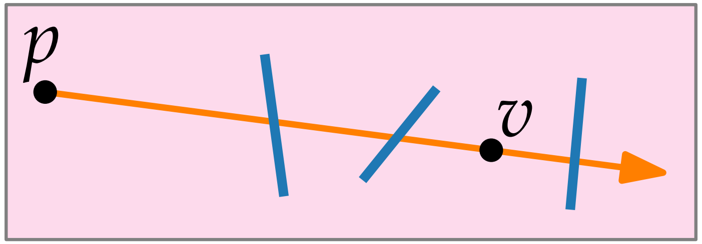


Let v' be the immediate predecessor of v according to $\prec$.

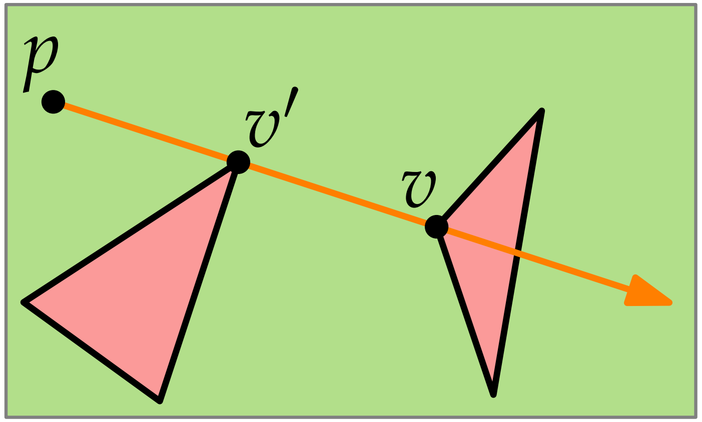


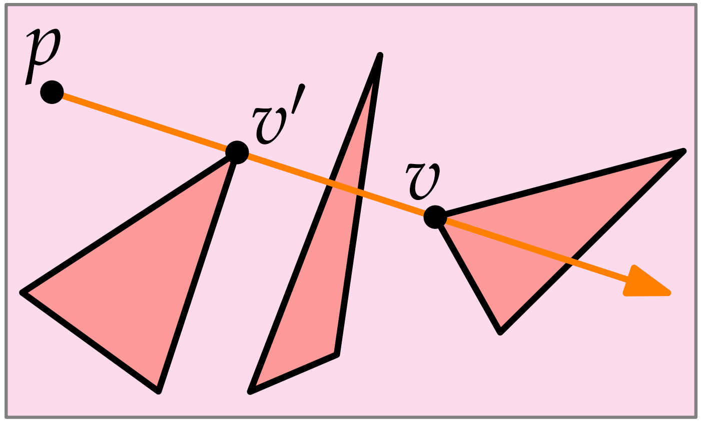
Cases

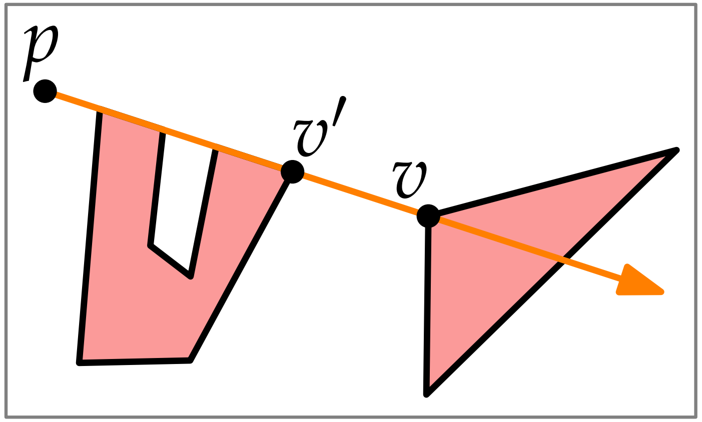




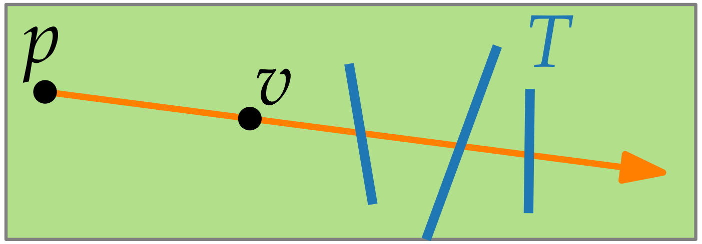
Let v' be the immediate predecessor of v according to $\prec$.

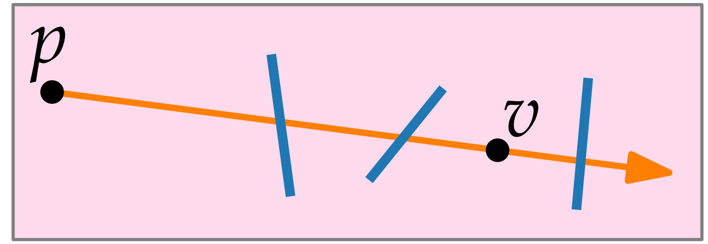






Cases





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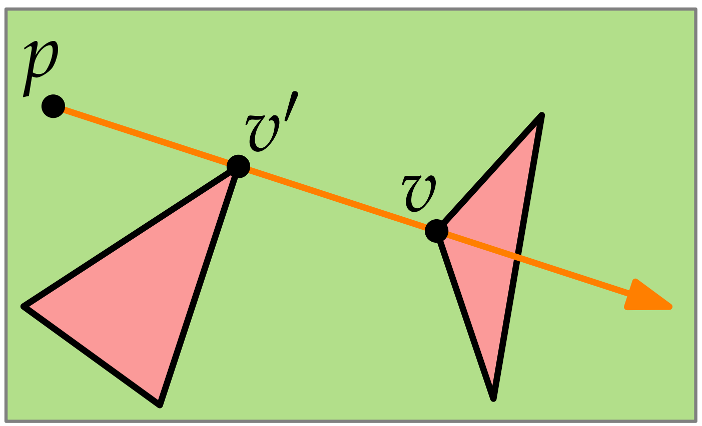
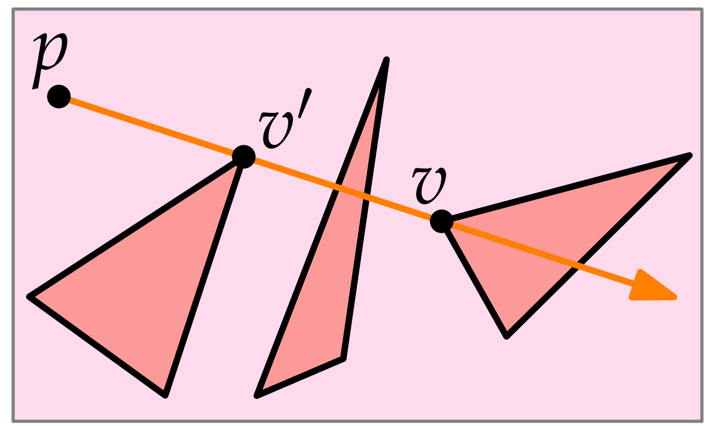
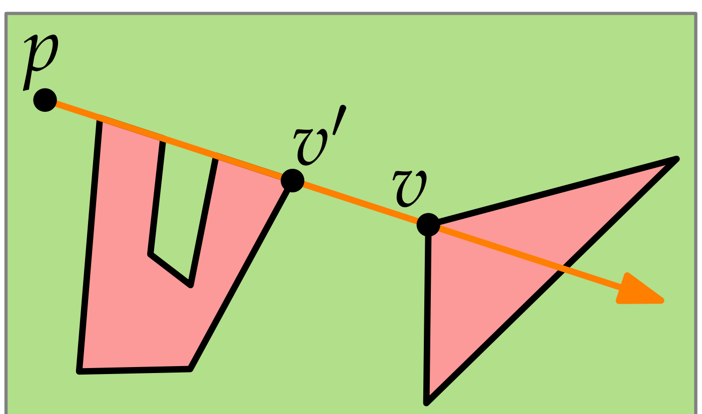
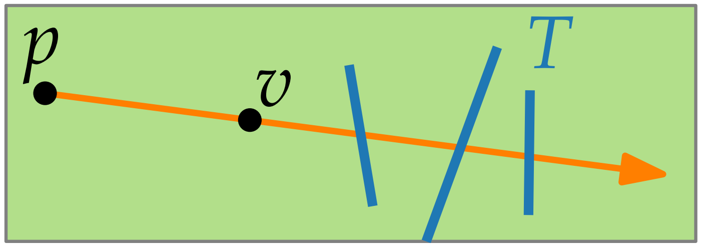
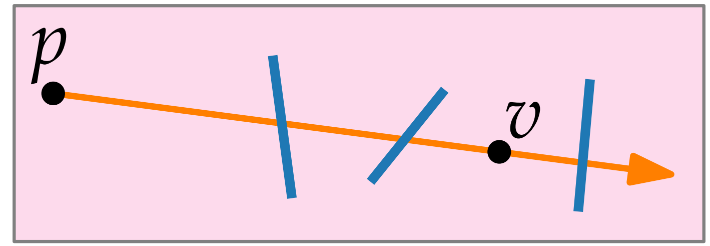




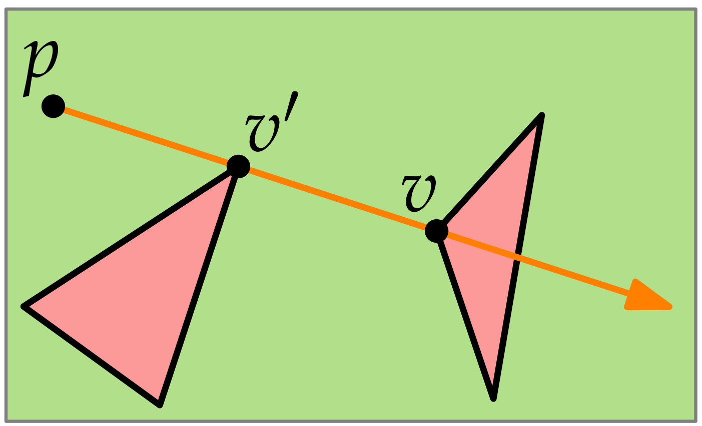


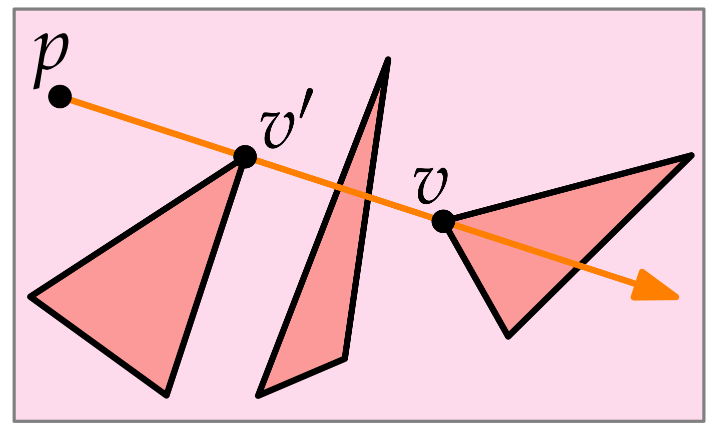
Cases

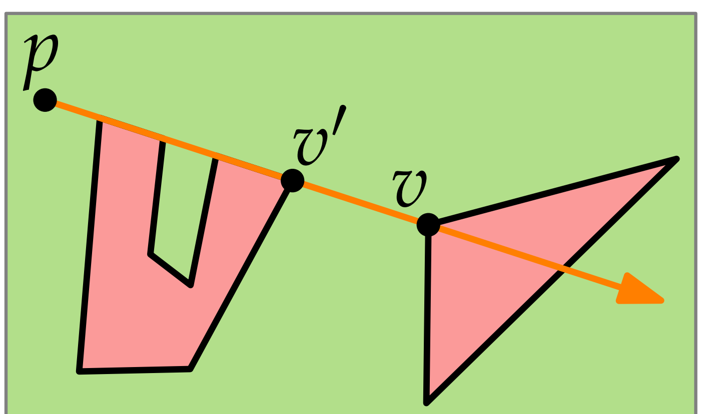


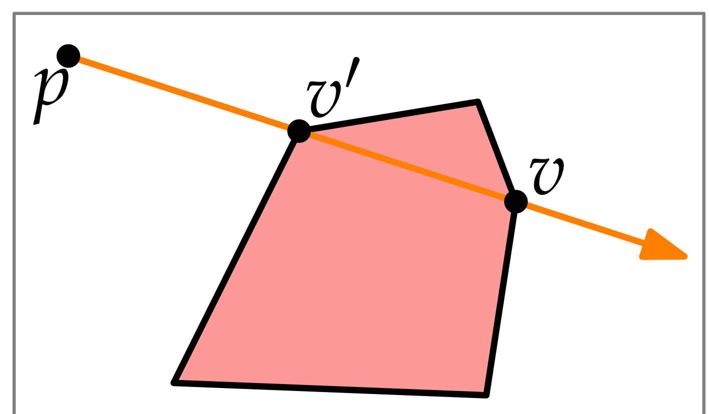


Let v' be the immediate predecessor of v according to $\prec$.

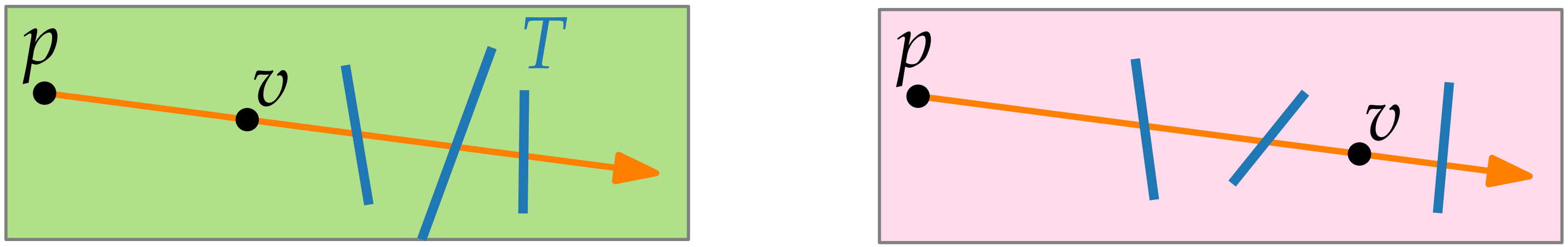




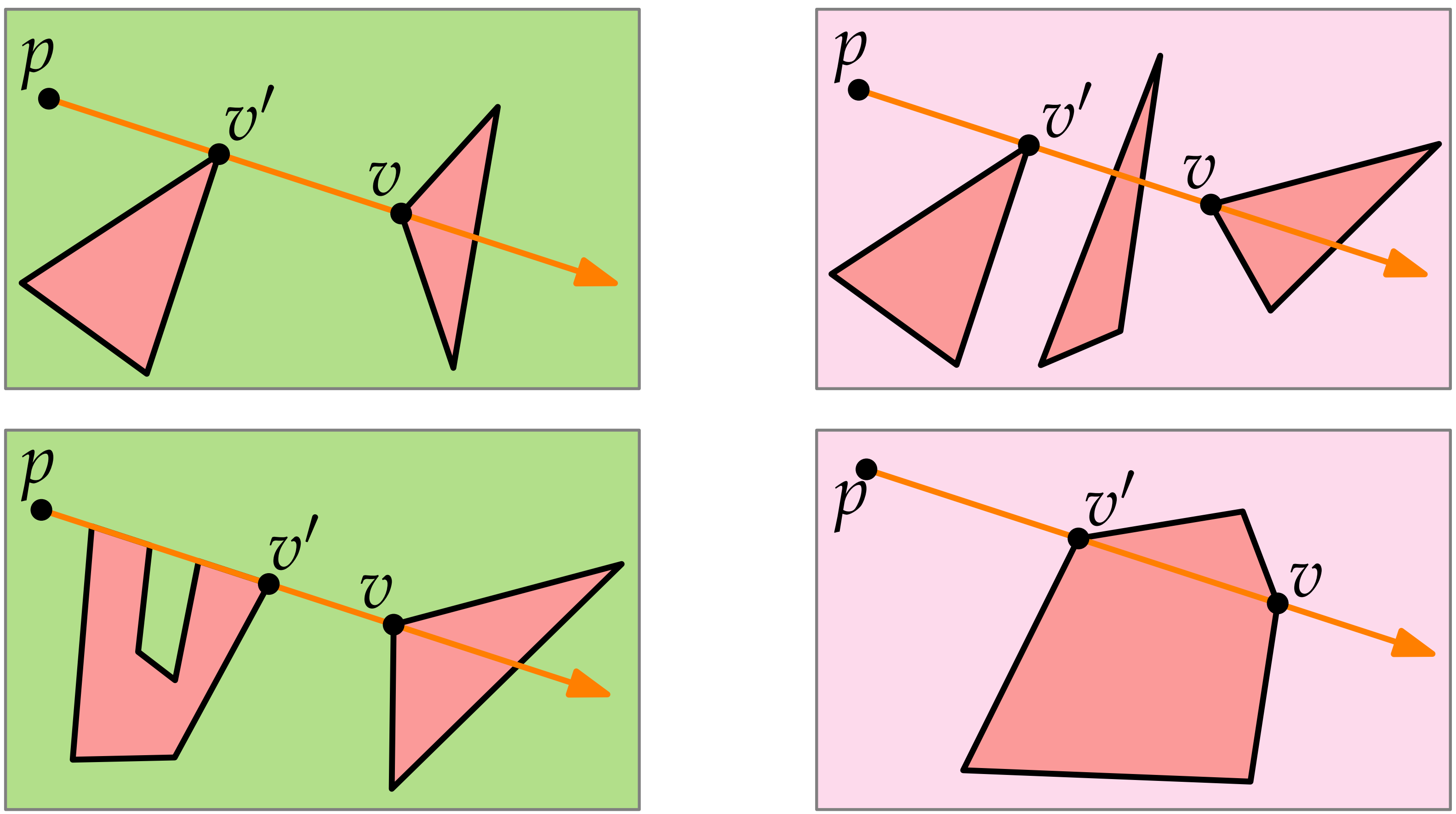




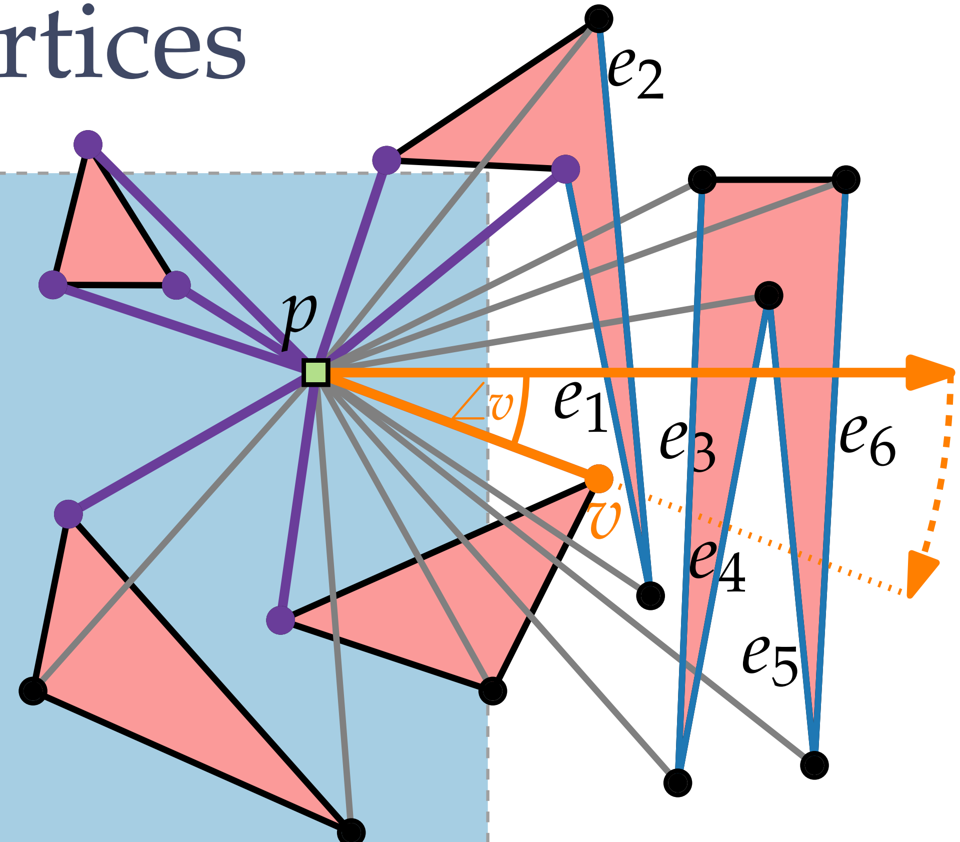
Cases



Let v' be the immediate predecessor of v according to $\prec$.



Computing Visible Vertices



VisibleVertices(p, S)

$r \leftarrow p + \mathbb{R}_{\geq 0}\binom{1}{0}$

$I \leftarrow \{e \in E(S) \mid e \cap r \neq \emptyset\}$

$\mathcal{T} \leftarrow \text{balancedBinaryTree}(I)$

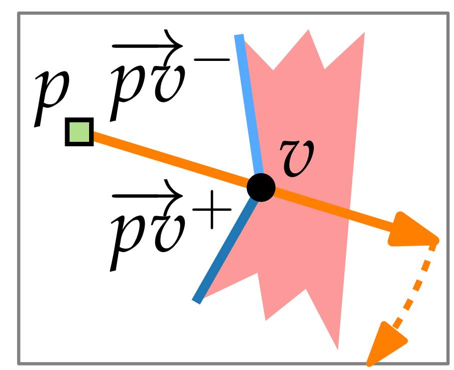
sort $V(S)$ $\quad$ $v \prec v' :\Leftrightarrow$ $\angle v < \angle v'$ or $(\angle v = \angle v'$ and $|pv| < |pv'|)$

$W \leftarrow \emptyset$

foreach $v \in V(S)$ **do**

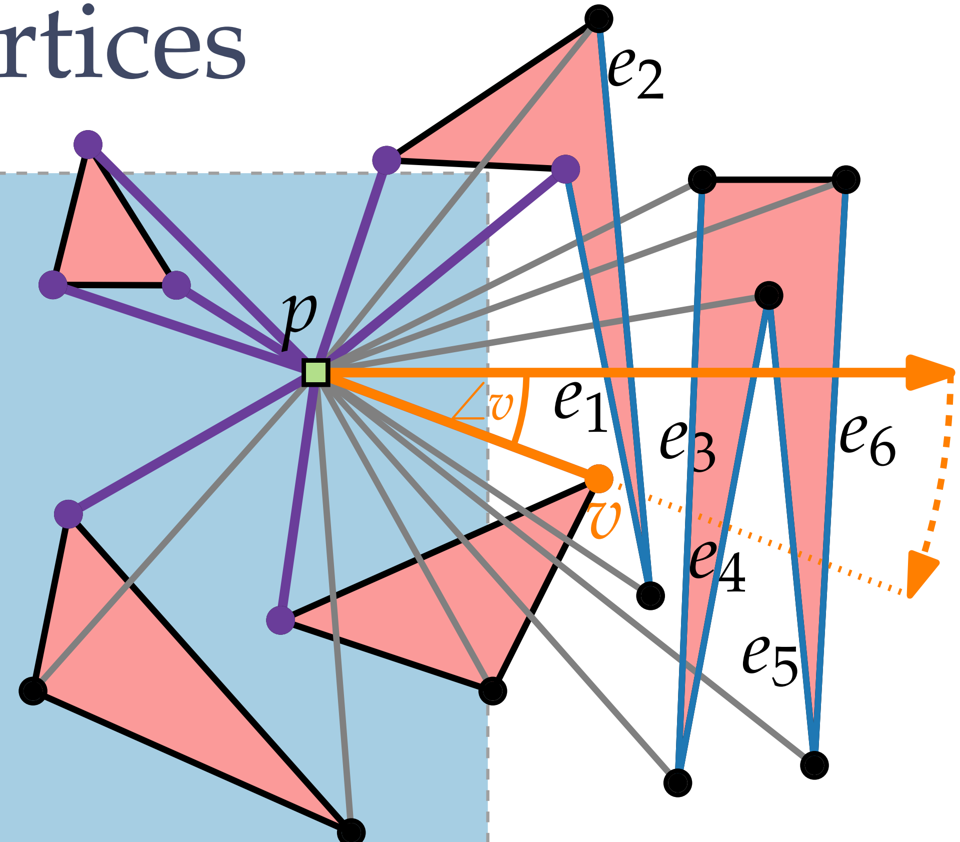
- **if** Visible(v) **then** ?
  - $W \leftarrow W \cup \{v\}$
- insert into $\mathcal{T}$ edges incident to v in $\overrightarrow{pv}^+$
- delete from $\mathcal{T}$ edges incident to v in $\overrightarrow{pv}^-$

return W



$O(n \log n)$

Computing Visible Vertices



VISIBLEVERTICES(p, S)

$r \leftarrow p + \mathbb{R}_{\geq 0}\binom{1}{0}$

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sort $V(S)$ $\qquad v \prec v' :\Leftrightarrow$ $\angle v < \angle v'$ or $(\angle v = \angle v'$ and $|pv| < |pv'|)$

$W \leftarrow \emptyset$

foreach $v \in V(S)$ **do**

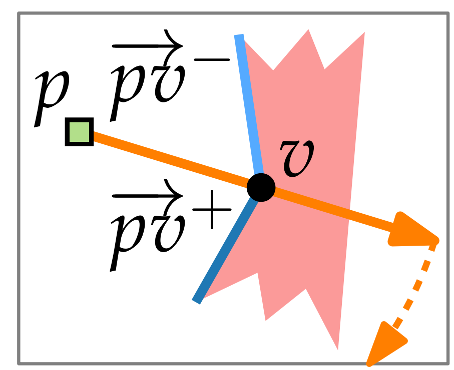
  if VISIBLE(v) **then** $\quad O(1)$

    $W \leftarrow W \cup \{v\}$

  insert into $\mathcal{T}$ edges incident to v in $\overrightarrow{pv}^{+}$

  delete from $\mathcal{T}$ edges incident to v in $\overrightarrow{pv}^{-}$

return W



$O(n \log n)$

Computing the Visibility Graph

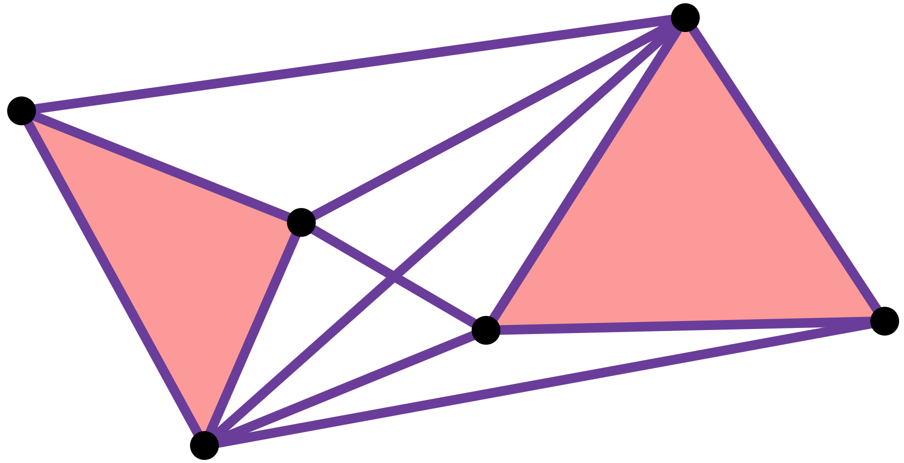
VisibilityGraph(S)

Input: a set S of disjoint polygons
Output: $G_{\text{vis}}(S)$

$E \leftarrow \emptyset$
foreach $v \in V(S)$ **do**
  $W =$ VisibleVertices(v, S)
  $E \leftarrow E \cup \{vw \mid w \in W\}$
return $(V(S), E)$

$O(n) \cdot$

?



Computing the Visibility Graph

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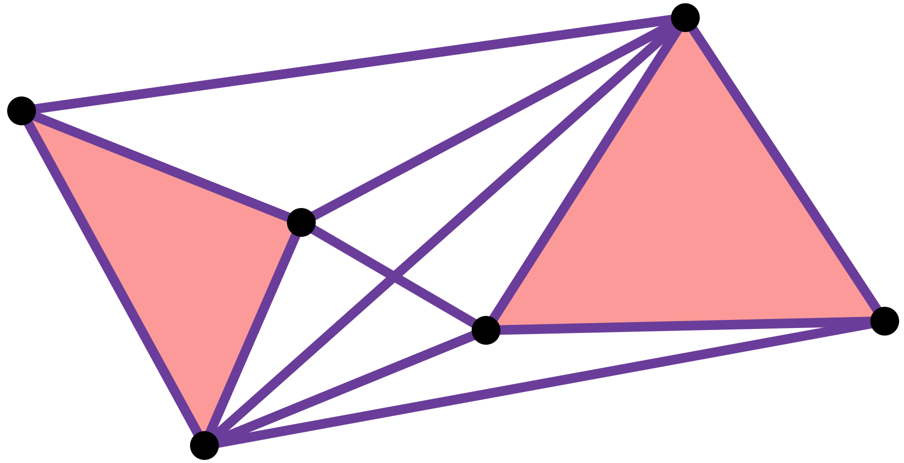
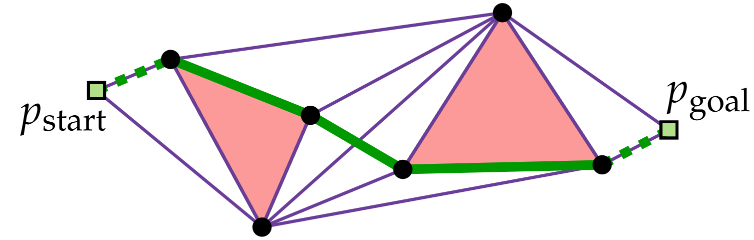
$O(n) \cdot$

$O(n \log n)$



Algorithm



$\textsc{ShortestPath}(S, p_{\text{start}}, p_{\text{goal}})$ $\quad n = |V(S)|, m = |E_{\text{vis}}(S)|$

$G_{\text{vis}} \leftarrow \textsc{VisibilityGraph}(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$?

foreach $uv \in E_{\text{vis}}$ **do** $\quad O(m)$

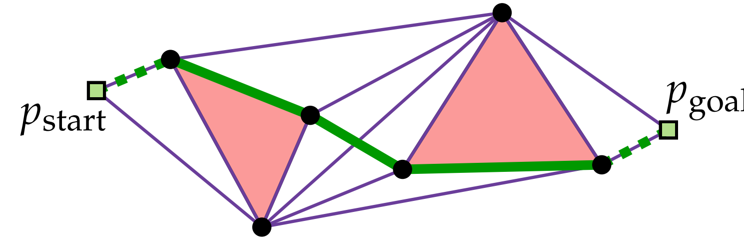
$w(uv) = d_{\text{Eucl.}}(u, v)$

$\pi \leftarrow \textsc{Dijkstra}(G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}})$ $\quad O(m + n \log n)$

return π

Running time?

Algorithm



ShortestPath$(S, p_{\text{start}}, p_{\text{goal}})$ $\quad n = |V(S)|, m = |E_{\text{vis}}(S)|$

$G_{\text{vis}} \leftarrow$ VisibilityGraph$(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$ $\quad O(n^2 \log n)$

foreach $uv \in E_{\text{vis}}$ **do** $\quad O(m)$

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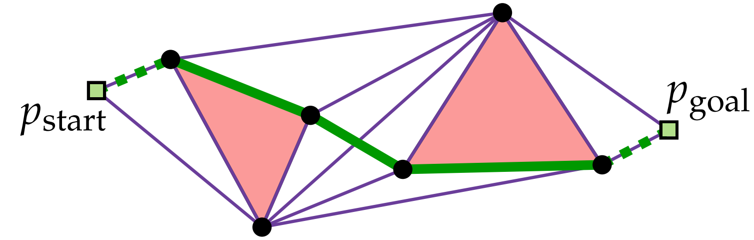
Running time?

Algorithm



$\textsc{ShortestPath}(S, p_{\text{start}}, p_{\text{goal}})$ $n = |V(S)|$, $m = |E_{\text{vis}}(S)|$

$G_{\text{vis}} \leftarrow \textsc{VisibilityGraph}(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$ $O(n^2 \log n)$

foreach $uv \in E_{\text{vis}}$ **do** $O(m)$

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$\pi \leftarrow \textsc{Dijkstra}(G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}})$ $O(m + n \log n)$

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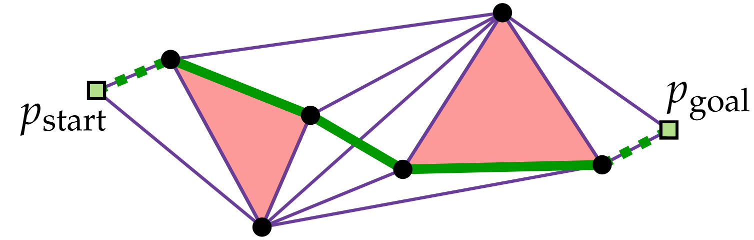
Running time?

Algorithm



ShortestPath$(S, p_{\text{start}}, p_{\text{goal}})$ $\quad n = |V(S)|, m = |E_{\text{vis}}(S)|$

$G_{\text{vis}} \leftarrow$ **VisibilityGraph**$(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$ $\quad O(n^2 \log n)$

foreach $uv \in E_{\text{vis}}$ **do** $\quad O(m)$

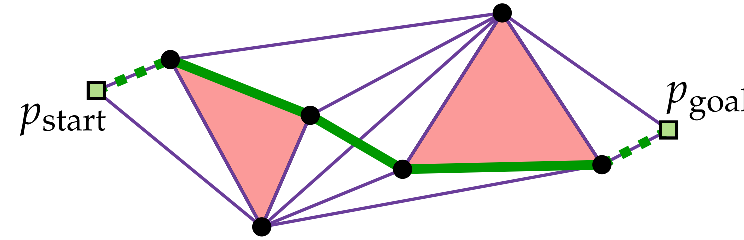
$\quad w(uv) = d_{\text{Eucl.}}(u, v)$

$\pi \leftarrow$ **Dijkstra**$(G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}})$ $\quad O(m + n \log n)$

return π

Running time? $\quad O(n^2 \log n)$

Algorithm



$\textsc{ShortestPath}(S, p_{\text{start}}, p_{\text{goal}})$ $n = |V(S)|$, $m = |E_{\text{vis}}(S)|$

$G_{\text{vis}} \leftarrow \textsc{VisibilityGraph}(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$ $O(n^2 \log n)$

foreach $uv \in E_{\text{vis}}$ **do** $O(m)$

$w(uv) = d_{\text{Eucl.}}(u, v)$

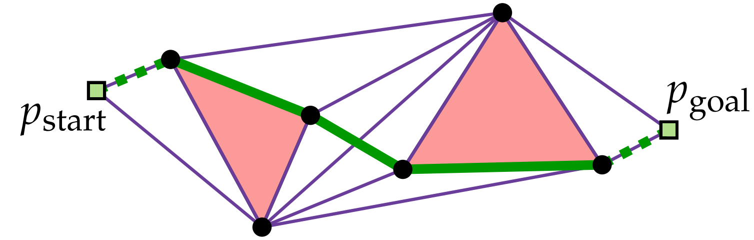
$\pi \leftarrow \textsc{Dijkstra}(G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}})$ $O(m + n \log n)$

return π

Running time? $O(n^2 \log n)$

Theorem. The visibility graph of a set of disjoint polygonal obstacles with n edges in total can be computed in $O(n^2 \log n)$ time.

Algorithm



ShortestPath$(S, p_{\text{start}}, p_{\text{goal}})$ $\quad n = |V(S)|, m = |E_{\text{vis}}(S)|$

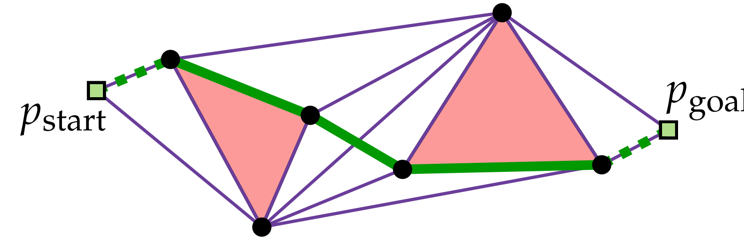
- $G_{\text{vis}} \leftarrow$ VisibilityGraph$(S \cup \{p_{\text{start}}, p_{\text{goal}}\})$ $\quad O(n^2 \log n)$
- **foreach** $uv \in E_{\text{vis}}$ **do** $\quad O(m)$
  - $w(uv) = d_{\text{Eucl.}}(u, v)$
- $\pi \leftarrow$ Dijkstra$(G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}})$ $\quad O(m + n \log n)$
- **return** π

Running time? $\quad O(n^2 \log n)$

Theorem. The visibility graph of a set of disjoint polygonal obstacles with n edges in total can be computed in $O(n^2 \log n)$ time.

Theorem. A shortest path between two points among a set of [...] can be computed in $O(n \log n + m)$ time with $O(n^2 \log n)$ preproc.

Algorithm



ShortestPath($S, p_{\text{start}}, p_{\text{goal}}$) $\quad n = |V(S)|,\ m = |E_{\text{vis}}(S)|$

$G_{\text{vis}} \leftarrow$ VisibilityGraph($S \cup \{p_{\text{start}}, p_{\text{goal}}\}$) $\quad O(n^2 \log n)$

foreach $uv \in E_{\text{vis}}$ **do** $\quad O(m)$

  $w(uv) = d_{\text{Eucl.}}(u, v)$

$\pi \leftarrow$ Dijkstra($G_{\text{vis}}, w, p_{\text{start}}, p_{\text{goal}}$) $\quad O(m + n \log n)$

return π

Running time? $\quad O(n^2 \log n)$

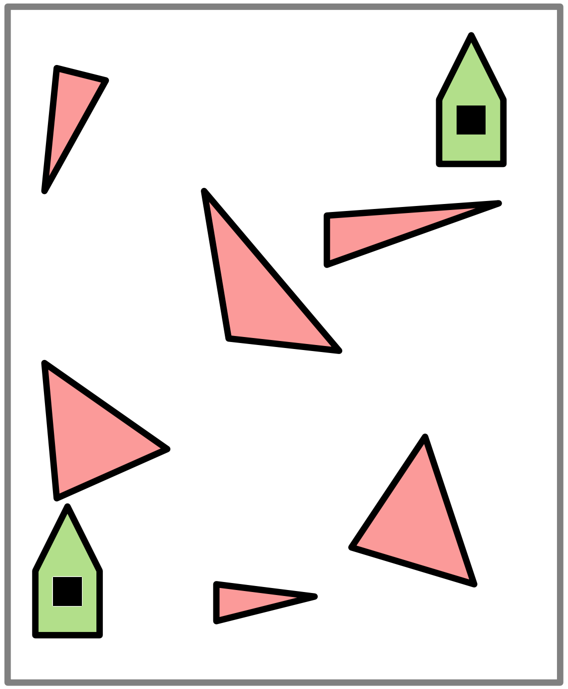
Theorem. The visibility graph of a set of disjoint polygonal obstacles with n edges in total can be computed in $O(n^2 \log n)$ time. [Ghosh & Mount]

($n \log n + m$)

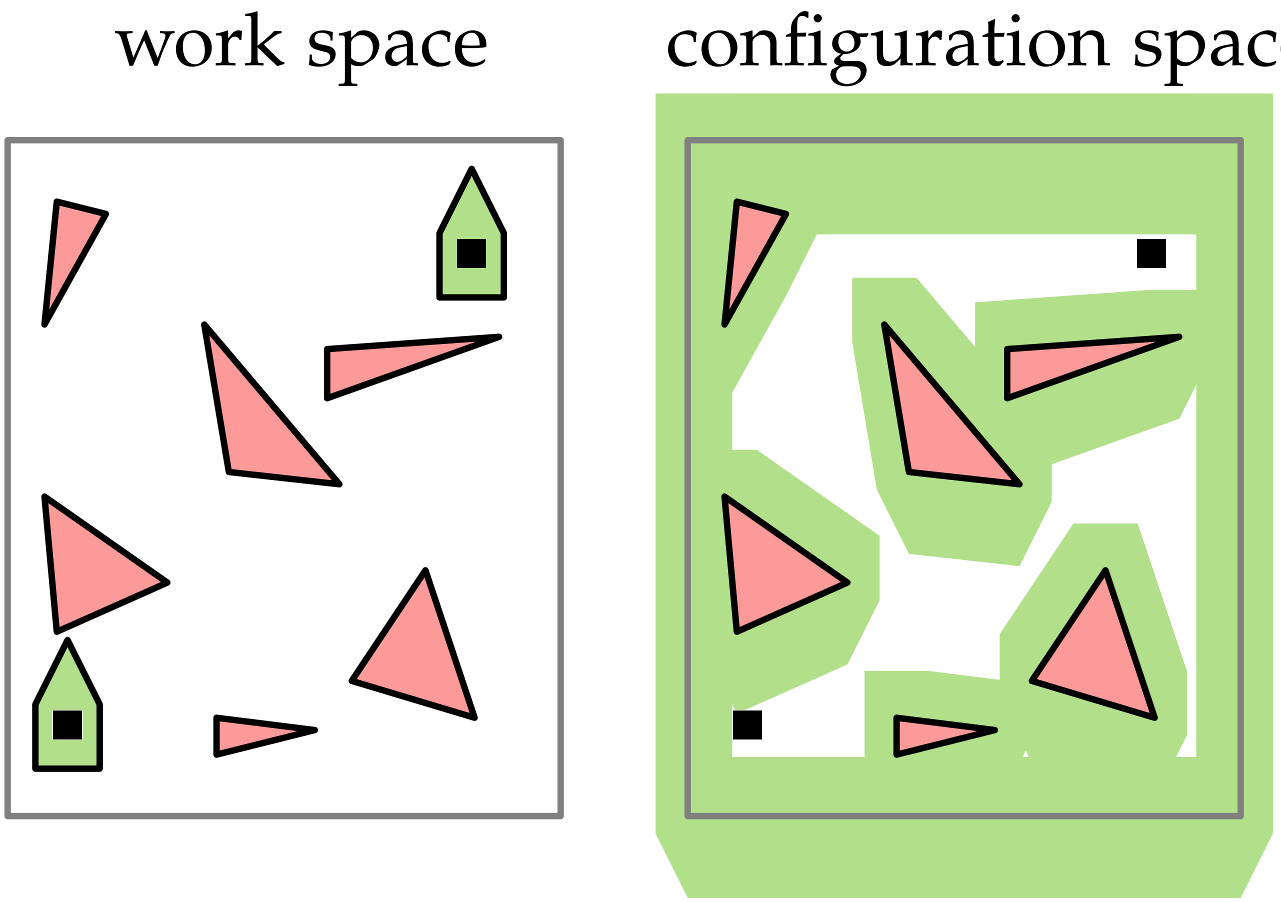
Theorem. A shortest path between two points among a set of [...] can be computed in $O(n \log n + m)$ time ~~with $O(n^2 \log n)$ preproc.~~

Translating Polygonal Robots

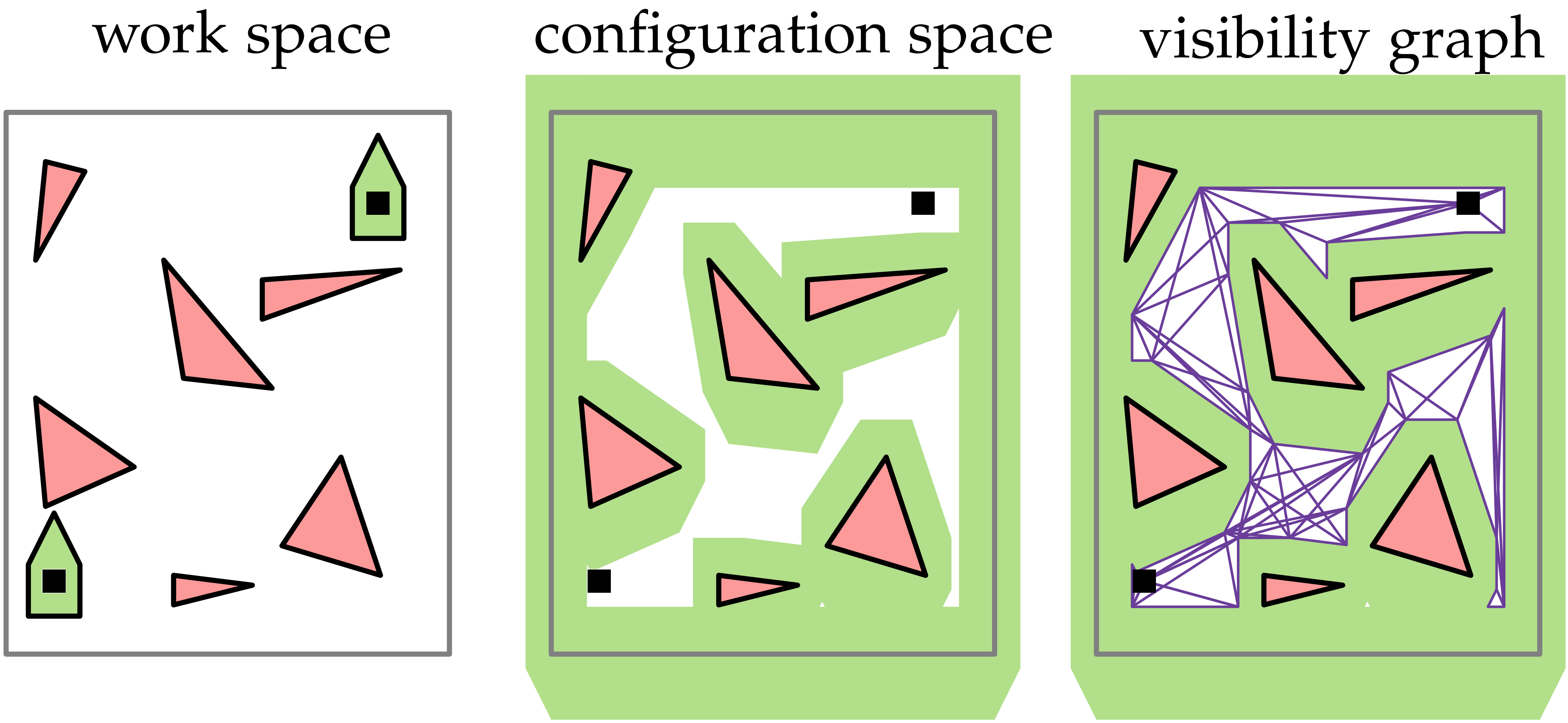
work space



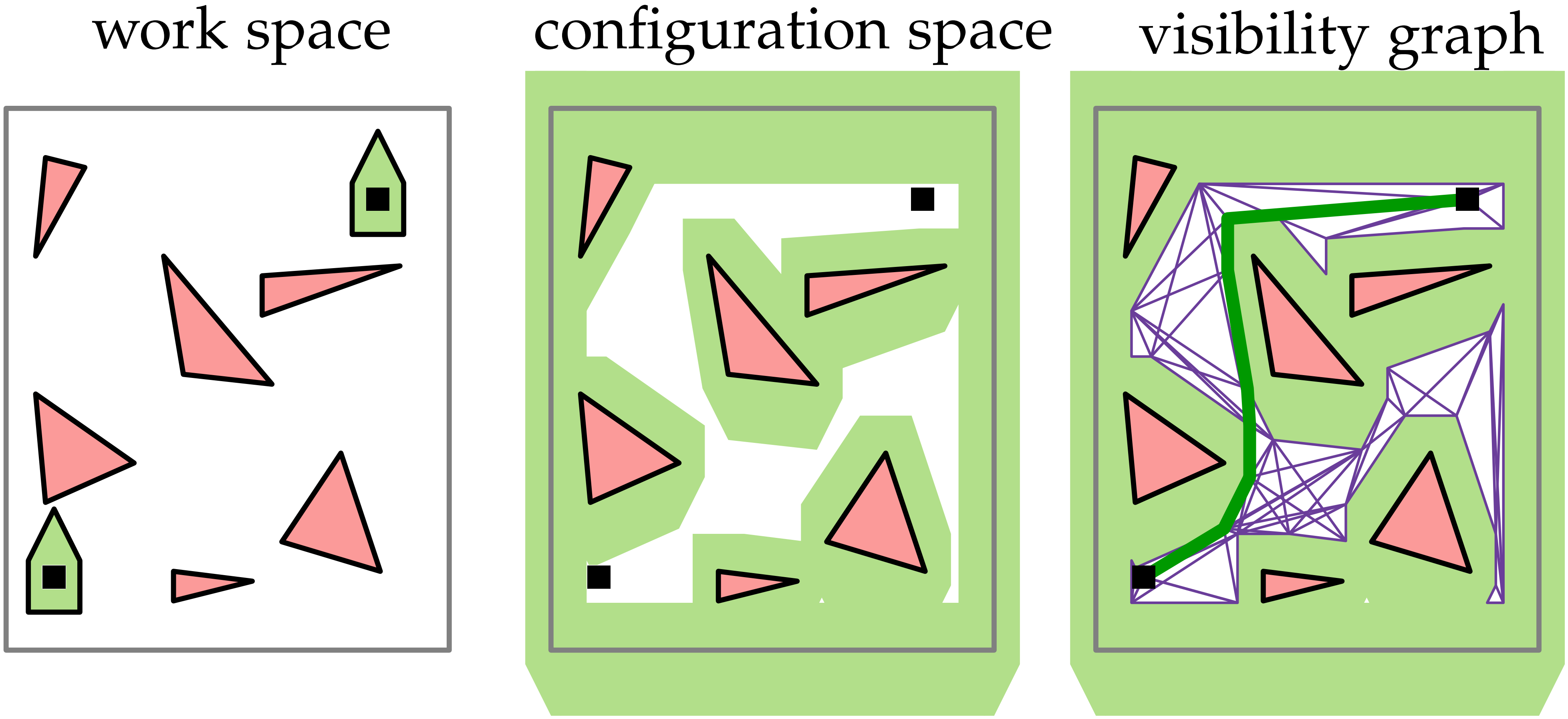
Translating Polygonal Robots



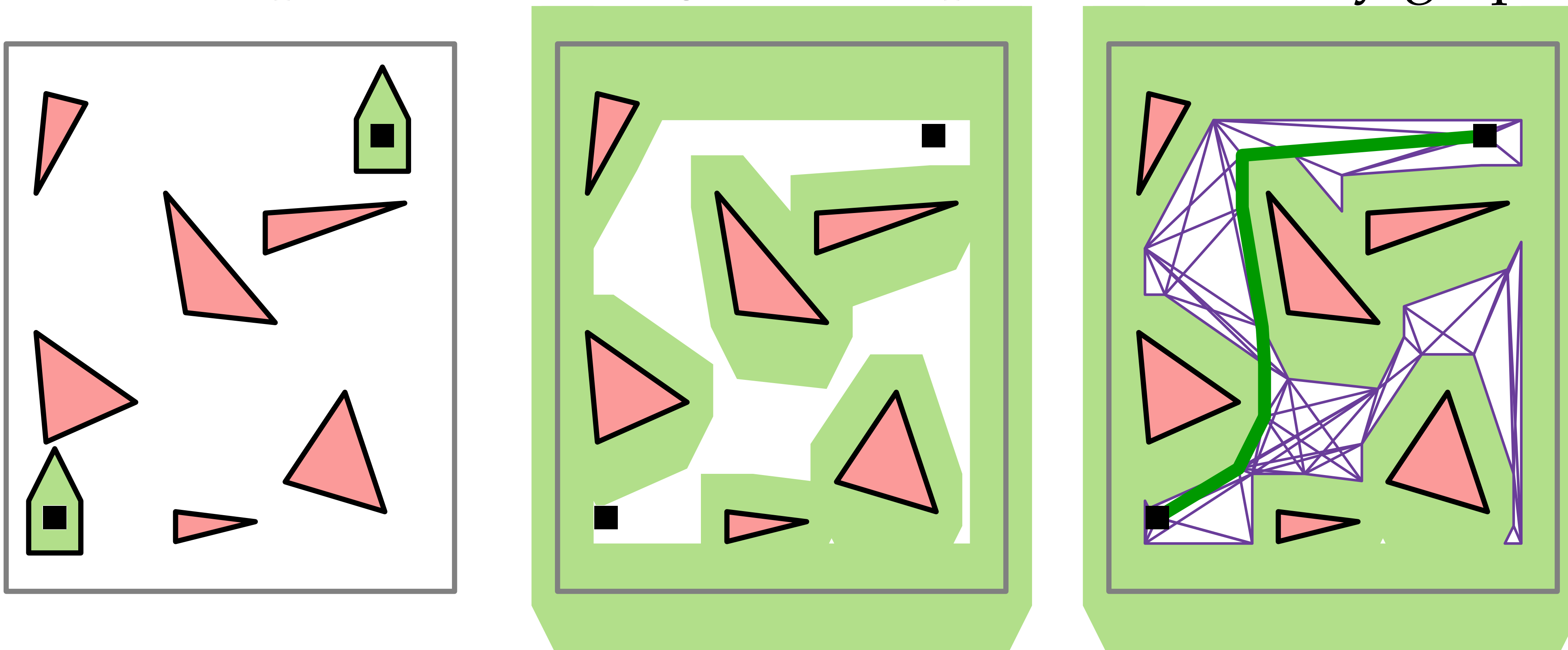
Translating Polygonal Robots



Translating Polygonal Robots

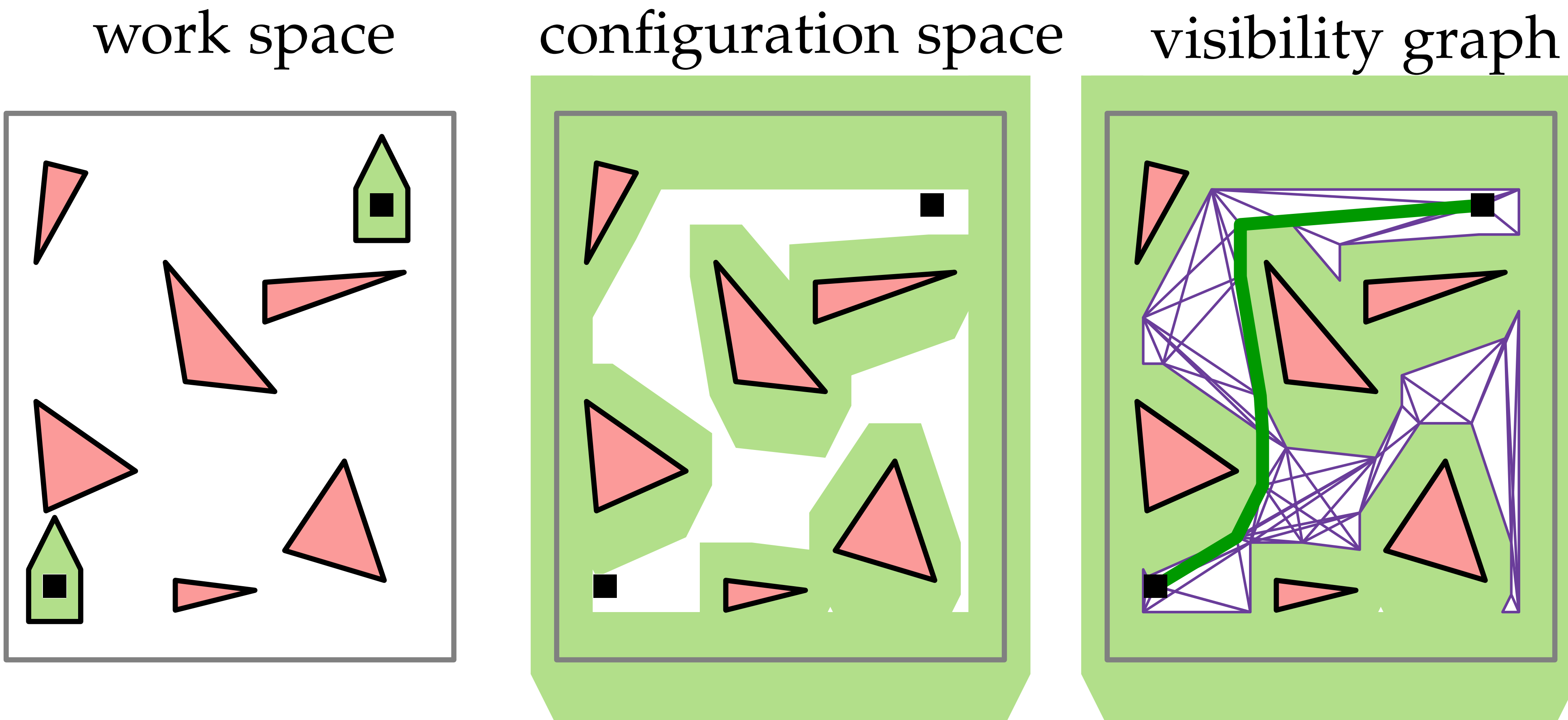


Translating Polygonal Robots



Theorem: For a convex constant-complexity translating robot, a shortest collision-free path among a set of polygonal obstacles with n edges in total can be computed in $O(n^2 \log n)$ time.

Translating Polygonal Robots



Theorem: For a convex constant-complexity translating robot, a shortest collision-free path among a set of polygonal obstacles with n edges in total can be computed in $O(n^{\cancel{2}} \log n)$ time.

[Hershberger & Suri]