The number Pi (π)

Pi might be the most iconic symbol of mathematics nowadays having its own annual celebration day with Pi Day on 3/14 where people hold competitions in memorizing as many digits of Pi as possible.[1]

In [Beckmann, 1982] on page 9, it is said that "By 2,000 B.C. men had grasped the significance of the constant that is today denoted by π, and that they had found a rough approximation of its value." Like the Pythagorean Theorem, the concept of Pi was known in every civilised culture independently (even in the Maya culture, see [Beckmann, 1982, p. 29-35]). Humans noticed the shape of the circle and its beauty (for example, its infinitely many symmetry axes) everywhere in nature, for instance when they looked at the sun or the moon. They also recognized the concepts of magnitude, qualitative reasoning and proportions ("more prey gives more food" [Beckmann, 1982, p. 10]), discovering that "the wider a circle is 'across', the longer it is 'around' " [Beckmann, 1982, p. 11]) early on. At some point in time, some humans must have arrived at the fact that "no matter how the two proportional quantities [for example a circle's diameter and its circumference] are varied, their ratio remains constant" [Beckmann, 1982, p. 11].

In early times, Babylonians and Egyptians were aware of the existence of such a constant and had their own approximate values for it. In the Old Testament in the Book of Kings (about 550 B.C.), it states that "also, he made a molten sea of ten cubits from brim to brim [the diameter measures 10 cubits], round in compass, and five cubits the height thereof; and a line of thirty cubits did compass it round about [the circumference measures 30 cubits]" [Beckmann, 1982, p. 15]. This leads to the biblical value of π, which can be computed to be exactly $\frac{30}{10} = 3$. So, in the Bible we have π approximated by 3. The Babylonians and the Egyptians had even better approximations (Babylonians: $3\frac{1}{8}$; Egyptians: $4(\frac{8}{9})^2$).

How were they able to come up with such good approximations without any calibrated measuring tapes, compasses, pencils and so on? An explanation for getting such an approximation with just stakes, sand and ropes is given in [Beckmann, 1982] on page 13ff. (see also Figure 1): We first put a stick in the sand to mark the origin of the circle. Then we knot a rope to it and draw a circle in the ground by keeping the rope taut. Now we take another rope and choose a point A on the circle and stretch the rope from point A through the

[1] This 'sport' is called *piphilology* and the current record holder is Rajveer Meena with 70.000 digits memorized.

origin O until it reaches another point B at the other side of the circle. We mark the length from point A to B (this is the diameter of the circle), and we set this as our unit (this is some kind of measuring tape). Now we look at how many times we can put this unit into the circumference of the circle. We will get that it fits in three times and a bit more (now we have 3 as an approximate value for π). But we can still refine that. We measure the little bit that is left-over in the circumference and look how often this length fits into our unit (the length from A to B) and we see that it fits in between 7 and 8 times. Therefore, we get $3\frac{1}{8}$ (or $3\frac{1}{7}$) for an approximation of π.

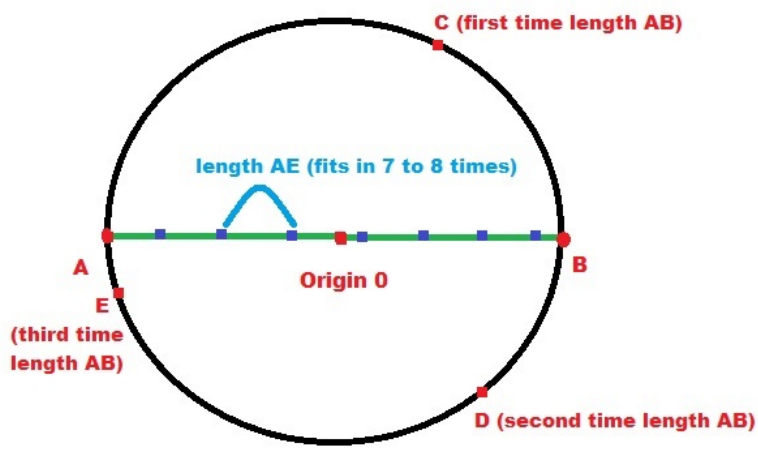

Figure 1: Illustration of getting an approximation of Pi, following [Beckmann, 1982, p. 13].

The Indian wise man Siddhanta (in a work published in 380 A.D.) had another approximate value for Pi. He had 3.1416, the same value that Aryabhata states in his *Aryabhatiya* from 499 A.D. and even Bashkara (born 1114 A.D.) had the same value that is a way better approximation than the ones from the Egyptians and Babylonians. Not only did the Indians try to find an approximation for π, but the Chinese also worked on it. In 264 A.D. Hou Han Shu used 3.1622 as an approximation, and in 718. A.D. Liu Hui found out that $3.141024 < \pi < 3.142704$ using the method of inscribing a polygon into a given circle (see [Beckmann, 1982, p. 27 & 62-73]). Moreover, Tsu Chungh-Chih (in another spelling: Zu Chongzhi) was able to improve these boundaries together along with his son: $3.1415926 < \pi < 3.1415927$. It lasted until the 16th century, when an even better approximation for π was found in Europe, and even now there are ongoing calculations in order to approach the constant π. Furthermore, over time some secrets about π were discovered, for example that it is irrational (first proven by Lambert in 1761, see [Beckmann, 1982, p. 100]) and transcendent (first proven by Lindemann in 1882). However, there are still some

mysteries about π that need to be examined, for example whether or not it is normal.
The last interesting fact to mention about Pi is that the ratio of a circle's circumference to its diameter was not always denoted by π. It was only in the mid-18th century that this was commonly established by Leonhard Euler. Presumably, one of the first to use the Greek letter π in today's sense was the English mathematician William Jones, who used it in his S*ynopsis Palmariorum Matheseos*, published in 1706. Euler later adopted this in his work *Mechanica sive motus scientia analytice exposita* from 1736. But before, Euler had used other abbreviations for today's π; he had used the letters p and c earlier to abbreviate this special ratio (cf. [Arndt, Haenel 2001, p. 165f.]).

References

[Katz, 2009] Victor Katz, A history of Mathematics - an introduction, Addison-Wesley, Boston, 2009

[Arndt, Haenel 2001] Jörg Arndt, Christoph Haenel, Pi - Unleashed, Springer-Verlag, Berlin Heidelberg 2001

[Beckmann, 1982] Petr Beckmann, A History Of Pi, Fifth Edition, The Golem Press, Boulder, Colorado, 1982