CLAY TABLETS

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Maybe one of the main reason why Babylonians are now widely known is because of their clay tablets. Sumerians and Babylonians were people who lived in Mesopotamia between 3000 B.C and 1000 B.C. They used to write and record all computations on dump clay tablets with a stick, they then let them dry in the hot sun or they backed them in oven. So till nowadays we can deal with a lot of tablets, as we could look at an ancient library. They used the so called cuneiform writing (wedge-shaped writing) and a positional sexagesimal (base 60) number system. Many of the mathematical tablets concern practical topics related to management of the reign or palaces. Some are related to commerce and other report geometrical problems related to irrigation systems or architectural problems; finally some are recorded by archeologists as simple writing exercises of scribes.

In this assay will be presented only few examples, enought to let be understand babylonian mathematical skills. First need to be specified that, differently from the modern (greek) way of studying mathematics, racking one's brain over theorems and abstract problems, Babylonians used mathematics only to deal with concrete problems. Their tools to find answers to problems, instead of theorems, were algorithms with the aid of tables which let them repeat quickly standard computations. As J J O'Connor and E F Robertson say "Perhaps the most amazing aspect of the Babylonian's calculating skills was their construction of tables to aid calculation." [JOC-EFR, 2000]

The most easy example of that, is the use of tables to compute the product of two numbers. They used the property that $a \cdot b = \frac{(a+b)^2 - a^2 - b^2}{2}$ or better that $a \cdot b = \frac{(a+b)^2 - (a-b)^2}{4}$, so instead of our classic column calculation, they only needed to be able to combine results from tables of squares. The most complete tablets with squares table had been founded at Senkerah on the Euphratesare, they are from around 2000 BC. The two tablets contains the squares of all the numbers from 1 to 59 and the cubes from 1 to 32.

About division we know that Babylonians even did not consider it as an operation, they just read $\frac{a}{b}$ as the product of a and $\frac{1}{b}$, so, all what they needed, was reciprocals tables which listed the inverse of integer numbers[1]. Reciprocal tables, even more than the square tables, were the most important to be own by a Babylonian scribe. It has been found reciprocal tables going up to the reciprocals of numbers up to several billion. In Figure 0.1 there is an example of such table.

Very useful for computations were also the single multiplication tables, which list the multiples of a single number. If we call that number p, the table gave all the multiples from $1p$ up to $20p$, and then it went up in steps of 10, writing the

[1]Number as $\frac{1}{7}$ or $\frac{1}{13}$ are missing in this table since, as Babylonian scribes used to write in computation, "7 does not divide". Instead of periodical or illimitate fraction they just used approximations of the required number.

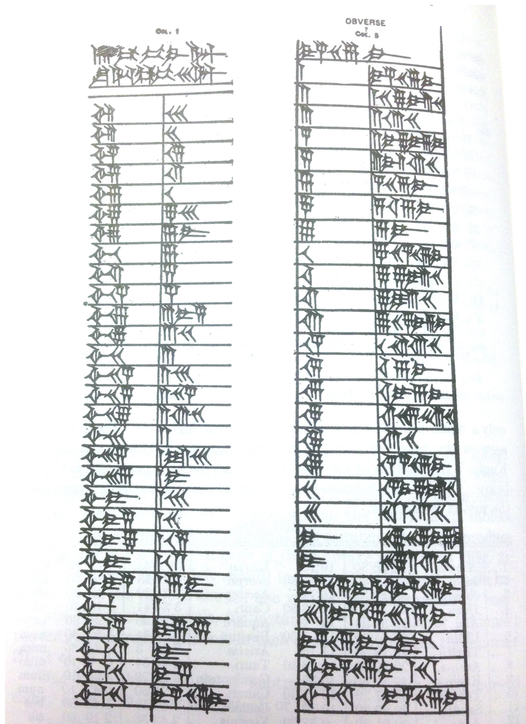

FIG. 4.—Mathematical cuneiform tablet, CBS 8536, in the Museum of the University of Pennsylvania.

(A)

First Column
. . . . gal (?) -bi 40 -ám
šu *a- na* gal-bi 30 -ám

igi 2	30
igi 3	20
igi 4	15
igi 5	12
igi 6	10
igi 8	7(30)
igi 9	6(40)
igi 10	6
igi 12	5
igi 15	4
igi 16	3(45)
igi 18	3(20)
igi 20	3
igi 24	2(30)
igi 25	2(24)
igi 28*	2(13)(20)
igi 30	2
igi 35*	1(52)(30)
igi 36	1(40)
igi 40	1(30)
igi 45	1(20)
igi 48	1(15)
igi 50	1(12)
igi 54	1(6)(40)
igi 60	1
igi 64	(56)(15)
igi 72	(50)
igi 80	(45)
igi 81	(44)(26)(40)

Fifth Column
44(26)(40)

1	44(26)(40)
2	[1]28(53)(20)
3	[2]13(20)
4	[2]48(56)(40)*
5	[3]42(13)(20)
6	[4]26(40)
7	[5]11(6)(40)
9	[6]40
10	[7]24(26)(40)
11	[8]8(53)(20)
12	[8]53(20)
13	[9]27(46)(40)*
14	[10]22(13)(20)
15	[11]6(40)
16	[11]51(6)(40)
17	[12]35(33)(20)
18	[13]20
19	[14]4(26)(40)
20	[14]48(53)(20)
30	[22]13(20)
40	[29]37(46)(40)
50	[38]2(13)(20)*

44(26)(40)a-na 44(26)(40)
[32]55(18)(31)(6)(40)
44(26)(40) square

igi 44(26)(40)	81
igi 81	44(26)(40)

Numbers that are incorrect are marked by an asterisk (*).

(B)

FIGURE 0.1. Tablet CBS 8536, in the museum of the university of Pennsylvania, from the book [Cajori,1928]

products $30p$, $40p$ and $50p$. So if one needed to know, for example, $38p$, he added $30p$ with $8p$. Sometimes single multiplication tables finished by giving also p^2.

FIGURE 0.2. Tablet MS 3048 on clay, Babylonia, 19th c. BC, 1 tablet, 7,6x4,4x2,3 cm [Moyer]

Beside single computations, tables were used also to solve problems and equations.[2]

For example it has be found tablets giving a list of the sums $n^3 + n^2$. This list permited to solve several equations of the form $ax^3 + bx^2 = c$ doing the following steps:

* multiply the equation by a^2 and by $\frac{1}{b^3}$ to get $(\frac{ax}{b})^3 + (\frac{ax}{b})^2 = \frac{ca^2}{b^3}$; which, calling $y = \frac{ax}{b}$, is an equation of the form $y^3 + y^2 = \frac{ca^2}{b^3}$
* look up in the $n^3 + n^2$ table, to find the value which satisfies $n^3 + n^2 = \frac{ca^2}{b^3}$;
* if such number is found, then the solution x could be easily computed multiplying that number y by $\frac{b}{a}$.

Cubic problems were generally related to volumes. Another study of cubic problems can be seen in the tablet in Figure 0.2. Its description reports: "Every line of this tablet says, "m has the root n". The numbers n at right take the values from 1 to30. The numbers m at left take the corresponding values $n{\cdot}(n{+}1){\cdot}(n{+}2) = n^3{+}3n^2{+}2n$. In the 6^{th} line, for instance, $n = 6$ and $m = 6 \cdot 7 \cdot 8 = 336 = 5 \cdot 60 + 36$." Here "the problems would have been of the form "An excavated room. Its length equals its width plus 1 cubit. Its height equals its length. Its volume plus its bottom area is ... (a given number).""[Moyer]

[2]For clarity in the following explanations we will use moden notation and terminology, althought formally is not correct. In fact, in Babylonian times, there was nothing similar to our modern notation, which uses algebraic symbolic representation of questions. This means that Babylonians did not deal with equations. Nevertheless Babilonians used to study typical methods, useful to solve typical problems, which is 'de facto' the aim of modern equations.

Surely Babylonians could also solve all linear equation as $ax = b$ looking at reciprocals and multiples tables. About quadratic equations we know that they considered separately two types of equation, namely $x^2 + bx = c$ and $x^2 - bx = c$ where here b, c have to be thought positive but not necessarily integers. To solve them they basically used the standard formula for the solutions $x = \sqrt{\left(\frac{b}{2}\right)^2 + c} - \frac{b}{2}$ and $x = \sqrt{\left(\frac{b}{2}\right)^2 + c} + \frac{b}{2}$ respectively. Notice that they would take only the positives root from the two roots of quadratic equations, since it is the only which makes sense while tolking about areas and lenght in buildings. Effectively quadratic problems were usually related to computations with areas. On an old babylonian tablet, we can, for example, read the attempt to solve the equation $x^2 + 7x = 1$ using the formula $x = \sqrt{\left(\frac{b}{2}\right)^2 + c} - \frac{b}{2}$. The aim of this tablet was to explain how to find the dimensions of a rectangle knowing that its length exceeds its breadth by 7 and its area is $1, 0$. The answer is written as follows: "Compute half of 7, namely $3; 30$, square it to get $12; 15$. To this the scribe adds $1, 0$ to get $1; 12, 15$. Take its square root (from a table of squares) to get $8; 30$. From this subtract $3; 30$ to give the answer 5 for the breadth of the triangle."[JOC-EFR, 2000]

As final example of the use of tabes we report the problem from the Tell Dhibayi tablet and its method to find a solution. This tablet is significative because is one of the examples that let us believe that Babylonians beside squares and roots knew also the Pythagorean rule. The table asks for the sides of a rectangle whose diagonal is $1, 15$ and whose area is $0, 45$. To make the explanation more understandable let us continue to use modern notation calling x and y the rectangle's dimensions. The scribe now reports the following steps for the computation:

* "Compute the product $2xy = 1; 30$.
* Subtract it from$x^2 + y^2 = 1; 33, 45$ to get$x^2 + y^2 - 2xy = 0; 3, 45$.
* Take the square root to obtain $x - y = 0; 15$.
* Divide by 2 to get$\frac{(x-y)}{2} = 0; 7, 30$.
* Divide $x^2 + y^2 - 2xy = 0; 3, 45$ by 4 to get $\frac{x^2}{4} + \frac{y^2}{4} - \frac{xy}{2} = 0; 0, 56, 15$.
* Add $xy = 0; 45$ to get $\frac{x^2}{4} + \frac{y^2}{4} + \frac{xy}{2} = 0; 45, 56, 15$.
* Take the square root to obtain $\frac{(x+y)}{2} = 0; 52, 30$.
* Add $\frac{(x+y)}{2} = 0; 52, 30$ to $\frac{(x-y)}{2} = 0; 7, 30$ to get $x = 1$.
* Subtract $\frac{(x-y)}{2} = 0; 7, 30$ from $\frac{(x+y)}{2} = 0; 52, 30$ to get $y = 0; 45$.
* Hence the rectangle has sides$x = 1$ and $y = 0; 45$."[JOC-EFR, 2000]

Let conclude this short excursus about clay tablets and its use quoting again J J O'Connor and E F Robertson "Remember that this is 3750 years old. We should be grateful to the Semitic scribe for recording this little masterpiece on tablets of clay for us to appreciate today."[JOC-EFR, 2000]

References

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